First published in:

PAMM · Proc. Appl. Math. Mech. **3**, 290–291 (2003) / **DOI** 10.1002/pamm.200310417

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Description for smooth contact conditions based on the internal geometry of contact surfaces

A kinematical approach, based on the consideration of the contact conditions in the local coordinate system, is proposed for the contact description and for consistent linearization. This leads to a simple structure of the tangent matrix, which is subdivided into main, rotational and curvature parts. Various alternatives neglecting parts of the contact tangent matrix are considered. Representative examples show the effectiveness of the proposed approach for contact problems with arbitrary large deformation.

1. Definition of the local coordinate system

The master-slave concept, based on the definition of penetration of a "slave" point into a master contact surface is known as a robust technique to describe contact conditions. This penetration is then used to construct a contact functional, and, also, for the constitutive model of the contact forces. The functional, defined over the unknown contact surface, is a nonlinear function of current geometry, therefore, the whole problem requires an iterative solution, e.g. by a Newton type iterative scheme. For this a linearization of the contact functional is necessary. The correct tangent matrix as a result of this linearization process is called the consistent tangent matrix. Within the penalty method such tangent matrices were derived by Wriggers and Simo (1985) for the 2D case, and by Parisch (1989) for the 3D case; in both papers the penalty functional has been linearized in the global coordinate system directly. The linearization procedure was generalized by Simo and Laursen (1992), where the local surface coordinate system has been introduced to get convective velocities for further linearization in the global coordinate system. In our contribution, only the local coordinate system is used to describe the kinematics of the deformation as well as the differential operations. During the linearization the coordinate increment vector can be treated as a velocity vector, and the linearization itself can be treated as a covariant differential operation in the local surface coordinate system. The important aspect of the proposed kinematical approach is to consider the global linearization separate from the local "slave" node searching procedure and to derive linearized equations from the kinematic equations in the local surface coordinate system. The idea is based on the a priori knowledge of the value of the penetration, which is assumed to be small. This fact allows to treat the contact conditions from the surface geometry point of view, which is very similar to shell theory. The local coordinate system is defined as follows

$$\mathbf{r}_s(\xi^1,\xi^2,\xi^3) = \mathbf{r}(\xi^1,\xi^2) + \xi^3\mathbf{n}, \tag{1}$$

where $\mathbf{r}_s$ is a position of a "slave" point, $\mathbf{r}$ is a projection of the "slave" point onto the contact surface, and ξ^3 is a value of the penetration. Consideration of the convective velocities in the form

$$\dot{\xi}^j = a^{ij}(\mathbf{v}_s - \mathbf{v}) \cdot \mathbf{r}_i, \tag{2}$$

allows to construct the contact integral, which is consistent with a coordinate ξ^3, in the following form

$$\delta W_c = \int_s N\delta g ds + \int_s T_i \delta\xi^i ds \tag{3}$$

2. Linearization of the contact integral

For this contribution only a frictionless case is considered. The linearization procedure of the contact integral (3), taking into account the equation for the convective velocities (2), leads to the tangent matrix contained in (4)-(6)

$$D(\delta W_c^N) = \epsilon_N \int_S H(-g)\ (\delta\mathbf{r}_s - \delta\mathbf{r}) \cdot (\mathbf{n}\otimes\mathbf{n})(\mathbf{v}_s - \mathbf{v}) dS - \tag{4}$$

$$-\epsilon_N \int_S H(-g)\ g\ \left[\delta\mathbf{r}_{,j} \cdot a^{ij}(\mathbf{n}\otimes\mathbf{r}_i)(\mathbf{v}_s - \mathbf{v})dS + (\delta\mathbf{r}_s - \delta\mathbf{r}) \cdot a^{ij}(\mathbf{r}_j \otimes \mathbf{n})\mathbf{v}_{,i}\right] dS - \tag{5}$$

$$- \int_S \epsilon_N \; H(-g) \; g \; (\delta \mathbf{r}_s - \delta \mathbf{r}) \cdot h^{ij} (\mathbf{r}_i \otimes \mathbf{r}_j)(\mathbf{v}_s - \mathbf{v}) dS \tag{6}$$

The full contact tangent matrix is subdivided into the "main" part eq. (4), the "rotational" (5) and the "pure curvature" part (6). Each part preserves the symmetry. Several expressions to compute the contact integral (3), besides the well known nodal collocation formulae, which leads to "node-to-surface" contact element, can be considered. If the contact with a rigid surface, described by analytical function, is considered, to which we restrict ourselves in this contribution, the searching procedure is reduced to the direct computation of the value of penetration from eq. (1). A family of solid-shell elements, see Hauptmann et. al. and references herein (2000), was used to model the shell structures in all numerical examples.

3. Numerical examples

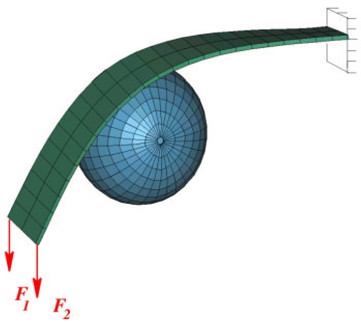

Figure 1. Bending over a sphere

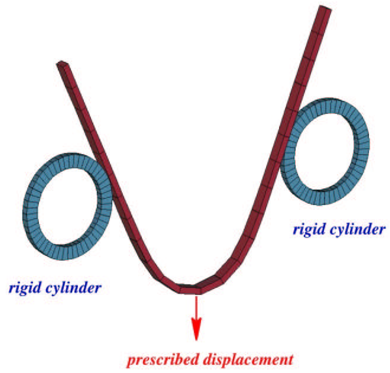

Figure 2. Bending + sliding of thin sheet

Full matrix				Main + rotational parts			Only main part		
No l.s.	No.	Cum. No	10^{-2} %	No l.s.	No.	Cum. No	No l.s.	No.	Cum. No
1	27	27	0.731	1	27	27	1	20	20
2-23	4	115	7.382	2-22	4	111	2-8	4	48
24	6	121	18.08	23	6	117	9-23	5	123
25-33	4	157	16.26	24-33	4	157	24	6	129
34	5	162	12.20	34	6	163	...	...	...
35-100	4	426	17.32	35-100	4	427	86-100	8	613

Table 1: Bending over a sphere. Biquadratic element. Influence of various contact tangent stiffness parts on convergence. Comparison of no. iterations for all load steps (l.s).

The numerical examples, comparing the number of iterations in each load step in table 1 for the bending of a shell over a sphere (Fig. 1), show that in both linear and quadratic approximations of the contact surfaces keeping the "pure curvature" part is meaningless. It even appears to be sufficient to keep only the main part as a contact tangent matrix for the linear case. In the case of contact surfaces described by analytical functions (Fig. 2), it was proved that increasing the number of Gauss points checked for the contact leads to an improved smoothness of the force-displacement curve.

4. References

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