Electron transport and current fluctuations in short coherent conductors

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Employing a real time effective action formalism we analyze electron transport and current fluctuations in comparatively short coherent conductors in the presence of electron-electron interactions. We demonstrate that, while Coulomb interaction tends to suppress electron transport, it may *strongly enhance* shot noise in scatterers with highly transparent conducting channels. This effect of excess noise is governed by the Coulomb gap observed in the current-voltage characteristics of such scatterers. We also analyze the frequency dispersion of higher current cumulants and emphasize a direct relation between electron-electron interaction effects and current fluctuations in disordered mesoscopic conductors.

I. INTRODUCTION

Recently considerable progress has been reached in understanding of an interplay between scattering effects and electron-electron interactions in low-dimensional disordered conductors. In particular, a profound relation[1,2,3,4] between full counting statistics[5] (FCS) and electron-electron interaction effects in mesoscopic conductors has been discovered and explored. This relation follows directly from the effective action of the system which can be conveniently derived by combining the scattering matrix technique with the path integral description of interactions[1,2,6].

An important simplification of the formalism amounts to neglecting the energy independence of scattering matrix. This approximation is applicable to rather short coherent scatterers with Thouless energy exceeding all other relevant energy scales. Besides that, all inelastic processes are assumed to occur in large reservoirs but not inside the scatterer. For this model an exact expression for the effective action was derived[3,4,8] and studied under various approximations, such as regular expansion[1,2,8] of the action in powers of the "quantum" part of the fluctuating field describing interactions as well as renormalization group (RG) analysis[3,4]. Both approaches are justified in the metallic regime, i.e. provided the effective system conductance is much larger than the quantum unit e^2/h. Further generalizations of the model allowed to consider spatially extended metallic conductors and to describe inelastic processes inside the system[9].

The main goal of the present paper is to investigate the problem employing a different set of approximations in order to go beyond the regimes already studied in the literature. Having in mind the relation between FCS and interaction effects in mesoscopic conductors, it would be useful to develop a straightforward and handy formalism enabling one to analyze the frequency dependence of higher order current correlation functions. In other words, the task at hand is to generalize the FCS-type of approach[5] (which is valid only in the low frequency limit) to arbitrary frequencies. In addition, it is desirable to extent the description of interaction effects in mesoscopic coherent conductors beyond the metallic regime. This step would allow, for instance, for a more detailed analysis of electron-electron interactions effects in conductors with few conducting channels, such as, e.g., single-wall carbon nanotubes and organic molecules.

Here we will demonstrate that the above goals can be accomplished assuming transmissions T_n of conducting channels to be either sufficiently small or, on the contrary, sufficiently close to unity. In these situations significant simplifications of the exact effective action can be worked out making the whole formalism more convenient for practical calculations.

Perhaps the most striking result of our analysis concerns the effect of electron-electron interactions on the shot noise in conductors with several highly transmitting channels. We will demonstrate that in this case Coulomb interaction yields *strong enhancement* of the shot noise. Specifically, at sufficiently small voltages V the low frequency noise power spectrum $\mathcal{S}_I(0)$ in weakly reflecting scatterers takes the form

$$\mathcal{S}_I(0) \propto |V|^{1-\frac{2}{g}}, \tag{1}$$

where g is the dimensionless conductance of a scatterer. Provided g is not much bigger than 2 the result (1) strongly exceeds the non-interacting dependence[10] $\mathcal{S}_I(0) \propto |V|$. In the limit of large voltages the noise spectrum is offset by the value of the Coulomb gap,

$$\mathcal{S}_I(0) \propto V + e/2C, \tag{2}$$

where C is the effective capacitance of the scatterer. We also note that interaction-induced excess noise in weakly reflecting scatterers is observed only at sufficiently low frequencies $\omega \lesssim eV$, while at larger ω Coulomb interaction – on the contrary – suppresses the shot noise.

