

Electron transport through interacting quantum dots

Dmitri S. Golubev^{1,3}, and Andrei D. Zaikin^{2,3}

¹*Institut für Theoretische Festkörperphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany*

²*Forschungszentrum Karlsruhe, Institut für Nanotechnologie, 76021, Karlsruhe, Germany*

³*I.E. Tamm Department of Theoretical Physics, P.N. Lebedev Physics Institute, 119991 Moscow, Russia*

We present a detailed theoretical investigation of the effect of Coulomb interactions on electron transport through quantum dots and double barrier structures connected to a voltage source via an arbitrary linear impedance. Combining real time path integral techniques with the scattering matrix approach we derive the effective action and evaluate the current-voltage characteristics of quantum dots at sufficiently large conductances. Our analysis reveals a rich variety of different regimes which we specify in details for the case of chaotic quantum dots. At sufficiently low energies the interaction correction to the current depends logarithmically on temperature and voltage. We identify two different logarithmic regimes with the crossover between them occurring at energies of order of the inverse dwell time of electrons in the dot. We also analyze the frequency-dependent shot noise in chaotic quantum dots and elucidate its direct relation to interaction effects in mesoscopic electron transport.

I. INTRODUCTION

Low temperature electron transport in disordered conductors with electron-electron interactions remains one of the most intriguing topics in modern mesoscopic physics. Interplay between charge discreteness, quantum coherence, scattering and interactions yields a rich variety of non-trivial effects, many of which can adequately be described only within a rather complicated theoretical framework.

One of the most successful theoretical approaches in mesoscopic physics is the scattering matrix formalism^{1,2}. In the absence of interactions this method allows for a complete and physically transparent analysis of electron dynamics in coherent conductors which includes not only electrical conductance but also shot noise³ and eventually all higher cumulants of the current operator⁴. Formally it is possible to generalize the Landauer formalism to systems with interactions⁵. Within this approach, however, the electron Green functions in the interacting region still remain to be evaluated diagrammatically or by other means.

An alternative way to tackle the problems with disorder and interactions is to employ non-perturbative path-integral-based techniques^{6,7}. Recently it was demonstrated^{8,9,10} that the advantages of the path integral methods and the scattering matrix approach can be conveniently combined within one formalism, thereby providing a powerful tool to theoretically describe interaction effects in mesoscopic conductors. Several important results have already been obtained in this way. For instance, with the aid of the instanton technique it was shown⁸ that – similarly to tunnel junctions^{11,12} – arbitrary coherent conductors can exhibit the phenomenon of weak charge quantization. Unfortunately, in the limit of large dimensionless conductances $g \gg 1$ this effect is rather weak and it gains importance only at exponentially small temperatures and voltages. Furthermore, weak charge quantization vanishes completely even at $T = 0$ provided at least one of the conducting channels is

fully transparent^{8,13}. The results⁸ were later confirmed and extended in Refs. 14 and 15.

Another – much more robust – effect of electron-electron interactions is the so-called interaction correction to the $I - V$ curve. This interaction correction is negative and its magnitude scales linearly with the parameter⁹

$$\beta = \frac{\sum_n T_n(1 - T_n)}{\sum_n T_n}, \quad (1)$$

where T_n is the transmission of the n -th conducting channel. In contrast to the effect of weak charge quantization, even for large conductances $g \gg 1$ the interaction correction remains clearly observable up to high temperatures and it vanishes only for $\beta \rightarrow 0$, i.e. provided *all* the conducting channels in the system are fully transparent. Moreover, as both temperature and voltage get lowered, the interaction correction grows logarithmically^{9,16}. As a result, at sufficiently low energies the smallness $\sim 1/g$ gets compensated by a large logarithm and the system enters a non-perturbative regime where terms of all orders in the interaction need to be evaluated. Let us also recall that for a particular case of highly conducting tunnel junctions this logarithmic enhancement of the interaction correction is already well known for single- and double-junction systems^{17,18} as well as for tunnel junction arrays^{19,20}.

It is interesting to observe that both interaction correction to the conductance and the shot noise spectrum³ are proportional to the same parameter β (1). This observation illustrates a close relation between quantum noise and interaction effects in coherent conductors^{9,16}. Proceeding further along these lines one can investigate the effect of electron-electron interactions on current noise¹⁰. The interaction correction to the Nyquist noise was again found to scale with the parameter (1) (in accordance with the fluctuation-dissipation theorem) while the same correction to the shot noise turned out to be proportional to the third cumulant of the current operator. It was conjectured¹⁰ that the same rule should apply for higher

cumulants as well, i.e. the interaction correction to the n -th cumulant should be proportional to the $(n+1)$ -th cumulant for all values of n . A general proof of this conjecture was very recently provided in Refs. 21,22.

Throughout the analysis^{8,9,10} it was assumed that in the absence of interactions the conductor is described by an *energy independent* (though otherwise general) scattering matrix²³. While in a number of important cases the above assumption indeed applies, in various other physical situations it turns out to be insufficient. Therefore it would be highly desirable to develop a generalization of the path integral technique^{9,10} to conductors described by energy dependent scattering amplitudes. This generalization is the primary goal of our present paper.

In physical terms our analysis should now effectively account for internal dynamics of coherent conductors or, in other words, for a finite *dwell time* of electrons. This effect should be combined with that of electron-electron interactions. Below we will accomplish this program for an important and widely studied class of conductors – the so-called quantum dots^{2,24}.

The structure of the paper and our main results are as follows.

In Sec. 2 we describe our general real time path integral formalism and derive the effective action of interacting quantum dots, Eqs. (17-22) of our paper. The effect of electron-electron interactions is treated in a standard manner by the Hubbard-Stratonovich decoupling of the Coulomb term in the Hamiltonian and reducing the problem to that of an electron interacting with the fluctuating quantum electromagnetic field defined on the Keldysh contour. In order to handle the fermionic part of the problem we combine our path integral analysis with the scattering matrix approach. For the model of a quantum dot adopted here the latter approach allows to *exactly* integrate out all the electron paths and express the effective action only in terms of the fluctuating fields which are then treated within an effective quasiclassical approximation suitable for highly conducting quantum dots with $g \gg 1$.

In Sec. 3 we derive a general expression for the current noise in chaotic quantum dots as a function of frequency, voltage and temperature. Although this expression itself does not include interactions, it turns out to be very useful for better understanding of the relation between shot noise and interaction effects in mesoscopic conductors.

Sec. 4 is devoted to a detailed description of the current-voltage characteristics of interacting quantum dots. In the most simple voltage-biased limit the corresponding general expressions are presented in Sec. 4A by Eqs. (53)-(61). Further analysis of these expressions for chaotic quantum dots in the leading non-trivial order in $1/g$ is carried out in Sec. 4B and 4C. A more general case of an arbitrary external impedance is considered in Sec. 4D. It is demonstrated that, as one goes away from the voltage-biased limit, the interaction correction to the current gets substantially modified and new regimes become possible.

Further discussion of our results and their comparison to several recent experiments can be found in Sec. 5. In Appendices we present various technical details of our derivation of the effective action (Appendices A and B), details of our averaging procedure (Appendix C) and general expressions for the current in interacting quantum dots (Appendix D).

II. EFFECTIVE ACTION AND CURRENT OPERATOR

A. General formalism

Let us consider a system of interacting electrons described by the Hamiltonian

$$H = \int d\mathbf{r} \psi_\sigma^\dagger(\mathbf{r}) \left[-\frac{\nabla^2}{2m} - \mu + U(\mathbf{r}) \right] \psi_\sigma(\mathbf{r}) + \frac{1}{2} \int d\mathbf{r} \int d\mathbf{r}' \psi_\sigma^\dagger(\mathbf{r}) \psi_\sigma^\dagger(\mathbf{r}') \frac{e^2}{|\mathbf{r} - \mathbf{r}'|} \psi_{\sigma'}(\mathbf{r}') \psi_\sigma(\mathbf{r}). \quad (2)$$

Here the operator $\psi_\sigma^\dagger(\mathbf{r})$ ($\psi_\sigma(\mathbf{r})$) creates (annihilates) an electron with the coordinate \mathbf{r} and the spin projection σ . In Eq. (2) the summation over σ is assumed. The time evolution of the density matrix of the whole system $\rho(t)$ is determined by the equation

$$\rho(t) = T \exp[-iHt] \rho(0) \tilde{T} \exp[iHt]. \quad (3)$$

Applying the Hubbard-Stratonovich transformation we rewrite this equation in the form

$$\rho(t) = \frac{\int \mathcal{D}V_j \bar{T} e^{-i \int_0^t dt' H_1(t')} \rho(0) \tilde{T} e^{i \int_0^t dt' H_2(t')} e^{iS_{\text{em}}}}{\int \mathcal{D}V_j e^{iS_{\text{em}}}}, \quad (4)$$

where

$$S_{\text{em}} = \sum_{j=1,2} (-1)^{j+1} \int_0^t dt' \int d\mathbf{r} \frac{(\nabla V_j(t', \mathbf{r}))^2}{8\pi}. \quad (5)$$

is the electromagnetic contribution to the action, the symbols T and \tilde{T} imply ordering respectively in the forward and backward time directions and

$$H_j = \int d\mathbf{r} \psi_\sigma^\dagger(\mathbf{r}) \left[-\frac{\nabla^2}{2m} - \mu + U(\mathbf{r}) - eV_j(t', \mathbf{r}) \right] \psi_\sigma(\mathbf{r}).$$

As usually, the expectation value of any physical operator is obtained by taking the trace of this operator multiplied by $\rho(t)$. Here we are mainly interested in the expectation value of the current operator. This operator, which we will specify later, can be obtained by taking the functional derivatives of the combination $T(\tilde{T}) \exp[\pm i \int_0^t dt' H_{1,2}(t')]$ with respect to the fluctuating fields $V_{1,2}(t)$. For this reason there is no need to

deal with the whole expression (4), the kernel of the current operator can be obtained from the effective action S , which is defined by the equation

$$e^{iS} = \text{Tr} \left\{ \text{Te}^{-i \int_0^t dt' H_1(t')} \rho(0) \tilde{\text{T}}e^{i \int_0^t dt' H_2(t')} \right\} e^{iS_{\text{em}}}. \quad (6)$$

The effective action S can be expressed through the Keldysh Green function matrix $\check{G}_{V_1, V_2}^{-1}$,

$$iS = 2\text{Tr} \ln \check{G}_{V_1, V_2}^{-1} + iS_{\text{em}}, \quad (7)$$

and the matrix $\check{G}_{V_1, V_2}^{-1}$ satisfies the equation

$$\left[\left(i \frac{\partial}{\partial t} + \frac{\nabla^2}{2m} + \mu - U(\mathbf{r}) + eV^+(t, \mathbf{r}) \right) \check{\mathbb{1}} + \delta \check{G}^{-1} \right] \times \check{G}_{V_1, V_2}(tt', \mathbf{r}\mathbf{r}') = \check{\sigma}_z \delta(t-t') \delta(\mathbf{r}-\mathbf{r}'), \quad (8)$$

Here we defined $\delta \check{G}^{-1} = (eV^-(t', \mathbf{r})/2) \check{\sigma}_z$ (where $\check{\sigma}_z$ is one of the Pauli matrices) and introduced symmetric and anti-symmetric combinations of the fluctuating fields:

$$V^+ = (V_1 + V_2)/2, \quad V^- = V_1 - V_2. \quad (9)$$

As we have already discussed, in the interesting for us limit of large dot conductances fluctuations of the fields $V_{1,2}$ remain relatively small. In this case one can expand the exact effective action in V^- keeping only the first and the second orders. Then one finds

$$iS = iS_{\text{em}} + 2\text{Tr}[\check{G}_{V^+} \delta \check{G}^{-1}] - \text{Tr}[(\check{G}_{V^+} \delta \check{G}^{-1})^2], \quad (10)$$

where \check{G}_{V^+} is the solution of Eq. (8) with $\delta \check{G}^{-1} = 0$. From (10) we obtain

$$\begin{aligned} iS = & iS_{\text{em}} + e \int_0^t dt' \int d^3\mathbf{r} (G_{V^+, 11}(t' - 0, t', \mathbf{r}, \mathbf{r}) \\ & + G_{V^+, 22}(t' + 0, t', \mathbf{r}, \mathbf{r})) V^-(t', \mathbf{r}) \\ & - e^2 \int_0^t dt' \int_0^t dt'' \int d^3\mathbf{r}' d^3\mathbf{r}'' G_{V^+, 12}(t', t'', \mathbf{r}', \mathbf{r}'') \\ & \times V^-(t'', \mathbf{r}'') G_{V^+, 21}(t'', t', \mathbf{r}'', \mathbf{r}') V^-(t', \mathbf{r}'). \end{aligned} \quad (11)$$

Note that at this stage it is important *not* to expand in V^+ keeping the exact nonlinear dependence of (10) on this field.

The components of the Green function \check{G}_{V^+} can be expressed through the initial single particle electron density matrix $\hat{\rho}_0$ and the single particle evolution operator \hat{U}^{φ^+} . These are the matrices in the channel space denoted by a hat here and below. The operator \hat{U}^{φ^+} is evaluated in the Appendix A. We have

$$\begin{aligned} \hat{G}_{V^+, 11}(t_1, t_2) &= -i\theta(t_1 - t_2) \hat{U}^{\varphi^+}(t_1, t_2) \\ &\quad + i\hat{U}^{\varphi^+}(t_1, 0) \hat{\rho}_0 \hat{U}_{\varphi^+}(0, t_2), \\ \hat{G}_{V^+, 22}(t_1, t_2) &= -i\theta(t_2 - t_1) \hat{U}^{\varphi^+}(t_1, t_2) \\ &\quad + i\hat{U}^{\varphi^+}(t_1, 0) \hat{\rho}_0 \hat{U}_{\varphi^+}(0, t_2), \\ \hat{G}_{V^+, 12}(t_1, t_2) &= i\hat{U}^{\varphi^+}(t_1, 0) \hat{\rho}_0 \hat{U}^{\varphi^+}(0, t_2), \\ \hat{G}_{V^+, 21}(t_1, t_2) &= -i\hat{U}^{\varphi^+}(t_1, 0) [\hat{1} - \hat{\rho}_0] \hat{U}^{\varphi^+}(0, t_2) \end{aligned} \quad (12)$$

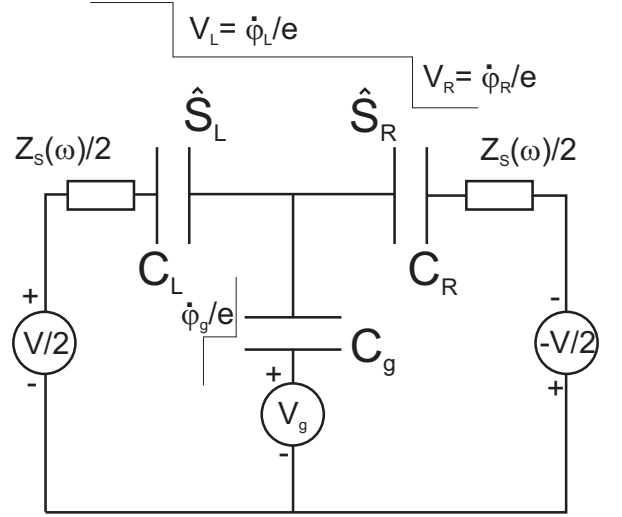


FIG. 1: Schematics of the quantum dot.

Here for future purposes we have defined

$$\varphi^\pm(t') = \int_0^{t'} dt'' eV^\pm(t''). \quad (13)$$

B. Effective action

The above analysis is rather general and can be applied to a variety of mesoscopic structures with disorder and interactions. Our primary goal here is to consider electron transport through a system which contains an interacting quantum dot. This system is schematically displayed in Fig. 1.

A quantum dot can be viewed as an island in-between two barriers connected to a voltage source via metallic leads with an arbitrary impedance $Z(\omega)$. Electrons can enter the dot through one of the barriers, spend some time there propagating between the barriers and being reflected, and finally leave the dot through another barrier. Details of the electron motion inside the dot will not be important for us: Electrons can either propagate ballistically from one barrier to the other or suffer additional scattering inside the dot, e.g. from the outer walls or otherwise.

Electron-electron interactions are taken into account by means of the effective capacitance model. Here we introduce capacitances of the left and right barriers $C_{L,R}$ and the gate capacitance C_g . Making use of this model we can specify the expression for the effective action further. Namely, we will assume that strong changes of the fluctuating voltage fields V^\pm in space are allowed only in the vicinity of the barriers where they suffer jumps V_L^\pm and V_R^\pm (see Fig. 1). Additional voltage drop inside the dot is neglected, i.e. there only the time dependence of the fields V^\pm is taken into account. In the leads these fields are assumed to vary slowly in space.

