

# Nondemolition measurements of a single quantum spin using Josephson oscillations.

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We consider a Josephson junction containing a single localized spin 1/2 between conventional singlet superconducting electrodes. We study the spin dynamics and measurements when a dc-magnetic field  $\mathbf{B} \parallel z$  acts on the spin and the junction is embedded into a dissipative circuit. We show that when tunneling or a voltage are turned on at time  $t = 0$  the Josephson current starts to oscillate with an amplitude depending on the initial ( $t = 0$ ) value of the spin  $z$ -component,  $S_z = \pm 1/2$ . At low temperatures, when effects of quasiparticles may be neglected, this procedure realizes a quantum-non-demolition (QND) measurement of  $S_z$ .

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Quantum measurements in mesoscopic systems by use of tunneling attracted recently great interest. One of the motivations is the challenge of the single spin detection [1–3]. The other motivation comes from the quantum computing where the final state (after the computation) of a qubit must be measured. The meter couples, usually, to a single observable of the qubit, e.g.,  $\sigma_z$ . If this observable commutes with the spin Hamiltonian, the QND regime is realized, i.e., the two possible eigenvalues of  $\sigma_z$  can be measured with proper probabilities even by a weakly coupled meter. Otherwise the initial state is quickly destroyed and one can only observe the steady state properties of the qubit and the meter performing continuous measurements [4–6]. In the spin detecting tunneling schemes all components of the localized spin are coupled to the tunneling electrons via the exchange interaction. Thus QND measurements seem impossible.

In this paper we show that in fact QND measurements of the spin projection on the direction of the applied field  $\mathbf{B} \parallel z$  are possible with the use of spin dependent Josephson tunneling at low temperatures, when the effect of quasiparticles is negligible. In this case the amplitude of Josephson oscillations depends on the state of the spin just before the measurement. The amplitude can be measured, e.g., in a circuit containing Josephson junction with the spin, a dc-voltage source  $V$  and a resistor  $R$ , see Fig. 1. The dissipative spin-dependent current in the circuit appears when the voltage exceeds a threshold and, effectively, it measures the squared amplitude of the Josephson-current oscillations.

The measurement can begin either when the voltage exceeding a threshold [12] or the tunneling in the presence of such a voltage is switched on. The first possibility appears more natural for mesoscopic circuits, while the second could be realized with the use of the STM with a superconducting tip and a molecule with spin on the superconducting substrate. As the tip approaches the molecule, tunneling is turned on.

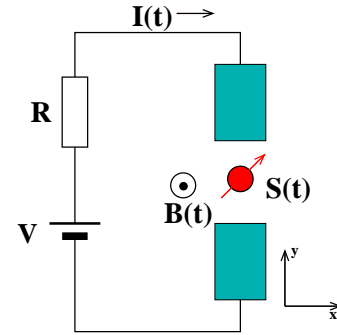


FIG. 1. The circuit containing Josephson junction with a spin.

The system we consider, i.e. a spin in a Josephson junction with applied voltage, is interesting also from the general viewpoint of quantum measurements. The entanglement of the spin with the measurement apparatus is realized here by use of the non-dissipative macroscopic quantum system as an intermediate step, namely with the superconducting phase degree of freedom. Remarkably, in such a system the amplitude of Josephson oscillations at frequency  $\omega_J = 2eV/\hbar$  at finite bias  $V$  carries information on the initial state of the spin even for junctions made of singlet superconductors. Due to the spin conservation in a singlet-Cooper-pair tunneling in the absence of quasiparticles ( $T \rightarrow 0$ ) the average spin component,  $\langle S_z \rangle$ , is preserved to second order in the tunneling amplitude at  $V < 2\Delta_0$  ( $\Delta_0$  being the superconducting gap), though the spin operator  $\hat{S}_z$  does not commute with the Hamiltonian  $\hat{\mathcal{H}}$  of the system. The noncommutativity of  $\hat{\mathcal{H}}$  and  $\hat{\mathbf{S}}$  enables the tunneling measurement of the  $z$ -component of the spin, while due to the preservation of  $\langle S_z \rangle$  it is a QND measurement.

