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Adjungierte Sensitivitätsanalyseverfahren für Markovketten mit Anwendung auf Zuverlässigkeit von IFMIF Beschleuniger-System Anlagen

Kurzfassung

Das Markovketten Verfahren und sein mathematisches Modell wurde über Jahre als ein leistungsfähiges Werkzeug benutzt, um die Entwicklung und das Verhalten physikalischer Systeme zu analysieren. Der Grad der Abstraktion des physikalischen Systems, die im mathematischen Modell verwendeten statistischen Daten, sowie die numerischen Näherungen zur Lösung der Gleichungen sind nur einige Quellen von Unsicherheit, die in den Zuverlässigkeitsresultaten enthalten sind.
Durch Verwenden der Sensitivitätsanalyse kann der Einfluss von Unsicherheitsdaten in Systembauteilen auf das Gesamtverhalten der Systemzuverlässigkeit analysiert werden und die Schwachpunkte im Modell können identifiziert werden. Mit den Ergebnissen der Sensitivitätsanalyse erhält man den Vertrauensgrad der Zuverlässigkeitsresultate. Somit können neue Verbesserungen oder Neuentwürfe des physikalischen Systems durchgeführt werden.

Diese Arbeit stellt die Implementierung der Adjungierten Sensitivitätsanalyseverfahren (*Adjoint Sensitivity Analysis Procedure* - ASAP) für die *Continuous Time Discrete Space Markovkette* (CTMC) als eine Alternative zu anderen rechenintensiven Methoden dar. Um dieses Verfahren als Endprodukt in Zuverlässigkeitsstudien zu entwickeln, wird die Zuverlässigkeit der physikalischen Systeme mit einer gekoppelten Fehlerbaum-Markovketten Technik analysiert, d.h. die Abstraktion des physikalischen Systems erfolgt, indem als Schnittstelle der oberen Ebene ein Fehlerbaum benutzt wird, der danach automatisch in eine Markovkette umgewandelt wird. Die resultierenden Differenzialgleichungen, die auf Markovkettenmodellen basieren, werden danach gelöst, um die Systemzuverlässigkeit zu erhalten. Weitere Sensitivitätsanalysen mit ASAP, die auf die CTMC Gleichungen angewendet werden, werden genutzt, um den Einfluss von Änderungen in den Eingabedaten auf das Zuverlässigkeitsmaß zu erkennen und das Vertrauen in die abschließenden Zuverlässigkeitsresultate zu erhalten.

Die Methoden zum Erzeugen der Markovketten und der ASAP für die Markovkettengleichung sind in dem neuen Computercodesystem QUEFT/MARKOMAG-S/MCADJSEN für Zuverlässigkeit und Sensitivitätsanalyse von physikalischen Systemen eingeführt worden. Die Validierung dieses Codesystems wurde mit einfachen Problemen durchgeführt, für die es analytische Lösungen gibt. Typische Sensitivitätsresultate zeigen, dass die mit ASAP erzielten numerischen Lösungen widerstandsfähig, stabil und genau sind. Die Methode und das Codesystem, die während dieser Arbeit entwickelt wurden, können als ein leistungsfähiges und stabiles Werkzeug weiter genutzt werden, um die Sensitivität von Zuverlässigkeitsmaßen für jedes physikalische System mit Markovketten zu analysieren.

Zuverlässigkeit und Sensitivitätsanalyse sind für die IFMIF Beschleuniger-System Anlagen mit diesen Methoden während dieser Arbeit durchgeführt worden. Die Zuverlässigkeitsresultate werden um die Verfügbarkeit der Hauptuntersysteme dieses komplizierten physikalischen Systems während einer typischen Missionszeit konzentriert.
Die Sensitivität Studien für zwei typische Antworten sind mit ASAP durchgeführt worden. Der Vergleich der Sensitivitätsresultate zwischen ASAP und den klassischen Methoden zeigt eine gute Übereinstimmung, aber mit dem Vorteil der Berechnungszeit im Fall von ASAP.

Abstract

The Markov chain technique and its mathematical model have been demonstrated over years to be a powerful tool to analyze the evolution and performance of physical systems. The degree level of abstraction for the physical system, the statistical data used in the mathematical model, the numerical approximations used to solve the equations, are only some sources of uncertainties in reliability results. By applying the sensitivity analysis, the influence of uncertainty data in system components to the overall behavior of the system reliability can be analyzed and the weak points in the model can be identified. Using the sensitivity results the confidence level of reliability results is obtained. Thus, new improvements or redesigning of the physical system can be performed.

This work presents the implementation of the Adjoint Sensitivity Analysis Procedure (ASAP) for the Continuous Time, Discrete Space Markov chains (CTMC), as an alternative to the other computational expensive methods. In order to develop this procedure as an end product in reliability studies, the reliability of the physical systems is analyzed using a coupled Fault-Tree – Markov chain technique, i.e. the abstraction of the physical system is performed using as the high level interface the Fault-Tree and afterwards this one is automatically converted into a Markov chain. The resulting differential equations based on the Markov chain model are solved in order to evaluate the system reliability. Further sensitivity analyses using ASAP applied to CTMC equations are performed to study the influence of uncertainties in input data to the reliability measures and to get the confidence in the final reliability results.

The methods to generate the Markov chain and the ASAP for the Markov chain equations have been implemented into the new computer code system QUEFT/MARKOMAG-S/MCADJSEN for reliability and sensitivity analysis of physical systems. The validation of this code system has been carried out by using simple problems for which analytical solutions can be obtained. Typical sensitivity results show that the numerical solution using ASAP is robust, stable and accurate. The method and the code system developed during this work can be used further as an efficient and flexible tool to evaluate the sensitivities of reliability measures for any physical system analyzed using the Markov chain.

Reliability and sensitivity analyses using these methods have been performed during this work for the IFMIF Accelerator System Facilities. The reliability studies using Markov chain have been concentrated around the availability of the main subsystems of this complex physical system for a typical mission time.
The sensitivity studies for two typical responses using ASAP have been performed. The results given by ASAP with those obtained using the classical methods have been compared, showing a good agreement but with the advantage of computational time in the case of ASAP.

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List of Abbreviations

ASAP	- Adjoint Sensitivity Analysis Procedure
ASE	- Adjoint Sensitivity Equations
BDD	- Binary Decision Diagram
BDF	- Backward Differential Formula
BKE	- Backward Kolmogorov Equations
CTMC	- Continuous-Time Discrete-Space Markov Chain
DASAP	- Discrete Adjoint Sensitivity Analysis Procedure
DASE	- Discrete Adjoint Sensitivity Equations
DDR	- Discrete response's sensitivity
DFSE	- Discrete Forward Sensitivity Equations
DR	- Response's sensitivity
DTL	- Drift Tube LINAC
DTMC	- Discrete-Time Discrete-Space Markov Chain
FSAP	- Forward Sensitivity Analysis Procedure
FKE	- Forward Kolmogorov Equations
FSE	- Forward Sensitivity System
FT	- Fault Tree
GMRES	- Generalized Minimum Residual method
HEBT	- High Energy Beam Transport system
HV	- High Voltage
IFMIF	- International Fusion Materials Irradiation Facility
LEBT	- Low Energy Beam Transport system
LINAC	- Linear Accelerator
LP	- Low Pressure
MTBF	- Mean Time Between Failures
MTTF	- Mean Time To Failure
MTTR	- Mean Time To Repair
ODE	- Ordinary Differential Equations
R	- System's response
RF	- Radio Frequency
RFQ	- Radio Frequency Quadrupole
TMVP	- Turbo Mechanical Vacuum Pump

1 Introduction

Reliability analysis of physical systems is widely applied today in many engineering fields including energy, aerospace, automotive, chemical processing, and even in project planning and financial management. The goals and sizes of this analysis vary from sector to sector and many methods have been developed and standardized over years. Nowadays these methods are growing in sophistication and are trying to depict as much as possible the real situations. The computer codes that perform this complex analysis have known a rapid development in the last decade as well.

Despite of all these methods and their complexity in analysis, all of them are using *a priori* statistical/experimental data, which are not error free. Experience has shown that no measurement, however carefully made, can be completely free of uncertainties.[1] These uncertainties are trying to be minimized, but even in these conditions they have to be considered in system reliability evaluation. These input data unfortunately are not the only source of uncertainties, even the methods themselves can be a source of uncertainty since they are an abstraction of the real-world problem. The numerical methods used for solving the mathematics behind of these methods are also affected by numerical errors. Other sources of uncertainties are due to not all parameters influencing the system are considered in analysis because of more or less, insufficient information, or that the investigated phenomena is not completely known or understood,[2] etc.

Based on these uncertainties, at the end of reliability evaluation, further questions arise. What confidence can one have that the numerical results produced by the reliability model are correct? What are the implications of these uncertainties to the reliability results? How these results are changing with the changing in input parameters? Answers to these questions are provided by sensitivity and uncertainty analysis.

The results of sensitivity analysis are used further for focusing on areas which need greater reliability and quality control and more careful design considerations. Based on sensitivity data, one can formulate system improvement suggestions and to reassess system reliability assuming the incorporation of the suggestions to support simulation studies, to guide or validate the establishment of cost-effective performance criteria and test methods.[3] But as one can see, the sensitivity analysis is applied at the very end and, therefore, one should perform before the reliability analysis of the physical system.

The study of reliability for a physical system is based on its behavior during operation to the various conditions, external or internal, dependent or independent by itself. Therefore, the mathematical model that describes this evolution should be a dynamical model rather than a static one. The mathematical model used in reliability analysis to depict dynamically the evolution of the physical system is the Markov chain. In the past years, it has been demonstrated the applicability of the Markov chains for the reliability analysis for real-world problems in various branches of engineering.[4-8]

Markov chains have also shown themselves to be a valuable analyses tool in a variety of other branches of science as from economic models, population forecasting, biology, etc., to financial planning. They have been and continue to be the method of choice for modeling many other systems.[9]

Over years, once the number of applications using Markov chains has grown up, and the dynamic reliability analysis has started to be used more and more, some useful computer packages has been developed[10-15] into the academic area and applied in various engineering fields such as aerospace, computing, and nuclear power plants. The commercial packages with Markov modeling capabilities[16-19] have known also a noticeable development in the last decade and they are used now extensively in many branches of industry, from cell phones, computers, communication and networks, health equipment, etc. until aviation, aerospace, military and nuclear sectors, to analyse and predict system reliability and availability.

Markov chains are used in reliability analyses whenever statistical dependences among failures and repairs or both must be considered. The mathematical model of Markov chain comprises independent and dependent variables, input parameters, and the relationship among these quantities through a set of ordinary differential equations. The input parameters are not known precisely, but may vary within some ranges that reflect the incomplete knowledge or uncertainty regarding them. The numerical methods used to solve the Markov chain set of equations introduce themselves numerical errors.[20-23] The effects of such errors and parameter variations must be quantified in order to assess the reliability range validity. This quantification is made by *sensitivity analysis*. Afterwards, the rank of parameters importance in affecting the reliability measure analyzed is performed based on the sensitivity results, the larger sensitivities the bigger influence on the final results. Further, the effect of parameter uncertainties to the uncertainty about computed system reliability is performed by *uncertainty analysis*[24, 25] using the previously computed sensitivities. Therefore, the components whose parameter's uncertainties give large sensitivities will have severe impact on the system reliability and its uncertainty.

The sensitivity analysis usually implies the derivatives of the reliability function with respect to system parameters. In reliability theory, the sensitivity analysis has been associated with the importance analysis and has been defined in the context with the combinatorial reliability models as a way to assess the relative importance of a component to the reliability of a system. Several measures of importance have been defined, but these measures often give counterintuitive or inconsistent results and they have generally fallen out of favor.[4, 26, 27] None of these methods does imply partial derivatives in their evaluation. The oldest proposed (Birnbaum, 1969)[28] and the only measure of importance for components in reliability analysis which is defined as partial derivative of system reliability function with respect to component reliability (the classical sensitivity analysis) is Birnbaum structural importance.[4, 26, 28]

Sensitivity studies on Markov chains using this classical approach have been performed emphasizing the difficulties that arise in the cases of systems with many components as it exists in real situations. The practical utility of such analyses has been pointed as well. During these studies has been concluded that the classical sensitivity analysis is impractical for large systems. Therefore, the approximate methods has been proposed and developed to avoid this drawback.[27] An overview of these studies will be presented into the next section.

In early eighties, Cacuci has developed a deterministic sensitivity analysis theory based on adjoint operators avoiding the disadvantages which the classical methods imply.[29, 30] This theory also known as the Adjoint Sensitivity Analysis Procedure (ASAP), has not been considered until now on sensitivity studies in reliability engineering.

1.1 Background and previous work

In this section an overview of previous work on sensitivity studies and approaches using Markov chains in reliability engineering is presented.

Markov chain represents an analytical model which is widely used for reliability studies of complex systems, to predict analytically or numerically measures as reliability, availability, or performance. As the systems become more complex, this technique becomes more difficult to apply. Simulation using Monte Carlo technique[31, 32] can be used to perform reliability analysis. A representation of the physical system with its relevant parts is constructed and a series of random events to which the system must respond is generated. Observations are made of the system reaction to the events. The method is powerful for evaluating reliability measures because the system representation can be made to virtually any level of detail. But the main drawback of this method is that the whole process of simulation should be repeated numerous times in order to obtain a statistically significant number of trials at which the simulated system behavior to be close with the behavior of the real system. If each trial requires an appreciable amount of computing time, then the total executing time may be excessive.[33, 34] Therefore, the simulation method is further quite expensive for sensitivity analysis. The computation time required for sensitivity studies using simulation (Markov Chain Monte Carlo)[9, 31] increase prohibitive especially when numerical difficulties occur, as the stiffness of the Markov chain,[34] which usually appear in simulation of high reliable systems. Therefore, the new analytical approaches have been proposed for sensitivity analysis using the mathematical model of Markov chain, rather than simulation.

The Markov chain models the physical system as a set of states in which the system can be during its life period and the transitions which can occur between these states. A state is a unique configuration of failed and operational components or subsystems. Based on these states and transitions a set of ordinary differential equations is defined. Considering the state space and the system evolution time, the Markov chain can be theoretically of the next four types:

- discrete-time discrete-state Markov chain,
- discrete-time continuous-state Markov chain,
- continuous-time discrete-state Markov chain,
- continuous-time continuous-state Markov chain.

It should be mentioned here that the cases in which the state space of Markov chain are continuous, are called Markov processes and this term is generally used. The term Markov chain is used for the Markov processes in which the state space is discrete. In reliability engineering, the state space is considered as a countable finite space in which the system can be only in one of these states once at a time. Thus, it is usually met in literature the Markov chain as a discrete time Markov chain (DTMC), or as continuous time Markov chain (CTMC), for the 1st and the 3rd type, respectively. DTMC only can make transitions from one state to another at discrete specified intervals, while CTMC can change states at any time.[7, 9]

Reliability analysis of complex systems is performed using CTMC, and has been widely used to predict the performance and reliability of a variety of systems. Solving the CTMC system of differential equations it is obtained the transient probabilities of the states in which the system can be during its evolution. Using these probabilities the measures of interest as availability, reliability, performability, mean time to failure, are obtained. Further, the sensitivity studies are performed to analyze the effect of uncertainties in input parameters to these measures. These measures of interest, i.e. the reliability functions, are named generically in the sensitivity theory the system responses.

Since the importance concept of components in reliability engineering has been proposed in 1969 by Birnbaum[28] in the way of classical sensitivity analysis which implies the derivatives of reliability function with respect to input parameters of components (e.g. mean time to failure/repair, failure rates), the subject has been treated poorly in literature until the middle of eighties. An alternative approach is to solve the model once with all its parameters considered at the nominal value, and then to repeat the all calculations modifying the input parameters one at a time to see the effect in results. For models which imply many parameters this method is time consuming.

Due to the wide use of Markov chains to model performance and reliability of the physical systems, the sensitivity analysis of Markov chains started to become a field of interest for scientists in reliability engineering, especially into electronic and computer science, at the middle of eighties. Early numerical sensitivities studies on stationary probabilities of Markov chains have been performed by Golub et al.[35] and Stewart et al.[36] Monte Carlo simulation has been used in the same time for sensitivity analysis on stationary and transient distribution of Markov chain by Glynn[37] and Reiman et al.[38] and has been proved to be a computational expensive alternative. Therefore, further numerical solutions to solve the Markov chain and the sensitivities have been proposed. Heidelberger and Goyal[39] performed sensitivities studies on transient distribution of CTMC using a numerical method based on uniformization technique. The method to compute the exact solution of the sensitivities measures directly from Markov Chain is based on that from Frank.[40] The uniformization technique is based on an infinite series representation of the transient distribution. The advantages of this iterative technique on Markov chains has been demonstrated and extended to the computation of derivative of the transient distribution with respect to the input parameters. Numerical example on a fault-tolerant database system has been performed using the SAVE[14] package and has been illustrated the need of the sensitivity analysis in reliability study of the system. The optimization of the system using sensitivity results is different depending on the objective, i.e. system reliability or availability.

Blake et al.[41] have discussed the extension of the Markov chains to include parametric sensitivity analysis for reliability and performability of multiprocessor systems. They performed reliability studies using Markov chains and then sensitivity analysis on the influence of input parameters to some measures of interest as mean time to failure (*MTTF*). They made a comparison of numerical methods as uniformization and Runge-Kutta for solving the original Markov chain and the system sensitivities as well as the number of computation operation that each method implies. There has been emphasized the degradation of the numerical methods performance for stiff problems which are common in systems with repair or reconfiguration. Alternative to such numerical problems has been proposed.[23] Further applications of the sensitivity results has been enumerated as to provide error bounds on the solution when are given bounds on the input parameters (uncertainty analysis), to identify the portions of the model that need refinement, to optimize the system, e.g. to maximize the MTTF with minimal cost, etc.

To the standard uniformization method used to compute the sensitivities of reliability function,[39, 41] Abdallah[34] proposed a new method called the uniformization power with the aim to reduce the computation time for stiff cases. He performed sensitivities studies using this method for stiff problems highlighting the advantage of uniformization power approach from point of view of computational cost. But he also pointed that for the cases when the Markov chain is non-stiff the sensitivities are computed more efficient using the standard uniformization method.

The Markov chain for a complex system is often large and complex and its construction is difficult. Hence high level interfaces which can be used to describe the system have been developed. The underlying Markov chain is then automatically generated from this description. As high level interfaces have been used the combinatorial techniques used in reliability as Block Diagram (BD), Fault-Tree (FT),[42-47] and Stochastic Petri Nets (SPN).[7] The dependability between components/events is described using one of these techniques and afterwards the Markov chain is generated and solved. Conversion algorithms between these combinatorial models and Markov chain are depicted in literature.[33, 48, 49] These methods have been implemented in computer packages as SAVE[14] which uses as input BD, HARP[10] and Galileo[50, 51] which use FT as input, SPNP[52] which uses as input SPN, HIMAP[12] which uses as input either FT, or SPN, or SHARPE[13] which uses as input BD, FT, SPN.

Muppala and Trivedi[53] have extended the sensitivity analysis to Generalized SPN (GSPN)[7] and implemented the method in SPNP[52] package. Their extension is made on the sensitivity analysis when are changing in the structure of the GSPN which is used further to generate the Markov chain. The uniformization method proposed by Heidelberger et al.[39] is used to obtain the sensitivities of CTMC and DTMC. Sensitivity studies using as high level interface Deterministic and Stochastic Petri Nets (DSPN) for scheduled maintenance systems has been performed by Choi et al.[54] for steady-state distribution and Bondavalli et al.[55] for transient distribution.

Boyd[33] performed sensitivity studies using HARP package using the recalculation method on fault tolerant hypercube computer architecture. Further sensitivities studies have been performed by Meyer[56] for stationary distribution of Markov chain and Ramesh and Trivedi[57] for transient distribution. They focused in principal on the bounds of different sensitivities measures, emphasizing the usefulness of the sensitivity results in assessing the effects of stiffness on the Markov chain that is characteristic in reliability studies of high reliable systems with repairs.

Sensitivity and uncertainty analysis using both Monte-Carlo simulation and Taylor-series method have been made by Haverkort et al.[58] for stationary solution and Papazoglou et al.[59] and Yin et al.[60] for transient distribution of Markov chain. To the limitation of the simulation due the intensive computation are proposed the analytical approaches which are faster and cheaper from computational point of view. Yet, the analytical methods are limited to few models and some constrains in input parameters.

As it can be seen, all the sensitivities studies proposed refined numerical algorithms to simplify the computation due the increasing size of the problem.

For studying the sensitivity of the steady-state distribution of CTMC, Cao et al.[61] proposed a new deterministic method based on the idea that the effect of parameter change can be decomposed into a sum of the effects of many individual paths of Markov chain. They introduced two concepts, namely realization factor and performance potential for Markov chain, and based on either two quantities they estimated the sensitivity of steady state solution of the Markov chain. They proposed further algorithms for sensitivity studies on steady state performance of Markov chain based on these concepts.[62]

To avoid the typical approach of the sensitivity analysis of Markov chain[40] which consists in solving a set of ordinary differential equations for the transient solution of state probabilities and a much larger set of differential equations for sensitivities, Ou and Dugan[27, 63] proposed an approximate approach to estimate the sensitivity without increasing the size of the problem, using the solution of the original Markov chain. Their approach consists in definition of state subsets according to the failure or operational status of the system and a specific component whose importance is studied. This differentiation is made during the Markov chain generation from a Dynamic Fault Tree. Different ways to compute the sensitivities for components depending on their dependencies (individual components or spare components) are provided. The approximate sensitivities are based only on the solution of the original Markov chain with the reliability functions obtained using the previous defined subset of states. They implemented the method into Galileo[50, 64] package and compared the numerical results with the exact solution which is based on recalculations.

From this survey on the sensitivity analysis of Markov chains, one can see that the method using adjoint operators originally developed by Cacuci[29,30] has not been considered in reliability studies yet, and represents the main purpose of this work.

1.2 Goals of this work

The goal of this work is the applicability and implementation of the Adjoint Sensitivity Analysis to Markov chains with the purpose to perform sensitivity studies using this approach, emphasizing the advantages of this method over the others. The role of the sensitivity analysis during the reliability studies is well known. Analysis of the sensitivity results will help to improve and optimize the physical systems to achieve the required reliability, to reduce the costs and to establish maintenance policies. Also the sensitivities can be used further for uncertainty studies and therefore to establish the confidence level of reliability results.

One necessary step in achieving this goal is to develop a methodology and code system for performing such an analysis. The Adjoint Sensitivity Analysis Procedure together with the Markov chain approach for reliability analyses purposes has been implemented into a stand-alone computer code system , i.e. QUEFT/MARKOMAG-S/MCADJSEN, and applied further on a real-world problem, namely on IFMIF Accelerator System Facilities. The numerical results on sensitivity analysis using the new code-system based on the theory developed in this work can be used further as basis for new uncertainty studies and optimizations, cost reduction, and maintenance policies, of this complex system.

Chapter 2 presents the mathematical model of Markov chain and the concepts behind it. The Kolmogorov system of ordinary differential equations and the properties of this system are described. Due to problem complexity the automated generation of this system is performed based on abstraction of the physical system in terms of Markov chain. Difficulties to generate straightforward this system of equations are emphasized and alternatives to generate automatically the Markov chain equations using combinatorial models are presented. A simple algorithm implemented to generate the Markov chain is described and an example of its application on a simple problem is performed. In close of this section the reliability measures and the equivalent relations among them are presented as well.

In Chapter 3, the main aspects of the local adjoint sensitivity applied to differential equations which describes the Markov chain are presented. As it is known the sensitivity theory based on adjoint functions developed by Cacuci[29, 30] comprises two aspects in development, namely the Forward Sensitivity Analysis Procedure (FSAP), and the Adjoint Sensitivity Analysis Procedure (ASAP). The first two sections highlight with FSAP and ASAP applied to Kolmogorov differential equations. It is emphasized that the FSAP should be used for the less usual situations when the number of perturbations in the considered problem is smaller than the number of measures of interest (responses) for sensitivity analysis. On the other hand, in the more common real situations when the number of perturbations exceeds the number of responses for sensitivity analysis, ASAP must be used since from the view of computational cost this procedure is the only practical way to perform a complete and systematic sensitivity analysis for the reliability of complex systems. During the ASAP procedure the following fundamental characteristics have been highlighted, namely that the adjoint function are independent of perturbations in parameters, the adjoint functions must be computed again only if the system response is changed, the adjoint sensitivity system of equations can be solved independently using other numerical methods than the original differential system of equations, since it is linear in the adjoint functions, and the adjoint functions are dependent only on the base case solution which should be available before to solve the adjoint system.

These two procedures applied to Markov chain are exemplified on a simple problem of a binary component where it is possible to have an analytical solution, since for more complex cases the number of differential equations is growing exponentially with the number of components and an analytical solution is hard or cannot be obtained. The validation of numerical results with the analytical solution of this simple problem is performed for various perturbations in system parameters. Also a brief description of the methodology which has been followed using the computer code system developed is presented.

Chapter 4 presents typical results obtained using the developed code system QUEFT/MARKOMAG-S/MCADJSEN for transient reliability analysis of the IFMIF-Accelerator System Facilities for a specific mission time and comparison with the reliability results from literature which have been obtained using a different method than Markov chain. Further, for two types of responses the typical sensitivity results obtained using ASAP and the traditional approach using recalculations are presented to illustrate the verification of numerical solution. The parameters importance ranking is performed afterwards based on relative sensitivities of each type of response highlighting the possibility of changing priority of the components depending of the type of response.

In closing are presented conclusions of this work by highlighting further possible developments for future research.

2 Reliability analysis using Markov chain

As it has been mentioned in the previous chapter, in reliability engineering the discrete space Markov chain is used to perform studies of reliability, and availability for physical systems. In this chapter, the basic concepts and the mathematical description of Markov chain[7, 9, 20, 65, 66] are presented to prepare the field for the adjoint sensitivity analysis procedure which will be applied on Markov chain into the next chapter. Automatic generation of Markov chain considering the abstraction of the physical system in terms of Fault Tree and an algorithm to generate the Markov chain is presented as well. The reliability measures which can be computed using the results of Markov chain analysis are summarized at the end of the chapter.

The Markov chain has been largely treated in literature, any reliability book dedicating a special chapter to this subject. The basic concept behind of Markov chain is that of states and the transitions which are occur between these states. It is possible to represent the behavior of a physical system by describing all the different states the system may occupy by indicating how the system moves from one state to another in time. The states of Markov chain are classified in *transient*, *recurrent*, and *absorbing* states. A transient state is a state in which the system enters, leaves this state, and never returns during its evolution. A recurrent state is opposite, i.e. the system can return in that state during its evolution. An *absorbing* state is a state in which the system enters and never leaves that state.
The structure of the physical system consists in subsystems and components which are connected each other. These subsystems and components, depending on the requests, can be operational or failed. A state for the physical system is a unique combination of failed and operational components or subsystems. The components and subsystems are characterized by a probability or a frequency to be failed or operational. The transitions between states are defined by these probabilities or frequencies. A graphical representation of this behavior is usually made using a *state transition diagram* which illustrates the transitions from one state to another.

2.1 Mathematical representation of Markov chain

In this section the concepts behind of Markov chain and the mathematical equations which describe it are presented mainly as in Trivedi [1982, 2002][7], Stewart [1994][20], and Norris [1997][9].

The Markov chain is a stochastic process of random variables. The characteristic property of this kind of process is that it retains no memory where it has been in the past which means in other words that only the current state of the process influence where it goes next. This lack of memory property makes it possible to predict how a Markov chain may behave, and to compute the probabilities of that behavior. Mathematically, this can be expressed as follows.[6,7,9,20]

Considering $\{X(t), t \in T\}$ as a discrete set of random variables which denotes the states of the system at time *t*, *T* being the time range, for all integers *n* and for any sequence t_0, t_1,…, t_n such that $t_0 < t_1 < \ldots < t_n < t$, the conditional probability distribution of the Markov chain is:

$$P\{X(t) \le x \mid X(t_0) = x_0, X(t_1) = x_1, \ldots, X(t_n) = x_n\} = P\{X(t) \le x \mid X(t_n) = x_n\} \qquad (2.1)$$

Thus, the fact that the system was in state x_0 at time t_0, in state x_1 at time t_1, and so on, up to state x_{n-1} at time t_{n-1} is irrelevant. The state in which the system finds itself at time *t* depends only on where it was at time t_n. The state $X(t_n) = x_n$ contains all the relevant information regarding the history of the process. This does not imply that the transitions are not allowed to depend on the actual time at which they occur. The state space of a Markov chain is usually taken to be a set of natural integers {0, 1, 2,…} or a subset of it.

When the time range *T* is discrete, i.e. *T* = {0, 1, 2,…}, The Markov chain is called *discrete time Markov chain* (DTMC), otherwise the time range is considered continuous $T = \{t : 0 \le t \le +\infty\}$ and the Markov chain is a *continuous time Markov chain* (CTMC). When the transitions out of state *X*(*t*) depend on time *t*, the Markov chain is said to be *nonhomogeneous* and the transition probabilities are said to be *transient*, otherwise if they are independent of time the Markov chain is said to be *homogeneous* and the transition probabilities are *stationary*. For nonhomogeneous case, the time is considered from the start time since the system operates, i.e. the global time.

The transition probabilities have the following properties[7]

$$\begin{aligned} &a)\ p_X(x) \in [0,1], \text{ for all } x \in \mathbb{R} \\ &b)\ \sum_{\text{all } i} p_X(x_i) = 1 \end{aligned} \qquad (2.2)$$

To satisfy the memoryless property, the time spent in a state of a Markov chain (*the sojourn time*) must be independent of the time already spent in that state. This implies that the times between state transitions must be exponentially distributed for a homogeneous CTMC and geometrically distributed for DTMC. In practical situations, this restriction may not be hold for nonhomogeneous CTMC. That leads to a generalization of Markov chain, i.e. semi-Markov process,[7] where the distribution of time the process spends in a given state is allowed to be general. The difference between a nonhomogeneous CTMC, and a semi-Markov process is that the time dependent transitions are considered for a global time since the system began to operate in the first case, while the time dependent transitions are considered for a local time, time measured since the system entered into a current state, in the second case.

Taking in account whether it is considered the discrete or continuous time, the DTMC and CTMC are treated in similar way, with the peculiarities which are specific for each type.[7, 9, 20] This work focuses on the CTMC and this type of Markov chain is further analyzed.

Let $I = \{0, 1, 2, \ldots\}$ to denote the state space of the chain, and $T = [0, +\infty)$ be the time range. The conditional probability that a transition occurs from the state i at time $s \geq 0$ to the state j at time $t \geq s$, is denoted by

$$p_{ij}(s,t) = P\{X(t) = j \mid X(s) = i\} \tag{2.3}$$

where $i, j \in I$, $s, t \in T$, and,

$$p_{ij}(t,t) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \tag{2.4}$$

These transition probabilities depend on the time interval of length $\Delta t = t - s$, but not on s or t. It follows that for all values of Δt and i,

$$\sum_{\text{all } j} p_{ij}(\Delta t) = 1 \tag{2.5}$$

where has been used the simplified notation $p_{ij}(\Delta t) = p_{ij}(s,t)$.

In studying the Markov chain it is often required to determine the probability that the chain is in a given state at a particular time t, i.e. the state probabilities at time t. Let $\pi_j(t)$ to be the probability that the system is in state j at time t,

$$\pi_j(t) = P\{X(t) = j\}, \quad j = 0,1,2,\ldots; \quad t \geq 0 \tag{2.6}$$

Since at any given time the chain must be in one state, then for any $t \geq 0$,

$$\sum_{\text{all } j} \pi_j(t) = 1 \tag{2.7}$$

By using the theorem of total probability (i.e. $P(A)=\sum_{i=1}^{n} P(A \mid B_i)P(B_i),\ A, B$ events),[7] the state probability can be expressed in terms of transition probabilities:

$$\begin{aligned} \pi_j(t) &= P\{X(t)=j\} \\ &= \sum_{i\in I} P\{X(t)=j \mid X(s)=i\} P\{X(s)=i\} \\ &= \sum_{i\in I} p_{ij}(s,t)\pi_i(s) \end{aligned} \tag{2.8}$$

If the time s is considered as an initial time, i.e. $s = 0$, then

$$\pi_j(t)=\sum_{i\in I} p_{ij}(0,t)\pi_i(0) \tag{2.9}$$

Hence, the probabilistic behavior of a CTMC is completely determined once the transition probabilities $p_{ij}(s, t)$ and the initial probability vector $\pi(0)$ are specified.

The Chapman-Kolmogorov equations which describe the Markov chain may be obtained directly from the *Markov property* (2.1). They are specified by

$$p_{ij}(s,t)=\sum_{k\in I} p_{ik}(s,u)p_{kj}(u,t) \quad , \quad i, j\in I \quad , \quad 0\le s\le u\le t \tag{2.10}$$

That means that in passing from state i at time s to state j at time t, the chain must pass through some intermediate state k at some intermediate time u.

The direct use of the equation (2.9) is difficult. The transition probabilities are usually obtained by solving a system of differential equations that are derived next.[7, 20]

As it has been mentioned previous, the probability of a transition $p_{ij}(t, t+\Delta t)$ to occur from a given source state i to a destination state j in the interval of time $[t, t+\Delta t]$ depends on the length of interval of observation Δt. As the duration of this interval becomes very small (continuous-time), the probability to observe a transition also becomes very small. Thus, when $\Delta t \to 0$ the probability $p_{ij}(t,t+\Delta t)\to 0$, for $i \neq j$. From conservation of probability (2.2.b) follows that as $\Delta t \to 0$, $p_{jj}(t,t+\Delta t)\to 1$ (Eq.2.4). In reverse, when Δt becomes large the probability to observe a transition increase, and once Δt becomes larger and larger the probability to observe multiple transitions becomes non negligible. In CTMC the purpose is to ensure that the observation interval Δt is sufficient small that the probability to observe two or more transitions within this interval is negligible, instantaneously, i.e. the probability of observing multiple transitions is a function of Δt that tends to zero faster than Δt.

Further it is defined the *transition rate* which, unlike the transition probability, it does not depend on a time interval Δt. It is an *instantaneously* defined quantity that denotes the number of transitions that occur per unit time.

For each state *j* does exist a nonnegative function $q_{ii}(t)$ called transition rate and defined by

$$\begin{aligned} q_{ii}(t) &= \frac{d}{dt} p_{ii}(s,t)|_{s=t} \\ &= \lim_{\Delta t \to 0} \frac{p_{ii}(t,t+\Delta) - p_{ii}(t,t)}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{p_{ii}(t,t+\Delta t) - 1}{\Delta t} \end{aligned} \tag{2.11}$$

for $i = j$, and similarly, for each *i* and *j*, $i \neq j$, there is a nonnegative function $q_{ij}(t)$ known as the transition rate at which the transitions occur from state *i* to state *j* at time *t*, defined by

$$\begin{aligned} q_{ij}(t) &= \frac{d}{dt} p_{ij}(s,t)|_{s=t} \\ &= \lim_{\Delta t \to 0} \frac{p_{ij}(t,t+\Delta t) - p_{ij}(t,t)}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{p_{ij}(t,t+\Delta t)}{\Delta t} \end{aligned} \tag{2.12}$$

Given that the system is in a state *i* at time *t*, the probability that it will remains in state *i* must decrease with time, whereas the probability that it will transfer to a different state *j* increase with time. Thus, it is appropriate that the derivative at time *t* to be negative in the first case, and positive in the second.

Then, from Eqs.(2.11) and (2.12), the transition probabilities and the transitions rates are related to each other through

$$\begin{cases} p_{ij}(t,t+\Delta t) = q_{ij}(t)\Delta t + o(\Delta t) & i \neq j \\ p_{ii}(t,t+\Delta t) = 1 + q_{ii}(t)\Delta t + o(\Delta t) & i = j \end{cases} \tag{2.13}$$

where $o(\Delta t)$ is any function of Δt that tends to zero faster than Δt,

$$\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$$

Making the substitution $t \to t + \Delta t$ into Eq.(2.10) yields

$$p_{ij}(s,t+\Delta t) = \sum_{k \in I} p_{ik}(s,u) p_{kj}(u,t+\Delta t)$$

This implies

$$p_{ij}(s,t+\Delta t)-p_{ij}(s,t)=\sum_{k\in I}p_{ik}(s,u)p_{kj}(u,t+\Delta t)-\sum_{k\in I}p_{ik}(s,u)p_{kj}(u,t)$$

$$=\sum_{k\in I}p_{ik}(s,u)[p_{kj}(u,t+\Delta t)-p_{kj}(u,t)]$$

Dividing both sides by Δt and taking the limits as $\Delta t\to 0$ and $u\to t$, it is obtained the differential equations called the *Kolmogorov forward equations*:

$$\frac{\partial p_{ij}(s,t)}{\partial t}=\sum_{i,j,k\in I}p_{ik}(s,t)q_{kj}(t)$$
$$=\sum_{\substack{i,j,k\in I\\ k\neq j}}p_{ik}(s,t)q_{kj}(t)-p_{ij}(s,t)q_{jj}(t) \tag{2.14}$$

In a similar way are obtained the *Kolmogorov backward equations*:

$$\frac{\partial p_{ij}(s,t)}{\partial t}=\sum_{i,j,k\in I}p_{kj}(s,t)q_{ik}(t)$$
$$=\sum_{\substack{i,j,k\in I\\ k\neq i}}p_{kj}(s,t)q_{ik}(t)-p_{ij}(s,t)q_{ij}(t) \tag{2.15}$$

Written in matrix form, the Eqs.(2.14) and (2.15) are as follows,

$$\frac{\partial P(s,t)}{\partial t}=P(s,t)Q(t) \tag{2.16}$$

for forward equations, and

$$\frac{\partial P(s,t)}{\partial t}=Q(t)P(s,t) \tag{2.17}$$

for backward equations, where have been defined the matrices $P(s,t)=[p_{ij}(s,t)]$ and $Q(t)=[q_{ij}(t)]$. The $P(s,t)$ and $Q(t)$ are square matrices of dimension the number of states into the Markov chain. The $Q(t)$ matrix is called *infinitesimal generator matrix* or *transition rate matrix* and has the property that the diagonal entry $q_{ii}(t)$ is the negated sum of the other elements on row or column depending on either forward or backward form is used. Thus, the sum of elements for $Q(t)$ matrix on rows for forward equations, or on columns for backward equations is zero. This property of *transition rate matrix* is coming from the conservation of probability (i.e. $\sum_{i,j\in I}p_{ij}(t,t+\Delta t)=1$, Eq.(2.2.b)), (2.11), and (2.13), as follows

$$1-p_{ii}(t,t+\Delta t)=\sum_{\substack{i,j\in I\\ i\neq j}}p_{ij}(t,t+\Delta t)$$

$$=\sum_{\substack{i,j\in I\\ i\neq j}}q_{ij}(t)\Delta t+o(\Delta t)$$

Dividing by Δt, taking the limit as $\Delta t \to 0$, and considering (2.11), yields

$$\lim_{\Delta t \to 0} \frac{1 - p_{ii}(t, t + \Delta t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\sum_{\substack{i,j \in I \\ i \neq j}} q_{ij}(t)\Delta t + o(\Delta t)}{\Delta t}$$

$$q_{ii}(t) = -\sum_{\substack{i,j \in I \\ i \neq j}} q_{ij}(t) \tag{2.18}$$

For the state probability distribution (unconditional probability), using the Eqs.(2.8) and (2.14), it is obtained

$$\begin{aligned} \frac{d\pi_j(t)}{dt} &= \sum_{i,j \in I} \pi_i(t) q_{ij}(t) \\ &= \sum_{\substack{i,j \in I \\ i \neq j}} \pi_i(t) q_{ij}(t) - \pi_j(t) q_{jj}(t) \end{aligned} \tag{2.19}$$

and from Eqs.(2.8) and (2.15), respectively

$$\begin{aligned} \frac{d\pi_j(t)}{dt} &= \sum_{i,j \in I} \pi_i(t) q_{ji}(t) \\ &= \sum_{\substack{i,j \in I \\ i \neq j}} \pi_i(t) q_{ji}(t) - \pi_j(t) q_{ii}(t) \end{aligned} \tag{2.20}$$

Written in matrix notation, the above expression is as follows

$$\frac{d\Pi(t)}{dt} = \Pi(t) Q(t) \tag{2.21}$$

for forward form, and

$$\frac{d\Pi(t)}{dt} = Q(t) \Pi(t) \tag{2.22}$$

for backward form, respectively, where the state probabilities vector $\Pi(t) = [\pi_i(t)]$ has been defined as a row vector for forward equations, or as a column vector for backward equations, of dimension the number of states into the Markov chain. In the case of backward form, the elements on main diagonal of transition rate matrix have the property (2.18) as follows,

$$q_{ii}(t) = -\sum_{\substack{i,j \in I \\ i \neq j}} q_{ji}(t) \tag{2.23}$$

or in other words the transition rate matrix in forward form is transpose for the backward form.

The equations (2.16) or (2.17) are used when one wants specifically to show the initial state, while the equations (2.21) or (2.22) are used when the initial state is implied. These last two equations (2.21), and (2.22), are widely used in reliability engineering for reliability analysis of physical system, where the states of Markov chain are identified with the states of the physical system. Solving this set of ordinary differential equations it is obtained the transient solution of probability state vector which is used further to analyze the reliability, availability and performance of the physical system. The steady-state solution is used also and is obtained solving the linear system of equations defined by either (2.21) or (2.22) in which the left hand side is considered zero. For instance from Eq.(2.22) yields,

$$Q(t)\Pi(t) = 0 \tag{2.24}$$

where $\Pi(t) = [\pi_1(t), \pi_2(t), ...]^T$ is the steady-state probability vector. The conservation of probability (i.e. $\sum_{i \in I} \pi_i = 1$) is used together with the initial state vector to obtain a nonzero unique solution.

The measures of reliability and the mathematical expression to compute them using the solution of Markov chain will be presented into the next sections of this chapter.

The transient solution of CTMC solving the Kolmogorov set of ordinary differential equations has a closed-form only in the cases of very small CTMC or highly structured CTMC. In the most other cases the numerical methods are used. The general solution of backward Kolmogorov equations (2.22) (for forward case it is similarly), together with an initial vector of state probabilities

$$\Pi(t_0) = [\pi_1(t_0), \pi_2(t_0), ..., \pi_n(t_0)]^T,\ 0 \le t_0 < t \tag{2.25}$$

is given by

$$\Pi(t) = \Pi(t_0)\exp[tQ(t)] \tag{2.26}$$

where the matrix exponential function is defined by the Taylor series

$$\exp[tQ(t)] = \sum_{i=0}^{\infty} \frac{[tQ(t)]^i}{i!} \tag{2.27}$$

A survey of numerical algorithms for solving the matrix exponential has been made by Moler and Van Loan,[67] and Golub and Van Loan.[68]

The numerical solution of (2.22) with (2.25) based on (2.27) for reliability studies has been treated by Reibman and Trivedi,[23] Stewart,[20] Tombuyses and Devooght,[21] Sidje and Stewart,[69] Rauzy.[22] Comparison between various algorithms has been performed from point of view of accuracy and computation time. For instance, Marca package[15] developed by Stewart incorporate all majority numerical methods for such problems. Depending on problems some numerical methods are preferably than others, especially due to the characteristics of Markov chains which imply large sparse matrices that usually are touched by stiffness in the case of high reliable systems. The numerical algorithms based on Krylov space methods are used in such cases. It has been shown[23] that for stiff problems is required a very small time-step to obtain accurate results. Computer packages which are dealing with this kind of numerical problems have been developed and tested in literature, and are freely distributed such as *Expokit* package,[70] or various variants of *LSODE*, the Livermore Solver for Ordinary Differential Equations.[71-73]

2.2 Automated generation of Markov chain

When reliability studies are performed using Markov chain technique, the basic procedure consists in abstracting the physical system first, then to construct the Markov chain, and to build and solve the Kolmogorov set of differential equations in the case of CTMC. The physical system is abstracted into a set of possible states in which it can be, from the perfect functioning state and until the failure state. Markov chain is used then to describe and analyze the movement of the system among the various states. This movement can be described graphically using a state diagram. Based on this diagram the set of ordinary differential equations is built. The Markov chain analysis is primarily a quantitative analysis technique, but the construction of the state diagram gives us increased system knowledge. Usually this state diagram is constructed manually. Unfortunately, once the number of states in Markov chain increases, to build such a state diagram is more difficult for analyst. To avoid this drawback, it is used a high level interface for abstraction of the physical system and afterwards the underlying Markov chain is constructed automatically based on that abstraction. Several computer packages have been developed based on this concept.[10-15, 50]
As it has been mentioned in the previous chapter, the abstraction of physical system is made using the high level interfaces based on combinatorial techniques as Block Diagram, Fault Tree, and Stochastic Petri Nets. These combinatorial techniques are based on graphical representation and easier to be understood. The first two have been also widely used as stand alone techniques for reliability analysis. The third one is a formalism specially developed as the interface for the automated generation and solution of Markov chain. Description of these techniques can be found in McCormick [1981][44], Dhillon [1981][5], Trivedi [1982, 2002][7], Aven [1992][6].

In this section, the steps in constructing the transition rate matrix for a Markov chain are presented. A developed algorithm for automated generation of Markov chain system of differential equation is presented using the Fault Tree abstraction of physical system as interface. This algorithm has been implemented into a stand alone computer code which is called MARKOMAG-S (Markov Chain Matrix Generator and Solver) and which is used further for numerical examples along this work. The implemented algorithm generates the backward Kolmogorov differential equations (2.22), and afterwards using an Ordinary Differential Equation (ODE) solver this set of equations is solved to get the transient state probability distribution. The solver used for numerical solution is based on *LSODE* package,[71] namely *the variable-coefficient ordinary differential equation solver with the preconditioned Krylov method GMRES for the solution of linear system, VODPK*.[72, 73] The backward Kolmogorov differential equations are generated in matrix form, i.e. during this process the transition rate matrix is generated.

To describe this algorithm, it will be shown first how the differential equations are generated for the trivial case of a binary component. Let be a component with two states, namely operational state (*Up*) and fail state (*Down*), which has the transition diagram as below.

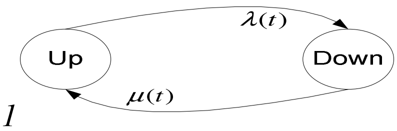


Figure 2.1: The state transition diagram for a binary component with repair

The operational state is labeled with *1* and the fail state with *2*. The component passes from the state *1* to the state *2* with the transition rate $\lambda(t)$, and from the state *2* to the state *1* with the transition rate $\mu(t)$. This behavior is depicted using labeled arcs which are arrowed from the source state to the destination state. The label of an arc is the transition rate. The transition rate $\lambda(t)$ is called failure rate, and $\mu(t)$ repair rate. If it is considered the transition rate matrix $Q(t) = [q_{ij}(t)]_{n\times n}$ with *n* the number of possible states, in this case *n* = *2*, the ordinary differential set of equations, written in matrix form is as follows,

$$\begin{cases} \dfrac{d}{dt}\begin{bmatrix} \pi_1(t) \\ \pi_2(t) \end{bmatrix} = \begin{bmatrix} q_{11}(t) & q_{12}(t) \\ q_{21}(t) & q_{22}(t) \end{bmatrix} \begin{bmatrix} \pi_1(t) \\ \pi_2(t) \end{bmatrix} \\ \begin{bmatrix} \pi_1(t_0) \\ \pi_2(t_0) \end{bmatrix} = \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix}_{t_0} \end{cases}$$

where the column vector $\Pi(t)=[\pi_i(t)]_n$ is the probability state vector, $\pi_i(t)$ being the probability that the component to be into the state *i* at time *t*. $\Pi(t_0)=[\pi_i(t_0)]_n$ represents the initial probability state vector at $t=t_0$. As it has been showed in the previous section, for the backward form, the elements of main diagonal have the property (2.23), where in this case the state space is $I=\{1,2\}$, and *n* = *2*. The elements in matrix are generated on columns which mean that for a transition from the source state *i* to the destination state *j* with a transition rate $\alpha(t)$, the element in matrix is $q_{ji}(t)=\alpha(t)$. In the case of binary component yields:

$$Q(t)=\begin{bmatrix}-\lambda(t) & \mu(t)\\ \lambda(t) & -\mu(t)\end{bmatrix}, \qquad \lambda(t),\mu(t),t\geq 0$$

Assuming that the initial time $t_0=0$, the component is in the state *1*, and taking into account the conservation of probability (2.2.b), it is constructed the next ODE system for a binary component.

$$\begin{cases}\dfrac{d}{dt}\begin{bmatrix}\pi_1(t)\\ \pi_2(t)\end{bmatrix}=\begin{bmatrix}-\lambda(t) & \mu(t)\\ \lambda(t) & -\mu(t)\end{bmatrix}\begin{bmatrix}\pi_1(t)\\ \pi_2(t)\end{bmatrix}\\ \begin{bmatrix}\pi_1(t_0=0)\\ \pi_2(t_0=0)\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}\end{cases}$$

This simple ODE system is solved analytically using usually the Laplace transform method.[7] For more complex systems where the analytical solution is not possible to be obtained, the numerical methods are used to find the transient solution of the state probability vector.

Next, an algorithm to build the transition state diagram considering the Fault Tree abstraction of the physical system is proposed. Similar methodology is met in HARP package.[10, 33]

Based on various scenarios, the abstraction of the physical system in terms of Fault Tree is performed first. The Fault Tree gates and the procedure to perform qualitative analysis are presented in Appendix A. The Fault Tree qualitative analysis is performed to find the minimal-cut-sets which are shortly the minimal combination of failed and operational components which can lead to the failure behavior of the physical system. Based on the abstraction of physical system the Markov chain is build. The states of Markov chain are identified and classified as operational or fail states using the previous Fault Tree qualitative analysis. At the end of this process the transition rate matrix is generated. The set of differential equations are solved afterwards to get the state transient probability distribution for a mission time. Using these results, the quantification of reliability function for the physical system is performed. These steps are presented into a flow diagram as follows,

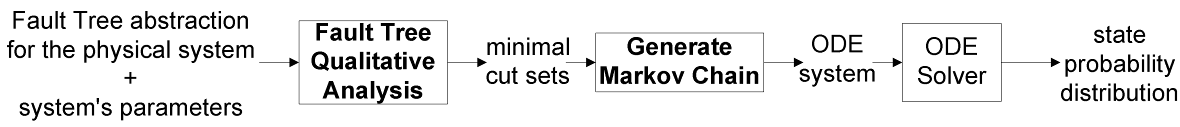


Figure 2.2: The Fault-Tree Markov chain technique flow diagram

Further, it is presented how the state diagram and the system of differential equations for an AND gate and an OR gate with two predecessors is generated. The gates with more than two predecessors can be reduced to the case with two predecessors. It is considered that these predecessors are basic events that consist in failure behavior of the respective components.

The AND gate and OR gate are graphically represented as follows.

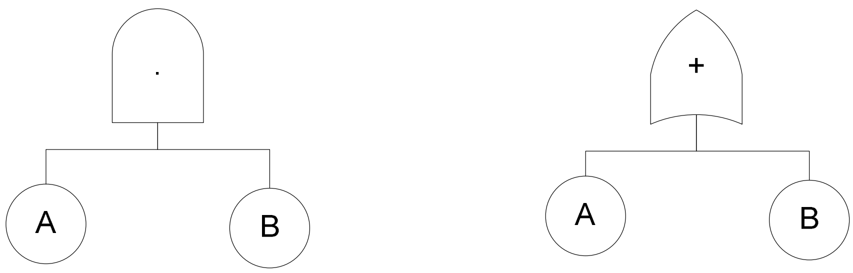


Figure 2.3 Fault Tree gates

a) AND gate b) OR gate

The next simplifying hypotheses have been considered:

- The components are binary, i.e. they have only two sates, namely operational and fail state;
- The behavior of a component is independent of the behavior of the other components;
- Two or more components do not have failure behavior at the same time ($\Delta t \to 0$), which means that the generated Markov chain will not be irreducible (in an *irreducible Markov chain* every state can be reached from every other state);
- A repaired/replaced component is *as good as new*, i.e. the component parameters (transition rates) will be the same as for a new component.

For a physical system that consists in two binary components with repair and which can be abstracted as an AND gate, that AND gate usually is defined as the failure behavior of the system. This behavior is reached only if both of the system components have failure behavior. This abstraction of the system can be represented into a Markov state diagram as follows.

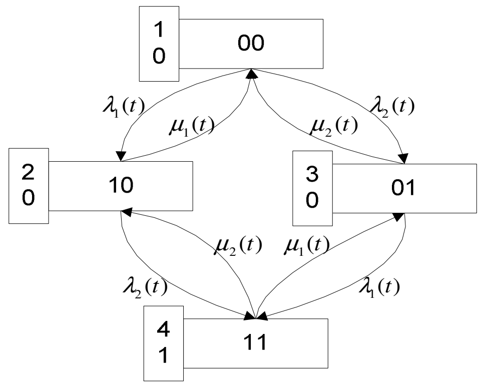


Figure 2.4: The state transition diagram for an AND gate with two successors

In representation of a state diagram the next conventions have been made:

- A state is represented as a number of digits either *0* or *1*. The number of digits into a state represents how many binary components are in system. The order of components in state is from left to right as in Fault-Tree representation.
- The failure behavior of a binary component is represented with *1*, otherwise with *0*;
- Each state is labeled with two indexes, an upper index which is an order number for the state, and a lower index which is either *0* or *1*, if the state is an operational one or a failure one, respectively. The order numbers of states are associated with the row and column indexes into transition rate matrix.
- Each component is characterized by failure and repair rate ($\lambda_i(t),\ \mu_i(t),\ i=1,2$)

Following the same procedure as for a component, the transition rate matrix becomes

$$Q(t)=\begin{bmatrix} q_{11}(t) & q_{12}(t) & q_{13}(t) & q_{14}(t) \\ q_{21}(t) & q_{22}(t) & q_{23}(t) & q_{24}(t) \\ q_{31}(t) & q_{32}(t) & q_{33}(t) & q_{34}(t) \\ q_{41}(t) & q_{42}(t) & q_{43}(t) & q_{44}(t) \end{bmatrix}=\begin{bmatrix} -\sum\limits_{\substack{j=1\\ j\neq 1}}^{4} q_{j1}(t) & q_{12}(t) & q_{13}(t) & q_{14}(t) \\ q_{21}(t) & -\sum\limits_{\substack{j=1\\ j\neq 2}}^{4} q_{j2}(t) & q_{23}(t) & q_{24}(t) \\ q_{31}(t) & q_{32}(t) & -\sum\limits_{\substack{j=1\\ j\neq 3}}^{4} q_{j3}(t) & q_{34}(t) \\ q_{41}(t) & q_{42}(t) & q_{43}(t) & -\sum\limits_{\substack{j=1\\ j\neq 4}}^{4} q_{j4}(t) \end{bmatrix}$$

$$Q(t)=\begin{bmatrix} -[\lambda_1(t)+\lambda_2(t)] & \mu_1(t) & \mu_2(t) & 0 \\ \lambda_1(t) & -[\mu_1(t)+\lambda_2(t)] & 0 & \mu_2(t) \\ \lambda_2(t) & 0 & -[\lambda_1(t)+\mu_2(t)] & \mu_1(t) \\ 0 & \lambda_2(t) & \lambda_1(t) & -[\mu_1(t)+\mu_2(t)] \end{bmatrix}$$

For a physical system abstracted as an OR gate, the failure behavior is reached if either one of its two components is failed, or both of them. Taking into account the hypotheses which have been made before, a state diagram in this case is as follows.

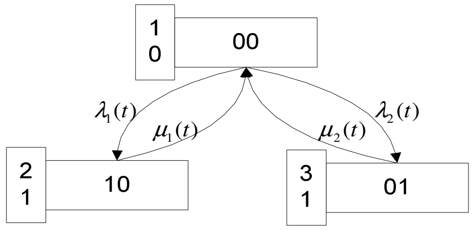


Figure 2.5 The state transition diagram for an OR gate with two successors

Following the same procedure as previous, the next transition matrix is obtained.

$$Q(t)=\begin{bmatrix} q_{11}(t) & q_{12}(t) & q_{13}(t) \\ q_{21}(t) & q_{22}(t) & q_{23}(t) \\ q_{31}(t) & q_{32}(t) & q_{33}(t) \end{bmatrix}=\begin{bmatrix} -[\lambda_1(t)+\lambda_2(t)] & \mu_1(t) & \mu_2(t) \\ \lambda_1(t) & -\mu_1(t) & 0 \\ \lambda_2(t) & 0 & -\mu_2(t)] \end{bmatrix}$$

The procedure can be extended to more complex configurations. To show further how the qualitative analysis of Fault-Tree is used in automated generation of Markov chain process, it is consider the example of the simple 2-Out-of-3 system presented and analyzed in Appendix A and B. The Fault-Tree of this system is presented in Figure 2.6 and consists in five components, i.e. *A*, *B*, *C*, *D*, and *E*, respectively. The minimal-cut-sets are {D E}, {A B}, {A C}, {B C}. Based on these minimal-cut-sets the *generic fault states* {`11xxx`}, {`1x1xx`}, {`x11xx`}, and {`xxx11`}, respectively, are defined as is explained in Appendix A.

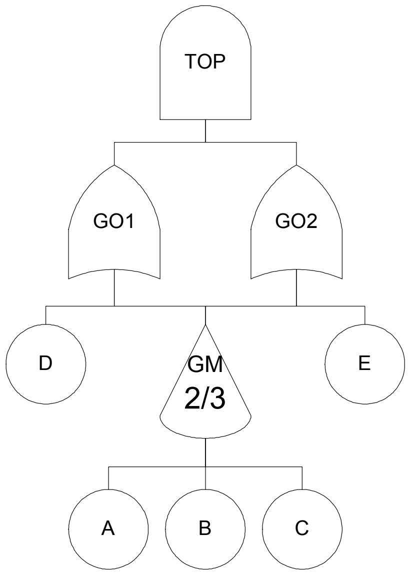


Figure 2.6 The Fault-Tree for the 2-Out-of-3 system

Then, based on the Fault Tree abstraction of the physical system, its associated Markov chain is generated. It is not known from the beginning how many possible states can exist. That means that it is not generated the complete Markov chain, but the Markov chain that contains only the possible states of the physical system. If a failure state is identified, the Markov chain from that state is not developed anymore. The number of states depends by the number of components in system and the system architecture. The fault tree analysis that has been performed before is used in this stage, i.e. the generic fault states, to identify the possible failure states of the physical system. For the case of no repairs, the failure states of the system are absorbing states.

