VIBRATION SUPPRESSION FOR TELESCOPIC SYSTEMS OF STRUCTURAL MEMBERS WITH CLEARANCE

Pierre Barthels and Jörg Wauer
Institut für Technische Mechanik
Martin Mittwollen and Dieter Arnold
Institut für Fördertechnik und Logistiksysteme
Universität Karlsruhe
D-76128 Karlsruhe Germany

Abstract

The dynamic behavior of multi-section constructions with clearance during extending and retracting motion of the sections is analyzed. For an appropriate physical model the governing boundary value problem is derived by applying Hamilton’s principle and a classical discretization procedure is used to generate a coupled system of nonlinear ordinary differential equations as the corresponding truncated mathematical model. On the basis of this model, a controller concept for preventing harmful vibrations is developed. The direct access to the system equations allows the application of established control strategies. A concept of state control via pole placement is designed which exhibits the desired effects.

1. INTRODUCTION

Graduated multi-section systems of structural components extending and retracting inside each other are interesting technical systems, e.g., mobile cranes, rack feeder, etc. (see [1], for example). Due to overall rigid body translation or slew maneuvers combined with the extending and retracting motion of the sections, bending vibrations of the system perpendicular to the telescopic axis occur. In technical applications these vibrations lead to a reduction of the efficiency and to safety problems so that a controlled vibration suppression seems to be useful.

A first step to develop efficient and safe multi-section constructions is an appropriate modeling of such systems and the examination of the vibrational behavior, which is done in [2] in all detail and briefly recapitulated in this contribution.

The objective of the present paper is to develop a controller concept for preventing harmful vibrations. First, a system without clearance and with a fixed telescopic length which can be characterized by a time-invariant system of linear differential equations, is reduced to its dominating modes. Using this reduced model, a concept of state control via pole placement is designed which exhibits the desired effects. Introducing a so-called Luenberger observer, straightforward measurements of the motion of the telescope base and of the control variable of the actuator are sufficient to operate the controller. For real telescopic operations an adaptive controller and observer are introduced. The controller, developed for the reduced linear system model, is applied to the significantly more complicated system with clearance for studying the influence of clearance on the vibration suppression during telescopic motions.

2. PHYSICAL MODEL

From the viewpoint of mechanics, a non-linear field problem of vibrating structural members with variable geometry has to be considered. Material surface areas of particular components move along surface areas of other components and define complicated boundary and transition conditions. The clearance produces non-linear effects. In many applications the different segments are slender and can be modeled as Bernoulli/Euler beams mounted on a rigid vehicle unit and carrying at some location, e.g., at the end of the last section, a load unit assumed to be rigid. The vehicle unit together with the first deformable segment and all the other segments (one of them together with the load) perform transverse motions and the extending or retracting motion of the sections is supplemented. The contact regions between two sections are modeled as discrete point contacts. A special feature of the modeling is to introduce the reaction forces at the contact points in the form of distributed line loads (by using Dirac impulse functions), so that for the contacting sections elementary boundary conditions remain. The contact formulation itself takes place via one-sided spring-damper elements.

The procedure is illustrated in **Fig. 1a** for a two-section telescopic beam system mounted on a rigid traverse performing a translational motion accompanied by an extending motion of the two beam segments with defined clearance between them. Beam 1 is fixed at a rigid vehicle unit; beam 2 carries a point load at its end. The vehicle is driven by a horizontal force F as excitation. The deformation of the beams (including vehicle mass and load) is represented by the absolute displacements $w(x_1,t)$ and $v(x_2,t)$. The model is defined by the following parameters: beam lengths $l_{1,2}$, constant cross-sectional areas $A_{1,2}$, constant cross-sectional moments of inertia $I_{1,2}$, density ρ and Young's modulus E of the two flexible components, masses of load and vehicle m_L and m_T, respectively, and telescopic length $l_A(t)$. The contact between the beams is realized (see **Fig. 1b**) via discrete spring-damper systems in the form of a so-called displacement condition (not a force condition) [3], the given number n of contact points, the clearance l_S, spring stiffness c, and damping coefficient d. c can be estimated from the geometry and the material of the contact partners whereas the estimation of d is more complicated. As the purpose of the model is the creation of a control concept for vibration suppression, it is important that the equations of motion stay as simple as possible. In the controlled system the clearance plays the role of an external disturbance and as the controller has to work for every kind of contact, a very accurate estimation of d is not necessary. In the axial direction it is assumed that there is no friction. This assumption is justifiable as the bearing between the different segments is realized as roller bearing in many applications. It is assumed here that the force flow leads from the upper part into the lower part.