We consider a Josephson junction where the spin-independent tunneling is described by amplitude  $T_0$  and the spin-dependent tunneling by  $T_s \lesssim T_0$ . The Hamiltonian of the system is

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_T, \quad \mathcal{H}_0 = \mathcal{H}_a + \mathcal{H}_b - \mu B_z S_z, \quad (1)$$

$$\mathcal{H}_T = \sum_{n,m,\alpha,\alpha'} a_{n\alpha}^\dagger [T_0 \delta_{\alpha,\alpha'} + T_s(\boldsymbol{\sigma})_{\alpha,\alpha'} \cdot \mathbf{S}] b_{m\alpha'} + h.c., \quad (2)$$

where  $\mathcal{H}_a$  and  $\mathcal{H}_b$  are the Hamiltonians of the superconducting leads a and b, and  $a_{n\alpha}^\dagger$  ( $a_{n\alpha}$ ) creates (annihilates) an electron in the lead a in the state  $n$  with the spin  $\alpha$ . Further,  $\boldsymbol{\sigma}$  represents the three Pauli matrices, the localized spin 1/2 is described by the operator  $\hat{\mathbf{S}}$ , and  $\mu$  is the magnetic moment of the spin. In the following we use notations  $B$  for  $\mu B_z$  and  $\omega_L = B/\hbar$ .

As we will show in the following, the Josephson current  $I(t)$  oscillates at the frequency  $\omega_J = 2eV/\hbar$ , and the average oscillation amplitude depends on the state of the spin at  $t = 0$ , i.e. on  $|\alpha|^2$  for the spin wave function  $|\Psi_s(0)\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$ . We show also that the dc-component of the current in the circuit with a resistor depends quadratically on the oscillation amplitude, and thus is sensitive to the value  $S_z$  at time  $t = 0$ .

First, we derive the expression for the Josephson current across the junction as a functional of the phase difference  $\varphi(t)$  across the junction and the spin wave function  $|\Psi_s(0)\rangle$  at low temperatures, neglecting the effect of quasiparticles, which will be estimated later. We neglect quantum fluctuations of the phase difference across the junction, assuming that the charging energy  $(2e)^2/C$  is lower than the Josephson energy  $\sim T_0^2 \rho^2 \Delta_0$ , where  $C$  is the junction capacitance and  $\rho$  is the density of states per spin in the leads. We assume that  $T_0^2 \rho^2, T_s^2 \rho^2 \ll 1$  and  $2eV, B \ll \Delta_0$ , and that all relevant frequencies are much lower than  $\Delta_0/\hbar$ . We find the Heisenberg operator for the current using the operator perturbation theory [13] with respect to the tunneling Hamiltonian  $\mathcal{H}_T$ , and then we average this operator over the electron ground state of the Hamiltonian  $\mathcal{H}_a + \mathcal{H}_b$ . The current operator  $\langle I(t) \rangle_e$  obtained in this way is an operator in the spin space and  $\langle \Psi_s^*(0) | \langle I(t) \rangle_e | \Psi_s(0) \rangle$  defines the average current for any initial state  $|\Psi_s(0)\rangle$ . Yet the measurement produces one of the two values, corresponding to the spin up/down states,  $S_z = \pm 1/2$ , rather than the average.

At time  $t > 0$  the current is given by

$$\hat{I}(t) = \exp\left(\frac{i}{\hbar} \hat{\mathcal{H}}t\right) \hat{I} \exp\left(-\frac{i}{\hbar} \hat{\mathcal{H}}t\right), \quad (3)$$

$$\exp\left(-\frac{i}{\hbar} \hat{\mathcal{H}}t\right) = \exp\left(-\frac{i}{\hbar} \hat{\mathcal{H}}_0 t\right) \mathbb{T} \exp\left[-\frac{i}{\hbar} \int_0^t \tilde{\mathcal{H}}_T(\tau) d\tau\right], \quad (4)$$

