Discounts in Auctions

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Abbreviations

$\bar{D}a$	treatment of the second-price auction with asymmetric bidders
$\bar{D}s$	treatment of the second-price auction with symmetric bidders
$\bar{D}$	setting of the second-price auction
π_i	payoff of bidder i
b_d	dominant strategy
b_i	bid of bidder i
Da	treatment of the DA with asymmetric bidders
Ds	treatment of the DA with symmetric bidders
D	setting of the DA
d_t	threshold discount
$D_{\overline{disc}}$	setting of the DA – bids without discount
D_{disc}	setting of the DA – bids with discount
$E[P]$	expected payment of winning bidder
$E[R]$	seller's expected revenue
$E[W]$	expected social welfare (expected social surplus)
$E\pi_i$	expected payoff of bidder i
m	number of weak bidders
N	$= \{1, \ldots, n\}$ set of bidders

n	number of bidders
$n-m$	number of strong bidders
p	price to pay
q	$= \frac{\#strong}{\#weak}$ proportion of the number of groups with a strong designated bidder and the number of groups with a weak designated bidder
r_d	relative deviation of a bid from the respective dominant strategy
u_i	utility of bidder i
v_i	valuation of bidder i
$F^n(x)$	$\equiv (F(x))^n, \forall x \in \mathbb{R}, n \geq 0$
AMASE	agent-based market simulation environment
CAME	computer-aided market engineering
cdf	cumulative probability distribution function
DA	discount auction
EA	second-price (sealed-bid) auction
FCC	Federal Communications Commission
iid	independent and identically distributed
IPV	independent private values auction model
ME	market engineering
MES	meet2trade experimental system
pdf	probability density function
SIPV	symmetric independent private values auction model
w.l.o.g.	without loss of generality

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Chapter 1

Introduction

1.1 Motivation and research questions

Affirmative actions in auctions have received more and more attention in business practice, in particular in public-sector procurement (Rothkopf et al. 2003). In auctions with affirmative actions, certain classes of competitors are given explicit advantages in competitions. Such classes include economically disadvantaged competitors, i.e. less effective competitors, which are given advantages in form of special terms of payment or compensations. Examples of economically disadvantaged competitors in the context of business practice are small businesses or businesses owned by minorities or women. These businesses are mostly favored by different forms of subsidies, discounts, or special payments. The rationale for these affirmative actions stems at a first glance from non-economic aspects – anti-discrimination, notions of fairness, and populism. At a second glance, economic aspects appear to be more relevant, as affirmative actions may increase auction revenue or decrease procurement cost (Rothkopf et al. 2003). A successful implementation of an auction with discounts is observed in the Federal Communications Commission (FCC) in the regional narrowband auction of radio spectrum rights in 1994 (Ayres and Cramton 1996). In this specific auction, the FCC has granted businesses owned by economically disadvantaged competitors a bidding credit of 40 percent. This affirmative action increased the government's revenue by 12 percent. That is, giving bidding preferences to weak (economically disadvantaged) bidders can increase auction revenues by inducing more competitive and aggressive bidding behavior to advantaged bidders. This behavior is also observed in procurement auctions, in which affirmative actions are used to subsidize minorities and decrease the cost of government procurement. Supportive evidence is derived from a laboratory experiment on procurement auctions in which a price-preference auction is employed (Corns and Schotter 1999).

In offering discounts in auctions to certain classes of competitors, it is presumed that economically disadvantaged or advantaged bidders can be identified in advance. That means that for example, in the procurement sector, detailed knowledge of industry conditions and business partners is needed to identify the designated competitors to whom an explicit advantage will be given. However, in general such knowledge about the auction participants or the different classes of competitors and the participant's economic background is difficult to derive in advance.

Surprisingly, discounts as affirmative actions are also observed in Internet auctions for consumer-to-consumer businesses, where the discount is not given explicitly to a certain class of economically disadvantaged competitors.

Regarding the Amazon Internet marketplace, Amazon offers an affirmative action subsidizing a single bidder on its auction platform in addition to the offered selling mechanisms. Basically, a seller on Amazon can sell an item by initiating and conducting an English auction with proxy bidding. Additionally, the feature *first bidder discount* allows the seller to add a first bidder discount of 10 percent to her auction when conducting it. The first bidder discount is a discount the high bidder receives at the end of the auction on the closing current price of the auction if having submitted the first valid bid. A winning bidder who has not submitted the first bid purchases the item at the price equal to the final price of the auction. The affirmative action of the discount is displayed in the auction by a symbol saying 10% *OFF 1st Bidder* as long as no bid has been entered. Upon submission of the first valid bid, the discount symbol is deleted and the discount is no longer available for subsequent bidders.

In the case of the Amazon first bidder discount auction, the surprising fact is that the discount is not given explicitly to a particular bidder; it is assigned by random to a bidder – the visitor of the Amazon auction platform being aware of the listed auction and the first to submit a valid bid. That is, no class of economically disadvantaged bidders (inefficient bidders) is detected in advance and explicitly given the discount. It is even more astonishing that Amazon announces the following information to sellers on its auction site:[1]

> *"For our Auctions customers, the First Bidder Discount* – 10% OFF 1st Bidder – *is an excellent way to entice bidders to bid early, and to keep on bidding. We've found that sellers who take advantage of the First Bidder Discount sell at a rate 15 percent higher than average."*

This assumes that assigning the discount to an economically advantaged or economically disadvantaged bidder is not decisive for the seller. When conducting an auction, on average a

[1] See http://www.amazon.com/exec/obidos/tg/browse/-/1161360/104-8669052-8371101#first-bidder-discount.

seller can extract an additional revenue by adding a first bidder discount to the auction.

An interesting question that accompanies the two pricing schemes and Amazon's statement is how the ex-ante expected revenues of both institutions – the pure auction and the first bidder discount auction – are related. Moreover, the question arises if the statement above holds, meaning whether on average a seller can extract an additional revenue by offering a discount when conducting an auction. This would also answer the question why some sellers decide in favor of the pricing scheme of a pure auction where the payment equals the final price of the auction, while other sellers offer the discounted pricing scheme.

In general, the desire is to find explanations for the research questions concerning the impact of a discount on bidding behavior and auction outcomes, as well as to contrast the respective results of the discount auction format to a benchmark auction format. Therefore, in this study a game theoretic model of an auction with discount (discount auction, DA) is developed and a laboratory experiment employing the DA and a benchmark auction is conducted. Regarding bidding behavior, the aim of this work is to answer the following research questions:

- When comparing the strategic behavior of bidders in the DA to the respective behavior in the benchmark auction, do bidders in the respective auction formats behave differently?

- When focussing solely on the DA, do bidders with discount behave differently than bidders without discount?

Additionally, this study deepens understanding on how the discount affects the auction outcome, in particular the auction revenue:

- By introducing a discount, how does this discount affect the auction outcome, i.e. the seller's revenue, the winning bidder's payoff, and the social surplus?

- Does an additional discount pay for the seller when conducting an auction?

- When focussing on bidders' characteristics and distinguishing between the case of symmetric bidders and the case of asymmetric bidders: In which case can the seller extract an additional revenue and raise her revenue?

- To what extent does the seller's revenue in the DA depend on to whom the discount is assigned: (i) an economically advantaged bidder or (ii) an economically disadvantaged bidder?

Firstly, in the game theoretic model, the seller's expected revenue, the winning bidder's expected payoff and the expected welfare (social surplus) are calculated. Secondly, human bidding behavior in a particular scenario of the DA is investigated in the laboratory experiment. Finally, the findings of the theoretical analysis are compared with the results of the laboratory experiment.

1.2 Methodological approach

Designing electronic markets has become an important issue for electronic commerce. Unlike traditional markets, electronic markets are supported by electronic media; they must be consciously designed since they are strongly affected by the technical infrastructure.

So far, there is little knowledge which institutions are suitable for certain situations or how the outcome of an electronic market should be measured and evaluated. Furthermore, as Roth (1999) points out, the practical design of an electronic market has to deal with complexities, mainly in the economic environment itself, as well as the participants' strategic behavior. In designing the institutional rules, one aims at achieving certain effects as well as efficiency of the market. At the same time, the strategic behavior of the agents and their reactions have to be predicted since they strongly influence the outcome. However, anticipating the agents' behavior is a difficult task. Dealing with such complexities requires more than simply attention to the institutional rules of a market. Additional approaches and tools from other disciplines are needed to supplement traditional methods. For example, experimental and computational economics are supplementary approaches that help in understanding complexities and show how to deal with them. In this context, market engineering focuses on a holistic and theoretically founded approach towards the design and operation of electronic markets (Weinhardt et al. 2003). In particular, the main objective in market engineering is to solve the design problem, or to consciously design electronic markets and redesign existing electronic markets (Neumann 2004).

Within this body of research, the market engineering approach is applied to the analysis and evaluation of a particular market institution with a discount mechanism, i.e. the DA market institution. The evaluation of the DA follows a two-step approach: first, a game theoretic model is developed to calculate equilibrium bidding strategies and the resulting auction outcomes; and second, a laboratory experiment is conducted to analyze human bidding behavior in the DA. The theoretical findings are compared to the experimental results. In other words, the study addresses the evaluation problem of market institutions by an *axiomatic approach* and an *experimental approach* (Neumann 2004).

The analysis of the DA market institution by means of game theory and experimental economics gives valuable advice concerning the design and improvement of auction mechanisms with discounts in a specific auction environment.

1.3 Overview and structure

This study is divided into two parts. In the theoretical part a game theoretic model of a market institution with a discount mechanism, denoted as DA, is developed. As benchmark of the DA the second-price sealed-bid auction is chosen. The second part presents a laboratory experiment in which human bidding behavior in the DA and the respective benchmark auction are investigated. Both the game theoretic model as well as the experimental analysis assume an independent private values auction model. The structure of this study is outlined in the following and illustrated in Figure 1.1.

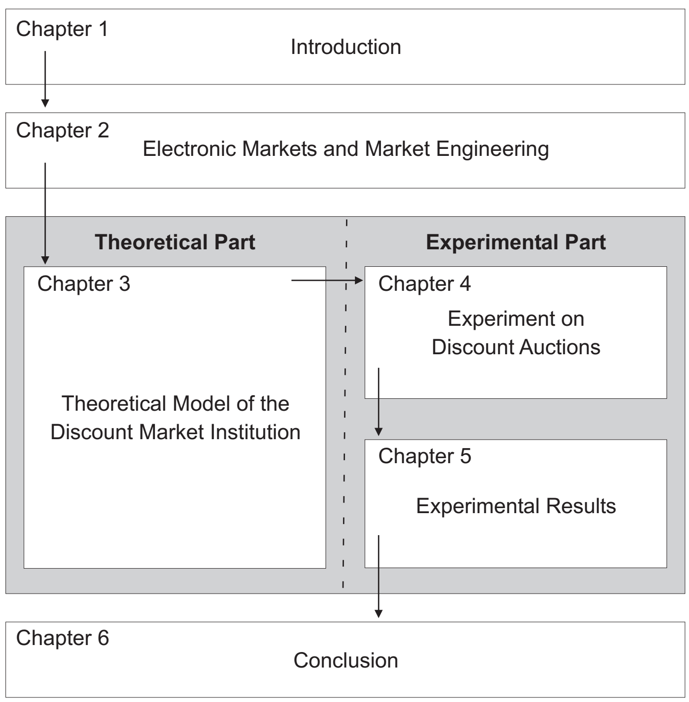


Figure 1.1: Structural overview of the study

Chapter 2 introduces the concept of market engineering and provides insights concerning the implementation of electronic markets. Firstly, a market is characterized as a microeconomic system consisting of two distinct components: (i) the market environment and (ii) the market institution. Secondly, a review on negotiations and auctions is presented – they are mostly understood as price-discovery mechanisms for market institutions and regarded as the dominating coordination mechanism in traditional markets. Thirdly, the microeconomic system is extended to electronic markets. Electronic markets comprise a technical infrastructure component since they are supported by electronic media. In the context of market engineering, methods for designing and analyzing electronic markets, e.g. methods from game theory, mechanism design, and experimental economics, are summarized. Finally, market engineering as a structured, systematic and theoretically founded approach towards the design and operation of electronic markets is presented.

In Chapter 3 the DA and the benchmark second-price auction are analyzed under the assumptions of an independent private values auction model. The model of the DA market institution is developed and insights into the discount mechanism are given. Furthermore, dominant strategies in equilibrium are identified in the DA: a designated bidder – a bidder to whom the discount is assigned – submits a bid slightly above his valuation. That is, he submits a bid equal to $\frac{1}{1-d}$ times his valuation ($d \in [0, 1)$ denotes the discount). Bidders without discount submit their valuations truthfully. In addition, the DA and the benchmark second-price auction are analyzed under two different assumptions: first, bidders are assumed to be symmetric, meaning that bidders are characterized by the same distribution function of valuations; and second, bidders are assumed to be asymmetric, meaning that bidders are distinguished into two groups – a group of weak bidders and a group of strong bidders – characterized by different distribution functions of valuations. The expected outcomes of the DA and the respective expected outcomes in the benchmark auction are calculated and compared in the symmetric case as well as in the asymmetric case, i.e. the seller's expected revenue, the winning bidder's expected payoff, and the expected welfare.

Chapter 4 describes the details of the laboratory experiment on the DA. The experiment design and its design parameters are introduced. Basically, the design follows a between subjects design. All experimental sessions are set up and conducted by the meet2trade system and the connected meet2trade Experimental System (MES). Additionally, the four treatments isolated from the observed experimental data are described, i.e. the treatments of the DA in the symmetric case, the second-price auction in the symmetric case, the DA in the asymmetric case, and the second-price auction in the asymmetric. Regarding the statistical analysis of the experimental data, the deployed statistical tests and the prespecified level of significance are briefly introduced.

Chapter 5 presents the statistical analysis of the experimental data. The analysis focuses on the behavior of human bidders as well as on the outcomes of the auctions conducted in the different settings, i.e. the auction revenue, the winning bidder's payoff, and the social surplus;

and finally on the auction revenue in the two symmetric and two asymmetric treatments. The experimental results of the DA and the benchmark auction are contrasted and discussed with respect to the research questions.

Chapter 6 reviews this body of research and summarizes its main findings. Limitations of this study are discussed and future research directions are suggested.

In literature, auctions with discounts, or more generally spoken auctions with affirmative actions, have been only rarely discussed so far: Both, theoretical and experimental studies of auctions with discounts are scarce. By bridging this gap, this study contributes to existing research and also supplements interesting research aspects from a market engineering perspective.

Chapter 2

Electronic Markets and Market Engineering

The main emphasis of this chapter is to create a consistent terminology and common understanding underlying the research field of electronic markets and their conscious design. Moreover, a structured approach for the design, development and implementation of electronic markets is suggested. The following sections introduce the basic terminology and concepts from economic literature used throughout the remainder of this book. Section 2.1 introduces the term 'market' and presents a framework which defines a market by environmental and institutional rules. In particular, negotiations and auctions are considered as an example of market institutions of practical importance. In section 2.2 both institutions are discussed and definitions from negotiation research and auction theory are presented. Section 2.3 extends the market framework to electronic markets focussing on their support by information technologies. An overview of methods from game theory, mechanism design theory and experimental economics for the design and analysis of electronic markets is given in section 2.4. Distinct aspects of market engineering as a holistic approach for the structured design, development and implementation of electronic markets are presented in section 2.5. Later on, this section introduces computer-aided market engineering, which employs tools to assist market engineers in the design, development and testing of electronic markets.

This specific groundwork is necessary to create a common terminology for understanding the theoretical model and experimental investigation presented in this work.

2.1 Markets and institutions

Markets play a central role in the economy. They facilitate the exchange of information, goods, services, and payments. Moreover, they create economic value for buyers, sellers, market intermediaries, and for society at large (Bakos 1998). In modern economies, exchange means trading tangible goods, services, and rights in substitution for money (Smith 2003). The involved transaction partners, e.g. buyers and sellers, are autonomous: they decide with whom they want to interact and whether they want to buy or sell (Schmid 1997). More generally, supply and demand are represented by subjects or organizational entities who participate in the transaction process to fulfill their needs. In the following, subjects and organizational entities are defined as *economic agents*:

Definition 2.1.1 Economic Agent
An economic agent[1] *is an organizational or individual entity participating in an economy*.

When considering the exchange and interaction activities between agents, the main task lies in the coordination of the agents' activities. Smith (1776) realizes that agents, having private information and striving for their own gains, are more successful in allocating their resources than a central entity is. The outcome is produced in a decentralized way without any explicit agreement between the acting agents. The process is not intentional and the agents' aims are not coordinated: the process even works while agents hold only private information ("invisible hand process"). Based on the human selfish and greedy motives, this process explains the competition in a market which tends to benefit the society as a whole.

Markets represent one organizational form for coordinating economic activities (Coase 1937). A market can be regarded as a coordinated price-based mechanism where price movements direct production. These price movements refer to a series of transactions on the market. Building on Coase's basic insight, Williamson (1975) suggests two basic organizational forms for coordinating the flow of objects through adjacent steps in the value-added chain: markets and hierarchies (Malone et al. 1987).[2] In addition to the two polar organizational forms, networks are suggested as an intermediated form of coordination (Powell 1990).

Focussing on the term *market*, a market is the location where an institutional framework is provided which allows buyers and sellers to announce their buying and selling intentions, exchange information, negotiate about the object and the conditions of the transaction, and

[1] In the following *agent* will be used synonym to *economic agent*. Human agents and software agents will be distinguished where it is necessary.

[2] An object is either a tangible, an intangible good or a right to a service. Input, output, resource, goods, and services are examples of objects.

complete a contractual agreement for an exchange of goods and services in an most efficient manner (Schmid and Lindemann 1998). In essence, markets have three main functions: (i) matching buyers and sellers, (ii) facilitating the exchange of objects, and (iii) providing an institutional framework that enables the efficient functioning of the market (Bakos 1998). Price discovery plays a central role within the matching of buyers and sellers: the prices at which demand and supply clear and trade occurs are determined (Bakos 1998). The exchange of objects between sellers and buyers is referred to as transaction (Williamson 1975).

Definition 2.1.2 Market
A market is the location (physical or virtual) where a transaction between buyers and sellers is facilitated and where the activities of buyers and sellers are governed by price and competition.

Historically, in ancient Greece around 600 B.C. the "agora" or market place was an economic association in the center of the polis – it was the social and economic center of the town. During the twelfth century the term market entered the English language. Later, in the eighteenth century the term market was separated from a physical and social place, comprising the activities buying and selling (Powell 1990). These activities comply with an institutional framework, which defines the rules and the process of the exchange.

The problem of designing an efficient economic system is emphasized by Hayek (1945), based on the observations that information and knowledge in a society is dispersed among economic agents and can of course be contradictory. The questions how to ensure the best use of resources and how to utilize private information is answered by his understanding of the price system. Hayek understands the price system as a communication network in which information from one part of the market is transmitted to another. Prices are used as signals that can act to coordinate the separate actions of the economic agents. Thus, prices can be seen as "carrier of all that the individual need know about others, and of the social and physical constraints on all the activities underlying those prices" (Smith 2003).

Another key function of an economic system is to support agents in decisions determining the flow of resources (Hurwicz 1973). The resource allocation mechanism should guide the agents towards actions which are at least feasible or even more efficient. The difficulty lies in designing the rules of the mechanism such that certain properties are achieved and agents behave in the desired way. Thus, agents should have an incentive to follow these rules and not to depart from compatible behavior patterns (Hurwicz 1973).[3] Reiter (1977) considers an economic system as a "kind of machine" which determines the allocation of resources among

[3]In mechanism design theory such mechanisms have the property of being incentive-compatible.

agents (the output of the machine) on the basis of available data of an economy (the input of the machine). He presents a formal structure for the design and evaluation of allocation mechanisms, which are also called *economic variables*.

Based on these works, Smith (2003) presents the framework of market (microeconomic) systems theory for analyzing market processes. This framework consists of two distinct components: (i) the market environment and (ii) the market institution. The environment describes the set of all individual circumstances in a market that can not be changed or influenced by the agents. Examples are individual preferences or characteristics of the agents, commodity endowments, or technology endowments. Moreover, some of the individual circumstances are private in nature, meaning that they are not known publicly. For instance, such individual circumstances are individual taste, information, knowledge, or individual skills.

Definition 2.1.3 Market Environment

Given a list of n economic agents $i \in N = \{1, \ldots, n\}$, a list of $K + 1$ commodities $k \in \{0, 1, \ldots, K\}$. Agent i's characteristics such as the utility function u_i, technology endowment T_i, and a commodity endowment ω_i are defined over a $K + 1$ dimensional commodity spaces R^{K+1}. A market environment (synonym: microeconomic environment) $e = (e_1, \ldots, e_n)$ is defined by the collection of characteristics $e_i = (u_i, T_i, \omega_i)$ of economic agents $i \in \{1, \ldots, n\}$.

The rules under which agents communicate and exchange information and property rights of commodities are defined by the institution. It specifies the resource allocation mechanism. According to Smith (1982 and 2003) the market institution is defined as follows:

Definition 2.1.4 Market Institution

A market institution (microeconomic institution, institution) defines (i) the language of the market, (ii) the rules that govern the exchange of messages and (iii) the rules that define the conditions under which messages lead to allocation and prices. Given a list of n economic agents $i \in N = \{1, \ldots, n\}$, the market institution comprises:

(i) A language or message space $M = M_1 \times \ldots \times M_n$ which consists of messages $m = (m_1, \ldots, m_n)$ and where $m_i \in M_i$ is a message which can be sent by agent i.

(ii) A set $G = (g_1(t_0, t, T), \ldots, g_n(t_0, t, T))$ of adjustment process rules, where for each agent i the adjustment rule $g_i(t_0, t, T)$ consists of

- *$g_i(t_0, \cdot, \cdot)$ as a starting rule, determining the start of the message exchange,*
- *$g_i(\cdot, t, \cdot)$ as a transition rule, governing the sequence and exchange of messages,*
- *$g_i(\cdot, \cdot, T)$ as a stopping rule, specifying the termination of the message exchange and the start of the allocation.*

(iii) *A set* $H = (h_1(m), \ldots, h_n(m))$ *of allocation rules with* $h_i : M \rightarrow X$ *and* $h_i(m) = x_i$ *specifying the final allocation to each agent* i *based on the messages sent by all agents.* m *is the final message determining the allocation and* X *is the set of all possible outcomes of agent* i.

(iv) *A set* $C = (c_1(m), \ldots, c_n(m))$ *of cost imputation rules where the rule* $c_i(m)$ *determines the payment to be made by agent* i *as a function of the messages sent by all agents.*

A market institution $I = (I_1, \ldots, I_n)$ is defined by the collection of the individual property right characteristics I_i of agent i with $I_i = (M_i, h_i(m), c_i(m), g_i(t_0, t, T))$. Each agent i's property right I_i specifies the messages, the commodities being allocated to the agent, the agent's payment, and the rules governing the process of the message exchange. Examples of such messages are bids, offers, counteroffers or acceptances.

With the above specified definitions the market system can be defined as:

Definition 2.1.5 Market System

A market system S *(microeconomic system) is the tupel consisting of the market environment* e *and the market institution* I *with* $S = (e, I)$.

In a market system, agents' behavior is motivated by their individual circumstances, preferences, and objectives. Moreover, it is strongly related to the underlying institutional rules. Note that the agents' characteristics are inherently private – only their consequences, i.e. their messages, are observable. Agents exchange messages based on their circumstances. The bidding language and the rules of the market are known to each bidder in advance. Thus, the behavioral actions of agents can be expressed by a function defined on the agents' economic environments and the institution.

Definition 2.1.6 Agent Behavior

Given a list of N *agents* $i \in N = \{1, \ldots, n\}$. *Agent's* i *outcome behavior is defined by a function* $b_i(e_i|\, I)$. *Behavior* b_i *is a mapping from the agent's environment* e_i*, conditional on the institution* I *into the set of messages* M_i.

Note that the function $b_i(e_i|\, I)$ is equal to the message m_i sent by agent i. The institution which determines the outcome (resource allocation and prices) is based on the agents' messages.

The operation of a market is regarded as the process comprising the pathway from the economic environment to agent behavior, to the institution and finally to the outcome. In essence, two central components should be mentioned: (i) markets are defined by rules that select, process and order messages of agents and (ii) agents have private information about

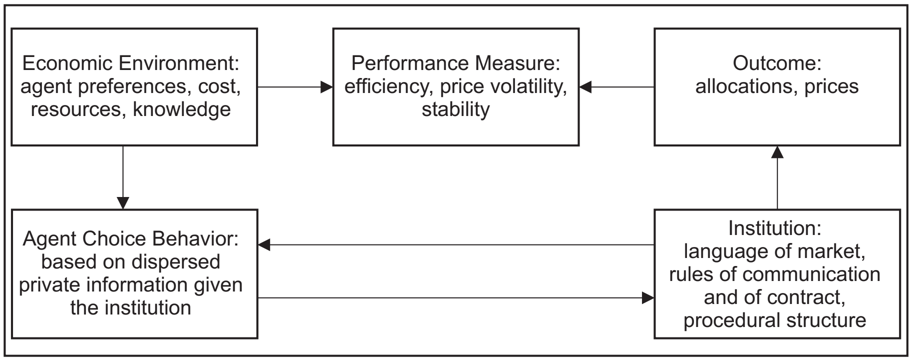


Figure 2.1: Components of a microeconomic system (Smith 2003)

their personal circumstances (Smith 2003). Figure 2.1 illustrates the components of a market and shows how these components are related.

The presented market system framework allows the comparison of different environments while using the same institution. It also allows for keeping the environment constant while changing the institution. As the rules of the market affect incentives, institutions influence the observable behavior of agents and thus the outcomes that result (Smith 2003).

2.2 Negotiations and auctions as market institutions

In economic theory, negotiations are mostly understood as price-discovery mechanisms for market institutions such as auctions. They are the dominating coordination mechanism in traditional markets (Ströbel 2003). Negotiations appear in various forms and situations and have been influenced by ethical, cultural and social circumstances. Moreover, negotiations are widely analyzed from different research areas and perspectives such as computer science, economic sciences and management, information systems, as well as law and social sciences. All these research disciplines address specific aspects of negotiation situations. Economics and management science concentrate on the construction of formal models and procedures of negotiations, rational strategies and outcomes influenced by bargaining theory, game theory, auction theory, and negotiation analysis (Bichler et al. 2003). Negotiation media and systems as well as software platforms for bidding and auctioning are the focus of computer science and information systems. These systems are conceptually based on the results from studies in economic and social sciences.

The change and development in information technology has enabled new ways of negotiations. The technology supports negotiations at nearly every stage: at the stage of information exchange, matching, comparison of data, decision support etc. These new possibilities have led to the emergence of innovative negotiation protocols. Examples range from combinatorial auctions to multi-attribute auctions, as well as automated negotiations among software agents. Focussing on these developments, the difference between negotiations and auctions seems to diminish and the questions arises whether negotiations are auctions, i.e. "Are all e-commerce negotiations auctions?" (Kersten et al. 2000).

2.2.1 Negotiations

The meaning and definition of the term negotiation is not clear-cut. Generally, a negotiation can be defined as the "key decision-making approach used to reach consensus whenever a person, organization or another entity cannot achieve its goal unilaterally" (Bichler et al. 2003). But as negotiations have been the focus of research in many different disciplines, the context in which the term negotiation is used or the definition of the term itself can vary. In game theory, bargaining is a synonym for negotiation (Ströbel 2003).[4] Bargaining situations are competitive situations where two or more agents who have different information negotiate the terms of possible cooperations (Harsanyi 1967a).[5] Decision and negotiation analysis focuses on the negotiation process. Negotiation analysis seeks to develop useful advice to involved parties and aims at situations that are not fully specified in advance.

In general, there are many reasons why negotiations take place: (1) they create something new that neither party could do on his or her own, or (2) they resolve a problem or dispute between the parties (Lewicki et al. 1999). Thus, negotiations are begun in order to come to an agreement that would not be found without negotiation or that the involved parties expect to come to a better agreement than by merely accepting fixed offers (Ströbel 2003). However, negotiations are a vehicle for agents to communicate and find a compromise reaching mutually beneficial agreements (Fatima et al. 2004).

In essence, the activity of negotiation can be characterized by a set of common core properties (Ströbel 2003; Strecker 2001). A negotiation is a communication and decision-making process in which two or more parties are searching for a solution to a problem which may

[4]In game theory, bargaining situations are analyzed as non-cooperative zero sum games (Aggarwal and Dupont 2001).

[5]The formal theory of bargaining defines a "bargaining situation" as a situation in which (i) agents have the possibility of concluding a mutually beneficial agreement, (ii) there is a conflict of interests about which agreement to conclude, and (iii) no agreement may be imposed on any individual without his approval (Carraro et al. 2005). Harsanyi (1967a, 1967b, and 1967c) models these situations as a game with incomplete information, meaning that at the beginning of the game some players have incomplete information about what other players know or believe (Myerson 2004).

involve a conflict of interest. Since the solution cannot be reached through unilateral action and each party is not willing to accept what the counter party is voluntarily offering, parties have to negotiate. The search for a solution as well as the decisions made are based on individual preferable solutions of each party. Note that parties are mutually dependent by finding a consensus. With these properties the term negotiation applies to a wide range of situations from simple bargaining to complex auction mechanisms.

The following definition of negotiation is given by Bichler et al. (2003):

Definition 2.2.1 Negotiation

Negotiation is an iterative communication and decision- making process between two or more agents who

(i) cannot achieve their objectives through unilateral actions,

(ii) exchange information comprising offers, counter-offers and arguments,

(iii) deal with interdependent tasks, and

(iv) search for a consensus which is a compromise decision.

The way offers, counter-offers, and messages are exchanged between the parties is governed by communication rules. The result of the negotiation itself can be either a compromise or a disagreement. If all agents are willing to accept the compromise and transact it according to its specifications, then the negotiation is complete.

The characteristic of a negotiation strongly depends on the parties' positions in the negotiation. Parties can be (i) more competitive and claim value or (ii) more open to create value (Kersten et al. 2000). In *distributive* negotiations each party engaged in the negotiation process strives to achieve the best possible settlement for themselves. The interest in the other party is only insofar that the other party affects the achievement of its own objectives. That is, one party can only gain at the other party's expense, and each party hides its objectives and preferences, revealing them only indirectly through their messages. In *integrative* negotiations new issues and options are added to the set of feasible alternatives during the negotiation process. Thus the dimension of the negotiation changes. There are two key characteristics of integrative negotiations: Firstly, integrative negotiations aim at the creation of value during the process. Secondly, parties are not selfish in integrative negotiations. They focus on needs and interests and not positions, exchange relevant information and ideas, and are interested in learning and restructuring the problem (Kersten et al. 2000; Lewicki et al. 1999).

The process a negotiation follows is characterized by the rules defining the *negotiation arena* and *agenda*, as well as permissible decision-making and communication activities. The arena is the place where the negotiator communicates and the agenda specifies the negotiation framework, including the specifications of the issues to be negotiated and the format in which they are presented.

The *negotiation protocol* includes the rules and thus specifies possible actions, allowable messages and offers as well as their sequences and timing. Moreover, it defines mechanisms that select alternatives during the negotiation process, constructing offers and making concessions. According to Bichler et al. (2003), negotiations are distinguished by three different levels of structuring: *unstructured*, *semi-structured*, and *structured* negotiations.

1. *Unstructured negotiations* do not follow any rules that limit the exchange of offers, counter-offers, or messages between the parties.

2. *Semi-structured negotiation* protocols leave some flexibility in decision-making and message exchange by the parties. The activities of the parties are not fully defined by the protocol.

3. *Structured negotiations* define all possible activities of the parties including their decision-making and information exchange.

Face-to-face negotiations are mostly considered to be unstructured negotiations as there are no explicitly defined rules to follow. Auctions are structured negotiations. In essence, they are resource allocation mechanisms based on the exchange of messages about a single issue, the price of a single well-defined object. Combinations of unstructured and structured negotiations can be viewed as semi-structured negotiations. For example, first an auction is used to identify potential parties, and second, a bilateral negotiation is used to find a bilateral agreement in a face-to-face negotiation with the identified parties.

In general, negotiations involve cooperation in order to create value. The objects being negotiated are not well-defined and the intention within the process is to define the object and specify the issues in order to obtain a common definition.

2.2.2 Auctions

Auctions are market institutions of practical importance.[6] They are considered as an important vehicle in conducting market transactions and have been used since antiquity for selling a variety of objects.[7] Art objects, antiques, agricultural produce, houses, etc. have been sold by

[6] The word "auction" [latin: *augere*] means to increase or augment. In the oldest auction form, the *English* auction, the price to pay for the single-item object auctioned is successively raised by the auctioneer.

[7] See Shubik (1983) for an historical sketch of auctions.

auctions for centuries. Nowadays, auctions are used to conduct a huge volume of economic transactions. For example, governments use auctions to sell rights to use electromagnetic spectrums for telecommunication and natural resources such as timber rights or off-shore oil licences, or to sell permits for energy, pollution, and transport.

The range of objects being sold in auctions has been greatly increased by e-commerce. In the private sector, Internet auction web-sites are used to sell all kinds of consumer goods (Lucking-Reiley 2000). In the business sector, procurement processes are supported by competitive bidding processes where participants compete for the right to sell their products or services (Krishna 2002).

In an auction, participants (*bidders*) submit *bids* that represent their demand or supply function (Moldovanu and Jehiel 2003). Bids can be interpreted as half-contracts, indicating the amount of money the bidder is willing to pay for a single-item object (or combination of items) he may get (Nisan 2000).

Definition 2.2.2 Bid

Bids are messages with which agents express their resource requirements and elicit information or give price signals within the auction.

The bid structure defines the flexibility with which agents can express their requirements and preferences (Kalagnanam and Parkes 2003). It is defined within the auction rules. Standard auction rules support bids allowing the specification of price or price and quantity.

The auction rules determine the way in which resources are allocated and the winner is determined, as well as the price at which the auction market clears and trade occurs (Wolfstetter 1996). Auctions represent a specific set of institutional rules. These rules determine what bids can be submitted and define how these bids are aggregated to yield allocations and prices (Moldovanu and Jehiel 2003). McAfee and McMillan (1987) define an auction as follows:

Definition 2.2.3 Auction

"An auction is a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants."

Thus, auctions are bidding mechanisms.[8] The bids are the only input – there is no need for additional input.

For most practical applications, the institutional rules must be relatively simple: they should be independent of the specific environment as well as the private information which is not available to the seller (Moldovanu and Jehiel 2003). In addition, the rules of an auction

[8] The term *dynamic pricing* is often used as a term for all kinds of auctions (Milgrom 2000).

are given to the bidders in advance and are not changed once the auction has begun (Ehrhart and Ott 2003). A salient feature of auctions is that "they elicit information, in the form of bids, from potential buyers regarding their willingness to pay and the outcome – that is, who wins what and pays how much – is determined solely on the basis of the received information" (Krishna 2002). Thus, auctions are universal, i.e. they "may be used to sell any good" (Krishna 2002). For example, a car can be auctioned under the same rules as a painting. Another feature of auctions is that they are anonymous – the identities of the bidders do not affect the outcome of the auction (Krishna 2002).

The performance of different auction formats under explicit consideration of strategic and behavioral aspects is the main question that guides auction theory toward a good and proper design of auction rules. The achievement of an allocative efficient outcome – the item is awarded to the bidder who values it the most – is the main goal in auction theory. From the perspective of the society as a whole, allocative efficiency is one goal to achieve in designing the auction rules.[9] Another goal in an auction is the maximization of revenue in the auction. This performance criteria is especially considered by the seller – she desires to receive as much revenue as possible for the auctioned good. Additionally, there are several other goals that justify the use and implementation of auctions in particular instances. In the following the most relevant auction goals are summarized (cf. for example to Krishna 2002; Moldovanu and Jehiel 2003; Wolfstetter 1996):

- *Allocative efficiency*: The outcome of an allocation is allocative efficient if the social welfare is maximized. This is the case when the object is awarded to the bidder with the highest valuation.

- *Revenue-maximization*: Selling the object at the possible price and gaining as much revenue as possible is a potential seller's desired goal. Competition among bidders has a clear positive affect on this goal.

- *Information aggregation and revelation*: Bidders elicit information by submitting bids during the auction. These bids are based on the private information of the bidders. The resulting prices reflect the aggregated information during the auction process.

- *Valuation and price discovery*: In many situations, the value of an object at the time of an auction is unknown. Signals that are related to the true value of the object are privately

[9] Economic or allocative efficiency means "the maximization of the (possibly weighted) sum of consumer and producer surplus" (Jehiel and Moldovanu 2001). It is measured in terms of total monetary surplus, the social welfare. Social welfare is the sum of bidders' and the bid-takers' monetary surpluses.

known. If these signals are publicly known, they would affect the value attached to the object by a particular bidder. Thus, during an auction process bidders can learn their valuation from the signals of rival bidders and adjust their valuation.

- *Transparency and fairness*: Common auction forms have the virtue of simplicity: the rules are precise, fixed in advance, applied equally to all participants, and transparent. This transparency limits possible bidder corruption, collusion or dishonesty in the auction process, thus making auctions more fair.

- *Speed of sale*: The speed of sale is important for perishable goods such as fish, flowers or vegetables.

These goals are only a few examples mentioned within the context of auction design. Auction goals can be manifold and the goals themselves can contradict each other. The achievement of a single goal strongly depends on the environment and the institutional rules as well as on the behavior of the participants.

In an auction, only rarely does a seller have incomplete information about the buyers' valuations in advance. The difficulty a seller faces is in finding a pricing scheme that performs well under incomplete information and finds the revenue-maximizing price of an object (Wolfstetter 1996). If the seller knew the buyers and their values attached to the object being sold, the seller could offer the object to the bidder with the highest value at or just below the amount the bidder is willing to pay. The value each bidder assigns to the object is the maximum amount he is willing to pay for that object.

In a *private value* situation this value is known only to the bidder himself. This value is private information; no bidder has information about the values of the rivals bidders and information about the other bidders' values would not affect the private valuation of a bidder assigned to an object. Private value situations are situations where paintings, stamps or antique furniture are auctioned – bidders assign different values to the object derived from the consumption or the usage of this object. The paradigm of *symmetric independent private values (SIPV)* assumes that each bidder's valuation of an object is independently drawn from an identical distribution (iid) (Myerson 1981; Riley and Samuelson 1981). Each bidder observes his own valuation and has no information about the opponent's valuation except for the distribution from which it is drawn. The SIPV model with risk-neutral agents is a model which makes assumptions allowing a thorough analysis of auctions. It assumes (Wolfstetter 1999):

- *single-unit auction*: a single indivisible object (a single-item) is offered for sale to one of several bidders;

- *private values*: the value for the object is only known to the bidder himself;

- *symmetry*: all bidders are indistinguishable;

- *independence, symmetry, continuity*: unknown valuations are independent and identically distributed (iid) and continuous random variable;

- *risk neutrality*: bidders are risk neutral.

In a *common value* situation, the value of an object is the same to each bidder but unknown to all bidders at the time of bidding. Bidders may have estimates about the value of the object but the object's true value is only observed after the auction has taken place. Common value models are used in situations where the value of an object is derived from a market price that is unknown at the time of the auction and determined throughout bidding. An example is given by the auction of land with an unknown amount of oil underground. The final value of the land is determined by future oil sales. Based on signals (bidders have different information in the form of e.g. geological tests or expert's estimate) throughout the bidding process, bidders learn the object's value and adapt their valuations throughout the bidding process. Note that in a private value model signals would not affect a bidder's private value.

A key characteristic of the bidding process in auctions with common value components is the *winner's curse*. Since the object for sale is the same to all bidders and its true value unknown during the auction, bidders have only estimates of the object's true value. For example, when bidders submit bids on their estimates, then the bidder with the most optimistic estimate will win the auction. But if the high bidder overestimates the true value of the object, then the winner suffers a loss. This is denoted as the winner's curse (cf. for example to Krishna 2002; Wolfstetter 1996).

Generally, one distinguishes between oral (outcry, open) and written (sealed, sealed-bid, closed) auctions. In open auctions bidders make offers and counter-offers which are visible to all bidders. In written auctions bidders most often submit a single bid which is not revealed to the other bidders. The most common auction forms in which a single-item object is being offered for sale are the following four: (i) the *English* auction, (ii) the *Dutch* auction, (iii) the *first-price* sealed-bid auction, and the *second-price* sealed-bid auction.

(i) *English auction*: In the English auction (also called the open, oral, or English auction or ascending-bid auction) the price gradually increases, typically in small increments, as long as there are at least two interested bidders. By raising the price, bidders drop out of the auction in succession. The auction stops when only a single bidder is interested; that bidder then wins the auction. The item is awarded to the bidder and the amount to pay equals the price at which the auction stopped, that is the price at which the second-to-last bidder dropped out. In variants of the English auction, the sale is conducted by an auctioneer who calls out the prices while raising them.

(ii) *Dutch auction*: In the Dutch auction (also called descending-bid auction, Dutch clock auction) the price is gradually lowered by small increments, starting at a high price. The price is lowered until a bidder accepts the current price. The auction then stops and the item is awarded to that bidder at that price. The price at which the auction starts is chosen high enough such that no bidder is interested in buying the item at that price. In variants of the Dutch auction, the auction is conducted by an auctioneer calling out the prices or using a mechanical device, or a clock. The clock ticks down the price until a bidder accepts the current price indicated by the clock.

(iii) *first-price sealed-bid auction*: In the first-price sealed-bid auction, bidders independently submit a single bid without seeing others' bids, e.g. bidders submit bids in sealed envelopes. The bidder with the highest submitted bid wins the auction and the item is awarded to him at the bid price.

(iv) *second-price sealed-bid auction*: In the second-price sealed-bid auction (also called Vickrey auction) bidders submit sealed bids not visible to the other bidders. The item is awarded to the highest bidder, the bidder with the highest submitted bid, and the bidder pays the second highest bid.

The English and Dutch auctions are open auctions, while the first-price and second-price auctions are sealed-bid auctions. Furthermore, the two following relations hold: Firstly, the Dutch auction and the first-price sealed-bid auction are called *strategically equivalent* and the bidder's bidding functions are exactly the same in both auctions (Krishna 2002; Klemperer 2004). Secondly, when taking private values into consideration, the English auction and the second-price sealed-bid auction are equivalent. In both auctions, submitting the true value is optimal for bidders independent of whatever other players do. With a common value component the information available would be relevant for the bidders – they would learn their valuation and condition their behavior (Krishna 2002; Klemperer 2004).

The auction formats considered so far are single-sided auctions, i.e. there is one seller and multiple buyers (forward auction) or there is one buyer and multiple sellers (reverse auction).[10]

[10]Double-sided auctions (or double auctions) are settings with multiple buyers and sellers submitting bids simultaneously. The two main institutions for double-sided auctions are the continuous double auction (CDA) and the call auction (call market) (Kalagnanam and Parkes 2003).

In a forward auction, buyers submit bids to the seller, meaning that the seller is the bid-taker, whereas in the reverse auction sellers submit bids to the buyer, meaning that the buyer is the bid-taker. The following classification scheme of single-sided forward auctions presented in Figure 2.2 distinguishes the standard auction types according to the following criteria: (i) open vs. sealed-bid and (ii) ascending-bid vs. descending-bid (Wurman et al. 2001).

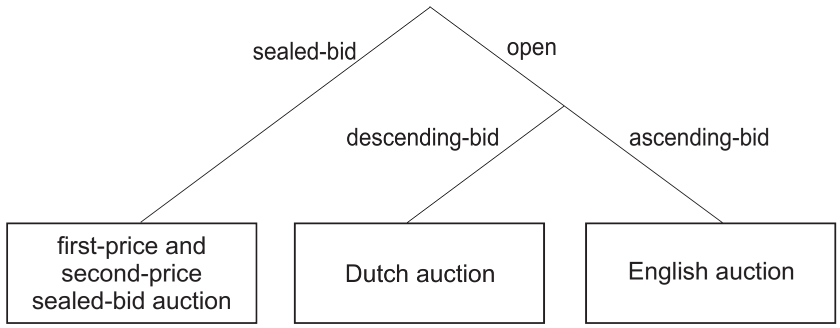


Figure 2.2: Classification scheme of the standard single-sided auction types

An equally important criterion for the classification of auctions is the type of object, and more specifically the structure of bids. Traditionally, microeconomic theory distinguishes between homogeneous and heterogeneous goods. In the common standard auction formats for single-sided auctions presented above, it is assumed that the object being auctioned is a single-item, single-unit and single-attribute object. For the actual design of an advanced auction institution, one needs to consider generalizations of the considered object – the rules depend on different dimensions of the object being traded (Bichler et al. 2002a,b). The auction types deployed are determined by the issues of the object: (i) multiple homogenous objects are auctioned in multi-unit auctions (Vickrey 1961; Kalagnanam and Parkes 2003), (ii) multiple heterogenous objects are auctioned in multi-item auctions (de Vries and Vohra 2003; Pekec and Rothkopf 2003), and (iii) objects with multiple attributes in addition to the attribute "price" are auctioned in multi-attribute auctions (Bichler 2001; Kalagnanam and Parkes 2003).

The efficient allocation of resources is common to all advanced auction mechanisms. One major factor influencing the complexity of this task is bid representation. Bid representation has not been an issue in auction research until now. "More elaborate bid representation schemes enable higher expressiveness and flexibility in an negotiation at the expense of increased complexity of bid evaluation" (Bichler et al. 2002b). For example, offers in multi-attribute auctions are presented as sets of attribute-value pairs, whereas combinatorial bids allow bids on packages of objects. The bidding language specifies the bids which are provided and is defined in the following.

Definition 2.2.4 Bidding Language
The bidding language specifies the bid space, i.e. the bids which can be expressed within an auction.

In essence, the bidding language specifies the syntax and semantics for bids and determines which bids can be expressed and how efficiently (Nisan 2000). Thus, a bidding language should be (i) expressive, i.e. able to express any desired vector of bids, and (ii) simple enough to understand and work with as well as simple in handling the allocation. The bidding language is closely related to the auction format and thus to the structure of the objects being auctioned (Kalagnanam and Parkes 2003). However, from a computational perspective, the winner determination problem, formulated as an optimization problem, can become computationally complex depending on the market and bid structure. Thus, by specifying the auction institution, the choices made for the bidding structure may have an impact on the desirable economic and computational properties (Kalagnanam and Parkes 2003).

2.2.3 Computerization of negotiations and auctions

Information technologies provide the means to transport information over space and time; they have established a universal service for representing and communicating information. Electronic commerce has highly benefited from advances in information technologies: it uses the Internet for purchasing and selling goods and services, including service and support after the sale (Kauffman and Walden 2001). Hence, the Internet is a medium for business transactions.

The rapid development of these technologies has affected the use of negotiations and traditional auctions – innovative negotiation protocols and auction institutions based on new technological possibilities have been applied to complex negotiation situations. In particular, electronic negotiations take advantage of this technological progress. Their processes are fully or partially conducted using electronic media, e.g. negotiation support systems, decision support systems, knowledge based systems, and systems for communication support. As electronic media provide the basis for information processing activities and communication activities, they enable negotiators to communicate and coordinate their activities via electronic channels. These electronic channels have to be thoroughly designed, developed and introduced into a functioning system. In traditional negotiations the medium need not be specifically designed to help the negotiators and support the communication process. In this case, a medium is the platform where transactions are coordinated through agent interactions (Ströbel 2003; Ströbel and Weinhardt 2003). The *media reference model* (MRM) presents a framework for the definition of a medium (Lechner and Schmid 2000; Schmid and Lechner 1999). It is described by a channel, transporting information, a language employed in communication and an organization describing the roles of the agents and the protocol defining all allowed interaction activities.

Definition 2.2.5 Medium
A medium facilitates transactions through (1) a channel system to process and communicate information over space and time, (2) a logical space which determines the syntax and the semantics of the information, and (3) an organization which describes the agents' behavior (their roles) and the interaction among the agents (protocol) (Klose et al. 2000).

According to the definition of medium, Bichler et al. (2003) define *e-negotiation media* as "information systems comprising electronic channels that process and transport data among the participants involved in a negotiation and provide a platform where transactions are coordinated through agent interaction. They implement the rules of communication in a negotiation protocol." The e-negotiation protocol is a model of the electronic negotiation process, governing all the rules concerning the process, permissible activities, their sequencing and timing as well as information exchange. An electronic negotiation itself can be defined as a negotiation where the interaction and information exchange occurs via electronic media (Bichler et al. 2003; Kersten 2003; Ströbel 2003):

Definition 2.2.6 Electronic Negotiation
Electronic negotiation (e-negotiation) is the negotiation process that is fully or partially conducted with the use of electronic media.

Electronic negotiation processes can be considered from two perspectives: (1) information processing and (2) interaction and communication activities (Kersten 2003). The first perspective focuses on the construction, implementation and use of models and systems to process information; the second perspective is related to electronic media. The processing and storage of information and production of knowledge is enabled by the use of electronic media. Moreover, all communication is performed with electronic (digital) channels that transport data. Thus, in electronic negotiations the design of the electronic medium and the relationship to other related components gain importance. The electronic medium has to be constructed for the specific purpose of facilitating the communication activities.

When considering electronic auctions, these can be defined as auctions which use electronic media for the exchange of bids.

Definition 2.2.7 Electronic Auction

An electronic auction (e-auction) is an auction which is fully or partially conducted with the use of electronic media.

The processes which are supported by electronic media range from the design and creation of an electronic auction to the bidding process, the winner determination as well as the determination of prices. The use of electronic media enables more complex bidding processes, allowing bids over multiple, homo- or heterogenous objects, and even bids on single or multiple qualitative issues. In the case of multiple qualitative issues, the measured utility indicates preferences over combinations of issues instead of prices. These innovative auction protocols extend the capabilities of standard auction protocols and allow dealing with common negotiation situations.

Through electronic media, bidders have access to auctions and can submit their bids from nearly every location. Participation occurs independent of the location: bidders do not have to meet at a certain place (or a certain time), thereby allowing a large group of bidders and potential buyers to be reached (Ehrhart and Ott 2003). Thus, electronic auctions are no longer constrained to physical locations. Furthermore, they are more flexible concerning duration or timing of bids (Lucking-Reiley 2000).

An example of electronic auctions are online auctions that use the Internet as electronic medium. When considering for example the consumer-to-consumer (C2C)[11] sector, the most common and popular auction listings in the Internet are offered by eBay[12], Amazon[13], or Yahoo![14]. These companies have announced their own auction platforms or auction-listing services which provide various trading mechanisms for selling new or used items from consumer to consumer. Auction sites sort their auction listings by categories such as "Collectibles", "Electronic&Computers", "Books", etc. and even subcategories such as "Stamps", "Coins", or "Toys" under the "Collectibles" category. A seller can easily deploy an auction and assign it to one of the auction listings in a selected (sub)category. Search engines and hierarchies of auction listings make it convenient for bidders to find the items they are looking for. Additionally, the auction listings help the seller to reach a large group of bidders or enter into new markets.

The eBay auction, the Amazon auction and the Yahoo! auction employ the same proxy bidding mechanism, i.e. an English auction with proxy-bidding. Differences can be observed

[11]The participants in an auction are private sellers and buyers, which is what is meant by the expression consumer-to-consumer (Ehrhart and Ott 2003).

[12]See http://www.ebay.com.

[13]See http://www.amazon.com.

[14]See http://www.yahoo.com.

particularly in the list of bid increments, the ending rules as well as particular auction features such as Amazon's First Bidder Discount or Take-it price, or eBay's Buy-it-now feature. Theoretical and empirical evidence concerning bidder behavior and strategies in online auctions have been presented by e.g. Ariely et al. (2002), Ockenfels and Roth (2002), and Ockenfels and Roth (2006). In auctions with a hard close a great activity of bidding at the very last moment before the auction ends is observed. That is, bidders refrain from bidding as long as there is time for rival bidders to react, thus avoiding a bidding war which might raise the final transaction price. Ockenfels and Roth (2006) analyze such sniping effects: in the eBay auction with hard close, the frequency of late bidding is higher compared to the soft close auction on Amazon.[15]

Lucking-Reiley (2000), Gupta and Bapna (2002), and Ehrhart and Ott (2003), for example, review auction mechanisms that are generally used for business transactions on the Internet. The auction mechanisms range from the standard auction types to advanced auction types such as multi-unit auctions, multi-item auctions (combinatorial auctions), multi-attribute auctions or reverse auctions. These advanced auction types are used for transactions in the business-to-business (B2B) sector. Moreover, auctions have been compared to posted price mechanisms for the sale of identical items or name-your-price mechanisms, both of which are used for selling via Internet.[16]

A key factor of auctions using information technologies is "the potential for achieving higher efficiency" than traditional auctions (Gupta and Bapna 2002). Electronic auctions improve efficiency in two ways (Milgrom 2000): (i) electronic auctions enable bidders to discover prices for (unique or rare) objects quickly and at low cost and (ii) electronic auctions may respond more quickly to changes in supply and demand. However, electronic auctions have also their limitations: the inability to see the object physically, the objects are described

[15]In their strategic model, Ockenfels and Roth (2006) give an example of an equilibrium strategy in eBay auctions with private values. Moreover, in the strategic model there are other equilibriums yielding even higher expected payoffs than the equilibrium presented by Ockenfels and Roth (cf. Seifert 2006).

[16]Auctions as competitive bidding procedures are dynamic pricing mechanisms, meaning that the price of an item emerges dynamically throughout the negotiation process. Traders of both sides (demand and supply) compete against each other by bidding on the item. Posted price mechanisms are static mechanisms that price out items in advance and do not provide a competitive bidding procedure. Posted price mechanisms offer the item at a fixed price, the posted price or take-it-or-leave-it price: the seller posts offer prices for the item and the buyers respond by taking or leaving the item at the announced price (Wang 1993). In name-your-price mechanisms consumers are asked to set the price for the good being offered. If the named price matches or exceeds the price set by the seller, the named price is accepted and the transaction is completed. Otherwise, the consumer's price is rejected. Such a mechanism is also called reverse pricing: agents are asked to name their price without an explicitly available reference point (Chernev 2003).

on-line by electronic images or text descriptions, and the possibility of fraud, e.g. no transaction of object and payment, shill bidding or collusion (Levine 2005; Lucking-Reiley 2000; Turban 1997).[17]

Therefore, researchers have recently started to model aspects of trust and reputation in auction mechanisms (cf. Brandt 2003; Rolli et al. 2006). For both, buyers and sellers, reputation mechanisms in the form of feedback and rating systems have been developed.

Electronic auctions have proliferated on the Internet, especially for use in business transactions; their usability for complex interactions has increased and they are more suited for traditional negotiation situations. But, as Wurman (2004) states, "the successful deployment and operation of an online auction system requires knowledge of mechanism design, system architecture, and successful Internet business practices."

2.2.4 Comparison of negotiations and auctions

The question "Are all e-commerce negotiations auctions?" was answered with "no" by Kersten et al. (2000). The authors argue that electronic auctions are an important vehicle in conducting business transactions and can be viewed as negotiations. But at the same time they state that "there is more to negotiation than can be addressed within auction frameworks" (Kersten et al. 2000).

Traditionally, negotiations are applied to situations where the creation of an object's value by competition or cooperation is the major objective. The participants negotiate about a single issue or multiple issues of one or more well, partially, or ill-defined objects. Moreover, negotiations occur between parties and are either (i) bilateral, (ii) multi-bilateral, or (iii) multilateral negotiations.[18] For example, multi-bilateral negotiations are common in business.

Auctions play a major role in situations where the determination of value is the main objective. Traditionally, auctions are resource allocation and price discovery mechanisms for standardized and well-defined objects. The determination of price is solely based on the bids submitted by the bidders during the bidding process – there is no other input. Hence, the

[17] Shill bids are "false-name bids" by a buyer with a false identity. In situations (such as Internet auctions) where the auctioneer cannot completely determine the identities of bidders, bidders can profit by submitting additional bids under false identities (Ausubel and Milgrom 2002). In most cases of collusion, buyers form coalitions such as bidding rings whose members agree not to bid against or outbid each other. They avoid the auction or place phantom bids.

[18] A bilateral negotiation means that two parties who compete and/or cooperate in order to achieve a compromise are involved. Multi-bilateral is defined as one party simultaneously engaged in multiple negotiations with selected counterparts. Multilateral involves more than two parties engaged in the process (Bichler et al. 2003; Ströbel 2003).

rules of the auctions are defined a priori and known to each participant; during the auction the rules are fixed. In most traditional auctions, the object to be auctioned is a single-item, single-attribute and homogenous object, and the standard auction types of single-sided and double-sided auctions focus on a single issue of the object: the price.

Auction theory interprets negotiations as price-discovery mechanisms for markets. Moreover, through the use of auctions in electronic commerce, an auction-centric perspective on negotiations is created such that every structured message exchange used in negotiations is regarded as an auction (cf. for example to Beam and Segev 1997; Benyoucef et al. 2000; Wurman et al. 2001; Bartolini et al. 2005). In electronic commerce, business transactions deal with complex objects and entailing circumstances such as interpersonal dynamics, social factors, cultural backgrounds etc. (Kersten et al. 2000). The use of information technologies enables the trading of complex objects such as multi-item, multi-attribute or multi-unit objects, applying innovative auction protocols. These protocols have extended the capabilities of traditional auctions to handle negotiation situations. The difference between auctions and negotiations begins to diminish with the presence of two or more issues. Nevertheless, different types of negotiation protocols are needed to account for the developments in information technologies, their implications for the negotiation process themselves, as well as personal relationships and social factors. "The presence of two or more issues begins to blur the difference between auctions and negotiations. [...] while auctions can be viewed as negotiations, there is more to negotiation than can be addressed within the auction framework" (Kersten et al. 2000).

In Table 2.1 characteristics of traditional auctions, traditional negotiations and electronic auctions are compared. The characteristics correspond to e.g. the participants, the objects and their issues, the communication process, the information exchange as well as the protocol (Bichler et al. 2003). However, by moving traditional negotiations and auctions online, both converge towards each other.

2.3 Electronic markets

The recent development in information technologies increased the number and functionality of information systems that involve organizations. Basic functions that are common to inter-organizational information systems are (1) input functions that accept input data from outside the system, (2) storage functions that retain input data and retrieve stored data, (3) processing functions that calculate and manipulate the input and stored data in other ways, and (4) output functions that produce processing results for use outside the system. Such inter-organizational

Characteristics of negotiations and auctions

Characteristic	Traditional Auction	Traditional Negotiation	Electronic Auction
Number of participants	multi-bilateral, single or double-sided	bilateral, multilateral or multi-bilateral; arbitrary number of sides	multi-bilateral, single or double-sided
Participation	open or restricted	restricted	open, restricted or rule-defined
Consensus required	bid-taker and selected bidder	selected or for all participants	selected participants
Number of objects	single, homogenous	single or multiple, homo- or heterogenous	single or multiple, homo- or heterogenous
Number of issues	single	single or multiple	single
Issues structure	well-defined	well-defined, partially defined, or ill-defined	well-defined
Offer space	fixed	may be unknown and modified	fixed
Exchange and knowledge of offers and concession-making	yes	yes	yes
Logrolling (conditional concessions)	no	yes	yes
Knowledge of offers and concessions	public or private	private (rarely public)	public or private
Exchange of opinions, arguments, threats	no	yes	no
Interdependence	between bid-taker and bidders (single-sided) or between but not within sides (double-sided)	full interdependence except multi-bilateral negotiations	between bid-taker and bidders (single-sided) or between but not within sides (double-sided)
Protocol	a priori defined, explicit and fixed	well-defined or partially defined; explicit or implicit	a priori defined, explicit and fixed
Competition versus cooperation	competition among bidders on at least one of the possibly two sides; cooperation prohibited	competition or cooperation among the agents	competition among bidders on at least one of possibly the two sides; cooperation prohibited
Process control	defined a priori	ill-defined, modifiable by participants	defined a priori

Table 2.1: Characteristics of negotiations and auctions (Bichler et al. 2003)

information systems (IOS) are used to characterize electronic markets (Bakos 1991). Bakos describes electronic markets as "inter-organizational information systems that allow buyers and vendors to exchange information about prices and product offerings." The definition given in Levecq and Weber (2002) states that "electronic markets are based on technology and are highly automated, providing different types of services for investors." Common to these definitions is that an electronic market carries out a market with technical aids to fulfil the needs of buyers, sellers and other information carriers concerning information dissemination and transaction. Electronic markets support the transaction processes mentioned above, enabling multiple buyers and sellers to interact, and provide additional services and tools. A transaction is considered as the exchange of objects between sellers and buyers. In particular, the ownership of objects is transferred from one agent to another and vice versa (Ströbel 2003). Thus, the goal of a transaction is to initiate, arrange and complete an agreement for an efficient exchange of objects. Hence, the number of agents engaged in a transaction is limited and the number of interaction processes between the agents corresponds to a finite number.

Definition 2.3.1 Transaction
A transaction is the exchange of objects between two agents. In a transaction the property rights of objects are transferred between the engaged agents (Ströbel and Weinhardt 2003).

An electronic medium which facilitates the transaction of objects between agents constitutes an electronic market (Ströbel 2003; Ströbel and Weinhardt 2003). The electronic market allows the agents to exchange information, goods, services, etc. according to pre-specified rules or protocols. The main functions are the same as those of a market: (i) matching buyers and sellers, (ii) facilitating the exchange of objects, and (iii) providing an institutional framework that enables the efficient functioning of the market (Bakos 1998). A key characteristic of electronic media and thus of electronic markets is that they are independent of time and space, as well as being ubiquitous and available globally (Schmid and Lindemann 1998). Furthermore, both human and software agents have access to electronic markets and can participate in the transaction. The market institution defines the coordination mechanism for the exchange of objects as well as the information and communication process. Thereby, the distinct phases of the electronic transaction are supported by electronic media and therefore electronic market services.

Definition 2.3.2 Electronic Market
An electronic market is a market that uses electronic media for transaction. The phases of transaction are fully or partially supported by electronic media.

A transaction can be grouped into logical sub-processes. These sub-processes form the phases of a transaction. Schmid and Lindemann (1998) and Schmid (1999) identify four interaction phases of an electronic transaction: (i) the *knowledge phase*, (ii) the *information phase*, (iii) the *agreement phase* , and (iv) the *settlement phase*. Based on the knowledge and information received in each phase, agents choose different actions.

(i) In the *knowledge phase* agents gather information about the characteristics of the object and the profiles of engaged agents, as well as trade conditions and juridical aspects. Based on their knowledge, agents advertise their willingness to interact about an object to a target group of potential transaction partners (Ströbel 2003).

(ii) In the *intention phase* agents specify their intention to buy or sell based on their individual supply and demand function. For instance, agents submit their offers.

(iii) In the *agreement phase* the terms and conditions of the transaction are negotiated. A successful negotiation results in a consensus or mutual agreement, leading to a deal.[19] A legal-binding contract, determines the conditions of the consensus and the involved parties and manifests the agreement.[20]

(iv) In the *settlement phase* the agreed-upon contract is executed according to the conditions determined including all processes (e.g. exchange of goods and money) and negotiated services (e.g. delivery services, warranties).

The agreement process represents the complete agent interaction in the intention and agreement phases. The agreement is based on the bids (offers and counter-offers) submitted by the agents; the bids are the interface between the intention and the agreement phase. The agreement process follows the implemented protocol – this can either be a negotiation or an auction protocol. In both cases the goal is to achieve a mutual agreement and execute the contract made. Note that an agreement can also be reached without a negotiation process, e.g. if agents merely accept the bids of their counterparts (Ströbel and Weinhardt 2003).

Both, traditional and electronic markets, use media to facilitate transactions deploying negotiation or auction protocols. The facilitation of information exchange, the negotiation about an object, the finding of an agreement, the settlement of a transaction, and lastly the economic exchange are major purposes and benefits of markets, which are independent of the underlying medium (Strecker 2004).

[19] A deal is the result of a mutual agreement among agents for the exchange of objects (Ströbel 2003).

[20] The legal aspects and conditions of a deal are fixed in a contract. A contract is the legal basis for the deal and specifies the conditions, the involved parties, the object of interest, and the rights and responsibilities of contract partners (Schmid 1999).

2.4 Methods for the design and analysis of electronic markets

Game theory, the theory of mechanism design, and experimental economics are basic economic methodologies used to design and analyze electronic markets. Most research in game theory focuses on the interaction of groups or rational agents. From an economic perspective, "game theory is the part of economics that deals with the rules of the game that define market operations" (Roth 1999). In particular, market institutions can be modeled by methods from game theory, e.g. auctions can be described as games with incomplete information. Closely connected to game theory is mechanism design theory. While in game theory the rules of the game are taken as given, the "rules of a game" in mechanism design theory should be designed (Levine 2006). Mechanism design regards the consequences of different rule types: Firstly, assumptions concerning agents' preferences, behavior, and information are made, secondly, the mechanism allowing agent strategies to produce outcomes is designed. The mechanism should thereby fulfill desirable economic properties and achieve a desired outcome. The standard approach to mechanism design is to formulate the design problem as an analytical optimization problem subject to the assumptions made. By introducing the *revelation principle*, mechanism design is reduced to the optimization of *incentive-compatible direct* mechanisms. Nevertheless, there is no guarantee that the mechanism design problem will be solved or that the desired properties will be achieved. Mechanism design can fail due to e.g. problem difficulty or computational considerations (Kalagnanam and Parkes 2003). Therefore, methods for testing and evaluating the designed mechanism are necessary. Experimental methodologies are one example for testing theoretical predictions and the robustness of mechanisms to unmodeled human behavior. In particular, laboratory experiments with human subjects allow for the control of carefully designed variables, the observation of predicted behavior and the outcome for the designed mechanism.

2.4.1 Game theory

Game Theory forms the basis for various areas of research, such as mechanism design theory, auction theory, experimental economics, negotiation analysis etc. In essence, game theory is a formal way of analyzing interaction between rational agents who behave strategically.[21] One way to describe a game is by listing the agents participating in the game, the possible choices

[21] A *rational* agent chooses his best action to play in a game. Note that the aim of a rational agent is to maximize his payoff.

(called actions, strategies) available to each agent and an (expected) utility of each agent which agents strive to maximize.[22]

Definition 2.4.1 Game
A game $\Gamma = (N, \Sigma, u)$ *is defined by*

(i) a set of $N = \{1, \ldots, n\}$ *agents,*

(ii) the strategy space $\Sigma = \Sigma_1 \times \ldots \times \Sigma_n$ *where* Σ_i *is the strategy space for each agent* $i \in N$*, and*

(iii) the payoffs of all agents $u : \Sigma \rightarrow \mathbb{R}^n$*,* $u = u(s) = (u_1(s), \ldots, u_n(s))$*, where agent* i*'s payoff functions is given by* $u_i : \Sigma \rightarrow \mathbb{R}$ *with* $u_i(s) = u_i(s_i, s_{-i})$ *for each strategy-profile* $s = (s_1, \ldots, s_n) \in \Sigma$*.*

The strategy choices of all agents except agent i is denoted with $s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n) \in \Sigma_{-i}$ where Σ_{-i} is the strategy space of all agents but agent i. In a game the agents' decisions determine the outcome, and players having preferences over the outcome strive to maximize their payoff. In particular, as an agent's payoff is influenced by the strategies of the other agents as well as by his own strategy, agents do well in predicting each other's actions.

Definition 2.4.2 Dominant Strategy
Given a game $\Gamma = (N, \Sigma, u)$*. Agent* i*'s,* $i \in N$ *strategy* $s_i^* \in \Sigma_i$ *is called a dominant strategy of agent* i *if and only if strategy* s_i^* *is the best response to any strategy profile* $s_{-i} \in \Sigma_{-i}$ *the other agents may play:*

$$\begin{aligned} & u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}) \quad \forall s_i \in \Sigma_i, s_{-i} \in \Sigma_{-i} \\ \exists s_{-i} \in \Sigma_{-i} : \quad & u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_i \neq s_i^*, s_i \in \Sigma_i \end{aligned}$$

One property of a dominant strategy is that it maximizes the agent's payoff no matter what the strategies of other agents are (Fudenberg and Tirole 1991). Note that in mechanism design literature, a dominant strategy is formulated in a weaker sense: Strategy s_i^* is a dominant strategy if it (weakly) maximizes the agent's expected utility for all possible strategies of other agents, $u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$ for all $s_i^* \neq s_i, s_{-i} \in \Sigma_{-i}$ (Jackson 2003; Parkes 2001).

A solution concept to a game is given by an equilibrium, a strategy combination consisting of a best strategy for each player. Solution concepts compute the outcome of the game (the

[22] In general, more than one decision maker exists in a game (also called player); if there is only one decision maker, the game becomes a decision problem.

payoffs) with self-interested agents. The most well-known solution concept is the Nash equilibrium, a profile of strategies such that each agent's strategy is a best response to the other players' strategies. A stronger solution concept is the equilibrium in dominant strategies. It is a strategy combination consisting of each player's dominant strategy and thus makes no assumptions about the available information agents may have about each other. Beliefs about the rational behavior of other agents in selecting one's own strategy are not required. The following definitions are according to Fudenberg and Tirole (1991):

Definition 2.4.3 Nash Equilibrium and Equilibrium in Dominant Strategies
Given a game $\Gamma = (N, \Sigma, u)$*. A strategy profile* $s^* = (s_1^*, \ldots, s_n^*) \in \Sigma$ *is called*

(i) a Nash equilibrium if and only if $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \quad \forall s_i \in \Sigma_i \, \forall i \in N$

(ii) an equilibrium in dominant strategies if and only if s_i^* *is a dominant strategy* $\forall i \in N$

A third solution concept is the *Bayesian-Nash* equilibrium. It is a Nash equilibrium in a game of incomplete information.[23] In a game with incomplete information at least one agent is uncertain about the other agents' types. These types are private information only known by the agents themselves. Every agent has some beliefs about the types of agents given by a probability distribution of the agent types. This may also be expressed by nature's move right at the beginning, choosing the agents' types with a certain probability. Thus, every agent has incomplete information about the agents types, i.e. the strategies of the agents as functions of their types, information partition and agents' payoff functions.

In an Bayesian-Nash equilibrium, every agent selects a strategy to maximize its expected payoff in equilibrium given the expected utility-maximizing strategies of other agents. In equilibrium, agent i's strategy is a best response to the distribution over strategies of other agents, given distributional information about the types of other agents.

In game theory, two leading frameworks are distinguished: (1) *cooperative* game theory and (2) *non-cooperative* game theory. Cooperative game theory analyzes optimal strategies for agents and assumes that agreements between them can be made and cooperative behavior

[23] In game theory the following information concepts are distinguished: Information is *common knowledge* if it is known to all players and each player knows that all of them know, that all of them know, and so on. Information is *complete* if nature does not move first or her initial move is observable by all agents. It is *incomplete* if nature moves first and her move is not observed by at least one agent. In a game with *perfect* information each player has knowledge about previous actions and thus knows at which stage he is in the game. No moves are simultaneous and even nature's moves are observable. Incomplete information implies *imperfect* information. Note that *nature* in a game is considered a non-person player which chooses actions at a certain point in the game with a certain probability.

may be enforced. A cooperative game is a competition between coalitions of players rather than between individual players. Non-cooperative game theory involves games in which players can cooperate, but any cooperation must be self-enforced.
Within the non-cooperative literature, normal form games (static) and extensive form games (dynamic) are distinguished. Focussing on games with *complete* information, a game in normal form is also called a game in strategic form. In particular, agents choose their actions simultaneously and their payoff functions are common knowledge. An extensive form game models dynamic situations and analyzes the dynamics of a game. A game in extensive form specifies the complete sequence of actions played by each agent, i.e. each agent's path through the tree, the complete list of agents' payoffs, and the available information at each node in the tree.

Auctions are examples of games with incomplete information. The rules of the auction fix the rules of the games – they are common knowledge to the bidders. Auctions can be described as an extensive form game where the rules of the auction, the number of sellers and buyers, the bidders' types as well as any choices by nature determine the extensive form (Wilson 1991). The rules as well as the bidding process determine the available information to each bidder. If all information is common knowledge, the types of the bidders, the information partition as well as the bidders' payoff can be specified, i.e. nature's moves can be observed by all bidders. However, in principle, the types of the bidders, e.g. their preferences, characteristics, endowments, are rarely known by other bidders. Thus, auctions are studied as extensive games under the constraint that nature chooses an assignment of types for bidders that is not observable by all agents – information is incomplete. For example, a static single-item auction might be described by a game tree with simultaneous moves by all bidders, corresponding to a normal form representation. The number of bidders might be common knowledge, whereas the valuations of the bidders assigned to the item are private information. The actions of the bidders are bid functions based on their valuations.

2.4.2 Theory of mechanism design

The design of mechanisms may be described as studying the design of institutional rules under the assumption that each agent behaves according to his own goal while having only private information. Consumers have information about their respective preferences, producers about their technologies, and resource holders about the resources. According to Hurwicz (1973) mechanisms should guide the agents towards actions which are at least feasible and should have certain desired properties such as allocative efficiency. The main difficulty lies in the proper integration of information and incentives: agents have different beliefs which reflect differences in their information and may not reveal their true valuations. Mechanisms need to employ incentives or enforcement schemes such that agents share their private information (Hurwicz 1973). Mechanisms that apply such concepts are called *incentive-compatible*.

In essence, a mechanism is a set of rules that governs agent interaction (Milgrom 2004). The design of the institutional rules through which the agents interact can have a profound impact on the results of that interaction (Jackson 2003). Once the rules of the mechanism and the designer's objective have all been specified, the designer applies a solution concept to predict the outcome and evaluates that outcome (cf. Milgrom 2004; Hurwicz 1987). In a mechanism, each agent has a message (strategy, action) space, and the outcome of the mechanism results as a function of the messages chosen.

Definition 2.4.4 Mechanism

Given a list of n agents $i \in N = \{1, \ldots, n\}$. A mechanism $M = (\Sigma_1, \ldots, \Sigma_n, g(\cdot))$ is composed of two elements: the defined set of messages, and an output function $g(\cdot)$ such that the mechanism

(i) defines for each agent i a family of messages (actions) Σ_i available with action $s_i \in \Sigma_i$,

(ii) provides the outcome $g : \Sigma_1 \times \ldots \times \Sigma_n \rightarrow O$ with $g(s) = g(s_1, \ldots, s_n)$ for the strategy profile $s = (s_1, \ldots, s_n)$.

This definition of a mechanism is based on Parkes (2001).[24]

Focussing on auctions as mechanisms, the message space are bids, the outcome is the allocation determining who gets the object, and the payment rule specifies how much each bidder pays – it is a function of the submitted bids.

The main objective of mechanism theory is to have a systematic look at the design of institutions and how these affect the outcomes of interaction (Jackson 2003). Mechanism design focuses on the institutional design under certain objectives and considers the consequences of different rule types (Bichler 2001). The basic assumptions are that (i) agents behave strategically and (ii) agents base their behavior and decisions on their private information at hand. An ideal mechanism provides agents with a dominant strategy and implements a solution to the allocation problem.[25] The solution of the allocation problem, also called winner-determination

[24] For similar definitions of a mechanism please refer to Nisan and Ronen (2001) or Jackson (2003). Nisan and Ronen (2001) define a mechanism as a tupel of the output function $g(\cdot)$ and an n-tupel of payments $(p_1, \ldots, p_n)$ where the mechanism provides each agent with a payment $p_i = p_i(s_1, \ldots, s_n)$, $i \in N$.

[25] A dominant strategy maximizes the agent's expected utility, whatever the strategies of other agents are.

problem, is achieved through an algorithmic approach, determining the winner and the outcomes.

A mechanism design problem consists of two components: the algorithmic output and the specification of the agent's private objective (Nisan and Ronen 2001). The output specification is a function called the *social choice function* f, which selects the optimal outcome given the agents' types. Each agent $i \in N$ has some private input (its type) $\theta_i \in \Theta_i$. Everything outside its type is common knowledge. The function $f : \Theta_1 \times \ldots \times \Theta_n \rightarrow O$ chooses an outcome $f(\theta) = o \in O$. That is, the output specification f maps a set of allowed outcomes $o \in O$ to each type vector $\theta = (\theta_1, \ldots, \theta_n)$. The mechanism design problem involves designing a mechanism M, so that individuals interacting through the mechanism have incentives to choose messages as a function of their private types θ_i that leads to socially desired outcomes $o \in O$ (cf. Nisan and Ronen 2001; Jackson 2003). Agents choose their strategies to maximize their own selfish utilities, which can be influenced by payments to be made. Thus, the mechanism needs to ensure that the agent's utilities are compatible with the institutional rules. In other words, the mechanism design problem involves implementing "rules of a game" to implement the solution to the social choice function despite the agent's self-interest (Parkes 2001). Given a mechanism with the output function g, a mechanism implements a social choice function $f(\theta)$ if the computed outcome o is a solution to the social choice function: $o = g(s(\theta)) = f(\theta)$.

In general, the number of possible mechanisms, which for example allow multiple rounds of interaction or complex resource allocation, is very large. Principles are necessary in order to restrict the mechanisms to a smaller or particular set of mechanisms and simplify the search for best mechanisms. The *revelation principle* states that any mechanism can be transformed into an equivalent *incentive-compatible direct-revelation mechanism* that implements the same social choice function (Jackson 2003). To explain this principle the terms *direct mechanism* and *incentive-compatible mechanism* have to be clarified: In a direct (direct-revelation) mechanism each agent is asked simultaneously to report his type. The only action available to the agents is to make claims about their types. An incentive-compatible mechanism is a direct mechanism where each agent truthfully reports his type. Incentive compatibility implies that agents, behaving selfishly, choose to report their private information truthfully out of their own self-interest. The revelation principle allows the transfer of results established in the space of direct mechanisms to all mechanisms. An example of an incentive-compatible direct-revelation mechanism for the single-item allocation is the second-price sealed-bid auction (Vickrey auction). Actually, the Vickrey auction is a *strategy-proof* mechanism, meaning that truth telling is the most profitable strategy for each agent, no matter what the other agents' strategies are. Neither bidding above or below the true valuation of the object benefits the agent.[26]

One of the most prominent mechanisms in mechanism theory is the so-called Vickrey-Clarke-Groves (VCG), pivotal, or Groves mechanisms (Vickrey 1961; Groves 1973) for problems in which agents have quasi-linear utilities.[27] Groves mechanisms are efficient and strategy-proof. In fact, the Groves family of mechanisms are the only mechanisms that are allocatively efficient and strategy-proof amongst all direct-revelation mechanisms. A special type of VCG mechanism is the Generalized Vickrey Auction (GVA), which denotes the application of the VCG mechanism to combinatorial allocation problems (Vickrey 1961; Varian 1995).[28] The implementation of the GVA is a sealed-bid combinatorial auction. Nevertheless, Groves mechanisms have a few bad computational properties: (i) agents must report complete information about their preferences to the mechanism and (ii) the optimization problem is solved centrally based on the submitted preferences. Note that in combinatorial domains these burdens are difficult to overcome.

To summarize, mechanism design theory discusses several properties of mechanisms. These properties are helpful for designing market mechanisms. The following lists the most desirable properties of mechanisms (Parkes 2001):

(i) *Allocative efficiency*: An efficient allocation of resources maximizes the sum of individual profits.

(ii) *Strategy-proofness*: Achieving an allocative efficient allocation of the resources requires that all agents truthfully report their valuations. The direct mechanism should thus induce incentive compatibility, i.e. all agents report their preferences truthfully in equilibrium. In the optimal case, truth telling is a dominant strategy, since the agents have no incentive to untruthfully report their preferences in order to increase their individual utility. In this case, the direct mechanism is strategy-proof.

(iii) *Individual rationality*: Another requirement is that the agents voluntarily join the mechanism. This in turn requires that the profit the agents derive from participation is greater

[26]A strategy-proof mechanism is also called a dominant-strategy incentive-compatible mechanism (Parkes 2001). It is a direct-revelation mechanism where truth-revelation is a dominant strategy equilibrium.

[27]Common assumptions in mechanism theory are that agents are risk-neutral and have quasi-linear utility functions (Parkes 2001). Each agent i's preferences are given by a valuation function $v_i(\theta_i, x)$. Its quasi-linear utility will be $u_i = u_i(\theta_i, o) = v_i(\theta_i, x) - p_i$, where outcome o defines a choice x from a discrete choice set and a payment p_i by the agent. This is the utility the agent aims to maximize.

[28]The Generalized Vickrey Auction (GVA) is a generalization of the Vickrey auction involving more complex problems, e.g. combinatorial allocation problems (Varian 1995).

or equal to that from non-participation, since the agents would otherwise decide to opt out.

(iv) *Budget balance*: A mechanism is said to be strictly budget-balanced if the amount of prices sum up to zero over all agents. In this case funds are neither removed from the system nor is the system subsidized from outside. Strict budget balance is an important property since the resource allocation can be performed at no cost. In case the mechanism runs a deficit, the agents running a deficit have to be subsidized. Such a situation cannot be sustained for an extended time period.

(v) *Computational tractability*: Computational tractability considers the complexity of computing the outcome of a mechanism from the agents' strategies. With an increasing size in bids, the allocation problem can become very demanding. Thus, computational constraints may delimit the design of the proper mechanism.

The theory of mechanism design provides a theoretical toolbox for the design of institutions in yielding desired properties or a desired outcome. Mechanism design uses methods from economics and game theory to design the rules of interaction for economic transaction. In particular, mechanism design theory "bridge[s] the gap between theoretic microeconomic implications and practical applicability" (Neumann 2004).

2.4.3 Experimental economics

Experimental economics provides methods to test game theoretic models and observe behavior in a controlled environment. Experimental methods have been used in many research disciplines, ranging from physics and chemistry to psychology and economics. In economics the introduction of experimental methods was motivated by theories concerning industrial organization and market performance (Plott 1982). In essence, the methodology of experimental economics is twofold: first, to motivate behavior in laboratory economic environments whose equilibrium properties are known to the experimenter or designer, and second, to use the experimental observations to test predictive hypotheses derived from one or more formal or informal models of these equilibrium properties (Smith 2002). To be more precise, from a formal point of view, experiments are used to test theory. The tested theory consists of a set of axioms or assumptions and definitions, and the logical conclusions that follow from them (Friedmann and Sunder 1994). The aim is to test whether the theory is formally valid and internally consistent and if conclusions are provable from the assumptions. The methodological ideal of experimentalists is to derive a testable hypothesis from a well-specified theory, to implement experiments with a specific design and contextualize specific auxiliary hypotheses (Smith 2002). However, the primary purpose of scientific experiments is to find regularities in the observed data or behavior within various environments and see which theories best fit these regularities. To test theoretical models, economists map the model in a laboratory experiment that captures the essence of the relevant theory. This mapping requires the experimenter to design institutional details, i.e. the degree of information provided in the instructions, the way information is presented, whether valuations are induced to the subjects, the communication allowed between subjects, etc. Such institutional details are important to design since they might affect the result of the experiment. For instance, experiments are applied in order to analyze market mechanisms such as auctions and one-to-one bargaining situations, to better understand and improve the features of market mechanisms, and to test newly-designed market mechanisms before introducing them as operating markets.

In essence, experimental work "[. . .] includes experiments designed to test the predictions of well-articulated formal theories and observe unpredicted regularities in a controlled environment that allows these observations to be unambiguously interpreted in relation to theory" (Kagel and Roth 1995). Controlled environment means that the experimenter has complete information about the economic data and that the institutional rules as well as the informational conditions are under the experimenter's control. Only subjective aspects of agents, such as agent's risk attitude, cannot be controlled.

Experiments are based on the principle of varying independent variables while holding all other influences constant. The objective then is to measure the effect of the variation of the variables. Important variables are controlled, that is that they are held constant at a convenient level. Such variables are also called *treatment* variables. By varying all treatment variables independently, the clearest possible evidence on their effects is obtained (Friedmann and Sunder 1994). Focussing on experimental economics and laboratory resource allocation experiments, the market institution appears as treatment variable while the market environment is kept constant (Smith 2003). Smith (2003) introduces microeconomic system theory as a conceptual framework in conducting market experiments. The market institution is controlled by imposing and enforcing institutional rules on the experimental subjects; the market environment, i.e. the agents' characteristics, knowledge endowments, and message behavior cannot be observed and thus cannot be directly controlled. Therefore, incentives schemes in the form of a monetary reward structure are imposed on the agents to control their characteristics. In particular, in order to achieve control over the agents' characteristics and thereby achieve a controlled environment, the reward function must satisfy the conditions of non-satiation (monotonicity), saliency, dominance and privacy (Smith 2003).

One natural question in experimental theory is whether replicable results from experiments are transferable to field data, or data which is derived from field observations. It is a question of whether the general principle of induction is applied in experimental research: observed behavioral regularities will persist in new situations as long as the relevant underlying conditions remain substantially unchanged (Friedmann and Sunder 1994). Propositions about behavior and performance of institutions that have been tested in one laboratory microeconomy also apply to other (laboratory) microeconomies where similar conditions are found (parallelism).

In laboratory experiments scientific data is achieved through controlled processes that should be replicable by other experimentalists. The interest in data replicability stems from the desire to answer the question "Do you see what I see?" (Smith 1987). This question confers on three aspects which should be fulfilled for experiments that other experimenters successfully replicate. Experimenters should be able to reproduce the result of the original experiment, i.e. the observations made, the way the observed data is interpreted, and the conclusions drawn should be the same. Documentation standards that have been developed to enhance the replicability of experiments comprise four aspects: (i) subjects, i.e. the scripts (instructions) handed out to subjects that supply descriptions of players, their action choices, and the possible payoffs, (ii) the laboratory environment, e.g. copies of the deployed software and descriptions of hardware used, (iii) raw data, e.g. copies of all valid data received in the experiment, and (iv) data processing, e.g. keeping records of specific procedures used for data analysis (Friedmann and Sunder 1994).
Replicability and control are two major means which support the attempt to reduce errors in the common knowledge of economic processes.

Nowadays, experiments are widely used in game theory, finance, and e-commerce. Particularly in the field of e-commerce, the experimental analysis of an electronic market is only possible if the electronic market institution is implemented in a running information system. Such an information system is often represented by a workable prototype which helps in conducting laboratory experiments. However, in dealing with these unsolved questions, laboratory experiments provide the best way to test a theory and explore the effects of variables that are difficult to observe, measure or control without experimental analysis.

Roth (1991) states that bringing together the knowledge concerning practical questions on market systems, the appropriate design of mechanisms for price formation such as auctions, etc. is the most demanding task in the long run. Experiments help in learning about economic environments as a function of size and complexity as well as the robustness of these environments, and in understanding which kind of environments facilitate which kind of learning.

