

Tunneling Spectroscopy of Two-level Systems Inside Josephson Junction

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(Dated: February 2, 2008)

We consider a two-level (TL) system with energy level separation $\hbar\Omega_0$ inside a Josephson junction. The junction is shunted by a resistor R and is current I (or voltage $V = RI$) biased. If the TL system modulates the Josephson energy and/or is optically active, it is Rabi driven by the Josephson oscillations in the running phase regime near the resonance $2eV = \hbar\Omega_0$. The Rabi oscillations, in turn, translate into oscillations of current and voltage which can be detected in noise measurements. This effect provides an option to fully characterize the TL systems and to find the TL's contribution to the decoherence when the junction is used as a qubit.

Josephson junctions are promising candidates for qubits in quantum computing. However, to ensure long decoherence time, the phase difference across the junction should be coupled very weakly with other low-energy degrees of freedom. Meanwhile, even small-area superconductor-insulator-superconductor (S-I-S) Josephson junctions, as solid-state mesoscopic systems, have many low-energy degrees of freedom inside amorphous insulating layer. Generally, those are phonons (i.e., variables with oscillator-type equidistant energy spectrum) and the two-level (TL) systems (i.e., systems with non-equidistant energy spectrum so that only excitations from the ground state to the lowest excited state are involved at a given frequency). Previously it was shown that optical phonons inside intrinsic Josephson junctions in cuprate layered superconductors cause anomalies in the DC current-voltage characteristics. Namely, peaks in the tunneling current at the voltages corresponding to the phonon frequencies, $V = \hbar\omega_{ph}/2e$, were observed and mechanisms of their coupling with the junction phase difference were identified, see Refs. 1–4.

While low-energy acoustic phonons are coupled weakly with the phase difference due to small statistical weight and negligible optical activity, the TL systems may be much more effective [5]. A TL system may be, for example, an ion having two possible positions inside potential wells with tunneling between them. These degrees of freedom interact with the junction phase difference if they modulate the Josephson energy and/or, if they are optically active. In the latter case they are coupled to the phase difference, $\phi(t)$ (we measure the phase difference in units of magnetic flux), via the electric field inside the junction, $\mathcal{E} = \dot{\phi}/d$, where d is the effective thickness of the junction.

Here we propose a method to characterize the TL systems inside Josephson junctions. Similar to phonons, the TL systems can cause anomalies in the DC I-V characteristics. But what is more, a TL system can precess at its Rabi frequency Ω_R when the Josephson oscillation frequency matches the level splitting, $\hbar\omega_J \equiv 2eV = \hbar\Omega_0$. This is similar to well known behavior of atom in the presence of a resonant electromagnetic wave, see Ref. 6.

However, unlike the optical systems, where the Rabi oscillation are observed as sidebands ($\Omega_0 \pm \Omega_R$) in the emission, in our system, the Josephson current oscillates at the Rabi frequency itself.

The transport and noise properties of the current and voltage biased Josephson junctions have been a subject of extensive studies. The techniques range from the semi-classical Langevin equation [7] which was generalized using the quantum effective action [8], to the infinite series summation [9, 10]. For $V \gg I_c R$ the Coulomb Blockade regime is realized, where the Cooper pairs tunnel incoherently and give rise to the shot noise. We will consider this regime for a point-like Josephson junction (its size much smaller than Josephson length). We will also assume that $2eV \gg k_B T$ and that ω_J is higher than all the frequencies at which we want to measure the voltage noise.

First we consider a junction with the Josephson energy modified by the interaction with TL system, described by the pseudospin-1/2 operator \mathbf{S}

$$H_J = -E_J(1 + \mathbf{j} \cdot \mathbf{S}) \cos 2\pi\phi/\Phi_0, \quad (1)$$

where $E_J = \Phi_0 I_c / 2\pi$, I_c is the Josephson critical current without TL system, and $\Phi_0 = h/(2e)$ is the flux quantum. Coupling constants $\mathbf{j} = (j_x, j_y, j_z)$ characterize the modulation of the Josephson critical current by the TL system. We assume a small shunt resistance, $R \ll R_N$, where R_N is the junction's normal state resistance. This ensures that voltage V on the junction can be small compared with the superconducting gap Δ in the running phase regime. The Hamiltonian of the voltage biased system reads [9, 11]