$$\hat{I} = \sum_{n,m,\alpha,\alpha'} \frac{i e}{\hbar} \hat{a}_{n\alpha}^\dagger [T_0 \delta_{\alpha,\alpha'} + T_s(\hat{\boldsymbol{\sigma}})_{\alpha,\alpha'} \cdot \hat{\mathbf{S}}] \hat{b}_{m\alpha'} + h.c., \quad (5)$$

where we accounted for isotropic exchange interaction of tunneling electrons with localized spin,  $\mathbb{T}$  is the time ordering operator and  $\tilde{A}(t) = \exp(i\mathcal{H}_0 t/\hbar) \hat{A} \exp(-i\mathcal{H}_0 t/\hbar)$  is the operator in the interaction representation,  $\hat{A}$  is the Schroedinger operator and  $\hat{A}(t)$  is the Heisenberg operator. After averaging over electronic degrees of freedom

in the absence of quasiparticles and assuming a classical phase difference on the junction  $\varphi(t)$  we find

$$\langle \hat{I}(t) \rangle_e = \frac{4ie}{\hbar^2} \int_0^t dt' \sin\left[\frac{\varphi(t) + \varphi(t')}{2}\right] \times \left\{ F^+(t-t') F(t-t') [T_0^2 - T_s^2 \tilde{\mathbf{S}}(t) \cdot \tilde{\mathbf{S}}(t')] - h.c. \right\}, \quad (6)$$

where  $\tilde{\mathbf{S}}(t) = \exp(-i\omega_L \hat{S}_z t) \mathbf{S} \exp(i\omega_L \hat{S}_z t)$ , and the Gor'kov Green functions are related to the Bessel functions as

$$F^+(t) = -F(t) = (\pi \Delta_0 \rho / 2) [J_0(\Delta_0 t / \hbar) - i N_0(\Delta_0 t / \hbar)].$$

For singlet Cooper pairing the dependence of the current on the spin is isotropic due to the isotropic exchange coupling of the spin and the tunneling electrons. We see that the average current is expressed via the spin correlation function  $\langle \Psi_s^*(0) | \tilde{\mathbf{S}}(t) \cdot \tilde{\mathbf{S}}(t') | \Psi_s(0) \rangle$ . This function depends on the initial spin state, thus leading to the corresponding dependence for the supercurrent, because

$$\tilde{\mathbf{S}}(t) \cdot \tilde{\mathbf{S}}(0) = (2 \cos \omega_L t + 1) \hat{\mathbf{1}} / 4 - i \hat{S}_z \sin \omega_L t, \quad (7)$$

where  $\hat{\mathbf{1}}$  is the unit matrix in the spin space. The functions  $F^+(t)F(t)$ ,  $F^+(t)F(t) \cos \omega_L t$  and  $F^+(t)F(t) \sin \omega_L t$  oscillate and drop on the scale  $\Delta_0^{-1}$  at  $B \ll \Delta_0$ . For the function  $\varphi(t)$  with characteristic frequencies well below  $\Delta_0$ , we take  $\varphi(t) \approx \varphi(t')$  and for  $t \gg \Delta_0^{-1}$  we obtain

$$\langle \hat{I}(t) \rangle_e = \hat{I}_c \sin \varphi(t), \quad \hat{I}_c = I_0 \hat{\mathbf{1}} + I_s \hat{S}_z. \quad (8)$$

$$I_0 = \frac{2\pi^2 e}{\hbar} \left( T_0^2 - \frac{3}{4} T_s^2 \right) \rho^2 \Delta_0, \quad I_s = \frac{4e}{\hbar} T_s^2 \rho^2 B.$$

$\langle \hat{I}(t) \rangle_e$  is the renormalized current via the Josephson junction obtained after integration over all high frequencies  $\gtrsim \Delta_0$ . It corresponds to the Josephson energy

$$\hat{E}_J\{\varphi(t), \hat{S}_z\} = (\hbar/2e)(I_0 \hat{\mathbf{1}} + I_s \hat{S}_z)[1 - \cos \varphi(t)], \quad (9)$$

which can be considered as an effective Hamiltonian for the Josephson junction with the spin. The spin suppresses the Josephson energy due to the negative contribution  $(3/4)T_s^2$  to  $I_0$  which can lead to the formation of a  $\pi$ -junction when  $I_0$  changes sign [7].

