Parity-Affected Persistent Currents in Superconducting Nanorings

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We argue that at sufficiently low temperatures T superconducting parity effect may strongly influence equilibrium persistent currents (PC) in isolated superconducting nanorings containing a weak link with few conducting modes. An odd electron, being added to the ring, occupies the lowest available Andreev state and produces a countercurrent circulating inside the ring. For a single channel quantum point contact at $T = 0$ this countercurrent exactly compensates the supercurrent I_e produced by all other electrons and, hence, yields complete *blocking* of PC for any value of the external magnetic flux. In superconducting nanorings with embedded normal metal the odd electron countercurrent can "overcompensate" I_e and a novel "π/N-junction" state occurs in the system. Changing the electron parity number from even to odd results in *spontaneous* supercurrent in the ground state of such rings without any externally applied magnetic flux.

Thermodynamic properties of isolated superconducting systems are sensitive to the parity of the total number of electrons [1, 2] even though this number $\mathcal{N}$ is macroscopically large. This parity effect is a fundamental property of a superconducting ground state described by the condensate of Cooper pairs. The number of electrons in the condensate is necessarily even, hence, for odd $\mathcal{N}$ at least one electron remains unpaired having an extra energy equal to the superconducting energy gap Δ. This effect makes thermodynamic properties of the ground states with even and odd $\mathcal{N}$ differ. Clear evidence for such parity effect was demonstrated experimentally in small superconducting islands [2, 3].

Can the supercurrent be affected by this parity effect? In many structures the answer to this question is negative because of the fundamental uncertainty relation $\delta\mathcal{N}\delta\varphi \gtrsim 1$. Should the electron number $\mathcal{N}$ be fixed, fluctuations of the superconducting phase φ become large disrupting the supercurrent. On the other hand, in transport experiments with fluctuations of φ being suppressed the parity effect cannot be observed because of large fluctuations of $\mathcal{N}$.

Nonetheless, the parity effect *can* coexist with non-vanishing supercurrent. The way out is to consider systems supporting circular persistent currents (PC), such as, e.g., isolated superconducting rings pierced by the magnetic flux Φ. In accordance with the number-phase uncertainty relation the global superconducting phase of the ring fluctuates strongly in this case, however these fluctuations are decoupled from the supercurrent and therefore cannot influence the latter. The aim of this paper is to demonstrate that the parity effect may substantially modify PC in superconducting nanorings, in particular if the total number of electrons is odd.

Consider, for instance, a single mode quantum point contact (QPC) embedded in a superconducting ring.

Provided the number of electrons in the system is not fixed the Josephson current across this contact is given by the well known expression [4]

$$I(\varphi) = -\frac{2e}{\hbar}\frac{\partial\varepsilon(\varphi)}{\partial\varphi}\tanh\frac{\varepsilon(\varphi)}{2T}, \qquad (1)$$

where φ is the phase difference across QPC, $\varepsilon(\varphi) = \Delta\sqrt{1 - \mathcal{T}\sin^2(\varphi/2)}$ and $\mathcal{T}$ is the contact transmission. This result has a very transparent physical interpretation being related [5] to contributions of discrete Andreev energy states $E_\pm = \pm\epsilon(\varphi)$ inside QPC via

$$I(\varphi) = \frac{2e}{\hbar}\left[\frac{\partial E_-}{\partial\varphi}f_-(E_-) + \frac{\partial E_+}{\partial\varphi}f_+(E_+)\right]. \qquad (2)$$

Using the Fermi filling factors for these states $f_\pm(E_\pm) = [1 + \exp(\pm\epsilon(\varphi)/T)]^{-1}$ one arrives at Eq. (1).

Let us now fix the number of electrons inside the ring and consider the limit $T \to 0$. For even $\mathcal{N}$ all electrons are paired occupying available states with energies below the Fermi level (see Fig. 1a). In this case one has
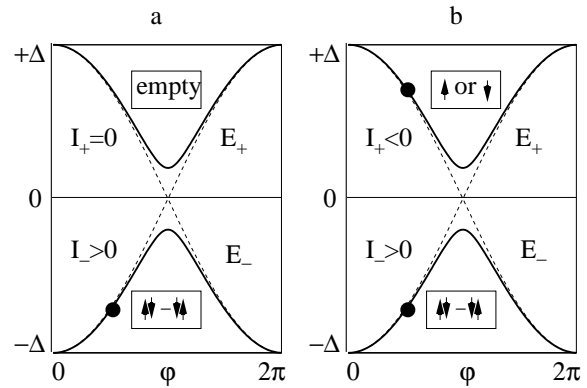

FIG. 1: Andreev levels inside QPC and their occupation at $T = 0$ for even (a) and odd (b) ensembles.