Let us introduce the fluctuating potentials of the left and right leads $V_{l,r}^{\pm}$ and of the dot V_d^{\pm} . Then we define the (time dependent) voltages across the barriers, $V_L^{\pm} = V_l^{\pm} - V_d^{\pm}$ and $V_R^{\pm} = V_d^{\pm} - V_r^{\pm}$, and find the electromagnetic potentials in the rest of our system by minimizing the action (11). Afterwards we substitute the result back into Eq. (11) and arrive at the expression which depends only on the phase jumps $\varphi_{L,R}^{\pm}(t') = \int_0^{t'} dt'' eV_{L,R}^{\pm}(t'')$. In this way two contributions to the total effective action can be identified,

$$iS = iS_{\text{dot}} + iS_{\text{ext}}. \quad (14)$$

Here the term iS_{dot} comes from the last two terms of Eq. (11) where the space integrals run over the inner

part of the dot and the barrier area. The second contribution iS_{ext} comes from the integrals over the remote parts of the system (leads). Making use of a slow coordinate dependence of the fields V^{\pm} in the leads one can expand the action iS_{ext} in these fields and keep the first and the second order terms in this expansion. We arrive at the standard form of the action²⁵ describing fluctuations produced by an arbitrary environment with a linear impedance. Above we introduced the two leads with identical impedances $Z_S(\omega)/2$. The fluctuating part of the voltage across the left lead is $(\dot{\varphi}_L^+ - \dot{\varphi}_g^+)/e$, the corresponding value for the right lead is $(\dot{\varphi}_R^+ + \dot{\varphi}_g^+)/e$. The corresponding contributions to the action need to be added and we arrive at the following expression

$$\begin{aligned} iS_{\text{ext}} = & -\frac{i}{e^2} \int_0^t dt' (C_L \ddot{\varphi}_L^+ \varphi_L^- + C_R \ddot{\varphi}_R^+ \varphi_R^- + C_g \ddot{\varphi}_g^+ \varphi_g^-) \\ & - \frac{2i}{e^2} \int_0^t dt_1 dt_2 Z_S^{-1}(t_1 - t_2) [(\varphi_L^-(t_1) - \varphi_g^-(t_1))(\dot{\varphi}_L^+(t_2) - \dot{\varphi}_g^+(t_2)) + (\varphi_R^-(t_1) + \varphi_g^-(t_1))(\dot{\varphi}_R^+(t_2) + \dot{\varphi}_g^+(t_2))] \\ & - \frac{1}{e^2} \int_0^t dt_1 dt_2 Z_S(t_1 - t_2) [(\varphi_L^-(t_1) - \varphi_g^-(t_1))(\varphi_L^-(t_2) - \varphi_g^-(t_2)) + (\varphi_R^-(t_1) + \varphi_g^-(t_1))(\varphi_R^-(t_2) + \varphi_g^-(t_2))] , \\ & Z_S^{-1}(t) = \int \frac{d\omega}{2\pi} \frac{e^{-i\omega t}}{Z_S(\omega)}, \quad Z_S(t) = \int \frac{d\omega}{2\pi} \omega \coth \frac{\omega}{2T} e^{-i\omega t} \text{Re} \left(\frac{1}{Z_S(\omega)} \right). \end{aligned} \quad (15)$$

The term iS_{em} is evaluated within the capacitance model and included in the above expression for iS_{ext} (15).

Now let us specify the effective action of the quantum dot. In what follows we will assume that the left (right) lead contains N_L (N_R) conducting channels. Every channel is characterized by the velocity v_n (which may also be different in different leads). In the absence of electron-electron interactions quantum transport through the dot can be fully described by the scattering matrix^{1,2}. As we have already pointed out in Sec. 1 a convenient way to include interaction effects is to combine the scattering matrix approach with the path integral technique. As compared to Refs. 8,9,10 here we are dealing with a more complicated situation because the scattering matrix of the dot can now depend on the energy E of incoming electrons. Therefore, the action^{9,10} cannot be directly used and a proper generalization is required for our problem.

Let us define the scattering matrix describing electron transport through our quantum dot in the absence of interactions:

$$\hat{S}(E) = \begin{pmatrix} \hat{r}(E) & \hat{t}'(E) \\ \hat{t}(E) & \hat{r}'(E) \end{pmatrix}. \quad (16)$$

For our derivation it is also convenient to perform the

Fourier transformation of the S -matrix. We introduce

$$\hat{S}(t) = \int \frac{dE}{2\pi} \hat{S}(E) e^{-iEt}, \quad \hat{S}^\dagger(t) = \int \frac{dE}{2\pi} \hat{S}^\dagger(E) e^{iEt},$$

and the functions $\hat{r}(t)$, $\hat{r}'(t)$, $\hat{t}(t)$, $\hat{t}'(t)$ defined analogously. In accordance with the causality principle one has $\hat{S}(t < 0) = \hat{S}^\dagger(t < 0) \equiv 0$.

It turns out that one can find an explicit expression for the evolution operator \hat{U}^{φ^+} in terms of the scattering matrix, construct the Green functions (12) and then derive the effective action of the quantum dot S_{dot} from Eq. (11). This program is carried out in Appendices A and B. The final result can be expressed in the form

$$iS_{\text{dot}} = iS_R - S_I. \quad (17)$$

As usually, the part S_R describes dissipative effects. It reads (see Appendix B):

$$\begin{aligned} iS_R = & -2i \int_0^t dz \int_0^\infty dx dy \text{tr} \{ [\delta(z-y) \hat{\varphi}^-(z) \delta(z-x) \\ & - \hat{S}^\dagger(z-y) \hat{\varphi}^-(z) \hat{S}(z-x)] \hat{\rho}(y,x) \}. \end{aligned} \quad (18)$$

where

$$\hat{\varphi}^-(z) = \begin{pmatrix} -\hat{1}\varphi_L^-(z) & 0 \\ 0 & \hat{1}\varphi_R^-(z) \end{pmatrix},$$

$$\hat{\rho}(y, x) = \rho_0(y-x) \begin{pmatrix} \hat{1}e^{i[\varphi_L^+(y)-\varphi_L^+(x)]} & 0 \\ 0 & \hat{1}e^{i[\varphi_R^+(x)-\varphi_R^+(y)]} \end{pmatrix} \quad (19)$$

($\hat{1}$ is the unity matrix), and

$$\rho_0(x) = \int \frac{dE}{2\pi} \frac{e^{iEx}}{1+e^{E/T}} = \frac{1}{2}\delta(x) - \frac{iT}{2\sinh[\pi Tx]} \quad (20)$$

is the equilibrium density matrix of non-interacting electrons.

The second term S_I in (17) accounts for quantum noise. This term is also evaluated in Appendix B. As a result we obtain

$$S_I = \int_0^t dx_1 dx_2 \int_0^\infty dy_1 dy_2 \int_0^\infty dz_1 dz_2$$

$$\times \text{tr} \{ [\delta(x_1 - z_1) \hat{\varphi}^-(x_1) \delta(x_1 - y_1)$$

$$- \hat{S}^\dagger(x_1 - z_1) \hat{\varphi}^-(x_1) \hat{S}(x_1 - y_1)] \hat{\rho}(y_2, y_1)$$

$$\times [\delta(x_2 - y_2) \hat{\varphi}^-(x_2) \delta(x_2 - z_2)$$

$$- \hat{S}^\dagger(x_2 - y_2) \hat{\varphi}^-(x_2) \hat{S}(x_2 - z_2)] \hat{h}(z_1, z_2) \} \quad (21)$$

Here we defined

$$\hat{h}(z_1, z_2) = h_0(z_1 - z_2) \begin{pmatrix} e^{i[\varphi_L^+(z_1)-\varphi_L^+(z_2)]} & 0 \\ 0 & e^{i[\varphi_R^+(z_2)-\varphi_R^+(z_1)]} \end{pmatrix}, \quad (22)$$

and $h_0(z) = \delta(z) - \rho_0(z)$. We also note that in the long time limit the integration over x, y in Eq. (18) and over $x_{1,2}, y_{1,2}$ in Eq. (21) can be extended to the interval $(-\infty, +\infty)$.

The expressions (17-22) for the effective action of the quantum dot represent the main technical result of our paper. This effective action is defined by essentially nonlocal in time expressions which account for electron-electron interaction effects in the presence of a non-zero dwell time τ_D of electrons in the quantum dot. Should τ_D be much shorter than any other relevant time scale in our problem, the time dependence of the scattering matrices $\hat{S}(t)$ and $\hat{S}^\dagger(t)$ can be approximated by the δ -function, $\hat{S}(t) \propto \hat{S}^\dagger(t) \propto \delta(t)$, in which case the action (17-22) reduces to one derived in Ref. 9.

C. Averaging of the action

If one is not interested in mesoscopic fluctuations of the effective action one can simplify the above expressions by averaging Eqs. (18) and (21) over energy intervals exceeding the dot level spacing δ . Consider first the term S_R (18). Let us illustrate the main idea by treating the average $\text{tr}\langle \hat{t}^\dagger(x) \hat{t}(y) \rangle$. We have

$$\text{tr}\langle \hat{t}^\dagger(x) \hat{t}(y) \rangle = \int \frac{dE_{1,2}}{(2\pi)^2} \text{tr}\langle \hat{t}^\dagger(E_1) \hat{t}(E_2) \rangle e^{iE_1 x - iE_2 y}. \quad (23)$$

In a broad interval of energies the average $\text{tr}\langle \hat{t}^\dagger(E_1) \hat{t}(E_2) \rangle$ should depend only the energy difference $E_1 - E_2$. Making use of this observation let us define the function $u_\omega^{RL} = \text{tr}\langle \hat{t}^\dagger(E) \hat{t}(E + \omega) \rangle$ and its Fourier transform $u_{RL}(t) = \int \frac{d\omega}{2\pi} u_\omega^{RL} e^{-i\omega t}$. The function u_ω^{RL} satisfies the property $u_{-\omega}^{RL} = u_\omega^{RL*}$, therefore $u_{RL}(t)$ is real. Other averages are defined analogously. We find

$$\begin{aligned} \text{tr}\langle [\delta(x)\delta(y)\hat{1} - \hat{r}^\dagger(x)\hat{r}(y)] \rangle &= \delta(x-y)u_{LL}(y), \\ \text{tr}\langle \hat{t}^\dagger(x)\hat{t}'(y) \rangle &= \delta(x-y)u_{LR}(y), \\ \text{tr}\langle [\delta(x)\delta(y)\hat{1} - \hat{r}'^\dagger(x)\hat{r}(y)] \rangle &= \delta(x-y)u_{RR}(y), \\ \text{tr}\langle \hat{t}^\dagger(x)\hat{t}(y) \rangle &= \delta(x-y)u_{RL}(y). \end{aligned} \quad (24)$$

Averaging of Eq. (18) with the aid of (24) yields

$$iS_R^{\text{av}} = -\frac{i}{\pi} \sum_{i,j=L,R} \int_0^t dz \int_0^z dx \varphi_i^-(z) u_{ij}(z-x) \varphi_j^+(x) \quad (25)$$

We note that S_R^{av} – unlike the non-averaged action S_R (18) – is bilinear in both φ^+ and φ^- .

Now let us average the term S_I . We first notice that, since this term is already quadratic φ^- , in the averaged version of S_I one can neglect fluctuations of the phases $\varphi_{L,R}^\pm$ and set $\varphi_{L,R}^\pm(t) = eV_{L,R}t$. After that the voltages $V_{L,R}$ can be absorbed as energy shifts of the Fermi distribution functions in the leads. Averaging of the components of the scattering matrix entering into Eq. (21) is carried out as above. We first proceed to the energy representation. Then we will get a sum of terms containing combinations of a similar structure, such as, e.g.,

$$\int \frac{dE}{2\pi} f(E + \omega - eV_L)(1 - f(E - eV_L))$$

$$\times \text{tr}\langle [\hat{1} - \hat{r}^\dagger(E)\hat{r}(E + \omega)] [\hat{1} - \hat{r}^\dagger(E + \omega)\hat{r}(E)] \rangle, \quad (26)$$

where $f(E) = (1 + \exp(E/T))^{-1}$ is the Fermi function. Since the averages should not depend on E , in all these combinations one can integrate over this variable. Collecting all terms we arrive at the final result

$$S_I^{\text{av}} = \sum_{i,j} \int_0^t dx_1 \int_0^t dx_2 \varphi_i^-(x_1) v_{ij}(x_1 - x_2) \varphi_j^-(x_2) \quad (27)$$

Here the kernels v are defined as $v_{ij}(t) = \int \frac{d\omega}{2\pi} v_\omega^{ij} e^{-i\omega t}$, where

$$v_\omega^{ij} = \frac{\omega}{2\pi} \coth \frac{\omega}{2T} \text{Re} u_\omega^{ij} + \left[\frac{\omega - eV}{4\pi} \coth \frac{\omega - eV}{2T} \right.$$

$$\left. + \frac{\omega + eV}{4\pi} \coth \frac{\omega + eV}{2T} - \frac{\omega}{2\pi} \coth \frac{\omega}{2T} \right] \tilde{v}_\omega^{ij} \quad (28)$$

$V = V_L + V_R$ and

$$\begin{aligned} \tilde{v}_\omega^{LL} &= \text{tr}\langle \hat{r}(E)\hat{r}^\dagger(E)\hat{t}'(E + \omega)\hat{t}^\dagger(E + \omega) \rangle, \\ \tilde{v}_\omega^{RL} &= -\text{tr}\langle \hat{r}(E)\hat{t}^\dagger(E)\hat{r}'(E + \omega)\hat{t}^\dagger(E + \omega) \rangle, \\ \tilde{v}_\omega^{LR} &= -\text{tr}\langle \hat{t}(E)\hat{r}^\dagger(E)\hat{t}'(E + \omega)\hat{r}'^\dagger(E + \omega) \rangle, \\ \tilde{v}_\omega^{RR} &= \text{tr}\langle \hat{t}(E)\hat{t}^\dagger(E)\hat{r}'(E + \omega)\hat{r}'^\dagger(E + \omega) \rangle. \end{aligned} \quad (29)$$

This concludes our derivation of the effective action for interacting quantum dots.

D. Current operator

In order to complete our general analysis let us define the kernel of the current operator for our problem. One can choose calculating the current either in the left or in the right junction, obviously in the stationary limit the result should remain the same in both cases. One can also use a symmetrized version of the current operator. One finds

$$I = \frac{\int \mathcal{D}\varphi^- \mathcal{D}\varphi^+ I(t, \varphi^\pm) e^{iS_{\text{ext}} + iS_R - S_I}}{\int \mathcal{D}\varphi^- \mathcal{D}\varphi^+ e^{iS_{\text{ext}} + iS_R - S_I}}, \quad (30)$$

where

$$\begin{aligned} I(t, \varphi^\pm) &= \frac{1}{2} \left(-ie \frac{\delta(iS_R - S_I)}{\delta\varphi_L^-(t)} - ie \frac{\delta(iS_R - S_I)}{\delta\varphi_R^-(t)} \right) \\ &= -\frac{ie}{2} \frac{\delta(iS_R - S_I)}{\delta \left(\frac{\varphi_L^-(t) + \varphi_R^-(t)}{2} \right)} \Bigg|_{\varphi_L^- = \varphi_R^- = \text{const}}. \end{aligned} \quad (31)$$

Since here we are interested in a stationary situation, the result for the current should not depend on time. Therefore one can choose an arbitrary value of t at which the above functional derivative is taken, with the only requirement that t should be sufficiently large. Inserting Eqs. (18) and (21) into (31), and defining the matrix

$$\hat{\Lambda} = \begin{pmatrix} \hat{1} & 0 \\ 0 & -\hat{1} \end{pmatrix},$$

we obtain

$$I(t, \varphi^\pm) = I_0(t, \varphi^+) + \delta I(t, \varphi^\pm), \quad (32)$$

where

$$\begin{aligned} I_0(t, \varphi^+) &= e \int dx dy \text{tr} \left\{ [\delta(t-x)\hat{\Lambda}\delta(t-y) \right. \\ &\quad \left. - \hat{S}^\dagger(t-x)\hat{\Lambda}\hat{S}(t-y)] \hat{\rho}(x, y) \right\}, \end{aligned} \quad (33)$$

and

$$\begin{aligned} \delta I(t, \varphi^\pm) &= -\frac{ie}{2} \int_0^t dx \int_0^\infty dy_1 dy_2 \int_0^\infty dz_1 dz_2 \text{Tr} \left\{ [\delta(t-z_1)\hat{\Lambda}\delta(t-y_1) - \hat{S}^\dagger(t-z_1)\hat{\Lambda}\hat{S}(t-y_1)] \hat{\rho}(y_2, y_1) \right. \\ &\quad \left. \times [\delta(x-y_2)\hat{\varphi}^-(x)\delta(x-z_2) - \hat{S}^\dagger(x-y_2)\hat{\varphi}^-(x)\hat{S}(x-z_2)] \hat{h}(z_1, z_2) \right\} \\ &\quad - \frac{ie}{2} \int_0^t dx \int_0^\infty dy_1 dy_2 \int_0^\infty dz_1 dz_2 \text{Tr} \left\{ [\delta(x-z_1)\hat{\varphi}^-(x)\delta(x-y_1) - \hat{S}^\dagger(x-z_1)\hat{\varphi}^-(x)\hat{S}(x-y_1)] \hat{\rho}(y_2, y_1) \right. \\ &\quad \left. \times [\delta(t-y_2)\hat{\Lambda}\delta(t-z_2) - \hat{S}^\dagger(t-y_2)\hat{\Lambda}\hat{S}(t-z_2)] \hat{h}(z_1, z_2) \right\}. \end{aligned} \quad (34)$$

III. NOISE OF A QUANTUM DOT

The general expression for the term S_I (21) in the action enables one to easily evaluate the current correlators at the barriers in the absence of interaction:

$$\mathcal{S}_{ij}(\omega) = \int dt e^{i\omega t} \langle \delta I_i(t) \delta I_j(0) + \delta I_j(0) \delta I_i(t) \rangle, \quad (35)$$

where $i, j = L, R$. One finds

$$\mathcal{S}_{ij}(\omega) = 2e^2 \int dt e^{i\omega t} \frac{\delta^2 S_I[\varphi_\alpha^-, \varphi_\beta^+ = eV_\beta t]}{\delta\varphi_i^-(t) \delta\varphi_j^-(0)}. \quad (36)$$

In this way one recovers the well known general expression for the noise in terms of the scattering amplitudes³. Averaging the result over mesoscopic fluctuations we get

$$\begin{aligned} \mathcal{S}_{ij}(\omega) &= \frac{2e^2}{\pi} (\text{Re } u_\omega^{ij} - \tilde{v}_\omega^{ij}) \omega \coth \frac{\omega}{2T} + \frac{e^2}{\pi} \tilde{v}_\omega^{ij} \\ &\quad \times \left((\omega - eV) \coth \frac{\omega - eV}{2T} + (\omega + eV) \coth \frac{\omega + eV}{2T} \right) \end{aligned} \quad (37)$$