The algorithm is starting from an initial state which is usually the state in which all components are operational. New states are generated considering the failure possibilities of components, one at a time. Using the generic fault states, it is decided if a new state is either a failure state or an operational state for the physical system. Afterwards, it is performed the checking step to see if the state is a possible state in which the system can exist. Depending on the failure behavior of the components from a state to another, the transition between states are defined and based on them and the states the transition rate matrix is generated. The algorithm is finished when all possible states are generated. The algorithm flow to generate the Markov chain is presented in Fig.2.7.

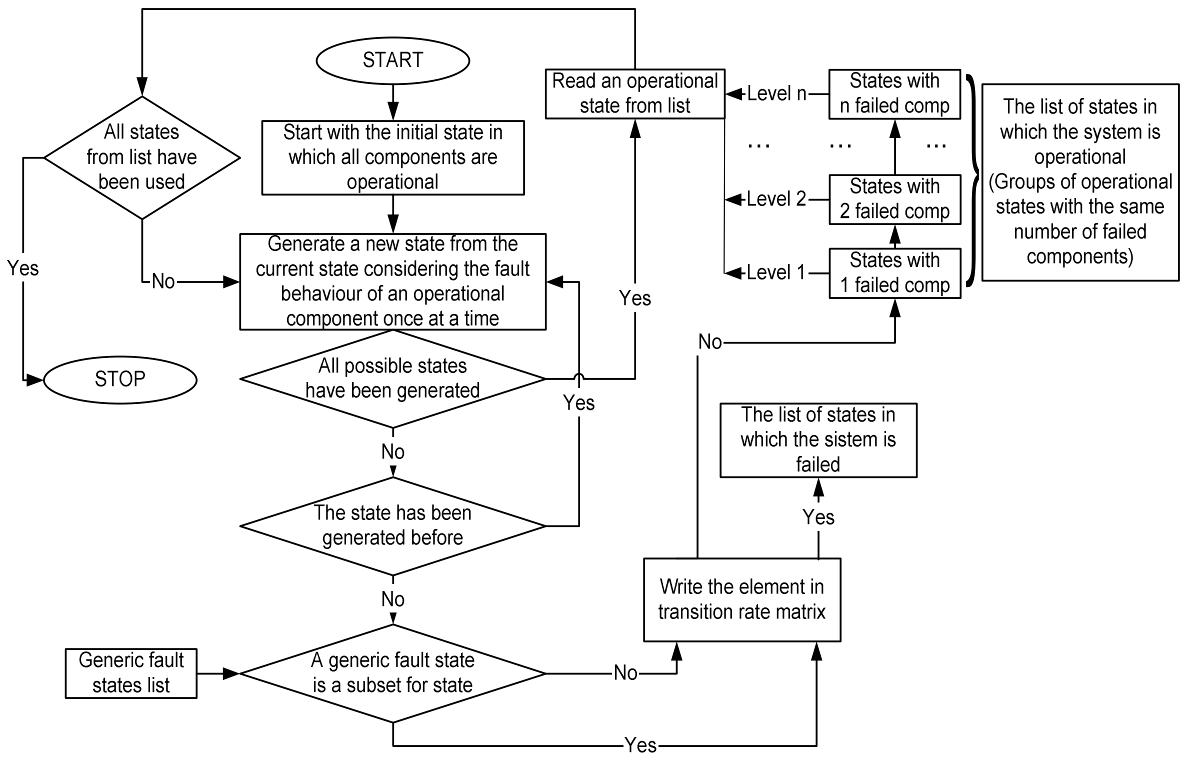


Figure 2.7 The algorithm flow for automated generation of Markov chain

The state transition diagram for 2-Out-of-3 system is represented in Fig.2.8. For sake of simplicity have been drawn only the failure rates that occur between states. This development has been made on levels, where it has been considered on level 0 the state without any failed component. On the level *1* are generated further the states with only one component failed. On the next level, i.e. the level *2*, are generated from operational states from previous level the states with two components failed and so forth. On the last level will be only the states that describe the failure behavior of physical system. A transition occurs only between the states that belong to two successive levels. For instance, if it is considered a transition between two states from two successive levels *n* and *n+1*, the state from level *n+1* has the same combination of failure components as the state from the level *n* plus an additional failure component. Between the failure states from a level (if they exists) and the states from the next level does not occur any transition. Therefore, one can see that in general the transition rate matrix is a sparse matrix.

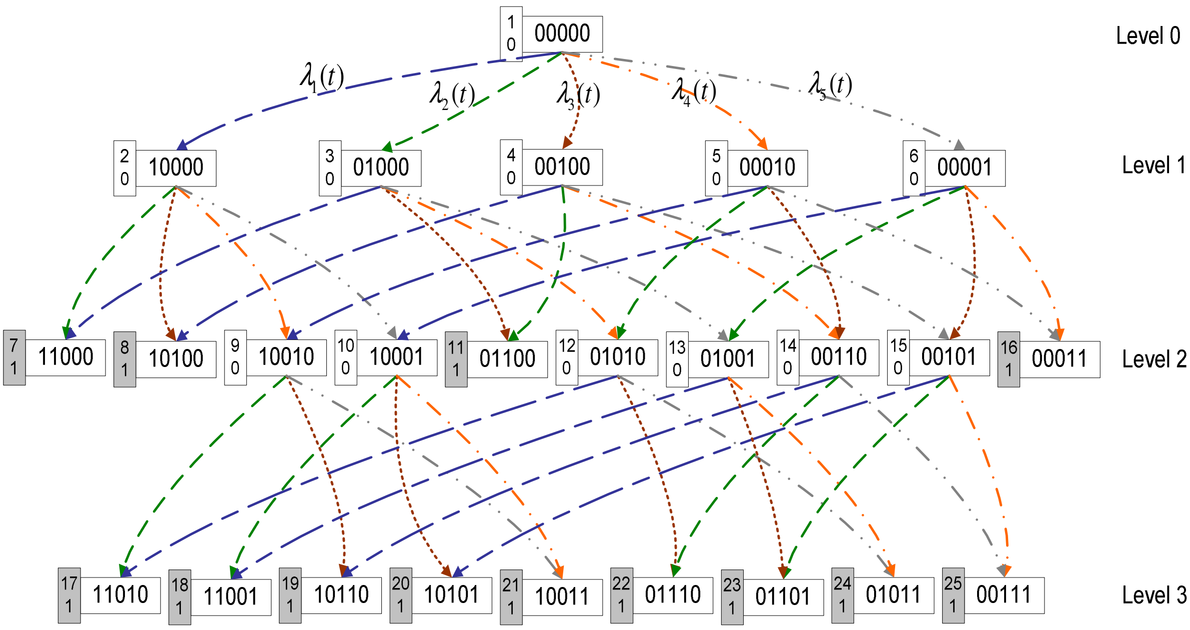


Figure 2.8 The state transition diagram for the 2-Out-of-3 system

For the considered example it is found 25 possible states, i.e. 13 states in which the system is failed and 12 states in which the system is operational. The system failure states are the states labeled with 7, 8, 11, and 16 to 25. These states were identified using the generic fault states `{11xxx}`, `{1x1xx}`, `{x11xx}`, and `{xxx11}`, all the system failure states including one of these generic fault states, i.e. the generic fault states are subsets of the system failure states. The transition rate matrix for this case will be of order 25. If the system components are characterized only by the failure rate $\lambda_i(t)$, $i = 1,...,5$, i.e. it does exist in system only non-

repairable components, the Markov chain consists in 25 states with 43 transitions. Considering the elements on the main diagonal, the number of nonzero elements in transition rate matrix is 55. If all system components are repairable, then the Markov chain has 25 states with 86 transitions, and the transition matrix has 111 elements.
Either the case with repairable or non-repairable components, or various other scenarios for this system are considered, the Kolmogorov equations consists of a set of 25 coupled ordinary differential equations which give the transient solution of probability state vector. This system of equation is presented in matrix form in Appendix B.

The number of states in Markov chain is growing exponentially with increasing of system complexity. For instance, for a system with n binary components, the Markov chain can have 2^n states. To avoid this drawback, methods of state space reduction has been proposed and used in literature. These methods consists into a more concise and smaller model specifications, reduction of states space considering only the relevant states, truncation of the number of failed components in state, decomposition of the system in subsystems and separate analysis of these subsystems, afterwards the results being combined to get system solution, etc. The last method is allowed only if the subsystems behavior is independent of each other.
For systems with repairable\replaceable components which are regularly inspected at a specific time, each time when a certain component is found as defected, it is repaired or replaced, and considered as good as new. Depending on the time period between two inspections, the physical system will not operate with more than a certain number of failed components. Therefore, based on this assumption and if it is considered the previous abstraction on levels, one can impose a limit at which the levels in Markov chain generating algorithm can be developed, i.e. the maximum number of failed components into a state. Also the probability that the system will be in one of the states on lower levels (e.g. the level 2 or level 3) will decrease and usually will be more relevant the probabilities of the states on first levels (e.g. the level 0 or level 1). This truncation method is not recommendable to be used for systems without repairs, where the probability of the states from lower levels will increase in time. A numerical example is presented on 2-Out-of-3 system in Appendix B. This method of reducing the number of states in Markov chain has been implemented in the MARKOMAG-S code.

Validation of this proposed algorithm to generate the Markov chain from Fault Tree abstraction of physical system considering reliability studies on a complex system and comparisons with the results from literature are presented in Chapter 4.

2.3 Reliability measures

Solving the Kolmogorov equations (2.22) together with an initial probability state vector (2.25), the transient probability state distribution is obtained. These results are used further to compute various reliability measures as reliability and availability for the physical system which has been analyzed using Markov chain technique.

Reliability of a component is the probability that the component survives until some time t considering that at a time t_0 the component was operating properly, $0 \le t_0 \le t \le \infty$. Extended to a system, the reliability $R(t)$ of a physical system is the probability that the system is functional until a time t given that the system was operating correctly at an initial time t_0. The reliability implies that the system was not under repair until the time t. In Markov analysis for reliability quantification, the failure states of the system are considered absorbing states, and the other states transient or recurrent.

Redundancy is used to achieve high reliability. Due to redundancy, the high reliable systems usually are fault tolerant and they continue to function even if one or more components have failed. For instance in the 2-Out-of-3 example, the system is operational even when one of its components is defect (the states from level *1* in transition state diagram from Fig.2.8), or in some cases when two of its components are failed (the operational states on level *2*).

The complementary function of reliability is called unreliability of the system ($1-R(t)$).

For repairable systems a fundamental quantity of interest is availability. Availability $A(t)$ of a physical system is the probability that the system is operating correctly at a requested time t. The complementary function of availability is unavailability of the system ($1-A(t)$). In this case, the failure states in Markov chain are not necessary absorbing states, since during the time period from t_0 to t the system could have been repaired or components replaced. The operational states could be in this case either transient or recurrent states.

Taking in account the peculiarities of the states in Markov chain, the availability and reliability are computed in the same way. If the states of Markov chain are considered $I = \{1, 2, ...\}$ as a set of states in which the system is failed $\{Down\}$ and operational $\{Up\}$, the transient solution of Kolmogorov equations gives us the reliability/availability of the system,

$$R(t) = \sum_{i \in Up} \pi_i(t) \tag{2.28}$$

and for complementary function,

$$U(t) = \sum_{i \in Down} \pi_i(t) = 1 - R(t) \tag{2.29}$$

For the previous example of 2-Out-of-3 system these two sets are $Down = \{7,8,11,16,17,...,25\}$, and $Up = \{1,2,...,6,9,10,12,13,14,15\}$.

Usually it is analyzed the reliability of the system when all its components are functioning properly, in this case the transient probability of the state in which all components are operational.

The *instantaneous* or *point availability* $A(t)$ of a component or a system is defined as the probability that the component or system is properly functioning at time *t*. In absence of a repair or a replacement, availability $A(t)$ is simply equal with reliability $R(t)$ for a component. The instantaneous availability is always greater than or equal to the reliability.[7]

For a component instantaneous availability is,

$$A(t) = \pi_i(t), i \in \{Up\} \tag{2.30}$$

The *limiting* or *steady-state availability* is the limiting value of $A(t)$ as *t* approaches infinity. This measure is usually nonzero in contrast with limiting reliability which is always zero.

$$\begin{aligned} A &= \lim_{t \to \infty} A(t) \neq 0 \\ R &= \lim_{t \to \infty} R(t) = 0 \end{aligned} \tag{2.31}$$

The *average* or *interval availability* represents the expected fraction of time the system is up in a given interval of time $[t_0, t]$, $0 \le t_0 < t \le \infty$,

$$A_I \equiv \frac{1}{t_f} \int_{t_0}^{t_f} A(t)dt \tag{2.32}$$

The time interval $[t_0, t]$ may be for instance the design life of the system or the time to accomplish some particular mission.

In CTMC the transitions between various states describe the system behavior. Considering the previous assumptions, a transition is related to the failure behavior of a component. The repairing or replacing of a component from system is characterized also by a transition. These transitions are instantaneous *failure rates* of components and represent the expected number of failures in a given time period. The failure rate is also known as *hazard rate* $\lambda(t)$.

The failure behavior of components is an experimental result statistically obtained and is described by statistical distributions. In reliability analysis using CTMC and semi-Markov models are used distribution functions of continuous random variables, where the continuous variable of interest is the time *t*.

In practice, after Dhilon and Singh,[5] the continuous statistical distributions used for mechanical devices to approximate experimental failure numbers are the exponential distribution, the extreme value distribution, the Weibull distribution, the normal distribution, the log-normal distribution, the gamma distribution. The electric and electronic components follow usually an exponential distribution. Selection of the appropriate distribution for the analyzed devices serve two different purposes, namely to fit the experimental results, and to represent a mechanism-based description of the device's failure.

The distribution of failure numbers versus time is defined as probability density function $f(t)$. The cumulative distribution function represents the integration of probability density function over a time interval $[t_0, t]$. The cumulative distribution function is defined as the probability that the component will not operate successfully for a required mission time. The probability density functions are usually used in reliability analysis considering their functions of two parameters (mean and variance) that are tabulated in reliability databases or estimated from data collected during component\system operation.

The equivalent relations between the reliability functions are as follows.[44]

Failure probability function (Probability density function – pdf)	$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt} = \lambda(t)R(t)$
Cumulative failure probability (Cumulative distribution function – cdf)	$F(t) = \int_0^t f(\tau)d\tau = 1 - R(t) = 1 - e^{-\int_0^t \lambda(\tau)d\tau}$ $F(0) = 1$, $F(\infty) = 0$
Reliability	$R(t) = \int_t^\infty f(\tau)d\tau = 1 - F(t) = e^{-\int_0^t \lambda(\tau)d\tau}$ $R(0) = 1$, $R(\infty) = 0$
Transition rate (Hazard rate)	$\lambda(t) = -\frac{1}{R}\frac{dR}{dt} = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{R(t)}$

Table 2.1: Equivalent relations between functions used in reliability analysis

For components and systems are also provided a mean time to failure (*MTTF*) and a mean time to repair (*MTTR*). The mean time between failures (*MTBF*) is defined as the sum of the previous two, i.e. $MTBF = MTTF + MTTR$. Mean time to failure can be computed using reliability as follows,

$$MTTF = \int_0^\infty R(t)dt, \tag{2.33}$$

while the mean time to repair is obtained usually statistically, since in reality repairs take different leght of time, depending by many factors as the skill level, the circumstances in which the repairing process take place, etc.

For the exponentially distribution, from the hazard rate formula given in Table 2.1 and the Eq.(2.33) it follows that the failure rates are constants, they being the parameters of exponential distribution, and equal with $(MTTF)^{-1}$. A reasonable approximation of repairing rates are given using this distribution, assuming that the repairing rate is constant and equal with $(MTTR)^{-1}$. The repairing rates are associated with the maintainability of the system which is a measure of how fast the system may be repaired following failure.

For such exponentially distributed failure rates, the steady-state availability is defined as follows,[7]

$$A = \frac{MTTF}{MTTF + MTTR} \tag{2.34}$$

The reliability measures can be written more general as a functional depending on transient probability distribution $\pi_i(t), i = 1,...,n$ and transition rate matrix whose elements are depending on some parameters $\alpha_k(t)$, $(q_{ij}(t) = f(\alpha_k(t)),\quad i, j = 1,...,n,\quad k = 1,...,m)$, as follows,

$$R(\pi_1,...,\pi_n;\alpha_1,...,\alpha_m;t) \equiv \int_{t_0}^{t_f} F(\pi_1,...,\pi_n;\alpha_1,...,\alpha_m;t)dt, \quad 0 \le t_0 < t \tag{2.35}$$

where n is the number of states in Markov chain, m the number of components parameters, and $F(\pi_i;\alpha_k;t)$ a nonlinear function of indicated arguments.

The level of knowledge of the physical system which is analyzed, the abstraction of the physical system as Fault Tree, how this Fault Tree is converted into a Markov chain, numerical method for obtaining the solution of Kolmogorov equations, the input parameters in mathematical model, i.e. the transition rates of components, etc., are only some sources of uncertainties to the final reliability results. Thus, further analysis is necessary to get more confidence in results. Sensitivity analysis is used to see the influence of changing in input parameters to reliability results, to provide new knowledge about the model and to optimize its performance. A new approach to sensitivity studies using Markov chain is presented into the next chapter.

2.4 Summary

The reliability of physical systems is studied dynamically using Markov chain. The physical system is abstracted in a set of states and its behavior is depicted using transitions which are occurring between these states. This behavior is mathematically described by the Kolmogorov set of differential equations whose solution is used to quantify the reliability measures for the physical system. For complex systems the number of states are increasing exponentially with the number of components in the system, and therefore to construct straightforward the attached Markov chain is difficult. In such cases combinatorial models are used as high level interface to abstract the physical system and based on these descriptions the Markov chain is generated automatically in a computer program. Based on several assumptions, an algorithm which can convert a Fault Tree into a Markov chain has been presented and implemented into a code called MARKOMAG-S. The resulted Kolmogorov equations are solved afterwards using VODPK[71, 72] ODE solver which is embedded in MARKOMAG-S code. The numerical solution of transient state probability vector is used to obtain the reliability and availability of the analyzed system. The abstractions of the physical system or the input data in the mathematical model are only some of the uncertainty sources in the reliability results. How these results are changing with changing in the input parameters or in the abstraction of physical system, or how it can be improved the system to get better results request further analysis. Sensitivity analysis provides answers to such questions. Ranking of input parameters based on their relative importance in reliability results, optimization and redesigning of critical parts for better performances, or further uncertainties studies can be performed using the sensitivities results. In Chapter 3 a new deterministic approach for the sensitivity analysis of system's reliability is developed.

3 Adjoint sensitivity analysis of Markov chains

The sensitivity analysis of reliability measures using Markov chains has been performed in literature using traditional approaches. In this chapter a new approach for sensitivity analysis based on adjoint functions is presented. The theory originally developed by Cacuci[29, 30] and known as Adjoint Sensitivity Analysis Procedure is applied to the mathematical model of the Markov chain in continuous time, discrete state space.

The aim of sensitivity analysis is to analyze the behavior of system responses to variations in input data, i.e. how the reliability measures are changing to perturbations in the transition rates and the initial states probabilities. The input data used in the mathematical model of Markov chain are not known exactly but they vary within some boundaries and further analysis is required to see the effect of these uncertainties to the final results.

3.1 Continuous-Time Markov Chains

As in the previous chapter has been presented, for a CTMC the time dependent behavior is described by the Kolmogorov set of ordinary differential equations written generally in matrix form as follows,

$$\begin{cases} \frac{d}{dt}\left[\Pi(t)\right]_{n\times 1} = \left[Q(t)\right]_{n\times n}\left[\Pi(t)\right]_{n\times 1} \\ \left[\Pi(t_0)\right]_{n\times 1} = \left[\Pi_0\right]_{n\times 1} \end{cases}, \quad \text{for } t \ge 0 \tag{3.1}$$

where the subscripts denote the respective dimensions of the probability vector and the transition rate matrix. Here, it has been considered the backward form and this set of equations will be used further for analysis in this chapter. For forward form the procedure is similar taking into account the additional transpositions for vectors and matrices.

Let n to be the number of states of Markov chain $I = \{1, 2, ..., n\}$, then the column vector $\Pi(t)$ is the state probability vector.

$$\Pi(t) = [\pi_1(t), \pi_2(t), ..., \pi_n(t)]^T \tag{3.2}$$

The i^{th} component of the state probability vector represents the probability that the system is in the state i at the time t. The elements of the state probability vector have the following properties

$$\pi_i(t) \in [0,1] \text{ for all } i \in I = \{1, 2, .., n\} \tag{3.3}$$

$$\sum_{i=1}^{n} \pi_i(t) = 1 \tag{3.4}$$

The column vector Π_0 represents the initial state probability vector at initial time t_0, $0 \le t_0 < t$, time which usually is considered zero ($t_0 = 0$). If it is assumed that the starting point in analysis is from the initial time in which the system is in the state with all components operational, then the initial state probability vector is as follows

$$\Pi_0 = [1, 0, ..., 0]^T \tag{3.5}$$

The transition rate matrix $Q(t) = [q_{ij}(t)]_{n \times n}$, $i, j \in I$, is a square matrix of order *n*, with the property that all its elements out of main diagonal are positive and the element on main diagonal negative and equal with minus sum of all other elements on column, i.e.,

$$q_{ii}(t) = -\sum_{\substack{i=1 \\ i \neq j}}^{n} q_{ji}(t) \tag{3.6}$$

Transition rate matrix elements $q_{ij}(t)$ are considered to depend on the parameters $\alpha_k(t)$,

$$q_{ij}(t) = f(\alpha_k(t)), \quad i, j = 1, ..., n, \quad k = 1, ..., m$$

where *m* is the number of components' parameters.

3.2 Sensitivity analysis of Markov chains

To perform sensitivity analysis one must first to have well defined the system's response whose sensitivity to changes in input parameters is analyzed.

The system's response R, i.e. the reliability measure of interest, is a functional of transient probability distribution $\pi_i(t),\ i = 1, ..., n$ and parameters $\alpha_k(t),\ k = 1, ..., m$, of the form

$$R(\pi_1, ..., \pi_n; \alpha_1, ..., \alpha_m; t) \equiv \int_{t_0}^{t_f} F(\pi_1, ..., \pi_n; \alpha_1, ..., \alpha_m; t) dt, \quad 0 \le t_0 < t \tag{3.7}$$

where $F(\pi_i; \alpha_k; t)$ is a nonlinear function of indicated arguments, t_0 and t_f are the initial-time and final-time considered in analysis, respectively.

Further it will be referred as system parameters the parameters $\alpha_k(t),\ k = 1, ..., m$ and as system variables the transient probability distribution $\pi_i(t),\ i = 1, ..., n$. Performing sensitivity studies, one wants to analyze the effect of variation in system parameters α_k to system response R.

Let $\mathrm{A}=\{\alpha_1(t),...,\alpha_m(t)\}$ to denote the set of system parameters. Conceptually, to perform the sensitivity analysis, the set of equations (3.1) is solved using the base case parameter values (also called nominal case), denoted by $\mathrm{A}^0=\{\alpha_1^0(t),...,\alpha_m^0(t)\}$, and the nominal initial probability vector, denoted by $\Pi^0(t_0)=[\pi_1^0(t_0),...,\pi_n^0(t_0)]^T$, to get the base case solution $\Pi^0(t)$ which is used further to obtain the base case response value $R^0(\Pi^0,\mathrm{A}^0)$. The nominal parameters values A^0 are statistical data based on experiments and their numerical values are not known exactly but within some error bounds as tolerances, variations, etc. The bounds in system parameters can be represented by a perturbation set $\delta\mathrm{A}=\{\delta\alpha_1,...,\delta\alpha_m\}$ whose elements are the respective parameter variations.

When the perturbation set $\delta\mathrm{A}$ is introduced in the original Markov chain equation (3.1) and response's equation (3.7), the corresponding perturbed solution becomes $\left(\Pi^0+\Phi\right)$ satisfying the perturbed system

$$\begin{cases}\dfrac{d}{dt}\left[\Pi^0(t)+\Phi(t)\right]_{n\times 1}=\left[Q(t)+\delta Q\right]_{n\times n}\left[\Pi^0(t)+\Phi(t)\right]_{n\times 1}\\ \left[\Pi(t_0)+\Phi(t_0)\right]_{n\times 1}=\left[\Pi_0+\Phi_0\right]_{n\times 1}\end{cases} \tag{3.8}$$

where the square matrix δQ of order n contains the variations $\delta\mathrm{A}$. The perturbed response would become in this case $R(\Pi^0+\Phi,\mathrm{A}^0+\delta\mathrm{A})$, where $\Phi=[\phi_1,...,\phi_n]^T=[\delta\pi_1,...,\delta\pi_n]^T$ denotes the variations in the respective components of transition probabilities vector.

The simplest way to obtain the perturbed response implies to solve repeatedly the perturbed system for each variation of in system's parameters once at a time. Such procedure becomes time consuming and even impractical when many variations $\delta\alpha_k$ are considered.

The typical deterministic approach for sensitivities studies using Markov chain used in literature[39, 41, 57, 58] is to solve the original system for the base case, and a larger set of differential equations for sensitivities, i.e. for each variation in system parameters must be solved an ODE system. This can be represented compact in matrix form as follows,

$$\begin{cases}\left[\dfrac{d}{dt}\left[\Pi(t)\right]_{n\times 1},\dfrac{\partial}{\partial\alpha_k}\left(\dfrac{d}{dt}\left[\Pi(t)\right]_{n\times 1}\right)\right]=\begin{bmatrix}\left[Q(t)\right]_{n\times n} & \left[0\right]_{n\times n}\\ \dfrac{\partial}{\partial\alpha_k}\left[Q(t)\right]_{n\times n} & \left[Q(t)\right]_{n\times n}\end{bmatrix}\begin{bmatrix}\left[\Pi(t)\right]_{n\times 1}\\ \dfrac{\partial}{\partial\alpha_k}\left[\Pi(t)\right]_{n\times 1}\end{bmatrix}\\ \left[\left[\Pi(t_0)\right]_{n\times 1},\dfrac{\partial}{\partial\alpha_k}\left[\Pi(t_0)\right]_{n\times 1}\right]=\left[\left[\Pi_0\right]_{n\times 1},\dfrac{\partial}{\partial\alpha_k}\left[\Pi_0\right]_{n\times 1}\right],\quad k=1,...,m\end{cases} \tag{3.9}$$

where the vector/matrix subscript denotes its respective order.

This method becomes also expensive from computational costs and effort point of view, even impractical when many parameters α_k exist in the analyzed problem.

A more effective method to avoid the drawback of the traditional sensitivity methods has been developed in early eighties by Cacuci,[29, 30] and is based on the adjoint functions.

The deterministic sensitivity theory developed by Cacuci[24,29,30] comprises two complementary aspects in sensitivity analysis development, namely the Forward Sensitivity Analysis Procedure (FSAP), and the Adjoint Sensitivity Analysis Procedure (ASAP). The scope of these procedures is to calculate exactly and efficiently the system response sensitivity to variations in system parameters around their nominal values using adjoint functions. These sensitivities are obtained by calculating the first *Gâteaux* *G*-differential of the system responses (reliability measures) at the nominal value of the system dependent variables (transient state probabilities) and system parameters (the transition rates of components). Following these procedures applied to Kolmogorov ordinary differential equations that describe the Markov chain, the sensitivities to all parameters are obtained with the advantage that in this case there must not be performed repetitive calculations that the other methods imply.

The *G-differential* denoted by $DF(x^0,h)$ of an operator $F(x)$ at x^0 with increment h is defined as follows,[24]

$$\begin{aligned} DF(x^0,h) &\equiv \lim_{\varepsilon\to 0}\frac{1}{\varepsilon}\left\{F(x^0+\varepsilon h)-F(x^0)\right\} \\ &= \frac{d}{d\varepsilon}\left\{F(x^0+\varepsilon h)\right\}_{\varepsilon=0} \end{aligned} \tag{3.10}$$

for all vectors h and scalar ε. The superscript "0" denotes the nominal values. The *G*-differential $DF(x^0,h)$ is related to the total variation $\left[F(x^0+\varepsilon h)-F(x^0)\right]$ of $F(x)$ at x^0 through the relation

$$F(x^0+\varepsilon h)-F(x^0) = DF(x^0,h)+\Delta(h), \quad \text{with} \quad \lim_{\varepsilon\to 0}\frac{\Delta(\varepsilon h)}{\varepsilon}=0 \tag{3.11}$$

In most practical cases the *G*-differential $DF(x^0,h)$ is linear in h and, therefore, the Eq.(3.11) indicates that the terms in $\Delta(h)$ are of second or higher order in $\|h\|$.

Rewritten Eq.(3.7) as follows,

$$R(\Pi,\mathrm{A}) \equiv \int_{t_0}^{t_f} F(\Pi,\mathrm{A})dt, \quad 0\le t_0 < t \tag{3.12}$$

and applying the G-differential, the response's sensitivity $DR(\Pi^0, \mathrm{A}^0; \Phi, \delta\mathrm{A})$ is obtained, where the following notations have been used: $x = (\Pi, \mathrm{A})$, $x^0 = (\Pi^0, \mathrm{A}^0)$, for the nominal variables and parameters, and $h = (\Phi, \delta\mathrm{A})$ for the variations in vector variables $\Phi \equiv \delta\Pi$ and parameters, respectively.

$$\begin{aligned} DR(\Pi^0, \mathrm{A}^0; \Phi, \delta\mathrm{A}) &\equiv \left\{ \frac{d}{d\varepsilon} \left[\int_{t_0}^{t_f} F(\Pi^0 + \varepsilon\Phi, \mathrm{A} + \varepsilon\delta\mathrm{A}) \right] \right\}_{\varepsilon=0} \\ &= R_{\mathrm{A}}^{'}(\Pi^0, \mathrm{A}^0)\delta\mathrm{A} + R_{\Pi}^{'}(\Pi^0, \mathrm{A}^0)\Phi \\ &= \sum_{k=1}^{m} \int_{t_0}^{t_f} \left(\frac{\partial F}{\partial \alpha_k} \right)_{(\Pi^0, \mathrm{A}^0)} \delta\alpha_k dt + \int_{t_0}^{t_f} \left(\frac{\partial F}{\partial \Pi} \right)^T_{(\Pi^0, \mathrm{A}^0)} \Phi(t) dt \end{aligned} \tag{3.13}$$

where the column vector has been considered,

$$\frac{\partial F}{\partial \Pi} = \left[\frac{\partial F}{\partial \pi_1}, \ldots, \frac{\partial F}{\partial \pi_n} \right]^T$$

For the system response defined as in Eq.(3.12), the Eq.(3.11) becomes

$$R(\Pi^0 + \Phi; \mathrm{A}^0 + \delta\mathrm{A}) = R(\Pi^0, \mathrm{A}^0) + DR(\Pi^0, \mathrm{A}^0; \Phi, \delta\mathrm{A}) + O(\|\Phi\|^2 + \|\delta\mathrm{A}\|^2) \tag{3.14}$$

which indicates that the exact value of the perturbed response using recalculations is predicted by the sensitivity $DR(\Pi^0, \mathrm{A}^0; \Phi, \delta\mathrm{A})$ to first order accuracy in $\|\Phi\|$ and $\|\delta\mathrm{A}\|$, respectively, i.e.,

$$R(\Pi^0 + \Phi; \mathrm{A}^0 + \delta\mathrm{A}) - R(\Pi^0, \mathrm{A}^0) = DR(\Pi^0, \mathrm{A}^0; \Phi, \delta\mathrm{A}) + O(\|\Phi\|^2 + \|\delta\mathrm{A}\|^2)$$

The sensitivity $DR(\Pi^0, \mathrm{A}^0; \Phi, \delta\mathrm{A})$ of response $R(\Pi, \mathrm{A})$ to variations Φ and $\delta\mathrm{A}$ contains two parts.[28] One part that is depending on the variations in systems parameters $\delta\mathrm{A}$ only, this is called the *direct-effect term* because it can be evaluated directly since the perturbations in system parameters $\delta\mathrm{A}$ are known, and another part which is depending on the variations in system variables $\Phi \equiv \delta\Pi$ and which is called *indirect-effect term* since the variations Φ are not known. From Eq.(3.13) the direct-effect term is

$$DR_d(\Pi^0, \mathrm{A}^0; \Phi, \delta\mathrm{A}) \equiv R_{\mathrm{A}}^{'}(\Pi^0, \mathrm{A}^0)\delta\mathrm{A} = \sum_{k=1}^{m} \int_{t_0}^{t_f} \left(\frac{\partial F}{\partial \alpha_k} \right)_{(\Pi^0, \mathrm{A}^0)} \delta\alpha_k dt \tag{3.15}$$

and the indirect-effect term

$$\begin{aligned} DR_i(\Pi^0, \mathrm{A}^0; \Phi, \delta\mathrm{A}) \equiv R_{\Pi}^{'}(\Pi^0, \mathrm{A}^0)\Phi &= \int_{t_0}^{t_f} \left(\frac{\partial F}{\partial \Pi} \right)^T_{(\Pi^0, \mathrm{A}^0)} \Phi(t) dt \\ &= \sum_{i=1}^{n} \int_{t_0}^{t_f} \left(\frac{\partial F}{\partial \pi_i} \right)_{(\Pi^0, \mathrm{A}^0)} \phi_i(t) dt \end{aligned} \tag{3.16}$$

The further analysis consists in evaluation of the indirect-effect term. Two procedures have been developed to evaluate this term, namely FSAP and ASAP. The application of these procedures to Markov chain equations is presented in the next sections.

3.2.1 Forward Sensitivity Analysis Procedure of Markov chain

In this section, the first procedure, namely FSAP, is applied to Kolmogorov set of ordinary differential equations which describes the Markov chain with the scope to obtain the response sensitivities as they are defined in Eq.(3.13). In practice, the set of system parameter variations is known, i.e. $\delta \mathrm{A}$, around the nominal values A^0. The systems parameters A and the system variables Π are related to each other through Eq.(3.1), and it follows that the variations $\delta \mathrm{A}$ and Φ are also related each other. Therefore, the response sensitivities can be calculated only after the variations in system variables, i.e. Φ, are determined.

FSAP consists in applying the *G*-differential to the Kolmogorov set of equations, and the resulted system of equations, namely the Forward Sensitivity Equations (FSE), has as solution the vector of variations in system parameters Φ.

Applying the *G*-differential to the Kolmogorov set of equations (3.1) yields

$$\begin{cases} \left\{ \frac{d}{d\varepsilon} \left[\frac{d}{dt} \left(\Pi(t) + \varepsilon \Phi(t) \right) \right] \right\}_{\varepsilon=0} = \left\{ \frac{d}{d\varepsilon} \left\{ \left[Q(t) + \varepsilon \delta Q \right] \left(\Pi(t) + \varepsilon \Phi(t) \right) \right\} \right\}_{\varepsilon=0} \\ \left\{ \frac{d}{d\varepsilon} \left[\Pi(t_0) + \varepsilon \Phi(t_0) \right] \right\}_{\varepsilon=0} = \left\{ \frac{d}{d\varepsilon} \left[\Pi_0 + \varepsilon \Phi_0 \right] \right\}_{\varepsilon=0} \end{cases} \tag{3.17}$$

Performing the additional operations and taking in account the condition that after differentiation all terms of second or higher order in ε vanish leads to the next differential system of equations,

$$\begin{cases} \frac{d}{dt} \Phi(t) - \left[Q^0(t) \right] \Phi(t) = \left[\delta Q \right] \Pi^0(t) \\ \Phi(t_0) = \Phi_0 \end{cases} \tag{3.18}$$

where the square matrix δQ of order *n* contains the parameter variations $\delta \mathrm{A}$.

The system of ordinary differential equations (3.18) represents the Forward Sensitivity Equations of Markov chain. This system of equations shows the dependency between the variations in vector variables and parameters. Solving this system using the same methods as for the original system (3.1) the vector of variations in system variables is evaluated, i.e. Φ, and afterwards the responses sensitivities.

This procedure is advantageous to be applied when the number of responses exceeds the number of system parameters and the parameters variations which should be considered in analysis. This case is rarely met in practice since it must be evaluated few responses against of many variations in system parameters (many transition rates vs. 2-3 reliability measures). That requires that the system of equations (3.18) to be solved repetitively for each variation in system parameters, since the vector Φ is dependent on parameters variations $\delta \mathrm{A}$ through the matrix δQ. Therefore, this procedure is just as expensive as performing repeatedly the exact recalculations by solving the Eq.(3.8), and then recalculating the perturbed response $R(\Pi^0 + \Phi, \mathrm{A}^0 + \delta\mathrm{A})$.

To avoid this dependence and the repetitive calculations of the system of equations (3.18), an alternative procedure has been developed, namely the Adjoint Sensitivity Analysis Procedure.

3.2.2 Adjoint Sensitivity Analysis Procedure of Markov chain

The Adjoint Sensitivity Analysis Procedure originally is applied further to the set of equations which describe the Markov chain. The practical motivation of this procedure is to avoid repetitive calculation of the Forward Sensitivity Equations of Markov chain (3.18), or of the original Kolmogorov set of equations (3.1). This goal is achieved by eliminating the dependency of the response's sensitivity $DR(\Pi^0, \mathrm{A}^0; \Phi, \delta\mathrm{A})$ to the unknown variations of system variables Φ. That is made by constructing an adjoint system of equations that is uniquely defined, independent of the vector Φ and the variations in system parameters $\delta\mathrm{A}$, and its solution is used to eliminate all unknown values Φ from the expression of $DR(\Pi^0, \mathrm{A}^0; \Phi, \delta\mathrm{A})$.

One of the main concepts used in ASAP is that of inner product which is defined for any two functions $f(x), g(x) \in \mathbb{R},\ x \in \partial\Omega$, as follows

$$\langle f(x), g(x) \rangle = \int_{\partial\Omega} f(x) g(x) dx \tag{3.19}$$

or for any two vector valued-functions $\boldsymbol{f}(x) \equiv [f_1(x), ..., f_n(x)]^T$, $\boldsymbol{g}(x) \equiv [g_1(x), ..., g_n(x)]^T$,

$$\begin{aligned} \langle \boldsymbol{f}(x), \boldsymbol{g}(x) \rangle &= \int_{\partial\Omega} [\boldsymbol{f}(x)]^T [\boldsymbol{g}(x)] dx \\ &= \sum_{i=1}^{n} \int_{\partial\Omega} f_i(x) g_i(x) dx \end{aligned} \tag{3.20}$$

where T denotes transposition.

The Adjoint Sensitivity Analysis Procedure[29] is based on the fact that the FSE represented by Eq.(3.18) is linear in Φ. That is obvious if the Eq.(3.18) is rewritten in the form

$$\begin{cases} \left(\dfrac{d}{dt}\boldsymbol{I} - \left[Q^0(t)\right]\right)\Phi(t) = \left[\delta Q\right]\Pi^0(t) \\ \Phi(t_0) = \Phi_0 \end{cases}$$

where $\boldsymbol{I}$ is the identity matrix of order n. Therefore, it is possible to introduce an arbitrary vector $\Psi(t) \equiv \left[\psi_1(t),...,\psi_n(t)\right]^T$ of adjoint functions by forming the inner product of Ψ and FSE given by Eq.(3.18), as follows

$$\int_{t_0}^{t_f} \Psi^T(t)\left(\frac{d}{dt}\boldsymbol{I} - \left[Q^0(t)\right]\right)\Phi(t)dt = \int_{t_0}^{t_f} \Psi^T(t)\left[\delta Q\right]\Pi^0(t)dt \tag{3.21}$$

to obtain,

$$\left\langle \Psi, \boldsymbol{L}(\Phi)\right\rangle = \left\langle \boldsymbol{L}^*(\Psi), \Phi\right\rangle + \left\{\mathbf{P}\left[\Psi,\Phi\right]\right\}_{\partial\Omega}, \quad \partial\Omega = [t_0, t_f], \;\; 0 \le t_0 < t_f \tag{3.22}$$

where $\boldsymbol{L}^*$ represents the formal adjoint operator of the operator $\boldsymbol{L}$, and $\left\{\mathrm{P}\left[\Psi,\Phi\right]\right\}_{\partial\Omega}$ denotes the bilinear concomitant evaluated in computational domain $\partial\Omega = [t_0, t_f]$.

Further are followed the ASAP guidelines[24, 29] in order to obtain the Adjoint Sensitivity Equations (ASE).

To get an expression for the right-hand side of the Eq.(3.22), using the left-hand side of Eq.(3.18), the operator $\boldsymbol{L}$ is defined and applied to any vector $(.)$ as follows,

$$\boldsymbol{L}(.) \equiv \left(\frac{d}{dt}\boldsymbol{I} - Q^0(t)\right)(.) \tag{3.23}$$

and applying Eq.(3.20) to the left-hand side of Eq.(3.22) yields

$$\left\langle \Psi, \boldsymbol{L}(\Phi)\right\rangle = \int_{t_0}^{t_f} \Psi^T(t)\left(\frac{d}{dt}\boldsymbol{I} - Q^0(t)\right)\Phi(t) \tag{3.24}$$

Performing the integration by parts over the domain $\partial\Omega = [t_0, t]$ to transfer all the differentiation operations from the vector of variation in system variables Φ to the vector of adjoint functions Ψ it is obtained an expression as follows

$$\begin{aligned} \left\langle \Psi, \boldsymbol{L}(\Phi)\right\rangle &= \int_{t_0}^{t_f}\left(-\frac{d}{dt}\Psi(t) - \left[Q^0(t)\right]^T\Psi(t)\right)^T\Phi(t)dt + \left(\Psi^T(t)\Phi(t)\right)\Big|_{t_0}^{t_f} \\ &= \left\langle \boldsymbol{L}^*(\Psi), \Phi\right\rangle + \left(\Psi^T(t)\Phi(t)\right)\Big|_{t_0}^{t_f} \end{aligned} \tag{3.25}$$

and identifying with the terms from the right-hand side of Eq.(3.22), the expressions for the formal adjoint operator $\boldsymbol{L}^*$ and for the bilinear concomitant are as follows,

$$\boldsymbol{L}^{*}(.) = \left(-\frac{d}{dt}\boldsymbol{I} - \left[Q^{0}(t) \right]^{T} \right)(.) \tag{3.26}$$

$$\begin{aligned} \left\{ \mathbf{P}\left[\Psi, \Phi\right] \right\}_{\partial\Omega} &= \left(\Psi^{T}(t)\Phi(t) \right)\Big|_{t_0}^{t_f} \\ &= \Psi^{T}(t_f)\Phi(t_f) - \Psi^{T}(t_0)\Phi(t_0) \end{aligned} \tag{3.27}$$

In the Eq.(3.27) the vector of variations in system parameters Φ is known for the initial time t_0, but is not known for the final time t_f. Therefore, the unknown values are eliminated by imposing the condition that the adjoint functions to vanish at the time $t = t_f$,

$$\Psi(t = t_f) = [0] \tag{3.28}$$

This condition reduces the expression (3.26) of bilinear concomitant to a quantity $\left\{ \hat{\mathbf{P}}\left[\Psi, \Phi\right] \right\}$ containing the boundary terms involving only known values of Φ

$$\left\{ \hat{\mathbf{P}}\left[\Psi, \Phi\right] \right\} = -\Psi^{T}(t_0)\Phi(t_0) \tag{3.29}$$

Hence, the Eq.(3.22) can be written as

$$\left\langle \Psi, \boldsymbol{L}(\Phi) \right\rangle = \left\langle \boldsymbol{L}^{*}(\Psi), \Phi \right\rangle + \left\{ \hat{\mathbf{P}}\left[\Psi, \Phi\right] \right\} \tag{3.30}$$

and further taking in account the Eq.(3.21) which has derived from the FSE (3.18), yields

$$\left\langle \boldsymbol{L}^{*}(\Psi), \Phi \right\rangle = \left\langle \Psi, [\delta Q]\Pi^{0}(t) \right\rangle - \left\{ \hat{\mathbf{P}}\left[\Psi, \Phi\right] \right\} \tag{3.31}$$

The right-hand side of this last equation does not contain any unknown value of the vector of variations in system variables Φ. Therefore the indirect-effect term from the expression of response's sensitivity $DR_i(\Pi^0, \mathrm{A}^0; \Phi, \delta\mathrm{A})$ should be expressed in terms of the left-hand side of the Eq.(3.31) to eliminate the unknown values Φ from its expression.

Further, it has to be found an expression to evaluate the vector of adjoint functions Ψ which is subject of condition (3.28).

The indirect effect term $DR_i(\Pi^0, \mathrm{A}^0; \Phi, \delta\mathrm{A})$ can be written in inner product form as follows

$$\begin{aligned} DR_i(\Pi^0, \mathrm{A}^0; \Phi, \delta\mathrm{A}) &= \int_{t_0}^{t_f} \left(\frac{\partial F}{\partial \Pi} \right)^{T}_{(\Pi^0, \mathrm{A}^0)} \Phi(t)\,dt \\ &= \left\langle \left(\frac{\partial F}{\partial \Pi} \right)_{(\Pi^0, \mathrm{A}^0)}, \Phi \right\rangle \end{aligned} \tag{3.32}$$

The Riesz representation theorem ensures that exists an unique vector $\left(\frac{\partial F}{\partial \Pi} \right)$ that satisfies the Eq.(3.32).[24,29] Requiring that the right-hand side of Eq.(3.32) and the left-hand side of

Eq.(3.31) to represent the same functional, since the vector of adjoint functions is still arbitrary at this stage, i.e.,

$$\left\langle \boldsymbol{L}^*(\Psi),\Phi\right\rangle = \left\langle \left(\frac{\partial F}{\partial \Pi}\right)_{(\Pi^0,\mathrm{A}^0)},\Phi\right\rangle \tag{3.33}$$

and identifying the first term from the left-side with the first term from right-side of the Eq.(3.33), yields

$$\boldsymbol{L}^*(\Psi) = \left(\frac{\partial F}{\partial \Pi}\right)_{(\Pi^0,\mathrm{A}^0)} \tag{3.34}$$

which holds uniquely in view of the Riesz representation theorem.

The adjoint operator $\boldsymbol{L}^*$ has been identified in Eq.(3.26) and applied to vector Ψ yields

$$\left(-\frac{d}{dt}\boldsymbol{I} - \left[Q^0(t)\right]^T\right)\Psi(t) = \left(\frac{\partial F}{\partial \Pi}\right)_{(\Pi^0,\mathrm{A}^0)} \tag{3.35}$$

The Eq.(3.35) together with Eq.(3.28) constitute the Adjoint Sensitivity Equations of Markov chain, i.e.,

$$\begin{cases} \dfrac{d\Psi(t)}{dt} + \left[Q^0(t)\right]^T \Psi(t) = -\left(\dfrac{\partial F}{\partial \Pi}\right)_{(\Pi^0,\mathrm{A}^0)} \\ \Psi(t=t_f) = [0] \end{cases} \tag{3.36}$$

The indirect effect-term can be written now in terms of adjoint functions, considering the Eqs.(3.32), (3.33), and (3.31), respectively,

$$DR_i(\Pi^0,\mathrm{A}^0;\Phi,\delta\mathrm{A}) = \left\langle \Psi,[\delta Q]\Pi^0(t)\right\rangle - \left\{\hat{\mathbf{P}}[\Psi,\Phi]\right\}$$

i.e.,

$$DR_i(\Pi^0,\mathrm{A}^0;\Phi,\delta\mathrm{A}) = \int_{t_0}^{t_f} \Psi^T(t)[\delta Q]\Pi^0(t)dt + \Psi^T(t_0)\Phi(t_0) \tag{3.37}$$

and replacing into Eq.(3.13), the expression for the response sensitivity is obtained,

$$\begin{aligned} DR(\Pi^0,\mathrm{A}^0;\Phi,\delta\mathrm{A}) &\equiv \frac{d}{d\varepsilon}\left\{\int_{t_0}^{t_f} F(\Pi^0+\varepsilon\Phi,\mathrm{A}+\varepsilon\delta\mathrm{A})\right\}_{\varepsilon=0} \\ &= \sum_{k=1}^{m}\int_{t_0}^{t_f}\left(\frac{\partial F}{\partial \alpha_k}\right)_{(\Pi^0,\mathrm{A}^0)}\delta\alpha_k dt + \int_{t_0}^{t_f}\Psi^T(t)[\delta Q]\Pi^0(t)dt + \Psi^T(t_0)\Phi(t_0) \end{aligned} \tag{3.38}$$

Using ASAP, the dependency of response sensitivity on unknown vector of variations in system variables Φ has been eliminated. An equivalent expression for the indirect-effect has been found, and depends only on the vector of adjoint functions Ψ. The vector of adjoint functions is the solution of ASE which does not depend on parameter variations $\delta\mathrm{A}$, and therefore Ψ is also independent of $\delta\mathrm{A}$.

The FSAP and ASAP procedures are similarly for the case in which the Markov chain is described by the forward Kolmogorov equations taking in account that in this case the vectors are row vectors and the transition rate matrix is the transpose of the transition matrix from backwards equations. The final equations for the forward case are presented in Appendix C.

Note that the ASE given in Eq.(3.36) is a final-time value problem and must be solved backward in time. It can be transformed into an initial value problem by means of changing the independent variable t to $\tau \equiv t_f - t$. This change of variable leads to the equivalent adjoint system

$$\begin{cases} \dfrac{d\Psi(\tau)}{d\tau} + \left[Q^0(t_f - \tau) \right]^T \Psi(\tau) = -\left(\dfrac{\partial F}{\partial \Pi} \right)_{(\Pi^{t_f - \tau}, \mathrm{A}^{t_f - \tau})} \\ \Psi(\tau = 0) = [0] \end{cases} \tag{3.39}$$

As it can be seen from the ASE (3.36), the advantages of ASAP is that it is obtained by an adjoint system which is linear in Ψ, does not depend on variation in system parameters $\delta\mathrm{A}$, but depends on functional F from response. As computational cost, for a given response R, one must evaluate the base-case solution Π^0 solving the original system of equations (3.1) once, and to solve only once the ASE (3.36) to evaluate the adjoint functions Ψ. Afterwards, every time when changes in system parameters are performed it should be evaluated only the response's sensitivity in terms of adjoint functions given by Eq.(3.38). Compared with the FSAP, this procedure is much cheaper than solve the FSE every time a change in input parameters is considered to obtain the variations vector in system's variables Φ and afterwards to evaluate system's sensitivity by using Eq.(3.13).

In addition, the ASE is independent of base-case solution Π^0, and therefore, could be solved independent of it, since the original system of Markov chain (3.1) is linear.

From these characteristics, it follows that the Adjoint Sensitivity Analysis procedure should be employed whenever the variations in system parameters exceed the number of responses of interest, case generally met in practice. Into reverse case should be used Forward Sensitivity Analysis Procedure, or the method which implies recalculation of original system each time a variation occurs in system parameters.

Further, the ranking of system parameters uncertainty importance is performed based on the absolute value of relative sensitivities of response with respect to the variations in system parameters given by formula,

$$\left|\frac{\partial R}{\partial \alpha_i}\frac{\alpha_i^0}{R^0}\right| \tag{3.40}$$

The relative sensitivities are used to rank the uncertainty importance in system's parameters due to it not depends on the scale of uncertainty, i.e. for the same analyzed parameter, different changes in parameter's value give the same relative sensitivity; the larger sensitivities, the bigger influence of parameter's uncertainty to system's response.

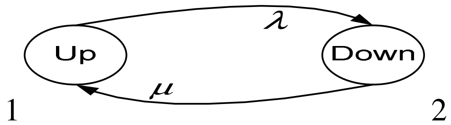
Based on the computed sensitivities, further uncertainties of system's response can be performed. For instance the use of sensitivities for uncertainty analysis can be illustrated by recalling that the linear approximation of the variance of a response[2] is given by $\mathrm{var}\langle \mathrm{R}\rangle = \sum_{i,j=1}^{k} S_i S_j \,\mathrm{cov}(\alpha_i,\alpha_j)$, where $S_i = \partial R/\partial\alpha_i$ is the response sensitivity to parameter α_i, and $\mathrm{cov}(\alpha_i,\alpha_j)$ is the covariance matrix of parameters α_i and α_j.

If all parameters are uncorrelated then $\mathrm{var}\langle \mathrm{R}\rangle = \sum_{i=1}^{k} S_i^2\sigma_i^2$, where σ_i^2 is distribution uncertainty (variance) of the parameter α_i, considering that these uncertainties are small. These formulas show the role of parameter sensitivities and uncertainties to the response uncertainty given by $\mathrm{var}\langle \mathrm{R}\rangle$. Therefore, if the sensitivity S_i and uncertainty σ_i^2 of parameter α_i are large, their contribution to response uncertainty is obvious larger to response uncertainty than the case in which either sensitivity S_i or uncertainty σ_i^2 is small.

3.3 ASAP applied to a simple Markov chain: A binary component

In this section, the previous procedures are applied to a simple Markov chain of a binary component as it has been presented in Section 2.2 during the automated generation algorithm. In this simple case it is possible to obtain analytical formulae for the solution of the original Kolmogorov system of equations, FSE, and ASE of the Markov chain, respectively, as is presented in Cacuci [2003],[24] and to compare them with the numerical solutions, in order to validate the implementation of these methods in the developed computer code QUEFT/MARKOMAG-S/MCADJSEN.

Considering a repairable component which can be either into an operational state (*Up* state) or failed state (*Down* state), its transition diagram is as follows (Fig.2.1),



where the instantaneous failure rate λ and the instantaneous repair rate μ are constant parameters.

The Kolmogorov differential equations which describes mathematically this behavior are as they have been presented in Section 2.2, i.e.,

$$\begin{cases} \dfrac{d}{dt}\begin{bmatrix}\pi_1(t)\\ \pi_2(t)\end{bmatrix} = \begin{bmatrix}-\lambda & \mu \\ \lambda & -\mu\end{bmatrix}\begin{bmatrix}\pi_1(t)\\ \pi_2(t)\end{bmatrix} \\ \begin{bmatrix}\pi_1(t_0)\\ \pi_2(t_0)\end{bmatrix} = \begin{bmatrix}\pi_{1_0}\\ \pi_{2_0}\end{bmatrix} \end{cases} \tag{3.41}$$

with additional conservation law of probability

$$\pi_{1_0} + \pi_{2_0} = 1 \tag{3.42}$$

If it is assumed that at the initial time $t_0 = 0$ the component is in the state 1, the initial state vector will be $\Pi_0 \equiv [\pi_{1_0}, \pi_{2_0}]^T = [1,0]^T$.

The average availability given by Eq.(2.29) is considered further as the system response for this problem,

$$A \equiv \frac{1}{t_f}\int_{t_0}^{t_f} \pi_1(t)dt \tag{3.43}$$

Using the Laplace transform method defining the next transforms

$$L\left(\pi_i^0(t)\right) = \pi_i^0(z) \equiv \int_0^\infty e^{-zt}\pi_i^0(t), \quad i = 1,2$$

$$L^{-1}\left(\frac{1}{z-a}\right) = e^{at}$$

and taking in account the Eq.(3.41), the analytical solution of ODE system (3.40) for the base-case is as follows,

$$\begin{cases} \pi_1^0(t) = \dfrac{\mu^0}{\lambda^0+\mu^0} + \left(\pi_{1_0} - \dfrac{\mu^0}{\lambda^0+\mu^0}\right)e^{-\left(\lambda^0+\mu^0\right)(t-t_0)} \\ \pi_2^0(t) = \dfrac{\lambda^0}{\lambda^0+\mu^0} - \left(\pi_{1_0} - \dfrac{\mu^0}{\lambda^0+\mu^0}\right)e^{-\left(\lambda^0+\mu^0\right)(t-t_0)} \end{cases} \tag{3.44}$$

where the superscript "0" denotes the nominal values. One can see that the conservation law of probability is verified, i.e. $\pi_1^0(t) + \pi_2^0(t) = 1$.

To obtain the system response sensitivities using the recalculation method, the procedure implies to solve the original system (3.40) once for the base-case and to solve again the same system but with some variations in parameters how many times a variation is considered. The system response sensitivities are evaluated in such case using the solution (3.43) as follows

$$\delta A_{REC} = A_{REC} - A^0 \tag{3.45}$$

where the system response for the base-case is

$$\begin{aligned} A^0 &= \frac{1}{t_f}\int_{t_0}^{t_f} \pi_1^0(t)dt \\ &= \frac{1}{t_f\left(\lambda^0+\mu^0\right)}\left[\mu^0\left(t_f - t_0\right) + \left(\pi_{1_0} - \frac{\mu^0}{\lambda^0+\mu^0}\right)\left(1-e^{-\left(\lambda^0+\mu^0\right)\left(t_f-t_0\right)}\right)\right] \end{aligned} \tag{3.46}$$

and for the perturbed parameters

$$A_{REC} = \frac{1}{t_f\left(\lambda^\delta+\mu^\delta\right)}\left[\mu^\delta\left(t_f - t_0\right) + \left(\pi_{1_0}^\delta - \frac{\mu^\delta}{\lambda^\delta+\mu^\delta}\right)\left(1-e^{-\left(\lambda^\delta+\mu^\delta\right)\left(t_f-t_0\right)}\right)\right] \tag{3.47}$$

where

$$\begin{aligned} \lambda^\delta &= \lambda^0 + \delta\lambda \\ \mu^\delta &= \mu^0 + \delta\mu \\ \pi_{1_0}^\delta &= \pi_{1_0} + \delta\pi_{1_0} \end{aligned}$$

To avoid the repetitive calculations for each variation in system parameters the FSAP and ASAP are used further. For this problem the system parameters are defined by $\alpha \equiv \left(\lambda, \mu, \pi_{1_0}, \pi_{2_0}\right)$, and system variables by $u \equiv \left(\pi_1, \pi_2\right)$.