$f_-(E_-) = 1$, $f_+(E_+) = 0$, the current is entirely determined by the contribution of the quasiparticle state E_- and Eq. (2) yields the same result as one for the grand canonical ensemble. By contrast, in the case of odd $\mathcal{N}$ one electron always remains unpaired and occupies the lowest available energy state – in our case E_+ – above the Fermi level. Hence, for odd $\mathcal{N}$ one has $f_\pm(E_\pm) = 1$ (Fig. 1b), the contributions of the two Andreev states in Eq. (2) exactly cancel each other, and the current across QPC remains zero for any φ or the magnetic flux Φ. Thus, we predict *a novel mesoscopic effect* – parity-induced blocking of PC in superconducting nanorings with embedded QPC [6].

It is important to emphasize that the above physical picture is based on the assumption that the supercurrent is determined *only* by the contributions of two discrete Andreev levels $E_\pm(\varphi)$. This is the case provided (a) quasiparticle states of the continuum do not contribute to the supercurrent and (b) there are no more discrete Andreev levels inside the junction. Both these conditions are met only for symmetric and extremely short QPC. In a general case of SNS junctions with a non-vanishing thickness of the normal layer $d \neq 0$ these conditions are violated, and no exact compensation of the supercurrent by the odd electron countercurrent is anymore possible.

Further analysis reveals even richer physics in the latter case. In particular, below we will demonstrate that at low T and for odd number of electrons a novel π/N-junction state should occur in such SNS rings. This effect in turn leads to *spontaneous* supercurrent flowing in the ring in its ground state. Under certain conditions the magnitude of this current can be as high as $I_{sp} \sim e\Delta/\hbar$ reaching the values up to $10 \div 100$ nA for generic BCS superconductors. Spontaneous currents of this magnitude can reliably be detected in modern experiments.

Parity projection formalism. In order to systematically investigate an interplay between the parity effect and PC in superconducting nanorings we will employ the parity projection formalism [7, 8, 9].

The grand canonical partition function $\mathcal{Z}(T, \mu) = \mathrm{Tr}\, e^{-\beta(\mathcal{H} - \mu\mathcal{N})}$ is linked to the canonical one $Z(T, \mathcal{N})$ as

$$\mathcal{Z}(T, \mu) = \sum_{\mathcal{N}=0}^{\infty} Z(T, \mathcal{N}) \exp\left(\frac{\mu\mathcal{N}}{T}\right). \quad (3)$$

Here and below $\mathcal{H}$ is the system Hamiltonian and $\beta \equiv 1/T$. Inverting this relation and defining the canonical partition functions Z_e and Z_o respectively for even ($\mathcal{N} \equiv \mathcal{N}_e$) and odd ($\mathcal{N} \equiv \mathcal{N}_o$) ensembles, one gets

$$Z_{e/o}(T) = \frac{1}{2\pi} \int_{-\pi}^{\pi} du\, e^{-i\mathcal{N}_{e/o}u} \mathcal{Z}_{e/o}(T, iTu), \quad (4)$$

where

$$\mathcal{Z}_{e/o}(T, \mu) = \frac{1}{2}\mathrm{Tr}\left\{ [1 \pm (-1)^{\mathcal{N}}] e^{-\beta(\mathcal{H} - \mu\mathcal{N})} \right\}$$

$$= \frac{1}{2}\big(\mathcal{Z}(T, \mu) \pm \mathcal{Z}(T, \mu + i\pi T)\big) \quad (5)$$

are the parity projected grand canonical partition functions. For $\mathcal{N} \gg 1$ it is sufficient to evaluate the integral in (4) within the saddle point approximation

$$Z_{e/o}(T) \sim e^{-\beta(\Omega_{e/o} - \mu_{e/o}\mathcal{N}_{e/o})}, \quad (6)$$

where $\Omega_{e/o} = -T\ln \mathcal{Z}_{e/o}(T, \mu)$ are the parity projected thermodynamic potentials,

$$\Omega_{e/o} = \Omega_f - T\ln\left[\frac{1}{2}\Big(1 \pm e^{-\beta(\Omega_b - \Omega_f)}\Big)\right] \quad (7)$$

and $\Omega_{f/b} = -T\ln\left[\mathrm{Tr}\left\{(\pm 1)^{\mathcal{N}} e^{-\beta(\mathcal{H} - \mu\mathcal{N})}\right\}\right]$. "Chemical potentials" $\mu_{e/o}$ are defined by the saddle point condition $\mathcal{N}_{e/o} = -\partial\Omega_{e/o}(T, \mu_{e/o})/\partial\mu_{e/o}$.