$$H = \frac{q^2}{2C} + H_R(Vt - \phi) + H_J(\phi) - \hbar\Omega_0 S_z, \quad (2)$$

where $H_R(\xi)$ is the Hamiltonian of the resistor on which the phase ξ drops, and q is the charge on the capacitor C , q being conjugate to ϕ . After the transformation $\tilde{\phi} = \phi - Vt$ and $\tilde{H} = H - Vq$ we obtain

$$\tilde{H} = \frac{\tilde{q}^2}{2C} + H_R(-\tilde{\phi}) + H_J(\tilde{\phi} + Vt) - \hbar\Omega_0 S_z, \quad (3)$$

where $\tilde{q} \equiv q - CV$. The Josephson current operator is

$$I_J \equiv I_c(1 + \mathbf{j} \cdot \mathbf{S}) \sin \frac{2\pi(\tilde{\phi} + Vt)}{\Phi_0}. \quad (4)$$

For frequencies such that $\hbar\omega \ll 2eV$ and in the running phase regime, $V \gg I_c R$, the *symmetrized* phase autocorrelator $S_\phi(\omega)$ is related to the *symmetrized* current autocorrelator, $S_I(\omega)$, via

$$S_\phi \approx S_{\phi,0} + \frac{R^2}{\omega^2 Y(\omega)} S_I, \quad S_{\phi,0} = \frac{\hbar R}{\omega Y(\omega)} \coth \frac{\hbar\omega}{2k_B T}, \quad (5)$$

where $Y(\omega) \equiv 1 + C^2 R^2 \omega^2$. In Eq. (5) the equilibrium correlator $S_{\phi,0}$ is due to the first two terms of Hamiltonian (3), while the next (S_I) term accounts for the Josephson coupling. Eq. (5) can be obtained either from an exact relation between the phase and current Green's functions or from the quasiclassical Langevin equation. In typical experiments, $RC\omega \ll 1$ for all relevant frequencies ω , and, therefore, $Y(\omega) \approx 1$. We, however, keep the factor $Y(\omega)$ to be able to discuss the regime of a junction shunted by a big capacitor.

Next, we calculate the correlator of the Josephson current operators. We are particularly interested in a near-resonance situation, $\Omega_0 \approx \omega_J$. We will use the full quantum Hamiltonian (3) to calculate the spin's contribution to the correlator S_I . As we can see from Eq. (3), the spin is subject to an ac driving at frequency ω_J , "broadened" by the fluctuating phase $\tilde{\phi}(t)$. Thus it is convenient to transform to the frame rotating with the angular velocity $(2\pi/\Phi_0)[V + (d/dt)\tilde{\phi}]$. Formally this amounts to performing canonical transformation $H' = U\tilde{H}U^{-1} + i\hbar\dot{U}U^{-1}$, with

$$U = \exp \left[\frac{2\pi i}{\Phi_0} (\tilde{\phi} + Vt) S_z \right]. \quad (6)$$

Without loss of generality we take $\mathbf{j} = (j_\perp, 0, j_\parallel)$. The result is

$$H' = \frac{\tilde{q}^2}{2C} + H_R(-\tilde{\phi}) - \hbar(\Omega_0 - \omega_J)S_z - \frac{2e\tilde{q}S_z}{C} - E_J(1 + j_\parallel S_z) \cos \frac{2\pi(\tilde{\phi} + Vt)}{\Phi_0} - \hbar\Omega_R S_x, \quad (7)$$

where $\Omega_R \equiv j_\perp I_c / (4e)$ is the Rabi frequency of the spin. The counter-rotating term ($\propto \exp \pm 4\pi(\tilde{\phi} + Vt)/\Phi_0$) can be shown to be not important. The resonance is reached when $\omega_J = \Omega_0$. Then the spin rotates around the x -axis (of the rotating frame) with the Rabi frequency Ω_R , but its dynamics is affected by the noise due to the charge and phase fluctuations.