2.5 Market engineering and computer-aided market engineering

"Markets evolve, but they are also designed" (Roth 1999). Designing electronic markets has become an important issue for electronic commerce.[29] Unlike traditional markets, electronic markets are supported by electronic media; they must be consciously designed since they are limited by the technical infrastructure.

Although there are many scientific approaches for analyzing and designing market institutions, a solid engineering practice for electronic markets is essential. An understanding and deep knowledge of various research disciplines such as economics, computer science and jurisprudence is necessary as these disciplines are at least indirectly involved in the creation, design, evaluation and introduction of electronic markets (Roth 1999). So far, there is little knowledge on which institutions are suitable for certain situations or how the outcome of an electronic market should be measured and evaluated. Furthermore, as Roth (1999) points out, the practical design of electronic markets has to deal with complexities, mainly of the economic environment itself, and the participants' strategic behavior. Dealing with such complexities requires more than simply attention to the institutional rules of a market. Furthermore, additional methods and tools from other disciplines are needed to supplement traditional approaches. For example, experimental and computational economics are supplementary theories that help in understanding complexities and show how to deal with them.

Economic design has become an engineering task (Varian 2002). More and more, an economist is regarded as an "engineer" (Roth 2002; Varian 2002) who has extensive knowledge and a solid foundation in theory and methodology. "Economists are increasingly called out to give advice about how to design new economic institutions" (Roth 2002), as in the case of auctioning telecommunication spectrum licences in the US.

The approach of market engineering and computer-aided market engineering presented in the next sections is mainly based on contributions of Weinhardt et al. (2003), Neumann (2004), Holtmann et al. (2002) and Holtmann (2004).

2.5.1 Market engineering

Market designers face a multitude of unsolved issues while designing electronic markets. The main objective of designing (traditional or electronic) markets is to improve market efficiency.

[29]Market design or mechanism design is related to the design of markets and its rules; it is a sub-field of the design of economics. In essence, the design of economics involves designing and maintaining economic institutions (Roth 2002).

For example, markets need to be designed to maximize the bid-taker's revenue, send the right price signals, mitigate collusive behavior, provide precise and accurate information to all participants, or reduce entry barriers (Babin et al. 2001). These objectives are achieved through (i) the specification of the problem's structure, the process and further requirements, (ii) the specification of the institutional rules and feasible activities, as well as their sequencing and timing, (iii) the reasoning, whether the institutional design and information exchange satisfy the required properties and whether the form and content of information exchange has to be redefined (Bichler et al. 2003).

From an implementation point of view, an electronic market must guarantee efficient and reliable communication, provide safe and trustworthy exchanges, ensure the correctness and reproducibility of market decisions, and provide efficient computation of market decisions, among other aspects (Babin et al. 2001). Thus the person who develops and programs an electronic medium for a particular environment requires precise specifications to design and implement this system. The software engineer use traditional software engineering methods to design and implement an information system. Moreover, the requirements have to be clearly defined such that the software engineer has a clear understanding of the goals of the system, the functionalities, as well as the processes that have to be embedded.

In general, the development and implementation of information systems follows the software engineering approach that is based on two principles: it uses mathematical results to design and construct systems, and behavioral results that determine the needs and requirements of the users.[30] Software engineering makes use of methods, concepts, tools and procedures such as prototyping, rapid application development, and object orientation to create, build and deploy systems. Thereby, software engineering follows a process comprising three phases: (i) definition phase, (ii) development phase and (iii) maintenance phase (Pressman 2001). In the definition phase, the basic requirements of the system are defined. These requirements encompass the specification of the problem, what the system should do and which solution the system would benefit from, the identification of users' requirements, the identification of information to be processed and activities to be supported, and procedures to produce an outcome. The development phase includes the design and implementation of the procedures and functionalities to which the requirements are mapped. In the design phase, an answer to the question "how the system is doing it" is given. The system is designed from a high level to a more detailed level, meaning that the problem structures are broken down into components to which solutions can be applied. The result is a complete software code or program

[30]The objective of the engineering approach is to find solutions to practical problems in a systematic way with fundamental knowledge of mathematical and natural sciences (Pahl and Beitz 1984).

which has to be evaluated, tested and integrated in order to form a system. Additionally, in the maintenance phase the product is deployed and launched to an operating system.

Electronic markets are markets which use electronic media for transaction. In essence, electronic markets are information systems that process and transport data, and provide communication for agent interaction. Thus, "designing electronic markets is consequently also a software engineering task" (Holtmann et al. 2002). The engineering of electronic markets requires extensive knowledge of economics and computer sciences: the institutional rules of the electronic market must be implemented in an information system and result in a functioning system, such that several economic desiderata are attained. As the relationship between institutional rules, agent behavior and market outcome is hardly known, electronic markets require conscious design. The design of market institutions shifts from a pure science to engineering – market engineering (Weinhardt et al. 2003). The purpose of market engineering is "to develop economically founded approaches and methods that support the designers in facing the difficulties associated with the design problem" (Neumann 2004). Market engineering is a structured, systematic and theoretically founded approach towards the design and operation of electronic markets: (1) the design is directed towards the definition of all institutional rules and creation of an electronic market as well as its deployment; (2) the operation is directed towards the maintenance of an electronic market as operating system, and (3) the theoretical foundation is directed towards a deep understanding and knowledge of electronic markets (Weinhardt et al. 2003; Neumann 2004). The institutional rules of an electronic market comprise not only rules concerning the microstructure (market institution) but also the IT-infrastructure as well as the business structure (Holtmann et al. 2002; Holtmann 2004). The market structure (institutional rules) is a combination of these three perspectives which exist independently.

The main objective in market engineering is to solve the design problem, or to consciously design electronic markets. While designing the institutional rules, the market engineer wants to achieve a certain effect and economic performance of the market. At the same time, the market engineer has to predict the strategic behavior of the agents and their reactions since they strongly influence the outcome. But the anticipation of agents' future behavior is a very difficulty task. To overcome these burdens, market engineering suggests a discursive approach: to break down the complex design problem in smaller, less complex problems that can be solved or computed. In general, discursive methods are formal design methods which specify strategies for solving design problems and derive reproducible and consistent solutions (Neumann 2004; Schnizler et al. 2005). Later on, market engineering turns from "the abstract to the concrete" (Neumann 2004). In this context, abstraction means extracting the essential features without including unnecessary details or redundant information. Thus the market engineer can concentrate on what to design instead of how to design (Mäkiö and Weber 2005).

The following definition of market engineering as well as the market engineering process with its phases is given by Neumann (2004):

Definition 2.5.1 Market Engineering
"*Market engineering is the engineering design of all institutional rules of an electronic market.*"

The market engineering process is generally structured from a problem-oriented perspective such that, first, requirements of a problem with scientific and theoretically founded methods are identified, and second, a concept is created that provides a solution to the problem and a desired outcome. For example, behavioral and cognitive models are used to determine the market participants' needs and requirements, and economic models are applied to design the institutional rules. Moreover, the process is based on a problem-oriented, abstract-to-the-concrete approach. The two core activities of market engineering, the design and the operation of electronic markets, define the phases of the process (Neumann 2004):

(i) *Environmental analysis*: Requirements and constraints of the object to be designed are identified in the environmental analysis phase. The environmental analysis concerns the definition of relevant markets, the identification of a promising market segment, as well as the evaluation of the target market segments. Additionally, the requirements of the new market mechanism are deduced. For instance, requirements concerning the environment are the number of agents, their characteristics and endowments, as well as the resources to be traded.

(ii) *Design and implementation*: The design phase consists of three sub-phases: (1) the conceptual design phase, (2) the embodiment design phase, and (3) the detail design phase. In the design phases, the market mechanism is conceptually designed, abstracted to a resource allocation mechanism and a payment function, mapped into an auction or negotiation protocol, and refined by modelling implementation details of the electronic market as a system. In the implementation phase, the institutional rules are fully implemented as a software code and running information system.

(iii) *Testing*: The functioning of the electronic market as well as its performance are tested in the testing phase. Before introducing the electronic market, the system has to go through functionality tests as well as economic and computational performance tests, meaning that the quality of the information system and service are tested. For example, economic tests are done in laboratory experiments, numerical simulations or agent-based simulations. Later on, pilot tests are done.

(iv) *Introduction*: In the introduction phase, the electronic market which was successfully tested rolls out and is launched as an operating electronic market.

The market engineering process is covered by feedback loops – at any phase of the process a decision is to be made whether to proceed with the next stage or to repeat the prior one. The decision is based on the quality of the output derived at each phase and whether this output should be improved. The output of a phase is the input of the phase to follow. Figure 2.3 presents the market engineering process with its four phases according to Neumann (2004).

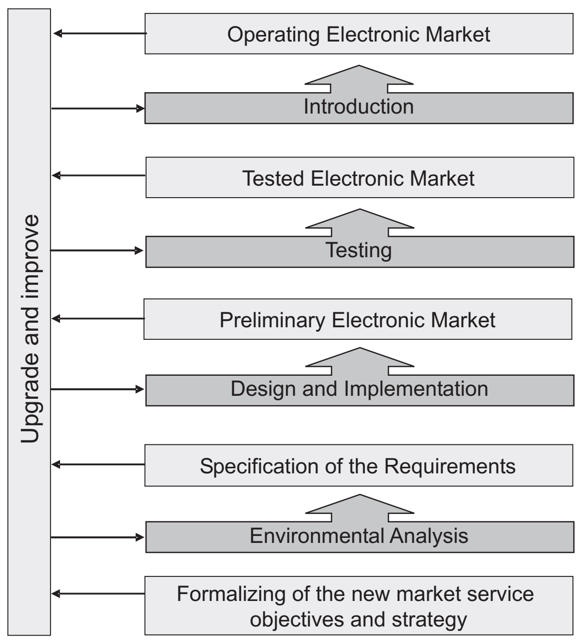


Figure 2.3: The market engineering process (Neumann 2004)

The market mechanism is fully designed and implemented in the design and implementation phase: the institutional rules are conceptually designed and further elaborated. They are broken down into independent components. The basic components are first designed on an abstract level and second on a concrete level. In particular, the conceptual design phase and the embodiment design phase focus on the "original design of the institutions" (Neumann 2004). Both design phases build the bridge between environmental analysis and software engineering; the conceptual design phase is more closely related to the environmental analysis, while the embodiment design is related more closely to software engineering. The transformation of the designed institutions into a functioning information system is subject to the implementation phase. A more detailed description, particularly of the conceptual design phase and the embodiment design phase, can be found in Neumann (2004).

The market engineering process with its four phases suggests a systematic approach to the design problem. The design of the trading rules is strongly related to the agent behavior which affects the outcome. To achieve a desired outcome, a market engineer faces the task of predicting the agents' behavior. Depending on the designed trading rules, the market engineer may give incentives to the agents and thereby incite them and affect their behavior. The main difficulty that comes along with the conceptual and embodiment design is the limited availability of methods, tools or techniques supporting the phases of the market engineering process (Neumann 2004). Nonetheless, the market engineer requires methods and tools that help him to conceptualize and implement electronic markets quickly and thoroughly.

2.5.2 Computer-aided market engineering

The basic idea of computer-aided market engineering is to automate the market engineering process and provide tools that support phases of the market engineering process (Neumann 2004). Computer-aided market engineering closes the gap between a structured design of electronic markets and the absence of methods, tools and techniques to support the market engineer. It offers a toolbox for the market engineer to ease and speed up the design process, as well as to ensure a high level of quality of the design process and thereby the designed product. The tools are phase-specific tools supporting the requirements in each phase of the process. In Neumann et al. (2005) the authors recommend a computer-aided market engineering workbench (CAME workbench) that considers the automation of the design and implementation phase. In particular, the workbench encompass the three sub-phases of the design phase: (1) conceptual design, (2) embodiment design, and (3) detail design and implementation. The central component of the CAME workbench is the core server which provides various trading rules on auctions and allows a configuration of the trading rules.[31] The trading rules of an instantiated electronic market are provided within the core and can be easily designed and configured through a market description language, the Market Modeling Language (Mäkiö 2006).

(ii.1) *Conceptual design*: In the conceptual design phase, the CAME workbench focuses on

[31] Neumann et al. (2005) recommend the tool workbench CAME for a conscious design of auction-based electronic markets. The CAME workbench supports all design phases of the market engineering process: to design the appropriate auction rules and to implement these rules such that an auction can be directly instantiated and run in the auction platform.

the design and configuration of the core functionalities of an electronic market. Common market structures, particularly auction structures, are identified and described on an abstract level. The conceptual design thereby follows a parametric design: each phase of the market or transaction process is defined by a number of parameters that determine the activities of the phases. In essence, each parameter represents a set of trading rules. The abstract description of the trading rules does not include the flow of information. The process which depends on the trading rules is added within the next phase, the embodiment design phase.

(ii.2) *Embodiment design*: In the embodiment design phase, the CAME workbench supports the market engineer by means of a Market Modeling Language. The Market Modeling Language refines the parametric description of the trading rules into a computer readable language. The Market Modeling Language itself does not produce a software code; instead, the trading rules are mapped into an XML schema-based language. The language applies the idea of identifying common auction structures and defining a single generic auction process. Note that this generic process describes the basic process common to most auctions on an abstract level (Mäkiö et al. 2004).

(ii.3) *Detail design and implementation*: In the detail design phase, the parameters of the Market Modeling Language are transformed into the generic process to describe a concrete auction. Additionally, the refined description represented by a detailed XML schema-based description is mapped into a software code. Running this software code deploys an instance of an auction with the defined trading rules.

The CAME workbench is coupled with an experimental tool and a simulation tool, both of which support the testing of the deployed electronic market (Weinhardt et al. 2005). The experimental tool allows for setting up game theoretic experiments for the evaluation of human behavior and its impact on electronic market outcome. The designed trading rules of the deployed electronic markets are evaluated in laboratory experiments and can be tested to see if the designed electronic market achieves the desired outcome. Running simulations within the workbench is another way to study market behavior and the influence of the institutional and environmental rules on the market outcome. In Weinhardt et al. (2005) the authors suggest an agent-based runtime simulation for the testing of electronic markets: modeling individual strategies within software agents offers a new way of analyzing electronic markets.

CAME is an approach to automate the process of designing electronic markets in a systematic and structured manner by offering tools to the market engineer and can be defined as follows (Neumann et al. 2005):

Definition 2.5.2 Computer-Aided Market Engineering
Computer-aided market engineering (CAME) is the automated engineering of electronic markets. CAME is a tool workbench that supports phases of the market engineering process, ranging from the design to the introduction phase.

So far, auction platforms such as the Michigan Internet AuctionBot (Wurman et al. 1998), the Global Electronic Market (Reich and Ben-Shaul 1998), the Generic Negotiation Platform (Benyoucef et al. 2000) or the meet2trade platform (Weinhardt et al. 2005) have been developed as tools for market engineers to create, configure and test auctions. However, these prototypes do not give the market engineer advice in designing the rules – there is still a lack of decision support. A decision support system is suggested for the CAME workbench which assists the market engineer in choosing the appropriate market structure within the design phase. Depending on the environment, the decision support system proposes the market engineer trading rules in order to achieve a desired goal or market outcome. Underlying this decision support system is a knowledge database in which knowledge concerning rules and their effects on outcome is stored. The rules are simply recommendations for the market engineer to give him advice in designing the rules.

The emergence of strategic analysis of electronic markets with the help of game theory and experimental economics has contributed to the establishment of market design. Moreover, tools are provided that investigate the relative impact of different market rules on the outcome. In the future, more advanced tools are needed in contributing to the emergence of more structured electronic markets and thus advancing the research discipline of market engineering.

Chapter 3

Theoretical Model of the Discount Auction

3.1 Motivation

Amazon[1] offers sellers the possibility of selling items via the Internet – it is an online marketplace where a large number of sellers and buyers are involved in trading consumer goods. In essence, Amazon is an online auction market where trades are conducted by electronic auctions.[2] The auctions are initiated by the sellers and interested buyers compete in the auctions by entering bids for the item. The exchange as well as the physical delivery of the items for money is arranged between buyers and sellers. Amazon itself acts as an intermediary in these trades. Its revenue comes from the auctions' fixed listing fee and the completion fee based on the transaction values of the auctions.

All auctions on Amazon are sorted and listed by categories on its web pages . Each auction is listed for a predefined time period lasting 2, 3, 5, 7, 8, 9, 10, 11, 12, 13, or 14 days during which interested bidders are invited to submit their bids. An auction starts at a minimum price r, the posted *reserve price*[3], set by the seller. Bidders then enter their maximum bids – these

[1] See http://www.amazon.de or http://www.amazon.com.

[2] The basic selling mechanism on Amazon is an auction. In addition, Amazon offers sellers the opportunity to add several features and listing options. For a more detailed description of these features see http://www.amazon.com/exec/obidos/tg/browse/-/1161360/ref=br_bxß_c_1_2/103-0209819-9166267.

[3] Setting a reserve price means reserving the right to not sell the item below a predetermined price. The reserve price is a minimum bidding level. By setting a reserve price, a seller excludes bidders with valuations below the reserve price from the auction (Krishna 2002; Bajari and Hortascu 2003). On Amazon the posted reserve price is called *minimum bid* and is revealed to all bidders. In addition, when setting a minimum bid, sellers on Amazon may also specify a secret (or hidden) reserve price. The amount of the secret reserve price is not revealed to the bidders; bidders only know that a secret reserve price exists. Note that Amazon calls the secret reserve price

bids cannot be below the reserve price. A bidder's maximum bid is also called his *reservation price*. At any time during the auction, the name[4] of the *highest bidder* as well as the *current price* are publicly revealed on the auction site. Moreover, Amazon sends e-mail to the highest bidder confirming his top position in the auction, as well as to the former highest bidder informing him of his displacement. The highest bidder is the bidder who has submitted the highest reservation price up to that time in the auction. If no or only a single bid has been placed, the current price equals the reserve price. If two or more bids have been placed, the current price is the minimum of the highest reservation price and the second highest reservation price plus a given *bid increment*.[5] When two or more bidders have submitted the same highest reservation price, a tie occurs. The tie-breaking rule states that the bidder who has submitted his bid first is the highest bidder and the current price is equal to these bids. The amount of the bid increment depends on the current price and thus changes dynamically throughout the auction. On the German Amazon auction sites the bid increment ranges from 0.05 euros for low current prices between 0.01 euros and 0.99 euros to 100.00 euros for current prices at or above 5,000.00 euros. On Amazon's US platform the bid increment ranges from 0.05 US dollars to 100.00 US dollars. The amount of the current price step and the bid increment are identical for both auction platforms, i.e. Amazon's German and US auction platforms – only differing in the currency. The detailed list of bid increments is given in Section A.1 in the Appendix A. All bids must exceed the current price by at least the bid increment. Bids below the current price plus the bid increment are invalid and thus are rejected. At the end of the auction, the item is awarded to the bidder who is the highest bidder at that time. The current price at the end of the auction is the closing current price or final price of the auction.

On Amazon, a seller who initiates an auction of an item can choose between two payment policies: first, the seller can decide to conduct the described auction, where at the end of the auction the highest bidder wins the auction and the price to pay is the closing current price, or the final price of the auction. Second, the seller can conduct an auction with a *first bidder discount*. A first bidder discount is a discount the high bidder receives at the end of the auction on the closing current price if he has submitted the first valid bid in the auction. That is, two conditions must hold for a bidder in order to receive the discount: (i) the bidder has to submit

simply *reserve price* and the posted reserve price *minimum bid*.

[4]Bidders in the auction often have pseudonyms as their (user)names. The username can be a fictive name which is used to log onto the platform.

[5]There is one exception: if the current high bidder submits a new reservation price above his actual reservation price, the current price does not change, although the number of submitted bids increases by one. The current price however is raised to the increment above the second highest reservation price, if the current high bid was below the second highest reservation price plus one increment and the highest bidder submits a new valid reservation price.

the first valid bid in the auction, and (ii) the bidder has to submit the highest reservation price in the auction. If both conditions hold, the highest bidder receives a discount on the final price of the auction. That is, the payment equals the discounted final price of the auction. If the highest bidder in the auction has not submitted the first valid bid, i.e. condition (i) is hurt, than, the highest bidder will not receive the discount and must pay the final price of the auction. On Amazon the first bidder discount equals 10 percent.

Besides the description of the item being sold, the name of the seller, the current price, the number of bids already placed and the ending time of the auction are publicly revealed to the bidders. Whether a first bidder discount is offered in an auction is indicated to all bidders only at the beginning of an auction: as long as no bid has been entered, the availability of the discount is indicated by a symbol saying 10% *OFF 1st Bidder*. With the submission of the first valid bid, the discount is assigned to this bidder. The symbol is deleted and the discount is no longer available for subsequent bidders. The first bidder is informed about his top position in the auction, the auction's current price, as well as about receiving the discount. Subsequent bidders are not informed about the discount in the auction which is already assigned to the first bidder. Those bidders do not know whether a first bidder discount was initially offered.

An interesting question that develops with the two pricing schemes is when and why a rational seller decides for one or the other scheme – the pricing scheme of a pure auction, where the payment equals the final price of the auction; or the discounted pricing scheme, where the price to pay either equals the discounted final price of the auction or the final price, depending on whether the winning bidder has entered the first bid or not. In an auction, sellers intend to sell their items at a price as high a price as possible and thus maximize their revenues. Thus, each seller would prefer to sell her item via a first bidder discount auction if this auction format generates higher revenues than a pure auction. The following example shows that this, may be the case, but does not hold in general.

Example 3.1.1 Suppose two bidders, bidder 1 and bidder 2, participate in a pure Amazon auction. Bidder 1 has a private valuation of 30 euros of the item, and bidder 2 a private valuation of 26 euros. The seller sets the reserve price to $r = 25$ euros; thus the bidding increment equals 1 euro. In the pure auction, both bidders enter their reservation prices equal to their valuations of the item. Bidder 1 is the highest bidder and the auction ends at the final price of 26 euros + 1 euro = 27 euros. This is the seller's revenue.

Now consider the case where the seller offers her item in a first bidder discount auction with a discount of 10%. First, suppose that bidder 1 places the first bid in the auction. Thus, the first bidder discount is assigned to bidder 1. At the end of the auction, bidder 1 is the highest bidder and receives a discount of 10% on the final price of 27 euros of the auction. Thus the

seller's revenue ends up at 24.30 euros, which is less than in the pure auction.
Suppose that bidder 2 places the first bid in the auction. Due to the first bidder discount which is assigned to bidder 2, bidder 2 can enter a reservation price above his valuation – he submits a bid of $\frac{1}{1-0.1}26$ euros = 28.89 euros. Still, bidder 2 does not outbid bidder 1. Again, bidder 1 is the high bidder and the final price of the auction is 28.89 euros + 1 euro = 29.89 euros. Hence, the seller's revenue is higher than in the pure auction.

Example 3.1.1 shows that there are cases in which a seller can raise the revenue by adding a first bidder discount to the pure auction format. However, there are also cases where the discount does not pay – the seller's revenue remains the same or is even lower than in a pure auction. Thus the question arises how the ex-ante expected revenues of both institutions – the pure auction and the first bidder discount auction – are related. In particular the following questions are addressed:

- Does it pay for the seller to offer a discount when conducting an auction?
- Under what conditions does the discount pay?

The subsequent sections focus on distinct aspects of the Amazon auction. Its two pricing schemes are analyzed in more detail and the questions mentioned above are answered.

3.2 Amazon auction

The objective of this section is to gain insight into the auction mechanisms of the pure Amazon auction and the Amazon first bidder discount auction. In both auction institutions the bidding mechanism is the same – the only difference is the pricing scheme. Regarding the bidding process, the pure Amazon auction has similarities to both, the English auction and the second-price sealed-bid auction. First, like the English auction, the pure Amazon auction is an iterative and open auction. Throughout the auction process, bidders are allowed to submit more than one bid – the number of bids per bidder is unlimited. The current price increases successively as long as there are are two or more interested bidders, and the current price in the auction is publicly announced at all times to the bidders. Bidders drop out of the auction if the current price exceeds the bidders' reservation prices. If there is only a single interested bidder left in the auction, the incremental bidding ends. Second, as in the second-price sealed-bid auction, bidders submit their reservation prices sealed, i.e. not visible for the other bidders. The difference lies in the required bid increment and the way the current price is determined. If it is assumed, that the bid increment is very small such that it can be neglected and further,

that each bidder is allowed to submit at most one bid, then the Amazon auction is equivalent to a second-price sealed-bid auction. Note that in a second-price auction with private values, the bidders' dominant strategy is to bid their respective true valuation. Thus, the high-value bidder will win the second-price sealed-bid auction and the price to pay is the second highest valuation. But in the pure Amazon auction, in general, the final price is higher than the second highest bid – the reason for this is that the final price in the Amazon auction equals the minimum of the second highest reservation price plus increment and the highest reservation price. Nevertheless, the Amazon auction is referred to as a dynamic variant of the second-price sealed-bid auction (Bajari and Hortascu 2003).
On its auction site Amazon describes the bidding procedure to bidders as follows:[6]

> "**Your automatic proxy: How bidding works**
>
> 1. Each time you enter a bid your automatic proxy goes to work for you.[7] Your proxy lets you set an upper bidding limit – that's your maximum bid – while keeping your actual bids as low as possible. (Your maximum is private – we don't disclose it to anyone.)
> 2. If another party beats your initial bid, the proxy raises your bid by one single increment more than the challenging bid. This pattern continues until another bidder exceeds your maximum bid, or until you win the auction."

This bidding technique, where a bidder's reservation price is used by proxy to subsequently place bids on behalf of the bidder, is also called *proxy bidding*: the automatic bidding proxy submits the bids up to the reservation price to become the high bidder at any point during the auction. That is, the proxy only submits a bid if its current high bid was outbid by a rival's bidding proxy while keeping the bid price as low as possible.

In auctions with fixed ending times (*hard close*), a phenomenon called *sniping* can be observed: many bids are submitted in the closing minutes or seconds of the auction (Ockenfels and Roth 2002). In order to undergo such sniping effects, Amazon offers a so-called *soft close* ending in addition to the fixed ending.[8] That is, the auction is automatically extended by ten minutes whenever a bid is submitted within the last ten minutes of the auction. The auction is terminated if no valid bid is entered within the last ten minutes.
The example below illustrates a pure auction as it is offered on Amazon's auction site.

[6] See http://www.amazon.com/exec/obidos/tg/browse/-/537852/104-8669052-8371101.
[7] On the Amazon auction sites the automatic proxy is called *Bid-Click*.
[8] Amazon also calls this soft close ending *Going, Going, Gone*.

Example 3.2.1 A seller conducts an auction on the Amazon platform in order to sell an item. The set $N = \{1, 2\}$ denotes the set of bidders each having a private valuation of the item: bidder 1 is willing to pay up to 30 euros for the item and bidder 2 up to 28 euros. The auction starts at a reserve price of $r =$ 25 euros with an Amazon bidding increment of 1 euro; the auction has a fixed ending time. Suppose that both bidders enter their reservation prices – their valuations. In the pure Amazon auction, bidder 1 enters 30 euros, thereby becoming the current highest bidder at the reserve price of 25 euros. Bidder 2 is the second to place a bid of at least 25 euros + 1 euro = 26 euros. Bidder 2 enters his reservation price of 28 euros. Thus, the current price is increased to 28 euros + 1 euro = 29 euros and the auction ends at this price. Bidder 1 is the highest bidder and receives a payoff of 30 euros $-$ 29 euros $=$ 1 euro, which is equal to the difference of his valuation and payment.
However, bidder 1 can do better. First let w.l.o.g. bidder 2 submit a bid of 28 euros. The current price, then, is the reserve price of 25 euros. Now bidder 1 enters a reservation price of 28.10 euros. The current price is raised to 28.10 euros which is also the final price of the auction. Bidder 1's payoff is 1.90 euros.
Suppose to the contrary, that bidder 1 is the first to submit a reservation price of 28 euros. Again, the current price is the reserve price. Bidder 2 enters his reservation price of 28 euros. Since both bidders have submitted the same reservation price, a tie occurs. Bidder 1, who was the first to submit his bid, is the highest bidder and the final price is 28 euros. Thus, bidder 1's payoff equals 2 euros.

In the following, the focus is set on Amazon's first bidder discount auctions, i.e. auctions with an additional first bidder discount feature. Sellers find the following information about the first bidder discount on the Amazon auction site:[9]

> "**First Bidder Discount**
>
> For our Auctions customers, the First Bidder Discount – *10% OFF 1st Bidder* – is an excellent way to entice bidders to bid early, and to keep on bidding. We've found that sellers who take advantage of the First Bidder Discount sell at a rate 15 percent higher than average.
>
> By offering something special to the first bidder – 10 percent off if they win the auction – you encourage people who visit your auction to bid at the first opportunity. Confident in the knowledge that they'll get a better price than subsequent bidders, first bidders are likely to keep topping competing bids.

[9] See http://www.amazon.com/exec/obidos/tg/browse/-/1161360/104-8669052-8371101#first-bidder-discount.

> You can offer this powerful incentive whenever you like. Interested? Consider:
>
> - **It attracts bidders.** First Bidder Discount auctions are highlighted with eye-catching icons. When bidders spot these icons, they will be motivated to act fast and capture the advantage over all competitors.
> - **It's effortless.** Amazon.com handles the accounting. We'll track the bidding and calculate the final amount due to you (final bid less 10 percent) if the first bidder wins.
>
> When you make a sale, we'll assess the completion fee based on the actual closing price – the discounted amount – not on the "high bid". You'll find this "Closing fee adjustment" on your invoice.
>
> Please note: if you offer a First Bidder Discount and you establish a Take-It Price[10], first bidders receive no discount on the Take-It Price."

The discount is assigned to the bidder who (i) has submitted the first valid bid in the auction and (ii) has entered the highest reservation price among all bidders. In essence, the first bidder discount states the following:

> *A bidder receives the first bidder discount if and only if the bidder is the first and highest bidder in the auction.*

If both conditions hold, the highest bidder in the auction receives a discount of 10 percent on the final price of the auction.

As noted above, Amazon gives two main reasons for offering the discount. On their web page, Amazon claims that first, the discount encourages bidders to submit a bid at the first opportunity, and second, first bidders are likely to keep topping competing bids. Moreover, Amazon states that sellers who offer a first bidder discount in their auction will on average raise their revenues by 15 percent.

The question which arises from the perspective of the seller is whether on average the first bidder discount will pay for him when conducting an auction. That is, is the expected auction revenue in an auction with discount greater than the respective expected auction revenue in a standard auction? As already shown in Example 3.1.1 there are cases where the discount

[10] Amazon offers different selling mechanisms. Besides using an auction, the seller may sell her item via a Take-It Price, i.e. the seller offers the item at a fixed or posted price. Bidders respond to the offer by taking or leaving the item at the announced price. Further on, an auction can be combined with a Take-It Price: bidders can decide to take the item immediately or participate in the auction. As long as the current price of the auction remains under the Take-It Price, the Take-It Price does not affect the auction.

feature positively affects the seller's revenue as well as cases where the discount feature does not pay for the seller but even yields a lower revenue.

From the buyer's perspective, the question arises why a bidder should submit a first bid and take the first bidder discount. On its auction sites Amazon describes the first bidder discount to the bidders as follows:[11]

> "**First Bidder Discount**
>
> The First Bidder Discount icon means savings for bidders. If you're the first to bid on an item, you lock in a 10% discount from the seller if you win the auction.
>
> There's no catch. The seller is offering this discount to attract an early bid – and it usually works. No wonder, because as the first bidder, you earn a substantial advantage over rival bidders. You can leave yourself room to bid an extra 10% higher, or just enjoy the winning discount. By pouncing on auctions early, you assure yourself an opportunity to win at a better price than your rivals.
>
> There's nothing to it – just bid as you ordinarily would. If you bid first and go on to win the auction, you'll pay 10% less than your winning bid. (Please note that if the seller is offering a Take-It Price, the First Bidder Discount does not apply to the Take-It Price.)"

In essence, Amazon gives two reasons why bidders should take advantage of the first bidder discount feature: first, winning bidders with the discount receive a greater gain than without the discount, because they receive a 10 percent discount on the final price (if the final prices are equal in both cases). Second, first bidders have an advantage over rival bidders and can use the 10 percent discount to enter a higher maximum reservation price. Indeed, first bidders benefit from the discount feature by submitting higher reservation prices. The discount enables first bidders to add an additional premium to the original reservation price: the original reservation price can be multiplied with a factor of $\frac{1}{1-10\%} \approx 1.11$.[12] Submitting a higher reservation price increases the probability of winning the auction. Yet the discount ensures that the first bidder pays at most his true reservation price.

The following example shows the rules of an auction with the first bidder discount feature from a bidder's perspective.

[11] See http://www.amazon.com/exec/obidos/tg/browse/-/537842/ref=br_bx_c_1_1/104-8669052-8371101.

[12] On its auction site Amazon states, that a first bidder can "bid an extra 10% higher". In fact, a first bidder can submit a bid of $\frac{1}{1-10\%} \approx 1.11$ times his original reservation price due to the discount, that is approximately 11 percent above the reservation price.

Example 3.2.2 (Extension of the Example 3.2.1) Consider an Amazon auction with a first bidder discount $d = 10\%$. The discount is offered to the highest bidder at the end of the auction if he has placed the first valid bid. Both bidders participate in the auction, bidder 1 with a private valuation of 30 euros of the item and bidder 2 with a private valuation of 28 euros. Suppose that bidders enter their valuations as reservation prices without discount. If the first bidder discount is available, two cases can be distinguished: (1) bidder 1 is the first to place a bid and (2) bidder 2 is the first bidder. In case (1) bidder 1 with the highest reservation price receives a discount on the closing current price at the end of the auction. His payment p is the discounted final price of the auction of 29 euros: $p = (1 - 0.1)\ 29$ euros $=$ 26.10 euros. Bidder 1's payoff increases from 1 euro to 3.90 euros. In case (2), bidder 2 is the first to place a bid. Because of the discount, bidder 2 can raise his original reservation price and submit a bid up to $\frac{1}{1-0.1}\ 28$ euros $=$ 31.11 euros, which is above his valuation of the item. By entering the reservation price of 31.11 euros, bidder 2 outbids bidder 1. At the end of the auction the closing price is 30 euros $+$ 1 euro $=$ 31 euros. Bidder 2 is the highest bidder; the price he has to pay is the discounted final price. Bidder 2 pays $(1 - 0.1)\ 31$ euros $=$ 27.90 euros and his payoff is 0.10 euros.

The first bidder can raise his reservation price due to the discount and increase the probability of winning the auction. However, the first bidder discount is available to a bidder only in two cases:

(i) That bidder is the first bidder being aware of the auction and available discount having a valuation of the item greater than or equal to the discounted reserve price: $v \geq (1 - d)r > 0$ with discount $d = 10\%$ and reserve price $r > 0$.

(ii) Other interested bidders who become aware of the auction have a valuation of the item below the discounted reserve price, that is $v < (1 - d)r$ with valuation $v > 0$, discount $d = 10\%$ and posted reserve price $r > 0$.

Consider a bidder to which the *10% OFF 1st bidder* symbol is indicated. This bidder has to ask and answer the following questions: *when* to bid and *what* to bid. *When* to bid asks for the timing of bids. Suppose that bidding immediately means directly placing a bid once the discount is available. Thus, a bidder can increase his original reservation price, i.e. his valuation, and bid up to $\frac{1}{1-d}$ times his valuation. Not bidding directly and placing a bid later in the auction means running the risk of not receiving the discount and reducing the probability of winning the auction. A rival bidder might place his bid and thereby benefit from the assigned discount. Thus, for a bidder interested in the item which is auctioned in a first bidder discount auction, submitting the first bid is an optimal action.

Consider the following situation: two bidders, bidder i and bidder j, participate in a first bidder discount auction offered by Amazon. Both bidders have private valuations $v_i, v_j \geq r+s$ of the item with $v_j < v_i$, reserve price $r > 0$ and bid increment $s > 0$. The duration of the auction is limited: it starts at $t = 0$ and ends at $t = T$. First, suppose that bidder i is the first to be aware of the auction and immediately submits the first bid with a reservation price of $b_i \in [r, \frac{1}{1-d}v_i]$ at $t \in [0, T]$ while the bid history h_t is empty.[13] The discount is assigned to bidder i, and the current price p_t is set to the reserve price: $p_t = r$. Thus, if bidder j does not enter the auction, bidder i wins the auction and pays the discounted final price. His payoff is $v_i - (1-d)r \geq 0$ with $v_i \geq (1-d)r$. If bidder j enters the auction at $t' > t$, $t' \in [0, T]$ and submits a bid $b_j \in [p_t + s, v_j]$ with $b_j < b_i$, bidder i's gain will be lowered. His gain is at most $v_i - (1-d)p_{t'}$ with $p_{t'} = \min\{b_j + s, b_i\}$.

Second, suppose that bidder i does not bid immediately. Instead, bidder i waits and bidder j submits his bid in $t \in [0, T]$. Then the discount is assigned to bidder j. Now, bidder i can react only if $v_i \geq r+s$. Assume that bidder i enters a bid of $b_i \in [r+s, v_i]$. In $t' > t$ the bidder with the highest submitted reservation price is the provisional highest bidder, and the current price is denoted by $p_{t'}$. Suppose further that bidder i is the highest bidder at time t' at a current price of $p_{t'} = \min\{b_j + s, b_i\}$. If no further bids are entered, the auction terminates at that final price and bidder's i payoff is $v_i - p_{t'}$. This is the highest payoff bidder i can achieve. In fact, bidder i's payoff is less than the payoff achieved with the first bidder discount. Particularly whenever the bid history is an empty set at any point t, the best action for a bidder is to submit a bid immediately with $b_i \in [r, \frac{1}{1-d}v_i]$ and take the discount.

What to bid in the first bidder discount auction, e.g. which action to play, depends on multiple factors such as the rival bidders' valuations of the item or the bid history h_t at time t. A bidder may play at time $t \in [0, T]$ any bid $b_i \in \mathbb{R}_+$ with $b_i \geq \max\{r, p_t + s\}$. More specifically, at any time t in the auction, bidders can choose between the following actions:

- If the discount is still available at $t \in [0, T]$ with $h_t = \emptyset$, then ***submit a bid*** with $b_i \in [r, \frac{1}{1-d}v_i]$.

- Choose one of the following actions at any time $t \in (0, T]$ with $h_t \neq \emptyset$:

  (*i*) If bidder i is the first bidder and if $\frac{1}{1-d}v_i \geq p_t + s$, then ***submit a bid*** b_i with $b_i \in [p_t + s, \frac{1}{1-d}v_i]$.

  (*ii*) If bidder i is not the first bidder and if $v_i \geq p_t + s$, then ***submit a bid*** b_i with $b_i \in [p_t + s, v_i]$.

[13] The bid history h_t lists all bids submitted up to time $t \in [0, T]$. At the very beginning of the auction the bid history is empty: $h_t = \emptyset$ for $t = 0$.

Based on the Amazon auction and its first bidder discount feature, the following sections present a formal model of an auction with a discount in a private values setting. Moreover, the Amazon auction format with its proxy bidding procedure, bid increment, reserve price and fixed ending rule are substituted by a standard second-price sealed-bid auction. Thus, the minimum bid increment as well as the reserve price are neglected. The auction is a one-shot auction and bidders are allowed to enter only a single bid – their reservation prices.

3.3 Preliminary steps

3.3.1 Basic assumptions

Throughout the following sections, the discount auction market institution is introduced and its theoretical foundation is laid out. In fact, the auction is a second-price auction augmented with a discount, where a single indivisible item is offered for sale. For the analysis of this auction, an *independent private values (IPV)* setting with risk neutral bidders is assumed (cf. Section 2.2.2):

- Bidders have private valuations.
- Bidders' valuations are independent and identically distributed (iid). All valuations are considered as continuous random variables.
- Bidders are risk neutral.

Assuming also symmetry between bidders, the IPV is extended to the SIPV auction model. The symmetry condition says:

- Bidders are symmetric or indistinguishable; all bidders are characterized by the same probability distribution function from which their valuations are drawn.

It is further assumed that the payment is a function of bids alone and each bidder has sufficient resources to pay the seller up to his valuation (no budget constraints).

Within an SIPV setting with risk neutral bidders, the revenue equivalence theorem holds: all auctions that award the item to the high value bidder and lead to the same bidder participation are revenue equivalent (cf. for example to Krishna 2002).

3.3.2 The discount mechanism

Assume now that a second-price auction is augmented with a discount. The idea of this auction is that exactly one seller is randomly selected and assigned a discount. This bidder is called the *designated* bidder.[14] The bidding procedures in both auctions, the second-price auction and the second-price auction with discount, are the same. Only the pricing policies are different. In the second-price auction with discount, the pricing policy states the following: if the winning bidder is not the designated bidder, then the price to pay is the final price of the auction, or the second highest bid. If the designated bidder wins the auction, then the payment is the discounted final price of the auction. In essence, the discount states:

(i) Exactly one bidder, the *designated* bidder, is randomly selected.

(ii) The discount is assigned only to the designated bidder. If and only if the designated bidder is the highest bidder in the auction, the discount applies and the designated bidder acquires the item at the discounted final price.

(iii) If the designated bidder does not win the auction, then the winning bidder purchases the item at the final price of the auction.

The rules presented above differ slightly from Amazon's first bidder discount feature. First, the discount is assigned to exactly one bidder, independent of whether or not this bidder is the first to bid. The timing of bids does not play a decisive role in obtaining the discount. In particular, bidding immediately in order to ensure the discount, as is the case in the Amazon first bidder discount auction, is not necessary. Second, once the discount is assigned to a bidder, the bidder cannot reject the discount. In the Amazon first bidder discount auction, bidders are free in deciding whether to submit the first bid and receive the first bidder discount, or alternatively not to enter the first bid and wait. Nevertheless, throughout the auction the designated bidder is in an advantageous position: he can enter a bid slightly above his valuation, i.e. submit a bid equal to $\frac{1}{1-d}$ times his valuation with discount $d \in [0, 1)$, but in case of winning the auction he pays at most a price equal to his valuation.

The definition of the discount auction or DA market institution in an IPV setting with risk neutral bidders is as follows:

[14]"Designated bidders" is an official term of the Federal Government and was used for example in the FCC auction as a term for subsidized bidders (Rothkopf et al. 2003).

Definition 3.3.1 DA market Institution
*In an **auction with discount** (discount auction, DA), a seller offers a single indivisible item to n ($n \in \mathbb{N}$) participating bidders. The seller augments the auction with a **discount** $d \in [0, 1)$.*[15] *There is no reserve price. Exactly one of the n participating bidders is randomly selected. This bidder is called the **designated bidder**. If the designated bidder wins the auction, the discount is realized on the second highest bid, the final price p of the auction, and the designated bidder pays $(1 - d)p$. If a rival bidder (unequal to the designated bidder) is the highest bidder, then the price to pay is the second highest bid, i.e. the final price p of the auction.*

Note that the corresponding auction to the DA with no discount is the second-price (sealed-bid) auction.

Although the DA market institution is based on the auction mechanism of a second-price auction, the strategic behavior in a DA is slightly different: the weakly dominant strategy of bidders who are not selected to receive the discount is to bid truthfully. On the contrary, the designated bidder submits a bid of $\frac{1}{1-d}$ times his valuation due to the discount (cf. Proposition 3.3.1). Note that the equilibrium strategies in a second-price auction and the defined DA are independent with respect to the bidders' risk attitudes.

Let $N = \{1, \ldots, n\}$ be a set of n risk neutral bidders participating in an auction, either the DA or the corresponding second-price auction, where a single indivisible item is offered for sale. Each of the n bidders participating in the auction has a private valuation $v_i, i \in N$ of the item. Bids entered throughout the auction are based on the valuations. Let u_i denote the *utility* (von Neumann-Morgenstern) of bidder $i \in N$, which in the case of risk neutral bidders is linear and therefore equal to the *payoff* π_i. If bidder i wins the auction his payoff π_i, which derives from participation in the auction, depends on his valuation v_i as well as on his rivals' bids $b_j, j \in N \setminus \{i\}$.

In a second-price auction, the payoff π_i of winning bidder i is $\pi_i = v_i - p$ with $p = \max_{j \in N \setminus \{i\}} b_j$ being the price the high bidder i has to pay for the item. Should that bidder i not be the winning bidder in the second-price auction, the payoff equals zero: $\pi_i = 0$. The bidder's payoff π_i is given by the following equation:

$$\pi_i = \begin{cases} v_i - \max_{j \in N \setminus \{i\}} b_j, & \text{if } b_i > \max_{j \in N \setminus \{i\}} b_j \\ 0, & \text{if } b_i < \max_{j \in N \setminus \{i\}} b_j \end{cases} \qquad \forall\, i \in N \tag{3.1}$$

[15] In case of $d = 1$ a rational designated bidder bids ∞ independent of his valuation, wins the auction, and purchases the item at a price of 0. Thus, a discount of $d = 1$ is neglected in the further analysis. When the discount equals zero, $d = 0$, the DA is a second-price auction: the price to pay is the second highest bid, independent of whether or not the highest bidder is the designated bidder.

Should two or more bidders have submitted the same highest bid, i.e. the highest bid and second highest bid are equal and $b_i = \max_{j \in N \setminus \{i\}} b_j$, the item goes to each of these bidders with equal probability. The winning bidder's payoff is equal to the difference of his valuation and the price to pay, i.e. the second-highest bid; the loosing has a payoff of zero.

In the DA, the payoffs are slightly different. As in the second-price auction the final price of the DA is the second highest bid, i.e. $p = \max_{j \in N \setminus \{i\}} b_j$. However, should the designated bidder $\hat{\imath} \in N$ purchase the item, the price to pay is the discounted final price of the auction $(1-d)p = (1-d) \max_{j \in N \setminus \{\hat{\imath}\}} b_j, d \in [0,1)$ and the designated bidder receives a payoff of $\pi_{\hat{\imath}} = v_{\hat{\imath}} - (1-d)p$. All other bidders receive a payoff of zero. Should a bidder $i \in N \setminus \{\hat{\imath}\}$, who is not the designated bidder, win the auction, the price to pay is the final price of the auction. His payoff equals $\pi_i = v_i - p$ and all other bidders have a payoff of zero. More specifically, a bidder i's payoff in the DA, in which bidder $\hat{\imath} \in N$ is selected as the designated bidder, is defined by

$$\pi_i = \begin{cases} v_{\hat{\imath}} - (1-d) \max_{j \in N \setminus \{\hat{\imath}\}} b_j, & \text{if } b_{\hat{\imath}} > \max_{j \in N \setminus \{\hat{\imath}\}} b_j \text{ and } i = \hat{\imath} \\ v_i - \max_{j \in N \setminus \{i\}} b_j, & \text{if } b_i > \max_{j \in N \setminus \{i\}} b_j \text{ and } i \neq \hat{\imath} \\ 0, & \text{if } b_i < \max_{j \in N \setminus \{i\}} b_j \end{cases} \tag{3.2}$$

Again, should two or more bidders have submitted the same highest bid, the item goes to each of these bidders with equal probability. The winning bidder's payoff is the difference of his valuation and the second highest bid ($i \neq \hat{\imath}$) or respectively the discounted second highest bid ($i = \hat{\imath}$); the loosing bidder has a payoff of zero.

In a second-price auction the bidding behavior is straightforward: bidding the private valuation is a weakly dominant strategy. Since a discount is added to a second-price auction, the strategies differ slightly from the weakly dominant strategy in a second-price auction. As already mentioned, the designated bidder is in an advantageous position: he bids $b_{\hat{\imath}} = \tilde{v}_{\hat{\imath}}$ with

$$\tilde{v}_{\hat{\imath}} = \frac{1}{1-d} v_{\hat{\imath}}$$

By bidding $\tilde{v}_{\hat{\imath}}$ the designated bidder pays at most his valuation $v_{\hat{\imath}}$ for the item when being the highest bidder in the DA. Should the second highest bid equal the highest bid $b_{\hat{\imath}}$, the designated bidder pays the discounted second highest bid, i.e. his valuation: $(1-d)b_{\hat{\imath}} = (1-d)\tilde{v}_{\hat{\imath}} = (1-d)\frac{1}{1-d} v_{\hat{\imath}} = v_{\hat{\imath}}$. Obviously, for the designated bidder $\hat{\imath}$ bidding $\tilde{v}_{\hat{\imath}}$ is a (weakly) dominant strategy and for all other bidders $j \in N \setminus \{\hat{\imath}\}$, bidding truthfully is a (weakly) dominant strategy.

Proposition 3.3.1 Consider n bidders participating in a DA with discount $d \in [0,1)$ where a single item is offered for sale. Each bidder $i \in N$ values the item at v_i. It is then a weakly dominant strategy for the designated bidder $\hat{\imath}$ to bid $b_{\hat{\imath}} = \tilde{v}_{\hat{\imath}} = \frac{1}{1-d}v_{\hat{\imath}}$, $\hat{\imath} \in N$ and for all bidders except bidder $\hat{\imath}$ to bid $b_j = v_j$, $j \in N \setminus \{\hat{\imath}\}$.

Proof: Consider bidder 1 and suppose $p_1 = \max_{j \in N \setminus \{1\}} b_j$ is the highest competing bid. Suppose bidder 1 is not the designated bidder, submitting a bid of $b_1 = v_1$. Bidder 1 wins the auction if $v_1 > p_1$ gaining a payoff of $\pi_1 = v_1 - p_1 > 0$ and does not win if $v_1 < p_1$ with a payoff of $\pi_1 = 0$. Note that in the case of $v_1 = p_1$ a tie-breaking rule decides to whom the item will be awarded. The item goes to each of the highest bidders with equal probability. Thus, bidder 1 is indifferent between winning and losing the auction.

Assume however that bidder 1 submits a bid z_1 below his valuation: $z_1 < v_1$. If $z_1 > p_1$ bidder 1 wins and his profit is $v_1 - p_1$. If $p_1 > v_1 > z_1$ or if $v_1 > p_1 > z_1$ bidder 1 does not win the auction; in both cases he loses the auction with a payoff of zero. However, in the latter case ($v_1 > p_1 > z_1$) bidder 1 would have made a positive profit of $v_1 - p_1 > 0$ by bidding v_1 instead of z_1. If $z_1 = p_1$ a tie-breaking rule decides to whom the item is awarded: (i) in the case that bidder 1 is the winning bidder, his profit is $\pi_1 = v_1 - p_1 > 0$; (ii) in the case that bidder 1 does not win the auction, his payoff is zero. In that case, bidding v_1 instead of z_1 would have improved his profit from zero to $v_1 - p_1 > 0$. Thus, bidding less than v_1 can never increase the payoff, whereas in some cases the payoff may decrease. A similar argumentation shows that bidding above the valuation v_1 is not profitable. Thus, bidding the valuation v_1 is a weakly dominant strategy for bidder 1, a non-designated bidder.

Assume that bidder 1 is the designated bidder who realizes the discount on the second highest bid in case of winning the auction. Again, p_1 is the highest competing bid. Suppose, that bidder 1 submits a bid of $b_1 = \tilde{v}_1 = \frac{1}{1-d}v_1$, $d \in [0, 1)$. Again, bidder 1 wins if $\frac{1}{1-d}v_1 > p_1$ and loses if $\frac{1}{1-d}v_1 < p_1$. In the case of winning the auction, the price to pay is the discounted second highest bid $(1-d)p_1$ and bidder 1 receives a payoff of $\pi_1 = v_1 - (1-d)p_1$. In the case of loosing the auction the payoff of bidder 1 is zero. Again, in the case of $\frac{1}{1-d}v_1 = p_1$ a tie-breaking rule decides to whom the item is allocated. Bidder 1 is indifferent between winning and losing the auction. The item is assigned to each winning bidder with equal probability.

Assume now, however, that bidder 1 submits a bid of z_1 below $b_1 = \frac{1}{1-d}v_1$, $d \in (0, 1)$:
If $z_1 > p_1$ bidder 1 wins and receives a positive payoff of $v_1 - (1-d)p_1$.
If $\frac{1}{1-d}v_1 > p_1 > z_1$ bidder 1 loses with a zero payoff. Bidding $\frac{1}{1-d}v_1$ would have increased his profit from zero to $v_1 - (1-d)p_1 > 0$, a positive payoff.
If $p_1 > \frac{1}{1-d}v_1 > z_1$ bidder 1 loses with a payoff of 0. If $z_1 = p_1$ a tie-breaking rule

again decides to whom the item is purchased: (i) if bidder 1 is the winning bidder, then his payoff is $v_1 - (1-d)p_1 > 0$; (ii) if bidder 1 is not the winning bidder, then bidder 1 could do better by bidding $\frac{1}{1-d}v_1$, which is above p_1 and gaining a positive payoff. However, bidding less than $\frac{1}{1-d}v_1$ can never increase the payoff of the designated bidder, while in some cases the payoff may decrease. A similar argumentation shows that bidding $z_1 > \frac{1}{1-d}v_1$ is not profitable. Thus, bidding $\frac{1}{1-d}v_1$ is a weakly dominant strategy for the designated bidder. q.e.d.

Note that for $d = 0$ with $\frac{1}{1-d}v = v$ the second-price auction with discount mechanism is equal to the corresponding second-price auction, and truthful bidding is a weakly dominant strategy.

The following analysis shows to what extent the discount affects the bidders' payoffs in the DA (cf. Weber 2005). Suppose the valuations $v_i,\ i \in N$ are independent drawings of a random variable V. The cumulative probability distribution function (cdf) is given by $F : \mathbb{R} \rightarrow [0,1]$. Each bidder knows his realization v_i and that bidders' values are identical and independently distributed according to F. Assume w.l.o.g. that bidder 1 has the highest valuation of the item, bidder 2 the second highest valuation, and bidder k the k^{th} highest valuation ($k \in N$): $v_1 \geq v_2 \geq \ldots \geq v_k \geq \ldots \geq v_n \geq 0$. Denote the designated bidder by bidder $\hat{\imath} \in N$. Let $z := (1-d)v_1$ and separate the set N into the two disjoint subsets $\overline{N} := \{i \in N |\, v_i \geq z\}$ and $\underline{N} := \{i \in N |\, v_i < z\}$. In equilibrium in the DA, the following two cases can then be distinguished: *Case 1* the designated bidder $\hat{\imath}$ outbids bidder 1 ($\hat{\imath} \in \overline{N}$) and *Case 2* the designated bidder $\hat{\imath}$ places a bid below the bid of the high bidder 1 ($\hat{\imath} \in \underline{N}$).

Case 1: $\hat{\imath} \in \overline{N}$

According to the assumption, the designated bidder $\hat{\imath} \in \overline{N}$ submits a bid above v_1:

$$\tilde{v}_{\hat{\imath}} = \frac{1}{1-d} v_{\hat{\imath}} \geq v_1$$

Hence, if $\hat{\imath} \neq 1$, bidding $\tilde{v}_{\hat{\imath}}$ means outbidding bidder 1 with the highest valuation v_1 and paying a price equal to $(1-d)v_1$. If bidder 1 is the designated bidder, then of course bidder 1 is the winning bidder purchasing the item at the discounted second highest bid, i.e. at a price of $(1-d)v_2$. Should $\tilde{v}_{\hat{\imath}}$ equal v_1 the tie-breaking rule is decisive: the item is awarded to both bidders, bidder $\hat{\imath}$ and bidder 1, with equal probability. However, should the designated bidder $\hat{\imath}$ be the winning bidder, bidder $\hat{\imath}$ purchases the item for the price of the discounted second highest bid:

(i) If bidder 1 ($\hat{\imath} = 1$) is the highest bidder, then the second highest bid is equal to v_2. Bidder 1 receives a discount on the second highest bid and pays $(1-d)v_2$. Bidder 1's payoff is

$\pi_1 = v_1 - (1-d)v_2 \geq 0$.
(ii) If bidder $\hat{\imath}$, $\hat{\imath} \neq 1$ is the highest bidder, then the second highest bid is v_1. The price to pay is $(1-d)v_1$ and bidder $\hat{\imath}$ gains a non-negative payoff of $\pi_{\hat{\imath}} = v_{\hat{\imath}} - (1-d)v_1 \geq 0$, as $v_{\hat{\imath}} \geq z = (1-d)v_1$.
Should a tie occur and the bid $\tilde{v}_{\hat{\imath}}$ of the designated bidder $\hat{\imath} \neq 1$ is equal to the valuation v_1 of bidder 1, the tie-breaking rule holds and the respective bidders receive a payoff of zero.

Case 2: $\hat{\imath} \in \underline{N}$
The designated bidder $\hat{\imath}$ cannot profitably outbid bidder 1. Submitting a bid of $v_{\hat{\imath}}$ according to his dominant strategy is not sufficient to outbid bidder 1 and win the auction:

$$\tilde{v}_{\hat{\imath}} = \frac{1}{1-d} v_{\hat{\imath}} < v_1$$

Bidder 1 is the highest bidder in the DA, and the price to pay is the maximum of the second highest bid and the bid $\tilde{v}_{\hat{\imath}}$: the price equals $\max\{v_2, \tilde{v}_{\hat{\imath}}\}$. The high bidder receives a non-negative payoff of $\pi_1 = v_1 - \max\{v_2, \tilde{v}_{\hat{\imath}}\}$ and the designated bidder has a payoff equal to zero.

Winning the DA by submitting the highest bid depends on the valuation of the bidders as well as on the discount, i.e. the valuation of the designated bidder $\tilde{v}_{\hat{\imath}}$, the highest valuation v_1 assigned to bidder 1, and the level of the discount.

Lemma 3.3.1 In equilibrium a bidder $\hat{\imath} \in N \setminus \{1\}$ with valuation $v_{\hat{\imath}}$ has a positive payoff if bidder $\hat{\imath}$ is the designated bidder, $v_1 > 0$ and the given discount $d \in (0,1)$ satisfies

$$d > 1 - \frac{v_{\hat{\imath}}}{v_1}$$

If bidder 1 is the designated bidder, he receives a positive payoff if the discount satisfies $d > 0$ or if $d = 0$ and $v_1 > v_2$.

Proof: The designated bidder $\hat{\imath}$, $\hat{\imath} \in N \backslash \{1\}$, places a bid of $\tilde{v}_{\hat{\imath}} = \frac{1}{1-d} v_{\hat{\imath}}$ and bidder 1 a bid of v_1. To win the auction, $\tilde{v}_{\hat{\imath}} > v_1$ must be fulfilled. As $d > 1 - \frac{v_{\hat{\imath}}}{v_1} \Leftrightarrow v_{\hat{\imath}} > (1-d)v_1 \Leftrightarrow \tilde{v}_{\hat{\imath}} = \frac{1}{1-d} v_{\hat{\imath}} > v_1$ holds, bidder $\hat{\imath}$ is the highest bidder. Further, as $d > 1 - \frac{v_{\hat{\imath}}}{v_1}$ assume an $\epsilon > 0$ such that $d = 1 - \frac{v_{\hat{\imath}}}{v_1} + \epsilon$. Then bidder $\hat{\imath}$ receives a payoff of $\pi_{\hat{\imath}} = v_{\hat{\imath}} - (1-d)v_1 = v_{\hat{\imath}} - (1 - (1 - \frac{v_{\hat{\imath}}}{v_1} + \epsilon))v_1 = v_{\hat{\imath}} - v_{\hat{\imath}} + \epsilon v_1 > 0$ as $\epsilon > 0$ and $v_1 > 0$.
Consider the case that bidder 1 is the designated bidder with the highest valuation. Bidder 1 cannot be outbid by any other bidder. With $v_1 \geq v_2$ and $d > 0$ bidder 1's payoff is equal to $v_1 - (1-d)v_2 > 0$ or with $v_1 > v_2$ and $d = 0$ bidder 1's payoff is $v_1 - v_2 > 0$.

Note that in the case of $v_1 = v_2$ and $d = 0$ the payoff of bidder 1 is equal to 0. q.e.d.

Remark 3.3.1 In equilibrium, a designated bidder $\hat{\imath} \in N$ wins the auction and has a non-negative payoff if the discount $d \in [0, 1)$ satisfies $d \geq 1 - \frac{v_{\hat{\imath}}}{v_1}$. In the case of bidding $\tilde{v}_{\hat{\imath}} = v_1$ a tie occurs. The item is awarded to each highest bidder with equal probability and the winning bidder's payoff is zero.

Lemma 3.3.1 defines a *threshold discount* d_t with $d_t = 1 - \frac{v_{\hat{\imath}}}{v_1}$. The designated bidder $\hat{\imath}$, $\hat{\imath} \in N \setminus \{1\}$ with valuation $v_{\hat{\imath}}$ receiving the discount, wins the auction only if the discount is greater than or equal to the threshold discount d_t. Consider a discount d below this threshold discount $d < d_t$. Then bidder $\hat{\imath}$ cannot outbid the high value bidder 1: $\tilde{v}_{\hat{\imath}} = \frac{1}{1-d} v_{\hat{\imath}} < \frac{1}{1-d_t} v_{\hat{\imath}} = \frac{1}{1-(1-\frac{v_{\hat{\imath}}}{v_1})} v_{\hat{\imath}} = v_1$.

Regarding the seller's revenue, the following can be stated: the revenue of the seller is positively affected only if the designated bidder $\hat{\imath} \neq 1$ submits a bid above the second highest valuation and below the highest valuation (see *Case 2*): $b_{\hat{\imath}} \in (v_2, v_1), \hat{\imath} \neq 1$. The price to pay for bidder 1 being the high value bidder in the auction equals $p = \max\{v_2, \tilde{v}_{\hat{\imath}}\} = \tilde{v}_{\hat{\imath}} > v_2$ with $\tilde{v}_{\hat{\imath}} < v_1$.

Lemma 3.3.2 In equilibrium the seller's revenue in the DA compared to the respective revenue in the corresponding second-price auction is increased only if the designated bidder's bid $\tilde{v}_{\hat{\imath}}$, $\hat{\imath} \in N \setminus \{1\}$, satisfies the equation:

$$v_2 < \tilde{v}_{\hat{\imath}} < v_1 \quad \Leftrightarrow \quad (1-d)v_2 < v_{\hat{\imath}} < (1-d)v_1$$

The discount $d \in [0, 1)$ satisfies the equation:

$$1 - \frac{v_{\hat{\imath}}}{v_2} < d < 1 - \frac{v_{\hat{\imath}}}{v_1}$$

with v_1 being the highest valuation and v_2 the second highest valuation.

Proof: The designated bidder $\hat{\imath}$, $\hat{\imath} \neq 1$, receives the discount and bids $\tilde{v}_{\hat{\imath}} = \frac{1}{1-d} v_{\hat{\imath}}$. The seller's revenue in the DA is greater than the respective revenue in the corresponding second-price auction only if the high value bidder 1 wins the auction and the price to pay is greater than v_2 (see *Case 2*). As denoted in *Case 2*, the price to pay is equal to $\max\{v_2, v_{\hat{\imath}}\}$. Only if $v_2 < \tilde{v}_{\hat{\imath}} < v_1$ the price the winning bidder 1 has to pay equals $v_{\hat{\imath}}$, which is greater than v_2, the payment of the corresponding second-price auction.

Thus, the seller's revenue in the DA is greater than the respective revenue in the corresponding second-price auction. Moreover, as $v_2 < \tilde{v}_{\hat{\imath}} < v_1$, the following inequality is derived immediately $v_2 < \frac{1}{1-d} v_{\hat{\imath}} < v_1 \Leftrightarrow (1 - \frac{v_{\hat{\imath}}}{v_2} < d) \wedge (d < 1 - \frac{v_{\hat{\imath}}}{v_1})$. q.e.d.

Thus a bid from the designated bidder influences the auction revenue in the DA in comparison to the auction revenue in the corresponding second-price auction only if $\tilde{v}_{\hat{\imath}} > v_2$: (i) in the case of $\tilde{v}_{\hat{\imath}} \geq v_1$, the designated bidder wins the auction and pays the discounted highest value v_1: $(1-d)v_1$ and (ii) in the case of $v_1 > \tilde{v}_{\hat{\imath}} > v_2$, bidder 1 purchases the item a price of $\tilde{v}_{\hat{\imath}}$, which is above the second highest valuation v_2.

In the following, from the seller's perspective, an interval or *corridor* of discounts can be defined, which ensures, that the bid of the designated bidder $\hat{\imath}$ has a positive impact on the revenue of the seller. That is, $\underline{d}_{\hat{\imath}} = 1 - \frac{v_{\hat{\imath}}}{v_2}$ defines the *minimum level* of that corridor and $\overline{d}_{\hat{\imath}} = 1 - \frac{v_{\hat{\imath}}}{v_1}$ the *maximum level*. The maximum level is equal to the threshold discount d_t. Each discount in the *corridor* $d \in (\underline{d}_{\hat{\imath}}, \overline{d}_{\hat{\imath}})$ ensures that the designated bidder $\hat{\imath} \in N \setminus \{1\}$ with valuation $v_{\hat{\imath}}$ increases the seller's revenue in the DA compared to the respective revenue in the corresponding second-price auction.

The following example illustrates the two cases mentioned above (*Case 1* and *Case 2*) as well as the discount threshold and the discount corridor.

Example 3.3.1 First, assume that three risk neutral bidders are competing in a second-price auction with private values. Bidder 1 values the item at $v_1 = 30$ euros, bidder 2 at $v_2 = 28$ euros and bidder 3 at $v_3 = 26$ euros. The dominant strategy in a second-price auction is to bid truthfully. Thus, the second-price auction ends at a final price of $p = 28$ euros and the item is awarded to bidder 1, the high value bidder.
Assume now that the item is auctioned in the DA with a discount of $d = 10\%$. The three bidders with the given private valuations for the item participate in the DA. If the discount is assigned to bidder 1, bidder 1 places a bid of $\tilde{v}_1 = 33.33$ euros; if bidder 2 is selected as designated bidder, bidder 2 submits a bid of $\tilde{v}_2 = 31.11$ euros; if bidder 3 is awarded the discount, bidder 3 bids $\tilde{v}_3 = 28.89$ euros.
Consider *Case 1* and *Case 2* with the sets $\overline{N} = \{v_1, v_2\}$ and $\underline{N} = \{v_3\}$. That is, should either bidder 1 or bidder 2 be awarded the discount, the winning bidder is the designated bidder and the price to pay is the discounted second highest bid. If bidder 3 is awarded the discount, bidder 3 cannot outbid the highest bidder.
Case 1: If bidder 1 is awarded the discount, bidder 1 bids 33.33 euros, wins the auction, and pays the discounted second highest bid $(1-0.1)28$ euros $= 25.20$ euros. If bidder 2 is selected

as the designated bidder, bidder 2 places a bid of 31.11 euros and wins the auction. The price to pay is the discounted second highest bid and equals $(1-0.1)30$ euros $=27$ euros.
Case 2: If bidder 3 receives the discount, bidder 3 bids 28.89 euros and thus cannot outbid bidder 1. Bidder 1 wins the auction with a bid at 30 euros and pays $p=\max\{v_2,\tilde{v}_3\}=28.89$ euros.
Regarding the threshold discount, bidder 3 as the designated bidder wins the auction and receives a positive payoff only if the discount d is greater than the threshold discount $d_t = 1-\frac{v_3}{v_1}=\frac{2}{15}=13.33\%$ (Lemma 3.3.1). Moreover, in equilibrium a bid of bidder 3 increases the revenue of the auctioneer only if if the discount is chosen in the bidding corridor of $(1-\frac{v_3}{v_2}, 1-\frac{v_3}{v_1})=(7.14\%, 13.33\%)$. That is, if in the DA the discount d is selected out of the interval $(7.14\%, 13.33\%)$, then the seller's revenue in the DA is greater than the respective revenue in the corresponding second-price auction.

In a second-price auction, an equilibrium in dominant strategy exists and the expected revenue of the seller is equal to the expected value of the second highest bid. Furthermore, it is a well known result from auction theory that in an SIPV setting with risk neutral bidders, the *revenue equivalence theorem* holds: the four standard auctions, i.e. the ascending, descending, first-price sealed-bid and second-price sealed-bid auction yield the same expected revenue (cf. for example to Wolfstetter 1999; Krishna 2002; Klemperer 2004). When introducing bidding credits in the form of a *discount* to a second-price auction, the revenue equivalence theorem does not apply. Consider the DA in an SIPV setting. A low value bidder with an assigned discount is able to outbid the bidder with the highest valuation, win the auction and purchase the item due to the discount. Thus, it is no longer assured that the item is awarded to the high value bidder. In particular, the DA and the second-price auction are neither revenue-equivalent nor strategically-equivalent.

When dropping the symmetry assumption, bidders are characterized by different distribution functions of valuations. From literature it is known that under these conditions, the seller's revenue in a second-price auction strongly depends on the types of the distribution functions (cf. Cantillon (2005) or Maskin and Riley (2000)).

The DA and its corresponding second-price auction are modelled and analyzed in the following sections: firstly in an SIPV setting (symmetric case) and secondly in an independent private values setting, in which bidders draw their valuations from different distribution functions (asymmetric case). In both cases bidders are assumed to be risk neutral.

When considering the expected revenue of the seller, then one major conclusion from this model is that in the symmetric case, the discount does not pay for the seller, while in the asymmetric case the discount may raise the seller's expected revenue. In addition, the bidders' payoffs as well as the social welfare are of interest. To give greater insights into the DA and deepen the understanding of how the discount affects the outcome, the following questions will be answered in both, the symmetric and the asymmetric case from a theoretical perspective:

- What is the expected outcome in the DA, i.e. the seller's expected revenue, the expected payoff of the designated bidder as well as a the expected payoff of a non-designated bidder, and the expected social welfare?

- What is the expected outcome in the corresponding second-price auction, i.e. the seller's expected revenue, the winning bidder's expected payoff, and the expected social welfare?

- To what extend does the discount affect the seller's expected revenue in the DA compared to the respective expected revenue in the corresponding second-price auction?

- Focussing on bidders' characteristics and distinguishing between the case of symmetric bidders and the case of asymmetric bidders: In which case can the seller extract an additional revenue and raise her revenue?

The aim of the following sections is to present a formal model of the DA and its corresponding second-price auction as well as to address the questions mentioned above.

3.4 Notation

Throughout this chapter it is assumed that a finite number of bidders participate in the DA and its corresponding second-price auction in an IPV setting. Let $N = \{1, \dots, n\}$ be the set of bidders and $n \in \mathbb{N}$, $n \geq 1$ be the number of bidders. Bidders are assumed to be risk neutral. The number of bidders as well as their risk attitudes are common knowledge. A bid of bidder $i \in N$ which he submits in the auction is denoted by b_i. At the time a bidder submits his bid, he knows how much he values the item for which he is bidding. Furthermore, in the DA $d \in [0, 1)$ denotes the discount, which one randomly selected bidder is granted. Additionally, dependent on the discount, the factor $\delta \in \mathbb{R}$ is defined by $\delta = \frac{1}{1-d} \geq 1$.

In the following, two cases are distinguished: first, the *symmetric case*, where bidders are of the same type and their valuations are independently distributed according to the same cumulative probability distribution function F; and second, the *asymmetric case*, where bidders are of different types and their valuations are characterized by two different cumulative probability distribution functions F_w and F_s.

1. In the *symmetric case*, the valuations $v_1, \ldots, v_n$ are realizations of independent random draws of the random variable V with the cumulative probability distribution function (cdf) $F : \mathbb{R} \rightarrow [0,1]$ which has the convex support $\mathcal{M} \subset \mathbb{R}$. The corresponding probability density function (pdf) is given by $f(v)$, $f : \mathbb{R} \rightarrow \mathbb{R}_+$, which is the derivative of $F(v)$ with $f(v) \equiv F'(v)$.
   Further, let $v_1, \ldots, v_n \in \mathbb{R}$ be n independent draws from the random variable V with the probability distribution function $F(v)$. Then, the n realizations $v_1, \ldots, v_n$ can be sorted and rearranged in decreasing order as $v_{(1)} \geq v_{(2)} \geq \ldots \geq v_{(n)}$. Each $v_{(k)}$, $k \in N$, is defined as the realization of the random variable $V_{(k),n}$, the kth order statistic. The kth order statistic $V_{(k),n}$ assigns to each of the n realization of V the kth highest value $v_{(k)}$. If the number n of drawings is known in advance, then the distribution and density of $V_{(1),n}$ and $V_{(2),n}$ is denoted by $F_{(1),n}$, $f_{(1),n}$ and $F_{(2),n}$, $f_{(2),n}$ (cf. Appendix B.2).
   Suppose that the n bidders participate in a DA. There is exactly one designated bidder in the DA. Let bidder $\hat{\imath} \in N$ denote the designated bidder to whom the discount is assigned. Bidder $\hat{\imath}$ has a valuation of $v_{\hat{\imath}}$ for the item; his weakly dominant strategy in the DA is to bid $b_{\hat{\imath}} = \tilde{v}_{\hat{\imath}} = \frac{1}{1-d} v_{\hat{\imath}}$, $d \in [0,1)$. The bid $\tilde{v}_{\hat{\imath}}$ is a realization of a random draw of the random variable $\tilde{V}$. $\tilde{F} : \mathbb{R} \rightarrow [0,1]$ with the convex support $\mathcal{S} \subset \mathbb{R}$ is the cdf of $\tilde{V}$ and $\tilde{f} : \mathbb{R} \rightarrow \mathbb{R}_+$ the respective pdf. $\tilde{F}$ is defined by $\tilde{F}(v) = F(\frac{1}{\delta}v) \, \forall v \in \mathbb{R}$ and its associated pdf is given by $\tilde{f}(v) = \frac{1}{\delta} f(\frac{1}{\delta}v) \, \forall v \in \mathbb{R}$
   More specifically, $\tilde{V}$ is a linear transformation of V: $t : \mathbb{R} \rightarrow \mathbb{R}$ with $t(v) = \tilde{v} = \delta v = \frac{1}{1-d} v$ defines the linear transformation function of the random variable V (cf. also to Appendix B.1).

2. In the asymmetric case, the n bidders are divided into two groups: a group of m weak bidders, $m \in \mathbb{N}, n > m > 1$, and a group of ($n - m$) strong bidders. Suppose w.l.o.g. bidder 1, $\ldots$, bidder m are the weak bidders in the auction and bidders $m+1$, $\ldots$, bidders n are the strong bidders. Let W denote the set of the weak bidders and S the set of the strong bidders with $N = S \cup W$ and $\emptyset = S \cap W$. Each weak bidder assigns a valuation $v_i, i \in W$ to the item. The valuations $v_1, \ldots, v_m$ are realizations of independent draws of the random variable V_w. The distribution function of V_w is given by $F_w : \mathbb{R} \rightarrow [0,1]$ which has the convex support $\mathcal{W}$ and its respective positive pdf is given by $f_w : \mathbb{R} \rightarrow \mathbb{R}_+$. The valuations $v_{m+1}, \ldots, v_n$ of the high value bidders are independently drawn from the random variable V_s which is distributed by F_s. F_s :

$\mathbb{R} \rightarrow [0,1]$ with the convex support $\mathcal{S}$ is the cdf of V_s and $f_s : \mathbb{R} \rightarrow \mathbb{R}_+$ its respective pdf. However, the valuations of the weak and strong bidders are distributed by different distribution functions, F_w and F_s with $F_w \neq F_s$.

Further, let $v_1, \ldots, v_n \in \mathbb{R}$ be m independent draws from the random variable V_w and $n - m$ independent draws of V_s with the probability distribution functions $F_w(v)$ and $F_s(v)$. Then, the vector $(v_{(1),n,m}, \ldots, v_{(n),n,m})$ denotes the sorted and rearranged order of the n realizations $v_1, \ldots, v_n$ by $v_{(1),n,m} \geq v_{(2),n,m} \geq \ldots \geq v_{(n),n,m}$. Each $v_{(k),n,m}$, $k \in N$, is defined as the realization of the random variable $V_{(k),n,m}$, the kth order statistic which assigns to each of the n realization of V_w and V_s the kth highest value $v_{(k),n,m}$. The distribution and density of $V_{(1),n,m}$ and $V_{(2),n,m}$ are denoted by $F_{(1),n,m}$, $f_{(1),n,m}$ and $F_{(2),n,m}$, $f_{(2),n,m}$.

Suppose that the m weak bidders and the $n - m$ strong bidders participate in the DA. Then the designated bidder $\hat{\imath} \in N$ may either belong to the group of weak bidders or the group of strong bidders. Should the designated bidder be a weak bidder, then w.l.o.g. the weak bidder is denoted by bidder $\hat{\imath} \in W$. If the designated bidder is a high value bidder, then w.l.o.g. the high value bidder is denoted by bidder $\hat{\jmath} \in S$. Independent of whether the designated bidder is a low value bidder $\hat{\imath}$ or a high value bidder $\hat{\jmath}$, the designated bidder has a (weakly) dominant strategy in the DA. The designated bidder to whom a valuation of v of the item is attached submits a bid of $\tilde{v} = \frac{1}{1-d}v$.

If the designated bidder is a low value bidder, his bid $\tilde{v}_{\hat{\imath}}$ is a realization of a random draw of the random variable $\tilde{V}_w$. The random variable $\tilde{V}_w$ is distributed according to $\tilde{F}_w$ with the convex support $\tilde{\mathcal{W}}$. More precisely, the random variable $\tilde{V}_w$ is a linear transformation of the random variable V_w with $\tilde{V}_w = t(V_w) = \delta V_w = \frac{1}{1-d}V_w$, $\delta \in \mathbb{R}$, $\delta = \frac{1}{1-d}$. $\tilde{F}_w : \mathbb{R} \rightarrow [0,1]$ is the cdf of $\tilde{V}_w$ defined by $\tilde{F}_w(v) = F_w(\frac{1}{\delta}v) \, \forall v \in \mathbb{R}$ and $\tilde{f}_w : \mathbb{R} \rightarrow \mathbb{R}$ its respective pdf with $\tilde{f}_w(v) = \frac{1}{\delta} f_w(\frac{1}{\delta}v) \, \forall v \in \mathbb{R}$. Hence, the same holds in the case, that the high value bidder $\hat{\jmath} \in S$ is the designated bidder to whom a valuation of $v_{\hat{\jmath}}$ is assigned. His bid $b_{\hat{\jmath}} = \tilde{v}_{\hat{\jmath}}$ is a realization of a random draw of the random variable $\tilde{V}_s$. Again, the random variable $\tilde{V}_s$ is a linear transformation of the random variable V_s. The cdf of $\tilde{V}_s$ is denoted by $\tilde{F}_s : \mathbb{R} \rightarrow [0,1]$ with $\tilde{F}_s(v) = F_s(\frac{1}{\delta}v) \, \forall v \in \mathbb{R}$ and the convex support $\tilde{\mathcal{S}}$. Its respective pdf is given by $\tilde{f}_s : \mathbb{R} \rightarrow \mathbb{R}$ with $\tilde{f}_s(v) = \frac{1}{\delta} f_s(\frac{1}{\delta}v) \, \forall v \in \mathbb{R}$.

3.5 Second-price auction

3.5.1 Symmetric distribution functions

The corresponding auction to the DA is the second-price auction. In the second-price auction, the high bidder wins the auction and the price to pay is the second highest bid. Recall, that in the second-price auction in an IPV setting truthful bidding $b_i \equiv v_i, i \in N$ is a weakly dominant strategy. The bidder with the highest valuation purchases the item at the price of the second highest valuation.

In the following, the second-price auction is analyzed in a SIPV setting with risk neutral bidders. More precisely, (i) the expected revenue of the seller, (ii) the expected payoff of the winning bidder, and (iii) the expected social welfare in the second-price auction are calculated. As the second-price auction is thoroughly studied in the literature, for example in Wilson (1991), Wolfstetter (1999), Krishna (2002), or Klemperer (2004), this section only briefly summarizes the most important results.

The price the high value bidder in a second-price auction pays is a realization of a random draw of the second order statistic. The price equals $v_{(2)}$ which denotes the second highest valuation among the realizations $v_1, \ldots, v_n$ of n independent draws of random variable V (cf. Appendix B.2). As noted, random variable V is distributed according to the distribution function F. The valuation $v_{(2)}$, the second highest valuation, is an independent draw of the second order statistic $V_{(2),n}$ with its distribution function

$$F_{(2),n} = nF^{n-1}(v) - (n-1)F^n(v) \tag{3.3}$$

(Equation B.9 with $k = 2$) and its respective pdf $f_{(2),n}(v) \equiv F'_{(2),n}(v)$ (Equation B.10 with $k = 2$)

$$f_{(2),n}(v) = n(n-1)F^{n-2}(v)f(v)(1 - F(v)) \tag{3.4}$$

With the equations above the **seller's expected revenue** in a second-price auction (EA) can easily be derived: the expected revenue is just the expectation of the second highest value, or the second order statistic, and is calculated by

$$E[R_{EA}] = E[V_{(2),n}] = \int_0^\infty v f_{(2),n}(v)dv \tag{3.5}$$

Further, fix a bidder $i \in N$ and let $y = \max_{j \in N \setminus \{i\}} v_j$ be the highest bid among the $n-1$ rival bidders. y is a realization of a draw of the random variable Y, which denotes the highest value among the $n-1$ remaining valuations except the valuation of the fixed bidder i. The random variable $Y \equiv V_{(1),n-1}$ is distributed according to the distribution function $F_{(1),n-1}$ with

$$F_{(1),n-1}(v) = F^{n-1}(v) \tag{3.6}$$

and its respective pdf

$$f_{(1),n-1}(v) = (n-1)F^{n-2}(v)f(v) \tag{3.7}$$

The **expected payoff** π_i **of the winning bidder** $i \in N$ is

$$E\pi_{i,EA} = \int_0^\infty \int_0^v (v-y)f_{(1),n-1}(y)dyf(v)dv \quad i \in N \tag{3.8}$$

The expected social welfare in the second-price auction equals the expectation of the highest valuation, i.e. the expected value of the first order statistic (Appendix B.2). $F_{(1),n}$ is the distribution function of the first order statistic $V_{(1),n}$ (Equation B.7) and $f_{(1),n}$ is the corresponding density function (Equation B.8)

$$F_{(1),n}(v) = F^n(v) \text{ and } f_{(1),n}(v) = nF^{n-1}(v)f(v) \tag{3.9}$$

The **expected social welfare** in the second-price auction calculates to

$$E[W_{EA}] = E[V_{(1),n}] = \int_0^\infty vf_{(1),n}(v)dv \tag{3.10}$$

An alternative way to calculate the expected welfare is to calculate the expected gains of all auction participants – the expected revenue of the seller as well as the expected payoff of all bidders.

$$E[W_{EA}] = E[R_{EA}] + nE\pi_{i,EA} \tag{3.11}$$

Note that in the equations above the subscript EA denotes the second-price auction.

Example 3.5.1 Suppose four risk neutral bidders participate in a second-price auction in a SIPV setting. Bidders have private valuations $v_i, i \in N = \{1, \dots, 4\}$ which are realizations of independent draws of random variable V. V is distributed according to the exponential distribution function $F : \mathbb{R} \to [0,1]$ with

$$F(v) = 1 - e^{-\lambda v} \quad \lambda > 0$$

and the corresponding density function $f : \mathbb{R} \to \mathbb{R}_+$ with

$$f(v) = \lambda e^{-\lambda v}$$

For $\lambda = \frac{1}{100}$ the expected value of V equals $E[V] = \frac{1}{\lambda} = 100$. The seller's expected revenue, the bidder's expected payoff and the expected welfare are then calculated and result as follows. The seller has an expected revenue of

$$E[R_{EA}] = 108.33$$

and each bidder has an expected payoff of

$$E\pi_{i,EA} = 25 \quad i = 1, \ldots, 4$$

(cf. Equation 3.5 and Equation 3.8). The expected welfare in the second-price auction is calculated by Equation 3.10 and equals

$$E[W_{EA}] = 208.33$$

For $\lambda = \frac{1}{150}$ the seller's expected revenue, the expected payoff of a bidder and the expected welfare equal

$$E[R_{EA}] = 162.50, \quad E\pi_{i,EA} = 37.50\,(i = 1, \ldots, 4) \quad \text{and} \quad E[W_{EA}] = 312.50$$

This example is calculated with Maple 9.5, a programming environment for mathematical problem-solving.[16]

3.5.2 Asymmetric distribution functions

In this section, the corresponding second-price auction of the DA is analyzed in an asymmetric IPV environment with risk neutral bidders. In contrast to the SIPV setting, the symmetry assumption is dropped, such that bidders are characterized by different probability distribution functions of valuations. The aim is to calculate the seller's expected revenue, the bidder's expected payoff as well as the expected welfare in the corresponding second-price auction under the given conditions.

Assume an independent private values setting. Suppose n risk neutral bidders participate in a second-price auction ($N = \{1, \ldots, n\}$). In particular, there are m weak bidders (low value bidders) with private valuations $v_1, \ldots, v_m$ of the item being auctioned. Each valuation $v_i, i \in W$ is a realization of an independent draw of the random variable V_w distributed by $F_w(v)$. The valuations $v_{m+1}, \ldots, v_n$ of the $n - m$ strong bidders (high value bidders) of the item are realizations of $n - m$ independent random draws of the random variable V_s with the distribution function $F_s(v)$.

[16] See http://www.maplesoft.com.

As noted $(v_{(1),n,m},\ldots,v_{(n),n,m})$ is the ordered vector of the valuations $v_i, i\in W$, of the m weak bidders and the valuations $v_j, j\in S$, of the $n-m$ strong bidders. $v_{(1),n,m}$ and $v_{(2),n,m}$ are the realizations of the random variables $V_{(1),n,m}$ and $V_{(2),n,m}$, i.e. the first and second order statistic. $F_{(k),n,m}$ denotes the distribution function of the kth order statistic $V_{(k),n,m}$ and $f_{(k),n,m}$ its corresponding density function. Note that the density function $f_{(k),n,m}$ is the derivative of the distribution function $F_{(k),n,m}$, $k\in N$.