The main advantage of the above formalism is that it allows to express the canonical partition functions and thermodynamical potentials in terms of the parity projected grand canonical ones thereby enormously simplifying the whole calculation. We further note that Ω_f is just the standard grand canonical thermodynamic potential and Ω_b represents the corresponding potential linked to the partition function $\mathcal{Z}(T, \mu + i\pi T)$. It is easy to see [8] that in order to recover this function one can evaluate the true grand canonical partition function $\mathcal{Z}(T, \mu)$, express the result as a sum over the Fermi Matsubara frequencies $\omega_f = 2\pi T(l + 1/2)$ and then substitute the Bose Matsubara frequencies $\omega_b = 2\pi Tl$ ($l = 0, \pm 1, ...$) instead of ω_f. This procedure automatically yields the correct expression for $\mathcal{Z}(T, \mu + i\pi T)$ and, hence, for Ω_b.

Having found the thermodynamic potentials for the even and odd ensembles one can easily determine the equilibrium current $I_{e/o}$. Consider, as before, isolated superconducting rings pierced by the magnetic flux Φ. Making use of the above expressions one finds PC circulating inside the ring:

$$I_{e/o} = I_f \pm \frac{I_b - I_f}{e^{\beta(\Omega_b - \Omega_f)} \pm 1}, \quad (8)$$

where the upper/lower sign corresponds to the even/odd ensemble and we have defined

$$I_{e/o} = -c\left(\frac{\partial\Omega_{e/o}}{\partial\Phi}\right)_{\mu(\Phi)}, \quad I_{f/b} = -c\left(\frac{\partial\Omega_{f/b}}{\partial\Phi}\right)_{\mu(\Phi)}.$$

Homogeneous superconducting rings. Turning to concrete calculations we first treat homogeneous nanorings with cross section s and perimeter $L = 2\pi R$. Throughout this paper rings will be assumed sufficiently thin, $\sqrt{s} < \lambda_L$, where λ_L is the London penetration length. Superconducting properties of such rings will be described within the (parity projected) mean field BCS theory. At low temperatures this description is justified provided quantum fluctuations of the order parameter, quantum phase slips (QPS) [10], can be neglected. This requirement can

be fulfilled only provided the total number of conducting channels in the ring remains large [11] $N_{\rm r} \sim p_F^2 s \gg 1$. In addition, the perimeter L should not be too large in order to disregard the QPS-induced reduction of PC [13]. Finally, we will neglect the difference between the mean field values of the BCS order parameter for the even and odd ensembles [7, 8]. This is legitimate provided the ring volume is large enough, $\mathcal{V} = Ls \gg 1/\nu\Delta$, where ν is the density of states at the Fermi level and Δ is the BCS order parameter for a bulk superconductor at $T = 0$.

The task at hand is now to evaluate the thermodynamic potentials $\Omega_{f/b}$. Within the mean field approach these quantities can be expressed in terms of the excitation energies ε_k and the order parameter $\Delta(\boldsymbol{r})$. One finds [8]

$$\Omega_f = \tilde{\Omega} \; - \; 2T \sum_k \ln\left(2\cosh\frac{\varepsilon_k}{2T}\right), \qquad (9)$$

$$\Omega_b = \tilde{\Omega} \; - \; 2T \sum_k \ln\left(2\sinh\frac{\varepsilon_k}{2T}\right), \qquad (10)$$

where $\tilde{\Omega} = \int d^3\boldsymbol{r}|\Delta(\boldsymbol{r})|^2/g + \mathrm{Tr}\{\hat{\xi}\}$, g is the BCS coupling constant and $\hat{\xi}$ is the single-particle energy operator:

$$\hat{\xi} = \frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial\boldsymbol{r}} - \frac{e}{c}\boldsymbol{A}(\boldsymbol{r})\right)^2 + U(\boldsymbol{r}) - \mu, \qquad (11)$$

$\boldsymbol{A}(\boldsymbol{r})$ is the vector potential and $U(\boldsymbol{r})$ describes the potential profile due to disorder and interfaces.