The operator of the charge on the capacitor is transformed in the rotating frame as

$$q' = U\tilde{q}U^{-1} = \tilde{q} - 2eS_z. \quad (8)$$

The Josephson current operator transforms as

$$I'_J = UI_JU^{-1} = I_c(1 + j_\parallel S_z) \sin \frac{2\pi(\tilde{\phi} + Vt)}{\Phi_0} - \frac{j_\perp I_c}{2} S_y, \quad (9)$$

which shows that dynamics of the spin translates into dynamics of the current.

For $|\mathbf{j}| \ll 1$, the spin's contribution to the average current is negligible (unless spin's energy relaxation is much faster than that of the Cooper pairs). This, however, is not always the case for the current near the Rabi frequency as described by the correlator $S_I(\omega) \equiv \langle I_J(t)I_J(t') \rangle_\omega$ at $\omega \approx \Omega_R$. Indeed, as we shall see, the spin-dependent part of I_J gives rise to a singular (peaked) contribution to the correlator S_I . We calculate $S_I(\omega)$ in the rotating frame using Eqs. (7) and (9). The smooth part of $S_I(\omega)$ is dominated by the shot noise of the Cooper pairs which tunnel incoherently. The lowest order in $RI_c/[VY(\omega)]$ approximation gives good results for S_I^{shot} if $VY(\omega) \gg I_c R$, $\omega \ll \omega_J$ and $R \ll R_Q \equiv h/4e^2$. Then $S_I^{\text{shot}}(\omega) = (I_c^2/4)[P(\omega + \omega_J) + P(\omega - \omega_J)]$, where $P(t) = \exp[J(t)]$, and $J(t) = (2\pi/\Phi_0)^2 \langle (\tilde{\phi}(t) - \tilde{\phi}(0))\tilde{\phi}(0) \rangle$ [12]. At high voltages $2eV \gg k_B T$ we obtain for symmetrized correlator

$$S_I^{\text{shot}}(\Omega_R) \approx S_I^{\text{shot}}(0) \approx \frac{I_c^2}{4} P(\omega_J) \approx \frac{eRI_c^2}{VY(\omega_J)}. \quad (10)$$

In addition, there exists a peak-shaped contribution to $S_I(\omega)$ near $\omega \approx \Omega_R$. Using $\langle \sin 2\pi(\tilde{\phi} + Vt)/\Phi_0 \rangle = RI_c/[2VY(\omega_J)] \equiv I_{av}$ we take into account the j_\parallel term by defining $S_* \equiv j_\perp S_y + (I_{av}/I_c)j_\parallel S_z$. Then

$$S_I^{\text{spin}}(t, t') = \frac{I_c^2}{8} \langle \{S_*(t)S_*(t')\}_+ \rangle. \quad (11)$$

To calculate this correlator we use Hamiltonian (7). Exactly at resonance the effective magnetic field $\hbar\Omega_R$ is directed along the x axis. Assuming the Rabi oscillations are under-damped (to be checked for self-consistency) we obtain

$$S_I^{\text{spin}} = \frac{j_{\text{eff}}^2 I_c^2}{16} \left[\frac{\Gamma_2}{(\omega - \Omega_R)^2 + \Gamma_2^2} + (\omega \rightarrow -\omega) \right], \quad (12)$$

where $j_{\text{eff}}^2 \equiv j_\perp^2 + (I_{av}/I_c)^2 j_\parallel^2$.

To calculate the dephasing rate Γ_2 we note that in (7) both the voltage noise $\delta V \equiv \tilde{q}/C$ and the shot noise (the j_\parallel term) are coupled to S_z , i.e., transversely to the Rabi field. Thus they contribute to Γ_2 through the longitudinal relaxation rate Γ_1 , with $\Gamma_2 = (1/2)\Gamma_1$.

The voltage noise consists of two (uncorrelated to the lowest order) contributions: the equilibrium Johnson-Nyquist one and the one due to the shot noise of the Cooper pairs. Using $\delta V = (d/dt)\tilde{\phi}$ (when no spin is present) and the expression for $S_{\phi,0}$, we obtain for the

Johnson-Nyquist noise

$$S_V^{\text{JN}}(\Omega_R) = \frac{\hbar\Omega_R R}{Y(\Omega_R)} \coth \frac{\hbar\Omega_R}{2k_B T}. \quad (13)$$

The corresponding rate is $\Gamma_2^{\text{JN}} = (e/\hbar)^2 S_V^{\text{JN}}(\Omega_R)$.