As the seller's expected revenue is the expectation of the second order statistic, the distribution function $F_{(2),n,m}$ as well as the density function $f_{(2),n,m}$ have to be calculated. $F_{(2),n,m}$ defines the probability of the event that $V_{(2),n,m}\le v$. In fact, this is equal to the event that (i) all n independent random draws of V_w and V_s are less than or equal to v, or (ii) $m-1$ independent random draws of V_w are less than or equal to v, one random draw of V_w is greater than v, and all $n-m$ random draws of V_s are less than or equal to v, or (iii) all m random draws of V_w are less than or equal to v, $n-m-1$ random draws of V_s are less than or equal to v, and one random draw of V_s is greater than v. There are m different ways in which (ii) can occur and $n-m$ ways in which (iii) can occur. So, the distribution function of the second order statistic equals

$$\begin{aligned} F_{(2),n,m}(v) &= F_w^m(v)F_s^{n-m}(v)+mF_w^{m-1}(v)\left(1-F_w(v)\right)F_s^{n-m}(v) \\ &\quad +(n-m)F_w^m(v)F_s^{n-m-1}(v)\left(1-F_s(v)\right) \\ &= mF_w^{m-1}(v)F_s^{n-m}(v)+(n-m)F_w^m(v)F_s^{n-m-1}(v) \\ &\quad -(n-1)F_w^m(v)F_s^{n-m}(v) \end{aligned} \tag{3.12}$$

and the associated density function is

$$\begin{aligned} f_{(2),n,m}(v) &= m(m-1)F_w^{m-2}(v)f_w(v)F_s^{n-m}(v) \\ &\quad +m(n-m)F_w^{m-1}(v)F_s^{n-m-1}(v)f_s(v) \\ &\quad +(n-m)mF_w^{m-1}(v)f_w(v)F_s^{n-m-1}(v) \\ &\quad +(n-m)(n-m-1)F_w^m(v)F_s^{n-m-2}(v)f_s(v) \\ &\quad -(n-1)mF_w^{m-1}(v)f_w(v)F_s^{n-m}(v) \\ &\quad -(n-1)(n-m)F_w^m(v)F_s^{n-m-1}(v)f_s(v) \end{aligned} \tag{3.13}$$

Using Equation 3.13 the expected value of the second highest valuation yields the **seller's expected revenue** in a second-price auction in which bidders' valuations are characterized by asymmetric distribution functions.

$$E[R_{EA}^{as}]=E[V_{(2),n,m}]=\int_0^\infty v f_{(2),n,m}(v)dv \tag{3.14}$$

Note that the superscript *as* indicates the asymmetries among the distribution functions and the subscript *EA* denotes the second-price auction. The expected revenue of the seller is just the sum of the ex-ante (prior to knowing their valuations) expected payments of the bidders.

To calculate the bidder's expected payoff, the ex-ante expected payment of a particular bidder in the auction is determined. Recall, that in case of winning the second-price auction the high bidder i's payoff is $\pi_i = v_i - y, i \in N$, where y denotes the final price in the auction, i.e. $y = \max_{j \in N \setminus \{i\}} b_j$. In case of bidder i loosing the auction, his payoff equals zero. Thus, to determine the expected payment of the high bidder, the highest value among the $n-1$ remaining bidders has to be determined. If a weak bidder wins the auction, the final price is determined by the bids of the $m-1$ remaining weak bidders and all $n-m$ strong bidders. If a strong bidder is the high bidder in the auction, the payment equals the highest valuation of all m weak bidders and $n-m-1$ remaining strong bidders. In both cases, y, which denotes the final price in the auction, is a realization of a random draw of the random variable $Y \equiv V_{(1),n-1,r}$, $r \in \{m-1, m\}$. The distribution function of Y is obtained by

$$F_{(1),n-1,r}(v) = F_w^r(v) F_s^{n-r-1}(v) \tag{3.15}$$

with

$$r = \begin{cases} m-1, & \text{if the high bidder is a weak bidder} \\ m, & \text{if the high bidder is a strong bidder} \end{cases}$$

The associated density function is

$$f_{(1),n-1,r}(v) = rF_w^{r-1}(v) f_w(v) F_s^{n-r-1}(v) + (n-r-1) F_w^r(v) F_s^{n-r-2} f_s(v) \tag{3.16}$$

Now, the expected payoff of a bidder can be written as follows:
The **expected payoff of a weak bidder** equals

$$E\pi_{i,w,EA}^{as} = \int_0^\infty \int_0^v (v-y) f_{(1),n-1,m-1}(y) dy f_w(v) dv \quad i \in W \tag{3.17}$$

with

$$f_{(1),n-1,m-1}(y) = (m-1) F_w^{m-2}(y) f_w(y) F_s^{n-m}(y) + (n-m) F_w^{m-1}(y) F_s^{n-m-1} f_s(y)$$

The **expected payoff of a strong bidder** is calculated by

$$E\pi_{j,s,EA}^{as} = \int_0^\infty \int_0^v (v-y) f_{(1),n-1,m}(y) dy f_s(v) dv \quad j \in S \tag{3.18}$$

with

$$f_{(1),n-1,m}(y) = mF_w^{m-1}(y) f_w(y) F_s^{n-m-1}(y) + (n-m-1) F_w^m(y) F_s^{n-m-2} f_s(y)$$

Note that in the equations above subscript w indicates "*weak*" and subscript s names "*strong*". Again, superscript as denotes the asymmetric case and subscript EA the second-price auction.

The expected welfare in the auction is the expectation of the first order statistic $V_{(1),n,m}$. The event that $V_{(1),n,m} \leq v$ is equal to the event that all m independent random draws from V_w (distributed by F_w) are below v and that all $n-m$ independent random draws from V_s (distributed by F_s) are below v. Thus, the distribution function of the first order statistic $V_{(1),n,m}$ equals

$$F_{(1),n,m}(v) = F_w^m(v)F_s^{n-m}(v) \tag{3.19}$$

The associated probability density function is the derivative of $F_{(1),n,m}(v)$ and obtained by

$$\begin{aligned} f_{(1),n,m}(v) &= mF_w^{m-1}(v)f_w(v)F_s^{n-m}(v) + (n-m)F_w^m(v)F_s^{n-m-1}(v)f_s(v) \quad (3.20) \\ &= F_w^{m-1}(v)F_s^{n-m-1}(v)\left(mf_w(v)F_s(v) + (n-m)f_s(v)F_w(v)\right) \end{aligned}$$

As the **expected welfare** is just the expectation of the highest valuation the expected welfare calculates to

$$E[W_{EA}^{as}] = E[V_{(1),n,m}] = \int_0^\infty v f_{(1),n,m}(v)dv \tag{3.21}$$

Consider, that the expected welfare equals the expected revenue of all players in the auction – the expected revenue of the seller and the expected revenues of all n bidders. So, the expected welfare in the second-price auction with m weak bidders and $n-m$ strong bidders can be rewritten as

$$E[W_{EA}^{as}] = E[R_{EA}^{as}] + mE\pi_{i,w,EA}^{as} + (n-m)E\pi_{j,s,EA}^{as} \tag{3.22}$$

Example 3.5.2 Suppose four risk neutral bidders participate in a second-price auction under the conditions of the IPV. Further, it is supposed, that bidders are asymmetric, i.e. bidders' valuations are distributed according to two different distribution functions. Bidders 1, 2 and 3 are weak bidders with valuations, which are drawn from the random variable V_w with the distribution function $F_w : \mathbb{R} \rightarrow [0, 1]$,

$$F_w(v) = 1 - e^{-\lambda_w v} \quad \lambda_w > 0$$

The valuations of bidder 4, the strong bidder, are drawings of random variable V_s, which is distributed by $F_s : \mathbb{R} \rightarrow [0, 1]$ with

$$F_s(v) = 1 - e^{-\lambda_s v} \quad \lambda_s > 0$$

Let $\lambda_w = \frac{1}{100}$ and $\lambda_s = \frac{1}{200}$. Then, the expected value of a weak bidder is $E[V_w] = \frac{1}{\lambda_w} = 100$ and the expected value of the strong bidder equals $E[V_s] = \frac{1}{\lambda_s} = 200$.

The associated density function to the distribution function $F_k(v)$ is given by $f_k : \mathbb{R} \to \mathbb{R}_+$, $f_k(v) = \lambda e^{-\lambda_k v}$, $\lambda_k > 0$, $k = w, s$.
Under the above assumptions, the seller in the second-price auction has an expected revenue of

$$E[R^{as}_{EA}] = 129.05$$

The expected payoff of a weak bidder i, $i = 1, 2, 3$, is denoted by $E\pi^{as}_{i,w,EA}$ and of the strong bidder by $E\pi^{as}_{4,s,EA}$ with

$$E\pi^{as}_{i,w,EA} = 18.10 \quad i = 1, \ldots, 3 \quad \text{and} \quad E\pi^{as}_{4,s,EA} = 91.43$$

The expected welfare in the second-price auction is either the sum of the expected revenue of all participants in the auction, or the expected value of the first order statistic and equals

$$E[W^{as}_{EA}] = 274.76$$

Increasing solely the expected value of the weak bidders from $E[V_w] = 100$ ($\lambda_w = \frac{1}{100}$) to $E[V_w] = 150$ ($\lambda_w = \frac{1}{150}$) while keeping the expected value of the strong bidder constant, then the following expectations are derived:

$$E[R^{as}_{EA}] = 174.87, \quad E[W^{as}_{EA}] = 341.49$$

$$E\pi^{as}_{i,w,EA} = 33.38 \; i = 1, \ldots, 3 \quad \text{and} \quad E\pi^{as}_{4,s,EA} = 66.49$$

3.6 Discount auction

3.6.1 Symmetric distribution functions

Consider the DA market institution as defined in Definition 3.3.1 in the SIPV setting with risk neutral bidders. In the following the expected outcomes in the DA, i.e. the seller's expected revenue, the bidder's expected payoff and the expected welfare are analyzed.

Suppose n bidders compete in the DA for an item offered for sale. Bidders are characterized by the same probability distribution function of valuations. That is, each valuation v_i of bidder $i \in N$ of the item being auctioned is a realizations of an independent draw of random variable V. As noted, bidders' valuations are independent and identically distributed according to the distribution function F. Suppose further that the designated bidder who is granted the discount is denoted by $\hat{\imath} \in N$. Recall that in the DA each bidder has a weakly dominant strategy (Section 3.3.2, Proposition 3.3.1): the designated bidder submits a bid of $b_{\hat{\imath}} = \frac{1}{1-d} v_{\hat{\imath}} = \delta v_{\hat{\imath}}$ with $\delta = \frac{1}{1-d}$ and $d \in [0, 1)$; all rival bidders $i \in N \setminus \{\hat{\imath}\}$ submit their values truthful and bid $b_i = v_i$. As already noted, the bids $b_{\hat{\imath}}$ of the designated bidders are realizations of a random draw of random variable $\tilde{V}$ which is distributed according to $\tilde{F}$ with $\tilde{F}(v) = F(\frac{1}{\delta}v)$.

Before calculating the expected outcomes in the DA, remember, that in principle in the DA two cases are to distinguish:

Case I: The designated bidder $\hat{\imath}$ wins the DA.

Case II: A bidder $i \in N \setminus \{\hat{\imath}\}$ is the high bidder in the DA.

In the following, the two cases are explained more precisely.
Case I: Consider the case that the designated bidder $\hat{\imath} \in N$ wins the auction with a bid of $b_{\hat{\imath}} = \delta v_{\hat{\imath}}$. His payoff is $\pi_{\hat{\imath}} = v_{\hat{\imath}} - \frac{1}{\delta}y = v_{\hat{\imath}} - (1-d)y$ with $\delta = \frac{1}{1-d}$, $d \in [0,1)$ and payment y being the highest bid among the $n-1$ rival bidders' bids: $y = \max_{j \in N \setminus \{\hat{\imath}\}} v_j$. Y denotes the highest valuation among the $n-1$ rival bidders' valuations and is distributed according to $G_{(1),n-1}(y)$ with

$$G_{(1),n-1}(y) = F^{n-1}(y) \tag{3.23}$$

Case II: Consider the case that a non-designated bidder $i \in N$, $i \neq \hat{\imath}$ is the high bidder in the auction. The payoff of that bidder equals $\pi_i = v_i - y$. The payment y is the highest bid of the $n-2$ rival bidders' valuations, to whom the discount is not assigned, and of the designated bidder's bid: $y = \max\{\max_{j \in N \setminus \{i,\hat{\imath}\}} v_j, \delta v_{\hat{\imath}}\}$. That is the price at which bidder i purchases the item. Y is distributed according to $G_{(1),n-2}(y)$ with

$$G_{(1),n-2}(y) = F^{n-2}(y)\tilde{F}(y) = F^{n-2}(y)F(\frac{1}{\delta}y) \tag{3.24}$$

To be more general, fix one bidder. The final price y which is the highest bid of the remaining $n-1$ bidders is a realization of a draw of random variable Y. The random variable Y is distributed according to the distribution function $G_{(1),r}(y), r = n-1, n-2$ with

$$G_{(1),r}(y) = F^r(y)\tilde{F}^{n-1-r}(y) = F^r(y)F^{n-1-r}(\frac{1}{\delta}y) \tag{3.25}$$

with

$$r = \begin{cases} n-1, & \text{if the high bidder is the designated bidder} \\ n-2, & \text{if the high bidder is a non-designated bidder} \end{cases}$$

For $r = n-1$, $G_{(1),n-1}(y)$ is the distribution function of the first order statistic which assigns to each realization of the $n-1$ independent draws of random variable V the highest value, that is **Case I**. For $r = n-2$, $G_{(1),n-2}(y)$ is the distribution function of the first order statistic which assigns to the $n-2$ independent draws of random variable V and a single draw of the random variable $\tilde{V}$ the highest value, that is **Case II**. The associated density function is given by

$$\begin{aligned} g_{(1),r}(y) &= rF^{r-1}(y)f(y)\tilde{F}^{n-1-r}(y) + (n-r-1)F^{r}(y)\tilde{F}^{n-2-r}(y)\tilde{f}(y) \\ &= F^{r-1}(y)F^{n-2-r}(\frac{1}{\delta}y)\left(rf(y)F(\frac{1}{\delta}y) + (n-r-1)F(y)\frac{1}{\delta}f(\frac{1}{\delta}y)\right) \end{aligned} \tag{3.26}$$

with $r = n-1$ or $r = n-2$.

Expected revenue of the seller

The expected revenue of the seller is just the sum of the ex-ante (prior to bidders knowing their valuations) expected payments of the bidders. Thus, the ex-ante expected payment of a particular bidder $i \in N$ with valuation v_i in a DA is to be derived. In the following, first, the ex-ante expected payment of the designated bidder $\hat{\imath}$ is calculated, **Case I**, and second, the ex-ante expected payment of a bidder i, $i \neq \hat{\imath}$ is determined, **Case II**.

Case I: Consider the case that the designated bidder $\hat{\imath} \in N$ is the winning bidder. The ex-ante **expected payment of the designated bidder** $\hat{\imath} \in N$ is obtained by

$$\begin{aligned} E[P_{\hat{\imath},DA}] &= \int_0^\infty \int_0^v \frac{1}{\delta} y g_{(1),n-1}(y) dy \tilde{f}(v) dv \\ &= \int_0^\infty \int_0^v \frac{1}{\delta} y (n-1)F^{n-2}(y)f(y)dy \frac{1}{\delta} f(\frac{1}{\delta}v)dv \\ &= \int_0^\infty \int_0^{\delta v} \frac{1}{\delta} y(n-1)F^{n-2}(y)f(y)dy f(v)dv \end{aligned} \tag{3.27}$$

Case II: Consider the case, that a bidder $i \in N \setminus \{\hat{\imath}\}$ wins the DA. The ex-ante **expected payment of a non-designated bidder** $i \in N \setminus \{\hat{\imath}\}$ is obtained by

$$\begin{aligned} E[P_{i,DA}] &= \int_0^\infty \int_0^v y g_{(1),n-2}(y) dy f(v) dv \\ &= \int_0^\infty \int_0^v y\Big[(n-2)F^{n-3}(y)f(y)F(\frac{1}{\delta}y) + F^{n-2}(y)\frac{1}{\delta}f(\frac{1}{\delta}y)\Big] dy f(v) dv \end{aligned} \tag{3.28}$$

The **expected revenue of the seller** is the sum of the ex-ante expected payment of the designated bidder $\hat{\imath}$ and $n-1$ times the ex-ante expected payment of a bidder i ($i \neq \hat{\imath}$), obtained by

$$\begin{aligned} E[R_{DA}] &= E[P_{\hat{\imath},DA}] + (n-1)E[P_{i,DA}] \\ &= \int_0^\infty \int_0^{\delta v} \frac{1}{\delta} y(n-1)F^{n-2}(y)f(y)dy f(v)dv \\ &+ (n-1)\int_0^\infty \int_0^v y\Big[(n-2)F^{n-3}(y)f(y)F(\frac{1}{\delta}y) + F^{n-2}(y)\frac{1}{\delta}f(\frac{1}{\delta}y)\Big] dy f(v) dv \end{aligned} \tag{3.29}$$

Expected payoff of a bidder

As the payoff functions are separable in money, they are linear, the expected payoff of a bidder is derived by

$$\begin{aligned}\text{Expected Payoff} &= \text{Expected [Valuation - Payment]}\\ &= \text{Expected [Valuation] - Expected [Payment]}\end{aligned}$$

To calculate the expected payoff of a bidder the two cases, **Case I** and **Case II** mentioned above, are again distinguished:

Case I: Consider the case that the designated bidder $\hat{\imath} \in N$ wins the auction. The **expected payoff of the designated bidder** $\hat{\imath} \in N$ is calculated by

$$\begin{aligned}E\pi_{\hat{\imath},DA} &= \int_0^\infty \int_0^{\delta v} \left(v - \frac{1}{\delta}y\right) g_{(1),n-1}(y)dyf(v)dv \\ &= \int_0^\infty \int_0^{\delta v} \left(v - \frac{1}{\delta}y\right) (n-1)F^{n-2}(y)f(y)dyf(v)dv\end{aligned} \tag{3.30}$$

Case II: Consider the case that a non-designated bidder $i \in N$, $i \neq \hat{\imath}$ is the high bidder in the auction. The **expected payoff of a non-designated bidder** $i \in N \setminus \{\hat{\imath}\}$ is obtained by

$$\begin{aligned}E\pi_{i,DA} &= \int_0^\infty \int_0^{v} (v-y)\, g_{(1),n-2}(y)dyf(v)dv \\ &= \int_0^\infty \int_0^{v} (v-y) \left[(n-2)F^{n-3}(y)f(y)F(\frac{1}{\delta}y) + F^{n-2}(y)\frac{1}{\delta}f(\frac{1}{\delta}y)\right] dyf(v)dv\end{aligned} \tag{3.31}$$

Bringing both cases together, the **expected payoff of all bidders** is the sum of the individual expected payoffs and can be written as

$$E\pi_{DA} = E\pi_{\hat{\imath},DA} + (n-1)E\pi_{i,DA} \tag{3.32}$$

Expected welfare

The expected welfare is the expected value of the highest value, i.e. the first order statistic. To calculate the expected welfare, the following two cases are again distinguished: in the first case the designated bidder is the winning bidder in the auction, that is **Case I**, and in the second case, it is not the designated bidder who wins the auction, that is **Case II**.

Case I: Consider the case that the designated bidder $\hat{\imath} \in N$ wins the DA. Then, the expected welfare is given by

$$E[W_{\hat{\imath},DA}] = \int_0^\infty vG_{(1),n-1}(\delta v)f(v)dv = \int_0^\infty vF^{n-1}(\delta v)f(v)dv \tag{3.33}$$

Case II: Consider the case that a non-designated bidder $i \in N \setminus \{\hat{\imath}\}$ wins the DA. Then, the expected welfare is calculated by

$$E[W_{i,DA}] = \int_0^\infty vG_{(1),n-2}(v)f(v)dv = \int_0^\infty vF^{n-2}(v)F(\frac{1}{\delta}v)f(v)dv \tag{3.34}$$

The **expected welfare of the DA** is the sum of the expected welfare derived from both cases, $E[W_{\hat{\imath},DA}]$ in **Case I** and $(n-1)$ times $E[W_{i,DA}]$ in **Case II**, and can be written as

$$\begin{aligned} E[W_{DA}] &= E[W_{\hat{\imath},DA}] + (n-1)E[W_{i,DA}] \\ &= \int_0^\infty vF^{n-1}(\delta v)f(v)dv + (n-1)\int_0^\infty vF^{n-2}(v)F(\frac{1}{\delta}v)f(v)dv \\ &= \int_0^\infty v\left(F^{n-1}(\delta v) + (n-1)F^{n-2}(v)F(\frac{1}{\delta}v)\right)f(v)dv \end{aligned} \tag{3.35}$$

Remark 3.6.1 The expected welfare is the sum of the expected revenues of all players, i.e. the expected revenue of the seller, the expected payoff of the designated bidder and $n-1$ tines the expected payoff of a non-designated bidders. The **expected welfare of the DA** is obtained with Equations 3.29 and 3.32, and equals

$$\begin{aligned} E[W_{DA}] &= E[R_{DA}] + E\pi_{DA} \\ &= (E[P_{\hat{\imath},DA}] + E\pi_{\hat{\imath},DA}) + (n-1)(E[P_{i,DA}] + E\pi_{i,DA}) \end{aligned} \tag{3.36}$$

Note that the expected welfare can be separated into two parts: first, $E[W_{\hat{\imath},DA}] = E[P_{\hat{\imath},DA}] + E\pi_{\hat{\imath},DA}$ is the designated bidder's expected payment and expected payoff in the case that the designated bidder wins the auction (**Case I**), and second, $E[W_{i,DA}] = E[P_{i,DA}] + E\pi_{i,DA}$ is the expected payment and expected payoff of a non-designated bidder i in the case that bidder i wins the auction (**Case II**).

Example 3.6.1 Assume a SIPV setting with risk neutral bidders. A seller initiates a DA and offers a single-item to four bidders for sale. Bidders' valuations are private and independently drawn from the random variable V, which is distributed according to the exponential distribution function. The exponential distribution function is defined by $F : \mathbb{R} \to [0,1]$,

$$F(v) = 1 - e^{-\lambda v}, \quad \lambda > 0$$

and the associated pdf is given by $f : \mathbb{R} \to \mathbb{R}_+$,

$$f(v) = \lambda e^{-\lambda v}$$

Suppose bidder 1 is the designated bidder, to whom the discount is assigned. Then, the bids of bidder 1 are distributed according to $\tilde{F} : \mathbb{R} \to [0, 1]$,

$$\tilde{F}(v) = F(\frac{1}{\delta}v) = 1 - e^{-\frac{1}{\delta}\lambda v}$$

$\delta = \frac{1}{1-d} > 1, d \in [0, 1)$, and $\lambda > 0$ with density $\tilde{f} : \mathbb{R} \to \mathbb{R}_+$,

$$\tilde{f}(v) = \frac{1}{\delta}f(\frac{1}{\delta}v) = \frac{1}{\delta}\lambda e^{-\frac{1}{\delta}\lambda v}$$

Bidder 1, being the designated bidder, submits a bid above his valuation due to the discount; bidder 2, bidder 3 and bidder 4 submit their valuations truthfully.
Let $E[V] = 100$ with $\lambda = \frac{1}{100}$ and the discount be equal to $d = 10\%$, i.e. $\delta = \frac{1}{1-0.1}$.
Then, the expected revenue of the seller is calculated by Equation 3.29 and computes to

$$E[R_{DA}] = 108.10$$

The expected payoff of the designated bidder (bidder 1) and the expected payoff of a non-designated bidder (bidder 2, bidder 3, and bidder 4) can be computed by Equations 3.30 and 3.31 and are

$$E\pi_{1,DA} = 27.92 \quad \text{and} \quad E\pi_{i,DA} = 24.02 \quad i = 2, 3, 4$$

The expected welfare is obtained by Equation 3.35 and equals

$$E[W_{DA}] = 208.11$$

For a discount of 20% and $\delta = \frac{1}{1-0.2}$ the seller's expected revenue, the bidders' expected payoffs and the expected welfare are given by

$$E[R_{DA}] = 107.29, \quad E\pi_{1,DA} = 31.33, \quad E\pi_{i,DA} = 22.89, \; i = 2, 3, 4 \quad \text{and} \quad E[W_{DA}] = 207.29$$

All calculations are performed by Maple 9.5.

3.6.2 Asymmetric distribution functions

The scope of this section is to analyze the DA in an independent private values setting with risk neutral bidders. Bidders are of different types and characterized by different probability distribution functions of valuations. More precisely bidders are either of the type *weak* and their valuations are independently drawn from the random variable V_w, which is distributed according to the distribution function $F_w(v)$, or bidders are of the type *strong* and their valuations are independently drawn from the random variable V_s, which is distributed according to the distribution function $F_s(v)$. There are m weak and $(n-m)$ strong bidders. Denote the set of weak bidders by W and the set of strong bidders by S. In the DA, a single bidder to whom the discount is assigned is randomly selected. That is the designated bidder. In the case that the designated bidder is of type weak, the designated bidder will be denoted by $\hat{\imath} \in W$; in the case that the designated bidder is a strong bidder, the designated bidder will be denoted by $\hat{\jmath} \in S$. To determine the expected outcomes in the DA, i.e. the seller's expected revenue, the expected payment and the expected payoff of a bidder, as well as the expected welfare, the following cases are distinguished:

- The designated bidder $\hat{\imath} \in W$, a weak bidder, wins the auction and pays the discounted final price of the DA.
- The designated bidder $\hat{\jmath} \in S$, a strong bidder, is the high bidder in the auction and receives a discount on the final price of the DA.
- A weak bidder $i \in W \setminus \{\hat{\imath}\}$, who is not the designated bidder, wins the auction and acquires the item at a price equal to the final price of the DA.
- A strong bidder $j \in S \setminus \{\hat{\jmath}\}$, who is not the designated bidder, wins the auction and purchases the item at the final price of the DA.

Assume bidder $i \in N$ is the winning bidder. Then, the final price of the DA is $y = \max_{k \in N \setminus \{i\}} b_k$. The price at which the item is purchased by the high bidder i is either the final price y or the discounted final price $\frac{1}{\delta}y = (1-d)y$ with $\delta = \frac{1}{1-d}$ and $d \in [0,1)$ depending on whether or not the designated bidder is the high bidder. Let y denote the highest bid among the $n-1$ bids of all bidders, except bidder i. Then y can be interpreted as a random draw of a random variable Y which is distributed according to a distribution function. In the following the two cases are distinguished:

Case I: the designated bidder is a weak bidder and

Case II: the designated bidder is a strong bidder.

To be more precise each case is distinguished in three subcases and for each subcase the distribution function of the random variable Y is determined. The associated probability density function is simply the derivative of the distribution function.

Case I: *the designated bidder is a weak bidder*

Assume a weak bidder $\hat{\imath} \in W$ is the designated bidder, who is randomly selected to receive the discount.

(a) Consider the case that the designated bidder $\hat{\imath} \in W$ wins the auction with a bid of $\delta v_{\hat{\imath}}$. Bidder $\hat{\imath}$ purchases the item at the discounted final price of the auction: $\frac{1}{\delta}y = (1-d)y$. His payoff is equal to $\pi_{\hat{\imath}} = v_{\hat{\imath}} - \frac{1}{\delta}y = v_{\hat{\imath}} - (1-d)y$. The final price $y = \max_{k \in N\setminus\{\hat{\imath}\}} v_k$, or the second highest bid, is a realization of a draw of the random variable Y, which is the first order statistic of $m-1$ random variables V_w and $n-m$ random variables V_s. Y is distributed according to the distribution function

$$G_{(1),m-1,n-m,w}(y) = F_w^{m-1}(y)F_s^{n-m}(y) \tag{3.37}$$

(b) Consider the case that a weak bidder $i \in W \setminus \{\hat{\imath}\}$, who is not the designated bidder, wins the auction. The price to pay for the item equals $y = \max_{k \in N\setminus\{i\}} b_k = \max\{\max_{k \in N\setminus\{i,\hat{\imath}\}} v_k, \delta v_{\hat{\imath}}\}$. The weak bidder's payoff is $\pi_i = v_i - y$. The final price y can be interpreted as a draw of a random variable Y, which is the first order statistic of random variable $\tilde{V}_w$, $m-2$ random variables V_w and $n-m$ random variables V_s. Y is distributed according to the distribution function

$$G_{(1),m-2,n-m,w}(y) = F_w^{m-2}(y)F_s^{n-m}(y)\tilde{F}_w(y) \tag{3.38}$$

(c) Consider the case that a strong bidder $j \in S$ wins the auction. The final price of the auction is $y = \max_{k \in N\setminus\{j\}} b_k = \max\{\max_{k \in N\setminus\{j,\hat{\imath}\}} v_k, \delta v_{\hat{\imath}}\}$ and he gains a payoff of $\pi_j = v_j - y$. y is a random draw of the random variable Y, the first order statistic of random variable $\tilde{V}_w$, $m-1$ random variables V_w and $n-m-1$ random variables V_s. The distribution function of Y is given by

$$G_{(1),m-1,n-m-1,w}(y) = F_w^{m-1}(y)F_s^{n-m-1}(y)\tilde{F}_w(y) \tag{3.39}$$

Summarizing the three cases **(a)** to **(c)** mentioned above, in which the designated bidder is a weak bidder, then y, is a realization of a draw of the random variable Y, distributed according to

$$G_{(1),m-r-1,n-m-u,w}(y) = F_w^{m-r-1}(y)F_s^{n-m-u}(y)\tilde{F}_w^{r+u}(y) \tag{3.40}$$

with $r, u \in \{0,1\} \wedge 0 \leq r+u \leq 1$. That is,

(a) $r = 0 \wedge u = 0$, if the high bidder is the designated weak bidder

(b) $r = 1 \wedge u = 0$, if the high bidder is a weak bidder, but the designated bidder

(c) $r = 0 \wedge u = 1$, if the high bidder is a strong bidder

The corresponding density function is the derivative of $G_{(1),m-r-1,n-m-u,w}(y)$ and obtained by

$$g_{(1),m-r-1,n-m-u,w}(y) = (m-r-1)F_w^{m-r-2}(y)f_w(y)F_s^{n-m-u}(y)\tilde{F}_w^{r+u}(y) \quad (3.41)$$
$$+(n-m-u)F_w^{m-r-1}(y)F_s^{n-m-u-1}(y)f_s(y)\tilde{F}_w^{r+u}(y)$$
$$+(r+u)F_w^{m-r-1}(y)F_s^{n-m-u}(y)\tilde{F}_w^{r+u-1}\tilde{f}_w(y)$$

Consider in the following the case, that the designated bidder is a strong bidder, who is granted the discount. As in **Case I** similar subcases can be distinguished.

Case II: *the designated bidder is a strong bidder*
Assume the discount is randomly assigned to a strong bidder $\hat{\jmath} \in S$, being the designated bidder.

(a) Consider the case that the designated bidder $\hat{\jmath}$ wins the auction with a bid of $\delta v_{\hat{\jmath}}$. He purchases the item at the discounted final price of the auction: $\frac{1}{\delta}y = (1-d)y$. The strong designated bidder has a payoff of $\pi_{\hat{\jmath}} = v_{\hat{\jmath}} - \frac{1}{\delta}y = v_{\hat{\jmath}} - (1-d)y$. The final price $y = \max_{k \in N \setminus \{\hat{\jmath}\}} v_k$, or the second-highest bid, is a realization of a random draw of the random variable Y, which is the first order statistic of m random variables V_w and $n-m-1$ random variables V_s. The distribution function of Y is defined by

$$G_{(1),m,n-m-1,s}(y) = F_w^m(y)F_s^{n-m-1}(y) \quad (3.42)$$

(b) Consider the case that a strong bidder $j \in S \setminus \{\hat{\jmath}\}$, who is not the designated bidder, wins the auction. The price to pay is the final price of the DA and equals $y = \max_{k \in N \setminus \{j\}} b_k = \max\{\max_{k \in N \setminus \{j,\hat{\jmath}\}} v_k, \delta v_{\hat{\jmath}}\}$. The strong bidder's payoff is $\pi_j = v_j - y$. y is a realization of a random draw of a random variable Y, which can be interpreted as the first order statistic of m random variables V_w, $n-m-2$ random variables V_s and random variable $\tilde{V}_s$. The distribution function of Y is defined by

$$G_{(1),m,n-m-2,s}(y) = F_w^m(y)F_s^{n-m-2}(y)\tilde{F}_s(y) \quad (3.43)$$

(c) Consider the case that a weak bidder $i \in W$ wins the auction. The final price of the DA at which bidder i acquires the item equals $y = \max_{k \in N \setminus \{i\}} b_k = \max\{\max_{k \in N \setminus \{i,\hat{\jmath}\}} v_k, \delta v_{\hat{\jmath}}\}$. The weak bidder gains a payoff of $\pi_i = v_i - y$. The final price y is a realization of the random variable Y which denotes the first order statistic of $m-1$ random variables V_w, $n-m-1$ random variables V_s and random variable $\tilde{V}_s$. The distribution function of the first order statistic Y is given by

$$G_{(1),m-1,n-m-1,s}(y) = F_w^{m-1}(y)F_s^{n-m-1}(y)\tilde{F}_s(y) \quad (3.44)$$

Referring to the three subcases **(a)** to **(c)** above in which the designated bidder is a strong bidder, then the distribution function of the random variable Y can be more generalized and calculates to

$$G_{(1),m-r,n-m-u-1,s}(y) = F_w^{m-r}(y)F_s^{n-m-u-1}(y)\tilde{F}_s^{r+u}(y) \tag{3.45}$$

with $r, u \in \{0, 1\} \wedge 0 \leq r + u \leq 1$. That is,

- **(a)** $r = 0 \wedge u = 0,$ if the high bidder is the designated strong bidder
- **(b)** $r = 0 \wedge u = 1,$ if the high bidder is a strong bidder, but the designated bidder
- **(c)** $r = 1 \wedge u = 0,$ if the high bidder is a weak bidder

The associated density function is the derivative of the distribution function and given by

$$\begin{aligned} g_{(1),m-r,n-m-u-1,s}(y) &= (m-r)F_w^{m-r-1}(y)f_w(y)F_s^{n-m-u-1}(y)\tilde{F}_s^{r+u}(y) \\ &\quad +(n-m-u-1)F_w^{m-r}(y)F_s^{n-m-u-2}(y)f_s(y)\tilde{F}_s^{r+u}(y) \\ &\quad +(r+u)F_w^{m-r}(y)F_s^{n-m-u-1}(y)\tilde{F}_s^{r+u-1}(y)\tilde{f}_s(y) \end{aligned} \tag{3.46}$$

With the above given distribution functions, Equations 3.40 and 3.45, and its respective density functions, Equations 3.41 and 3.46, first a bidder's (ex-ante) expected payment as well as the seller's expected revenue, second the expected payoff of a bidder, and third the expected welfare of the DA are calculated.

Expected revenue of the seller

The expected revenue of the seller in the DA is just the sum of the ex-ante expected payments of all bidders. To calculate the ex-ante expected payments of the bidders, the two cases **Case I** and **Case II** with its three subcases **(a)** to **(c)** are considered. First, with Equations 3.41 and 3.46 the bidders' expected payments in the six subcases are calculated, and second, the seller's expected revenue is derived.

Case I: *the designated bidder is a weak bidder*

(a) The ex-ante **expected payment of the designated bidder** $\hat{\imath} \in W$**, a weak bidder**, is obtained by

$$\begin{aligned} E[P^{as}_{\hat{\imath},w,DA}] &= \int_0^\infty \int_0^v \frac{1}{\delta} y g_{(1),m-1,n-m,w}(y)dy\tilde{f}_w(v)dv \\ &= \int_0^\infty \int_0^v \frac{1}{\delta} y g_{(1),m-1,n-m,w}(y)dy\frac{1}{\delta}f_w(\frac{1}{\delta}v)dv \\ &= \int_0^\infty \int_0^{\delta v} \frac{1}{\delta} y g_{(1),m-1,n-m,w}(y)dyf_w(v)dv \end{aligned} \tag{3.47}$$

(b) The ex-ante **expected payment of a non-designated bidder** $i \in W \setminus \{\hat{\imath}\}$**, a weak bidder**, is derived by

$$E[P^{as}_{i,w,DA}] = \int_0^\infty \int_0^v y g_{(1),m-2,n-m,w}(y) dy f_w(v) dv \tag{3.48}$$

(c) The ex-ante **expected payment of a non-designated bidder** $j \in S$**, a strong bidder**, is given by

$$E[P^{as}_{j,w,DA}] = \int_0^\infty \int_0^v y g_{(1),m-1,n-m-1,w}(y) dy f_s(v) dv \tag{3.49}$$

Under the assumption that a weak bidder $\hat{\imath} \in W$ is the designated bidder, the **seller's expected revenue** is just the sum of ex-ante expected payments of all bidders in the auction.

$$E[R^{as}_{w,DA}] = E[P^{as}_{\hat{\imath},w,DA}] + (m-1)E[P^{as}_{i,w,DA}] + (n-m)E[P^{as}_{j,w,DA}] \tag{3.50}$$

Superscript as indicates the asymmetries among bidders, subscript w depicts, that the designated bidder in the auction is a weak bidder, and DA denotes the auction format. In particular, in case that a weak bidder is the designated bidder, the seller's expected revenue is the sum of the expected payment of the designated bidder $\hat{\imath} \in W$ (**(a)** with Equation 3.47), $m-1$ times the expected payment of a weak bidder, but not the designated bidder (**(b)** with Equation 3.48) and $n-m$ times the expected payment of a strong bidder (**(c)** with Equation 3.49).

Case II: *the designated bidder is a strong bidder*

(a) The ex-ante **expected payment of the designated bidder** $\hat{\jmath} \in S$**, a strong bidder**, equals

$$\begin{aligned} E[P^{as}_{\hat{\jmath},s,DA}] &= \int_0^\infty \int_0^v \frac{1}{\delta} y g_{(1),m,n-m-1,s}(y) dy \tilde{f}_s(v) dv \\ &= \int_0^\infty \int_0^v \frac{1}{\delta} y g_{(1),m,n-m-1,s}(y) dy \frac{1}{\delta} f_s(\frac{1}{\delta} v) dv \\ &= \int_0^\infty \int_0^{\delta v} \frac{1}{\delta} y g_{(1),m,n-m-1,s}(y) dy f_s(v) dv \end{aligned} \tag{3.51}$$

(b) The ex-ante **expected payment of a non-designated bidder** $j \in S \setminus \{\hat{\jmath}\}$**, a strong bidder**, is

$$E[P^{as}_{j,s,DA}] = \int_0^\infty \int_0^v y g_{(1),m,n-m-2,s}(y) dy f_s(v) dv \tag{3.52}$$

(c) The ex-ante **expected payment of a non-designated bidder** $i \in W$**, a weak bidder**, is obtained by

$$E[P^{as}_{i,s,DA}] = \int_0^\infty \int_0^v y g_{(1),m-1,n-m-1,s}(y) dy f_w(v) dv \tag{3.53}$$

Under the assumption that a strong bidder $\hat{\jmath} \in S$ is the designated bidder, the **seller's expected revenue** is the sum of the expected payments of all individual bidders and can be written as

$$E[R^{as}_{s,DA}] = E[P^{as}_{\hat{j},s,DA}] + (n - m - 1)E[P^{as}_{j,s,DA}] + mE[P^{as}_{i,s,DA}] \tag{3.54}$$

Note that superscript as indicates the asymmetries among bidders preferences, subscript s indicates the case of a strong designated bidder, and subscript DA the chosen auction institution. The equation above presents the equation of the seller's expected revenue under the assumption that a strong bidder obtains the discount. Thus, the seller's expected revenue is the sum of the expected payment of the designated strong bidder $\hat{j} \in S$ (**(a)** with Equation 3.51), the $n - m - 1$ times the expected payment of a strong bidder, but not the designated bidder (**(b)** with Equation 3.52), and m times the expected payment of a weak bidder (**(c)** with Equation 3.53).

The seller's expected revenue in the DA with bidders characterized by asymmetric distribution functions of valuations is the sum of the expected payments of all individual bidders in case, that a weak bidder is the designated bidder (**Case I** with Equation 3.50) and in case, that a strong bidder is the designated bidder (**Case II** with Equation 3.54). Both partial expected revenues have to be weighted with the probability, that either a weak bidder is randomly selected to receive the discount, that happens with a probability of $\frac{m}{n}$, or that a strong bidder is randomly selected as the designated bidder, that happens with a probability of $\frac{n-m}{n}$.

The **seller's expected revenue** in the DA with asymmetric bidders is

$$E[R^{as}_{DA}] = \frac{m}{n}E[R^{as}_{w,DA}] + \frac{n-m}{n}E[R^{as}_{s,DA}] \tag{3.55}$$

Expected payoff of a bidder

The following turns to the expected payoff of a bidder participating in the DA with asymmetries among bidders. The expected payoff or equivalent the expected payoff a bidder $i \in N$ achieves from a DA depends on his type, either being of type weak or of type strong, on being a designated bidder to whom the discount is assigned, on his valuation and the rival bidders' valuations as well as on the assigned distribution functions of valuations.
As the payoff functions are separable in money, they are linear, the expected payoff of a bidder is derived by

$$\begin{aligned} \text{Expected Payoff} &= \text{Expected [Valuation - Payment]} \\ &= \text{Expected [Valuation] - Expected [Payment]} \end{aligned}$$

To calculate the expected payoff of a bidder the two cases mentioned above, **Case I** and **Case II** with its subcases **(a)** to **(c)** and the given distribution functions, Equations 3.40 and 3.45, with its respective density functions, Equations 3.41 and 3.46, are distinguished.

Case I: *the designated bidder is a weak bidder*

(a) The **expected payoff of the designated bidder** $\hat{\imath} \in W$**, a weak bidder**, is given by

$$E\pi^{as}_{\hat{\imath},w,DA} = \int_0^\infty \int_0^{\delta v} (v - \frac{1}{\delta} y) g_{(1),m-1,n-m,w}(y) dy f_w(v) dv \tag{3.56}$$

(b) The **expected payoff of a non-designated bidder** $i \in W \setminus \{\hat{\imath}\}$**, a weak bidder**, equals

$$E\pi^{as}_{i,w,DA} = \int_0^\infty \int_0^{v} (v - y) g_{(1),m-2,n-m,w}(y) dy f_w(v) dv \tag{3.57}$$

(c) The **expected payoff of a non-designated bidder** $j \in S$**, a strong bidder**, is obtained by

$$E\pi^{as}_{j,w,DA} = \int_0^\infty \int_0^{v} (v - y) g_{(1),m-1,n-m-1,w}(y) dy f_s(v) dv \tag{3.58}$$

Under the assumption, that a weak bidder is the designated bidder the expected payoff of all bidders equals the sum of the expected payoff of the designated weak bidder $\hat{\imath} \in W$ (**(a)** with Equation 3.56), $m - 1$ times the expected payoff of a weak bidder $i \in W \setminus \{\hat{\imath}\}$, but the designated bidder (**(b)** with Equation 3.57), and $n - m$ times the expected payoff of a strong bidder $j \in S$ (**(c)** with Equation 3.58).

$$E\pi^{as}_{w,DA} = E\pi^{as}_{\hat{\imath},w,DA} + (m-1)E\pi^{as}_{i,w,DA} + (n-m)E\pi^{as}_{j,w,DA} \tag{3.59}$$

Case II: *the designated bidder is a strong bidder*

(a) The **expected payoff of the designated bidder** $\hat{\jmath} \in S$**, a strong bidder**, is derived by

$$E\pi^{as}_{\hat{\jmath},s,DA} = \int_0^\infty \int_0^{\delta v} (v - \frac{1}{\delta} y) g_{(1),m,n-m-1,s}(y) dy f_s(v) dv \tag{3.60}$$

(b) The **expected payoff of a non-designated bidder** $j \in S \setminus \{\hat{\jmath}\}$**, a strong bidder**, calculates to

$$E\pi^{as}_{j,s,DA} = \int_0^\infty \int_0^{v} (v - y) g_{(1),m,n-m-2,s}(y) dy f_s(v) dv \tag{3.61}$$

(c) The **expected payoff of a non-designated bidder** $i \in W$**, a weak bidder**, is determined by

$$E\pi^{as}_{i,s,DA} = \int_0^\infty \int_0^{v} (v - y) g_{(1),m-1,n-m-1,s}(y) dy f_w(v) dv \tag{3.62}$$

Under the assumption that a strong bidder is the designated bidder, the expected payoff of all bidders equals the sum of the expected payoff of the designated bidder $\hat{\jmath} \in S$ (**(a)** with

Equation 3.60), $n-m-1$ times the expected payoff of a strong bidder $j \in S \setminus \{\hat{\jmath}\}$, but the designated bidder $\hat{\jmath}$ (**(b)** with Equation 3.61), and m times the expected payoff of a weak bidder $i \in W$ (**(c)** with Equation 3.62).

$$E\pi^{as}_{s,DA} = E\pi^{as}_{\hat{\jmath},s,DA} + (n-m-1)E\pi^{as}_{j,s,DA} + mE\pi^{as}_{i,s,DA} \tag{3.63}$$

The sum of the expected payoff of all n bidders in the DA is the sum of the bidders' expected payoffs under the assumption that a weak bidder is the designated bidder (**Case I** with Equation 3.59), and the sum of the bidders' expected payoffs under the assumption that a strong bidder is the designated bidder (**Case II** with Equation 3.63).

$$E\pi^{as}_{DA} = \frac{m}{n}E\pi^{as}_{w,DA} + \frac{n-m}{n}E\pi^{as}_{s,DA} \tag{3.64}$$

As the group of the weak bidders consists of m bidders, the probability that a weak bidder is the designated bidder is $\frac{m}{n}$ (**Case I**) and as the group of strong bidders consist of $n-m$ bidders, the probability that a strong bidder is the designated bidder equals $\frac{n-m}{n}$ (**Case II**).

Focussing on the expected payoff of a weak bidder or a strong bidder, the following can be derived.

Remark 3.6.2
The **expected payoff of a weak bidder** $i \in W \setminus \{\hat{\imath}\}$**, who is not the designated bidder**, is given with Equation 3.57 and Equation 3.62 by

$$E\pi^{as}_{i,DA} = \frac{m}{n}E\pi^{as}_{i,w,DA} + \frac{n-m}{n}E\pi^{as}_{i,s,DA}$$

The **expected payoff of a strong bidder** $j \in S \setminus \{\hat{\jmath}\}$**, who is not the designated bidder**, is obtained with Equation 3.58 and Equation 3.61 and calculated by

$$E\pi^{as}_{j,DA} = \frac{m}{n}E\pi^{as}_{j,w,DA} + \frac{n-m}{n}E\pi^{as}_{j,s,DA}$$

Expected welfare

The expected welfare equals the expectation of the highest value among all valuations assigned to the n bidders. In the following the expected welfare is calculated again for the already mentioned two cases, **Case I** and **Case II** with its subcases and Equations 3.40 and 3.45.

Case I: *the designated bidder is a weak bidder*
(a) In the case that the designated bidder $\hat{\imath} \in W$, a weak bidder, wins the auction, the expected

welfare is given by

$$E[W^{as}_{\hat{\imath},w,DA}] = \int_0^\infty vG_{(1),m-1,n-m,w}(\delta v)f_w(v)dv = \int_0^\infty vF_w^{m-1}(\delta v)F_s^{n-m}(\delta v)f_w(v)dv \tag{3.65}$$

(b) In the case that a weak bidder $i \in W \setminus \{\hat{\imath}\}$ wins the auction, the expected welfare is determined by

$$E[W^{as}_{i,w,DA}] = \int_0^\infty vG_{(1),m-2,n-m,w}(v)f_w(v)dv = \int_0^\infty vF_w^{m-2}(v)F_s^{n-m}(v)F_w(\frac{1}{\delta}v)f_w(v)dv \tag{3.66}$$

(c) In the case that a strong bidder $j \in S$ wins the auction, the expected welfare is equal to

$$E[W^{as}_{j,w,DA}] = \int_0^\infty vG_{(1),m-1,n-m-1,w}(v)f_s(v)dv = \int_0^\infty vF_w^{m-1}(v)F_s^{n-m-1}(v)F_w(\frac{1}{\delta}v)f_s(v)dv \tag{3.67}$$

In the case that **the designated bidder is a weak bidder** $\hat{\imath} \in W$**, the expected welfare** is obtained by

$$E[W^{as}_{w,DA}] = E[W^{as}_{\hat{\imath},w,DA}] + (m-1)E[W^{as}_{i,w,DA}] + (n-m)E[W^{as}_{j,w,DA}] \tag{3.68}$$

Case II: *the designated bidder is a strong bidder*
(a) In the case that the designated bidder $\hat{\jmath} \in S$, a strong bidder, wins the auction, the expected welfare is obtained by

$$E[W^{as}_{\hat{\jmath},s,DA}] = \int_0^\infty vG_{(1),m,n-m-1,s}(\delta v)f_s(v)dv = \int_0^\infty vF_w^{m}(\delta v)F_s^{n-m-1}(\delta v)f_s(v)dv \tag{3.69}$$

(b) In the case that a strong bidder $j \in S \setminus \{\hat{\jmath}\}$, who is not the designated bidder, wins the auction, the expected welfare is calculated by

$$E[W^{as}_{j,s,DA}] = \int_0^\infty vG_{(1),m,n-m-2,s}(v)f_s(v)dv = \int_0^\infty vF_w^{m}(v)F_s^{n-m-2}(v)F_s(\frac{1}{\delta}v)f_s(v)dv \tag{3.70}$$

(c) In the case that a weak bidder $i \in W$ wins the auction, the expected welfare is

$$\begin{aligned} E[W^{as}_{i,s,DA}] &= \int_0^\infty vG_{(1),m-1,n-m-1,s}(v)f_w(v)dv \\ &= \int_0^\infty vF_w^{m-1}(v)F_s^{n-m-1}(v)F_s(\frac{1}{\delta}v)f_w(v)dv \end{aligned} \tag{3.71}$$

In the case that a **strong bidder** $\hat{j} \in S$ **is the designated bidder, the expected welfare** is obtained by

$$E[W^{as}_{s,DA}] = E[W^{as}_{\hat{j},s,DA}] + (n-m-1)E[W^{as}_{j,s,DA}] + mE[W^{as}_{i,s,DA}] \tag{3.72}$$

As **Case I** appears with a probability of $\frac{m}{n}$ and **Case II** with a probability of $\frac{n-m}{n}$, the **expected welfare in the asymmetric case** is

$$E[W^{as}_{DA}] = \frac{m}{n}E[W^{as}_{w,DA}] + \frac{n-m}{n}E[W^{as}_{s,DA}] \tag{3.73}$$

Remark 3.6.3 The expected welfare of the DA with asymmetries among bidders equals the sum of the seller's expected revenue and the expected payoff of all bidders. With the equations above the expected welfare of the DA in the asymmetric case can be written as

$$\begin{aligned} E[W^{as}_{DA}] &= E[R^{as}_{DA}] + E\pi^{as}_{DA} \\ &= \frac{m}{n}\left(E[R^{as}_{w,DA}] + E\pi^{as}_{w,DA}\right) + \frac{n-m}{n}\left(E[R^{as}_{s,DA}] + E\pi^{as}_{s,DA}\right) \end{aligned} \tag{3.74}$$

Note that the expected welfare is divided in two parts: $E[W^{as}_{w,DA}] = E[R^{as}_{w,DA}] + E\pi^{as}_{w,DA}$ is the expected welfare in case that the designated bidder is a weak bidder (**Case I**) and $E[W^{as}_{s,DA}] = E[R^{as}_{s,DA}] + E\pi^{as}_{s,DA}$ is the expected welfare in case, that the designated bidder is a strong bidder (**Case II**).

Example 3.6.2 Consider the DA market institution in an independent private values setting with risk neutral bidders. The discount which is assigned to the designated bidder is 10%. Bidder 1, bidder 2, and bidder 3 are weak bidders and bidder 4 is a strong bidder. That is, the valuations of the four bidders of the single-item being offered in the DA for sale, are distributed according to different distribution functions. The weak bidders' valuations are distributed according to $F_w(v) = 1 - e^{-\lambda_w v}$, $\lambda_w > 0$ and the strong bidder's valuations are distributed according to $F_s(v) = 1 - e^{-\lambda_s v}$, $\lambda_s > 0$. Each weak bidder has an expected valuation of $E[V_w] = \frac{1}{\lambda_w} = 100$ with $\lambda_w = \frac{1}{100}$ and the strong bidder, bidder 4, has an expected valuation of $E[V_s] = \frac{1}{\lambda_s} = 200$ with $\lambda_s = \frac{1}{200}$.

The seller's expected revenue of the auction and the expected welfare equals for $n = 4$ and $m = 3$

$$E[R_{DA}^{as}] = 128.84 \quad E[W_{DA}^{as}] = 274.52$$

The expected payoff of a weak designated bidder and a weak bidder are given by

$$E\pi_{\hat{\imath},w,DA}^{as} = 20.54 \quad E\pi_{i,DA}^{as} = 17.33 \quad i \in W \setminus \{\hat{\imath}\}$$

and the expected payoff of a strong designated bidder and a strong bidder are equal to

$$E\pi_{\hat{\jmath},s,DA}^{as} = 97.91 \quad E\pi_{j,DA}^{as} = 88.64 \quad j \in S \setminus \{\hat{\jmath}\}$$

Changing the expected value of all weak bidders and replacing them by (i) $E[V_w] = 1$ ($\lambda_w = 1$) and (ii) $E[V_w] = 150$ ($\lambda_w = \frac{1}{150}$) while holding the expected value of the strong bidder constant, the seller's expected revenue changes in case (i) to 1.83 ($\lambda_w = 1$) and in case (ii) to 174.52 ($\lambda_w = \frac{1}{150}$).

The computation of the numbers was performed by Maple 9.5.

3.7 Comparison of the second-price auction and the discount auction

3.7.1 Symmetric distribution functions

The scope of this section is to compare the DA and its corresponding second-price auction under the conditions of the SIPV model with risk neutral bidders. In particular, the expected outcomes of the DA are related to the expected outcomes of the second-price auction, i.e. the seller's expected revenue and the expected payoff of an individual bidder. Note, in Section 3.5.1 and in Section 3.6.1 a detailed analysis of the respective auction outcomes in the SIPV setting is presented.

By contrasting the theoretical models of the DA and the second-price auction in the symmetric case, the following research questions mentioned in the introductory chapter are addressed:

- By introducing a discount, how does this discount affect the auction outcomes, i.e. the seller's revenue, the winning bidder's payoff, and the social welfare?

- Does the discount pay for the seller when conducting an auction?

To answer the questions above, the theoretical findings of both auction formats are compared, leading to the following central results: First, it is proven, that in the symmetric case, the seller's expected revenue obtained in a DA is less than or equal to the seller's expected revenue in the corresponding second-price auction. That is, under the symmetry assumption, the seller cannot extract an additional revenue by offering a discount (with respect to the expected revenues). Second, the expected payoff of a bidder in the DA is compared to the expected payoff of that bidder in the second-price auction. Distinguishing between the expected payoff of a designated bidder and the expected payoff of a non-designated bidder, then the major result is, that (i) for the designated bidder the DA yields a higher expected payoff than the second-price auction, and that (ii)for a non-designated bidder the expected payoff in the DA is lower than the expected payoff in the corresponding second-price auction.

The following proposition summarises the central results concerning the expected auction revenues in the DA and the second-price auction.

Proposition 3.7.1 If the discount is positive and lower than 1, $d \in (0,1)$, then the seller's expected revenue in the DA is lower than the seller's expected revenue in the corresponding second-price auction.

$$E[R_{DA}] < E[R_{EA}] \tag{3.75}$$

If the discount is zero $d = 0$, then the seller's expected revenue in the DA is equal to the seller's expected revenue in the corresponding second-price auction.

Proof: cf. Theorem C.1.1 in Appendix C.1. q.e.d.

Now, the expected payoff of an individual bidder $i \in N$ in both auction formats is considered. In the DA, the expected payoff of the designated bidder $\hat{\imath} \in N$ and the expected payoff of a non-designated bidder $i \in N \setminus \{\hat{\imath}\}$ are distinguished. In the following both expected payoffs are related to the respective expected payoff in the corresponding second-price auction. The following holds:

Proposition 3.7.2

(i) If the discount in the DA is positive and lower than 1, $d \in (0,1)$, then the expected payoff of the designated bidder $\hat{\imath} \in N$ in the DA is greater than the expected payoff of bidder $\hat{\imath}$ in the corresponding second-price auction.

$$E\pi_{\hat{\imath},DA} > E\pi_{\hat{\imath},EA} \tag{3.76}$$

(ii) If the discount in the DA is positive and lower than 1, $d \in (0,1)$, then the expected payoff of a bidder $i \in N \setminus \{\hat{\imath}\}$, but the designated bidder, in the DA is lower than the expected payoff of bidder i in the corresponding second-price auction.

$$E\pi_{i,DA} < E\pi_{i,EA} \quad \forall i \in N \setminus \{\hat{\imath}\} \tag{3.77}$$

(iii) If the discount in the DA is zero $d = 0$, then bidder i's expected payoff derived in the DA equals bidder i's expected payoff in the corresponding second price auction ($i \in N$).

Proof: ad (i): Consider the expected payoff of the designated bidder $\hat{\imath}$ in the DA defined by Equation 3.30 and a given $\delta = \frac{1}{1-d} > 1$ and $d \in (0,1)$.

$$\begin{aligned}
E\pi_{\hat{\imath},DA} &= \int_0^\infty \int_0^{\delta v} \left(v - \frac{1}{\delta}y\right)(n-1)F^{n-2}(y)f(y)dyf(v)dv \\
&= \int_0^\infty \int_0^{v} \left(v - \frac{1}{\delta}y\right)(n-1)F^{n-2}(y)f(y)dyf(v)dv \\
&+ \int_0^\infty \int_v^{\delta v} \left(v - \frac{1}{\delta}y\right)(n-1)F^{n-2}(y)f(y)dyf(v)dv
\end{aligned}$$

The second term $\int_0^\infty \int_v^{\delta v} \pi_{\hat{\imath}} \left(v - \frac{1}{\delta}y\right)(n-1)F^{n-2}(y)f(y)dyf(v)dv \geq 0$ is greater than or equal to zero. Moreover, the following holds:

$$\begin{aligned}
E\pi_{\hat{\imath},DA} &\geq \int_0^\infty \int_0^{v} \left(v - \frac{1}{\delta}y\right)(n-1)F^{n-2}(y)f(y)dyf(v)dv \\
&> \int_0^\infty \int_0^{v} (v-y)(n-1)F^{n-2}(y)f(y)dyf(v)dv \\
&= E\pi_{\hat{\imath},EA}
\end{aligned}$$

with $d \in (0,1)$ and $1 > \frac{1}{\delta} > 0$.

ad (ii): Consider the expected payoff of the bidder $i \in N \setminus \{\hat{\imath}\}$ in the DA given by Equation 3.31

$$E\pi_{i,DA} = \int_0^\infty \int_0^{v} (v-y)\left[(n-2)F^{n-3}(y)f(y)F(\frac{1}{\delta}y) + F^{n-2}(y)\frac{1}{\delta}f(\frac{1}{\delta}y)\right]dyf(v)dv$$

and in the corresponding second-price auction according to Equation 3.8

$$E\pi_{i,EA} = \int_0^\infty \int_0^{v} (v-y)(n-1)F^{n-2}(y)f(y)dyf(v)dv$$

The following has to be proven

$$\begin{aligned}
E\pi_{i,DA} &= \int_0^\infty \int_0^{v} (v-y)\left[(n-2)F^{n-3}(y)f(y)F(\frac{1}{\delta}y) + F^{n-2}(y)\frac{1}{\delta}f(\frac{1}{\delta}y)\right]dyf(v)dv \\
&< \int_0^\infty \int_0^{v} (v-y)(n-1)F^{n-2}(y)f(y)dyf(v)dv \\
&= E\pi_{i,EA}
\end{aligned}$$

Note, the term $(v - y)$ appears on both sides in the equation above and can therefore be neglected. Note, that with $d \in (0, 1)$, $0 < \frac{1}{\delta} < 1$, and with $F(\frac{1}{\delta}y) \leq F(y) \,\forall y \in \mathbb{R}$ and $F(\frac{1}{\delta}y) < F(y) \,\forall y \in (a, \delta b)$ (Equations B.2 and B.3) as well as with $\frac{1}{\delta}f(\frac{1}{\delta}y) \leq f(y) \,\forall y \in \mathbb{R}$ and $\frac{1}{\delta}f(\frac{1}{\delta}y) < f(y) \,\forall y \in (a, \delta b)$ (Equations B.5 and B.6) as denoted in Appendix B.1 the inequality holds.

ad (iii): For $d = 0$, δ is set to 1. The auction format of the DA and the auction format of the corresponding second-price auction are identical. Moreover, the bids of the designated bidder $\hat{\imath}$ are random draws of the random variable $\tilde{V} = V$ which is distributed according to $\tilde{F} = F$. Thus, the bidders' expected payoffs in both auction formats are equal. q.e.d.

Example 3.7.1 Consider again the DA and its corresponding second-price auction in the SIPV setting. Suppose four risk neutral bidders participate in the DA or in the second-price auction. The discount offered to the designated bidder is 10%. The valuations of the four bidders are distributed according to the exponential distribution function: bidders' expected valuations are given by $E[V] = \frac{1}{\lambda}$. The bidders' expected values are all equal to $E[V] = 1$ for $\lambda = 1$. Respectively the bidders' values are all varied stepwise to $E[V] = \frac{1}{50}$ for $\lambda = \frac{1}{50}$, or $E[V] = \frac{1}{100}$ for $\lambda = \frac{1}{100}$, or $E[V] = \frac{1}{150}$ for $\lambda = \frac{1}{150}$, or $E[V] = \frac{1}{190}$ for $\lambda = \frac{1}{190}$, or finally to $E[V] = \frac{1}{200}$ for $\lambda = \frac{1}{200}$.

With the given bidders' expected values the second-price auction and the DA are compared in case of symmetric bidders. The expected welfare ($E[W_{EA}], E[W_{DA}]$), the seller's expected revenue ($E[R_{EA}], E[R_{DA}]$), and the expected payoff of a bidder ($E\pi_{i,EA}, E\pi_{\hat{\imath},DA}, E\pi_{i,DA}$) are indicated in Table 3.1. In the DA the expected payoff of a designated bidder is displayed by $E\pi_{\hat{\imath},DA}$ and the expected payoff of a bidder who is not the designated bidder by $E\pi_{i,DA}$.

The central results of Proposition 3.7.1 and Proposition 3.7.2 are confirmed and illustrated in Table 3.1. With symmetric bidders, the seller's expected revenue in the DA is lower than the seller's expected revenue in the second-price auction for a discount of $d = 10\%$ and the different expected values $E[V]$: $E[W_{EA}] > E[W_{DA}]$. That is, the discount does not pay for the seller. Referring to the bidder's expected payoff, the data in Table 3.1 confirm, that a bidder with an assigned discount yields a higher expected payoff in the DA than in the second-price auction, while for a non-designated bidder the reverse holds: $E\pi_{\hat{\imath},DA} > E\pi_{\hat{\imath},EA}$, $i = \hat{\imath}$ and $E\pi_{i,EA} > E\pi_{i,DA}$, $i \neq \hat{\imath}$.

Moreover, Figures 3.1 – 3.3 present the graphs of the expected welfare, the seller's expected revenue, and the bidders' expected payoffs in the symmetric case, plotted over the discount. The functions depend solely on the discount $d \in [0, 1)$ – all other parameters are

Symmetric case: comparison of

second-price auction and discount auction

	2^{nd}-price auction			DA with $d = 10\%$			
λ	$E[W_{EA}]$	$E[R_{EA}]$	$E\pi_{i,EA}$	$E[W_{DA}]$	$E[R_{DA}]$	$E\pi_{\hat{\imath},DA}$	$E\pi_{i,DA}$
1	2.083	1.083	0.250	2.081	1.081	0.279	0.240
$\frac{1}{50}$	104.167	54.167	12.500	104.053	54.051	13.960	12.013
$\frac{1}{100}$	208.333	108.333	25.000	208.106	108.106	27.921	24.026
$\frac{1}{150}$	312.500	162.500	37.500	312.159	162.159	41.882	36.039
$\frac{1}{190}$	395.833	205.833	47.500	395.402	205.402	53.050	45.650
$\frac{1}{200}$	416.667	216.667	50.000	416.213	216.213	55.843	48.053

Table 3.1: Expected values in the discount auction and in the corresponding second-price auction with symmetric bidders (case: four bidders)

hold constant. The expected value of all bidders is set to $E[V] = 100$ ($\lambda = \frac{1}{100}$). The graphs illustrate the central results of Proposition 3.7.1 and Proposition 3.7.2. In Figure 3.3 the expected payoff of a bidder in the second-price auction is denoted by $E\pi i\_EA$, the expected payoff of a bidder, but the designated bidder, in the DA by $E\pi i\_DA$, and the expected payoff of the designated bidder by $E\pi d\_DA$. Interestingly to note is, that the expected welfare of the DA is always lower than the expected welfare of the second-price auction and with increasing discount the expected welfare of the DA decreases (cf. Figure 3.1).

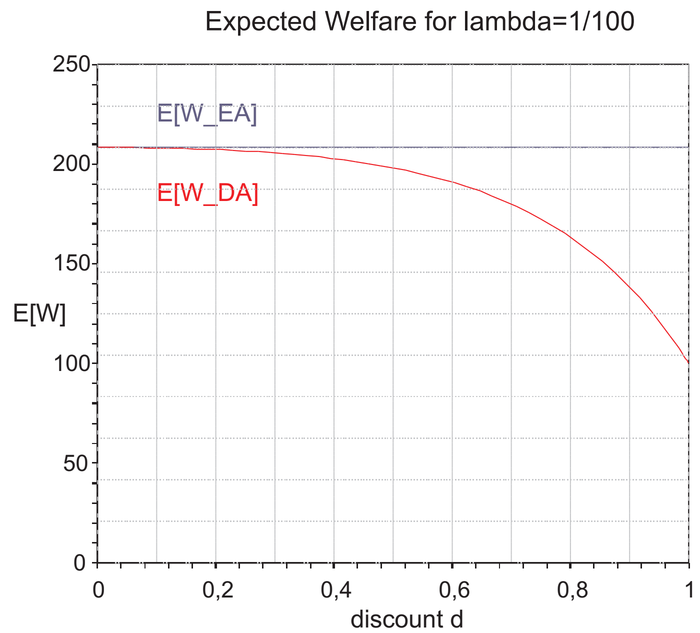


Figure 3.1: Expected welfare in the second-price auction and in the DA in the symmetric case

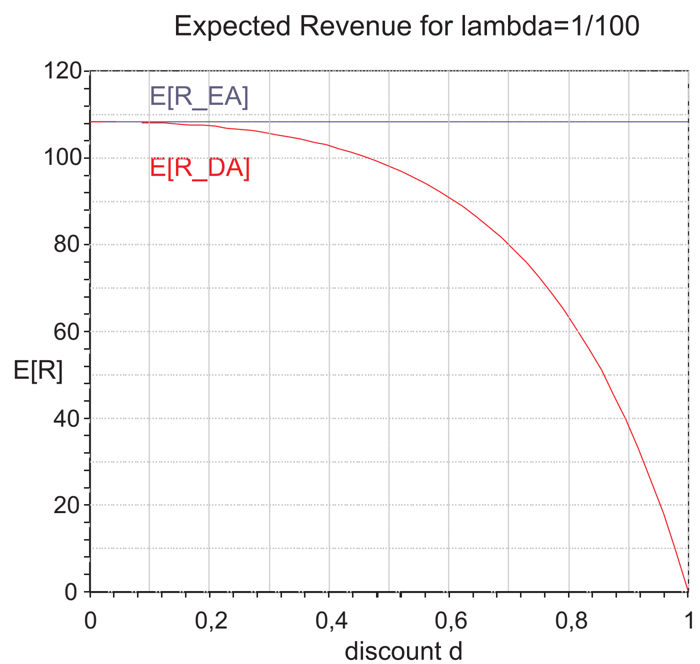


Figure 3.2: Seller's expected revenue in the second-price auction and in the DA in the symmetric case

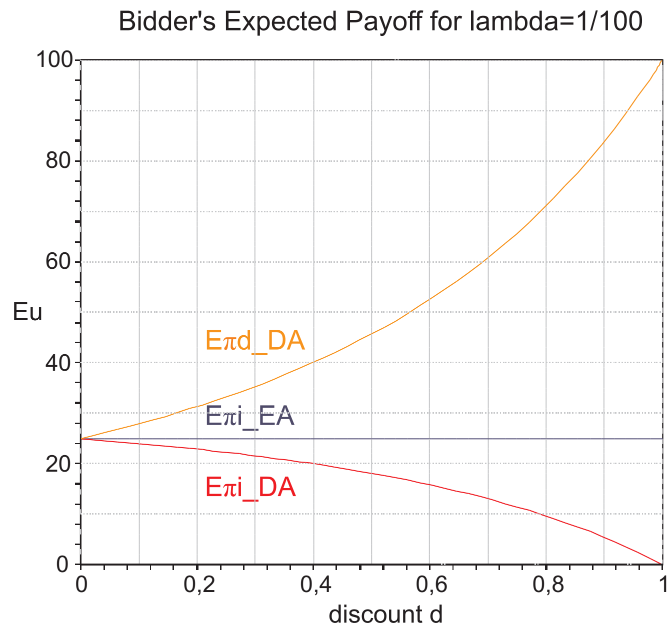


Figure 3.3: Bidder's expected payoff in the second-price auction and in the DA in the symmetric case

3.7.2 Asymmetric distribution functions

The comparison of the DA and its corresponding second-price auction under the assumptions of the IPV model and risk neutral bidders is more complicated in the asymmetric case than in the symmetric case. Recall, that in the symmetric case, the comparison of both models is directly derived and it is shown for example, that the seller's expected revenue in the DA is lower than the seller's expected revenue in the corresponding second-price auction (Section 3.7.1). In the asymmetric case, such generalizations are difficult to derive. A comparison of the seller's expected revenue as well as the expected payoff of an individual bidder between the two auction formats is more complex. For instance, whether the seller's expected revenue in the DA is greater than, lower than or equal to the seller's expected revenue in the corresponding second-price auction strongly depends on the nature of the bidders' heterogeneity, the assigned probability distribution functions, the number of strong and weak bidders in the auction, as well as on the level of the discount. Under different assumptions the seller's expected revenue in the DA may be higher or lower than the seller's expected revenue in a second-price auction.

The following example presents a case where the seller's expected revenue in the DA is greater than the seller's expected revenue in the second-price auction.

Example 3.7.2 Assume an IPV setting with risk neutral bidders. A seller offers a single, indivisible item for sale in a second-price auction. In the second-price auction two bidders compete for the item: a strong bidder and a weak bidder. The weak bidder is denoted by $Weak$ and the strong bidder denoted by $Strong$. The bidders' valuations are uniformly distributed according to different distribution functions. The weak bidder's valuations are drawings of the random variable V_w with the support $[a, b]$, $0 \leq a < b$, $a, b \in \mathbb{R}$ and distributed according to F_w; the strong bidder's valuations are drawings of the random variable V_s with support $[e, f]$, $0 \leq e < f$, $e, f \in \mathbb{R}$ and distributed according to F_s.[17] The drawn valuations are private information but their distribution function is common knowledge. The valuation for the item attached to the weak bidder is denoted by v_w, the valuation assigned to the strong bidder by v_s. For the support of both distribution functions the following condition is assumed:

$$\delta b < e$$

with $d \in (0, 1)$ and $\delta = \frac{1}{1-d} > 1$, i.e. the intersection of the supports is empty. Thus, the valuation of the strong bidder always dominates the valuation of the weak bidder.

With the existence of dominant strategies in the second-price auction, the auction revenue in the second-price auction always equals v_w, the valuation of the weak bidder. Moreover, the seller's expected revenue is the expected valuation of the second highest valuation in the second-price auction (EA), i.e. the expected valuation of the weak bidder

$$E[R_{EA}^{as}] = E[V_w] = \int_a^b v f_w(v) dv = \frac{1}{2}(b + a)$$

Consider now the DA. Suppose, that the above assumptions hold. Additionally, suppose the discount offered in the DA is positive $d \in (0, 1)$. In the DA one bidder is randomly selected, called the designated bidder. That bidder receives the discount. Recall that in the DA an equilibrium in dominant strategies exist: the designated bidder bids δv_i with $\delta = \frac{1}{1-d} > 1, i = w \vee i = s$, whereas the rival bidder submits his valuation truthfully. The bids of the designated bidder, either the bidder *Weak* ($i = w$) or the bidder *Strong* ($i = s$), are drawings of the random variable $\tilde{V}_i$ distributed by $\tilde{F}_i$, $i = w \vee i = s$. Now, the following two cases can be distinguished:

[17]The uniform distribution function on an interval $[a, b]$ with $a, b \in \mathbb{R}$ and $a \leq b$ is given by
$F(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b < x \end{cases}$. The density function is the derivative of the uniform distribution function on $[a, b]$ and is given by $f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{others} \end{cases}$.

1. The *weak* bidder is the designated bidder:
   the weak bidder bids $\delta v_w = \frac{1}{1-d} v_w$. Bidder $Weak$ can not outbid the strong bidder, who places a bid of v_s. The strong bidder wins and purchases the item at a price of δv_w. Thus, under the assumption, that a weak bidder is the designated bidder, the expected revenue is the expectation of the bid δv_w of the weak bidder (the second highest bid) in the auction.

$$\begin{aligned} E[R^{as}_{w,DA}] &= E[\delta V_w] = \delta E[V_w] \\ &= \frac{1}{2}\delta(b+a) \end{aligned}$$

2. The *strong* bidder is the designated bidder:
   the weak bidder places a bid of v_w according to his dominant strategy. As the weak bidder can not bid outbid either the bid δv_s nor the valuation of bidder *Strong*, bidder *Strong* wins the auction. The item is awarded to the strong bidder at the discounted second highest bid, i.e. the discounted valuation of the weak bidder: $\frac{1}{\delta} v_w = (1-d)v_w$. Under the assumption, that the strong bidder receives the discount in the DA, the seller's expected revenue is the expectation of the discounted valuation of the weak bidder.

$$\begin{aligned} E[R^{as}_{s,DA}] &= E[\frac{1}{\delta} V_w] = \frac{1}{\delta} E[V_w] \\ &= \frac{1}{2}\frac{1}{\delta}(b+a) \end{aligned}$$

As both case may appear with equal probability of $\frac{1}{2}$, the seller's expected revenue in the DA is equal to

$$\begin{aligned} E[R^{as}_{DA}] &= \frac{1}{2}E[R^{as}_{w,DA}] + \frac{1}{2}E[R^{as}_{s,DA}] = \frac{1}{2}\left(\frac{1}{2}\delta(b+a) + \frac{1}{2}\frac{1}{\delta}(b+a)\right) \\ &= \frac{1}{2}\left(\frac{1}{2}(b+a)\right)\left(\delta + \frac{1}{\delta}\right) = \frac{1}{2}E[V_w]\left(\frac{\delta^2+1}{\delta}\right) \end{aligned}$$

It is assumed, that $d > 0$, such that $\delta > 1$ and $\frac{\delta^2+1}{\delta} > 2$ holds. Now, the following inequality is derived immediately

$$E[R^{as}_{DA}] = \frac{1}{2}E[V_w]\left(\frac{\delta^2+1}{\delta}\right) > E[V_w] = E[R^{as}_{EA}]$$

Based on the assumption $\delta b < e$, the introduction of a positive discount enhances the seller's expected revenue. In particular, the seller's expected revenue in the DA is higher than the seller's expected revenue in the second-price auction.

Assume that bidders valuations are uniformly distributed on the interval $[0, 1]$ (weak bidder) and $[2, 3]$ (strong bidder) and the discount is set to $d = 0.1$ and $\delta b < e$, i.e. $\delta 1 < 2$ holds. Then, the following is obtained immediately:
the seller's expected revenue in the second-price auction is $E[R^{as}_{EA}] = \frac{1}{2} = 0.5$. With a discount of $d = 0.1$, the seller has an expected revenue of $E[R^{as}_{DA}] = 0.503$ in the DA.

Assume now that the bidder's valuations are uniformly distributed on $[0, 1]$ (weak bidder) and on $[10, 11]$ (strong bidder) and the discount is set to $d = 0.1$, $d = 0.2$, $d = 0.5$ or $d = 0.75$ respectively. Note, that the inequality $\delta b < e$, i.e. $\delta 1 < 10$, must holt. Then, the seller's expected revenue in the DA raises from $E[R^{as}_{DA}] = 0.503$ ($d = 0.1$) to $E[R^{as}_{DA}] = 0.513$ ($d = 0.2$), $E[R^{as}_{DA}] = 0.625$ ($d = 0.5$), or even $E[R^{as}_{DA}] = 1.063$ ($d = 0.75$).

Note that in case of different assumptions in the given example, i.e. changing the condition of $\delta b < e$, the seller's expected revenue in the DA changes. Particularly, the result derived in the example, that the seller's expected revenue in the DA auction is greater than the seller's expected revenue in the second-price auction may not necessarily hold. In fact, the outcome of the DA strongly depends on the nature of heterogeneity in bidders' preferences (Maskin and Riley 2000). As long as the valuation v_s of the strong bidder is higher than the bid of the designated weak bidder δv_w, i.e. $v_s > \delta v_w$, the auction is efficient. Thus, in the two bidder case, the auction revenue achieved in the DA is greater than the achieved revenue in the corresponding auction. In the case that the weak bidder may outbid the high value bidder with $\delta v_w \geq v_s$, because of $\delta b \geq e$ and the affirmative action of the discount, efficiency as well as the seller's revenue are reduced. However, the presented example is a special case of two bidders with asymmetric preferences.

So far, the analytical model of the DA (Section 3.6.2) and the corresponding second-price auction (Section 3.5.2) under the conditions of independent private values, risk neutral bidders, and bidders characterized by asymmetric distribution functions of valuations are presented. In both models the seller's expected revenue, the expected payoff of an individual bidder as well as the expectation about the auction welfare are derived.

As in the Example above, the focus is set now on a comparison of both auction models in which bidders are characterized by different distribution functions of valuations. Assume an IPV setting with risk neutral bidders in which the DA and the corresponding second-price auction are conducted. Assume further that four bidders – one strong bidder and three weak bidders – participate in the auction, which is either conducted as a DA or a second-price auction. Bidders are asymmetric and characterized by two different distribution functions of valuations. The shape of the distribution – here the exponential function – is the same, but the expected values differ. Bidder 4 is the strong bidder and his expectation about his valuation

Asymmetric case: comparison of expected values

in the second-price auction and in the DA

	$\lambda_s = \frac{1}{200}$								
	2^{nd}-price auction			DA with $d = 10\%$			DA with $d = 20\%$		
λ_w	$E[W^{as}_{EA}]$	$E[R^{as}_{EA}]$	$E\pi^{as}_{EA}$	$E[W^{as}_{DA}]$	$E[R^{as}_{DA}]$	$E\pi^{as}_{DA}$	$E[W^{as}_{DA}]$	$E[R^{as}_{DA}]$	$E\pi^{as}_{DA}$
1	200.012	**1.824**	198.188	200.011	**1.831**	198.181	200.011	**1.853**	198.158
$\frac{1}{10}$	201.117	**17.472**	183.645	201.110	**17.516**	183.594	201.085	**17.672**	183.413
$\frac{1}{50}$	222.949	74.487	148.462	222.846	74.471	148.375	222.475	74.410	148.066
$\frac{1}{100}$	274.762	129.048	145.714	274.518	128.844	145.674	273.644	128.115	145.529
$\frac{1}{150}$	341.494	174.870	166.623	341.133	174.518	166.615	339.844	173.260	166.584
$\frac{1}{190}$	401.145	208.504	192.641	400.708	208.068	192.640	399.146	206.506	192.639

Table 3.2: Expected values in the discount auction and in the corresponding second-price auction with asymmetric bidders (case: three weak bidders and one strong bidder)

is $E[V_s] = \frac{1}{\lambda_s} = 200$. Bidder 1, bidder 2 and bidder 3 are all weak bidders with the same expected valuation of $E[V_w] = \frac{1}{\lambda_w}$. First, for all weak bidders λ_w is set to 1; then, stepwise, λ_w is varied to different levels for all weak bidders ($\lambda_w = 10$, or $\lambda_w = 50$, or $\lambda_w = 100$, or $\lambda_w = 150$, or $\lambda_w = 190$) such that their expectations of the valuations change while the strong bidder's expectation of his valuation is hold constant. Now, the four bidders, three weak bidders and a strong bidder, participate in the DA or respectively in the corresponding second-price auction. In the DA the discount is assumed first at a level of $d = 10\%$ and second at a level of $d = 20\%$.

Table 3.2 depicts the expected outcomes of the DA and the second-price auction (EA), i.e. the expected welfare, the seller's expected revenue, as well as the sum of the expected payoffs of all individual bidders. $E[W^{as}_{EA}]$ resp. $E[W^{as}_{DA}]$ denotes the expected welfare, $E[R^{as}_{EA}]$ resp. $E[R^{as}_{DA}]$ the seller's expected revenue, and $E\pi^{as}_{EA}$ resp. $E\pi^{as}_{DA}$ the sum of the expected payoffs of all bidders in the DA resp. in the corresponding second-price auction.

The main result derived from this comparison is that **there are cases in which the discount pays for the seller**, meaning that the seller's expected revenue in the DA is greater than the seller's expected revenue in the second-price auction. In Table 3.2, these cases are printed bold. More precisely, under the conditions, that a single strong bidder and one or more weak bidders participate in the DA, and that the expected value of the strong bidder is much higher than the expected values of the weak bidder, the seller's expected revenue in the DA exceeds the seller's expected revenue in the corresponding second-price auction. Thus, for $\lambda_w = 1$ or $\lambda_w = \frac{1}{10}$ and $\lambda_s = \frac{1}{200}$ with a discount of $d = 10\%$ and $d = 20\%$ the following holds:

$$E[R^{as}_{EA}] < E[R^{as}_{DA}]$$

A weak bidder can outbid the strong bidder only with small probability, even in case of receiving the discount, which is similar to the case given in Example 3.7.2. If the difference of the expected values of the strong and the weak bidder is getting smaller, the probability of a weak bidder to outbid the strong bidder raises. Additionally, the probability of a weak bidder, to whom the discount is assigned, to win the auction raises, and thus, the seller's revenue is decreased. For $\lambda_w = \frac{1}{50}, \frac{1}{100}, \frac{1}{150}, \frac{1}{190}$ and $\lambda_s = \frac{1}{200}$ the expected revenue in the second-price auction is higher than the expected revenue in the DA:

$$E[R^{as}_{EA}] > E[R^{as}_{DA}]$$

Table 3.3 indicates the expected payoffs of the bidders in both auctions. The expected payoff of a bidder of type weak, indicated by subscript *w*, is given by $E\pi^{as}_{i,w,EA}$ resp. $E\pi^{as}_{i,DA}$ (the payoff of a weak bidder i except the designated bidder, $i \in W \setminus \{\hat{\imath}\}$), and the expected payoff of a strong bidder, indicated by subscript *s*, by $E\pi^{as}_{j,s,EA}$ resp. $E\pi^{as}_{j,DA}$ (the payoff of a strong bidder j, except the designated bidder, $j \in S \setminus \{\hat{\jmath}\}$). The expected payoff of a weak designated bidder is given by $E\pi^{as}_{\hat{\imath},w,DA}$ and the expected payoff of a strong designated bidder by $E\pi^{as}_{\hat{\jmath},s,DA}$. The subscript DA resp. EA indicates the auction format, either a DA or the corresponding second-price auction (EA).

Asymmetric case: comparison of bidder's expected payoff

in the second-price auction and in the DA

	$\lambda_s = \frac{1}{200}$									
	2^{nd}-price auction		DA with $d = 10\%$				DA with $d = 20\%$			
λ_w	$E\pi^{as}_{i,w,EA}$	$E\pi^{as}_{j,s,EA}$	$E\pi^{as}_{i,DA}$	$E\pi^{as}_{\hat{\imath},w,DA}$	$E\pi^{as}_{j,DA}$	$E\pi^{as}_{\hat{\jmath},s,DA}$	$E\pi^{as}_{i,DA}$	$E\pi^{as}_{\hat{\imath},w,DA}$	$E\pi^{as}_{j,DA}$	$E\pi^{as}_{\hat{\jmath},s,DA}$
1	0.003	198.178	0.003	0.004	194.842	198.360	0.003	0.004	191.259	198.541
$\frac{1}{10}$	0.287	182.784	0.273	0.336	179.413	184.410	0.258	0.398	175.672	186.056
$\frac{1}{50}$	5.726	131.282	5.471	6.593	128.058	136.517	5.181	7.656	124.271	142.045
$\frac{1}{100}$	18.095	91.429	17.333	20.541	88.641	97.910	16.457	23.467	85.338	105.042
$\frac{1}{150}$	33.377	66.494	32.032	37.528	64.156	72.876	30.471	42.419	61.404	80.128
$\frac{1}{190}$	46.610	52.811	44.783	52.118	50.791	58.777	42.655	58.552	48.431	65.701

Table 3.3: Bidders' expected payoffs in the discount auction and in the corresponding second-price auction with asymmetric bidders (case: three weak bidders and one strong bidder)

3.8 Related work

In the literature, second-price auctions in a SIPV setting have been thoroughly analyzed and discussed. One important result is that within a SIPV setting and risk neutral bidders, the revenue equivalence theorem holds. If one of the assumptions of the SIPV setting or the risk neutrality of bidders are dropped, the revenue equivalence no longer holds (Maskin and Riley 2000).

One central assumption of the SIPV is the symmetry assumption: bidders are characterized by the same probability distribution functions of valuations, i.e. their preference parameters are drawn from the same probability distribution function. In particular, if bidders are of the same type (their preference parameters are equal), they will have the same beliefs about the rival bidders. However, this symmetry assumption is violated in many real-life auction environments. For example, in art auctions bidders' tastes are known to be quite idiosyncratic.

Many results of the symmetric auction framework do not extend to asymmetric auctions. There is limited literature dealing with asymmetries between commonly known distribution functions from which valuations are independently drawn. Theoretical and experimental analysis of asymmetric auction models which focus on second-price sealed-bid auctions or first-price sealed-bid auctions are presented for example by Maskin and Riley (2000), Cantillon (2005), Güth et al. (2005), or Elbittar (2002).