The excitation spectrum ε_k has the form

$$\varepsilon_k = \varepsilon(\boldsymbol{p}) = \boldsymbol{p}\boldsymbol{v}_S + \sqrt{\xi^2 + \Delta^2}, \qquad (12)$$

where $\boldsymbol{p}$ is a quasiparticle momentum, $\xi = (p^2 - \tilde{\mu})/2m$, $\tilde{\mu} = \mu(\Phi) - mv_S{}^2/2$ and

$$v_S = \frac{\hbar}{2mR}\min_n\left(n - \frac{\Phi}{\Phi_0}\right) \qquad (13)$$

is the superconducting velocity. Both $\varepsilon(\boldsymbol{p})$ and v_S are periodic functions of the flux Φ with the period equal to the superconducting flux quantum $\Phi_0 = hc/2e$.

Consider the most interesting case $T \ll \hbar v_F/L$ [14]. Making use of the above expressions one obtains

$$I_e = ev_S\varrho_e s, \quad I_o = ev_S\varrho_o s - e\frac{v_F}{L}\mathrm{sgn}v_S, \qquad (14)$$

where $\varrho_{e/o} = \mathcal{N}_{e/o}/\mathcal{V}$. The first Eq. (14) – together with (13) – coincides with that for the grand canonical ensemble. In contrast, for odd ensembles (second Eq. (14)) there exists an additional flux-dependent term which, however, remains rather small in multichannel rings [15]. Estimating the main contribution to $I_{e/o}$ as $I \sim ev_F N_{\rm r}/L$, one gets $(I_e - I_o)/I \sim 1/N_{\rm r} \ll 1$.

Now we proceed to the situations in which parity effect gains more importance and leads to new physical effects. Namely, we consider superconducting rings with
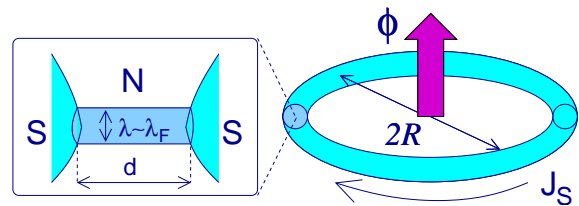

FIG. 2: Superconducting ring with embedded SNS junction of length d.

$N_{\rm r} \gg 1$ interrupted by a weak link with only few conducting channels N. In such systems the mean field BCS description remains applicable and, on the other hand, the parity effect can be large due to the condition $N \sim 1$. Quantum point contacts (QPC) or, more generally, SNS junctions (of length d) can be used for practical realization of such weak links. The corresponding structure is depicted in Fig. 2.

Superconducting rings with QPC. As a first example we consider a ring interrupted by QPC. The optimal value of the phase difference φ across QPC is fixed by the equation $\partial\Omega_{e/o}/\partial\varphi = 0$, which yields the current conservation condition $I^{(c)} = I^{(r)}$. Here $I^{(c)} \sim 2e\mathcal{T}\Delta/\hbar$ is the current through the contact and $I^{(r)} \simeq (ev_F N_{\rm r}/L)(\varphi - 2\pi\Phi/\Phi_0)$ is the supercurrent inside the ring. From the current conservation one easily finds [16]

$$\varphi \simeq 2\pi\Phi/\Phi_0, \quad \text{if} \quad L \ll L^*, \qquad (15)$$

$$\varphi \simeq 2\pi n, \qquad \text{if} \quad L \gg L^*, \qquad (16)$$

where $L^* = \xi_0 N_{\rm r}/\mathcal{T} \gg \xi_0 \sim \hbar v_F/\Delta$. In a more general case of QPC with N conducting channels in the expression for L^* one should set $\mathcal{T} \to \sum_i^N \mathcal{T}_i$.

In the limit (16) one recovers the results identical to ones for homogeneous rings. Below we will concentrate on the opposite limit $L \ll L^*$ and also assume $N \ll N_{\rm r}$. Due to Eq. (15) in this case the dependence $I_{e/o}(\Phi)$ is fully determined by the current-phase relation for QPC which can be found by means of Eq. (8). Expressing $I_{f/b}$ via φ it is convenient to employ the general formula [17]

$$I_{f/b} = \frac{2e}{\hbar}\sum_{i=1}^N T\sum_{\omega_{f/b}}\frac{\sin\varphi}{\cos\varphi + W_i(\omega_{f/b})}. \qquad (17)$$

