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On sensitivity investigations of thin-walled shell structures using transient finite element analysis and finite perturbations

In standard stability investigations of structures applying the finite element method usually the bifurcation and snap-through points are detected. For practical design purposes not only the equilibrium state itself is significant but also the safety of the stable equilibrium state. The sensitivity, which quantifies the safety, can be investigated by introducing finite perturbations at a certain load level and considering the perturbed motion. In this contribution the application of Liapunov Characteristic Exponents (LCE) for the judgement of the perturbed motion is investigated.

1. Sensitivity analysis

For computation of a sensitivity measure a static nonlinear analysis is performed first, in order to reach the equilibrium state to be analysed. Then, at this state a perturbation energy is introduced by setting velocity initial conditions $\dot{\mathbf{u}}_{\mathbf{0}}$, thus the initial kinetic energy $W_{kin} = \frac{1}{2}\dot{\mathbf{u}}_{\mathbf{0}}\mathbf{M}\dot{\mathbf{u}}_{\mathbf{0}} = W_{pert}$ with $\mathbf{M}$ as the mass matrix. As next step the transient analysis is performed in order to obtain the motion of the structure. For a small perturbation energy the structure vibrates in the vicinity of the original equilibrium state. Increasing the perturbation energy a certain critical energy can be obtained, that just transfers the system out of the basin of attraction of the original equilibrium state (for further Information see e.g. [1]). The minimum kinetic energy defines the sensitivity:

$$S = \frac{1}{W_{kin,min}} \tag{1}$$

For a rational judgement of the motion an indicator is needed, that allows an automatical and efficient decision, whether the critical perturbation energy is reached. The Liapunov Characteristic Exponent appears to be a good choice.

2. Liapunov Characteristic Exponent (LCE)

Liapunov Characteristic Exponents allow the judgement whether a motion is stationary or unstable and are defined by the following equation:

$$\lambda = \lim_{\substack{d_0 \to 0 \\ t \to \infty}} \left(\frac{1}{t} \ln \frac{||\mathbf{d}(t)||}{||\mathbf{d_0}||} \right) \tag{2}$$

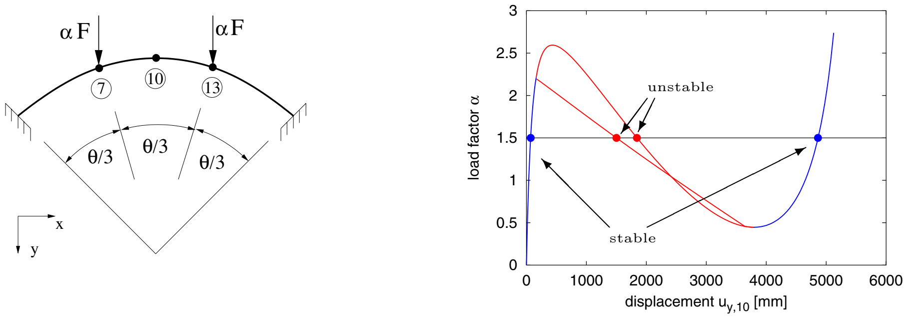
in which λ denotes the LCE, $\mathbf{d_0}$ denotes the initial perturbation and $\mathbf{d}(t)$ denotes the perturbation at the time point t, $||\ldots||$ is some norm. λ gets the same value, if the structure vibrates around the original equilibrium state, independent of the chosen reference and perturbed motions (see [2]). In this case the reference and the perturbed motions converge to the same attractor and λ gets negative. Furthermore λ changes, if the structure leaves the basin of attraction of the original equilibrium state. In this case the reference motion converges to the original equilibrium state and the perturbed motion converges to another attractor ($\lambda = 0$) or diverges ($\lambda > 0$). Therfore a change of motion can be indicated by the change of the value of LCE. In order to keep the computational effort low, a simple maximum norm of a selected characteristic degree of freedom (DOF) is used. Thus only the maximum values of a certain displacement u, computed only at time points, for which $\dot{u} = 0$, are taken into account. This leads to the following formulation of LCE:

$$\lambda = \lim_{t\to\infty} \left(\frac{1}{t} \ln \frac{|u_{ref}(t^{**}) - u_{per}(t^{*})|}{|u_{ref,0} - u_{per,0}|} \right); \quad \dot{u}_{ref}(t^{**}) = 0, \dot{u}_{per}(t^{*}) = 0 \tag{3}$$

where the subscript *ref* denotes the reference motion and subscript *per* denotes the perturbed motion.