3. FORMULATION

Boundary value problem

Applying Hamilton's principle

$$\delta\int_{t_0}^{t_1}\left(T-U\right)dt+\int_{t_0}^{t_1}W_{virt}dt=0, \tag{3.1}$$

the governing boundary value problem can be derived. T is the kinetic energy, U the potential energy and W_{virt} the virtual work of forces without potential of the considered system. The kinetic energy reads

$$T=\frac{1}{2}\int_0^{l_1}\rho A_1^* w_t^2 dx_1+\frac{1}{2}\int_0^{l_2}\rho A_2^* v_t^2 dx_2, \tag{3.2}$$

where $\rho A_1^*=\rho A_1+m_T\delta\left(x_1\right)$ and $\rho A_2^*=\rho A_2+m_L\delta\left(x_2-l_2\right)$ and the symbol $\delta\left(.\right)$ represents Dirac's delta-function. If the action of the spring-damper systems is completely included into the virtual work, for the remaining potential energy one obtains

$$\begin{aligned}U=&\frac{1}{2}\int_0^{l_1}\left[EI_1 w_{x_1x_1}^2-\rho g\left(\int_{x_1}^{l_1}A_1^* d\overline{x}_1+\int_0^{l_2}A_2^* dx_2\right)w_{x_1}^2\right]dx_1\\&+\frac{1}{2}\int_0^{l_2}\left[EI_2 v_{x_2x_2}^2-\rho g\int_{x_2}^{l_2}A_2^* d\overline{x}_2 v_{x_2}^2\right]dx_2.\end{aligned} \tag{3.3}$$

Since no internal damping of the beam segments will be taken into consideration, as the worst case for control, the virtual work contains all the contact forces between the beams and the locally concentrated driving force of the vehicle as distributed loads $f_1\left(x_1,t\right)$ and $f_2\left(x_2,t\right)$ which couple the resulting field equations:

$$W_{virt}=\int_0^{l_1}f_1\delta w dx_1+\int_0^{l_2}f_2\delta v dx_2. \tag{3.4}$$

Due to the formulation of all these locally concentrated forces by distributed loads using Dirac impulses, the boundary conditions will be homogeneous. Evaluating Hamilton's principle (3.1) introducing T, U and W_{virt} according to eqs. (3.2), (3.3) and (3.4), respectively, yields the governing field equations

$$\begin{aligned}&\rho A_1^* w_{tt}+EI_1 w_{x_1x_1x_1x_1}+\rho A_1 g\left[\left(l_1-x_1\right)w_{x_1}\right]_{x_1}+g\left(m_L+\rho A_2 l_2\right)w_{x_1x_1}\\&=f_1\left(x_1,t\right)+\delta\left(x_1-l_1\right)g\left(m_L+\rho A_2 l_2\right)w_{x_1},\end{aligned} \tag{3.5}$$

$$\rho A_2^* v_{tt} + EI_2 v_{x_2x_2x_2x_2} + \rho A_2 g \left[(l_2 - x_2) v_{x_2} \right]_{x_2} + g m_L v_{x_2x_2}$$
$$= f_2(x_2,t) - \delta(x_2) g (m_L + \rho A_2 l_2) v_{x_2} + \delta(x_2 - l_2) g m_L v_{x_2} \tag{3.6}$$

and the corresponding boundary conditions

$$w_{x_1}(0,t) = 0, w_{x_1x_1x_1}(0,t) = 0, w_{x_1x_1}(l_1,t) = 0, w_{x_1x_1x_1}(l_1,t) = 0, \tag{3.7}$$

$$v_{x_2x_2}(0,t) = 0, v_{x_2x_2x_2}(0,t) = 0, v_{x_2x_2}(l_2,t) = 0, v_{x_2x_2x_2}(l_2,t) = 0 \tag{3.8}$$

for the two bodies.