For QPC one has $W_i(\omega) = (2/\mathcal{T}_i)(1 + \hbar^2\omega^2/\Delta^2) - 1$. Substituting this function into (17) and summing over ω_f one rederives the standard result [4], while the same summation over ω_b yields

$$I_b = -\frac{2e}{\hbar}\sum_{i=1}^N \frac{\partial\varepsilon_i(\varphi)}{\partial\varphi}\coth\frac{\varepsilon_i(\varphi)}{2T}, \qquad (18)$$

where $\varepsilon_i(\varphi) = \Delta\sqrt{1 - \mathcal{T}_i\sin^2(\varphi/2)}$. Finally, the difference $\Omega_b - \Omega_f \equiv \Omega_{bf}$ is evaluated as a sum of the ring

$(\Omega_{bf}^{(r)})$ and QPC $(\Omega_{bf}^{(c)})$ contributions. The latter is found by integrating $I_{f/b}(\varphi)$ over the phase φ with the result

$$\Omega_{bf}^{(c)} = 2T \sum_{i=1}^{N} \ln \coth \left(\frac{\varepsilon_i(\varphi)}{2T} \right), \qquad (19)$$

while the former is defined by the standard expression

$$\beta \Omega_{bf}^{(r)} = 2\mathcal{V} \int \frac{d^3 \boldsymbol{p}}{(2\pi\hbar)^3} \ln \left(\coth \frac{\varepsilon(\boldsymbol{p})}{2T} \right) \simeq \nu \mathcal{V} \sqrt{\Delta T} e^{-\frac{\Delta}{T}}.$$

Combining all these results with Eq. (8) we get

$$I_{e/o} = -\frac{2e}{\hbar} \sum_{i=1}^{N} \frac{\partial \varepsilon_i(\varphi)}{\partial \varphi} \tanh \frac{\varepsilon_i(\varphi)}{2T}$$

$$\times \left[1 \pm \frac{(\coth \frac{\varepsilon_i(\varphi)}{2T})^2 - 1}{e^{\beta \Omega_{bf}^{(r)}} \prod_{j=1}^{N} (\coth \frac{\varepsilon_j(\varphi)}{2T})^2 \pm 1} \right]. \qquad (20)$$

The term in the square brackets accounts for the parity effect in our system. For $N = 1$ and at $T = 0$ this term reduces to unity for even ensembles and to zero for odd ones, thus confirming our intuitive picture of parity-induced blocking of PC discussed above. For $T > 0$ Eq. (20) demonstrates that both for even and especially for odd $\mathcal{N}$ the current-phase relation for QPC may substantially deviate from that derived for the grand canonical ensemble [4], see Fig. 3. For even ensembles the super-current *increases* above its grand canonical value. This effect is mainly pronounced for phases φ not very far
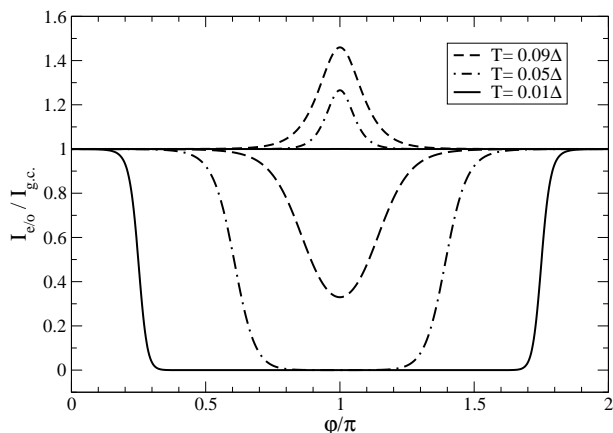

FIG. 3: The ratio between canonical and grand canonical values of PC $I_{e/o}/I_{g.c.}$ (represented by the term in the square brackets in Eq. (20)) versus φ in a single mode QPC at different temperatures for even (three upper curves) and odd (three lower curves) ensembles. Here we have chosen $T_S^* = 0.1\Delta$ and $\mathcal{T} = 0.99$.

from $\varphi = \pi$ and – at sufficiently low T – it becomes progressively more important with increasing temperature. On the contrary, for odd ensembles the supercurrent is always suppressed below its grand canonical value. This suppression is gradually lifted with increasing temperature, though at phases φ in the vicinity of the point $\varphi = \pi$ blocking of PC may persist up to sufficiently high T. Eq. (20) also shows that in QPC with several conducting channels and at $T \to 0$ the current through the most transparent channel will be blocked by the odd electron. Hence, though blocking of PC remains incomplete in this case, it may nevertheless be important also for QPC with $N > 1$.

