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Brake vibrations at very low driving velocities

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Vibrations in vehicle brake systems have been subject to research for a long time. One simple phenomenological model ist the classical friction oscillator, consisting of a mass on a belt. A closer look to a real brake system reveals that from the tyre's contact on the street to the disc the brake torque is transmitted via the tyre, which also possesses elastic, dissipative and inertial properties. Motivated by this fact an extended 2-DOF friction oscillator model is developed featuring the rim's inertia and the dissipative and elastic behaviour of the tyre. In this article the model is given and simulation results are presented. It is found, that additional torsional elasticity und damping can induce very low frequent vibrations. In addition an outlook on further studies is given.

1 Model

The model consists of two identical masses m which are pressed onto a disc (moment of inertia J) by the brake force F_B. The disc is connected to a ring by a torsional damper d_1 and spring c_1. The mass m is connected to the car by a spring c_2 and a damper d_2. Herein the masses m represent the brake pads and partially the moved mass of the saddle (as a "reduced mass"), the disc represents the brake disc and the rim and finally the ring stands for the tyre's tread. The elastic and dissipative elements c_1, d_1 are introduced in order to model the behaviour of the rubber tyre by means of a linearized tyre model [1]. The system is assumed to be symmetric, so the coordinate x describes the position of both pads, further φ is the angle of the disc and ψ ($\dot{\psi} = const$) the angle of the tread.

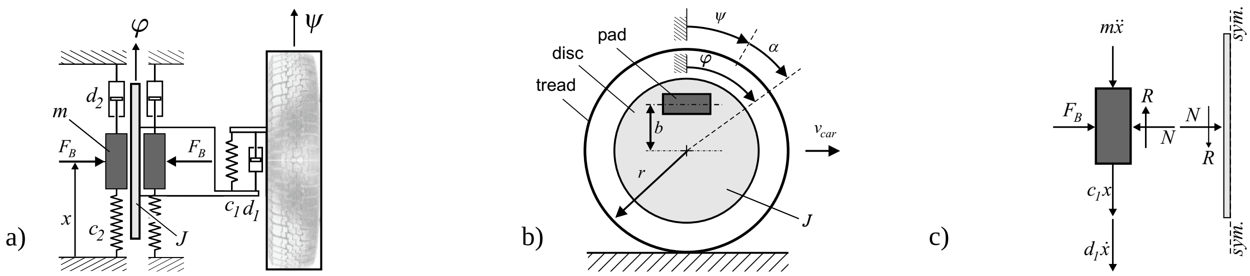

Fig. 1 a) overall modell consisting of two identical masses, a disc and the tyre. – b) Sideview, further kinematics of the modell. – c) Forces on pad mass and disc

Introducing the angle $\alpha = \psi - \varphi$ and considering the symmetry, the model comprising three rigid bodies can be described by a system of two nonlinear coupled differential equations. Assuming the friction force to be only dependent on the relative velocity $v_{rel} = b(\dot{\alpha} + \dot{\psi}) - \dot{x}$ and the normal force N between the pads and the disc, the equations of motion are

$$\ddot{\alpha} + \left(2D_1\omega_1\dot{\alpha} + \frac{2b}{J}R(\dot{\alpha}, \dot{x}; N)\right) + \omega_1^2\alpha = 0, \tag{1}$$
$$\ddot{x} + \left(2D_2\omega_2\dot{x} - \frac{1}{m}R(\dot{\alpha}, \dot{x}; N)\right) + \omega_2^2 x = 0.$$

With μ as the sliding friction coefficient, μ_0 the sticking friction coefficient and λ the constrained force in the sticking case, the friction force is given by

$$R = \begin{cases} \min(\lambda, \mu_0 N) : v_{rel} = 0 \\ \mu N \, \mathrm{sign}(v_{rel}) : v_{rel} \neq 0 \end{cases} \qquad \text{with } \mu = \begin{cases} const & : \text{Coulomb} \\ f(v_{rel}) & : \text{Stribeck} \end{cases}. \tag{2}$$

In the sticking case $v_{rel} = 0$ the number of DOF reduces from two to one and the system's motion is constrained by the kinematic sticking condition and its holonomic constraint equation

$$0 = v_{rel} = b\left(\dot{\alpha} + \dot{\psi}\right) - \dot{x} \quad (\dot{\psi} = \text{const}) \qquad \rightarrow \qquad 0 = b\,(\alpha + \psi) - x + C_0, \tag{3}$$

wherein C_0 represents the initial conditions at the onset of sticking. The constraint force λ is determined as Lagrange multiplier introducing (3) into the equations of motion (1).

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2 Simulation results

With the state vector $\vec{z} = (\alpha, x, \dot{\alpha}, \dot{x})^T$, the system can be written as $\dot{\vec{z}} = \mathbf{A}\vec{z} + \vec{b}R(\dot{\alpha}, \dot{x}; N), \quad \vec{z}, \vec{b} \in \mathbb{R}^{(4\times1)}, \mathbf{A} \in \mathbb{R}^{(4\times4)}$. Since the number of dimensions exceeds three, only subspaces of the four dimensional state space can be plotted. In order to visualize the stick-condition (3) – which describes a hyper plane $\mathcal{H}$ in the state space – the subspace $(x, \dot{x}, \dot{\alpha})$ is chosen. Figure 2 and 3 show results only for Coulomb friction characteristic since the application of a Stribeck characteristic showed no fundamentally new phenomena. The choice of the friction characteristic mainly effects the stability of the steady state, which is not discussed here.