Moreover, literature on affirmative actions in asymmetric auctions subsidizing a class of bidders is rare. Often such classes include economically disadvantaged, less effective bidders. There are different forms of subsidizing such groups – advantages can be given in the form of set-asides, discounts, bidding credits, or special payment terms (Rothkopf et al. 2003). Ayres and Cramton (1996) present an example from the Federal Communications Commission (FCC) where 30 telecommunication spectrum licences were auctioned among asymmetric bidders. In the auction, businesses owned by minorities or women were subsidized, meaning that they received a bidding credit of 40 percent. Milgrom (2004) gives a more theoretical example of an auction with asymmetric bidders and bidding credits. Both examples show that with asymmetric bidders, bidding credits might pay for the seller and that auction revenue can be increased. Such affirmative actions are also applied in procurement auctions in which contracts are auctioned. The policy of subsidizing inefficient competitors can lower project costs and enhance cost effectiveness. Corns and Schotter (1999) present an experiment on price-preference auctions with asymmetric bidders and show that choosing the right degree of price preference leads to cost effectiveness. The given examples are useful in understanding how bidding credits can positively affect the seller's expected revenue in auction models, where the symmetry assumption is dropped, and thus are strongly related to the presented study of the DA market institution.

3.8.1 Asymmetric auctions

In their model, Maskin and Riley (2000) assume an independent private values auction model with risk neutral bidders. The authors drop the symmetry assumption, such that the bidders' valuations are distributed according to different probability distribution functions. In essence, the authors show that the revenue equivalence between the high-bid auction, the first-price sealed-bid auction, and the open auction, an English auction (equivalent to a second-price sealed-bid auction), is hurt. Furthermore, they show that the revenue ranking of the first-price sealed-bid auction and the second-price sealed-bid auction depends on the kind of asymmetries of bidders, i.e. on the nature of the bidders' heterogeneity. That is, under different assumptions the seller's expected revenue in a first-price sealed-bid auction may be higher or lower than in an open auction. Example 3.8.1 is derived from Maskin and Riley (2000) and presents the case of two asymmetric bidders participating in a high bid auction (first-price sealed-bid auction) and an open auction (second-price auction). It is shown that the first-price auction yields higher revenues than the second-price auction.

Example 3.8.1 Suppose that a strong bidder and a weak bidder participate in a first-price sealed-bid auction with valuations uniformly distributed on different supports. The weak bidder's valuation is uniformly distributed on $[0, 1]$ and the strong bidder's valuation on $[2, 3]$. Assume the weak bidder bids his valuation $b_w = v_w \in [0, 1]$. The best response of a strong bidder in a first-price auction is to then bid $b_s = 1$. The strong buyer tries to maximize his payoff while making sure to outbid the weaker bidder. In fact, the expected revenue from the auction is 1.
Turning to the second-price auction, the highest bidder wins the auction since he can always outbid the weak bidder. The price to pay is equal to the second highest bid; the expected revenue equals $\frac{1}{2}$, the expected value of the weak bidder's valuation.

In general, it can be stated that whenever the strong bidder's distribution function is such that with a high probability, the strong bidder's valuation is much greater than the valuation of a weak bidder (with a different distribution function), the first-price auction will tend to achieve a higher revenue. In a first-price auction the strong bidder will enter a bid equal to the maximum support of the weak bidder, whereas in the second-price auction, the strong bidder will pay the expected value of the weak bidder's valuation. In the literature this principle is referred as *Getty Effect* (Maskin and Riley 2000).

Considering the strategic behavior of bidders with asymmetric preferences, Maskin and Riley show quite generally that *strong* buyers prefer the second-price sealed-bid auction, whereas *weak* buyers prefer the first-price sealed-bid auction. Strong buyers are considered to be buyers who are more likely to have a high valuation for the item to be auctioned.

Cantillon (2005) sheds more light on the impact of bidders' asymmetries in the first-price and second-price auction and analyzes how asymmetries affect the bidders' behavior and in turn the expected revenue as well as the profit. In order to address these questions, a benchmark environment is defined: the distribution of valuations in this environment is the geometric average of the distributions in the original environment with asymmetric preferences among the bidders. In the benchmark environment bidders draw their valuations independently from the same distribution function – the geometric average of the distribution functions in the asymmetric case. By construction of the distribution in the benchmark environment, the expected value of the highest valuation among bidders is the same as in the original auction. The main result derived from this auction model is that the introduction of asymmetries lowers the revenue in the first-price and second-price auctions compared to the benchmark symmetric auction format.

Recall the basic property of the benchmark auction (BA): the expected value of the highest valuation among bidders is the same as in the original asymmetric auction. With this property, the question of how asymmetries affect the revenue of the auction while keeping the welfare constant can be extracted. In other words, the two auction environments are compared for which the potential social surplus, or the expected welfare, is the same.
Cantillon considers an independent private values auction environment. n bidders participate in an auction (a second-price sealed-bid auction or a first-price sealed-bid auction), where a single indivisible item is offered for sale. Each bidder i's valuation is independently distributed according to the distribution function F_i with support $[\underline{v_i}, \overline{v_i}]$, $i \in N = \{1, \ldots, n\}$. Bidders are asymmetric, if $F_i(v) \neq F_j(v)$, for $i, j \in N \; i \neq j$, and for a non-zero measure of valuations. Given the distribution functions $F_1, \ldots, F_n$, their benchmark distribution function is denoted by F for all v with

$$F(v) = \left(\prod_{i=1}^{n} F_i(v) \right)^{\frac{1}{n}}$$

F is the geometric average of the distributions in the original environment and defined on the support $[\max_{i \in N} \underline{v_i}, \min_{i \in N} \overline{v_i}]$. Given the asymmetric distribution functions $F_i(v), i \in N$ and the geometric average distribution function $F(v) \forall i \in N$ the seller's expected revenue in a second-price sealed-bid auction, i.e. the expected value of the second order statistic, can be derived. In the second-price auction with symmetric bidders, the distribution function of the second order statistic of the configuration $(F,\ldots,F)$ is given by

$$F_{(2),n,s}(v) = nF^{n-1}(v) - (n-1)F^{n}(v)$$

(subscript *s* denotes the symmetric case) and the distribution function of the second order statistic in the asymmetric case of the configuration $(F_1,\ldots,F_n)$ by

$$F_{(2),n,as}(v) = \sum_{i=1}^{n}\left((1-F_i(v))\prod_{j\neq i}F_j(v)\right) + \prod_{i=1}^{n}F_i(v)$$

(subscript *as* denotes the asymmetric case). With the given distribution functions of the second order statistic in both cases, Cantillon proves that $F_{(2),n,as}(v) - F_{(2),n,s}(v) \geq 0$ for all v with some strict inequality.

$$F_{(2),n,s}(v) \leq F_{(2),n,as}(v) \quad \Leftrightarrow \quad Prob(V_{(2),n,s} > v) \geq Prob(V_{(2),n,as} > v)\,\forall v$$

means that the distribution of the second order statistic in the benchmark environment first-order stochastically dominates the distribution in the asymmetric auction (there are more "high values").[18] Then the expected revenue $E[R_s]$ of the second-price auction in the symmetric case (the benchmark auction) is greater than the expected revenue $E[R_{as}]$ in the asymmetric auction:

$$E[R_s] > E[R_{as}]$$

Additionally, consider two asymmetric second-price sealed-bid auctions, where in one auction bidders are considered more asymmetric than in the other. Cantillon shows that the more asymmetric the configuration, the lower the expected revenue. However, Cantillon gives a detailed analysis of a second-price auction in the asymmetric case and proves that asymmetries hurt revenue (compared to the defined benchmark environment). Moreover, the analysis is extended to the first-price sealed-bid auction. Here, the result that asymmetries also hurt auction revenue is derived for three classes of distributional asymmetries.

Güth et al. (2005) present a laboratory experiment on first-price and second-price auctions in an independent private values auction environment. Bidders' valuations are independently drawn from distinct but commonly known distribution functions, meaning that bidders are asymmetric. In particular, Güth et al. (2005) conducted an experiment in which either the weak or strong type was randomly assigned to a subject, and subjects played both auctions,

[18] A random variable X first-order stochastically dominates (FSD) the random variable Y, if $Prob(X > z) \geq Prob(Y > z)\,\forall z$. This is written in the form $F(z) \leq G(z)\,\forall z$ with F and G being the distribution functions of X and Y. If X first-order stochastically dominates Y, then the following holds: $E[X] \geq E[Y]$ (Wolfstetter 1999, ch. 4.3).

a first-price and a second-price auction, with a randomly chosen partner of the opposite type. Moreover, bidders were given the chance to select the auction format before and after they were informed about their own valuations. In particular, in the experiment bidders had to determine a price they were willing to pay for the right to dictate the auction format (first-price or second-price auction).
Bidders' valuations were independently drawn from uniform distribution functions: the weak bidders' valuations were uniformly distributed on the interval $[50, 150]$, while the strong bidders' valuations were uniformly distributed on the interval $[50, 200]$.

In the experiment each subject was either of the weak or strong type and participated in an auction with a bidder of the opposite type. Each experimental session was divided into three phases. In the first phase, six first-price auctions followed by six second-price auctions were conducted. Bidders' valuations were drawn in each round before the bids were made. In the second phase sixteen bidding rounds were conducted. In each of the sixteen rounds, the auction mechanism was chosen as follows: first, the two bidders in the auction were asked to state their maximum willingness to pay for the dictatorship for the auction mechanism. Second, one of the two bidders was randomly selected to dictate the auction mechanism. The selected bidder's determined price for the dictatorship was then compared to a random number between $[0, 30]$. If the bidder's price was greater or equal to the selected random number, then that bidder determined the auction rule. In all other cases, the auction mechanism was randomly selected by flipping a coin. The selected auction mechanism was subsequently played as in the first phase. The third phase was conducted as the second phase, except that this time bidders knew their valuations before any decision was made. The experiment was conducted in eight sessions – seven sessions with 14 participants and one session with 12 participants.

Summarizing the main results, Güth et al. (2005) observe that (i) prices achieved in the first-price auction are higher than prices achieved in the second-price auction; (ii) in the second-price auction bidders bid close to their valuations; and (iii) strong bidders tend to pay more for the dictatorship of the auction rule.

Another experimental approach on bidding behavior in asymmetric auctions has been taken by Elbittar (2002). In his experiment on first-price auctions in an independent private values environment, the impact of revealing information about bidding behavior as well the seller's revenue and the efficiency of the auction is evaluated. Two bidders participated in a first-price sealed-bid auction in which a single item was offered for sale. The bidders decided what to bid under two different information conditions. In the symmetric condition, bidders had no information about the rank order of the valuations. In the asymmetric condition, the two bidders were informed about the rank order, i.e. whether they were a high value or low value bidder respectively, but not about the size of difference in valuations. By revealing the rankings of the valuations, the ex-ante symmetric auction becomes asymmetric. In essence, this experiment is based on a theoretical model of Landsberger et al. (2001), where bidders draw their valuations from the same distribution function but the ranking of these valuations is common knowledge. The aim of the experiment is to confirm and evaluate the key predictions derived from the theoretical model: (i) under the asymmetric condition, the low value bidder will bid more aggressively than the high value bidder; (ii) under the asymmetric condition, bidders will bid more aggressively than under the symmetric condition; and (iii) the seller's expected revenue is higher when rankings of valuations are unknown.

The experiment was designed to measure the information impact of revealing valuation rankings on bidding behavior in the first-price sealed-bid auction. Therefore, a dual-market consisting of two phases was employed. In the first phase, bidders submitted a bid under the symmetric condition based on their assigned valuation. In the second phase, bidders were asked to submit a bid again based on their assigned valuation (that of the first phase) and under the information of being a high or low value bidder. Afterwards, by flipping a coin, it was determined which of the two markets was selected to determine the allocation, the highest bidder and the price to pay.

Within the experiment, two bidders were randomly selected who participated in both phases of the dual-market process. The bidders' valuations were randomly drawn from the same commonly-known uniform distribution function. Additionally, the bidders' positions as high or low value bidders were randomly determined.

Overall four sessions were conducted. In each session ten periods of a single-auction market were followed by twenty periods of a dual-auction market. During the initial period, bidders remained in their positions as high or low value bidders. In two of the four sessions, the single-auction market was conducted under symmetric conditions and under asymmetric conditions in two sessions. Eighteen subjects participated in session one and 20 subjects in sessions two to four.

The main results derived from the experiment according to Elbittar (2002) are as follows: as expected, the low value bidder bid more aggressively since the ranking order of valuations was revealed. Contrary to the expectation, revealing information about the ranking of valuations might not produce higher revenues.

3.8.2 Asymmetric auctions with bidding credits

An interesting fact was observed in the regional narrowband auction of the Federal Communications Commission (FCC) in 1994 (Ayres and Cramton 1996). The FCC offered thirty regional narrowband licences for sale. These licences were sold in a simultaneous multiple-round auction, also called a simultaneous ascending auction (Cramton 2004; Milgrom 2004). The simultaneous ascending auction is similar to the traditional English auction, in which a group of items with strong interdependencies is sold. Throughout the auction, bidding is conducted in rounds. All licences are on block at the same time and bidders can bid on any of the items in each round. The bidding process continues until bidding has stopped on all licences and no bidder is willing to raise the price on a single licence, i.e. a single round passes in which no new bid on any item was submitted. The auction then ends with the highest bidder on each licence as the winning bidder, paying the price equal to his high bid. To assure that the auction ends in a reasonable amount of time, minimum bid increments are specified and adjusted within the bidding process. During the auction each bidder is fully informed about the bidders' identities, and after each auction round the bid history, high bids, and identities of the highest bidders are announced. In the focused spectrum auction, the FCC has granted businesses owned by minorities and women, or so-called designated bidders, bidding credits of 40 percent. The affirmative action increased the government's revenue by 12 percent. Ayres and Cramton (1996) analyze this specific auction and find that giving *bidding preferences* to weak bidders, i.e. subsidizing weak bidders, who have lower expected valuations, can increase auction revenues. Strong bidders are forced to bid more aggressively and to compete with weak bidders. Thus, introducing bidding preferences can enhance on the one hand *intragroup* competition and on the other hand *intergroup* competition. For example, bidding preferences such as set-asides that reduce the quantity of items available to strong bidders may cause them to bid more aggressively (intragroup competition); subsidizing policies enhancing economic-disadvantaged bidders, e.g. by bidding credits, may put these bidders in a challenging position competing with the strong bidders (intragroup competition). Further, according to Ayres and Cramton (1996), bidding preferences are likely to enhance revenue only if the following conditions hold: (i) there is insufficient competition among the high value bidders; and (ii) the seller is able to identify stable classes of economic-advantaged and economic-disadvantaged bidders, i.e. bidders with high valuations and bidders with low valuations. In the latter case, the difficulty for the seller lies in the estimation of the expected difference of at least the two groups of strong and weak bidders in order to identify the subsidy policy and the level of the subsidy to enhance revenue. Note that subsidizing high-value bidders will in general lead to reduced competition and to lower expected revenues. However, the difficulty lies in meeting the two conditions. Regarding real world auctions, it can be observed that few sellers introduce a reserve price, i.e. a minimum bidding level, above their valuations. By this sellers subsidize themselves as a weak bidder to increase revenue. The following example is taken from Ayres and Cramton (1996).

Example 3.8.2 Assume that four bidders participate in a simultaneous multi-round auction where two licences are auctioned. Remember the simultaneous multi-round auction is similar to a traditional English auction and that the two licences are auctioned simultaneously. In any round, a bidder may bid on any of the licences, and switching between licences is allowed. The auction ends if all licences can be allocated and no one raises the prevailing bid on any licence. Each bidder is interested in one single licence. The bidders are grouped in two groups – a group of high value bidders and a group of low value bidders with the following reservation prices: two strong bidders $Strong_1, Strong_2$ with valuations $s_1 = 110$ and $s_2 = 90$, and two weak bidders $Weak_1, Weak_2$ willing to bid up to $w_1 = 60$ and $w_2 = 40$ respectively. These valuations are independently drawn from distinct but commonly known distribution functions. If the increment within the auction is neglected, then the seller's expected revenue equals 120. In each auction, the strong bidders have to outbid the weak bidders, dropping out at an announced price of 60. Since the number of offered licences is equal to the number of strong bidders, there is no need for the strong bidders to compete with each other, only with the weak bidders within the auction. Taking this as benchmark, set-asides or bidding credits offered to the group of weak bidders are considered in the following.

Set-Asides can create Intragroup Competition

Consider one licence to be set aside. The set-aside licence is to be auctioned only among the weak bidders. Weak bidder $Weak_1$ purchases the licence at a price of 40, the valuation of $Weak_2$. The remaining licence is to be auctioned among the strong bidders with a revenue of 90, paid by the strong bidder $Strong_1$. Thus, setting aside one licence raises the government's expected revenue by 10 to 130. In this case, intra-group competition among the strong bidders is increased, raising the governmental revenue to 130 at the cost of reduced efficiency. One licence is purchased by a weak bidder instead of a high value bidder.

Bidding Credits can create Set-Asides

Consider a bidding credit of 50 percent to weak bidders, i.e. in case of winning the auction, bidders receive a credit of 50 percent on the winning bid. Due to this credit weak bidders can raise their bids up to $\tilde{w}_1 = 120$ and $\tilde{w}_2 = 80$. Thus, the strong bidder $Strong_1$ and the weak bidder $Weak_1$ each win a licence with a winning bid of 90. The revenue of the seller would be 135: the strong bidder has to pay the winning bid of 90 and the weak bidder, receiving a credit of 50 percent on the winning bid, has to pay 45. Note that the strong bidder $Strong_1$ competes with the bidder $Weak_1$ to win the licence. Introducing a bidding credit of 50 percent raises the revenue of the seller from 120 in the benchmark auction to 135 in this auction. At the same time the bidding credit leads to inefficiency by allowing the weak bidder $Weak_1$ to win instead of the higher value bidder $Strong_2$. Note that the bidding credit achieves higher revenues than the set-asides due to the fact that strong bidders must compete with weak bidders in order to win a licence.

Bidding Credits can create Intergroup Competition

Assume now that the bidding credit of 50 percent is calibrated to the level allowing the weak bidder $Weak_1$ to bid up to an epsilon below the valuation of bidder $Strong_2$, i.e. $\tilde{w}_1 + \epsilon = s_2 = 90$ with $\epsilon \geq 0, \epsilon \rightarrow 0$. By this, a maximum at inter-group competition is achieved, leading to a maximum achievable seller revenue with 180. Assume that the credit is calibrated to 25 percent. With a bidding credit of 25 percent bidder $Weak_1$ would bid $\tilde{w}_1 = 80$. The strong bidders would win the auction with a winning bid of 80. The total auction revenue would be 160, achieved by an inter-group bidding competition. Note that giving the weak bidders a credit of 33.33% would force the strong bidders to bid 90 and thus result in a total revenue of 180. At the same time efficiency is kept – the licences are awarded to the high value bidders.

Milgrom (2004) presents several tactics used in auctions to increase participation. Such tactics are mentioned above: set-asides and bidding credits can encourage bidders to enter an auction. Another tactic which is not focussed upon in this discussion is to allow losing bidders to earn some profits, e.g. such as in the so-called premium auction (Milgrom 2004). Further Milgrom (2004) gives a special example of an auction in which a single-item is offered to a strong and a weak bidder for sale. In the auction, the weak bidder is granted a bidding credit and thus, encouraged to enter the auction. Then, it appears, that the seller's expected revenue is raised: the seller's expected revenue with bidding credits is greater than the seller's expected revenue without bidding credits.

According to Rothkopf et al. (2003), subsidizing a class of competitors believed to be at an economic disadvantage in an auction is a widespread practice, particularly in public-sector procurement. Reasons for such policies stem from thoughts about non-economic aspects such as fairness, anti-discrimination, populism etc. Additionally, it is widely presumed that a preferential treatment "is costly for the bid taker and economically inefficient" (Rothkopf et al. 2003). That this presumption is not necessarily correct is shown for example by an experimental evaluation of an auction model with an affirmative action subsidizing a class of disadvantaged competitors by Corns and Schotter (1999). More specifically, Corns and Schotter conducted a laboratory experiment in which a price-preference auction is employed: high-cost minority firms are given preferential treatment in the awarding of contracts in such a way that these firms can win the auction without having submitted the lowest bid. One prediction from theory is that if the initiator of the auction has accurate prior information about the distribution of the firms' costs, then a price preference can be determined which will be cost effective. In their experiment Corns and Schotter (1999) demonstrate that in procurement auctions, affirmative actions can be used to subsidize minorities and decrease the cost of government procurement. In particular, price preferences make high-cost firms look like low-cost firms and thus increase competition. Low-cost firms tend to bid more aggressively and closer to their cost. At the same time, high-cost firms face less competition and bid less aggressively.

The experiment performed by Corns and Schotter (1999) was a straight-forward implementation of a price-preference auction in an independent private values auction environment: a procurement auction was employed using the auction mechanism of a reverse first-price sealed-bid auction with an additional preference rule. The preference rule states that after having collected all bids in the auction round, the bids of the low-cost bidders are increased by a given percentage. In particular, the bids of the low-cost bidders are adjusted by multiplying each bid by one plus the amount of the preference, whereas the bids of the high-cost bidders remain unchanged. Afterwards, the contract is awarded to the firm with the lowest comparison bid at the price of its submitted bid. By these means, the adjustment procedure affects only the winner determination but not the payoff of the winning firm. The preference rule has mainly two effects: first, all bids are made comparable for the purpose of winner determination; and second, high-cost firms are given a bidding advantage in the auction for purchasing the contract.

In the laboratory experiment students participated in 20 auction rounds. In each auction round, a single contract was auctioned in a price-preference auction among a fixed number of firms represented by students. Each firm was either of type A – a high-cost bidder – or of type B – a low-cost bidder. The costs of the type A bidders were uniformly distributed on the interval $[110, 220]$ while the costs of type B bidders were uniformly distributed on the interval $[100, 200]$. In each price-preference auction two type A bidders competed against four type B bidders for the contract. Before the auction round started, the bidders' costs were randomly drawn and assigned to the bidders. Each bidder knew his own type, how many bidders of each type participated in the auction, his own private costs, and the distributions of the costs presented to all bidders. Based on their given costs, bidders had to submit one sealed bid in the auction. After each bidder had submitted his bid, the winner of the auction, the price to pay as well as the type of the winning bidder was announced. The payoff the winner received in each auction was equal to the difference between the submitted bid and the winner's cost.

Before the first auction round started, the types – type A or type B – were randomly assigned to the bidders. Two high-cost bidders and four low-cost bidders formed one group and competed against each other in all auction rounds. At the end of all rounds subjects were paid a show-up fee and the sum of their payoffs earned in all the rounds played.

Overall in the experiment, four treatments were conducted. In each treatment five bidding groups participated with a total of 30 bidders. The treatments differed in the definition of the preference rule: a 0-percent, 5-percent, 10-percent and 15-percent preference for Type A was given, meaning that the bids from type B bidders were adjusted depending on the degree of the price-preference percentage. The bidder with the lowest comparison bid won the auction at the price of his submitted bid.

The lowest average price achieved in all treatments was in the 5-percent treatment – this rule outperforms those in which no preference as well as a 10-percent or 15-percent preference was given. Moreover, bidders bid more aggressively, or closer to the cost, as the auction proceeded. In general, the laboratory experiment showed that an affirmative action in the form of the price-preference rule favors disadvantaged bidders, leading to a higher, more aggressive competition and cost effectiveness.

Chapter 4

An Experiment on Discount Auctions

4.1 Motivation and research questions

The interest in auctions with discounts brings up the desire to find explanations to the questions "Why does a seller offer a discount in an auction?" and "Can one identify cases where the discount may pay for the seller?". In response to these questions, the theoretical model of the DA market institution was developed. One explanation derived from the theoretical analysis of the DA model is that asymmetries between bidders can be a driver for discounts in auctions. Asymmetries between bidders means that bidders differ in their nature as well as their preference structures and do not have the same beliefs concerning rival bidders. In particular, asymmetries can be observed in real-world auction environments such as art auctions – bidders' tastes are known to be quite idiosyncratic or bidders have different budget constraints. Offering discounts to economically disadvantaged bidders may increase competition in an auction and thus raise the seller's revenue. More generally, the asymmetries between bidders and the nature of their heterogeneity might be a driver for a seller to offer such an affirmative action in an auction. Whether such an affirmative action pays for the seller strongly depends on the kind of action, the degree of such an action, and the strategic impact on bidders' decisions and bidding behavior.

The theoretical analysis of the DA is carried out under very strong artificial assumptions. Participants are assumed to be rational and risk-neutral in an independent private values auction environment. The main findings of the theoretical analysis can be summarized as follows:

1. In the DA the designated bidder submits a bid above his valuation and all other bidders submit their valuations truthfully (Proposition 3.3.1).

2. If bidders are symmetric the discount does not pay for the seller (Proposition 3.7.1)

3. If bidders are asymmetric the discount might pay for the seller, especially if one strong bidder competes with one or more weak bidders (e.g. Example 3.7.2 and Table 3.2).

Because of the restrictive assumptions made and the complexity of the theoretical model in the asymmetric case, some unresolved questions still exist. When comparing the findings to real-world auction environments, it can be stated that attitudes of practical relevance such as risk aversion, risk lovingness, impatience, or uncertainty have not been modeled in the DA. When focussing on Internet auctions, these different attitudes can be observed and thus are of practical relevance: some bidders consider bidding as fun or gambling and are often risk-loving, whereas other bidders are uncertain about their own valuations throughout the bidding process, adapting their valuations throughout the auction. Moreover, some assumptions made such as the rationality assumption are sometimes hurt in real-world settings. Thus, a seller initiating an Internet auction in order to sell an item not only has to anticipate different types of bidders interested in purchasing the offered item and their bidding strategies when choosing the right selling mechanism; she must also face irrational behavior. Anticipation of the different bidders' characteristics, prediction of impacts on the bidding behavior in the selected selling mechanism, as well as predictions of effects on the auction outcome are difficult to derive.

Generally, the question arises if a theoretical model can sufficiently explain the behavior of the seller and bidders in a real-world discount auction. That is, can we infer real-world discount auctions from the DA? Can we transfer theoretical predictions derived from the DA directly to reality?

For the DA the following issues are fundamental:

1. The DA model is based on the assumptions that bidders behave rationally. However, from real-world auctions it is known that bidders do not always behave rationally as theoretically assumed.

2. The rules of the DA are similar to those of real-world discount auctions but not exactly the same. For instance, when taking an Amazon first bidder discount auction into consideration, bidders may behave differently in that auction than in the DA.

Unfortunately, the DA model makes very strong artificial assumptions and thus limits the bidders and the seller in their decisions. Such limitations are necessary since real-world auctions are not that easy to model. Besides the limitations in the bidders' attitudes, the auction mechanism of the DA idealizes and eases the rules of real-world auctions. Recall that in the Amazon first bidder discount auction, a bidder receives the discount by submitting the first valid bid. Thus, when becoming aware of the auction and the available first bidder discount, bidding immediately is essential for receiving the discount. This aspect of the Amazon first bidder discount auction is relaxed in the DA and eased by determining the designated bidder in advance: one bidder is randomly selected before the DA starts and the discount assigned to that bidder. Moreover, in the Amazon auction the number of bidders and their arrival time is not commonly known in advance. In the DA, the number of bidders participating is fixed and commonly known to all bidders in advance.

To shed more light on the discount and explain bidding behavior in the DA, a laboratory experiment has been conducted. The goal of this experiment is to evaluate the bidders' strategic behavior and the impact of that bidding behavior on the auction revenue in a controlled environment. Additionally, the experiment aims at validating the theoretical predictions, guided by the following key questions:

1. Is the predicted behavior of bidders in the DA consistent with the observed behavior in the experiment?

2. In the symmetric case, is a seller able to extract an additional revenue by offering a discount (contrary to the prediction)?

3. In the asymmetric case, can the expectation of a seller – to raise her revenue by offering a discount – be confirmed?

Based on the theoretical predictions, the conducted experiment gives deeper insights into the discount's impact on the strategic behavior of bidders and its effect on the auction revenue. In the context of the market engineering approach, the theoretical and experimental findings of the DA model may be used (i) to advise the seller in choosing the proper auction mechanism and (ii) to indicate to the seller that a discount should be offered when knowing information about the bidders' characteristics.

4.2 Experimental design

The research questions require an experimental design that allows a comparison of the DA to the second-price auction in (i) the symmetric case and (ii) the asymmetric case. In principle, both market institutions follow the rules of the underlying second-price auction mechanism and only differ in the existence of the discount – not in any other design parameter.

The experiment follows a *between subjects* design; it focuses on the isolated effect of levels of variables. The level of a treatment variable is only varied between single treatments and across subjects but not within one trial. The experimental design describes the nature and number of variables focused upon in the experiment. The variable *institutional rules* has two levels:

- $\bar{D}$: second-price auction
- D: DA market institution

The variable *bidders' characterization* determines the distribution functions of bidders' valuations and has two levels

- s (*symmetric case*): ex-ante identical probability distribution functions
- a (*asymmetric case*): ex-ante different probability distribution functions

For the selected variables, two different levels are chosen that produce sharply different outcomes. The variation of the single variable at different levels while the other variable stays constant leads to two different treatments. Overall, the variation of both variables results in the four treatments: $\bar{D}s$, $\bar{D}a$, Ds and Da. Figure 4.1 presents a schematic view of the four treatments.

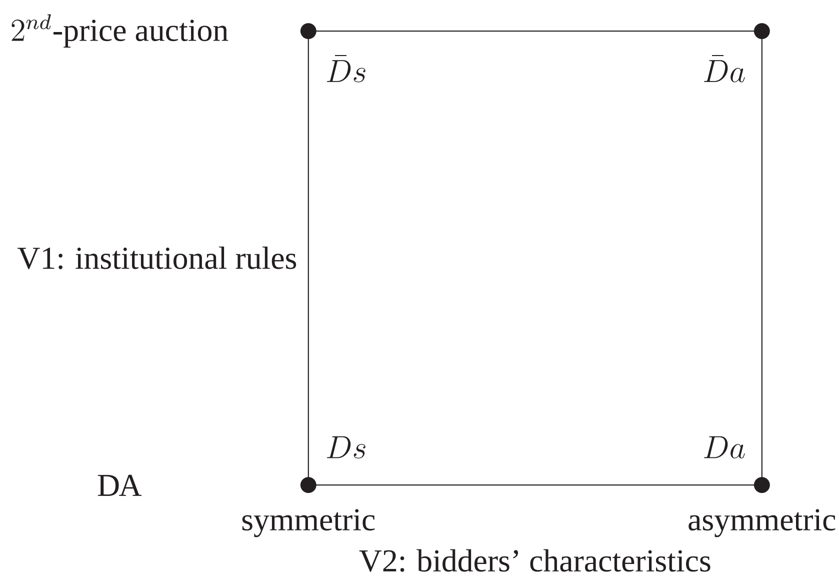


Figure 4.1: Schematic view of the treatments

Treatments with between subjects design

Treatments $\bar{D}s$ and $\bar{D}a$ present the pure second-price auction in case of symmetries (treatment $\bar{D}s$) and asymmetries (treatment $\bar{D}a$) among bidders. The variable *institutional rules* is constant. Treatments Ds and Da focus on the DA (holding *institutional rules* constant on the level

Setting	Institutional rules	# Sessions
$\bar{D}$	2^{nd}-price auction	3
D	DA market institution	3

Table 4.1: Conducted settings in the experiment

DA). Bidders participating in the DA have valuations which are realizations of random draws according to (i) the same distribution function (s) or (ii) two different distribution functions (a).

The experiment is conducted in order to analyze strategic bidding behavior of bidders and auction outcomes in the DA and the respective benchmark auction, i.e. the second-price auction, in the symmetric and asymmetric case. In the experiment, only the *institutional rules* are varied across subjects; the difference in bidders' valuations is derived by rearranging and regrouping the data. Thus, instead of conducting the four treatments, two *settings* – setting $\bar{D}$ and setting D – are played in the experiment based on the same induced valuations (cf. Table 4.1). Three sessions of the corresponding second-price auction are conducted (setting $\bar{D}$) as well as three sessions of the DA (setting D). Throughout the experiment, only the institutional rules changed while all other parameters were kept on a constant level and the environmental parameters left unchanged. The setting $\bar{D}$ constitutes the benchmark case: auctions without a discount, i.e. pure second-price auctions, are conducted and bidding behavior in these auctions is observed. In setting D, a discount is introduced by employing the DA – in each auction the discount is offered to a randomly selected bidder.
The sessions of both settings are conducted separately and each subject participates only once in the experiment.

To derive the data from all four treatments for a statistical analysis, the observed data from the experimental sessions are rearranged (cf. Section 5.3). When regrouping the observed data, the four different treatments indicated in Figure 4.1 are derived: (i) treatment $\bar{D}s$ – the second-price auction with symmetric bidders, (ii) treatment $\bar{D}a$ – the second-price auction with asymmetric bidders, (iii) Ds – the DA with symmetric bidders, and (iv) Da – the DA with asymmetric bidders. That is, both treatment variables *institutional rules* and *bidders' characterization* are artificially varied and combined in a computer simulation based on the observed data leading to the four treatments.
As benchmark auction, the second-price auction is used in the symmetric and the asymmetric case. The experimental design and the application of the regrouping method on the observed

data allows the isolated treatment effects to be derived and investigated by pairwise comparison of single treatments. In particular, the pairwise comparison of treatments $\bar{D}s$ and Ds as well as treatments $\bar{D}a$ and Da isolates the effect of the discount mechanism. Comparing treatments $\bar{D}s$ and $\bar{D}a$ as well as treatments Ds and Da focuses on the effect of the change in the bidders' characterization, i.e. switching from symmetries among bidders to asymmetries. It should be noted that since the scope of this study involves the analysis of the auction mechanisms, in particular of the discount mechanism, treatments $\bar{D}s$ and treatment Ds as well as treatment $\bar{D}a$ and treatment Da are compared with respect to bidding behavior and auction outcomes.

4.3 Design parameters

Settings $\bar{D}$ and D are conducted in several sessions with 15 subjects each. There are three sessions of setting $\bar{D}$, denoted by $\bar{D}1, \ldots, \bar{D}3$, and three sessions of setting D, denoted by $D1, \ldots, D3$ (cf. Table 4.1). Thus, a total of 90 subjects participated.

Directly at the beginning of each session the 15 subjects are grouped randomly into 5 groups with 3 subjects each. Each group participates in six consecutive auctions, also referred to as (auction) rounds. The five groups remain unchanged throughout these six rounds. The grouping is not revealed to the participants, and participants have no information about their counterparts in an auction. Since the groups themselves are independent of each other, the auctions within one round are independent of each other as well. In each of the five groups, participants are designated as player 1, player 2, and player 3. The names are randomly assigned to each group member at the beginning of the first round. Name assignments for group members are not changed within the session. In each session 30 auctions are conducted overall by five groups playing 6 auction rounds each.

In each auction conducted, a virtual seller is employed in the experimental software. The item purchased in an auction is not a real item. A virtual single-item object is offered to the participants within an auction (i.e. one group in a single round) and the item is awarded to exactly one subject within this auction.[1] Before an auction round, each participant is informed about his maximum willingness to pay for the item, i.e. his valuation of that item. In each round the 15 valuations assigned to the 15 subjects are randomly selected between $[100, 109]$ (10 integer values, low values) and $[146, 150]$ (5 integer values, high values). Each valuation is an integer number and each value from the two intervals is assigned exactly once to the

[1] In fact, the item is awarded to at most one subject within a group participating in an auction. Should none of the bidders submit a bid, none will be awarded the item.

participants per round. In essence, each valuation of the 15 valuations is assigned exactly to one participant per round; the 15 valuations are uniformly distributed over the 15 subjects per round. [2] Participants are informed that the valuations are integer numbers randomly drawn from the interval $[100, 150]$. Information about the probability distribution function is not revealed to the participants. In addition, each subject only has private information about his own valuation and no information about the other subjects' valuations.
When giving information about the probability distribution functions to the participants, the low value participants immediately recognize that they may never outbid the high value participants in either the second-price auction or in the DA. So, focussing on the strategic impact on bidders' behavior, one has to ask the questions: How will the low value bidders behave? Will they submit bids truthfully according to their dominant strategies? Or will the low-value bidders tend to behave irrationally and submit bids above or below their dominant strategies? Thus, to prevent distortions and irrational behavior, or participants from deviating from the dominant strategies and performing poorly in the auction, participants are not informed about the probability distribution functions of the valuations.

The experiment is conducted using a fictive currency, denoted by GE (Geldeinheiten), the experimental currency unit. 1 GE is equivalent to 0.10 euro or 10 GE are worth 1 euro. Note that in each round, participants are allowed to submit a bid between 0 GE and 999.99 GE- that is, under and overbidding is allowed. In an auction round, the participant to whom the item is awarded has to pay the announced price for the item. The gain between his valuation and the paid price is credited on the participant's experimental account. Participants who are not awarded the item receive a gain of zero and the total account balance is not changed.

The valuations induced to the participating subjects in each auction round are the same for each session, meaning the same table of valuations is used in all sessions. This allows a pairwise comparison of the strategies chosen by the participants in setting $\bar{D}$ and setting D. The table of valuations is represented by a (6×15)-matrix given in Table D.1 in Appendix D.

The benchmark setting $\bar{D}$ employs the pure auction mechanism of a second-price auction with a reserve price of zero, whereas setting D employs the discount auction mechanism of the DA. In each of the three sessions played from setting D, the discount is assigned to participants with different assigned names: in all auctions played in session $D1$ in each group, the discount is assigned to the participant with number 1; in all auctions conducted in session $D2$ in each group, player 2 is selected as designated bidder; and in all auctions of session $D3$ the participant with number 3 is the designated bidder within each group, receiving the

[2] As in each session in each round, the bidder's valuations are uniformly distributed the conducted settings reflect the symmetric case.

discount. In each auction played in setting D the discount is set to 20%.

By construction, in each session in setting $\bar{D}$ 15 bids are observed per round; that is, in all three sessions 45 bids are observed in a single round. Of these 45 bids, 15 bids stem from strong bidders, each bidder with a high induced valuation, and 30 bids from weak bidders, each bidder with a low induced valuation. In setting D in each round each induced valuation assigned to the subjects is played once by a designated bidder and two times by a non-designated bidder. Here, in all three sessions the bids of the designated bidder and the two non-designated bidders are based on the same assigned valuation. That is, in setting D, out of the 45 submitted bids per round, 15 bids are submitted by designated bidders – 5 strong designated bidders and 10 weak designated bidders – and 30 bids by non-designated bidders – 10 strong non-designated bidders and 20 weak non-designated bidders. Note that in all three sessions, the group number and the name of the designated bidder and the two non-designated bidders are equal. To compare the bids made in all three sessions to the benchmark auction of the pure second-price auction, setting $\bar{D}$ is conducted within three sessions.[3]

Recall that a method to rearrange the data is used to generate the four treatments $\bar{D}s$, $\bar{D}a$, Ds and Da out of the observed data of the settings $\bar{D}$ and D. More precisely, with the rearranging method applied to the observed data, the symmetric case is derived by regrouping the observed bids from the first round (or by regrouping the observed bids from all rounds). In the symmetric case the fifteen valuations of the intervals $[100, 109]$ and $[146, 150]$ are equally distributed and assigned to the subjects in a single round and a single session. The method now reassigns the observed data derived in a particular round of all three sessions in such a way that (i) bidders with low valuations and their respective bids are always grouped and (ii) bidders with high valuations and their respective bids are always grouped. Rearranging the 45 observed data leads to 15 homogenous groups, each with three bidders: 10 groups with weak bidders and 5 groups with strong bidders.[4] Thus, a symmetric uniform distribution function is artificially achieved. The asymmetric case is more complex – as in the symmetric case, a differentiation is necessary between weak bidders with assigned values derived from the interval $[100, 109]$ and strong bidders with assigned valuation derived from the interval $[146, 150]$. The 45 valuations derived from the three sessions of a particular round are assigned randomly to the 45 bidders – virtually 15 groups each with three bidders are created under the condition that in each group, one strong bidder competes with two weak bidders. Applying this method

[3]To receive the benchmark bids to the two bids of non-designated bidders in setting D, based on the same valuation in each round, it is sufficient to conduct two sessions of setting $\bar{D}$.

[4]Setting $\bar{D}$ and setting D reflect the symmetric case indirectly; the symmetry effect is strengthened by rearranging the data and regrouping the valuations and the respective bids, creating homogenous groups of weak and strong bidders.

to the observed data, the observed bid of a strong value bidder is then recombined with the observed bid of two weak bidders.
Rearranging the data has the advantage of being able to artificially generate the four treatments. Moreover, this method reduces costs, since not all treatments have to be conducted with experimental subjects in a laboratory.

Focussing on the bidding process and the auction rules except the payment rules in both settings, the underlying mechanism is a second-price auction. The second-price auction is implemented internally in the experimental software as a second-price sealed-bid auction and explained to bidders as English proxy-auction. It is known from literature that in experimental settings the second-price sealed-bid auction performs poorly, whereas bidders in an English auction behave more closely to their dominant strategies. A reason for this is that the two institutions create differences in feedback, such that bidders adopt and learn their dominant strategies differently (Harstad 2000). In both auction formats a bidder receives a negative feedback and a negative gain when his bid and the second-highest bid exceeds the bidder's valuation. However, in a second-price sealed-bid auction, bidders might overbid, win, and still make a gain, meaning that these bidders receive positive feedback. This is the case when no rival bid between the bidder's overbid and his valuation is submitted. In an English auction this is not possible; here overbidding occurs only when the price has reached a bidder's valuation and rival bidders remain in the auction. Thus, in a second-price sealed-bid auction, bidders tend to deviate from their dominant strategies and submit bids above their valuations (cf. Kagel and Roth 1995; Harstad 2000). To prevent this effect in the experiment, the implemented second-price auction is explained as an English proxy-auction. Participants are informed when entering a bidding limit that this limit is transmitted to a bidding automata that bids on behalf of the participants up to their respective bidding limit. As in a second-price sealed-bid auction participants enter only one single bid which is only visible to them. Other participants are unaware of the amount of that bid. Hence, with these rules the English proxy-auction is equivalent to the second-price auction.

Comparing the settings $\bar{D}$ and D, the experiment set-up is equal for both settings. In particular, the design parameters are the same and settings only differ in the auction institution or more specifically in the offered discount. Table 4.2 and the following list summarize the most important experiment parameters common to both settings:

- Model framework: independent private values auction environment with induced valuations.
- Bidder valuations: 15 integer values taken from the interval $[100, 109]$ and $[146, 150]$ are induced to the participants in each round. Each of the 15 integer numbers is assigned to

Experiment design parameters

sett.	♯sessions	♯bids/ session	♯auctions/ session	♯rounds/ session	♯auctions/ round	♯subjects/ treatm.	♯subjects/ session	♯subjects/ auction
$\bar{D}$	3	90	30	6	5	45	15	3
D	3	90	30	6	5	45	15	3
	♯sessions	♯bids	♯auctions			♯subjects		
Total	6	540	150			90		

sett.: setting. ♯: number of. treatm.:treatment

Table 4.2: Summary of the experiment set-up

exactly one participant per round.

- Auction institution: The bidding process is explained as an English proxy-auction with a reserve price of 0 GE and minimum bid increment of 0.01 GE. Internally a second-price sealed-bid auction is implemented in the experimental software.

- Feasible bids: All bids between 0 GE and 999.99 GE (with up to two digits) are feasible. Each participant is allowed to submit at most one sealed bid per round.

4.4 Conducting the experiment

The experiment was conducted at the experimental laboratory of the Institute of Information Systems and Management at Universität Karlsruhe (TH) from December 14^{th} to December 16^{th}, 2005. Participants were randomly selected from a database with more than 3,000 volunteers. All participants were undergraduate or graduate students mostly from the School of Economics and Business Engineering. Only a few subjects invited to participate had previously participated in a negotiation or auction experiment, and only a few participants were experienced in negotiations or auctions. None of the subjects participated repeatedly.

The experiment was conducted by computer with meet2trade and the meet2trade experimental system (cf. Section 4.5). All decisions of the participants as well as answers to questionnaires were entered into a computer terminal.

Before entering the laboratory, participants had to randomly draw a letter from a sealed envelope indicating a certain cabin seat assignment. In the laboratory, the 15 visually isolated cabins were labelled with letters from ’A’ to ’O’. Participants were seated at the computer terminal with the corresponding letter.[5] In addition, all cabins were supplied with a pencil, a calculator, the experimental instructions,[6] a sheet of paper, a feedback form, as well as a contract. Participants had the opportunity to fill out the feedback form and give remarks on the conducted experiment, the played auction mechanism, as well as the chosen strategies in the auctions. After finishing the experiment and receiving payment, participants had to sign a contract and confirm their received payments. The instructions, the sheet of paper, the feedback form as well as the contract was collected at the end by the experimenter. Throughout the experiment, communication between the subjects was not permitted. They were only allowed to privately ask the experimenter for clarifications but not for advice.

After all subjects had been seated, the instructions were read aloud to all participants. This took approximately 15 minutes. Then, each participant had to fill out a questionnaire about the rules of the experiment and the auction mechanism explained in the instructions. The questionnaire was a screen-based questionnaire and all participants had to answer the questions correctly. Fourteen questions were asked in setting $\bar{D}$ and 17 questions in setting D. It took the participants approximately 10 minutes to answer the questions. After all participants had filled out the questionnaire, the first auction round began.

All subjects played six consecutive auction rounds – there were no trial rounds. In each auction round five independent auctions were conducted at the same time by different groups of subjects. Recall that before the first round started the 15 participants were randomly assigned to one of five groups. Each group consisted of three subjects participating in the same auction. The assignment of participants to groups was fixed and did not change throughout the experiment. Furthermore, participants were numbered from 1 to 3 within each group, serving as the participants' name throughout the auction and all consecutive auction rounds. Before each auction round, participants were informed about their valuations of the item being auctioned in the current round, as well as their actual experimental account on the computer screen. This screen had to be confirmed by each participant before starting the auction of the current round.

Throughout the auction, the participant's valuation of the item was displayed on the screen. Based on his valuation, each bidder had to decide how much to bid for the item and type the value of the bid in the bidding screen. By confirming this value, the bid was submitted and entered into the experimental software. Participants received a notification of the bid submission. Additionally, at the end of the auction, participants received a notification of the auction result displayed on the screen. Information about being the winning bidder, the name

[5]Photographs of the experimental laboratory and the single cabins can be found in Section D.6, Appendix D.

[6]The experiment was carried out in German. The original instructions for all settings are provided in German in Section D.5, Appendix D.

of the winning bidder, the final price of the auction, and the price to pay in case of being the winning bidder was indicated on the screen.

In setting D subjects were also informed about whether they were the designated bidder by displaying the information ’Discount: 20%’ or whether they were a non-designated bidder by indicating the information ’Discount: no Discount’ on the bidding screen. Concerning the auction result, a designated bidder, being the winner in an auction, was informed that the price to pay for the item was a discounted price.

At the end of an auction round, participants were shown a computer screen with information about their payoff in the last round, the old experimental account balance before that round, as well as the new account balance. Moreover, the valuation of the item in the forthcoming round was displayed. Again, participants had to confirm the information by clicking the ’Confirm’ button.

Playing the six consecutive auction rounds lasted about 20 minutes. Within each auction round the time for bid submission was limited to 2 minutes.

After the six auction rounds, participants were asked to fill out a screen-based questionnaire containing 48 questions by entering the answers on the computer. The questionnaire comprised questions about the participants’ background, their behavior in conflict situations, their attitudes concerning auction systems, as well as questions on the system and user interface design. It took the participants approximately 15 minutes to answer these questions.

At the end of the experiment, subjects remained seated at their computer terminals and were then called individually to be paid privately. The instructions, the sheet of paper, the filled-out feedback form as well as the signed contract had to be given to the experimenter. The signed contract contained confirmation of subject participation in the experiment as well as reception of payment for participation.

An experimental session lasted about one hour and ten minutes. Table 4.3 summarizes the approximate duration of the different phases in an experimental session.

Overall, 6 sessions were performed with 15 participants in each session, three sessions of setting $\bar{D}$ and three sessions of setting D. The data of all conducted sessions are used for a general analysis of bidding behavior.

Table 4.2 in Section 4.3 summarizes the parameters of the sessions. Overall, 88 bids out of 90 possible bids in session $\bar{D}1$ and 89 bids out of 90 bids in session $\bar{D}2$ were observed. In session $\bar{D}3$ as well as sessions $D1, \ldots, D3$ all bidders transmitted their bids.[7] A complete

[7] When conducting the experiment, subjects did not submit bids in either session of setting $\bar{D}$. In the first session of setting $\bar{D}$ player 3 in group 2 did not submit his bid in round 1 and round 2; in the second session of setting $\bar{D}$ player 1 in group 5 did not transmit his bid in round 5. Reasons for not submitting bids are manifold: subjects might have thought too long about the amount to bid while time was running out, or they did not submit

Phases of experimental session	Approximate duration
Reading instructions	15 min
Questionnaire on instruction: 14 questions in setting $\bar{D}$ 17 questions in setting D	10 min
6 consecutive auction rounds	20 min
Questionnaire: 48 questions on background, system design, etc.	15 min
Payment of subjects	10 min
Total	1h 10min

Table 4.3: Phases of a conducted experimental session

listing of all experimental observations is provided in Section D.2, Appendix D.

In the experiment, a fictitious currency called ’Geldeinheiten (GE)’ was used. The cash rate of the GE earned by each subject was: 1 GE = 0.1 euro (or 10 GE = 1 euro). Subjects received a show-up fee of 80 GE paid on their experimental account. The average participant earnings in the experiment were 13.29 euros. In setting $\bar{D}$ the average payoff was 12.83 euros and 13.74 euros in setting D. Bidders’ earnings in setting $\bar{D}$ ranged from 3.90 to 19.40 euros and from 8.00 to 25.59 euros in setting D. In total, all participants received a positive payoff and no participant went bankrupt – in setting $\bar{D}$ only two participants suffered losses, whereas in setting D none of the participants suffered a loss. In setting $\bar{D}$ the two participants received a payoff of 6.10 and 3.90 euros, that is, both payoffs were lower than the show-up fee of 8.00 euros (80 GE).

4.5 Experimental system

The experiment was conducted with the meet2trade toolbox – a workbench for CAME (Weinhardt et al. 2005). As described earlier (cf. Section 2.5.2), the aim of CAME is to provide users with a toolbox for conducting research on electronic markets. In particular, the toolbox enables the market engineer to design and evaluate electronic markets by simulations and experiments in a structured manner.

a bid due to the client design, or they refused to submit a bid, etc.

meet2trade is a generic, flexible trading platform facilitating easy creation and automation of auction-based markets. The platform is flexible enough to host markets from a large variety of domains and support various market mechanisms. Furthermore, the platform is configurable regarding the user perspective: it meets the users' individual trading needs and supports users in selecting and configuring markets by adapting the client to their individual preferences for these tasks. Beside its flexibility and configurability, the most powerful advantage is the facilitation of designing markets by a Market Modelling Language (MML). This language was developed in order to describe electronic market parameters and enhance easy development and creation of electronic auctions (Mäkiö and Weber 2004). The innovative trading concepts offered in this system – e.g. market configuration and platform flexibility – offer a starting point for a vast area of economic research. The meet2trade system delivers not only the platform to host these concepts but also provides a toolbox for their examination. The tools offered by meet2trade consist of an experimental system (meet2trade Experimental System, MES) and an agent-based simulation environment (Agent-based Market Simulation Environment, AMASE) (cf. Kolitz and Weinhardt 2006; Czernohous 2005).

Essentially, the intention of MES is to conduct economic experiments on electronic markets on the meet2trade system instead of deploying experimental standard software. Thus, MES and meet2trade are strongly interwoven – MES is integrated in the underlying platform using components of meet2trade (Kolitz and Weinhardt 2006). Regarding the conducted experiment, the meet2trade market core was used to configure and employ the institutional rules of the DA and the corresponding second-price auction; each session of the experiment was configured, conducted and settled with the experimental system.

The workbench meet2trade follows a client server architecture with a central server. The server provides the running platform for all available markets as well as the hosting of all data (e.g. user data, account data, product information, protocol data) and data preparation. The clients connected to this central server display this data and provide an interface for submission of bids and displaying relevant information.
The meet2trade system is based on Java technology. The server uses the Enterprise Java Beans (EJB) concept. EJB is a server side component architecture for distributed computing developed by SUN Microsystems. meet2trade uses a JBoss application server and the MaxDB database system for data storage.

The server follows a 3-tier architecture and therefore consists of three layers: The *communication layer* prepares data for client presentation, provides for communication and administrates all connected clients; the *business layer* consists of a core market environment called ARTE (Auction Runtime Environment) where all auctions are run and all bids are processed;

the *database layer* encapsulates all database access and therefore provides for the logging of all data as well as the management of user and depot data. Client-server communication is carried out through the Java Messaging Service (JMS), which provides a reliable, queue-based and asynchronous means of communication. All data exchanged between client and server is encapsulated in the XML format.

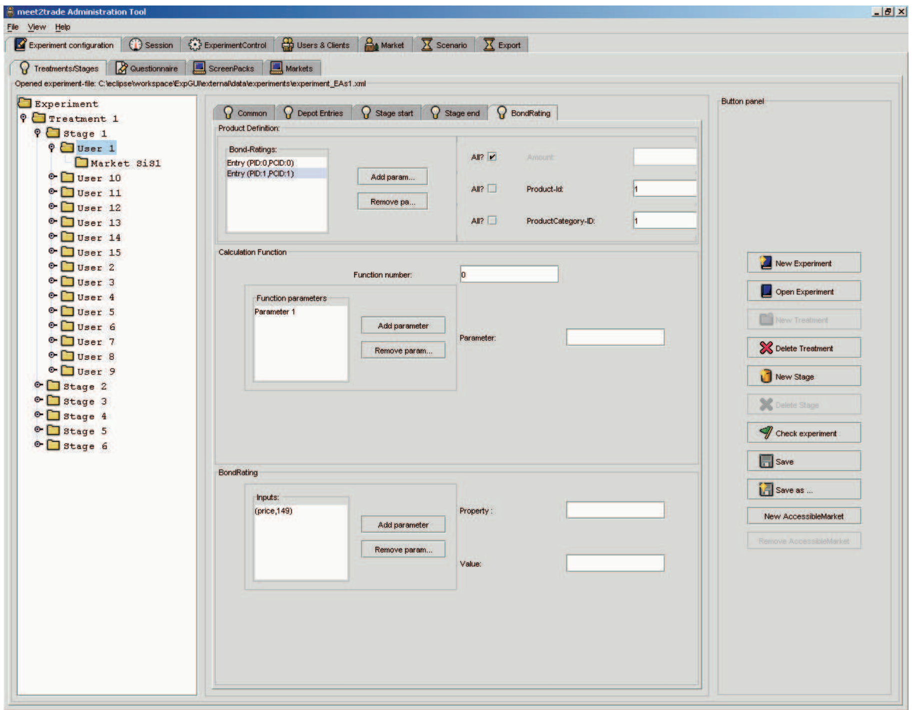


Figure 4.2: Administration tool of MES: design and configuration of a session

The experimental system is integrated into the meet2trade platform using central components of the platform such as the created and configured markets running in the system, the generic trading client running in an experimental modus, and the database to store the data. Moreover, the MES provides a graphical user interface, the administration tool as experimenter interface, for designing and configuring experiments as well as monitoring sessions during run-time. With this tool the experimenter can design and set up a session by specifying the design parameters. Figure 4.2 shows the graphical user interface of the administration tool when designing a session. The session configuration is described by XML documents (the description of the experiment, the trading clients, etc.). The XML documents are submitted to a server and stored in the database. When conducting an experimental session, the XML documents are read, analyzed and administrated by the central experimental control component. Additionally, this component has various functions such as the controlling and monitoring of the experimental sessions, the loading of markets from the database, the control over the subjects' accounts, the collection and storage of the experimental data in the database, as well as the control and administration of the bidder clients. Throughout a session the server sends information to the administration tool about the state of subscribed clients as well as the state of the session. In a session the rounds played are called stages, with each stage consisting of two phases: a pre-stage phase and a trade phase. In the pre-stage phase participants see a screen with information such as their experimental account and their payoff gained in the previous round. In the trade phase participants were shown the bidding client and asked to enter their bids.

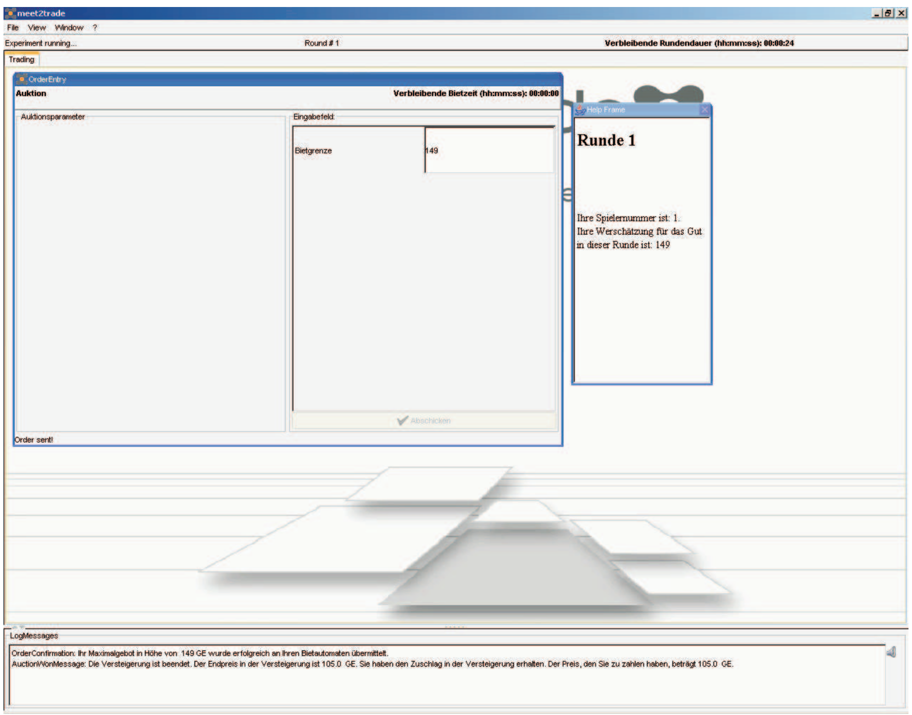


Figure 4.3: Generic bidder client – screen in the trade phase

After a session has been set up, the experimenter starts the session with the administration tool. With this tool the experimenter operates, controls and monitors the session. Each participant has to log into the system. When all participants have logged in, the experimenter receives a confirmation and the experimental system automatically sends an electronic questionnaire to all clients. After participants have successfully filled out the questionnaire, the experimenter is notified. Then, by clicking on a button, the first auction round starts. In the pre-stage phase of the auction round an information screen is displayed – this screen has to be confirmed by each participant. The trade phase then starts and participants see the bidding

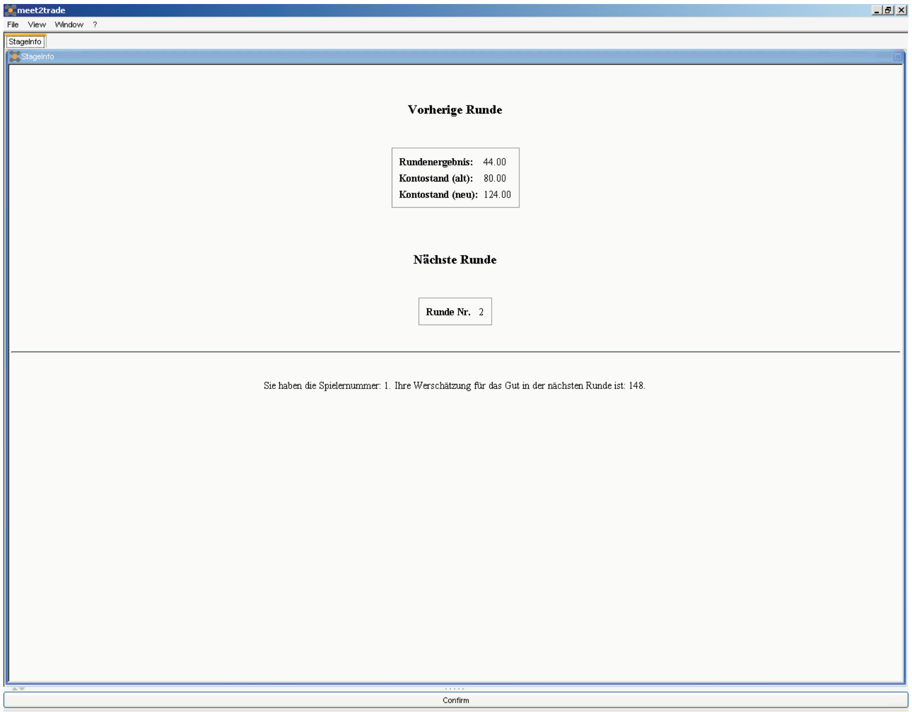


Figure 4.4: Information screen – screen in the pre-stage phase

client. Throughout the trade phase, all collected data are stored in the database. After a round is finished participants are again shown the information screen, giving them all relevant information. When this screen is confirmed, the next trade phase starts – this process continues until all rounds are conducted. After the final stage (auction round) information on the final account balance is given to the participants and an optional questionnaire may be given to the participants before ending the experiment. Both the bidding screen as well as the information screen are illustrated in Figure 4.3 and Figure 4.4.

Throughout the experiment all data are stored in the database, i.e. data on starting and ending stages, data collected from the questionnaires, data received in each auction round, etc.

4.6 Statistical analysis of the experimental data

Throughout the experiment all observed data were collected and stored to a database. Of particular interest is the bidding behavior of the participants and the auction outcomes, i.e. the seller's revenue, the winning bidder's payoff, and the social surplus, of the conducted auctions. Bidding behavior and auction outcomes are analyzed on the level of the conducted settings – aggregated over all sessions within one setting. Additionally, the study focuses on the seller's revenue (auction revenue) in the four treatments.

The data analysis is based on observations received from six sessions. For each setting, either setting $\bar{D}$ or setting D, three experimental sessions are available. In every session five groups, each with three subjects, played six consecutive auction rounds under the same rules. Overall the subjects submitted 537 bids from 540 possible bids in 180 auctions (2 settings $\times$ 3 sessions per setting $\times$ 6 auction rounds per session $\times$ 5 auctions per auction round).[8] In each setting the individual auction outcomes of 90 auctions are recorded.

Moreover, to receive the data for the four treatments, (i) solely experimental data from the first round of all sessions as well as (ii) data from all six rounds and sessions are considered. Recall that no trial rounds were conducted in the experiment. Thus, the observations from the first round are not dependent on pre-rounds and not affected by learning effects – they are independent. The 45 observations from the first round of setting $\bar{D}$, as well as the 45 observed bids from setting D, are randomly rearranged or reordered. Here, rearranging means that subjects with induced valuations and bids are randomly reassigned to the groups: in the symmetric setting homogenous groups of solely weak bidders and solely strong bidders are virtually created; in the asymmetric case one strong bidder is always virtually grouped with two weak bidders. Rearranging the data of setting $\bar{D}$ brings up the treatments $\bar{D}s$ and $\bar{D}a$, and by rearranging the data from setting D the treatments Ds and Da are virtually constructed. In particular, in setting D and thus in treatment Ds and treatment Da, the observations have to be rearranged in such a way that exactly one observation of a designated bidder is arranged in each newly created group. Additionally, a complete exploration of all experimental data is given by considering all observed bids. Based on the 270 bids in setting $\bar{D}$ – with three default-bids equal to 0 – the treatments $\bar{D}s$ and $\bar{D}a$ are virtually created as described above.[9] The same applies to the 270 observations derived from setting D leading to the treatments Ds and Da. As the experimental data includes learning effects over the rounds and bidders adapt their behavior throughout the experiment, learning their dominant strategy, the bids from a single bidder are not independent. In consideration of this effect, the data is used for the comprehensive analysis of the single treatments. Overall, when considering data from the first round, 15 groups – each with three players – are virtually created for each treatment, and when considering data from all rounds, 90 groups – each with three players – are constructed.

[8] In the first session $\bar{D}1$ player 3 in group 2 did not submit a bid in the first and second auction round. Also, in session $\bar{D}2$ player 1 in group 5 did not submit a bid in round 5. Thus, the number of observed bids was reduced from 540 to 537. For further analysis, the non-submitted bids are treated by default as 0.

[9] When rearranging the observed data from a single round, it is considered that each of the 15 valuations was induced three times in the respective round – once in each session. Thus, by creating virtually new groups in that round, only groups with mutual different valuations are created.

Various statistical tests are used to identify major characteristics of the experimental results and measure differences between settings and treatments concerning bidding behavior and auction outcomes. All statistical computations are run with the software package R – A Language and Environment (Version R.2.0.0, R Development Core Team 2006).[10] On the one hand, R is a programming language; on the other, it is a software tool for statistical analysis. The statistical functions and methods provided by R are used for analysing experimental data. In particular, the functions `shapiro.test` (Shapiro-Wilk test), `t.test` (t-test), `wilcox.test` (Wilcoxon rank sum test and Wilcoxon signed-ranks test), `ks.test` (Kolmogorov-Smirnov test), and `chisq.test` (Chi-squared test) are used in the analysis.

For testing samples on normality the *Shapiro-Wilk test* is used when the true mean or true variance of the population is unknown.

To measure differences in central tendency between two samples, different test are offered: the *t-test* and the *Wilcoxon rank sum test* (also referred to as *Mann-Whitney U test*).

The *t-test* investigates the difference between the means of two populations: it tests the hypothesis that the difference between the population means of the two samples is zero. However, the t-test assumes that the samples are drawn from a normally distributed population.

In case of distribution-free data, differences between two independent samples, i.e. the two settings or the treatments, can be measured by the *Wilcoxon rank sum test*. The test pools the two independent samples into one sample, ranks the data within the pooled sample, and computes a rank sum for each sample. Then it tests the null hypothesis that the rank sums are equal and that there is no systematic difference between the samples.

In comparing samples which can be combined by matched pairs of observed data, both independent, the *Wilcoxon matched-pairs signed-ranks test*, or *Wilcoxon signed-ranks test*, can be used, generating more powerful results. The Wilcoxon signed-ranks test calculates the differences between the pairs, ranking them from smallest to largest by absolute values. The test investigates differences in central tendency: it is used to test the null hypothesis that the population median of the paired differences between the two samples is zero.

Another test procedure to measure differences between two independent samples is the *Kolmogorov-Smirnov test* for two independent samples. The *Kolmogorov-Smirnov test* investigates differences between the distribution functions of two independent samples. meaning the cumulative frequency distribution of two independent samples are compared. If, in fact, the two samples are derived from the same population, then the distributions would be expected to be identical or rather similar to each other. If at any point the difference between

[10] See http://cran.r-project.org.

the two cumulative frequency distributions is significant, then there is a great likelihood that samples are derived from different populations.

The *chi-square goodness-of-fit test* is used to test differences in proportions of count data. It is also referred to as the chi-square test for a single sample. The procedure assigns each observation of n observations derived from a single sample to one of k categories. The hypothesis evaluates whether there is a difference between the observed frequencies of the k categories and the expected frequencies. The *chi-square test for* $r \times c$ *tables* is an extension of the chi-square goodness-of-fit test to two dimensional tables. The test procedure assumes that r independent random samples are taken from the same population distribution or from r identical population distribution and that c categories exist in each of the r independent samples. It then calculates the expected numbers in each category and compares the result to the observed number. The *chi-square test for homogeneity* evaluates whether or not the r samples are homogenous with respect to the proportions of observations in each category. That is, the procedure tests the null hypothesis, that there are no differences in proportion of the observed frequencies of the c categories and their expected frequencies for all samples – the observed frequency of a cell is equal to the expected frequency of the respective cell.

When applying the statistical tests, the null hypotheses are tested on a significance level of 5 percent ($\alpha = 0.05$) for significance and on a level of 10 percent ($\alpha = 0.1$) for weak significance. Exact probabilities are indicated by the p-value.

A comprehensive discussion of parametric and nonparametric tests for statistical analysis can be found in Spiegel (1976) or Sheskin (2004).

Chapter 5

Experimental Results

This chapter presents the experimental results. First, a detailed analysis of the bidding behavior in both settings is given in Section 5.1. Section 5.2 presents a descriptive overview of the experimental results with respect to the auction outcomes. In addition, a more detailed discussion of the significance of the observed differences in data with respect to the auction outcomes is presented. In section 5.3 the focus is set on the four treatments and a comparison of the institutional rules in the symmetric and the asymmetric cases with respect to the auction revenues. Section 5.4 summarizes the main findings. All experimental results are listed in Appendix D: the tables in Section D.2 list the observed bids, the auction revenues, and the bidders' payoffs; the tables indicating the strategic bidding behavior of subjects in each experimental session are presented in Section D.3. Additionally, the auction revenues achieved in the isolated treatments are listed in Section D.4.

5.1 Bidding behavior

One motivation for conducting the experiment was the research question whether bidders, depending on the institutional rules, behave as predicted in theory (Section 3.3.2, Proposition 3.3.1). More specifically, it is asked:

1. Do bidders in the second-price auction (setting $\bar{D}$) submit their valuations truthfully?

2. In the DA (setting D), do designated bidders submit bids above their valuations according to their dominant strategy and all non-designated bidders submit their valuations truthfully?

3. Are there any (significant) differences between the bidding behavior observed in the second-price auction (setting $\bar{D}$) and the DA (setting D)?

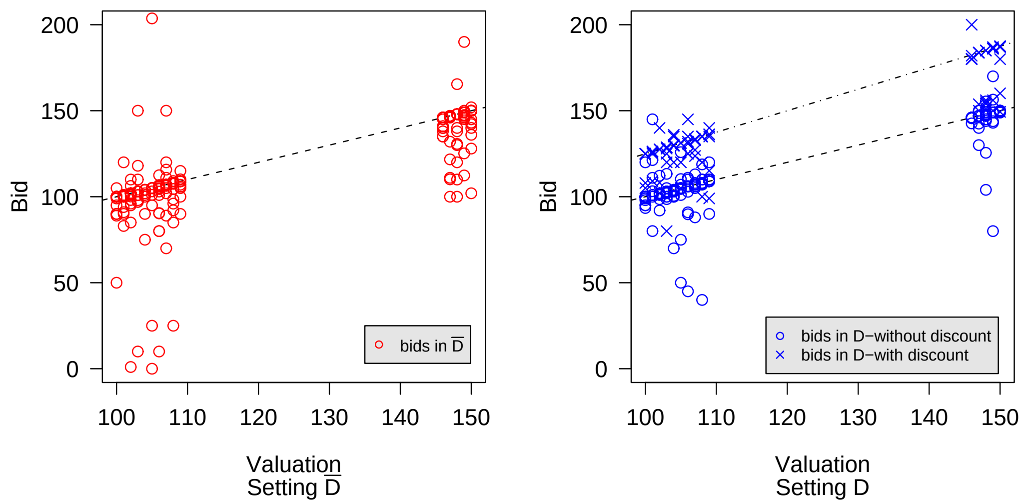


Figure 5.1: Bids in setting $\bar{D}$ and setting D

5.1.1 Observed bids

In order to answer these questions, bidders' behavior in the experiment is analyzed in the following. Figure 5.1 and Figure 5.2 display graphs of submitted bids in setting $\bar{D}$ and setting D. Recall that in setting $\bar{D}$ 267 bids are observed, with 270 bids in setting D (in each setting, three sessions with 6 rounds and 15 subjects were conducted). The first graph in Figure 5.1 indicates the bids observed in setting $\bar{D}$ and the conducted second-price auction. The plot is based on 265 out of 267 data points; two data points are not plotted since they represent outliers.[1] The dashed line in the graph indicates the dominant strategy: in the second-price auction, bidding truthfully is a dominant strategy. As the graph illustrates, bids are close to the dominant strategy; that is, many bids are on the dashed line or in an ϵ-surrounding of that line. Additionally, more data points are plotted below the dashed line than above – a general tendency for underbidding can be observed. The second graph in Figure 5.1 illustrates the 270 bids derived from the discount auction. Again, the dashed lines indicate the theoretical benchmark: (i) the upper dashed line indicates the dominant strategy of designated bidders and is the benchmark of bids with discount (ii) the lower dashed line displays the behavior of non-designated bidders in equilibrium and is the benchmark of bids without discount. Similar to the first graph, many data points in the second graph lie on the dashed lines or in an

[1] The data point (104,250) indicating a bid of 250 based on a valuation of 104 (player 2 in group 4, round 4 in session $\bar{D}$1) is not plotted, nor is the data point (148,400) (player 2 in group 1, round 3 in session $\bar{D}$).

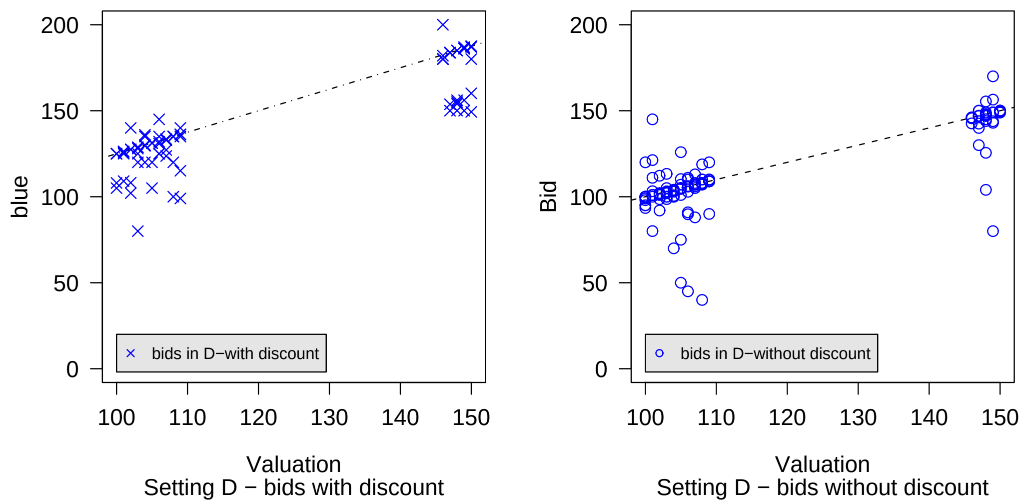


Figure 5.2: Bids in setting D – bids with and without discount

ϵ-surrounding of the lines. It seems that there is a general tendency towards underbidding. In Figure 5.2 the bids observed in the discount auction (setting D) are plotted slightly differently than presented in Figure 5.1 – in Figure 5.2 the first graph solely displays the bids of the designated bidders (90 bids) and the second graph solely the bids of the non-designated bidders (180 bids). From the first graph it can be concluded that many bids are submitted close to or below the dominant strategy. This also holds true for non-designated bidders: they generally follow their dominant strategy or submit bids close to the theoretical benchmark. However, it seems that non-designated bidders have a higher tendency for overbidding than designated bidders. Non-designated bidders are more likely to submit a bid above the theoretical benchmark.

The observed bids plotted in Figures 5.1 and 5.2 can be summarized by indicating the average bid of each setting, the average dominant strategy as well as the average deviation from the dominant strategy (Table 5.1). The average deviation simply calculates the average mean of the differences between the observed bids and the respective dominant strategies in a particular setting. Independent of the conducted setting, i.e. the employed institutional rules, the average submitted bid is below the average theoretical benchmark; in all settings a trend towards underbidding is observed. Note that setting D can be split into two mutual different settings: sub-setting D_{disc} which considers only bids of designated bidders (bids with discount, subscript $disc$ denotes 'discount') and sub-setting $D_{\overline{disc}}$ which considers only bids of non-designated bidders (bids without discount, subscript $\overline{disc}$ denotes 'no discount').

Setting	Mean dominant strategy	Mean bid	Mean deviation
$\bar{D}$	119.00	115.40	-3.60
D	128.92	125.19	-3.73
D_{disc}	148.75	141.69	-7.06
$D_{\overline{disc}}$	119.00	116.95	-2.05

Table 5.1: Average bids in setting $\bar{D}$ and setting D

Attention is drawn to the deviation of the bids from the dominant strategy measured on average, i.e. the difference between the average bid and the average dominant strategy. For example, in the DA the average deviation (-3.73) from the dominant strategy is slightly higher than in the second-price auction (-3.60). Furthermore, Table 5.2 summarizes the mean, the standard deviation, the median as well as the minimum and maximum of all bids within a particular setting. The values derived from theory as well as those observed in the experiment are indicated.

Setting	Mean bid	Std. dev	Median	Min	Max
Theory					
$\bar{D}$	119	20.7	107	100	150
D	128.9	26.6	127.5	100	187.5
D_{disc}	148.8	20.8	133.8	125	187.5
$D_{\overline{disc}}$	119	20.7	107	100	150
Experiment					
$\bar{D}$	115.4	33.3	106.3	0	400
D	125.2	27.3	124.3	40	200
D_{disc}	141.7	26.6	133.0	80	200
$D_{\overline{disc}}$	116.9	23.6	107.4	40	170

Table 5.2: Setting $\bar{D}$ and setting D – summary of bids (theoretical benchmark and experiment)

In the following, it is interesting to analyze the question whether the differences between bids submitted in the different settings are significant. Intuitively, it seems that the bids submitted in the discount auction are slightly higher than those in the second-price auction. This is due to the institutional rule of the discount, enabling designated bidders in equilibrium to bid above their valuations. Moreover, since rational bidders follow the same dominant strategy in setting $\bar{D}$ and setting $D_{\overline{disc}}$, the mean, the standard deviation, the median as well as the minimum and maximum of all bids predicted from theory are equal (Table 5.2). Nevertheless, a slight difference between the observed bids in the second-price auction and the respective bids in the discount auction can be measured in the experiment. To test differences in central tendency between the (sub-)settings, the Wilcoxon signed-ranks test (WSR, matched-pairs) is applied. The results of the WSR test procedure are summarized in Table 5.3.[2] A pairwise comparison of the settings yields that the differences in central tendency between the mutual different settings are significant. An extraordinary result is that the effect of the discount rule is so strong that when bids with discount (subsetting D_{disc}) are compared to bids without discount (subsetting $D_{\overline{disc}}$), a significant difference can be observed. In the DA the observed bids with discount are significantly higher than the observed bids without discount; in addition, the observed bids of designated bidders in setting D are greater than those submitted in setting $\bar{D}$ (WSR, one-sided, with p-value < 0.001). Surprisingly, a significant difference is observed when comparing the second-price auction bids to the bids without discount in the discount auction (WSR, two-sided, p-value < 0.001).[3] In fact, a general tendency towards bids without discount being higher than the bids in the second-price auction can be observed. This is astonishing, since bidders in the second-price auction follow the same dominant strategy as non-designated bidders in the discount auction. Thus, the deviations of bids from dominant strategy in the different settings must be analyzed more thoroughly.

Setting	Wilcoxon signed-ranks test (matched-pairs)	Interpretation	♯ pairs
$\bar{D}$ vs. D	Hypothesis $H_0 : b_{\bar{D}} \geq b_D$ $V = 5671.5$, p-value < 0.001	bids in $\bar{D}$ are lower than bids in D	267
$\bar{D}$ vs. D_{disc}	Hypothesis $H_0 : b_{\bar{D}} \geq b_{D_{disc}}$ $V = 301$, p-value < 0.001	bids in $\bar{D}$ are lower than bids with discount in D	90
$\bar{D}$ vs. $D_{\overline{disc}}$	Hypothesis $H_0 : b_{\bar{D}} = b_{D_{\overline{disc}}}$ $V = 3590.5$, p-value < 0.001	bids in $\bar{D}$ differ from bids without discount in D	177
$D_{\overline{disc}}$ vs. D_{disc}	Hypothesis $H_0 : b_{D_{\overline{disc}}} \geq b_{D_{disc}}$ V = 71, p-value < 0.001	in D bids without discount are lower than bids with discount	90

Table 5.3: Setting $\bar{D}$ and setting D – comparison of observed bids

When distinguishing the bids of designated bidders (setting D_{disc}) from the bids of non-

[2]To compare the 180 bids without discount submitted in sub-setting $D_{\overline{disc}}$ to the 90 bids with discount submitted in sub-setting D_{disc} by the Wilcoxon signed-ranks test (matched-pairs) procedure, a single bid with discount is paired with the average of two bids without discount based on the same induced valuation in the same group and round. Thus, the WSR is applied to 90 matched pairs.

[3]In fact, the bids submitted in the second-price auction are lower than the respective bids without discount in the second-price auction. This difference is significant. Applying the WSR (matched-pairs, one-sided) to test whether the null hypothesis $H_0 : b_{\bar{D}} \geq b_{D_{\overline{disc}}}$ can be rejected results in $V = 3590.5$, p-value < 0.001.

designated bidders (setting $D_{\overline{disc}}$), it can be observed that designated bidders deviate significantly more from the theoretical benchmark (-7.06) than non-designated bidders (-2.05). Moreover, little difference between the bidding behavior of non-designated bidders in setting D (-2.05) and that of bidders in setting $\bar{D}$ (-3.60) can be measured. Note that for both groups, truthful bidding is the dominant strategy. Thus, it has to be clarified whether the deviations of bids from dominant strategies are significant as well as whether these differences occur due to noise caused by the additional discount rule.

5.1.2 Deviation of bids from dominant strategy

In this section the performance of bids in regards to the dominant strategy is measured. Therefore, first the absolute and then the relative deviation of bids from the respective dominant strategy is calculated and categorized. Starting with the absolute deviation of bids from the dominant strategy, the difference $b-b_d$ between a bid b and the respective dominant strategy b_d is classified in five categories.[4] Table 5.4 presents the frequencies of differences belonging to one category: category $(-\infty, -1)$ and category $[-1, 0)$ measure the frequencies of underbidding, category 0 counts the number of bids equal to the theoretical benchmark, while category $(0, 1]$ and category $(1, \infty)$ comprise the bids above the respective dominant strategy, i.e. the frequencies of overbidding. At the same time, category $[-1, 0)$ and category $(0, 1]$ indicate bids close to the dominant strategy. That is, these categories as well as category 0 represent the frequencies of bids which are close to or equal to the dominant strategy. Again, as in Table 5.1 the two settings – setting $\bar{D}$ and setting D – as well as the two sub-settings – sub-setting D_{disc} and sub-setting $D_{\overline{disc}}$ – are distinguished, and for each (sub-)setting the frequencies of bids falling in one of the categories are indicated. In setting $\bar{D}$ and D the frequencies of bids following the dominant strategy are approximately equal: the number of rational bids in setting $\bar{D}$ (91 bids or 34.08%) is slightly lower than the number of rational bids in setting D (98 bids or 36.30%). Also, the number of bids close to the dominant strategy falling in category $[-1, 0)$ and $(0, 1]$ in setting $\bar{D}$ (67 bids or 25.09%) is slightly lower than in setting D (69 bids or 25.56%). Focussing on the average bid of each bidder, in setting $\bar{D}$ 11 bidders on average behave according to the dominant strategy, while in setting D 9 bidders on average played the dominant strategy. However, there are more bidders who submitted a bid below the dominant strategy than above. In setting $\bar{D}$ 29 out of 45 bidders on average follow the strategy of

[4] In setting $\bar{D}$ the difference between a bid b and the theoretical benchmark v equals $b - v$ with v being the induced valuation. In setting D the dominant strategy of a designated bidder is to bid $\delta v = \frac{1}{1-d} v = \frac{1}{1-0.2} v$, whereas for a non-designated bidder it is to submit his valuation v. Thus, for a designated bidder the difference between bid and dominant strategy equals $b - \delta v$ and $b - v$ for a non-designated bidder.

underbidding and only 5 bidders the strategy of overbidding; in setting D the proportion of bidders is very similar – out of 45 bidders, 30 bidders tend to underbid while only 6 bidders overbid. These numbers address the first and second questions which were presented at the beginning of the section. The trend of underbidding observed in Figure 5.1 and in Table 5.1 can be confirmed.

To test whether the differences between observed bids and the theoretical benchmark in setting $\bar{D}$ and setting D are homogeneous with respect to the proportions of observations in each of the five categories, the chi-square test for homogeneity is applied (Table 5.4). The null hypothesis that in the underlying populations the samples represent, all of the proportions in the same category are equal, cannot be rejected (chi-square test for homogeneity with a p-value equal to 0.501).

Setting	Underbidding		Dominant Strategy	Overbidding		Total
	$(-\infty, -1)$	$[-1, 0)$	0	$(0, 1]$	$(1, \infty)$	
$\bar{D}$	88 (32.96%)	49 (18.35%)	91 (34.08%)	18 (6.74%)	21 (7.87%)	267 (100%)
D	73 (27.04%)	52 (19.26%)	98 (36.30%)	17 (6.30%)	30 (11.11%)	270 (100%)
Total	161 (29.98%)	101 (18.81%)	189 (35.20%)	35 (6.52%)	51 (9.50%)	537 (100%)
	$\chi^2 = 3.35$		$df = 4$	p-value = 0.501		
D_{disc}	37 (41.11%)	18 (20.00%)	24 (26.67%)	4 (4.44%)	7 (7.78%)	90 (100%)
$D_{\overline{disc}}$	36 (20.00%)	34 (18.89%)	74 (41.11%)	13 (7.22%)	23 (12.78%)	180 (100%)
Total	73 (27.04%)	52 (19.26%)	98 (36.30%)	17 (6.30%)	30 (11.11%)	270 (100%)
	$\chi^2 = 15.4631$		$df = 4$	p-value= 0.004		

Table 5.4: Setting $\bar{D}$ and setting D – deviation of bids from dominant strategy

In order to measure the deviations from the dominant strategy within setting D the observations of the sub-settings D_{disc} and $D_{\overline{disc}}$ are classified according to the five categories: the differences between observed bids and the respective dominant strategies of designated bidders and of non-designated bidders are assigned to one of the categories. It is surprising that only 24 bids (26.67%) from designated bidders equal the predictions from theory, whereas 74 bids (41.11%) from non-designated bidders are in accordance with the dominant strategy. Moreover, on average only 1 designated bidder out of 15 followed the dominant strategy over all rounds in setting D, whereas 8 non-designated bidders (out of 30 non-designated bidders) on average submitted their valuations truthfully over all rounds and thus followed the dominant strategy. However, in both settings a trend towards underbidding is clearly visible: 61.11% (55 bids) of all bids from designated bidders are below the dominant strategy, while 12.22%

(11 bids) are above it. The same holds true for bids without discount observed in setting D: 38.89% (70 bids) of all bids without discount are below the respective valuation, while 20% (36 bids) are above the respective valuation. That is, a significant difference between the bids of designated bidders and non-designated bidders in setting D is apparent (chi-squared test for homogeneity with a p-value equal to 0.004).[5] A direct comparison of bidders with and without assigned discount leads to the conclusion that bidders with discount show greater activity of underbidding, whereas bidders without discount show greater activity of overbidding.