SNS rings. Finally, we turn to superconducting rings containing a piece of a normal metal with non-zero thickness d. In contrast to the situation of QPC considered above, the Josephson current in SNS structures cannot anymore be attributed only to the discrete Andreev states inside a weak link, and an additional contribution from the states in the continuum should also be taken into account. Furthermore, for any non-zero d there are always more than two discrete Andreev levels in the system. Accordingly, significant modifications in the physical picture of the parity effect in SNS rings can be expected.

The key difference can be understood already by comparing the typical structure of discrete Andreev levels in SNS junctions (Fig. 4) with that of QPC (Fig. 1). As before, in the limit $T \to 0$ all states below (above) the Fermi level are occupied (empty) provided the total number of electrons in the system is even (Fig. 4a). If, on the other hand, this number is odd the lowest Andreev state above the Fermi energy is occupied as well (Fig. 4b) thus providing an additional contribution to the Josephson current. This contribution, however, cancels only that of a symmetric Andreev level below the Fermi energy, while the contributions of all other occupied Andreev levels and of the continuum states remain uncompensated. Hence, unlike in the QPC limit displayed in Fig. 1, in SNS rings
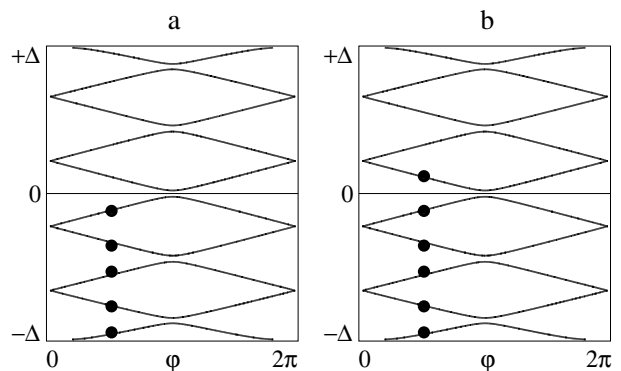

FIG. 4: Andreev levels in a single mode SNS junction with $d = 6\hbar v_F/\Delta$ and their occupation at $T = 0$ for even (a) and odd (b) ensembles.

one should not anymore expect the effect of PC blocking by the odd electron, but rather some other non-trivial features of the parity effect.

In order to construct a quantitative description of the parity effect in SNS rings we will again make use of the parity projection technique formulated above. Here we only restrict our attention to transparent SNS junctions where the effects under consideration are most pronounced. In this case the function $W_i(\omega) \equiv W(\omega)$ is the same for all channels. It reads

$$W(\omega) = \left(\frac{2\hbar^2\omega^2}{\Delta^2} + 1 \right) \cosh\left(\frac{2\omega d}{v_F} \right)$$

$$+ \frac{2\hbar\omega}{\Delta} \sqrt{1 + \frac{\hbar^2\omega^2}{\Delta^2}} \sinh\left(\frac{2\omega d}{v_F} \right). \qquad (21)$$

Substituting this function into (17) and repeating the whole calculation as above, we arrive at the final result which cannot in general be represented in a tractable analytic form. Significant simplifications occur in the most interesting limit [18] $T \to 0$, in which case we obtain

$$I_e = \frac{e\Delta N}{\hbar} \left(\sin\frac{\varphi}{2} - \frac{2y\sin\varphi}{\pi} \ln\frac{1}{y} \right), \qquad (22)$$

$$I_o = I_e - \frac{e\Delta}{\hbar} \left(\sin\frac{\varphi}{2} + y\,\mathrm{sgn}\varphi \cos\varphi \right) \qquad (23)$$

for short SNS junctions $y \equiv d\Delta/\hbar v_F \ll 1$ and

$$I_e = \frac{ev_F N}{\pi d}\varphi, \quad I_o = \frac{ev_F N}{\pi d}\left(\varphi - \frac{\pi\,\mathrm{sgn}\varphi}{N} \right) \qquad (24)$$

for long ones $d \gg \xi_0 \sim \hbar v_F/\Delta$. These results apply for $-\pi < \varphi < \pi$ and should be 2π-periodically continued otherwise. The term containing $\ln(1/y)$ in Eq. (23) for I_e is written with the logarithmic accuracy and is valid for φ not too close to $\varphi = \pm\pi$.