The *G*-differential to Eq.(3.41) is applied further to get the response sensitivity

$$\begin{aligned} \delta A(u^0, \alpha^0; \Phi, \delta\alpha) &= \frac{1}{t_f}\int_{t_0}^{t_f}\left\{\frac{d}{d\varepsilon}\left[\pi_1^0(t) + \varepsilon\delta\pi_1(t)\right]\right\}_{\varepsilon=0} \\ &= \frac{1}{t_f}\int_{t_0}^{t_f}\delta\pi_1(t)dt = \frac{1}{t_f}\int_{t_0}^{t_f}\phi_1(t)dt \end{aligned} \tag{3.48}$$

where $\Phi = [\phi_1(t), \phi_2(t)]^T = [\delta\pi_1(t), \pi_2(t)]^T$ is the vector of variations in system variables, and $\delta\alpha = (\delta\lambda, \delta\mu, \delta\pi_{1_0}, \delta\pi_{2_0})$ is the set of variations in system parameters.

3.3.1 FSAP for a binary component

The variation δA in system response represents the solution of FSE of Markov chain which is obtained by applying the *G*-differential to the ODE system (3.41). Applying *G*-differential and carrying out the additional operations yields

$$\begin{cases} \frac{d}{dt}\begin{bmatrix}\phi_1(t)\\ \phi_2(t)\end{bmatrix} - \begin{bmatrix}-\lambda^0 & \mu^0\\ \lambda^0 & -\mu^0\end{bmatrix}\begin{bmatrix}\phi_1(t)\\ \phi_2(t)\end{bmatrix} = \begin{bmatrix}-\delta\lambda & \delta\mu\\ \delta\lambda & -\delta\mu\end{bmatrix}\begin{bmatrix}\pi_1^0(t)\\ \pi_2^0(t)\end{bmatrix} \\ \begin{bmatrix}\phi_1(t_0)\\ \phi_2(t_0)\end{bmatrix} = \begin{bmatrix}\phi_{1_0}\\ \phi_{2_0}\end{bmatrix} \end{cases} \tag{3.49}$$

Taking in account the conservation law of probability (3.41) by applying the *G*-differential, for the initial conditions of FSE (3.47) stands the equality

$$\phi_{2_0} = -\phi_{1_0} \tag{3.50}$$

The analytical solution of FSE (3.48) considering the Eqs.(3.41) and (3.49) is as below,

$$\begin{cases} \phi_1(t) = \phi_{1_0} e^{-\left(\lambda^0+\mu^0\right)(t-t_0)} - \dfrac{(\delta\lambda+\delta\mu)\pi_1^0(t) - \delta\mu}{\lambda^0+\mu^0}\left(1-e^{-\left(\lambda^0+\mu^0\right)(t-t_0)}\right) \\ \phi_2(t) = -\phi_{1_0} e^{-\left(\lambda^0+\mu^0\right)(t-t_0)} + \dfrac{(\delta\lambda+\delta\mu)\pi_1^0(t) - \delta\mu}{\lambda^0+\mu^0}\left(1-e^{-\left(\lambda^0+\mu^0\right)(t-t_0)}\right) \end{cases} \tag{3.51}$$

One can see from Eq.(3.51) that $\phi_2(t) = -\phi_1(t)$. If the analytical solution of the original system of equations is replaced, i.e. Eq.(3.44) in FSE (3.49), and separate the terms after variations in system parameters, yields the analytical solution of FSE as follows,

$$\begin{cases} \phi_1(t) = \phi_{1_0} e^{-\left(\lambda^0+\mu^0\right)(t-t_0)} + \\ \quad + \dfrac{\delta\lambda}{\left(\lambda^0+\mu^0\right)^2}\left[\left(\lambda^0+\mu^0\right)\left[\mu^0 - \left(\lambda^0+\mu^0\right)\pi_{1_0}\right](t-t_0)e^{-\left(\lambda^0+\mu^0\right)(t-t_0)} - \mu^0\left(1-e^{-\left(\lambda^0+\mu^0\right)(t-t_0)}\right)\right] + \\ \quad + \dfrac{\delta\mu}{\left(\lambda^0+\mu^0\right)^2}\left[\left(\lambda^0+\mu^0\right)\left[\mu^0 - \left(\lambda^0+\mu^0\right)\pi_{1_0}\right](t-t_0)e^{-\left(\lambda^0+\mu^0\right)(t-t_0)} + \lambda^0\left(1-e^{-\left(\lambda^0+\mu^0\right)(t-t_0)}\right)\right] \\ \phi_2(t) = -\phi_{1_0} e^{-\left(\lambda^0+\mu^0\right)(t-t_0)} - \\ \quad - \dfrac{\delta\lambda}{\left(\lambda^0+\mu^0\right)^2}\left[\left(\lambda^0+\mu^0\right)\left[\mu^0 - \left(\lambda^0+\mu^0\right)\pi_{1_0}\right](t-t_0)e^{-\left(\lambda^0+\mu^0\right)(t-t_0)} - \mu^0\left(1-e^{-\left(\lambda^0+\mu^0\right)(t-t_0)}\right)\right] - \\ \quad - \dfrac{\delta\mu}{\left(\lambda^0+\mu^0\right)^2}\left[\left(\lambda^0+\mu^0\right)\left[\mu^0 - \left(\lambda^0+\mu^0\right)\pi_{1_0}\right](t-t_0)e^{-\left(\lambda^0+\mu^0\right)(t-t_0)} + \lambda^0\left(1-e^{-\left(\lambda^0+\mu^0\right)(t-t_0)}\right)\right] \end{cases}$$

(3.52)

and using the Eq.(3.48) an analytical expression for the response sensitivity using the FSAP is obtained, i.e.

$$
\begin{aligned}
\delta A_{FSAP} &= \frac{1}{t_f}\int_{t_0}^{t_f} \phi_1(t)dt \\
&= \frac{1}{t_f}\left\{ \frac{\phi_{1_0}}{\lambda^0+\mu^0}\left(1-e^{-(\lambda^0+\mu^0)(t_f-t_0)}\right) - \right. \\
&\quad -\frac{\delta\lambda}{(\lambda^0+\mu^0)^3}\left\{\left(\pi_{1_0}(\lambda^0+\mu^0)-2\mu^0\right)\left(1-e^{-(\lambda^0+\mu^0)(t_f-t_0)}\right)+\right. \\
&\qquad \left. +(t_f-t_0)(\lambda^0+\mu^0)\left[\left(\mu^0-\pi_{1_0}(\lambda^0+\mu^0)\right)e^{-(\lambda^0+\mu^0)(t_f-t_0)}+\mu^0\right]\right\} - \\
&\quad -\frac{\delta\mu(t)}{(\lambda^0+\mu^0)^3}\left\{\left(\pi_{1_0}(\lambda^0+\mu^0)+\lambda^0-\mu^0\right)\left(1-e^{-(\lambda^0+\mu^0)(t_f-t_0)}\right)+\right. \\
&\qquad \left.\left. +(t_f-t_0)(\lambda^0+\mu^0)\left[\left(\mu^0-\pi_{1_0}(\lambda^0+\mu^0)\right)e^{-(\lambda^0+\mu^0)(t_f-t_0)}-\lambda^0\right]\right\}\right\}
\end{aligned}
\tag{3.53}
$$

3.3.2 ASAP for a binary component

The ASE are obtained following the ASAP, namely forming the inner product of the FSE together with a vector of adjoint functions $\Psi \equiv [\psi_1(t), \psi_2(t)]^T$ to obtain Eq.(3.22).

$$\langle \Psi, \boldsymbol{L}(\Phi)\rangle = \langle \boldsymbol{L}^*(\Psi), \Phi\rangle + \{\mathbf{P}[\Psi,\Phi]\}\Big|_{t_0}^{t_f}$$

Afterwards, the next sequence of operations as presented in previous section are performed, namely the adjoint operator is set to $\boldsymbol{L}^*(\Psi) = (\partial F/\partial \Pi^0)$ which leads to $\boldsymbol{L}^*(\Psi) = [\partial\pi_1(t)/\partial\pi_1, \partial\pi_1(t)/\partial\pi_2]^T = [1,0]^T$, the unknown values $\Phi(t_f)$ are eliminated by imposing $\Psi(t_f) = [0]$, and the known initial conditions for Φ is used. That sequence of operations transforms the Eq(3.22) to

$$\langle \partial F/\partial \Pi^0, \Phi\rangle = \langle \Psi, [\delta Q]\Pi^0(t)\rangle + \Psi^T(t_0)\Phi(t_0)$$

where the vector of adjoint functions satisfies the ASE

$$
\begin{cases}
\dfrac{d}{dt}\begin{bmatrix}\psi_1(t)\\ \psi_2(t)\end{bmatrix} + \begin{bmatrix}-\lambda^0 & \mu^0\\ \lambda^0 & -\mu^0\end{bmatrix}^T \begin{bmatrix}\psi_1(t)\\ \psi_2(t)\end{bmatrix} = -\begin{bmatrix}\partial F/\partial\pi_1\\ \partial F/\partial\pi_2\end{bmatrix} \\
\begin{bmatrix}\psi_1(t_f)\\ \psi_2(t_f)\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}
\end{cases}
\tag{3.54}
$$

The response sensitivity can be written now in terms of adjoint functions, i.e.,

$$\begin{aligned}\delta A(u^0,\alpha^0;\Phi,\delta\alpha) &= \frac{1}{t_f}\left(\int_{t_0}^{t_f}\Psi^T(t)\left[\delta Q\right]\Pi^0(t)dt+\Psi^T(t_0)\Phi(t_0)\right)\\ &= \frac{1}{t_f}\left(\int_{t_0}^{t_f}\left(\left[\psi_1(t),\psi_2(t)\right]\begin{bmatrix}-\delta\lambda & \delta\mu\\ \delta\lambda & -\delta\mu\end{bmatrix}\begin{bmatrix}\pi_1^0(t)\\ \pi_2^0(t)\end{bmatrix}\right)dt+\left[\psi_1(t_0),\psi_2(t_0)\right]\begin{bmatrix}\phi_1(t_0)\\ \phi_2(t_0)\end{bmatrix}\right)\\ &= \frac{1}{t_f}\left(\int_{t_0}^{t_f}\sum_{i=1}^{2}\psi_i(t)\sum_{j=1}^{2}\delta q_{ij}\pi_j^0(t)dt+\sum_{i=1}^{2}\psi_i(t_0)\phi_i(t_0)\right)\end{aligned}$$

(3.55)

The analytical solution of equation (3.54) is as below

$$\begin{cases}\psi_1(t)=\dfrac{\lambda^0}{\left(\lambda^0+\mu^0\right)^2}\left(\dfrac{\partial F}{\partial\pi_1}-\dfrac{\partial F}{\partial\pi_2}\right)\left(1-e^{-\left(\lambda^0+\mu^0\right)\left(t_f-t\right)}\right)+\\ \qquad+\dfrac{1}{\lambda^0+\mu^0}\left(\mu^0\dfrac{\partial F}{\partial\pi_1}+\lambda^0\dfrac{\partial F}{\partial\pi_2}\right)\left(t_f-t\right)\\ \psi_2(t)=\dfrac{\mu^0}{\left(\lambda^0+\mu^0\right)^2}\left(\dfrac{\partial F}{\partial\pi_1}-\dfrac{\partial F}{\partial\pi_2}\right)\left(e^{-\left(\lambda^0+\mu^0\right)\left(t_f-t\right)}-1\right)+\\ \qquad+\dfrac{1}{\lambda^0+\mu^0}\left(\mu^0\dfrac{\partial F}{\partial\pi_1}+\lambda^0\dfrac{\partial F}{\partial\pi_2}\right)\left(t_f-t\right)\end{cases} \tag{3.56}$$

Once the analytical solutions for the original Markov chain, FSE, and ASE are available, i.e. the Eqs.(3.44), (3.51), and (3.56), respectively, the response sensitivity obtained by using FSAP and ASAP can be compared.

Performing additional operations in Eq.(3.55) and taking in account the Eqs.(3.42), and (3.50), the response sensitivity using ASAP becomes

$$\begin{aligned}\delta A_{ASAP} = \frac{1}{t_f}\Big[\int_{t_0}^{t_f}\left(\psi_1(t)-\psi_2(t)\right)\left[\delta\mu\left(1-\pi_1^0(t)\right)-\delta\lambda\pi_1^0(t)\right]dt+\\ +\phi_1(t_0)\left(\psi_1(t_0)-\psi_2(t_0)\right)\Big]\end{aligned} \tag{3.57}$$

It follows from the above equation that the system response sensitivities to variations in system parameters are

$$\frac{\delta A_{ASAP}}{\partial\lambda}=\frac{1}{t_f}\int_{t_0}^{t_f}\left(\psi_1(t)-\psi_2(t)\right)\left(-\pi_1^0(t)\right)dt \tag{3.58}$$

$$\frac{\delta A_{ASAP}}{\partial\mu}=\frac{1}{t_f}\int_{t_0}^{t_f}\left(\psi_1(t)-\psi_2(t)\right)\left(1-\pi_1^0(t)\right)dt \tag{3.59}$$

$$\frac{\delta A_{ASAP}}{\partial\pi_{1_0}}=-\frac{\delta A_{ASAP}}{\partial\pi_{2_0}}=\frac{1}{t_f}\left(\psi_1(t_0)-\psi_2(t_0)\right) \tag{3.60}$$

Replacing the analytical solutions of $\pi_1^0(t)$, and $\psi_i(t), i=1,2$, from Eqs.(3.44), and (3.56), into Eq.(3.57) yields

$$\begin{aligned}\delta A_{ASAP} = \frac{1}{t_f}\Bigg\{ & \int_{t_0}^{t_f} \frac{1}{\lambda^0+\mu^0}\left(\frac{\partial F}{\partial \pi_1}-\frac{\partial F}{\partial \pi_2}\right)\left(1-e^{-(\lambda^0+\mu^0)(t_f-t)}\right)\Bigg[- \\ & -\delta\lambda\left(\frac{\mu^0}{\lambda^0+\mu^0}+\left(\pi_{1_0}-\frac{\mu^0}{\lambda^0+\mu^0}\right)e^{-(\lambda^0+\mu^0)(t-t_0)}\right)- \\ & -\delta\mu\left(-\frac{\lambda^0}{\lambda^0+\mu^0}+\left(\pi_{1_0}-\frac{\mu^0}{\lambda^0+\mu^0}\right)e^{-(\lambda^0+\mu^0)(t-t_0)}\right)\Bigg]dt+ \\ & +\frac{\phi_{1_0}}{\lambda^0+\mu^0}\left(\frac{\partial F}{\partial \pi_1}-\frac{\partial F}{\partial \pi_2}\right)\left(1-e^{-(\lambda^0+\mu^0)(t_f-t_0)}\right)\Bigg\}\end{aligned} \tag{3.61}$$

The final form of response sensitivity using ASAP is obtained after the integration, i.e.

$$\begin{aligned}\delta A_{ASAP} = \frac{1}{t_f}\left(\frac{\partial F}{\partial \pi_1}-\frac{\partial F}{\partial \pi_2}\right)\Bigg\{ & \frac{\phi_{1_0}}{\lambda^0+\mu^0}\left(1-e^{-(\lambda^0+\mu^0)(t_f-t_0)}\right)- \\ & -\frac{\delta\lambda}{(\lambda^0+\mu^0)^3}\Big\{\left(\pi_{1_0}(\lambda^0+\mu^0)-2\mu^0\right)\left(1-e^{-(\lambda^0+\mu^0)(t_f-t_0)}\right)+ \\ & +(t_f-t_0)(\lambda^0+\mu^0)\left[\left(\mu^0-\pi_{1_0}(\lambda^0+\mu^0)\right)e^{-(\lambda^0+\mu^0)(t_f-t_0)}+\mu^0\right]\Big\}- \\ & -\frac{\delta\mu}{(\lambda^0+\mu^0)^3}\Big\{\left(\pi_{1_0}(\lambda^0+\mu^0)+\lambda^0-\mu^0\right)\left(1-e^{-(\lambda^0+\mu^0)(t_f-t_0)}\right)+ \\ & +(t_f-t_0)(\lambda^0+\mu^0)\left[\left(\mu^0-\pi_{1_0}(\lambda^0+\mu^0)\right)e^{-(\lambda^0+\mu^0)(t_f-t_0)}-\lambda^0\right]\Big\}\Bigg\}\end{aligned} \tag{3.62}$$

Replacing the value vector $[\partial F/\partial \Pi] \equiv [\partial F/\partial \pi_1, \partial F/\partial \pi_2]^T$ in Eq.(3.62) considering the system response as was defined in Eq.(3.43), $[\partial F/\partial \Pi] = [\partial \pi_1(t)/\partial \pi_1, \partial \pi_1(t)/\partial \pi_2]^T = [1,0]^T$, it is obtained an identical expression for system sensitivity using ASAP as in Eq.(3.53) where it have been used FSAP, i.e.

$$\delta A_{ASAP} = \delta A_{FSAP} \tag{3.63}$$

Either from the Eq.(3.43) or from Eq.(3.62), it follows that the response sensitivities to system parameters are into a final form as follows,

$$\frac{\partial A}{\partial \lambda} = -\frac{1}{t_f}\frac{1}{\left(\lambda^0+\mu^0\right)^3}\left\{\left(\pi_{1_0}\left(\lambda^0+\mu^0\right)-2\mu^0\right)\left(1-e^{-\left(\lambda^0+\mu^0\right)\left(t_f-t_0\right)}\right)+\right.$$
$$\left.+\left(t_f-t_0\right)\left(\lambda^0+\mu^0\right)\left[\left(\mu^0-\pi_{1_0}\left(\lambda^0+\mu^0\right)\right)e^{-\left(\lambda^0+\mu^0\right)\left(t_f-t_0\right)}+\mu^0\right]\right\} \tag{3.64}$$

$$\frac{\partial A}{\partial \mu} = -\frac{1}{t_f}\frac{1}{\left(\lambda^0+\mu^0\right)^3}\left\{\left(\pi_{1_0}\left(\lambda^0+\mu^0\right)+\lambda^0-\mu^0\right)\left(1-e^{-\left(\lambda^0+\mu^0\right)\left(t_f-t_0\right)}\right)+\right.$$
$$\left.+\left(t_f-t_0\right)\left(\lambda^0+\mu^0\right)\left[\left(\mu^0-\pi_{1_0}\left(\lambda^0+\mu^0\right)\right)e^{-\left(\lambda^0+\mu^0\right)\left(t_f-t_0\right)}-\lambda^0\right]\right\} \tag{3.65}$$

$$\frac{\partial A}{\partial \pi_{1_0}} = -\frac{\partial A}{\partial \pi_{2_0}} = \frac{1}{t_f}\frac{1}{\lambda^0+\mu^0}\left(1-e^{-\left(\lambda^0+\mu^0\right)\left(t_f-t_0\right)}\right) \tag{3.66}$$

It can be seen from the above sensitivities that for a large mission time, i.e. $t_f \rightarrow \infty$, the sensitivities to the initial conditions (3.66) vanish as they should for a well-posed problem, i.e.

$$\lim_{t_f\rightarrow\infty}\frac{\partial A}{\partial \pi_{1_0}} = 0 = \lim_{t_f\rightarrow\infty}\frac{\partial A}{\partial \pi_{2_0}} \tag{3.67}$$

Furthermore, for $t_f \rightarrow \infty$ the transient solution (3.44) trends to the stationary solution which is the solution of the algebraic system of equations defined by Eq.(2.24) together with the conservation law of probability, and the initial state vector, i.e. the first term from the right-hand side of the Eq.(3.44). For the response sensitivities, taking the limit as $t_f \rightarrow \infty$ yields the response sensitivities as for the stationary solution case, i.e.,

$$\lim_{t_f\rightarrow\infty}\frac{\partial A}{\partial \lambda} = -\frac{\mu^0}{\left(\lambda^0+\mu^0\right)^2} \quad \text{and} \quad \lim_{t_f\rightarrow\infty}\frac{\partial A}{\partial \mu} = \frac{\lambda^0}{\left(\lambda^0+\mu^0\right)^2} \tag{3.68}$$

3.3.3 Numerical validation

Further in this section, the numerical comparisons between the analytical solution of the response sensitivity obtained previously and the numerical solution obtained using MARKOMAG code with VODPK ODE solver integrated are presented. The input data have been taken from IFMIF (International Fusion Materials Irradiation Facilities)-Accelerator System Facilities reliability study.[75, 76] The transient reliability and sensitivity study of this physical system will be presented into the next chapter. In this section are considered some of its facilities as binary systems which can be either in operational or not-operational state. For each facility is performed using the procedures presented previous, sensitivity studies of the response given by Eq.(3.43). The response sensitivities to the variations in system parameters are given by the Eqs.(3.64) to (3.66). The transient analysis has been performed for a mission time of 168 hours which is considered to be the period of time between two scheduled maintenance operations. After a maintenance operation the physical system analyzed is considered to be as good as new. Therefore it is set further the initial time $t_0 = 0$ and the final time $t_f = 168$ hours. The initial probability state vector is set to $[\pi_{1_0}, \pi_{2_0}]^T = [1,0]^T$, i.e. the physical system is in the full operational state at the initial moment of time. The input data are constants as mean time to failure and repair, and therefore, the failures are considered exponentially distributed. Cumulative distribution function is in this case given by $f(t) = \lambda e^{-\lambda t}$, $0 \le t < \infty$, where λ is the distribution parameter. It follows from the equivalent relations given in Table 2.1 that the transition rate $\lambda(t)$ is constant and equal with the distribution parameter.

Facility	***MTTF***(hours)	***MTTR***(hours)	$\lambda = 1/MTTF$	$\mu = 1/MTTR$
Injector	156.7	2.2	6.38162E-03	4.54545E-01
LINAC	465.0	19.0	2.15054E-03	5.26316E-02
Cooling System	499996.0	4.0	2.00002E-06	2.5E-01
RF System	225.5	9.0	4.43459E-03	1.11111E-01
HEBT	224.5	7.7	4.45434E-03	1.29870E-01

Table 3.1 The nominal values of system parameters for binary components

In the above table LINAC stands for Linear Accelerator System, RF for Radio Frequency System, and HEBT for High Energy Beam Transport System.

The perturbations considered further are introduced numerically and not correspond to actual physical reliability parameter's uncertainties. They are used as mathematical means to verify the accuracy and stability of the numerical solution of ASE. These variations in system parameters have been performed as follows. In practice one wishes that the *MTTF* for a component to be large and *MTTR* to be small. Therefore the *MTTF* has been perturbed by increasing it with a percentage of 0.1, 1.0, 5.0, and 10.0, respectively, of its base-case value, and the *MTTR* has been varied in the same way by decreasing it with the same percents from its nominal value as before. That leads to a decreasing of the failure rate, and to an increasing of repair rate. For the perturbations in the initial conditions does exist a linear dependency due to the conservation law of probability. Therefore, a perturbation in one of the initial state probability is equal and opposite in sign with the other such as the conservation of probability to be satisfied. It has been considered variations in initial state probability by 0.1, 5.0, and 10.0 percents from their nominal values. That means that for 100% probability the system to be in the *Up* state at the initial time, the perturbation with 5% in initial conditions means that at the initial time it is a 95% probability that the system to be in the *Up* state and 5% probability that the system to be in the *Down* state. That can be resumed for the perturbed parameters as follows:

$$\begin{matrix} MTTF = MTTF^0 + \delta MTTF, & \delta MTTF = x \cdot MTTF^0 \\ MTTR = MTTR^0 + \delta MTTR, & \delta MTTR = -x \cdot MTTR^0 \end{matrix}, \quad x = \{0.1\%, 1.0\%, 5.0\%, 10\%\}$$

$$\pi_{1_0} = \pi_{1_0}^0 + \delta\pi_{1_0}, \quad \delta\pi_{1_0} = y \cdot \pi_{1_0}^0, \quad y = \{-0.1\%, -5.0\%, -10\%\}$$
$$\pi_{2_0} = \pi_{2_0}^0 - \delta\pi_{1_0}$$
$$\pi_{1_0} + \pi_{2_0} = 1$$
$$\pi_{i_0} \in [0,1], \; i = 1,2$$

For each of the previous systems considered as binary components, the response sensitivities to the perturbation in initial conditions are presented into the Tables 3.2 - 3.6. The time step has been chosen small ($\Delta t = 0.6\,\text{min}$) for the numerical solution by stiffness considerations. The comparison of ASAP numerical solution has been performed with the analytical solution, the numerical solution using recalculations and numerical solution given by FSAP. In this case the analytical solution is as in Eq.(3.66).

Table 3.2 The influence of perturbations in initial state probabilities for Injector

Perturbation in initial conditions	Transient Duration (h) /No. of time steps	Nominal Value R^0	Relative sensitivity $\frac{\Delta R}{\Delta \alpha_i} \cdot \frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	Analytical DR	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
-0.1% of π_{1_0} and 0.1% of π_{2_0}	1 / 100	.9972E+00	.8034E+00	-.8012E-03	-.8012E-03	-.8012E-03	-.8012E-03
	6 / 600	.9908E+00	.3420E+00	-.3388E-03	-.3388E-03	-.3388E-03	-.3388E-03
	12 / 1200	.9886E+00	.1821E+00	-.1801E-03	-.1801E-03	-.1801E-03	-.1801E-03
	24 / 2400	.9874E+00	.9155E-01	-.9040E-04	-.9040E-04	-.9040E-04	-.9040E-04
	168 / 16800	.9863E+00	.1309E-01	-.1291E-04	-.1291E-04	-.1291E-04	-.1291E-04
-5% of π_{1_0} and 5% of π_{2_0}	1 / 100	.9972E+00	.8034E+00	-.4006E-01	-.4006E-01	-.4006E-01	-.4006E-01
	6 / 600	.9908E+00	.3420E+00	-.1694E-01	-.1694E-01	-.1694E-01	-.1694E-01
	12 / 1200	.9886E+00	.1821E+00	-.9004E-02	-.9004E-02	-.9004E-02	-.9004E-02
	24 / 2400	.9874E+00	.9155E-01	-.4520E-02	-.4520E-02	-.4520E-02	-.4520E-02
	168 / 16800	.9863E+00	.1309E-01	-.6457E-03	-.6457E-03	-.6457E-03	-.6457E-03
-10% of π_{1_0} and 10% of π_{2_0}	1 / 100	.9972E+00	.8034E+00	-.8012E-01	-.8012E-01	-.8012E-01	-.8012E-01
	6 / 600	.9908E+00	.3420E+00	-.3388E-01	-.3388E-01	-.3388E-01	-.3388E-01
	12 / 1200	.9886E+00	.1821E+00	-.1801E-01	-.1801E-01	-.1801E-01	-.1801E-01
	24 / 2400	.9874E+00	.9155E-01	-.9040E-02	-.9040E-02	-.9040E-02	-.9040E-02
	168 / 16800	.9863E+00	.1309E-01	-.1291E-02	-.1291E-02	-.1291E-02	-.1291E-02

Table 3.3 The influence of perturbations in initial state probabilities for LINAC

Perturbation in initial conditions	Transient Duration (h) /No. of time steps	Nominal Value R^0	Relative sensitivity $\frac{\Delta R}{\Delta \alpha_i} \cdot \frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	Analytical DR	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
-0.1% of π_{1_0} and 0.1% of π_{2_0}	1 / 100	.9989E+00	.9741E+00	-.9731E-03	-.9731E-03	-.9731E-03	-.9731E-03
	6 / 600	.9942E+00	.8572E+00	-.8523E-03	-.8523E-03	-.8523E-03	-.8523E-03
	12 / 1200	.9895E+00	.7407E+00	-.7329E-03	-.7329E-03	-.7329E-03	-.7329E-03
	24 / 2400	.9826E+00	.5662E+00	-.5563E-03	-.5563E-03	-.5563E-03	-.5563E-03
	168 / 16800	.9650E+00	.1126E+00	-.1086E-03	-.1086E-03	-.1086E-03	-.1086E-03
-5% of π_{1_0} and 5% of π_{2_0}	1 / 100	.9989E+00	.9741E+00	-.4866E-01	-.4866E-01	-.4866E-01	-.4866E-01
	6 / 600	.9942E+00	.8572E+00	-.4261E-01	-.4261E-01	-.4261E-01	-.4261E-01
	12 / 1200	.9895E+00	.7407E+00	-.3664E-01	-.3664E-01	-.3664E-01	-.3664E-01
	24 / 2400	.9826E+00	.5662E+00	-.2782E-01	-.2782E-01	-.2782E-01	-.2782E-01
	168 / 16800	.9650E+00	.1126E+00	-.5432E-02	-.5432E-02	-.5432E-02	-.5432E-02
-10% of π_{1_0} and 10% of π_{2_0}	1 / 100	.9989E+00	.9741E+00	-.9731E-01	-.9731E-01	-.9731E-01	-.9731E-01
	6 / 600	.9942E+00	.8572E+00	-.8523E-01	-.8523E-01	-.8523E-01	-.8523E-01
	12 / 1200	.9895E+00	.7407E+00	-.7329E-01	-.7329E-01	-.7329E-01	-.7329E-01
	24 / 2400	.9826E+00	.5662E+00	-.5563E-01	-.5563E-01	-.5563E-01	-.5563E-01
	168 / 16800	.9650E+00	.1126E+00	-.1086E-01	-.1086E-01	-.1086E-01	-.1086E-01

Table 3.4 The influence of perturbations in initial state probabilities for Cooling System

Perturbation in initial conditions	Transient Duration (h) /No. of time steps	Nominal Value R^0	Relative sensitivity $\frac{\Delta R}{\Delta \alpha_i} \cdot \frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	Analytical DR	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
-0.1% of π_{1_0} and 0.1% of π_{2_0}	1 / 100	9.99999E-01	.8848E+00	-.8848E-03	-.8848E-03	-.8848E-03	-.8848E-03
	6 / 600	9.99996E-01	.5179E+00	-.5179E-03	-.5179E-03	-.5179E-03	-.5179E-03
	12 / 1200	9.99995E-01	.3167E+00	-.3167E-03	-.3167E-03	-.3167E-03	-.3167E-03
	24 / 2400	9.99993E-01	.1663E+00	-.1663E-03	-.1663E-03	-.1663E-03	-.1663E-03
	168 / 16800	9.99992E-01	.2381E-01	-.2381E-04	-.2381E-04	-.2381E-04	-.2381E-04
-5% of π_{1_0} and 5% of π_{2_0}	1 / 100	9.99999E-01	.8848E+00	-.4424E-01	-.4424E-01	-.4424E-01	-.4424E-01
	6 / 600	9.99996E-01	.5179E+00	-.2590E-01	-.2590E-01	-.2590E-01	-.2590E-01
	12 / 1200	9.99995E-01	.3167E+00	-.1584E-01	-.1584E-01	-.1584E-01	-.1584E-01
	24 / 2400	9.99993E-01	.1663E+00	-.8313E-02	-.8313E-02	-.8313E-02	-.8313E-02
	168 / 16800	9.99992E-01	.2381E-01	-.1190E-02	-.1190E-02	-.1190E-02	-.1190E-02
-10% of π_{1_0} and 10% of π_{2_0}	1 / 100	9.99999E-01	.8848E+00	-.8848E-01	-.8848E-01	-.8848E-01	-.8848E-01
	6 / 600	9.99996E-01	.5179E+00	-.5179E-01	-.5179E-01	-.5179E-01	-.5179E-01
	12 / 1200	9.99995E-01	.3167E+00	-.3167E-01	-.3167E-01	-.3167E-01	-.3167E-01
	24 / 2400	9.99993E-01	.1663E+00	-.1663E-01	-.1663E-01	-.1663E-01	-.1663E-01
	168 / 16800	9.99992E-01	.2381E-01	-.2381E-02	-.2381E-02	-.2381E-02	-.2381E-02

Table 3.5 The influence of perturbations in initial state probabilities for RF System

Perturbation in initial conditions	Transient Duration (h) /No. of time steps	Nominal Value R^0	Relative sensitivity $\frac{\Delta R}{\Delta \alpha_i} \cdot \frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	Analytical DR	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
-0.1% of π_{1_0} and 0.1% of π_{2_0}	1 / 100	.9979E+00	.9464E+00	-.9444E-03	-.9444E-03	-.9444E-03	-.9444E-03
	6 / 600	.9893E+00	.7291E+00	-.7213E-03	-.7213E-03	-.7213E-03	-.7213E-03
	12 / 1200	.9824E+00	.5507E+00	-.5410E-03	-.5410E-03	-.5410E-03	-.5410E-03
	24 / 2400	.9746E+00	.3469E+00	-.3381E-03	-.3381E-03	-.3381E-03	-.3381E-03
	168 / 16800	.9636E+00	.5346E-01	-.5152E-04	-.5152E-04	-.5152E-04	-.5152E-04
-5% of π_{1_0} and 5% of π_{2_0}	1 / 100	.9979E+00	.9464E+00	-.4722E-01	-.4722E-01	-.4722E-01	-.4722E-01
	6 / 600	.9893E+00	.7291E+00	-.3607E-01	-.3607E-01	-.3607E-01	-.3607E-01
	12 / 1200	.9824E+00	.5507E+00	-.2705E-01	-.2705E-01	-.2705E-01	-.2705E-01
	24 / 2400	.9746E+00	.3469E+00	-.1690E-01	-.1690E-01	-.1690E-01	-.1690E-01
	168 / 16800	.9636E+00	.5346E-01	-.2576E-02	-.2576E-02	-.2576E-02	-.2576E-02
-10% of π_{1_0} and 10% of π_{2_0}	1 / 100	.9979E+00	.9464E+00	-.9444E-01	-.9444E-01	-.9444E-01	-.9444E-01
	6 / 600	.9893E+00	.7291E+00	-.7213E-01	-.7213E-01	-.7213E-01	-.7213E-01
	12 / 1200	.9824E+00	.5507E+00	-.5410E-01	-.5410E-01	-.5410E-01	-.5410E-01
	24 / 2400	.9746E+00	.3469E+00	-.3381E-01	-.3381E-01	-.3381E-01	-.3381E-01
	168 / 16800	.9636E+00	.5346E-01	-.5152E-02	-.5152E-02	-.5152E-02	-.5152E-02

Table 3.6 The influence of perturbations in initial state probabilities for HEBT

Perturbation in initial conditions	Transient Duration (h) /No. of time steps	Nominal Value R^0	Relative sensitivity $\frac{\Delta R}{\Delta \alpha_i} \cdot \frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	Analytical DR	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
-0.1% of π_{1_0} and 0.1% of π_{2_0}	1 / 100	.9979E+00	.9377E+00	-.9357E-03	-.9357E-03	-.9357E-03	-.9357E-03
	6 / 600	.9896E+00	.6938E+00	-.6866E-03	-.6866E-03	-.6866E-03	-.6866E-03
	12 / 1200	.9833E+00	.5050E+00	-.4966E-03	-.4966E-03	-.4966E-03	-.4966E-03
	24 / 2400	.9767E+00	.3049E+00	-.2978E-03	-.2978E-03	-.2978E-03	-.2978E-03
	168 / 16800	.9683E+00	.4576E-01	-.4431E-04	-.4431E-04	-.4431E-04	-.4431E-04
-5% of π_{1_0} and 5% of π_{2_0}	1 / 100	.9979E+00	.9377E+00	-.4679E-01	-.4679E-01	-.4679E-01	-.4679E-01
	6 / 600	.9896E+00	.6938E+00	-.3433E-01	-.3433E-01	-.3433E-01	-.3433E-01
	12 / 1200	.9833E+00	.5050E+00	-.2483E-01	-.2483E-01	-.2483E-01	-.2483E-01
	24 / 2400	.9767E+00	.3049E+00	-.1489E-01	-.1489E-01	-.1489E-01	-.1489E-01
	168 / 16800	.9683E+00	.4576E-01	-.2216E-02	-.2216E-02	-.2216E-02	-.2216E-02
-10% of π_{1_0} and 10% of π_{2_0}	1 / 100	.9979E+00	.9377E+00	-.9357E-01	-.9357E-01	-.9357E-01	-.9357E-01
	6 / 600	.9896E+00	.6938E+00	-.6866E-01	-.6866E-01	-.6866E-01	-.6866E-01
	12 / 1200	.9833E+00	.5050E+00	-.4966E-01	-.4966E-01	-.4966E-01	-.4966E-01
	24 / 2400	.9767E+00	.3049E+00	-.2978E-01	-.2978E-01	-.2978E-01	-.2978E-01
	168 / 16800	.9683E+00	.4576E-01	-.4431E-02	-.4431E-02	-.4431E-02	-.4431E-02

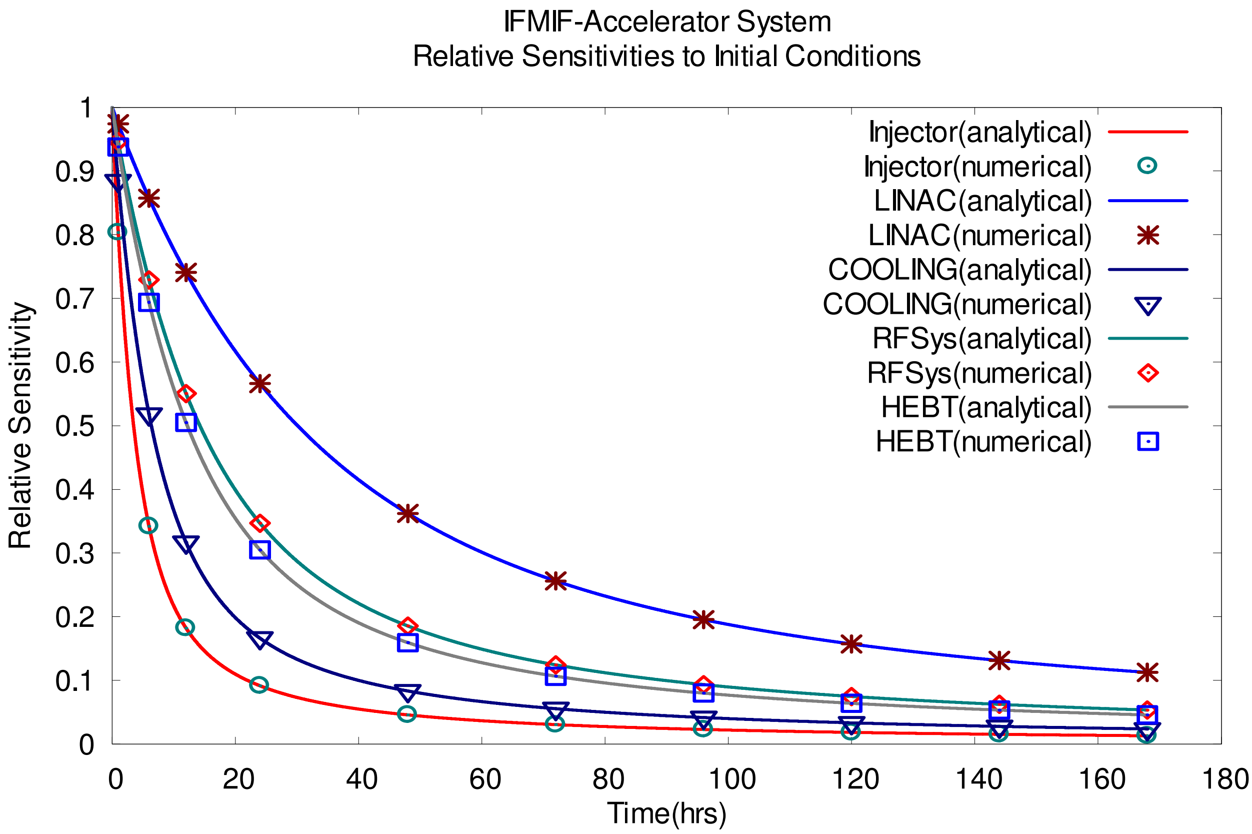


Figure 3.1 Relatives sensitivities to perturbations in initial state probabilities for binary components

In Fig.3.1 it is shown the relative sensitivities, numerical and analytical solutions, for each binary component considering perturbations in initial state probabilities. It can be seen a good agreement between the numerical and analytical solution.

For the perturbations in system parameters the response sensitivities are presented in the Tables 3.7 to 3.11. For this case the analytical solution is as in Eq.(3.64) for perturbation in *MTTF* and as in Eq.(3.65) for perturbations in *MTTR*. One can be see that the analytical and numerical sensitivity solutions given by ASAP and FSAP agree each other but for some cases these results are different from the exact recalculations, especially when the percentage of variation in system parameters is large. Also the stiffness of the problem can influence this difference, and that can be observed especially when are performed perturbations in *MTTR* which have small value in comparison with the perurbations in *MTTF*. This is a reason for which the sensitivities using exact recalculations agree in general better with sensitivities computed for perturbations in *MTTF* versus the perturbations in *MTTR*.

Table 3.7 The influence of perturbations in system parameters at t_f =168h for Injector

Par α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Relative sensitivity $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	Analytical DR	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ	-0.1%	.9863E+00	-.1367E-01	.1348E-04	.1348E-04	.1348E-04	.1348E-04
	-1%			.1348E-03	.1348E-03	.1348E-03	.1348E-03
	-5%			.6740E-03	.6740E-03	.6744E-03	.6740E-03
	-10%			.1348E-02	.1348E-02	.1350E-02	.1348E-02
μ	0.1%		.1349E-01	.1330E-04	.1330E-04	.1329E-04	.1330E-04
	1%			.1330E-03	.1330E-03	.1317E-03	.1330E-03
	5%			.6650E-03	.6650E-03	.6342E-03	.6651E-03
	10%			.1330E-02	.1330E-02	.1212E-02	.1330E-02

Table 3.8 The influence of perturbations in system parameters at t_f =168h for LINAC

Par α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Relative sensitivity $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	Analytical DR	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ	-0.1%	.9650E+00	-.3501E-01	.3378E-04	.3379E-04	.3379E-04	.3378E-04
	-1%			.3378E-03	.3379E-03	.3380E-03	.3378E-03
	-5%			.1689E-02	.1689E-02	.1692E-02	.1689E-02
	-10%			.3378E-02	.3379E-02	.3390E-02	.3378E-02
μ	0.1%		.3059E-01	.2952E-04	.2952E-04	.2950E-04	.2952E-04
	1%			.2952E-03	.2952E-03	.2928E-03	.2952E-03
	5%			.1476E-02	.1476E-02	.1417E-02	.1476E-02
	10%			.2952E-02	.2952E-02	.2726E-02	.2952E-02

Table 3.9 The influence of perturbation in system parameters at t_f =168h for RF System

Par α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Relative sensitivity $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	Analytical DR	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ	-0.1%	.9636E+00	-.3641E-01	.3508E-04	.3508E-04	.3508E-04	.3508E-04
	-1%			.3508E-03	.3508E-03	.3508E-03	.3508E-03
	-5%			.1754E-02	.1754E-02	.1757E-02	.1754E-02
	-10%			.3508E-02	.3508E-02	.3514E-02	.3508E-02
μ	0.1%		.3435E-01	.3310E-04	.3310E-04	.3311E-04	.3310E-04
	1%			.3310E-03	.3310E-03	.3314E-03	.3310E-03
	5%			.1655E-02	.1655E-02	.1663E-02	.1655E-02
	10%			.3310E-02	.3310E-02	.3342E-02	.3310E-02

Table 3.10 The influence of perturbation in system parameters at t_f =168h for Cooling System

Par α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Relative sensitivity $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	Analytical DR	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ	-0.1%	9.99992E-1	-.7810E-05	.7809E-08	.7809E-08	.7810E-08	.7809E-08
	-1%			.7809E-07	.7809E-07	.7810E-07	.7809E-07
	-5%			.3905E-06	.3905E-06	.3905E-06	.3905E-06
	-10%			.7809E-06	.7809E-06	.7809E-06	.7809E-06
μ	0.1%		.7619E-05	.7619E-08	.7619E-08	.7612E-08	.7619E-08
	1%			.7619E-07	.7619E-07	.7545E-07	.7619E-07
	5%			.3809E-06	.3809E-06	.3632E-06	.3810E-06
	10%			.7619E-06	.7619E-06	.6942E-06	.7619E-06

Table 3.11 The influence of perturbation in system parameters at t_f =168h for HEBT

Par α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Relative sensitivity $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	Analytical DR	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ	-0.1%	.9683E+00	-.3169E-01	.3069E-04	.3069E-04	.3069E-04	.3069E-04
	-1%			.3069E-03	.3069E-03	.3070E-03	.3069E-03
	-5%			.1534E-02	.1534E-02	.1537E-02	.1534E-02
	-10%			.3069E-02	.3069E-02	.3079E-02	.3069E-02
μ	0.1%		.3018E-01	.2922E-04	.2922E-04	.2919E-04	.2922E-04
	1%			.2922E-03	.2922E-03	.2895E-03	.2922E-03
	5%			.1461E-02	.1461E-02	.1397E-02	.1461E-02
	10%			.2922E-02	.2922E-02	.2946E-02	.2922E-02

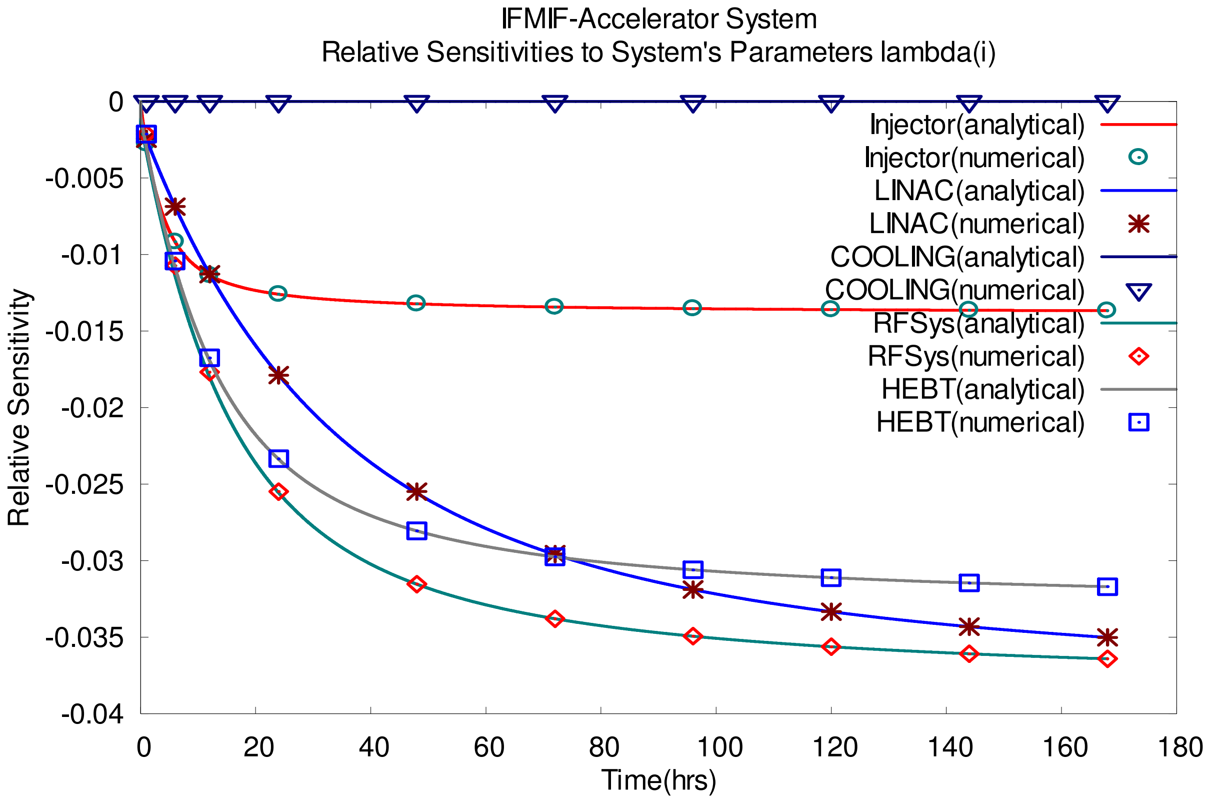


Figure 3.2 Relatives sensitivities to perturbations in system parameters λ for binary components

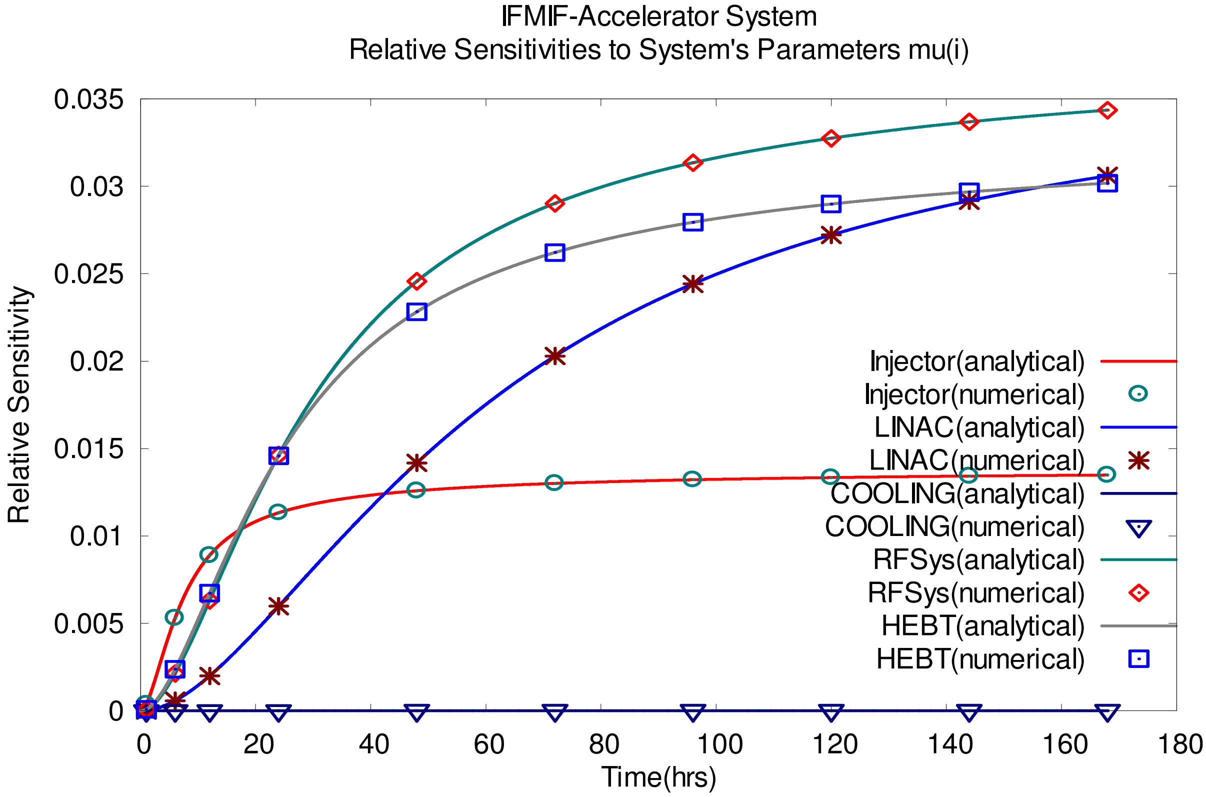


Figure 3.3 Relatives sensitivities to perturbations in system parameters μ for binary components

3.4 Implementation considerations

Standard techniques for solving the Kolmogorov equations which describe the Markov chain are based on discretization of this set of ordinary differential equations and approximate the solution numerically. The methods discretize the time interval into a finite number of subintervals and compute the solution step by step.

The system of Eqs.(3.1) is currently solved numerically using available differential equation solvers based on multistep methods such as Adams formulae and backward differential formulae (BDF) combined with Krylov subspace methods, since Markov chain equations are often very large and sparse.[20,74] These methods have been proved to be applicable in various studies of transient solution of Markov chain.[23,36] These studies have shown that the Adams and BDF methods are suitable for problems with long interval of integration and high accuracy requirements. Adams methods are employed in the case of nonstiff problems, while BDF methods are recommended when the system of equations is stiff.

For $i=1,...,n$ the differential backward Kolmogorov equations (3.1), FSE (3.18), and ASE (3.36), can be written in component form as follows.

Backward Kolmogorov equations (BKE):

$$\begin{cases} \dfrac{d\pi_i(t)}{dt} = \sum_{j=1}^{n} q_{ij}(t)\pi_j(t) \\ \pi_i(t_0) = \pi_{i_0} \end{cases} \tag{3.69}$$

Forward sensitivity equations (FSE) of Markov chain:

$$\begin{cases} \dfrac{d\phi_i(t)}{dt} - \sum_{j=1}^{n} q_{ij}^0(t)\phi_j(t) = \sum_{j=1}^{n} \delta q_{ij}\pi_j^0(t) \\ \phi_i(t_0) = \phi_{i_0} \end{cases} \tag{3.70}$$

Adjoint sensitivity equations (ASE) of Markov chain:

$$\begin{cases} \dfrac{d\psi_i(t)}{dt} + \sum_{j=1}^{n} q_{ji}^0(t)\psi_j(t) = -\dfrac{\partial F}{\partial \pi_i^0} \\ \psi_i(t = t_f) = 0 \end{cases} \tag{3.71}$$

All these linear ODE systems can be written generally in component form for all $i=1,...,n$ as follows

$$\begin{cases} \dfrac{dy_i}{dt} = f_i(t, y_1, ..., y_n) \\ y_i(t_0) = y_{i_0} \end{cases} \tag{3.72}$$

Where for all $i=1,...,n$

$$f_i(t, y_1, ..., y_n) \equiv \begin{cases} f_i(t, \pi_1, ..., \pi_n) = \sum_{j=1}^{n} q_{ij}(t)\pi_j(t), \quad \text{for BKE} \\ f_i(t, \phi_1, ..., \phi_n) = \sum_{j=1}^{n} q_{ij}^0(t)\phi_j(t) + \sum_{j=1}^{n} \delta q_{ij}\pi_j^0(t), \quad \text{for FSE} \\ f_i(t, \psi_1, ..., \psi_n) = -\sum_{j=1}^{n} q_{ji}^0(t)\psi_j(t) - \dfrac{\partial F}{\partial \pi_i^0}, \quad \text{for ASE} \end{cases} \tag{3.73}$$

The system of equations (3.72) is a linear system of form $[\mathrm{A}]_{n\times n}[x]_{n\times 1} = [b]_{n\times 1}$, or in component form as $\sum_{j=1}^{n} a_{ij}x_j = b_i$, $i=1,...n$, where the square matrix A is $[\mathrm{A}]_{n\times n} = [\boldsymbol{I}]_{n\times n} - h[\boldsymbol{J}]_{n\times n}$, with $\boldsymbol{I}$ the identity matrix, h a scalar, and $\boldsymbol{J}$ the *Jacobian-matrix* defined as below

$$
\boldsymbol{J} \equiv \begin{bmatrix} \frac{\partial f_1}{\partial y_1} & \cdots & \frac{\partial f_1}{\partial y_j} & \cdots & \frac{\partial f_1}{d y_n} \\ \vdots & \ddots & \vdots & & \vdots \\ \frac{\partial f_i}{\partial y_1} & \cdots & \frac{\partial f_i}{\partial y_j} & \cdots & \frac{\partial f_i}{\partial y_n} \\ \vdots & & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial y_1} & \cdots & \frac{\partial f_n}{\partial y_j} & \cdots & \frac{\partial f_n}{\partial y_n} \end{bmatrix}, \text{ with } y_i = \begin{cases} \pi_i, & \text{for BKE} \\ \phi_i, & \text{for FSE} \\ \psi_i, & \text{for ASE} \end{cases}, \; i = 1,...,n
$$

One can observe that the Jacobian-matrix is the same for the backward Kolmogorov equations (BKE) and FSE, i.e. the transition rate matrix $Q(t)$, and minus transpose of $Q(t)$ for ASE, and therefore the same routine for Jacobian matrix evaluation can be used for all three cases. The developed computer code MARKOMAG-S is using for the transient solution of the original system of Eqs.(3.1), FSE (3.18), and ASE (3.36), respectively, the VODPK solver developed by Brown, Byrne, and Hindmarsh[72,73] together with the incomplete LU factorization for preconditioning.[20] This solver uses both methods Adams and BDF with preconditioned Krylov method GMRES[68,73] to evaluate the transient solution of stiff and nonstiff systems of ordinary differential equations $dy/dt = f(t, y)$. The discretization for this type of equations and the numerical solution approximation using the methods mentioned previous are treated amply in Radhakrisnan and Hindmarsh [1993],[71] Brown et al.[1989],[72] and Brown and Hindmarsh [1989].[73]

System response sensitivity (3.38) written in component form for all $i = 1,...,n$ is as below

$$
\begin{aligned} & DR(\pi_1^0,...,\pi_n^0,\alpha_1^0,...,\alpha_n^0;\phi_1,...,\phi_n,\delta\alpha_1,...,\delta\alpha_n) = \\ & = \int_{t_0}^{t_f} \sum_{k=1}^{m} \left(\frac{\partial F}{\partial \alpha_k} \right) \delta\alpha_k dt + \int_{t_0}^{t_f} \sum_{i=1}^{n} \psi_i(t) \sum_{j=1}^{n} \left(\delta q_{ij} \pi_j(t) \right) dt + \sum_{i=1}^{n} \left(\psi_i(t_0) \phi_{i_0} \right) \end{aligned} \tag{3.74}
$$

The numerical integration of system response and response sensitivities are performed using one of the Newton-Cotes quadrature formulae. Applying a quadrature rule to discretize the above integral form of the response sensitivity would yield

$$
\begin{aligned} & DR(\pi_1^0,...,\pi_n^0,\alpha_1^0,...,\alpha_n^0;\phi_1,...,\phi_n,\delta\alpha_1,...,\delta\alpha_n) = \\ & = \sum_{l=t_0}^{N} \sum_{k=1}^{m} \left(\frac{\partial F}{\partial \alpha_k} \right)^l \delta\alpha_k \Delta t_l + \sum_{l=t_0}^{N} \sum_{i=1}^{n} \psi_i(t_l) \sum_{k=1}^{m} \left(\frac{\partial f_i}{\partial \alpha_k} \right)^l \delta\alpha_k \Delta t_l + \sum_{i=1}^{n} \left(\psi_i(t_0) \phi_{i_0} \right) \end{aligned} \tag{3.75}
$$

where the superscript l denotes that the respective term is evaluated at the time t_l, and

$$
\sum_{j=1}^{n} \delta q_{ij}{}^l \pi_j(t_l) = \sum_{k=1}^{m} \left(\frac{\partial f_i}{\partial \alpha_k} \right)^l \delta\alpha_k, \quad i = 1,...,n \text{, with } f_i(t) = \sum_{j=1}^{n} q_{ij}(t) \pi_j(t). \tag{3.76}
$$

$N = (t_f - t_0)/\Delta t_l$ represents the number of time steps in quadrature and should be equal with the total number of time steps used in numerical evaluation of the solution of BKE (3.1), and ASE (3.36). That implies that the time step Δt_l in both cases should be the same to assure the consistency with the derivatives $\partial f_i / \partial \alpha_k$ which are evaluated during the solving process of ASE. Here one can see again the advantages of the ASAP, i.e. after the BKE and ASE are solved, in Eq.(3.75) all parameters are known, and considering the same system response, for a new variation $\delta\alpha_k$ in system parameters should be performed only integrations based on quadrature formulae which is cheaper than to solve many times either BKE or FSE.