The structure of the paper is as follows. In Sec. II we will briefly summarize the main steps of our derivation of the exact effective action for the conductor described by an arbitrary energy independent scattering matrix. This general expression will be analyzed in Sec. III in various limits. The limiting forms of the effective action will then be used to describe electron transport and current fluctuations in coherent conductors. In Sec. IV we will use them to obtain the frequency and voltage dependence of current cumulants. In Sec. V we will consider a scatterer shunted by a linear Ohmic conductor and, assuming that the system conductance is sufficiently large, discuss perturbative logarithmic interaction corrections to the $I-V$ curve as well as their RG analysis. In Sec. VI and VII we will go beyond the metallic limit and study the effect of electron-electron interactions respectively on the $I-V$ curve and on the shot noise of highly transmitting conductors. Further extensions of our results for the case of quantum dots will be considered in Sec. VIII. Some formal manipulations and further details of our derivations are presented in Appendices A, B and C.

II. EFFECTIVE ACTION

For our derivation we will make use of the Keldysh path integral formalism describing the system of interacting electrons by the action which depends on the Grassmann electron fields $\Psi_\sigma^{1,2}(t,\boldsymbol{r}), \Psi_\sigma^{1,2\,\dagger}(t,\boldsymbol{r})$. The labels 1 and 2 correspond to the forward and the backward branches of the Keldysh contour and σ is the spin index. Performing the standard Hubbard-Stratonovich decoupling of the Coulomb interaction terms in the action, one introduces the two fluctuating electric potentials $V_1(t,\boldsymbol{r})$ and $V_2(t,\boldsymbol{r})$. Afterwards the electron fields are integrated out and one arrives at a formally exact effective action

$$iS = 2\mathrm{Tr}\ln \check{G}_V^{-1} + i\int_0^t dt' \int d^3\boldsymbol{r}\frac{(\nabla V_1)^2-(\nabla V_2)^2}{8\pi}. \quad (3)$$

Here we assumed the spin degeneracy and defined

$$\check{G}_V^{-1} = \begin{pmatrix} i\frac{\partial}{\partial t}+\frac{\nabla^2}{2m}+\mu-U(\boldsymbol{r})+eV_1 & 0 \\ 0 & i\frac{\partial}{\partial t}+\frac{\nabla^2}{2m}+\mu-U(\boldsymbol{r})+eV_2 \end{pmatrix}, (4)$$

where $U(\boldsymbol{r})$ is the external static potential.

The action (3) can also be expressed in a different way, which is convenient for generating correlation functions of the current:

$$iS = \ln\mathrm{Tr}\left[\mathcal{T}\mathrm{e}^{-i\int_0^t dt' \boldsymbol{H}_1(t')}\hat{\boldsymbol{\rho}}_0\tilde{\mathcal{T}}\mathrm{e}^{i\int_0^t dt'\boldsymbol{H}_2(t')}\right] + i\int_0^t dt'\int d^3\boldsymbol{r}\frac{(\nabla V_1)^2-(\nabla V_2)^2}{8\pi}, \quad (5)$$

with the trace taken over the fermionic variables. Here $\hat{\boldsymbol{\rho}}_0$ is the initial $N-$particle density matrix of electrons,

$$\boldsymbol{H}_{1,2} = \sum_\sigma \int d^3\boldsymbol{r}\,\hat{\Psi}^\dagger_\sigma(\boldsymbol{r})\hat{H}_{1,2}(t)\hat{\Psi}_\sigma(\boldsymbol{r}),$$
$$\hat{H}_{1,2}(t) = -\frac{\nabla^2}{2m}+U(\boldsymbol{r})-eV_{1,2}(t,\boldsymbol{r}) \quad (6)$$

are the effective Hamiltonians on the forward and backward parts of the Keldysh contour, $\mathcal{T}$ and $\tilde{\mathcal{T}}$ are respectively the forward and backward time ordering operators.