Let us concentrate on the results (24) applicable for sufficiently long SNS junctions. We observe that at $T = 0$ the current I_e – as in the QPC case – coincides with that for the grand canonical ensembles [19], while for odd $\mathcal{N}$ the current-phase relation is shifted by the value π/N. This shift has a simple interpretation being related to the odd electron contribution $(2e/\hbar)\partial E_1/\partial\varphi$ from the lowest (above the Fermi level) Andreev state $E_1(\varphi)$ inside the SNS junction. As we have expected, this contribution indeed does not compensate for the current from other quasiparticle states. Rather it provides a possibility for a parity-induced π-junction state [20] in our system: According to Eq. (24) for single mode SNS junctions the "saw tooth" current-phase relation will be shifted exactly by π, see Fig. 5. More generally, we can talk about a novel π/N-junction state, because in the odd case the minimum Josephson energy (zero current) state is reached at $\varphi = \pm\pi/N$, see Fig. 6. For any $N > 1$ this is a twofold degenerate state within the interval $-\pi < \varphi < \pi$ [21]. In the particular case $N = 2$ the current-phase relation $I_o(\varphi)$ turns π-periodic.

Let us recall that the π-junction state can be realized in SNS structures by driving the electron distribution function in the contact area out of equilibrium [22, 23, 24]. Here, in contrast, the situation of a π- or π/N-junction is achieved in thermodynamic equilibrium. Despite this drastic difference, there also exists a certain physical similarity between the effects discussed here and in Refs. 22, 23, 24: In both cases the electron distribution function in the weak link deviates substantially from the Fermi function. It is this deviation which is responsible for the appearance of the π-junction state in both physical situations.

It is also important to emphasize that the countercurrent produced by the odd electron causes a jump on the current-phase dependence at $\varphi = 0$, and the direction of I_o at sufficiently small φ is always opposite to that of I_e. This is a generic feature of odd ensembles which
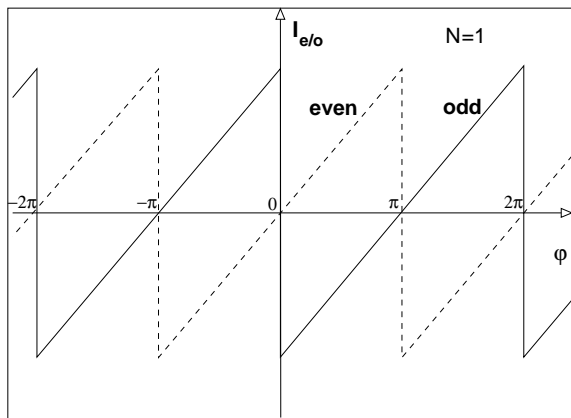

FIG. 5: The zero temperature current-phase dependence (24) for SNS rings with $N = 1$: $I_e(\varphi)$ (dashed) line and $I_o(\varphi)$ (solid) line.
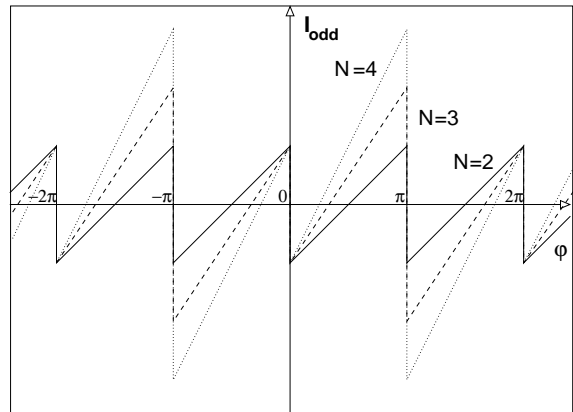