The current amplitude depends on the initial state of the spin with different values for  $S_z = 1/2$  and  $S_z = -1/2$  until the spin flips due to quasiparticles or due to relaxation. Otherwise, on average, the spin does not flip in Josephson tunneling because the first electron of the Cooper pair may flip it, but then the second one restores the spin since the Cooper pair cannot carry spin; thus  $S_z$  on average is an integral of motion.

The effect of the Josephson current on the spin can also be understood from the following argument. One sees from Eq. (2) that the spin feels the fluctuating exchange field induced by tunneling electrons,

$$\mu\hat{\mathbf{h}}(t) = T_s \sum_{n,m,\alpha,\alpha'} a_{n\alpha}^\dagger(t)(\boldsymbol{\sigma})_{\alpha,\alpha'} b_{m\alpha'}(t) + h.c. \quad (10)$$

This field causes deviations of spin from the  $z$ -axis orientation due to presence of transverse components  $\sigma_x$  and  $\sigma_y$  when the first electron of a Cooper pair tunnels via spin. This spin response interferes with the spin dependent part of the current operator (5) giving rise to the spin dependent contribution in the current.

Having obtained linear dependence of the Josephson critical current on the spin  $z$ -component we have reduced the problem to the well studied case of a dc-SQUID measuring a flux qubit [4]. Here, for completeness, we provide a simplified version of the derivation of the dephasing and measurement times, valid for  $R \ll h/e^2$  and  $T \ll V$ .

Without the resistor ( $R = 0$ ) the system is dissipationless,  $\varphi(t) = \omega_J t$  and no measurement is performed. Introducing dissipation allows us to measure the amplitude of the Josephson-current oscillations, and, thus, the initial state of the spin. We consider the circuit in Fig. 1 with  $R \neq 0$ . The circuit dynamics is described by

$$\frac{\hbar}{2e} C \ddot{\varphi} + \frac{\hbar}{2e} \frac{\dot{\varphi}}{R} + I_c \sin \varphi = \frac{V}{R} + \xi(t), \quad (11)$$

where the Langevin force  $\xi(t)$  is the random current (Nyquist noise) generated by the resistor. Let us consider the simplest case when the junction capacitance can be neglected. This is possible if  $RC\omega_J \ll 1$ . When  $V \ll I_c R$  no voltage drops on the junction and the dc-current is not sensitive to the value of the Josephson critical current and, thus, to the state of the spin. For  $V \gg I_c R$ , solving Eq. (11) and averaging one obtains  $\varphi(t) = \omega_J t + (I_c R/V) \cos(\omega_J t)$  (see e.g., [8]) and

$$I = I_c \sin \varphi \approx I_c \sin(\omega_J t) + (I_c^2 R/V) \cos^2(\omega_J t). \quad (12)$$

The dc-component of the current is given by

$$\hat{I}_{dc} = \frac{\hat{I}_c^2 R}{2V} = \frac{(I_0^2 + I_s^2/4)R}{2V} \hat{\mathbf{1}} + \frac{I_0 I_s R}{V} \hat{S}_z. \quad (13)$$

Now the dc-current is sensitive to the state of the spin. For a single measurement one of the two possible values of the current,  $(I_0 \pm I_s/2)^2 R/2V$ , is realized. In a multiple set of measurements these values occur with probabilities  $P_+ = |\alpha|^2$  and  $P_- = 1 - |\alpha|^2$ .