One can also show (see Appendix A for details) that the action (3) can be expressed in the following equivalent form[8]

$$iS = 2\mathrm{tr}\ln\{1+(\mathcal{U}_2(0,t)\mathcal{U}_1(t,0)-1)\hat{\rho}_0\} + i\int_0^t dt'\int d^3\boldsymbol{r}\frac{(\nabla V_1)^2-(\nabla V_2)^2}{8\pi}, \quad (7)$$

where

$$\mathcal{U}_{1,2}(t_1,t_2) = \mathcal{T}e^{-i\int_{t_1}^{t_2}dt'\left(-\frac{\nabla^2}{2m}-\mu+U(\boldsymbol{r})+eV_{1,2}(t',\boldsymbol{r})\right)} \quad (8)$$

are the evolution operators. Here we distinguish between the "full" trace ("Tr", Eq. (3)) implying the integration over both the coordinates and the time variable, and the "reduced" trace ("tr", Eq. (7)) which denotes the coordinate integration only.

In this paper we will analyze the properties of a short coherent conductor placed in-between two bulk reservoirs. Electron transport across such a conductor can be described by the scattering matrix:

$$\hat{S} = \begin{pmatrix} \hat{r} & \hat{t}' \\ \hat{t} & \hat{r}' \end{pmatrix}, \quad (9)$$

Below we will assume that this matrix does not depend on the energy of incoming electrons. This assumption is justified provided the dwell time of an electron inside the scatterer is shorter than any other relevant time scale. In addition, we assume that an external circuit, which also includes the leads, is linear and characterized by an effective impedance $Z_S(\omega)$. The fluctuating electric potential is assumed to slowly depend on the coordinates in the reservoirs, but to drop sharply across the scatterer. Denoting this voltage drop as $V_{1,2}(t')$, one can also introduce the corresponding Keldysh phase fields $\varphi_{1,2}(t) = \int_0^t dt' eV_{1,2}(t')$ as well as their combinations $\varphi^+ = (\varphi_1+\varphi_2)/2$ and $\varphi^- = \varphi_1-\varphi_2$. The total action of our system reads

$$iS[\varphi^\pm] = iS_S[\varphi^\pm] + i\int_0^t dt'\, C\frac{\dot{\varphi}^+\dot{\varphi}^-}{e^2} + iS_{\rm sc}[\varphi^\pm]. \quad (10)$$

Here $iS_S[\varphi^\pm]$ describes the external circuit:

$$iS_S[\varphi^\pm] = -\frac{1}{2e^2}\int_0^t dt_1 dt_2\,\varphi^-(t_1)\alpha_S(t_1-t_2)\varphi^-(t_2) + \frac{i}{e^2}\int_0^t dt_1dt_2\,\varphi^-(t_1)Z_S^{-1}(t_1-t_2)\big(eV_x-\dot{\varphi}^+(t_2)\big), (11)$$

where $Z_S^{-1}(t) = \int\frac{d\omega}{2\pi}\frac{\mathrm{e}^{-i\omega t}}{Z_S(\omega)}$ is the response function of the external shunt, $\alpha_S(t) = \int\frac{d\omega}{2\pi}\mathrm{e}^{-i\omega t}\mathrm{Re}\left(\frac{\omega\coth\frac{\omega}{2T}}{Z_S(\omega)}\right)$, and V_x is the total voltage drop on the system "scatterer + external shunt".

The second term in Eq. (10) originates from the last term of Eq. (7) and describes the energy of the fluctuating fields. This term contains the scatterer capacitance C.

Finally, the most interesting for us contribution $iS_{\rm sc}$ is given by the first term of Eq. (7) with the evolution operators $\mathcal{U}_{1,2}$ describing the scattering of electrons in the conductor. Following the standard procedure we introduce the scattering channels (labeled by indices) and denote the coordinate and the group velocity in the $k-$th channel respectively as y_k and v_k. The matrix corresponding to the operator $\mathcal{U}_1(t,0)$ then takes the form

$$\mathcal{U}_1^{nk}(t,0;y_2,y_1) = e^{i\alpha_n\varphi_1(t)-i\alpha_k\varphi_1(0)}\frac{\delta\left(\frac{y_2}{v_n}-\frac{y_1}{v_k}-t\right)}{\sqrt{v_nv_k}}\times \left[\delta_{nk}+\theta(y_2)\theta(-y_1)e^{-i\alpha_n\varphi_1(-y_1)+i\alpha_k\varphi_1(-y_1)}(S_{nk}-\delta_{nk})\right], \tag{12}$$