For the special case in which the beam segments contact each other at the two points $x_1 = l_1$ and $x_2 = 0$ only, the distributed forces are specified as

$$f_1 = \delta(x_1) F + \delta(x_1 - l_A(t)) \left(F_K(\xi_1(t)) + \frac{d}{dt}\xi_1(t) \cdot D_K(\xi_1(t)) \right)$$
$$+ \delta(x_1 - l_1) \left(F_K(\xi_2(t)) + \frac{d}{dt}\xi_2(t) \cdot D_K(\xi_2(t)) \right), \tag{3.9}$$

$$f_2 = -\delta(x_2) \left(F_K(\xi_1(t)) + \frac{d}{dt}\xi_1(t) \cdot D_K(\xi_1(t)) \right)$$
$$- \delta(x_2 - (l_1 - l_A(t))) \left(F_K(\xi_2(t)) + \frac{d}{dt}\xi_2(t) \cdot D_K(\xi_2(t)) \right). \tag{3.10}$$

The non-linear characteristic of the spring force $F_K(\xi(t))$ takes into account the fact that in the range of backlash no forces can be transferred. The same is valid for the assumed damping coefficient $D_K(\xi(t))$:

$$F_K(\xi(t)) = c \left[\xi(t) - \frac{1}{2}\left(\xi(t) + \frac{l_S}{2} \right) \mathrm{sign}\left(\xi(t) + \frac{l_S}{2} \right) \right.$$
$$\left. + \frac{1}{2}\left(\xi(t) - \frac{l_S}{2} \right) \mathrm{sign}\left(\xi(t) - \frac{l_S}{2} \right) \right], \tag{3.11}$$

$$D_K(\xi(t)) = d \left[1 - \frac{1}{2}\mathrm{sign}\left(\xi(t) + \frac{l_S}{2} \right) + \frac{1}{2}\mathrm{sign}\left(\xi(t) - \frac{l_S}{2} \right) \right], \tag{3.12}$$

$$\xi_1(t) = v(0,t) - w(l_A(t),t), \xi_2(t) = v((l_1 - l_A(t)),t) - w(l_1,t). \tag{3.13}$$

Discretization

The discretization of the coupled partial differential equations (3.5) and (3.6) (nonlinear and time-variant in general) together with the corresponding boundary conditions (3.7) and (3.8) is based on Galerkin's method. For that, the approximate solutions $\overline{w}(x_1,t)$ and $\overline{v}(x_2,t)$ are represented by a series expansion

$$\overline{w}(x_1,t)=\sum_{i=1}^{N}u_i(t)\left(\cos(\lambda_i x_1)+\frac{\cos(\lambda_i l_1)}{\cosh(\lambda_i l_1)}\cosh(\lambda_i x_1)\right), \tag{3.14}$$

$$\begin{aligned}\overline{v}(x_2,t)=u_{N+1}(t)+u_{N+2}(t)x_2+\sum_{i=3}^{N}u_{N+i}(t)\Big[\cosh(\kappa_i x_2)+\cos(\kappa_i x_2)\\ -\frac{\cosh(\kappa_i l_2)-\cos(\kappa_i l_2)}{\sinh(\kappa_i l_2)-\sin(\kappa_i l_2)}\left(\sinh(\kappa_i x_2)+\sin(\kappa_i x_2)\right)\Big]\end{aligned} \tag{3.15}$$

fulfilling all boundary conditions (3.7) and (3.8).

Galerkin's averaging leads to a system of ordinary differential equations of the type

$$\mathbf{M}\ddot{\mathbf{u}}=\mathbf{F}(\mathbf{u},\dot{\mathbf{u}},t). \tag{3.16}$$

4. VIBRATION SUPPRESSION CONCEPT

To suppress vibrations, a state space control concept is introduced. For a system without clearance and with a fixed telescopic length eq. (3.16) represents a time-invariant system of linear ordinary differential equations which can be reformulated as

$$\mathbf{M}\ddot{\mathbf{u}}+\mathbf{C}\mathbf{u}=\mathbf{b}^{*}F(t) \tag{4.1}$$

where $\mathbf{b}^{*}$ is a $2N$-dimensional vector, $\mathbf{M}$ is the mass matrix and $\mathbf{C}$ is the stiffness matrix of the system.