Now we are in a position to derive the measurement rate  $\Gamma_m$  needed to resolve two values of the current corresponding to  $S_z = \pm 1/2$  on the background of shot noise of Cooper pairs tunneling incoherently due to the resistor. The signal is  $\delta I \equiv I_0 I_s R/V$ . The noise power of the background current  $S_I(\omega)$  is defined as

$$S_I(\omega) = \frac{1}{2} \int_{-\infty}^{\infty} dt \exp(i\omega t) \langle [I(t)I(0) + I(0)I(t)] \rangle. \quad (14)$$

At  $V \gg I_c R$  it is given by

$$S_I(\omega) \approx 2eI_{dc} \approx eI_0^2 R/V, \quad \omega < V \ll \Delta_0, \quad (15)$$

assuming  $I_0 \gg I_s$ . This expression can be obtained [8] from the Langevin equation (11) with the appropriate high frequency spectrum of  $\xi(t)$  [9]. It describes the shot noise of individual Cooper pairs tunneling incoherently through the junction and dissipating each energy  $2eV$  into the microscopic modes of the resistor [10]. We define the measurement rate as:

$$\Gamma_m \equiv (\delta I)^2 / (8S_I) \approx (I_s^2 R) / (8eV). \quad (16)$$

Note that the current correlation function for the shot noise is expressed via the phase correlation function as

$$\langle I(t)I(0) \rangle = I_0^2 \langle \sin \varphi(t) \sin \varphi(0) \rangle. \quad (17)$$

Next we discuss the back-action effect of the measurement, i.e. the effect of incoherent Cooper pair tunneling, on the spin dynamics. It leads to the dephasing of transverse spin components, i.e. to the destruction of coherent superposition of states  $S_z = \pm 1/2$ . To derive the dephasing rate,  $\Gamma_d$ , we consider the  $x$ -component of spin averaged over the electronic degrees of freedom. We obtain

$$\langle S_x(t) \rangle_e = \eta(t) \tilde{S}_x(t) + \frac{\hbar}{2e} I_s \tilde{S}_y(t) \int_0^t \cos \varphi(\tau) d\tau, \quad (18)$$

where  $\eta(t) < 1$  is an oscillating function close to unity [see Eq. (20)], describing the reduction of the spin amplitude. Its origin will be discussed later. Here we focus on the second term in the right hand side. This term describes the phase accumulation due to the effect of a random magnetic field acting on spin,  $h_{z\tau}(t) = [\hbar I_s / (2e)] \cos \varphi(t)$ , in agreement with the effective Hamiltonian (9). The dephasing rate is given by

$$\Gamma_d = \frac{1}{2\hbar^2} \left( \frac{\hbar I_s}{2e} \right)^2 \int_{-\infty}^{\infty} d\tau \langle \cos \varphi(\tau) \cos \varphi(0) \rangle. \quad (19)$$

By use of Eqs. (15), (17) we get  $\Gamma_d \approx I_s^2 R / (8eV) \approx \Gamma_m$ .

The inequality  $\Gamma_d \geq \Gamma_m$  is the fundamental property of quantum mechanics as it follows from the uncertainty relations (see, e.g., review [11] and references therein). It was proven for the case of a linear detector with the direct coupling of the detector to the measured observable, see e.g., Ref. [4,14]. In our setup the interaction of spin with tunneling electrons is nonzero only in the second order of perturbation theory. However, effectively we obtain a linear relation between the spin  $z$ -component and the superconducting current accessible experimentally, see Eqs. (8), (9). Consequently, the optimal relation  $\Gamma_d \approx \Gamma_m$  is satisfied for our measurements.

Now we discuss the effects which are beyond the accuracy of the effective Hamiltonian (9). We calculate the

$z$ -component of the spin,  $\langle S_z(t) \rangle_e$  averaged over the electronic degrees of freedom, as was done in Eqs. (3), (4). We derive for  $t \gg \hbar/\Delta_0$ :

$$\begin{aligned} \langle \hat{S}_z(t) \rangle_e &= \langle \exp(i\mathcal{H}t) \hat{S}_z \exp(-i\mathcal{H}t) \rangle_e = \eta(t) \hat{S}_z, \\ \eta(t) &= 1 - 2T_s^2 \rho^2 [1 + \cos \varphi(t)]. \end{aligned} \quad (20)$$