where S_{nk} is the matrix element of the $S-$matrix (9), $\alpha_k=1$ provided the $k-$th channel is in the left lead and $\alpha_k=0$ otherwise. The operator $\mathcal{U}_2(0,t)$ is constructed from $\mathcal{U}_1(t,0)$ by means of the Hermitian conjugation together with the replacement $\varphi_1\to\varphi_2$. We point out that Eq. (12) is similar to the well known relation between the Green function and the $S-$matrix established by Fisher and Lee[11].

Having found the expressions for the evolution operators, we make use of Eq. (7) and arrive at the final result:

$$iS_{\rm sc} = 2{\rm tr}\ln\left\{\hat 1\delta(x-y)+\theta(t-x)\theta(x)\begin{bmatrix}\hat t^\dagger\hat t({\rm e}^{i\varphi^-(x)}-1) & 2i\hat t^\dagger\hat r'\sin\frac{\varphi^-(x)}{2}\\ 2i\hat r'^\dagger\hat t\sin\frac{\varphi^-(x)}{2} & \hat t'^\dagger\hat t'({\rm e}^{-i\varphi^-(x)}-1)\end{bmatrix}\begin{bmatrix}\rho_0(y-x){\rm e}^{i\frac{\varphi^+(x)-\varphi^+(y)}{2}} & 0\\ 0 & \rho_0(y-x){\rm e}^{i\frac{\varphi^+(y)-\varphi^+(x)}{2}}\end{bmatrix}\right\}, \tag{13}$$

where

$$\rho_0(x)=\int\frac{dx}{2\pi}\frac{{\rm e}^{iEx}}{1+{\rm e}^{E/T}}=\frac{1}{2}\delta(x)-\frac{iT}{2\sinh\pi Tx} \tag{14}$$

is the Fourier transform of the Fermi function.

III. LIMITING FORMS OF THE ACTION

The general expression for the effective action (13) contains a great deal of information and can be used in order to describe a variety of different phenomena. At the same time this expression still remains rather complex, and further simplifications are highly desirable. These simplifications can be achieved in various limiting cases to be discussed below in this section.

Before we turn to concrete calculations let us note that several important physical limits of Eq. (13) are already well known and have been studied in details. For instance, setting $\varphi^-=$ const and $\dot\varphi^+=eV$ one reduces the action (13) to the FCS cumulant generating function[5] for a coherent scatterer in the absence of electron-electron interactions. In this case φ^- plays the role of the so-called counting field.

If one allows for the time-dependent voltage $V(t)$ across the scatterer and at the same time keeps φ^- constant, with the aid of Eq. (13) one can describe FCS at an ac bias, adiabatic pumping through the conductor and related effects[12]. In addition, neglecting the interactions but keeping the full time dependence for the phase φ^-, with the aid of Eq. (13) one can fully describe frequency dispersion of all current cumulants. For this purpose it suffices to perform a regular expansion of the action (13) in powers of the field φ^-. Frequency dispersion of the third current cumulant was analyzed in this way in Ref. 8.

The same expansion allows to obtain valuable information about the effect of electron-electron interactions on transport properties, shot noise as well as higher cumulants of the current operator. Assuming the dimensionless conductance of a scatterer g and/or that of the external circuit g_S are/is large, one can obtain the interaction correction to the current[1] of order $1/(g+g_S)$ expanding the action (13) up to the second order in φ^-. In order to derive the interaction correction to the shot noise[2] one should expand the action up to the third order in φ^-.

Finally, keeping the exact non-linear dependence of the action on the fields $\varphi^\pm$ but expanding (13) to the first order in the transmission matrix $\hat t^\dagger\hat t$, one immediately reproduces the well known AES effective action[13,14]. The latter action, being combined with iS_S (11) and the capacitance term, describes Coulomb blockade effects in tunnel junctions embedded in a linear electromagnetic environment[14,15].