The shot noise contributes twice: as part of the voltage noise δV and via the j_{\parallel} term in (7). Using Eq. (5) for the δV part we obtain

$$S_V^{\text{shot}}(\Omega_R) = \frac{R^2}{Y(\Omega_R)} S_I^{\text{shot}}(\Omega_R). \quad (14)$$

The corresponding rate is $\Gamma_2^{\text{shot},\delta V} = (e/\hbar)^2 S_V^{\text{shot}}(\Omega_R)$. Finally the j_{\parallel} term contributes the rate

$$\Gamma_2^{\parallel} = (j_{\parallel}/4e)^2 S_I^{\text{shot}}(\Omega_R). \quad (15)$$

The last equation follows from $\langle\langle \sin \varphi(t) \sin \varphi(t') \rangle\rangle = \langle\langle \cos \varphi(t) \cos \varphi(t') \rangle\rangle$, where $\varphi(t) \equiv 2\pi(\phi(t) + Vt)/\Phi_0$. One can check that all three rates are to be added ‘‘incoherently’’, that is the noise cross-terms vanish. In particular, the cross-term between the two shot noise contributions vanishes because $\langle\langle \sin \varphi(t) \cos \varphi(t') \rangle\rangle = 0$. Thus, accounting for intrinsic TL dephasing Γ_0 ,

$$\Gamma_2 = \Gamma_2^{\text{JN}} + \Gamma_2^{\text{shot},\delta V} + \Gamma_2^{\parallel} + \Gamma_0. \quad (16)$$

For the ‘‘signal’’, i.e., the height of the voltage peak we obtain

$$S_V^{\text{peak}}(\Omega_R) = \frac{j_{\text{eff}}^2 R^2 I_c^2}{16\Gamma_2 Y(\Omega_R)}, \quad (17)$$

while the ‘‘noise’’, i.e., the smooth background is given by

$$S_V^{\text{bg}} = \frac{R^2}{Y(\Omega_R)} \left(S_I^{\text{shot}} + \frac{\hbar\Omega_R}{R} \coth \frac{\hbar\Omega_R}{2k_B T} \right). \quad (18)$$

Collecting all the terms we obtain the signal-to-noise ratio

$$\mathcal{R} = \frac{Y(\Omega_R) A_{\parallel} \left[\coth \frac{\hbar\Omega_R}{2k_B T} + \frac{eR^2 I_c^2}{\hbar\Omega_R V Y(\omega_J)} \right]^{-1}}{\left[\coth \frac{\hbar\Omega_R}{2k_B T} + \frac{eR^2 I_c^2 B_{\parallel}}{\hbar\Omega_R V Y(\omega_J)} + \frac{2R_Q Y(\Omega_R) \Gamma_0}{\pi R \Omega_R} \right]}, \quad (19)$$

where $A_{\parallel} \equiv 1 + \left(\frac{j_{\parallel}}{j_{\perp}} \frac{R I_c}{V Y(\omega_J)} \right)^2$, $B_{\parallel} \equiv 1 + Y(\Omega_R) \left(\frac{j_{\parallel}}{2\pi} \frac{R_Q}{R} \right)^2$, and $R_Q \equiv h/(4e^2)$.

For purely transverse coupling, $j_{\parallel} = 0$, the essential physics is the following: we illuminate the spin with the ‘‘magnetic’’ field $2\hbar\Omega_R \cos 2\pi(Vt + \phi)/\Phi_0$. This field can be thought of as having a sharp peak (a line) near $\omega = \omega_J$. The width of this Josephson line is given by the total voltage noise at zero frequency, $\hbar\Delta\omega = \pi S_V(0)/R_Q = (\pi/R_Q)(S_V^{\text{JN}}(\omega = 0) + R^2 S_I^{\text{shot}}(\omega = 0))$. This relation between the width of the Josephson line and the total

voltage noise was obtained in Refs. [13, 14]. The Rabi oscillation produced by this ‘‘line’’ are, in turn, also broadened by the same amount (in addition to the intrinsic broadening) $\Gamma_2 = \Delta\omega/2 + \Gamma_0$. Finally, the spin’s (broadened) Rabi precession leads to broadened oscillations of the Josephson current and voltage at Ω_R on top of the background of the JN and shot noise.