Following the standard procedure²⁶ let us express the scattering matrix of the dot in the form

$$\hat{S}(E) = \hat{R} + \hat{T}' [1 - \hat{U}(E)\hat{R}']^{-1} \hat{U}(E)\hat{T}, \quad (38)$$

where the energy independent matrices

$$\hat{R} = \begin{pmatrix} \hat{r}_L & 0 \\ 0 & \hat{r}'_R \end{pmatrix}, \quad \hat{R}' = \begin{pmatrix} \hat{r}'_L & 0 \\ 0 & \hat{r}_R \end{pmatrix} \quad (39)$$

and

$$\hat{T} = \begin{pmatrix} \hat{t}_L & 0 \\ 0 & \hat{t}'_R \end{pmatrix}, \quad \hat{T}' = \begin{pmatrix} \hat{t}'_L & 0 \\ 0 & \hat{t}_R \end{pmatrix} \quad (40)$$

account for scattering properties of the left and right barriers while $\hat{U}(E)$ is the unitary matrix which effectively describes scattering in the ‘‘internal’’ part of the dot. Further analysis requires specifying the model for our quantum dot. Here we will mainly address *chaotic* quantum dots in which case $\hat{U}(E)$ belongs to the circular

ensemble. The averages u_ω^{ij} and v_ω^{ij} are evaluated with the aid of the diagrammatic technique^{26,28}. We obtain

$$\begin{aligned} u_\omega^{LL,RR} &= \frac{g_{L,R}}{2} - \frac{g_{L,R}^2}{2g(1-i\omega\tau_D)}, \\ u_\omega^{LR} &= u_\omega^{RL} = \frac{g_L g_R}{2g(1-i\omega\tau_D)}. \end{aligned} \quad (41)$$

The details of the derivation are presented in Appendix C. Here we defined the dimensionless conductances of the left and right barriers $g_{L,R} = 2\text{tr}(\hat{t}_{L,R}^\dagger(E)t_{L,R}(E)) = 2\pi/e^2 R_{L,R}$, their sum $g = g_L + g_R$, and the electron dwell time in the quantum dot $\tau_D = 4\pi/g\delta$. The general expressions for \tilde{v}_ω^{ij} can also be obtained with the aid of the results derived in Appendix C. One finds

$$\begin{aligned} \tilde{v}_\omega^{LL} &= \frac{g_L^2 g_R}{2g^3} \left(g_L \frac{\omega^2 \tau_D^2}{1 + \omega^2 \tau_D^2} + g_R \right) \\ &+ \frac{g_L g_R^2}{2g^4} \frac{(g_L + g_R)^2 \omega^2 \tau_D^2 + g_R^2 \beta_L}{1 + \omega^2 \tau_D^2} \\ &+ \frac{g_L^4 g_R}{2g^4(1 + \omega^2 \tau_D^2)} \beta_R, \\ \tilde{v}_\omega^{RL} &= \frac{g_L^2 g_R^2}{2g^3(1 + \omega^2 \tau_D^2)} \\ &+ \frac{g_L g_R^3 (i\omega\tau_D g_L + (1 + i\omega\tau_D)g_R)}{2g^4(1 + \omega^2 \tau_D^2)} \beta_L \\ &+ \frac{g_L^3 g_R ((1 - i\omega\tau_D)g_L - i\omega\tau_D g_R)}{2g^4(1 + \omega^2 \tau_D^2)} \beta_R. \end{aligned} \quad (42)$$

Here we introduced the Fano factors of the barriers

$$\beta_j = \frac{\text{tr}(\hat{t}_j^\dagger \hat{r}_j' \hat{r}_j'^\dagger)}{\text{tr}(\hat{t}_j \hat{t}_j^\dagger)}, \quad j = L, R.$$

The functions \tilde{v}_ω^{RR} and \tilde{v}_ω^{LR} are recovered by interchanging the indices $L \leftrightarrow R$ in the expressions (42) for \tilde{v}_ω^{LL} and \tilde{v}_ω^{RL} .

In the low frequency limit $\omega \rightarrow 0$ one finds $\tilde{v}_\omega^{LL} = \tilde{v}_\omega^{RR} = \tilde{v}_\omega^{LR} = \tilde{v}_\omega^{RL}$ and, hence, $\mathcal{S}_{LL}(0) = \mathcal{S}_{RR}(0) = \mathcal{S}$, where \mathcal{S} is the noise spectrum of the dot,

$$\begin{aligned} \mathcal{S} &= \frac{4T}{R_q} \left(\frac{g_L g_R (g^2 - g_L g_R)}{g^3} - \frac{g_L g_R^4 \beta_L + g_L^4 g_R \beta_R}{g^4} \right) \\ &+ \frac{2eV}{R_q} \coth \frac{eV}{2T} \left(\frac{g_L^2 g_R^2}{g^3} + \frac{g_L g_R^4 \beta_L + g_L^4 g_R \beta_R}{g^4} \right). \end{aligned} \quad (43)$$

Here and below $R_q = h/e^2$ is the quantum resistance unit.

In the leading approximation the dot conductance is $G_0 = g_L g_R / g R_q$. Hence, for chaotic quantum dots the total Fano factor $\tilde{\beta}$ (defined in a standard manner as a ratio between the shot noise spectrum and its Schottky value $2eI$) reads

$$\tilde{\beta} = \frac{g_L g_R}{(g_L + g_R)^2} + \frac{g_R^3 \beta_L + g_L^3 \beta_R}{(g_L + g_R)^3}. \quad (44)$$

We note that Eq. (43) agrees with the results for zero frequency noise of chaotic cavities previously derived in different limits by various authors^{3,29}. It is also satisfactory to observe that in the particular case of two diffusive conductors $\beta_L = \beta_R = 1/3$ Eq. (44) again yields $\tilde{\beta} = 1/3$ for any g_L and g_R .

Our analysis also allows to generalize the results for the current noise in chaotic cavities to the case of finite frequencies. In order to find the noise correlator $\mathcal{S}(\omega)$ it is in general necessary to account for the barrier capacitances. Assuming for simplicity $C_g = 0$, we obtain

$$\begin{aligned} \mathcal{S}(\omega) &= |Y_L(\omega)|^2 \mathcal{S}_{LL}(\omega) + |Y_R(\omega)|^2 \mathcal{S}_{RR}(\omega) \\ &+ 2\text{Re}(Y_L(\omega) \mathcal{S}_{LR}(\omega) Y_R(\omega)), \end{aligned} \quad (45)$$

where

$$\begin{aligned} Y_L(\omega) &= \frac{-i\omega C_R + \frac{e^2}{\pi}(u_\omega^{RR} - u_\omega^{RL})}{-i\omega(C_L + C_R) + \frac{e^2}{\pi}(u_\omega^{LL} + u_\omega^{RR} - u_\omega^{LR} - u_\omega^{RL})}, \\ Y_R(\omega) &= \frac{-i\omega C_L + \frac{e^2}{\pi}(u_\omega^{LL} - u_\omega^{LR})}{-i\omega(C_L + C_R) + \frac{e^2}{\pi}(u_\omega^{LL} + u_\omega^{RR} - u_\omega^{LR} - u_\omega^{RL})} \end{aligned}$$

and u_ω^{ij} are defined in (41). The above equations demonstrate that in a general case the noise spectrum (45) depends on frequency in a complicated manner. However, for fully symmetric quantum dots, i.e. for $C_L = C_R$, $g_L = g_R$ and $\beta_L = \beta_R = \beta$, one finds $Y_L(\omega) = Y_R(\omega) = 1/2$. In this case all frequency dependent contributions contained in the functions \tilde{v}_ω^{ij} (42) cancel out and Eq. (45) reduces to the standard form³

$$\begin{aligned} \mathcal{S}(\omega) &= 2(1 - \tilde{\beta})G_0\omega \coth \frac{\omega}{2T} \\ &+ \tilde{\beta}G_0 \sum_{\pm} (\omega \pm eV) \coth \frac{\omega \pm eV}{2T} \end{aligned} \quad (46)$$

with the Fano factor $\tilde{\beta} = (1 + \beta)/4$ in accordance with Eq. (44).

IV. CURRENT-VOLTAGE CHARACTERISTICS AND CONDUCTANCE

Let us now turn to the interaction effects. In order to evaluate the current-voltage characteristics we should average the above expressions for the phase-dependent current $I(t, \varphi^\pm)$ (32)-(34) over the fluctuating phase fields φ^\pm . In addition to that – provided one is not interested in the effect of mesoscopic fluctuations – one can also average the result over such fluctuations. We proceed exactly in this order and first perform averaging over the phase fluctuations.

A. Averaging over fluctuating phase fields

Let us combine Eqs. (32)-(34) with (30) and carry out functional integration over φ^\pm . We notice that in

the interesting for us limit of large dot conductances it is parametrically justified to perform this integration with the averaged effective action

$$S^{\text{av}} = S_{\text{ext}} + S_R^{\text{av}} + iS_I^{\text{av}}$$

instead of the exact one. This observation simplifies the whole procedure enormously because the action S^{av} is quadratic in φ^\pm . Hence, the integrals become Gaussian and can be handled exactly.

Since in all the integrals only linear combinations of the phases φ^+ enter into the exponent, it is convenient to first integrate out this variable. This integration yields functional δ -functions. For example,

one of the terms in Eq. (34) contains the exponent $e^{i[\varphi_L^+(y_2) - \varphi_L^+(y_1) + \varphi_R^+(z_2) - \varphi_R^+(z_1)]}$, in which case the averaging over $\varphi_{L,R}^+$ fixes the variables φ^- in the form

$$\begin{aligned} \varphi_j^-(\tau) = & -K_{jL}(z_1 - \tau) + K_{jL}(z_2 - \tau) - K_{jR}(y_1 - \tau) \\ & + K_{jR}(y_2 - \tau), \quad j = L, R, g. \end{aligned} \quad (47)$$

The Fourier transform of the functions $K_{ij}(t)$ reads

$$K_\omega^{ij} = \frac{e^2}{-i\omega + 0} \mathcal{A}_{ij}^{-1}(\omega), \quad (48)$$

where the matrix $\mathcal{A}(\omega)$ has the form

$$\mathcal{A}(\omega) = \begin{pmatrix} -i\omega C_L + \frac{2}{Z_S(\omega)} + \frac{e^2}{\pi} u_{LL}(\omega) & \frac{e^2}{\pi} u_{RL}(\omega) & -\frac{2}{Z_S(\omega)} \\ \frac{e^2}{\pi} u_{LR}(\omega) & -i\omega C_R + \frac{2}{Z_S(\omega)} + \frac{e^2}{\pi} u_{RR}(\omega) & \frac{2}{Z_S(\omega)} \\ -\frac{2}{Z_S(\omega)} & \frac{2}{Z_S(\omega)} & -i\omega C_g + \frac{4}{Z_S(\omega)} \end{pmatrix}. \quad (49)$$

Other terms of Eq. (34) are treated analogously. The above expressions hold for an arbitrary external impedance $Z_S(\omega)$. Further general expressions for the current are presented in Appendix D.

Let us now focus our attention on the important limit of vanishing external impedance $Z_S(\omega) \rightarrow 0$. In this limit one finds

$$K_\omega^{ij} = K_\omega \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}, \quad (50)$$

where we defined

$$\begin{aligned} K_\omega &= \frac{e^2}{-i\omega + 0} \frac{1}{-i\omega C_\Sigma + \frac{e^2}{\pi} u(\omega)}, \quad (51) \\ u(\omega) &= u_\omega^{LL} + u_\omega^{RR} - u_\omega^{LR} - u_\omega^{RL} \\ &= \text{tr}\langle \hat{1} - \hat{S}^\dagger(E) \hat{S}(E + \omega) \rangle. \end{aligned} \quad (52)$$

and $C_\Sigma = C_L + C_R + C_g$. What remains is to integrate out the variable φ^- . Because of the obtained functional δ -functions this integration becomes trivial as well. One should just substitute the trajectories (47) into S_I^{av} , into the imaginary part of S_{ext} and into the term δI . Taking the unitarity of the S -matrix into account, in the

voltage-biased limit $Z_S(\omega) \rightarrow 0$ one finds

$$I(V) = I_0(V) + \delta I(V). \quad (53)$$

For the sake of convenience we present the expression for the term $I_0(V)$ in two equivalent forms:

$$\begin{aligned} I_0(V) &= 2e \int dx \rho_0(x) \mathcal{T}(x) e^{-F(x)} (e^{ieV_L x} - e^{-ieV_R x}) \\ &= e \int \frac{dE d\omega}{(2\pi)^2} \text{tr}\{[\hat{\Lambda} - \hat{S}^\dagger(E + \omega) \hat{\Lambda} \hat{S}(E + \omega)] \hat{\rho}_E\} \\ &\quad \times \left(\int dx e^{i\omega x - F(x)} \right). \end{aligned} \quad (54)$$

Here we have defined $\mathcal{T}(x) = \int \frac{dE}{2\pi} \text{tr}[\hat{t}^\dagger(E) \hat{t}(E)] e^{-iEx}$, and

$$\hat{\rho}_E = \begin{pmatrix} \hat{1} f(E - eV_L) & 0 \\ 0 & \hat{1} f(E + eV_R) \end{pmatrix}, \quad (55)$$

where $f(E) = 1/(1 + \exp(E/T))$ is the Fermi function.

The interaction correction to the current $\delta I(V)$ takes the form

$$\begin{aligned} \delta I &= -e \text{Im} \int dx \int dy_1 dy_2 \int dz_1 dz_2 \mathcal{F}(y_1, y_2, z_1, z_2) (K(-y_1) - K(-z_1)) \\ &\quad \times \text{tr}\{[\delta(z_1 + x) \hat{\Lambda} \delta(y_1 + x) - \hat{S}^\dagger(z_1 + x) \hat{\Lambda} \hat{S}(y_1 + x)] \hat{\rho}_0(y_1 - y_2) [\delta(y_2) \delta(z_2) - \hat{S}^\dagger(y_2) \hat{S}(z_2)] \hat{h}_0(z_2 - z_1)\}, \end{aligned} \quad (56)$$

where we have set

$$\begin{aligned} \mathcal{F}(y_1, y_2, z_1, z_2) &= e^{-F(y_1 - y_2) - F(z_1 - z_2) - F(y_1 - z_1)} \\ &\quad \times e^{-F(y_2 - z_2) + F(y_1 - z_2) + F(y_2 - z_1)}. \end{aligned} \quad (57)$$

In the limit $Z_S(\omega) \rightarrow 0$ we have $\text{Re}(iS_{\text{ext}}) = 0$ and the

function $F(x)$ is simply equal to $F(x) = S_I^{\text{av}}$ evaluated for $\varphi_L^-(\tau) = -\varphi_R^-(\tau) = K(x - \tau) - K(-\tau)$. In this case we find

$$F(x) = 2 \int \frac{d\omega}{2\pi} |K_\omega|^2 (1 - \cos \omega x) (v_\omega^{LL} + v_\omega^{RR} - v_\omega^{LR} - v_\omega^{RL}), \quad (58)$$

where the functions v_ω^{ij} are defined in Eqs. (28)-(29).

As we shall see below, for a wide range of parameters the function (57) in Eq. (56) can be approximated by unity $\mathcal{F} \rightarrow 1$. Under this approximation one can conveniently perform the Fourier transformation and reduce the expression (56) to the form

$$\delta I = e \text{Im} \int \frac{dE d\omega}{(2\pi)^2} K_\omega \text{tr} \{ [1 - \hat{S}^\dagger(E) \hat{S}(E + \omega)] \hat{h}_{E+\omega} \times [\hat{S}^\dagger(E + \omega) \hat{\Lambda} \hat{S}(E + \omega) - \hat{S}^\dagger(E) \hat{\Lambda} \hat{S}(E)] \hat{\rho}_E \}, \quad (59)$$

where $\hat{h}_E = \hat{1} - \hat{\rho}_E$.

Employing the above general expressions one can also define the zero bias conductance of the dot. In doing so, we express the voltages $V_{L,R}$ in the form $V_{L,R} = (V \pm V_G)/2$, where V_G is the effective potential of the dot tuned through the gate, and find the linear in V correction to the current. The voltage V_G can be removed by shifting the energy and performing a proper re-definition of the S -matrix. Thus we find $G = G_F + \delta G$, where

$$G_F = -\frac{e^2}{2} \int \frac{dE d\omega}{(2\pi)^2} \text{Tr} \left\{ \hat{1} - \hat{S}^\dagger(E + \omega) \hat{\Lambda} \hat{S}(E + \omega) \hat{\Lambda} \right\} \times \left(\int dx e^{-i\omega x - F(x)} \right) \frac{\partial f}{\partial E}, \quad (60)$$

and

$$\delta G = \frac{e^2}{2} \text{Im} \int \frac{dE d\omega}{(2\pi)^2} K_\omega [1 - 2f(E + \omega)] \frac{\partial f(E)}{\partial E} \times \text{tr} \{ [\hat{S}^\dagger(E) \hat{\Lambda} \hat{S}(E) \hat{\Lambda} - \hat{S}^\dagger(E + \omega) \hat{\Lambda} \hat{S}(E + \omega) \hat{\Lambda}] \times [1 - \hat{S}^\dagger(E) \hat{S}(E + \omega)] \}. \quad (61)$$

In Eq. (61) we again set $\mathcal{F} = 1$. This is sufficient in the perturbative regime to be studied below.