Further, the steps of the computer code system QUEFT/MARKOMAG-S/MCADJSEN developed with this work for performing reliability and sensitivity analysis using Markov chain are presented. The flow diagram of this code system is presented in Fig.3.4. The abstraction of physical system as a Fault Tree together with components data (failure rates, distribution types, distribution parameters) are the input for code. The first module QUEFT is getting these data and performs a qualitative Fault-Tree Analysis using the MOCUS algorithm described in Appendix A to find the minimal cut sets. The output of this module represents the input for the next module MARKOMAG-S. This module is generating the Markov chain of the physical system based on the number of system components and generic fault states which are constructed using the minimal cut sets found by QUEFT, as it has been described in Section 2.2. At the end of this process the backward Kolmogorov differential system of Eqs. (3.69) is built symbolically. Further, depending on the type of Markov chain, the values in transition rate matrix are computed once for the homogeneous case, and each time step for the nonhomogeneous case. All the elements of the Kolmogorov system of equations are stored in vector form, and for transition rate matrix is used compressed row storage. All the vector matrix multiplications during of solving process are performed in sparse form. The VODPK ODE Solver is used afterwards to solve the system of differential equations and to obtain the transient probability distribution of the system state vector. Using the transient solution, the system response is computed for the base-case. At this step the reliability analysis phase is done.

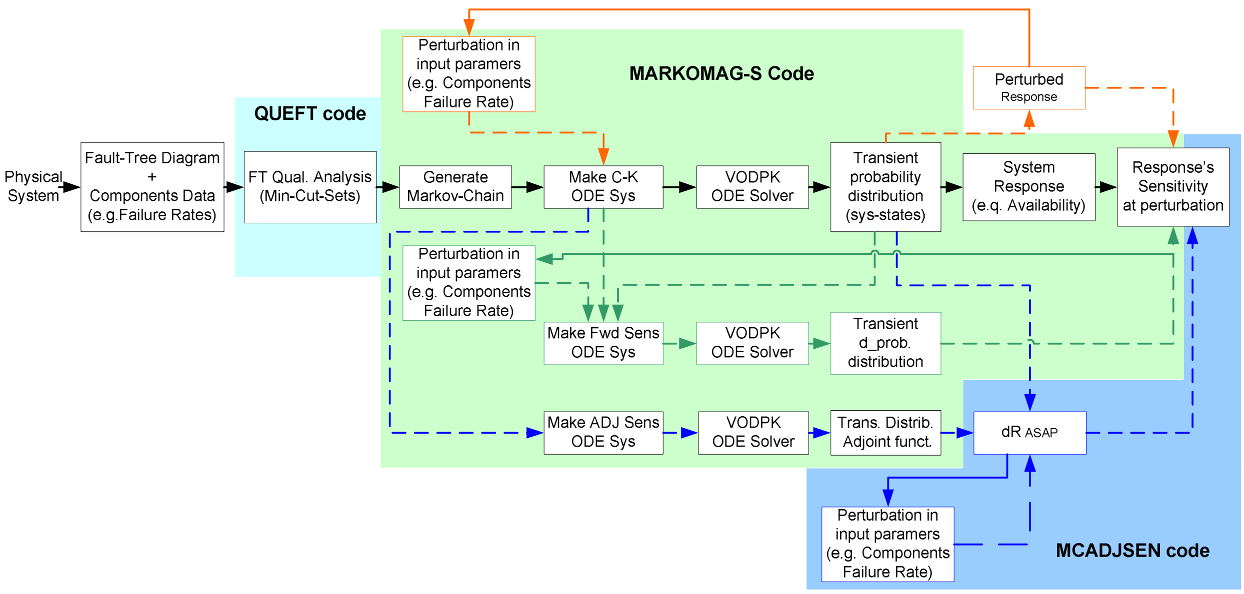


Figure 3.4 The flow diagram of the QUEFT/MARKOMAG-S/MCADJSEN code system

To perform sensitivity analysis of the response computed during the reliability phase, three options are available. First option is to make changes in input parameters once at a time and to run the code again with the perturbed parameters so many times how many perturbations exist, and afterwards to perform the difference between the recalculated responses and the base-case solution to get the response sensitivity to those perturbations. This way to compute response sensitivities implies $n+1$ computations of the Kolmogorov ODE system for n perturbations in system parameters.

The second choice is to compute the response sensitivities using FSAP. Using this option, the forward sensitivity equations (3.70) are generated using the transition rate matrix and numerical solution obtained for the base case of Eq.(3.69) . The FSE is solved using the same solver in order to obtain the transient probability distribution of perturbation state vector. The response sensitivities are computed afterwards using the solution of FSE. For new perturbations in system parameters the FSE must be solved again. This way is advantageous when in the considered problem are less parameters than responses, situation rarely met in Markov chain analysis.

The third option is to perform sensitivity analysis using the adjoint system of equations (3.71). As in the FSE case, these equations are constructed during the generation of Markov-chain algorithm since the same transition matrix is used, but transposed. The nonhomogeneous term depending on the system response (the right-side of the ASE) is evaluated during the computation process as well. The ASE are solved using the same VODPK solver in order to obtain the transient vector of adjoint functions. The derivatives implied in evaluation of the indirect effect term of response sensitivities by Eq.(3.75) are automatically evaluated and stored. The transient solution of the adjoint system and the derivatives (3.76) are given as outputs at the end of the computation process of ASE.

These values are used further into the next module MCADJSEN which is performing the evaluation of the Eq.(3.75). Every time new variations in system parameters are considered, the system sensitivities are calculated based only on the parameter perturbations, the adjoint functions and the derivatives evaluated previous, without being necessary to be solved again the ODE system. For a given response, the response sensitivities evaluation using the ASE option implies to solve the ODE system only two times, namely once for the base case solution of Markov chain equations (3.69), and once for the adjoint system of equations (3.75). This way is advantageous for the common case met in Markov chain analysis where the number of parameters exceeds by far the number of responses.

The FSAP and ASAP formalisms can be applied further for the discretized set of BKE by one of discretization methods used for ODE systems, to obtain the *discrete FSE* (DFSE) and *discrete ASE* (DASE). This formalism is called as discrete adjoint sensitivity analysis procedures (DASAP). The discrete system response sensitivity (*DDR*) can be obtained similar applying the G-differential to the discretized system response *R* which is represented in general form as follows

$$\begin{aligned} R &= \int_{t_0}^{t_f = t_0 + N\Delta t_l^q} F(t, \pi_1, ..., \pi_n, \alpha_1, ..., \alpha_m) dt \\ &= \sum_{l=t_0}^{N} F(t_l, \pi_1^l, ..., \pi_1^l, \alpha_1, ..., \alpha_m) \Delta t_l^q \end{aligned} \tag{3.77}$$

where Δt_l^q denotes the quadrature time step.

For ODE systems of form (3.72) has been shown by Cacuci[24] using for discretization the one-step Euler formula, that the sensitivities results given by *DR* and *DDR* are not identical with one another, but

$$DR = DDR + O(\Delta t_l^q) \tag{3.78}$$

How long the consistency between ASE and DASE solution is ensured which implies that the quadrature time step to be identical with the time-step used to discretize the equations, either DASAP or discretized equations produced by ASAP can be used to evaluate response sensitivities. If the consistency is not ensured, it is recommended to use differential equations given by ASAP as it has been presented previous, and afterwards to discretize the resulting equations in order to solve them numerically. The DASE makes it possible to analyze the response sensitivity to the step size and truncation error, or some certain parameters introduced by discretization process in numerical evaluation which is beyond the scope of this work. It must be mentioned that the differential BKE, ASE and the integral form of response represent the forms that contain the behaviour of the physical system, and these equations must be discretized and solved consistently.

3.5 Summary

In this chapter the Adjoint Sensitivity Analysis Procedure has been applied to Markov chain system. During this procedure the adjoint sensitivity system of equations of Markov chain has been constructed. A form in terms of adjoint functions for the indirect-effect term of the response's sensitivity using the ASAP guidelines has been obtained. This form is independent on the perturbations in system parameters (i.e. the transition rates).

The ASAP fundamental characteristics have been highlighted:

a) The ASE does not depend on the variations in system parameters, thus nor the solution of this system, i.e. the adjoint functions;
b) From computational point of view, the sensitivities evaluation of a given response with respect to variations in system parameters implies only to solve once the original system of Markov chain, and once the ASE. Afterwards, each time a variation in system parameters occurs, the expression of the indirect-effect term of response sensitivities using the adjoint functions should be evaluated.
c) The ASE depends on the system's response. Therefore, the ASE must be solved anew with response changing.

The validation of this method by comparison of numerical results with traditional approach that implies recalculations and analytical solution, has been performed for a simple Markov chain of a binary component. Further reliability and sensitivity studies are performed in Chapter 4 using the computer code-system QUEFT/MARKOMAG-S/MCADJSEN that has been developed for reliability and sensitivity analysis purposes using the Markov chain technique.

4 Sensitivity studies on reliability of IFMIF-Accelerator System Facilities: Illustrative Example

In this chapter, the methods and the code system described previous have been used for reliability and sensitivity analysis of the accelerator system of the International Fusion Irradiation Materials Facility (IFMIF).[75] First, the reliability analysis using Markov chains is performed for the all subsystems of accelerator system facility in order to obtain the transient availability of this system and its subsystems for the considered mission time. Afterwards, the sensitivity analysis on the subsystems is performed using ASAP of Markov chains. The primary purpose of this analysis is to study the effect of changes in the reliability parameters of components/subsystems to the overall availability of the accelerator system, ranking afterwards, based on the computed sensitivities, the uncertainty importance of components/subsystems parameters in affecting accelerator's availability.

The IFMIF project has been proposed and developed to provide the necessary irradiation field for testing present materials, to develop new materials, and to generate a materials database for the design, construction, licensing and safe operation of the future fusion power reactors. A greater understanding of the behavior of such materials when exposed to high levels of irradiations is required to ensure a safe and reliable fusion reactor design. An intense source of radiation should provide the necessary environment to study the effects of such radiations on these materials. This source of irradiation is given by a linear accelerator-based neutron source which must provide a continuous wave of high radiation. For such continuous high irradiative wave, the reliability of this system is essentially.

By the end of 1996, in the final report of IFMIF Conceptual Design Activity,[75] the overall requirements for IFMIF has been established at 70% online performance per year, i.e. the system should be operational 6132 hours from a total amount of 8760 hours. In one year it has been allocated for scheduled maintenance one month, plus eight hours every week, which represents a total of 1160 hours. Thus, the scheduled operation has been established to 7600 hours/year. The difference between the scheduled operation time and the required operation time, i.e. 1468 hours/year, has been allocated for unscheduled repairs due to random failures which can occur during the operation of IFMIF. The IFMIF comprises five main modules, namely test facilities, target facilities, accelerator facilities, conventional facilities, and central control and common instrumentation system, respectively. After reliability studies, the total availability of the IFMIF to 80.7% as the result of the availabilities of its subsystems has been established:

Subsystem	Availability (%)
Test facilities	97.5
Target facilities	95.0
Accelerator facilities	88.0
Conventional facilities	99.5
Central control system and common instrumentation	99.5
Total availability (product)	**80.7**

Table 4.1 Allocated availability for IFMIF

The most complex subsystem and the main contributor to the total availability of IFMIF system is the accelerator system facilities. To provide the continuous wave of high radiation, and to avoid interruptions into the radiation beam, this system consists in two linear accelerators which operate in parallel, and which are represented schematically as in Fig.4. This configuration will allow also operation to continue in providing the necessary radiation field when one or the other of the two accelerators is temporarily removed from service for repair

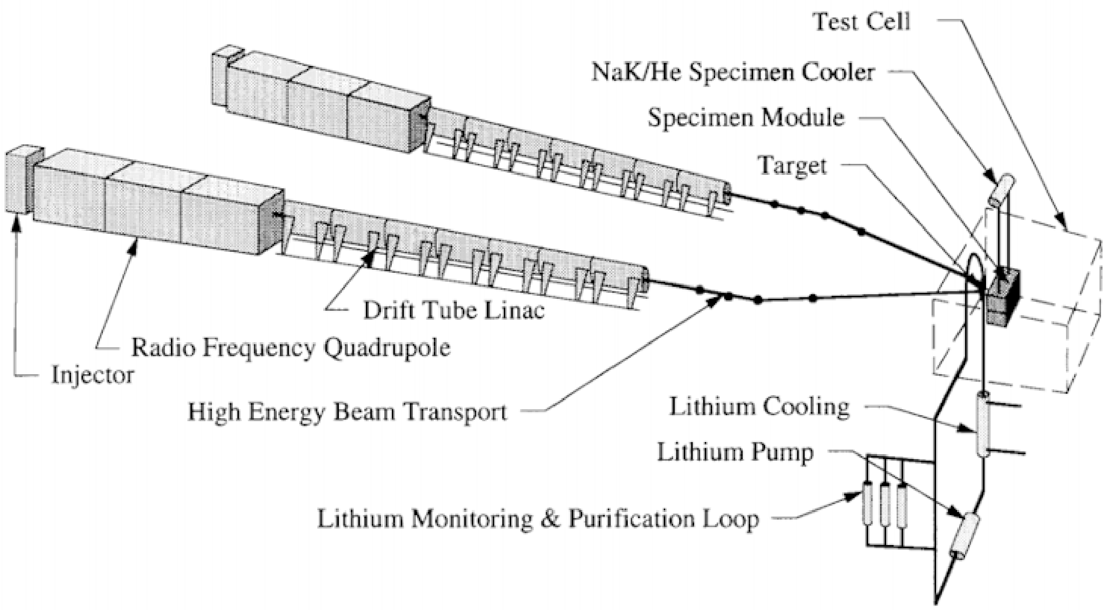


Figure 4 IFMIF Accelerator System Facilities

The availability of 88% for accelerator system facilities has been obtained using the Fault-Tree technique for a mission time of 168 hours (7 days) which is considered to be the period of time between two scheduled maintenance operations. Since this system is not yet physically built and operational, but still under design, all the input data for its subsystems and components used in reliability analysis have been taken from similar installations and facilities. Therefore, sensitivities studies of availability and other types of responses must be performed on accelerator system facilities in order to see the influence of changes in components reliability to the reliability of this system, since in the final stage of physical building of this complex system the components parameters can be changed due to various reasons, as costs, easiness in maintenance, replacement or repairing, the repairing or replacing time which can modify the operation time of the whole system, the components life time, etc. Apart of that, the specified values are statistically obtained and therefore are not known precisely but with some uncertainty bounds. The sensitivities results performed in this chapter can be used further for uncertainty studies of the analyzed types of responses for the accelerator system using for instance the moments matching technique as is described in Hahn and Shapiro [1967][83], Papazoglou and Gyftopoulos [1980][25], Ronen [1988].[2]

The IFMIF accelerator system comprises a sequence of acceleration and beam transport stages. These stages are presented in more details into IFMIF-CDA final report.[75] The main subsystems of the accelerator system which assure these stages until the target are presented in Fig.4.1.

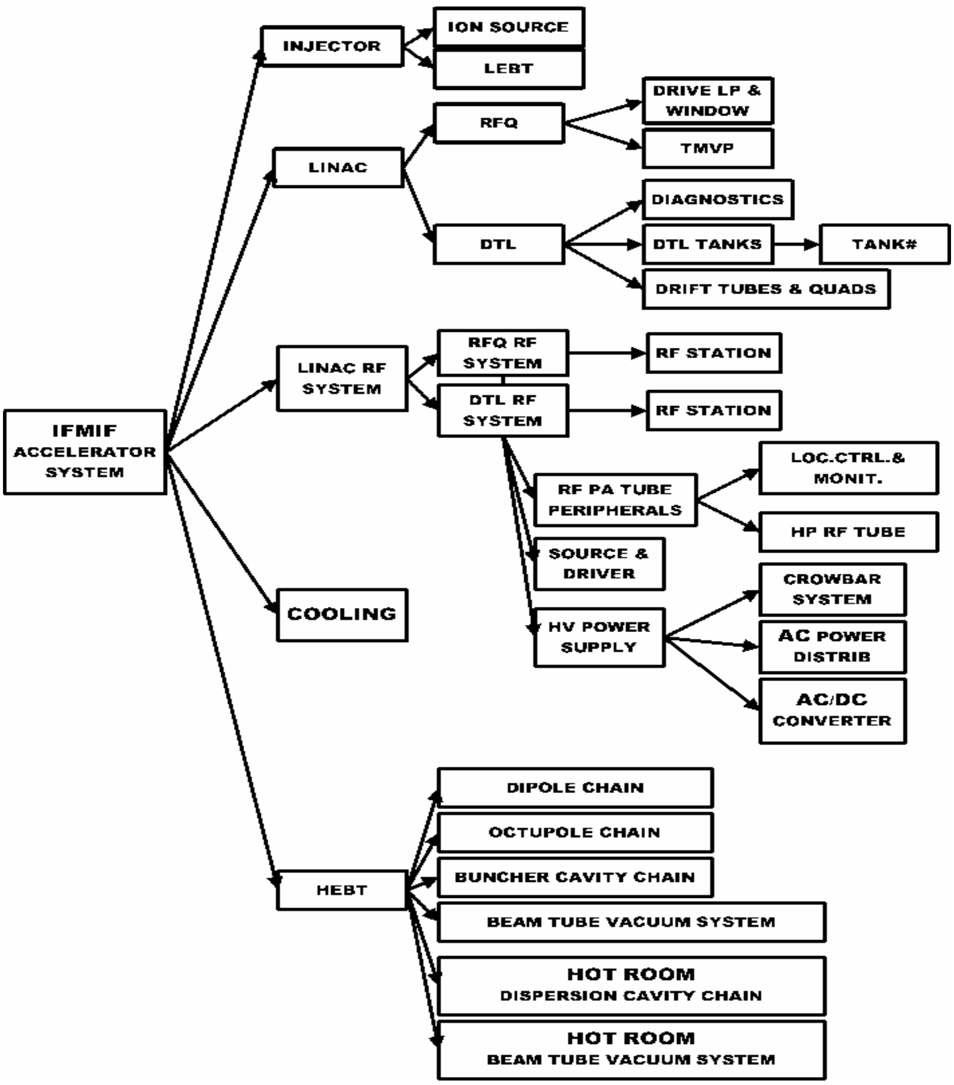


Figure 4.1 The main subsystems of accelerator system

The transient reliability analysis for each of these subsystems is performed before sensitivity analysis. A simplified representation of the Fault-Tree for the Accelerator system built based on IFMIF-CDA,[75] and Piaszczyk,[76] is presented in Fig.4.2. The numbered gates with triangle shape represent transfer gates which mean that below part of fault tree from this gate is developed separately in a different part. This type of gate has been introduced only with purpose to simplify the representation. An AND or an OR gate having as predecessor an event which contains in description box the multiplication sign x followed by a number means that the gate has as predecessors the indicated number of identical events. The subsystems behaviors are mutually independent and, therefore, a structural decomposition of the accelerator system into subsystems, separate analysis of subsystems, and aggregation of the intermediate results to obtain the final solution is possible. A transient availability analysis for each subsystem is performed and the stationary solution is compared afterwards with the solution given by Piaszczyk [1996][66] to check the results validity. The transient solution is graphically represented for each subsystem for a time interval of 168 hours (7 days) between two scheduled maintenance operations. The initial state probability vector has been considered to be of form [1,0,…,0], i.e. at the initial time the system is starting from the state in which all its subsystems/components are perfect operational. Further, during the reliability analysis the evolution of this state is studied, i.e. the state in which all components/subsystems are operational, since this is the interested state for the IFMIF project.

The input reliability data given as mean time to failure/repair for components and subsystem are presented in Table 4.2. In this analysis using the MARKOMAG-S code the complete Markov chain is generated and its attached differential equations for each of the above subsystems are solved as it has been presented in previous chapters. The results are presented from the top towards the base of the fault tree. In general as can be seen from the availability graphs, the subsystems availabilities reach the stationary solution at the end of mission time, but do exist some cases in which the stationary solution is reached after a longer period of time. In these cases the time scale is extended. A comparison with the numerical solution given by IFMIF-CDA,[75] and Piaszczyck,[76] at the end of the seven days mission time gives a good agreement and proves that the Markov chain generating algorithm and the assumptions that have been made in Section 2.2 are good enough for the purposes of this analysis. The numerical solutions after 168 hours given by Piaszczyk[76] and those obtained by using the method described in Chapter 2 are present in Table 4.3.

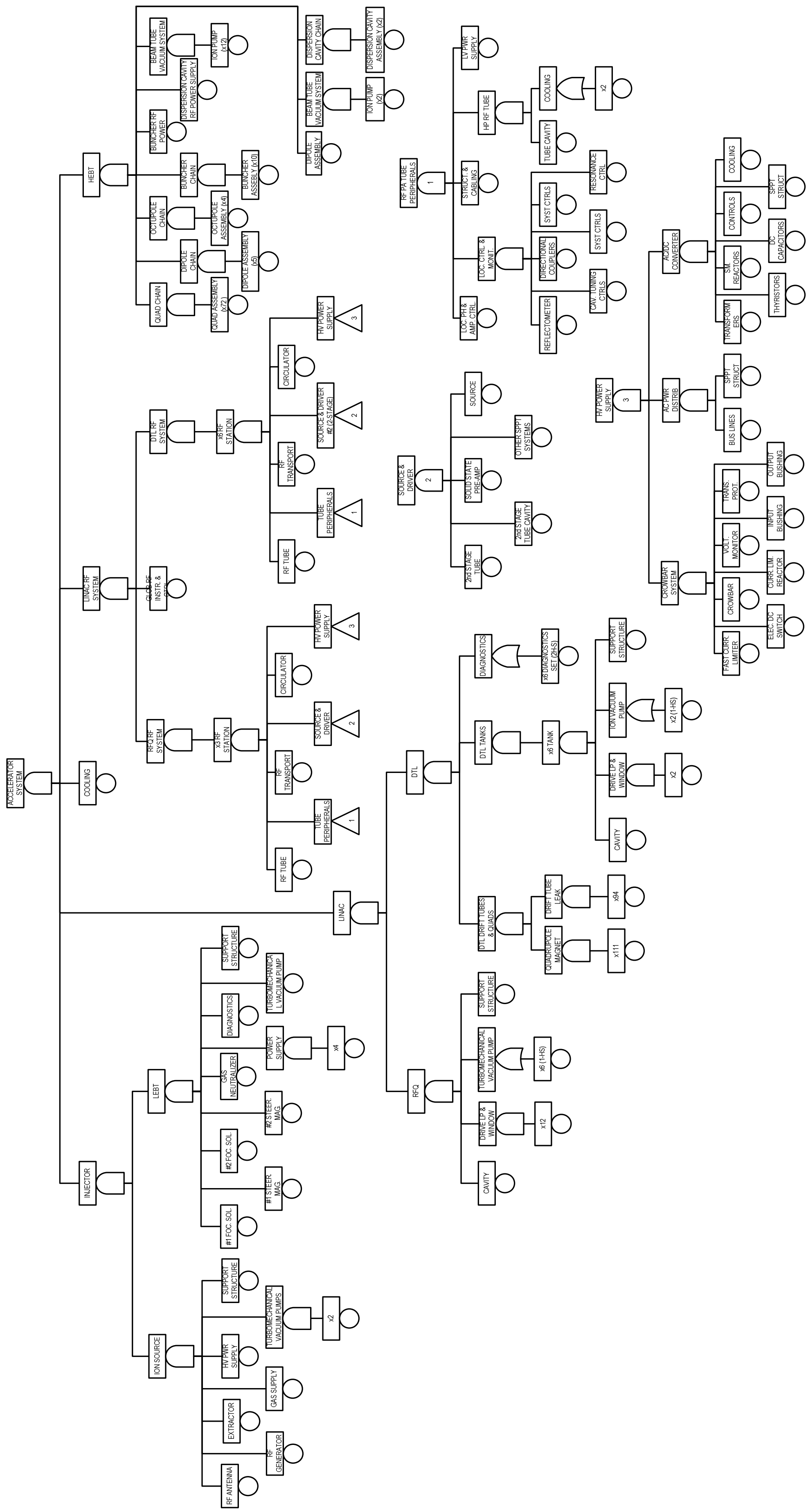


Figure 4.2 The Fault Tree of IFMIF-Accelerator System Facilities

Table 4.2 Mean time to failure/repair and the equivalent failure rates for the subsystems and components of accelerator system facilities

No.crt.	Subsystem/component of Accelerator System Facilities	MTTF (h)	MTTR (h)	λ (failures/h)	μ (rep./h)
1	**Injector**	156.70	2.20	6.3816E-03	4.5455E-01
2	Ion Source	160.78	2.07	6.2197E-03	4.8309E-01
3	RF Antenna	166.00	2.00	6.0241E-03	5.0000E-01
4	RF Generator	49999.00	1.00	2.0000E-05	1.0000E+00
5	Extractor	8756.00	4.00	1.1421E-04	2.5000E-01
6	Gas Supply	23998.00	2.00	4.1670E-05	5.0000E-01
7	High Voltage Power Supply	99976.00	24.00	1.0002E-05	4.1667E-02
8	Turbomechanical Vacuum Pump	19996.00	4.00	5.0010E-05	2.5000E-01
9	Support Structure	4999832.00	168.00	2.0001E-07	5.9524E-03
10	Low Energy Beam Transport	6680.43	5.49	1.4969E-04	1.8215E-01
11	Focusing Solenoid #1	499996.00	4.00	2.0000E-06	2.5000E-01
12	Steering Magnet #1	499996.00	4.00	2.0000E-06	2.5000E-01
13	Focusing Solenoid #2	499997.00	3.00	2.0000E-06	3.3333E-01
14	Steering Magnet #2	499992.00	8.00	2.0000E-06	1.2500E-01
15	Gas Neutralizer	23999.00	1.00	4.1668E-05	1.0000E+00
16	Magnet Power Supply	499990.00	10.00	2.0000E-06	1.0000E-01
17	Diagnostics	23988.00	12.00	4.1688E-05	8.3333E-02
18	**LINAC**	465.00	19.00	2.1505E-03	5.2632E-02
19	Radio Frequency Quadrupole	1178.09	10.48	8.4883E-04	9.5420E-02
20	Cavity	4995000.00	5000.00	2.0020E-07	2.0000E-04
21	*Drive LP&Window Set.*	1654.67	12.00	6.0435E-04	8.3333E-02
22	Drive LP&Window	19988.00	12.00	5.0030E-05	8.3333E-02
23	*Turbomechanical Vacuum Pump Set.*	3328.80	4.00	3.0041E-04	2.5000E-01
24	Turbomechanical Vacuum Pump	19996.00	4.00	5.0010E-05	2.5000E-01
25	Drift Tube LINAC	792.50	24.58	1.2618E-03	4.0683E-02
26	*Diagnostics Set.*	23984.00	16.00	4.1694E-05	6.2500E-02
27	*DTL Tanks*	1148.58	16.18	8.7064E-04	6.1805E-02
28	*Tank#*	7012.71	16.18	1.4260E-04	6.1805E-02
29	Ion Vacuum Pump	39996.00	4.00	2.5003E-05	2.5000E-01
30	*DTL Drift Tubes & Quads*	4770.31	72.00	2.0963E-04	1.3889E-02
31	Quadrupole Magnet Set	8901.38	72.00	1.1234E-04	1.3889E-02
32	Drift Tube Leak Set	10530.72	72.00	9.4960E-05	1.3889E-02
33	**Cooling System**	499996.00	4.00	2.0000E-06	2.5000E-01
34	**Radio Frequency System**	225.50	9.00	4.4346E-03	1.1111E-01
35	RFQ RFSys	706.54	9.03	1.4153E-03	1.1074E-01
36	*RF Station#*	2146.70	9.00	4.6583E-04	1.1111E-01
37	RF Tube	9986.00	14.00	1.0014E-04	7.1429E-02
38	RF Transport	999990.00	10.00	1.0000E-06	1.0000E-01
39	Circulator	49999.00	1.00	2.0000E-05	1.0000E+00
40	Global RF Instrumentation & Controls	49998.75	1.00	2.0001E-05	1.0000E+00
41	DTL RF Sys	346.51	9.03	2.8859E-03	1.1074E-01
42	*RF PA Tube Peripherals*	7399.64	5.36	1.3514E-04	1.8657E-01
43	Local Ph.&Amp. Controls	99963.40	10.00	1.0004E-05	1.0000E-01
44	Struct. & Cablig	299999.00	10.00	3.3333E-06	1.0000E-01
45	Low Voltage Power Supply	99990.00	10.00	1.0001E-05	1.0000E-01
46	*Loc Ctrl&Monit*	23787.60	10.00	4.2039E-05	1.0000E-01

No.crt.	Subsystem/component of Accelerator System Facility	MTTF (h)	MTTR (h)	λ (failures/h)	μ (rep./h)
47	Reflectometer	999990.00	10.00	1.0000E-06	1.0000E-01
48	Cav. Tuning Controls	99990.00	10.00	1.0001E-05	1.0000E-01
49	System Controls	99990.00	10.00	1.0001E-05	1.0000E-01
50	Directional Couplers	999990.00	10.00	1.0000E-06	1.0000E-01
51	Resonance Controls	99990.00	10.00	1.0001E-05	1.0000E-01
52	*HP RF Tube*	27268.63	2.73	3.6672E-05	3.6630E-01
53	Cooling	19998.00	2.00	5.0005E-05	5.0000E-01
54	Tube Cavity	9990.00	10.00	1.0010E-04	1.0000E-01
55	*Source & Driver*	8169.76	10.24	1.2240E-04	9.7656E-02
56	2nd Stage Tube	10987.00	13.00	9.1017E-05	7.6923E-02
57	2nd Stage Tube Cavity	999991.00	9.00	1.0000E-06	1.1111E-01
58	Solid State Pre-Amp.	49998.00	2.00	2.0001E-05	5.0000E-01
59	Other SPPT Systems	2999998.00	2.00	3.3333E-07	5.0000E-01
60	Source	99998.00	2.00	1.0000E-05	5.0000E-01
61	*High Voltage Power Supply*	11393.91	9.09	8.7766E-05	1.1001E-01
62	*Crowbar Sys*	18167.44	10.00	5.5044E-05	1.0000E-01
63	Fast Current Limiter	199990.00	10.00	5.0003E-06	1.0000E-01
64	Electrical DC Switch	199990.00	10.00	5.0003E-06	1.0000E-01
65	Crowbar	99990.00	10.00	1.0001E-05	1.0000E-01
66	Current Lim. Reactor	199990.00	10.00	5.0003E-06	1.0000E-01
67	Volt Monitor	199990.00	10.00	5.0003E-06	1.0000E-01
68	Input Bushing	199990.00	10.00	5.0003E-06	1.0000E-01
69	Transp. Prot.	99990.00	10.00	1.0001E-05	1.0000E-01
70	Output Bushing	199990.00	10.00	5.0003E-06	1.0000E-01
71	*AC Power Distr.*	749988.12	10.00	1.3334E-06	1.0000E-01
72	*AC/DC Converter*	31904.69	7.45	3.1343E-05	1.3423E-01
73	Transformers	499990.00	10.00	2.0000E-06	1.0000E-01
74	Thyristors	499990.00	10.00	2.0000E-06	1.0000E-01
75	SM Reactors	499990.00	10.00	2.0000E-06	1.0000E-01
76	DC Capacitors	199990.00	10.00	5.0003E-06	1.0000E-01
77	Controls	99990.00	10.00	1.0001E-05	1.0000E-01
78	Support Structure	2999990.00	10.00	3.3333E-07	1.0000E-01
79	Cooling	99998.00	2.00	1.0000E-05	5.0000E-01
80	**High Energy Beam Transport**	224.50	7.70	4.4543E-03	1.2987E-01
81	*Quadrupole Chain*	575.71	2.00	1.7370E-03	5.0000E-01
82	*Dipole Chain*	8330.48	2.00	1.2004E-04	5.0000E-01
83	Dipole Assmbly	41664.38	2.00	2.4001E-05	5.0000E-01
84	*Octupole Chain*	10413.84	2.00	9.6026E-05	5.0000E-01
85	Octupole Assembly	41664.38	2.00	2.4001E-05	5.0000E-01
86	*Buncher Cavity Chain*	1317.83	8.66	7.5882E-04	1.1547E-01
87	Buncher Cavity Assembly	13288.58	8.66	7.5253E-05	1.1547E-01
88	*Beam Tube Vacuum Sys.*	3318.75	10.00	3.0132E-04	1.0000E-01
89	Ion Pump Assembly	39990.00	10.00	2.5006E-05	1.0000E-01
90	*BTVS-Hot Room*	19940.01	48.00	5.0150E-05	2.0833E-02
91	Ion Pump Assembly	39952.00	48.00	2.5030E-05	2.0833E-02
92	*Dispersion Cavity Chain -Hot Room*	4924.03	48.00	2.0309E-04	2.0833E-02
93	Dispersion Cavity Assembly	9720.01	48.00	1.0288E-04	2.0833E-02

For the 1st level of the fault tree (Fig.4.3) the associated Markov chain is generated completely as in Fig.4.4. The system of equations consists in 32 coupled equations. The structure of transition rate matrix is shown in Fig.4.5; the sparse matrix has 193 nonzero elements.

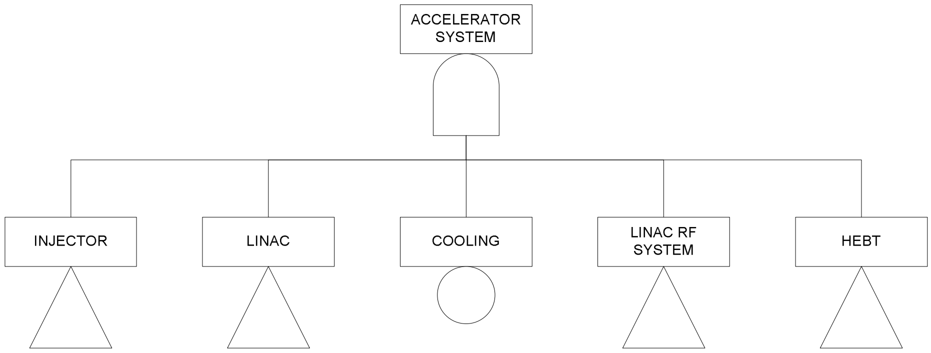


Figure 4.3 The first level of the Fault-Tree for the Accelerator system

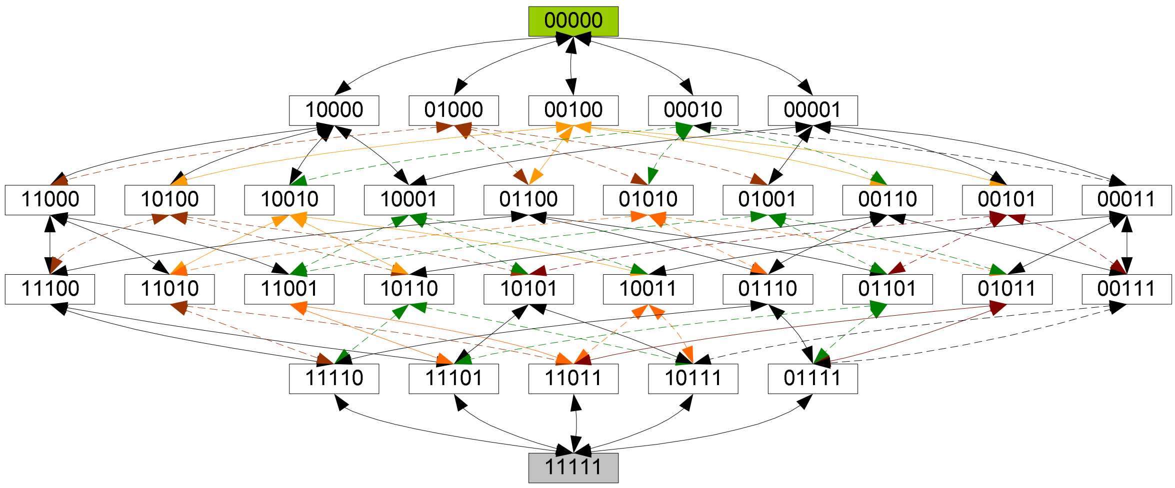


Figure 4.4 The Markov chain for the first level of accelerator system

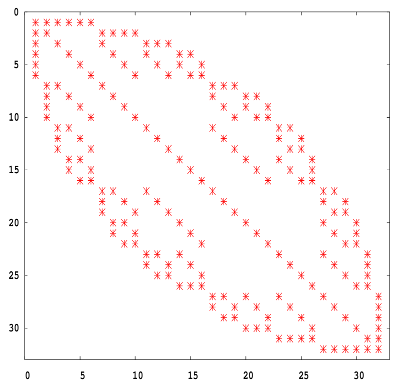


Figure 4.5 The transition rate matrix structure for the first level of accelerator system

The Figs 4.6 and 4.7 show the transient availabilities for the accelerator systems and its main subsystems. It is important to note that at the end of mission time the HEBT's availability did not reach the stationary solution.

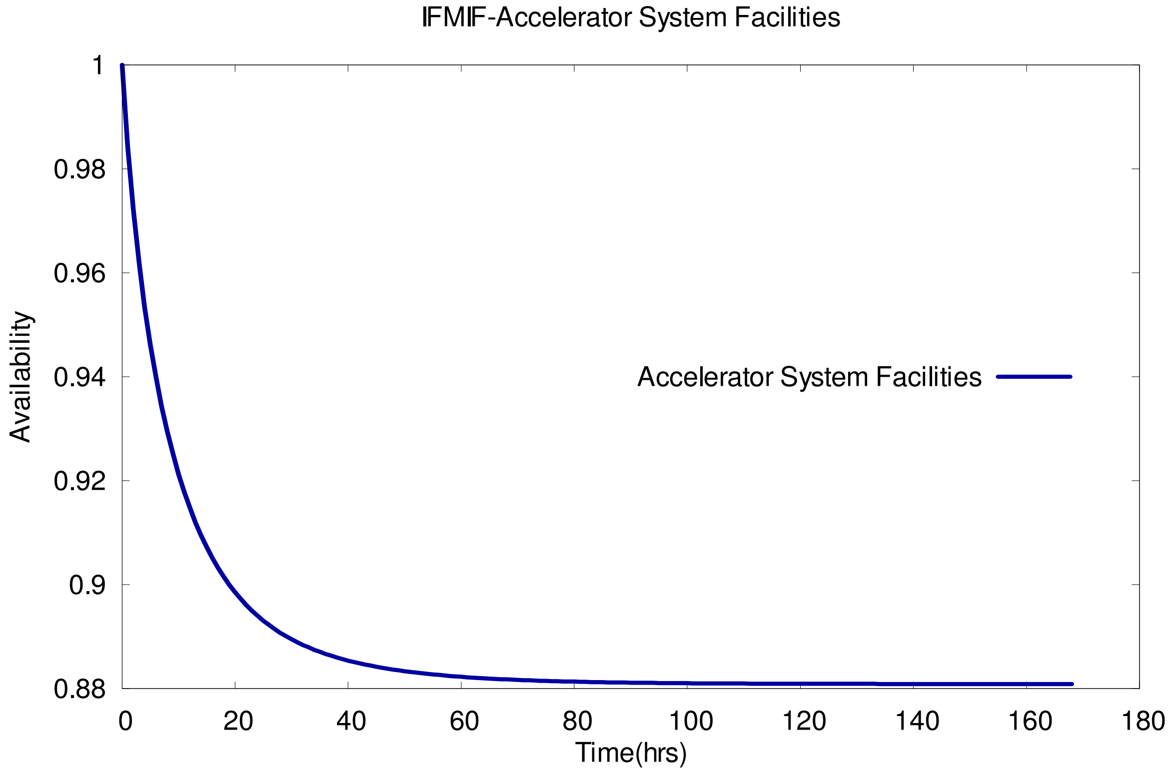


Figure 4.6 Transient availability of the accelerator system

It can be seen that the 88% availability of the accelerator system for the considered initial state probability vector is reached after about 120 hours. But other situations in which due to different reasons the system does not start with all its subsystems fully operational may exist. For a scenario that at starting point the accelerator system has a probability of 15% failure behavior in one or all of its main subsystems, its transient availability changes as in Fig.4.7. Considering that the failures causes are discovered and eliminated, one can see that at the end of mission time the availability is 88%, due to the fact that the stationary solution which in this case is reached at the end of the considered mission time, does not depend on the initial conditions. One can see also that the expected availability in the first hours of operation with considered failures has various behaviors depending on the affected subsystem. In the failure behavior of the injector or all the subsystems case the availability is decreasing and afterwards is increasing until the expected value. In such situations based on various scenarios should be studied the minimum acceptable availability as such in minimum number of hours the overall system to recover and to reach the expected availability of 88%. Special maintenance policies for such cases should be considered during operation of this system.

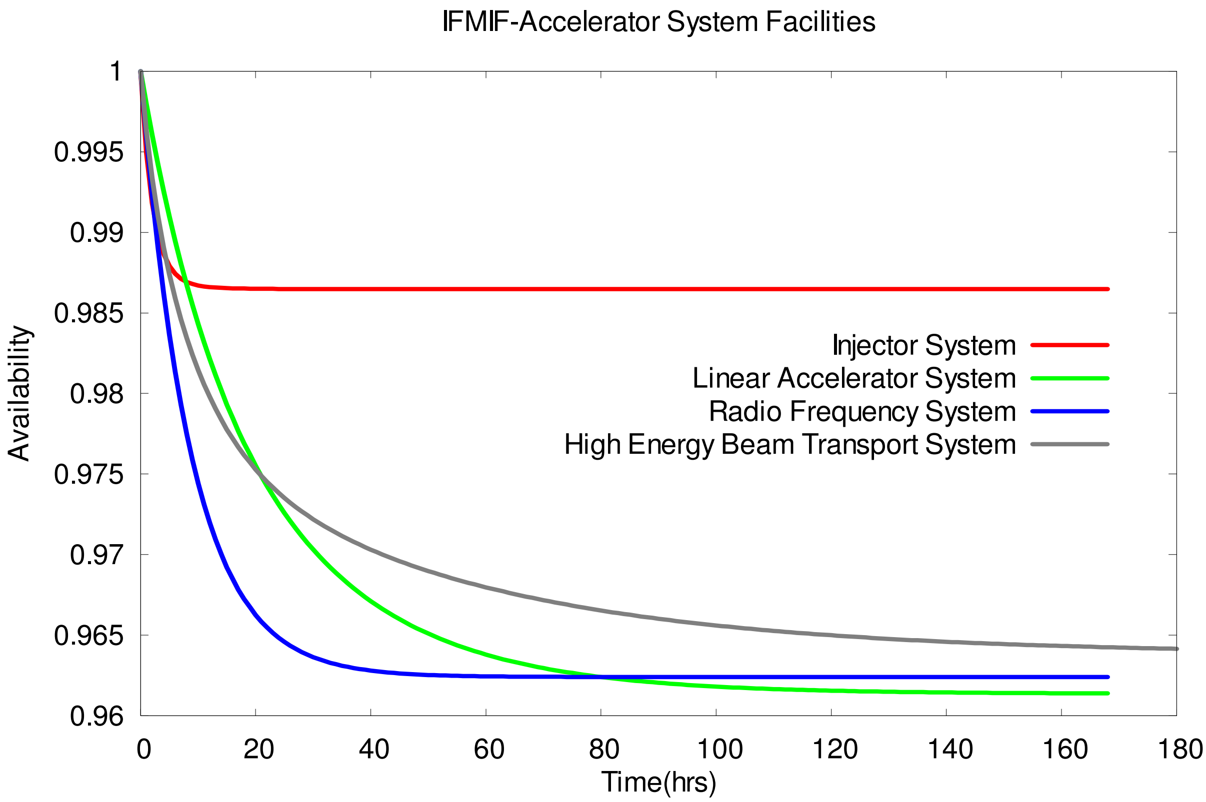


Figure 4.6.1 Transient availability for the main subsystems of the accelerator system

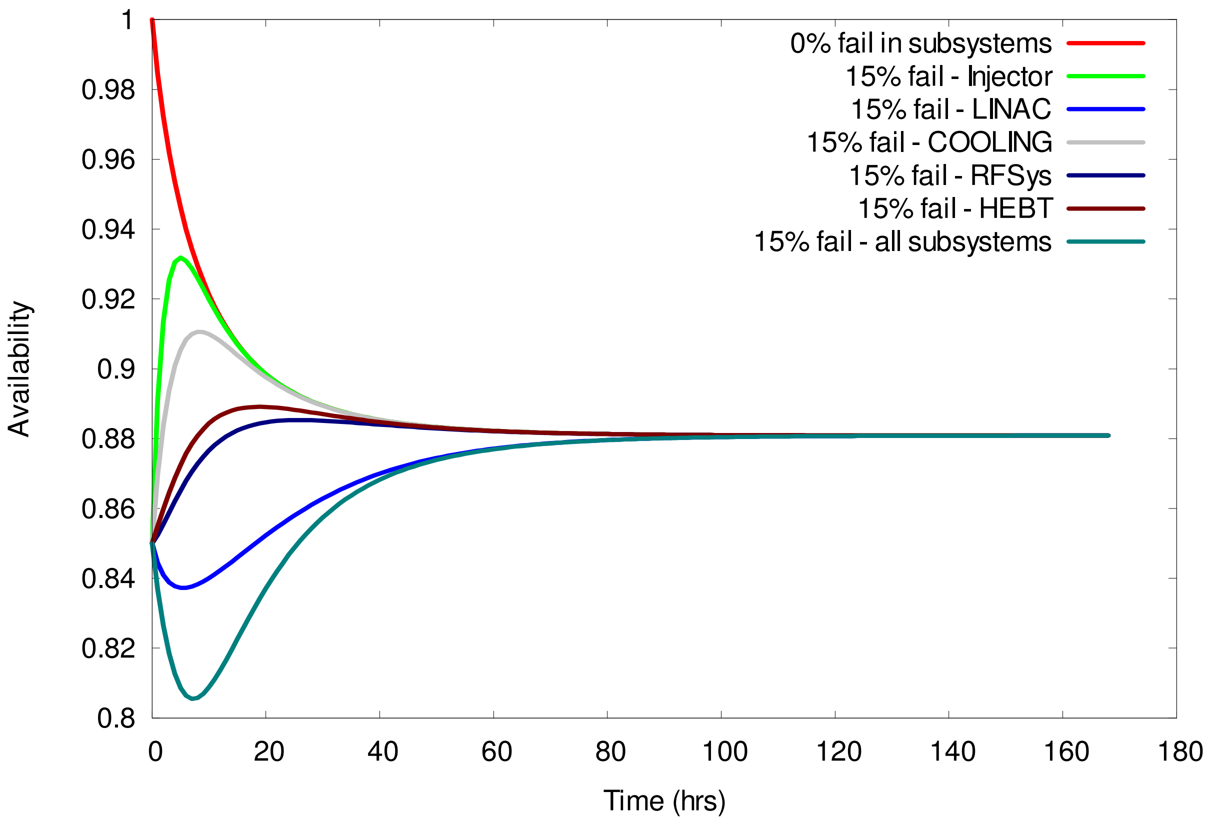


Figure 4.7 The availability of accelerator system considering a probability of 15% failure at starting point

Further, the transient availability for each subsystem together with the fault tree used to generate the Markov chain for transient analysis is presented as follows:

a) Figs.4.8, and 4.9 for injector system,
b) Figs.4.10 to 4.12 for linear accelerator system (LINAC),
c) Figs.4.13 to 4.16 for radio frequency system (RF System),
d) Figs.4.17 to 4.20 for high energy beam transport system (HEBT).