The maximum average value of the spin  $z$ -component is slightly smaller than  $1/2$ . This is because spin flips can happen after one electron of a Cooper pair passes the spin. The time of passing is  $\sim \hbar/\Delta_0$  and probability to scatter on the spin per unit time is  $\sim T_s^2 \rho^2 (\Delta_0/\hbar) \cos \varphi(t)$ . Also the second term in Eq. (18), which corresponds to an extra phase in the spin rotation, can be understood in terms of the spin flips. Each passage of a Cooper pair changes this phase by  $\Delta\alpha \sim B/\Delta_0$  and in the case of coherent tunneling during the time  $t$  this phase changes by  $\alpha(t) \sim T_s^2 \rho^2 (\Delta_0/\hbar) (B/\Delta_0) t \sim T_s^2 \rho^2 \omega_L t$ . Hence, in addition to  $B$  there is the effective magnetic field acting on spin  $h_z \sim T_s^2 \rho^2 B$  as follows also from the effective Hamiltonian (9). For uncorrelated tunneling of Cooper pairs this field is random and in addition to regular change it leads to phase diffusion as  $\langle [\alpha(t) - \alpha(0)] \alpha(0) \rangle \propto (\Delta\alpha)^2 t \propto B^2 t$  and  $\Gamma_d \propto B^2$ .

For comparison, we consider a normal tunnel junction with unpolarized electrons in the leads. We find the current sensitivity to the spin  $z$  component also,

$$\begin{aligned} \langle \hat{I}(t) \rangle_e &= \frac{2\pi e}{\hbar} \rho^2 [(2T_0^2 + 3T_s^2) V \hat{\mathbf{1}} - \\ &T_s^2 (|V+B| - |V-B|) \hat{S}_z]. \end{aligned} \quad (21)$$

Now, however, the spin flips are allowed because tunneling electrons are not paired. For short times  $\hbar [T_s^2 \rho_0^2 V]^{-1} \gg t \gg \hbar V^{-1}$  we obtain

$$\langle \hat{S}_z(t) \rangle_e = \hat{S}_z - 2\pi T_s^2 \rho^2 [2\hat{S}_z \max(|V|, |B|) - B] t, \quad (22)$$

which gives at  $eV \gg B$  the spin-flip (demolition) rate  $\Gamma_n^\dagger \sim T_s^2 \rho^2 eV/\hbar$ . The dephasing rate is about the same. The measurement rate given by  $\hbar \Gamma_m \sim T_s^4 \rho^2 B^2 / (T_0^2 eV)$  is smaller than the spin-flip rate. Hence, tunneling measurement of spin is impossible in the normal state.

Thermal quasiparticles cause spin flips as the electrons do in the normal state. The demolition rate is proportional to the fraction of quasiparticles. At  $T \ll \Delta_0$  it is exponentially small,  $\hbar \Gamma_s^\dagger \lesssim T_s^2 \rho^2 eV \exp(-\Delta_0/T)$ .

In this paper we have considered spin  $S = 1/2$ . We note that also for arbitrary  $S$  the current operator  $\langle \hat{I}(t) \rangle_e$  is a linear combination of operators  $\hat{S}_z^n$ ,  $0 \leq n \leq 2S$ . It depends on the spin initial state and differs for the states  $\langle \hat{S}_z \rangle = \pm S$  in contrast to the classical spin (discussed in Ref. [3]) for which it depends only on  $S_z^2$  because  $\tilde{\mathbf{S}}(t) \cdot \tilde{\mathbf{S}}(0) = (S^2 - S_z^2) \cos \omega_L t + S_z^2$ .

We note, that other strategies of spin measurement could also be considered. E.g., a Josephson junction with a spin could be inserted into a superconducting ring, a

voltage across the junction could be induced by changing magnetic flux across the ring and the alternating flux induced by Josephson oscillations could be measured by a secondary circuit coupled inductively to the superconducting ring. This will be studied elsewhere.

In conclusion, we have shown that the nondemolition measurements of a quantum spin are possible using the Josephson oscillations. The average amplitude of Josephson oscillations depends on the initial state of the spin after the tunneling or voltage is switched on and the voltage exceeds the dissipative threshold. This amplitude can be found measuring the dc-current in the circuit with a resistor. Use of the singlet-pair tunneling as an intermediate nondissipative system allows us to obtain information on the initial value of the  $z$  component of the spin without flipping the spin. Thus a QND tunneling measurement of spin is performed.

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