In what follows we will investigate the properties of the action (13) in some other limiting cases.

A. Weakly transmitting barriers

Let us first consider the case of weakly transmitting barriers and expand the action in powers of the matrix $\hat t^\dagger\hat t$. As we have already pointed out, the first order terms of this expansion yield the AES effective action[13,14]. Here we proceed further and expand the action (13) up to $(\hat t^\dagger\hat t)^2$ keeping the complete nonlinear dependence on the fluctuating phases $\varphi^\pm$. It is easy to see that in order to recover all such terms it is necessary to expand the logarithm in Eq. (13) up to the fourth order in the term containing the density matrix ρ_0. Higher order terms of this expansion can be omitted within the required accuracy since they do not contain contributions proportional to T_n and T_n^2, where T_n represent the channel transmissions defined in a standard manner as the eigenvalues of the matrix $\hat{t}^\dagger\hat{t}$.

The whole calculation is performed in a straightforward manner, although requires some care. The final result reads

$$
\begin{aligned}
iS \;=\; & -\frac{i}{\pi}\int_0^t dx\, \dot{\varphi}^+(x)\sin\varphi^-(x)\left[\mathrm{tr}[\hat{t}^\dagger\hat{t}] + \frac{2}{3}\mathrm{tr}[(\hat{t}^\dagger\hat{t})^2]\sin^2\frac{\varphi^-(x)}{2}\right] \\
& -\frac{2}{\pi}\int_0^t dxdy\;\alpha(x-y)\sin\frac{\varphi^-(x)}{2}\sin\frac{\varphi^-(y)}{2}\left\{\mathrm{tr}[\hat{t}^\dagger\hat{t}(1-\hat{t}^\dagger\hat{t})]\cos[\varphi^+(x)-\varphi^+(y)] + \mathrm{tr}[(\hat{t}^\dagger\hat{t})^2]\cos\frac{\varphi^-(x)-\varphi^-(y)}{2}\right\} \\
& +\frac{4i}{3}\mathrm{tr}[(\hat{t}^+\hat{t})^2]\int_0^t dxdydz\;\frac{T^3\sin\frac{\varphi^-(x)}{2}\sin\frac{\varphi^-(y)}{2}\sin\frac{\varphi^-(z)}{2}}{\sinh[\pi T(y-x)]\sinh[\pi T(x-z)]\sinh[\pi T(z-y)]} \\
& \times\left\{\sin[\varphi^+(y)-\varphi^+(x)]\cos\frac{\varphi^-(z)}{2} + \sin[\varphi^+(z)-\varphi^+(y)]\cos\frac{\varphi^-(x)}{2} + \sin[\varphi^+(x)-\varphi^+(z)]\cos\frac{\varphi^-(y)}{2}\right\} \\
& -16\mathrm{tr}[(\hat{t}^\dagger\hat{t})^2]\int_0^t dxdydzdw\;\rho_0(y-x)\rho_0^*(x-w)\rho_0(w-z)\rho_0^*(z-y) \\
& \times\sin\frac{\varphi^-(x)}{2}\sin\frac{\varphi^-(y)}{2}\sin\frac{\varphi^-(z)}{2}\sin\frac{\varphi^-(w)}{2}\cos\left[\varphi^+(x)-\varphi^+(y)+\varphi^+(z)-\varphi^+(w)\right]. \qquad (15)
\end{aligned}
$$

where

$$\alpha(x) = \int\frac{d\omega}{2\pi}\,e^{-i\omega x}\omega\coth\frac{\omega}{2T} = -\frac{\pi T^2}{\sinh^2\pi Tx}. \qquad (16)$$

Eq. (15) represents the complete expression for the effective action valid up to the second order in the transmissions T_n. This expression involves no further approximations and fully accounts for the non-linear dependence on the fluctuating phase fields $\varphi^\pm$.