Reduction of order

Eq. (4.1) represents a $2N$-degree-of-freedom system. The objective of an order reduction is to find, for a given model of high order, a model of significantly lower order whose dynamic behavior approximates the original behavior as well as possible. This means that the approximate model has to contain the essential modes of the original system, since they dominate the dynamic behavior of the original system (see [4], for instance). For this purpose, the system equations (4.1) are transformed to principal coordinates $\mathbf{y}$ by

$$\mathbf{u}=\mathbf{M}_R\mathbf{y}. \tag{4.2}$$

The columns of the $(2N,2N)$-matrix $\mathbf{M}_R$ will be composed of the right eigenvectors $\mathbf{m}_{Ri}$ $(i=1,2,...,2N)$ of the system. $\mathbf{M}_L$ is a matrix which contains the left eigenvectors of the system $\left(\mathbf{M}_L^T=\mathbf{M}_R^{-1}\right)$. There is

$$\mathbf{M}_L^T\mathbf{M}^{-1}\mathbf{C}\mathbf{M}_R=-\mathrm{diag}\left(\omega_1^2,\omega_2^2,...,\omega_{2N}^2\right)=\mathbf{D} \tag{4.3}$$

and ω_i^2 $(i=1,2,...,2N)$ are the eigenvalues of the system.
From eq. (4.1), using (4.2) and (4.3), it follows that

$$\ddot{\mathbf{y}}=-\mathbf{D}\mathbf{y}+\mathbf{M}_L^T\mathbf{M}^{-1}\mathbf{b}^*F(t). \tag{4.4}$$

If then the eigenvectors corresponding to the large eigenvalues are removed from $\mathbf{M}_R$ and $\mathbf{M}_L$, one obtains the reduced $(2N,N_r)$-matrices $\mathbf{M}_{Rr}$ and $\mathbf{M}_{Lr}$, and with that the approximate model is

$$\ddot{\mathbf{y}}_r=-\mathbf{D}_r\mathbf{y}_r+\mathbf{M}_{Lr}^T\mathbf{M}^{-1}\mathbf{b}^*F(t). \tag{4.5}$$

This type of order reduction is justified since the state control should control the rigid body motion and the lower modal vibrations of the system. The high-frequency oscillations possess a small magnitude and will diminish strongly by material damping effects. The number of higher-order modes to be included into the reduced model has to be determined depending on the application and the quality of control desired.

Driving unit

The driving unit of the telescope will be represented by a scalar system of first order

$$T_A\dot{F}+F=K_AU \tag{4.6}$$

with time constant T_A and amplification factor K_A. U is the control voltage of the motor. Introducing the state variables

$$z_1=y_{r1},...,z_{N_r}=y_{rN_r},z_{N_r+1}=\dot{y}_{r1},...,z_{2N_r}=\dot{y}_{rN_r},z_{2N_r+1}=F, \tag{4.7}$$

instead of (4.5) and (4.6) one obtains

$$\dot{\mathbf{z}}=\mathbf{A}\mathbf{z}+\mathbf{b}U. \tag{4.8}$$

Controllability and observability

Controllability and observability are checked by computing the so-called controllability matrix $\mathbf{Q}_S$ and observability matrix $\mathbf{Q}_B$ (see [5], for instance):

$$\mathbf{Q}_S = \left[\mathbf{q}_{S1}, \mathbf{q}_{S2}, ..., \mathbf{q}_{SN^*}\right], \mathbf{q}_{S1} = \mathbf{b}, \mathbf{q}_{Si+1} = \mathbf{A}\mathbf{q}_{Si}, \tag{4.9}$$