B. Perturbation theory and renormalization of the S-matrix

The above general expressions allow to conveniently proceed with the perturbation theory in the interaction. In order to obtain the first order correction to the conductance one should expand G_F (60) in $F(x)$ and combine linear in $F(x)$ terms of this expansion with the interaction correction δG (61). Then one finds $G = G_L + \delta G_1 + \delta G_2$, where $G = G_L$ is the Landauer conductance of the dot in the absence of interactions,

$$G_L = -\frac{e^2}{2} \int \frac{dE}{2\pi} \text{tr} [\hat{1} - \hat{S}^\dagger(E) \hat{\Lambda} \hat{S}(E) \hat{\Lambda}] \frac{\partial f}{\partial E}, \quad (62)$$

and the two corrections $\delta G_{1,2}$ read

$$\delta G_1 = \frac{e^2}{2} \text{Im} \int \frac{dE d\omega}{(2\pi)^2} K_\omega \left(\coth \frac{\omega}{2T} - \tanh \frac{E + \omega}{2T} \right) \times \text{tr} \{ \hat{S}^\dagger(E + \omega) \hat{\Lambda} \hat{S}(E + \omega) \hat{\Lambda} - \hat{S}^\dagger(E) \hat{\Lambda} \hat{S}(E) \hat{\Lambda} \} \frac{\partial f(E)}{\partial E}, \quad (63)$$

$$\delta G_2 = -\frac{e^2}{2} \text{Im} \int \frac{dE d\omega}{(2\pi)^2} K_\omega \tanh \frac{E + \omega}{2T} \frac{\partial f(E)}{\partial E} \times \text{tr} \{ \hat{S}^\dagger(E) \hat{\Lambda} \hat{S}(E) \hat{\Lambda} \hat{S}^\dagger(E) \hat{S}(E + \omega) - \hat{S}^\dagger(E) \hat{\Lambda} \hat{S}(E + \omega) \hat{\Lambda} \}. \quad (64)$$

We note that in Eq. (63) the terms containing $\coth(\omega/2T)$ come from the expansion of (60) while all the \tanh -terms originate from Eq. (61).

It is easy to see that $\delta G_1 \equiv 0$ for any $\hat{S}(E)$ and at any temperature. Indeed, making a shift $E + \omega \rightarrow E$ in the term involving the product $\hat{S}^\dagger(E + \omega) \hat{\Lambda} \hat{S}(E + \omega) \hat{\Lambda}$ one reduces the expression under the integral to the form $\text{Im} K_\omega \mathcal{B}(E, \omega)$, where

$$\mathcal{B}(E, \omega) = \left(\coth \frac{\omega}{2T} - \tanh \frac{E}{2T} \right) \frac{\partial f(E - \omega)}{\partial E} - \left(\coth \frac{\omega}{2T} - \tanh \frac{E + \omega}{2T} \right) \frac{\partial f(E)}{\partial E}. \quad (65)$$

Since $f(E)$ is the Fermi function, one has $\mathcal{B}(E, \omega) = \mathcal{B}(E, -\omega)$. At the same time, as follows from Eq. (51), $\text{Im} K_\omega$ is an odd function of ω . Hence, the integral over ω vanishes identically, the term δG_1 drops out and only the combination (64) needs to be taken into consideration.

Let us make a shift $\hat{S}(E) \rightarrow \hat{S}(E) + \delta \hat{S}(E)$ in the Landauer formula (62). Then the corresponding linear in $\delta \hat{S}(E)$ correction to the conductance becomes

$$\delta G_L = \frac{e^2}{2} \int \frac{dE}{2\pi} \text{tr} [\delta \hat{S}^\dagger(E) \hat{\Lambda} \hat{S}(E) \hat{\Lambda} + \hat{S}^\dagger(E) \hat{\Lambda} \delta \hat{S}(E) \hat{\Lambda}] \frac{\partial f}{\partial E}. \quad (66)$$

Comparing this formula to the first order correction (64) we can choose $\delta \hat{S}(E)$ and $\delta \hat{S}^\dagger(E)$ in the form:

$$\delta \hat{S} = \frac{1}{2i} \int \frac{d\omega}{2\pi} K_\omega \left[\tanh \frac{E + \omega}{2T} [\hat{S}(E + \omega) - \hat{S}(E)] + \tanh \frac{E - \omega}{2T} \hat{S}(E) [\hat{S}^\dagger(E - \omega) - \hat{S}^\dagger(E)] \hat{S}(E) \right], \\ \delta \hat{S}^\dagger = \frac{-1}{2i} \int \frac{d\omega}{2\pi} K_\omega^* \left[\tanh \frac{E + \omega}{2T} [\hat{S}^\dagger(E + \omega) - \hat{S}^\dagger(E)] + \tanh \frac{E - \omega}{2T} \hat{S}^\dagger(E) [\hat{S}(E - \omega) - \hat{S}(E)] \hat{S}^\dagger(E) \right]. \quad (67)$$

Although this choice is obviously not a unique one, we will stick to it for the reasons which will become clear below. Making use of the property $K_{-\omega} = K_\omega^*$, it is

easy to check that the conditions $(\delta\hat{S}^\dagger(E))^\dagger = \delta\hat{S}(E)$ and $\delta\hat{S}^\dagger(E)\hat{S}(E) + \hat{S}^\dagger(E)\delta\hat{S}(E) = 0$ are identically fulfilled, i.e. the first order renormalization (67) preserves the unitarity of the S -matrix.

In order to find the function K_ω (51) for chaotic quantum dots one needs to perform the average of the product of two S -matrices at different energies (52). Neglecting small localization corrections, and applying the formula (C3) we obtain

$$\text{tr}(\hat{1} - \hat{S}^\dagger(E)\hat{S}(E + \omega)) = \frac{-i\omega\tau_D g}{2(1 - i\omega\tau_D)}. \quad (68)$$

Combining (68) with (51,52), we get

$$K_\omega = \frac{e^2}{(-i\omega + 0) \left(-i\omega C_\Sigma + \frac{e^2}{2\pi} g \frac{-i\omega\tau_D}{1 - i\omega\tau_D} \right)} \quad (69)$$

and after the the Fourier transformation we arrive at the expression

$$K(t) = \frac{2\pi}{g} \theta(t) \left(\frac{\tau_D^2 (1 - e^{-t/\tau})}{(\tau_D + \tau_0)^2} + \frac{t}{\tau_0 + \tau_D} \right). \quad (70)$$

Here we have defined the classical RC -time of the central island $\tau_0 = 2\pi C_\Sigma / (e^2 g)$ and $\tau = \tau_D \tau_0 / (\tau_D + \tau_0)$. For large quantum dots one typically has $\tau_0 \ll \tau_D$ and, hence, $\tau \approx \tau_0$.

Eq. (70) allows to specify the necessary condition of applicability for our analysis. Let us recall that during our derivation we have expanded the effective action in powers of the fluctuating phase field φ^- . The least action condition fixes this field to be equal to $\varphi^-(t) = K(t)$. Since all the available time integrals are effectively cut at times exceeding τ_D , it is sufficient to require $\varphi^-(\tau_D) = K(\tau_D) \ll 2\pi$. Then from Eq. (70) we conclude that our expansion of the exact effective action in powers of the field $\varphi^-(t)$ is justified in the “metallic” limit

$$g \gg 1. \quad (71)$$

We also note that this condition is necessary but in general not a sufficient one in order to justify the perturbative expansion in the interaction employed in this subsection. An additional condition will be established below in Sec. 4C.

Consider the most relevant physical limit $\tau_D \gg \tau_0$. In this case one has $K_\omega \approx \frac{2\pi}{g} \left(\frac{1}{-i\omega} - \frac{1}{1/\tau_0 - i\omega} \right)$. Since the integral in Eqs. (67) is taken over a wide range of energies, the main logarithmic correction to the scattering matrix can be obtained if one replaces $\hat{S}(E + \omega)$ by its energy independent average $\langle \hat{S}(E) \rangle = \hat{R}$. Then one finds

$$\delta\hat{S}(E) = \frac{\ln(1/E\tau_0)}{g} [\langle \hat{S} \rangle - \hat{S}(E) - \hat{S}(E)(\langle \hat{S}^\dagger \rangle - \hat{S}^\dagger(E))\hat{S}(E)]. \quad (72)$$

Eq. (72) can be used to derive renormalization group (RG) equations for the energy dependent scattering matrix of a quantum dot. Following the standard procedure

let us fix an infinitesimal energy interval between E and $E - dE$. From Eq. (72) one easily finds the corresponding correction $d\hat{S}(E)$ to the scattering matrix. Lowering the energy and repeating this procedure many times one arrives at the following RG equations

$$\frac{d\hat{S}(E)}{d \ln(1/E\tau_0)} = \frac{\langle \hat{S} \rangle - \hat{S}(E)\langle \hat{S}^\dagger \rangle \hat{S}(E)}{g}, \quad (73)$$

where the effective conductance g is expressed via the renormalized scattering matrix at a given energy by means of the Landauer formula. Making use of Eqs. (38-40) we define

$$g = \text{tr} \hat{T}^\dagger \hat{T}. \quad (74)$$

Hence, g itself gets renormalized and becomes energy dependent. Finally, substituting $\hat{S}(E)$ in the form (38) we observe that $\hat{U}(E)$ remains unchanged in the course of renormalization. Thus Eq. (73) describes the renormalization of the barrier transmissions connecting the chaotic cavity to the ideal leads. After simple manipulations and making use of Eq. (74) we arrive at the scaling equations for the transmission matrices $\hat{T}^\dagger \hat{T}$:

$$\frac{d\hat{T}^\dagger \hat{T}}{d \ln(E\tau_0)} = \frac{2(\hat{1} - \hat{T}^\dagger \hat{T})\hat{T}^\dagger \hat{T}}{\text{tr} \hat{T}^\dagger \hat{T}}. \quad (75)$$

Eq. (75) coincides with that recently derived in Ref. 22 by means of a different approach.

Let us emphasize again that the whole analysis of the present subsection is valid within the first order perturbation theory in the interaction. Hence, the above RG equation can be applied only as long as the renormalized conductance $g(E)$ remains large.

C. I-V curve in the voltage-biased limit

Provided temperature and/or voltage exceed the level spacing in the dot and provided one is not interested in resolving subtle details of the $I-V$ curve related to mesoscopic fluctuations, it is convenient to perform averaging of the above general expressions for the current over such fluctuations. The average values of the products of reflection and transmission matrices depend only on ω , not on E . Hence, one can integrate over E and derive the current-voltage characteristics of a quantum dot in the presence of interactions. In particular, the term I_0 in Eq. (54) acquires the standard Landauer form

$$I_0 = \frac{e^2}{\pi} \text{tr}(\hat{t}^\dagger(E)\hat{t}(E))V = G_0 V. \quad (76)$$

In the case of chaotic quantum dots one finds²⁶:

$$\begin{aligned} \text{tr}(\hat{t}^\dagger(E)\hat{t}(E)) &= \frac{g_L g_R}{2g} \\ &+ \nu \frac{g_L g_R^2 (1 - \beta_L) + g_L^2 g_R (1 - \beta_R)}{g^3}, \end{aligned} \quad (77)$$

where $\nu = -1, 0$ or $1/2$ respectively for circular orthogonal, unitary and symplectic ensembles. The second term in Eq. (77) represents the weak localization (WL) correction. For the model under consideration this correction is parametrically ($\sim 1/g$) small as compared to the first term in (77). Below we will concentrate on the interaction correction to the current δI , which is of the same order in $1/g$, but nevertheless strongly exceeds the WL correction at sufficiently low energies.

Let us consider the perturbative in the interaction regime. After averaging over mesoscopic fluctuations the interaction correction (59) takes the form

$$\delta I = -\frac{e}{\pi} \int \frac{d\omega}{2\pi} \text{Im}(K_\omega D(\omega)) (\omega - eV) \coth \frac{\omega - eV}{2T}, \quad (78)$$

where we defined

$$D(\omega) = \text{tr} \left\{ [\hat{r}^+(E + \omega) \hat{t}'(E + \omega) - \hat{r}^+(E) \hat{t}'(E)] \times [\hat{t}'^+(E) \hat{r}(E + \omega) + \hat{r}'^+(E) \hat{t}(E + \omega)] \right\} \quad (79)$$

All other contributions to the interaction correction vanish upon averaging. An explicit evaluation of the function $D(\omega)$ (79) is easily performed with the aid of Eq. (C9). Here we only quote the result:

$$D(\omega) = \frac{g_L g_R (g_R \beta_L + g_L \beta_R)}{2(g_L + g_R)^2} \frac{\omega^2 \tau_D^2}{(1 - i\omega \tau_D)^2}. \quad (80)$$

Combining Eqs. (78), (69) and (80), we obtain

$$\delta I = -\frac{eB}{2} \int \frac{d\omega}{2\pi} \frac{1}{\omega} \left(\frac{1}{1 + \omega^2 \tau^2} - \frac{1}{1 + \omega^2 \tau_D^2} \right) \times \left[(\omega + eV) \coth \frac{\omega + eV}{2T} - (\omega - eV) \coth \frac{\omega - eV}{2T} \right], \quad (81)$$

where we defined

$$B = g_L g_R (g_L \beta_R + g_R \beta_L) / (g_L + g_R)^3. \quad (82)$$

One can also rewrite Eq. (81) in the form

$$\delta I = -\frac{eB}{\pi} \int_0^\infty dt \frac{\pi^2 T^2}{\sinh^2 \pi T t} e^{-t/\tau_D} (1 - e^{-t/\tau_0}) \sin eV t. \quad (83)$$

The integral can be evaluated analytically. We find

$$\delta I = -\frac{eB}{\pi} \text{Im} \left[\left(\frac{1}{\tau} + ieV \right) \Psi \left(1 + \frac{1}{2\pi T \tau} + i \frac{eV}{2\pi T} \right) - \left(\frac{1}{\tau_D} + ieV \right) \Psi \left(1 + \frac{1}{2\pi T \tau_D} + i \frac{eV}{2\pi T} \right) \right], \quad (84)$$

where $\Psi(x)$ is the digamma function. This is a complete expression for the first order interaction correction to the current in chaotic quantum dots with large conductances in the voltage-biased regime.

We will now analyze the above general expression in various specific limits. At $T \rightarrow 0$ the $I - V$ curve reduces to the following simple form

$$\frac{dI}{dV} = G_0 - \frac{B}{R_q} \ln \frac{\tau_D^2 (1 + (eV\tau)^2)}{\tau^2 (1 + (eV\tau_D)^2)}. \quad (85)$$

Provided the voltage is large $eV\tau \gg 1$ the interaction correction turns out to be small. In the limit $\tau_0 \ll \tau_D$ one finds

$$\frac{dI}{dV} = G_0 - \frac{Bg^2 E_C^2}{\pi R_q e^2 V^2}, \quad (86)$$

where $E_C = e^2/2C_\Sigma$ is the charging energy. In the intermediate range of voltages $1/\tau_D \ll eV \ll 1/\tau_0$ the interaction correction becomes logarithmic in V

$$\frac{dI}{dV} = G_0 + \frac{2B}{R_q} \ln(eV\tau_0) \quad (87)$$

and at high voltages and temperatures $eV, T \ll 1/\tau_D$ we obtain

$$\frac{dI}{dV} = G_0 - \frac{2B}{R_q} \left[\ln \frac{\tau_D}{\tau} - \left(\frac{(eV)^2}{2} + \frac{\pi^2 T^2}{3} \right) (\tau_D^2 - \tau^2) \right]. \quad (88)$$

The latter expression demonstrates that in the regime under consideration the linear conductance of the quantum dot remains non-zero down to zero temperature.

Taking the limit $V \rightarrow 0$ in the general results (83,84) one can derive the linear conductance of the dot at arbitrary temperatures. Evaluating the integrals, we obtain

$$G = G_0 - \frac{2B}{R_q} [L(T\tau) - L(T\tau_D)], \quad (89)$$

where we defined

$$L(x) = \Psi \left(1 + \frac{1}{2\pi x} \right) + \gamma + \frac{1}{2\pi x} \Psi' \left(1 + \frac{1}{2\pi x} \right) \quad (90)$$

and $\gamma \simeq 0.577$ is the Euler constant. As before, this general expression can be simplified further in various limits. At high temperatures $T \gg gE_C$ we find

$$G = G_0 - \frac{Bg}{R_q} \left\{ \frac{E_C}{3T} - \frac{3\zeta(3)g}{2\pi^4} \left(\frac{E_C}{T} \right)^2 \right\}, \quad (91)$$

where $\zeta(3) \simeq 1.202$. In the regime $1/\tau_D \ll T \ll 1/\tau_0$ we again arrive at the logarithmic interaction correction to the conductance

$$G = G_0 + \frac{2B}{R_q} \ln(T\tau_0) \quad (92)$$

which crosses over to Eq. (88) as T becomes smaller than the inverse dwell time $1/\tau_D$.

The above results completely describe perturbative interaction correction to the current-voltage characteristics of highly conducting chaotic quantum dots in the voltage-biased regime. We also note that in certain limits these results reduce to ones obtained previously^{9,22,24,30} by means of different techniques. For instance, the logarithmic behavior of the interaction correction (92) can easily be recovered from the RG approach²² which we also discussed in Sec. 4B. On the other hand, some other

results, e.g., Eqs. (86,91), cannot be obtained within this RG scheme.

Under which conditions do the above perturbative results remain valid? At high temperatures the validity of the result (91) is guaranteed by the inequality $T \gg gE_C$. In this limit no additional requirement (71) is needed, i.e. Eq. (91) applies also for $g \sim 1$. The same is true for the result (86) derived in the limit of large voltages. On the other hand, at lower temperatures and voltages $T, eV \ll gE_C$ the condition (71) should be satisfied ensuring that fluctuations of the phase $\varphi^-(t)$ remain weak.

An additional applicability condition for the above results is obtained if one requires fluctuations of the phase $\varphi^+(t)$ to be small. Let us recall that in the course of our derivation we have approximated the function \mathcal{F} (57) by unity. This approximation is appropriate as soon as the function $F(t) = \langle (\varphi^+(t) - \varphi^+(0))^2 \rangle / 2$ remains much smaller than one and fluctuations of the phase $\varphi^+(t)$ can be disregarded. Hence, the condition

$$F(\tau_F) \sim 1 \quad (93)$$

defines yet one more time scale in our problem, τ_F , which restricts the applicability of the above perturbative results.