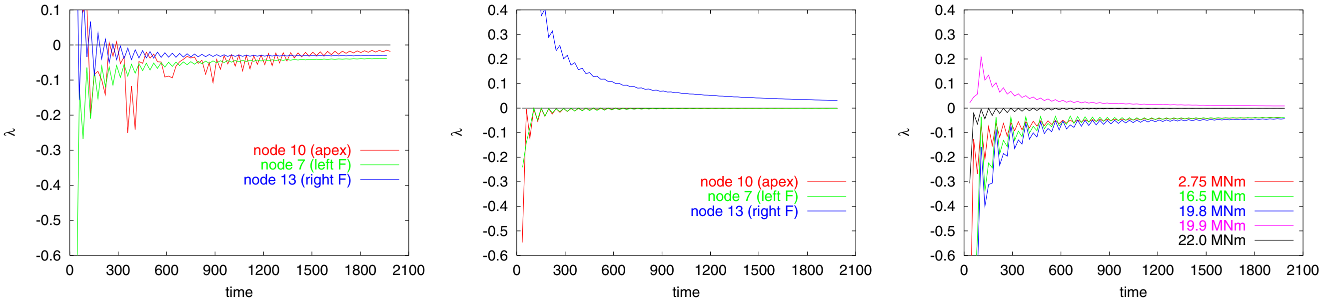
3. Example: Circular Arch

The proposed procedure is executed for the circular arch example shown in Fig.1a) with the following properties: $E = 0.1373\ \ MN/mm^2$, $\nu = 0$, $R = 10\ \ m$, $t = 0.3\ \ m$, $\theta = 90^o$, $F = 25\ \ MN$. The arch is discretized with 18 4-node bilinear degenerated shell elements. A static analysis using the arc-length method leads to the load deflection curve



a) b)

Figure 1: a) Outline of investigated circular arch, b) Load deflection curve for $u_{y,10}$



a) b) c)

Figure 2: Time evolution of λ_7, λ_{10} and λ_{13} a) for $W_{kin,1} = 2.75$ MNm, b) for $W_{kin,2} = 27.5$ MNm, c) time evolution of λ_7 for different W_{kin}

shown in Fig. 1 b), where the blue lines denote the stable and the red lines the unstable equilibrium states. In the following the sensitivity of the stable equilibrium state at the load level $\alpha = 1.5$ is investigated. At this load level the structure has two stable equilibrium states and two unstable states in between. In a first step the influence of the choice of the characteristic DOF on the value of λ is investigated. Three different displacements – vertical displacements of the nodes 7, 10 and 13 – were selected for this purpose. The reference motion is obtained for small perturbation energy of $W_{kin,ref} = 0.275\ MNm$, and the perturbed motions in case 1 for $W_{kin,1} = 2.75\ MNm$ and in case 2 for $W_{kin,2} = 27.5\ MNm$. In case 1 the structure vibrates around the original equilibrium state. The LCEs for all DOFs converge to negative values indicating asymptotic stability (see Fig. 2a)). It must be mentioned, that the converged values and the convergence behavior are slightly different. In case 2 the structure moves to the stable equilibrium state in the postbuckling region. All λ_i converge to "0" indicating that the reference and the perturbed motions converge to different attractors (Fig. 2b)). The convergence behavior is significantly different in this case. In the next step the LCE for node 7 is computed for different perturbation energies. As expected, λ converges to almost the same negative value for smaller perturbation energies, see the three curves with negative λ in Fig.2c). For a slightly larger energy the structure leaves the basin of attraction of the original stable equilibrium state and moves towards the other state, then λ converges to "0", see the curves for 19.9 and 22.0 MNm in Fig. 2c).

4. Conclusions

In the present work the application of LCE for the judgement of the perturbed motion in sensitivity analysis is investigated. For the computation of LCE a simple formulation using the maximum norm for a chosen characteristic DOF is proposed. The calculations with different DOFs – displacements – show, that the value of the LCEs depends slightly on the choice of the characteristic DOF. Nevertheless, the change of the value of the LCE in the proposed procedure allows always the correct judgement of the motion for all chosen DOF.

5. References

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