B. Reflectionless limit

Another physically important limit is that of reflectionless barriers $\hat{r} = 0$. In this limit the action (13) can be significantly simplified by means of the exact procedure which we outline below.

To begin with, we notice that in the case of reflectionless barriers the action (13) reveals significant similarity to the Luttinger model of 1D interacting electron gas. The latter model is usually treated by means of the bosonization technique. In the case of quantum dots and point contacts this method was applied in Refs. 16,17,18,19. One can also evaluate an effective action for reflectionless barriers directly without employing the bosonization technique. Actually an important feature of the final result can be guessed even before doing the calculation. Indeed, since the RPA approximation is known[20] as an exact procedure for the Luttinger model, one can expect that in the limit $\hat{r}' \to 0$ the action (13) should become quadratic in $\varphi^\pm$, at least for not very large values of the phase. We will demonstrate that this is indeed the case if $|\varphi^-| < \pi$.

Let us put $\hat{r}' = 0$ in Eq. (13). Then the action can be split into two parts

$$
\begin{aligned}
iS_{sc}^{(0)} &= iN_{\rm ch}S_0\big[\theta(t-x)\theta(x)(\mathrm{e}^{i\varphi^-(x)}-1),\; -\varphi^+\big] \\
&\quad + iN_{\rm ch}S_0\big[\theta(t-x)\theta(x)(\mathrm{e}^{-i\varphi^-(x)}-1),\; \varphi^+\big], \quad (17)
\end{aligned}
$$

where $N_{\rm ch}$ is the number of open channels and

$$iS_0[a,\varphi^+] = 2\mathrm{tr}\ln\left[1 + a(x)\rho_0(y-x)\mathrm{e}^{i\frac{\varphi^+(y)-\varphi^+(x)}{2}}\right]. \qquad (18)$$

The action (18) can be evaluated exactly. The details of our derivation are summarized in Appendix B. Here we only present the final result:

$$
\begin{aligned}
iS_0[a,\varphi^+] &= 2\int dx\left(\rho_0(0) + \frac{\dot{\varphi}^+(x)}{4\pi}\right)\ln[1+a(x)] \\
&\quad + \int dxdy\,\alpha(x-y)\frac{\ln[1+a(x)]\ln[1+a(y)]}{4\pi}, \quad (19)
\end{aligned}
$$

where $\rho_0(0)$ is a large constant, which has a meaning of the electron density, and which is later canceled by the corresponding contribution of ions. Combining this formula with Eq. (17) we arrive at the main result of this subsection

$$
\begin{aligned}
iS_{\rm sc}^{(0)} &= -\frac{iN_{\rm ch}}{\pi}\int_0^t dx\,W(\varphi^-(x))\dot{\varphi}^+(x) \\
&\quad -\frac{N_{\rm ch}}{2\pi}\int_0^t dxdy\,W(\varphi^-(x))\alpha(x-y)W(\varphi^-(y)), \quad (20)
\end{aligned}
$$

where $W(\varphi)$ is the $2\pi-$periodic function of φ^-, which equals to φ^- in the interval $-\pi < \varphi^- < \pi$. Under certain conditions the contribution of large phase values, $|\varphi^-| > \pi$, can be disregarded, and one can use the Gaussian action

$$iS_{\rm sc}^{(0)} \to -\frac{iN_{\rm ch}}{\pi}\int_0^t dx\, \varphi^-(x)\dot\varphi^+(x) - \frac{N_{\rm ch}}{2\pi}\int_0^t dxdy\, \varphi^-(x)\alpha(x-y)\varphi^-(y) \quad (21)$$

instead of the exact one (20). Within this approximation the action for the scatterer with $N_{\rm ch}$ perfectly transmitting channels coincides with that for a linear Ohmic resistor with the conductance $e^2N_{\rm ch}/\pi$. It is important to emphasize, however, that this approximation ignores the action periodicity in the phase space and, hence, becomes inadequate as soon as electron charge discreteness turns out to be an important effect.