In order to determine the time scale τ_F it suffices to evaluate the function $F(t)$ in the limit of low voltages $V \rightarrow 0$. In this limit Eq. (58) reduces to a simple expression

$$F(t) = \int \frac{d\omega}{2\pi} \text{Im}(K_\omega) \coth \frac{\omega}{2T} (1 - \cos \omega t). \quad (94)$$

Combining this expression with Eq. (69) one finds

$$F(t) = \frac{\pi T t^2}{g(\tau_0 + \tau_D)} + \frac{2\kappa}{g} \int_0^\infty d\omega \frac{\omega \coth \frac{\omega}{2T}}{1 + \omega^2 \tau^2} \frac{1 - \cos \omega t}{\omega^2}, \quad (95)$$

where $\kappa = \tau_D^2 / (\tau_D + \tau_0)^2$. In the most interesting limit $T \rightarrow 0$ the integral in Eq. (95) yields within the logarithmic accuracy $F(t) \simeq (2\kappa/g) \ln(t/\tau)$. Then from Eq. (93) we obtain

$$\tau_F \sim \tau \exp(g/2\kappa). \quad (96)$$

Thus fluctuations of the phase $\varphi^+(t)$ can be neglected at all temperatures and voltages only provided the dwell time τ_D is much smaller than the parameter τ_F (96). More generally, the results presented in this subsection are correct under the condition

$$\max(T, eV, \tau_D^{-1}) \gg \tau_F^{-1}. \quad (97)$$

It is easy to check that in this case the interaction correction in Eqs. (87), (88) and (92) remains small as compared to G_0 . If the condition (97) is violated the system enters an essentially non-perturbative regime in which case more accurate analysis becomes necessary. This analysis is beyond the scope of the present paper. Let us also point out that, while the non-perturbative regime is important from a theoretical point of view, it does not seem to be of much practical relevance for quantum dots with $g \gg 1$ considered here.

D. Effect of external impedance

So far all our final results have been formulated for the case of ideally conducting external leads $Z_S(\omega) \rightarrow 0$. In many experimental situations, however, this voltage-biased model is not appropriate since the impedance of external leads remains non-zero in the relevant frequency range. For this reason it is important to find out how the above results for the $I - V$ curve are modified in the case $Z_S(\omega) \neq 0$.

In order to answer this question we will make use of the general expressions for the current derived in Appendix D. Averaging these expressions over mesoscopic fluctuations we arrive at the result

$$I(V) = I_0(V) + \delta I_{\text{tot}}(V), \quad (98)$$

where $I_0(V)$ remains the same and is given by Eq. (76), while $\delta I_{\text{tot}}(V)$ represents the total interaction correction which takes the form:

$$\delta I_{\text{tot}}(V) = -\frac{e}{2\pi} \int \frac{d\omega}{2\pi} \left((\omega + eV) \coth \frac{\omega + eV}{2T} - (\omega - eV) \coth \frac{\omega - eV}{2T} \right) \text{Im} \sum_{ij} K_{ij}(\omega) D_{ji}(\omega), \quad (99)$$

where

$$\begin{aligned} D_{RL}(\omega) &= \text{tr} \langle \hat{t}'(E + \omega) \hat{t}'^\dagger(E) \hat{r}(E + \omega) \hat{r}^\dagger(E + \omega) \rangle, \\ D_{LL}(\omega) &= \text{tr} \langle \hat{r}^\dagger(E) \hat{t}'(E) \hat{t}'^\dagger(E) \hat{r}(E + \omega) \rangle, \\ D_{RR}(\omega) &= \text{tr} \langle \hat{t}(E + \omega) \hat{t}^\dagger(E + \omega) \hat{r}'(E + \omega) \hat{r}'^\dagger(E) \rangle, \\ D_{LR}(\omega) &= \text{tr} \langle \hat{t}(E + \omega) \hat{t}^\dagger(E) \hat{r}'(E) \hat{r}'^\dagger(E) \rangle. \end{aligned} \quad (100)$$

The above expressions are still rather cumbersome. For the sake of simplicity below we will analyze the case of symmetric quantum dots with $g_L = g_R = g/2$, $\beta_L = \beta_R = \beta$, $R_L = R_R = R$ and $C_L = C_R = C$. We will also assume the external impedance to be Ohmic $Z_S(\omega) = R_S$ at all relevant frequencies. In this case by virtue of Eq. (C9) we find

$$\begin{aligned} D_{LL} = D_{RR} &= \frac{g\beta}{16} + \frac{g(1-2\beta)}{32(1-i\omega\tau_D)} + \frac{g\beta}{32(1-i\omega\tau_D)^2}, \\ D_{LR} = D_{RL} &= \frac{g(1+2\beta)}{32(1-i\omega\tau_D)} - \frac{g\beta}{32(1-i\omega\tau_D)^2}. \end{aligned} \quad (101)$$

From Eqs. (48, 49) we also obtain $K_{LL} = K_{RR} = K_S + K$, $K_{LR} = K_{RL} = K_S - K$, where

$$\begin{aligned} K &= \frac{e^2}{(-i\omega + 0) \left(-2i\omega C + \frac{-i\omega C_g}{1-i\omega R_S C_g/4} + \frac{2}{R} \frac{-i\omega\tau_D}{1-i\omega\tau_D} \right)}, \\ K_S &= \frac{e^2}{(-i\omega + 0) \left(-2i\omega C + \frac{4}{R_S} + \frac{2}{R} \right)}. \end{aligned} \quad (102)$$

To simplify the analysis further let us replace $1 - i\omega R_S C_g/4$ by 1 in the denominator of the expression for

K (102). This is appropriate either for $C_g \rightarrow 0$ or in the limit $R_S \ll 2R$. Then we get

$$\delta I_{\text{tot}}(V) = \delta I(V) + \delta I_S(V), \quad (103)$$

where $\delta I(V)$ is the interaction correction evaluated for $Z_S = 0$ and $\delta I_S(V)$ represents an additional contribution due to the external shunt. The term $\delta I(V)$ is defined by Eqs. (81,83,84), whereas for $\delta I_S(V)$ one finds

$$\delta I_S(V) = -\frac{eg}{16\pi} \int \frac{d\omega}{2\pi} \text{Im} \left(K_S(\omega) \left(\beta + \frac{1}{1 - i\omega\tau_D} \right) \right) \times \left[(\omega + eV) \coth \frac{\omega + eV}{2T} - (\omega - eV) \coth \frac{\omega - eV}{2T} \right]. \quad (104)$$

This formula can also be transformed to the following expression

$$\delta I_S(V) = -\frac{e}{4\pi} \frac{g}{g + 4g_S} \int_0^\infty dt \frac{\pi^2 T^2}{\sinh^2 \pi T t} \sin eVt \times \left[\beta(1 - e^{-t/\tau_S}) + 1 - \frac{\tau_D e^{-t/\tau_D} - \tau_S e^{-t/\tau_S}}{\tau_D - \tau_S} \right], \quad (105)$$

where we defined $\tau_S = RR_S C / (R_S + 2R)$. As before, this integral can be expressed in terms of the Ψ -functions. For the sake of brevity we will omit the corresponding expressions here.

Let us concentrate on the linear in voltage regime. Making use of the above general results one can easily determine the total linear conductance G of the dot in the presence of an external Ohmic shunt. It reads

$$G = G_0 - \frac{1}{2R_q} \left\{ \beta(L(T\tau) - L(T\tau_D)) + \frac{g}{g + 4g_S} \times \left[\frac{\tau_D(L(T\tau_D) - L(T\tau_S))}{\tau_D - \tau_S} + (\beta + 1)L(T\tau_S) \right] \right\}, \quad (106)$$

where the function $L(x)$ was defined in Eq. (90).

For large quantum dots one typically has $\tau_D \gg \tau_0 > \tau_S$. In this case at sufficiently low temperatures $T\tau_D \ll 1$ the whole quantum dot can be considered as a single scatterer, and we find

$$G = \tilde{G}_0 - \frac{2\tilde{\beta}}{R_q} \frac{g}{g + 4g_S} \left(\gamma + 1 + \ln \frac{1}{2\pi T\tau_D} \right), \quad (107)$$

where

$$\tilde{G}_0 = G_0 - \frac{\beta}{2R_q} \left(\ln \frac{\tau_D}{\tau} + \frac{g}{g + 4g_S} \ln \frac{\tau_D}{\tau_S} \right) \quad (108)$$

and $\tilde{\beta} = (\beta + 1)/4$ is the effective total Fano factor of a symmetric chaotic quantum dot, see also Eq. (44). Eqs. (107), (108) smoothly match with the result⁹ derived for a single coherent scatterer with $\tau_D \lesssim \tau_0$.

In the opposite high temperature limit $T\tau_D \gg 1$, albeit $T\tau_0 \ll 1$, we obtain

$$G \approx G_0 - \frac{\beta}{2R_q} \frac{g}{g + 4g_S} \ln \frac{\tau_0}{\tau_S} - \frac{\beta}{R_q} \frac{g + 2g_S}{g + 4g_S} \left(\gamma + 1 + \ln \frac{1}{2\pi T\tau_0} \right). \quad (109)$$

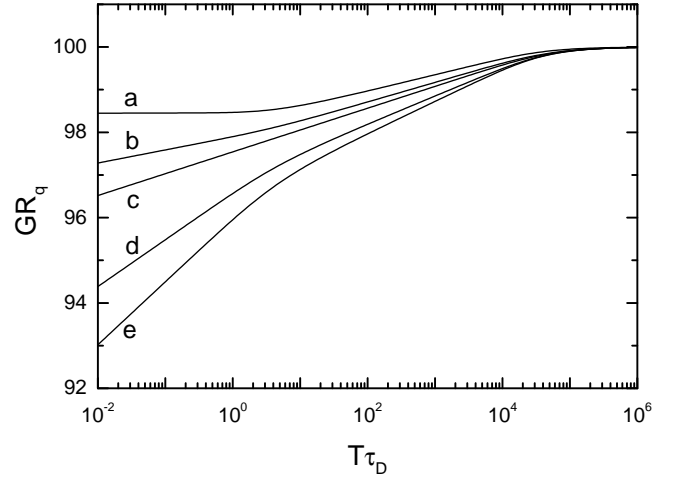


FIG. 2: Conductance of a symmetric quantum dot, as given by Eq. (106). The Fano factor of each barrier is chosen to be $\beta = 1/3$, its conductance is $200/R_q$ (hence $G_0 = 100/R_q$) and $\tau_D/\tau_0 = 10^4$. Different curves correspond to different values of the shunt resistor: (a) $R_S = 2.5 \times 10^{-5} R_q$, (b) $R_S = 0.0025 R_q$, (c) $R_S = 0.005 R_q$, (d) $R_S = 0.025 R_q$, and (e) $R_S = 0.25 R_q$. A crossover between the two logarithmic regimes is clearly observed at $T\tau_D \sim 1$.

Let us compare the results (107) and (109). In both temperature regimes we observe that the interaction correction depends logarithmically on temperature, though with different prefactors in front of the logarithm. At high temperatures $T\tau_D \gg 1$ this prefactor is determined by a sum of two different contributions originating from the terms δI and δI_S . In this regime the two barriers of the quantum dot behave as independent ones. In the other regime $T\tau_D < 1$ the first logarithmic term saturates and does not depend on T anymore providing effective renormalization of the non-interacting conductance (108). On the contrary, the logarithmic temperature dependence (107) in the second term survives down to exponentially small temperatures. In this regime the quantum dot behaves as a single coherent scatterer. Its internal structure becomes insignificant in this case and the interaction correction scales with the total Fano factor $\tilde{\beta}$. A crossover between the two logarithmic regimes is clearly observed in Fig. 2. It is also important to emphasize that, while the logarithmic in T correction (107) vanishes for $R_S \rightarrow 0$, in the opposite limit $R_S \gg R$ it becomes practically independent of the shunt resistance.

The presence of two different logarithmic regimes can also be understood bearing in mind a close relation between shot noise and interaction effects in mesoscopic conductors. At high temperatures $T\tau_D \gg 1$, i.e. in the regime of independent barriers, the noise at each of them determines the corresponding contribution to the interaction correction. Hence, the latter is proportional to the parameter β . In the opposite limit $T\tau_D \ll 1$ the barriers are not anymore independent. In this regime the temperature dependent part of the interaction correction

should be related to the total shot noise of the quantum dot which, as we discussed in Sec. 3, depends on the parameter $\tilde{\beta}$. The high frequency noise $\omega\tau_D \gtrsim 1$ now only provides the renormalization of G_0 which is again proportional to β .

We will continue the discussion of the above results and their experimental relevance in the next section.

V. DISCUSSION AND CONCLUSIONS

The analysis presented here provides a general theoretical description of electron transport through disordered interacting quantum dots in the “metallic” limit of large conductances. Our formalism combines real time path-integral-based influence functional technique with the scattering matrix approach. The main technical result of the present work is the derivation of the effective action for quantum dots described by *arbitrary* energy dependent scattering matrices. Due to this particular feature our technique allows to investigate quantum transport of interacting electrons in a very wide class of conductors which includes, e.g., structures with resonant transmission and many others. This formalism is a direct generalization of our earlier approach^{9,10} which embraced structures described by energy independent scattering matrices and did not account for internal dynamics of disordered conductors.

Although our analysis is mainly aimed at interaction effects, the first and immediate result of our derivation is the expression for frequency-dependent current noise in non-interacting quantum dots. In fully symmetric dots the shot noise spectrum turns out to be defined by the standard expression (46) with the total Fano factor $\tilde{\beta} = (\beta + 1)/4$. For asymmetric quantum dots the corresponding expression becomes far more complicated.

Turning to interaction effects, let us briefly summarize our main results for the linear conductance G of chaotic quantum dots (characterized by dimensionless conductance $g \gg 1$ and charging energy E_C) connected to the voltage source via an Ohmic resistor R_S . The conductance G is expressed in a general form

$$G = G_0 + \delta G(T),$$

where G_0 is the conductance of a non-interacting dot and δG is the interaction correction. This correction is negative, i.e. electron-electron interactions tend to suppress the conductance of quantum dots.

For the situation under consideration one typically has $\tau_D \gg \tau_0 \sim 1/gE_C > \tau_S = R_S C/2(1 + G_0 R_S)$. For simplicity we quote the results for symmetric quantum dots. At high temperatures $T\tau_0 \gg 1$ in the leading approximation one finds

$$\delta G/G_0 \simeq -\frac{\beta\chi E_C}{3T}, \quad (110)$$

where β is the Fano factor of a single barrier (1) and the parameter χ is defined below. At lower temperatures

$T\tau_0 \ll 1$ the power law dependence (110) crosses over into the logarithmic one

$$\delta G \simeq \frac{\beta\chi}{2R_q} \ln(T\tau_0). \quad (111)$$

The parameter χ in Eqs. (110), (111) depends on the relation between R_S and the dot resistance $R_d \equiv 1/G_0$. For $R_S \rightarrow 0$ one has $\chi = 1$, while in the opposite limit $R_S \gg R_d$ this parameter is $\chi = 2$. Thus, in both limits (110) and (111) the magnitudes of the interaction correction δG evaluated in the current- and voltage-biased regimes differ by the factor 2. The magnitude of δG is smaller in the latter regime because voltage fluctuations across the dot are suppressed in this case.

Eq. (111) applies down to temperatures $T \sim 1/\tau_D$. At lower temperatures another logarithmic regime sets in and the difference between current- and voltage-biased situations becomes more dramatic. In the limit $T\tau_D \ll 1$ one finds

$$G \simeq \tilde{G}_0 + \frac{2\tilde{\beta}}{R_q} \frac{1}{1 + 4R_d/R_S} \ln(T\tau_D), \quad (112)$$

where

$$\tilde{G}_0 \simeq G_0 - \frac{\beta\chi}{2R_q} \ln \frac{\tau_D}{\tau_0}. \quad (113)$$

Eq. (113) describes an effective renormalization of G_0 by electron-electron interactions. This renormalization is again twice as big in the current-biased regime as it is in the voltage-biased one. More importantly, as follows from Eq. (112), for any non-zero R_S *no saturation* of the logarithmic dependence of the interaction correction on temperature is expected at $T\tau_D \lesssim 1$. We also observe that at $T\tau_D \gtrsim 1$ the interaction correction δG always scales with β and, hence, vanishes completely for fully transparent barriers $\beta \rightarrow 0$. By contrast, at lower temperatures $T\tau_D \lesssim 1$ only the T -independent renormalization of G_0 disappears in this limit, while the correction (112) scales with $\tilde{\beta}$ and remains finite even for $\beta = 0$. This fact illustrates a direct relation between shot noise and interaction effects in mesoscopic conductors, cf. Eqs. (46) and (112). We also observe that in the limit $\beta \rightarrow 0$ the conductance (112) becomes completely independent of the charging energy E_C .

Similar results can be obtained for a non-linear $I - V$ characteristics of our system. In particular, at $T \rightarrow 0$ the corresponding expressions are reproduced if one substitutes $G \rightarrow dI/dV$ and $T \rightarrow eV$ in Eqs. (110-113).

Both logarithmic regimes in the dependence of the interaction correction on temperature and voltage discussed above were observed in many experiments on various mesoscopic structures. Here we will briefly discuss only a few examples. In the experiment³¹ this dependence was found in short diffusive metallic bridges at temperatures (voltages) $T \lesssim 1$ K. No saturation was observed down to the lowest temperature < 100 mK. Estimating the Thouless energy $\sim 1/\tau_D$ for the parameters of Ref. 31 one arrives at the value of order few

Kelvin. Hence, this experiment was performed in the regime $T\tau_D < 1$ (112).