FIG. 6: The same as in Fig. 5 only for the odd ensembles (second Eq. (24)) and for $N = 2, 3$ and 4.
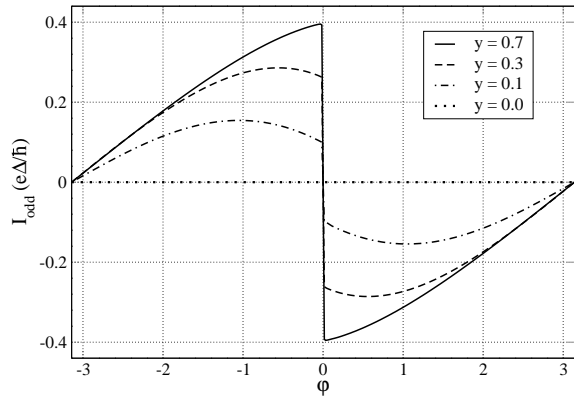
FIG. 7: The zero temperature current-phase relation $I_o(\varphi)$ $(-\pi < \varphi < \pi)$ for $N = 1$ and different values of the parameter $y = d\Delta/\hbar v_F$.
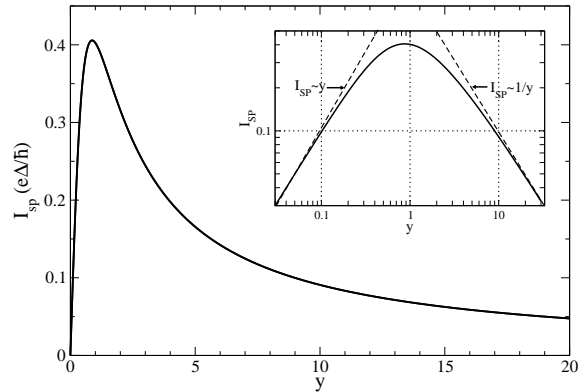

FIG. 8: The spontaneous current amplitude I_{sp} as a function of the parameter y at $T = 0$. In the inset, the same function is shown on the $log - log$ scale. Dashed lines indicate the asymptotic behavior of $I_{sp}(y)$ in the limits of small and large y.

persists for all non-zero values of y. The current-phase relation $I_o(\varphi)$ for arbitrary values of the parameter y can be computed numerically. Examples are presented in Fig. 7. One clearly observes the current jump at $\varphi = 0$. As it is obvious from Eq. (23), this feature disappears only in the QPC limit $y \to 0$.

BCS ground state with spontaneous current. Perhaps the most spectacular physical consequence of this current jump is the presence of spontaneous supercurrents in the ground state of SNS rings with odd number of electrons. Similarly to the case of standard π-junctions [20] such spontaneous supercurrents should flow even in the absence of an externally applied magnetic flux. Unlike in Ref. 20, however, here the spontaneous current state occurs for *any* inductance of the ring because of the non-sinusoidal dependence $I_o(\varphi)$.

Consider, for instance, the limit $d \gg \xi_0$. In the case of odd number of electrons the ground state energy of an SNS ring can be written in a simple form

$$E = \frac{\Phi^2}{2c\mathcal{L}} + \frac{\pi\hbar v_F N}{\Phi_0^2 d}\left(\Phi - \frac{\Phi_0 \mathrm{sgn}\Phi}{2N}\right)^2, \qquad (25)$$

where the first term is the magnetic energy of the ring ($\mathcal{L}$ is the ring inductance and c is the speed of light) while the second term represents the Josephson energy of the SNS junction. Minimizing (25) with respect to the flux Φ one immediately concludes that the ground state of the ring is a twofold degenerate state with a non-vanishing *spontaneous* current

$$I = \pm\frac{ev_F}{d}\left[1 + \frac{2ev_F N}{d}\frac{\mathcal{L}}{\Phi_0}\right]^{-1} \qquad (26)$$

flowing either clockwise or counterclockwise. In the limit of small inductances $\mathcal{L} \to 0$ this current does not vanish and its amplitude just reduces to that of the odd electron current at $\varphi \to 0$. Hence, at small ring inductances the

magnitude I_{sp} of this spontaneous current can easily be obtained from Eqs. (23-24):

$$I_{sp} = e\Delta^2 d/\hbar^2 v_F, \qquad \text{if } d \ll \xi_0, \qquad (27)$$

$$I_{sp} = ev_F/\pi d, \qquad \text{if } d \gg \xi_0. \qquad (28)$$

For intermediate values of the parameter y the amplitude of the current I_{sp} can be evaluated numerically. The results are displayed in Fig. 8. One observes that – in agreement with Eq. (27) – I_{sp} increases linearly with d at small d, reaches its maximum value $I_{\max} \sim 0.4e\Delta/\hbar$ at $d \sim \xi_0$ and then decreases with further increase of d approaching the dependence (28) in the limit of large d. For generic BCS superconductors the magnitude of this maximum current can be estimated as $I_{\max} \sim 10 \div 100$ nA. These values might be considered as surprisingly large ones having in mind that this current is associated with only one Andreev electron state.

In summary, new physical effects emerge from an interplay between the electron parity number and persistent currents in superconducting nanorings. Perhaps the most striking observation is that the BCS ground state of such rings with the odd number of electrons is the state with non-zero *spontaneous* supercurrent. This and other novel features predicted here can be directly tested in modern experiments.

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