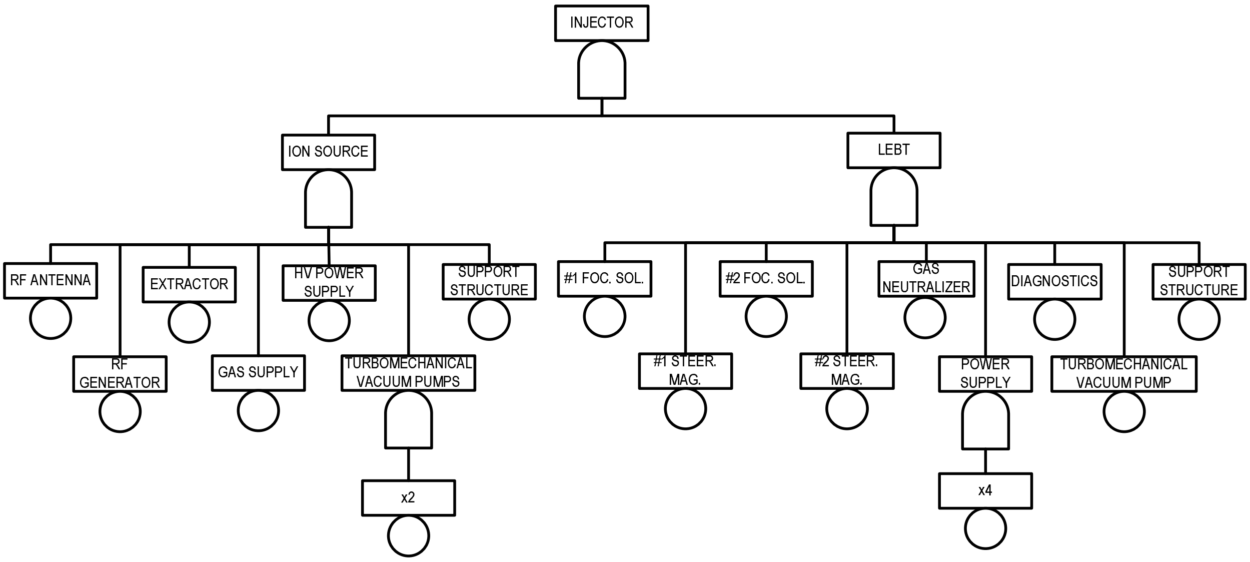


Figure 4.8 The Fault Tree of Injector System

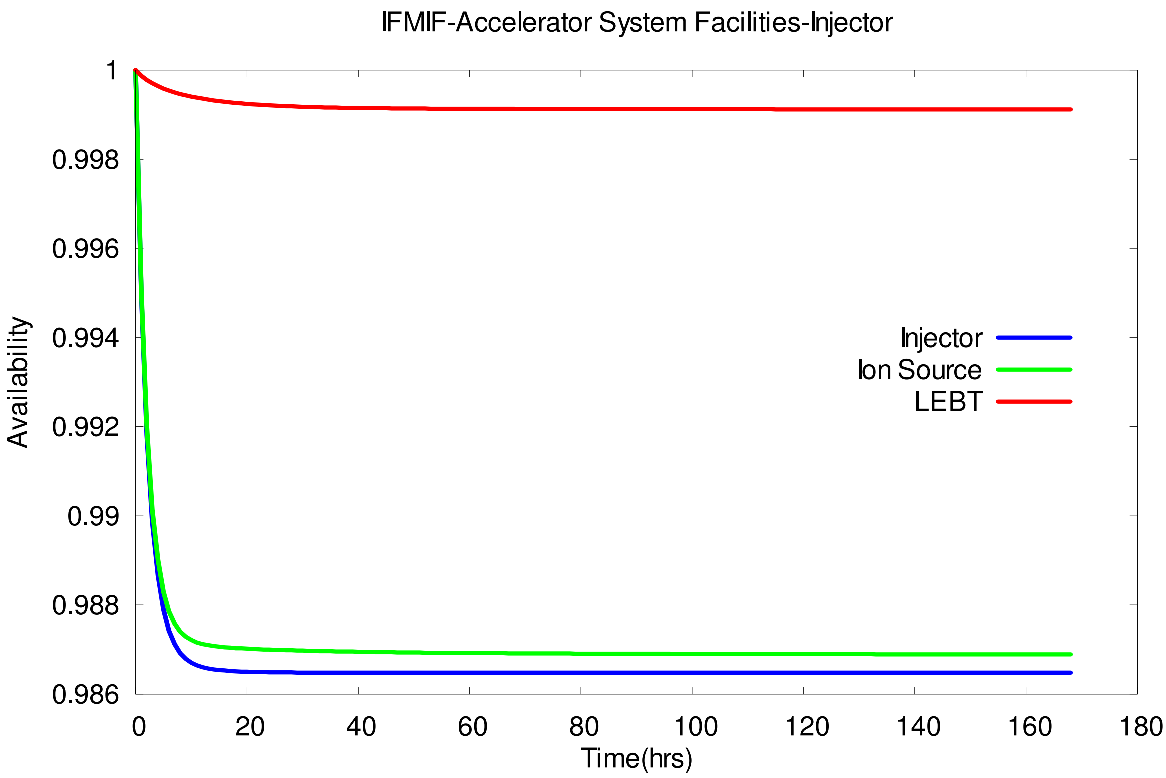


Figure 4.9 Transient availability of Injector and its subsystems

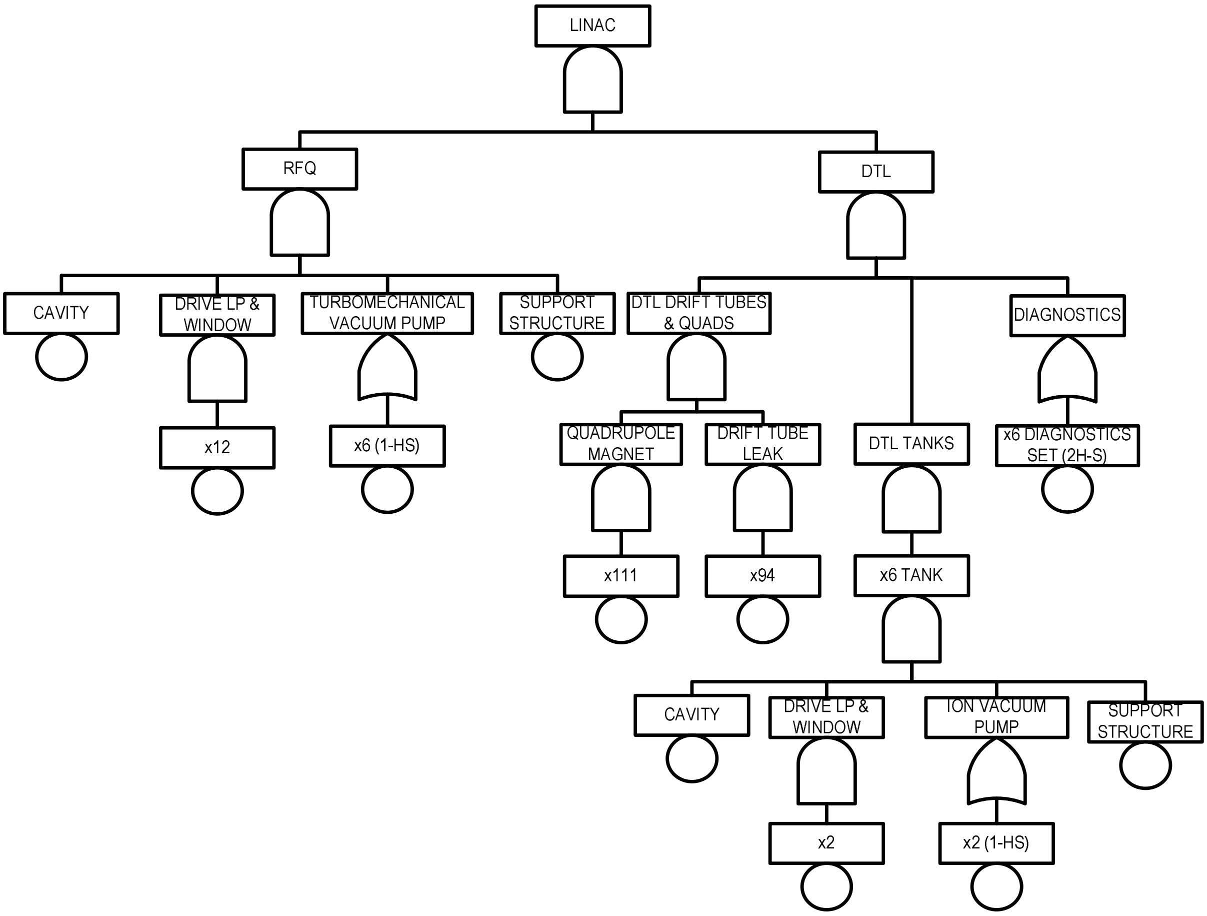


Figure 4.10 The Fault Tree of LINAC Subsystem

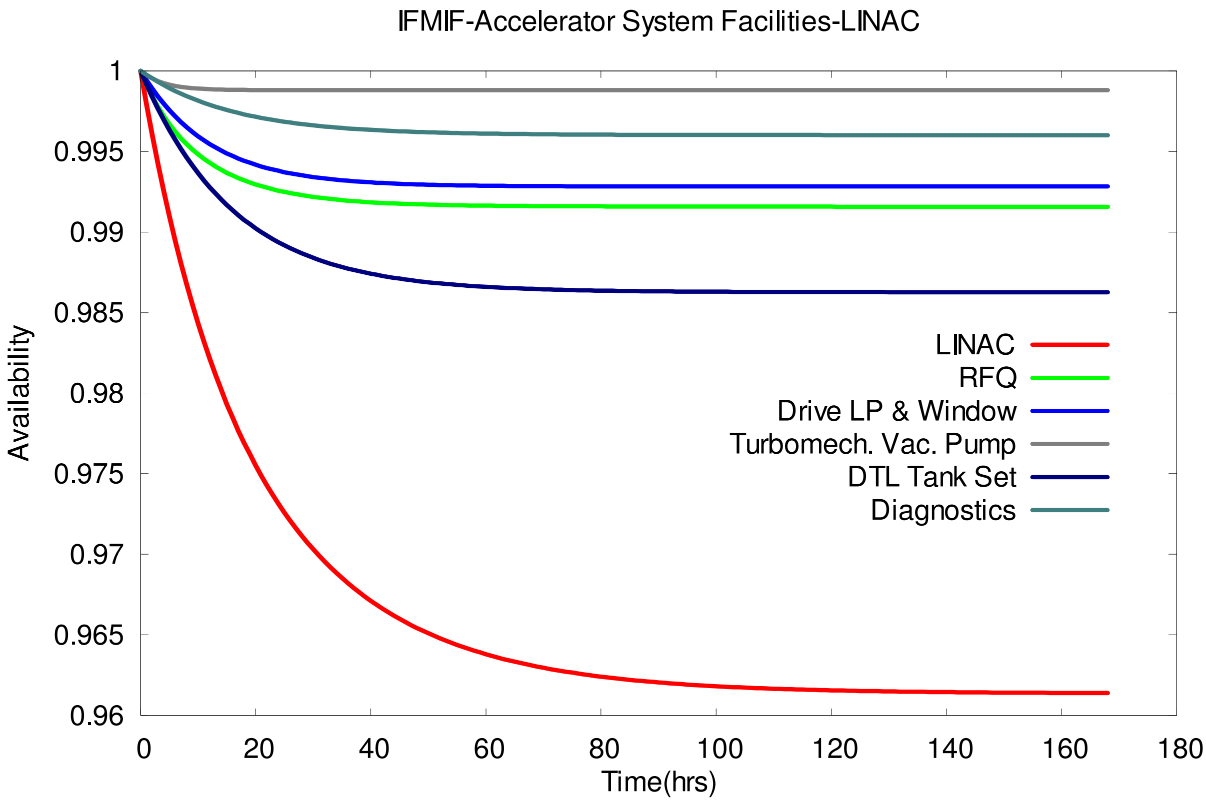


Figure 4.11 Transient availability of LINAC and its subsystems

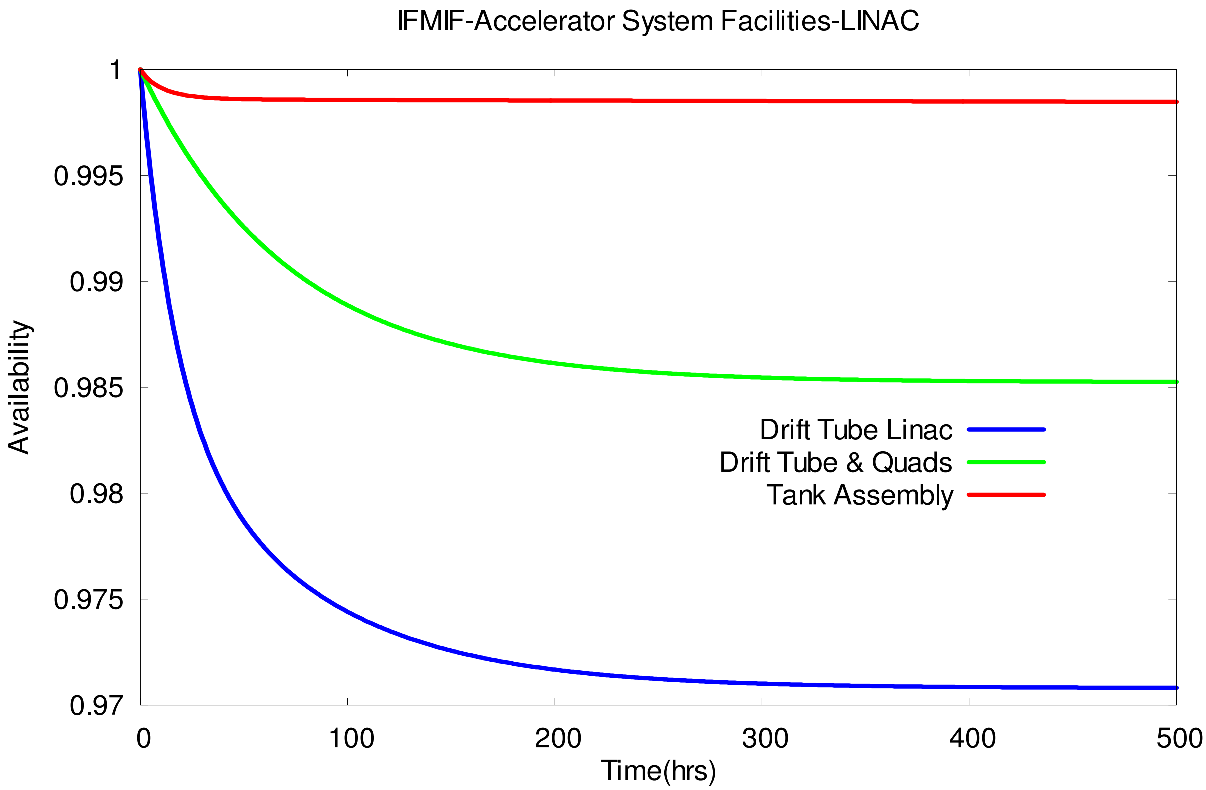


Figure 4.12 Transient availability of LINAC subsystems

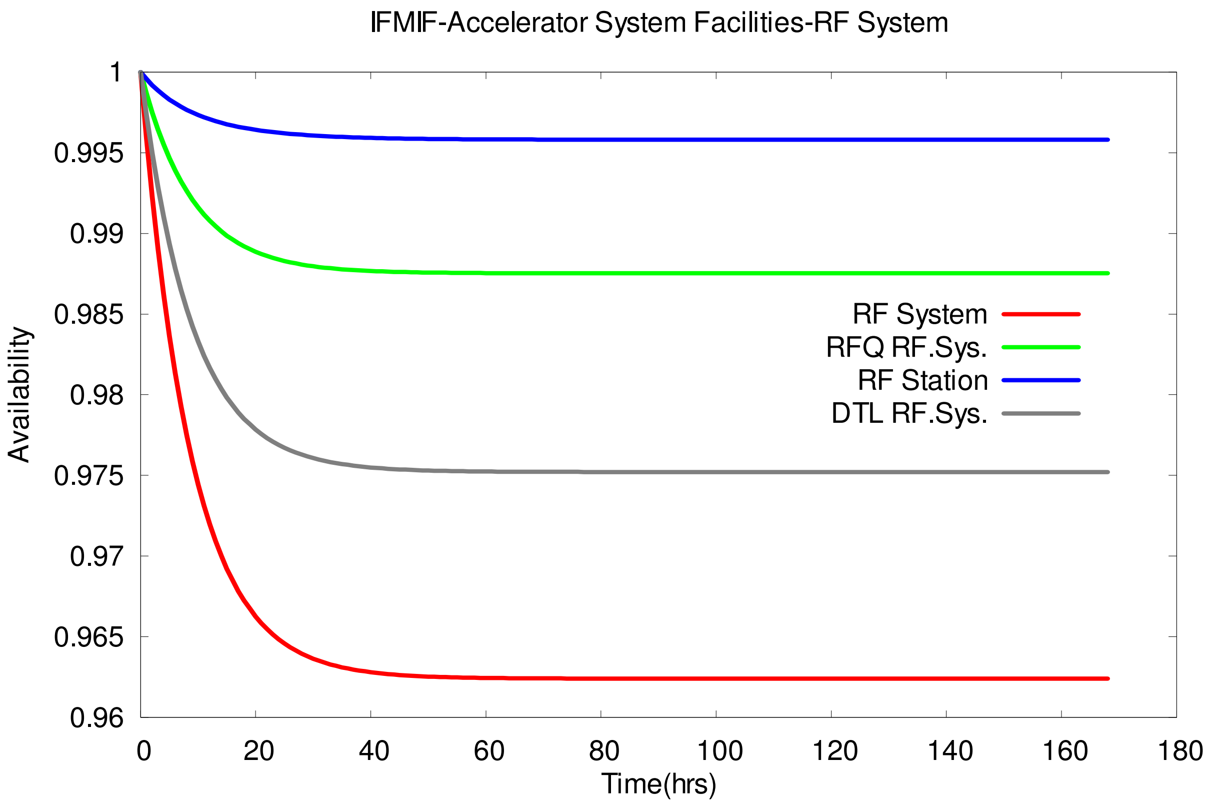


Figure 4.14 Transient availability of RF System and its subsystems

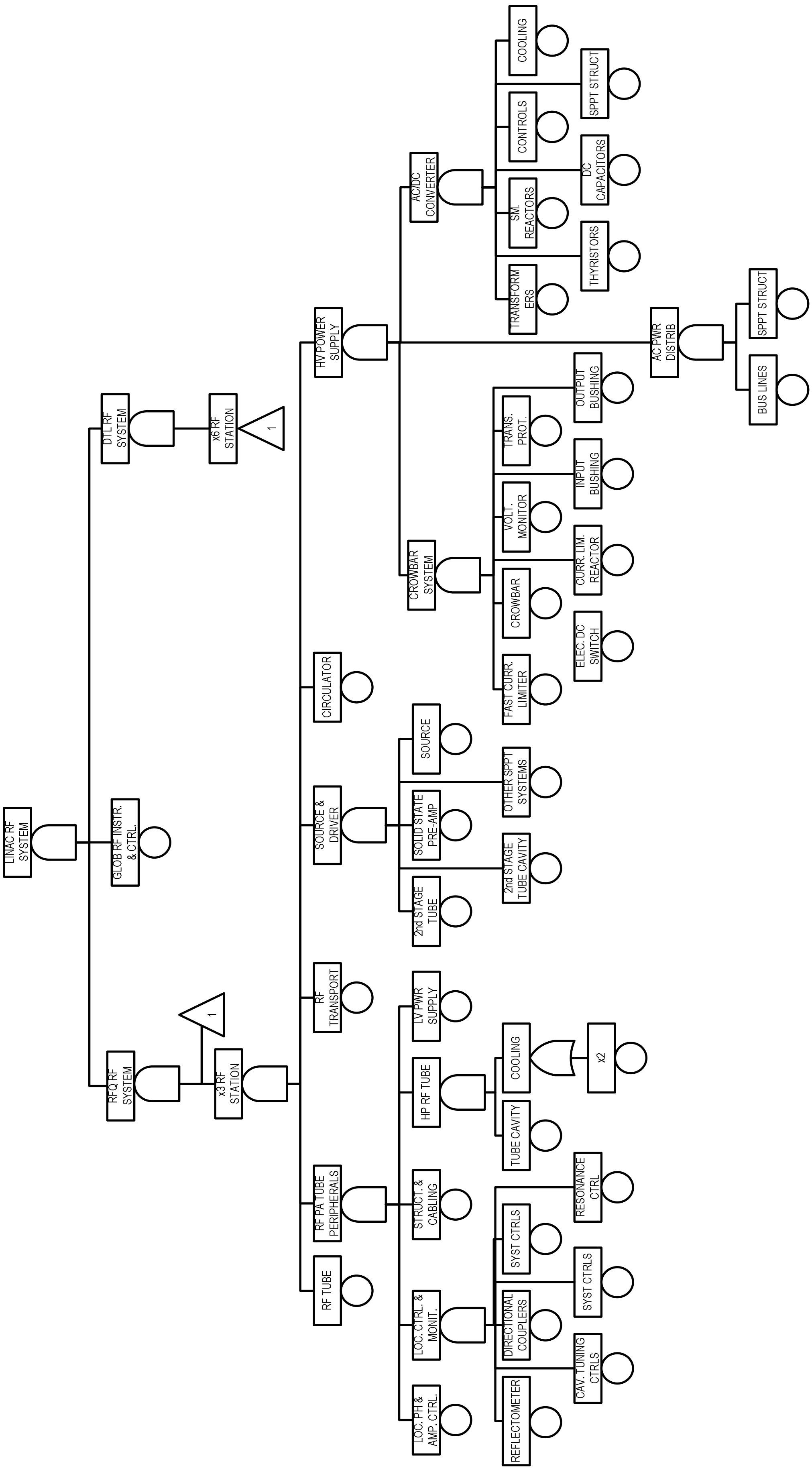


Figure 4.13 The Fault Tree of RF System

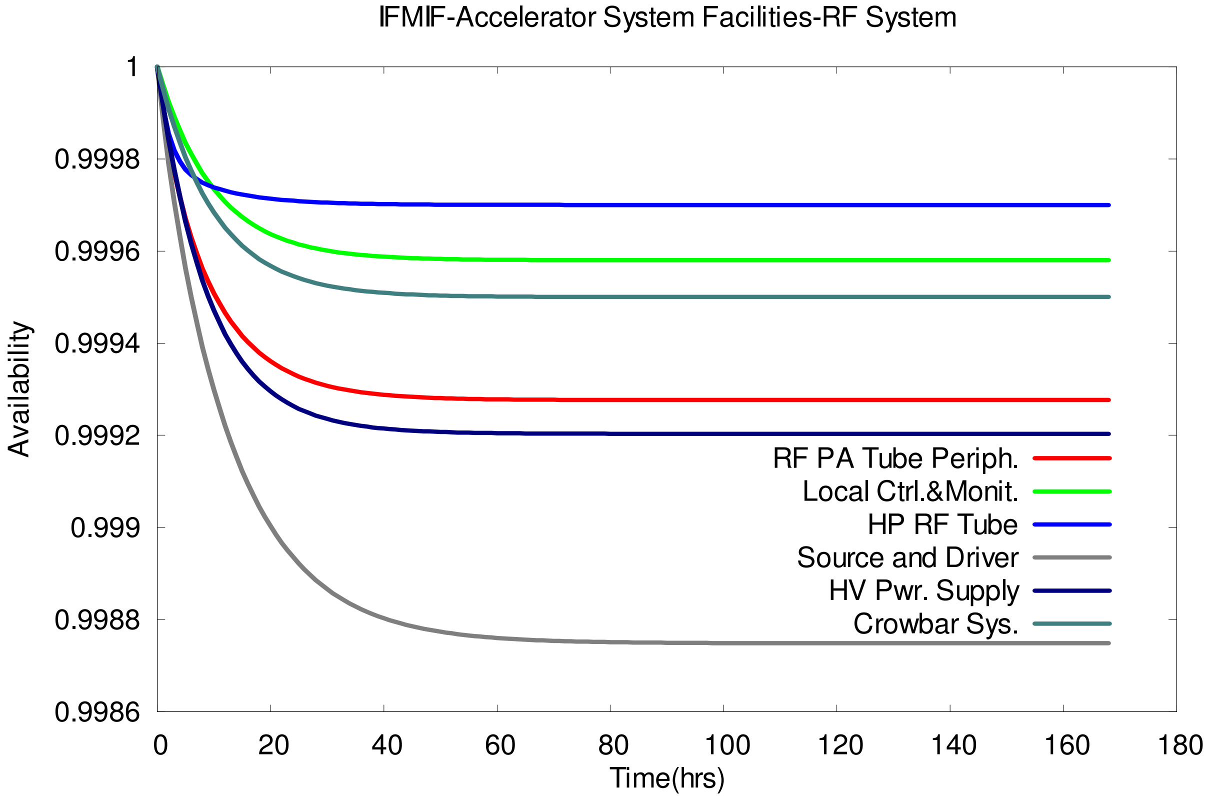


Figure 4.15 Transient availability of RF subsystems (1)

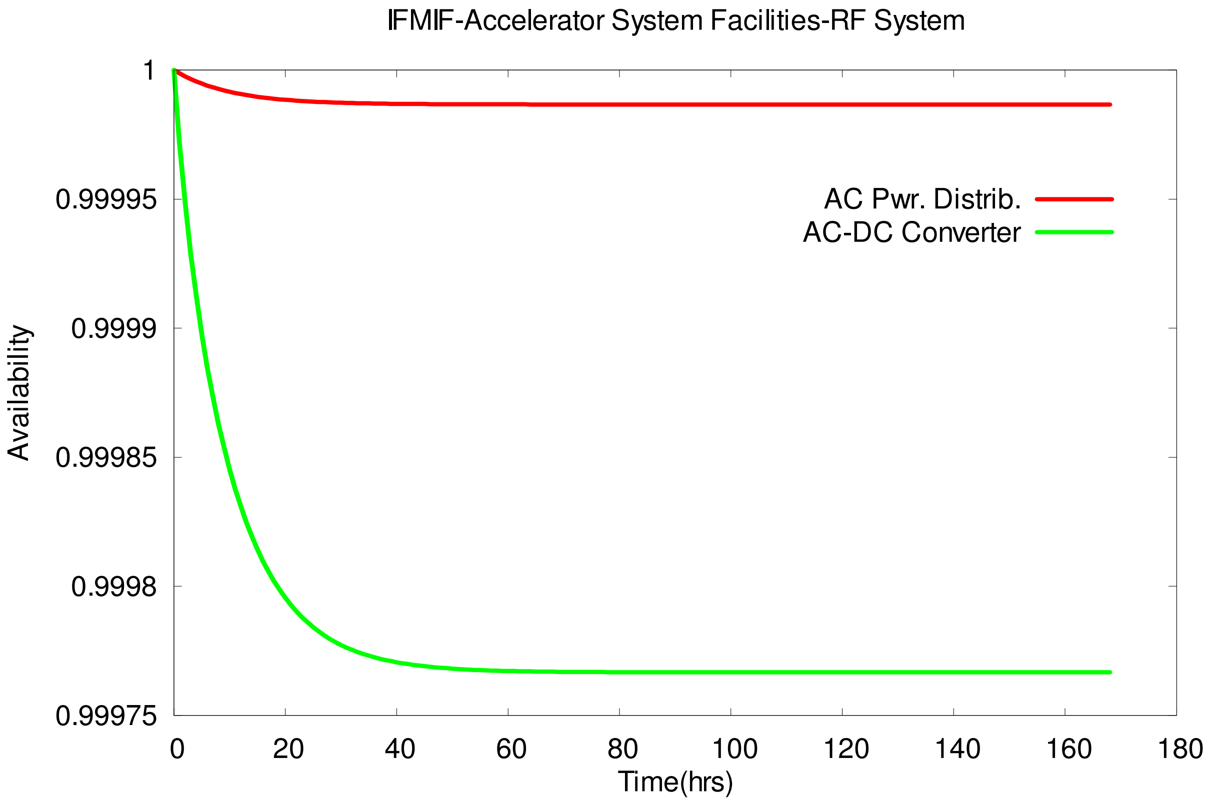


Figure 4.16 Transient availability of RF subsystems (2)

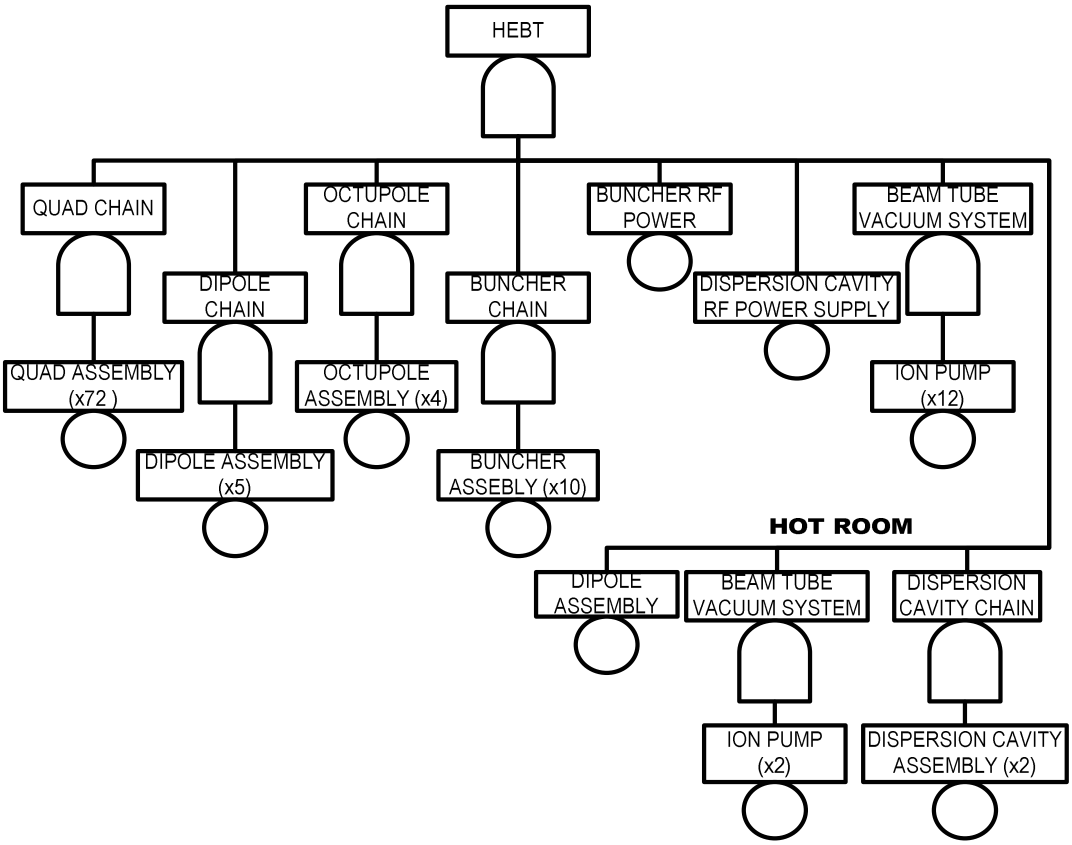


Figure 4.17 The Fault Tree of HEBT System

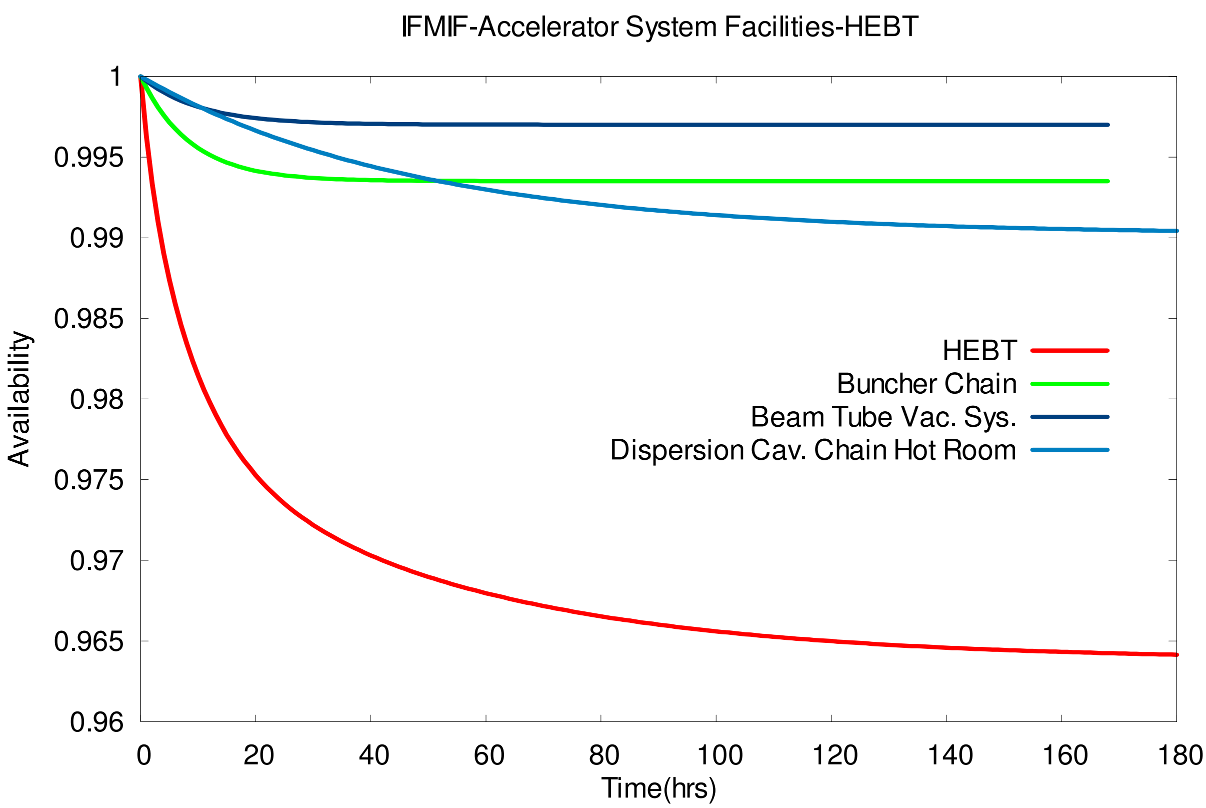


Figure 4.18 Transient availability of HEBT and its subsystems

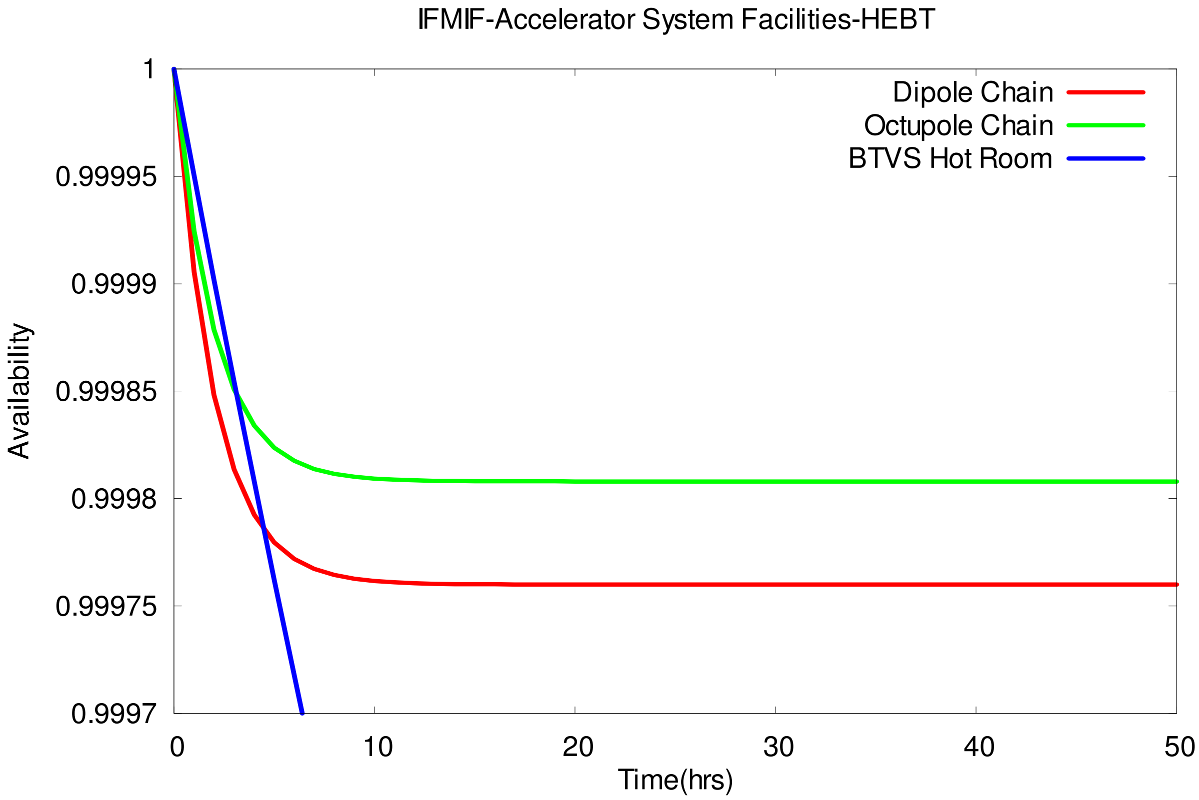


Figure 4.19 Transient availability of HEBT subsystems (1)

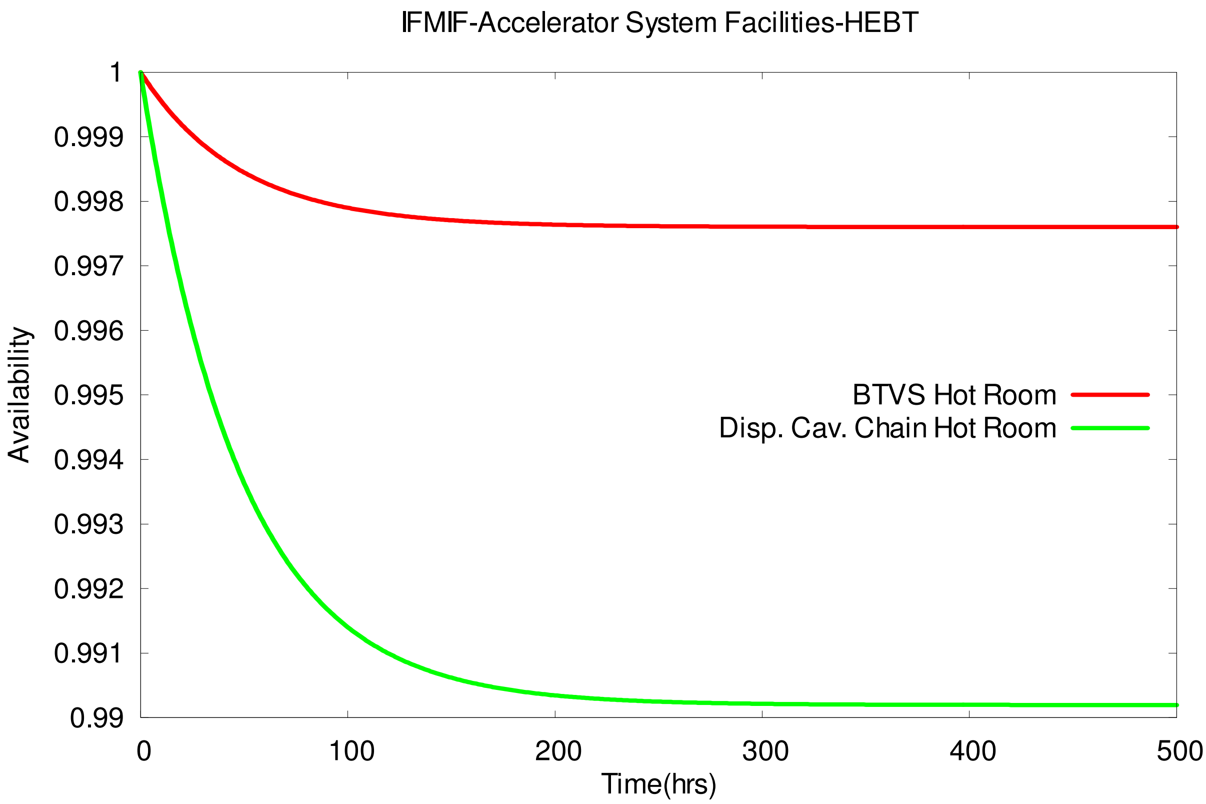


Figure 4.20 Transient availability of HEBT subsystems (2)

One can see from the Table 4.3 that at the end of the seven day mission time the stationary distribution where it is available agrees with the stationary solution, this mission time being sufficient to obtain the steady-state solution. The steady-state solutions are compared with ones given by Piaszczyk[76] in which it has been used the quantitative Fault Tree analysis for availability evaluation. For the cases in which the stationary solution has not been reached after the seven days mission time, it is displayed the availability computed at the end of the mission time with the next column stationary solution and the time this solution has been reached.

IFMIF Accelerator System Facilities - Availability (steady-state solution) (Mission Time = 168h) Top-Down				
No.	***Subsystem***	***Ref.*[66]**	***MARKOMAG-S***	
1	**Accelerator Sys.**	0.8814	0.8809	
2	**Injector**	0.9866	0.9865	
3	Ion Source	0.9874	0.9869	
4	Low Energy Beam Transport	0.9992	0.9991	
5	**LINAC**	0.9623	0.9614	
6	Radio Frequency Quadrupole	0.9913	0.9916	
7	*Drive LP&Window*	0.9928	0.9928	
8	*Turbomechanical Vacuum Pump*	0.9995	0.9988	
9	Drift Tube LINAC	0.9708	0.9722	0.9708 after 500h
10	*Diagnostics*	0.9989	0.9960	
11	*DTL Tanks*	0.9863	0.9863	
12	*Tank#*	0.9977	0.9985	0.9976 after 2600h
13	*DTL Drift Tubes & Quads*	0.9853	0.9866	0.9853 after 500h
14	**Radio Frequency System**	0.963	0.9623	
15	RFQ RFSys	0.9875	0.9875	
16	*RF Station#*	0.9958	0.9958	
17	DTL RF Sys	0.9752	0.9752	
18	*RF Station#*	0.9958	0.9958	
19	*RF PA Tube Peripherals*	0.9993	0.9993	
20	*Local Control&Monitor*	0.9996	0.9996	
21	*HP RF Tube*	0.9999	0.9997	
22	*Source & Driver*	0.9987	0.9987	
23	*High Voltage Power Supply*	0.9992	0.9992	
24	*Crowbar System*	0.9995	0.9995	
25	*AC Power Distribution*	0.9999	0.9999	
26	*AC/DC Converter*	0.9998	0.9998	
27	**High Energy Beam Transport**	0.964	0.9643	0.9638 after 300h
28	*Dipole Chain*	0.9998	0.9998	
29	*Octupole Chain*	0.9998	0.9998	
30	*Buncher Cavity Chain*	0.9935	0.9935	
31	*Beam Tube Vacuum System.*	0.997	0.9970	
32	*BTVS-Hot Room*	0.9976	0.9977	0.9976 after 350h
33	*Dispersion Cavity Chain -Hot Room*	0.9904	0.9905	0.9902 after 350h

Table 4.3 Steady-state availability of accelerator system and its subsystems

Next step in analysis is to perform sensitivity calculations of system's responses to variations in system parameters. The base-case parameters are given in Table 4.2 and the local sensitivity analysis is performed for each system and subsystem of accelerator system presented before. Two types of responses have been considered in analysis, namely the interval and steady-state availability. The importance of parameters for each subsystem and component of the accelerator system based on the relative sensitivities taken in their absolute value of considered response to perturbations in parameters has been ranked. The relative sensitivities have been computed using the following formula,

$$\left| \frac{\Delta \mathrm{R}_i}{\mathrm{R}_i^0} \frac{\alpha_j^0}{\Delta \alpha_j} \right| \tag{4.1}$$

where R_i^0 is the system response for the base case, $i = 1,2$, $\Delta \mathrm{R}_i = \mathrm{R}_{\mathrm{pred}} - \mathrm{R}^0$ is the sensitivity of the considered response using *ASAP*, α_j^0 is the base case system parameter for which is analyzed the response sensitivity, $j = 1,...,k$, with k being the number of parameters, and $\Delta \alpha_j = \alpha_j - \alpha_j^0$ is the perturbation into parameter α_j^0.

4.1 Sensitivities of the interval availability for IFMIF-Accelerator system facilities

The first type of response considered in sensitivity analysis is the interval availability as defined by Eq.(2.32) and which for this case is of form,

$$R = \frac{1}{t_f} \int_{t_0}^{t_f} \pi_1(t)dt \tag{4.2}$$

This type of response represents the expected fraction of time the accelerator and its subsystems are up with all subsystems/components operational in the given interval of time $\left[t_0 = 0, t_f = 168h\right]$ which is the considered mission time of 7 days between two scheduled maintenance operations.

As it has been presented in Chapter 3, for this type of response the source term in the adjoint sensitivity system (3.36), or (3.70) is of form $\partial F / \partial \Pi = \left[\partial F / \partial \pi_1, ..., \partial F / \partial \pi_n\right]^T = [1,0,...,0]^T$ for all time steps. As in the numerical example given in Section 3.3, because the uncertainties in input parameters are not available, the perturbations in these parameters for which the sensitivities have been computed are 0.1%, 1.0%, 5.0%, and 10.0%, respectively from their nominal values with the note that the *MTTF* has been increased with these percentages and the *MTTR* have been decreased as is the trend in practical situations. Into the Tables 4.4 to 4.24, for each subsystem of accelerator system facilities, the sensitivities of the interval availability (4.2) to variations in system parameters are presented. The sensitivities has been performed for comparison purposes using direct recalculations, FSAP, and ASAP, respectively.

Tables 4.4 and 4.5 display numerical results of the first level of accelerator system for the cases in which perturbations in the initial state probability vector, and in system parameters, i.e. the failure/repair rates, are considered. For the perturbations in initial conditions the methods give exact numerical results. The sensitivities computed at variations in initial conditions are used in general for verification purposes of the implemented method. It can be seen that the sensitivities of the response decrease with the increasing of time, the trend being that for infinity time the sensitivities to variations into the initial probability vector to vanish as has been shown in the previous chapter (e.g. Eq.3.66).

Of practical interest are the sensitivities computed at variations in system's parameters, i.e. in the failure rates of components, which give the impact of changes in their values in affecting the system's response, in this case the interval availability.

Comparison of the relative sensitivities in Table 4.5 shows that the parameters with largest impact in affecting interval availability of accelerator system within their variations are those of RF System, namely the failure rate λ_{RFSys}, followed by those of LINAC, HEBT and Injector. The parameter's variations with the smallest impact in interval availability of accelerator system are those of the Cooling system.

For the perturbations in system's parameters, in general, the differences in numerical results appears with increasing the scale of variation from the base-case value of the considered parameter. As it has been explained previous in Section 3.3, it can be seen that for variations in $MTTF = 1/\lambda$ it is a good agreement between numerical results given by either ASAP or FSAP and the sensitivities using recalculation method. Differences appears in general for sensitivities at variations in $MTTR = 1/\mu$ larger than 5.0% from the base-case value, but still in close agreement.

The computed sensitivities of the interval availability (4.2) at variations in system's parameters λ_i and μ_i for the subsystems of accelerator systems facilities are displayed as follows:

a) Table 4.6 through 4.8 the sensitivities for the Injector system and its subsystems,
b) Table 4.9 through 4.13 the sensitivities for Linear Accelerator and its subsystems,

c) Table 4.14 through 4.23 the sensitivities for Radio Frequency System, and

d) Table 4.24 for High Energy Beam Transport System.

For the Injector system and its subsystems the largest sensitivities in absolute value are given by the variations in the parameters of Ion Source with RF Antenna, i.e. the variations in RF Antenna parameters affect the most the interval availability of Ion Source, which are affecting further the interval availability of the Injector.

For LINAC, the interval availability is the most affected by perturbations in DTL's parameters. In the case of RF System whose variations in reliability parameters affect the most the interval availability of the Accelerator System facilities, the largest impact in its interval availability is given by variations in parameters of DTL RF System followed by RFQ RF System.

The complete rank of uncertainty importance of reliability parameters of systems, subsystems, and components of IFMIF Accelerator-System Facilities in affecting interval availability based on the computed relative sensitivities in absolute value is shown in Fig.4.21.

Accelerator System

Table 4.4 **Sensitivities to perturbations in initial state probability vector**

Perturbation in initial conditions (linear dependency)	Transient Duration (h)/ No. of time steps	Nominal Value R^0	Relative Sensitivity $\frac{\Delta R}{\Delta \alpha_i} \cdot \frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
-0.1% of $\pi_1(t_0)$ and 0.1% of $\pi_{32}(t_0)$	1 / 100	.9920E+00	.1000E+01	-.9919E-03	-.9919E-03	-.9919E-03
	6 / 600	.9645E+00	.9875E+00	-.9525E-03	-.9525E-03	-.9525E-03
	12 / 1200	.9452E+00	.9197E+00	-.8693E-03	-.8693E-03	-.8693E-03
	24 / 2400	.9238E+00	.7281E+00	-.6726E-03	-.6726E-03	-.6726E-03
	168 / 16800	.8882E+00	.1425E+00	-.1266E-03	-.1266E-03	-.1266E-03
-5% of $\pi_1(t_0)$ and 5% of $\pi_{32}(t_0)$	1 / 100	.9920E+00	.1000E+01	-.4960E-01	-.4960E-01	-.4960E-01
	6 / 600	.9645E+00	.9875E+00	-.4762E-01	-.4762E-01	-.4762E-01
	12 / 1200	.9452E+00	.9197E+00	-.4347E-01	-.4347E-01	-.4347E-01
	24 / 2400	.9238E+00	.7281E+00	-.3363E-01	-.3363E-01	-.3363E-01
	168 / 16800	.8882E+00	.1425E+00	-.6329E-02	-.6329E-02	-.6329E-02
-10% of $\pi_1(t_0)$ and 10% of $\pi_{32}(t_0)$	1 / 100	.9920E+00	.1000E+01	-.9919E-01	-.9919E-01	-.9919E-01
	6 / 600	.9645E+00	.9875E+00	-.9525E-01	-.9525E-01	-.9525E-01
	12 / 1200	.9452E+00	.9197E+00	-.8693E-01	-.8693E-01	-.8693E-01
	24 / 2400	.9238E+00	.7281E+00	-.6726E-01	-.6726E-01	-.6726E-01
	168 / 16800	.8882E+00	.1425E+00	-.1266E-01	-.1266E-01	-.1266E-01

Table 4.5 **Sensitivities to perturbations in system parameters** (t_f =168h)

Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
$\lambda_{INJECTOR}$	-0.1%			.1212E-04	.1212E-04	.1212E-04
	-1%		-.1365E-01	.1212E-03	.1213E-03	.1212E-03
	-5%			.6062E-03	.6066E-03	.6062E-03
	-10%	.8882E+00		.1212E-02	.1214E-02	.1212E-02
$\mu_{INJECTOR}$	0.1%			.1195E-04	.1194E-04	.1195E-04
	1%		.1346E-01	.1195E-03	.1184E-03	.1195E-03
	5%			.5976E-03	.5699E-03	.5976E-03
	10%			.1195E-02	.1089E-02	.1195E-02
λ_{LINAC}	-0.1%			.3102E-04	.3102E-04	.3102E-04
	-1%		-.3492E-01	.3102E-03	.3103E-03	.3102E-03
	-5%			.1551E-02	.1553E-02	.1551E-02
	-10%	.8882E+00		.3102E-02	.3112E-02	.3102E-02
μ_{LINAC}	0.1%			.2708E-04	.2706E-04	.2708E-04
	1%		.3049E-01	.2708E-03	.2686E-03	.2708E-03
	5%			.1354E-02	.1300E-02	.1354E-02
	10%			.2708E-02	.2501E-02	.2708E-02
$\lambda_{COOLING}$	-0.1%			.6923E-08	.6923E-08	.6923E-08
	-1%		-.7794E-05	.6923E-07	.6923E-07	.6923E-07
	-5%			.3461E-06	.3461E-07	.3461E-06
	-10%	.8882E+00		.6923E-06	.6923E-06	.6923E-06
$\mu_{COOLING}$	0.1%			.6744E-08	.6744E-08	.6744E-08
	1%		.7593E-05	.6744E-07	.6746E-07	.6744E-07
	5%			.3372E-06	.3376E-06	.3372E-06
	10%			.6744E-06	.6762E-06	.6744E-06
λ_{RFSys}	-0.1%			.3226E-04	.3227E-04	.3226E-04
	-1%		-.3632E-01	.3226E-03	.3228E-03	.3226E-03
	-5%			.1613E-02	.1616E-02	.1613E-02
	-10%	.8882E+00		.3226E-02	.3238E-02	.3226E-02
μ_{RFSys}	0.1%			.3040E-04	.3041E-04	.3040E-04
	1%		.3423E-01	.3040E-03	.3043E-03	.3040E-03
	5%			.1520E-02	.1454E-02	.1520E-02
	10%			.3040E-02	.2788E-02	.3041E-02
λ_{HEBT}	-0.1%			.2809E-04	.2809E-04	.2809E-04
	-1%		-.3162E-01	.2809E-03	.2810E-03	.2809E-03
	-5%			.1404E-02	.1407E-02	.1404E-02
	-10%	.8882E+00		.2809E-02	.2818E-02	.2809E-02
μ_{HEBT}	0.1%			.2670E-04	.2668E-04	.2671E-04
	1%		.3007E-01	.2670E-03	.2646E-03	.2671E-03
	5%			.1335E-02	.1277E-02	.1335E-02
	10%			.2670E-02	.2446E-02	.2671E-02

The computed sensitivities can be used further for uncertainty analysis of this type of response. For example, assuming that all parameters are uncorrelated, the variance of the average availability R_1 is given by $\mathrm{var}\langle R\rangle = \sum_{i=1}^{k} S_i^2\sigma_i^2$, where σ_i^2 is the uncertainty (variance) of the parameter α_i, and $S_i = \partial R/\partial\alpha_i$ is the response sensitivity to changes in parameter α_i. Considering the top level of the IFMIF Accelerator-System (Table 4.5) is as follows,

$$
\begin{aligned}
\operatorname{var}\langle R_1\rangle = {} & (-0.1365\cdot 10^{-1})^2\sigma^2_{\lambda_{INJECTOR}} + (0.1346\cdot 10^{-1})^2\sigma^2_{\mu_{INJECTOR}} + \\
& (-0.3492\cdot 10^{-1})^2\sigma^2_{\lambda_{LINAC}} + (0.3049\cdot 10^{-1})^2\sigma^2_{\mu_{LINAC}} + \\
& (-0.7794\cdot 10^{-5})^2\sigma^2_{\lambda_{COOLING}} + (0.7593\cdot 10^{-5})^2\sigma^2_{\mu_{COOLING}} + \\
& (-0.3632\cdot 10^{-1})^2\sigma^2_{\lambda_{RFSys}} + (0.3423\cdot 10^{-1})^2\sigma^2_{\mu_{RFSys}} + \\
& (-0.3162\cdot 10^{-1})^2\sigma^2_{\lambda_{HEBT}} + (0.3007\cdot 10^{-1})^2\sigma^2_{\mu_{HEBT}}
\end{aligned}
$$

Further, knowing the uncertainties of reliability parameters for all analyzed subsystems such uncertainty analyses can be performed.

This example shows the role of parameter sensitivities and uncertainties in parameters to the response uncertainty given by $\operatorname{var}\langle \mathrm{R}_1\rangle$. Therefore, if the sensitivity S_i and uncertainty σ_i^2 of parameter α_i are large, their contribution to response uncertainty is obvious larger to response uncertainty than the case in which either sensitivity S_i or uncertainty σ_i^2 is small. One can see that all sensitivities contribute to system's response uncertainty.

INJECTOR

Table 4.6 **Sensitivities to perturbations in system parameters** (t_f =168h)

Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel. Sens. $\frac{\Delta \mathrm{R}}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{\mathrm{R}^0}$	ASAP $\mathrm{R}_{pred}-\mathrm{R}^0$	REC $\mathrm{R}_{recal}-\mathrm{R}^0$	FSAP $\mathrm{R}_{pred}-\mathrm{R}^0$
λ *Ion Source*	-0.1%	.9867E+00	-.1256E-01	.1239E-04	.1239E-04	.1239E-04
	-1%			.1239E-03	.1239E-03	.1239E-03
	-5%			.6194E-03	.6198E-03	.6194E-03
	-10%			.1239E-02	.1240E-02	.1239E-02
μ *Ion Source*	0.1%		.1240E-01	.1223E-04	.1222E-04	.1223E-04
	1%			.1223E-03	.1212E-03	.1223E-03
	5%			.6117E-03	.5833E-03	.6117E-03
	10%			.1223E-02	.1115E-02	.1223E-02
λ *LEBT*	-0.1%	.9867E+00	-.7942E-03	.7836E-06	.7836E-06	.7836E-06
	-1%			.7836E-05	.7836E-05	.7836E-05
	-5%			.3918E-04	.3918E-04	.3918E-04
	-10%			.7836E-04	.7837E-04	.7836E-04
μ *LEBT*	0.1%		.7674E-03	.7571E-06	.7564E-06	.7572E-06
	1%			.7571E-05	.7499E-05	.7571E-05
	5%			.3786E-04	.3612E-04	.3786E-04
	10%			.7571E-04	.6905E-04	.7572E-04

Table 4.7 **Sensitivities to perturbations in system parameters** (t_f =168h)

Ion Source						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
$\lambda_{RF\ Ant.}$	-0.1%	.9871E+00	-.1176E-01	.1161E-04	.1161E-04	.1161E-04
	-1%			.1161E-03	.1161E-03	.1161E-03
	-5%			.5806E-03	.5810E-03	.5806E-03
	-10%			.1161E-02	.1163E-02	.1161E-02
$\mu_{RF\ Ant.}$	0.1%		.1162E-01	.1147E-04	.1146E-04	.1146E-04
	1%			.1147E-03	.1136E-03	.1147E-03
	5%			.5736E-03	.5469E-03	.5736E-03
	10%			.1147E-02	.1045E-02	.1147E-02
$\lambda_{RF\ Gen.}$	-0.1%	.9871E+00	-.1988E-04	.1962E-07	.1962E-07	.1962E-07
	-1%			.1962E-06	.1962E-06	.1962E-06
	-5%			.9812E-06	.9818E-06	.9812E-06
	-10%			.1962E-05	.1963E-05	.1962E-05
$\mu_{RF\ Gen.}$	0.1%		.1976E-04	.1951E-07	.2010E-07	.1939E-07
	1%			.1951E-06	.1938E-06	.1947E-06
	5%			.9753E-06	.9292E-06	.9740E-06
	10%			.1951E-05	.1774E-05	.1949E-05
$\lambda_{Extractor}$	-0.1%	.9871E+00	-.4457E-03	.4400E-06	.4400E-06	.4400E-06
	-1%			.4400E-05	.4400E-05	.4400E-05
	-5%			.2200E-04	.2200E-04	.2200E-04
	-10%			.4400E-04	.4400E-04	.4400E-04
$\mu_{Extractor}$	0.1%		.4348E-03	.4292E-06	.4288E-06	.4256E-06
	1%			.4292E-05	.4251E-05	.4280E-05
	5%			.2146E-04	.2046E-04	.2143E-04
	10%			.4292E-04	.3911E-04	.4288E-04
$\lambda_{Gas\ Supp.}$	-0.1%	.9871E+00	-.8233E-04	.8127E-07	.8127E-07	.8127E-07
	-1%			.8127E-06	.8127E-06	.8127E-06
	-5%			.4064E-05	.4064E-05	.4064E-05
	-10%			.8127E-05	.8128E-05	.8127E-05
$\mu_{Gas\ Supp.}$	0.1%		.8134E-04	.8029E-07	.8021E-07	.7956E-07
	1%			.8029E-06	.7957E-06	.8011E-06
	5%			.4014E-05	.3825E-05	.4009E-05
	10%			.8029E-05	.7307E-05	.8019E-05
$\lambda_{HVPSupp}$	-0.1%	.9871E+00	-.2057E-03	.2031E-06	.2031E-06	.2031E-06
	-1%			.2031E-05	.2031E-05	.2031E-05
	-5%			.1015E-04	.1015E-04	.1015E-04
	-10%			.2031E-04	.2031E-04	.2031E-04
$\mu_{HVPSupp}$	0.1%		.1717E-03	.1695E-06	.1693E-06	.1578E-06
	1%			.1695E-05	.1681E-05	.1643E-05
	5%			.8474E-05	.8146E-05	.8342E-05
	10%			.1695E-04	.1568E-04	.1674E-04
$\lambda_{TVP1\text{-}2}$	-0.1%	.9871E+00	-.1952E-03	.1927E-06	.1927E-06	.1927E-06
	-1%			.1927E-05	.1927E-05	.1927E-05
	-5%			.9635E-05	.9635E-05	.9635E-05
	-10%			.1927E-04	.1927E-04	.1927E-04
$\mu_{TVP1\text{-}2}$	0.1%		.1904E-03	.1880E-06	.1878E-06	.1865E-06
	1%			.1880E-05	.1862E-05	.1871E-05
	5%			.9400E-05	.8964E-05	.9376E-05
	10%			.1880E-04	.1713E-04	.1877E-04
$\lambda_{SuppStr}$	-0.1%	.9871E+00	-.1236E-04	.1220E-07	.1220E-07	.1220E-07
	-1%			.1220E-06	.1220E-06	.1220E-06
	-5%			.6100E-06	.6100E-06	.6100E-06
	-10%			.1220E-05	.1220E-05	.1220E-05
$\mu_{SuppStr.}$	0.1%		.3482E-05	.3437E-08	.3436E-08	.1382E-08
	1%			.3437E-07	.3429E-07	.2520E-07
	5%			.1718E-06	.1699E-06	.1305E-06
	10%			.3437E-06	.3361E-06	.2757E-06

Table 4.8 **Sensitivities to perturbation in system parameters** (t_f =168h)

LEBT						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
$\lambda_{\#1.Foc.Sol}$, $\lambda_{\#1.Str.Mg}$	-0.1%	.9992E+00	-.7809E-05	.7803E-08	.7803E-08	.7803E-08
	-1%			.7803E-07	.7803E-07	.7803E-07
	-5%			.3901E-06	.3901E-06	.3901E-06
	-10%			.7803E-06	.7803E-06	.7803E-06
$\mu_{\#1.Foc.Sol}$, $\mu_{\#1.Str.Mgn}$	0.1%		.7619E-05	.7613E-08	.7803E-08	.7608E-08
	1%			.7613E-07	.7539E-07	.7612E-07
	5%			.3806E-06	.3629E-06	.3806E-06
	10%			.7613E-06	.6936E-06	.7613E-06
$\lambda_{\#2.Foc.Sol}$	-0.1%	9992E+00	-.5893E-05	.5888E-08	.5888E-08	.5888E-08
	-1%			.5888E-07	.5888E-07	.5888E-07
	-5%			.2944E-06	.2944E-06	.2944E-06
	-10%			.5888E-06	.5888E-06	.5888E-06
$\mu_{\#2.Foc.Sol}$	0.1%		.5786E-05	.5781E-08	.5775E-08	.5778E-08
	1%			.5781E-07	.5725E-07	.5780E-07
	5%			.2890E-06	.2755E-06	.2890E-06
	10%			.5781E-06	.5264E-06	.5781E-06
$\lambda_{\#2.StrMgn}$	-0.1%	9992E+00	-.1524E-04	.1523E-07	.1523E-07	.1523E-07
	-1%			.1523E-06	.1523E-06	.1523E-06
	-5%			.7613E-06	.7613E-06	.7613E-06
	-10%			.1523E-05	.1523E-05	.1523E-05
$\mu_{\#2.StrMgn.}$	0.1%		.1448E-04	.1446E-07	.1445E-07	.1445E-07
	1%			.1446E-06	.1433E-06	.1446E-06
	5%			.7232E-06	.6905E-06	.7231E-06
	10%			.1446E-05	.1321E-05	.1446E-05
$\lambda_{GasNeutr}$	-0.1%	9992E+00	-.4142E-04	.4138E-07	.4138E-07	.4138E-07
	-1%			.4138E-06	.4138E-06	.4138E-06
	-5%			.2069E-05	.2072E-05	.2069E-05
	-10%			.4138E-05	.4138E-05	.4138E-05
$\mu_{GasNeutr}$	0.1%		.4117E-04	.4114E-07	.4110E-07	.4114E-07
	1%			.4114E-06	.4073E-06	.4114E-06
	5%			.2057E-05	.1959E-05	.2057E-05
	10%			.4114E-05	.3742E-05	.4114E-05
$\lambda_{PwrSup1-4}$	-0.1%	9992E+00	-.1881E-04	.1879E-07	.1879E-07	.1879E-07
	-1%			.1879E-06	.1879E-06	.1879E-06
	-5%			.9397E-06	.9397E-06	.9397E-06
	-10%			.1879E-05	.1879E-05	.1879E-05
$\mu_{PwrSup1-4}$	0.1%		.1762E-04	.1760E-07	.1759E-07	.1759E-07
	1%			.1760E-06	.1744E-06	.1760E-06
	5%			.8802E-06	.8410E-06	.8801E-06
	10%			.1760E-05	.1610E-05	.1760E-05
λ_{Diags}	-0.1%	9992E+00	-.4643E-03	.4639E-06	.4639E-06	.4639E-06
	-1%			.4639E-05	.4639E-05	.4639E-05
	-5%			.2320E-04	.2320E-04	.2320E-04
	-10%			.4639E-04	.4639E-04	.4639E-04
μ_{Diags}	0.1%		.4286E-03	.4282E-06	.4278E-06	.4282E-06
	1%			.4282E-05	.4243E-05	.4282E-05
	5%			.2141E-04	.2047E-04	.2141E-04
	10%			.4282E-04	.3923E-04	.4282E-04
λ_{TVP}	-0.1%	9992E+00	-.1952E-03	.1951E-06	.1951E-06	.1951E-06
	-1%			.1951E-05	.1951E-05	.1951E-05
	-5%			.9754E-05	.9754E-05	.9754E-05
	-10%			.1951E-04	.1951E-04	.1951E-04
μ_{TVP}	0.1%		.1905E-03	.1903E-06	.1901E-06	.1903E-06
	1%			.1903E-05	.1885E-05	.1903E-05
	5%			.9516E-05	.9074E-05	.9516E-05
	10%			.1903E-04	.1734E-04	.1903E-04

$\lambda_{SuppStr}$	-0.1%	9992E+00	-.1236E-04	.1235E-07	.1235E-07	.1235E-07
	-1%			.1235E-06	.1235E-06	.1235E-06
	-5%			.6175E-06	.6175E-06	.6175E-06
	-10%			.1235E-05	.1235E-05	.1235E-05
$\mu_{SuppStr}$	0.1%		.3482E-05	.3479E-08	.3479E-08	.3421E-08
	1%			.3479E-07	.3472E-07	.3461E-07
	5%			.1740E-06	.1720E-06	.1735E-06
	10%			.3479E-06	.3402E-06	.3474E-06