Recently it was argued³⁰ that in order to explain the experimental results³¹ it is necessary to assume the presence of insulating barriers at interfaces between the bridge and the reservoirs. We note, however, that this assumption can hardly be justified because for the samples³¹ it would inevitably lead to values of g 10 ÷ 100 times smaller than actually measured. The presence of insulating barriers can also be ruled out on the experimental grounds³². This controversy is eliminated if one recalls that the calculation³⁰ was performed within the voltage-biased model $R_S \rightarrow 0$. For this reason the authors³⁰ have overlooked the logarithmic dependence (112) and attributed the observations³¹ to the regime $T\tau_D \gg 1$, hence, requiring much smaller values of $1/\tau_D < 100$ mK. Since the voltage-biased model does not strictly apply to the experiments³¹, no such requirement is actually needed.

Note, that in some other experiments, e.g., ones with multiwalled carbon nanotubes^{33,34,35}, the voltage-biased model appears to be applicable. Both the logarithmic temperature dependence³⁶ of δG at larger T and its saturation at temperatures of order few Kelvin were clearly observed. The latter temperature range is consistent with the estimates of the parameter $1/\tau_D$ for the nanotubes^{33,34,35}.

More recently, the logarithmic temperature dependence of the conductance was observed in strongly disordered multiwalled carbon nanotubes^{37,38}. No saturation of this dependence was found down to the lowest measurement temperature $T \sim 100$ mK. Although estimates of the parameter $1/\tau_D$ for the samples^{37,38} yield values below 100 mK, it is not easy to interpret these observations in terms of the high temperature logarithmic regime (111) because $L_T \sim \sqrt{D/T}$ (D is the diffusion coefficient) remains shorter than the length of the nanotube L at all relevant temperatures $T \gtrsim 0.1$ K. In this situation our zero-dimensional description cannot be applied to the nanotube as a whole.

An alternative explanation is based on viewing the nanotube as a chain of N connected in series statistically independent segments of the length $L_s \approx L/N$. For $L_T \gtrsim L_s$ each of the N segments should behave as a zero-dimensional coherent scatterer shunted by an external impedance effectively produced by the remaining $N - 1$ segments. This scenario applies provided the scale L_s exceeds an effective transversal dimension of the system L_{tr} . In the opposite case $L_s < L_{tr}$ a slightly more general picture of an array of $M \times N$ independent scatterers can be introduced, where the number M depends on the ratio L_{tr}/L_s . Applying the result (112) for each of the segments and assuming $N \gg 1$, at temperatures $T\tau_D \lesssim 1$ one finds the conductance of the system in the form³⁹

$$G(T) \approx \tilde{G}_0 + \frac{2\tilde{\beta}}{R_q} \frac{M}{N} \ln(T\tau_D), \quad (114)$$

where \tilde{G}_0 represents its conductance at $T \sim 1/\tau_D$, $M = 1$

for $L_s > L_{tr}$ and $M > 1$ otherwise. For diffusive nanotubes one has $\tilde{\beta} \approx 1/3$ and $\tau_D \sim L_s^2/D$. The crucial point of this scenario is that at sufficiently low T the scale L_s does not depend on temperature and is set by interactions.

Note that a similar effect was also observed in disordered metallic wires⁴⁰. Clear deviations from the standard 1d behavior of the interaction correction $\delta G \propto 1/\sqrt{T}$ were reported at temperatures $T \lesssim 1/\tau_\varphi$, where τ_φ is the low temperature dephasing time measured in the same experiment. For the parameters⁴⁰ one can verify that both the observed magnitude and temperature dependence of the interaction correction are consistent with Eq. (114), where $L_s \sim L_\varphi = \sqrt{D\tau_\varphi}$ and $M = 1$.

Turning again to the data³⁸ let us recall that in this case the observed values of L_φ were considerably smaller than the nanotube circumference πd . Therefore we can assume $L_s < \pi d$ and define⁴¹ $M \sim \pi d/L_s$. Then we obtain $M/N \simeq \pi d/L$, i.e. L_s drops out and for $\tilde{\beta} = 1/3$ the prefactor in front of the logarithmic term in Eq. (114) becomes $\simeq 2.1d/R_q L$. This universal value fits well with the observations³⁸ at $T \lesssim 10$ K. Finally, we note that the latter inequality is also consistent with the condition $T\tau_\varphi \lesssim 1$ because the measured values of τ_φ were found to be in the range $\tau_\varphi \sim 10^{-12}$ sek. Hence, the assumption $L_s \sim L_\varphi$ and $\tau_D \sim \tau_\varphi$ appears to be supported by the data³⁸ as well.

Further experimental and theoretical investigations of the effect of interactions on low temperature electron transport in disordered conductors are warranted.

We would like to thank D.A. Bagrets, A.V. Galaktionov, Yu.V. Nazarov and S.V. Sharov for interesting discussions and P. Hakonen, P. Mohanty, M. Paalanen and H. Weber for patiently explaining us various details of their data and experiments^{31,37,40}. This work is part of the Kompetenznetz "Funktionelle Nanostrukturen" supported by the Landestiftung Baden-Württemberg gGmbH.

APPENDIX A: EVOLUTION OPERATOR

For non-interacting systems the relation between the S -matrix and the Green functions, or, which is the same, the evolution operator is known since the work by Fisher and Lee⁴². Here we use the inverse form of this relation, namely we express the evolution operator in terms of the S -matrix. We introduce the components of the evolution operator $U_{ij}(t, 0; r, r')$, where i, j label the left (l) and the right (r) leads as well as the dot (d). The component U_{ll} reads

$$\begin{aligned} U_{ll}^{\alpha\beta}(t, 0; r, r') &= \frac{1}{\sqrt{v_\alpha^l v_\beta^l}} \int \frac{dE}{2\pi} e^{-iEt + iE(r/v_\alpha^l - r'/v_\beta^l)} \\ &\times [\delta_{\alpha\beta} + \theta(r)\theta(-r')(r_{\alpha\beta}(E) - \delta_{\alpha\beta}) \\ &+ \theta(-r)\theta(r')(r_{\alpha\beta}^+(E) - \delta_{\alpha\beta})]. \end{aligned}$$

Here α, β are the channel indices, v_α^j is the velocity in the α -th channel of j -th lead. Three other components of the evolution operator U_{rr} , U_{lr} and U_{rl} are expressed analogously. It is convenient to introduce the “flight times” in the α -th channel $x = r/v_\alpha^j$, $y = r'/v_\alpha^j$ and express the wave functions and the evolution operator via these new variables. The velocities v_α^j then disappear, and we can conveniently rewrite the above relation in the matrix form

$$\begin{aligned} \hat{U}(t, 0; x, y) &= \hat{1}\delta(t - x + y) \\ &+ \theta(x)\theta(-y)[\hat{S}(t - x + y) - \hat{1}\delta(t - x + y)] \\ &+ \theta(-x)\theta(y)[\hat{S}^+(-t + x - y) - \hat{1}\delta(t - x + y)]. \end{aligned} \quad (\text{A1})$$

Note that here we present \hat{U} in the form of a 2×2 (rather than 3×3) matrix leaving out the components describing transitions (e.g. from the dot to the dot and some others) which turn out to be unimportant for our analysis. We will return to this point in Appendix B.

The operator (A1) has the following properties: (i) $\hat{U}^\dagger(t, 0) = \hat{U}(0, t)$, and (ii) $\lim_{t \rightarrow +\infty} \hat{U}^\dagger(t, 0)\hat{U}(t, 0) = \hat{1}$. At any finite time t this operator does not obey the unitarity condition. This is quite natural because the components describing the evolution in the dot are missing in Eq. (A1). However the operator $\hat{U}(t, 0; x, y)$ becomes unitary in the limit $t \rightarrow \infty$.

Let us now introduce the fluctuating potential $V(r, t)$ and, as we have already discussed in Sec. 3, assume that this potential is spatially constant both inside the metallic leads and inside the dot, and it suffers jumps at the junctions between the dot and the leads. In other words, we consider three different fluctuating in time potentials $V_j(t)$, where the index j labels the left lead, the dot and the right lead.

The wave functions are then modified in two ways. First, they acquire additional phase factors $\exp\left\{i \int_0^t dt' eV_j(t')\right\} = \exp\{\varphi_j(t)\}$. These phase factors can be eliminated by a gauge transformation, and we will omit them in what follows. In addition to that, the phase of the wave function changes every time the electron crosses one of the junctions. The corresponding additional phase factors acquired by the electron are:

$$\begin{aligned} e^{-i\varphi_L^+} &\text{ left junction, from left to right,} \\ e^{i\varphi_L^+} &\text{ left junction, from right to left,} \\ e^{-i\varphi_R^+} &\text{ right junction, from left to right,} \\ e^{i\varphi_R^+} &\text{ right junction, from right to left.} \end{aligned} \quad (\text{A2})$$

Below we will restrict ourselves to the case $t > 0$. This is sufficient, since the evolution operator at negative times is just a conjugate operator. Taking the phase factors (A2) into account we find \hat{U} in the presence of fluctuating voltages $V_j^+(t)$:

$$\hat{U}^{\varphi^+}(t, 0; x, y) = \hat{A}(t - x)\hat{U}(t, 0; x, y)\hat{A}^*(-y) \quad (\text{A3})$$

where the matrix \hat{A} reads

$$\hat{A}(t) = \begin{pmatrix} e^{i\varphi_L^+(t)}\hat{1} & 0 \\ 0 & e^{-i\varphi_R^+(t)}\hat{1} \end{pmatrix}. \quad (\text{A4})$$

The relation (A3) turns out to be remarkably simple since every classical path – though being scattered arbitrarily many times inside the dot and from the outer barriers – can cross these barriers only twice: when it enters and leaves the dot. The times of these two events are related in a trivial way to the total time of the evolution and to the initial and final coordinates: For any x and y the path enters the dot at a time $-y$ and leaves it at a time $t - x$. We also note that this property holds only for structures with two barriers. In systems with three and more barriers no such a simple relation between $\hat{U}^{\varphi^+}(t, 0; x, y)$ and $\hat{U}(t, 0; x, y)$ exist.

APPENDIX B: EFFECTIVE ACTION

Let us now use the above expressions for the evolution operator and derive the effective action of a quantum dot. We start from the term S_R which is defined by the second term in Eq. (11) as

$$\begin{aligned} iS_R &= e \int_0^t dt' \int d^3\mathbf{r} (G_{V^+,11}(t' - 0, t', \mathbf{r}, \mathbf{r}) \\ &+ G_{V^+,22}(t' + 0, t', \mathbf{r}, \mathbf{r}))V^-(t', \mathbf{r}). \end{aligned} \quad (\text{B1})$$

Here the space integrals are taken over the dot and both leads. As we have already discussed, we will adopt the model assuming that the fluctuating voltage fields inside the leads do not depend on coordinates. Making use of Eqs. (12,13) let us transform Eq. (B1) to the following form

$$\begin{aligned} iS_R &= 2i \int_0^t dt' \int d^3\mathbf{r} d^3\mathbf{r}' d^3\mathbf{r}'' U^{\varphi^+}(t', 0, \mathbf{r}\mathbf{r}') \\ &\times \rho_0(\mathbf{r}', \mathbf{r}'')U^{\varphi^+}(0t', \mathbf{r}''\mathbf{r})\dot{\varphi}^-(t', \mathbf{r}). \end{aligned} \quad (\text{B2})$$

Let us split this expression by distinguishing each of the three coordinates \mathbf{r} , \mathbf{r}' and \mathbf{r}'' to be in the left lead, in the dot or in the right lead. Altogether we obtain $3^3 = 27$ terms. Without loss of generality this number can be reduced substantially by means of the following steps. First, we choose the initial density matrix ρ_0 in the form corresponding to zero transmissions of both junctions. For such a choice initially we have three isolated systems, in which case the coordinates \mathbf{r}' and \mathbf{r}'' always belong to the same electrode. This step already reduces the total number of terms down to nine. Second, we restrict ourselves to the limit of sufficiently long times, which is of a primary interest in our problem. In this limit the initial form of the density matrix inside the dot does not matter at all, electron transfer and relaxation processes will eventually lead to some final density matrix in the dot which will be *independent* of the initial one. Hence,

in the interesting limit of long times our effective action should not depend on the initial density matrix inside the dot and we can safely exclude the dot region from the integration over \mathbf{r}' and \mathbf{r}'' extending this integration only at the two reservoirs. After that we are left with only six terms⁴³, and the action S_R takes the form

$$iS_R = 2i \sum_{s=l,d,r} \sum_{s'=l,r} \int_0^t dt' \int_{s'} d^3\mathbf{r}' d^3\mathbf{r}'' \rho_0^{s'}(\mathbf{r}', \mathbf{r}'') \times \int_s d^3\mathbf{r} U_{ss'}^{\varphi^+}(t'0, \mathbf{r}\mathbf{r}') U_{s's}^{\varphi^+}(0t', \mathbf{r}''\mathbf{r}) \hat{\varphi}_s^-(t'). \quad (\text{B3})$$

In order to proceed further, we will make use of the unitarity of the evolution operator. It yields

$$\sum_{s=l,d,r} \int_s d^3\mathbf{r} U_{ss'}^{\varphi^+}(t'0, \mathbf{r}\mathbf{r}') U_{s's}^{\varphi^+}(0t', \mathbf{r}''\mathbf{r}) \equiv \delta(\mathbf{r}' - \mathbf{r}'') \quad (\text{B4})$$

With the aid of this identity one can express the integrals over \mathbf{r} in the dot region through the integrals over the coordinate in the leads, where an explicit form of the evolution operator (A3) is already available. We find

$$iS_R = 2i \int_0^t dt' \hat{\varphi}_d^-(t') \sum_{s'=l,r} \int_{s'} d^3\mathbf{r}' \rho_0^{s'}(\mathbf{r}', \mathbf{r}') + 2i \sum_{s,s'=l,r} \int_0^t dt' \int_{s'} d^3\mathbf{r}' d^3\mathbf{r}'' \rho_0^{s'}(\mathbf{r}', \mathbf{r}'') \times \left[\int_l d^3\mathbf{r} U_{s'l}^{\varphi^+}(0t', \mathbf{r}''\mathbf{r}) U_{l's'}^{\varphi^+}(t'0, \mathbf{r}\mathbf{r}') \hat{\varphi}_L^-(t') - \int_r d^3\mathbf{r} U_{s'r}^{\varphi^+}(0t', \mathbf{r}''\mathbf{r}) U_{r's'}^{\varphi^+}(t'0, \mathbf{r}\mathbf{r}') \hat{\varphi}_R^-(t') \right],$$

where $\varphi_L^- = \varphi_l^- - \varphi_d^-$, $\varphi_R^- = \varphi_d^- - \varphi_r^-$. The first term in this equation is proportional to the total number of electrons in the leads. Because of the charge neutrality this term should be exactly canceled by the analogous term from the ion background. For this reason we will omit this term in our further consideration.

In order to evaluate the remaining terms we switch to the channel representation. Since we assume that the density matrix in the leads corresponds to the equilibrium Fermi distribution of electrons, both this density matrix and the matrix of the phases φ^- are diagonal in the channel space

$$\hat{\rho}_0(x-y) = \rho_0(x-y) \hat{1}, \quad \hat{\varphi}^-(t) = \begin{pmatrix} -\hat{1} \hat{\varphi}_L^-(t) & 0 \\ 0 & \hat{1} \hat{\varphi}_R^-(t) \end{pmatrix}. \quad (\text{B5})$$

The action S_R acquires the following form:

$$iS_R = -2i \int dx dy \text{tr} \left[\hat{\rho}_0(x-y) \int_0^t dt' \int dz \hat{A}(-y) \times \hat{U}(0, t'; y, z) \hat{A}^*(t-z) \hat{\varphi}^-(t') \hat{A}(t-z) \times \hat{U}(t', 0; z, x) \hat{A}^*(-x) \right]. \quad (\text{B6})$$

We also notice that the matrices $\hat{\varphi}^-(t')$ (B5) and \hat{A} (A4) commute, hence $\hat{A}^*(t-z)$ and $\hat{A}(t-z)$ drop out. Performing the change of the integration variables $x \rightarrow -x$, $y \rightarrow -y$, we obtain

$$iS_R = -2i \int dx dy \text{tr} \left[\hat{A}^*(x) \hat{\rho}_0(y-x) \hat{A}(y) \hat{B}(t, y, x) \right]. \quad (\text{B7})$$

Here we defined the matrix

$$\hat{B}(t, y, x) = \int_0^t dt' \int dz \hat{U}(0, t'; -y, z) \hat{\varphi}^-(t') \hat{U}(t', 0; z, -x). \quad (\text{B8})$$

With the aid of Eq. (A1) this matrix can be evaluated explicitly. For our purposes it is sufficient to set $\hat{\varphi}^-(t) = \hat{\varphi}^-(0) = 0$. Then we get

$$\hat{B}(t, y, x) = \theta(x)\theta(y) \int_0^t dz [\delta(z-y) \hat{\varphi}^-(z) \delta(z-x) - \hat{S}^+(z-y) \hat{\varphi}^-(z) \hat{S}(z-x)]. \quad (\text{B9})$$

Making use of this expression together with (A4) and (B5) one can now directly multiply matrices in Eq. (B7) and arrive at the final result for S_R presented in Eq. (18).