It is also important to note that the Rabi oscillations of the pseudospin correspond to exactly one Cooper pair going back and forth across the junction. This can be seen from Eq. (8), or from the fact that for the Rabi oscillations to occur exactly one ‘‘Josephson photon’’ with the energy $\hbar\omega_J$ must be absorbed and reemitted by the spin, i.e., exactly one Cooper pair must go through the voltage drop V .

To get a feeling for the relevant numbers we take the data obtained by Simmonds *et al.* [5], where the two lowest levels of a junction (phase qubit) in the superconducting (phase-non-running) regime were driven resonantly. The level splitting $\omega_{01}(I)$ was varied by the bias current I in the frequency interval 8.6-9.1 GHz. At some values of $\omega_{01}(I)$ appreciable splittings (avoided level crossings) were observed. This was suggested to originate from TL systems with $\Omega_0 \approx \omega_{01}(I)$, and the interaction with the phase difference of the type (1). The splitting is caused by the j_{\perp} term, while j_{\parallel} term is inessential as long as $j_{\parallel} E_J \ll \hbar\Omega_0$. Thus, the strongest impurity had $j_{\perp} \approx 6.5 \cdot 10^{-5}$, while we have an upper bound for the strength of the longitudinal coupling $j_{\parallel} < 10^{-3}$. This gives $\Omega_R \approx 2\pi \times 200$ MHz (the splitting of 25 MHz in [5] is due to the reduction factors corresponding to the zero-point motion of the phase degree of freedom in the potential well). In Ref. [5] the critical current is $I_c \approx 10 \mu\text{A}$, the normal resistance of the junction is $R_N \sim 30 \Omega$, while $C \sim 1$ pF. Using these parameters we estimate the signal strength and the signal-to-noise ratio \mathcal{R} . For the temperature we assume $T \approx 10$ mK, or $k_B T/\hbar = 2\pi \times 200$ MHz. The minimum voltage is given by $I_c R$. We assume the shunting resistance of order $R \sim 0.1\Omega \ll R_N$. Shunts of this magnitude have been used in [15]. Hence, we have $\omega_J > (2e/\hbar)I_c R \approx 2\pi \times 0.5$ GHz. From above ω_J is restricted by the gap which gives $\omega_J < (2e/\hbar)I_c R_N \approx 2\pi \times 150$ GHz. Thus we can take $\omega_J \sim 2\pi \times 10$ GHz to be in resonance with the observed TLS. For $j_{\parallel} = 0$ and assuming $\Gamma_0 = 0$ we obtain the signal-to-noise ratio $\mathcal{R} \approx 0.25$. For the maximally allowed $j_{\parallel} = 10^{-3}$ the ratio \mathcal{R} does not change considerably.

For the integrated signal (signal amplitude) we obtain $\left[\int_{\Omega_R} \frac{d\omega}{2\pi} S_V^{\text{peak}} \right]^{1/2} \approx \frac{j_{\text{eff}} R I_c}{4\sqrt{2Y(\Omega_R)}} \approx 10^{-2}$ nV. The Rabi line width is dominated by the Johnson-Nyquist noise, $\Gamma_2 \approx 2\pi \times 5$ kHz. If Γ_0 exceeds considerably this value the signal-to-noise ratio \mathcal{R} will be reduced.

We also note that for the above introduced parameters we have $1/(CR) \sim 10^{13} \text{ s}^{-1}$. Thus $Y(\Omega_R) \approx Y(\omega_J) \approx 1$. One has to increase C by at least three order of magni-

tude in order to start having $Y(\Omega_R) > 1$.