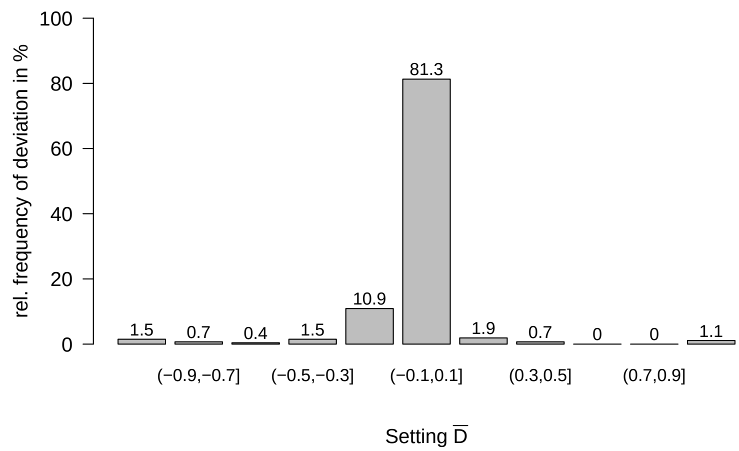
The question arises whether this effect stems from the institutional rule of the discount, meaning that for example bidders without discount consciously submit higher bids, either to increase their chances of winning the auction or to increase the price the winning bidder must pay in that auction. To gain a deeper insight into this observed effect, the relative deviation r_d of a bid from the dominant strategy is introduced as a measure:

$$r_d = \frac{b - b_d}{b_d}$$

where b_d is the dominant strategy based on the respective induced valuation. In case of setting $\bar{D}$ and sub-setting $D_{\overline{disc}}$ b_d equals the induced valuation v on which the submitted bid b is based; in case of setting D_{disc}, the subset of all bids in setting D submitted by a designated bidder, b_d equals $\frac{1}{1-d}v = \frac{1}{0.8}v = 1.25\ v$ with discount $d = 20\%$.

Figure 5.3 displays the frequencies of the relative deviations in percent of the bids: the first graph plots the frequencies counted in setting $\bar{D}$ while the second plots the frequencies derived from setting D. For all bids, the relative deviation $r_d = \frac{b-b_d}{b_d}$ is calculated and eleven categories are considered in the first step. In both graphs the x-axis denotes the eleven categories and the y-axis the relative deviation in percent. The eleven categories range from the categories $(-\infty, -0.9], (-0.9, -0.7], \ldots, (-0.1, 0.1], \ldots, (0.7, 0.9], (0.9, \infty)$. Each category has an interval size of 0.2. For example, taking the bids observed in the second-price auction, it is observed that 81.3% of all 267 submitted bids deviate (relative) from the dominant strategy up to ± 10%. More specifically, 81.3% of all bids in the second-price auction have a relative deviation from the dominant strategy ranging between -10% and 10% (Figure 5.3, graph 'Setting $\bar{D}$'). Furthermore, it is observed that about 11% of the bids in both settings fall in the category $(-0.3, -0.1]$, meaning that the relative deviation of these bids from the dominant strategies ranges from -30% to -10%.

[5] In applying the chi-squared test for homogeneity to the frequencies of observed bids in the second-price auction (setting $\bar{D}$) as well as to the respective frequencies of the observed bids without discount (setting $D_{\overline{disc}}$), the difference between both distributions is then significant. The chi-squared test results in $\chi^2 = 10.6363, df = 2$ and p-value =0.031. Thus, the observed behavior of bidders towards the dominant strategy differs significantly for both settings – setting $\bar{D}$ and setting $D_{\overline{disc}}$, although the dominant strategy is the same.



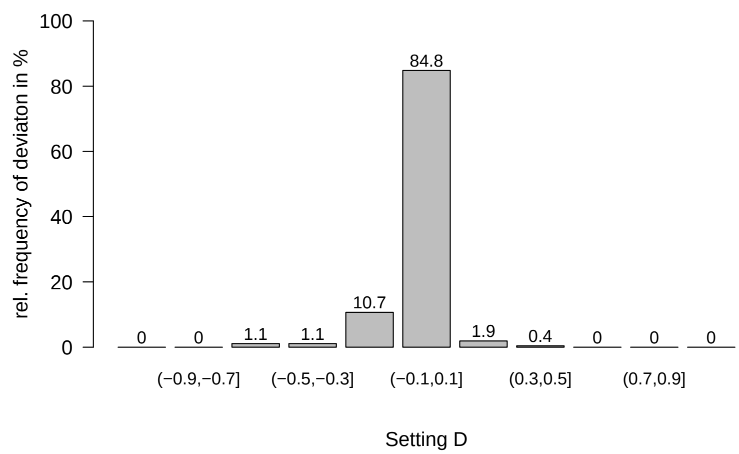


Figure 5.3: Relative deviation of bids from dominant strategy in setting $\bar{D}$ and setting D

Testing the relative deviations in setting $\bar{D}$ and setting D on homogeneity, no significant difference between the underlying population of the samples can be found.[6]

Distinguishing the bids observed in setting D – bids with and without an assigned discount – it can in fact be observed that in the category $(-0.3, -0.1]$ (a relative deviation of -30% to -10% from the dominant strategy) 22.2% of the bids with discount are identified, whereas only 5% of the bids without discount fall in this category (Figure 5.4). Hence, this category reflects that designated bidders submit their valuations instead of submitting $\frac{1}{1-d}v$ ac-

[6]The Wilcoxon signed-ranks test (WSR, matched-pairs) is applied to the distribution of the relative deviations in the eleven categories. There is no indication of a significant difference in central tendency between the distribution of the frequencies of relative deviations in setting $\bar{D}$ and the respective distribution of frequencies of relative deviations in setting D (WSR, two-sided, with V = 17.5, p-value = 0.618, n=270).

cording to their dominant strategy. Reasons for the increased activity from designated bidders submitting their valuation instead of their dominant strategy are manifold: intuitively one reason might be that designated bidders with an assigned discount are not aware of this discount and do not include it when calculating their dominant strategy, or it could be that calculating the dominant strategy is too complex. For example, being unaware of the discount might be due to the screen design of the bidding screen employed in the experimental system.

It is interesting to note that the frequencies of relative deviations between 10% and 30% from the dominant strategy differ between the settings: for bids without discount the frequency in that category is higher than the frequency of bids without discount in the same category. This fact is demonstrated in Figure 5.4.

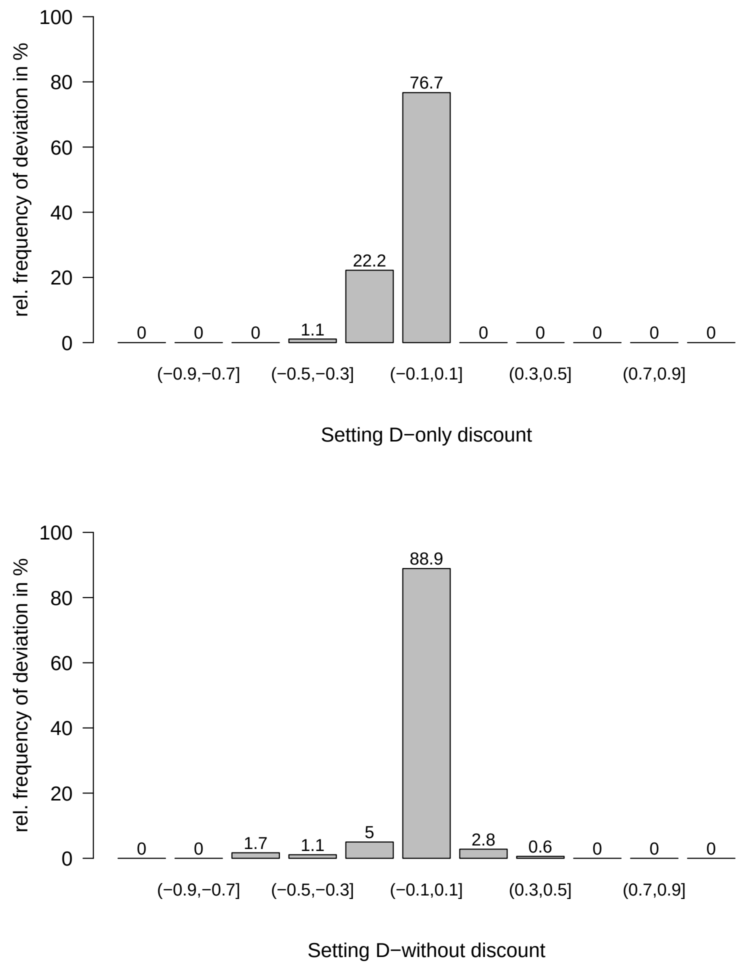


Figure 5.4: Relative deviation of bids from dominant strategy in setting D – bids with and without discount

In particular, 2.8% of the bids without discount fall in the category $(0.1, 0.3]$ while none of the bids with discount belong to that category. One can conclude that this effect is caused by the discount: bidders without discount tend to overbid and submit bids close to $\frac{1}{1-d}v$ either to increase their probability of winning the auction or increase the second-highest bid and thereby the price the winning bidder must pay.

Figure 5.3 and Figure 5.4 strengthen the presumption that the discount directly affects bidding behavior: although at a first glance the difference between the bidding behavior in regard to the dominant strategies in setting $\bar{D}$ and setting D might be neglected, a more thorough examination indicates a significant difference concerning bidding behavior (cf. Table 5.3 and Table 5.4).

In the following the eleven categories indicated in Figures 5.3 and 5.4 are condensed into five categories. The aggregated level of the relative deviations of bids from the dominant strategy in setting $\bar{D}$ and D measured by $r_d = \frac{b-b_d}{b_d}$ is summarized in Table 5.5. The five categories represent ϵ-surroundings of the dominant strategy (category $0 \pm \epsilon$), the relative deviation of -20% from the dominant strategy (category $-0.2 \pm \epsilon$), the relative deviation of 25% from the dominant strategy (category $0.25 \pm \epsilon$), as well as two categories comprising all negative and positive deviations from dominant strategies of bids which have not been sorted into one of the first three categories. In the following ϵ is set to 0.1. For example, category $-0.2 \pm \epsilon$ comprises all bids with a relative deviation of the dominant strategy from a minimum of -30% to a maximum of -10%. The ϵ-surrounding of 0 is of particular interest: for example, in the second-price auction a high frequency in this category is expected, since bidders in setting $\bar{D}$ have the dominant strategy to submit their valuation truthfully. Additionally, in setting D a high frequency of relative deviations close to -0.2 in case of designated bidders is assumed, as some of these bidders have not included the discount in calculating the dominant strategy. Further, in setting D a high frequency of relative deviations close to +0.25 in case of non-designated bidders is also assumed, as some of these bidders include the discount assigned to rival bidders in their strategic bidding behavior.

According to Table 5.5 217 (81.27%) bids submitted in the second-price auction (setting $\bar{D}$), 69 (76.67%) bids with discount (subsetting D_{disc}) and 160 (88.89%) bids without discount (subsetting $D_{\overline{disc}}$) fall in the category $0 \pm \epsilon$. Furthermore, it is apparent that the frequency of relative deviations in the categories 'underbidding' is greater than in the categories 'overbidding'. In particular, designated bidders show a high frequency of submitting bids close to their induced valuation instead of submitting a bid close to their dominant strategy (22.22%). The frequency of this category is much higher compared to the respective frequency of the same category measured in setting $\bar{D}$ or sub-setting $D_{\overline{disc}}$. Moreover, overbidding in the discount auction is only observed for non-designated bidders: the observed frequency of bids falling into the category $0.25 \pm \epsilon$ equals 1.67% and the frequency of bids falling into the category 'others' (above the dominant strategy) also equals 1.67%. Applying the chi-square test for homogeneity indicates that the distribution functions of the underlying populations the samples represent are not equal for at least one category (chi-square test for homogeneity with $\chi^2 = 24.9358$ and p-value=0.001).[7] The difference between the relative deviations belonging to the five categories is significant.

Setting	Underbidding		Dominant Strategy	Overbidding		Total
	others	$-0.2 \pm \epsilon$	$0 \pm \epsilon$	$0.25 \pm \epsilon$	others	
			$\epsilon = 0.1$			
$\bar{D}$	11	29	217	2	8	267
	4.12%	10.86%	81.27%	0.75%	3.00%	100%
D_{disc}	1	20	69	0	0	90
	1.11%	22.22%	76.67%	0.00%	0.00%	100%
$D_{\overline{disc}}$	5	9	160	3	3	180
	2.78%	5.00%	88.89%	1.67%	1.67%	100%
	$\chi^2 = 24.9358$		$df = 8$	p-value=0.002		

Table 5.5: Setting $\bar{D}$ and setting D – relative deviation of bids from dominant strategy

Recall that the induced valuations as well as the sequence for both settings are identical. Thus, the paired differences of the relative deviations between the settings can be considered. In applying the Wilcoxon signed-ranks test (matched-pairs) to the data, significant differences between the central tendency of the relative deviations can be measured. Table 5.6 displays the results of the WSR. Significant differences between the relative deviations of bids from the dominant strategy between two settings are measured between setting $\bar{D}$ and setting D (two-sided, significant with p-value of 0.042), setting $\bar{D}$ and sub-setting $D_{\overline{disc}}$ (two-sided, significant with p-value < 0.001) as well as sub-setting $D_{\overline{disc}}$ and sub-setting D_{disc} (two-sided, significant with p-value $= 0.013$). The following results can be summarized: (i) the relative deviations of bids from dominant strategy in the second-price auction differ significantly from

[7] Because the chi-square test is not strong enough for the frequencies given in Table 5.5 (some frequencies are ≤ 2), the p-value is simulated based on 10^5 replicates: $\chi^2 = 24.9358$, p-value=0.001. Furthermore, applying the chi-square for homogeneity to the bids observed in setting D_{disc} and $D_{\overline{disc}}$ results in χ^2=21.3757 and a p-value < 0.001, simulation based on 10^5 replicates. Thus, the distribution functions of the underlying populations are not equal for at least one category. Comparing the distribution functions of the bids in settings $\bar{D}$ and $D_{\overline{disc}}$, the null hypothesis cannot be rejected. The differences between the frequencies are insignificant for all categories: $\chi^2 = 7.2072$, p-value = 0.122 (simulated with 10^5 replicates.)

the respective relative deviations in the discount auction; (ii) the difference between the relative deviations measured in the second-price auction and the respective relative deviations in the discount auctions are significant, whereby only non-discounted bids are considered; and (iii) comparing the relative deviations of bids from dominant strategy in setting D, the difference between relative deviations of bids with and without discount is significant. In particular conclusion (ii) is interesting, since bids in both settings follow the same dominant strategy.

Setting	Wilcoxon signed-ranks test (matched-pairs)	Interpretation	♯ pairs
$\bar{D}$ vs. D	Hypothesis $H_0 : r_{d\bar{D}} = r_{dD}$ $V = 11021$, p-value = 0.042	relative deviations from dominant strategies in $\bar{D}$ and in D differ	267
$\bar{D}$ vs. D_{disc}	Hypothesis $H_0 : r_{d\bar{D}} = r_{dD_{disc}}$ V = 1858.5, p-value = 0.352	–	90
$\bar{D}$ vs. $D_{\overline{disc}}$	Hypothesis $H_0 : r_{d\bar{D}} = r_{dD_{\overline{disc}}}$ V = 3629.5, p-value < 0.001	relative deviations from dominant strategies in $\bar{D}$ and in $D_{\overline{disc}}$ differ	180
$D_{\overline{disc}}$ vs. D_{disc}	Hypothesis $H_0 : r_{dD_{\overline{disc}}} = r_{dD_{disc}}$ $V = 2397$, p-value=0.013	relative deviations from dominant strategies in $D_{\overline{disc}}$ and in D_{disc} differ	90

Table 5.6: Setting $\bar{D}$ and setting D – comparison of relative deviation of bids from dominant strategy

Obviously, together with the results indicated in Table 5.4 and Table 5.5, bidders in the second-price auction have a greater difficulty in playing their dominant strategy than non-designated bidders in the discount auction. Note that because the discount is an additional rule which causes the auction mechanism to become more complex, it is expected that bidders in the discount auction are likely to have more problems in playing the dominant strategy. It seems that now although complexity has increased, non-designated bidders in the discount auction perform better than bidders in the pure second-price auction. One explanation of this phenomenon might be that non-designated bidders have to think more about their dominant strategy due to the additional discount rule. In fact, the discount rule does not affect the strategic behavior in equilibrium of non-designated bidders (compared to the strategic behavior in equilibrium in the second-price auction). However, the rule forces non-designated bidders to think harder about what to bid in the discount auction. Thus, non-designated bidders perform better than bidders in the pure second-price auction.

5.1.3 Bidding behavior over auction rounds

The following discusses the bidding behavior of subjects throughout the conducted auction rounds in the experiment. Therefore, learning effects as well as adaptations in bidding behavior over rounds are analyzed. In this context, the following addresses whether subjects learn their dominant strategy throughout the conducted auction rounds, how well they adapt their bidding behavior towards the dominant strategy, and how fast they learn. In order to shed light on these issues, the average bid per round, the deviation of the average bid from the theoretical benchmark over the six conducted auction rounds, as well as the relative deviation of the average bid per round from the theoretical benchmark are analyzed. Figures 5.5 – 5.6 present the average bids per round as well as the average deviations of bids from dominant strategies per round observed in the played second-price auction (setting $\bar{D}$) and the employed discount auction (setting D). The graphs plotted in these figures present a dynamic perspective on the average bids and deviations over rounds, focussing on learning effects of bidders over the six auction rounds. Additionally, the data points plotted in the graphs are summarized in Table 5.7. Table 5.8 indicates the relative deviations ("dev.") of the average bids ("bids") from the dominant strategy per round for each setting.

Round	$\bar{D}$		D		D_{disc}		$D_{\overline{disc}}$	
	bid	dev.	bid	dev.	bid	dev.	bid	dev.
1	105.96	-12.37	118.93	-9.99	137.38	-11.37	109.71	-9.29
2	107.74	-10.55	124.58	-4.33	137.16	-11.59	118.30	-0.70
3	119.51	0.51	126.40	-2.52	143.05	-5.70	118.08	-0.92
4	119.35	0.35	125.71	-3.21	142.32	-6.43	117.41	-1.59
5	121.06	1.78	126.84	-2.08	143.62	-5.13	118.44	-0.56
6	118.19	-0.81	128.69	-0.23	146.61	-2.14	119.73	0.73
mean	115.40	-3.60	125.19	-3.73	141.69	-7.06	116.95	-2.05

Table 5.7: Setting $\bar{D}$ and setting D – average bid and average deviation of bids from dominant strategy by rounds

In particular Table 5.8 addresses the questions how well and how fast subjects learn and adapt their behavior towards the dominant strategy throughout the conducted auction rounds. It is quite obvious that, independent of the setting and the conducted auction mechanism, bidders have a general tendency to underbid in the first two rounds: this tendency is greater in the first than in the second round. However, a general trend towards underbidding cannot be identified in the second-price auction. With an increasing number of rounds the bidding behavior converges towards the predicted bidding behavior, and on average, bidders are close to the dominant strategy. In the discount auction, the trend for underbidding is apparent; even here, a clear convergence of the average bids towards the dominant strategy can be observed (for an increasing number of conducted auction rounds).

Round	$\bar{D}$ rel. dev.	D rel. dev.	D_{disc} rel. dev.	$D_{\overline{disc}}$ rel. dev.
1	-10.39%	-7.75%	-7.64%	-7.81%
2	-8.87%	-3.36%	-7.79%	-0.59%
3	0.43%	-1.95%	-3.83%	-0.77%
4	0.29%	-2.49%	-4.32%	-1.34%
5	1.50%	-1.61%	-3.45%	-0.47%
6	-0.68%	-0.18%	-1.44%	0.61%
mean	-2.95%	-2.89%	-4.75%	-1.73%
std. dev.	5.24%	2.60%	2.50%	3.05%

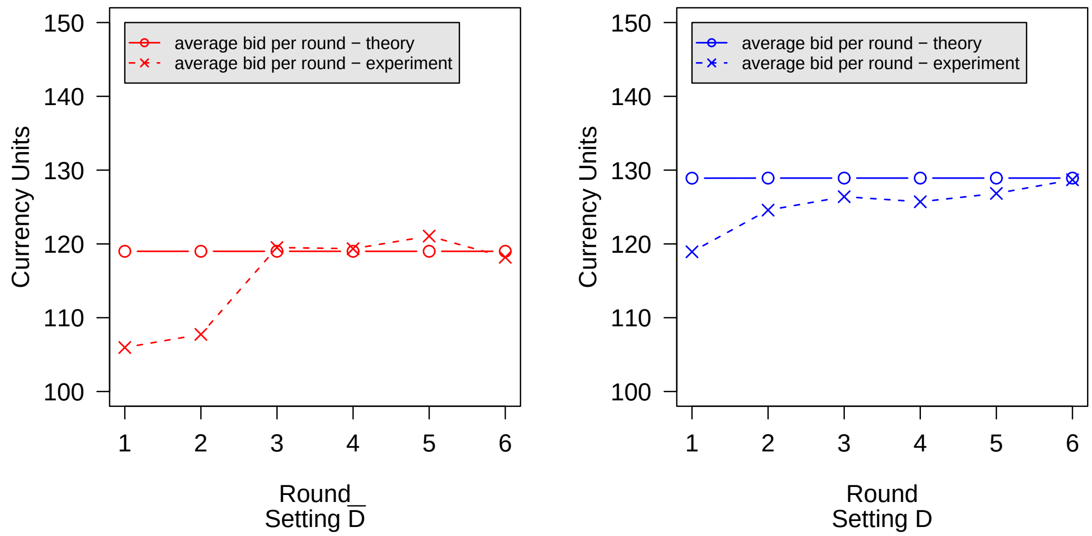
Table 5.8: Setting $\bar{D}$ and setting D – relative deviation of bids from dominant strategy by rounds

From the literature and experimental investigations, it is known that in the second-price (sealed-bid) auction, bidders tend to deviate from their dominant strategies and submit bids above their valuations (cf. Harstad 2000; Kagel and Roth 1995). That is, overbidding is observed in the second-price auction, whereas in the English auction this happens only rarely. Additionally, in comparing the performance of the second-price auction to an English auction, it is known that the second-price auction performs poorly and bidders in an English auction behave closer to their dominant strategy. Recall, that the underlying auction mechanism of the second-price auction employed in the conducted experiment was explained to the experimental subjects as an English-proxy auction (Chapter 4, Section 4.3). As displayed in the figures, subjects in the second-price auction (setting $\bar{D}$) learn their dominant strategy faster than subjects in the discount auction (setting D). From rounds 3 to 6 the relative deviation of bids from the dominant strategy in setting $\bar{D}$ is close to the dominant strategy ranging between -0.68% and 1.5%, whereas in setting D the respective relative deviation ranges between -2.49% and -0.18%. In the very last round, bids submitted to the discount auction are on average closer to the theoretical benchmark (-0.18%) than those submitted in the second-price auction (-0.68%).[8] In looking more closely at the behavior of subjects over rounds in

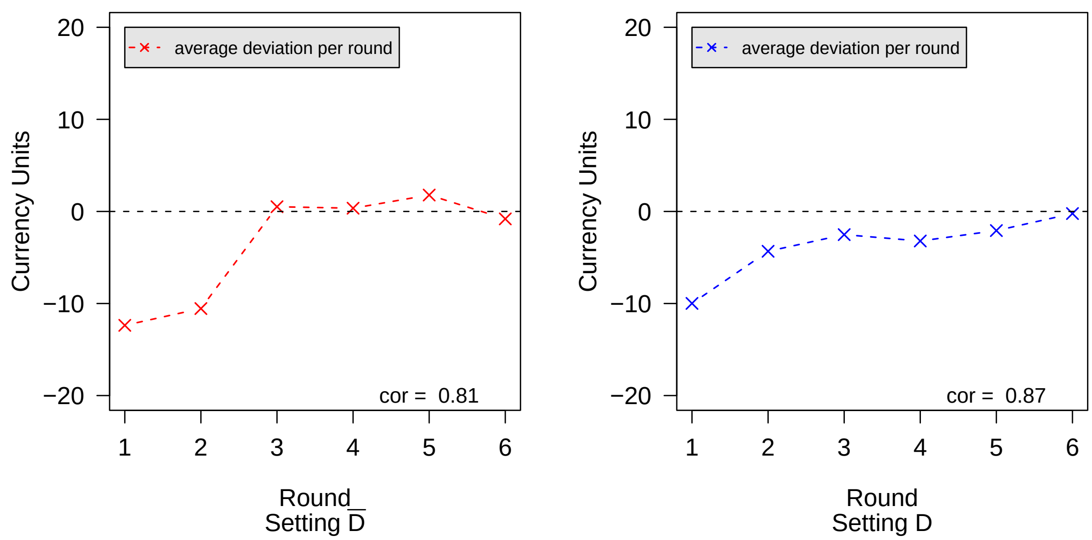
[8] The Spearmean's rank correlation coefficient of the observed differences over the six rounds is positive in setting $\bar{D}$ ($cor = 0.81$) as well as setting D ($cor = 0.87$). Thus, in both settings an upward trend towards the dominant strategy can be measured. The same holds true for bids with and without discount in setting D – the correlation coefficient for bids with discount equals $cor = 0.93$ and 0.74 for bids without discount.

the discount auction, it becomes apparent that non-designated bidders behave close to the behavior predicted from theory over all rounds except in the first round: from rounds 2 to 6 non-designated bidders bid close to the dominant strategy (relative deviation between -1.34% and 0.61%) with a tendency towards underbidding. Designated bidders need more rounds to adapt their behavior towards the theoretical benchmark – only in the very last round is the average bid close to the dominant strategy (relative deviation of -1.44%). In addition, designated bidders have a general tendency towards underbidding.

To summarize these observations, in regards to bidding behavior over the conducted auction rounds, it can be stated that bidders in the second-price auction and non-designated bidders in the discount auction adapt their behavior towards the dominant strategy much faster than bidders with an assigned discount. Thus, it seems that the institutional rule of the discount brings in noise such that designated bidders have more difficulties in adapting their behavior and finding their dominant strategy.



(a)



(b)

Figure 5.5: Setting $\bar{D}$ and setting D – (a) average bid per round and (b) average deviation of bids from dominant strategy per round

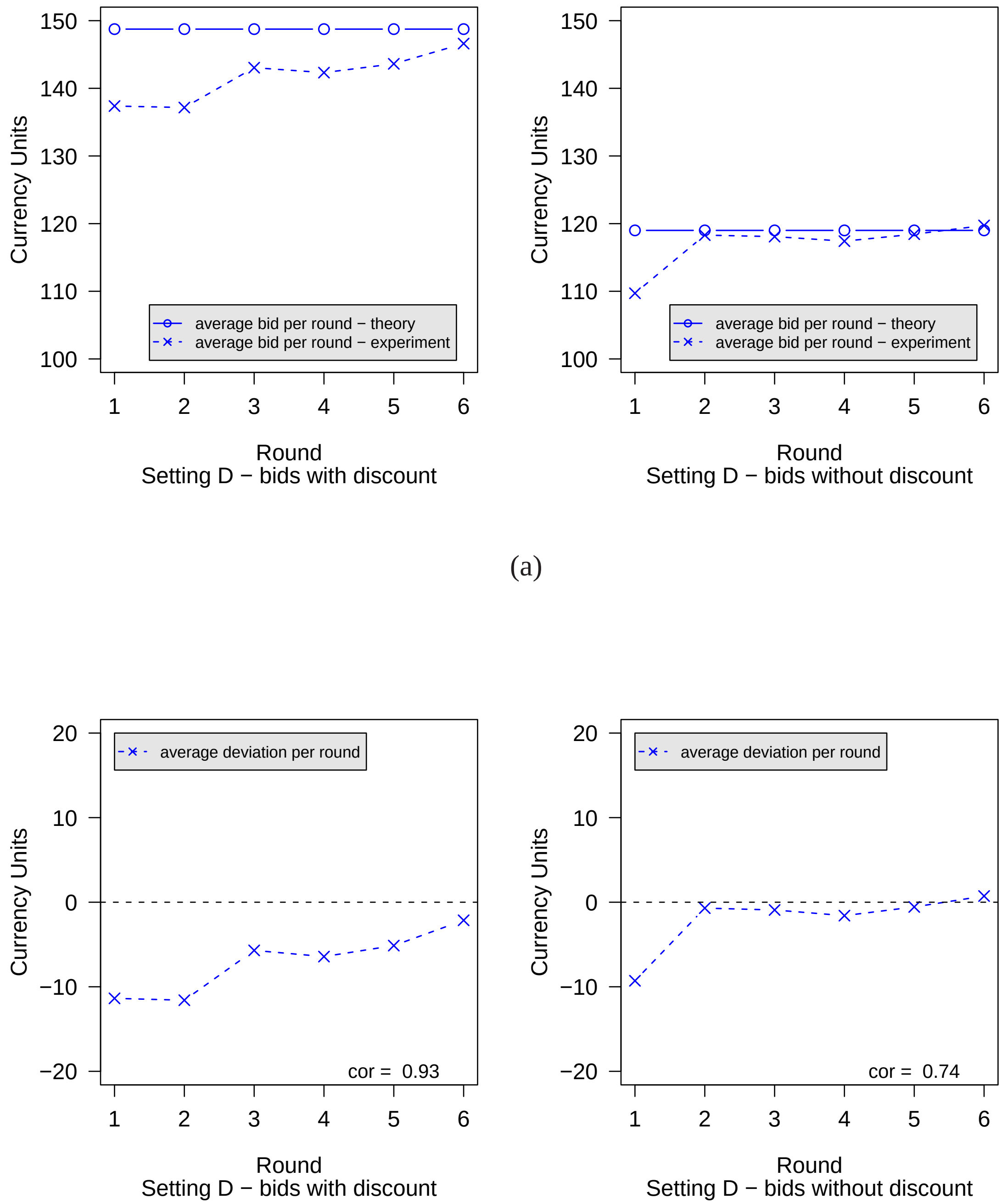


Figure 5.6: Setting D_{disc} and setting $D_{\overline{disc}}$ – (a) average bid per round and (b) average deviation of bids from dominant strategy per round

5.2 Auction outcomes

5.2.1 Descriptive analysis of the average outcomes

In order to analyze the auction outcomes observed in the first step of the experiment, a descriptive overview on the average outcomes is given in both settings – setting $\bar{D}$ and setting D. More specifically, the seller's revenue (auction revenue), the average winning bidder's payoff and the average surplus of both settings derived from theory and observed in the experiment are compared. Figures 5.7 – 5.8 illustrate the data graphically. All data are average values over all conducted auction rounds, i.e. 90 auctions are conducted in each setting (90 auctions = 3 sessions $\times$ 6 auction rounds per session $\times$ 5 auctions per auction round). Thus, each average outcome is the average over 90 auction outcomes. Recall that in both auction mechanisms, the second-price auction employed in setting $\bar{D}$ and the DA employed in setting D, strategies in dominant equilibrium exist. For each auction the equilibrium outcome is calculated based on the induced valuations to bidders participating in that auction, the auction revenue, the winning bidder's payoff and the social surplus (welfare) are determined as well. The equilibrium outcomes are derived from 90 conducted auctions in each setting. In figures 5.7 – 5.8 the equilibrium outcomes are indicated by "Theory", whereas the average outcomes observed in the experiment are indicated by "Experiment". Note that the settings reflect the symmetric case, since bidders' valuations are characterized by an ex-ante identical probability distribution function. The valuations induced to bidders in each session and auction round are given in Appendix D, Table D.1.

In examining the average auction revenues, Figure 5.7 (a) shows that the average revenues achieved in the experimental settings are both below the predicted theoretical results. In the case of the second-price auction (setting $\bar{D}$), the average auction revenue decreases from 114.23 to 112.41, while in the DA (setting D) the average auction revenue decreases from 112.17 to 109.66. On average, both the revenues achieved in the discount auctions are below the revenues achieved in the second-price auction, in theory as well as in the experiment. The lower revenues in both settings (compared to the theoretical benchmark) are a direct result of the bidding behavior of bidders in both settings – a general tendency of underbidding is observed in both settings. This behavior was outlined in Section 5.1.

Concerning the average winning bidder's payoff depicted in Figure 5.7 (b), the following can be concluded: In the pure second-price auction (setting $\bar{D}$), the winning bidder's payoff in the experiment (24.17) is on average lower than in the theoretical benchmark (25.77). In the discount auction, the opposite can be observed – here, the average winning bidder's payoff in the experiment (28.7) is above the winning bidder's payoff in equilibrium (27.01). When

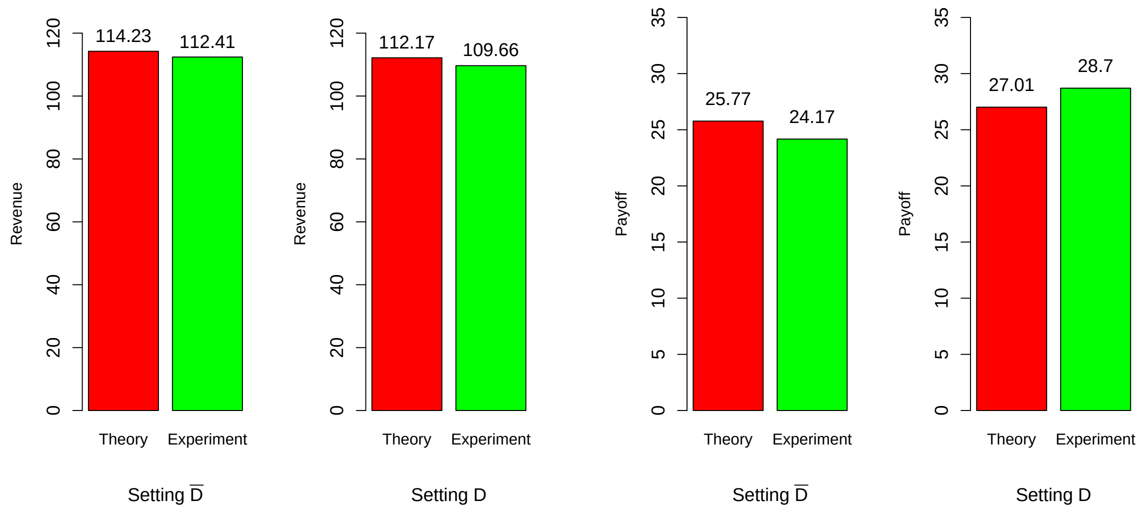


Figure 5.7: Setting $\bar{D}$ and setting D – (a) average revenue of seller and (b) average winning bidder's payoff

taking a closer look at the average winning bidder's payoff achieved in the discount auction, the average payoff can be distinguished in the average payoff of a designated bidder with the assigned discount of 20% and the average payoff of a non-designated bidder. The results are intuitive: winning bidders with an assigned discount achieve higher payoffs than bidders without discount. More specifically, Figure 5.8 shows the following results: based on the induced valuations and the observed bids in the experiment, (i) the average payoff of designated bidders (38.59) is greater than the average payoff of bidders without discount (16.03), (ii) the average payoff of designated bidders (38.59) is greater than the overall average bidder payoff (28.7) in setting D, and (iii) the average payoff of non-designated bidders (16.03) is below the overall average bidder payoff (28.7). These relations also hold true for the theoretical benchmark. In comparing the average payoffs observed in the experiment to the those predicted in theory, the average payoffs of the designated bidders (38.59) are nearly equal to those in equilibrium (38.79), whereas the average payoffs of non-designated bidders (16.03) are greater than those in equilibrium (14.09). Focussing on the conducted sessions of setting D, it can be observed from the data that only in session $D2$ the average payoff of a designated bidder is greater than the average payoff achieved in the theoretical benchmark, whereas in sessions $D1$ and $D3$ the observed average payoff of the designated bidder is lower than the respective average payoff

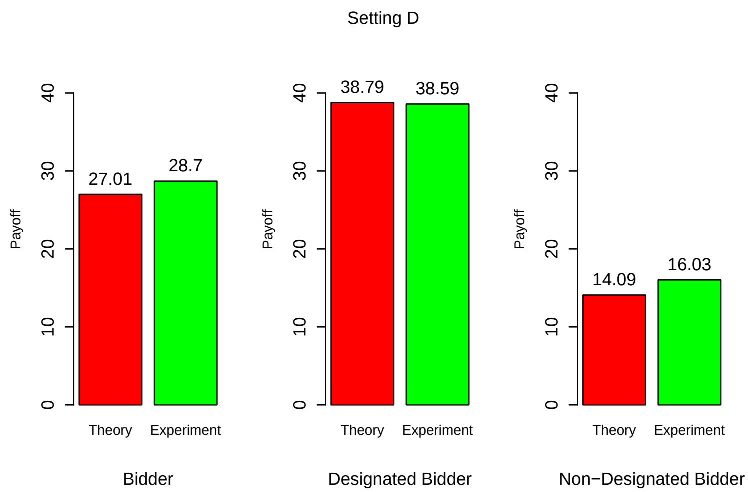


Figure 5.8: Average winning bidder's payoff in setting D

of the theoretical benchmark. The average payoff of the non-designated bidders dominates the average payoff in equilibrium in all sessions.

Similar to setting D, the average auction revenue is about 6% below the theoretical benchmark and the average winning bidder's payoff is about 6% above the theoretical benchmark, meaning the average social surplus in the experiment and in theory are approximately the same (Figure 5.9). The average social surplus observed in the experiment is 138.35, while in theory the average social surplus equals 139.18. In setting $\bar{D}$ the observed average social surplus (136.58) is lower than the theoretical average social surplus (140). Note that for each auction, the sum of auction revenue and winning bidder's payoff is equal to the social surplus.

Table 5.9 indicates the frequency of a designated and non-designated bidder being the high bidder in the discount auction. Again, the observations from the experiment are compared to respective outcomes predicted from theory. Of the 90 auctions conducted in setting D, a designated bidder was the winner 49 times (54.44%), while a non-designated bidder purchased the virtual object being auctioned 41 times (45.55%). The difference between the benchmark and the experimental setting occurred due to the fact that in one auction, a designated bidder won the auction, although according to equilibrium strategies, that bidder was not the one to win the auction. This was the case in session $D3$ in round 1, group 1: player 3 with a valuation of 107 received a discount of 20% and submitted a bid of 127.20, which is below his dominant strategy of 133.75. He outbid bidder 1, the bidder with the highest valuation in that auction who submitted a bid of 80 based on a induced valuation of 149, as well as bidder 2 with a bid of 105 according to his dominant strategy. Thus, bidder 3 was the highest bidder in that

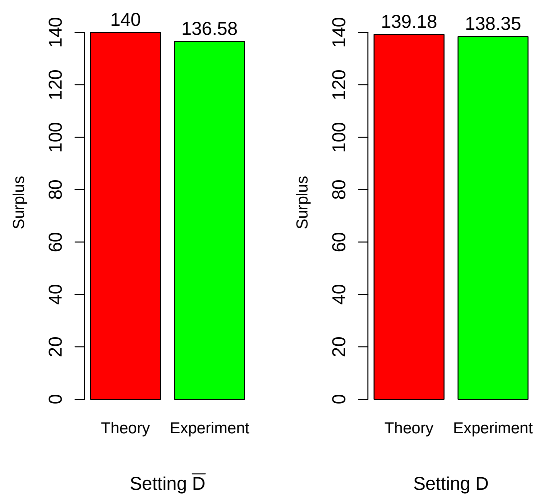


Figure 5.9: Average social surplus in setting $\bar{D}$ and setting D

auction, receiving a discount on the second-highest bid of 105 and a payoff equal to 23. In all other sessions – session $D1$ and session $D2$ – the observed frequencies of designated and non-designated winning bidders are equal to those predicted from theory.

Distribution of winning bidders in setting D					
Setting		Theory		Experiment	
		Designated	Non-Designated	Designated	Non-Designated
D	rel.	53.33%	46.67%	54.44%	45.55%
	abs.	48	42	49	41

Table 5.9: Distribution of winning bidders in setting D

Table 5.10 summarizes the information given in Figure 5.7 – 5.9. Again, for both settings the experimental results are compared to the respective results of the theoretical benchmark. For setting $\bar{D}$ the first row gives the average outcomes in equilibrium according to the induced valuations of the bidders in the experiment. Since this is the benchmark, the average outcomes are equal to 100%. The second row indicates the experimental results in that setting and presents the average outcomes as well as the relative performance to the benchmark. Rows four and five summarize the average outcomes for setting D: row four indicates the benchmark case and row five the observed outcomes. In setting $\bar{D}$ all average outcomes derived from the experiment are below the average outcomes derived from theory: the average auction revenue decreases by 1.59%, the average payoff by 6.21%, and the social surplus by 2.44%. In setting D the observed average revenue decreases from the theoretical average revenue by 2.24%, while the average social surplus decreases only by 0.6%. Only the observed average payoff dominates the theoretical solution by 6.26%. This is due to the fact that non-designated bidders profited more than predicted from the discount auction and increased their average payoff by 13.77%. One explanation for this quite surprising fact is that designated bidders have a general tendency to underbid: 61.11% submit bids below their dominant strategy (cf. Table 5.4), while about 22.22% of designated bidders do not use the discount and submit bids close to their valuation (cf. Table 5.5). Thus, the probability of non-designated bidders winning the auction increases, as well as the revenue of the non-designated bidders. Note that the revenue achieved by the seller decreases.

Setting	Description	Revenue	Payoff	Surplus	Payoff	
					Designated	Non-Designated
$\bar{D}$	Theoretic solution (mean)	114.23	25.77	140	-	-
	with induced valuations	100%	100%	100%	-	-
	Experimental results (mean)	112.41	24.17	136.58	-	-
		98.41%	93.79%	97.56%	-	-
D	Theoretic solution (mean)	112.17	27.01	139.18	38.79	14.09
	with induced valuations	100%	100%	100%	100%	100%
	Experimental results (mean)	109.66	28.7	138.35	38.59	16.03
		97.76%	106.26%	99.40%	99.48%	113.77%

Table 5.10: Setting $\bar{D}$ and setting D – auction outcomes (theoretical benchmark and experiment)

Note that the social surplus in an auction is equal to the sum of auction revenue and the winning bidder's payoff in that auction. The social surplus is at maximum when the auction is efficient. This is the case in equilibrium. Thus, inefficiency leads to a decrease in social surplus. However, the auction revenue and the payoff can be lower or higher than in theory. Regarding the social surplus and the efficiency of the auctions in settings $\bar{D}$ and D, the following is observed: in setting $\bar{D}$, 73 (81.11%) auctions out of 90 (100%) auctions are efficient, meaning that the bidder with the highest induced valuation wins the auction. Seventeen (18.89%) auctions are inefficient. In setting D 71 out of 90 auctions are efficient and

19 inefficient. From theory it is known that by adding a discount to an auction, inefficiency can be induced. In this case, adding a discount of 20% to the second-price auction leads to inefficiency. From the theoretical solution one can predict that in setting D 18 out of 90 auctions are inefficient due to the discount. In equilibrium 18 times the object is awarded to a bidder not having the highest valuation; that is, the winning bidder is a designated bidder who submits a bid above his valuation due to the discount purchasing the object, although not the bidder with the highest valuation. However, in the experiment 19 instead of 18 inefficient auctions are observed. The increase in inefficiency is due to the fact that as mentioned above, a designated bidder won the auction in session $D3$, instead of player 1 from group 1 with the highest induced valuation in that group. Player 1 with a valuation of 147 placed a bid equal to 80; he was outbid by the designated bidder (player 3) who submitted a bid of 127.20. Player 3 received a valuation of 107 in that round. Thus, the number of the predicted inefficient auctions increased by 1 to 19 inefficient auctions observed in the experiment.

Bidders' earnings in setting $\bar{D}$ and setting D			
Setting/Session	Av. Earning	Min. Earning in euros	Max. Earning
$\bar{D}1$	12.52	6.10	19.05
$\bar{D}2$	13.08	3.90	19.40
$\bar{D}3$	12.90	8.20	17.10
$\bar{D}$	12.83	3.90	19.40
$D1$	13.28	8.20	22.44
$D2$	13.92	8.00	22.36
$D3$	14.02	8.00	25.59
D	13.74	8.00	25.59

Table 5.11: Setting $\bar{D}$ and setting D – average bidders' earnings in euros

The sum of all payoffs received by a bidder throughout the experiment, including the paid show-up fee, was converted to euros at the end of the experiment. The cash rate of 1 GE (GE = 'Geldeinheiten' used as experimental currency unit) was 0.10 euro. The bidders' earnings in the experiment are summarized in Table 5.11. The average payment of all subjects was 13.29 euros. In the sessions of the discount auction the earnings (13.74 euros, predicted from theory: 13.40 euros) are slightly higher on average than in the the second-price auction sessions (12.83 euros, predicted from theory: 13.15 euros). Although none of the subjects went bankrupt, two suffered losses: in session $\bar{D}1$ player 2 in group 4 received a payment of 6.10 euros while in session $\bar{D}2$ player 1 in group 2 earned 3.90 euros. Both payments are below the show-up fee of 8.00 euros (80 GE), which were paid to the experimental account directly at the beginning of the experiment. A more detailed listing of all bidder's earnings can be found in Appendix D, Table D.6.

5.2.2 Individual auction outcomes

Experimental results are based on six conducted sessions with 30 auctions in each session. Overall, in setting $\bar{D}$ data from 90 conducted second-price auctions are recorded, while in setting D data from 90 conducted discount auctions are recorded. In the following, for each played auction, the auction revenue R, the winning bidder's payoff P as well as the social surplus V are displayed. Note that the social surplus (welfare) is equal to the sum of auction revenue and winning bidder's payoff: $V = R + P$. That is, V is the induced valuation of the winning bidder to whom the virtual object of that auction is awarded. The winning bidder pays a price of R to the seller and receives a payoff of P, which is the difference between his valuation and the price of the object. The individual auction outcomes are displayed in Tables 5.12 – 5.17. More specifically, Tables 5.12 – 5.14 display the outcomes from the second-price auctions conducted in the sessions of setting $\bar{D}$, while Tables 5.15 – 5.17 present the auction outcomes of the discount auctions derived from the sessions of setting D. In each table, the auction outcomes of the 30 auctions conducted in a particular session are given: in each round a single auction was conducted within each group. Overall, five groups played six consecutive auction rounds in each session. The 3-tuple (R,P,V) indicates the auction results which were derived from one of the five groups in a particular auction round. For each of the five groups, the average results of that group are indicated, meaning the minimum and maximum outcome over all rounds. Within each session, the overall average values as well as the standard deviations of the results are displayed. Note that the average values over all sessions are indicated in Table 5.10.

As in the experiment in setting D the discount mechanism was employed, with the winning bidder in a DA being either a designated bidder, a bidder with discount, or a non-designated bidder. In Tables 5.15 – 5.17 individual auction outcomes caused by designated bidders are printed in bold. To illustrate an example, the individual outcome of group three in the first round of session $D1$ presented in Table 5.15 is illustrated: the 3-tuple (R,P,V) is given by (**87.20**,**58.80**,**146**). Player one, who is the designated bidder in that group, wins the auction and purchases the object at a price of 87.20, that is, the second-highest discounted bid in that auction. The payoff is equal to the difference between his valuation of the item and the price to pay: $58.80 = 146 - 87.20$. Because the winning bidder is the designated bidder, the 3-tuple is printed in bold.

Outcomes of auctions in session $\bar{D}1$

Round	Group 1			Group 2			Group 3			Group 4			Group 5		
	R	P	V	R	P	V	R	P	V	R	P	V	R	P	V
1	106.99	42.01	149	102.99	45.01	148	104	42	146	80	21	101	98.50	51.50	150
2	103.99	4.01	108	103.01	46.99	150	107	39	146	100	1	101	102	45	147
3	138	12	150	103.99	1.01	105	146	0	146	120	27	147	98	10	108
4	118	32	150	107.01	40.99	148	108	39	147	145.99	-41.99	104	100	49	149
5	149	-44	105	102.99	3.01	106	109	38	147	148	2	150	105	41	146
6	150	-2	148	107.01	41.99	149	108	39	147	102	7	109	100.50	45.50	146
mean	127.66	7.34	135	104.50	29.83	134.33	113.67	32.83	146.5	116	2.67	118.67	100.67	40.33	141
min	103.99	-44	105	102.99	1.01	105	104	0	146	80	-41.99	101	98	10	108
max	150	42.01	150	107.01	46.99	150	146	42	147	148	27	150	105	51.50	150
			overall mean R: 112.5					overall mean P: 22.6					overall mean V: 135.1		
			std. dev. R: 24.9					std. dev. P: 18.1					std. dev. V: 16.0		

Table 5.12: Session $\bar{D}1$ – individual auction outcomes in the experiment

Outcomes of auctions in session $\bar{D}2$

Round	Group 1			Group 2			Group 3			Group 4			Group 5		
	R	P	V	R	P	V	R	P	V	R	P	V	R	P	V
1	107	42	149	147.02	-44.02	103	109	37	146	98.10	2.90	101	85	65	150
2	105	3	108	102	47	149	109	37	146	100.90	47.10	148	100	2	102
3	150	-2	148	104	5	109	140	9	149	107	40	147	97	4	101
4	115	35	150	106.30	41.70	148	108	39	147	104.90	41.10	146	101.99	47.01	149
5	105	44	149	103	3	106	109	38	147	147.90	2.10	150	102	44	146
6	148.01	1.99	150	106.50	42.50	149	107.8	39.2	147	101.90	7.10	109	104	42	146
mean	121.67	20.67	142.33	111.47	15.86	127.33	113.8	33.2	147	110.12	23.38	133.5	98.33	34	132.33
min	105	-2	108	102	-44.02	103	107.8	9	146	98.10	2.10	101	85	2	101
max	150	44	150	147.02	47	149	140	39.2	149	147.90	47.10	150	104	65	150
			overall mean R: 111.08					overall mean P: 25.42					overall mean V: 136.50		
			std. dev. R: 15.5					std. dev. P: 23.1					std. dev. V: 17.5		

Table 5.13: Session $\bar{D}2$ – individual auction outcomes in the experiment

Outcomes of auctions in session $\bar{D}3$

Round	Group 1			Group 2			Group 3			Group 4			Group 5		
	R	P	V	R	P	V	R	P	V	R	P	V	R	P	V
1	148.99	-41.99	107	146.50	1.50	148	109.01	36.99	146	90.01	11.99	102	99	51	150
2	95	13	108	143.05	5.95	149	109.01	36.99	146	101	47	148	95	52	147
3	136	12	148	105.01	3.99	109	146	3	149	107	40	147	97	4	101
4	105	45	150	107.14	40.86	148	108	39	147	104	42	146	120	29	149
5	104.99	44.01	149	104	2	106	109	38	147	130.70	19.30	150	111	35	146
6	147.99	2.01	150	115.76	33.24	149	108	39	147	102	7	109	104	42	146
mean	123	12.34	135.33	120.24	14.59	134.83	114.84	32.16	147	105.79	27.88	133.67	104.33	35.5	139.83
min	95	-41.99	107	104	1.50	106	108	3	146	90.01	7	102	95	4	101
max	148.99	45	150	146.50	40.86	149	146	39	149	130.70	47	150	120	52	150

overall mean R: 113.64 — overall mean P: 24.50 — overall mean V: 138.13

std. dev. R: 16.4 — std. dev. P: 19.8 — std. dev. V: 17.0

Table 5.14: Session $\bar{D}3$ – individual auction outcomes in the experiment

Outcomes of auctions in session $D1$

Round	Group 1			Group 2			Group 3			Group 4			Group 5		
	R	P	V	R	P	V	R	P	V	R	P	V	R	P	V
1	**85.60**	**63.40**	**149**	147	1	148	**87.20**	**58.80**	**146**	**88.8**	**13.20**	**102**	120	30	150
2	**82.40**	**25.60**	**108**	**119.20**	**30.80**	**150**	**87.20**	**58.80**	**146**	120	28	148	133	14	147
3	148	2	150	**87.20**	**17.80**	**105**	**119.20**	**26.80**	**146**	145	2	147	**86.4**	**16.60**	**103**
4	128.75	21.25	150	123.67	24.33	148	120	27	147	**85.6**	**60.40**	**146**	125.5	23.50	149
5	131.26	17.74	149	**84**	**22**	**106**	101	46	147	148	2	150	133.5	12.50	146
6	**120**	**28**	**148**	132.91	16.09	149	134.99	12.01	147	**81.6**	**27.40**	**109**	**83.2**	**62.80**	**146**
mean	116	26.33	142.33	115.66	18.67	134.33	108.27	38.23	146.5	111.5	22.17	133.67	113.6	26.57	140.17
min	82.40	2	108	84	1	105	87.20	12.01	146	81.6	2	102	83.2	12.50	103
max	148	63.40	150	147	30.80	150	134.99	58.80	147	148	60.40	150	133.5	62.80	150

overall mean R: 113.00 — overall mean P: 26.39 — overall mean V: 139.40 — freq. discount: 46.66%

std. dev. R: 24.9 — std. dev. P: 18.1 — std. dev. V: 16.0

Table 5.15: Session $D1$ – individual auction outcomes in the experiment

Outcomes of auctions in session $D2$

Round	Group 1			Group 2			Group 3			Group 4			Group 5		
	R	P	V	R	P	V	R	P	V	R	P	V	R	P	V
1	107	42	149	**112**	**36**	**148**	136.24	9.76	146	**78.86**	**22.14**	**101**	108	42	150
2	105	3	108	143	7	150	136.24	9.76	146	**116**	**32**	**148**	**85.61**	**61.39**	**147**
3	**120**	**28**	**148**	**84**	**25**	**109**	**116.53**	**32.47**	**149**	**104**	**3**	**107**	**86.40**	**14.60**	**101**
4	115.10	34.90	150	135	13	148	**86.39**	**60.61**	**147**	135	11	146	108.14	40.86	149
5	**88.20**	**60.80**	**149**	**84.8**	**19.20**	**104**	126.24	20.76	147	**85.86**	**64.14**	**150**	**85.81**	**60.19**	**146**
6	**124.32**	**25.68**	**150**	**85.6**	**63.40**	**149**	131.24	15.76	147	**87.19**	**12.81**	**100**	129.57	16.43	146
mean	109.94	32.40	142.33	107.4	27.27	134.67	122.15	24.85	147	101.15	24.18	125.33	100.59	39.25	139.83
min	88.20	3	108	84	7	104	86.39	9.76	146	78.86	3	100	85.61	14.60	101
max	124.32	60.80	150	143	63.40	150	136.24	60.61	149	135	64.14	150	129.57	61.39	150
overall mean R: 108.25				overall mean P: 29.59			overall mean V: 137.83			freq. discount: 56.67%					
std. dev. R: 19.6				std. dev. P: 20.2			std. dev. V: 16.8								

Table 5.16: Session $D2$ – individual auction outcomes in the experiment

Outcomes of auctions in session $D3$

Round	Group 1			Group 2			Group 3			Group 4			Group 5		
	R	P	V	R	P	V	R	P	V	R	P	V	R	P	V
1	**84**	**23**	**107**	**100.42**	**46.58**	**147**	130	16	146	**80.80**	**25.20**	**106**	**80**	**70**	**150**
2	**85.61**	**18.39**	**104**	**120**	**29**	**149**	131.25	14.75	146	126.25	21.75	148	127.50	19.50	147
3	**117.60**	**32.40**	**150**	**84**	**20**	**104**	146	3	149	**84.80**	**62.20**	**147**	**80.80**	**27.20**	**108**
4	**87.20**	**62.80**	**150**	**85.72**	**62.28**	**148**	125	22	147	131.25	14.75	146	**82.56**	**66.44**	**149**
5	125	24	149	**84.94**	**18.06**	**103**	**97.04**	**49.96**	**147**	**119.62**	**28.38**	**148**	127.50	18.50	146
6	147.51	2.49	150	125	24	149	**100.72**	**46.28**	**147**	**87.20**	**14.80**	**102**	126.25	19.75	146
mean	107.82	27.18	135	100.01	33.32	133.33	121.67	25.33	147	104.99	27.85	132.83	104.10	36.90	141
min	84	2.49	104	84	18.06	103	97.04	3	146	80.80	14.75	102	80	18.50	108
max	147.51	62.80	150	125	62.28	149	146	49.96	149	131.25	62.20	148	127.50	70	150
overall mean R: 107.72				overall mean P: 30.11			overall mean V: 137.83			freq. discount: 60%					
std. dev. R: 22.4				std. dev. P: 19.7			std. dev. V: 17.1								

Table 5.17: Session $D3$ – individual auction outcomes in the experiment

In Tables 5.15 – 5.17 the frequency of designated bidders being winning bidders in each single session is indicated by **freq. discount**. For example, in session $D1$ this frequency is equal to 46.66% (cf. Table 5.15). The distribution of winning bidders being designated or non-designated bidders over all sessions of setting D is displayed in Table 5.9.

The empirical distributions of the individual auction outcomes presented in Tables 5.12 – 5.17 are discussed in the following. The empirical distributions of the auction revenues, the winning bidder's payoff and the social surplus in setting $\bar{D}$ and D give deeper insight into the differences between both settings and the effects of the discount mechanism. They allow a direct comparison between both settings. The empirical distributions of the auction revenues are displayed in Figure 5.10, the distributions of the payoffs in Figure 5.11 and the distributions of the surplus in Figure 5.12. In each figure, the graphs indicate the empirical distributions of the second-price auction (setting $\bar{D}$) and the discount auction (setting D): the first graphs plots the empirical distributions based on the equilibrium outcomes ('Theory'), and the second graph the respective empirical distributions based on the experimental data ('Experiment').

To explain the empirical distributions of the auction revenue, firstly the theoretical benchmark of both settings and secondly the experimental results of both settings are discussed. The predicted distributions of the revenues are displayed in the first graph in Figure 5.10. It is shown that for revenues below 105 and revenues between 133 and 148 the distribution of the revenues of the discount auction is greater than the respective distribution of the second-price auction; for revenues between 105 and 133 and in an ϵ-surrounding of 149, the distribution of the revenues in setting $\bar{D}$ are above the respective curve of setting D. However, there is no indication of the central tendency of the revenues in setting D and $\bar{D}$. Recall that the induced valuations ranged between [100,109] and [146,150]. Based on these valuations the revenues in the second-price auction (setting $\bar{D}$) range between (i) 102 and 109 as well as (ii) 146 and 149. Thus, in case (i) a weak bidder determines the payment of the winning bidder by his bid, whereas in case (ii) a strong bidder, being the second-highest bidder, determines the price to pay for the object. In the discount auction (setting D) the auction revenues are slightly different since they are directly affected by the discount. More specifically, one of the following cases might appear in each discount auction: (i) the price to pay is not affected by the discount since the winning bidder as well as the second-highest bidder are both non-designated bidders; (ii) the price to pay is determined by a designated bidder being the second-highest bidder; and (iii) the price to pay is the second-highest discounted bid since the winning bidder is the designated bidder. Thus, there is a wider range of realized auction revenues in setting D than in setting $\bar{D}$. From theory it can be predicted that in the discount auction, auction revenues range from 81.60 to 87.20, 116.80 to 136.25 and 146 to 149. Overall, in equilibrium the empirical distribution of the revenue from the discount auction is based on 27 different measured values, whereas the distribution of the second-price auction is based on 11 different measured values.

Regarding the conducted experiment, the distribution functions are similar in shape to those predicted from theory. For low auction revenues as well as for high auction revenues, the distribution of auction revenues in setting $\bar{D}$ is below the respective curve in setting D; for auction revenues between 105 and 136, the distribution of the auction revenues in setting $\bar{D}$ lies upon the respective distribution function of setting D. In fact, the difference between the revenues in both settings is significant (Kolmogorv-Smirnov test: significant with p-value< 0.001, cf. Table 5.19). Nevertheless, a central tendency towards a discount significantly lowering or raising the revenues in the two focussed settings cannot be concluded.

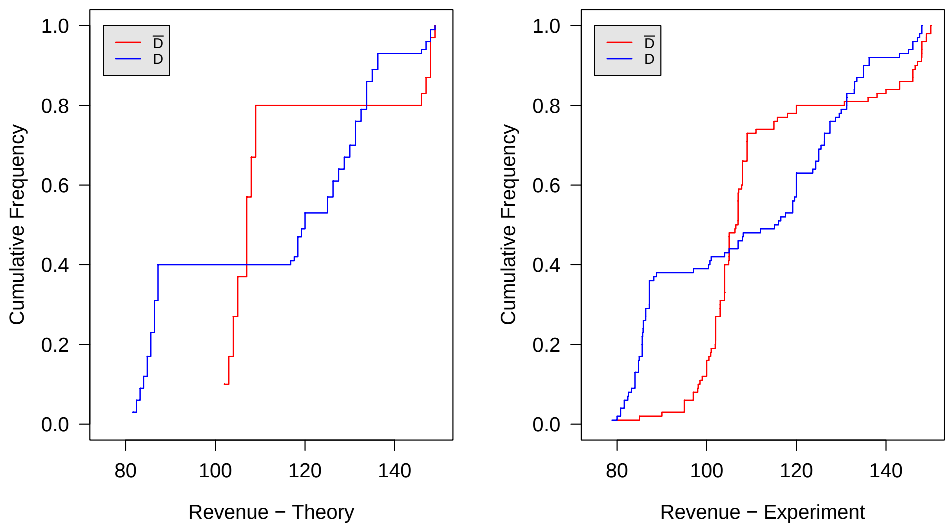


Figure 5.10: Empirical distribution of auction revenues in setting $\bar{D}$ and setting D – theoretical benchmark and experiment

Similar observations can be made with respect to the winning bidder's payoff. No indication concerning the central tendency of the bidder's payoff in setting $\bar{D}$ and D is given. Figure 5.11 displays the empirical distributions of the payoff for both settings (i) in the case of the theoretical benchmark and (ii) in the case of the experimental results. The shape of the distribution function of the payoffs is similar to those of the auction revenue – only the order has been reversed. For small and high payoffs, the distribution of the payoffs in setting $\bar{D}$ lies upon the distribution of the payoffs in setting D, while for payoffs between approximately 20 and 40 the distribution of the payoffs in setting D is greater than the respective distribution of

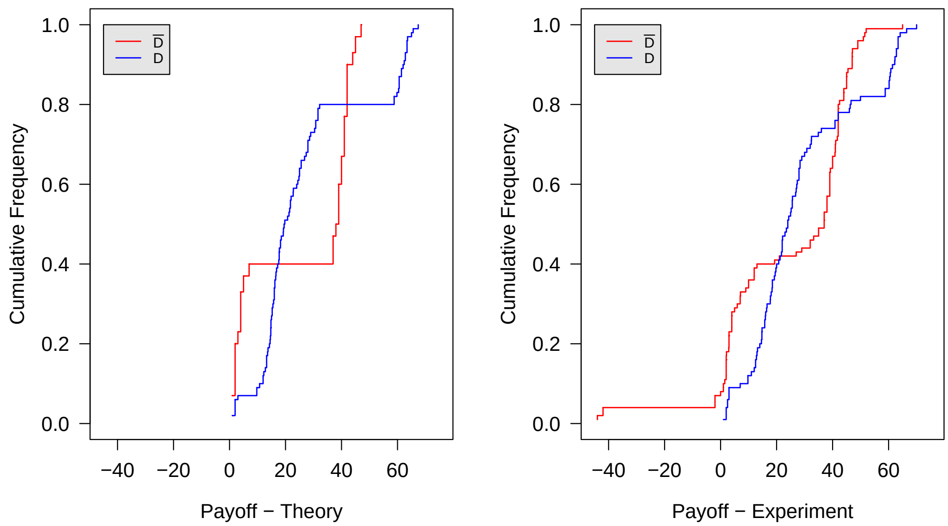


Figure 5.11: Empirical distribution of winning bidder's payoff in setting $\bar{D}$ and setting D – theoretical benchmark and experiment

setting $\bar{D}$ (this holds for the predictions from theory as well as the experimental results). The difference between the curves is significant as indicated in Table 5.20 (Kolmogorov-Smirnov test: significant with p-value $= 0.003$).

When comparing the empirical distributions of the social surplus in settings $\bar{D}$ and D, no significant differences in central tendency can be observed. There is only a small difference between the two curves, both in theory and in the experiment. It is notable that when comparing the empirical distributions of the social surplus in the experiment with those predicted from theory, the distributions show a reverse order. In equilibrium, the distribution of the surplus in setting D is greater than or equal to the respective distribution in setting $\bar{D}$, while in the experiment, the distribution of the surplus in setting D is equal to or below the respective curve of setting $\bar{D}$ (with the exception of a single point in that curve). That is, in theory it is predicted that the discount lowers the welfare, while in the experiment it is observed that the discount increases the welfare (with respect to the welfare of the benchmark second-price auction).

Regarding the differences between the auction outcomes predicted from theory and those resulting from the experiment, the main results are indicated and discussed in Section 5.2.1 (cf. Table 5.10). In the following the differences between predictions from theory and observations in the experiment are explained in more detail. The differences are measured by means of

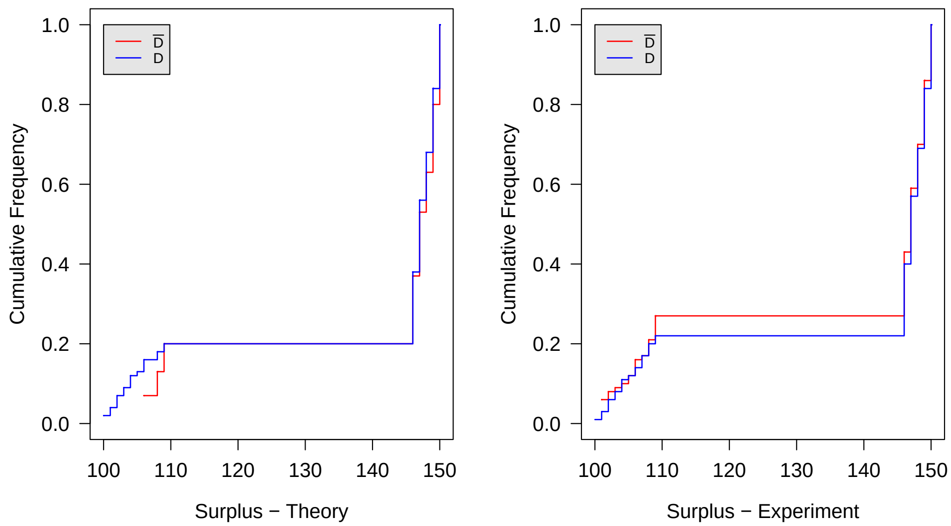


Figure 5.12: Empirical distribution of the social surplus in setting $\bar{D}$ and setting D – theoretical benchmark and experiment

statistical analysis to determine whether indications with respect to the central tendency of the differences between theory and experiment for both settings are significant. Table 5.18 displays the differences between outcomes in equilibrium and observed outcomes. The mean, the standard deviation, the median as well as the minimum and maximum of the differences between the theoretical benchmark and the experimental results are displayed for both settings (setting $\bar{D}$ and setting D): the differences are measured with respect to the auction revenues, the winning bidder's payoff and the social surplus.

Applying the Wilcoxon signed-ranks test (WSR, matched-pairs) to test for differences in the central tendencies between the equilibrium outcomes and the observed outcomes in the experiment, the following holds true (cf. Table 5.10): Comparing the outcomes in setting $\bar{D}$ derived from theory with those derived from the experiment, the differences are statistically (weak) significant. The auction revenues predicted from theory are significantly greater than the respective observed outcomes (WSR, one-sided, significant with $V = 1707$ and p-value < 0.001), the payoffs received in equilibrium are below the respective payoffs derived from the experiment (WSR, one-sided, weak significant with V = 975 and p-value = 0.083), and the predicted social surpluses are above the respective surpluses observed in the experiment (WSR, one-sided, significant with V = 171 and p-value < 0.001). Similar observations are made for setting D: the auction revenues predicted from theory are greater than the respective;

in the experiment observed, revenues (WSR, one-sided, significant with V = 1190.5 and p-value = 0.005) and the payoffs calculated in theory are below the respective experimental results (WSR, one-sided, significant with V = 573, p-value = 0.015). Only the measured differences between theory and experimental results with respect to the social surplus in the case of the DA are not significant – the null hypothesis that the surpluses in equilibrium are above the respective surpluses in the experiment cannot be rejected (one-sided, no significant difference in central tendency: V = 5, p-value = 0.211). Recall that in Section 5.2.1 the average individual auction outcomes in the two settings were graphically displayed in Figures 5.7 – 5.9. The WSR test partially confirms the significance in differences between theory and experiment for both settings and the respective auction outcomes.

It is found that the standard deviations between differences in auction revenue, winning bidder's payoff as well as the social surplus in setting $\bar{D}$ are greater than the respective standard deviations of differences in setting D (cf. Table 5.18). However, it was expected that whenever there is a difference between predicted and observed outcomes, the differences between the theoretical benchmark and experimental observations should be higher in the discount auction than in the second-price auction.

Deviation of experimental results from results of theoretical benchmark										
	setting $\bar{D}$					setting D				
outcomes	mean	std. dev.	median	min	max	mean	std. dev.	median	min	max
revenues	1.8	12.8	0.01	-44.0	-47.0	2.5	10.6	0.0	-33.6	49.8
payoff	1.6	20.1	0.0	-46.0	88.0	-1.7	8.8	0.0	-35.3	33.6
social surplus	3.4	11.1	0.0	0.0	47.0	0.8	6.2	0.0	-8.0	42.0

Table 5.18: Setting $\bar{D}$ and setting D – comparison of equilibrium auction outcomes and observed auction outcomes

The following focuses on a comparison of the auction revenues in the conducted auctions in setting $\bar{D}$ and setting D for different cases. That is, the auction revenues, the winning bidder's payoff and the social surplus are compared by means of statistical analysis. Tables 5.19 – 5.21 summarize the results of the statistical analysis indicating the mean, the standard deviation, the median, the minimum and maximum of the observed data derived from setting $\bar{D}$ and D, as well as the differences in central tendency between the data sets from settings $\bar{D}$ and D. Depending on the distribution of the outcomes to be compared, different tests are applied for measuring significant differences between the data sets: the t-test, the Wilcoxon singed-ranks test (WSR, matched-pairs), and the Kolmogorov Smirnov test (KS) (cf. Section

4.6).[9] The observed data used as database for the statistical analysis are obtained from different rounds or groups in each setting. More specifically, the data sets are derived from the following cases: (i) the 1^{st} auction round, (ii) the 6^{th} auction round, (iii) the average auction outcomes per group, (iv) the average auction outcomes per round, and (v) the auction outcomes over all rounds and groups. In case (i) only observed data from the first round are considered. Since there are no trial rounds, the data are independent. Overall, as in each session in the first round, five auctions were played with 15 paired observations (3 sessions per setting x 5 auctions per round and per session) – 15 observations derived from setting $\bar{D}$ and 15 observations derived from setting D – and then compared, i.e. 15 matched-pairs. The same holds for case (ii): in the analysis, 15 matched pairs of the auction outcomes derived from the 6^{th} round are compared. Because the observations are derived from the last auction round, learning effects cannot be excluded from the observed data. Case (iii) and case (iv) focus on average auction outcomes; that is, in case (iii) the average auction outcomes of a single group over all rounds (15 observations per setting) and in case (iv) the average auction outcomes over all groups per round (18 observations per setting) are analyzed. Considering the individual auction outcomes in each round and session, 90 observations derived from setting $\bar{D}$ are compared to 90 observations derived from setting D (case (v)).

Table 5.19 presents the results of the statistical analysis of the data derived from each setting observed in the experiment for each case with respect to the auction revenue. When applying the t-test or the WSR from case (i) to case (iv) in order to measure differences in central tendency between the revenues of both settings, the following is derived: the null hypothesis that the auction revenues in the second-price auction ($R_{\bar{D}}$) are equal to the auction revenues in the discount auction (R_D) cannot be rejected. In testing difference between the distribution ($F_{R_{\bar{D}}}$) of the revenues in setting $\bar{D}$ and the respective distribution (F_{R_D}) in setting D, both significantly differ from each other (KS, significant with p-value < 0.001, cf. Figure 5.10). Although the average revenues in the discount auction are below the average revenues in the second-price auction (case (i) to case (v)), one cannot conclude that the discount significantly hurts auction revenue. A significant difference in central tendency cannot be concluded. Nevertheless, the difference between the distribution functions of the auction revenues obtained in the second-price and derived from the discount auction is significant.

[9] All data are tested for normality with the Shapiro-Wilk test. When the null hypothesis that the data follows a normal distribution is rejected (on a significance level of 5 percent), then the t-test is not applied. Instead, the Wilcoxon (matched-pairs) signed-ranks test or the Kolmogorov-Smirnov test is used to measure difference in central tendency or between the different distribution functions. The Kolmogorov-Smirnov test is applied in case the difference scores of the matched-pairs is not symmetric to the median of the population of difference scores (Sheskin 2004).

Auction revenues in setting $\bar{D}$ and Setting D						
Setting	Mean	Std. dev.	Median	Min	Max	# pairs
(i) auction revenues in 1st auction round						
$\bar{D}$	108.8	21.8	104.0	80.0	149.0	15
D	103.1	22.2	100.4	78.9	147.0	15
$\bar{D}$ vs. D	$H_0 : R_{\bar{D}} = R_D$, WSR: $V = 59$, p-value = 0.706					
(ii) auction revenues in 6th auction round						
$\bar{D}$	114.2	18.2	107.0	100.5	150.0	15
D	113.2	22.8	124.3	81.6	147.5	15
$\bar{D}$ vs. D	$H_0 : R_{\bar{D}} = R_D$, WSR: $V = 59$, p-value= 0.978					
(iii) average auction revenues per groups						
$\bar{D}$	112.4	8.6	113.7	98.3	127.7	15
D	109.7	7.1	108.3	100.0	122.2	15
$\bar{D}$ vs. D	$H_0 : R_{\bar{D}} = R_D$, t-test: $t = 1.080$, $df = 14$, p-value = 0.298					
(iv) average auction revenues per rounds						
$\bar{D}$	112.4	6.7	113.4	98.5	122.8	18
D	109.7	7.9	110.7	94.18	119.6	18
$\bar{D}$ vs. D	$H_0 : R_{\bar{D}} = R_D$, t-test (paired): $t = 1.115$, $df = 17$, p-value = 0.280					
(v) individual auction revenues						
$\bar{D}$	112.4	17.5	112.4	80.0	150.0	90
D	109.7	22.0	115.6	78.9	148.0	90
$\bar{D}$ vs. D	$H_0 : F_{R_{\bar{D}}} = F_{R_D}$, KS: D = 0.356, p-value < 0.001					

KS:Kolmogorov-Smirnov Test. WSR: Wilcoxon singed-ranks test (matched-pairs, two-sided).

Table 5.19: Setting $\bar{D}$ and setting D – comparison of auction revenues

In the experiment, the payoffs of the winning bidders received from each auction were recorded and stored in the database. What can be observed from the data is that in all cases, the average payoff in the discount auction is equal to or greater than the average payoff in the second-price auction, indicated in Table 5.20. This statement is true for the payoffs – denoted by $P_{\bar{D}}$ and P_D – to be compared between both settings for the first auction round, the sixth auction round, the average payoffs per groups, the average payoffs over all groups per round, as well as all observed payoffs. However, a significant difference in central tendency between the payoffs of both settings can only be observed in case (iv). The average payoffs per round obtained in the second-price auction differ weakly significant from those observed in the discount auction (t-test, weak significance with p-value $= 0.062$).

When comparing all payoffs and measuring differences between the distribution (F_{P_D}) of the payoffs in setting D and the respective distribution ($F_{P_{\bar{D}}}$) in setting $\bar{D}$, the difference is then significant (KS, significant with p-value $= 0.003$). This indicates a great likelihood that the winning bidder's payoff from both settings is drawn from different populations.

Winning bidder's payoff in setting $\bar{D}$ and setting D						
Setting	Mean	Std. dev.	Median	Min	Max	# pairs
(i) payoff in 1st auction round						
$\bar{D}$	24.3	32.7	37	-44	65	15
D	33.3	20.4	30	1	70	15
$\bar{D}$ vs. D	$H_0 : P_{\bar{D}} = P_D$, WSR: V = 45, p-value = 0.660					
(ii) payoff in 6th auction round						
$\bar{D}$	25.8	18.9	39	-2	45.5	15
D	25.8	18.0	19.7	2.5	63.4	15
$\bar{D}$ vs. D	$H_0 : P_{\bar{D}} = P_D$, WSR: $V = 63$, p-value = 0.890					
(iii) average payoffs per groups						
$\bar{D}$	24.2	11.4	27.9	2.7	40.3	15
D	28.7	6.0	27.2	18.7	39.3	15
$\bar{D}$ vs. D	$H_0 : P_{\bar{D}} = P_D$, t-test: t = -1.563, df = 14, p-value = 0.140					
(iv) average payoffs per rounds						
$\bar{D}$	24.2	10.2	26.26	8	40.8	18
D	28.7	8.4	29.11	13	45.7	18
$\bar{D}$ vs. D	$H_0 : P_{\bar{D}} = P_D$, t-test: t = -2, df = 17, p-value = 0.062					
(v) winning bidder's payoff						
$\bar{D}$	24.2	23.4	37	-44	65	90
D	28.7	18.9	24	1	70	90
$\bar{D}$ vs. D	$H_0 : F_{P_{\bar{D}}} = F_{P_D}$, KS: D = 0.267, p-value = 0.003					

KS:Kolmogorov-Smirnov Test. WSR: Wilcoxon singed-ranks test (matched-pairs, two-sided).

Table 5.20: Setting $\bar{D}$ and setting D – comparison of winning bidder's payoff

From theory it is predicted that in equilibrium the average social surplus in the second-price auction is greater than the average social surplus in the discount auction. This does not hold in general for the observed data in the experiment. Depending on the specific case (e.g., the social surplus in the first round (case (i)) or all social surpluses (case (v)), the average social surplus $V_{\bar{D}}$ observed in the second-price auction is either higher or lower than the respective average social surplus V_D in the discount auction (cf. Table 5.21).

Social surplus in setting $\bar{D}$ and setting D						
Setting	Mean	Std. dev.	Median	Min	Max	# pairs
(i) social surplus in 1st auction round						
$\bar{D}$	133.1	22.2	146	101	150	15
D	136.3	20.3	147	101	150	15
$\bar{D}$ vs. D	$H_0 : V_{\bar{D}} = V_D$, WSR: V = 1.5, p-value = 0.269					
(ii) social surplus in 6th auction round						
$\bar{D}$	140.1	16.1	147	109	150	15
D	139	18.4	147	100	150	15
$\bar{D}$ vs. D	$H_0 : V_{\bar{D}} = V_D$, WSR:$V = 3$, p-value = 0.371					
(iii) average social surplus per groups						
$\bar{D}$	136.6	7.7	135.0	118.7	147	15
D	138.4	6.2	139.8	125.3	147	15
$\bar{D}$ vs. D	$H_0 : V_{\bar{D}} = V_D$, t-test: t = -1.2674, df = 14, p-value = 0.226					
(iv) average social surplus per rounds						
$\bar{D}$	136.6	6.1	139.2	129.8	148	18
D	138.4	6.3	138.9	122.8	148	18
$\bar{D}$ vs. D	$H_0 : V_{\bar{D}} = V_D$, WSR: V= 46.5, p-value = 0.460					
(v) social surplus						
$\bar{D}$	136.6	19.1	147	101	150	90
D	138.4	18.1	147	100	150	90
$\bar{D}$ vs. D	$H_0 : V_{\bar{D}} = V_D$, WSR: V = 90, p-value = 0.588					

WSR: Wilcoxon signed-ranks test (matched-pairs,two-sided).

Table 5.21: Setting $\bar{D}$ and setting D – comparison of social surplus

In measuring the central tendency between differences of the two samples – the data observed in setting $\bar{D}$ and the respective data observed in setting D – in none of the cases are the differences between the average social surplus in setting $\bar{D}$ and the respective one in setting D significant. The null hypotheses that the social surplus in setting $\bar{D}$ is equal to that in setting D (WSR, matched-pairs, applied in case (i), case (ii), case (iv), and case (v)) can be rejected; the p-value exceeds the weak significance level of $\alpha = 0.1$. Also, in case (iii) applying the t-test (p-value of 0.113) brings out no significant results: the social surpluses observed in the different groups in setting $\bar{D}$ do not significantly differ from the respective social surpluses observed in setting D.

Payoff of designated and non-designated winning bidders in Setting D						
Setting	Mean	Std. dev.	Median	Min	Max	# pairs
average payoff per round						
D	28.7	8.4	29.1	13.0	45.65	18
D_{disc}	38.6	13.8	36.9	20.4	63.84	18
D vs. D_{disc}	$H_0 : P_D \geq P_{D_{disc}}$, t-test (paired): t = -4.578, df = 17, p-value < 0.001					
average payoff per round						
D	28.7	8.4	29.1	13.0	45.65	18
$D_{\overline{disc}}$	16.0	8.3	17.24	0	31.3	18
D vs. $D_{\overline{disc}}$	$H_0 : P_D \leq P_{D_{\overline{disc}}}$, t-test (paired): t = -6.184, df = 17, p-value < 0.001					
average payoff per round						
$D_{\overline{disc}}$	16.0	8.3	17.24	0	31.3	18
D_{disc}	38.6	13.8	36.9	20.4	63.84	18
$D_{\overline{disc}}$ vs. D_{disc}	$H_0 : P_{D_{\overline{disc}}} \leq P_{D_{disc}}$, t-test (paired): t = 7.201, df = 17, p-value < 0.001					

Table 5.22: Setting D_{disc} and setting $D_{\overline{disc}}$ – comparison of winning bidder's payoff

In the following attention is drawn solely to the winning bidder's payoff in setting D (cf. Table 5.22). When comparing the average payoffs per round in the discount auction to the respective payoffs of designated bidders and then to the average payoffs of non-designated bidders per round in the discount auction, the following trend is observed: the average payoff of a designated bidder in setting D is significantly greater than the average payoff of a winning bidder in setting D (t-test with p-value < 0.001). This is quite obvious, since a designated winning bidder receives an additional premium due to the discount and thus can increase his payoff. The null hypothesis that the average payoff of a bidder in the discount auction is lower than the average payoff of a non-designated bidder can be rejected. The difference between the average payoffs is significant (t-test: significant with p-value < 0.001). The same holds when comparing the average payoffs of non-designated bidders to the respective average payoffs of designated bidders; the average payoff of non-designated bidders is significantly below the average payoff of designated bidders (t-test: significant with p-value < 0.001).

5.3 Auction revenues in the symmetric and asymmetric case

Having analyzed the outcomes of the auctions in the previous sections, i.e. the auction revenue, the winning bidder's payoff, and the social surplus in setting $\bar{D}$ and setting D, the focus is now set solely on the auction revenues in the different treatments. Recall that in Section 4.1 the following research questions based on the theoretical findings in Chapter 3 were inspired:

1. In the symmetric case, is a seller able to extract an additional revenue by offering a discount (contrary to the prediction)?

2. In the asymmetric case, can the expectation of a seller – to raise her revenue by offering a discount – be confirmed?

Thus, the question of interest focuses on the single treatments in the symmetric case – treatment $\bar{D}s$ and treatment Ds – and the single treatments in the asymmetric case – treatment $\bar{D}a$ and treatment Da (cf. Section 4.2). To isolate the different treatments, the bids observed in the experimental sessions have to be rearranged and resorted. In the first step, only data from the very first round of all sessions are considered since these observations are independent of each other. In the second step, the analysis is extended and all collected data (observed bids) throughout the conducted auction rounds are considered. The motivation for this step is to include more data in the analysis in order to receive more significant results. Note that this step includes some cost – the cost of losing independence. The observations are not independent since the 540 data points stem from 90 bidders. Thus, the analysis including data from rounds two to six should be interpreted cautiously.

The four treatments $\bar{D}s$, Ds, $\bar{D}a$ and Da are isolated by rearranging the data received in the conducted sessions of either setting $\bar{D}$ or setting D. More specifically, consider a particular round: the 45 bids observed in that round, either in the three sessions of setting $\bar{D}$ or in the respective sessions of setting D, are rearranged: (i) In the symmetric case, 15 homogeneous groups of either solely weak or strong bidders are randomly created, each group consisting of three bidders.[10] Thus in each round, ten weak groups and five strong groups are produced. (ii) In the asymmetric case, 15 heterogeneous groups are virtually created by random: each group consists of one strong bidder and two weak bidders. (iii) In the treatments of the second-price auction – treatment $\bar{D}s$ and treatment $\bar{D}a$ – bids solely derived from the particular auction

[10] A weak bidder is a bidder with an induced low valuation, that is, an integer value between $[100, 109]$. Analogously, a strong bidder is a bidder with a high induced valuation – an integer value drawn from the interval $[146, 150]$ (cf. Section 4.3).

round in all the three conducted sessions of setting $\bar{D}$ are considered.[11] (iv) Treatments Ds and Da are virtually created by regrouping the bids observed in the respective auction round from the three sessions in setting D. Note that in a single auction round over all sessions in setting D, the discount was assigned to a bidder 15 times. That is, out of the 45 observed bids per round, 15 bids are based on a discount and 30 are without discount.[12]

Rearranging the bids allows us to virtually create new groups. In each group a second-price auction and a discount auction are virtually conducted based on the observed bids. In the following sections, the obtained auction revenues in case of symmetries – treatment $\bar{D}s$ and treatment Ds – as well as the auction revenues in case of asymmetries – treatment $\bar{D}a$ and treatment Da – are analyzed.

5.3.1 Treatment $\bar{D}s$ and treatment Ds

To begin of the analysis, Table 5.23 displays the auction revenues of treatment $\bar{D}s$ and treatment Ds achieved in the first round. That is, the average, the standard deviation, the median, the minimum, and the maximum of the revenues of the conducted auctions – 15 conducted second-price auctions in treatment $\bar{D}s$ and 15 conducted discount auctions in treatment Ds with a discount of 20% – are summarized. The observed bids, which are regrouped and used to virtually conduct these auctions, are derived from the first round: observations of setting $\bar{D}$ are used to determine the auction revenues in treatment $\bar{D}s$ and observations of setting D to determine the respective revenues in treatment Ds (cf. Table D.13 for Round 1, Appendix D.4). Furthermore, the auction revenues of the following two cases are contrasted: (i) the revenues based on the experimental observations (’Experiment’) and (ii) the equilibrium revenues as predicted from theory (’Theory’). According to Table 5.23, the discount appears to lower the auction revenue. Focussing on the revenues achieved in the experiment, the average revenue decreases from 111.1 to 97.3 by introducing the discount. This difference is equal to a decrease of 12.42% due to the discount and the strategic behavior of bidders (with a general tendency of underbidding, cf. Section 5.1). Theory predicts that the discount effects the revenue in such a way that the average revenue in the first round decreases by 18.98% when introducing a discount (cf. Table 5.23, row ’Theory’).