C. Weakly reflecting barriers

Let us now assume that the reflection probabilities are small, $R_n = 1 - T_n \ll 1$, but not equal to zero. In this case it is convenient to proceed perturbatively expanding the action (13) in powers of R_n or, which is the same, in powers of the matrices $\hat r', \hat r'^\dagger$. The details of our derivation are provided in Appendix C. Expanding the action (13) to the first order in R_n, one finds

$$iS_{\rm sc} = iS_{\rm sc}^{(0)} + iS_{\rm sc}^{(1)}, \quad (22)$$

where $iS_{\rm sc}^{(0)}$ is defined in Eq. (20) and

$$\begin{aligned} iS_{\rm sc}^{(1)} \; = \; & \frac{i\mathcal{R}}{\pi}\int_0^t dx\,\dot\varphi^+(x)\sin\varphi^-(x) \\ & + \frac{\mathcal{R}}{\pi}\int dxdy\,\alpha(x-y)\,\varphi^-(x)\sin\varphi^-(y) \\ & - \frac{2\mathcal{R}}{\pi}\int_0^t dxdy\,\alpha(x-y)\,e^{i[\varphi^+(x)-\varphi^+(y)]} \\ & \times e^{\int_0^t dz[T\coth\pi T(x-z) - T\coth\pi T(y-z)]\varphi^-(z)} \\ & \times \sin\frac{\varphi^-(x)}{2}\sin\frac{\varphi^-(y)}{2}. \end{aligned} \quad (23)$$

As we have already pointed out, this expression is justified provided all the channels are weakly reflecting, $R_n \ll 1$. At the same time the parameter $\mathcal{R} = \sum_n R_n$ needs not to be small should the total number of channels in the system be large. We also note that different limiting forms of the action (15) and (20)-(23) can be combined if some channels are weakly transmitting $T_n \ll 1$ while the others have small reflection coefficients $R_m \ll 1$.

IV. FREQUENCY DISPERSION OF HIGHER CURRENT CUMULANTS

Let us now make use of the above limiting expressions for the effective action in order to describe various properties of short coherent conductors. In this section we neglect the effects of electron-electron interactions and set $\varphi^+(t) = eVt$, where V is the time-independent bias voltage.

Complete information about all correlation functions of the current operator in the zero frequency limit is contained in the FCS cumulant generating function[5]. This function, however, becomes insufficient if one is interested in the frequency dependence of the current cumulants. This dependence can in general be recovered only from the complete effective action (13). Unfortunately the latter appears too complicated to directly proceed with the analysis of the n-th current cumulant. The situation is significantly simplified in the two complementary limits of small and almost perfect channel transmissions, respectively $T_n \ll 1$ and $1 - T_n \ll 1$. In these two cases one can make use of the limiting forms of the effective action derived in the previous section. The corresponding analysis is presented below.

In the absence of interaction effects one can treat the action $iS_{\rm sc}$ as a generating functional for the current correlation functions

$$\langle I(t_1)\dots I(t_m)\rangle = \left.\frac{(ie)^m\delta^m\, e^{iS_{\rm sc}[eVt,\varphi^-]}}{\delta\varphi^-(t_1)\dots\delta\varphi^-(t_m)}\right|_{\varphi^-=0}. \quad (24)$$

Here $I(t_j)$ are measurable classical currents which commute with each other[7]. On a quantum level, the same correlator can be written in terms of the current operators $\hat I(t_j)$, however the choice of the time ordering becomes important in this case.

One can also define the m-th current cumulant $\tilde{\mathcal{S}}_m(t_1,\dots t_m)$ as an irreducible part of the correlator (24):

$$\tilde{\mathcal{S}}_m = (ie)^m \left.\frac{\delta^m\, iS_{\rm sc}[eVt,\varphi^-]}{\delta\varphi^-(t_1)\dots\delta\varphi^-(t_m)}\right|_{\varphi^-=0}. \quad (25)$$