For some (extreme) parameters, such that $Y(\Omega_R) \gg 1$, we obtain $\mathcal{R} \gg 1$ which is in contrast with the limitation $\mathcal{R} \leq 4$ found for the measurements of the peak in the current noise at the frequency Ω_0 [16–18] using a normal-state tunnel junction (broad band amplifier). In that case the voltage $V \gg \hbar\Omega_0/e$ (broad band) is applied. It *incoherently* excites the TL system but also introduces the relaxation due to dissipation necessary for measurement procedure. The relaxation is determined by the noise at frequency Ω_0 and the signal is measured on the background of the noise at the same frequency. As a result, \mathcal{R} is a universal number. In the case considered here, spin is excited at (high) frequency Ω_0 , but the signal is observed at low frequency, due to nonlinearity of coupled spin and Josephson junction system. It is worth mentioning, however, that in the regime with large signal-to-noise \mathcal{R} the single Cooper pair mainly charges and discharges the capacitor, barely going through the resistor; thus, the integrated signal in this regime is reduced.

Now let's consider the mechanism where spin couples to the junction via the electric field. The Hamiltonian is

$$H = \frac{q^2}{2C} + H_R(Vt - \phi) - E_J \cos \frac{2\pi\phi}{\Phi_0} - \hbar\Omega_0 S_z - \frac{Q_{TL}q}{C} S_x, \quad (20)$$

where Q_{TL} is the effective charge of the TL system, given by $Q_{TL} = d_{TL}/L$, where d_{TL} is the TL's dipole moment, while L is the junction's width. For simplicity we assumed purely transverse coupling. Remarkably, the splitting observed in Ref. [5] could also be explained by (20) with $Q_{TL} \sim e$. In this mechanism the spin is coupled to the variable q , which in zeroth order in tunneling (E_J) does not oscillate and has only the Johnson-Nyquist noise spectrum. At $V > RI_c$ this variable acquires an oscillating part due to the Josephson oscillations of the current. Thus the Rabi driving becomes possible. The width of the Rabi line is again determined by the full width of the Josephson line $\Gamma_2^{JN} + \Gamma_2^{\text{shot}, \delta V}$. From the integral (weight) of the Josephson peak in the $S_q(\omega)$ correlator we obtain the Rabi frequency $\Omega_R = RI_c Q_{TL} / (2\hbar\sqrt{Y(\omega_J)})$. Then, analysis similar to the one presented above again gives Eq. (19) for the signal-to-noise ratio (with $A_{\parallel} = B_{\parallel} = 1$ as we assumed purely transverse coupling). For $Q_{TL} \sim e$, we obtain a similar to the previous mechanism Ω_R and a similar value of \mathcal{R} . For the integrated signal we obtain $\left[\int_{\Omega_R} \frac{d\omega}{2\pi} S_V^{\text{peak}} \right]^{1/2} \sim \frac{Q_{TL} R^2 I_c}{e R_Q} \approx 10^{-2}$ nV. Note that the Rabi frequency Ω_R and the integrated signal depend differently on R in two coupling mechanisms. This may allow to distinguish between the two, while they are undistinguishable in measurements of type [5].

In this letter we discussed what happens when the Josephson oscillations are in resonance with one TLS. Let us mention another interesting possibility to manipulate the system. By changing the applied voltage slowly, one can create the regime of the “adiabatic passage” when

$\omega_J(t)$ passes slowly via Ω_0 and exactly one additional Cooper pair is transferred through the junction. Varying $\omega_J(t)$ in a wide enough interval one can “touch” many TL systems and create a measureable additional current.

In conclusion, we propose that the measurements of the low frequency voltage noise in a Josephson junction in the dissipative (running phase) regime may be used to characterize the TL systems inside the junctions, i.e., energy splitting Ω_0 , coupling strength j_{\perp} from the Rabi frequency, and intrinsic dephasing rate Γ_0 from the height of the voltage peak, Eq. (17). We predict a peak at the Rabi frequency when a TL system is resonantly driven by the Josephson oscillations, $\omega_J = \Omega_0$, with the Rabi frequency proportional to the interaction strength between the TL system and the Josephson phase. The peak intensity (signal-to-noise ratio) can be controlled by the shunt resistor and capacitor.

This work was supported by the US DOE and by the ESF “Vortex” Program. AS acknowledges support of CFN (DFG) and EU IST Project SQUBIT.

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