[11] In a single round from setting $\bar{D}$, each integer value between $[100, 109]$ and $[146, 150]$ is induced three times. Thus, out of the 45 observed bids per round, 30 bids are based on induced low valuations (weak bids) while 15 are based on high induced valuations (strong bids)

[12] By construction, out of the 15 bids with discount, 10 bids are weak bids with discount and 5 bids are strong bids with discount. Moreover, out of the 30 bids without discount, 20 bids are weak bids and 10 are strong bids. That is, each integer value between $[100, 109]$ and $[146, 150]$ is played once with discount and two times without discount.

The theoretical benchmark 'Theory' refers to the equilibrium revenue of an auction when bidders follow their dominant strategy. When comparing the experimental results to those predicted from theory, the average revenue in the second-price auction in the experiment decreases from the predicted average revenue of 119.1 (100%) to the observed average revenue of 111.1 (93.28%) by 6.72%, whereas the average auction revenue in the discount auction is slightly higher than the predicted average auction revenue: the average revenue increases from the predicted average revenue of 96.5 (100%) to the observed average revenue of 97.3 (100.82%).

Description	Treatment	Mean	Std. dev.	Median	Min	Max
			Revenue			
Experiment	$\bar{D}s$	111.1	22.4	105	80	148
	Ds	97.3	16.4	96	78.9	125.2
Theory	$\bar{D}s$	119.1	21.2	96	102	149
	Ds	96.5	16.6	86.40	81.6	120

Table 5.23: Treatment $\bar{D}s$ and treatment Ds – auction revenues from round 1 (experiment and theoretical benchmark)

In addition, the Wilcoxon signed-ranks test (WSR, matched-pairs, one-sided) is applied to investigate differences in central tendency between the revenues in treatments $\bar{D}s$ and Ds. The test is used to evaluate the null hypothesis that the population median of the paired differences of the auction revenues in the two samples of treatment $\bar{D}s$ and treatment Ds is below zero. Table 5.24 lists the results of the WSR for each conducted auction round.

			Experiment					Theory		
Round	$R_{\bar{D}s}$	R_{Ds}	p-value	V	pairs	$R_{\bar{D}s}$	R_{Ds}	p-value	V	pairs
1	111.1	97.3	0.001586	100	15	119.1	96.5	0.000361	120	15
2	112.5	99.9	0.017670	97	15	118.9	96.6	0.000361	120	15
3	116.8	98.5	0.000031	120	15	118.9	96.6	0.000361	120	15
4	118.0	99.6	0.000092	118	15	119.1	96.2	0.000361	120	15
5	119.0	98.7	0.000031	120	15	119.3	96.7	0.000031	120	15
6	118.8	99.5	0.000427	114	15	118.9	96.3	0.000361	120	15
Total	116.0	98.9	4.652e-14	3824	90	119.0	96.5	$< 2.2\text{e-}16$	4095	90

Wilcoxon signed-ranks test (matched-pairs) with Hypothesis $H_0 : R_{\bar{D}s} \leq R_{Ds}$

Table 5.24: Treatment $\bar{D}s$ and treatment Ds – auction revenues from round 1 to round 6 (experiment and theoretical benchmark)

The test shows that a significant difference between the auction revenues from the second-price auction and the respective auction revenues of the discount auction in the symmetric case can be identified: in each round the auction revenues of treatment $\bar{D}s$ are significantly greater than the respective auction revenues in treatment Ds. More specifically, the WSR shows that in the first round, the p-value is equal to 0.002 (V=100), in round 2 the p-value is equal to 0.02 (V=97) and in all consecutive rounds the p-value is below 0.001 ($V \geq 114$).

The central result of this analysis is that the discount does not pay for the seller: the revenues of the seller in treatment Ds, the discount auction, are significantly lower than the respective revenues in treatment $\bar{D}s$, the second-price auction. This result is also predicted from theory; in equilibrium, offering the discount hurts revenue (significant differences in each round with a p-value < 0.001).

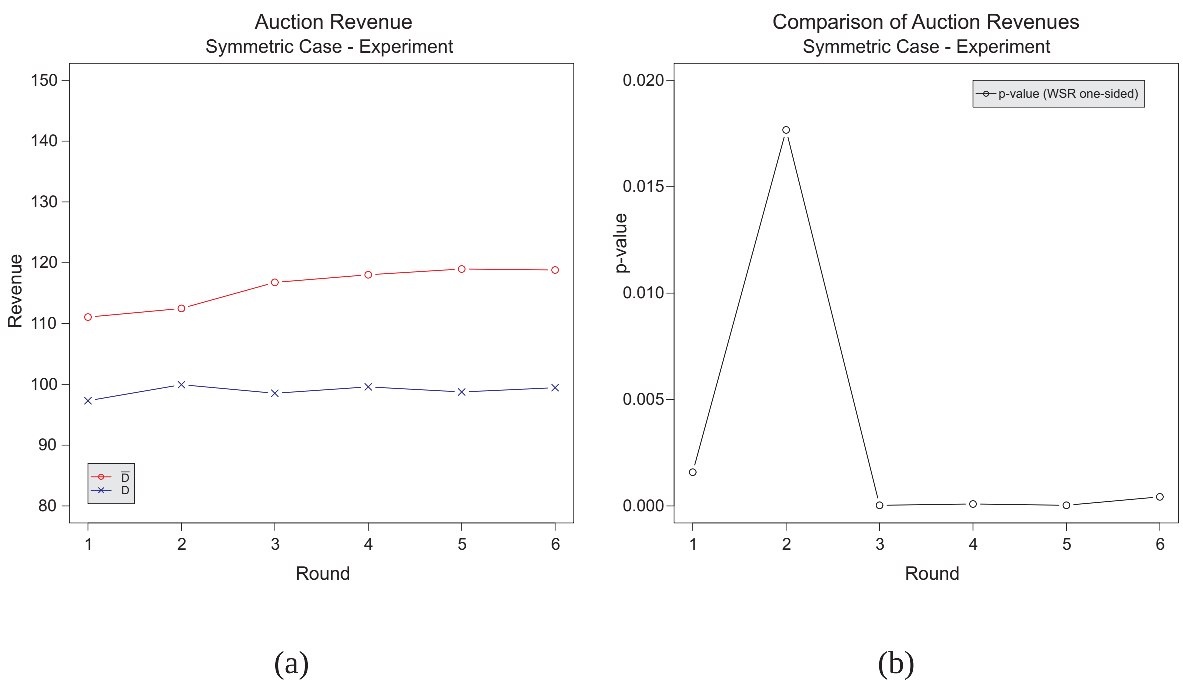
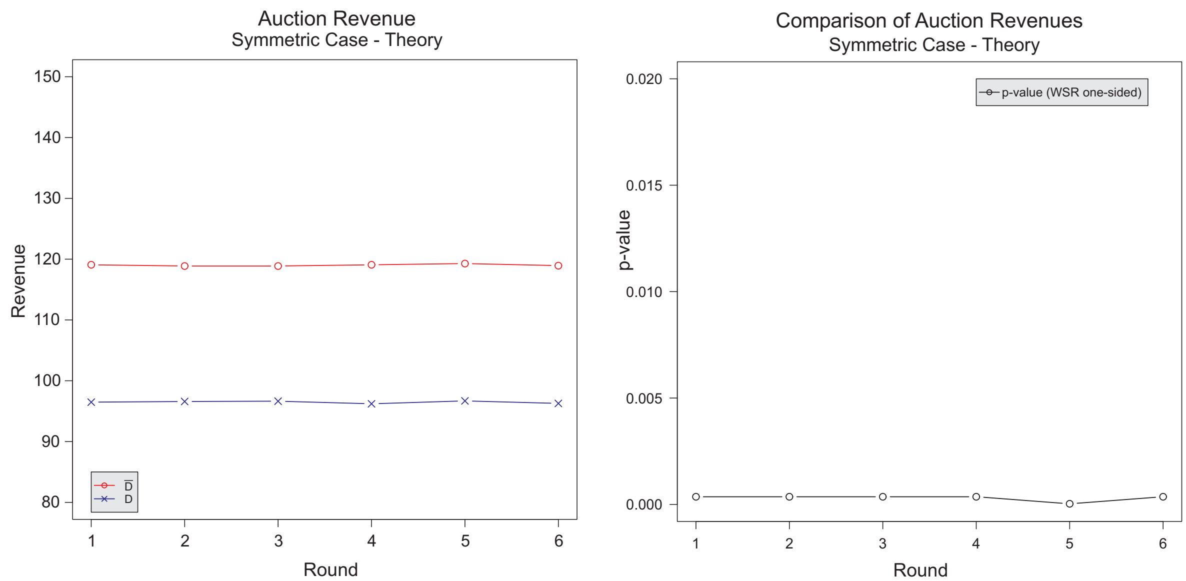


Figure 5.13: Treatment $\bar{D}s$ and treatment Ds – development of auction revenues from round 1 to round 6 (experiment)

The development of the average auction revenues during the course of the experiment and the conducted auction rounds is illustrated in Figure 5.13 and Figure 5.14.[13] Figure 5.13 depicts the development of the average auction revenues over the rounds for the second-price and discount auctions based on the experimental observations. Graph 5.13 (a) illustrates the average auction revenues over the rounds and graph 5.13 (b) plots the results, i.e. the p-value,

[13]The database of Figure 5.13 and Figure 5.14 is listed in Table D.13, in Appendix D.4.



(a) (b)

Figure 5.14: Treatment $\bar{D}s$ and treatment Ds – development of auction revenues from round 1 to round 6 (theoretical benchmark)

derived from the WSR, indicating the significance of the difference in central tendency between the auction revenues of the second-price and discount auctions. Similar results are illustrated in Figure 5.14 focussing on the equilibrium revenues predicted from theory, when bidders behave according to their dominant strategy.

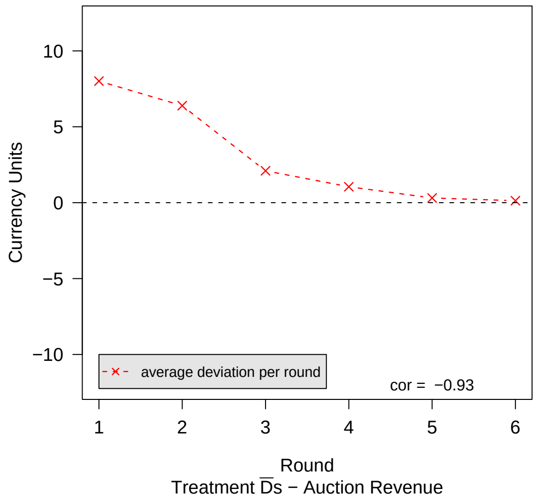
Finally, Table 5.25 depicts the differences between the average auction revenues predicted from theory and the respective observed average auction revenues in the experiment for both treatments – treatment $\bar{D}s$ and treatment Ds – in each round: (i) columns 2-5 list the average revenues of the second-price auction in equilibrium as well as from the experiment and measures the difference between both average revenues (absolute and relative difference); (ii) columns 6-9 indicate the respective results of the discount auction – the average revenues in equilibrium, the observed average revenues, and the absolute and relative differences between both. In treatment $\bar{D}s$ it is observed that in each round the average auction revenue in equilibrium is greater than the respective average revenue derived in the experiment. Moreover, in treatment $\bar{D}s$, the average auction revenue observed in the first round starts at a level above the equilibrium revenue; with an increasing number of conducted auction rounds, the observed average auction revenue converges toward the predicted average auction revenue. This effect results from the general tendency of bidders towards underbidding, that is, to submit bids below their dominant strategy which then lowers the achieved revenues in the experiment. However, throughout the course of the experiment bidders learn their dominant strategy and adapt their behavior. In the treatment of the discount auction, the relation between predicted and observed average auction outcome is the opposite: the average revenue observed in the experiment is greater than the predicted average revenue. This holds for all rounds. Thus, in treatment Ds the respective average auction revenue starts at a level below the equilibrium revenue, and throughout the conducted auction revenues no trend towards the average revenue in equilibrium can be observed. In particular, over all rounds, the average auction revenue is below the average revenue in equilibrium. Naturally this effect is also a direct result of the general tendency to underbid observed in setting D throughout the course of the experiment. In setting D, however, bidders with discount deviated from the dominant strategy such that the probability to win the auction increased for bidders without discount, leading at the same time to higher revenues.

	$R_{\bar{D}s}$		Deviation		R_{Ds}		Deviation	
Round	Theo.	Exp.	abs.	rel.	Theo.	Exp.	abs.	rel.
1	119.1	111.1	8.01	6.73%	96.5	97.3	-0.82	-0.85%
2	118.9	112.5	6.39	5.38%	96.6	99.9	-3.35	-3.47%
3	118.9	116.8	2.10	1.77%	96.6	98.5	-1.89	-1.96%
4	119.1	118.0	1.05	0.88%	96.2	99.6	-3.37	-3.50%
5	119.3	119.0	0.31	0.26%	96.7	98.7	-2.05	-2.12%
6	118.9	118.8	0.13	0.11%	96.3	99.5	-3.18	-3.30%
mean	119.0	116.0	3.0	2.52%	96.5	98.9	-2.44	-2.53%

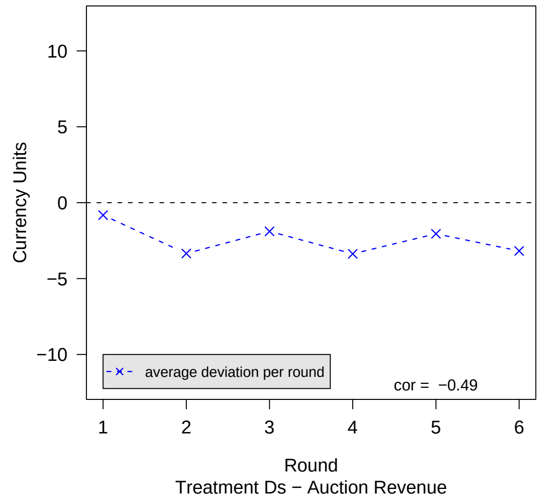
Table 5.25: Treatment $\bar{D}s$ and treatment Ds – average deviation of auction revenues from theory in round 1 to round 6

Figure 5.15 displays the difference between the average revenue in equilibrium and the observed average revenue per round. In addition, the Spearman's rank correlation coefficient cor of the observed differences versus the round number is indicated. This coefficient is negative for both auction formats – thus, there is an downward trend. From intuition a general tendency towards the average revenue in equilibrium with an increasing number of conducted auction rounds was expected for both auction formats. In fact, over the consecutive rounds, in treatment $\bar{D}s$ the average revenue appears to converge towards the average revenue in equilibrium, whereas in treatment Ds such a trend is not observed.

So far the auction revenues in treatment $\bar{D}s$ and the respective ones in treatment Ds based on the 45 bids observed in each conducted auction round of setting $\bar{D}$ and of setting D have been analyzed. Moreover, the focus was placed on the development of the average auction



(a)



(b)

Figure 5.15: Treatment $\bar{D}s$ and treatment Ds – average deviation of auction revenues from theoretical benchmark in round 1 to round 6

revenues over the course of the experiment – significance in difference between the observed average revenues and the predicted average revenues in each round was measured. In the analysis, the available data of each auction round, i.e. the 45 observed bids from either setting $\bar{D}$ or setting D, were fully exhausted. In each round, 15 groups were virtually created: each group consists of three bidders (bidders with their induced valuations and their submitted bids). In each group a second-price auction and a discount auction was conducted. That is, for each auction format (i) five auctions with solely strong bidders (high value bidders) and (ii) ten auctions with solely weak bidders (low value bidders) were virtually conducted, leading to the auction revenues of treatment $\bar{D}s$ and treatment Ds. Moreover, in case of treatment Ds, out of the 15 conducted auctions per round, the discount was awarded five times to a strong bidder and ten times to a weak bidder.

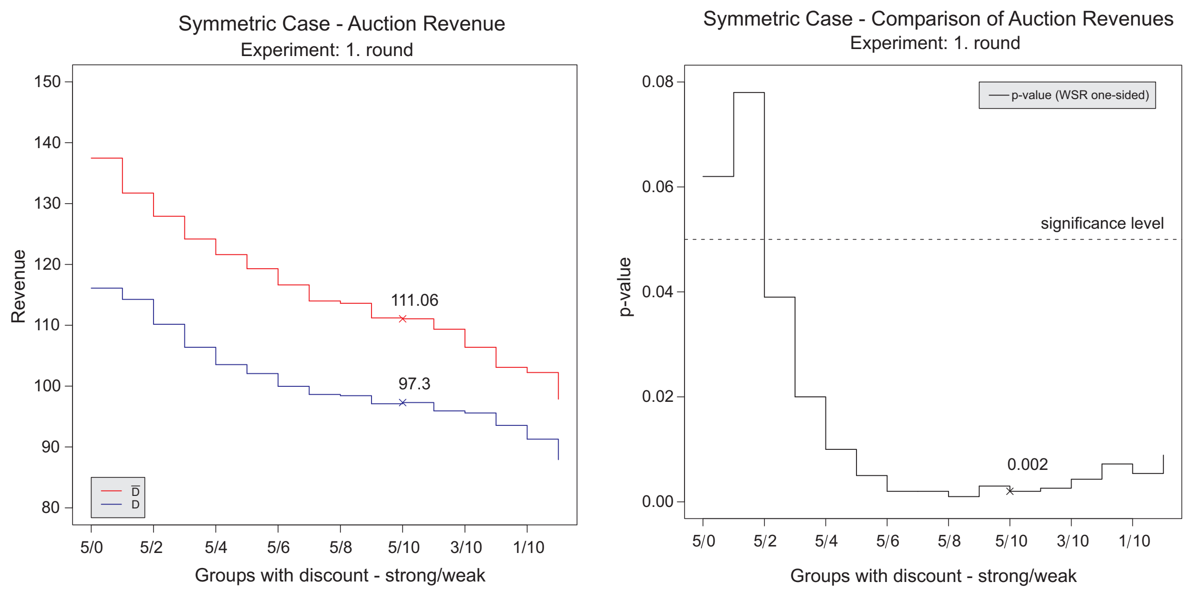
In the following, '$\#strong$' denotes the number of strong bidders and '$\#weak$' the number of weak bidders to whom the discount is assigned. Note that in each group solely one bidder is chosen as designated bidder to whom the discount is assigned. The proportion of $\#strong$ to $\#weak$ is denoted by $q = \frac{\#strong}{\#weak}$. So far, the proportion q was held constantly to 5/10 = 1/2. In the following analysis, the proportion is changed and the impact on the auction revenues in treatment $\bar{D}s$ and treatment Ds will be measured.

In the first step, the number of strong designated bidders is held constantly to 5 while the number of weak designated bidders increases from 0 to 10; in the second step, the number of weak designated bidders is held constantly to 10 while the number of strong designated bidders decreases from 4 to 0. In the first step, q decreases from 5/0 to 5/10. In q=5/0 the discount auction is conducted five times, each time with a group consisting of strong bidders, and within each group a strong bidder is awarded the discount. The average revenue over the 5 conducted auctions is calculated. The number of groups in which a weak bidder is chosen as designated bidder is increased successively from 1 to 10, and in point 5/10 the average revenue over 15 conducted discount auctions is calculated. In the second step, q decreases from 4/10 to 0/10. For q=4/10 the discount auction is conducted 14 times: four times with solely strong groups in which a strong bidder is selected as designated bidder, and ten times with only weak groups in which a weak bidder is selected as designated bidder. Again, the average auction revenue over the 14 auction revenues is calculated. The number of groups with a strong designated bidder successively decreases from 4 to 0, resulting in point $q =$0/10.[14] Note that the constellation of the groups for conducting the discount auction by varying q is the same constellation of groups for conducting the second-price auction. This means that for each q the induced valuations to the bidders within the groups are the same for both treatments – for the virtually conducted discount auction and the virtually conducted second-price auction.

Figure 5.16 and Figure 5.17 display the average auction revenues of the discount auction by varying q and also display the respective average auction revenues of the second-price auction.[15] The x-axis indicates the proportion q varying from 5/0 to 5/10 and then varying from 4/10 to 0/10; altogether, there are 16 q-values. Figure 5.16 displays the average auction revenues based on the experimental results (graph (a)) and plots the p-value derived from the application of the WSR, testing the null hypotheses whether the revenues in the second-price auction are lower than or equal to the revenues in the discount auction. The graphical representation is helpful for a direct comparison of the revenues in treatment $\bar{D}s$ and treatment Ds in dependence of q. Figure 5.16 (a) shows that the curve of the average auction revenues in the discount auction lies below the curve of the average auction revenues of the second-price auction. This suggests that the discount lowers the revenue of the seller, independent of

[14] In point q=5/10 the number of groups with a weak designated bidder is twice high as the number of groups with a strong designated bidder (cf. Table 5.24). In this case, all observed data from a single auction round are exhausted. In all other cases, not all observations available from the particular round are fully used in the analysis.

[15] The average auction revenues in treatment $\bar{D}s$ and treatment Ds are calculated for different proportions of q. The experimental as well as the equilibrium revenues of the first round are listed in Tables D.15 and D.16, Appendix D.4.



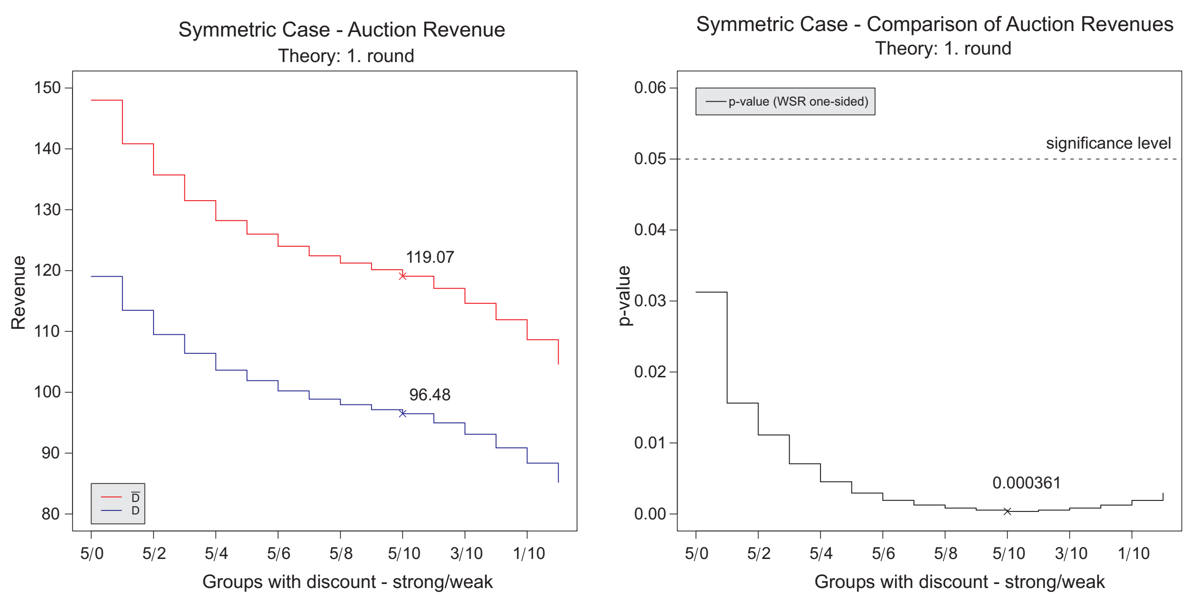
(a) (b)

Figure 5.16: Treatment $\bar{D}s$ and treatment Ds – development of auction revenues in round 1 for different proportions q (experiment)

the proportion q. In fact, the difference is (weakly) significant: the revenues of the discount auction are (weakly) significantly lower than the revenues achieved in the second-price auction (WSR, one-sided, p-value < 0.08). Figure 5.16 (b) plots the results of the WSR. The difference between the revenues of the second-price auction and the respective revenues of the discount auctions is weakly significant for all proportions q. For q below 5/3 the revenues of the second-price auction are significantly higher than the respective revenues of the discount auction.[16] This result indicates that, for the case at hand, the discount hurts revenue whenever bidders are symmetric and indistinguishable. Even when changing the proportion of strong designated and weak designated bidders, the major result is not affected: the revenues achieved in treatment Ds are below the revenues achieved in treatment $\bar{D}s$. The difference between the revenues strongly depends on the assigned discount. Note that in our case, a discount of 20% was awarded and the average revenue of the discount auction is 14.74% below the average revenue of the second-price auction (cf. Table 5.24, row 'Total').

Similar observations can be made with respect to the equilibrium outcomes. In Figure 5.17 the equilibrium outcomes (graph (a)) as well as the results of the WSR (graph (b)) are displayed. Graphs (a) and (b) indicate that when bidders behave according to their domi-

[16] Note that for $q = 5/10$ the results have already been discussed and presented in Table 5.24, round 1.



(a) (b)

Figure 5.17: Treatment $\bar{D}s$ and treatment Ds – development of auction revenues in round 1 for different proportions q (theoretical benchmark)

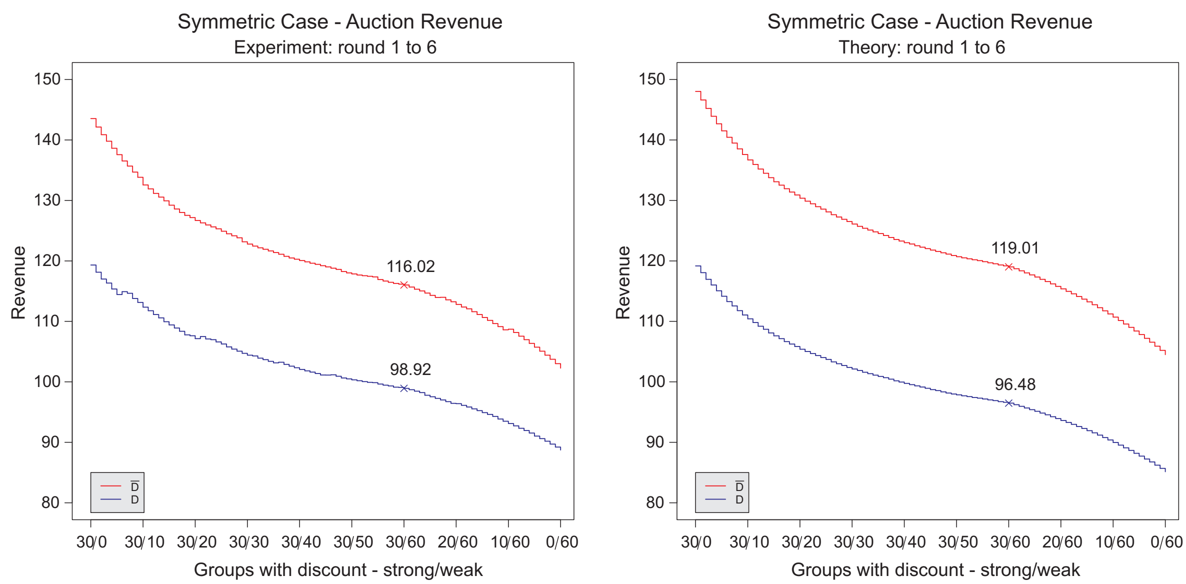
nant strategy, significance for the difference between the auction revenues of the second-price auction and the respective revenues of the discount auction increases.[17]

When considering the observed bids in all conducted auction rounds, the number of groups and virtually conducted auctions increases from 15 groups and 15 conducted auctions to 90 groups and 90 conducted auctions. That is, in treatment $\bar{D}s$, the second-price auction is virtually conducted 90 times based on the observations, and in treatment Ds, the discount auction is virtually conducted 90 times. With the increasing number of groups, the number of proportion also increases: $\#strong$ ranges between 0 and 30, that is, the discount can be assigned at most 30 times to a strong bidder; and $\#weak$ ranges between 0 and 60, that is, at most 60 groups exist in which the discount is assigned to a weak bidder. Thus, the proportion ranges from q=30/0 to q=30/60 and then from q=29/60 to q=0/60. Overall, 91 proportions have to be considered. Note that in point q=30/60 all bids observed in the experiment are fully exhausted. Figure 5.18 presents the graphs of the auction revenues achieved in the experiment and the equilibrium auction revenues.

As displayed in Figure 5.18 the curve of the auction revenues in treatment Ds lies below the curve of the auction revenues in treatment $\bar{D}s$. In fact, testing the difference in auction

[17] For $q = 5/10$ the results have already been listed in Table 5.24, round 1.

revenues of both treatments with the WSR shows that the difference is significant, with a p-value < 0.001.[18] Again, the difference between the achieved average revenues in treatment $\bar{D}s$ and treatment Ds strongly depends on the amount of the discount. In equilibrium, the average revenue of treatment Ds is 18.91% below the average revenue of treatment $\bar{D}s$ (cf. Table 5.24, row 'Total').



(a) (b)

Figure 5.18: Treatment $\bar{D}s$ and treatment Ds – development of auction revenues from round 1 to round 6 for different proportions q (experiment and theoretical benchmark)

[18]The average auction revenues in treatment $\bar{D}s$ and treatment Ds are calculated for the different proportions of q. The experimental revenues as well as the equilibrium revenues of all conducted auction rounds are listed in Tables D.17 and D.18, Appendix D.4.

5.3.2 Treatment $\bar{D}a$ and treatment Da

After the analysis of the symmetric case, this section presents the results of the (virtually) conducted second-price auctions and the respective discount auctions under asymmetries. That is, the auction revenues of the conducted auctions in treatment $\bar{D}a$ and treatment Da are investigated and compared. Furthermore, as in the symmetric case, the auction results based on the observed bids in the experiment as well as the auction results in equilibrium are analyzed.

In order to derive the auction revenues from the experimental data for each conducted round, the 45 observations (the observed bids) from setting $\bar{D}$ and the 45 observations from setting D are randomly rearranged. For each round, this results in 15 groups in treatment $\bar{D}a$ and 15 groups in treatment Da, each group consisting of three bidders: one strong bidder and two weak bidders with the respective observations. In each group a second-price auction is conducted, based on the observations of setting $\bar{D}$, and a discount auction is virtually conducted, based on the observations of setting D. Table 5.26 summarizes the results of the 15 conducted auctions derived in the very first auction round in both treatments – treatment $\bar{D}a$ and treatment Da. More specifically, the experimental results as well as the results in equilibrium are listed: the mean, the standard deviation, the median, the minimum, and the maximum over the 15 conducted second-price auctions and the 15 conducted discount auctions in the very first round (cf. Table D.14 for Round 1, Appendix D.4).

Description	Treatment	Mean	Std. dev.	Median	Min	Max
			Revenue			
Experiment	$\bar{D}a$	103.8	14.5	103	85	149
	Da	104.1	22.4	107	64.0	140
Theory	$\bar{D}a$	106.8	1.8	107	104	109
	Da	115.6	22.3	127.5	83.2	136.6

Table 5.26: Treatment $\bar{D}a$ and treatment Da – auction revenues from round 1

Table 5.26 shows that in the first round conducted, the average revenue of 103.8 achieved in the second-price auction is lower than the average revenue of 104.1 in the discount auction. Even when comparing the median of the 15 auction revenues in both auction formats, the same result is achieved: the median (103) of the 15 revenues of the second-price auction is below the median (107) of the 15 revenues of the discount auction. This suggests that there is a difference in central tendency between the auction revenues achieved in the different treatments. In fact, the difference in central tendency is insignificant for the first round. When applying the Wilcoxon signed-ranks test (matched-pairs, one-sided), a short WSR reveals that the null hypothesis cannot be rejected with a p-value of 0.402 and V equal to 55 (cf. Table 5.27). In equilibrium, the average revenue of 106.8 (and the median of 107) over the 15 conducted second-price auctions is lower than the respective average revenue of 115.6 (and the median of 127.5) of the 15 conducted discount auctions. The difference in central tendency is significant (WSR, matched-pairs, one-sided with p-value = 0.047 and $V = 30$), meaning that in equilibrium the discount pays for the seller (cf. Table 5.27).

Comparing the experimental results and the results predicted from theory, it then appears that in both treatments the average revenues are below the average revenues predicted from theory. In treatment $\bar{D}a$ the average revenue in the experiment is 2.81% below the average predicted revenue, while in treatment Da the average revenue in the experiment is almost 10% (9.95%) below the predicted average revenue. In both treatments, these differences can be explained by the fact that in both settings of the conducted experiment, bidders tend to underbid. The effect of underbidding was even higher in setting D than in setting $\bar{D}$ (cf. Section 5.1). However, when focussing on the experimental results of the average auction revenues achieved in the first round, the extraordinary result is that the average revenue derived in the discount auction is higher than the average revenue received in the second-price auction.

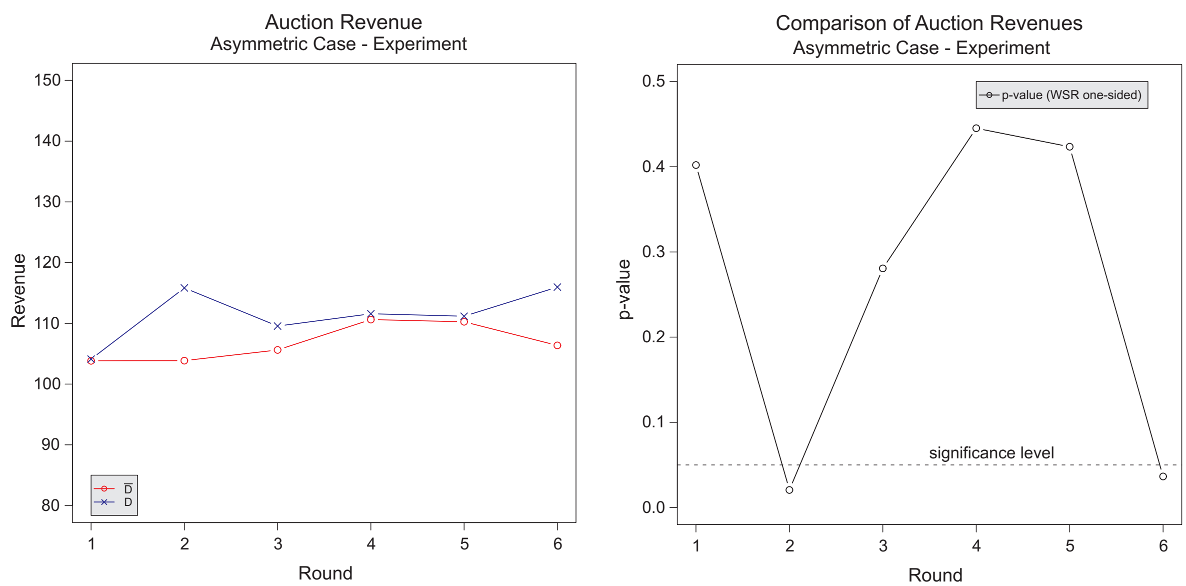
	Experiment					Theory				
Round	$R_{\bar{D}a}$	R_{Da}	p-value	V	pairs	$R_{\bar{D}a}$	R_{Da}	p-value	V	pairs
1	103.8	104.1	0.402	55	15	106.8	115.6	0.047	30.0	15
2	103.8	115.8	0.021	24	15	106.6	115.5	0.024	25.0	15
3	105.6	109.6	0.281	49	15	107.0	115.6	0.050	30.5	15
4	110.6	111.6	0.445	57	15	106.2	115.4	0.025	25.0	15
5	110.3	111.2	0.423	56	15	107.0	115.6	0.084	35.0	15
6	106.4	116.0	0.036	28	15	106.0	15.4	0.007	16.0	15
Total	106.8	111.4	0.012	1482.5	90	106.6	115.5	1.500e-06	886.5	90

Wilcoxon signed-ranks test (matched-pairs) with Hypothesis $H_0 : R_{\bar{D}a} \geq R_{Da}$

Table 5.27: Treatment $\bar{D}a$ and treatment Da – auction revenues from round 1 to round 6

Table 5.27 shows the development of the auction revenues in treatment $\bar{D}a$ and treatment Da throughout the course of the experiment and lists the average revenues of both auction formats. Furthermore, the respective average revenues in equilibrium achieved in each round are indicated. The following experimental results are interesting: (i) in each round the average revenue in treatment $\bar{D}a$ is lower than the average revenue in treatment Da; (ii) the difference in revenues is significant only in round two and round six; and (iii) in total, when comparing all 90 revenues from treatment $\bar{D}a$ and treatment Da, the difference is significant. When applying the WSR (matched-pairs, one-sided), then the difference in round two between the revenues of the two auction formats is significant, with a p-value equal to 0.021 and V=24; in round six the WSR (matched-pairs, one-sided) results in a p-value of 0.036 and V=36. Consequently, the discount auction achieves higher revenues than the second-price auction in case of asymmetries. When comparing the revenues in round two and round six as well as comparing all revenues over all rounds, significant differences in central tendency between the observed average revenues in treatment $\bar{D}a$ and treatment Da are observed.

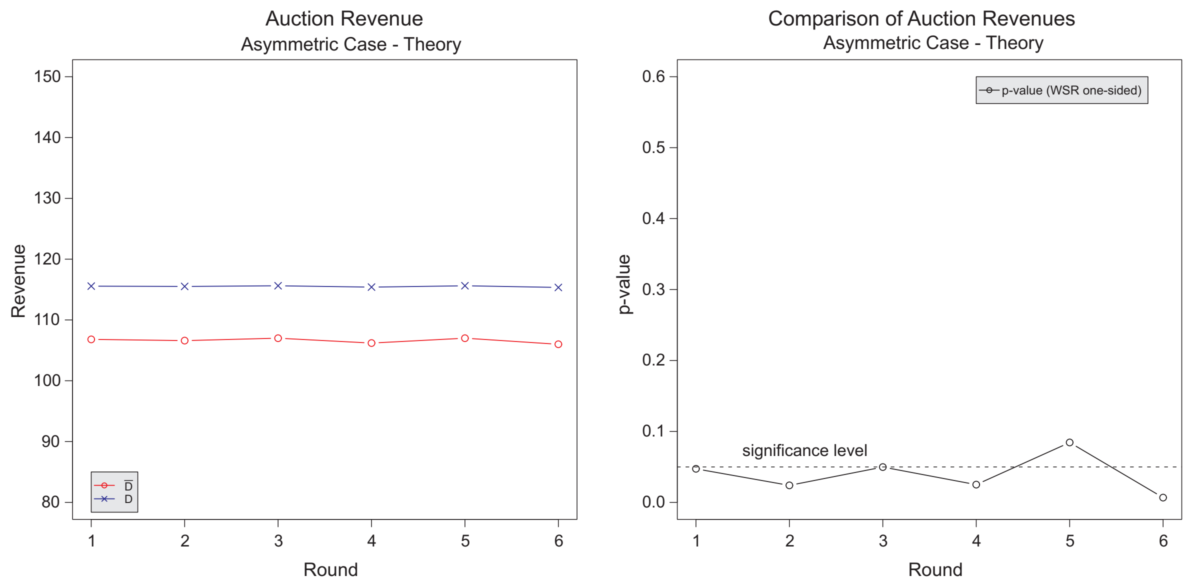
Similar observations are made with respect to the auction revenues in equilibrium as shown in Table 5.27. Indeed, the average revenue in treatment Da is higher than the average revenue achieved in treatment $\bar{D}a$ in each round. Furthermore, as the statistic analysis shows, the difference for each round is (weakly) significant: In all rounds with the exception of round five, the WSR (matched-pairs, one-sided) results in a p-value below the significance level of 0.05; only in round five is the p-value below the weak significance level of 0.1.



(a) (b)

Figure 5.19: Treatment $\bar{D}a$ and treatment Da – development of auction revenues from round 1 to round 6 (experiment)

To illustrate the development of the auction revenues throughout the course of the experiment, the average revenues as well as the results of the WSR (matched-pairs, one-sided), i.e. the p-value, are plotted against the auction rounds. Figure 5.19 illustrates in graph (a) the curves of the average auction revenues of treatment $\bar{D}a$ and the curve of the respective revenues of treatment Da over the rounds, and in graph (b) the curve of the p-value, indicating



(a) (b)

Figure 5.20: Treatment $\bar{D}a$ and treatment Da – development of auction revenues from round 1 to round 6 (theoretical benchmark)

the significance of the difference in central tendency for each round. Analogously, in equilibrium the respective curves are plotted and indicated in Figure 5.20. As can be easily seen, the curves are more smooth in equilibrium, without any distortions. That is, the absolute difference between the average revenues of both treatments ranges between 8.5 and 9.5, whereas in the experiment the absolute difference ranges between 0.4 and 12. It is interesting that in the last round of the experiment, bidders submit bids close to equilibrium such that the revenues achieved in the single treatments are close to equilibrium (cf. Section 5.1.3).

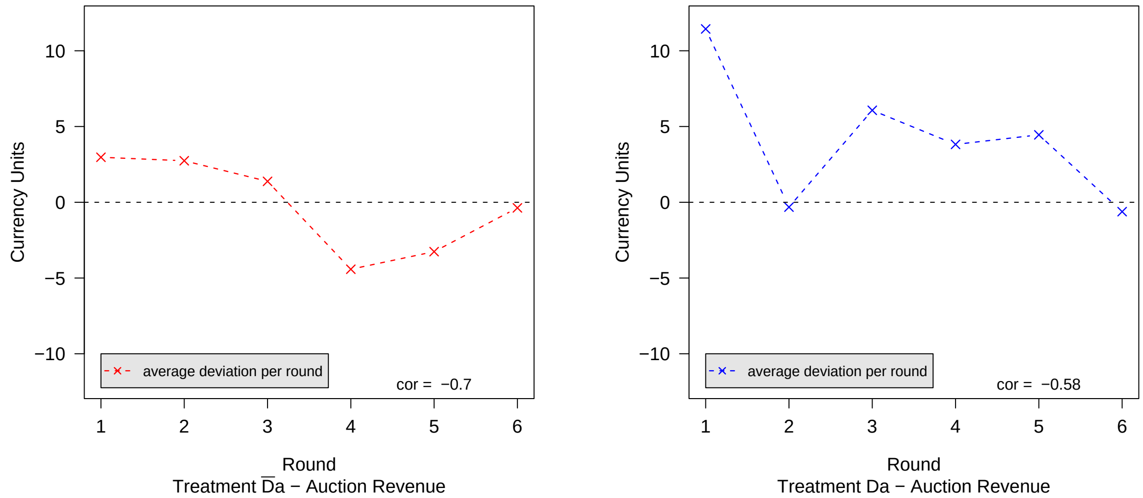
In fact, when measuring the absolute (for short 'abs.') and relative (for short 'rel.') deviations of the achieved revenues from the respective revenues in equilibrium, the absolute (relative) deviation in the last round in treatment $\bar{D}a$ is on average equal to -0.37 (-0.35%) and -0.62 (-0.54%) in treatment Da. When looking more closely at the absolute deviation, Table 5.28 reveals that in treatment $\bar{D}a$ the equilibrium average revenue starts above the experimental average revenue with a negative trend over the first four rounds; over the last two rounds a positive trend is identified. Thus, in treatment $\bar{D}a$, from round one to round four the average auction revenues derived in the experiment are below the respective average revenues in equilibrium. Recall that especially in the first rounds of the conducted second-price auction, a high frequency of underbidding was identified, while throughout the course of the experiment, learning effects were identified. In round six the deviation is below zero, meaning that in the very last round the average revenue of the second-price auction in the experiment is below the respective average revenue in equilibrium. The Spearman's rank correlation coefficient is negative: $cor = -0.7$ (treatment $\bar{D}a$).

	$R_{\bar{D}a}$		Deviation		R_{Da}		Deviation	
Round	Theo.	Exp.	abs.	rel.	Theo.	Exp.	abs.	rel.
1	106.8	103.8	2.97	2.78%	115.6	104.1	11.44	9.90%
2	106.6	103.8	2.74	2.57%	115.5	115.8	-0.32	-0.28%
3	107.0	105.6	1.38	1.29%	115.6	109.6	6.07	5.25%
4	106.2	110.6	-4.42	-4.16%	115.4	111.6	3.82	3.31%
5	107.0	110.3	-3.26	-3.05%	115.6	111.2	4.45	3.85%
6	106.0	106.4	-0.37	-0.35%	115.4	116.0	-0.62	-0.54%
mean	106.6	106.8	-0.16	-0.15%	115.5	111.4	4.41	3.58%

Table 5.28: Treatment $\bar{D}a$ and treatment Da – average deviation of auction revenues from theory in round 1 to round 6

The curve of the absolute deviations over the six rounds and the correlation coefficient in treatment $\bar{D}a$ are displayed in Figure 5.21 (a). In analogy, Figure 5.21 (b) shows the development of the absolute deviation over rounds in the case of treatment Da. In the first, third, fourth and fifth rounds the average auction revenue in equilibrium is above the experimental average auction revenue of the respective round. In rounds two and six, the opposite is observed: in both rounds, the average auction revenue derived in the experiment is slightly higher than the average auction revenue in equilibrium. The Spearman's rank correlation coefficient is equal to -0.56 as indicated in Figure 5.21 (b).

As in the symmetric case discussed in the previous section, the proportion of groups with a strong designated bidder and groups with a weak designated bidder will be varied in treatment Da as well. Recall that from the 45 observed bids in a particular round in setting D, 15 bids are obtained from designated bidders – 5 bids are submitted by strong designated bidders and 10 bids are submitted by weak designated bidders – while 30 bids are obtained from non-designated bidders – 10 bids care made by strong bidders and 20 bids are submitted by weak bidders. The bids with the respective induced valuations are rearranged and new groups are created; regarding groups of bidders who submit these bids, each group comprises a strong bidder and two weak bidders. This results in at most 5 groups with a strong designated bidder and 10 groups with a weak designated bidder. When focussing on a particular auction round in treatment Da, 15 groups are randomly created, each group conducting a discount auction. The same grouping was then used for conducting the fifteen respective auctions in treatment



(a) (b)

Figure 5.21: Treatment $\bar{D}a$ and treatment Da – average deviation of auction revenues from theoretical benchmark in round 1 to round 6

$\bar{D}a$ for that particular round. The only difference is that the auctions conducted in treatment $\bar{D}a$ are based solely on observations from setting $\bar{D}$, while the auctions conducted in treatment Da are solely based on the observed bids in setting D. In the analysis so far, the data obtained in each round from either setting $\bar{D}$ or setting D were fully exhausted; in each round 15 groups were virtually created.

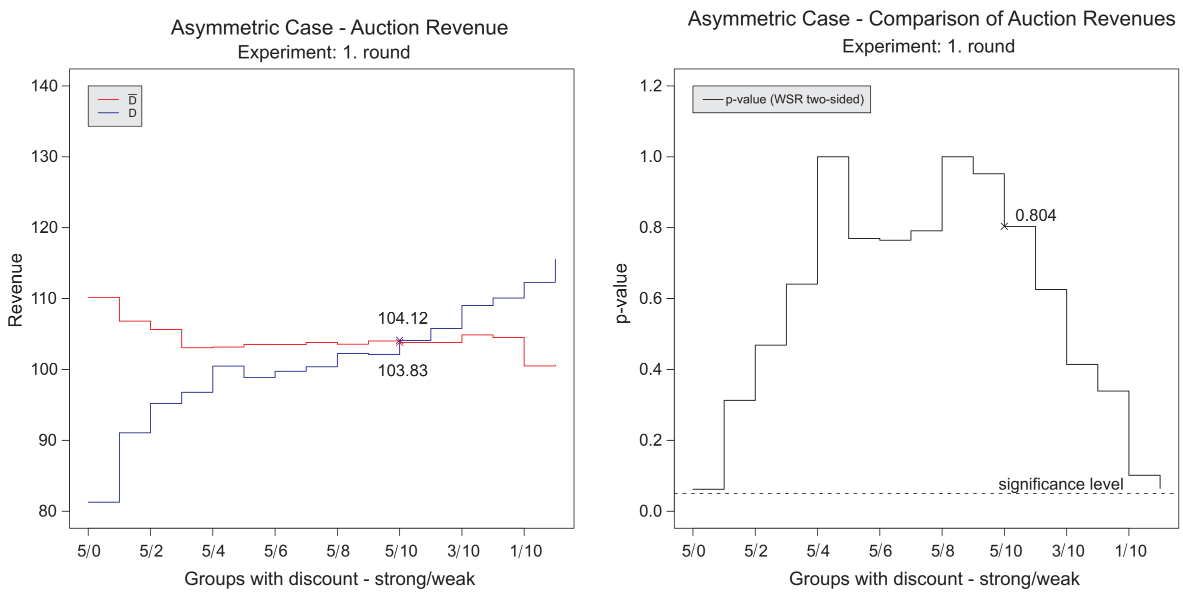
In the following, the proportion of groups with strong designated bidders and groups with weak designated bidders in the case of treatment Da will be varied. So far this proportion was equal to 5/10=1/2 for each round, particularly for the first round, meaning 5 groups with strong designated bidders and 10 groups with weak designated bidders were created. $q = \frac{\#strong}{\#weak}$ defines this proportion with '$\#strong'$ being the number of groups with a strong designated bidder, and '$\#weak$' being the number of groups with a weak designated bidder. In each group one bidder at most is selected to whom the discount is assigned. In subsequent steps, the proportion is changed, and a discount and second-price auction for the received constellation of groups will be conducted. For each proportion and each group, the achieved revenues for both auctions are calculated; the average auction revenue over all groups for the played auction format is also measured.

Varying $\#weak$ in a first step from 0 to 10 while holding $\#strong$ constant at 5, and then reducing $\#strong$ in a second step from 4 to 0 while holding $\#weak$ constant at the level 10, results in the following data points for q: $5/0, 5/1, 5/2, \ldots, 5/10, 4/10, 3/10, \ldots, 0/10$. Overall, 16 data points are calculated. For example, in point 5/0, five groups are randomly created (each group with one strong bidder and two weak bidders): in treatment Da a discount auction is conducted in each group and the discount awarded to a strong designated bidder in each group. In each discount auction, the strong bidder wins the auction in equilibrium and pays the price for the discounted second-highest bid, which is submitted by a weak bidder. The revenues as well as the average revenues achieved in treatment Da are measured. Then, five second-price auctions are conducted with the same groups – these results are measured as well. In the case of the second-price auction, the strong bidder is the high bidder, purchasing the object at the price of the second-highest bid submitted by a weak bidder. Increasing now the number of groups with a weak designated bidder from 0 to 10 raises the overall number of groups from 5 to 15. The proportion q decreases from 5/0=Inf. ('Infinity') to 5/10=1/2. Point q=5/10 is equal to the case indicated in Table 5.27. The number of groups with a weak designated bidder is twice as high as the number of groups with a strong designated bidder. Reducing the number '$\#strong$' from 5 to 0 while the number $\#weak$ is constantly held at 10 results in the proportion q =0/10=0, that is, the point in which the discount is only assigned to weak bidders in each of the ten heterogeneous groups.

Focussing solely on the first auction round in treatments $\bar{D}a$ and Da and now varying the proportion of q, the auction revenues in both treatments can be calculated for each q. Additionally, the average revenues achieved over all groups, i.e. the average revenue of the conducted discount auctions and the average revenue of the conducted second-price auctions, are calculated for each q. Then the revenues of treatment $\bar{D}a$ are contrasted to the revenues of treatment Da for each q: the WSR (matched-pairs, two-sided) is applied to test differences in central tendency between the revenues in treatment $\bar{D}a$ and treatment Da. Figure 5.22 and Figure 5.23 illustrate the experimental results as well as the results in equilibrium based on the data of the first conducted auction round. In both figures, graph (a) displays the average auction revenue of the second-price auction and the discount auction over q, while graph (b) plots the results of the applied WSR (matched-pairs, two-sided), i.e. the p-value, showing significance in the difference in central tendency between the revenues of the second-price auction and the respective revenues of the discount auction for each q.[19]

The graphical representation allows a direct comparison of the revenues in treatment $\bar{D}a$

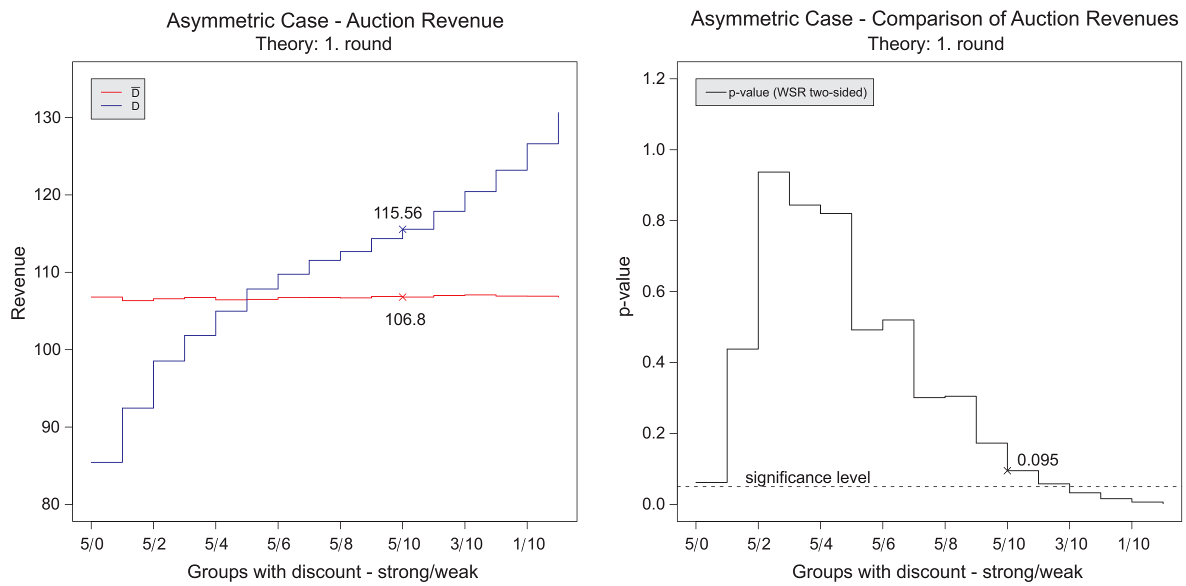
[19] The experimental revenues as well as the revenues in equilibrium from the first round achieved in treatment $\bar{D}a$ and treatment Da are listed in Table D.19, Appendix D.4. Furthermore, the average auction revenues received in the first round from treatment $\bar{D}a$ and treatment Da are calculated for different proportions of q (Table D.20, Appendix Appendix D.4).



(a) (b)

Figure 5.22: Treatment $\bar{D}a$ and treatment Da – development of auction revenues in the round 1 for different proportions q (experiment)

and treatment Da dependent on q. Figure 5.22 shows that for all q >5/10 the average auction revenue of the discount auction is below the average auction revenue of the second-price auction. That is, for each q >5/10, the discount on average does not pay for the seller. In particular, assigning the discount solely to strong bidders and not weak bidders (q =5/0) at most lowers the average revenue in the discount auction compared to the second-price auction. Increasing the number of groups in which the discount is awarded to weak bidders increases the average revenue of the discount auction. Note that in the case of the second-price auction, it is predicted that the high value bidder, i.e. the strong bidder, wins the auction and pays the price of the second-highest bid submitted by a weak bidder. Thus, the achieved average revenue in the second-price auction is almost constant (equal to the first order statistic of integer values [100,109]). Deviations from the predicted value of about 106.6 stem from noise in bidding behavior; in addition, a tendency to underbid was observed in the second-price auction. However, measuring significance shows that the difference in central tendency is not significant; only for q =5/0 is the difference weakly significant (cf. Figure 5.22 (b)). It is interesting that when the number of weak bidders with an assigned discount increases (and the number of strong bidders with an assigned discount decreases) the curve of the discount auction rises: for $q \leq 5/10$ the curve lies above the respective curve of the second-price auction. That is,



(a) (b)

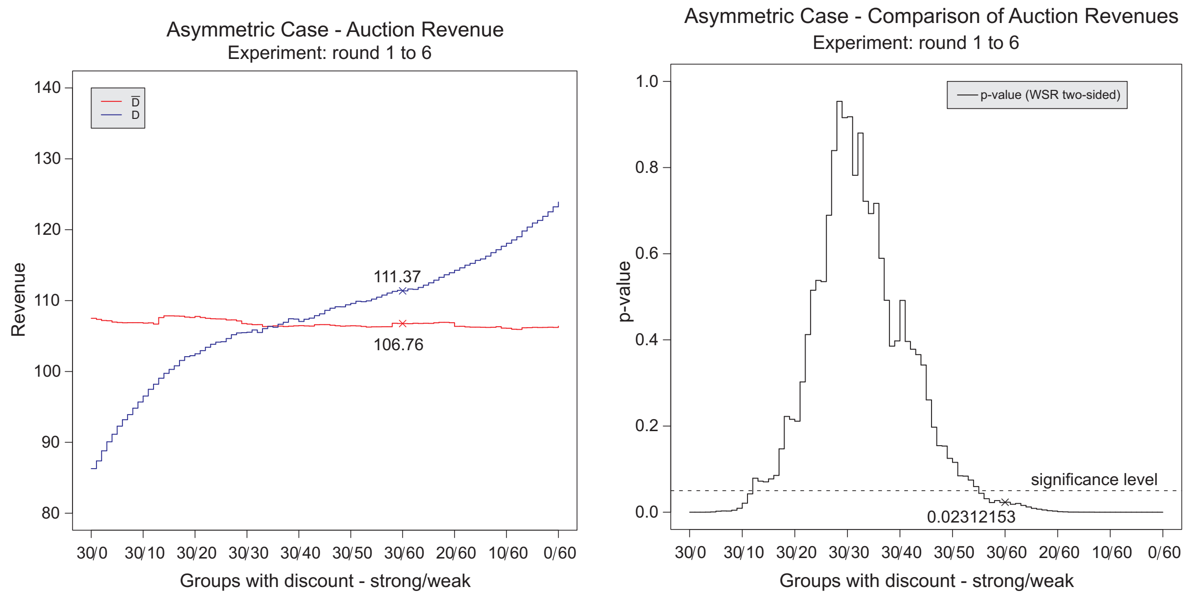
Figure 5.23: Treatment $\bar{D}a$ and treatment Da – development of auction revenues in round 1 for different proportions q (theoretical benchmark)

awarding the discount more often to weak bidders or economic disadvantaged people than to strong bidders results in the discount paying. This is exactly the case illustrated here. Particularly in q =0/10, the difference between the revenues of both treatments is weakly significant. Note that the average auction revenues as well as the result of the WSR as depicted in Figure 5.22 and Figure 5.23 for q =5/10 have already been indicated in Table 5.27.

In equilibrium, the average auction revenue achieved for each proportion q in the first round in treatment $\bar{D}a$ is approximately 106.8 (cf. Figure 5.23 and Table 5.27). Similar to the experiment, a general trend towards underbidding is observed, with the curve of the average revenues of treatment $\bar{D}a$ above the respective curve based on the experimental observations. In the case of the discount auction, the curve of the auction revenues for treatment Da in equilibrium shows a stronger upward trend than the respective curve based on the experimental observation. Additionally, the difference between the revenues in the discount auction and those derived in the second-price auction is maximized for the proportions q =5/0 and q =0/10. In contrast to the revenues based in the experiment, the difference is now significant (WSR, matched-pairs, two-sided) for q <4/10 and weakly significant for q =5/0 and q <5/9. This indicates that under asymmetries the discount might pay for the seller, particularly when the discount is assigned to weak bidders.

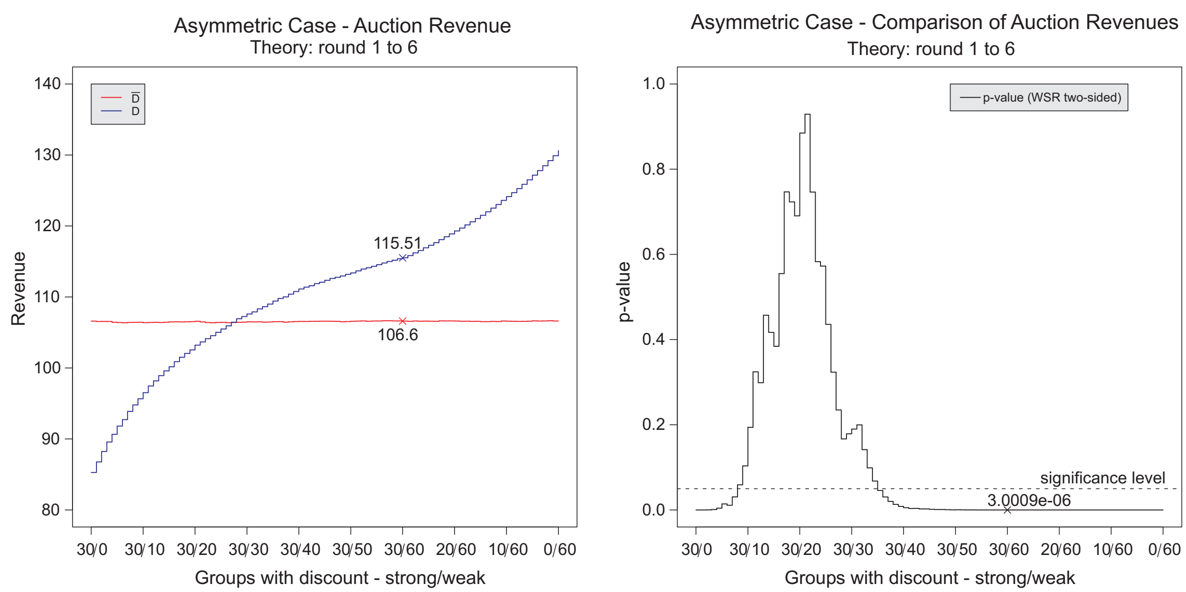
In the following analysis of treatment $\bar{D}a$ and Da, all observations derived from all conducted auction rounds for setting $\bar{D}$ and setting D are included. To isolate the treatments $\bar{D}a$ and Da, the 45 bids of setting $\bar{D}$ and the 45 bids derived from setting D in each round are randomly recombined: for each round 15 groups (each with three bidders) are randomly created, each consisting of bids submitted by a strong bidder and two weak bidders. Overall, this results in 90 created groups for each treatment (15 groups per round x 6 auction rounds). More specifically, in treatment $\bar{D}s$ the second-price auction is virtually conducted 90 times, once per group, based on the observations of setting $\bar{D}$, while in treatment Ds the discount auction is conducted 90 times based on the observations derived from setting D. By construction, in setting D the submitted 270 bids can be divided into 180 bids without discount and 90 bids with discount; of the 90 bids with discount, 30 bids are submitted by strong designated bidders and 60 bids by weak designated bidders. Thus, the proportion q can range at most from 30/0 to 30/60 and 29/60 to 0/60. Overall, in treatment Da 91 different proportions are regarded and the respective revenues for the virtually created groups are calculated for each proportion. Table D.21 in Appendix D.4 lists all the results for the different groups in treatment Da as well as for treatment $\bar{D}a$ with the same grouping. The curves of the average auction revenues in treatment $\bar{D}a$ and treatment Da for each q are plotted in Figure 5.24 and in Figure 5.25 (cf. Table D.22 in Appendix D.4). In Figure 5.24 the curves are plotted based on the experimental observations, while in Figure 5.25 the respective curves are predicted from theory. Both figures confirm the results derived from the first auction round: awarding the discount to strong bidders does not pay for the seller; assigning the discount to weak bidders instead increases the revenue of the seller. In particular, as indicated in Figure 5.24, when applying the WSR (matched-pairs, two-sided) the difference in central tendency between the revenues of treatment $\bar{D}a$ and treatment Da is significant for q>30/11 and for q<30/54; in equilibrium significant difference between the auction revenues of both treatment is measured for $q >$30/8 and $q <$30/35.

For this particular case, the study shows that under asymmetries, the discount raises the revenue of the seller significantly when the discount is awarded to weak bidders; when the discount is awarded to strong bidders, the discount hurts revenue.



(a) (b)

Figure 5.24: Treatment $\bar{D}a$ and treatment Da – development of auction revenues from round 1 to round 6 for different proportions q (experiment)



(a) (b)

Figure 5.25: Treatment $\bar{D}a$ and treatment Da – development of auction revenues from round 1 to round 6 for different proportions q (theoretical benchmark)

5.4 Summary of the experimental results

The analysis of the experimental results was guided by research questions outlined in Section 4.1:

- Is the predicted behavior of bidders in the DA consistent with the observed behavior in the conducted experiment?
- In the symmetric case, is a seller able to extract an additional revenue by offering a discount (contrary to the prediction)?
- In the asymmetric case, can the expectation of a seller – to raise her revenue by offering a discount – be confirmed?

In order to address these questions, first the observed bids and the bidding behavior over the course of the experiment were analyzed. In particular, deviations from dominant bidding strategy and learning effects over rounds were investigated. Furthermore, the auction outcomes of the conducted experimental settings – setting $\bar{D}$ and setting D – were analyzed. Additionally, a comparative overview of the experimental findings for the isolated treatments in the symmetric case – treatment $\bar{D}s$ and treatment Ds – as well as for the isolated treatments in the asymmetric case – treatment $\bar{D}a$ and treatment Da – was given. The analysis of the auction revenues in the particular treatments was directed towards the last two research questions mentioned above.

Concerning the bidding behavior observed in the experimental sessions, the analysis presented in Section 5.1 shows that in the conducted auctions bidders tend to underbid – a general tendency for bidding below the dominant strategy is observed. The frequency of underbidding is higher in the case of the discount auction than the respective frequency in the second-price auction. In setting D particularly, significant differences in the deviation of the dominant strategy between bidders with discount and bidders without discount is observed. Bidders without discount more often submit bids closer to their dominant strategy. In particular, two behavioral patterns were analyzed: More than 22% of the bidders with an assigned discount do not make use of the discount. Instead of submitting bids slightly above their valuations close to the dominant strategy, these bidders submit bids close to their valuation and deviate about -20% from their dominant strategy. In contrast, only a few bidders without discount use the discount to submit higher bids – less than 2% of bidders without discount submit bids about 25% above their dominant strategy, i.e. above their valuation.

Comparing the bids submitted in setting $\bar{D}$ with those submitted in setting D, the analysis shows that the bids submitted in the discount auction are significantly higher than the respective bids in the second-price auction. This is of course due to the effect of the discount, which allows designated bidders to submit higher bids. Nevertheless, the additional discount raises the complexity of the rules, such that bidders with discount have more difficulties in calculating their dominant strategy.

A surprising result is derived when comparing the bids submitted in setting $\bar{D}$ and the bids without discount submitted in setting D. Here a significant difference in the bidding behavior is measured. Recall that the dominant strategy for bidders in setting $\bar{D}$, bidders in the second-price auction, and for bidders in setting $D_{\overline{disc}}$, bidders without discount in the DA, is the same. Moreover, the bids submitted in the second-price auction are significantly lower than the bids without discount in the discount auction. Indeed, bidders without discount in setting D perform better, or closer to the dominant strategy, than bidders in setting $\bar{D}$. This contradicts the expectation that bidders in setting D, in particular bidders without discount, should have more difficulties in finding their dominant strategies due to the additional complexity of the discount rule.

Focussing on the development of bidding behavior throughout the course of the experiment, the study shows that bidders adapt their strategic behavior towards the dominant strategy. In the second-price auction and in the discount auction, a general trend towards the dominant strategy is observed. However, in setting D, bidders with an assigned discount need more rounds to learn, whereas bidders without discount need solely one round to bid close to their dominant strategy.

Furthermore, the two settings – setting $\bar{D}$ and setting D – are experimentally investigated with respect to the auction outcomes, i.e. the auction revenue, the winning bidder's payoff, and the social surplus. When studying the experimental data derived from setting $\bar{D}$ and setting D, the following major results (cf. Section 5.2) are found: It is predicted from theory that introducing the discount leads to a loss of efficiency. That is, the social surplus in the discount auction is below the social surplus of the second-price auction. In addition, it is predicted in equilibrium that in setting D, the number of inefficient auctions are higher due to the discount. Analyzing the experimental data reveals that the predictions cannot be confirmed in general; the social surplus in the discount auction is higher than the social surplus in the second-price auction. The difference between the measured social surplus in setting $\bar{D}$ and setting D is not significant. Nevertheless, as predicted in setting D, the number of inefficient auctions is higher than the number of inefficient auctions in setting $\bar{D}$.

Comparing the winning bidder's payoff from both settings, the main finding is that the discount increases the payoff, a fact which is also predicted from theory. It is interesting to note that the average payoff in setting $\bar{D}$ is below the average payoff in equilibrium, while the overall payoff in setting D is above the respective average payoff in equilibrium. Particularly in setting D, the discount lowers the payoff of designated bidders while raising the payoff of non-designated bidders (compared to the respective payoff in equilibrium). A more thorough analysis of the winning bidder's payoff reveals that the decrease of the designated bidder's payoff is much lower than the increase of the non-designated bidder's payoff. The increase of the non-designated bidder's payoff is a direct result of the bidding behavior of designated bidders who do not use the discount to submit higher bids and thus increase the probability of winning the auction. In general, significance in difference between the payoffs derived in both settings cannot be measured. When solely comparing the average payoffs per round and the cumulative distribution functions of the payoffs in both settings, significance in difference is revealed.

Despite the social surplus and payoff, the analysis focuses on the revenue of the seller. The study finds that the observed revenues are lower than the revenues predicted from theory. This effect is caused by the general tendency for underbidding in both settings. A further analysis of the observed auction revenues confirms the predictions from theory, namely that the discount lowers the revenue of the seller: the average revenue in the discount auction is lower than the average revenue in the second-price auction. However, this result is not significant when solely considering the auction revenues from the first round, the sixth round, the average revenues per round, the average revenues per group, or all observed auction revenues. When solely comparing the cumulative distribution functions of the revenues in both settings, the difference between both distribution functions turns out to be significant.

Based on the conducted two settings, four treatments are virtually created by reordering the observed bids: treatment $\bar{D}s$ functions as benchmark to treatment Ds with symmetric bidders; analogously in the asymmetric case, treatment $\bar{D}a$ corresponds as benchmark to treatment Da (cf. Section 5.3). Both treatments – treatment $\bar{D}s$ and treatment $\bar{D}a$ – employ the second-price auction, while treatment Ds and treatment Da both employ the DA. The study of the four treatments includes a comparative analysis with respect to the auction revenues. The extraordinary result of this study is that in the symmetric case, the discount does not pay for the seller, whereas in the asymmetric case the discount increases the seller's revenue.

To be more precise, the revenues of the seller in treatment Ds are significantly lower than the respective revenues in treatment $\bar{D}s$. This not only holds for the revenues derived in the first round but even more for the revenues achieved in each conducted auction round. Additionally, in analyzing the development of the revenues over the course of the experiment, the study shows that in treatment $\bar{D}s$ the average revenue converges towards the average revenue in equilibrium, whereas in treatment Ds such a trend is not observed. It is interesting that in treatment $\bar{D}s$, the average revenues derived from the experiment per round are lower than the respective average revenues predicted from theory; in treatment Ds it is the reverse case, where the experimental average revenues per round are above the respective average revenues predicted from theory.

Furthermore, in varying the proportion of the number of groups in which a strong bidder is awarded the discount and the number of groups in which the discount is assigned to a weak bidder, the study shows that for all proportions, the revenues in treatment Ds are below the revenues in treatment $\bar{D}s$, with this difference being significant. In addition, when the number of strong groups decreases while the number of weak bidders increases, the revenues – the average revenues in treatment $\bar{D}s$ and the respective average revenues in treatment Ds – decrease.

The comparative analysis of the treatments in the asymmetric case shows the central result that under asymmetries, the seller raises her revenue by offering a discount. The average revenue achieved in the discount auction is higher than the average revenue in the second-price auction for each of the conducted rounds. At the same time, the study reveals that the difference with respect to the seller's revenue is significant for solely two out of the six conducted auction rounds. Nevertheless, when comparing all virtually played auctions, significance is measured. An analysis of the development of the auction revenues in treatment $\bar{D}a$ and Da throughout the course of the experiment shows that the revenues in both treatments are very close to the revenues predicted from theory, especially in the very last round.

Additionally, varying the proportion of the number of groups with a strong designated bidder and the number of groups with a weak designated bidder, then the central result of the study is that whenever the discount is assigned solely to strong bidders, the discount does not pay for the seller. In contrast, when awarding the discount to an increasing number of weak bidders or solely to weak bidders, then the discount raises the revenue of the seller. In particular, depending on the proportion of strong designated and weak designated bidders, the seller can raise her revenue from the discount auction above the revenue of the benchmark second-price auction. Significant difference with respect to the auction revenues are solely measured when considering all experimental data. As the study reveals, the difference is significant when the number of strong designated bidders is about more than three times higher than the number of weak designated bidders, or when the number of weak designated bidders is approximately more than two times higher than the number of strong designated bidders.

In the first case, the seller suffers a loss by offering a discount, while in the second case, the seller can extract an additional revenue by introducing the discount.

Chapter 6

Conclusion

The Internet marketplace Amazon offers sellers the possibility to sell objects via an English proxy-auction on its auction platform. Additionally, a seller can add a *first bidder discount* of 10 percent when conducting an auction. In an auction, the first bidder discount is assigned to the bidder who submits the first valid bid. If that bidder wins the auction, then the winning bidder purchases the object at the discounted final price of the auction. In all other cases, the winner pays the final price of the auction.

On the Amazon auction platform the first bidder discount is displayed by a symbol saying 10% *OFF 1st Bidder*. This symbol is visible as long as no bid has been entered. Upon submission of the first valid bid, the discount symbol is deleted. The discount is no longer available for subsequent bidders, and subsequent bidders are not informed about the discount assigned to the first bidder in the auction. Those bidders cannot distinguish whether they are participating in a standard auction or a first bidder discount auction.

What can be observed is that some participants sell their objects through the standard Amazon auction, while other sellers use the first bidder discount feature and add the first bidder discount to their auction. Thus, the question arises as to why some sellers add the first bidder discount to their auction while others do not, how the ex-ante expected revenues of both institutions – the pure auction and the first bidder discount auction – are related, as well as how the first bidder discount mechanism affects bidding behavior. To shed more light on the effect of the discount mechanism, a model of a discount auction (DA) was developed which simplifies the first bidder discount mechanism employed on the Amazon platform.

The DA basically follows the auction mechanism of the second-price sealed-bid auction, which is equivalent to the English-proxy auction. In the DA exactly one bidder out of n bidders is randomly selected as the designated bidder, i.e. the bidder to whom the discount is assigned. If the designated bidder wins the auction, the price to pay is the discounted second-highest bid;

in all other cases the high bidder purchases the object at the price of the second-highest bid.

The scope of the present study was to analyze the discount mechanism and its effect on auction outcome from a market engineering perspective. Firstly, a game theoretic model of the DA was developed to predict the strategic behavior of bidders. Secondly, since the underlying rules of the DA strongly influence the bidding behavior and therefore the auction outcome, a laboratory experiment with human subjects employing the DA in a controlled environment was conducted. The impact of the discount mechanism on the outcome was measured based on the experimental results.

6.1 Summary and review of the work

The interest in auctions with discounts brings up the desire to find explanations for research questions concerning the impact of the discount on bidding behavior and auction outcomes. First, regarding the bidding behavior the primary contribution of this study was to shed light on the following questions:

- When comparing the strategic behavior of bidders in the DA to the respective behavior in the benchmark second-price auction, can significant differences in bidding behavior be observed?

- When focussing solely on the DA institution, do bidders with discount behave significantly different from bidders without discount?

Second, this study offered deeper insights into how the discount affects the auction outcomes, in particular the auction revenue, thereby addressing the following research questions:

- When comparing the seller's revenue in the DA to the respective seller's revenue in the benchmark second-price auction, can a seller raise her revenue by introducing a discount?

- When focussing on bidders' characteristics and distinguishing between the case of symmetric bidders and the case of asymmetric bidders: In which case can the seller extract an additional revenue and raise her revenue?

- To what extent does the seller's revenue in the DA depend on to whom the discount is assigned: (i) an economically advantaged bidder (strong bidder) or (ii) an economically disadvantaged bidder (weak bidder)?

In the context of the market engineering approach, the theoretical and experimental findings of the DA model may be used to advise the seller to choose the proper auction mechanism and indicate that the seller should offer a discount when she has information about the bidders' characteristics.

The present study was done in several steps reflected by the following structure.

Chapter 1 set the overall research context and motivated the research questions mentioned above. Driven by these questions the introductory chapter outlined the methodological approach of this study: to investigate the theoretical model of the DA market institution based on the independent private values auction model, to conduct a laboratory experiment and to contrast the theoretical findings with the experimental results.

Chapter 2 introduced the market engineering approach, meaning the structured, systematic and theoretically founded approach towards the design and operation of electronic markets. In particular, the main objective in market engineering is to solve the design problem – the conscious design of electronic markets. As the design of the underlying trading rules is one part of the market engineering approach, methods from game theory, mechanism design as well as experimental economics for designing and analyzing electronic markets were briefly presented in that chapter. Furthermore, the given overview was meant to review basic concepts and create a common terminology used throughout this study.

Chapter 3 presented the theoretical part of this study. Motivated by the example of the Amazon Internet marketplace and the rule of the first bidder discount auction, the DA market institution in an independent private values auction model was developed. The theoretical analysis showed, that dominant strategies in equilibrium exist. Moreover, the DA market institution and the benchmark second-price auction were analysed for two cases: firstly, the symmetric case, when bidders are (ex-ante) symmetric, meaning that bidders are characterized by the same distribution function of valuations, and secondly, in the asymmetric case, when bidders are (ex-ante) asymmetric and characterized by two different distribution functions of valuations. For both cases, the seller's expected revenue, the winning bidder's expected payoff, and the expected welfare (social surplus) of the DA market institution as well as of the benchmark second-price auction were calculated. The expected outcomes of both auction formats were contrasted and compared in both cases – the symmetric case and the asymmetric case.

Chapter 4 described the experimental design and the set-up of the conducted laboratory experiment. The conducted settings of the two auction mechanisms – the benchmark second-price auction and the DA – as well as the four treatments – the two auction mechanisms in the symmetric case as well as the asymmetric case – were presented. The implementation of the two settings with the experimental system meet2trade and MES, as well as the execution of the single sessions were described. Additional, to analyse and evaluate the experimental observations, the basic test procedures of the statistical analysis were briefly introduced.

Chapter 5 presented the statistical analysis of the experimental data and discussed the experimental results. The analysis thereby followed the research questions posed in the introduction. First, the strategic behavior of bidders in the DA and the second-price auction were contrasted. Particular interest was taken in the analysis of the deviations of bids from dominant strategies from a static perspective as well as a dynamic perspective, i.e. over the conducted auction rounds. Additionally, it was analyzed whether behavioral patterns, different from the predicted behavior in equilibrium, could be detected. Second, a comparative overview of the auction outcomes derived in the settings was given. Differences in central tendency between the auction outcomes in the two different settings were analyzed with respect to the seller's revenue, the winning bidder's payoff and the social surplus. Another focus was set on the analysis of the four virtually created treatments based on the observed data. Within the symmetric as well as the asymmetric cases, the benchmark second-price auction and the DA with respect to the auction revenues were contrasted.

In summary, the present study investigated the DA and the second-price auction under symmetries and asymmetries theoretically and by means of a laboratory experiment. In both approaches the independent private values model was assumed.

In the theoretical portion of the study, the DA market institution was developed, and it was shown that dominant strategies in equilibrium exist. Moreover, bidders participating in the DA were distinguished to be first symmetric and second asymmetric. In both cases, the seller's expected revenue, the winning bidder's expected payoff, and the expected welfare were calculated and compared to the respective expected values derived in the second-price auction. The main findings can be summarized as follows:

1. In equilibrium, the designated bidder in the DA submits a bid above his valuation ($\frac{1}{1-d}$ times his valuation, $d \in [0, 1)$ denotes the discount) and all other bidders submit their valuations truthfully.

2. If bidders are symmetric, the seller cannot gain from offering an additional revenue when conducting an auction.

3. If bidders are asymmetric, the seller might extract an additional revenue by offering a discount when conducting an auction. This is especially the case, if one strong bidder competes with one or more weak bidders and if the difference between weak and strong bidders concerning their asymmetries is very strong.

In the second part of the study, a computer-based laboratory experiment with human subjects was presented. In the conducted experiment, human bidding behavior in the DA and the second-price auction was investigated. In the DA the discount was set to 20 percent. Concerning bidding behavior the presented experimental results show that:

1. In both auction formats bidders have a general tendency towards underbidding, i.e. bidding below their dominant strategy.

2. The bids in the DA are significantly higher than the bids in the second-price auction.

3. In particular, the bids without discount in the DA are significantly higher than the bids in the second-price auction, although in equilibrium bidders follow the same dominant strategy. In addition, the bids without discount in the DA are submitted closer to the dominant strategy than the respective bids in the second-price auction. However, this result is not significant.

4. Throughout the course of the experiment, designated bidders with an assigned discount in the DA have difficulties in adapting their behavior towards the dominant strategy. In contrast, bidders without discount in the DA and bidders in the second-price auction need only a few rounds to adapt their behavior and submit bids close to the dominant strategy.

In summarizing the results derived from the analysis of the four virtually created treatments – the second-price auction under symmetries, the DA under symmetries, the second-price auction under asymmetries, and the DA under asymmetries – based on the experimental results, the study shows that:

1. Under symmetries, the seller can not extract an extra revenue by offering a discount when conducting the auction. That is, on average, the seller's revenue in the DA is lower than the seller's revenue in the second-price auction. The differences in central tendency with respect to the revenues are significant.

2. Under asymmetries, the seller gains from offering an additional discount in the conducted auction. The average auction revenues achieved in the DA are higher than the respective average revenues in the second-price auction. However, the differences in central tendency are not significant.

Additionally, in the symmetric and asymmetric case the proportion of strong and weak designated bidders was varied. In other words, the study showed to what extent the revenue achieved in the DA and the respective second-price auction depends on to whom the discount is assigned: either to solely strong bidders (and no weak bidders), to solely weak bidders (and no strong bidders), or to strong and weak bidders with a certain proportion. When bidders are symmetric, the proportion of strong designated and weak designated bidders does not influence the main result, so that offering a discount in an auction does not pay for the seller. When bidders are asymmetric, then the proportion of strong designated and weak designated bidders is decisive – it has a strong impact on the seller's revenue in the DA. In particular, awarding the discount solely to strong bidders does not pay for the seller; in contrast, in offering a discount in the DA and assigning the discount mostly to weak bidders, the seller can extract an extra revenue.

6.2 Limitations of the present work

The presented theoretical and experimental study is done under very strong and artificial assumptions. Both the theoretical model of the DA and the conducted laboratory experiment are based on the independent private values auction model. Moreover, participants are assumed to be rational and risk neutral. When comparing the model of the DA with its assumptions to real-world auction formats and environments such as Internet auctions, it can be stated that attitudes of practical relevance such as risk aversion, risk lovingness, impatience, or uncertainty have not been modeled in the DA. In particular, the assumption of private values does not hold: bidders often do not know their distribution function of valuations, and often the value of an object is derived from a market price that is unknown at the time of the auction. Such auctions clearly include a common-value component. Moreover, some assumptions made, such as the rationality assumption, are sometimes hurt in real-world settings.

In comparing the auction model of the DA to the Amazon first bidder discount auction, the model of the DA is limited. Firstly, the DA does not account for the effect of the "first bidder". In the DA the discount is assigned to a single, randomly selected bidder and not to the bidder who has submitted the first valid bid in that auction. Participants are aware of the existence of the discount being assigned to a randomly selected participant. In the Amazon first bidder discount auction, only the first bidder to whom the discount is assigned or bidders who have observed the auction before the first bid was submitted have information about the discount. Secondly, the number of participants in the DA is set to a fixed number which is publicly known. In Internet auctions, the number of participants is not fixed – as long as the auction is listed and conducted, participants aware of the auction may participate in that auction. Note that in the conducted experiment the number of participants was set to three in each auction.

Another limitation of the present experimental study is the selection of the valuations induced to the bidders. In the experiment, the induced valuations were selected in advance, covering the integer values $\{100, 101, \ldots, 109\}$ and $\{146, 147, \ldots, 150\}$. Since each value was assigned to exactly one participant per round in each session, the fifteen integer values were uniformly distributed. By construction, symmetry and asymmetry were indirectly included in the laboratory examination and were isolated from the conducted settings afterwards. In the symmetric case, homogeneous groups were virtually created: participants with low valuations were grouped together, as well as participants with high valuations. In contrast, in the asymmetric case heterogeneous groups with one high value participant and two low value participants were created. However, this construction limited the presented experimental study to one particular case.

In the theoretical model, the discount was not limited to a particular amount – moreover, the discount was selected to be greater than or equal to zero and lower than one. In the laboratory the discount was set at the level of 20 percent for practical reasons, allowing for an easy calculation of the dominant strategy for designated bidders; in equilibrium they submitted a bid of 25 percent above their valuation. Nevertheless, especially in the asymmetric case the size of the discount has strong impacts on the auction outcomes, thus the experimental study presented a limited example of the DA.

6.3 Future work

The main objective in market engineering is to solve the design problem, that is to consciously design electronic markets. In the present study, the market engineering approach was applied to market institutions combining auctions with discounts. Following the market engineering process, a game theoretic model of the DA was developed and evaluated by means of experimental economics.

However, as the limitations discussed in the previous section point out, these findings contribute only partially to solving the design problem. To deepen understanding and knowledge of the market engineer concerning auctions with discount, a broader analysis of auctions with discounts by means of game theory, experimental economics, empirical studies, as well as computer or agent-based simulations is necessary.

Future work may consider distinct aspects as an extension of the presented game theoretic model and the laboratory experiment. Firstly, in Internet auctions, the assumption that the number of participants is fixed and publicly known in advance does not hold. In fact, bidders arrive randomly at the auction platform and even enter the listed auctions randomly. To extend the game theoretic model, the arrival and participation of bidders in auctions has to be modelled by a stochastic process.