$$\mathbf{Q}_B = \left[\mathbf{q}_{B1}, \mathbf{q}_{B2}, ..., \mathbf{q}_{BN^*}\right]^T, \mathbf{q}_{B1} = \mathbf{c}_{wF}, \mathbf{q}_{Bi+1} = \mathbf{A}^T\mathbf{q}_{Bi} \tag{4.10}$$

where

$$N^* = 2N_r + 1, \tag{4.11}$$

$$\mathbf{c}_{wF} = \left[\left[W_1(0), W_2(0), ..., W_N(0), 0, ..., 0\right]\mathbf{M}_{Rr}\mathbf{S}\right]^T \tag{4.12}$$

and $\mathbf{S}$ comes from

$$\mathbf{y}_r = \mathbf{S}\mathbf{z}. \tag{4.13}$$

The system is completely controllable if the determinant of $\mathbf{Q}_S$ does not vanish, and it is completely observable if the determinant of $\mathbf{Q}_B$ does not vanish. For real systems, both conditions are usually fulfilled, but it should be mentioned that for a very large mass m_T problems with the observability may occur since then the reaction of the telescope vibrations on the motion of the base is very weak.

Control design by pole placement

The control synthesis in the state space is directed to the goal of taking the state of the system from an initial state $\mathbf{z}_0$ to the state $\mathbf{z}_E = 0$ fulfilling demands on the dynamic behavior of the system. The poles of the feedback control loop determine the transfer behavior, and therefore they have to be selected in such a manner that the requirements on the dynamic behavior are fulfilled. This leads to the desired characteristic polynomial of the closed control loop:

$$p(s) = p_0 + p_1 s + ... + p_{N^*-1}s^{N^*-1} + s^{N^*}. \tag{4.14}$$

To achieve this aim, the N^*-dimensional vector $\mathbf{r}$ (see **Fig. 2**) as defined by *J. Ackermann* reads (see [5], for instance)

$$\mathbf{r}^T = p_0\mathbf{q}_S^T + ... + p_{N^*-1}\mathbf{q}_S^T\mathbf{A}^{N^*-1} + \mathbf{q}_S^T\mathbf{A}^{N^*} \tag{4.15}$$

where $\mathbf{q}_S^T$ is the last row of the inverse controllability matrix $\mathbf{Q}_S^{-1}$ (see (4.9)).

For a constant or a slowly changing (compared to the control loop) command variable w_{FS} (see **Fig. 2**), the pre-filter S (see **Fig. 2** and [5], for instance) reads

$$S = \frac{1}{\mathbf{c}_{wF}^T(\mathbf{b}\mathbf{r}^T - \mathbf{A})^{-1}\mathbf{b}}. \tag{4.16}$$

Luenberger observer

For the present problem, the state vector $\mathbf{z}$ cannot be measured directly, but the control voltage U of the motor and the displacement $w(0,t)$ of the base are measurable. The objective of the so-called Luenberger observer is to find from this information an approximate value $\hat{\mathbf{z}}$ of $\mathbf{z}$. The observer then is written as

$$\dot{\hat{\mathbf{z}}} = \mathbf{F}\hat{\mathbf{z}} + \mathbf{b}U + \mathbf{k}w(0,t). \tag{4.17}$$

The eigenvalues of $\mathbf{F}$ are prescribed and placed on the left of the eigenvalues of the closed control loop, which leads to the desired characteristic polynomial

$$f(s) = f_0 + f_1 s + \ldots + f_{N^*-1} s^{N^*-1} + s^{N^*}. \tag{4.18}$$

According to (4.15) $\mathbf{k}$ then reads

$$\mathbf{k} = f_0 \mathbf{q}_B + \ldots + f_{N^*-1}\mathbf{A}^{N^*-1}\mathbf{q}_B + \mathbf{A}^{N^*}\mathbf{q}_B \tag{4.19}$$

where $\mathbf{q_B}$ is the last column of the inverse observability matrix $\mathbf{Q}_B^{-1}$ (see (4.10)).
$\mathbf{F}$ reads (see [5], for instance)