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Table 4.9 **Sensitivities to perturbation in system parameters** (t_f =168h)

Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel. Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
λ_{RFQ}	-0.1%	.9661E+00	-.8263E-02	.7983E-05	.7983E-05	.7984E-05
	-1%			.7983E-04	.7984E-04	.7984E-04
	-5%			.3992E-03	.3993E-03	.3992E-03
	-10%			.7983E-03	.7990E-03	.7983E-03
μ_{RFQ}	0.1%		.7708E-02	.7446E-05	.7440E-05	.7442E-05
	1%			.7446E-04	.7379E-04	.7446E-04
	5%			.3723E-03	.3559E-03	.3723E-03
	10%			.7446E-03	.6818E-03	.7446E-03
λ_{DTL}	-0.1%	.9661E+00	-.2583E-01	.2495E-04	.2495E-04	.2495E-04
	-1%			.2495E-03	.2496E-03	.2495E-03
	-5%			.1248E-02	.1249E-02	.1248E-02
	-10%			.2495E-02	.2501E-02	.2495E-02
μ_{DTL}	0.1%		.2147E-01	.2075E-04	.2073E-04	.2073E-04
	1%			.2075E-03	.2058E-03	.2074E-03
	5%			.1037E-02	.9981E-03	.1037E-02
	10%			.2075E-02	.1923E-02	.2075E-02

Table 4.10 **Sensitivities to perturbation in system parameters** (t_f =168h)

RFQ						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
λ_{Cavity}	-0.1%	.9921E+00	-.1662E-04	.1649E-07	.1649E-07	.1649E-07
	-1%			.1649E-06	.1649E-06	.1649E-06
	-5%			.8246E-06	.8246E-06	.8246E-06
	-10%			.1649E-05	.1649E-05	.1649E-05
μ_{Cavity}	0.1%		.1851E-06	.1837E-09	.1837E-09	.9556E-10
	1%			.1837E-08	.1836E-08	.1216E-08
	5%			.9183E-08	.9179E-08	.7023E-08
	10%			.1837E-07	.1835E-07	.1481E-07
$\lambda_{Drive\ LP\ \&\ Window}$	-0.1%	.9921E+00	-.6690E-02	.6637E-05	.6637E-05	.6637E-05
	-1%			.6637E-04	.6637E-04	.6637E-04
	-5%			.3318E-03	.3320E-03	.3318E-03
	-10%			.6637E-03	.6641E-03	.6637E-03
$\mu_{Drive\ LP\ \&\ Window}$	0.1%		.6175E-02	.6127E-05	.6121E-05	.6114E-05
	1%			.6127E-04	.6072E-04	.6123E-04
	5%			.3063E-03	.2930E-03	.3063E-03
	10%			.6127E-03	.5615E-03	.6126E-03
$\lambda_{TMV\ Pump}$	-0.1%	.9921E+00	-.1172E-02	.1162E-05	.1162E-05	.1162E-05
	-1%			.1162E-04	.1162E-04	.1162E-04
	-5%			.5811E-04	.5812E-04	.5811E-04
	-10%			.1162E-03	.1162E-03	.1162E-03
$\mu_{TMV\ Pump}$	0.1%		.1143E-02	.1134E-05	.1133E-05	.1132E-05
	1%			.1134E-04	.1123E-04	.1133E-04
	5%			.5669E-04	.5406E-04	.5668E-04
	10%			.1134E-03	.1033E-03	.1134E-03

$\lambda_{SuppStr.}$	-0.1%	.9921E+00	-.1236E-04	.1226E-07	.1226E-07	.1231E-07
	-1%			.1226E-06	.1226E-06	.1227E-06
	-5%			.6129E-06	.6129E-06	.6130E-06
	-10%			.1226E-05	.1226E-05	.1226E-05
$\mu_{SuppStr}$	0.1%		.3481E-05	.3453E-08	.3452E-08	.2386E-08
	1%			.3453E-07	.3445E-07	.3009E-07
	5%			.1727E-06	.1707E-06	.1609E-06
	10%			.3453E-06	.3377E-06	.3286E-06

Table 4.11 **Sensitivities to perturbation in system parameters** (t_f =168h)

DTL						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
λ_{DTQ}	-0.1%	.9748E+00	-.9180E-02	.8949E-05	.8949E-05	.8954E-05
	-1%			.8949E-04	.8949E-04	.8952E-04
	-5%			.4474E-03	.4476E-03	.4475E-03
	-10%			.8949E-03	.8956E-03	.8950E-03
μ_{DTQ}	0.1%		.4845E-02	.4724E-05	.4721E-05	.4639E-05
	1%			.4724E-04	.4702E-04	.4693E-04
	5%			.2362E-03	.2310E-03	.2355E-03
	10%			.4724E-03	.4519E-03	.4712E-03
$\lambda_{DTL\ TSys}$	-0.1%	.9748E+00	-.1256E-01	.1225E-04	.1225E-04	.1225E-04
	-1%			.1225E-03	.1225E-03	.1225E-03
	-5%			.6123E-03	.6127E-03	.6124E-03
	-10%			.1225E-02	.1226E-02	.1225E-02
$\mu_{DTL\ TSys}$	0.1%		.1122E-01	.1094E-04	.1093E-04	.1089E-04
	1%			.1094E-03	.1084E-03	.1093E-03
	5%			.5469E-03	.5241E-03	.5466E-03
	10%			.1094E-02	.1006E-02	.1093E-02
$\lambda_{Diags1\text{-}6}$	-0.1%	.9748E+00	-.6024E-03	.5872E-06	.5872E-06	.5878E-06
	-1%			.5872E-05	.5872E-05	.5874E-05
	-5%			.2936E-04	.2936E-04	.2937E-04
	-10%			.5872E-04	.5872E-04	.5874E-04
$\mu_{Diags1\text{-}6}$	0.1%		.5384E-03	.5249E-06	.5244E-06	.5157E-06
	1%			.5249E-05	.5203E-05	.5215E-05
	5%			.2625E-04	.2514E-04	.2615E-04
	10%			.5249E-04	.4823E-04	.5240E-04

Table 4.12 **Sensitivities to perturbation in system parameters** (t_f =168h)

DTQ						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
λ_{QMSys}	-0.1%	.9909E+00	-.4937E-02	.4892E-05	.4892E-05	.4893E-05
	-1%			.4892E-04	.4892E-04	.4893E-04
	-5%			.2446E-03	.2447E-03	.2446E-03
	-10%			.4892E-03	.4894E-03	.4892E-03
μ_{QMSys}	0.1%		.2603E-02	.2580E-05	.2578E-05	.2572E-05
	1%			.2580E-04	.2568E-04	.2577E-04
	5%			.1290E-03	.1261E-03	.1289E-03
	10%			.2580E-03	.2468E-03	.2579E-03
λ_{DTLSys}	-0.1%	.9909E+00	-.4175E-02	.4137E-05	.4137E-05	.4138E-05
	-1%			.4137E-04	.4137E-04	.4137E-04
	-5%			.2069E-03	.2069E-03	.2069E-03
	-10%			.4137E-03	.4139E-03	.4137E-03
μ_{DTLSys}	0.1%		.2201E-02	.2181E-05	.2180E-05	.2175E-05
	1%			.2181E-04	.2171E-04	.2179E-04
	5%			.1090E-03	.1066E-03	.1090E-03
	10%			.2181E-03	.2086E-03	.2180E-03

Table 4.13 **Sensitivities to perturbation in system parameters** (t_f =168h)

Tank#						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ *Cavity*	-0.1%	.9987E+00	-.1663E-04	.1661E-07	.1661E-07	.1661E-07
	-1%			.1661E-06	.1661E-06	.1661E-06
	-5%			.8303E-06	.8303E-06	.8303E-06
	-10%			.1661E-05	.1661E-05	.1661E-05
μ *Cavity*	0.1%		.1852E-06	.1850E-09	.1850E-09	.7815E-10
	1%			.1850E-08	.1849E-08	.5809E-09
	5%			.9248E-08	.9244E-08	.6226E-08
	10%			.1850E-07	.1848E-07	.1312E-07
λ *Drv LP & Window1-2*	-0.1%	.9987E+00	-.5572E-03	.5564E-06	.5564E-06	.5567E-06
	-1%			.5564E-05	.5564E-05	.5564E-05
	-5%			.2782E-04	.2782E-04	.2782E-04
	-10%			.5564E-04	.5564E-04	.5564E-04
μ *Drv LP & Window1-2*	0.1%		.5143E-03	.5136E-06	.5131E-06	.5080E-06
	1%			.5136E-05	.5089E-05	.5115E-05
	5%			.2568E-04	.2455E-04	.2563E-04
	10%			.5136E-04	.4705E-04	.5130E-04
λ *Ion Vac Pump 1-2*	-0.1%	.9987E+00	-.9762E-04	.9749E-07	.9749E-07	.9744E-07
	-1%			.9749E-06	.9748E-06	.9749E-06
	-5%			.4874E-05	.4874E-05	.4874E-05
	-10%			.9749E-05	.9749E-05	.9749E-05
μ *Ion Vac Pump 1-2*	0.1%		.9523E-04	.9511E-07	.9501E-07	.9444E-07
	1%			.9511E-06	.9420E-06	.9486E-06
	5%			.4755E-05	.4534E-05	.4749E-05
	10%			.9511E-05	.8666E-05	.9502E-05
λ *Support Structure.*	-0.1%	.9987E+00	-.1236E-04	.1234E-07	.1234E-07	.1252E-07
	-1%			.1234E-06	.1234E-06	.1241E-06
	-5%			.6172E-06	.6172E-06	.6178E-06
	-10%			.1234E-05	.1234E-05	.1235E-05
μ *Support Structure*	0.1%		.3482E-05	.3477E-08	.3477E-08	.2533E-08
	1%			.3477E-07	.3470E-07	.2757E-07
	5%			.1739E-06	.1719E-06	.1525E-06
	10%			.3477E-06	.3400E-06	.3179E-06

RF System

Table 4.14 **Sensitivities to perturbation in system parameters** (t_f =168h)

Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ *RFQ RF Sys*	-0.1%	.9643E+00	-.1194E-01	.1152E-04	.1152E-04	.1152E-04
	-1%			.1152E-03	.1152E-03	.1152E-03
	-5%			.5758E-03	.5761E-03	.5758E-03
	-10%			.1152E-02	.1153E-02	.1152E-02
μ *RFQ RF Sys*	0.1%		.1126E-01	.1086E-04	.1085E-04	.1084E-04
	1%			.1086E-03	.1076E-03	.1085E-03
	5%			.5429E-03	.5188E-03	.5428E-03
	10%			.1086E-02	.9936E-03	.1086E-02
λ *RF Inst&Ctrl*	-0.1%	.9643E+00	-.1988E-04	.1917E-07	.1917E-07	.1919E-07
	-1%			.1917E-06	.1916E-06	.1917E-06
	-5%			.9583E-06	.9583E-06	.9585E-06
	-10%			.1917E-05	.1917E-05	.1917E-05
μ *RF Inst&Ctrl*	0.1%		.1975E-04	.1905E-07	.1903E-07	.1902E-07
	1%			.1905E-06	.1886E-06	.1903E-06
	5%			.9524E-06	.9073E-06	.9522E-06
	10%			.1905E-05	.1733E-05	.1904E-05

$\lambda_{DTL\ RF\ Sys}$	-0.1%	.9643E+00	-.2406E-01	.2320E-04	.2320E-04	.2321E-04
	-1%			.2320E-03	.2321E-03	.2320E-03
	-5%			.1160E-02	.1161E-02	.1160E-02
	-10%			.2320E-02	.2326E-02	.2320E-02
$\mu_{DTL\ RF\ Sys}$	0.1%		.2269E-01	.2188E-04	.2186E-04	.2186E-04
	1%			.2188E-03	.2168E-03	.2188E-03
	5%			.1094E-02	.1046E-02	.1094E-02
	10%			.2188E-02	.2004E-02	.2188E-02

Table 4.15 **Sensitivities to perturbation in system parameters** (t_f =168h)

RF Station						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
$\lambda_{RF\ Tube}$	-0.1%	.9961E+00	-.1283E-02	.1278E-05	.1278E-05	.1279E-05
	-1%			.1278E-04	.1278E-04	.1278E-04
	-5%			.6392E-04	.6392E-04	.6392E-04
	-10%			.1278E-03	.1279E-03	.1278E-03
$\mu_{RF\ Tube}$	0.1%		.1167E-02	.1162E-05	.1161E-05	.1147E-05
	1%			.1162E-04	.1152E-04	.1157E-04
	5%			.5811E-04	.5561E-04	.5801E-04
	10%			.1162E-03	.1066E-03	.1160E-03
$\lambda_{RF\ Transport}$	-0.1%	.9961E+00	-.7007E-03	.6980E-06	.6980E-06	.6980E-06
	-1%			.6980E-05	.6980E-05	.6980E-05
	-5%			.3490E-04	.3490E-04	.3490E-04
	-10%			.6980E-04	.6980E-04	.6980E-04
$\mu_{RF\ Transport}$	0.1%		.6776E-03	.6749E-06	.6743E-06	.6706E-06
	1%			.6749E-05	.6685E-05	.6734E-05
	5%			.3375E-04	.3219E-04	.3371E-04
	10%			.6749E-04	.6155E-04	.6744E-04
$\lambda_{Circulator}$	-0.1%	.9961E+00	-.1176E-02	.1171E-05	.1171E-05	.1171E-05
	-1%			.1171E-04	.1171E-04	.1171E-04
	-5%			.5855E-04	.5855E-04	.5855E-04
	-10%			.1171E-03	.1171E-03	.1171E-03
$\mu_{Circulator}$	0.1%		.1099E-02	.1095E-05	.1094E-05	.1085E-05
	1%			.1095E-04	.1085E-04	.1091E-04
	5%			.5474E-04	.5231E-04	.5466E-04
	10%			.1095E-03	.1002E-03	.1094E-03
$\lambda_{Tube\ Peripherals}$	-0.1%	.9961E+00	-.9404E-05	.9367E-08	.9367E-08	.9515E-08
	-1%			.9367E-07	.9367E-07	.9371E-07
	-5%			.4683E-06	.4683E-06	.4683E-06
	-10%			.9367E-06	.9367E-06	.9368E-06
$\mu_{Tube\ Peripherals}$	0.1%		.8808E-05	.8773E-08	.8765E-08	.8261E-08
	1%			.8773E-07	.8692E-07	.8624E-07
	5%			.4387E-06	.4191E-06	.4325E-06
	10%			.8773E-06	.8025E-06	.8682E-06
$\lambda_{Source\ \&Drv}$	-0.1%	.9961E+00	-.1988E-04	.1980E-07	.1980E-07	.1980E-07
	-1%			.1980E-06	.1980E-06	.1980E-06
	-5%			.9901E-06	.9901E-06	.9901E-06
	-10%			.1980E-05	.1980E-05	.1980E-05
$\mu_{Source\ \&Drv}$	0.1%		.1976E-04	.1968E-07	.1966E-07	.1962E-07
	1%			.1968E-06	.1949E-06	.1966E-06
	5%			.9842E-06	.9376E-06	.9835E-06
	10%			.1968E-05	.1790E-05	.1967E-05
$\lambda_{HV\ Pwr\ Suppl}$	-0.1%	.9961E+00	-.7540E-03	.7510E-06	.7510E-06	.7512E-06
	-1%			.7510E-05	.7510E-05	.7511E-05
	-5%			.3755E-04	.3755E-04	.3755E-04
	-10%			.7510E-04	.7511E-04	.7510E-04
$\mu_{HV\ Pwr\ Suppl}$	0.1%		.7108E-03	.7080E-06	.7074E-06	.7019E-06
	1%			.7080E-05	.7014E-05	.7051E-05
	5%			.3540E-04	.3381E-04	.3534E-04
	10%			.7080E-04	.6473E-04	.7072E-04

Table 4.16 **Sensitivities to perturbation in system parameters** (t_f =168h)

Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
RF PA Tube Peripherals						
λ *LC&Monit*	-0.1%	.9993E+00	-.3952E-03	.3949E-06	.3949E-06	.3950E-06
	-1%			.3949E-05	.3949E-05	.3949E-05
	-5%			.1975E-04	.1975E-04	.1975E-04
	-10%			.3949E-04	.3950E-04	.3949E-04
μ *LC&Monit*	0.1%		.3702E-03	.3699E-06	.3696E-06	.3666E-06
	1%			.3699E-05	.3665E-05	.3686E-05
	5%			.1850E-04	.1767E-04	.1847E-04
	10%			.3699E-04	.3384E-04	.3695E-04
λ *LP&AC*	-0.1%	.9993E+00	-.9407E-04	.9401E-07	.9401E-07	.9398E-07
	-1%			.9401E-06	.9401E-06	.9402E-06
	-5%			.4700E-05	.4700E-05	.4700E-05
	-10%			.9401E-05	.9401E-05	.9401E-05
μ *LP&AC*	0.1%		.8812E-04	.8806E-07	.8798E-07	.8649E-07
	1%			.8806E-06	.8724E-06	.8742E-06
	5%			.4403E-05	.4207E-05	.4388E-05
	10%			.8806E-05	.8054E-05	.8784E-05
λ *HP RF Tube*	-0.1%	.9993E+00	-.9848E-04	.9841E-07	.9841E-07	.9840E-07
	-1%			.9841E-06	.9841E-06	.9841E-06
	-5%			.4921E-05	.4920E-05	.4921E-05
	-10%			.9841E-05	.9841E-05	.9841E-05
μ *HP RF Tube*	0.1%		.9685E-04	.9678E-07	.9669E-07	.9641E-07
	1%			.9678E-06	.9584E-06	.9665E-06
	5%			.4839E-05	.4612E-05	.4835E-05
	10%			.9678E-05	.8812E-05	.9672E-05
λ *Struct&Cabl*	-0.1%	.9993E+00	-.3135E-05	.3133E-08	.3133E-08	.3183E-08
	-1%			.3133E-07	.3133E-07	.3136E-07
	-5%			.1566E-06	.1566E-06	.1566E-06
	-10%			.3133E-06	.3133E-06	.3134E-06
μ *Struct&Cabl*	0.1%		.2936E-05	.2934E-08	.2932E-08	.2751E-08
	1%			.2934E-07	.2908E-07	.2887E-07
	5%			.1467E-06	.1402E-06	.1448E-06
	10%			.2934E-06	.2684E-06	.2905E-06
λ *LV Pwr Supply*	-0.1%	.9993E+00	-.9405E-04	.9398E-07	.9398E-07	.9395E-07
	-1%			.9398E-06	.9398E-06	.9399E-06
	-5%			.4699E-05	.4699E-05	.4699E-05
	-10%			.9398E-05	.9398E-05	.9398E-05
μ *LV Pwr Supply*	0.1%		.8809E-04	.8803E-07	.8795E-07	.8650E-07
	1%			.8803E-06	.8722E-06	.8741E-06
	5%			.4402E-05	.4206E-05	.4389E-05
	10%			.8803E-05	.8052E-05	.8783E-05

Table 4.17 **Sensitivities to perturbation in system parameters** (t_f =168h)

Param. α_i	Perturb. $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
Loc Ctrl & Monit						
λ *Reflectometer,* λ *Directional Couplers.*	-0.1%	.9996E+00	-.9405E-05	.9401E-08	.9401E-08	.9503E-08
	-1%			.9401E-07	.9401E-07	.9396E-07
	-5%			.4700E-06	.4700E-06	.4702E-06
	-10%			.9401E-06	.9401E-06	.9401E-06
μ *Reflectometer,* μ *Directional Couplers*	0.1%		.8809E-05	.8806E-08	.8798E-08	.8542E-08
	1%			.8806E-07	.8725E-07	.8644E-07
	5%			.4403E-06	.4207E-06	.4361E-06
	10%			.8806E-06	.8055E-06	.8745E-06

$\lambda_{Cav\ Tunn\ Ctrl}$ $\lambda_{Syst\ Ctrls\ 1\text{-}2}$ $\lambda_{Resonance\ Ctrl}$	-0.1%	.9996E+00	-.9405E-04	.9401E-07	.9401E-07	.9396E-07
	-1%			.9401E-06	.9401E-06	.9401E-06
	-5%			.4701E-05	.4701E-05	.4701E-05
	-10%			.9401E-05	.9401E-05	.9401E-05
$\mu_{Cav\ Tunn\ Ctrl}$ $\mu_{Syst\ Ctrls\ 1\text{-}2}$ $\mu_{Resonance\ Ctrl}$	0.1%		.8809E-04	.8806E-07	.8798E-07	.8652E-07
	1%			.8806E-06	.8725E-06	.8745E-06
	5%			.4403E-05	.4207E-05	.4388E-05
	10%			.8806E-05	.8055E-05	.8786E-05

Table 4.18 **Sensitivities to perturbation in system parameters** (t_f =168h)

HP RF Tube						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
$\lambda_{Cooler1\text{-}2}$	-0.1%	.9997E+00	-.9881E-04	.9878E-07	.9878E-07	.9878E-07
	-1%			.9878E-06	.9878E-06	.9878E-06
	-5%			.4939E-05	.4939E-05	.4939E-05
	-10%			.9878E-05	.9878E-05	.9878E-05
$\mu_{Cooler1\text{-}2}$	0.1%		.9762E-04	.9759E-07	.9749E-07	.9740E-07
	1%			.9759E-06	.9664E-06	.9751E-06
	5%			.4880E-05	.4650E-05	.4878E-05
	10%			.9759E-05	.8882E-05	.9755E-05
$\lambda_{Tube\ Cavity}$	-0.1%	.9997E+00	-.9405E-04	.9402E-07	.9402E-07	.9405E-07
	-1%			.9402E-06	.9402E-06	.9403E-06
	-5%			.4701E-05	.4701E-05	.4701E-05
	-10%			.9402E-05	.9402E-05	.9402E-05
$\mu_{Tube\ Cavity}$	0.1%		.8810E-04	.8807E-07	.8799E-07	.8683E-07
	1%			.8807E-06	.8726E-06	.8751E-06
	5%			.4403E-05	.4207E-05	.4391E-05
	10%			.8807E-05	.8056E-05	.8789E-05

Table 4.19 **Perturbation in system's parameters** (t_f =168h)

Source & Driver						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
$\lambda_{2nd\ Stage\ Tube}$	-0.1%	.9988E+00	-.1090E-02	.1089E-05	.1089E-05	.1090E-05
	-1%			.1089E-04	.1089E-04	.1089E-04
	-5%			.5446E-04	.5446E-04	.5446E-04
	-10%			.1089E-03	.1089E-03	.1089E-03
$\mu_{2nd\ Stage\ Tube}$	0.1%		.9990E-03	.9979E-06	.9969E-06	.9853E-06
	1%			.9979E-05	.9889E-05	.9941E-05
	5%			.4989E-04	.4773E-04	.4981E-04
	10%			.9979E-04	.9148E-04	.9965E-04
$\lambda_{2nd\ St\ Tube\ Cavity}$	-0.1%	.9988E+00	-.8518E-05	.8508E-08	.8508E-08	.8610E-08
	-1%			.8508E-07	.8508E-07	.8503E-07
	-5%			.4254E-06	.4254E-06	.4255E-06
	-10%			.8508E-06	.8508E-06	.8510E-06
$\mu_{2nd\ St\ Tube\ Cavity}$	0.1%		.8035E-05	.8026E-08	.8019E-08	.7671E-08
	1%			.8026E-07	.7952E-07	.7917E-07
	5%			.4013E-06	.3833E-06	.3973E-06
	10%			.8026E-06	.7336E-06	.7955E-06
$\lambda_{Solid\ State\ Pre\text{-}Amp}$	-0.1%	.9988E+00	-.3952E-04	.3948E-07	.3948E-07	.3950E-07
	-1%			.3948E-06	.3947E-06	.3948E-06
	-5%			.1974E-05	.1974E-05	.1974E-05
	-10%			.3948E-05	.3948E-05	.3948E-05
$\mu_{Solid\ State\ Pre\text{-}Amp}$	0.1%		.3905E-04	.3900E-07	.3897E-07	.3885E-07
	1%			.3900E-06	.3864E-06	.3892E-06
	5%			.1950E-05	.1859E-05	.1948E-05
	10%			.3900E-05	.3550E-05	.3896E-05

$\lambda_{SPPT\ Sys.}$	-0.1%	.9988E+00	-.6587E-06	.6580E-09	.6579E-09	.6638E-09
	-1%			.6580E-08	.6579E-08	.6581E-08
	-5%			.3290E-07	.3290E-07	.3291E-07
	-10%			.6580E-07	.6579E-07	.6583E-07
$\mu_{SPPT\ Sys.}$	0.1%		.6508E-06	.6500E-09	.6516E-09	.6207E-09
	1%			.6500E-08	.6439E-08	.6462E-08
	5%			.3250E-07	.3098E-07	.3237E-07
	10%			.6500E-07	.5926E-07	.6477E-07
λ_{Source}	-0.1%	.9988E+00	-.1976E-04	.1974E-07	.1974E-07	.1977E-07
	-1%			.1974E-06	.1973E-06	.1974E-06
	-5%			.9869E-06	.9869E-06	.9870E-06
	-10%			.1974E-05	.1974E-05	.1974E-05
μ_{Source}	0.1%		.1952E-04	.1950E-07	.1949E-07	.1933E-07
	1%			.1950E-06	.1931E-06	.1945E-06
	5%			.9750E-06	.9291E-06	.9736E-06
	10%			.1950E-05	.1775E-05	.1948E-05

Table 4.20 **Sensitivities to perturbation in system parameters** (t_f =168h)

HV Power Supply						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
$\lambda_{Crowbar}$ System	-0.1%	.9992E+00	-.5174E-03	.5170E-06	.5170E-06	.5171E-06
	-1%			.5170E-05	.5170E-05	.5170E-05
	-5%			.2585E-04	.2585E-04	.2585E-04
	-10%			.5170E-04	.5170E-04	.5170E-04
$\mu_{Crowbar}$ System	0.1%		.4847E-03	.4843E-06	.4838E-06	.4820E-06
	1%			.4843E-05	.4798E-05	.4836E-05
	5%			.2421E-04	.2314E-04	.2420E-04
	10%			.4843E-04	.4430E-04	.4841E-04
$\lambda_{AC\ Pwr}$ Distrib	-0.1%	.9992E+00	-.1254E-04	.1253E-07	.1253E-07	.1252E-07
	-1%			.1253E-06	.1253E-06	.1253E-06
	-5%			.6265E-06	.6265E-06	.6265E-06
	-10%			.1253E-05	.1253E-05	.1253E-05
$\mu\ \mu_{AC\ Pwr}$ Distrib	0.1%		.1175E-04	.1174E-07	.1173E-07	.1155E-07
	1%			.1174E-06	.1163E-06	.1164E-06
	5%			.5868E-06	.5607E-06	.5845E-06
	10%			.1174E-05	.1074E-05	.1171E-05
$\lambda_{AC/DC}$ Converter	-0.1%	.9992E+00	-.2231E-03	.2229E-06	.2229E-06	.2230E-06
	-1%			.2229E-05	.2229E-05	.2229E-05
	-5%			.1115E-04	.1115E-04	.1115E-04
	-10%			.2229E-04	.2229E-04	.2229E-04
$\mu_{AC/DC}$ Converter	0.1%		.2127E-03	.2126E-06	.2124E-06	.2116E-06
	1%			.2126E-05	.2106E-05	.2123E-05
	5%			.1063E-04	.1015E-04	.1062E-04
	10%			.2126E-04	.1941E-04	.2125E-04

Table 4.21 **Sensitivities to perturbation in system parameters** (t_f =168h)

Crowbar System						
Param. α_i	Perturb. $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
λ *Fast Curr. Limit*	-0.1%			.4700E-07	.4700E-07	.4700E-07
λ *Elec DC Switch*	-1%		-.4702E-04	.4700E-06	.4700E-06	.4700E-06
λ *Curr Lim. React.*	-5%			.2350E-05	.2350E-05	.2350E-05
	-10%			.4700E-05	.4700E-05	.4700E-05
μ *Fast Curr. Limir*	0.1%	.9995E+00		.4403E-07	.4399E-07	.4402E-07
μ *Elec. DC Switch*	1%		.4405E-04	.4403E-06	.4362E-06	.4403E-06
μ *Curr Lim. React.*	5%			.2201E-05	.2103E-05	.2201E-05
	10%			.4403E-05	.4027E-05	.4403E-05
λ *Crowbar*	-0.1%			.9400E-07	.9400E-07	.9400E-07
λ *Transp. Prot.*	-1%		-.9405E-04	.9400E-06	.9400E-06	.9400E-06
	-5%			.4700E-05	.4700E-05	.4700E-05
	-10%	.9995E+00		.9400E-05	.9400E-05	.9400E-05
μ *Crowbar*	0.1%			.8805E-07	.8797E-07	.8805E-07
μ *Transp. Prot.*	1%		.8809E-04	.8805E-06	.8724E-06	.8805E-06
	5%			.4403E-05	.4207E-05	.4403E-05
	10%			.8805E-05	.8054E-05	.8805E-05
λ *Volt Monitor*	-0.1%			.4700E-07	.4700E-07	.4700E-07
λ *Input Bushing*	-1%		-.4702E-04	.4700E-06	.4700E-06	.4700E-06
λ *Output Bushing*	-5%			.2350E-05	.2350E-05	.2350E-05
	-10%			.4700E-05	.4700E-05	.4700E-05
μ *Volt Monitor*	0.1%	.9995E+00		.4403E-07	.4399E-07	.4402E-07
μ *Input Bushing*	1%		.4405E-04	.4403E-06	.4362E-06	.4403E-06
μ *Output Bushing*	5%			.2201E-05	.2103E-05	.2201E-05
	10%			.4403E-05	.4027E-05	.4403E-05

Table 4.22 **Sensitivities to perturbation in system parameters** (t_f =168h)

AC Power Distribut.						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
λ *Bus Lines*	-0.1%			.3135E-08	.3142E-08	.3136E-08
	-1%		-.3135E-05	.3135E-07	.3135E-07	.3136E-07
	-5%			.1567E-06	.1567E-06	.1568E-06
	-10%	9.99987E-1		.3135E-06	.3135E-06	.3135E-06
μ *Bus Lines*	0.1%			.2936E-08	.2940E-08	.2890E-08
	1%		.2936E-05	.2936E-07	.2903E-07	.2906E-07
	5%			.1468E-06	.1403E-06	.1460E-06
	10%			.2936E-06	.2686E-06	.2924E-06
λ *SPPT Structure*	-0.1%			.9405E-08	.9411E-08	.9403E-08
	-1%		-.9405E-05	.9405E-07	.9400E-07	.9406E-07
	-5%			.4702E-06	.4702E-06	.4702E-06
	-10%	9.99987E-1		.9405E-06	.9404E-06	.9405E-06
μ *SPPT Structure*	0.1%			.8809E-08	.8807E-08	.8659E-08
	1%		.8810E-05	.8809E-07	.8725E-07	.8757E-07
	5%			.4405E-06	.4208E-06	.4391E-06
	10%			.8809E-06	.8057E-06	.8790E-06

Table 4.23 **Sensitivities to perturbation in system parameters** (t_f =168h)

AC-DC Converter						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
$\lambda_{Transf.}$ $\lambda_{Thyrist.}$ $\lambda_{SM\ React}$	-0.1%	9.9978E-01	-.1881E-04	.1881E-07	.1881E-07	.1892E-07
	-1%			.1881E-06	.1881E-06	.1879E-06
	-5%			.9403E-06	.9403E-06	.9406E-06
	-10%			.1881E-05	.1880E-05	.1881E-05
$\mu_{Transf.}$ $\mu_{Thyrist.}$ $\mu_{SM\ React}$	0.1%		.1762E-04	.1762E-07	.1760E-07	.1701E-07
	1%			.1762E-06	.1745E-06	.1731E-06
	5%			.8808E-06	.8415E-06	.8708E-06
	10%			.1762E-05	.1611E-05	.1747E-05
$\lambda_{DC\ Cap.}$	-0.1%	9.9978E-01	-.4702E-04	.4701E-07	.4702E-07	.4721E-07
	-1%			.4701E-06	.4702E-06	.4701E-06
	-5%			.2351E-05	.2350E-05	.2351E-05
	-10%			.4701E-05	.4701E-05	.4701E-05
$\mu_{DC\ Cap.}$	0.1%		.4405E-04	.4404E-07	.4400E-07	.4282E-07
	1%			.4404E-06	.4363E-06	.4342E-06
	5%			.2202E-05	.2104E-05	.2185E-05
	10%			.4404E-05	.4028E-05	.4379E-05
$\lambda_{Controls}$	-0.1%	9.9978E-01	-.9405E-04	.9403E-07	.9404E-07	.9407E-07
	-1%			.9403E-06	.9403E-06	.9407E-06
	-5%			.4701E-05	.4701E-05	.4701E-05
	-10%			.9403E-05	.9402E-05	.9403E-05
$\mu_{Controls}$	0.1%		.8810E-04	.8808E-07	.8800E-07	.8669E-07
	1%			.8808E-06	.8727E-06	.8715E-06
	5%			.4404E-05	.4207E-05	.4380E-05
	10%			.8808E-05	.8056E-05	.8773E-05
$\lambda_{SPPT\ Str.}$	-0.1%	9.9978E-01	-.3135E-05	.3134E-08	.3134E-08	.3179E-08
	-1%			.3134E-07	.3134E-07	.3157E-07
	-5%			.1567E-06	.1567E-06	.1567E-06
	-10%			.3134E-06	.3134E-06	.3132E-06
$\mu_{SPPT\ Str.}$	0.1%		.2936E-05	.2936E-08	.2935E-08	.2465E-08
	1%			.2936E-07	.2910E-07	.2850E-07
	5%			.1468E-06	.1403E-06	.1444E-06
	10%			.2936E-06	.2682E-06	.2890E-06
$\lambda_{Cooling}$	-0.1%	9.9978E-01	-.1976E-04	.1976E-07	.1976E-07	.1978E-07
	-1%			.1976E-06	.1976E-06	.1976E-06
	-5%			.9879E-06	.9881E-06	.9880E-06
	-10%			.1976E-05	.1976E-05	.1976E-05
$\mu_{Cooling}$	0.1%		.1952E-04	.1952E-07	.1952E-07	.1943E-07
	1%			.1952E-06	.1929E-06	.1947E-06
	5%			.9760E-06	.9306E-06	.9745E-06
	10%			.1952E-05	.1776E-05	.1949E-05

HEBT

Table 4.24 **Sensitivities to perturbation in system parameters** (t_f =168h)

Param. α_i	Perturb. $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ_{QuadCh}	-0.1%	.9688E+00	-.3420E-02	.3313E-05	.3313E-05	.3313E-05
	-1%			.3313E-04	.3313E-04	.3313E-04
	-5%			.1657E-03	.1657E-03	.1657E-03
	-10%			.3313E-03	.3314E-03	.3313E-03
μ_{QuadCh}	0.1%		.3378E-02	.3272E-05	.3269E-05	.3273E-05
	1%			.3272E-04	.3241E-04	.3273E-04
	5%			.1636E-03	.1559E-03	.1636E-03
	10%			.3272E-03	.2979E-03	.3273E-03

	-0.1%			.2297E-06	.2297E-06	.2297E-06
$\lambda_{DipoleCh}$	-1%		-.2371E-03	.2297E-05	.2297E-05	.2297E-05
	-5%			.1149E-04	.1149E-04	.1149E-04
	-10%	.9688E+00		.2297E-04	.2297E-04	.2297E-04
	0.1%			.2269E-06	.2266E-06	.2269E-06
$\mu_{DipoleCh.}$	1%		.2342E-03	.2269E-05	.2247E-05	.2269E-05
	5%			.1134E-04	.1081E-04	.1134E-04
	10%			.2269E-04	.2065E-04	.2269E-04
	-0.1%			.1838E-06	.1838E-06	.1838E-06
$\lambda_{OctupCh}$	-1%		-.1897E-03	.1838E-05	.1838E-05	.1838E-05
	-5%			.9188E-05	.9188E-05	.9188E-05
	-10%	.9688E+00		.1838E-04	.1838E-04	.1838E-04
	0.1%			.1815E-06	.1813E-06	.1815E-06
$\mu_{OctupCh}$	1%		.1873E-03	.1815E-05	.1797E-05	.1815E-05
	5%			.9075E-05	.8648E-05	.9075E-05
	10%			.1815E-04	.1652E-04	.1815E-04
	-0.1%			.5996E-05	.5996E-05	.5996E-05
$\lambda_{BuncherCh}$	-1%		-.6189E-02	.5996E-04	.5997E-04	.5996E-04
	-5%			.2998E-03	.2999E-03	.2998E-03
	-10%	.9688E+00		.5996E-03	.6000E-03	.5996E-03
	0.1%			.5667E-05	.5662E-05	.5668E-05
$\mu_{BuncherCh}$	1%		.5850E-02	.5667E-04	.5615E-04	.5668E-04
	5%			.2834E-03	.2707E-03	.2834E-03
	10%			.5667E-03	.5182E-03	.5668E-03
$\lambda_{BunchRFPwr}$	-0.1%			.3837E-05	.3837E-05	.3837E-05
$\lambda_{DispCav\ RFPowSup}$	-1%		-.3961E-02	.3837E-04	.3837E-04	.3837E-04
	-5%			.1919E-03	.1919E-03	.1919E-03
	-10%	.9688E+00		.3837E-03	.3839E-03	.3837E-03
$\mu_{BunchRFPwr}$	0.1%			.3617E-05	.3614E-05	.3617E-05
	1%		.3734E-02	.3617E-04	.3584E-04	.3617E-04
$\mu_{DispCav\ RFPowSup}$	5%			.1809E-03	.1728E-03	.1809E-03
	10%			.3617E-03	.3308E-03	.3617E-03
	-0.1%			.2735E-05	.2735E-05	.2735E-05
λ_{BTVSys}	-1%		-.2823E-02	.2735E-04	.2735E-04	.2735E-04
	-5%			.1368E-03	.1368E-03	.1368E-03
	-10%	.9688E+00		.2735E-03	.2736E-03	.2735E-03
	0.1%			.2560E-05	.2558E-05	.2560E-05
μ_{BTVSys}	1%		.2643E-02	.2560E-04	.2537E-04	.2560E-04
	5%			.1280E-03	.1223E-03	.1280E-03
	10%			.2560E-03	.2343E-03	.2560E-03
	-0.1%			.2016E-05	.2016E-05	.2016E-05
$\lambda_{Dipole\ As\ HR}$	-1%		-.2081E-02	.2016E-04	.2016E-04	.2016E-04
	-5%			.1008E-03	.1008E-03	.1008E-03
	-10%	.9688E+00		.2016E-03	.2017E-03	.2016E-03
	0.1%			.1327E-05	.1326E-05	.1327E-05
$\mu_{Dipole\ As\ HR}$	1%		.1370E-02	.1327E-04	.1319E-04	.1327E-04
	5%			.6636E-04	.6445E-04	.6636E-04
	10%			.1327E-03	.1253E-03	.1327E-03
	-0.1%			.1680E-05	.1680E-05	.1680E-05
$\lambda_{BTVSys\ HR}$	-1%		-.1734E-02	.1680E-04	.1680E-04	.1680E-04
	-5%			.8401E-04	.8402E-04	.8401E-04
	-10%	.9688E+00		.1680E-03	.1681E-03	.1680E-03
	0.1%			.1106E-05	.1105E-05	.1106E-05
$\mu_{BTVSys\ HR}$	1%		.1141E-02	.1106E-04	.1099E-04	.1106E-04
	5%			.5529E-04	.5370E-04	.5529E-04
	10%			.1106E-03	.1044E-03	.1106E-03
	-0.1%			.6779E-05	.6779E-05	.6779E-05
$\lambda_{Disp\ Cav\ Ch}$	-1%		-.6997E-02	.6779E-04	.6780E-04	.6779E-04
	-5%			.3390E-03	.3391E-03	.3390E-03
	-10%	.9688E+00		.6779E-03	.6784E-03	.6779E-03
	0.1%			.4468E-05	.4465E-05	.4467E-05
$\mu_{Disp\ Cav\ Ch}$	1%		.4612E-02	.4468E-04	.4441E-04	.4468E-04
	5%			.2234E-03	.2170E-03	.2234E-03
	10%			.4468E-03	.4217E-03	.4468E-03

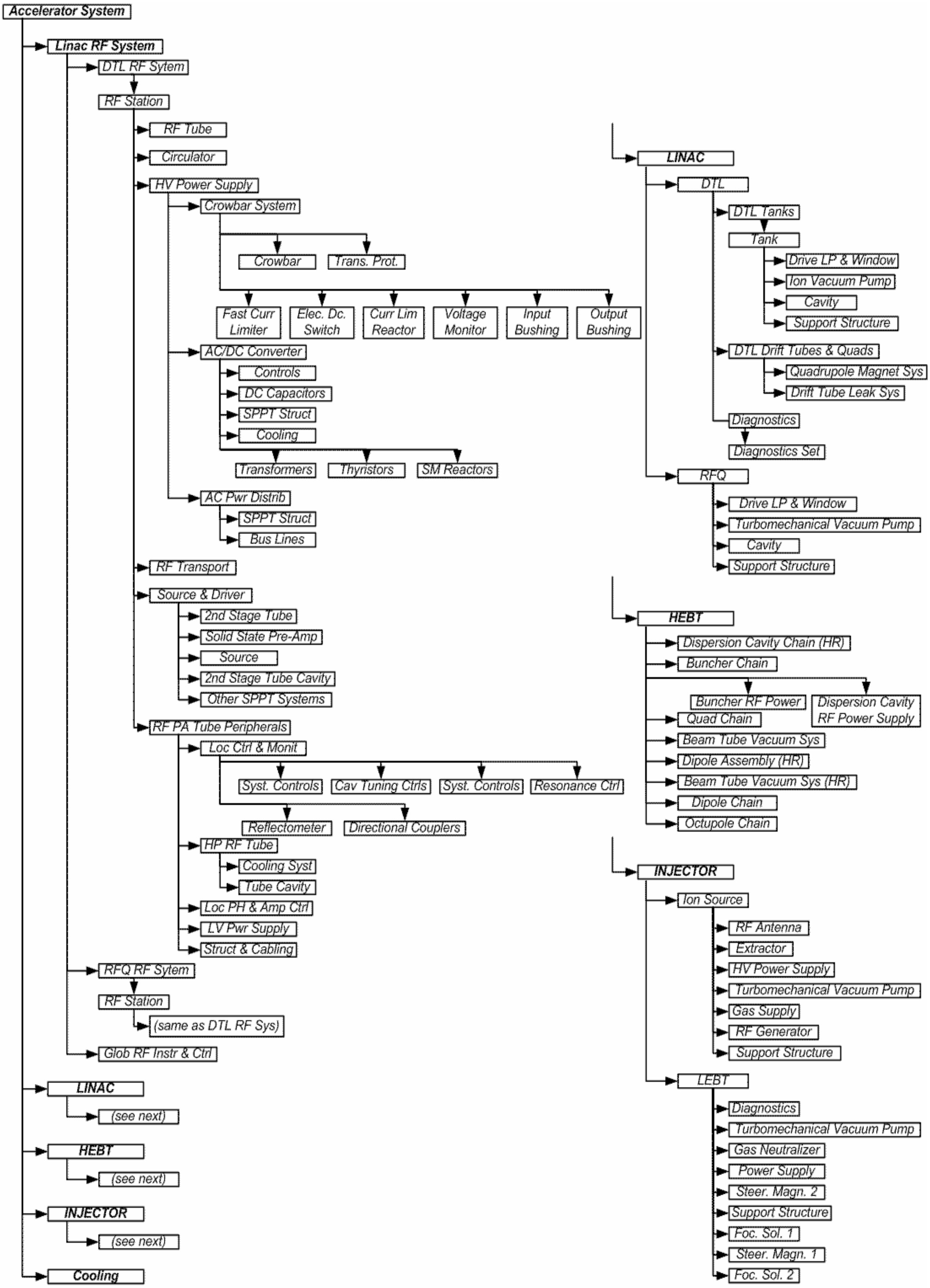

Figure 4.21 Components importance for IFMIF Accelerator System Facilities based on the sensitivities of interval availability to variations in input parameters *MTTF \ MTTR*

4.2 Sensitivities of the steady-state availability for IFMIF-Accelerator system facilities

The second type of response for which sensitivity studies of IFMIF Accelerator System Facilities have been performed is steady-state availability. Noting that for the most cases the considered mission time of seven days is enough that the stationary solution to be reached, the following defined type of response is similarly to the steady-state availability.

$$R = \int_{t_0}^{t_f} \pi_1(t)\delta(t - t_f)dt \tag{4.3}$$

where δ represents the Dirac-delta functional. Considering the same mission time of seven days, i.e. $t_f = 168$ hours, with the initial time $t_0 = 0$, performing the integration and taking in account the properties of delta function, the response becomes

$$R = \pi_1(t_f = 168h)$$

This type of responses has been computed before during the reliability phase and the numerical results have been presented in Table 4.3, and represents the considered response in IFMIF-CDA study.[75,76]

Replacing the source term in adjoint sensitivity equations (3.36) for this type of response, the source term will be $\partial F / \partial \Pi = \left[\partial F / \partial \pi_1, ..., \partial F / \partial \pi_n\right]^T = [1, 0, ..., 0]^T$ at the end of mission time, i.e. at $t_f = 168$ hours, and $\partial F / \partial \Pi = \left[\partial F / \partial \pi_1, ..., \partial F / \partial \pi_n\right]^T = [0, 0, ..., 0]^T$ for the rest of the time steps.

The sensitivities studies for all systems and subsystems for this type of response are displayed as previous case as follows:

a) Table 4.25 and 4.26 for the top level of Accelerator system,
b) Table 4.27 through 4.29 the sensitivities for the Injector system and its subsystems,
c) Table 4.30 through 4.34 the sensitivities for Linear Accelerator and its subsystems,
d) Table 4.35 through 4.44 the sensitivities for Radio Frequency System, and
e) Table 4.45 for High Energy Beam Transport System.

For the top level of accelerator system for the steady-state availability, one can see better the trend of vanishing of the sensitivities at variation in initial conditions presented in Table 4.25.

For this type of response it can be seen in general that for the same component the same variation in either failure or repair rate give the same effect in system response of type (4.3).

The components parameters importance ranking based on the sensitivities of steady-state availability (4.3) in absolute value is presented in Fig.4.22. It can be seen by comparison with

the component importance performed for the interval availability presented in the previous section that if on subsystems the importance ranking is in general the same, the overall importance has been changed. As one can see from the Table 4.26, and Figs.4.21 and 4.22, if for the first type of response the rank of parameter's importance was RF System, LINAC, HEBT, INJECTOR, and COOLING system, respectively, in this case LINAC and RF System changed the places, and therefore their subsystems and components as well. Thus, the reliability components which have the larger impact in affecting the steady-state availability of accelerator system facilities are those of LINAC followed by those of RF System.
Here can be highlighted another aspect of this analysis, namely that the components parameters importance can be different for different type of responses, and the different type of responses can behave different to the same variations in system parameters.

In closing, it must be mentioned that for designing of a stable reliable system, the system should contain components and subsystems with such characteristics that their perturbed parameters do not have to change significant the components priority from a type of response to another. If that is not possible, priority should have the importance given by the reliability measure of most interest for the considered system.

Accelerator System

Table 4.25 **Sensitivities to perturbation in initial conditions**

Perturbation in initial conditions (linear dependency)	Transient Duration (h)/ No. of time steps	Nominal Value R^0	Relative Sensitivity $\frac{\Delta R}{\Delta \alpha_i} \cdot \frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
-0.1% of $\pi_1(t_0)$ and 0.1% of $\pi_{32}(t_0)$	1 / 100	.9845E+00	.1000E+01	-.9845E-03	-.9845E-03	-.9845E-03
	6 / 600	.9399E+00	.9473E+00	-.8903E-03	-.8902E-03	-.8902E-03
	12 / 1200	.9148E+00	.7366E+00	-.6738E-03	-.6736E-03	-.6736E-03
	24 / 2400	.8940E+00	.3529E+00	-.3155E-03	-.3154E-03	-.3154E-03
	168 / 16800	.8809E+00	.1048E-03	-.9235E-07	-.9233E-07	-.9233E-07
-5% of $\pi_1(t_0)$ and 5% of $\pi_{32}(t_0)$	1 / 100	.9845E+00	.1000E+01	-.4923E-01	-.4922E-01	-.4922E-01
	6 / 600	.9399E+00	.9473E+00	-.4452E-01	-.4451E-01	-.4451E-01
	12 / 1200	.9148E+00	.7366E+00	-.3369E-01	-.3368E-01	-.3368E-01
	24 / 2400	.8940E+00	.3529E+00	-.1577E-01	-.1577E-01	-.1577E-01
	168 / 16800	.8809E+00	.1048E-03	-.4618E-05	-.4616E-05	-.4616E-05
-10% of $\pi_1(t_0)$ and 10% of $\pi_{32}(t_0)$	1 / 100	.9845E+00	.1000E+01	-.9845E-01	-.9845E-01	-.9845E-01
	6 / 600	.9399E+00	.9473E+00	-.8903E-01	-.8902E-01	-.8902E-01
	12 / 1200	.9148E+00	.7366E+00	-.6738E-01	-.6736E-01	-.6736E-01
	24 / 2400	.8940E+00	.3529E+00	-.3155E-01	-.3154E-01	-.3154E-01
	168 / 16800	.8809E+00	.1048E-03	-.9235E-05	-.9233E-05	-.9233E-05

Table 4.26 **Sensitivities to perturbation in system parameters** (t_f =168h)

Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
$\lambda_{INJECTOR}$	-0.1%	.8809E+00	-.1386E-01	.1221E-04	.1220E-04	.1220E-04
	-1%			.1221E-03	.1220E-03	.1220E-03
	-5%			.6103E-03	.6102E-03	.6098E-03
	-10%			.1221E-02	.1221E-02	.1220E-02
$\mu_{INJECTOR}$	0.1%		.1386E-01	.1221E-04	.1218E-04	.1220E-04
	1%			.1221E-03	.1208E-03	.1220E-03
	5%			.6103E-03	.5811E-03	.6098E-03
	10%			.1221E-02	.1110E-02	.1220E-02
λ_{LINAC}	-0.1%	.8809E+00	-.3926E-01	.3458E-04	.3458E-04	.3458E-04
	-1%			.3458E-03	.3459E-03	.3458E-03
	-5%			.1729E-02	.1732E-02	.1729E-02
	-10%			.3458E-02	.3471E-02	.3458E-02
μ_{LINAC}	0.1%		.3922E-01	.3455E-04	.3451E-04	.3454E-04
	1%			.3455E-03	.3422E-03	.3454E-03
	5%			.1727E-02	.1648E-02	.1727E-02
	10%			.3455E-02	.3153E-02	.3454E-02
$\lambda_{COOLING}$	-0.1%	.8809E+00	-.8003E-05	.7050E-08	.7047E-08	.7048E-08
	-1%			.7050E-07	.7047E-07	.7047E-07
	-5%			.3525E-06	.3523E-06	.3523E-06
	-10%			.7050E-06	.7047E-06	.7047E-06
$\mu_{COOLING}$	0.1%		.8003E-05	.7050E-08	.7025E-08	.7046E-08
	1%			.7050E-07	.6998E-07	.7046E-07
	5%			.3525E-06	.3356E-06	.3523E-06
	10%			.7050E-06	.6407E-06	.7047E-06
λ_{RFSys}	-0.1%	.8809E+00	-.3839E-01	.3381E-04	.3381E-04	.3381E-04
	-1%			.3381E-03	.3382E-03	.3381E-03
	-5%			.1691E-02	.1694E-02	.1690E-02
	-10%			.3381E-02	.3394E-02	.3381E-02
μ_{RFSys}	0.1%		.3839E-01	.3381E-04	.3377E-04	.3381E-04
	1%			.3381E-03	.3349E-03	.3381E-03
	5%			.1691E-02	.1613E-02	.1690E-02
	10%			.3381E-02	.3084E-02	.3381E-02
λ_{HEBT}	-0.1%	.8809E+00	-.3317E-01	.2922E-04	.2921E-04	.2921E-04
	-1%			.2922E-03	.2922E-03	.2921E-03
	-5%			.1461E-02	.1463E-02	.1461E-02
	-10%			.2922E-02	.2931E-02	.2921E-02
μ_{HEBT}	0.1%		.3317E-01	.2922E-04	.2918E-04	.2921E-04
	1%			.2922E-03	.2893E-03	.2921E-03
	5%			.1461E-02	.1393E-02	.1461E-02
	10%			.2922E-02	.2664E-02	.2921E-02

INJECTOR

Table 4.27 **Sensitivities to perturbation in system parameters** (t_f =168h)

Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel. Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ *Ion Source*	-0.1%	.9865E+00	-.1272E-01	.1255E-04	.1254E-04	.1254E-04
	-1%			.1255E-03	.1254E-03	.1254E-03
	-5%			.6275E-03	.6274E-03	.6270E-03
	-10%			.1255E-02	.1256E-02	.1254E-02
μ *Ion Source*	0.1%		.1272E-01	.1255E-04	.1253E-04	.1254E-04
	1%			.1255E-03	.1242E-03	.1254E-03
	5%			.6275E-03	.5975E-03	.6270E-03
	10%			.1255E-02	.1141E-02	.1254E-02
λ *LEBT*	-0.1%	.9865E+00	-.8214E-03	.8103E-06	.8100E-06	.8100E-06
	-1%			.8103E-05	.8100E-05	.8100E-05
	-5%			.4051E-04	.4050E-04	.4050E-04
	-10%			.8103E-04	.8101E-04	.8100E-04
μ *LEBT*	0.1%		.8214E-03	.8103E-06	.8092E-06	.8100E-06
	1%			.8103E-05	.8020E-05	.8100E-05
	5%			.4051E-04	.3857E-04	.4050E-04
	10%			.8103E-04	.7364E-04	.8100E-04

Table 4.28 **Sensitivities to perturbation in system parameters** (t_f =168h)

Ion Source						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ *RF Ant.*	-0.1%	.9869E+00	-.1192E-01	.1176E-04	.1175E-04	.1175E-04
	-1%			.1176E-03	.1175E-03	.1175E-03
	-5%			.5879E-03	.5878E-03	.5874E-03
	-10%			.1176E-02	.1176E-02	.1175E-02
μ *RF Ant.*	0.1%		.1192E-01	.1176E-04	.1174E-04	.1175E-04
	1%			.1176E-03	.1163E-03	.1175E-03
	5%			.5879E-03	.5598E-03	.5874E-03
	10%			.1176E-02	.1069E-02	.1175E-02
λ *RF Gen.*	-0.1%	.9869E+00	-.2003E-04	.1977E-07	.1974E-07	.1974E-07
	-1%			.1977E-06	.1974E-06	.1974E-06
	-5%			.9886E-06	.9899E-06	.9869E-06
	-10%			.1977E-05	.1977E-05	.1974E-05
μ *RF Gen.*	0.1%		.2003E-04	.1977E-07	.1972E-07	.1974E-07
	1%			.1977E-06	.1982E-06	.1974E-06
	5%			.9886E-06	.9443E-06	.9869E-06
	10%			.1977E-05	.1798E-05	.1974E-05
λ *Extractor*	-0.1%	.9869E+00	-.4568E-03	.4508E-06	.4506E-06	.4506E-06
	-1%			.4508E-05	.4506E-05	.4506E-05
	-5%			.2254E-04	.2253E-04	.2253E-04
	-10%			.4508E-04	.4507E-04	.4506E-04
μ *Extractor*	0.1%		.4568E-03	.4508E-06	.4502E-06	.4506E-06
	1%			.4508E-05	.4462E-05	.4507E-05
	5%			.2254E-04	.2146E-04	.2253E-04
	10%			.4508E-04	.4097E-04	.4506E-04
λ *Gas Supp.*	-0.1%	.9869E+00	-.8340E-04	.8231E-07	.8224E-07	.8220E-07
	-1%			.8231E-06	.8224E-06	.8224E-06
	-5%			.4116E-05	.4112E-05	.4112E-05
	-10%			.8231E-05	.8227E-05	.8224E-05
μ *Gas Supp.*	0.1%		.8340E-04	.8231E-07	.8216E-07	.8223E-07
	1%			.8231E-06	.8170E-06	.8224E-06
	5%			.4116E-05	.3924E-05	.4112E-05
	10%			.8231E-05	.7478E-05	.8224E-05

$\lambda_{HVPSupp}$	-0.1%	.9869E+00	-.2398E-03	.2367E-06	.2366E-06	.2363E-06
	-1%			.2367E-05	.2366E-05	.2369E-05
	-5%			.1183E-04	.1183E-04	.1183E-04
	-10%			.2367E-04	.2366E-04	.2367E-04
$\mu_{HVPSupp}$	0.1%		.2383E-03	.2351E-06	.2349E-06	.2347E-06
	1%			.2351E-05	.2329E-05	.2348E-05
	5%			.1176E-04	.1121E-04	.1173E-04
	10%			.2351E-04	.2142E-04	.2349E-04
$\lambda_{TVP1\text{-}2}$	-0.1%	.9869E+00	-.2001E-03	.1975E-06	.1974E-06	.1974E-06
	-1%			.1975E-05	.1974E-05	.1974E-05
	-5%			.9873E-05	.9869E-05	.9869E-05
	-10%			.1975E-04	.1974E-04	.1974E-04
$\mu_{TVP1\text{-}2}$	0.1%		.2001E-03	.1975E-06	.1972E-06	.1974E-06
	1%			.1975E-05	.1954E-05	.1974E-05
	5%			.9873E-05	.9402E-05	.9869E-05
	10%			.1975E-04	.1796E-04	.1974E-04
$\lambda_{SuppStr}$	-0.1%	.9869E+00	-.2124E-04	.2096E-07	.2096E-07	.2059E-07
	-1%			.2096E-06	.2096E-06	.2092E-06
	-5%			.1048E-05	.1048E-05	.1052E-05
	-10%			.2096E-05	.2096E-05	.2101E-05
$\mu_{SuppStr.}$	0.1%		.8879E-05	.8762E-08	.8760E-08	.4110E-08
	1%			.8762E-07	.8736E-07	.6549E-07
	5%			.4381E-06	.4315E-06	.3639E-06
	10%			.8762E-06	.8502E-06	.7572E-06