Secondly, the game theoretic model can be extended so that it more precisely reflects the Amazon first bidder discount auction. The model should consider the time process of bidding and assign the discount to the bidder submitting the first valid bid. It is presumed that the number of bidders is not fixed in advance and bidders may randomly enter the auction. Additionally, when the discount is already assigned to the first bidder, subsequent bidders should not be aware of the discount, meaning bidders with a later arrival time at the auction have no knowledge about the existence of the discount in that auction. The extension to the first bidder discount would explicitly account for the timing of bids, forcing bidders to submit an early bid or signal of their valuations. Early signals of a bidder's valuation are interesting in common-value auction models or when bidder's valuations are affiliated – it would be interesting to analyze an extension of these models.

Thirdly, it would be interesting to analyze a variant of the first bidder discount. Instead of assigning the discount to the first bidder, the discount could be attached to the first bid itself. In other words, similar to the first bidder discount mechanism, the bidder who has submitted the first valid bid is awarded the discount. Now, if the first bid is the winning bid, the bidder purchases the object at the discounted final price of the auction; in all other cases the winning bidder pays the final price. This first bid mechanism promotes much stronger an early first bid, and the time process of bidding becomes more decisive. Here, more complex first bid mechanisms could be conceived and should be analyzed theoretically and by experiments, e.g. mechanisms in which several discounts in decreasing order are attached to early bids, in particular to the first bid, second bid, third bid, etc. Thus, early bids as signals of bidders' valuations are promoted, which are of interest in common-value settings.

Fourthly, as pointed out in the previous section, a particular instance of the DA was employed in the laboratory experiment: the number of participants was set to three, the discount was set at the level of 20 percent, and the uniformly distributed induced valuations were limited to the set of 15 integer values per round and session, ranging from $[100, 109]$ and $[146, 150]$ in each round. Depending on the research questions and the underlying game theoretic model, the number of participants could be modelled by a stochastic process throughout the course of the experiment. Additionally, depending on the underlying market institution and the selected scenario, the distribution function of bidder's valuation needs to be refined. From an experimental perspective, either uniform or normal distribution functions are suggested, as they are easy for participants to understand. Moreover, the parameters of the distribution functions, i.e. the kind of asymmetries, and the level of the discount have to be selected carefully, as they have a strong impact on the auction outcomes. Particularly in auctions with discount, the interplay between bidders' asymmetries (kind of asymmetries) and the level of the discount is decisive.

Finally, auctions with discounts become more and more relevant in business practice, in particular in the public-procurement sector. As Rothkopf et al. (2003) state, affirmative actions in auctions have become a widespread practice particularly in public-sector procurement. A prominent example was given by the FCC in the regional narrowband auction of radio spectrum auctions in 1994 (Ayres and Cramton 1996). Businesses owned by minorities and women were subsidized by a 40 percent bidding credit, leading to an increase of 12 percent in government revenue. Additionally, there is evidence that governmental agencies in the US use more and more minority price preferences in their procurement programs (Corns and Schotter 1999). So far, theoretical research and experimental studies investigating price preferences in procurement auctions are limited: Research in this field is for example presented by Ayres and Cramton (1996), Corns and Schotter (1999), and Rothkopf et al. (2003). Ayres and Cramton (1996) present the case of the FCC auction, Corns and Schotter (1999) investigate a procurement auction with price preferences by means of a laboratory experiment, and Rothkopf et al. (2003) analyze a common-value model with asymmetric bidders in a procurement setting. Overall, these studies show that a policy of subsidizing economically disadvantaged (inefficient) competitors can lower project costs and raise governmental revenues. Nevertheless, further research in this field is necessary in order to achieve a comprehensive understanding of subsidizing policies from a market engineering perspective.

Appendix

Appendix A

Amazon Auction

A.1 Bid increments in the Amazon auction

Amazon's German[1] and US[2] auction platforms use bid increments that depend on the current price in the auction. As the current price changes dynamically throughout the auction also the bid increment changes. Table A.1 indicates the current prices and the related bid increments used by Amazon.

Bid increments of Amazon's auction platforms

Amazon's German auction platform		Amazon's US auction platform	
current price in euros	bid increment in euros	current price in US dollars	bid increment in US dollars
0.01 – 0.99	0.05	0.01 – 0.99	0.05
1.00 – 9.99	0.25	1.00 – 9.99	0.25
10.00 – 24.99	0.50	10.00 – 24.99	0.50
25.00 – 99.99	1.00	25.00 – 99.99	1.00
100.00 – 249.99	2.50	100.00 – 249.99	2.50
250.00 – 499.99	5.00	250.00 – 499.99	5.00
500.00 – 999.99	10.00	500.00 – 999.99	10.00
1,000.00 – 2,499.99	25.00	1,000.00 – 2,499.99	25.00
2,500.00 – 4,999.99	50.00	2,500.00 – 4,999.99	50.00
5,000.00 and up	100.00	5,000.00 and up	100.00

Table A.1: Bid increments from Amazon's German and US auction platforms

[1] See http://www.amazon.de .
[2] See http://www.amazon.com .

Appendix B

Linear transformation and order statistics

In the following basic concepts and terminology concerning linear transformation of random variables and order statistics are presented. A more detailed overview on these topics is given in for example Wolfstetter (1999), Balakrishnan and Rao (1998), Arnold et al. (1992), and David (1981).

B.1 Linear transformation

Let X be a random variable with the probability distribution function $F : \mathbb{R} \rightarrow [0,1]$ and the respective probability density function $f : \mathbb{R} \rightarrow \mathbb{R}_+$ with $f(x) = F'(x)$. Let $\tilde{X}$ be a linear transformation of X. The linear transformation of the random variable X is defined by $\tilde{X} = t(\delta, X) = \delta X$, $\delta \in \mathbb{R}, \delta \neq 0$. [1] In general, linear transformations are *distribution-preserving*, meaning that $\tilde{X}$ is a random variable with a distribution of the same form as X: $\tilde{F} : \mathbb{R} \rightarrow [0,1]$ is the probability distribution function of $\tilde{X}$ and $\tilde{f} : \mathbb{R} \rightarrow \mathbb{R}_+$ its corresponding probability density function. Further $\mathcal{M} \subset \mathbb{R}$ is the convex support of F with elements $a, b \in \mathcal{M}$, $a = \inf(\mathcal{M})$, $b = \sup(\mathcal{M})$ with $a < b$, such that $F(x) = 0 \, \forall x \leq a$, $F(x) = 1 \, \forall x \geq b$, and $F(x) \in (0,1) \, \forall x \in (a,b)$. Additionally, due to the linear transformation, $\mathcal{S} \subset \mathbb{R}$ defines the convex support of $\tilde{F}$: for each element $s \in \mathcal{S}$ the following holds: $\forall s \in \mathcal{S} \, \exists m \in \mathcal{M} : s = t(m) = \delta m$. Now, the relation of both distribution functions – F and $\tilde{F}$ – can be described as follows:

Proposition B.1.1 X and $\tilde{X}$ are two random variables with the associated distribution functions F with convex support $\mathcal{M}$ and $\tilde{F}$ with convex support $\mathcal{S}$. The random variable $\tilde{X}$ is

[1] A linear transformation t of a random variable X has the following form $t(X) = aX + b = Y$ where a and b are real numbers, and $a \neq 0$. $Y = aX + b$ is a again a random variable.

derived from a linear transformation of the random variable X with the linear transformation $\tilde{X} = \delta X$ and $\delta > 1, \delta \in \mathbb{R}$. Then, for F and $\tilde{F}$ the following holds:

(i)
$$\tilde{F}(x) = F(\frac{1}{\delta}x) \quad [\text{ or equivalently } \tilde{F}(\delta x) = F(x)] \quad \forall x \in \mathbb{R} \tag{B.1}$$

(ii)
$$F(x) \geq \tilde{F}(x) \quad \forall\, x \in \mathbb{R} \tag{B.2}$$

(iii)
$$F(x) > \tilde{F}(x) \quad \forall\, x \in (a, \delta b) \tag{B.3}$$

Proof: ad (i): With the definition of the cdf F and $\tilde{F}$ as well as the linear transformation the following equation is obvious:

$$\tilde{F}(x) = Prob(\tilde{X} \leq x) = Prob(\delta X \leq x) = Prob(X \leq \frac{1}{\delta}x) = F(\frac{1}{\delta}x)\,\forall x \in \mathbb{R}$$

and thus Equation B.1 holds. The alternative formulation of Equation B.1 can be directly derived with $y = \frac{1}{\delta}x$ or equivalently $\delta y = x$: $\tilde{F}(x) = \tilde{F}(\delta y) = F(y) = F(\frac{1}{\delta}x)$.
ad (ii): With Equation B.1 the following holds:

$$F(x) = \tilde{F}(\delta x) \overset{\delta>1}{\geq} \tilde{F}(x)\,\forall x \in \mathbb{R}$$

Because of $\delta X > X$, $f(.) \in \mathbb{R}_+$, $\int_x^{\delta x} \tilde{f}(t)dt \geq 0\,\forall x \in \mathbb{R}$, and $\delta > 1$ Equation B.2 is proven.
ad (iii): The cdf F is defined on the convex support $\mathcal{M}$ with

$$F(x) = \int_{-\infty}^{x} f(t)dt = \begin{cases} 0, & x \leq a \\ \in (0,1), & x \in (a,b) \\ 1, & x \geq b \end{cases}$$

Furthermore $F(x) \geq \tilde{F}(x) = F(\frac{1}{\delta}x)\,\forall x \in \mathbb{R}$. With the definition of F on the convex support $\mathcal{M}$ and $F(x) \geq F(\frac{1}{\delta}x)\,\forall x \in \mathbb{R}$ it is to show, that $F(x) > F(\frac{1}{\delta}x)\,\forall x \in (a, \delta b)$ or $\int_{-\infty}^{x} f(t)dt > \int_{-\infty}^{\frac{1}{\delta}x} f(t)dt\,\forall x \in (a, \delta b)$. As the integral $\int_{\frac{1}{\delta}x}^{x} f(t)dt > 0\,\forall x \in (a, \delta b)$ the Equation (B.3) holds. q.e.d.

Focussing on the relation between the probability density function $F'(x) \equiv f(x)$ of X and the respective probability density function $\tilde{F}'(x) \equiv \tilde{f}(x)$, then the following can be stated:

Proposition B.1.2 Let X be a random variable with the cumulative probability distribution function $F : \mathbb{R} \to [0, 1]$ and the probability density function $f : \mathbb{R} \to \mathbb{R}_+$. Further, let random variable $\tilde{X}$ be a linear transformation of random variable X with $t(X) = \delta X = \tilde{X}$, $\delta > 1$. Then, the probability density function $\tilde{f} : \mathbb{R} \to \mathbb{R}_+$ of the cumulative probability distribution function $\tilde{F} : \mathbb{R} \to [0, 1]$ of the random variable $\tilde{X}$ is given by

$$\tilde{f}(x) = \frac{1}{\delta} f(\frac{1}{\delta} x) \quad \forall x \in \mathbb{R} \tag{B.4}$$

Proof: For the cumulative probability distribution function of X and the cumulative probability distribution function of $\tilde{X}$ the following equation holds: $\tilde{F}(x) = F(\frac{1}{\delta} x)$. Differentiation of both sides with respect to x results in

$$\frac{d}{dx} \tilde{F}(x) = \frac{d}{dx} \int_{-\infty}^{x} \tilde{f}(t) dt = \tilde{f}(x)$$

and

$$\frac{d}{dx} F(\frac{1}{\delta} x) = f(\frac{1}{\delta} x) \frac{d}{dx} (\frac{1}{\delta} x) = f(\frac{1}{\delta} x) \frac{1}{\delta} \quad \forall x \in \mathbb{R}$$

q.e.d.

Proposition B.1.3 With the given relation of the probability density functions f and $\tilde{f}$, as well as with F and $\tilde{F}$ being the distribution functions, as defined above, and $\delta > 1$ the following holds:

(i)

$$\tilde{f}(x) \leq f(x) \quad \forall x \in \mathbb{R} \tag{B.5}$$

(ii)

$$\tilde{f}(x) < f(x) \quad \forall x \in (a, \delta b) \tag{B.6}$$

Proof: ad(i): Assume $\tilde{f}(x) > f(x)$ holds for all $x \in \mathbb{R}$. Integrate both functions on the interval $(-\infty, x)$ with $x \in \mathbb{R}$, then the following is derived: $\int_{-\infty}^{x} \tilde{f}(t) dt > \int_{-\infty}^{x} f(t) dt \, \forall x \in \mathbb{R}$. This is equivalent to $\tilde{F}(x) > F(x) \, \forall x \in \mathbb{R}$. Obviously, this is in contradiction to Equation B.2 $\tilde{F}(x) \leq F(x) \, \forall x \in \mathbb{R}$. Thus, the assumption is rejected and the contrary holds.

ad(ii): Assume $\tilde{f}(x) \geq f(x)$ holds for all $x \in (a, \delta b)$. With a similar argumentation as in the given proof of (i) as well as Equation B.3 $\tilde{F}(x) < F(x) \, \forall x \in (a, \delta b)$ the assumption can be rejected and the contrary holds. q.e.d.

B.2 Order statistics

Let $x_1, \ldots, x_n \in \mathbb{R}$ be n independent draws from a random variable X with the probability distribution function $F(x)$ with convex support $\mathcal{M} \subset \mathbb{R}$ and with the respective probability density function $f(x) = F'(x)$. Let N be a finite set of n numbers with $N = \{1, \ldots, n\}$. Then, the n realizations $x_1, \ldots, x_n$ can be sorted and rearranged in decreasing order as $x_{(1)} \geq x_{(2)} \geq \ldots \geq x_{(n)}$. Each $x_{(k)}$, $k \in N$, is defined as the realization of the random variable $X_{(k),n}$, the kth order statistic. The kth order statistic $X_{(k),n}$ assigns to each realization of the n random draws of X the kth highest value $x_{(k)}$. Altogether, there are n order statistics $X_{(1),n}, \ldots, X_{(n),n}$.

The distribution function $F_{(k),n}$ with density $f_{(k),n}$ determines the probability of the event that $X_{(k),n} \leq x$. More precisely, $X_{(k),n} \leq x$ means, that the kth smallest observation is not greater than x, or equivalently that at least $n-(k-1)$ observations are below or equal to x and at most $k-1$ observations are above x. Particularly $X_{(1),n}$ and $X_{(2),n}$ are the random variables of the highest and second-highest realizations $x_{(1),n}$ and $x_{(2),n}$ with the distribution functions $F_{(1),n}$, $F_{(2),n}$ and the respective probability density functions $f_{(1),n}$, $f_{(2),n}$. In particular, $F_{(1),n}$ is the distribution of the first order statistic $X_{(1),n}$ denoting the probability of the event that $X_{(1),n} \leq x$, $x \in \mathbb{R}$, which is equal to the event, that all realizations of the n independent draws are below x: $x_i \leq x \, \forall i \in N$ (cf. Krishna 2002; Wolfstetter 1999):

$$F_{(1),n}(x) = F^n(x) \tag{B.7}$$

The associated probability density function is

$$f_{(1),n}(x) = nF^{n-1}(x)f(x) \tag{B.8}$$

To be more general the distribution of the kth order statistic $X_{(k),n}$ is defined by

$$F_{(k),n}(x) = \sum_{j=0}^{k-1} \binom{n}{n-j} F^{n-j}(x)\,(1-F(x))^j \tag{B.9}$$

The probability density function $f_{(k),n}(x)$ is the derivative of the distribution function $F_{(k),n}(x)$ with $F'_{(k),n}(x) = f_{(k),n}(x)$. One possibility to obtain the derivative is to differentiate Equation B.9 with respect to x. An alternative formulation is given in the following:

$$f_{(k),n}(x) = \frac{n!}{(n-k)!(k-1)!} f(x)F^{n-k}(x)\,(1-F(x))^{k-1} \tag{B.10}$$

The two expectations of the kth order statistic $X_{(k),n}$ are given by

$$E[X_{(k),n}] = \int_{-\infty}^{+\infty} x f_{(k),n}(x)dx \quad \text{and} \quad Var[X_{(k),n}] = \int_{-\infty}^{+\infty} (x - E[X_{(k),n}])^2 f_{(k),n}(x)dx \tag{B.11}$$

Appendix C

Theoretical model of the DA

C.1 Expected revenues in the symmetric case

Assume a SIPV setting with risk neutral bidders. Assume further, that a seller offers a single indivisible item in the DA for sale. Each bidder $i \in N$ has a valuation of the item; all valuations are realizations of independent draws of the same random variable V which is distributed according to F. The associated density function is denoted by f. Further, let the random variable $\tilde{V}$ denote the linear transformation of V with $\tilde{V} = \delta V$, $\delta = \frac{1}{1-d}$ and $d \in [0,1)$. $\tilde{F}$ denotes the distribution function of $\tilde{V}$ with $\tilde{F}(v) = F(\frac{1}{\delta}v)$ and $\tilde{f}(v) = \frac{1}{\delta}f(\frac{1}{\delta}v)$ its respective density function.

Proposition C.1.1 If the discount is positive and smaller than 1, $d \in (0,1)$, then the seller's expected revenue in the DA is lower than the seller's expected revenue in the corresponding second-price auction.

$$E[R_{DA}] < E[R_{EA}] \tag{C.1}$$

If the discount is zero $d = 0$, then the seller's expected revenue in the DA is equal to the seller's expected revenue in the corresponding second-price auction.

Proof: The seller's expected revenue in the second-price auction is given by Equation 3.5

$$E[R_{EA}] = \int_0^\infty v \left[n(n-1)F^{n-2}(v)f(v) - n(n-1)F^{n-1}(v)f(v)\right] dv$$

and in the discount auction by Equation 3.29

$$\begin{aligned} E[R_{DA}] &= \int_0^\infty \int_0^v \frac{1}{\delta} y(n-1)F^{n-2}(y)f(y)dy\tilde{f}(v)dv \\ &+ (n-1)\int_0^\infty \int_0^v y\Big[(n-2)F^{n-3}(y)f(y)\tilde{F}(y) + F^{n-2}(y)\tilde{f}(y)\Big]dyf(v)dv \end{aligned}$$

To prove the inequality $E[R_{DA}] < E[R_{EA}]$ the following has to be shown:

$$\begin{aligned}
E[R_{DA}] &= \frac{1}{\delta}\int_0^\infty \int_0^v y(n-1)F^{n-2}(y)f(y)dy\tilde{f}(v)dv \\
&\quad +(n-1)\int_0^\infty \int_0^v y\Big[(n-2)F^{n-3}(y)f(y)\tilde{F}(y) + F^{n-2}(y)\tilde{f}(y)\Big]dyf(v)dv \\
&\overset{\tilde{F}(v)\le F(v), \tilde{f}(v)\le f(v), \frac{1}{\delta}<1}{<} \int_0^\infty \int_0^v y(n-1)F^{n-2}(y)f(y)dyf(v)dv \\
&\quad +(n-1)\int_0^\infty \int_0^v y\Big[(n-2)F^{n-3}(y)f(y)F(y) + F^{n-2}(y)f(y)\Big]dyf(v)dv \\
&= \int_0^\infty \int_0^v n(n-1)yF^{n-2}(y)f(y)dyf(v)dv \\
&= \int_0^\infty \big[nyF^{n-1}(y)\big]_0^v f(v)dv - \int_0^\infty \int_0^v nF^{n-1}(y)dyf(v)dv \\
&= \int_0^\infty nvF^{n-1}(v)f(v)dv - \int_0^\infty \int_0^v nF^{n-1}(y)dyf(v)dv \\
&\overset{(1)}{=} \int_0^\infty v\left[n(n-1)F^{n-2}(v)f(v) - n(n-1)F^{n-1}(v)f(v)\right]dv \\
&= \int_0^\infty vf_{(2),n}(v)dv \\
&= E[R_{EA}]
\end{aligned}$$

The equality (1) has to be proven; the validity of this equation is shown in several steps.

$$\begin{aligned}
&\int_0^\infty nvF^{n-1}(v)f(v)dv - \int_0^\infty \int_0^v nF^{n-1}(y)dyf(v)dv \\
\overset{(1)}{=}\ &\int_0^\infty v\left[n(n-1)F^{n-2}(v)f(v) - n(n-1)F^{n-1}(v)f(v)\right]dv
\end{aligned}$$

Step 1:

First, integral $\int_0^\infty \int_0^v nF^{n-1}(y)dyf(v)dv$ on the left side of the equation is analyzed more thoroughly. By changing the integral parts and substituting the integral borders the following is derived:

$$\begin{aligned}
\int_0^\infty \int_0^v nF^{n-1}(y)dyf(v)dv &= \int_0^\infty \left[\int_y^\infty nf(v)dv\right]F^{n-1}(y)dy \\
&= \int_0^\infty \left[nF(v)\right]_y^\infty F^{n-1}(y)dy \\
&= \int_0^\infty n\left(F(\infty) - F(y)\right)F^{n-1}(y)dy \\
&= \int_0^\infty n\left(1 - F(y)\right)F^{n-1}(y)dy \\
&= \int_0^\infty nF^{n-1}(y) - nF^{n}(y)dy
\end{aligned}$$

Step 2:
Furthermore focussing on the first term in the last equation of Step 1 $\int_0^\infty nF^{n-1}(y)dy$ and replacing y by v, then, the following holds

$$\int_0^\infty nF^{n-1}(v)dv = \left[nvF^{n-1}(v)\right]_0^\infty - \int_0^\infty n(n-1)vF^{n-2}(v)f(v)dv$$

Now, the second term of the last equation of Step 1 is considered, $\int_0^\infty nF^n(y)dy$ and again, y is replaced by v:

$$\int_0^\infty nF^n(v)dv = [nvF^n(v)]_0^\infty - \int_0^\infty n^2vF^{n-1}(v)f(v)dv$$

Bringing both together this results in

$$\begin{aligned}
\int_0^\infty nF^{n-1}(v) - nF^n(v)dv &= \left[nvF^{n-1}(v)\right]_0^\infty - \int_0^\infty n(n-1)vF^{n-2}(v)f(v) \\
&\quad - [nvF^n(v)]_0^\infty + \int_0^\infty n^2vF^{n-1}(v)f(v)dv \\
&= \left[nvF^{n-1}(v)\right]_0^\infty - [nvF^n(v)]_0^\infty \\
&\quad - \int_0^\infty n(n-1)vF^{n-2}(v)f(v)dv + \int_0^\infty n^2vF^{n-1}(v)f(v)dv \\
&= 0 - \int_0^\infty n(n-1)vF^{n-2}(v)f(v)dv + \int_0^\infty n^2vF^{n-1}(v)f(v)dv
\end{aligned}$$

Step 3:
With Step 1 and Step 2 the following holds

$$\begin{aligned}
E[R_{DA}] &< \int_0^\infty nvF^{n-1}(v)f(v)dv - \int_0^\infty \int_0^v nF^{n-1}(y)dyf(v)dv \\
&= \int_0^\infty nvF^{n-1}(v)f(v)dv + \int_0^\infty n(n-1)vF^{n-2}(v)f(v)dv \\
&\quad - \int_0^\infty n^2vF^{n-1}(v)f(v)dv \\
&= \int_0^\infty n(1-n)vF^{n-1}(v)f(v)dv + \int_0^\infty n(n-1)vF^{n-2}(v)f(v)dv \\
&= \int_0^\infty n(n-1)vF^{n-2}(v)f(v)\left(1-F(v)\right)dv \\
&= E[R_{EA}]
\end{aligned}$$

Assume the discount to be zero. Then the expected revenue in a second-price auction is equal to the expected revenue in the DA: $E[R_{EA}] = E[R_{DA}]$. With $d = 0$ and $\delta = \frac{1}{1-d} = 1$ the equation is derived immediately. Moreover, the expected revenues are the same, as both auction mechanisms are identical. q.e.d.

Appendix D

Experimental approach

D.1 Design parameters

R.: round. P1: player (bidder) with number 1. P2: player (bidder) with number 2. P3: player (bidder) with number 3.

Induced valuations in the experiment

R.	Group 1			Group 2			Group 3			Group 4			Group 5		
	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
1	149	105	107	103	148	147	146	109	104	102	101	106	108	100	150
2	108	100	104	150	106	149	146	109	105	103	148	101	107	147	102
3	102	148	150	105	109	104	146	149	100	106	107	147	103	101	108
4	103	109	150	107	106	148	108	147	100	146	104	105	101	102	149
5	105	149	100	106	104	103	109	101	147	108	150	148	107	146	102
6	148	150	103	107	149	106	108	105	147	109	100	102	146	104	101
Discount in Session	$D1$	$D2$	$D3$	$D1$	$D2$	$D3$	$D1$	$D2$	$D3$	$D1$	$D2$	$D3$	$D1$	$D2$	$D3$

Table D.1: Induced valuations and assignment of discounts to bidders in the experiment

D.2 Experimental observations

Set.:Setting. S.:session. R.: round. P1: player (bidder) with number 1. P2: player (bidder) with number 2. P3: player (bidder) with number 3. NA: not available. A1: auction conducted withing group 1. A2: auction conducted within group 2. A3: auction conducted within group 3. A4: auction conducted within group 4. A5: auction conducted within group 5. In each round a single auction is conducted within each group. Thus, in each auction round 5 auctions are played simultaneously. Av.: Average earnings of participants in a single session.

Bids in setting $\bar{D}$ and setting D

Set.	S.	R.	Group 1			Group 2			Group 3			Group 4			Group 5		
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
$\bar{D}$	1	1	112.37	25	106.99	102.99	120.00	NA	146	90	104.00	1.00	101	80	98.5	90	128.0
$\bar{D}$	1	2	108.00	90	103.99	149.99	103.01	NA	146	107	105.00	10.01	100	101	102.0	132	85.0
$\bar{D}$	1	3	106.27	138	149.99	105.01	100.01	103.99	146	146	100.00	10.01	120	147	98.0	91	104.5
$\bar{D}$	1	4	118.00	109	149.99	107.01	104.50	147.99	108	146	100.00	145.99	250	0	100.0	96	147.5
$\bar{D}$	1	5	203.67	149	99.99	106.01	102.01	102.99	109	100	147.00	107.99	150	148	105.0	141	101.5
$\bar{D}$	1	6	165.45	150	102.99	107.01	145.10	105.99	108	105	147.01	108.99	100	102	141.0	100	100.5
$\bar{D}$	2	1	148.99	105	107	150.0	110.0	147.02	135.0	109	90.0	98.1	101	90.5	85	50.00	142.2
$\bar{D}$	2	2	108.00	105	104	102.0	90.3	149.00	139.9	109	95.0	100.9	148	100.1	89	100.00	100.1
$\bar{D}$	2	3	102.00	400	150	103.1	105.9	104.00	140.0	149	90.0	106.5	107	146.9	97	100.00	92.1
$\bar{D}$	2	4	103.00	115	150	106.3	101.0	148.00	108.0	147	100.0	146.0	104	104.9	94	101.99	125.2
$\bar{D}$	2	5	105.00	190	100	106.8	101.0	103.00	109.0	101	111.0	109.1	150	147.9	NA	146.00	102.0
$\bar{D}$	2	6	148.01	152	103	106.5	146.1	106.00	107.8	105	110.1	110.1	100	101.9	138	104.00	101.0
$\bar{D}$	3	1	148.99	105	150	103.00	148	146.50	146	109.01	104	101.00	90.01	80.0	25	99	145
$\bar{D}$	3	2	107.99	95	75	143.05	106	148.90	146	109.01	105	101.00	130.01	83.0	70	147	95
$\bar{D}$	3	3	101.99	140	136	105.01	109	103.99	146	149.01	100	105.00	107.00	121.7	97	101	96
$\bar{D}$	3	4	102.99	105	140	107.14	106	147.99	108	147.01	100	145.00	104.00	101.1	120	102	150
$\bar{D}$	3	5	104.99	147	89	112.49	104	102.99	109	101.01	147	107.50	150.00	130.7	111	146	110
$\bar{D}$	3	6	147.99	150	110	115.76	149	105.99	108	105.01	147	108.99	100.00	102.0	135	104	101
D	1	1	186.25	104.99	107.00	80.00	147.99	147.00	180.00	109	70	140	100	111	120.0	93.44	150
D	1	2	135.01	100.00	103.00	149.33	106.00	149.00	180.00	109	75	120	147	111	133.0	147.00	102
D	1	3	102.09	148.00	149.00	120.00	109.00	104.01	180.00	149	98	145	106	147	128.0	101.00	108
D	1	4	128.75	109.00	149.00	123.67	106.00	149.00	100.00	147	120	200	103	107	125.5	102.00	149
D	1	5	131.26	149.00	99.90	130.83	104.00	105.00	99.00	101	150	135	149	148	133.5	146.00	102
D	1	6	185.01	150.00	102.99	132.91	149.00	110.00	134.99	105	150	140	99	102	182.0	104.00	101
D	2	1	156.45	105.00	107.00	103	154	140	145.66	136.24	100.00	98.57	125	45	107.52	108.00	150
D	2	2	118.80	105.00	104.00	150	125	143	145.66	136.24	101.01	98.62	155	145	107.01	153.76	102
D	2	3	112.20	156.10	150.00	105	135	100	145.66	186.24	95.01	102.87	133	130	103.06	109.07	108
D	2	4	113.30	115.10	149.99	107	135	144	107.99	183.74	95.01	142.63	135	50	100.99	108.14	149
D	2	5	110.25	156.11	100.00	106	136	103	108.99	126.24	142.50	107.33	180	104	107.26	180.18	102

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Set.	S.	R.	Group 1			Group 2			Group 3			Group 4			Group 5		
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
D	2	6	155.40	160.11	103.00	107	187	106	107.99	131.24	147.11	108.99	125	92	146.02	129.57	101
D	3	1	80.00	105	127.20	100.00	125.53	150.0	146	120.00	130.00	101	80.03	132.00	40.0	100	187.00
D	3	2	107.01	100	130.00	150.00	89.79	150.0	146	110.00	131.25	102	145.03	126.25	88.0	147	127.50
D	3	3	101.50	147	187.50	105.00	89.99	120.0	146	170.00	125.00	106	105.03	183.75	100.0	101	135.00
D	3	4	102.50	109	187.50	107.15	90.91	150.1	110	147.01	125.00	146	103.53	131.25	103.2	102	186.00
D	3	5	104.50	149	125.00	106.17	100.51	125.0	110	121.30	183.75	108	149.53	185.00	113.0	146	127.50
D	3	6	147.51	150	128.75	108.00	143.91	125.0	110	125.90	183.75	109	100.00	127.50	146.0	104	126.25

Table D.2: Setting $\bar{D}$ and setting D – observed bids in the experiment

Results of the conducted auctions of setting $\bar{D}$ and setting D

Set.	S.	R.	Auction revenue in					Winning player in				
			A1	A2	A3	A4	A5	A1	A2	A3	A4	A5
$\bar{D}$	1	1	106.99	102.99	104	80	98.5	1	2	1	2	3
$\bar{D}$	1	2	103.99	103.01	107	100	102	1	1	1	3	2
$\bar{D}$	1	3	138	103.99	146	120	98	3	1	1	3	3
$\bar{D}$	1	4	118	107.01	108	145.99	100	3	3	2	2	3
$\bar{D}$	1	5	149	102.99	109	148	105	1	1	3	2	2
$\bar{D}$	1	6	150	107.01	108	102	100	1	2	3	1	1
$\bar{D}$	2	1	107	147.02	109	98.1	85	1	1	1	2	3
$\bar{D}$	2	2	105	102	109	100.9	100	1	3	1	2	3
$\bar{D}$	2	3	150	104	140	107	97	2	2	2	3	2
$\bar{D}$	2	4	115	106.3	108	104.9	101.99	3	3	2	1	3
$\bar{D}$	2	5	105	103	109	147.9	102	2	1	3	2	2
$\bar{D}$	2	6	148.01	106.5	107.8	101.9	104	2	2	3	1	1
$\bar{D}$	3	1	148.99	146.5	109.01	90.01	99	3	2	1	1	3
$\bar{D}$	3	2	95	143.05	109.01	101	95	1	3	1	2	2
$\bar{D}$	3	3	136	105.01	146	107	97	2	2	2	3	2
$\bar{D}$	3	4	105	107.14	108	104	120	3	3	2	1	3
$\bar{D}$	3	5	104.99	104	109	130.7	111	2	1	3	2	2
$\bar{D}$	3	6	147.99	115.76	108	102	104	2	2	3	1	1
D	1	1	85.6	147	87.2	88.8	120	1	2	1	1	3
D	1	2	82.4	119.2	87.2	120	133	1	1	1	2	2
D	1	3	148	87.2	119.2	145	86.4	3	1	1	3	1
D	1	4	128.75	123.67	120	85.6	125.5	3	3	2	1	3
D	1	5	131.26	84	101	148	133.5	2	1	3	2	2
D	1	6	120	132.91	134.99	81.6	83.2	1	2	3	1	1
D	2	1	107	112	136.24	78.856	108	1	2	1	2	3
D	2	2	105	143	136.24	116	85.608	1	1	1	2	2
D	2	3	120	84	116.528	104	86.4	2	2	2	2	2
D	2	4	115.1	135	86.392	135	108.14	3	3	2	1	3
D	2	5	88.2	84.8	126.24	85.864	85.808	2	2	3	2	2
D	2	6	124.32	85.6	131.24	87.192	129.57	2	2	3	2	1
D	3	1	84	100.424	130	80.8	80	3	3	1	3	3
D	3	2	85.608	120	131.25	126.25	127.5	3	3	1	2	2
D	3	3	117.6	84	146	84.8	80.8	3	3	2	3	3
D	3	4	87.2	85.72	125	131.25	82.56	3	3	2	1	3
D	3	5	125	84.936	97.04	119.624	127.5	2	3	3	3	2
D	3	6	147.51	125	100.72	87.2	126.25	2	2	3	3	1

Table D.3: Setting $\bar{D}$ and setting D – individual auction revenues

Bidders' payoffs in setting $\bar{D}$ and setting D

Set.	S.	R.	Group 1			Group 2			Group 3			Group 4			Group 5		
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
$\bar{D}$	1	1	42.01	0	0	0	45.01	0	42	0	0	0	21	0	0	0	51.5
$\bar{D}$	1	2	4.01	0	0	46.99	0	0	39	0	0	0	0	1	0	45	0
$\bar{D}$	1	3	0	0	12	1.01	0	0	0	0	0	0	0	27	0	0	10
$\bar{D}$	1	4	0	0	32	0	0	40.99	0	39	0	0	-41.99	0	0	0	49
$\bar{D}$	1	5	-44	0	0	3.01	0	0	0	0	38	0	2	0	0	41	0
$\bar{D}$	1	6	-2	0	0	0	41.99	0	0	0	39	7	0	0	45.5	0	0
$\bar{D}$	2	1	42	0	0	-44.02	0	0	37	0	0	0	2.9	0	0	0	65
$\bar{D}$	2	2	3	0	0	0	0	47	37	0	0	0	47.1	0	0	0	2
$\bar{D}$	2	3	0	-2	0	0	5	0	0	9	0	0	0	40	0	4	0
$\bar{D}$	2	4	0	0	35	0	0	41.7	0	39	0	41.1	0	0	0	0	47.01
$\bar{D}$	2	5	0	44	0	3	0	0	0	0	38	0	2.1	0	0	44	0
$\bar{D}$	2	6	0	1.99	0	0	42.5	0	0	0	39.2	7.1	0	0	42	0	0
$\bar{D}$	3	1	0	0	-41.99	0	1.50	0	36.99	0	0	11.99	0	0	0	0	51
$\bar{D}$	3	2	13	0	0	0	0	5.95	36.99	0	0	0	47	0	0	52	0
$\bar{D}$	3	3	0	12	0	0	3.99	0	0	3	0	0	0	40	0	4	0
$\bar{D}$	3	4	0	0	45	0	0	40.86	0	39	0	42	0	0	0	0	29
$\bar{D}$	3	5	0	44.01	0	2	0	0	0	0	38	0	19.3	0	0	35	0
$\bar{D}$	3	6	0	2.01	0	0	33.24	0	0	0	39	7	0	0	42	0	0
D	1	1	63.4	0	0	0	1	0	58.8	0	0	13.2	0	0	0	0	30
D	1	2	25.6	0	0	30.8	0	0	58.8	0	0	0	28	0	0	14	0
D	1	3	0	0	2	17.8	0	0	26.8	0	0	0	0	2	16.6	0	0
D	1	4	0	0	21.25	0	0	24.33	0	27	0	60.4	0	0	0	0	23.5
D	1	5	0	17.74	0	22	0	0	0	0	46	0	2	0	0	12.5	0
D	1	6	28	0	0	0	16.09	0	0	0	12.01	27.4	0	0	62.8	0	0
D	2	1	42	0	0	0	36	0	9.76	0	0	0	22.144	0	0	0	42
D	2	2	3	0	0	7	0	0	9.76	0	0	0	32	0	0	61.392	0
D	2	3	0	28	0	0	25	0	0	32.472	0	0	3	0	0	14.6	0
D	2	4	0	0	34.9	0	0	13	0	60.608	0	11	0	0	0	0	40.86
D	2	5	0	60.80	0	0	19.2	0	0	0	20.76	0	64.136	0	0	60.192	0

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Set.	S.	R.	Group 1			Group 2			Group 3			Group 4			Group 5		
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
D	2	6	0	25.68	0	0	63.4	0	0	0	15.76	0	12.808	0	16.43	0	0
D	3	1	0	0	23	0	0	46.576	16	0	0	0	0	25.2	0	0	70
D	3	2	0	0	18.392	0	0	29	14.75	0	0	0	21.75	0	0	19.5	0
D	3	3	0	0	32.4	0	0	20	0	3	0	0	0	62.2	0	0	27.20
D	3	4	0	0	62.8	0	0	62.280	0	22	0	14.75	0	0	0	0	66.44
D	3	5	0	24	0	0	0	18.064	0	0	49.96	0	0	28.376	0	18.5	0
D	3	6	0	2.49	0	0	24	0	0	0	46.28	0	0	14.8	19.75	0	0

Table D.4: Setting $\bar{D}$ and setting D – bidders' payoffs by rounds

Bidders' aggregated payoffs in setting $\bar{D}$ and setting D

Set.	S.	R.	Group 1			Group 2			Group 3			Group 4			Group 5		
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
$\bar{D}$	1	1	122.01	80	80	80	125.01	80	122	80	80	80	101	80	80	80	131.5
$\bar{D}$	1	2	126.02	80	80	126.99	125.01	80	161	80	80	80	101	81	80	125	131.5
$\bar{D}$	1	3	126.02	80	92	128	125.01	80	161	80	80	80	101	108	80	125	141.5
$\bar{D}$	1	4	126.02	80	124	128	125.01	120.99	161	119	80	80	59.01	108	80	125	190.5
$\bar{D}$	1	5	82.02	80	124	131.01	125.01	120.99	161	119	118	80	61.01	108	80	166	190.5
$\bar{D}$	1	6	80.02	80	124	131.01	167	120.99	161	119	157	87	61.01	108	125.5	166	190.5
$\bar{D}$	2	1	122	80	80	35.98	80	80	117	80	80	80	82.9	80	80	80	145
$\bar{D}$	2	2	125	80	80	35.98	80	127	154	80	80	80	130	80	80	80	147
$\bar{D}$	2	3	125	78	80	35.98	85	127	154	89	80	80	130	120	80	84	147
$\bar{D}$	2	4	125	78	115	35.98	85	168.7	154	128	80	121.1	130	120	80	84	194.01
$\bar{D}$	2	5	125	122	115	38.98	85	168.7	154	128	118	121.1	132.1	120	80	128	194.01
$\bar{D}$	2	6	125	123.99	115	38.98	127.5	168.7	154	128	157.2	128.2	132.1	120	122	128	194.01
$\bar{D}$	3	1	80	80	38.01	80	81.50	80	116.99	80	80	91.99	80	80	80	80	131
$\bar{D}$	3	2	93	80	38.01	80	81.50	85.95	153.98	80	80	91.99	127	80	80	132	131
$\bar{D}$	3	3	93	92	38.01	80	85.49	85.95	153.98	83	80	91.99	127	120	80	136	131
$\bar{D}$	3	4	93	92	83.01	80	85.49	126.81	153.98	122	80	133.99	127	120	80	136	160
$\bar{D}$	3	5	93	136.01	83.01	82	85.49	126.81	153.98	122	118	133.99	146.3	120	80	171	160
$\bar{D}$	3	6	93	138.02	83.01	82	118.73	126.81	153.98	122	157	140.99	146.3	120	122	171	160
D	1	1	143.4	80	80	80	81	80	138.8	80	80	93.2	80	80	80	80	110
D	1	2	169	80	80	110.8	81	80	197.6	80	80	93.2	108	80	80	94	110
D	1	3	169	80	82	128.6	81	80	224.4	80	80	93.2	108	82	96.6	94	110
D	1	4	169	80	103.25	128.6	81	104.33	224.4	107	80	153.6	108	82	96.6	94	133.5
D	1	5	169	97.74	103.25	150.6	81	104.33	224.4	107	126	153.6	110	82	96.6	106.5	133.5
D	1	6	197	97.74	103.25	150.6	97.09	104.33	224.4	107	138.01	181	110	82	159.4	106.5	133.5
D	2	1	122	80	80	80	116	80	89.76	80	80	80	102.144	80	80	80	122
D	2	2	125	80	80	87	116	80	99.52	80	80	80	134.144	80	80	141.392	122
D	2	3	125	108	80	87	141	80	99.52	112.472	80	80	137.144	80	80	155.992	122
D	2	4	125	108	114.9	87	141	93	99.52	173.080	80	91	137.144	80	80	155.992	162.86
D	2	5	125	168.80	114.9	87	160.2	93	99.52	173.080	100.76	91	201.280	80	80	216.184	162.86

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Set.	S.	R.	Group 1			Group 2			Group 3			Group 4			Group 5		
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
D	2	6	125	194.48	114.9	87	223.6	93	99.52	173.080	116.52	91	214.088	80	96.43	216.184	162.86
D	3	1	80	80	103	80	80	126.576	96	80	80	80	80	105.2	80	80	150
D	3	2	80	80	121.392	80	80	155.576	110.75	80	80	80	101.75	105.2	80	99.5	150
D	3	3	80	80	153.792	80	80	175.576	110.75	83	80	80	101.75	167.4	80	99.5	177.20
D	3	4	80	80	216.592	80	80	237.856	110.75	105	80	94.75	101.75	167.4	80	99.5	243.64
D	3	5	80	104	216.592	80	80	255.920	110.75	105	129.96	94.75	101.75	195.776	80	118	243.64
D	3	6	80	106.49	216.592	80	104	255.920	110.75	105	176.24	94.75	101.75	210.576	99.75	118	243.64

Table D.5: Setting $\bar{D}$ and setting D – bidders' aggregated payoffs by rounds

Bidders' earnings (in euros) in setting $\bar{D}$ and setting D

Set.	S.		Group 1			Group 2			Group 3			Group 4			Group 5	
		Av.	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
$\bar{D}$	1	12.52	8.00	8.00	12.40	13.10	16.70	12.10	16.10	11.90	15.70	8.70	6.10	10.80	12.55	16.60	19.05
$\bar{D}$	2	13.08	12.50	12.40	11.50	3.90	12.75	16.87	15.40	12.80	15.72	12.82	13.21	12.00	12.20	12.80	19.40
$\bar{D}$	3	12.90	9.30	13.80	8.30	8.20	11.87	12.68	15.40	12.20	15.70	14.10	14.63	12.00	12.20	17.10	16.00
D	1	13.28	19.70	9.77	10.32	15.06	9.71	10.43	22.44	10.70	13.80	18.10	11.00	8.20	15.94	10.65	13.35
D	2	13.92	12.50	19.45	11.49	8.70	22.36	9.30	9.95	17.31	11.65	9.10	21.41	8.00	9.64	21.62	16.29
D	3	14.02	8.00	10.65	21.66	8.00	10.40	25.59	11.08	10.50	17.62	9.48	10.18	21.06	9.98	11.80	24.36

Table D.6: Setting $\bar{D}$ and setting D – bidders' earnings in euros

D.3 Behavior of bidders

R: round. P1: player (bidder) with number 1. P2: player (bidder) with number 2. P3: player (bidder) with number 3. NA: not available. b: bids submitted to the auction. v: induced valuation. Tupel (b, v) of a player indicates the submitted bid b of that player which is based on the induced valuation v. Tupel $(\boldsymbol{b}, \boldsymbol{v})$ indicates the bid-valuation tuple of a designated bidder to whom a discount is assigned. In setting $\boldsymbol{D}$ players received a discount of $\mathbf{20\%}$ – (i) in session $\boldsymbol{D1}$ the discount was assigned to player 1, (ii) in session $\boldsymbol{D2}$ to player 2 and (iii) in session $\boldsymbol{D3}$ to player 3.

Bidding behavior in session $\bar{D}1$

R	**Group 1**						**Group 2**						**Group 3**					
	P1		**P2**		**P3**		**P1**		**P2**		**P3**		**P1**		**P2**		**P3**	
	b	v	b	v	b	v	b	v	b	v	b	v	b	v	b	v	b	v
1	112.37	149	25	105	106.99	107	102.99	103	120	148	NA	147	146	146	90	109	104	104
2	108	108	90	100	103.99	104	149.99	150	103.01	106	NA	149	146	146	107	109	105	105
3	106.27	102	138	148	149.99	150	105.01	105	100.01	109	103.99	104	146	146	146	149	100	100
4	118	103	109	109	149.99	150	107.01	107	104.50	106	147.99	148	108	108	146	147	100	100
5	203.67	105	149	149	99.99	100	106.01	106	102.01	104	102.99	103	109	109	100	101	147	147
6	165.45	148	150	150	102.99	103	107.01	107	145.10	149	105.99	106	108	108	105	105	147.01	147
mean	135.63	119.17	110.17	126.83	118.99	119	113	130	112.44	120.33	115.24	126.17	127.17	127.17	115.67	120	117.17	117.17

R	**Group 4**						**Group 5**					
	P1		**P2**		**P3**		**P1**		**P2**		**P3**	
	b	v	b	v	b	v	b	v	b	v	b	v
1	1.00	102	101	101	80	106	98.5	108	90	100	128.0	150
2	10.01	103	100	148	101	101	102.0	107	132	147	85.0	102
3	10.01	106	120	107	147	147	98.0	103	91	101	104.5	108
4	145.99	146	250	104	0	105	100.0	101	96	102	147.5	149
5	107.99	108	150	150	148	148	105.0	107	141	146	101.5	102
6	108.99	109	100	100	102	102	141.0	146	100	104	100.5	101
mean	64	112.33	136.83	118.33	96.33	118.17	107.42	112	108.33	116.67	111.17	118.67

Table D.7: Bidders' strategies in session $\bar{D}1$

Bidding behavior in session $\bar{D}2$

R	Group 1						Group 2						Group 3					
	P1		P2		P3		P1		P2		P3		P1		P2		P3	
	b	v	b	v	b	v	b	v	b	v	b	v	b	v	b	v	b	v
1	148.99	149	105	105	107	107	150	103	110	148	147.02	147	135	146	109	109	90	104
2	108	108	105	100	104	104	102	150	90.30	106	149	149	139.90	146	109	109	95	105
3	102	102	400	148	150	150	103.10	105	105.90	109	104	104	140	146	149	149	90	100
4	103	103	115	109	150	150	106.30	107	101	106	148	148	108	108	147	147	100	100
5	105	105	190	149	100	100	106.80	106	101	104	103	103	109	109	101	101	111	147
6	148.01	148	152	150	103	103	106.50	107	146.10	149	106	106	107.80	108	105	105	110.10	147
mean	119.17	119.17	177.83	126.83	119	119	112.45	113	109.05	120.33	126.17	126.17	123.28	127.17	120	120	99.35	117.17

R	Group 4						Group 5					
	P1		P2		P3		P1		P2		P3	
	b	v	b	v	b	v	b	v	b	v	b	v
1	98.10	102	101	101	90.50	106	85	108	50	100	142.20	150
2	100.90	103	148	148	100.10	101	89	107	100	147	100.10	102
3	106.50	106	107	107	146.90	147	97	103	100	101	92.10	108
4	146	146	104	104	104.90	105	94	101	101.99	102	125.20	149
5	109.10	108	150	150	147.90	148	NA	107	146	146	102	102
6	110.10	109	100	100	101.90	102	138	146	104	104	101	101
mean	111.78	112.33	118.33	118.33	115.37	118.17	100.60	112	100.33	116.67	110.43	118.67

Table D.8: Bidders' strategies in session $\bar{D}2$

Bidding behavior in session $\bar{D}3$

R	Group 1						Group 2						Group 3					
	P1		P2		P3		P1		P2		P3		P1		P2		P3	
	b	v	b	v	b	v	b	v	b	v	b	v	b	v	b	v	b	v
1	148.99	149	105	105	150	107	103	103	148	148	146.50	147	146	146	109.01	109	104	104
2	107.99	108	95	100	75	104	143.05	150	106	106	148.90	149	146	146	109.01	109	105	105
3	101.99	102	140	148	136	150	105.01	105	109	109	103.99	104	146	146	149.01	149	100	100
4	102.99	103	105	109	140	150	107.14	107	106	106	147.99	148	108	108	147.01	147	100	100
5	104.99	105	147	149	89	100	112.49	106	104	104	102.99	103	109	109	101.01	101	147	147
6	147.99	148	150	150	110	103	115.76	107	149	149	105.99	106	108	108	105.01	105	147	147
mean	119.16	119.17	123.67	126.83	116.67	119	114.41	113	120.33	120.33	126.06	126.17	127.17	127.17	120.01	120	117.17	117.17

R	Group 4						Group 5					
	P1		P2		P3		P1		P2		P3	
	b	v	b	v	b	v	b	v	b	v	b	v
1	101	102	90.01	101	80	106	25	108	99	100	145	150
2	101	103	130.01	148	83	101	70	107	147	147	95	102
3	105	106	107	107	121.70	147	97	103	101	101	96	108
4	145	146	104	104	101.10	105	120	101	102	102	150	149
5	107.50	108	150	150	130.70	148	111	107	146	146	110	102
6	108.99	109	100	100	102	102	135	146	104	104	101	101
mean	111.42	112.33	113.50	118.33	103.08	118.17	93	112	116.5	116.67	116.17	118.67

Table D.9: Bidders' strategies in session $\bar{D}3$

Bidding behavior in session $D1$

R	Group 1						Group 2						Group 3					
	P1		P2		P3		P1		P2		P3		P1		P2		P3	
	b	v	b	v	b	v	b	v	b	v	b	v	b	v	b	v	b	v
1	**186.25**	**149**	104.99	105	107	107	**80**	**103**	147.99	148	147	147	**180**	**146**	109	109	70	104
2	**135.01**	**108**	100	100	103	104	**149.33**	**150**	106	106	149	149	**180**	**146**	109	109	75	105
3	**102.09**	**102**	148	148	149	150	**120**	**105**	109	109	104.01	104	**180**	**146**	149	149	98	100
4	**128.75**	**103**	109	109	149	150	**123.67**	**107**	106	106	149	148	**100**	**108**	147	147	120	100
5	**131.26**	**105**	149	149	99.90	100	**130.83**	**106**	104	104	105	103	**99**	**109**	101	101	150	147
6	**185.01**	**148**	150	150	102.99	103	**132.91**	**107**	149	149	110	106	**134.99**	**108**	105	105	150	147
mean	**144.73**	**119.17**	126.83	126.83	118.48	119	**122.79**	**113**	120.33	120.33	127.33	126.17	**145.66**	**127.17**	120	120	110.5	117.17

R	Group 4						Group 5					
	P1		P2		P3		P1		P2		P3	
	b	v	b	v	b	v	b	v	b	v	b	v
1	**140**	**102**	100	101	111	106	**120**	**108**	93.44	100	150	150
2	**120**	**103**	147	148	111	101	**133**	**107**	147	147	102	102
3	**145**	**106**	106	107	147	147	**128**	**103**	101	101	108	108
4	**200**	**146**	103	104	107	105	**125.5**	**101**	102	102	149	149
5	**135**	**108**	149	150	148	148	**133.5**	**107**	146	146	102	102
6	**140**	**109**	99	100	102	102	**182**	**146**	104	104	101	101
mean	**146.67**	**112.33**	117.33	118.33	121	118.17	**137**	**112**	115.57	116.67	118.67	118.67

Table D.10: Bidders' strategies in session $D1$

Bidding behavior in session $D2$

R	Group 1						Group 2						Group 3					
	P1		P2		P3		P1		P2		P3		P1		P2		P3	
	b	v	b	v	b	v	b	v	b	v	b	v	b	v	b	v	b	v
1	156.45	149	**105**	**105**	107	107	103	103	**154**	**148**	140	147	145.66	146	**136.24**	**109**	100	104
2	118.80	108	**105**	**100**	104	104	150	150	**125**	**106**	143	149	145.66	146	**136.24**	**109**	101.01	105
3	112.20	102	**156.10**	**148**	150	150	105	105	**135**	**109**	100	104	145.66	146	**186.24**	**149**	95.01	100
4	113.30	103	**115.10**	**109**	149.99	150	107	107	**135**	**106**	144	148	107.99	108	**183.74**	**147**	95.01	100
5	110.25	105	**156.11**	**149**	100	100	106	106	**136**	**104**	103	103	108.99	109	**126.24**	**101**	142.50	147
6	155.40	148	**160.11**	**150**	103	103	107	107	**187**	**149**	106	106	107.99	108	**131.24**	**105**	147.11	147
mean	127.73	119.17	**132.90**	**126.83**	119	119	113	113	**145.33**	**120.33**	122.67	126.17	126.99	127.17	**149.99**	**120**	113.44	117.17

R	Group 4						Group 5					
	P1		P2		P3		P1		P2		P3	
	b	v	b	v	b	v	b	v	b	v	b	v
1	98.57	102	**125**	**101**	45	106	107.52	108	**108**	**100**	150	150
2	98.62	103	**155**	**148**	145	101	107.01	107	**153.76**	**147**	102	102
3	102.87	106	**133**	**107**	130	147	103.06	103	**109.07**	**101**	108	108
4	142.63	146	**135**	**104**	50	105	100.99	101	**108.14**	**102**	149	149
5	107.33	108	**180**	**150**	104	148	107.26	107	**180.18**	**146**	102	102
6	108.99	109	**125**	**100**	92	102	146.02	146	**129.57**	**104**	101	101
mean	109.83	112.33	**142.17**	**118.33**	94.33	118.17	111.98	112	**131.45**	**116.67**	118.67	118.67

Table D.11: Bidders' strategies in session $D2$

Bidding behavior in session $D3$

R	Group 1						Group 2						Group 3					
	P1		P2		P3		P1		P2		P3		P1		P2		P3	
	b	v	b	v	b	v	b	v	b	v	b	v	b	v	b	v	b	v
1	80	149	105	105	**127.20**	**107**	100	103	125.53	148	**150**	**147**	146	146	120	109	**130**	**104**
2	107.01	108	100	100	**130**	**104**	150	150	89.79	106	**150**	**149**	146	146	110	109	**131.25**	**105**
3	101.50	102	147	148	**187.50**	**150**	105	105	89.99	109	**120**	**104**	146	146	170	149	**125**	**100**
4	102.50	103	109	109	**187.50**	**150**	107.15	107	90.91	106	**150.10**	**148**	110	108	147.01	147	**125**	**100**
5	104.50	105	149	149	**125**	**100**	106.17	106	100.51	104	**125**	**103**	110	109	121.30	101	**183.75**	**147**
6	147.51	148	150	150	**128.75**	**103**	108	107	143.91	149	**125**	**106**	110	108	125.90	105	**183.75**	**147**
mean	107.17	119.17	126.67	126.83	**147.66**	**119**	112.72	113	106.77	120.33	**136.68**	**126.17**	128	127.17	132.37	120	**146.46**	**117.17**

R	Group 4						Group 5					
	P1		P2		P3		P1		P2		P3	
	b	v	b	v	b	v	b	v	b	v	b	v
1	101	102	80.03	101	**132**	**106**	40	108	100	100	**187**	**150**
2	102	103	145.03	148	**126.25**	**101**	88	107	147	147	**127.50**	**102**
3	106	106	105.03	107	**183.75**	**147**	100	103	101	101	**135**	**108**
4	146	146	103.53	104	**131.25**	**105**	103.20	101	102	102	**186**	**149**
5	108	108	149.53	150	**185**	**148**	113	107	146	146	**127.50**	**102**
6	109	109	100	100	**127.50**	**102**	146	146	104	104	**126.25**	**101**
mean	112	112.33	113.86	118.33	**147.62**	**118.17**	98.37	112	116.67	116.67	**148.21**	**118.67**

Table D.12: Bidders' strategies in session $D3$

D.4 Auction revenues in the treatments

Notation: No.: number. v_i: valuation of bidder i. b_i: bid submitted by bidder i. Pi: Player i. $i = 1, 2, 3$. $R_{\bar{D}s}$: auction revenue in treatment $\bar{D}s$. R_{Ds}: auction revenue in treatment Ds. $R_{\bar{D}a}$: auction revenue in treatment $\bar{D}a$. R_{Da}: auction revenue in treatment Da. Exp.: Experiment. Theo.: Theory. $\#strong$: number of groups in which the discount is assigned to a strong bidder. $\#weak$: number of groups in which the discount is assigned to a weak bidder. q: $\frac{\#strong}{\#weak}$ proportion of $\#strong$ and $\#weak$. p-value: p-value of the Wilcoxon signed-ranks test (matched-pairs). V: result of the Wilcoxon singed-ranks test.

Auction revenues in treatment $\bar{D}s$ and treatment Ds by rounds

		Treatment $\bar{D}s$								Treatment Ds									
		Valuations			Bids in $\bar{D}s$			$R_{\bar{D}s}$		Discount			Bids in Ds			R_{Ds}	
Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
1	1	105	107	103	25.00	106.99	102.99	102.99	105	1	0	0	105.00	107.00	103.00	105.00	85.6
1	2	109	104	102	90.00	104.00	1.00	90.00	104	1	0	0	136.24	70.00	98.57	78.86	83.2
1	3	101	106	108	101.00	80.00	98.50	98.50	106	1	0	0	125.00	111.00	107.52	88.80	86.4
1	4	100	105	107	90.00	105.00	107.00	105.00	105	1	0	0	108.00	104.99	107.00	85.60	85.6
1	5	103	109	104	150.00	109.00	90.00	109.00	104	1	0	0	80.00	109.00	100.00	100.00	87.2
1	6	102	101	106	98.10	101.00	90.50	98.10	102	1	0	0	140.00	100.00	45.00	80.00	84.8
1	7	108	100	105	85.00	50.00	105.00	85.00	105	1	0	0	120.00	93.44	105.00	84.00	84.0
1	8	107	103	109	150.00	103.00	109.01	109.01	107	1	0	0	127.20	100.00	120.00	96.00	87.2
1	9	104	102	101	104.00	101.00	90.01	101.00	102	1	0	0	130.00	101.00	80.03	80.80	81.6
1	10	106	108	100	80.00	25.00	99.00	80.00	106	1	0	0	132.00	40.00	100.00	80.00	86.4
1	11	149	148	147	112.37	120.00	0.00	112.37	148	1	0	0	186.25	147.99	147.00	118.39	118.4
1	12	146	150	149	146.00	128.00	148.99	146.00	149	1	0	0	180.00	150.00	156.45	125.16	120.0
1	13	148	147	146	110.00	147.02	135.00	135.00	147	1	0	0	154.00	140.00	145.66	116.53	117.6
1	14	150	149	148	142.20	148.99	148.00	148.00	149	1	0	0	187.00	80.00	125.53	100.42	119.2
1	15	147	146	150	146.50	146.00	145.00	146.00	147	1	0	0	150.00	146.00	150.00	120.00	120.0
2	1	108	100	104	108.00	90.00	103.99	103.99	104	1	0	0	135.01	100.00	103.00	82.40	83.2
2	2	106	109	105	103.01	107.00	105.00	105.00	106	1	0	0	125.00	109.00	75.00	87.20	87.2
2	3	103	101	107	10.01	101.00	102.00	101.00	103	1	0	0	120.00	111.00	107.01	88.80	85.6
2	4	102	108	100	85.00	108.00	105.00	105.00	102	1	0	0	127.50	118.80	100.00	95.04	86.4
2	5	104	106	109	104.00	90.30	109.00	104.00	106	1	0	0	130.00	106.00	110.00	88.00	87.2
2	6	105	103	101	95.00	100.90	100.10	100.10	103	1	0	0	131.25	98.62	145.00	131.25	82.4
2	7	107	102	108	89.00	100.10	107.99	100.10	107	1	0	0	133.00	102.00	107.01	85.61	86.4

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		Treatment $\bar{D}s$								Treatment Ds							
		Valuations			Bids in $\bar{D}s$			$R_{\bar{D}s}$		Discount			Bids in Ds			R_{Ds}	
Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
2	8	100	104	106	95.00	75.00	106.00	95.00	104	1	0	0	105.00	104.00	89.79	83.20	84.8
2	9	109	105	103	109.01	105.00	101.00	105.00	105	1	0	0	136.24	101.01	102.00	81.60	84.0
2	10	101	107	102	83.00	70.00	95.00	83.00	102	1	0	0	126.25	88.00	102.00	81.60	85.6
2	11	150	149	146	149.99	0.00	146.00	146.00	149	1	0	0	149.33	149.00	145.66	119.20	119.2
2	12	148	147	150	100.00	132.00	102.00	102.00	148	1	0	0	155.00	147.00	150.00	120.00	120.0
2	13	149	146	148	149.00	139.90	148.00	148.00	148	1	0	0	150.00	146.00	147.00	117.60	118.4
2	14	147	150	149	100.00	143.05	148.90	143.05	149	1	0	0	153.76	150.00	143.00	120.00	120.0
2	15	146	148	147	146.00	130.01	147.00	146.00	147	1	0	0	180.00	145.03	147.00	117.60	118.4
3	1	102	105	109	106.27	105.01	100.01	105.01	105	1	0	0	102.09	105.00	109.00	105.00	87.2
3	2	104	100	106	103.99	100.00	10.01	100.00	104	1	0	0	120.00	98.00	102.87	82.30	84.8
3	3	107	103	101	120.00	98.00	91.00	98.00	103	1	0	0	133.00	103.06	101.00	82.45	82.4
3	4	108	102	105	104.50	102.00	103.10	103.10	105	1	0	0	135.00	112.20	105.00	89.76	84.0
3	5	109	104	100	105.90	104.00	90.00	104.00	104	1	0	0	135.00	104.01	95.01	83.20	83.2
3	6	106	107	103	106.50	107.00	97.00	106.50	106	1	0	0	145.00	106.00	100.00	84.80	85.6
3	7	101	108	102	100.00	92.10	101.99	100.00	102	1	0	0	109.07	108.00	101.50	86.40	86.4
3	8	105	109	104	105.01	109.00	103.99	105.01	105	1	0	0	120.00	89.99	100.00	80.00	87.2
3	9	100	106	107	100.00	105.00	107.00	105.00	106	1	0	0	125.00	106.00	105.03	84.80	85.6
3	10	103	101	108	97.00	101.00	96.00	97.00	103	1	0	0	128.00	101.00	108.00	86.40	86.4
3	11	148	150	146	138.00	149.99	146.00	146.00	148	1	0	0	156.10	149.00	145.66	119.20	120.0
3	12	149	147	148	146.00	147.00	400.00	147.00	148	1	0	0	186.24	147.00	148.00	118.40	118.4
3	13	150	146	149	150.00	140.00	149.00	149.00	149	1	0	0	187.50	146.00	149.00	119.20	119.2
3	14	147	148	150	146.90	140.00	136.00	140.00	148	1	0	0	183.75	147.00	150.00	120.00	120.0
3	15	146	149	147	146.00	149.01	121.70	146.00	147	1	0	0	180.00	170.00	130.00	136.00	119.2
4	1	103	109	107	118.00	109.00	107.01	109.00	107	1	0	0	128.75	109.00	107.00	87.20	87.2
4	2	106	108	100	104.50	108.00	100.00	104.50	106	1	0	0	135.00	107.99	120.00	96.00	86.4
4	3	104	105	101	250.00	0.00	100.00	100.00	104	1	0	0	135.00	107.00	100.99	85.60	84.0

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		Treatment $\bar{D}s$								Treatment Ds							
		Valuations			Bids in $\bar{D}s$			$R_{\bar{D}s}$		Discount			Bids in Ds			R_{Ds}	
Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
4	4	102	103	109	96.00	103.00	115.00	103.00	103	1	0	0	108.14	113.30	109.00	109.00	87.2
4	5	107	106	108	106.30	101.00	108.00	106.30	107	1	0	0	123.67	106.00	110.00	88.00	86.4
4	6	100	104	105	100.00	104.00	104.90	104.00	104	1	0	0	125.00	103.00	50.00	82.40	84.0
4	7	101	102	103	94.00	101.99	102.99	101.99	102	1	0	0	125.50	102.00	102.50	82.00	82.4
4	8	109	107	106	105.00	107.14	106.00	106.00	107	1	0	0	115.10	107.15	90.91	85.72	85.6
4	9	108	100	104	108.00	100.00	104.00	104.00	104	1	0	0	100.00	95.01	103.53	100.00	83.2
4	10	105	101	102	101.10	120.00	102.00	102.00	102	1	0	0	131.25	103.20	102.00	82.56	81.6
4	11	150	148	147	149.99	147.99	146.00	147.99	148	1	0	0	187.50	149.00	147.00	119.20	118.4
4	12	146	149	150	145.99	147.50	150.00	147.50	149	1	0	0	200.00	149.00	149.00	119.20	120.0
4	13	148	147	146	148.00	147.00	146.00	147.00	147	1	0	0	150.10	147.01	142.63	117.60	117.6
4	14	149	150	148	125.20	140.00	147.99	140.00	149	1	0	0	186.00	149.99	144.00	119.99	120.0
4	15	147	146	149	147.01	145.00	150.00	147.01	147	1	0	0	183.74	146.00	149.00	119.20	119.2
5	1	105	100	106	203.67	99.99	106.01	106.01	105	1	0	0	131.26	99.90	106.00	84.80	84.8
5	2	104	103	109	102.01	102.99	109.00	102.99	104	1	0	0	136.00	105.00	108.99	87.19	87.2
5	3	101	108	107	100.00	107.99	105.00	105.00	107	1	0	0	126.24	107.33	107.26	85.86	86.4
5	4	102	105	100	101.50	105.00	100.00	101.50	102	1	0	0	127.50	110.25	100.00	88.20	84.0
5	5	106	104	103	106.80	101.00	103.00	103.00	104	1	0	0	130.83	104.00	103.00	83.20	83.2
5	6	109	101	108	109.00	101.00	109.10	109.00	108	1	0	0	99.00	101.00	108.00	101.00	86.4
5	7	107	102	105	0.00	102.00	104.99	102.00	105	1	0	0	133.50	102.00	104.50	83.60	84.0
5	8	100	106	104	89.00	112.49	104.00	104.00	104	1	0	0	125.00	106.17	100.51	84.94	84.8
5	9	103	109	101	102.99	109.00	101.01	102.99	103	1	0	0	125.00	110.00	121.30	97.04	87.2
5	10	108	107	102	107.50	111.00	110.00	110.00	107	1	0	0	135.00	113.00	102.00	90.40	85.6
5	11	149	147	150	149.00	147.00	150.00	149.00	149	1	0	0	156.11	150.00	149.00	120.00	120.0
5	12	148	146	149	148.00	141.00	190.00	148.00	148	1	0	0	185.00	146.00	149.00	119.20	119.2
5	13	147	150	148	111.00	150.00	147.90	147.90	148	1	0	0	183.75	149.53	148.00	119.62	120.0
5	14	146	149	147	146.00	147.00	147.00	147.00	147	1	0	0	180.18	149.00	142.50	119.20	119.2

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					Treatment $\bar{D}s$								Treatment Ds				
		Valuations			Bids in $\bar{D}s$			$R_{\bar{D}s}$		Discount			Bids in Ds			R_{Ds}	
Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
5	15	150	148	146	150.00	130.70	146.00	146.00	148	1	0	0	180.00	104.00	146.00	116.80	118.4
6	1	103	107	106	102.99	107.01	105.99	105.99	106	1	0	0	128.75	107.00	110.00	88.00	85.6
6	2	108	105	109	108.00	105.00	108.99	108.00	108	1	0	0	134.99	105.00	108.99	87.19	87.2
6	3	100	102	104	100.00	102.00	100.00	100.00	102	1	0	0	125.00	102.00	104.00	83.20	83.2
6	4	101	103	107	100.50	103.00	106.50	103.00	103	1	0	0	126.25	102.99	108.00	86.40	85.6
6	5	106	108	105	106.00	107.80	105.00	106.00	106	1	0	0	125.00	107.99	125.90	125.00	86.4
6	6	109	100	102	110.10	100.00	101.90	101.90	102	1	0	0	140.00	99.00	92.00	79.20	81.6
6	7	104	101	103	104.00	101.00	110.00	104.00	103	1	0	0	129.57	101.00	103.00	82.40	82.4
6	8	107	106	108	115.76	105.99	108.00	108.00	107	1	0	0	132.91	106.00	110.00	88.00	86.4
6	9	105	109	100	105.01	108.99	100.00	105.01	105	1	0	0	131.24	109.00	100.00	87.20	87.2
6	10	102	104	101	102.00	104.00	101.00	102.00	102	1	0	0	127.50	104.00	101.00	83.20	83.2
6	11	148	150	149	165.45	150.00	145.10	150.00	149	1	0	0	185.01	150.00	149.00	120.00	120.0
6	12	147	146	148	147.01	141.00	148.01	147.01	147	1	0	0	183.75	146.02	155.40	124.32	118.4
6	13	150	149	147	152.00	146.10	110.10	146.10	149	1	0	0	160.11	143.91	150.00	120.00	119.2
6	14	146	148	150	138.00	147.99	150.00	147.99	148	1	0	0	182.00	147.51	150.00	120.00	120.0
6	15	149	147	146	149.00	147.00	135.00	147.00	147	1	0	0	187.00	147.11	146.00	117.69	117.6
mean								116.0	119.01							98.92	96.5

Table D.13: Treatment $\bar{D}s$ and treatment Ds – auction revenues by rounds

Auction revenues in treatment $\bar{D}a$ and treatment Da by rounds

		Treatment $\bar{D}a$								Treatment Da						
		Valuations			Bids in $\bar{D}a$			$R_{\bar{D}a}$		Discount			Bids in Da			R_{Da}	
Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
1	1	149	105	107	112.37	25.00	106.99	106.99	107	0	1	0	156.45	105.00	107.00	107.00	131.25
1	2	148	103	109	120.00	102.99	90.00	102.99	109	0	1	0	147.99	80.00	109.00	109.00	128.75
1	3	147	104	102	147.02	104.00	1.00	104.00	104	0	1	0	147.00	130.00	98.57	130.00	130.00
1	4	146	101	106	146.00	101.00	80.00	101.00	106	0	1	0	145.66	125.00	111.00	125.00	126.25
1	5	150	108	100	128.00	98.50	90.00	98.50	108	0	1	0	150.00	120.00	93.44	120.00	135.00
1	6	149	105	107	148.99	105.00	107.00	107.00	107	0	0	1	80.00	104.99	127.20	83.99	133.75
1	7	148	103	109	110.00	150.00	109.00	110.00	109	0	0	1	125.53	103.00	136.24	100.42	136.25
1	8	147	104	102	0.00	90.00	98.10	90.00	104	0	0	1	140.00	70.00	140.00	140.00	127.50
1	9	146	101	106	135.00	101.00	90.50	101.00	106	0	0	1	146.00	100.00	132.00	132.00	132.50
1	10	150	108	100	142.20	85.00	50.00	85.00	108	0	0	1	150.00	107.52	108.00	108.00	125.00
1	11	149	105	107	148.99	105.00	150.00	148.99	107	1	0	0	186.25	105.00	107.00	85.60	85.60
1	12	148	103	109	148.00	103.00	109.01	109.01	109	1	0	0	154.00	100.00	120.00	96.00	87.20
1	13	147	104	102	146.50	104.00	101.00	104.00	104	1	0	0	150.00	100.00	101.00	80.80	83.20
1	14	146	101	106	146.00	90.01	80.00	90.01	106	1	0	0	180.00	80.03	45.00	64.02	84.80
1	15	150	108	100	145.00	25.00	99.00	99.00	108	1	0	0	187.00	40.00	100.00	80.00	86.40
2	1	150	108	100	149.99	108.00	90.00	108.00	108	0	1	0	150.00	135.01	100.00	135.01	135.00
2	2	149	104	106	0.00	103.99	103.01	103.01	106	0	1	0	149.00	130.00	106.00	130.00	130.00
2	3	146	109	105	146.00	107.00	105.00	107.00	109	0	1	0	145.66	136.24	75.00	136.24	136.25
2	4	148	103	101	100.00	10.01	101.00	100.00	103	0	1	0	147.00	120.00	111.00	120.00	128.75
2	5	147	107	102	132.00	102.00	85.00	102.00	107	0	1	0	147.00	133.00	102.00	133.00	133.75
2	6	150	108	100	102.00	108.00	105.00	105.00	108	0	0	1	150.00	118.80	105.00	118.80	125.00
2	7	149	104	106	149.00	104.00	90.30	104.00	106	0	0	1	143.00	103.00	125.00	125.00	132.50

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					Treatment $\bar{D}a$								Treatment Da				
		Valuations			Bids in $\bar{D}a$			$R_{\bar{D}a}$		Discount			Bids in Da			R_{Da}	
Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
2	8	146	109	105	139.90	109.00	95.00	109.00	109	0	0	1	146.00	109.00	131.25	131.25	131.25
2	9	148	103	101	148.00	100.90	100.10	100.90	103	0	0	1	145.03	98.62	126.25	126.25	126.25
2	10	147	107	102	100.00	89.00	100.10	100.00	107	0	0	1	147.00	107.01	127.50	127.50	127.50
2	11	150	108	100	143.05	107.99	95.00	107.99	108	1	0	0	149.33	107.01	100.00	85.61	86.40
2	12	149	104	106	148.90	75.00	106.00	106.00	106	1	0	0	150.00	104.00	89.79	83.20	84.80
2	13	146	109	105	146.00	109.01	105.00	109.01	109	1	0	0	180.00	110.00	101.01	88.00	87.20
2	14	148	103	101	130.01	101.00	83.00	101.00	103	1	0	0	155.00	102.00	145.00	116.00	82.40
2	15	147	107	102	147.00	70.00	95.00	95.00	107	1	0	0	153.76	88.00	102.00	81.60	85.60
3	1	148	102	105	138.00	106.27	105.01	106.27	105	0	1	0	148.00	102.09	105.00	105.00	127.50
3	2	150	109	104	149.99	100.01	103.99	103.99	109	0	1	0	149.00	135.00	104.01	135.00	136.25
3	3	146	100	106	146.00	100.00	10.01	100.00	106	0	1	0	145.66	125.00	102.87	125.00	125.00
3	4	149	107	103	146.00	120.00	98.00	120.00	107	0	1	0	149.00	133.00	103.06	133.00	133.75
3	5	147	101	108	147.00	91.00	104.50	104.50	108	0	1	0	147.00	109.07	108.00	109.07	126.25
3	6	148	102	105	400.00	102.00	103.10	103.10	105	0	0	1	147.00	112.20	120.00	120.00	131.25
3	7	150	109	104	150.00	105.90	104.00	105.90	109	0	0	1	150.00	109.00	120.00	120.00	130.00
3	8	146	100	106	140.00	90.00	106.50	106.50	106	0	0	1	146.00	98.00	145.00	145.00	132.50
3	9	149	107	103	149.00	107.00	97.00	107.00	107	0	0	1	170.00	106.00	128.00	128.00	128.75
3	10	147	101	108	146.90	100.00	92.10	100.00	108	0	0	1	130.00	101.00	135.00	104.00	135.00
3	11	148	102	105	140.00	101.99	105.01	105.01	105	1	0	0	156.10	101.50	105.00	84.00	84.00
3	12	150	109	104	136.00	109.00	103.99	109.00	109	1	0	0	187.50	89.99	100.00	80.00	87.20
3	13	146	100	106	146.00	100.00	105.00	105.00	106	1	0	0	180.00	95.01	106.00	84.80	84.80
3	14	149	107	103	149.01	107.00	97.00	107.00	107	1	0	0	186.24	105.03	100.00	84.02	85.60
3	15	147	101	108	121.70	101.00	96.00	101.00	108	1	0	0	183.75	101.00	108.00	86.40	86.40
4	1	150	103	109	149.99	118.00	109.00	118.00	109	0	1	0	149.00	128.75	109.00	128.75	128.75
4	2	148	107	106	147.99	107.01	104.50	107.01	107	0	1	0	149.00	123.67	106.00	123.67	133.75
4	3	147	108	100	146.00	108.00	100.00	108.00	108	0	1	0	147.00	100.00	120.00	120.00	135.00

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					Treatment $\bar{D}a$								Treatment Da				
		Valuations			Bids in $\bar{D}a$			$R_{\bar{D}a}$		Discount			Bids in Da			R_{Da}	
Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
4	4	146	104	105	145.99	250.00	0.00	145.99	105	0	1	0	142.63	135.00	107.00	135.00	130.00
4	5	149	101	102	147.50	100.00	96.00	100.00	102	0	1	0	149.00	125.50	102.00	125.50	126.25
4	6	150	103	109	150.00	103.00	115.00	115.00	109	0	0	1	149.99	113.30	115.10	115.10	136.25
4	7	148	107	106	148.00	106.30	101.00	106.30	107	0	0	1	144.00	107.00	135.00	135.00	132.50
4	8	147	108	100	147.00	108.00	100.00	108.00	108	0	0	1	147.01	107.99	125.00	125.00	125.00
4	9	146	104	105	146.00	104.00	104.90	104.90	105	0	0	1	146.00	103.00	131.25	131.25	131.25
4	10	149	101	102	125.20	94.00	101.99	101.99	102	0	0	1	149.00	100.99	108.14	108.14	127.50
4	11	150	103	109	140.00	102.99	105.00	105.00	109	1	0	0	187.50	102.50	109.00	87.20	87.20
4	12	148	107	106	147.99	107.14	106.00	107.14	107	1	0	0	150.10	107.15	90.91	85.72	85.60
4	13	147	108	100	147.01	108.00	100.00	108.00	108	1	0	0	183.74	110.00	95.01	88.00	86.40
4	14	146	104	105	145.00	104.00	101.10	104.00	105	1	0	0	200.00	103.53	50.00	82.82	84.00
4	15	149	101	102	150.00	120.00	102.00	120.00	102	1	0	0	186.00	103.20	102.00	82.56	81.60
5	1	149	105	100	149.00	203.67	99.99	149.00	105	0	1	0	149.00	131.26	99.90	131.26	131.25
5	2	147	106	104	147.00	106.01	102.01	106.01	106	0	1	0	150.00	130.83	104.00	130.83	132.50
5	3	150	103	109	150.00	102.99	109.00	109.00	109	0	1	0	149.00	125.00	108.99	125.00	128.75
5	4	148	101	108	148.00	100.00	107.99	107.99	108	0	1	0	148.00	126.24	107.33	126.24	126.25
5	5	146	107	102	141.00	105.00	101.50	105.00	107	0	1	0	146.00	133.50	102.00	133.50	133.75
5	6	149	105	100	190.00	105.00	100.00	105.00	105	0	0	1	149.00	110.25	125.00	125.00	125.00
5	7	147	106	104	111.00	106.80	101.00	106.80	106	0	0	1	142.50	106.00	136.00	136.00	130.00
5	8	150	103	109	150.00	103.00	109.00	109.00	109	0	0	1	149.53	105.00	99.00	105.00	136.25
5	9	148	101	108	147.90	101.00	109.10	109.10	108	0	0	1	104.00	101.00	135.00	83.20	135.00
5	10	146	107	102	146.00	0.00	102.00	102.00	107	0	0	1	146.00	107.26	127.50	127.50	127.50
5	11	149	105	100	147.00	104.99	89.00	104.99	105	1	0	0	156.11	104.50	100.00	83.60	84.00
5	12	147	106	104	147.00	112.49	104.00	112.49	106	1	0	0	183.75	106.17	100.51	84.93	84.80
5	13	150	103	109	150.00	102.99	109.00	109.00	109	1	0	0	180.00	103.00	110.00	88.00	87.20
5	14	148	101	108	130.70	101.01	107.50	107.50	108	1	0	0	185.00	121.30	108.00	97.04	86.40

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					Treatment $\bar{D}a$								Treatment Da				
		Valuations			Bids in $\bar{D}a$			$R_{\bar{D}a}$		Discount			Bids in Da			R_{Da}	
Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
5	15	146	107	102	146.00	111.00	110.00	111.00	107	1	0	0	180.18	113.00	102.00	90.40	85.60
6	1	148	103	107	165.45	102.99	107.01	107.01	107	0	1	0	155.40	128.75	107.00	128.75	128.75
6	2	150	106	108	150.00	105.99	108.00	108.00	108	0	1	0	150.00	125.00	107.99	125.00	132.50
6	3	149	105	109	145.10	105.00	108.99	108.99	109	0	1	0	149.00	131.24	108.99	131.24	131.25
6	4	147	100	102	147.01	100.00	102.00	102.00	102	0	1	0	150.00	125.00	102.00	125.00	125.00
6	5	146	104	101	141.00	100.00	100.50	100.50	104	0	1	0	146.02	129.57	101.00	129.57	130.00
6	6	148	103	107	148.01	103.00	106.50	106.50	107	0	0	1	147.51	102.99	132.91	132.91	133.75
6	7	150	106	108	152.00	106.00	107.80	107.80	108	0	0	1	150.00	110.00	134.99	134.99	135.00
6	8	149	105	109	146.10	105.00	110.10	110.10	109	0	0	1	143.91	105.00	140.00	140.00	136.25
6	9	147	100	102	110.10	100.00	101.90	101.90	102	0	0	1	147.11	99.00	127.50	127.50	127.50
6	10	146	104	101	138.00	104.00	101.00	104.00	104	0	0	1	146.00	104.00	126.25	126.25	126.25
6	11	148	103	107	147.99	110.00	115.76	115.76	107	1	0	0	185.01	103.00	108.00	86.40	85.60
6	12	150	106	108	150.00	105.99	108.00	108.00	108	1	0	0	160.11	106.00	110.00	88.00	86.40
6	13	149	105	109	149.00	105.01	108.99	108.99	109	1	0	0	187.00	125.90	109.00	100.72	87.20
6	14	147	100	102	147.00	100.00	102.00	102.00	102	1	0	0	183.75	100.00	92.00	80.00	81.60
6	15	146	104	101	135.00	104.00	101.00	104.00	104	1	0	0	182.00	104.00	101.00	83.20	83.20
mean								106.8	106.6							111.37	115.51

Table D.14: Treatment $\bar{D}a$ and treatment Da – auction revenues by rounds

Auction revenues in treatment $\bar{D}s$ and treatment Ds: round 1

			Treatment $\bar{D}s$								Treatment Ds							
			Valuations			Bids in $\bar{D}s$			$R_{\bar{D}s}$		Discount			Bids in Ds			R_{Ds}	
No.	Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
1	1	13	148	147	146	110.00	147.02	135.00	135.00	147	1	0	0	154.00	140.00	145.66	116.53	117.6
2	1	14	150	149	148	142.20	148.99	148.00	148.00	149	1	0	0	187.00	80.00	125.53	100.42	119.2
3	1	15	147	146	150	146.50	146.00	145.00	146.00	147	1	0	0	150.00	146.00	150.00	120.00	120.0
4	1	11	149	148	147	112.37	120.00	0.00	112.37	148	1	0	0	186.25	147.99	147.00	118.39	118.4
5	1	12	146	150	149	146.00	128.00	148.99	146.00	149	1	0	0	180.00	150.00	156.45	125.16	120.0
6	1	1	105	107	103	25.00	106.99	102.99	102.99	105	1	0	0	105.00	107.00	103.00	105.00	85.6
7	1	4	100	105	107	90.00	105.00	107.00	105.00	105	1	0	0	108.00	104.99	107.00	85.60	85.6
8	1	6	102	101	106	98.10	101.00	90.50	98.10	102	1	0	0	140.00	100.00	45.00	80.00	84.8
9	1	9	104	102	101	104.00	101.00	90.01	101.00	102	1	0	0	130.00	101.00	80.03	80.80	81.6
10	1	3	101	106	108	101.00	80.00	98.50	98.50	106	1	0	0	125.00	111.00	107.52	88.80	86.4
11	1	2	109	104	102	90.00	104.00	1.00	90.00	104	1	0	0	136.24	70.00	98.57	78.86	83.2
12	1	7	108	100	105	85.00	50.00	105.00	85.00	105	1	0	0	120.00	93.44	105.00	84.00	84.0
13	1	8	107	103	109	150.00	103.00	109.01	109.01	107	1	0	0	127.20	100.00	120.00	96.00	87.2
14	1	10	106	108	100	80.00	25.00	99.00	80.00	106	1	0	0	132.00	40.00	100.00	80.00	86.4
15	1	5	103	109	104	150.00	109.00	90.00	109.00	104	1	0	0	80.00	109.00	100.00	100.00	87.2
mean									111.1	119.0							97.3	96.5

Table D.15: Treatment $\bar{D}s$ and treatment Ds – auction revenues from round 1

Comparison of auction revenues in treatment $\bar{D}s$ and treatment Ds: round 1

					Experiment				Theory			
No.	# pairs	# strong	# weak	q	$R_{\bar{D}s}$	R_{Ds}	p-value	V	$R_{\bar{D}s}$	R_{Ds}	p-value	V
1	5	5	0	Inf	137.47	116.10	0.0620	14	148.00	119.04	0.031250	15
2	6	5	1	5.0	131.73	114.25	0.0780	18	140.83	113.47	0.015625	21
3	7	5	2	2.5	127.91	110.16	0.0390	25	135.71	109.49	0.011127	28
4	8	5	3	1.7	124.18	106.39	0.0200	33	131.50	106.40	0.007074	36
5	9	5	4	1.2	121.61	103.54	0.0100	42	128.22	103.64	0.004545	45
6	10	5	5	1.0	119.30	102.07	0.0050	52	126.00	101.92	0.002945	55
7	11	5	6	0.8	116.63	99.96	0.0020	63	124.00	100.22	0.001920	66
8	12	5	7	0.7	114.00	98.63	0.0020	73	122.42	98.87	0.001258	78
9	13	5	8	0.6	113.61	98.43	0.0010	86	121.23	97.97	0.000828	91
10	14	5	9	0.6	111.21	97.11	0.0030	86	120.14	97.14	0.000545	105
11	15	5	10	0.5	111.06	97.30	0.0020	100	119.07	96.48	0.000361	120
12	14	4	10	0.4	109.36	95.93	0.0026	86	117.07	94.97	0.000545	105
13	13	3	10	0.3	106.38	95.59	0.0043	73	114.62	93.11	0.000825	91
14	12	2	10	0.2	103.08	93.55	0.0072	61	111.92	90.87	0.001253	78
15	11	1	10	0.1	102.24	91.29	0.0054	53	108.64	88.36	0.001911	66
16	10	0	10	0.0	97.86	87.91	0.0089	43	104.60	85.20	0.002929	55
mean					115.5	100.5			123.4	99.8		