Let us now turn to the expression for S_I which is defined by the last term in Eq. (11)

$$S_I = e^2 \int_0^t dt' \int_0^{t'} dt'' \int d^3\mathbf{r}' d^3\mathbf{r}'' G_{V^+, 12}(t', t'', \mathbf{r}', \mathbf{r}'') \times V^-(t'', \mathbf{r}'') G_{V^+, 21}(t'', t', \mathbf{r}'', \mathbf{r}') V^-(t', \mathbf{r}').$$

The whole consideration is completely analogous to that carried out above for the term S_R . Making use of Eq. (12), we find

$$S_I = e^2 \int_0^t dt_1 \int_0^{t_1} dt_2 \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 d^3\mathbf{y}_1 d^3\mathbf{y}_2 d^3\mathbf{z}_1 d^3\mathbf{z}_2 \times U_{\varphi^+}(t_1 0, \mathbf{x}_1 \mathbf{y}_1) \rho_0(\mathbf{y}_1, \mathbf{y}_2) U_{\varphi^+}(0 t_2, \mathbf{y}_2 \mathbf{x}_2) V^-(t_2, \mathbf{x}_2) \times U_{\varphi^+}(t_2 0, \mathbf{x}_2 \mathbf{z}_2) h_0(\mathbf{z}_2, \mathbf{z}_1) U_{\varphi^+}(0 t_1, \mathbf{z}_1, \mathbf{x}_1) V^-(t_1, \mathbf{x}_1).$$

where $h_0(\mathbf{z}_2, \mathbf{z}_1) = 1 - \rho_0(\mathbf{z}_2, \mathbf{z}_1)$. As before we identify the contributions containing the initial density matrix in the dot. Such terms can again be omitted in the interesting limit of long times. We again apply the unitarity condition (B4) and explicitly introduce the channel indices. Then after some straightforward manipulations we obtain

$$S_I = \int_0^t dt_1 \int_0^{t_1} dt_2 \int dx_1 dx_2 dy_1 dy_2 dz_1 dz_2 \times \text{tr} \left\{ \hat{U}(0, t_1; z_1, x_1) \hat{\varphi}^-(t_1) \hat{U}(t_1, 0; x_1, y_1) \hat{A}^*(-y_1) \times \hat{\rho}_0(y_1 - y_2) \hat{A}(-y_2) \hat{U}(0, t_2; y_2, x_2) \hat{\varphi}^-(t_2) \times \hat{U}(t_2, 0; x_2, z_2) \hat{A}^*(-z_2) \hat{h}_0(z_2 - z_1) \hat{A}(-z_1) \right\}.$$

With the aid of the matrix (B9) this expression can be transformed further and eventually takes the form

$$S_I = \int dy_1 dy_2 \int dz_1 dz_2 \text{tr} \left\{ \hat{B}(z_1, y_1) \hat{A}^*(y_1) \hat{\rho}_0(y_2 - y_1) \right. \\ \left. \times \hat{A}(y_2) \hat{B}(y_2, z_2) \hat{A}^*(z_2) \hat{h}_0(z_1 - z_2) \hat{A}(z_1) \right\}. \quad (\text{B10})$$

Multiplying matrices in Eqs. (B10) we arrive at the final expressions for S_I defined in Eq. (21).

APPENDIX C: AVERAGING OVER CIRCULAR ENSEMBLE OF S -MATRICES

Here we will evaluate the following average

$$W = \text{tr} \langle \hat{A} \hat{S}^\dagger(E_1) \hat{B} \hat{S}(E_2) \hat{C} \hat{S}^\dagger(E_3) \hat{D} \hat{S}(E_4) \rangle. \quad (\text{C1})$$

Let us use the representation (38) and split the S -matrix into two parts

$$\hat{S}(E) = \hat{R} + \delta \hat{S}(E), \quad \delta \hat{S}(E) = \hat{T}' [1 - \hat{U}(E)]^{-1} \hat{U}(E) \hat{T}. \quad (\text{C2})$$

One can show that $\langle \delta \hat{S}(E) \rangle = 0$. At any given energy E the matrix $\hat{U}(E)$ belongs to the circular ensemble.

In a semi-classical limit averaging of the products of such matrices taken at the same energy can be performed with the aid of the diagram technique²⁶. Here we need a more general version of this technique which would also allow to average the products of matrices taken at different energies. Appropriate modifications of the rules can be formulated with the aid of the results²⁸. In short, using the terminology²⁶, every U -cycle involving two matrices $\hat{U}(E_1)$ and $\hat{U}^\dagger(E_2)$ carries an additional factor $M/(M - iE_{12}t_0)$ as compared to the situation²⁶. Here $M = N_L + N_R$, where $N_{L(R)}$ is the number of

channels in the left (right) barrier/lead, $t_0 = 2\pi/\delta$ and $E_{ij} = E_i - E_j$. An U -cycle involving four matrices $\hat{U}^\dagger(E_1)$, $\hat{U}(E_2)$, $\hat{U}^\dagger(E_3)$ and $\hat{U}(E_4)$ gives an additional factor

$$\frac{M^3(M - i(E_{23} + E_{41})t_0)}{(M - iE_{41}t_0)(M - iE_{21}t_0)(M - iE_{23}t_0)(M - iE_{43}t_0)}.$$

Higher order U -cycles do not contribute in the semi-classical limit $M \rightarrow \infty$, and we will not consider them here.

Employing Eq. (C2) one can express the average (C1) as a sum of 10 non-vanishing terms. The first of them is trivial and equals $\text{tr}(\hat{A} \hat{R}^\dagger \hat{B} \hat{R} \hat{C} \hat{R}^\dagger \hat{D} \hat{R})$. Several terms are bilinear in the matrices $\delta \hat{S}$ and $\delta \hat{S}^\dagger$. Such terms are evaluated with the aid of the formula

$$\text{tr} \langle \hat{A} \delta \hat{S}(E_1) \hat{B} \delta \hat{S}^\dagger(E_2) \rangle = \frac{(\text{tr} \hat{T}'^\dagger \hat{A} \hat{T}') (\text{tr} \hat{T} \hat{B} \hat{T}^\dagger)}{M - \text{tr} \hat{R}' \hat{R}'^\dagger - iE_{12}t_0} \\ = \frac{2(\text{tr} \hat{T}'^\dagger \hat{A} \hat{T}') (\text{tr} \hat{T} \hat{B} \hat{T}^\dagger)}{g(1 - iE_{12}\tau_D)}. \quad (\text{C3})$$

Here we have used the identity $M - \text{tr} \hat{R}' \hat{R}'^\dagger = \text{tr} \hat{T} \hat{T}^\dagger = g$, and defined the dwell time $\tau_D = 2t_0/g$. Eq. (C3) is a direct generalization of the analogous formula²⁶ derived for the case $E_1 = E_2$. This formula allows to evaluate the averages u_ω^{ij} (24). For example, $u_\omega^{RL} = \text{tr} \langle \hat{t}^\dagger(E) \hat{t}(E + \omega) \rangle = \text{tr} \langle \hat{C}_1 \hat{S}^\dagger(E) \hat{C}_2 \hat{S}(E + \omega) \rangle$, where we defined the matrices

$$\hat{C}_1 = \begin{pmatrix} \hat{1} & 0 \\ 0 & 0 \end{pmatrix}, \quad \hat{C}_2 = \begin{pmatrix} 0 & 0 \\ 0 & \hat{1} \end{pmatrix}. \quad (\text{C4})$$

Employing Eq. (C3) we arrive at (41).

The terms involving three matrices $\delta \hat{S}$ and/or $\delta \hat{S}^\dagger$ are slightly more complicated. They read

$$\text{tr} \langle \hat{A} \delta \hat{S}(E_1) \hat{B} \delta \hat{S}(E_2) \hat{C} \delta \hat{S}^\dagger(E_3) \rangle = \frac{4(\text{tr} \hat{T}'^\dagger \hat{A} \hat{T}') (\text{tr} \hat{T} \hat{C} \hat{T}^\dagger) (\text{tr} \hat{T} \hat{B} \hat{T}'^\dagger \hat{R}'^\dagger)}{g^2(1 - iE_{13}\tau_D)(1 - iE_{23}\tau_D)}, \\ \text{tr} \langle \hat{A} \delta \hat{S}(E_1) \hat{B} \delta \hat{S}^\dagger(E_2) \hat{C} \delta \hat{S}^\dagger(E_3) \rangle = \frac{4(\text{tr} \hat{T}'^\dagger \hat{A} \hat{T}') (\text{tr} \hat{T} \hat{B} \hat{T}^\dagger) (\text{tr} \hat{R}' \hat{T}'^\dagger \hat{C} \hat{T}^\dagger)}{g^2(1 - iE_{12}\tau_D)(1 - iE_{13}\tau_D)}. \quad (\text{C5})$$

The derivation of Eqs. (C5) is straightforward, and we will not discuss the corresponding details here.

What remains is to evaluate the average

$$w = \text{tr} \langle \hat{A} \delta \hat{S}^\dagger(E_1) \hat{B} \delta \hat{S}(E_2) \hat{C} \delta \hat{S}^\dagger(E_3) \hat{D} \delta \hat{S}(E_4) \rangle. \quad (\text{C6})$$

This average is given by the combination of five diagrams shown in Fig. 3:

$$w = D_1 + D_2 + D_3 + D_4 - D_5. \quad (\text{C7})$$

The diagram D_5 is subtracted in order to compensate for double counting of certain graphs in the diagrams D_3 and

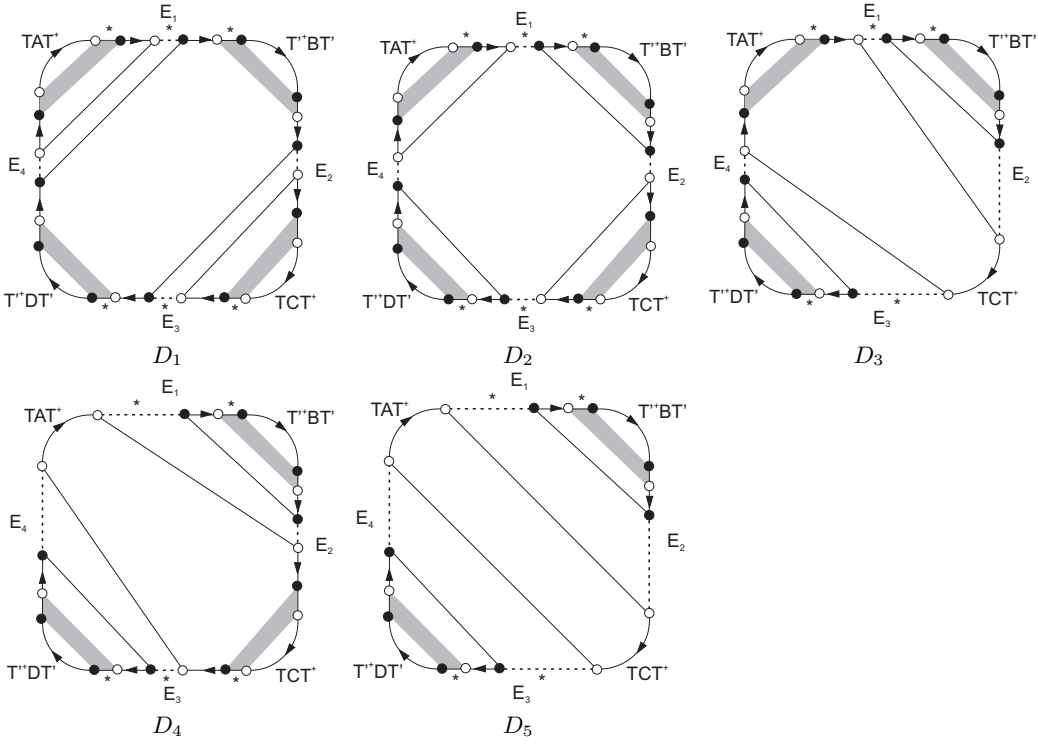


FIG. 3: Five diagrams contributing to $\text{tr}\langle \hat{A}\delta\hat{S}^\dagger(E_1)\hat{B}\delta\hat{S}(E_2)\hat{C}\delta\hat{S}^\dagger(E_3)\hat{D}\delta\hat{S}(E_4) \rangle$. We refer to Ref. 26 for the definition of the graphical objects presented here.

D_4 . The contributions of the diagrams read

$$\begin{aligned}
 D_1 &= \frac{(\text{tr}\hat{F}_A)(\text{tr}\hat{F}_C)(\text{tr}\hat{F}_B\hat{F}_D)}{(M - iE_{41}t_0)(M - iE_{23}t_0)}, & D_2 &= -\frac{(\text{tr}\hat{F}_A)(\text{tr}\hat{F}_B)(\text{tr}\hat{F}_C)(\text{tr}\hat{F}_D)(M - i(E_{23} + E_{41})t_0)}{(M - iE_{41}t_0)(M - iE_{21}t_0)(M - iE_{23}t_0)(M - iE_{43}t_0)}, \\
 D_3 &= \frac{(\text{tr}\hat{F}_A\hat{T}\hat{C}\hat{T}^\dagger)(\text{tr}\hat{F}_B)(\text{tr}\hat{F}_D)}{(M - iE_{21}t_0)(M - iE_{43}t_0)}, & D_4 &= \frac{(\text{tr}\hat{T}\hat{A}\hat{T}^\dagger\hat{F}_C)(\text{tr}\hat{F}_B)(\text{tr}\hat{F}_D)}{(M - iE_{21}t_0)(M - iE_{43}t_0)}, & D_5 &= \frac{(\text{tr}\hat{T}\hat{A}\hat{T}^\dagger\hat{T}\hat{C}\hat{T}^\dagger)(\text{tr}\hat{F}_B)(\text{tr}\hat{F}_D)}{(M - iE_{21}t_0)(M - iE_{43}t_0)}. \quad (\text{C8})
 \end{aligned}$$

Here we have introduced the ladder blocks defined in Fig. 4:

$$\begin{aligned}
 \hat{F}_A &= \hat{T}\hat{A}\hat{T}^\dagger + \sum_{n=0}^{\infty} (\text{tr}\hat{T}\hat{A}\hat{T}^\dagger)\hat{R}'\hat{R}'^\dagger \frac{(\text{tr}\hat{R}'\hat{R}'^\dagger)^n}{(M - iE_{41}t_0)^{n+1}} = \hat{T}\hat{A}\hat{T}^\dagger + \frac{2\text{tr}\hat{T}\hat{A}\hat{T}^\dagger}{g(1 - iE_{41}\tau_D)}\hat{R}'\hat{R}'^\dagger, \\
 \hat{F}_B &= \hat{T}'^\dagger\hat{B}\hat{T}' + \frac{2\text{tr}\hat{T}'^\dagger\hat{B}\hat{T}'}{g(1 - iE_{21}\tau_D)}\hat{R}'\hat{R}'^\dagger, & \hat{F}_C &= \hat{T}\hat{C}\hat{T}^\dagger + \frac{2\text{tr}\hat{T}\hat{C}\hat{T}^\dagger}{g(1 - iE_{23}\tau_D)}\hat{R}'\hat{R}'^\dagger, & \hat{F}_D &= \hat{T}'^\dagger\hat{D}\hat{T}' + \frac{2\text{tr}\hat{T}'^\dagger\hat{D}\hat{T}'}{g(1 - iE_{43}\tau_D)}\hat{R}'\hat{R}'^\dagger.
 \end{aligned}$$

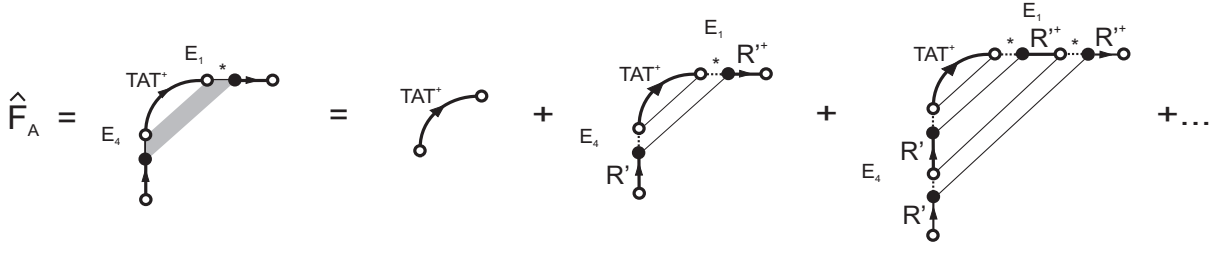


FIG. 4: The ladder block \hat{F}_A appearing in the diagrams shown in Fig 3. The blocks \hat{F}_B , \hat{F}_C and \hat{F}_D are defined analogously.