Table 4.29 **Sensitivities to perturbation in system parameters** (t_f =168h)

LEBT						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
$\lambda_{\#1.Foc.Sol}$, $\lambda_{\#1.Str.Mg}$	-0.1%	.9991E+00	-.8034E-05	.8027E-08	.7993E-08	.7993E-08
	-1%			.8027E-07	.7993E-07	.7993E-07
	-5%			.4013E-06	.3996E-06	.3996E-06
	-10%			.8027E-06	.7993E-06	.7993E-06
$\mu_{\#1.Foc.Sol}$, $\mu_{\#1.Str.Mgn}$	0.1%		.8034E-05	.8027E-08	.7985E-08	.7993E-08
	1%			.8027E-07	.7914E-07	.7993E-07
	5%			.4013E-06	.3806E-06	.3996E-06
	10%			.8027E-06	.7266E-06	.7993E-06
$\lambda_{\#2.Foc.Sol}$	-0.1%	.9991E+00	-.6034E-05	.6029E-08	.5995E-08	.5995E-08
	-1%			.6029E-07	.5995E-07	.5995E-07
	-5%			.3014E-06	.2997E-06	.2997E-06
	-10%			.6029E-06	.5995E-06	.5995E-06
$\mu_{\#2.Foc.Sol}$	0.1%		.6034E-05	.6029E-08	.5989E-08	.5995E-08
	1%			.6029E-07	.5935E-07	.5995E-07
	5%			.3014E-06	.2855E-06	.2997E-06
	10%			.6029E-06	.5450E-06	.5995E-06
$\lambda_{\#2.StrMgn}$	-0.1%	.9991E+00	-.1603E-04	.1602E-07	.1599E-07	.1599E-07
	-1%			.1602E-06	.1599E-06	.1599E-06
	-5%			.8010E-06	.7993E-06	.7993E-06
	-10%			.1602E-05	.1599E-05	.1599E-05
$\mu_{\#2.StrMgn.}$	0.1%		.1603E-04	.1602E-07	.1597E-07	.1599E-07
	1%			.1602E-06	.1583E-06	.1599E-06
	5%			.8010E-06	.7612E-06	.7993E-06
	10%			.1602E-05	.1453E-05	.1599E-05
$\lambda_{GasNeutr}$	-0.1%	.9991E+00	-.4238E-04	.4234E-07	.4163E-07	.4163E-07
	-1%			.4234E-06	.4163E-06	.4163E-06
	-5%			.2117E-05	.2082E-05	.2081E-05
	-10%			.4234E-05	.4163E-05	.4163E-05
$\mu_{GasNeutr}$	0.1%		.4238E-04	.4234E-07	.4159E-07	.4163E-07
	1%			.4234E-06	.4122E-06	.4163E-06
	5%			.2117E-05	.1982E-05	.2081E-05
	10%			.4234E-05	.3785E-05	.4163E-05

$\lambda_{PwrSup1\text{-}4}$	-0.1%	.9991E+00	-.2003E-04	.2002E-07	.1998E-07	.1998E-07
	-1%			.2002E-06	.1998E-06	.1998E-06
	-5%			.1001E-05	.9991E-06	.9991E-06
	-10%			.2002E-05	.1998E-05	.1998E-05
$\mu_{PwrSup1\text{-}4}$	0.1%		.2003E-04	.2002E-07	.1996E-07	.1998E-07
	1%			.2002E-06	.1978E-06	.1998E-06
	5%			.1001E-05	.9515E-06	.9991E-06
	10%			.2002E-05	.1817E-05	.1998E-05
λ_{Diags}	-0.1%	.9991E+00	-.5007E-03	.5003E-06	.4996E-06	.4996E-06
	-1%			.5003E-05	.4996E-05	.4996E-05
	-5%			.2501E-04	.2498E-04	.2498E-04
	-10%			.5003E-04	.4996E-04	.4996E-04
μ_{Diags}	0.1%		.5007E-03	.5003E-06	.4991E-06	.4996E-06
	1%			.5003E-05	.4946E-05	.4996E-05
	5%			.2501E-04	.2379E-04	.2498E-04
	10%			.5003E-04	.4542E-04	.4996E-04
λ_{TVP}	-0.1%	.9991E+00	-.2009E-03	.2007E-06	.1998E-06	.1998E-06
	-1%			.2007E-05	.1998E-05	.1998E-05
	-5%			.1003E-04	.9991E-05	.9991E-05
	-10%			.2007E-04	.1998E-04	.1998E-04
μ_{TVP}	0.1%		.2009E-03	.2007E-06	.1996E-06	.1998E-06
	1%			.2007E-05	.1978E-05	.1998E-05
	5%			.1003E-04	.9516E-05	.9991E-05
	10%			.2007E-04	.1817E-04	.1998E-04
$\lambda_{SuppStr}$	-0.1%	.9991E+00	-.2124E-04	.2122E-07	.2122E-07	.2122E-07
	-1%			.2122E-06	.2122E-06	.2122E-06
	-5%			.1061E-05	.1061E-05	.1061E-05
	-10%			.2122E-05	.2122E-05	.2122E-05
$\mu_{SuppStr}$	0.1%		.8877E-05	.8869E-08	.8868E-08	.8785E-08
	1%			.8869E-07	.8844E-07	.8844E-07
	5%			.4435E-06	.4369E-06	.4433E-06
	10%			.8869E-06	.8608E-06	.8870E-06

LINAC

Table 4.30 **Sensitivities to perturbation in system parameters** (t_f =168h)

Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel. Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ_{RFQ}	-0.1%	.9614E+00	-.8819E-02	.8478E-05	.8477E-05	.8477E-05
	-1%			.8478E-04	.8478E-04	.8477E-04
	-5%			.4239E-03	.4240E-03	.4238E-03
	-10%			.8478E-03	.8484E-03	.8477E-03
μ_{RFQ}	0.1%		.8819E-02	.8478E-05	.8468E-05	.8477E-05
	1%			.8478E-04	.8394E-04	.8477E-04
	5%			.4239E-03	.4038E-03	.4238E-03
	10%			.8478E-03	.7712E-03	.8477E-03
λ_{DTL}	-0.1%	.9614E+00	-.3006E-01	.2890E-04	.2890E-04	.2890E-04
	-1%			.2890E-03	.2891E-03	.2890E-03
	-5%			.1445E-02	.1447E-02	.1445E-02
	-10%			.2890E-02	.2899E-02	.2890E-02
μ_{DTL}	0.1%		.2987E-01	.2872E-04	.2869E-04	.2871E-04
	1%			.2872E-03	.2845E-03	.2872E-03
	5%			.1436E-02	.1371E-02	.1436E-02
	10%			.2872E-02	.2622E-02	.2872E-02

Table 4.31 **Sensitivities to perturbation in system parameters** (t_f =168h)

RFQ						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ_{Cavity}	-0.1%	.9916E+00	-.3307E-04	.3279E-07	.3280E-07	.3280E-07
	-1%			.3279E-06	.3280E-06	.3280E-06
	-5%			.1640E-05	.1640E-05	.1640E-05
	-10%			.3279E-05	.3280E-05	.3280E-05
μ_{Cavity}	0.1%		.5525E-06	.5479E-09	.5479E-09	.3474E-09
	1%			.5479E-08	.5478E-08	.3771E-08
	5%			.2739E-07	.2738E-07	.2287E-07
	10%			.5479E-07	.5473E-07	.4669E-07
$\lambda_{Drive\ LP\ \&\ Window}$	-0.1%	.9916E+00	-.7201E-02	.7140E-05	.7139E-05	.7139E-05
	-1%			.7140E-04	.7140E-04	.7139E-04
	-5%			.3570E-03	.3571E-03	.3570E-03
	-10%			.7140E-03	.7144E-03	.7139E-03
$\mu_{Drive\ LP\ \&\ Window}$	0.1%		.7201E-02	.7140E-05	.7132E-05	.7139E-05
	1%			.7140E-04	.7069E-04	.7139E-04
	5%			.3570E-03	.3401E-03	.3570E-03
	10%			.7140E-03	.6494E-03	.7139E-03
$\lambda_{TMV\ Pump}$	-0.1%	.9916E+00	-.1201E-02	.1191E-05	.1190E-05	.1190E-05
	-1%			.1191E-04	.1190E-04	.1190E-04
	-5%			.5953E-04	.5951E-04	.5950E-04
	-10%			.1191E-03	.1190E-03	.1190E-03
$\mu_{TMV\ Pump}$	0.1%		.1201E-02	.1191E-05	.1189E-05	.1190E-05
	1%			.1191E-04	.1178E-04	.1190E-04
	5%			.5953E-04	.5667E-04	.5950E-04
	10%			.1191E-03	.1082E-03	.1190E-03
$\lambda_{SuppStr.}$	-0.1%	.9916E+00	-.2124E-04	.2106E-07	.2106E-07	.2114E-07
	-1%			.2106E-06	.2106E-06	.2108E-06
	-5%			.1053E-05	.1053E-05	.1053E-05
	-10%			.2106E-05	.2106E-05	.2106E-05
$\mu_{SuppStr}$	0.1%		.8879E-05	.8804E-08	.8801E-08	.6969E-08
	1%			.8804E-07	.8777E-07	.8015E-07
	5%			.4402E-06	.4336E-06	.4193E-06
	10%			.8804E-06	.8542E-06	.8507E-06

Table 4.32 **Sensitivities to perturbation in system parameters** (t_f =168h)

DTL						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ_{DTQ}	-0.1%	.9722E+00	-.1351E-01	.1309E-04	.1309E-04	.1309E-04
	-1%			.1309E-03	.1309E-03	.1309E-03
	-5%			.6544E-03	.6548E-03	.6545E-03
	-10%			.1309E-02	.1310E-02	.1309E-02
μ_{DTQ}	0.1%		.1017E-01	.9850E-05	.9844E-05	.9743E-05
	1%			.9850E-04	.9790E-04	.9818E-04
	5%			.4925E-03	.4779E-03	.4917E-03
	10%			.9850E-03	.9280E-03	.9839E-03
$\lambda_{DTL\ TSys}$	-0.1%	9722E+00	-.1389E-01	.1346E-04	.1346E-04	.1346E-04
	-1%			.1346E-03	.1346E-03	.1346E-03
	-5%			.6730E-03	.6734E-03	.6730E-03
	-10%			.1346E-02	.1348E-02	.1346E-02
$\mu_{DTL\ TSys}$	0.1%		.1389E-01	.1346E-04	.1344E-04	.1346E-04
	1%			.1346E-03	.1332E-03	.1346E-03
	5%			.6729E-03	.6412E-03	.6728E-03
	10%			.1346E-02	.1225E-02	.1346E-02

$\lambda_{Diags1\text{-}6}$	-0.1%	9722E+00	-.6667E-03	.6460E-06	.6459E-06	.6461E-06
	-1%			.6460E-05	.6459E-05	.6461E-05
	-5%			.3230E-04	.3230E-04	.3230E-04
	-10%			.6460E-04	.6460E-04	.6460E-04
$\mu_{Diags1\text{-}6}$	0.1%		.6665E-03	.6458E-06	.6451E-06	.6456E-06
	1%			.6458E-05	.6394E-05	.6458E-05
	5%			.3229E-04	.3075E-04	.3229E-04
	10%			.6458E-04	.5871E-04	.6458E-04

Table 4.33 **Sensitivities to perturbation in system parameters** (t_f =168h)

DTQ						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
λ_{QMSys}	-0.1%	.9866E+00	-.7269E-02	.7172E-05	.7172E-05	.7173E-05
	-1%			.7172E-04	.7172E-04	.7172E-04
	-5%			.3586E-03	.3587E-03	.3586E-03
	-10%			.7172E-03	.7176E-03	.7172E-03
μ_{QMSys}	0.1%		.5460E-02	.5387E-05	.5384E-05	.5380E-05
	1%			.5387E-04	.5354E-04	.5385E-04
	5%			.2694E-03	.2614E-03	.2693E-03
	10%			.5387E-03	.5075E-03	.5387E-03
λ_{DTLSys}	-0.1%	.9866E+00	-.6149E-02	.6067E-05	.6067E-05	.6068E-05
	-1%			.6067E-04	.6067E-04	.6067E-04
	-5%			.3033E-03	.3034E-03	.3033E-03
	-10%			.6067E-03	.6070E-03	.6067E-03
μ_{DTLSys}	0.1%		.4617E-02	.4555E-05	.4553E-05	.4549E-05
	1%			.4555E-04	.4528E-04	.4554E-04
	5%			.2278E-03	.2210E-03	.2277E-03
	10%			.4555E-03	.4291E-03	.4555E-03

Table 4.34 **Sensitivities to perturbation in system parameters** (t_f =168h)

Tank#						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
λ_{Cavity}	-0.1%	.9985E+00	-.3307E-04	.3303E-07	.3303E-07	.3303E-07
	-1%			.3303E-06	.3303E-06	.3303E-06
	-5%			.1651E-05	.1651E-05	.1651E-05
	-10%			.3303E-05	.3303E-05	.3303E-05
μ_{Cavity}	0.1%		.5525E-06	.5517E-09	.5517E-09	.1956E-09
	1%			.5517E-08	.5517E-08	.2105E-08
	5%			.2759E-07	.2757E-07	.1977E-07
	10%			.5517E-07	.5511E-07	.4341E-07
$\lambda_{Drv\ LP\ \&\ Window1\text{-}2}$	-0.1%	.9985E+00	-.6001E-03	.5992E-06	.5991E-06	.5993E-06
	-1%			.5992E-05	.5992E-05	.5991E-05
	-5%			.2996E-04	.2996E-04	.2996E-04
	-10%			.5992E-04	.5992E-04	.5991E-04
$\mu_{Drv\ LP\ \&\ Window1\text{-}2}$	0.1%		.6001E-03	.5992E-06	.5985E-06	.5991E-06
	1%			.5992E-05	.5932E-05	.5991E-05
	5%			.2996E-04	.2853E-04	.2996E-04
	10%			.5992E-04	.5447E-04	.5991E-04
$\lambda_{Ion\ Vac\ Pump\ 1\text{-}2}$	-0.1%	.9985E+00	-.1000E-03	.9990E-07	.9985E-07	.9985E-07
	-1%			.9990E-06	.9995E-06	.9985E-06
	-5%			.4995E-05	.4993E-05	.4993E-05
	-10%			.9990E-05	.9986E-05	.9986E-05
$\mu_{Ion\ Vac\ Pump\ 1\text{-}2}$	0.1%		.1000E-03	.9990E-07	.9976E-07	.9986E-07
	1%			.9990E-06	.9887E-06	.9986E-06
	5%			.4995E-05	.4755E-05	.4993E-05
	10%			.9990E-05	.9078E-05	.9986E-05

λ *Support Structure.*	-0.1%	.9985E+00	-.2124E-04	.2121E-07	.2121E-07	.2156E-07
	-1%			.2121E-06	.2121E-06	.2131E-06
	-5%			.1060E-05	.1060E-05	.1061E-05
	-10%			.2121E-05	.2121E-05	.2121E-05
μ *Support Structure*	0.1%		.8879E-05	.8866E-08	.8863E-08	.6626E-08
	1%			.8866E-07	.8839E-07	.7578E-07
	5%			.4433E-06	.4366E-06	.4045E-06
	10%			.8866E-06	.8603E-06	.8299E-06

RF System

Table 4.35 **Sensitivities to perturbation in system parameters** (t_f =168h)

Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ *RFQ RF Sys*	-0.1%	.9623E+00	-.1262E-01	.1215E-04	.1214E-04	.1214E-04
	-1%			.1215E-03	.1214E-03	.1214E-03
	-5%			.6073E-03	.6076E-03	.6072E-03
	-10%			.1215E-02	.1216E-02	.1214E-02
μ *RFQ RF Sys*	0.1%		.1262E-01	.1215E-04	.1213E-04	.1214E-04
	1%			.1215E-03	.1202E-03	.1214E-03
	5%			.6073E-03	.5786E-03	.6072E-03
	10%			.1215E-02	.1105E-02	.1214E-02
λ *RF Inst&Ctrl*	-0.1%	9623E+00	-.2003E-04	.1928E-07	.1925E-07	.1925E-07
	-1%			.1928E-06	.1925E-06	.1925E-06
	-5%			.9639E-06	.9623E-06	.9623E-06
	-10%			.1928E-05	.1925E-05	.1925E-05
μ *RF Inst&Ctrl*	0.1%		.2003E-04	.1928E-07	.1923E-07	.1925E-07
	1%			.1928E-06	.1906E-06	.1925E-06
	5%			.9639E-06	.9165E-06	.9623E-06
	10%			.1928E-05	.1750E-05	.1925E-05
λ *DTL RF Sys*	-0.1%	9623E+00	-.2540E-01	.2444E-04	.2444E-04	.2444E-04
	-1%			.2444E-03	.2445E-03	.2444E-03
	-5%			.1222E-02	.1224E-02	.1222E-02
	-10%			.2444E-02	.2450E-02	.2444E-02
μ *DTL RF Sys*	0.1%		.2540E-01	.2444E-04	.2442E-04	.2444E-04
	1%			.2444E-03	.2420E-03	.2444E-03
	5%			.1222E-02	.1165E-02	.1222E-02
	10%			.2444E-02	.2227E-02	.2444E-02

Table 4.36 **Sensitivities to perturbation in system parameters** (t_f =168h)

RF Station						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ *RF Tube*	-0.1%	.9958E+00	-.1400E-02	.1394E-05	.1394E-05	.1394E-05
	-1%			.1394E-04	.1394E-04	.1394E-04
	-5%			.6971E-04	.6971E-04	.6971E-04
	-10%			.1394E-03	.1394E-03	.1394E-03
μ *RF Tube*	0.1%		.1400E-02	.1394E-05	.1393E-05	.1394E-05
	1%			.1394E-04	.1380E-04	.1394E-04
	5%			.6971E-04	.6639E-04	.6970E-04
	10%			.1394E-03	.1267E-03	.1394E-03

λ_{RF} *Transport*	-0.1%	.9958E+00	-.7241E-03	.7210E-06	.7208E-06	.7208E-06
	-1%			.7210E-05	.7208E-05	.7208E-05
	-5%			.3605E-04	.3604E-04	.3604E-04
	-10%			.7210E-04	.7208E-04	.7208E-04
μ_{RF} *Transport*	0.1%		.7241E-03	.7210E-06	.7201E-06	.7208E-06
	1%			.7210E-05	.7137E-05	.7208E-05
	5%			.3605E-04	.3432E-04	.3604E-04
	10%			.7210E-04	.6553E-04	.7208E-04
$\lambda_{Circulator}$	-0.1%	.9958E+00	-.1252E-02	.1247E-05	.1247E-05	.1247E-05
	-1%			.1247E-04	.1247E-04	.1247E-04
	-5%			.6234E-04	.6233E-04	.6233E-04
	-10%			.1247E-03	.1247E-03	.1247E-03
$\mu_{Circulator}$	0.1%		.1252E-02	.1247E-05	.1245E-05	.1247E-05
	1%			.1247E-04	.1234E-04	.1247E-04
	5%			.6234E-04	.5936E-04	.6233E-04
	10%			.1247E-03	.1133E-03	.1247E-03
λ_{Tube} *Peripherals*	-0.1%	.9958E+00	-.1000E-04	.9960E-08	.9958E-08	.9958E-08
	-1%			.9960E-07	.9958E-07	.9958E-07
	-5%			.4980E-06	.4979E-06	.4979E-06
	-10%			.9960E-06	.9958E-06	.9958E-06
μ_{Tube} *Peripherals*	0.1%		.1000E-04	.9960E-08	.9948E-08	.9958E-08
	1%			.9960E-07	.9859E-07	.9958E-07
	5%			.4980E-06	.4742E-06	.4979E-06
	10%			.9960E-06	.9053E-06	.9958E-06
λ_{Source} *&Drv*	-0.1%	.9958E+00	-.2003E-04	.1995E-07	.1992E-07	.1992E-07
	-1%			.1995E-06	.1992E-06	.1992E-06
	-5%			.9975E-06	.9958E-06	.9958E-06
	-10%			.1995E-05	.1992E-05	.1992E-05
μ_{Source} *&Drv*	0.1%		.2003E-04	.1995E-07	.1990E-07	.1992E-07
	1%			.1995E-06	.1972E-06	.1992E-06
	5%			.9975E-06	.9484E-06	.9958E-06
	10%			.1995E-05	.1811E-05	.1992E-05
$\lambda_{HV\ Pwr}$ *Suppl*	-0.1%	.9958E+00	-.7973E-03	.7940E-06	.7938E-06	.7938E-06
	-1%			.7940E-05	.7938E-05	.7938E-05
	-5%			.3970E-04	.3969E-04	.3969E-04
	-10%			.7940E-04	.7939E-04	.7938E-04
$\mu_{HV\ Pwr}$ *Suppl*	0.1%		.7973E-03	.7940E-06	.7930E-06	.7938E-06
	1%			.7940E-05	.7860E-05	.7938E-05
	5%			.3970E-04	.3780E-04	.3969E-04
	10%			.7940E-04	.7217E-04	.7938E-04

Table 4.37 **Sensitivities to perturbation in system parameters** (t_f =168h)

RF PA Tube Peripherals						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
$\lambda_{LC\&Monit}$	-0.1%	.9993E+00	-.4203E-03	.4200E-06	.4199E-06	.4199E-06
	-1%			.4200E-05	.4199E-05	.4199E-05
	-5%			.2100E-04	.2100E-04	.2100E-04
	-10%			.4200E-04	.4199E-04	.4199E-04
$\mu_{LC\&Monit}$	0.1%		.4203E-03	.4200E-06	.4195E-06	.4199E-06
	1%			.4200E-05	.4158E-05	.4199E-05
	5%			.2100E-04	.2000E-04	.2100E-04
	10%			.4200E-04	.3817E-04	.4199E-04
$\lambda_{LP\&AC}$	-0.1%	.9993E+00	-.1000E-03	.9997E-07	.9995E-07	.9995E-07
	-1%			.9997E-06	.9995E-06	.9995E-06
	-5%			.4999E-05	.4998E-05	.4998E-05
	-10%			.9997E-05	.9996E-05	.9995E-05
$\mu_{LP\&AC}$	0.1%		.1000E-03	.9997E-07	.9985E-07	.9995E-07
	1%			.9997E-06	.9896E-06	.9995E-06
	5%			.4999E-05	.4760E-05	.4998E-05
	10%			.9997E-05	.9087E-05	.9995E-05

$\lambda_{HP\ RF\ Tube}$	-0.1%	.9993E+00	-.1002E-03	.1001E-06	.1000E-06	.1000E-06
	-1%			.1001E-05	.1000E-05	.1000E-05
	-5%			.5005E-05	.5002E-05	.5002E-05
	-10%			.1001E-04	.1000E-04	.1000E-04
$\mu_{HP\ RF\ Tube}$	0.1%		.1002E-03	.1001E-06	.9993E-07	.1000E-06
	1%			.1001E-05	.9904E-06	.1000E-05
	5%			.5005E-05	.4763E-05	.5002E-05
	10%			.1001E-04	.9094E-05	.1000E-04
$\lambda_{Struct\&Cabl}$	-0.1%	.9993E+00	-.3334E-05	.3331E-08	.3331E-08	.3331E-08
	-1%			.3331E-07	.3331E-07	.3331E-07
	-5%			.1666E-06	.1665E-06	.1665E-06
	-10%			.3331E-06	.3331E-06	.3331E-06
$\mu_{Struct\&Cabl}$	0.1%		.3334E-05	.3331E-08	.3328E-08	.3331E-08
	1%			.3331E-07	.3298E-07	.3331E-07
	5%			.1666E-06	.1586E-06	.1665E-06
	10%			.3331E-06	.3028E-06	.3331E-06
$\lambda_{LV\ Pwr\ Supply}$	-0.1%	.9993E+00	-.1000E-03	.9994E-07	.9993E-07	.9993E-07
	-1%			.9994E-06	.9993E-06	.9993E-06
	-5%			.4997E-05	.4996E-05	.4996E-05
	-10%			.9994E-05	.9993E-05	.9993E-05
$\mu_{LV\ Pwr\ Supply}$	0.1%		.1000E-03	.9994E-07	.9983E-07	.9993E-07
	1%			.9994E-06	.9894E-06	.9993E-06
	5%			.4997E-05	.4758E-05	.4996E-05
	10%			.9994E-05	.9084E-05	.9993E-05

Table 4.38 **Sensitivities to perturbation in system parameters** (t_f =168h)

Loc Ctrl & Monit						
Param. α_i	Perturb. $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
$\lambda_{Reflectometer,}$ $\lambda_{Directional\ Couplers.}$	-0.1%	.9996E+00	-.1000E-04	.9998E-08	.9996E-08	.9996E-08
	-1%			.9998E-07	.9996E-07	.9996E-07
	-5%			.4999E-06	.4998E-06	.4998E-06
	-10%			.9998E-06	.9996E-06	.9996E-06
$\mu_{Reflectometer,}$ $\mu_{Directional\ Couplers}$	0.1%		.1000E-04	.9997E-08	.9986E-08	.9996E-08
	1%			.9997E-07	.9897E-07	.9996E-07
	5%			.4999E-06	.4760E-06	.4998E-06
	10%			.9997E-06	.9087E-06	.9996E-06
$\lambda_{Cav\ Tunn\ Ctrl}$ $\lambda_{Syst\ Ctrls\ 1\text{-}2}$ $\lambda_{Resonance\ Ctrl}$	-0.1%	.9996E+00	-.1000E-03	.9998E-07	.9996E-07	.9996E-07
	-1%			.9998E-06	.9996E-06	.9996E-06
	-5%			.4999E-05	.4998E-05	.4998E-05
	-10%			.9998E-05	.9996E-05	.9996E-05
$\mu_{Cav\ Tunn\ Ctrl}$ $\mu_{Syst\ Ctrls\ 1\text{-}2}$ $\mu_{Resonance\ Ctrl}$	0.1%		.1000E-03	.9997E-07	.9986E-07	.9996E-07
	1%			.9997E-06	.9897E-06	.9996E-06
	5%			.4999E-05	.4760E-05	.4998E-05
	10%			.9997E-05	.9087E-05	.9996E-05

Table 4.39 **Sensitivities to perturbation in system parameters** (t_f =168h)

HP RF Tube						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
$\lambda_{Cooler1\text{-}2}$	-0.1%	.9997E+00	-.1001E-03	.1001E-06	.9997E-07	.9997E-07
	-1%			.1001E-05	.9997E-06	.9997E-06
	-5%			.5003E-05	.4999E-05	.4999E-05
	-10%			.1001E-04	.9997E-05	.9997E-05
$\mu_{Cooler1\text{-}2}$	0.1%		.1001E-03	.1001E-06	.9987E-07	.9997E-07
	1%			.1001E-05	.9898E-06	.9997E-06
	5%			.5003E-05	.4760E-05	.4999E-05
	10%			.1001E-04	.9088E-05	.9997E-05

λ *Tube Cavity*	-0.1%	.9997E+00	-.1000E-03	.9999E-07	.9997E-07	.9997E-07
	-1%			.9999E-06	.9997E-06	.9997E-06
	-5%			.4999E-05	.4999E-05	.4998E-05
	-10%			.9999E-05	.9997E-05	.9997E-05
μ *Tube Cavity*	0.1%		.1000E-03	.9999E-07	.9987E-07	.9997E-07
	1%			.9999E-06	.9898E-06	.9997E-06
	5%			.4999E-05	.4760E-05	.4998E-05
	10%			.9999E-05	.9088E-05	.9997E-05

Table 4.40 **Sensitivities to perturbation in system parameters** (t_f =168h)

Source & Driver						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ *2nd Stage Tube*	-0.1%	.9987E+00	-.1182E-02	.1180E-05	.1180E-05	.1180E-05
	-1%			.1180E-04	.1180E-04	.1180E-04
	-5%			.5902E-04	.5902E-04	.5902E-04
	-10%			.1180E-03	.1180E-03	.1180E-03
μ *2nd Stage Tube*	0.1%		.1182E-02	.1180E-05	.1179E-05	.1180E-05
	1%			.1180E-04	.1169E-04	.1180E-04
	5%			.5902E-04	.5621E-04	.5901E-04
	10%			.1180E-03	.1073E-03	.1180E-03
λ *2nd St Tube Cavity*	-0.1%	.9987E+00	-.9002E-05	.8990E-08	.8989E-08	.8989E-08
	-1%			.8990E-07	.8989E-07	.8989E-07
	-5%			.4495E-06	.4494E-06	.4494E-06
	-10%			.8990E-06	.8989E-06	.8989E-06
μ *2nd St Tube Cavity*	0.1%		.9002E-05	.8990E-08	.8980E-08	.8989E-08
	1%			.8990E-07	.8900E-07	.8989E-07
	5%			.4495E-06	.4280E-06	.4494E-06
	10%			.8990E-06	.8172E-06	.8989E-06
λ *Solid State Pre-Amp*	-0.1%	.9987E+00	-.4003E-04	.3998E-07	.3995E-07	.3995E-07
	-1%			.3998E-06	.3995E-06	.3995E-06
	-5%			.1999E-05	.1998E-05	.1997E-05
	-10%			.3998E-05	.3995E-05	.3995E-05
μ *Solid State Pre-Amp*	0.1%		.4003E-04	.3998E-07	.3991E-07	.3995E-07
	1%			.3998E-06	.3955E-06	.3995E-06
	5%			.1999E-05	.1902E-05	.1997E-05
	10%			.3998E-05	.3632E-05	.3995E-05
λ *SPPT Sys.*	-0.1%	.9987E+00	-.6672E-06	.6664E-09	.6658E-09	.6658E-09
	-1%			.6664E-08	.6658E-08	.6658E-08
	-5%			.3332E-07	.3329E-07	.3329E-07
	-10%			.6664E-07	.6658E-07	.6658E-07
μ *SPPT Sys.*	0.1%		.6672E-06	.6664E-09	.6652E-09	.6658E-09
	1%			.6664E-08	.6592E-08	.6658E-08
	5%			.3332E-07	.3171E-07	.3329E-07
	10%			.6664E-07	.6053E-07	.6658E-07
λ *Source*	-0.1%	.9987E+00	-.2002E-04	.1999E-07	.1997E-07	.1997E-07
	-1%			.1999E-06	.1997E-06	.1997E-06
	-5%			.9996E-06	.9987E-06	.9987E-06
	-10%			.1999E-05	.1998E-05	.1997E-05
μ *Source*	0.1%		.2002E-04	.1999E-07	.1996E-07	.1997E-07
	1%			.1999E-06	.1978E-06	.1997E-06
	5%			.9996E-06	.9512E-06	.9987E-06
	10%			.1999E-05	.1816E-05	.1997E-05

Table 4.41 **Sensitivities to perturbation in system parameters** (t_f =168h)

Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
			HV Power Supply			
λ *Crowbar System*	-0.1%	.9992E+00	-.5502E-03	.5498E-06	.5497E-06	.5497E-06
	-1%			.5498E-05	.5497E-05	.5497E-05
	-5%			.2749E-04	.2749E-04	.2748E-04
	-10%			.5498E-04	.5497E-04	.5497E-04
μ *Crowbar System*	0.1%		.5502E-03	.5498E-06	.5491E-06	.5497E-06
	1%			.5498E-05	.5443E-05	.5497E-05
	5%			.2749E-04	.2618E-04	.2748E-04
	10%			.5498E-04	.4997E-04	.5497E-04
λ *AC Pwr Distrib*	-0.1%	9992E+00	-.1334E-04	.1333E-07	.1332E-07	.1332E-07
	-1%			.1333E-06	.1332E-06	.1332E-06
	-5%			.6663E-06	.6661E-06	.6661E-06
	-10%			.1333E-05	.1332E-05	.1332E-05
μ μ*AC Pwr Distrib*	0.1%		.1334E-04	.1332E-07	.1331E-07	.1332E-07
	1%			.1332E-06	.1319E-06	.1332E-06
	5%			.6662E-06	.6344E-06	.6661E-06
	10%			.1332E-05	.1211E-05	.1332E-05
λ *AC/DC Converter*	-0.1%	9992E+00	-.2335E-03	.2333E-06	.2333E-06	.2333E-06
	-1%			.2333E-05	.2333E-05	.2333E-05
	-5%			.1167E-04	.1166E-04	.1166E-04
	-10%			.2333E-04	.2333E-04	.2333E-04
μ *AC/DC Converter*	0.1%		.2335E-03	.2333E-06	.2330E-06	.2333E-06
	1%			.2333E-05	.2310E-05	.2333E-05
	5%			.1167E-04	.1111E-04	.1166E-04
	10%			.2333E-04	.2121E-04	.2333E-04

Table 4.42 **Sensitivities to perturbation in system parameters** (t_f =168h)

Param. α_i	Perturb. $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
			Crowbar System			
λ *Fast Curr. Limit* λ *Elec DC Switch* λ *Curr Lim. React.*	-0.1%	.9995E+00	-.5001E-04	.4998E-07	.4997E-07	.4998E-07
	-1%			.4998E-06	.4998E-06	.4998E-06
	-5%			.2499E-05	.2499E-05	.2499E-05
	-10%			.4998E-05	.4998E-05	.4998E-05
μ *Fast Curr. Limir* μ *Elec. DC Switch* μ *Curr Lim. React.*	0.1%		.5001E-04	.4998E-07	.4993E-07	.4997E-07
	1%			.4998E-06	.4948E-06	.4997E-06
	5%			.2499E-05	.2380E-05	.2499E-05
	10%			.4998E-05	.4543E-05	.4997E-05
λ *Crowbar* λ *Transp. Prot.*	-0.1%	.9995E+00	-.1000E-03	.9997E-07	.9995E-07	.9995E-07
	-1%			.9997E-06	.9995E-06	.9995E-06
	-5%			.4998E-05	.4998E-05	.4998E-05
	-10%			.9997E-05	.9995E-05	.9995E-05
μ *Crowbar* μ *Transp. Prot.*	0.1%		.1000E-03	.9997E-07	.9985E-07	.9995E-07
	1%			.9997E-06	.9896E-06	.9995E-06
	5%			.4998E-05	.4760E-05	.4997E-05
	10%			.9997E-05	.9086E-05	.9995E-05
λ *Volt Monitor* λ *Input Bushing* λ *Output Bushing*	-0.1%	.9995E+00	-.5001E-04	.4998E-07	.4997E-07	.4998E-07
	-1%			.4998E-06	.4998E-06	.4998E-06
	-5%			.2499E-05	.2499E-05	.2499E-05
	-10%			.4998E-05	.4998E-05	.4998E-05
μ *Volt Monitor* μ *Input Bushing* μ *Output Bushing*	0.1%		.5001E-04	.4998E-07	.4993E-07	.4997E-07
	1%			.4998E-06	.4948E-06	.4997E-06
	5%			.2499E-05	.2380E-05	.2499E-05
	10%			.4998E-05	.4543E-05	.4997E-05

Table 4.43 **Sensitivities to perturbation in system parameters** (t_f =168h)

AC Power Distribut.						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
λ *Bus Lines*	-0.1%	9.99987E-1	-.3334E-05	.3334E-08	.3333E-08	.3333E-08
	-1%			.3334E-07	.3333E-07	.3333E-07
	-5%			.1667E-06	.1667E-06	.1667E-06
	-10%			.3334E-06	.3333E-06	.3333E-06
μ *Bus Lines*	0.1%		.3334E-05	.3334E-08	.3330E-08	.3333E-08
	1%			.3334E-07	.3300E-07	.3333E-07
	5%			.1667E-06	.1587E-06	.1667E-06
	10%			.3334E-06	.3030E-06	.3333E-06
λ *SPPT Structure*	-0.1%	9.99987E-1	-.1000E-04	.1000E-07	.1000E-07	.1000E-07
	-1%			.1000E-06	.1000E-06	.1000E-06
	-5%			.5001E-06	.5000E-06	.5000E-06
	-10%			.1000E-05	.1000E-05	.1000E-05
μ *SPPT Structure*	0.1%		.1000E-04	.1000E-07	.9990E-08	.1000E-07
	1%			.1000E-06	.9901E-07	.1000E-06
	5%			.5001E-06	.4762E-06	.5000E-06
	10%			.1000E-05	.9091E-06	.1000E-05

Table 4.44 **Sensitivities to perturbation in system parameters** (t_f =168h)

AC-DC Converter						
Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred}-R^0$	REC $R_{recal}-R^0$	FSAP $R_{pred}-R^0$
λ *Transf.* λ *Thyrist.* λ *SM React*	-0.1%	.9998E+00	-.2000E-04	.2000E-07	.2000E-07	.2000E-07
	-1%			.2000E-06	.2000E-06	.2000E-06
	-5%			.9999E-06	.9998E-06	.9998E-06
	-10%			.2000E-05	.2000E-05	.2000E-05
μ *Transf..* μ *Thyrist.* μ *SM React*	0.1%		.2000E-04	.2000E-07	.1998E-07	.2000E-07
	1%			.2000E-06	.1980E-06	.2000E-06
	5%			.9999E-06	.9522E-06	.9998E-06
	10%			.2000E-05	.1818E-05	.2000E-05
λ *DC Cap.*	-0.1%	.9998E+00	-.5001E-04	.5000E-07	.4999E-07	.4999E-07
	-1%			.5000E-06	.4999E-06	.4999E-06
	-5%			.2500E-05	.2499E-05	.2499E-05
	-10%			.5000E-05	.4999E-05	.4999E-05
μ *DC Cap.*	0.1%		.5001E-04	.5000E-07	.4994E-07	.4999E-07
	1%			.5000E-06	.4949E-06	.4999E-06
	5%			.2500E-05	.2380E-05	.2499E-05
	10%			.5000E-05	.4544E-05	.4999E-05
λ *Controls*	-0.1%	.9998E+00	-.1000E-03	.9999E-07	.9998E-07	.9998E-07
	-1%			.9999E-06	.9998E-06	.9998E-06
	-5%			.5000E-05	.4999E-05	.4999E-05
	-10%			.9999E-05	.9998E-05	.9998E-05
μ *Controls*	0.1%		.1000E-03	.9999E-07	.9988E-07	.9998E-07
	1%			.9999E-06	.9899E-06	.9998E-06
	5%			.5000E-05	.4761E-05	.4999E-05
	10%			.9999E-05	.9089E-05	.9998E-05
λ *SPPT Str.*	-0.1%	.9998E+00	-.3334E-05	.3333E-08	.3333E-08	.3333E-08
	-1%			.3333E-07	.3333E-07	.3333E-07
	-5%			.1667E-06	.1666E-06	.1666E-06
	-10%			.3333E-06	.3333E-06	.3333E-06
μ *SPPT Str.*	0.1%		.3334E-05	.3333E-08	.3329E-08	.3333E-08
	1%			.3333E-07	.3300E-07	.3333E-07
	5%			.1667E-06	.1587E-06	.1666E-06
	10%			.3333E-06	.3030E-06	.3333E-06

$\lambda_{Cooling}$	-0.1%	.9998E+00	-.2002E-04	.2001E-07	.2000E-07	.2000E-07
	-1%			.2001E-06	.2000E-06	.2000E-06
	-5%			.1001E-05	.9998E-06	.9998E-06
	-10%			.2001E-05	.2000E-05	.2000E-05
$\mu_{Cooling}$	0.1%		.2002E-04	.2001E-07	.1998E-07	.2000E-07
	1%			.2001E-06	.1980E-06	.2000E-06
	5%			.1001E-05	.9522E-06	.9998E-06
	10%			.2001E-05	.1818E-05	.2000E-05

HEBT

Table 4.45 **Sensitivities to perturbation in system parameters** (t_f =168h)

Param. α_i	Perturbation $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Rel.Sens. $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	REC $R_{recal} - R^0$	FSAP $R_{pred} - R^0$
λ_{QuadCh}	-0.1%	.9643E+00	-.3465E-02	.3341E-05	.3338E-05	.3338E-05
	-1%			.3341E-04	.3338E-04	.3338E-04
	-5%			.1671E-03	.1669E-03	.1669E-03
	-10%			.3341E-03	.3339E-03	.3338E-03
μ_{QuadCh}	0.1%		.3465E-02	.3341E-05	.3335E-05	.3338E-05
	1%			.3341E-04	.3305E-04	.3338E-04
	5%			.1671E-03	.1590E-03	.1669E-03
	10%			.3341E-03	.3036E-03	.3338E-03
$\lambda_{DipoleCh}$	-0.1%	.9643E+00	-.2402E-03	.2316E-06	.2314E-06	.2314E-06
	-1%			.2316E-05	.2314E-05	.2314E-05
	-5%			.1158E-04	.1157E-04	.1157E-04
	-10%			.2316E-04	.2315E-04	.2314E-04
$\mu_{DipoleCh.}$	0.1%		.2402E-03	.2316E-06	.2312E-06	.2314E-06
	1%			.2316E-05	.2292E-05	.2314E-05
	5%			.1158E-04	.1102E-04	.1157E-04
	10%			.2316E-04	.2104E-04	.2314E-04
$\lambda_{OctupCh}$	-0.1%	.9643E+00	-.1922E-03	.1853E-06	.1852E-06	.1852E-06
	-1%			.1853E-05	.1852E-05	.1852E-05
	-5%			.9265E-05	.9258E-05	.9258E-05
	-10%			.1853E-04	.1852E-04	.1852E-04
$\mu_{OctupCh}$	0.1%		.1922E-03	.1853E-06	.1850E-06	.1852E-06
	1%			.1853E-05	.1833E-05	.1852E-05
	5%			.9265E-05	.8817E-05	.9258E-05
	10%			.1853E-04	.1683E-04	.1852E-04
$\lambda_{BuncherCh}$	-0.1%	.9643E+00	-.6530E-02	.6296E-05	.6295E-05	.6295E-05
	-1%			.6296E-04	.6296E-04	.6295E-04
	-5%			.3148E-03	.3149E-03	.3148E-03
	-10%			.6296E-03	.6299E-03	.6295E-03
$\mu_{BuncherCh}$	0.1%		.6530E-02	.6296E-05	.6289E-05	.6295E-05
	1%			.6296E-04	.6233E-04	.6295E-04
	5%			.3148E-03	.2999E-03	.3148E-03
	10%			.6296E-03	.5726E-03	.6295E-03
$\lambda_{BunchRFPwr}$	-0.1%	.9643E+00	-.4189E-02	.4039E-05	.4039E-05	.4039E-05
	-1%			.4039E-04	.4039E-04	.4039E-04
	-5%			.2020E-03	.2020E-03	.2019E-03
	-10%			.4039E-03	.4040E-03	.4039E-03
$\mu_{BunchRFPwr}$	0.1%		.4189E-02	.4039E-05	.4035E-05	.4039E-05
	1%			.4039E-04	.3999E-04	.4039E-04
	5%			.2020E-03	.1924E-03	.2019E-03
	10%			.4039E-03	.3673E-03	.4039E-03

$\lambda_{DispCav\ RFPowSup}$	-0.1%	.9643E+00	-.4189E-02	.4039E-05	.4039E-05	.4039E-05
	-1%			.4039E-04	.4039E-04	.4039E-04
	-5%			.2020E-03	.2020E-03	.2019E-03
	-10%			.4039E-03	.4040E-03	.4039E-03
$\mu_{DispCav\ RFPowSup}$	0.1%		.4189E-02	.4039E-05	.4035E-05	.4039E-05
	1%			.4039E-04	.3999E-04	.4039E-04
	5%			.2020E-03	.1924E-03	.2019E-03
	10%			.4039E-03	.3673E-03	.4039E-03
λ_{BTVSys}	-0.1%	.9643E+00	-.3005E-02	.2897E-05	.2897E-05	.2897E-05
	-1%			.2897E-04	.2897E-04	.2897E-04
	-5%			.1449E-03	.1449E-03	.1448E-03
	-10%			.2897E-03	.2898E-03	.2897E-03
μ_{BTVSys}	0.1%		.3005E-02	.2897E-05	.2894E-05	.2897E-05
	1%			.2897E-04	.2868E-04	.2897E-04
	5%			.1449E-03	.1380E-03	.1448E-03
	10%			.2897E-03	.2634E-03	.2897E-03
$\lambda_{Dipole\ As\ HR}$	-0.1%	.9643E+00	-.2796E-02	.2696E-05	.2696E-05	.2696E-05
	-1%			.2696E-04	.2696E-04	.2696E-04
	-5%			.1348E-03	.1348E-03	.1348E-03
	-10%			.2696E-03	.2696E-03	.2696E-03
$\mu_{Dipole\ As\ HR}$	0.1%		.2493E-02	.2404E-05	.2402E-05	.2403E-05
	1%			.2404E-04	.2385E-04	.2403E-04
	5%			.1202E-03	.1156E-03	.1202E-03
	10%			.2404E-03	.2227E-03	.2403E-03
$\lambda_{BTVSys\ HR}$	-0.1%	.9643E+00	-.2330E-02	.2247E-05	.2247E-05	.2247E-05
	-1%			.2247E-04	.2247E-04	.2247E-04
	-5%			.1123E-03	.1123E-03	.1123E-03
	-10%			.2247E-03	.2247E-03	.2247E-03
$\mu_{BTVSys\ HR}$	0.1%		.2077E-02	.2003E-05	.2001E-05	.2003E-05
	1%			.2003E-04	.1987E-04	.2003E-04
	5%			.1001E-03	.9634E-04	.1001E-03
	10%			.2003E-03	.1856E-03	.2003E-03
$\lambda_{Disp\ Cav\ Ch}$	-0.1%	.9643E+00	-.9380E-02	.9044E-05	.9044E-05	.9044E-05
	-1%			.9044E-04	.9045E-04	.9044E-04
	-5%			.4522E-03	.4524E-03	.4522E-03
	-10%			.9044E-03	.9052E-03	.9044E-03
$\mu_{Disp\ Cav\ Ch}$	0.1%		.8374E-02	.8075E-05	.8068E-05	.8074E-05
	1%			.8075E-04	.8012E-04	.8075E-04
	5%			.4038E-03	.3885E-03	.4037E-03
	10%			.8075E-03	.7484E-03	.8075E-03

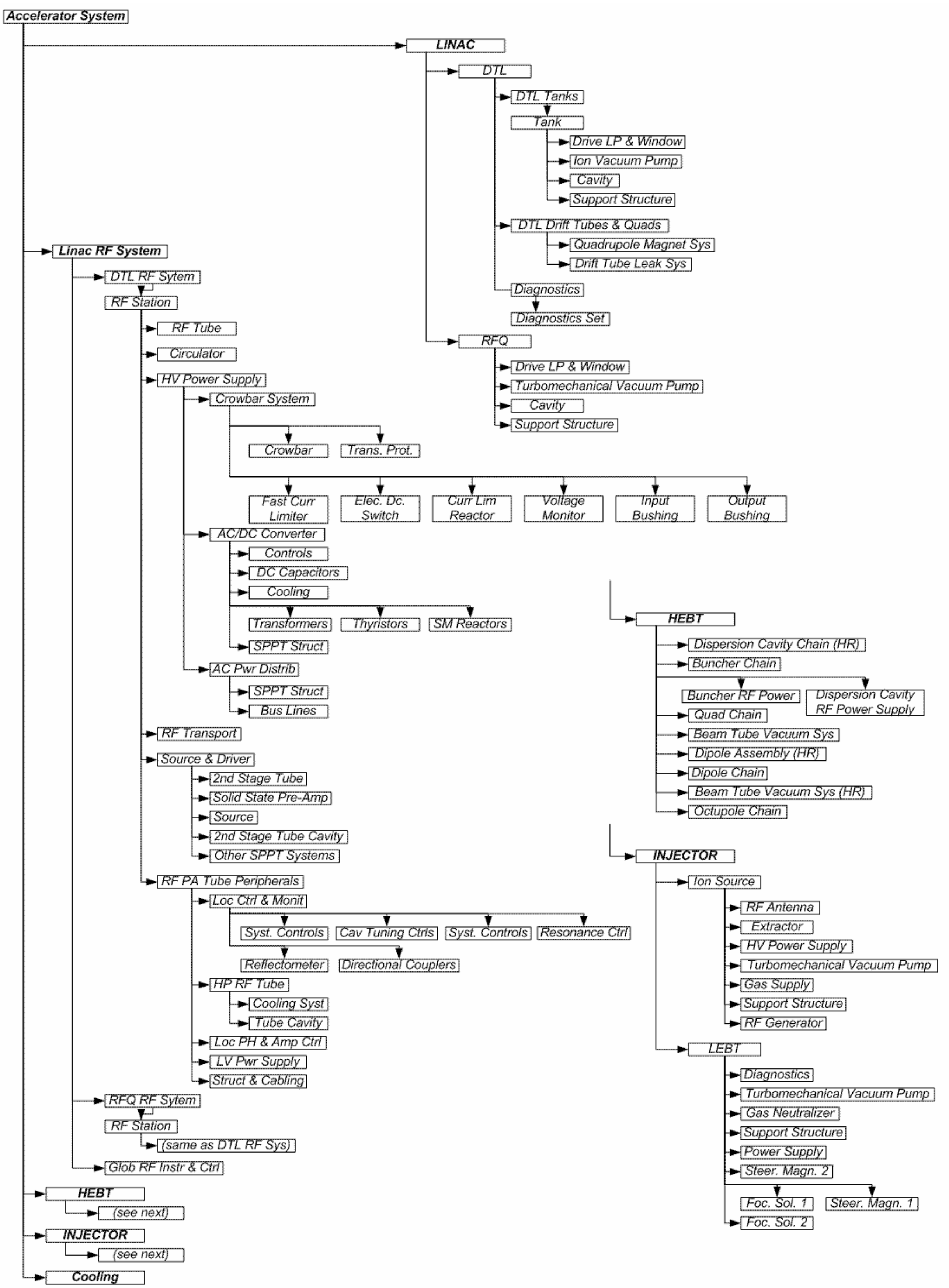

Figure 4.22 Components importance for IFMIF Accelerator System Facilities based on sensitivities of steady-state availability to variations in input parameters *MTTF* \ *MTTR*

5 Conclusions

This work has highlighted the development and implementation of the deterministic local sensitivity analysis theory originally developed by Cacuci,[29,30] for the mathematical model of Markov chains. During this implementation, the fundamental characteristics of Adjoint Sensitivity Analysis Procedure (ASAP) of Markov chains have been emphasized, namely that: a) the adjoint sensitivity system is linear in the adjoint functions; b) the adjoint functions resulted from the adjoint sensitivity system of Markov chains following ASAP guidelines are independent of variations in input parameters; c) the adjoint functions do not depend on the base-case solution of the Markov chain equations; d) the adjoint functions must be computed anew for every response since the adjoint sensitivity system of Markov chain depends on the system response by its nonhomogeneous term; e) this formalism is the practical way to perform inexpensively a complete and systematic sensitivity analysis of various reliability measures for physical systems using Markov chain technique, since the differential set of equations that describes Markov chain usually implies more parameters than responses.

With this work, the ASAP for Markov chain has been implemented into a new computer code system to perform reliability and sensitivity analysis of physical systems. The Fault-Tree abstraction of physical system as high-level interface for this code has been used. This initial representation is automatically converted into a Markov chain and the associated system of differential equations is generated. The developed code system QUEFT/MARKOMAG-S/MCADJSEN uses a coupled Fault-Tree Markov chain technique in order to asses dynamically the system's reliability, and FSAP/ASAP for sensitivity analysis. The accuracy and robustness of the numerical solution of the adjoint sensitivity system has been verified using the analytical solution on a simple problem of a binary component/system. The typical sensitivity results of reliability for considered examples to perturbations in initial conditions and system parameters indicate that the numerical solution of ASE is as robust, stable and accurate as the calculations of original Markov chain equations. From computational point of view, the response sensitivities for these examples have been obtained more efficient and with a smaller computational cost using ASAP than the traditional methods.

The developed code system has been used to perform a complete reliability and sensitivity analysis of the IFMIF Accelerator System facilities. The sensitivities of the systems and subsystems of IFMIF Accelerator System Facilities are important for further studies regarding to reliability, costs and maintenance policies of this complex system. As an intermediate step in assessing all system's sensitivities, the dynamical reliability analysis using Markov chain technique have been performed for the Accelerator System and all its subsystems showing for the first time in reliability studies of IFMIF-Accelerator system the evolution of availability of this complex system and its subsystems. The steady-state availability computed in this work for this complex system have been compared with similar results given in literature, showing that the algorithm developed for the conversion from Fault Tree to Markov chain is well implemented with the advantage that in this work the transient availability for the considered mission time have been analyzed for every of its subsystems.

Sensitivities studies using ASAP of Markov chains has been performed for two types of responses, namely for the interval and steady-state availability of the systems and subsystems of IFMIF-Accelerator System Facilities. The comparisons of results given by ASAP agree with the traditional sensitivity methods with the advantage of computational time. These studies have been performed in order to analyze the impact of changes in reliability parameters of components and subsystems to these reliability measures of accelerator system. Based on the sensitivity results, the rank of the importance of parameters uncertainties as they affect the analyzed availabilities has been performed.

The sensitivities computed in this work can be used to propose new improvements in order to increase the availability of the Accelerator System Facilities; to perform extensive uncertainty analysis of either interval or steady-state availability; to eliminate unimportant data for later considerations in a global analysis, to establish maintenance policies for critical parts and components in order to maximize the efficiency of this system, but also to prioritize the improvements in the developed computer code system. The final results show that the ASAP is a valuable tool for deterministic sensitivity analysis of reliability measures obtained using Markov chains.

Future work could concentrate in implementation of the ASAP considering other types of responses than those considered during numerical examples shown in this work for further sensitivity investigations of different reliability measures, and also to continue the validation of the developed code system, to perform reliability and sensitivity studies using the methods developed in this work for other systems than IFMIF-Accelerator System Facilities.

These methods can be used further for sensitivity studies of any phenomena that imply Markov chains in analysis.

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A. Fault-Tree: Qualitative analysis

In this section is presented the qualitative Fault-Tree analysis which has been used to generate the Markov chain.

Due to its simplicity, the Fault-Tree analysis has been widely applied for reliability and risk studies. The Fault-Tree is a combinatorial technique that abstracts the physical system through a logic diagram which depicts the component failure modes and other fault events. This description is made mainly using combinatorial logic AND/OR gates. During this analysis it can identify various combinations of events which lead to system fail. An event is called usually the failure behavior of a component or subsystem.

The principal steps in the Fault-Tree analysis are as follows. First, the physical system must be abstracted defining the TOP event which is often the failure behavior of the system, and to identify the other events which can lead to the TOP event. That is made usually through decomposition of the physical system in subsystems and components until the lowest level where further decomposition is not possible or useful for analysis. At the lowest level of decomposition are only components. The failure behavior of a component is called basic event. All the events are related each other using combinatorial gates until are reached the TOP event.

The qualitative Fault-Tree analysis consists in identifying the various sets of events that can result in system failure. These sets of events are called cut-sets. Because the cut-sets may contain repeated events, or events which are not basic events, using the rules of Boolean algebra further operations are performed to obtain the minimal-cut-sets. A minimal-cut-set is a minimal set of basic events which lead to the TOP event. If a basic event is eliminated, that minimal-cut-set ceases to be a cut-set.

To obtain the complete set of minimal-cut-sets some algorithms have been developed. These algorithms are either top-down, or bottom-up algorithms. Due their efficiency, the most used algorithms are the top-down algorithms proposed by Fussel and Vesely [1972][77] and called MOCUS algorithm (methodology for obtaining cut-sets), and by Akers [1978][78] named BDD (binary decision diagram). In the last decade BDD has been developed and improved based on the method proposed by Bryant [1986][79] and now is the most used algorithm due its efficiency, against of MOCUS algorithm. Until the middle of nineties majority software packages for Fault-Tree analyses used the MOCUS algorithm mainly because the BDD method was not sufficiently developed until that time. Nowadays the BDD algorithm is implemented, further developed, and is used in almost all Fault Tree software packages in principal because it can deal with very large Fault Trees into a quite small amount of computing time. In this analysis the MOCUS algorithm has been used and it is shortly described below.

The algorithm begins with the TOP event and systematically goes down through the fault tree until the basic events. The fault tree is developed into a matrix with the event elements of the matrix. Each row of the matrix is a cut-set. Each time an OR gate is encountered new rows are produced. Each time an AND gate is encountered the length of the rows in which the gate appear is increased. The algorithm stops when all the elements of the matrix are basic events. The rules of Boolean algebra are applied after each gate development. Repeated events in the same row are deleted (idempotent rule), and the row that contains all elements of another row is deleted (absorption rule). At the end of this algorithm the complete set of minimal-cut-sets as rows of this matrix is obtained.

For implementing this algorithm, the following assumptions are necessary:

- the fault tree is coherent, i.e. does not contain negated events,
- the components are binary, i.e. have only two states – operational or failed.

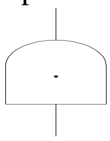
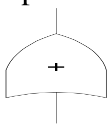
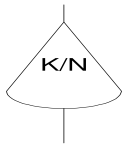
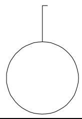
The algorithm has been improved to manage with negated gates/events and multi-state components.[80] The multi-state components are implemented using the Markov chain.

The MOCUS algorithm has been implemented in the computer code QUEFT (Qualitative Evaluator for Fault-Tree) to find all the minimal-cut-sets of the fault tree which is used as a high level interface for further Markov analysis. This code has been tested for various fault trees from literature. The time of finding the complete set of minimal-cut-sets depends on fault tree dimension (the number of basic events and gates), and how strong is the connected fault tree structure. The rules of Boolean algebra and the fault tree gates which have been used are presented below.