$$\mathbf{F} = \mathbf{A} - \mathbf{k}\mathbf{c}_{wF}^T. \tag{4.20}$$

Telescopic operations with clearance

For real telescopic operations the parameters of the controller and of the observer are determined for different telescopic lengths and approximated by polynomials which leads to an adaptive controller $\mathbf{r}(l_A(t)), S(l_A(t))$ and observer $\mathbf{F}(l_A(t)), \mathbf{b}(l_A(t))$, $\mathbf{k}(l_A(t))$. Due to the Luenberger observer, straightforward measurements of the motion of the telescope base and of the control variable of the actuator are sufficient to operate the controller. This makes it possible, to apply the controller, developed for the reduced linear system model, to the significantly more complicated system with clearance (3.16) for studying the influence of clearance on the vibration suppression during telescopic motions.

5. SIMULATION RESULTS

Quantitative results are presented here for a 2-sectional system. The results should illustrate the effect of the controller on a telescopic system with and without clearance and only represent a small extract of the existing results. The parameters originate from a test rig of the Institut für Fördertechnik und Logistiksysteme, Universität Karlsruhe (TH):

$l_1 = l_2 = 1.35\text{m}$, $A_1 = A_2 = 0.001\text{m}^2$, $I_1 = I_2 = 0.83 \cdot 10^{-8}\,\text{m}^4$, $\rho = 7850\text{kg/m}^3$,
$m_T = 100\text{kg}$ $m_L = 17.897\text{kg}$, $E = 2.1 \cdot 10^{11}\,\text{N/m}^2$, $c = 10^7\,\text{N/m}$, $d = 10^3\,\text{Ns/m}$, $n = 3$.

The calculation results are based on 4-term truncations (3.14) and (3.15). The controller influences the rigid body motion and the first modal vibration of the system. The poles of the closed control loop and of the Luenberger observer are placed on the points -8 and -12 in the complex plane.

The system starts from an initial point without any initial velocity and has to cover a straight distance of $8\,\text{m}$ before it stops after $8\,\text{s}$. **Fig. 3** shows the position of the base of the telescope versus time for a prescribed velocity of 1m/s and for a motion prescribed by the controller. Both simulations are done with $(l_S = 0.01\text{m})$ and without $(l_S = 0\text{m})$ clearance. During the simulations, the telescopic length increases from $l_A(0) = 0.15\text{m}$ to $l_A(8) = 1\text{m}$ with constant velocity. **Fig. 4** shows the position of the telescope tip relative to its base during the motion and illustrates the vibration suppression by state control. The remaining deflection of the relative position in the simulation with clearance comes from the tilted position of the upper segment in the lower segment due to clearance.

6. CONCLUSIONS

To improve efficiency and to overcome possible safety problems of multi-section constructions during extending and retracting motion of the sections, a vibration suppression in such structural systems of variable geometry seems to be useful. To achieve this, an appropriate modeling of the system together with the development of an efficient control strategy are the essential problems to be treated. For slender beam-shaped structural members, the present contribution has suggested an approach to find a good solution with a justifiable computational expense.

To suppress unavoidable vibrations, the concept of state control via pole placement seems to be very efficient. Based on a model reduction, it is possible to design a control approach which exhibits the desired effects without extensive effort. Introducing a so-called Luenberger observer, straightforward measurements of the motion of the telescope base and of the control variable of the actuator are sufficient to operate the controller. Additionally, this makes it possible to apply the controller developed for the reduced linear system model to the significantly more complicated system with clearance for studying the influence of clearance on the closed control loop. The straightforward handling opens the way for exhaustive parameter studies.

Acknowledgments

The authors thank Deutsche Forschungsgemeinschaft (DFG) for supporting this project under grant AR 160/15-1.

REFERENCES

1. K. Fujita, Y. Shiona, K. Koike, T. Koseki. *Multibody Dynamics of the 2 Section Ladder Model Extending and Retracting Each Other.* Proc. Of ASME-DETC' 03, Chicago (USA), Sept. 2-6, 2003, VIB-48367.