Eventually we arrive at an important – though rather lengthy – formula

$$\begin{aligned}
W = & \text{tr} \hat{A} \hat{R}^\dagger \hat{B} \hat{R} \hat{C} \hat{R}^\dagger \hat{D} \hat{R} + \frac{2(\text{tr} \hat{T}'^\dagger \hat{B} \hat{T}')(\text{tr} \hat{T} \hat{C} \hat{R}^\dagger \hat{D} \hat{R} \hat{A} \hat{T}^\dagger)}{g(1 - iE_{21}\tau_D)} + \frac{2(\text{tr} \hat{T}'^\dagger \hat{B} \hat{R} \hat{C} \hat{R}^\dagger \hat{D} \hat{T}')(\text{tr} \hat{T} \hat{A} \hat{T}^\dagger)}{g(1 - iE_{41}\tau_D)} \\
& + \frac{2(\text{tr} \hat{T}'^\dagger \hat{D} \hat{R} \hat{A} \hat{R}^\dagger \hat{B} \hat{T}')(\text{tr} \hat{T} \hat{C} \hat{T}^\dagger)}{g(1 - iE_{23}\tau_D)} + \frac{2(\text{tr} \hat{T}'^\dagger \hat{D} \hat{T}')(\text{tr} \hat{T} \hat{A} \hat{R}^\dagger \hat{B} \hat{R} \hat{C} \hat{T}^\dagger)}{g(1 - iE_{43}\tau_D)} \\
& + \frac{4(\text{tr} \hat{T}'^\dagger \hat{B} \hat{T}')(\text{tr} \hat{T} \hat{C} \hat{T}^\dagger)(\text{tr} \hat{R}' \hat{T}'^\dagger \hat{D} \hat{R} \hat{A} \hat{T}^\dagger)}{g^2(1 - iE_{21}\tau_D)(1 - iE_{23}\tau_D)} + \frac{4(\text{tr} \hat{T}'^\dagger \hat{B} \hat{T}')(\text{tr} \hat{T} \hat{A} \hat{T}^\dagger)(\text{tr} \hat{T} \hat{C} \hat{R}^\dagger \hat{D} \hat{T}' \hat{R}'^\dagger)}{g^2(1 - iE_{21}\tau_D)(1 - iE_{41}\tau_D)} \\
& + \frac{4(\text{tr} \hat{T}'^\dagger \hat{D} \hat{T}')(\text{tr} \hat{T} \hat{A} \hat{T}^\dagger)(\text{tr} \hat{R}' \hat{T}'^\dagger \hat{B} \hat{R} \hat{C} \hat{T}^\dagger)}{g^2(1 - iE_{41}\tau_D)(1 - iE_{43}\tau_D)} + \frac{4(\text{tr} \hat{T}'^\dagger \hat{D} \hat{T}')(\text{tr} \hat{T} \hat{C} \hat{T}^\dagger)(\text{tr} \hat{T} \hat{A} \hat{R}^\dagger \hat{B} \hat{T}' \hat{R}'^\dagger)}{g^2(1 - iE_{43}\tau_D)(1 - iE_{23}\tau_D)} \\
& + \frac{4(\text{tr} \hat{T} \hat{A} \hat{T}^\dagger)(\text{tr} \hat{T} \hat{C} \hat{T}^\dagger) \text{tr}(\hat{T}'^\dagger \hat{B} \hat{T}' \hat{T}'^\dagger \hat{D} \hat{T}')}{g^2(1 - iE_{41}\tau_D)(1 - iE_{23}\tau_D)} + \frac{4(\text{tr} \hat{T}'^\dagger \hat{B} \hat{T}')(\text{tr} \hat{T}'^\dagger \hat{D} \hat{T}')(\text{tr} \hat{T} \hat{A} \hat{T}^\dagger \hat{T} \hat{C} \hat{T}^\dagger)}{g^2(1 - iE_{21}\tau_D)(1 - iE_{43}\tau_D)} \\
& + \frac{8(\text{tr} \hat{T} \hat{A} \hat{T}^\dagger)(\text{tr} \hat{T} \hat{C} \hat{T}^\dagger)(\text{tr} \hat{T}'^\dagger \hat{D} \hat{T}')(\text{tr} \hat{T}'^\dagger \hat{B} \hat{T}' \hat{R}'^\dagger \hat{R}')}{g^3(1 - iE_{43}\tau_D)(1 - iE_{41}\tau_D)(1 - iE_{23}\tau_D)} + \frac{8(\text{tr} \hat{T} \hat{A} \hat{T}^\dagger)(\text{tr} \hat{T} \hat{C} \hat{T}^\dagger)(\text{tr} \hat{T}'^\dagger \hat{B} \hat{T}')(\text{tr} \hat{T}'^\dagger \hat{D} \hat{T}' \hat{R}'^\dagger \hat{R}')}{g^3(1 - iE_{21}\tau_D)(1 - iE_{41}\tau_D)(1 - iE_{23}\tau_D)} \\
& + \frac{8(\text{tr} \hat{T}'^\dagger \hat{B} \hat{T}')(\text{tr} \hat{T}'^\dagger \hat{D} \hat{T}')(\text{tr} \hat{T} \hat{A} \hat{T}^\dagger)(\text{tr} \hat{T} \hat{C} \hat{T}^\dagger \hat{R}' \hat{R}'^\dagger)}{g^3(1 - iE_{21}\tau_D)(1 - iE_{43}\tau_D)(1 - iE_{41}\tau_D)} + \frac{8(\text{tr} \hat{T}'^\dagger \hat{B} \hat{T}')(\text{tr} \hat{T}'^\dagger \hat{D} \hat{T}')(\text{tr} \hat{T} \hat{C} \hat{T}^\dagger)(\text{tr} \hat{T} \hat{A} \hat{T}^\dagger \hat{R}' \hat{R}'^\dagger)}{g^3(1 - iE_{21}\tau_D)(1 - iE_{43}\tau_D)(1 - iE_{23}\tau_D)} \\
& - \frac{16(\text{tr} \hat{T} \hat{A} \hat{T}^\dagger)(\text{tr} \hat{T} \hat{C} \hat{T}^\dagger)(\text{tr} \hat{T}'^\dagger \hat{B} \hat{T}')(\text{tr} \hat{T}'^\dagger \hat{D} \hat{T}') (M - \text{tr} \hat{R}' \hat{R}'^\dagger \hat{R}' \hat{R}'^\dagger - i(E_{23} + E_{41})t_0)}{g^4(1 - iE_{21}\tau_D)(1 - iE_{43}\tau_D)(1 - iE_{41}\tau_D)(1 - iE_{23}\tau_D)}. \tag{C9}
\end{aligned}$$

One can demonstrate that the result (C9) is consistent with the unitarity of the S -matrix. For this purpose let us put $E_4 = E_3$ and $\hat{D} = 1$. Due to the unitarity of S -matrices of the barriers almost all of the terms cancel out and the result reduces to

$$\begin{aligned}
W = & \text{tr} \hat{A} \hat{R}^\dagger \hat{B} \hat{R} \hat{C} + \frac{2(\text{tr} \hat{T}'^\dagger \hat{B} \hat{T}')(\text{tr} \hat{T} \hat{C} \hat{A} \hat{T}^\dagger)}{g(1 - iE_{21}\tau_D)} \\
= & \text{tr} \langle \hat{A} \hat{S}^\dagger(E_1) \hat{B} \hat{S}(E_2) \hat{C} \rangle. \tag{C10}
\end{aligned}$$

The last equation is a consequence of Eq. (C3). This is just the expression expected from the definition (C1) and the unitarity of the S -matrix.

With the aid of Eq. (C9) one can find the averages \tilde{v}_ω^{ij} defined in Eq. (29). We transform them to the form

$$\begin{aligned}
\tilde{v}_\omega^{LL} = & \text{tr} \langle \hat{C}_1 \hat{S}^\dagger(E) \hat{C}_1 \hat{S}(E + \omega) \hat{C}_2 \hat{S}^\dagger(E + \omega) \hat{C}_1 \hat{S}(E) \rangle \\
\tilde{v}_\omega^{RL} = & \text{tr} \langle \hat{C}_1 \hat{S}^\dagger(E) \hat{C}_2 \hat{S}(E + \omega) \hat{C}_1 \hat{S}^\dagger(E + \omega) \hat{C}_1 \hat{S}(E) \rangle \tag{C11} \quad \text{where}
\end{aligned}$$

and similarly for \tilde{v}_ω^{RR} and \tilde{v}_ω^{LR} . Making use of Eq. (C9) we arrive at Eqs. (42). Analogously, we derive the expressions (101).

APPENDIX D: GENERAL EXPRESSIONS FOR THE CURRENT

In Sec. 4A we focused on the expression for the current in the limit of zero external impedance $Z_S \rightarrow 0$. Our analysis also allows to establish more general expressions valid for arbitrary $Z_S(\omega)$. We obtain

$$I(V) = I_0(V) + \delta I_{LR} + \delta I_{LL} + \delta I_{RR}, \tag{D1}$$

$$I_0(V) = 2e \int dx \rho_0(x) \mathcal{T}(x) \left(e^{-F_{LL}(x)} e^{ieV_L x} - e^{-F_{RR}(x)} e^{-ieV_R x} \right), \quad (\text{D2})$$

$$\begin{aligned} \delta I_{LR} &= 2e \text{Im} \int dx \int dy_1 dy_2 dz_1 dz_2 [\rho_0(y_2 - y_1) h_0(z_1 - z_2) + \rho_0(y_1 - y_2) h_0(z_2 - z_1)] \\ &\quad \times \mathcal{F}_{LR}(y_1, y_2, z_1, z_2) e^{-i[eV_R(y_1 - y_2) + eV_L(z_1 - z_2)]} \text{Tr} \left\{ \hat{r}^\dagger(x + z_1) \hat{t}'(x + y_1) \right. \\ &\quad \left. \times [(K_{LR}(-y_1) + K_{LL}(-z_1)) \hat{t}^\dagger(y_2) \hat{r}(z_2) - (K_{RR}(-y_1) + K_{RL}(-z_1)) \hat{r}'^\dagger(y_2) \hat{t}(z_2)] \right\}, \end{aligned} \quad (\text{D3})$$

$$\begin{aligned} \delta I_{LL} &= -2e \text{Im} \int dx \int dy_1 dy_2 dz_1 dz_2 \rho_0(y_1 - y_2) h_0(z_2 - z_1) \mathcal{F}_{LL}(y_1, y_2, z_1, z_2) e^{i[eV_L(y_1 - y_2 - z_1 + z_2)]} \\ &\quad \times \text{Tr} \left\{ \hat{t}^\dagger(x + z_1) \hat{t}(x + y_1) [(K_{LL}(-y_1) - K_{LL}(-z_1)) (\delta(y_2) \delta(z_2) \hat{1} - \hat{r}^\dagger(y_2) \hat{r}(z_2)) \right. \\ &\quad \left. + (K_{RL}(-y_1) - K_{RL}(-z_1)) \hat{t}^\dagger(y_2) \hat{t}(z_2)] \right\}, \end{aligned} \quad (\text{D4})$$

$$\begin{aligned} \delta I_{RR} &= 2e \text{Im} \int dx \int dy_1 dy_2 dz_1 dz_2 \rho_0(y_1 - y_2) h_0(z_2 - z_1) \mathcal{F}_{RR}(y_1, y_2, z_1, z_2) e^{-i[eV_R(y_1 - y_2 - z_1 + z_2)]} \\ &\quad \times \text{Tr} \left\{ \hat{t}^\dagger(x + z_1) \hat{t}'(x + y_1) [(K_{RR}(-y_1) - K_{RR}(-z_1)) (\delta(y_2) \delta(z_2) \hat{1} - \hat{r}'^\dagger(y_2) \hat{r}'(z_2)) \right. \\ &\quad \left. + (K_{LR}(-y_1) - K_{LR}(-z_1)) \hat{t}^\dagger(y_2) \hat{t}'(z_2)] \right\}. \end{aligned} \quad (\text{D5})$$

Here we defined

$$\mathcal{F}_{LR}(y_1, y_2, z_1, z_2) = e^{-F_{RR}(y_1 - y_2) - F_{LL}(z_1 - z_2) + F_{LR}(y_1 - z_1) + F_{LR}(y_2 - z_2) - F_{LR}(y_1 - z_2) - F_{LR}(y_2 - z_1)}, \quad (\text{D6})$$

$$\mathcal{F}_{LL}(y_1, y_2, z_1, z_2) = e^{-F_{LL}(y_1 - y_2) - F_{LL}(z_1 - z_2) - F_{LL}(y_1 - z_1) - F_{LL}(y_2 - z_2) + F_{LL}(y_1 - z_2) + F_{LL}(y_2 - z_1)}, \quad (\text{D7})$$

$$\begin{aligned} F_{ij}(x) &= \int \frac{d\omega}{2\pi} \text{Im} K_{ij}(\omega) \coth \frac{\omega}{2T} (1 - \cos \omega x) \\ &\quad + \int \frac{d\omega}{(2\pi)^2} \left(\sum_{\pm} (\omega \pm eV) \coth \frac{\omega \pm eV}{2T} - 2\omega \coth \frac{\omega}{2T} \right) (1 - \cos \omega x) \sum_{k,n=L,R} K_{ik}^*(\omega) \delta v_{kn}(\omega) K_{nj}(\omega). \end{aligned} \quad (\text{D8})$$

$\delta v_{kn}(\omega)$ are defined in Eq. (29). The function \mathcal{F}_{RR} is obtained by substituting $F_{LL}(x) \rightarrow F_{RR}(x)$ into Eq. (D7).

- ¹ M. Büttiker, Phys. Rev. B **46**, 12485 (1992).
² C.W.J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).
³ Ya.M. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).
⁴ L.S. Levitov, H. Lee, and G.B. Lesovik, J. Math. Phys. **37**, 4845 (1996).
⁵ Y. Meir and N.S. Wingreen, Phys. Rev. Lett. **68**, 2512 (1992).
⁶ D.S. Golubev and A.D. Zaikin, Phys. Rev. B **59**, 9195 (1999).
⁷ A. Kamenev and A. Andreev, Phys. Rev. B **60**, 2218 (1999).
⁸ Yu.V. Nazarov, Phys. Rev. Lett. **82**, 1245 (1999).
⁹ D.S. Golubev and A.D. Zaikin, Phys. Rev. Lett. **86**, 4887 (2001).
¹⁰ A.V. Galaktionov, D.S. Golubev and A.D. Zaikin, cond-mat/0212494, to appear in Phys. Rev. B (2003).
¹¹ S.V. Panyukov and A.D. Zaikin, Phys. Rev. Lett. **67**, 3168 (1991).
¹² X. Wang and H. Grabert, Phys. Rev. B **53**, 12621 (1996).
¹³ K.A. Matveev, Phys. Rev. B **51**, 1743 (1995).
¹⁴ A. Kamenev, Phys. Rev. Lett. **85**, 4160 (2000).
¹⁵ D.S. Golubev and A.D. Zaikin, Europhys. Lett. **60**, 113 (2002).
¹⁶ A. Levy Yeyati, A. Martin-Rodero, D. Esteve, and C. Urbina, Phys. Rev. Lett. **87**, 046802 (2001).
¹⁷ A.A. Odintsov, Sov. Phys. JETP, **67**, 1265 (1988); D.S. Golubev, and A.D. Zaikin, Phys. Rev. B **46**, 10903 (1992); JETP Lett. **63**, 1007 (1996).
¹⁸ D.S. Golubev, J. König, H. Schoeller, G. Schön, and A.D. Zaikin, Phys. Rev. B **56**, 15782 (1997); G. Göppert and H. Grabert *ibid.*, **58**, R10155 (1998).

- ¹⁹ D.S. Golubev, and A.D. Zaikin, Phys. Lett. **A169**, 475 (1992); G. Göppert and H. Grabert, Eur. Phys. J. B **16**, 687 (2000); Sh. Farhangfar, R.S. Poikolainen, J.P. Pekola, D.S. Golubev, and A.D. Zaikin, Phys. Rev. B **63**, 075309 (2001).
- ²⁰ K.B. Efetov and A. Tschersich, Europhys. Lett. **59**, 114 (2002); I.S. Beloborodov, K.B. Efetov, A.V. Lopatin, and V.M. Vinokur, cond-mat/0304448; A. Altland, L.I. Glazman, and A. Kamenev, cond-mat/0305246.
- ²¹ M. Kinderman and Yu.V. Nazarov, cond-mat/0304078; M. Kinderman, Yu.V. Nazarov, and C.W.J. Beenakker, cond-mat/0306375.
- ²² D.A. Bagrets and Yu.V. Nazarov, cond-mat/0304339.
- ²³ Within this approach an effective energy dependence of (renormalized) transmissions T_n can only be generated by electron-electron interactions^{21,22}.
- ²⁴ I.L. Aleiner, P.W. Brouwer, and L.I. Glazman, Phys. Rep. **358**, 309 (2002).
- ²⁵ G. Schön and A.D. Zaikin, Phys. Rep. **198**, 237 (1990).
- ²⁶ P.W. Brouwer and C.W.J. Beenakker, J. Math. Phys. **37**, 4904 (1996).
- ²⁷ P.W. Brouwer and M. Büttiker, Europhys. Lett. **37**, 441 (1997).
- ²⁸ M.L. Polianski and P.W. Brouwer, cond-mat/0208408.
- ²⁹ K.E. Nagaev, P. Samuelsson, and S. Pilgram, Phys. Rev. B **66**, 195318 (2002); D.A. Bagrets and Yu.V. Nazarov, cond-mat/0301505.
- ³⁰ P. Schwab and R. Raimondi, Eur. Phys. J. B **30**, 5 (2002).
- ³¹ H. Weber, R. Häussler, H. von Löhneysen, and J. Kroha, Phys. Rev. B **63**, 165426 (2001).
- ³² H. Weber, private communication.
- ³³ L. Langer, V. Bayot, E. Grivei, J.-P. Issi, J. P. Heremans, C.H. Olk, L. Stockman, C. Van Haesendonck, and Y. Bruynseraede Phys. Rev. Lett. **76**, 479 (1996).
- ³⁴ C. Schönenberger, A. Bachtold, C. Strunk, J.-P. Salvetat, and L. Forro, Appl. Phys. **A 69**, 283 (1999).
- ³⁵ K. Liu, S. Roth, G.S. Düsberg, G.T. Kim, D. Popa, K. Mukhopadhyay, R. Doome, and J.B. Nagy, Phys. Rev. B **61**, 2375 (2000).
- ³⁶ The authors³⁴ have fitted their data to the power law $G \propto T^p$ with $p \sim 0.1 \div 0.2$. In the temperature range of about one decade such a dependence can hardly be distinguished from a logarithmic one.
- ³⁷ R. Tarkiainen, M. Ahlskog, L. Roschier, P. Hakonen, and M. Paalanen, in: *Electronic Correlations: From Meso- to Nano-Physics*, ed. by T. Martin, G. Montambaux and J. Tran Thanh Van, EDP Sciences, 2001, p. 437.
- ³⁸ R. Tarkiainen, M. Ahlskog, A. Zyuzin, P. Hakonen, and M. Paalanen, preprint. We are grateful to M. Paalanen and P. Hakonen for communicating the draft of this paper to us prior to its publication.
- ³⁹ The same result follows if one repeats the calculation¹⁹ for an array of $M \times N$ ($N \gg 1$) independent coherent scatterers with the Fano factor $\tilde{\beta}$.
- ⁴⁰ R.A. Webb, P. Mohanty, and E.M.Q. Jariwala, Fortsch. Phys. **46**, 779 (1998).
- ⁴¹ Following the authors³⁸ we assume that electrons are mainly conducted by the outer shell of the nanotube. In this case one can define $M = L_{\text{tr}}/L_s$ and $L_{\text{tr}} = \pi d$, where d is the nanotube diameter.
- ⁴² D.S. Fisher and P.A. Lee, Phys. Rev. B **23**, 6851 (1981).
- ⁴³ We have also verified our general arguments by directly calculating *all* the contributions to the effective action. This calculation also demonstrates that in the long time limit our results are insensitive to the terms omitted here.