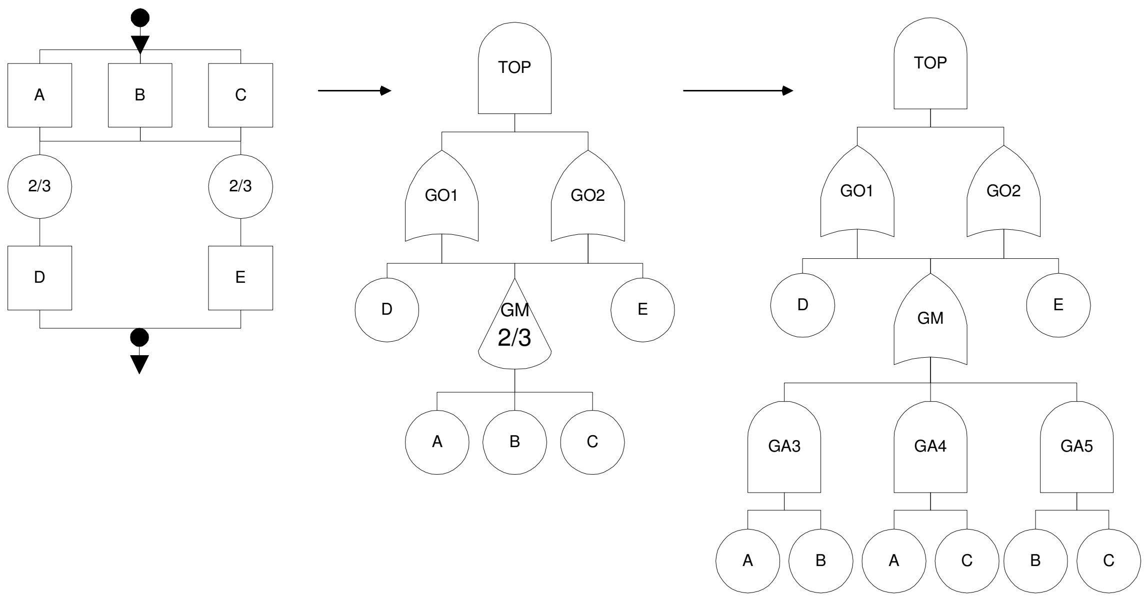
Let *X*, *Y*, and *Z* be boolean variables, the rules of boolean algebra are as follows.

Absorption rule:	$X(X+Y) = X$ $X+XY = X$
Idempotent rule:	$XX = X$ $X+X = X$
Commutative rule:	$XY = YX$ $X+Y = Y+X$
Associative rule:	$X(YZ) = (XY)Z$ $X+(Y+Z) = (X+Y)+Z$
Distributive rule:	$X(Y+Z) = XY+XZ$ $(X+Y)(X+Z)=X+YZ$

The fault tree gates and their graphical representation are as follows.

Output event  Input events	AND gate The output event occurs if and only if all the input events occur
Output event  Input events	OR gate The output event occurs if any one or more of the input events occur
Output event  Input events	K-Out-of-N gate (also known as majority gate, or voting gate) The output event occurs if *K* out of *N* input events occur. This gate is decomposed into an OR gate with $\binom{K}{N} = \frac{N!}{K!(N-K)!}$ AND gates as input. These AND gates have as input combinations of *K* events from the original set of input events.
	Basic event – it consists in failure behavior of the component with known failure parameters (failure probability, failure/repair rates)

For illustrative purposes it has been chosen a simple 2-Out-of-3 system.[81] This example is originally as a block diagram in Fig.A.1.a. It consists in five components *A*, *B*, *C*, *D*, and *E*, respectively. As basic events are considered the failure behavior of components, and the block diagram is converted into a fault tree as is depicted in Fig.A.1.b. Further, in Fig.A1c is represented the majority gate transformation.



a. b. c.

Fig.A.1 Simple 2-Out-of-3 system

The TOP event represents the system failure and is an AND gate. Additional gates were named *GO1*, *GO2* (OR gates), and *GM* (2-Out-of-3 gate), respectively. This fault tree (Fig.A.1.b) is introduced as input for the QUEFT code. All minimal-cut-sets are computed using the MOCUS algorithm, i.e. {D E}, {A B}, {A C}, {B C}.

In order to find the minimal-cut-sets, the QUEFT code converts the K-Out-of-N gates into a combinatorial structure of OR and AND gates as in Fig.A1c. It searches the minimal-cut-sets starting from the TOP event after the algorithm described previous.

{TOP (and)} →	{GO1(or) GO2(or)} →	{D GO2(or)} → {GM(or) GO2(or)}	{D GM(or)} → {D E } {GM(or) GM(or)} {GM(or) E }
TOP event is the initial matrix entry	TOP is an AND gate. Its inputs are GO1 & GO2. Replace it in row.	GO1 is an OR gate. Its inputs are D & GM. Replace it in row with D. Produce another row in which it is replaced with GM.	Apply the rules of Boolean algebra. Reduce 3rd row to GM (idempotent rule). Delete 1st and 4th row (absorption rule).

The algorithm continues in the same way until in matrix are only basic events.

{D E } → {GM(or)}	{D E } → {GA3(and)} {GA4(and)} {GA5(and)}	{D E} {A B} {A C} {B C}

Using the minimal-cut-sets, the *generic fault states* are defined and used to identify the fault states during the automated generation of Markov chain. The concept of *generic fault states* which has been introduced in this work is explained further. For instance, the system states are written in form {ABCDE} considering different behavior of the components. If it is assumed that into the system are binary components only, and if it is made the convention that the fault behavior to be assigned with `1` and the operational state of the component to be assigned with `0`, one can define based on the minimal-cut-sets {A B}, {A C}, {B C}, and {D E}, respectively, the next generic fault states: {`11xxx`}, {`1x1xx`}, {`x11xx`}, and {`xxx11`}. In the place of `x` can be either `1`, or `0`. Any state of Markov chain which is generated and contains in the indicated positions the failure behavior for components is considered a failure state, otherwise that state is an operational state. For example the state {`10010`} is an operational state in which the components *A* and *D* are not operational, but the state {`10110`} is a failure state because it includes the generic fault state {`1x1xx`} in which the components *A* and *C* have failed and leads to system failure.

B. 2-Out-of-3 System: Numerical Example

The transition rate matrix for the 2-Out-of-3 system presented in Section 2.2 and in Appendix A, whose transition state diagram is depicted in Fig.2.8, has the next $Q(t)=[q_{ij}]_{25\times 25}$ transition rate matrix for the case with repairable components,

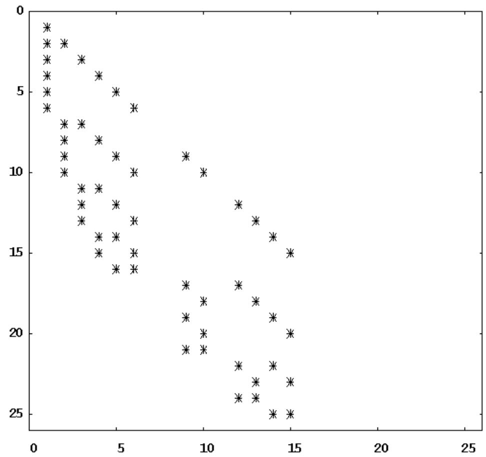
$$
\begin{bmatrix}
q_{11} & \mu_1 & \mu_2 & \mu_3 & \mu_4 & \mu_5 \\
\lambda_1 & q_{22} & & & & & \mu_2 & \mu_3 & \mu_4 & \mu_5 \\
\lambda_2 & & q_{33} & & & & \mu_1 & & & & \mu_3 & \mu_4 & \mu_5 \\
\lambda_3 & & & q_{44} & & & & \mu_1 & & & \mu_2 & & & \mu_4 & \mu_5 \\
\lambda_4 & & & & q_{55} & & & & \mu_1 & & & \mu_2 & & \mu_3 & & \mu_5 \\
\lambda_5 & & & & & q_{66} & & & & \mu_1 & & & \mu_2 & & \mu_3 & \mu_4 \\
 & \lambda_2 & \lambda_1 & & & & q_{77} \\
 & \lambda_3 & & \lambda_1 & & & & q_{88} \\
 & \lambda_4 & & & \lambda_1 & & & & q_{99} & & & & & & & & \mu_2 & & \mu_3 & & \mu_5 \\
 & \lambda_5 & & & & \lambda_1 & & & & q_{1010} & & & & & & & & \mu_2 & & \mu_3 & \mu_4 \\
 & & \lambda_3 & \lambda_2 & & & & & & & q_{1111} \\
 & & \lambda_4 & & \lambda_2 & & & & & & & q_{1212} & & & & & \mu_1 & & & & & \mu_3 & & \mu_5 \\
 & & \lambda_5 & & & \lambda_2 & & & & & & & q_{1313} & & & & & \mu_1 & & & & & \mu_3 & \mu_4 \\
 & & & \lambda_4 & \lambda_3 & & & & & & & & & q_{1414} & & & & & \mu_1 & & & \mu_2 & & & \mu_5 \\
 & & & \lambda_5 & & \lambda_3 & & & & & & & & & q_{1515} & & & & & \mu_1 & & & \mu_2 & & \mu_4 \\
 & & & & \lambda_5 & \lambda_4 & & & & & & & & & & q_{1616} \\
 & & & & & & & & \lambda_2 & & & \lambda_1 & & & & & q_{1717} \\
 & & & & & & & & & \lambda_2 & & & \lambda_1 & & & & & q_{1818} \\
 & & & & & & & & \lambda_3 & & & & & \lambda_1 & & & & & q_{1919} \\
 & & & & & & & & & \lambda_3 & & & & & \lambda_1 & & & & & q_{2020} \\
 & & & & & & & & \lambda_5 & \lambda_4 & & & & & & & & & & & q_{2121} \\
 & & & & & & & & & & & \lambda_3 & & \lambda_2 & & & & & & & & q_{2222} \\
 & & & & & & & & & & & & \lambda_3 & & \lambda_2 & & & & & & & & q_{2323} \\
 & & & & & & & & & & & \lambda_5 & \lambda_4 & & & & & & & & & & & q_{2424} \\
 & & & & & & & & & & & & & \lambda_5 & \lambda_4 & & & & & & & & & & q_{2525}
\end{bmatrix}(t)
$$

where the elements A to E have the failure rates $\lambda_i(t),\ i=1,...,5$, and the repair rates $\mu_i(t),\ i=1,...,5$. The elements on the main diagonal follow the property (2.23), i.e.,

$$q_{ii}(t) = -\sum_{\substack{i,j\in I \\ i\neq j}} q_{ji}(t), \quad I=\{1,...,25\}$$

The probability state vector is $\Pi(t)=[\pi_1(t),...,\pi_{25}(t)]^T$, $\pi_i(t)\in[0,1]$, and $\sum_{i=1}^{25}\pi_i(t)=1$.

For the case with no repairable components, the part of transition matrix above the main diagonal vanishes, and the elements on main diagonal are modified accordingly, as it is shown in the next figure.



For numerical example, considering the system structure Fig.A.1.a, it is assumed that the failure probabilities of components are exponentially distributed, and their performances are given in terms of mean time to failure and repair, as follow,

component	*A*	*B*	*C*	*D*	*E*
MTTF (h)	100	100	100	1000	1000
MTTR (h)	5	5	5	10	10

In this example are analyzed two extreme cases, namely one when the system contains all components with repairs, and the other one when none of its components are repairable. In the first case it is studied the availability of system and in the second case its reliability. At the initial time $t_0 = 0$ it is assumed that the system is in the state in which all its components are operational, i.e. $\Pi_0 = [1, 0, ..., 0]^T$.

Considering the system structure (Fig.A.1.a), it has been split its components in two groups, namely a group that contains the components *A*, *B*, and *C* assumed to be identically, and another group of components *D*, and *E* identically as well. The components from each group have the same characteristics. It follows that the failure\repair rates are

component	*A*	*B*	*C*	*D*	*E*
$\lambda = 1/MTTF$ (failure/h)	0.01	0.01	0.01	0.001	0.001
$\mu = 1/MTTR$ (repair/h)	0.2	0.2	0.2	0.1	0.1

Considering the transient analysis for a mission time of *T* = 500 hours, the transient availability and unavailability of the system, using the developed code QUEFT/MARKOMAG-S is obtained. As truncated model, the development of Markov chain until the level 2 (Fig.2.8) is considered. In the case of truncated model are 16 states, thus the transition rate matrix is of order 16, where four states are failure states, and the rest are operational states.

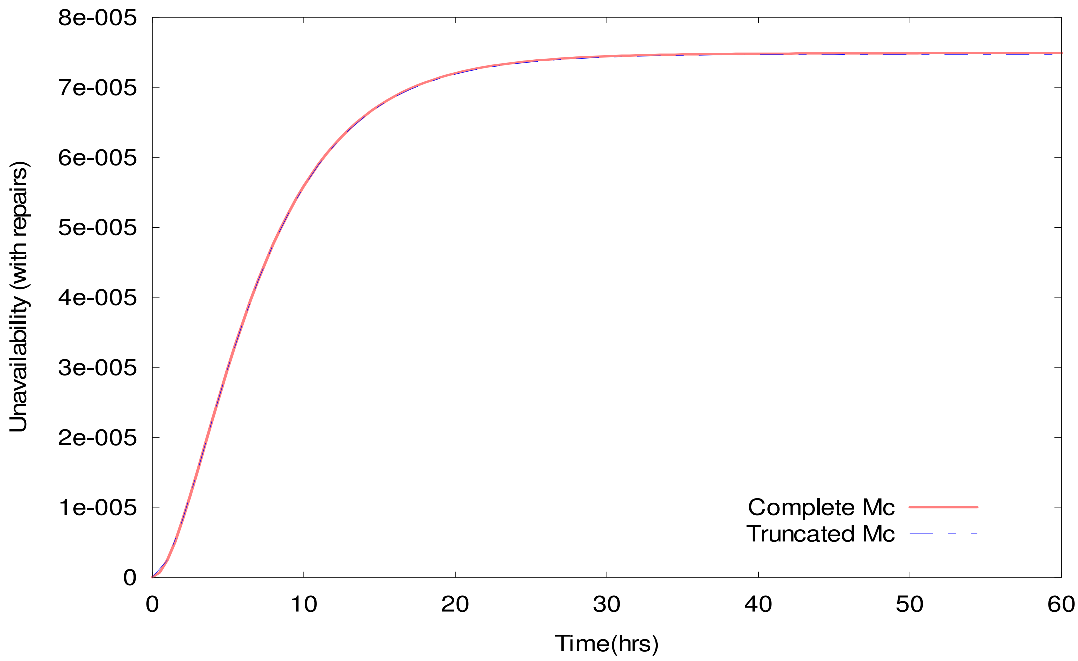


Figure B.1 Transient unavailability of 2-Out-of-3 System

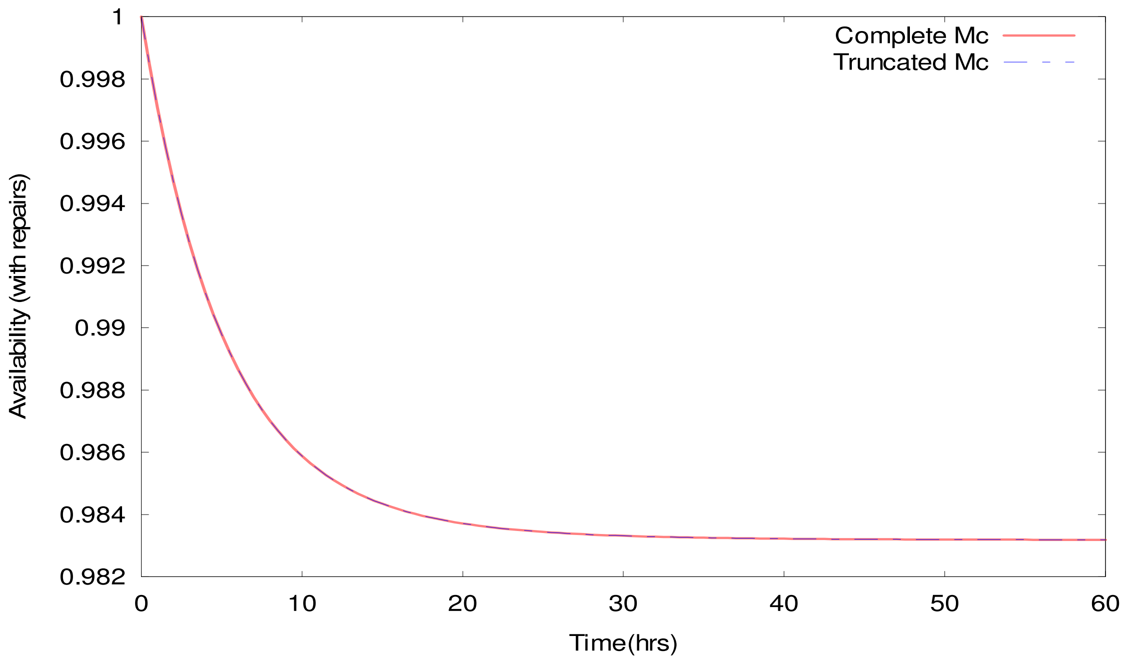
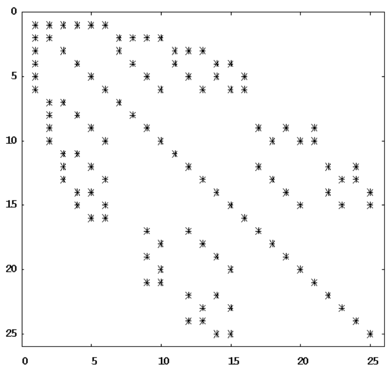


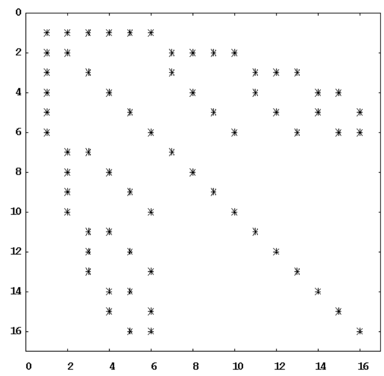
Figure B.2 Transient availability of 2-Out-of-3 System

where the graphs shows the transient until the stationary solution has been reached.

A comparison between the transition rate matrices in case of complete Markov chain and truncated Markov chain of 2-Out-of-3 system is presented below



Complete transition rate matrix
Order: 25
Non-zero elements: 111



Truncated transition rate matrix
Order: 16
Non-zero elements: 66

In the case of the complete Markov chain, the set of operational states are $Up = \{1,\ldots,6,9,10,12,\ldots,15\}$ and $Down = \{7,8,11,16,\ldots,25\}$ for the failure states, respectively. In the case of the truncated model the system states are $Up_{tr} = Up$, and $Down_{tr} = \{7,8,11,16\}$, respectively. The quantification of the transient unavailability and availability have been made using the formulae (2.25) and (2.26), respectively. The stationary solution of system unavailability has been compared with the solution obtained by MUSTAMO[82] code which uses the Fault-Tree method for reliability analysis.

MARKOMAG-S	
- Complete model	0.74884E-04
- Truncated model (lev.2)	0.74722E-04
MUSTAMO	0.75254E-04

For the availability it has been obtained

Complete model	0.9831815
Truncated model	0.9831817

For the case without repairable components, the next transient solutions have resulted:

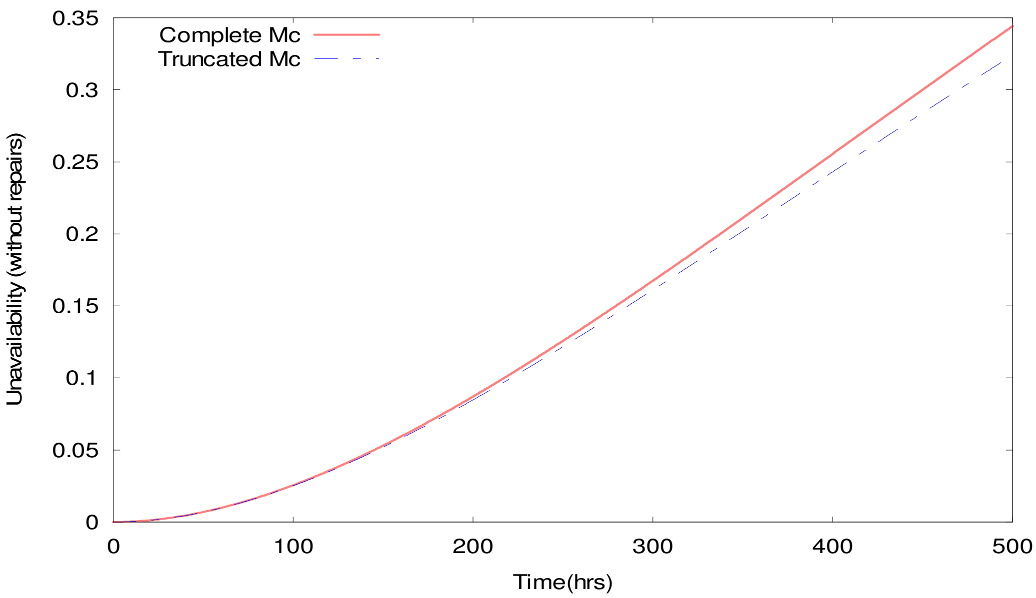


Figure B.3 Transient unreliability of 2-Out-of-3 System

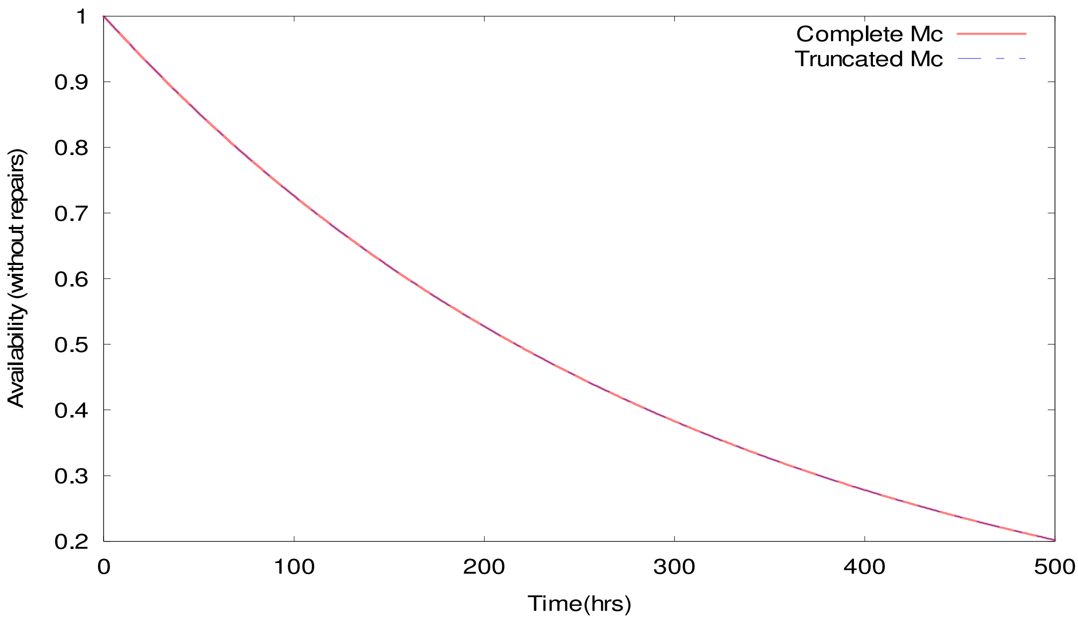


Figure B.4 Transient reliability of 2-Out-of-3 System

The stationary solution of the system with non repairable components is as follows,

Complete model	0.112535E-06
Truncated model	0.112535E-06

From this small example, one can see that in certain situations depending on the problem, instead to generate the complete Markov chain, a reduced model which contains the most relevant states that have the most contribution to the interested results can be used. Most situations requests to analyze the probability that system to be in the state without any component defect (the state indexed *1* in Fig.2.8), and the notion of availability is extended also to the components inside of the system.

In this case the numerical values at the end of the mission time are as follows,

$\pi_1(t = 500h)$	complete model	Truncated model
with repairs	0.98318153	0.98318169
without repairs	0.20189651	0.20189651

As it can be seen from these results, the truncated model considered trend to be more optimistic than the complete Markov chain.

From the results for the case with repairable components (Figs.B.2, and B.5), one can see that the probability of the state with all components available has the biggest weight to the availability of the system, the probabilities of the other operational state being very small in comparison by it.

Next, for this example, it is performed a sensitivity analysis which will provide the relative importance of the two groups of components, related to their performances. The system sensitivity is analyzed using adjoint method and for comparison the recalculation and forward method. The system response is assumed to be of form

$$R = \frac{1}{T}\int_{t_0=0}^{t_f=T} \pi_1(t)dt$$

Performing sensitivity analysis on this type of response using the developed code MARKOMAG-S/MCADJSEN, it is evaluated the relative importance of the performances for each group of components, i.e. {*A*, *B*, *C*}, and {*D*, *E*}, since the structure of the system is as in Fig.A1a. It is considered variations in system parameters, namely in the parameters $\lambda_{A,B,C}, \mu_{A,B,C}, \lambda_{D,E}, \mu_{D,E}$, for the case with repairs, and in the parameters $\lambda_{A,B,C}, \lambda_{D,E}$, for the case without repairs. Since the components *A*, *B*, and *C* are in parallel to assure a certain level of redundancy against of only two for the components *D*, and *E*, the parameters of the first group of components are more important to be considered than the others, because their effect on system behavior is bigger, for the considered response. This fact should be confirmed by sensitivity analysis.

Doing small variations in system parameters, the effect of each of these perturbations to system response is analyzed. In practice, one wants to increase *MTTF* and to decrease *MTTR* of components, so the perturbations has been made in this idea, namely the *MTTF* has been increased with a percent of 0.1, 1.0, 5.0, and 10.0, from its value, i.e. $MTTF + \delta MTTF$ where $\delta MTTF = \{0.1\% MTTF, 1\% MTTF, 5\% MTTF, 10\% MTTF\}$, and the *MTTR* has been decreased with the percents from its values, i.e. $MTTR - \delta MTTR$.

For the case with repairs it is obtained the next sensitivities for the mission time *T*=500h.

Perturbation in system's parameters (t_f =500h.)

Par. α_i	Perturbation. $\frac{\Delta\alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Relative sensitivity $\frac{\Delta R}{\Delta\alpha_i}\cdot\frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	$R_{recal} - R^0$	FSAP $R_{pred} - R^0$
$\lambda_{A,B,C}$	-0.1%	.9834E+00	-.4925E-02	.4843E-05	.4843E-05	.4843E-05
	-1%			.4843E-04	.4844E-04	.4843E-04
	-5%			.2422E-03	.2422E-03	.2422E-03
	-10%			.4843E-03	.4846E-03	.4843E-03
$\mu_{A,B,C}$	0.1%	.9834E+00	.4875E-02	.4794E-05	.4789E-05	.4795E-05
	1%			.4794E-04	.4747E-04	.4795E-04
	5%			.2397E-03	.2285E-03	.2397E-03
	10%			.4794E-03	.4364E-03	.4794E-03
$\lambda_{D,E}$	-0.1%	.9834E+00	-.9789E-03	.9627E-06	.9627E-06	.9626E-06
	-1%			.9627E-05	.9627E-05	.9627E-05
	-5%			.4813E-04	.4813E-04	.4813E-04
	-10%			.9627E-04	.9627E-04	.9627E-04
$\mu_{D,E}$	0.1%	.9834E+00	.9589E-03	.9430E-06	.9420E-06	.9430E-06
	1%			.9430E-05	.9338E-05	.9431E-05
	5%			.4715E-04	.4495E-04	.4715E-04
	10%			.9430E-04	.8589E-04	.9430E-04

The relative sensitivities in absolute value are used afterwards to rank the components importance for considered response.

Imp.	Par. α_i	Relative Sensitivity $\left\|\frac{\Delta R}{\Delta \alpha_i} \cdot \frac{\alpha_i^0}{R^0}\right\|$
1	$\lambda_{A,B,C}$	.4925E-02
2	$\mu_{A,B,C}$	.4875E-02
3	$\lambda_{D,E}$	.9789E-03
4	$\mu_{D,E}$	.9589E-03

For the case without repairs the results are shown below,

Perturbation in system's parameters (t_f =500h.)

Par. α_i	Perturbation. $\frac{\Delta \alpha_i}{\alpha_i^0}$ (%)	Nominal Value R^0	Relative sensitivity $\frac{\Delta R}{\Delta \alpha_i} \cdot \frac{\alpha_i^0}{R^0}$	ASAP $R_{pred} - R^0$	$R_{recal} - R^0$	FSAP $R_{pred} - R^0$
$\lambda_{A,B,C}$	-0.1%	.4988E+00	-.1860E+00	.9279E-04	.9280E-04	.9278E-04
	-1%			.9279E-03	.9292E-03	.9278E-03
	-5%			.4639E-02	.4673E-02	.4639E-02
	-10%			.9279E-02	.9412E-02	.9278E-02
$\lambda_{D,E}$	-0.1%	.4988E+00	-.1860E-01	.9279E-05	.9279E-05	.9278E-05
	-1%			.9279E-04	.9280E-04	.9278E-04
	-5%			.4639E-03	.4643E-03	.4639E-03
	-10%			.9279E-03	.9292E-03	.9278E-03

Imp.	Par. α_i	Relative Sensitivity $\left\|\frac{\Delta R}{\Delta \alpha_i} \cdot \frac{\alpha_i^0}{R^0}\right\|$
1	$\lambda_{A,B,C}$	.1860E+00
2	$\lambda_{D,E}$	.1860E-01

The results confirm the previous observation, but for more complex systems in which are involved tens or hundreds of components to do such observations are quite difficult. Using the sensitivity analysis, the relative importance of parameters for each component of considered response is ranked. Further, one can establish maintenance policies, or improve the system in its weak points by redundancy, or by replacement of the components with others with better performances (e.g. *MTTF* higher, *MTTR* lower) in order to get an imposed value of system response. These results may be used also for further uncertainity analysis of system response.

C. ASAP applied to Forward Kolmogorov ODE

When the Markov chain is described by the forward Kolmogorov ordinary differential equations as in Eq.(2.21), and making similarities with the ASAP applied to backward equations (3.1) from Chapter 3, the system of equations is as follows

$$\begin{cases} \dfrac{dP(t)}{dt} = P(t)\left[\Theta(t)\right] \\ P(t_0) = P_0 \end{cases} \quad \text{, for } t \geq 0 \tag{C.1}$$

where the probability transition vector is a row vector of order n -the number of states in Markov chain, i.e.,

$$P(t) \equiv \Pi^T(t)$$
$$P(t) = [p_1(t), ..., p_n(t)]$$

and transition rate matrix

$$\left[\Theta(t)\right]_{n\times n} \equiv \left[Q(t)\right]^T_{n\times n}$$

where the system responses is defined by Eq.(3.12)

$$R(P, \mathrm{A}) \equiv \int_{t_0}^{t_f} F(P, \mathrm{A})dt, \quad 0 \leq t_0 < t \tag{C.2}$$

with $\mathrm{A} = \{\alpha_1, ..., \alpha_m\}$ the set of system parameters included in transition rate matrix $\Theta(t) = \left[\theta_{ij}(t)\right]_{n\times n}$, $\theta_{ij}(t) = f(\alpha_k), \quad i, j = 1, ..., n; \quad k = 1, ..., m, \quad \theta_{ii}(t) = -\sum_{\substack{i=1 \\ i\neq j}}^{n} \theta_{ij}(t)$.

Then, the sensitivity DR of R to variations $\Gamma \equiv \delta P$ and $\delta\mathrm{A} = \{\delta\alpha_1, ..., \delta\alpha_m\}$ is given by the G-differential

$$DR(P^0, \mathrm{A}^0; \Gamma; \delta\mathrm{A}) \equiv \frac{d}{d\varepsilon}\left\{\int_0^{t_f} F(\Pi^0 + \varepsilon\Gamma; \mathrm{A}^0 + \varepsilon\delta\mathrm{A})dt\right\}_{\varepsilon=0} = DR_d + DR_i \tag{C.3}$$

where

$$DR_d \equiv \sum_{k=1}^{m} \int_0^{t_f} \left(\frac{\partial F}{\partial \alpha_k}\right) \delta\alpha_k dt = \text{"direct effect" term} \tag{C.4}$$

$$DR_i \equiv \int_{t_0}^{t_f} \left(\frac{\partial F}{\partial P}\right)_{(P^0, \alpha^0)} \left[\Gamma(t)\right]^T = \sum_{i=1}^{n} \int_{t_0}^{t_f} \left(\frac{\partial F}{\partial p_i}\right) \gamma_i(t)dt = \text{"indirect effect" term} \tag{C.5}$$

with $\dfrac{\partial F}{\partial P} = \left(\dfrac{\partial F}{\partial p_1}, ..., \dfrac{\partial F}{\partial p_n}\right)$ row vector

The variations Γ and $\delta\mathrm{A}$ are related each other through the G-differential of Eq.(C.1), namely

$$\begin{cases} \dfrac{d\Gamma(t)}{dt} - \Gamma(t)\left[\Theta^0(t)\right] = P^0(t)\left[\delta\Theta\right] \\ \Gamma(t = t_0) = \delta\Gamma_0 \end{cases} \tag{C.6}$$

where superscript "0" denotes nominal values.

The adjoint system to Eq.(C.6), following the ASAP as in section 3.2, is

$$\begin{cases} \dfrac{d\Lambda(t)}{dt} + \Lambda(t)\left[\Theta^0(t)\right]^T = -\left(\dfrac{\partial F}{\partial P}\right)_{(P^0,\mathrm{A}^0)} \\ \Lambda(t = t_f) = 0 \end{cases} \tag{C.7}$$

In terms of the adjoint function $\Lambda(t) = \left[\lambda_1(t), ..., \lambda_n(t)\right]$, the indirect-effect term DR_i is given by

$$DR_i = \int_{t_0}^{t_f} P^0(t)\left[\delta\Theta\right]\Lambda^T(t)dt + \Gamma(t_0)\Lambda^T(t_0) \tag{C.8}$$

and the response sensitivity using forward Kolmogorov equations is as follows

$$DR(P^0, \mathrm{A}^0; \Gamma; \delta\mathrm{A}) = \sum_{k=1}^{m}\int_{t_0}^{t_f}\left(\frac{\partial F}{\partial \alpha_k}\right)\delta\alpha_k dt + \int_{t_0}^{t_f} P^0(t)\left[\delta\Theta\right]\Lambda^T(t)dt + \Gamma(t_0)\Lambda^T(t_0) \tag{C.9}$$

D. Description of the computer-code system QUEFT/MARKOMAG-S/MCADJSEN

With this work, a computer code-system has been developed with the purposes to validate numerical solution of the adjoint sensitivity method applied to Markov chain. A description of the steps which are followed in order to asses the reliability and sensitivity for a physical system abstracted as a Fault-Tree has been presented in Section 3.4. The code system consists mainly in three separate modules, namely QUEFT (Qualitative Evaluator for Fault-Tree), MARKOMAG-S (Markov chain Matrix Generator and Solver), and MCADJSEN (Markov chain Adjoint Sensitivity Module). Each module is presented further during this section. The code is still under development and has been tested on x86 machine architecture under Linux\Windows OS, on Intel XEON cluster architecture with Linux OS, and IBM RS-6000 with Unix AIX OS. It is written in standard Fortran 77 programming language with extensions. It must be mentioned that this code is not into a final stage, but developed enough for the purposes of this work. Further developments and modifications of this code system are possible.

D.1 QUEFT module

The QUEFT code finds the minimal-cut-sets (MCS) of a fault-tree for a physical system, using the Top-Down MOCUS algorithm as has been described in appendix A, and using the hypothesis from Section 2.2. These MCS are used further by MARKOMAG-S code to generate the associated Markov chain. The input file for the QUEFT code consists in enumeration of the basic events and the logic structure among these events (the fault tree). It also has to be provided, the failure/repair rates or failure probability distribution for each basic event.

The structure of the input file for QUEFT code must be as follows:

```
1. output_file_name
    (max. char*20)
2. *end
3. basic_ev distrib_type parameter_1 parameter_2
   (char*5)   (char*6)     (real*8)   (real*8)
            <------ maximum 60 characters ----->
4. *end
5. gate_id  gate_type K_out_N no_predecessors predecessors_id
   (char*5) (char*5)  (int*4)     (int*4)         (char*5)
6. *end
7. top_gate
8. info
```

The input file consists of four main sections, each section being ended with an `*end` marker. First must be provided a name for the output file which will be used further as input file for MARKOMAG-S code. The name of the output file must be a maximum of 20 characters. The output file declaration is ended using the `*end` marker on the next line. Afterwards, (see the Section 3 in input file structure) must be written the basic components identifications (maximum 5 characters) with the distribution type (maximum 6 characters) and the distribution parameters (two times of `real*8` numbers). In the actual state of the code development, the continuous distribution type may be exponential, Weibull, extreme value, or as mean time to failure\repair. The declaration of basic components must be ended with an `*end` marker. It has been considered the probability distributions of two parameters for Weibull and extreme value functions. These distributions considering the equivalent relations given in table 2.1 are as follows:

Cumulative failure probability		Hazard rate
Exponential:	$f(t) = \lambda e^{-\lambda t}$	$\lambda(t) = \lambda$
Extreme Value:	$f(t) = e^{(t-\beta)/\gamma} e^{-e^{(t-\beta)/\gamma}} / \gamma$	$\lambda(t) = e^{(t-\beta)/\gamma} / \gamma$
Weibull:	$f(t) = \beta (t/\theta)^{\beta-1} e^{-(t/\theta)^{\beta}}$	$\lambda(t) = \beta t^{\beta-1} / \theta^{\beta}$

where $t \geq 0$, and $\lambda, \beta, \gamma, \theta,$ are distributions parameters that must be as follows in the input file:

- Parameter1: λ Exponential distribution – scale parameter

β Extreme value distribution – location parameter

θ Weibull distribution – scale parameter

MTTF

- Parameter 2: 0.0 for the exponential distribution

γ Extreme value distribution – scale parameter

β Weibull distribution – shape parameter

MTTR

The following section (see the Section 5 in input file structure) consists in definition of the structure of fault tree. First must be written the gate id (max. 5 characters) followed by its type (AND, OR, MAJ). Then must be introduced an `integer*4` that has to be 0 if the gate type is AND or OR, or a number great than 0 if the gate type is a majority gate MAJ. In this last case, the number represents K out of N events which have to occur simultaneously in order that the event defined by the majority gate to occurs. The next `integer*4` represents the number of predecessors which the gate contains. For the majority gate, this number (N) must be greater than the precedent number (K). Further in row must be written the predecessors ids, each of them being maximum 5 characters, and according to the number written before. The predecessors could be components and gates ids as well. After all structure of the fault tree has been defined, the next line has to contain the `*end` marker. The TOP gate id for which the fault-tree is analyzed will be written on the next line. Further in the input file (see section 8 in input file structure) could be written additional text containing information about the considered problem. The TOP gate id is the last field that is read by QUEFT code.

An input file could be as in the 2-Out-of-3 example presented along this work. Its fault-tree (Figures 2.6, and A.1.b) consists in a TOP AND gate which has as predecessors two OR gates with the ids *GO1* and *GO2*, respectively. Each OR gate has as predecessors a basic event and a 2-out-of-3 majority gate with three basic events.

The input file of QUEFT for this problem should be as follows (note that the distributions name and the gates type must be written using small case as `mtime`, `expon`, `extval`, `weibul`, for distributions, and `and`, `or`, `maj` for gate types; the numerical values have been chosen only for illustration purposes).

```
2out3.mcs
*end
A     mtime      200.0D+00   1.0D+00
B     expon      2.0D+00     0.0
C     extval     1.0D+00     1.0D-01
D     weibul     0.2D+00     0.5
E     mtime      1.0D+04     0.3D+01
*end
GM23  maj   2 3  A     B     C
GO1   or    0 2  D     GM23
GO2   or    0 2  E     GM23
TOP   and   0 2  GO1   GO2
*end
TOP
info:
An input file example for QUEFT code (simple 2-Out-of-3 system)
```

After a run of QUEFT code using this input file, the results will be written in the `2out3.mcs` file that will contain the basic components id and MCS which will be used further by MARKOMAG code to generate the Markov chain. The output file `2out3.mcs` is as follows,

```
5
A     mtime      200.0D+00   1.0D+00
B     expon      2.0D+00     0.0
C     extval     1.0D+00     1.0D-01
D     weibul     0.2D+00     0.5
E     mtime      1.0D+04     0.3D+01
4
2
```

```
4
6
8
4
5
1
2
1
3
2
3
```

The above output file structure is as follows:

- the number of basic events,
- the list of basic events id, distribution functions, rates/parameters
- the number of MCS
- the list of numbers of BE in each MCS
- the list of BE order numbers for each MCS

The numbers after the basic components id define the MCS of system, namely the combinations of basic events (usually the failure behavior of basic components) DE, AB, AC, BC. These are not important for the user as long as the MARKOMAG code use these data in generating the Markov chain algorithm. But, for checking the accuracy the meaning of the numbers are as follows: each basic event is counted and receive an index, i.e. A-1, B-2, C-3, D-4, and E-5, respectively. The first number in column after the list of basic events represents the number of MCS, i.e. 4. The next four numbers in column represent the identification for the MCS, i.e. 1st MCS has 2 basic events, the next MCS has 4-2=2 basic events, the next one has 6-4=2 basic events, and so on. The number of basic events in MCS is given by the difference between two successive numbers in column. Afterwards, each basic event is identifying following the basic components indexes, e.g. starting with the row 6 after the list of basic events, it is known that the 1st MCS has 2 basic events and these basic events are indexed with 4, and 5, which means that contains the basic events for the components D, and E, from row 8 it follows that the 2nd MCS have also two basic events indexed with 1, and 2, which correspond to the basic events A, and B, and so on.

D.1.1 QUEFT options

The QUEFT code can be run at this stage of development from command line only as follows:

```
$ QUEFT.exe arguments
```

It must be provided at least one argument that consists in the input file name, otherwise an error message is shown. An optional argument is `-v` which can be typed after the input file name argument when the user wants to check the results. In this case some additional output files are generated.

`check0ft.txt` – the list of basic events and the Fault Tree logic structure after the majority `maj` gates have been transformed in combinations of `or` and `and` gates; Fault Tree logic structure, each gate containing only 2 successors - automatic id for each new gate is assigned,

`check123nodup.txt` - this file contains a message that says that no duplicated subtrees/events have been found, otherwise, the next three files are generated instead:

`check1dup.txt` – the list of duplicated events and how many times appear into FT,

`check2nmcs.txt` – the list of duplicated events and the number of MCS for each of them,

`check3mcsdup.txt` – the list of MCS for duplicated events,

`check4mcs.txt` - the list of MCS for the FT that have as top gate the event defined into the input file with basic events id,

`check5gfst.txt` - the same list as before, but with basic events order numbers which represents the defined generic fault states for the next process of Markov chain construction.

For instance, for the considered example assuming that the input file name is `2out3`, the command line is as follows:

```
$ QUEFT.exe 2out3
```

or with additional files option for verification

```
$ QUEFT.exe 2out3 -v
```

In the 1st case will be generated only the output file, namely `2out3.mcs`. In the second case the additional files `check0ft.txt`, `check1dup.txt`, `check2nmcs.txt`, `check3mcsdup.txt`, `check4mcs.txt`, `check5gfst.txt` are generated. These files are as follows.

The file `check0ft.txt`

```
---------------------------------------------
basic events =  5
---------------------------------------------
A
B
C
D
E
---------------------------------------------
top gate considered: TOP
---------------------------------------------
no. of gates in FT after MAJ transf.= 7
---------------------------------------------
TOP   and   G01   G02
```

```
G01   or    D     GM
G02   or    GM    E
GM    or     1004  1005  1006
 1004 and   A     B
 1005 and   A     C
 1006 and   B     C
----------------------------------------------
no. of gates in FT with only 2 pred.= 8
----------------------------------------------
TOP   and   G01   G02
G01   or    D     GM
G02   or    GM    E
GM    or     1004 10005
10005 or     1005  1006
 1004 and   A     B
 1005 and   A     C
 1006 and   B     C
--end-of-FT-transf---------------------------
```

The file `check1dup.txt`

```
no. of duplicated events: 1
GM       2
```

The file `check2nmcs.txt`

```
top  no.MCS
GM    3
```

The file `check3mcsdup.txt`

```
2 A     B
2 A     C
2 B     C
```

The file `check4mcs.txt`

```
-Minimal-Cut-Sets-  4 -for-TOP-function- TOP   -
2 D     E
2 A     B
2 A     C
2 B     C
--MCS-end--
```

The file `check5gfst.txt`

```
4
2  4  5
2  1  2
2  1  3
2  2  3
```

Further, as it has been mentioned before, only the output file `2out3.mcs` is necessary for the next module MARKOMAG-S.

D.2 MARKOMAG-S module

The MARKOMAG code represents the core of the code-system. It generates the Markov chain for the considered problem using the data from the QUEFT code, builds the Kolmogorov system of ordinary differential equations for the Markov chain (3.68), solves this ODE system using the integrated VODPK ODE solver, builds and solves using the same solver the forward and adjoint sensitivity system of equations of the original Kolmogorov equations (3.69 and 3.70). Afterwards, depending on the chosen options, the computation of the defined system response is performed using the solution of original ODE system, or the computation of response sensitivity using the solution of forward sensitivity system. If the adjoint method is chosen, the additional data files comprising the adjoint functions and derivatives are generated to be used with the next module MCADJSEN.

Together with the output file of QUEFT module, the MARKOMAG-S module is requesting also a configuration file, namely `config.mc` where the additional parameters must be given in order to perform the reliability/sensitivity analysis. The file `config.mc` looks currently as follows, where the inputs are briefly explained

```
**Configuration file for MARKOMAG - MC generator/solver
*MCgenerator options
0         <- markov chain truncation level (integer*4) 0 - no truncation
mcmout    <- option to save the MCMatrix 'mcmout'
mcmin     <- option to use the MCMatrix generated previous 'mcmin'
*MCsolver options
0.0d0     <- initial-time for solver (real*8)
500.0d0   <- final-time for solver (real*8)
1.0d-0    <- time-step for solver (real*8)
1.0D-15   <- tolerance for vector solution (real*8)
5.0d+04   <- maximum no. of internal steps in solver (real*8>500)
norm      <- normalization solution vector - 'norm'
*Sensitivity analysis options
1         <- responses type (integer*4) 1, 2, ...
0.0       <- variation in initial conditions (%) (real*8)
1  2      <- the indexes of the states that are varied (2 x integer*4)
fwdss     <- solve the FSE - 'fwdss' (character*5) /or recalculate 'recal'
nc5288    <- integration method for response - closed Newton-Cotes formulae
```

This file contains three main parts, namely a part where are provided the options for the Markov chain construction algorithm, a 2nd part where are the options for the ODE solver, and a last part with options for sensitivity analysis using either the recalculation method or the forward sensitivity method. The 1st two lines, the 6th one, and the 13th line, starting with an asterisk are comment lines and are not read by the program.

In Markov chain options, the truncation level represents the level of markov chain as it has been explained in Section 2.2 during the automated generation algorithm. The next two options are optional and they cannot exist simultaneously. The `mcmout` option is used to save the Markov chain matrix into a binary file `03mcm.bin` for further calculations, in situations in which the program fails after Markov chain generation, or in situations of changing parameters, as long as the saved matrix does contain the symbolic form of transition rates , i.e. the transition rate matrix does not contain any value but only indexes of rows, columns and associated indexes of transition rates, without the main diagonal which is computed afterwards when the numerical values are introduced. Also a binary file `03failst.bin` containing the indexes of the fail states is generated. The `mcmin` option can be used only after the files `03mcm.bin` containing the transition rate matrix and `03failst.bin` containing the fail states indexes, have been generated before for the considered problem. These options have been considered for large Markov chains where the generation time of the transition matrix can be large.

In solver options the initial time is the time from which the analysis is started and for which does exist an initial state probability vector. The final time represents the end of mission time for considered problem. If `norm` option is active, then the normalization of vector solution at each time step is performed. Usually this option should not be used, but it has been introduced for special cases when the numerical difficulties for solving the Kolmogorov equations occur and the conservation law of probability is not satisfied.

The sensitivity analysis options part contains five fields and it is used for sensitivity studies using the recalculation and forward sensitivity method depending on which option have been chosen into the field four of this section, namely `recal` for recalculation or `fwdss` for forward sensitivity method. The 1st field of the sensitivity section contains the type response for which the sensitivity to variation in input parameters is analyzed. It has been implemented two types of responses for which the sensitivity studies has been performed along this work, namely the interval availability R_1, and the availability at the end of the considered mission time R_2 which for t_f large ($t_f \to \infty$) is the steady-state availability, as follows,

Response type	
1	$R_1 = \frac{1}{t_f}\int_{t_0}^{t_f} \pi_1(t)dt$
2	$R_2 = \int_{t_0}^{t_f} \pi_1(t)\delta(t - t_f)dt = \pi_1(t_f)$

where t_0 is the initial-time and t_f is the final-time for problem, $\pi_1(t)$ is the probability that the system is in the state with all components operational, and δ represents the Dirac-delta function.

The 2nd field contains the percentage of the variations in initial state probability vector. Usually but not necessary the initial state probability vector is considered of the form $[1,0,...,0]$. The next field contains the indexes of two states in which are perturbed the initial conditions, namely from the first state a percentage from its value is substracted and added to the state with the index 2, in order to keep up the probability conservation law.

The last field in `config.mc` file contains the numerical quadrature method used in response or in response sensitivity evaluation. Has been implemented the closed Newton-cotes formulae as follows,

Closed Newton-Cotes quadrature formula	
`nc12`	$\int_{t_0}^{t_1} f(t)dt = \frac{h}{2}\left(f_0 + f_1\right) + O(h^2)$
`nc13`	$\int_{t_0}^{t_2} f(t)dt = \frac{h}{3}\left(f_0 + 4f_1 + f_2\right) + O(h^4)$
`nc38`	$\int_{t_0}^{t_3} f(t)dt = \frac{3h}{8}\left(f_0 + 3f_1 + 3f_2 + f_3\right) + O(h^4)$
`nc245`	$\int_{t_0}^{t_4} f(t)dt = \frac{2h}{45}\left(7f_0 + 32f_1 + 12f_2 + 32f_3 + 7f_4\right) + O(h^6)$
`nc5288`	$\int_{t_0}^{t_5} f(t)dt = \frac{5h}{288}\left(19f_0 + 75f_1 + 50f_2 + 50f_3 + 75f_4 + 19f_5\right) + O(h^6)$

where h is the time-step, and $O(h^x)$ is the error's order of the quadrature formula. It has been used the notation f_i for $f(t_i)$.

If either `recal` or `fwdss` options are selected, an additional input file must be provided. This file should contain the percentage variations in input parameters in order to avoid the alteration of the base-case values. The structure of this file is similar as for the input file (i.e. the output file given by QUEFT). The name of this file must be `rec.{inputfilename}` if `recal` option is choose, and `var.{inputfilename}` for `fwdss` option, where `{inputfilename}` is the name of file given by QUEFT. For instance, for the considered example these files will be as follows, depending what option has been chosen.

The file `rec.2out3.mcs`:

```
5
A        mtime  0.1D+0   0.0
B        mtime  0.0D+0   0.0
C        mtime  0.0D+0   0.0
D        mtime  0.0D+0   0.0
E        mtime  0.0D+0   0.0
```

The file `var.2out3.mcs`:

```
5
A         mtime -0.1D+0   0.0
B         mtime  0.0D+0   0.0
C         mtime  0.0D+0   0.0
D         mtime  0.0D+0   0.0
E         mtime  0.0D+0   0.0
```

In the above example has been performed a variation in *MTTF* for the component A with 0.1% of its nominal value for recalculations, and in $\lambda = 1/MTTF$ with -0.1% of its nominal value for forward sensitivity case. Note that for this case the first file has variations in mean time to failure/repair, and in the second file the variations are in transfer rates, for the same input file. This is the explanation for which it is the opposite sign for the same variation of the parameter. If none of the options above is selected, then these files are not required by the program.

The modularized structure of this code-system is advantageous if for the same fault-tree the base-case values of system parameters are changed. In this case the MCS have been computed once and the new values should be written only in the output file generated by QUEFT, without running the QUEFT code again.

D.2.1 MARKOMAG-S options

The call of MARKOMAG-S code from command line is as follows,

`$ MARKOMAG.exe arguments`

where the arguments are the input file (i.e. the output file from QUEFT), and some other optional arguments. It must be provided at least one argument, namely the input file. The optional arguments in the command line are

`-v` for additional checking files at different steps in analysis,

`-r` for the case in which one analyses systems with repairs,

`-s` for the case in which the adjoint analysis is chosen.

If either none of the optional arguments or `-r` argument has been chosen, at the end of the run of this module, the next ASCII files are generated.

`00trsolhom.dat`	- file containing the state probabilities distribution for homogeneous case
`00trsolnoh.dat`	- file containing the state probabilities distribution for nonhomogeneous case

01unavaila.dat	- file containing the sum of down-states probabilities
01unavaila.dat	- file containing the sum of up-states probabilities

If the optional argument `-v` has been given, then the additional files are generated

02checkmcm.dat	- check file that contains the transition rates matrix in coordinate format, once for homogeneous case and at each time step for the nonhomogeneous case
vizmcm.mtx	- file which can be used to visualize the sparse transition rate matrix of the original equations of Markov chain, using OCTAVE/GNUPlot or other packages as MATLAB, MatView, mtxView

If in configuration file `config.mc` the option `fwdss` has been chosen, will be generated also the next files

02checkvar.dat	- check file that contains the variations in the system's parameters for forward sensitivity system
00fwdsp.dat	- file containing the transient solution for the forward sensitivity system when are variations in system's parameters
00fwdic.dat	- file containing the transient solution for the forward sensitivity system when are variations in initial probability state vector

If sensitivity analysis using ASAP have been chosen, i.e. the optional argument `-a`, then the next ASCII files are generated:

00trshomadj.dat	- file containing the transient solution for the adjoint sensitivity system in homogeneous case
00trsnohadj.dat	- file containing the transient solution for the adjoint sensitivity system in nonhomogeneous case
02checkdfda.dat	- check file that contains the numerical values of derivatives dF/d(alpha) matrix in coordinate format used in sensitivity response evaluation using ASAP, see the Eqs.(3.74), (3.75)

For this option, the next binary files are generated as well

00adjfun.bin	- file containing the transient solution of the adjoint sensitivity system
00derivdfda.bin	- file containing the derivative dF/d(alpha) matrix, Eq.(3.75)

The system response for the base case or for recalculation method is written in the file `10resp.dat`, and the response variation for the sensitivity analysis using FSAP is written in `10varrespfwd.dat`.

The response sensitivity using recalculations is evaluated running once the code with the nominal values, without `recal` option in configuration file. Afterwards, the system response evaluation for base-case is obtained. Choosing `recal` option, writing in `rec.{inputfilename}` the percent of variation for the interested parameter and running the code again, the evaluation of perturbed response is obtained. The difference between perturbed response and the response in base-case give the response sensitivity to variation in that parameter. In order to get the response sensitivity at perturbation in parameters using the FSAP, one must choose the `fwdss` option in configuration file, and to write the perturbation of interested parameter in `var.{inputfilename}`. This value should be equal with the response sensitivity evaluated using recalculation method.

For evaluation of the response sensitivities using ASAP the last two binary files are used in the next module MCADJSEN, namely the files containing the transient solution of the adjoint sensitivity system and the files containing the derivatives with respect to system's parameters.

D.3 MCADJSEN module

This last module represents the implementation of the response sensitivity formula using adjoint functions given by Eq.(3.74). The adjoint functions and the derivatives given by Eq.(3.75) have been evaluated using ASAP by MARKOMAG code and are available into the binary files `00adjfun.bin`, and `00derivdfda.bin`, respectively. These files can be used as library files for the considered problem for a given response. They must be computed again once the response is changed. To keep the consistency between the input data the same configuration file `config.mc` from MARKOMAG-S module is used together with above binary files and the file given by QUEFT. An additional file must be provided, namely `var.{inputfilename}` in which are written the variations in all input parameters. For instance for the 2-out-of-3 example the file `var.2out3.mcs` for MCADJSEN module is as below, where it has been considered 0.1% variations in all input parameters

```
5
A        mtime  -1.0D-1   1.0D-1
B        mtime  -1.0D-1   1.0D-1
C        mtime  -1.0D-1   1.0D-1
D        mtime  -1.0D-1   1.0D-1
E        mtime  -1.0D-1   1.0D-1
```

The code is called from command line as follows

```
$ MCADJSEN.exe argument
```

where the argument is the file given by QUEFT and which has been used with MARKOMAG-S as well. After the code is run, the all response sensitivities are evaluated and written in the `10sensit.dat` file. For instance, for the considered example assuming the input parameters given as in the case with reparation from Appendix B, with the variation file as above, the `10sensit.dat` file looks as follows,

```
Rpred-R0(   1) = 0.48432805312977E-05
Rpred-R0(   2) = 0.48432805312977E-05
Rpred-R0(   3) = 0.48432805312977E-05
Rpred-R0(   4) = 0.96265268416298E-06
Rpred-R0(   5) = 0.96265268416298E-06
Rpred-R0(   6) = 0.47942014594720E-05
Rpred-R0(   7) = 0.47942014594720E-05
Rpred-R0(   8) = 0.47942014594720E-05
Rpred-R0(   9) = 0.94297018538473E-06
Rpred-R0(  10) = 0.94297018538473E-06
```

where the first five sensitivities are for variations in first parameter of each component in considered order and the next sensitivities are for perturbations in the second parameter of components.

For other variations in system parameters should be modified only the values in `var.{inputfilename}` file, and the module to be run again.

For the variations in initial state vector, all the values in `var.{inputfilename}` file must be set to zero, and the perturbations entered in configuration file `config.mc` as it has been described in the previous section.

The evaluated sensitivities are used further to compute the relative sensitivities of system's response to variations in system's parameters and based on their absolute value to rank the importance of each parameter in affecting the system's response.

Until this stage the complete analysis for homogeneous Markov chain can be performed using the new developed code. It can be performed a complete analysis for the nonhomogeneous case for the systems without repairable/replaceable components only. The nonhomogeneous case considering repairs is planed for future development as well as a graphic user interface in which the management of input and output files between modules to be automated.