2. P. Barthels, J. Wauer. *Modeling and Dynamic Analysis of Telescopic Systems of Structural Members with Clearance.* Nonlinear Dynamics, 2005, in press.
3. B. Waltersberger, R. Keppler, J. Wauer. *Stoßsimulation mittels einseitiger Feder-Dämpfer Kontakte*. PAMM 3, 2004, 119-120.
4. J. Lunze. *Regelungstechnik 1*. Springer, Berlin, 2001.
5. 0. Föllinger. *Regelungstechnik*. Elitera, Berlin, 1978.

FIGURES

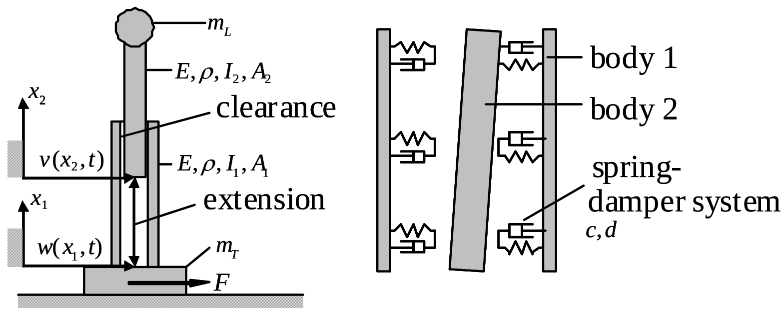


Fig. 1: a) System model, b) Contact formulation.

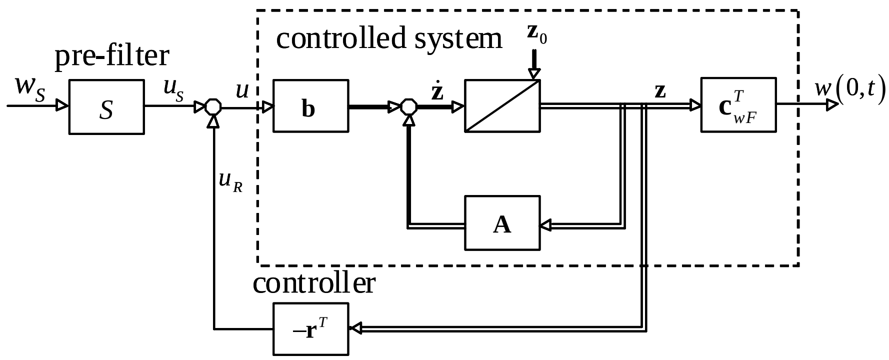
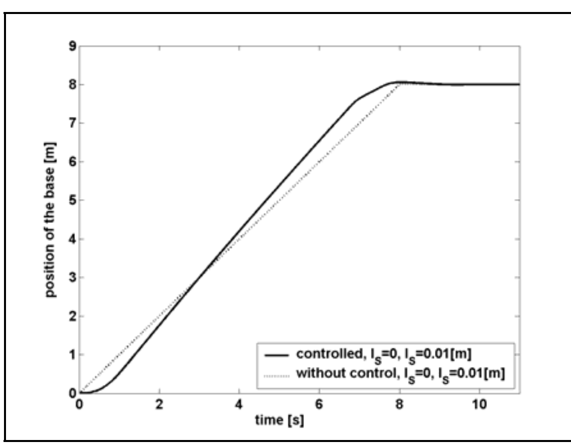


Fig. 2: State space control loop.



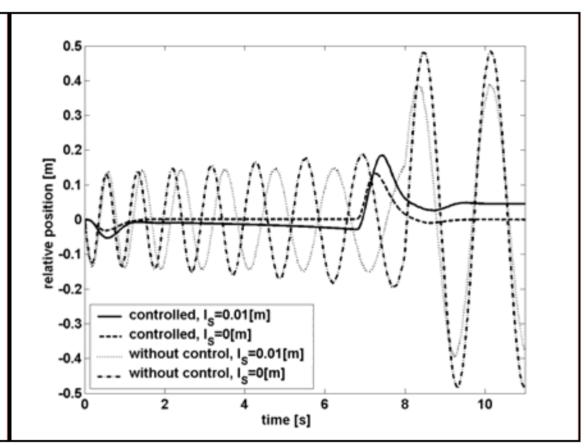


Fig. 3: Position of the base. Fig. 4: Relative position.