Wilcoxon signed-ranks test (matched-pairs) with Hypothesis $H_0 : R_{\bar{D}s} \leq R_{Ds}$

Table D.16: Treatment $\bar{D}s$ and treatment Ds – comparison of auction revenues from round 1

Auction revenues in treatment $\bar{D}s$ and treatment Ds: round 1 to round 6

						Treatment $\bar{D}s$								Treatment Ds				
			Valuations			Bids in $\bar{D}s$			$R_{\bar{D}s}$		Discount			Bids in Ds			R_{Ds}	
No.	Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
1	5	14	146	149	147	146.00	147.00	147.00	147.00	147	1	0	0	180.18	149.00	142.50	119.20	119.2
2	3	11	148	150	146	138.00	149.99	146.00	146.00	148	1	0	0	156.10	149.00	145.66	119.20	120.0
3	3	14	147	148	150	146.90	140.00	136.00	140.00	148	1	0	0	183.75	147.00	150.00	120.00	120.0
4	3	15	146	149	147	146.00	149.01	121.70	146.00	147	1	0	0	180.00	170.00	130.00	136.00	119.2
5	3	12	149	147	148	146.00	147.00	400.00	147.00	148	1	0	0	186.24	147.00	148.00	118.40	118.4
6	2	14	147	150	149	100.00	143.05	148.90	143.05	149	1	0	0	153.76	150.00	143.00	120.00	120.0
7	1	11	149	148	147	112.37	120.00	0.00	112.37	148	1	0	0	186.25	147.99	147.00	118.39	118.4
8	6	14	146	148	150	138.00	147.99	150.00	147.99	148	1	0	0	182.00	147.51	150.00	120.00	120.0
9	4	14	149	150	148	125.20	140.00	147.99	140.00	149	1	0	0	186.00	149.99	144.00	119.99	120.0
10	1	14	150	149	148	142.20	148.99	148.00	148.00	149	1	0	0	187.00	80.00	125.53	100.42	119.2
11	4	15	147	146	149	147.01	145.00	150.00	147.01	147	1	0	0	183.74	146.00	149.00	119.20	119.2
12	1	13	148	147	146	110.00	147.02	135.00	135.00	147	1	0	0	154.00	140.00	145.66	116.52	117.6
13	5	11	149	147	150	149.00	147.00	150.00	149.00	149	1	0	0	156.11	150.00	149.00	120.00	120.0
14	6	15	149	147	146	149.00	147.00	135.00	147.00	147	1	0	0	187.00	147.11	146.00	117.68	117.6
15	5	13	147	150	148	111.00	150.00	147.90	147.90	148	1	0	0	183.75	149.53	148.00	119.62	120.0
16	5	15	150	148	146	150.00	130.70	146.00	146.00	148	1	0	0	180.00	104.00	146.00	116.80	118.4
17	3	13	150	146	149	150.00	140.00	149.00	149.00	149	1	0	0	187.50	146.00	149.00	119.20	119.2
18	1	12	146	150	149	146.00	128.00	148.99	146.00	149	1	0	0	180.00	150.00	156.45	125.16	120.0
19	6	13	150	149	147	152.00	146.10	110.10	146.10	149	1	0	0	160.11	143.91	150.00	120.00	119.2
20	2	12	148	147	150	100.00	132.00	102.00	102.00	148	1	0	0	155.00	147.00	150.00	120.00	120.0
21	2	11	150	149	146	149.99	0.00	146.00	146.00	149	1	0	0	149.33	149.00	145.66	119.20	119.2
22	6	11	148	150	149	165.45	150.00	145.10	150.00	149	1	0	0	185.01	150.00	149.00	120.00	120.0

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			Treatment $\bar{D}s$								Treatment Ds								
			Valuations			Bids in $\bar{D}s$			$R_{\bar{D}s}$		Discount			Bids in Ds			R_{Ds}		
No.	Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.	
23	2	15	146	148	147	146.00	130.01	147.00	146.00	147	1	0	0	180.00	145.03	147.00	117.60	118.4	
24	4	12	146	149	150	145.99	147.50	150.00	147.50	149	1	0	0	200.00	149.00	149.00	119.20	120.0	
25	6	12	147	146	148	147.01	141.00	148.01	147.01	147	1	0	0	183.75	146.02	155.40	124.32	118.4	
26	2	13	149	146	148	149.00	139.90	148.00	148.00	148	1	0	0	150.00	146.00	147.00	117.60	118.4	
27	4	13	148	147	146	148.00	147.00	146.00	147.00	147	1	0	0	150.10	147.01	142.63	117.61	117.6	
28	1	15	147	146	150	146.50	146.00	145.00	146.00	147	1	0	0	150.00	146.00	150.00	120.00	120.0	
29	4	11	150	148	147	149.99	147.99	146.00	147.99	148	1	0	0	187.50	149.00	147.00	119.20	118.4	
30	5	12	148	146	149	148.00	141.00	190.00	148.00	148	1	0	0	185.00	146.00	149.00	119.20	119.2	
31	3	2	104	100	106	103.99	100.00	10.01	100.00	104	1	0	0	120.00	98.00	102.87	82.30	84.8	
32	4	7	101	102	103	94.00	101.99	102.99	101.99	102	1	0	0	125.50	102.00	102.50	82.00	82.4	
33	2	4	102	108	100	85.00	108.00	105.00	105.00	102	1	0	0	127.50	118.80	100.00	95.04	86.4	
34	6	3	100	102	104	100.00	102.00	100.00	100.00	102	1	0	0	125.00	102.00	104.00	83.20	83.2	
35	6	10	102	104	101	102.00	104.00	101.00	102.00	102	1	0	0	127.50	104.00	101.00	83.20	83.2	
36	2	6	105	103	101	95.00	100.90	100.10	100.10	103	1	0	0	131.25	98.62	145.00	131.25	82.4	
37	3	1	102	105	109	106.27	105.01	100.01	105.01	105	1	0	0	102.09	105.00	109.00	105.00	87.2	
38	3	3	107	103	101	120.00	98.00	91.00	98.00	103	1	0	0	133.00	103.06	101.00	82.45	82.4	
39	5	4	102	105	100	101.50	105.00	100.00	101.50	102	1	0	0	127.50	110.25	100.00	88.20	84.0	
40	2	10	101	107	102	83.00	70.00	95.00	83.00	102	1	0	0	126.25	88.00	102.00	81.60	85.6	
41	6	1	103	107	106	102.99	107.01	105.99	105.99	106	1	0	0	128.75	107.00	110.00	88.00	85.6	
42	4	3	104	105	101	250.00	0.00	100.00	100.00	104	1	0	0	135.00	107.00	100.99	85.60	84.0	
43	6	9	105	109	100	105.01	108.99	100.00	105.01	105	1	0	0	131.24	109.00	100.00	87.20	87.2	
44	6	7	104	101	103	104.00	101.00	110.00	104.00	103	1	0	0	129.57	101.00	103.00	82.40	82.4	
45	3	10	103	101	108	97.00	101.00	96.00	97.00	103	1	0	0	128.00	101.00	108.00	86.40	86.4	
46	2	7	107	102	108	89.00	100.10	107.99	100.10	107	1	0	0	133.00	102.00	107.01	85.60	86.4	
47	5	7	107	102	105	0.00	102.00	104.99	102.00	105	1	0	0	133.50	102.00	104.50	83.60	84.0	
48	3	8	105	109	104	105.01	109.00	103.99	105.01	105	1	0	0	120.00	89.99	100.00	80.00	87.2	

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			Treatment $\bar{D}s$								Treatment Ds							
			Valuations			Bids in $\bar{D}s$			$R_{\bar{D}s}$		Discount			Bids in Ds			R_{Ds}	
No.	Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
49	5	6	109	101	108	109.00	101.00	109.10	109.00	108	1	0	0	99.00	101.00	108.00	101.00	86.4
50	3	5	109	104	100	105.90	104.00	90.00	104.00	104	1	0	0	135.00	104.01	95.01	83.20	83.2
51	6	5	106	108	105	106.00	107.80	105.00	106.00	106	1	0	0	125.00	107.99	125.90	125.00	86.4
52	4	1	103	109	107	118.00	109.00	107.01	109.00	107	1	0	0	128.75	109.00	107.00	87.20	87.2
53	1	5	103	109	104	150.00	109.00	90.00	109.00	104	1	0	0	80.00	109.00	100.00	100.00	87.2
54	6	8	107	106	108	115.76	105.99	108.00	108.00	107	1	0	0	132.91	106.00	110.00	88.00	86.4
55	5	2	104	103	109	102.01	102.99	109.00	102.99	104	1	0	0	136.00	105.00	108.99	87.19	87.2
56	6	6	109	100	102	110.10	100.00	101.90	101.90	102	1	0	0	140.00	99.00	92.00	79.20	81.6
57	3	9	100	106	107	100.00	105.00	107.00	105.00	106	1	0	0	125.00	106.00	105.03	84.80	85.6
58	1	4	100	105	107	90.00	105.00	107.00	105.00	105	1	0	0	108.00	104.99	107.00	85.60	85.6
59	1	7	108	100	105	85.00	50.00	105.00	85.00	105	1	0	0	120.00	93.44	105.00	84.00	84.0
60	3	7	101	108	102	100.00	92.10	101.99	100.00	102	1	0	0	109.07	108.00	101.50	86.40	86.4
61	5	9	103	109	101	102.99	109.00	101.01	102.99	103	1	0	0	125.00	110.00	121.30	97.04	87.2
62	2	9	109	105	103	109.01	105.00	101.00	105.00	105	1	0	0	136.24	101.01	102.00	81.60	84.0
63	4	5	107	106	108	106.30	101.00	108.00	106.30	107	1	0	0	123.67	106.00	110.00	88.00	86.4
64	5	3	101	108	107	100.00	107.99	105.00	105.00	107	1	0	0	126.24	107.33	107.26	85.86	86.4
65	2	5	104	106	109	104.00	90.30	109.00	104.00	106	1	0	0	130.00	106.00	110.00	88.00	87.2
66	4	4	102	103	109	96.00	103.00	115.00	103.00	103	1	0	0	108.14	113.30	109.00	109.00	87.2
67	1	6	102	101	106	98.10	101.00	90.50	98.10	102	1	0	0	140.00	100.00	45.00	80.00	84.8
68	4	10	105	101	102	101.10	120.00	102.00	102.00	102	1	0	0	131.25	103.20	102.00	82.56	81.6
69	3	6	106	107	103	106.50	107.00	97.00	106.50	106	1	0	0	145.00	106.00	100.00	84.80	85.6
70	4	6	100	104	105	100.00	104.00	104.90	104.00	104	1	0	0	125.00	103.00	50.00	82.40	84.0
71	4	8	109	107	106	105.00	107.14	106.00	106.00	107	1	0	0	115.10	107.15	90.91	85.72	85.6
72	6	4	101	103	107	100.50	103.00	106.50	103.00	103	1	0	0	126.25	102.99	108.00	86.40	85.6
73	5	1	105	100	106	203.67	99.99	106.01	106.01	105	1	0	0	131.26	99.90	106.00	84.80	84.8
74	5	5	106	104	103	106.80	101.00	103.00	103.00	104	1	0	0	130.83	104.00	103.00	83.20	83.2

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			Treatment $\bar{D}s$								Treatment Ds								
			Valuations			Bids in $\bar{D}s$			$R_{\bar{D}s}$		Discount			Bids in Ds			R_{Ds}	
No.	Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
75	4	9	108	100	104	108.00	100.00	104.00	104.00	104	1	0	0	100.00	95.01	103.53	100.00	83.2
76	1	1	105	107	103	25.00	106.99	102.99	102.99	105	1	0	0	105.00	107.00	103.00	105.00	85.6
77	2	1	108	100	104	108.00	90.00	103.99	103.99	104	1	0	0	135.01	100.00	103.00	82.40	83.2
78	1	2	109	104	102	90.00	104.00	1.00	90.00	104	1	0	0	136.24	70.00	98.57	78.86	83.2
79	2	3	103	101	107	10.01	101.00	102.00	101.00	103	1	0	0	120.00	111.00	107.01	88.80	85.6
80	2	2	106	109	105	103.01	107.00	105.00	105.00	106	1	0	0	125.00	109.00	75.00	87.20	87.2
81	3	4	108	102	105	104.50	102.00	103.10	103.10	105	1	0	0	135.00	112.20	105.00	89.76	84.0
82	6	2	108	105	109	108.00	105.00	108.99	108.00	108	1	0	0	134.99	105.00	108.99	87.19	87.2
83	5	10	108	107	102	107.50	111.00	110.00	110.00	107	1	0	0	135.00	113.00	102.00	90.40	85.6
84	1	8	107	103	109	150.00	103.00	109.01	109.01	107	1	0	0	127.20	100.00	120.00	96.00	87.2
85	1	10	106	108	100	80.00	25.00	99.00	80.00	106	1	0	0	132.00	40.00	100.00	80.00	86.4
86	2	8	100	104	106	95.00	75.00	106.00	95.00	104	1	0	0	105.00	104.00	89.79	83.20	84.8
87	1	3	101	106	108	101.00	80.00	98.50	98.50	106	1	0	0	125.00	111.00	107.52	88.80	86.4
88	1	9	104	102	101	104.00	101.00	90.01	101.00	102	1	0	0	130.00	101.00	80.03	80.80	81.6
89	4	2	106	108	100	104.50	108.00	100.00	104.50	106	1	0	0	135.00	107.99	120.00	96.00	86.4
90	5	8	100	106	104	89.00	112.49	104.00	104.00	104	1	0	0	125.00	106.17	100.51	84.94	84.8
mean									116.0	119.0							98.9	96.5

Table D.17: Treatment $\bar{D}s$ and treatment Ds – auction revenues from round 1 to round 6

Comparison of auction revenues in treatment $\bar{D}s$ and treatment Ds: round 1 to round 6

					Experiment				Theory			
Number	# pairs	# strong	# weak	q	$R_{\bar{D}s}$	R_{Ds}	p-value	V	$R_{\bar{D}s}$	$R_{D}s$	p-value	V
1	30	30	0	Inf	143.53	119.32	1.367994e-06	461	148.03	119.17	8.602332e-07	465
2	31	30	1	30.0	142.13	118.13	1.005697e-06	491	146.61	118.06	5.836035e-07	496
3	32	30	2	15.0	140.87	117.00	6.698179e-07	523	145.22	116.95	3.960106e-07	528
4	33	30	3	10.0	139.79	116.34	4.893267e-07	555	143.91	116.02	2.687731e-07	561
5	34	30	4	7.5	138.62	115.36	3.559663e-07	588	142.68	115.06	1.824553e-07	595
6	35	30	5	6.0	137.57	114.44	2.368491e-07	623	141.51	114.15	1.238279e-07	630
7	36	30	6	5.0	136.53	114.91	2.509495e-06	624	140.44	113.27	8.409800e-08	666
8	37	30	7	4.3	135.68	114.64	1.951621e-06	658	139.49	112.56	5.712731e-08	703
9	38	30	8	3.8	134.68	113.79	1.404615e-06	694	138.53	111.77	3.879909e-08	741
10	39	30	9	3.3	133.83	113.14	1.008395e-06	731	137.59	111.06	2.636741e-08	780
11	40	30	10	3.0	132.56	112.35	7.725285e-07	768	136.70	110.42	1.792257e-08	820
12	41	30	11	2.7	131.91	111.76	5.512699e-07	807	135.95	109.81	1.218480e-08	861
13	42	30	12	2.5	131.16	111.13	3.926303e-07	847	135.19	109.20	8.285526e-09	903
14	43	30	13	2.3	130.55	110.58	2.791458e-07	888	134.49	108.69	5.633397e-09	946
15	44	30	14	2.1	129.94	109.94	1.863359e-07	931	133.77	108.09	3.828761e-09	990
16	45	30	15	2.0	129.21	109.41	1.322267e-07	974	133.09	107.61	2.605083e-09	1035
17	46	30	16	1.9	128.58	108.90	9.368914e-08	1018	132.52	107.15	1.769955e-09	1081
18	47	30	17	1.8	128.01	108.36	6.256965e-08	1064	131.94	106.66	1.204764e-09	1128
19	48	30	18	1.7	127.53	107.77	4.182339e-08	1111	131.38	106.25	8.195790e-10	1176
20	49	30	19	1.6	127.16	107.63	2.958208e-08	1158	130.90	105.84	5.580587e-10	1225
21	50	30	20	1.5	126.69	107.14	1.978641e-08	1207	130.36	105.39	3.800440e-10	1275
22	51	30	21	1.4	126.29	107.49	3.814016e-08	1237	129.88	105.02	2.587954e-10	1326
23	52	30	22	1.4	125.95	107.10	2.511167e-08	1288	129.44	104.68	1.762948e-10	1378

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					Experiment				Theory			
Number	# pairs	# strong	# weak	q	$R_{\bar{D}s}$	R_{Ds}	p-value	V	$R_{\bar{D}s}$	R_{Ds}	p-value	V
24	53	30	23	1.3	125.63	106.97	1.830745e-08	1338	128.96	104.35	1.201107e-10	1431
25	54	30	24	1.2	125.31	106.61	1.206242e-08	1391	128.56	104.01	8.168466e-11	1485
26	55	30	25	1.2	124.90	106.26	8.772079e-09	1443	128.11	103.71	5.566059e-11	1540
27	56	30	26	1.2	124.49	105.78	5.785545e-09	1498	127.64	103.31	3.793277e-11	1596
28	57	30	27	1.1	124.15	105.41	3.820058e-09	1554	127.26	103.00	2.584577e-11	1653
29	58	30	28	1.1	123.82	105.07	2.524994e-09	1611	126.88	102.70	1.762168e-11	1711
30	59	30	29	1.0	123.16	104.71	1.916574e-09	1666	126.51	102.39	1.201394e-11	1770
31	60	30	30	1.0	122.78	104.41	1.386216e-09	1723	126.10	102.12	8.191781e-12	1830
32	61	30	31	1.0	122.45	104.29	1.046155e-09	1780	125.72	101.88	5.587086e-12	1891
33	62	30	32	0.9	122.17	103.92	6.908190e-10	1841	125.39	101.59	3.809397e-12	1953
34	63	30	33	0.9	121.92	103.67	4.766780e-10	1902	125.10	101.35	2.593148e-12	2016
35	64	30	34	0.9	121.65	103.39	3.152060e-10	1965	124.81	101.11	1.764144e-12	2080
36	65	30	35	0.9	121.38	103.15	2.268110e-10	2027	124.52	100.90	1.203482e-12	2145
37	66	30	36	0.8	121.10	103.24	2.084370e-10	2084	124.20	100.69	8.212320e-13	2211
38	67	30	37	0.8	120.76	102.89	1.431510e-10	2149	123.87	100.45	5.605520e-13	2278
39	68	30	38	0.8	120.48	102.59	9.446900e-11	2216	123.54	100.18	3.823610e-13	2346
40	69	30	39	0.8	120.28	102.34	6.240100e-11	2284	123.29	99.97	2.606800e-13	2415
41	70	30	40	0.8	120.05	102.05	4.125200e-11	2353	123.01	99.74	1.779690e-13	2485
42	71	30	41	0.7	119.85	101.82	2.729700e-11	2423	122.79	99.54	1.215690e-13	2556
43	72	30	42	0.7	119.62	101.61	1.950300e-11	2492	122.51	99.34	8.304500e-14	2628
44	73	30	43	0.7	119.43	101.38	1.291500e-11	2564	122.27	99.15	5.673200e-14	2701
45	74	30	44	0.7	119.21	101.13	8.560000e-12	2637	122.03	98.93	3.874700e-14	2775
46	75	30	45	0.7	119.01	101.12	6.571000e-12	2707	121.79	98.72	2.642300e-14	2850
47	76	30	46	0.7	118.79	101.17	5.806000e-12	2774	121.57	98.55	1.809700e-14	2926
48	77	30	47	0.6	118.60	100.92	3.838000e-12	2850	121.34	98.35	1.232300e-14	3003
49	78	30	48	0.6	118.24	100.64	2.724000e-12	2925	121.12	98.15	8.438000e-15	3081

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					Experiment				Theory			
Number	# pairs	# strong	# weak	q	$R_{\bar{D}s}$	R_{Ds}	p-value	V	$R_{\bar{D}s}$	R_{Ds}	p-value	V
50	79	30	49	0.6	118.02	100.49	1.931000e-12	3001	120.89	97.99	5.773000e-15	3160
51	80	30	50	0.6	117.85	100.33	1.368000e-12	3078	120.70	97.86	3.886000e-15	3240
52	81	30	51	0.6	117.67	100.19	9.690000e-13	3156	120.51	97.69	2.665000e-15	3321
53	82	30	52	0.6	117.55	100.04	6.410000e-13	3237	120.35	97.56	1.776000e-15	3403
54	83	30	53	0.6	117.46	99.92	4.240000e-13	3319	120.19	97.42	1.221000e-15	3486
55	84	30	54	0.6	117.36	99.87	3.000000e-13	3400	120.04	97.30	8.880000e-16	3570
56	85	30	55	0.5	116.92	99.64	3.000000e-13	3400	119.87	97.17	5.550000e-16	3655
57	86	30	56	0.5	116.67	99.45	2.120000e-13	3482	119.69	97.02	4.440000e-16	3741
58	87	30	57	0.5	116.46	99.33	1.500000e-13	3565	119.53	96.90	2.220000e-16	3828
59	88	30	58	0.5	116.28	99.12	9.900000e-14	3651	119.33	96.73	2.220000e-16	3916
60	89	30	59	0.5	116.15	99.08	7.000000e-14	3736	119.18	96.61	1.110000e-16	4005
61	90	30	60	0.5	116.02	98.92	4.700000e-14	3824	119.01	96.48	1.110000e-16	4095
62	89	29	60	0.5	115.67	98.70	7.000000e-14	3736	118.70	96.22	1.110000e-16	4005
63	88	28	60	0.5	115.32	98.46	1.060000e-13	3649	118.36	95.95	2.220000e-16	3916
64	87	27	60	0.5	115.04	98.22	1.600000e-13	3563	118.02	95.68	2.220000e-16	3828
65	86	26	60	0.4	114.68	97.78	2.260000e-13	3480	117.69	95.40	4.440000e-16	3741
66	85	25	60	0.4	114.30	97.53	3.420000e-13	3396	117.33	95.13	5.550000e-16	3655
67	84	24	60	0.4	113.96	97.27	5.180000e-13	3313	116.95	94.84	8.880000e-16	3570
68	83	23	60	0.4	113.98	97.01	5.610000e-13	3241	116.58	94.55	1.221000e-15	3486
69	82	22	60	0.4	113.56	96.73	8.460000e-13	3160	116.20	94.24	1.776000e-15	3403
70	81	21	60	0.3	113.24	96.44	1.278000e-12	3080	115.79	93.93	2.665000e-15	3321
71	80	20	60	0.3	112.80	96.39	2.000000e-12	3000	115.38	93.61	3.886000e-15	3240
72	79	19	60	0.3	112.37	96.11	3.026000e-12	2922	114.97	93.29	5.773000e-15	3160
73	78	18	60	0.3	112.08	95.84	4.423000e-12	2846	114.56	92.97	8.438000e-15	3081
74	77	17	60	0.3	111.60	95.53	6.701000e-12	2770	114.12	92.62	1.232300e-14	3003
75	76	16	60	0.3	111.13	95.24	1.016100e-11	2695	113.68	92.29	1.809700e-14	2926

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					Experiment				Theory			
Number	# pairs	# strong	# weak	q	$R_{\bar{D}s}$	R_{Ds}	p-value	V	$R_{\bar{D}s}$	R_{Ds}	p-value	V
76	75	15	60	0.2	110.64	94.91	1.542200e-11	2621	113.23	91.93	2.642300e-14	2850
77	74	14	60	0.2	110.16	94.62	2.342800e-11	2548	112.76	91.57	3.863600e-14	2775
78	73	13	60	0.2	109.63	94.28	3.562400e-11	2476	112.26	91.19	5.662100e-14	2701
79	72	12	60	0.2	109.13	93.85	5.422200e-11	2405	111.75	90.79	8.304500e-14	2628
80	71	11	60	0.2	108.61	93.48	8.261200e-11	2335	111.23	90.39	1.215690e-13	2556
81	70	10	60	0.2	108.70	93.10	3.444400e-11	2299	110.70	89.97	1.780800e-13	2485
82	69	9	60	0.1	108.16	92.73	5.175700e-11	2231	110.14	89.54	2.607910e-13	2415
83	68	8	60	0.1	107.55	92.32	7.783100e-11	2164	109.57	89.09	3.820280e-13	2346
84	67	7	60	0.1	106.97	91.95	1.171340e-10	2098	109.01	88.66	5.596630e-13	2278
85	66	6	60	0.1	106.36	91.53	1.764290e-10	2033	108.41	88.18	8.199000e-13	2211
86	65	5	60	0.1	105.73	91.03	2.659650e-10	1969	107.82	87.72	1.201150e-12	2145
87	64	4	60	0.1	105.07	90.62	4.012950e-10	1906	107.19	87.24	1.760259e-12	2080
88	63	3	60	0.1	104.41	90.19	6.060390e-10	1844	106.56	86.76	2.579492e-12	2016
89	62	2	60	0.0	103.74	89.71	9.161170e-10	1783	105.90	86.22	3.780309e-12	1953
90	61	1	60	0.0	103.01	89.22	1.386216e-09	1723	105.21	85.69	5.540568e-12	1891
91	60	0	60	0.0	102.26	88.72	2.099702e-09	1664	104.50	85.13	8.121281e-12	1830
mean					120.2	102.1			123.1	99.7		

Wilcoxon signed-ranks test (matched-pairs) with Hypothesis $H_0 : R_{\bar{D}s} \leq R_{Ds}$

Table D.18: Treatment $\bar{D}s$ and treatment Ds – comparison of auction revenues from round 1 to round 6

Auction revenues in treatment $\bar{D}a$ and treatment Da: round 1

			Treatment $\bar{D}a$								Treatment Da							
			Valuations			Bids in $\bar{D}a$			$R_{\bar{D}a}$		Discount			Bids in Da			R_{Da}	
No.	Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
1	1	13	147	104	102	146.50	104.00	101.00	104.00	104	1	0	0	150.00	100.00	101.00	80.800	83.20
2	1	14	146	101	106	146.00	90.01	80.00	90.01	106	1	0	0	180.00	80.03	45.00	64.024	84.80
3	1	12	148	103	109	148.00	103.00	109.01	109.01	109	1	0	0	154.00	100.00	120.00	96.000	87.20
4	1	11	149	105	107	148.99	105.00	150.00	148.99	107	1	0	0	186.25	105.00	107.00	85.600	85.60
5	1	15	150	108	100	145.00	25.00	99.00	99.00	108	1	0	0	187.00	40.00	100.00	80.000	86.40
6	1	8	147	104	102	0.00	90.00	98.10	90.00	104	0	0	1	140.00	70.00	140.00	140.000	127.50
7	1	5	150	108	100	128.00	98.50	90.00	98.50	108	0	1	0	150.00	120.00	93.44	120.000	135.00
8	1	10	150	108	100	142.20	85.00	50.00	85.00	108	0	0	1	150.00	107.52	108.00	108.000	125.00
9	1	3	147	104	102	147.02	104.00	1.00	104.00	104	0	1	0	147.00	130.00	98.57	130.000	130.00
10	1	6	149	105	107	148.99	105.00	107.00	107.00	107	0	0	1	80.00	104.99	127.20	83.992	133.75
11	1	2	148	103	109	120.00	102.99	90.00	102.99	109	0	1	0	147.99	80.00	109.00	109.000	128.75
12	1	1	149	105	107	112.37	25.00	106.99	106.99	107	0	1	0	156.45	105.00	107.00	107.000	131.25
13	1	4	146	101	106	146.00	101.00	80.00	101.00	106	0	1	0	145.66	125.00	111.00	125.000	126.25
14	1	7	148	103	109	110.00	150.00	109.00	110.00	109	0	0	1	125.53	103.00	136.24	100.424	136.25
15	1	9	146	101	106	135.00	101.00	90.50	101.00	106	0	0	1	146.00	100.00	132.00	132.000	132.50
mean									103.8	106.8							104.1	115.6

Table D.19: Treatment $\bar{D}a$ and treatment Da – auction revenues from round 1

Comparison of auction revenues in treatment $\bar{D}a$ and treatment Da: round 1

					Experiment				Theory			
No.	# pairs	# strong	# weak	q	$R_{\bar{D}a}$	R_{Da}	p-value	V	$R_{\bar{D}a}$	R_{Da}	p-value	V
1	5	5	0	Inf	110.20	81.28	0.0620	15	106.80	85.44	0.0620	15
2	6	5	1	5.0	106.83	91.07	0.3130	16	106.33	92.45	0.4380	15
3	7	5	2	2.5	105.64	95.20	0.4690	19	106.57	98.53	0.9370	15
4	8	5	3	1.7	103.06	96.80	0.6410	22	106.75	101.84	0.8440	20
5	9	5	4	1.2	103.17	100.49	1.0000	23	106.44	104.97	0.8200	20
6	10	5	5	1.0	103.55	98.84	0.7700	31	106.50	107.84	0.4920	20
7	11	5	6	0.8	103.50	99.77	0.7650	37	106.73	109.75	0.5200	25
8	12	5	7	0.7	103.79	100.37	0.7910	43	106.75	111.54	0.3010	25
9	13	5	8	0.6	103.58	102.26	1.0000	45	106.69	112.67	0.3050	30
10	14	5	9	0.6	104.03	102.13	0.9520	54	106.86	114.35	0.1730	30
11	15	5	10	0.5	103.83	104.12	0.8040	55	106.80	115.56	0.0950	30
12	14	4	10	0.4	103.82	105.79	0.6257	44	107.00	117.88	0.0580	22
13	13	3	10	0.3	104.88	109.00	0.4143	33	107.08	120.42	0.0327	15
14	12	2	10	0.2	104.54	110.08	0.3394	26	106.92	123.19	0.0161	9
15	11	1	10	0.1	100.50	112.31	0.1016	14	106.91	126.60	0.0068	4
16	10	0	10	0.0	100.65	115.54	0.0645	9	106.80	130.62	0.0020	0
mean					104.1	101.6			106.7	110.9		

Wilcoxon signed-ranks test (matched-pairs) with Hypothesis $H_0 : R_{\bar{D}a} \geq R_{Da}$

Table D.20: Treatment $\bar{D}a$ and treatment Da – comparison of auction revenues from round 1

Auction Revenues in Treatment $\bar{D}a$ and Treatment Da: Round 1 to Round 6

						Treatment $\bar{D}a$								Treatment Da				
			Valuations			Bids in $\bar{D}a$			$R_{\bar{D}a}$		Discount			Bids in Da			R_{Da}	
No.	Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
1	5	13	150	103	109	150.00	102.99	109.00	109.00	109	1	0	0	180.00	103.00	110.00	88.00	87.20
2	2	14	148	103	101	130.01	101.00	83.00	101.00	103	1	0	0	155.00	102.00	145.00	116.00	82.40
3	2	13	146	109	105	146.00	109.01	105.00	109.01	109	1	0	0	180.00	110.00	101.01	88.00	87.20
4	5	11	149	105	100	147.00	104.99	89.00	104.99	105	1	0	0	156.11	104.50	100.00	83.60	84.00
5	2	11	150	108	100	143.05	107.99	95.00	107.99	108	1	0	0	149.33	107.01	100.00	85.61	86.40
6	1	15	150	108	100	145.00	25.00	99.00	99.00	108	1	0	0	187.00	40.00	100.00	80.00	86.40
7	6	14	147	100	102	147.00	100.00	102.00	102.00	102	1	0	0	183.75	100.00	92.00	80.00	81.60
8	3	14	149	107	103	149.01	107.00	97.00	107.00	107	1	0	0	186.24	105.03	100.00	84.02	85.60
9	5	15	146	107	102	146.00	111.00	110.00	111.00	107	1	0	0	180.18	113.00	102.00	90.40	85.60
10	1	11	149	105	107	148.99	105.00	150.00	148.99	107	1	0	0	186.25	105.00	107.00	85.60	85.60
11	4	11	150	103	109	140.00	102.99	105.00	105.00	109	1	0	0	187.50	102.50	109.00	87.20	87.20
12	5	12	147	106	104	147.00	112.49	104.00	112.49	106	1	0	0	183.75	106.17	100.51	84.94	84.80
13	1	12	148	103	109	148.00	103.00	109.01	109.01	109	1	0	0	154.00	100.00	120.00	96.00	87.20
14	2	12	149	104	106	148.90	75.00	106.00	106.00	106	1	0	0	150.00	104.00	89.79	83.20	84.80
15	6	13	149	105	109	149.00	105.01	108.99	108.99	109	1	0	0	187.00	125.90	109.00	100.72	87.20
16	4	12	148	107	106	147.99	107.14	106.00	107.14	107	1	0	0	150.10	107.15	90.91	85.72	85.60
17	1	13	147	104	102	146.50	104.00	101.00	104.00	104	1	0	0	150.00	100.00	101.00	80.80	83.20
18	3	15	147	101	108	121.70	101.00	96.00	101.00	108	1	0	0	183.75	101.00	108.00	86.40	86.40
19	4	15	149	101	102	150.00	120.00	102.00	120.00	102	1	0	0	186.00	103.20	102.00	82.56	81.60
20	6	12	150	106	108	150.00	105.99	108.00	108.00	108	1	0	0	160.11	106.00	110.00	88.00	86.40
21	6	11	148	103	107	147.99	110.00	115.76	115.76	107	1	0	0	185.01	103.00	108.00	86.40	85.60
22	4	13	147	108	100	147.01	108.00	100.00	108.00	108	1	0	0	183.74	110.00	95.01	88.00	86.40

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						Treatment $\bar{D}a$								Treatment Da				
			Valuations			Bids in $\bar{D}a$			$R_{\bar{D}a}$		Discount			Bids in Da			R_{Da}	
No.	Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
23	1	14	146	101	106	146.00	90.01	80.00	90.01	106	1	0	0	180.00	80.03	45.00	64.02	84.80
24	3	11	148	102	105	140.00	101.99	105.01	105.01	105	1	0	0	156.10	101.50	105.00	84.00	84.00
25	6	15	146	104	101	135.00	104.00	101.00	104.00	104	1	0	0	182.00	104.00	101.00	83.20	83.20
26	5	14	148	101	108	130.70	101.01	107.50	107.50	108	1	0	0	185.00	121.30	108.00	97.04	86.40
27	3	13	146	100	106	146.00	100.00	105.00	105.00	106	1	0	0	180.00	95.01	106.00	84.80	84.80
28	4	14	146	104	105	145.00	104.00	101.10	104.00	105	1	0	0	200.00	103.53	50.00	82.82	84.00
29	3	12	150	109	104	136.00	109.00	103.99	109.00	109	1	0	0	187.50	89.99	100.00	80.00	87.20
30	2	15	147	107	102	147.00	70.00	95.00	95.00	107	1	0	0	153.76	88.00	102.00	81.60	85.60
31	3	6	148	102	105	400.00	102.00	103.10	103.10	105	0	0	1	147.00	112.20	120.00	120.00	131.25
32	2	5	147	107	102	132.00	102.00	85.00	102.00	107	0	1	0	147.00	133.00	102.00	133.00	133.75
33	5	2	147	106	104	147.00	106.01	102.01	106.01	106	0	1	0	150.00	130.83	104.00	130.83	132.50
34	2	9	148	103	101	148.00	100.90	100.10	100.90	103	0	0	1	145.03	98.62	126.25	126.25	126.25
35	4	9	146	104	105	146.00	104.00	104.90	104.90	105	0	0	1	146.00	103.00	131.25	131.25	131.25
36	5	6	149	105	100	190.00	105.00	100.00	105.00	105	0	0	1	149.00	110.25	125.00	125.00	125.00
37	4	3	147	108	100	146.00	108.00	100.00	108.00	108	0	1	0	147.00	100.00	120.00	120.00	135.00
38	3	9	149	107	103	149.00	107.00	97.00	107.00	107	0	0	1	170.00	106.00	128.00	128.00	128.75
39	6	1	148	103	107	165.45	102.99	107.01	107.01	107	0	1	0	155.40	128.75	107.00	128.75	128.75
40	1	3	147	104	102	147.02	104.00	1.00	104.00	104	0	1	0	147.00	130.00	98.57	130.00	130.00
41	2	1	150	108	100	149.99	108.00	90.00	108.00	108	0	1	0	150.00	135.01	100.00	135.01	135.00
42	2	10	147	107	102	100.00	89.00	100.10	100.00	107	0	0	1	147.00	107.01	127.50	127.50	127.50
43	4	4	146	104	105	145.99	250.00	0.00	145.99	105	0	1	0	142.63	135.00	107.00	135.00	130.00
44	4	1	150	103	109	149.99	118.00	109.00	118.00	109	0	1	0	149.00	128.75	109.00	128.75	128.75
45	4	8	147	108	100	147.00	108.00	100.00	108.00	108	0	0	1	147.01	107.99	125.00	125.00	125.00
46	4	2	148	107	106	147.99	107.01	104.50	107.01	107	0	1	0	149.00	123.67	106.00	123.67	133.75
47	5	7	147	106	104	111.00	106.80	101.00	106.80	106	0	0	1	142.50	106.00	136.00	136.00	130.00
48	5	10	146	107	102	146.00	0.00	102.00	102.00	107	0	0	1	146.00	107.26	127.50	127.50	127.50

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						Treatment $\bar{D}a$					Treatment Da							
			Valuations			Bids in $\bar{D}a$			$R_{\bar{D}a}$		Discount			Bids in Da			R_{Da}	
No.	Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
49	3	5	147	101	108	147.00	91.00	104.50	104.50	108	0	1	0	147.00	109.07	108.00	109.07	126.25
50	4	6	150	103	109	150.00	103.00	115.00	115.00	109	0	0	1	149.99	113.30	115.10	115.10	136.25
51	4	5	149	101	102	147.50	100.00	96.00	100.00	102	0	1	0	149.00	125.50	102.00	125.50	126.25
52	6	9	147	100	102	110.10	100.00	101.90	101.90	102	0	0	1	147.11	99.00	127.50	127.50	127.50
53	6	10	146	104	101	138.00	104.00	101.00	104.00	104	0	0	1	146.00	104.00	126.25	126.25	126.25
54	3	7	150	109	104	150.00	105.90	104.00	105.90	109	0	0	1	150.00	109.00	120.00	120.00	130.00
55	1	1	149	105	107	112.37	25.00	106.99	106.99	107	0	1	0	156.45	105.00	107.00	107.00	131.25
56	6	5	146	104	101	141.00	100.00	100.50	100.50	104	0	1	0	146.02	129.57	101.00	129.57	130.00
57	6	7	150	106	108	152.00	106.00	107.80	107.80	108	0	0	1	150.00	110.00	134.99	134.99	135.00
58	1	5	150	108	100	128.00	98.50	90.00	98.50	108	0	1	0	150.00	120.00	93.44	120.00	135.00
59	1	10	150	108	100	142.20	85.00	50.00	85.00	108	0	0	1	150.00	107.52	108.00	108.00	125.00
60	1	2	148	103	109	120.00	102.99	90.00	102.99	109	0	1	0	147.99	80.00	109.00	109.00	128.75
61	1	4	146	101	106	146.00	101.00	80.00	101.00	106	0	1	0	145.66	125.00	111.00	125.00	126.25
62	1	6	149	105	107	148.99	105.00	107.00	107.00	107	0	0	1	80.00	104.99	127.20	83.99	133.75
63	1	8	147	104	102	0.00	90.00	98.10	90.00	104	0	0	1	140.00	70.00	140.00	140.00	127.50
64	6	2	150	106	108	150.00	105.99	108.00	108.00	108	0	1	0	150.00	125.00	107.99	125.00	132.50
65	1	7	148	103	109	110.00	150.00	109.00	110.00	109	0	0	1	125.53	103.00	136.24	100.42	136.25
66	5	5	146	107	102	141.00	105.00	101.50	105.00	107	0	1	0	146.00	133.50	102.00	133.50	133.75
67	6	4	147	100	102	147.01	100.00	102.00	102.00	102	0	1	0	150.00	125.00	102.00	125.00	125.00
68	6	8	149	105	109	146.10	105.00	110.10	110.10	109	0	0	1	143.91	105.00	140.00	140.00	136.25
69	5	8	150	103	109	150.00	103.00	109.00	109.00	109	0	0	1	149.53	105.00	99.00	105.00	136.25
70	5	9	148	101	108	147.90	101.00	109.10	109.10	108	0	0	1	104.00	101.00	135.00	83.20	135.00
71	2	2	149	104	106	0.00	103.99	103.01	103.01	106	0	1	0	149.00	130.00	106.00	130.00	130.00
72	2	6	150	108	100	102.00	108.00	105.00	105.00	108	0	0	1	150.00	118.80	105.00	118.80	125.00
73	3	4	149	107	103	146.00	120.00	98.00	120.00	107	0	1	0	149.00	133.00	103.06	133.00	133.75
74	5	4	148	101	108	148.00	100.00	107.99	107.99	108	0	1	0	148.00	126.24	107.33	126.24	126.25

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						Treatment $\bar{D}a$								Treatment Da				
			Valuations			Bids in $\bar{D}a$			$R_{\bar{D}a}$		Discount			Bids in Da			R_{Da}	
No.	Round	Group	v_1	v_2	v_3	b_1	b_2	b_3	Exp.	Theo.	P1	P2	P3	b_1	b_2	b_3	Exp.	Theo.
75	3	8	146	100	106	140.00	90.00	106.50	106.50	106	0	0	1	146.00	98.00	145.00	145.00	132.50
76	1	9	146	101	106	135.00	101.00	90.50	101.00	106	0	0	1	146.00	100.00	132.00	132.00	132.50
77	3	3	146	100	106	146.00	100.00	10.01	100.00	106	0	1	0	145.66	125.00	102.87	125.00	125.00
78	4	10	149	101	102	125.20	94.00	101.99	101.99	102	0	0	1	149.00	100.99	108.14	108.14	127.50
79	6	3	149	105	109	145.10	105.00	108.99	108.99	109	0	1	0	149.00	131.24	108.99	131.24	131.25
80	5	3	150	103	109	150.00	102.99	109.00	109.00	109	0	1	0	149.00	125.00	108.99	125.00	128.75
81	3	2	150	109	104	149.99	100.01	103.99	103.99	109	0	1	0	149.00	135.00	104.01	135.00	136.25
82	3	10	147	101	108	146.90	100.00	92.10	100.00	108	0	0	1	130.00	101.00	135.00	104.00	135.00
83	2	4	148	103	101	100.00	10.01	101.00	100.00	103	0	1	0	147.00	120.00	111.00	120.00	128.75
84	2	8	146	109	105	139.90	109.00	95.00	109.00	109	0	0	1	146.00	109.00	131.25	131.25	131.25
85	6	6	148	103	107	148.01	103.00	106.50	106.50	107	0	0	1	147.51	102.99	132.91	132.91	133.75
86	2	3	146	109	105	146.00	107.00	105.00	107.00	109	0	1	0	145.66	136.24	75.00	136.24	136.25
87	4	7	148	107	106	148.00	106.30	101.00	106.30	107	0	0	1	144.00	107.00	135.00	135.00	132.50
88	5	1	149	105	100	149.00	203.67	99.99	149.00	105	0	1	0	149.00	131.26	99.90	131.26	131.25
89	2	7	149	104	106	149.00	104.00	90.30	104.00	106	0	0	1	143.00	103.00	125.00	125.00	132.50
90	3	1	148	102	105	138.00	106.27	105.01	106.27	105	0	1	0	148.00	102.09	105.00	105.00	127.50
mean									106.8	106.6							111.4	115.5

Table D.21: Treatment $\bar{D}a$ and treatment Da – auction revenues from round 1 to round 6

Comparison of auction revenues in treatment $\bar{D}a$ and treatment Da: round 1 to round 6

					Experiment				Theory			
No.	# pairs	# strong	# weak	q	$R_{\bar{D}a}$	R_{Da}	p-value	V	$R_{\bar{D}a}$	$R_{D}a$	p-value	V
1	30	30	0	Inf	107.50	86.29	0.00000334	459.0	106.60	85.28	0.0000016913	465.0
2	31	30	1	30.0	107.35	87.38	0.00000431	483.0	106.55	86.76	0.0000209023	465.0
3	32	30	2	15.0	107.19	88.80	0.00003735	485.0	106.56	88.23	0.0001705321	465.0
4	33	30	3	10.0	107.15	90.08	0.00017515	491.0	106.55	89.57	0.0009819525	465.0
5	34	30	4	7.5	106.97	91.14	0.00066814	497.0	106.44	90.65	0.0042170785	465.0
6	35	30	5	6.0	106.91	92.29	0.00225201	502.0	106.40	91.81	0.0141392395	465.0
7	36	30	6	5.0	106.86	93.19	0.00290720	523.0	106.36	92.73	0.0110205498	495.0
8	37	30	7	4.3	106.89	93.92	0.00281456	550.0	106.41	93.88	0.0306881932	495.0
9	38	30	8	3.8	106.89	94.82	0.00457657	566.5	106.42	94.79	0.0589821315	501.0
10	39	30	9	3.3	106.89	95.69	0.00906017	577.5	106.44	95.66	0.1034832005	507.0
11	40	30	10	3.0	106.82	96.54	0.02077439	582.5	106.38	96.52	0.1939815243	507.0
12	41	30	11	2.7	106.85	97.48	0.04254893	587.5	106.41	97.46	0.3240745177	507.0
13	42	30	12	2.5	106.69	98.20	0.07894035	592.5	106.43	98.18	0.2987750507	535.0
14	43	30	13	2.3	107.60	99.05	0.07197657	622.5	106.40	98.92	0.4572091995	535.0
15	44	30	14	2.1	107.84	99.73	0.07045532	650.5	106.45	99.59	0.4168284515	565.0
16	45	30	15	2.0	107.84	100.29	0.07729755	674.5	106.49	100.16	0.3842998800	595.0
17	46	30	16	1.9	107.82	100.80	0.08528288	698.5	106.50	100.89	0.5548413330	595.0
18	47	30	17	1.8	107.80	101.55	0.14710966	701.5	106.49	101.51	0.7466660261	595.0
19	48	30	18	1.7	107.68	102.09	0.22224621	707.5	106.50	102.05	0.7232307365	623.0
20	49	30	19	1.6	107.61	102.23	0.21553660	737.5	106.53	102.54	0.6904811492	653.0
21	50	30	20	1.5	107.76	102.49	0.21124721	767.5	106.58	103.22	0.8847854394	653.0
22	51	30	21	1.4	107.61	102.94	0.30248408	773.5	106.49	103.67	0.9289947980	653.0
23	52	30	22	1.4	107.50	103.41	0.41241438	779.5	106.40	104.13	0.7463113818	653.0

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					Experiment				Theory			
No.	# pairs	# strong	# weak	q	$R_{\bar{D}a}$	R_{Da}	p-value	V	$R_{\bar{D}a}$	R_{Da}	p-value	V
24	53	30	23	1.3	107.43	103.84	0.51523983	789.5	106.36	104.55	0.5828610199	653.0
25	54	30	24	1.2	107.41	104.14	0.53812518	814.5	106.41	105.02	0.5725400940	676.5
26	55	30	25	1.2	107.40	104.19	0.53523698	844.5	106.42	105.49	0.4355909831	676.5
27	56	30	26	1.2	107.27	104.65	0.68937103	847.5	106.38	105.93	0.3233693150	676.5
28	57	30	27	1.1	107.28	105.18	0.83944112	852.5	106.40	106.44	0.2346557093	676.5
29	58	30	28	1.1	107.13	105.43	0.95369356	863.5	106.43	106.93	0.1667414975	676.5
30	59	30	29	1.0	106.76	105.48	0.91584077	870.5	106.46	107.24	0.1788742980	706.5
31	60	30	30	1.0	106.69	105.54	0.91791147	900.5	106.50	107.60	0.1898546003	736.5
32	61	30	31	1.0	106.60	105.85	0.78213096	906.5	106.49	107.90	0.1995883183	766.5
33	62	30	32	0.9	106.61	105.50	0.88017978	954.5	106.50	108.32	0.1417004419	766.5
34	63	30	33	0.9	106.34	106.05	0.72183704	955.5	106.46	108.63	0.0988147917	766.5
35	64	30	34	0.9	106.37	106.35	0.69315908	980.5	106.48	109.00	0.0677833321	766.5
36	65	30	35	0.9	106.43	106.25	0.71683148	1016.5	106.52	109.42	0.0458015789	766.5
37	66	30	36	0.8	106.40	106.67	0.58933229	1020.5	106.53	109.79	0.0305241978	766.5
38	67	30	37	0.8	106.34	106.94	0.49199066	1028.5	106.46	110.01	0.0200916927	766.5
39	68	30	38	0.8	106.39	107.43	0.38556493	1030.5	106.50	110.40	0.0130750452	766.5
40	69	30	39	0.8	106.43	107.39	0.39753159	1065.5	106.54	110.77	0.0084213803	766.5
41	70	30	40	0.8	106.47	107.05	0.49167655	1124.5	106.56	111.12	0.0053734462	766.5
42	71	30	41	0.7	106.42	107.37	0.39642050	1129.5	106.55	111.39	0.0034005653	766.5
43	72	30	42	0.7	106.40	107.53	0.37828995	1156.5	106.57	111.58	0.0037047798	796.5
44	73	30	43	0.7	106.59	107.88	0.36580277	1185.5	106.58	111.88	0.0023347188	796.5
45	74	30	44	0.7	106.61	108.13	0.34167410	1210.5	106.59	112.07	0.0025228984	826.5
46	75	30	45	0.7	106.61	108.62	0.26068386	1211.5	106.59	112.35	0.0015840978	826.5
47	76	30	46	0.7	106.53	108.93	0.19733468	1213.5	106.58	112.61	0.0009882119	826.5
48	77	30	47	0.6	106.45	109.13	0.15437341	1220.5	106.57	112.77	0.0010624484	856.5
49	78	30	48	0.6	106.39	109.12	0.15357768	1253.5	106.51	112.96	0.0006607478	856.5

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					Experiment				Theory			
No.	# pairs	# strong	# weak	q	$R_{\bar{D}a}$	R_{Da}	p-value	V	$R_{\bar{D}a}$	R_{Da}	p-value	V
50	79	30	49	0.6	106.42	109.40	0.12489127	1265.5	106.54	113.19	0.0004087326	856.5
51	80	30	50	0.6	106.45	109.60	0.11567140	1291.5	106.58	113.39	0.0004369806	886.5
52	81	30	51	0.6	106.42	109.91	0.08441779	1293.5	106.60	113.67	0.0002695779	886.5
53	82	30	52	0.6	106.35	109.84	0.08380862	1327.0	106.62	113.93	0.0001655510	886.5
54	83	30	53	0.6	106.27	109.96	0.07474243	1350.0	106.58	114.11	0.0001012937	886.5
55	84	30	54	0.6	106.30	110.21	0.05952467	1362.0	106.61	114.31	0.0000617542	886.5
56	85	30	55	0.5	106.30	110.48	0.04383612	1367.0	106.61	114.54	0.0000375205	886.5
57	86	30	56	0.5	106.31	110.78	0.03131527	1370.0	106.64	114.79	0.0000227329	886.5
58	87	30	57	0.5	106.31	111.06	0.02240563	1374.0	106.64	115.00	0.0000137432	886.5
59	88	30	58	0.5	106.80	111.29	0.02699861	1426.0	106.62	115.18	0.0000082905	886.5
60	89	30	59	0.5	106.77	111.44	0.02255094	1444.5	106.62	115.38	0.0000049916	886.5
61	90	30	60	0.5	106.76	111.37	0.02312153	1482.5	106.60	115.51	0.0000030009	886.5
62	89	29	60	0.5	106.74	111.63	0.01833971	1425.5	106.57	115.83	0.0000020010	840.5
63	88	28	60	0.5	106.80	111.58	0.02024356	1399.5	106.61	116.21	0.0000014088	798.5
64	87	27	60	0.5	106.77	111.85	0.01584319	1343.5	106.59	116.54	0.0000009263	754.5
65	86	26	60	0.4	106.80	112.18	0.01228027	1288.5	106.60	116.92	0.0000006388	714.0
66	85	25	60	0.4	106.78	112.50	0.00884062	1229.5	106.59	117.28	0.0000004337	674.0
67	84	24	60	0.4	106.87	112.88	0.00706505	1180.5	106.57	117.65	0.0000002930	635.0
68	83	23	60	0.4	106.93	113.28	0.00516140	1126.5	106.63	118.08	0.0000002120	600.0
69	82	22	60	0.4	106.93	113.64	0.00355887	1070.5	106.62	118.48	0.0000001423	563.0
70	81	21	60	0.3	106.88	113.92	0.00268565	1022.5	106.62	118.88	0.0000000951	527.0
71	80	20	60	0.3	106.36	114.28	0.00114611	941.5	106.61	119.30	0.0000000633	492.0
72	79	19	60	0.3	106.37	114.62	0.00087416	898.5	106.58	119.71	0.0000000397	456.0
73	78	18	60	0.3	106.29	115.00	0.00040942	830.5	106.59	120.15	0.0000000262	423.0
74	77	17	60	0.3	106.26	115.25	0.00034733	796.5	106.56	120.58	0.0000000163	389.0
75	76	16	60	0.3	106.26	115.67	0.00020965	746.5	106.57	121.05	0.0000000107	358.0

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					Experiment				Theory			
No.	# pairs	# strong	# weak	q	$R_{\bar{D}a}$	R_{Da}	p-value	V	$R_{\bar{D}a}$	R_{Da}	p-value	V
76	75	15	60	0.2	106.23	115.87	0.00018886	717.5	106.53	121.50	0.0000000065	326.0
77	74	14	60	0.2	106.21	116.27	0.00012378	674.5	106.53	121.99	0.0000000043	297.0
78	73	13	60	0.2	106.24	116.76	0.00006802	625.5	106.56	122.52	0.0000000029	270.0
79	72	12	60	0.2	106.32	117.18	0.00005462	594.5	106.54	123.02	0.0000000019	243.0
80	71	11	60	0.2	106.12	117.67	0.00001598	524.5	106.61	123.60	0.0000000014	220.0
81	70	10	60	0.2	106.10	118.09	0.00001035	488.5	106.59	124.14	0.0000000009	195.0
82	69	9	60	0.1	105.96	118.55	0.00000289	424.5	106.58	124.69	0.0000000006	171.0
83	68	8	60	0.1	105.93	119.00	0.00000177	390.5	106.56	125.26	0.0000000004	148.0
84	67	7	60	0.1	106.16	119.82	0.00000062	340.5	106.57	125.86	0.0000000003	126.0
85	66	6	60	0.1	106.18	120.37	0.00000033	305.5	106.59	126.50	0.0000000002	105.5
86	65	5	60	0.1	106.22	120.94	0.00000018	273.5	106.63	127.16	0.0000000001	86.5
87	64	4	60	0.1	106.20	121.31	0.00000016	255.5	106.61	127.80	0.0000000001	67.5
88	63	3	60	0.1	106.21	121.89	0.00000009	225.5	106.62	128.48	5.000000e-11	49.5
89	62	2	60	0.0	106.25	122.52	0.00000004	194.5	106.65	129.20	4.000000e-11	33.0
90	61	1	60	0.0	106.21	123.22	0.00000001	141.5	106.61	129.89	2.000000e-11	15.0
91	60	0	60	0.0	106.39	123.91	0.00000001	124.5	106.60	130.62	2.000000e-11	0.0
mean					106.7	107.9			106.5	111.0		

Wilcoxon signed-ranks test (matched-pairs) with Hypothesis $H_0 : R_{\bar{D}a} \geq R_{Da}$

Table D.22: Treatment $\bar{D}a$ and treatment Da – comparison of auction revenues from round 1 to round 6

D.5 Experimental instructions

Participants of the experiment were recruited at Universität Karlsruhe (TH). It was to secure, that all subjects may understand the experimental rules. To avoid misunderstandings and misinterpretations of the instructions, the rules, the questionnaires and the computer screens by the subjects, the experiment was conducted in German. The instructions of the experiment as presented to the participants in both settings, setting $\bar{D}$ and setting D, are given in the following.

Anleitung

Einführung

Sie nehmen an einem Experiment teil, in dem Entscheidungsverhalten untersucht wird. Sie können bei diesem Experiment Geld verdienen. Wie viel Sie verdienen, hängt sowohl von Ihren Entscheidungen als auch von den Entscheidungen der anderen Teilnehmer ab. Diese Anleitung erläutert Ihnen, wie Sie durch eigene Entscheidungen einen Geldbetrag verdienen können, der Ihnen nach dem Experiment in bar ausbezahlt wird. Lesen Sie daher die folgenden Absätze genau durch.

In diesem Experiment wird in Geldeinheiten (GE) gerechnet. Im Anschluss an das Experiment erhalten Sie für jede erspielte GE 10 Euro-Cent, d. h. für jeweils 10 GE einen Euro. Das Experimentsystem führt für Sie ein Konto, auf dem Ihre Gewinne und Verluste während des Experminents verrechnet werden. Als Anfangsausstattung erhalten Sie einen Geldbetrag von 80 GE, der Ihnen auf Ihrem Konto gutgeschrieben wird. Am Ende des Experiments wird der Endbetrag auf Ihrem Konto in Euro umgerechnet und an Sie ausbezahlt. Falls Sie Verluste machen, so werden diese Verluste von Ihrer Anfangsausstattung abgezogen. Sollten Ihre Verluste größer sein als Ihre Anfangsausstattung zuzüglich Ihrer Gewinne, d. h. Ihr Kontostand ist zum Ende des Experiments negativ, so erhalten Sie null Euro. Folglich können Sie Geld verdienen, aber kein Geld verlieren.

Jeder Teilnehmer trifft seine Entscheidungen isoliert von den anderen Teilnehmern an einem Computerterminal. Kommunikation zwischen den Teilnehmern ist nicht erlaubt. Wir bitten Sie außerdem, die Computer nur zur Eingabe Ihrer Entscheidungen und zur Beantwortung der Fragen am Bildschirm zu benutzen. Bitte verwenden Sie hierfür die vorgesehenen Bildschirmformulare und starten oder beenden Sie eigenmächtig keine Programme und ändern Sie keine Einstellungen.

Aufgabe

Im Experiment stehen Sie folgender Entscheidungssituation gegenüber: Sie nehmen als Bieter nacheinander an sechs Auktionen (Runden) teil. In jeder dieser Auktionsrunden wird jeweils genau ein Gut versteigert. Dieses Gut kann von genau einem Bieter (Teilnehmer) erworben werden. Weiterhin bekommen Sie mitgeteilt, wie viele Geldeinheiten Ihnen das Gut in dieser Auktionsrunde wert ist. Falls Sie den Zuschlag erhalten, bekommen Sie diesen Wert auf Ihrem Konto gutgeschrieben. Gleichzeitig müssen Sie aber auch einen Preis für das Gut bezahlen; dieser wird von Ihrem Konto abgezogen. Ihr Ertrag ist folglich die Differenz zwischen dem Wert des Gutes und dem dafür gezahlten Preis. Erhalten Sie den Zuschlag nicht, ist Ihr Ertrag null. Im Verlauf des Experiments nehmen Sie an genau sechs solchen Auktionsrunden teil.

1

In jeder Auktionsrunde bilden Sie zusammen mit zwei weiteren Teilnehmern eine Dreiergruppe. Die Zusammensetzung der Gruppen wird zu Beginn ausgelost und ändert sich während des Experiments nicht, d. h. die Mitbieter in Ihrer Gruppe sind von Runde zu Runde dieselben. Innerhalb einer Gruppe sind die Spieler mit den Spielernummern 1, 2 und 3 bezeichnet. Ihre Spielernummer wurde Ihnen zufällig zugelost und ändert sich im Verlauf des Experiments nicht. Ihre Gruppe ist vollkommen unabhängig von den anderen Gruppen im Raum und es gibt keine Interaktion zwischen den Gruppen.

Sobald eine Versteigerung zu Ende ist, erfahren Sie sowohl die Spielernummer desjenigen Bieters, der in Ihrer Gruppe den Zuschlag erhalten hat, als auch den Endpreis der Auktion.

Ablauf einer Auktionsrunde

Im Einzelnen läuft eine Auktionsrunde wie folgt ab:

1. Vor Beginn der Auktionsrunde wird Ihnen vom System mitgeteilt, wie viele Geldeinheiten Ihnen das Gut in dieser Runde wert ist. Dieser Wert wird im Folgenden auch als Ihre „Wertschätzung" bezeichnet. Die Wertschätzungen des Gutes sind zufällig und wurden vor dem Experiment für alle Spieler einzeln ausgelost. Die ausgelosten Werte sind ganzzahlige Werte zwischen 100 und 150.

   Bitte beachten Sie: Sie kennen jeweils nur die eigene Wertschätzung für das Gut. Die Wertschätzungen der anderen Spieler für das Gut kennen Sie nicht.

2. Zu Anfang werden Ihnen Ihre Spielernummer, Ihre Wertschätzung für das Gut in der Auktionsrunde und Ihr Kontostand angezeigt. Bitte bestätigen Sie diese Anzeige durch Klicken des „Confirm"-Knopfes am unteren Bildschirmrand.

   Ihre Spielernummer sowie Ihre Wertschätzung wird Ihnen auch weiterhin während der Auktion angezeigt.

3. Anschließend werden Sie von dem Experimentsystem aufgefordert, genau **einen** Wert in die Bildschirmmaske einzutragen. Dieser Wert wird als Ihre „Bietgrenze" bezeichnet. Bitte tragen Sie in das entsprechende Feld Ihr Maximalgebot ein. Ihr Maximalgebot ist der Betrag, bis zu dem Sie an einer Versteigerung des Gutes teilnehmen möchten.

   Bitte bestätigen Sie die Abgabe Ihres Maximalgebotes durch Klicken des „Abschicken"-Knopfes in der Bildschirmmaske.

   Sie müssen Ihr Maximalgebot innerhalb von 2 Minuten abgeben. Nach Ablauf dieser Zeit können keine Gebote mehr abgegeben werden – das Gut wird dann unter den Bietern Ihrer Gruppe versteigert.

   In diesem Experiment nehmen Sie nicht selber an der Versteigerung teil. Statt dessen bietet ein in das System eingebauter Bietautomat automatisch für Sie. Der Bietautomat zieht

2

Figure D.1: Experimental instruction of setting $\bar{D}$ (pp. 1–2) sich aus der Versteigerung zurück, sobald der Preis in der Versteigerung Ihr Maximalgebot (Bietgrenze) übersteigt.

Bitte beachten Sie: Als Gebote sind alle Beträge zwischen 0 GE und 999.99 GE mit bis zu zwei Nachkommastellen zulässig, wobei die Nachkommastellen durch einen Punkt „.“ zu separieren sind.

4. Ein in das System eingebauter Versteigerer überprüft nun für jeden Preis, wie viele Bieter noch an der Versteigerung teilnehmen. Der Versteigerer beginnt mit einem Preis von 0 GE und bietet das Gut den Bietautomaten an. Solange noch mindestens zwei Bieter – vertreten jeweils durch ihren Bietautomaten – an der Versteigerung teilnehmen, wird der Preis um 0.01 GE erhöht. Die Versteigerung endet, sobald nur noch ein Bieter übrig bleibt. Dieser Bieter erhält den Zuschlag in der Versteigerung, d. h. der Höchstbieter gewinnt die Versteigerung.

   Den Preis, den der Höchstbieter für das Gut zu zahlen hat, ist der Endpreis der Versteigerung: Der Endpreis einer Versteigerung ist der höchste Betrag, bei dem noch mindestens zwei Bieter an der Versteigerung teilgenommen haben. Scheiden die letzten beiden (oder mehr) Bieter bei demselben Betrag aus der Versteigerung aus, so entscheidet das Los, welcher der Bieter den Zuschlag erhält. In diesem Fall ist der Endpreis der Versteigerung genau die höchste Bietgrenze, bei der die letzten Bieter gerade noch bereit waren zu bieten.

   Das Ergebnis der Auktion, d. h. der Endpreis der Auktion sowie die Spielernummer des Bieters, der den Zuschlag in der Versteigerung erhalten hat, werden Ihnen in dem Fenster am unteren Bildschirmrand angezeigt.

   Beispiele:

   a) Angenommen, die drei Bieter einer Gruppe beauftragen das System für sie bis zu einem Maximalgebot von 138 GE (Bieter 1) bzw. 113 GE (Bieter 2) bzw. 145 GE (Bieter 3) an der Versteigerung teilzunehmen. Der Bietautomat von Bieter 2 steigt aus der Versteigerung aus, sobald der Preis 113 GE übersteigt. Steigt der Preis über 138 GE, steigt auch Bieter 1 aus der Versteigerung aus. Folglich erhält Bieter 3 den Zuschlag und der Endpreis der Versteigerung ist 138 GE.

   b) Lauten die Maximalgebote der drei Bieter 138 GE (Bieter 1) bzw. 113 GE (Bieter 2) bzw. 138 GE (Bieter 3), d. h. Bieter 1 und Bieter 3 haben dasselbe Maximalgebot (Bietgrenze) in die Bildschirmmaske eingegeben, so steigen die Bietautomaten von Bieter 1 und Bieter 3 gleichzeitig aus der Versteigerung aus. Das System lost, ob Bieter 1 oder Bieter 3 den Zuschlag erhält. Der Endpreis der Versteigerung ist 138 GE.

   Derjenige Bieter, der in einer Versteigerung den Zuschlag erhält, bekommt auf seinem Konto die Differenz zwischen seiner Wertschätzung und dem Preis, den er für das Gut bezahlt, gutgeschrieben. Ist der zu zahlende Preis höher als seine Wertschätzung, so wird

3

dieser Betrag von seinem Kontostand abgezogen. Diejenigen Bieter, die den Zuschlag nicht bekommen, erzielen einen Ertrag von null.

   Beispiele:

   Angenommen, Bieter 3 erhält den Zuschlag und der Endpreis, den Bieter 3 zu zahlen hat, ist 138 GE. Bieter 3 erzielt einen Ertrag, der sich aus seiner Wertschätzung und dem zu zahlenden Preis berechnet. Dieser Betrag wird Bieter 3 auf seinem Konto verbucht.

   (a) Angenommen, Bieter 3 hat eine Wertschätzung von 148 GE. Dann erzielt Bieter 3 einen positiven Ertrag in Höhe von 148 GE - 138 GE = 10 GE. Dieser Betrag wird Bieter 3 auf seinem Konto gutgeschrieben.

   (b) Angenommen, Bieter 3 hat eine Wertschätzung von 128 GE. Dann erzielt Bieter 3 einen Ertrag in Höhe von 128 GE - 138 GE = -10 GE, d. h. der Betrag von 10 GE wird ihm von seinem Kontostand abgezogen.

Eine Auktionsrunde dauert 3 Minuten. Ist die Auktionsrunde beendet, wird Ihnen der erzielte Ertrag in der eben gespielten Runde, der Kontostand der vorherigen Runde, sowie der aktuelle Kontostand insgesamt angezeigt. Bestätigen Sie bitte diesen Bildschirm erneut mit dem „Confirm“-Knopf am unteren Bildschirmrand. Die nächste Auktionsrunde wird gestartet, sobald alle Teilnehmer den „Confirm“-Knopf bestätigt haben.

Fragebogen

Im Anschluss an die gespielten Auktionsrunden werden Ihnen noch einige Fragen zum Experiment und zu dem Experimentsystem an Ihrem Bildschirm gestellt.

Bitte bleiben Sie nach Beantwortung des Fragebogens an Ihrem Platz sitzen und unterlassen Sie jede Form von Kommunikation mit anderen Teilnehmern. Auf Ihrem Platz finden Sie ein Formular „Erklärung des Vertragnehmers“. Bitte füllen Sie den oberen Teil des Formulars aus. Die genaue Auszahlung im unteren Teil wird vom Experimentleiter ausgefüllt. Sie werden anschließend einzeln nach Ihrem Sitzplatzbuchstaben zur Auszahlung aufgerufen. Bitte verlassen Sie dann leise den Raum. Alle Unterlagen, die Ihnen für das Experiment ausgeteilt wurden, nehmen Sie bitte mit und geben diese bei der Auszahlung wieder ab. Dies gilt insbesondere für die „Erklärung des Vertragnehmers“ und Ihren Sitzplatzbuchstaben.

... und noch eine Bemerkung zum Schluss

Sollten Sie während des Experiments eine Frage haben, bleiben Sie bitte ruhig an Ihrem Platz sitzen und geben Sie dem Experimentleiter durch Handzeichen ein Signal. Warten Sie bitte, bis der Experimentleiter an Ihrem Platz ist, und stellen Sie Ihre Frage so leise wie möglich.

Bevor das Experiment beginnt, werden Ihnen an Ihrem Bildschirm zunächst einige Fragen zu den Regeln dieses Experiments gestellt. Geben Sie bitte die jeweiligen Antworten an Ihrem Computer ein.

4

Figure D.2: Experimental instruction of setting $\bar{D}$ (pp. 3–4)

Anleitung

Einführung

Sie nehmen an einem Experiment teil, in dem Entscheidungsverhalten untersucht wird. Sie können bei diesem Experiment Geld verdienen. Wie viel Sie verdienen, hängt sowohl von Ihren Entscheidungen als auch von den Entscheidungen der anderen Teilnehmer ab. Diese Anleitung erläutert Ihnen, wie Sie durch eigene Entscheidungen einen Geldbetrag verdienen können, der Ihnen nach dem Experiment in bar ausbezahlt wird. Lesen Sie daher die folgenden Absätze genau durch.

In diesem Experiment wird in Geldeinheiten (GE) gerechnet. Im Anschluss an das Experiment erhalten Sie für jede erspielte GE 10 Euro-Cent, d. h. für jeweils 10 GE einen Euro. Das Experimentsystem führt für Sie ein Konto, auf dem Ihre Gewinne und Verluste während des Experminents verrechnet werden. Als Anfangsausstattung erhalten Sie einen Geldbetrag von 80 GE, der Ihnen auf Ihrem Konto gutgeschrieben wird. Am Ende des Experiments wird der Endbetrag auf Ihrem Konto in Euro umgerechnet und an Sie ausbezahlt. Falls Sie Verluste machen, so werden diese Verluste von Ihrer Anfangsausstattung abgezogen. Sollten Ihre Verluste größer sein als Ihre Anfangsausstattung zuzüglich Ihrer Gewinne, d. h. Ihr Kontostand ist zum Ende des Experiments negativ, so erhalten Sie null Euro. Folglich können Sie Geld verdienen, aber kein Geld verlieren.

Jeder Teilnehmer trifft seine Entscheidungen isoliert von den anderen Teilnehmern an einem Computerterminal. Kommunikation zwischen den Teilnehmern ist nicht erlaubt. Wir bitten Sie außerdem, die Computer nur zur Eingabe Ihrer Entscheidungen und zur Beantwortung der Fragen am Bildschirm zu benutzen. Bitte verwenden Sie hierfür die vorgesehenen Bildschirmformulare und starten oder beenden Sie eigenmächtig keine Programme und ändern Sie keine Einstellungen.

Aufgabe

Im Experiment stehen Sie folgender Entscheidungssituation gegenüber: Sie nehmen als Bieter nacheinander an sechs Auktionen (Runden) teil. In jeder dieser Auktionsrunden wird jeweils genau ein Gut versteigert. Dieses Gut kann von genau einem Bieter (Teilnehmer) erworben werden. Weiterhin bekommen Sie mitgeteilt, wie viele Geldeinheiten Ihnen das Gut in dieser Auktionsrunde wert ist. Falls Sie den Zuschlag erhalten, bekommen Sie diesen Wert auf Ihrem Konto gutgeschrieben. Gleichzeitig müssen Sie aber auch einen Preis für das Gut bezahlen; dieser wird von Ihrem Konto abgezogen. Ihr Ertrag ist folglich die Differenz zwischen dem Wert des Gutes und dem dafür gezahlten Preis. Erhalten Sie den Zuschlag nicht, ist Ihr Ertrag null. Im Verlauf des Experiments nehmen Sie an genau sechs solchen Auktionsrunden teil.

1

In jeder Auktionsrunde bilden Sie zusammen mit zwei weiteren Teilnehmern eine Dreiergruppe. Die Zusammensetzung der Gruppen wird zu Beginn ausgelost und ändert sich während des Experiments nicht, d. h. die Mitbieter in Ihrer Gruppe sind von Runde zu Runde dieselben. Innerhalb einer Gruppe sind die Spieler mit den Spielernummern 1, 2 und 3 bezeichnet. Ihre Spielernummer wurde Ihnen zufällig zugelost und ändert sich im Verlauf des Experiments nicht. Ihre Gruppe ist vollkommen unabhängig von den anderen Gruppen im Raum und es gibt keine Interaktion zwischen den Gruppen.

Zu Beginn der ersten Auktionsrunde wird vom System in jeder Gruppe genau ein Spieler ausgelost, dem in jeder der 6 Auktionsrunden ein **Rabatt („Discount“)** zugewiesen wird. Der **ausgewählte** Bieter erhält einen Rabatt in Höhe von 20% – dieser wird dem ausgewählten Bieter in der Versteigerung durch den Text „Discount 20%“ in der Bildschirmmaske, im Feld „Auktionsparameter“, angezeigt. Der Rabatt wird dem ausgewählten Bieter auf den Endpreis der Versteigerung gewährt, wenn er den Zuschlag in der Versteigerung erhält. Allen anderen Bietern, die keinen Rabatt erhalten, wird dies durch den Text „kein Discount“ in der Bildschirmmaske angezeigt.

Sobald eine Auktionsrunde zu Ende ist, erfahren Sie sowohl die Spielernummer desjenigen Bieters, der in Ihrer Gruppe den Zuschlag erhalten hat, als auch den Endpreis der Auktion. Falls Sie der ausgewählte Bieter sind, dem der Rabatt zugelost wurde, und den Zuschlag in einer Auktionsrunde erhalten, so wird Ihnen dies am Ende der Auktion ebenfalls angezeigt.

Ablauf einer Auktionsrunde

Im Einzelnen läuft eine Auktionsrunde wie folgt ab:

1. Vor Beginn der Auktionsrunde wird Ihnen vom System mitgeteilt, wie viele Geldeinheiten Ihnen das Gut in dieser Runde wert ist. Dieser Wert wird im Folgenden auch als Ihre „Wertschätzung“ bezeichnet. Die Wertschätzungen des Gutes sind zufällig und wurden vor dem Experiment für alle Spieler einzeln ausgelost. Die ausgelosten Werte sind ganzzahlige Werte zwischen 100 und 150.

   Bitte beachten Sie: Sie kennen jeweils nur die eigene Wertschätzung für das Gut. Die Wertschätzungen der anderen Spieler für das Gut kennen Sie nicht.

2. Zu Anfang werden Ihnen Ihre Spielernummer, Ihre Wertschätzung für das Gut in der Auktionsrunde und Ihr Kontostand angezeigt. Bitte bestätigen Sie diese Anzeige durch Klicken des „Confirm“-Knopfes am unteren Bildschirmrand.

   Ihre Spielernummer sowie Ihre Wertschätzung wird Ihnen auch weiterhin während der Auktion angezeigt.

3. Anschließend werden Sie von dem System aufgefordert, genau **einen** Wert in die Bildschirmmaske einzutragen. Dieser Wert wird als Ihre „Bietgrenze“ bezeichnet. Bitte tragen

2

Figure D.3: Experimental instruction of setting D (pp. 1–2)

Sie in das entsprechende Feld Ihr Maximalgebot ein. Ihr Maximalgebot ist der Betrag, bis zu dem Sie an einer Versteigerung des Gutes teilnehmen möchten.

Bitte bestätigen Sie die Abgabe Ihres Maximalgebotes durch Klicken des „Abschicken“-Knopfes in der Bildschirmmaske.

Sie müssen Ihr Maximalgebot innerhalb von 2 Minuten abgeben. Nach Ablauf dieser Zeit können keine Gebote mehr abgegeben werden – das Gut wird dann unter den Bietern Ihrer Gruppe versteigert.

In diesem Experiment nehmen Sie nicht selber an der Versteigerung teil. Statt dessen bietet ein in das System eingebauter Bietautomat automatisch für Sie. Der Bietautomat zieht sich aus der Versteigerung zurück, sobald der Preis in der Versteigerung Ihr Maximalgebot (Bietgrenze) übersteigt.

Bitte beachten Sie: Als Gebote sind alle Beträge zwischen 0 GE und 999.99 GE mit bis zu zwei Nachkommastellen zulässig, wobei die Nachkommastellen durch einen Punkt „.“ zu separieren sind.

4. Ein in das System eingebauter Versteigerer überprüft nun für jeden Preis, wie viele Bieter noch an der Versteigerung teilnehmen. Der Versteigerer beginnt mit einem Preis von 0 GE und bietet das Gut den Bietautomaten an. Solange noch mindestens zwei Bieter – vertreten jeweils durch ihren Bietautomaten – an der Versteigerung teilnehmen, wird der Preis um 0.01 GE erhöht. Die Versteigerung endet, sobald nur noch ein Bieter übrig bleibt. Dieser Bieter erhält den Zuschlag in der Versteigerung, d. h. der Höchstbieter gewinnt die Versteigerung.

   Der Endpreis einer Versteigerung ist der höchste Betrag, bei dem noch mindestens zwei Bieter an der Versteigerung teilgenommen haben. Scheiden die letzten beiden (oder mehr) Bieter bei demselben Betrag aus der Versteigerung aus, so entscheidet das Los, welcher der Bieter den Zuschlag erhält. In diesem Fall ist der Endpreis der Versteigerung genau die höchste Bietgrenze, bei der die letzten Bieter gerade noch bereit waren zu bieten.

   Ist ein Bieter, dem **nicht** der Rabatt zugelost wurde, Höchstbieter in der Auktion, so ist der Preis, den dieser für das Gut zu zahlen hat, der Endpreis der Versteigerung. Erhält der **ausgewählte** Bieter, dem der Rabatt zugelost wurde, den Zuschlag, so wird ihm ein Rabatt von 20% auf den Endpreis gewährt: Der Preis, den der ausgewählte Bieter zu zahlen hat, ist der Endpreis der Versteigerung abzüglich des Rabatts von 20%.

   Das Ergebnis der Auktion, d. h. der Endpreis der Auktion sowie die Spielernummer des Bieters, der den Zuschlag in der Versteigerung erhalten hat, werden Ihnen in dem Fenster am unteren Bildschirmrand angezeigt. Gewinnt ein ausgewählter Bieter die Versteigerung, so wird nur diesem Spieler zusätzlich der Preis, den er für das Gut zu zahlen hat, angezeigt.

3

Beispiele:

a) Angenommen, Bieter 1 ist der Bieter, dem der Rabatt von 20% zugelost wurde. Die drei Bieter einer Gruppe beauftragen das System für sie bis zu einem Maximalgebot (Bietgrenze) von 138 GE (Bieter 1) bzw. 113 GE (Bieter 2) bzw. 145 GE (Bieter 3) an der Versteigerung teilzunehmen. Der Bietautomat von Bieter 2 steigt aus der Versteigerung aus, sobald der Preis 113 GE übersteigt. Steigt der Preis über 138 GE, steigt auch Bieter 1 aus der Versteigerung aus. Folglich erhält Bieter 3 den Zuschlag und der Endpreis der Versteigerung ist 138 GE. Der Preis, den Bieter 3, dem kein Rabatt zugelost wurde, für das Gut bezahlt, ist gleich dem Endpreis von 138 GE.

*Wäre andererseits Bieter 3 der Rabatt zugesprochen worden, so hätte Bieter 3 einen Preis in Höhe des Endpreises abzüglich des Rabatts für das Gut zu zahlen, d. h. den Preis von 138 GE - 20%*138 GE = 110.4 GE.*

b) Angenommen, Bieter 3 wurde der Rabatt von 20% zugelost. Lauten die Maximalgebote der drei Bieter 138 GE (Bieter 1) bzw. 113 GE (Bieter 2) bzw. 138 GE (Bieter 3), d. h. Bieter 1 und Bieter 3 haben dasselbe Maximalgebot in die Bildschirmmaske eingegeben, so steigen die Bietautomaten von Bieter 1 und Bieter 3 gleichzeitig aus der Versteigerung aus. Der Endpreis der Versteigerung ist 138 GE. Das System lost, ob Bieter 1 oder Bieter 3 den Zuschlag erhält:
*Erhält Bieter 1 den Zuschlag, dann ist der Preis, den Bieter 1 zahlt, gleich dem Endpreis von 138 GE; erhält Bieter 3 den Zuschlag, dann wird Bieter 3 der Rabatt auf den Endpreis gewährt und er zahlt einen Preis von 138 GE - 20%*138 GE = 110.4 GE für das Gut.*

Derjenige Bieter, der in einer Versteigerung den Zuschlag erhält, bekommt auf seinem Konto die Differenz zwischen seiner Wertschätzung und dem Preis, den er für das Gut bezahlt, gutgeschrieben. Ist der zu zahlende Preis höher als seine Wertschätzung, so wird dieser Betrag von seinem Kontostand abgezogen. Diejenigen Bieter, die den Zuschlag nicht bekommen, erzielen einen Ertrag von null.

Beispiele:

Angenommen, Bieter 3 erhält den Zuschlag und der Preis, den Bieter 3 zu zahlen hat, ist 138 GE. Bieter 3 erzielt einen Ertrag, der sich aus seiner Wertschätzung und dem zu zahlenden Preis berechnet.

(a) Angenommen, Bieter 3 hat eine Wertschätzung von 148 GE. Dann erzielt Bieter 3 einen positiven Ertrag in Höhe von 148 GE - 138 GE = 10 GE. Dieser Betrag wird Bieter 3 auf seinem Konto gutgeschrieben.

(b) Angenommen, Bieter 3 hat eine Wertschätzung von 128 GE. Dann erzielt Bieter 3 einen Ertrag in Höhe von 128 GE - 138 GE = -10 GE, d. h. der Betrag von 10 GE wird ihm von seinem Kontostand abgezogen.

Eine Auktionsrunde dauert 3 Minuten. Ist die Auktionsrunde beendet, wird Ihnen der erzielte Ertrag in der eben gespielten Runde, der Kontostand der vorherigen Runde, sowie der aktuelle Kontostand insgesamt angezeigt. Bestätigen Sie bitte diesen Bildschirm erneut mit

4

Figure D.4: Experimental instruction of setting *D* (pp. 3–4) dem „Confirm“-Knopf am unteren Bildschirmrand. Die nächste Auktionsrunde wird gestartet, sobald alle Teilnehmer den „Confirm“-Knopf bestätigt haben.

Fragebogen

Im Anschluss an die gespielten Auktionsrunden werden Ihnen noch einige Fragen zum Experiment und zu dem Experimentsystem an Ihrem Bildschirm gestellt.

Bitte bleiben Sie nach Beantwortung des Fragebogens an Ihrem Platz sitzen und unterlassen Sie jede Form von Kommunikation mit anderen Teilnehmern. Auf Ihrem Platz finden Sie ein Formular „Erklärung des Vertragnehmers“. Bitte füllen Sie den oberen Teil des Formulars aus. Die genaue Auszahlung im unteren Teil wird vom Experimentleiter ausgefüllt. Sie werden anschließend einzeln nach Ihrem Sitzplatzbuchstaben zur Auszahlung aufgerufen. Bitte verlassen Sie dann leise den Raum. Alle Unterlagen, die Ihnen für das Experiment ausgeteilt wurden, nehmen Sie bitte mit und geben diese bei der Auszahlung wieder ab. Dies gilt insbesondere für die „Erklärung des Vertragnehmers“ und Ihren Sitzplatzbuchstaben.

... und noch eine Bemerkung zum Schluss

Sollten Sie während des Experiments eine Frage haben, bleiben Sie bitte ruhig an Ihrem Platz sitzen und geben Sie dem Experimentleiter durch Handzeichen ein Signal. Warten Sie bitte, bis der Experimentleiter an Ihrem Platz ist, und stellen Sie Ihre Frage so leise wie möglich.

Bevor das Experiment beginnt, werden Ihnen an Ihrem Bildschirm zunächst einige Fragen zu den Regeln dieses Experiments gestellt. Geben Sie bitte die jeweiligen Antworten an Ihrem Computer ein.

5

Figure D.5: Experimental instruction of setting D (p. 5)

D.6 Experimental laboratory

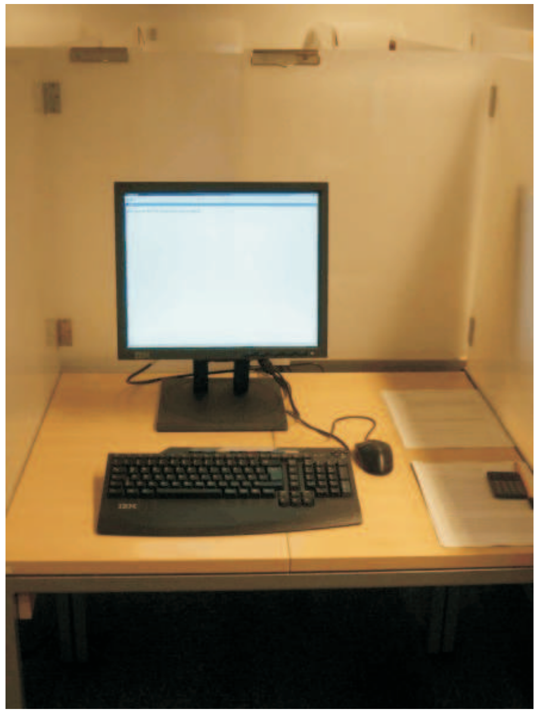

Figure D.6: Experimental laboratory – photography 1

Figure D.7: Experimental laboratory – photography 2

Figure D.8: Experimental laboratory – photography 3

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