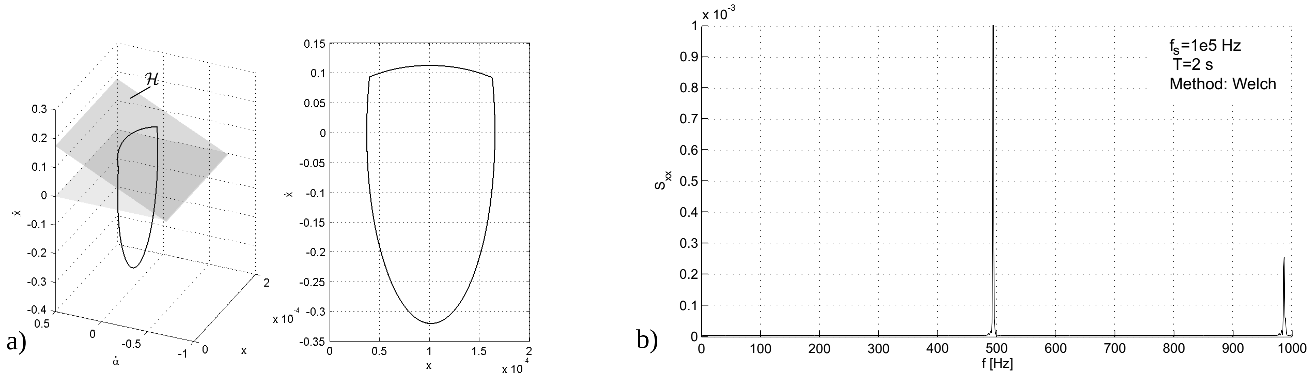

Fig. 2 a) Phase subspace with stick condition plane $\mathcal{H}$ (left) – b) Pad's motion power spectrum for $v_{car} = 0.2m/s$

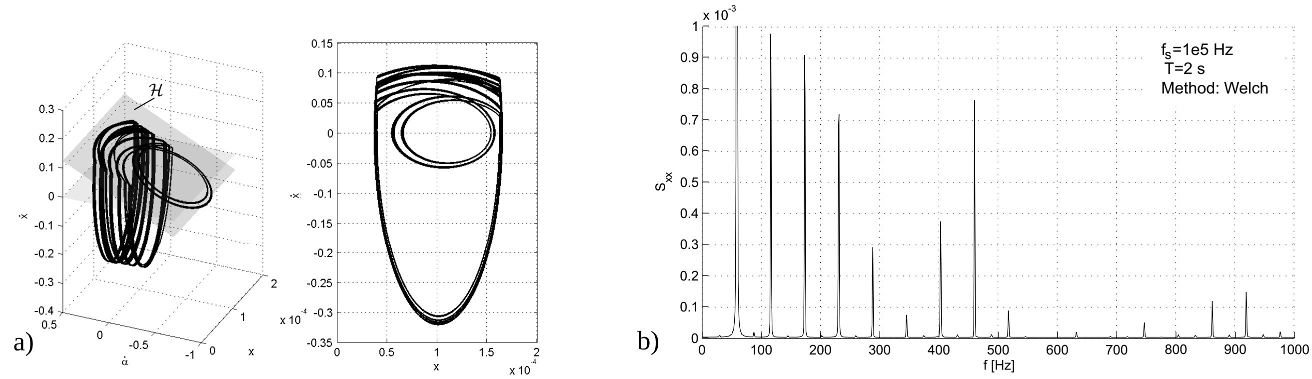

Fig. 3 a) Phase subspace with stick condition plane $\mathcal{H}$ – b) Pad's motion power spectrum for $v_{car} = 0.1m/s$

Figure 2 shows the phase subspace for a car velocity of $0.2m/s$; here the pad's behaviour is quite similar to that modelled by the one dimensional friction oscillator. But the behaviour is changed radically by reducing the car velocity (e.g. $v_{car} = 0.1m/s$, figure 3). For very low velocities, the pad's motion shows strong spectral contents at very low frequencies. Apparently brake vibrations in the low frequency band (chatter, judder) can also be induced by torsional elasticities of the wheel. (*Simulation parameters:* $F_B = 10\,\mathrm{kN}, \omega_1 = 5000\,\mathrm{rad/s}, \omega_2 = 330\,\mathrm{rad/s}, D_1 = 0.1, D_2 = 0.01, m = 1\,\mathrm{kg}, J = 1\,\mathrm{kgm}^2$ – similar to [2],[3].)

3 Further work

Beyond the presented results the stability of the steady state and the system's sensitivity to the parameters was inspected. Future work will comprise extensions of the model, more thorough modelling of friction and experimental studies.

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