For classical currents, the first three cumulants are defined as

$$\begin{aligned} &\tilde{\mathcal{S}}_1 = \langle I\rangle\,, \\ &\tilde{\mathcal{S}}_2(t_1,t_2) = \langle \delta I(t_1)\delta I(t_2)\rangle\,, \\ &\tilde{\mathcal{S}}_3(t_1,t_2,t_3) = \langle \delta I(t_1)\delta I(t_2)\delta I(t_3)\rangle\,, \end{aligned} \quad (26)$$

where $\delta I(t) = I(t) - \langle I\rangle$. The fourth and higher cumulants take a more complicated form, e.g.,

$$\begin{aligned} &\tilde{\mathcal{S}}_4(t_1,t_2,t_3,t_4) = \langle \delta I(t_1)\delta I(t_2)\delta I(t_3)\delta I(t_4)\rangle \\ &-\tilde{\mathcal{S}}_2(t_1,t_2)\tilde{\mathcal{S}}_2(t_3,t_4) - \tilde{\mathcal{S}}_2(t_1,t_3)\tilde{\mathcal{S}}_2(t_2,t_4) \\ &-\tilde{\mathcal{S}}_2(t_1,t_4)\tilde{\mathcal{S}}_2(t_2,t_3). \end{aligned} \quad (27)$$

In what follows we will use Eq. (25) as a definition of the current cumulants also in the quantum case. This definition unambiguously fixes time ordering of the current operators. The corresponding expression for the third cumulant ($m = 3$) in terms of the time-ordered current operators has been specified in Ref. 8. Analogous expressions for higher cumulants $m > 3$ are cumbersome and we do not present them here.

We also define the Fourier transform of the current cumulants:

$$\tilde{\mathcal{S}}_m = \int dt_1 \ldots dt_m \frac{e^{i\omega_1 t_1 + \ldots + i\omega_m t_m}}{2\pi} \tilde{\mathcal{S}}_m(t_1, \ldots t_m). \quad (28)$$

We begin with the limit of weakly transmitting barriers in which case in the lowest order in T_n one can evaluate the current cumulants making use of the AES effective action[13,14]. Combining Eqs. (15) and (25) and dropping the terms $\propto T_n^2$ for the odd ($m = 2l + 1$) cumulants one finds

$$\tilde{\mathcal{S}}_m = \frac{e^{m+1} g V}{2\pi} \delta(\omega_1 + \ldots + \omega_m), \quad (29)$$

where $g = 2\sum_n T_n$ is the dimensionless conductance of the scatterer. Analogously one can evaluate the even ($m = 2l$) current cumulants which read

$$\tilde{\mathcal{S}}_m = \frac{e^m g}{2^m \pi} \delta(\omega_1 + \ldots + \omega_m) \times \sum_{\nu_j = \pm 1}{}' \left(eV + \sum_{j=1}^m \frac{\nu_j \omega_j}{2} \right) \coth\left(\frac{eV}{2T} + \sum_{j=1}^m \frac{\nu_j \omega_j}{4T} \right). \quad (30)$$

Here the prime in the sum implies the summation over "charge" configurations $\nu_j = \pm 1$ with the odd number of positive (negative) "charges" ν_j. This result is also valid in the lowest order in T_n.

Keeping the terms $\propto T_n^2$ in Eq. (15) and repeating the same calculation one can evaluate the second order corrections to Eqs. (29) and (30). The corresponding expressions turn out to be rather complicated and for this reason are omitted here. We note, however, that in the limit of low voltages (or high frequencies) $eV \ll T, \omega_1, \ldots, \omega_m$ the non-local in time terms in the action (15) do not contribute to the odd current cumulants. Hence, in this limit the whole analysis gets much simpler and the odd cumulants are fully determined by the remaining (local in time) part of the action. In other words, in the limit $eV \ll T, \omega_1, \ldots, \omega_m$ the odd current cumulants can be evaluated (up to the terms $\sim T_n^2$) with the aid of the generating function

$$iS_D = -\frac{ieV}{\pi} \int_0^t dx \ \sin\varphi^-(x) \times \left[\sum_n T_n + \frac{2}{3} \sum_n T_n^2 \sin^2 \frac{\varphi^-(x)}{2} \right], \quad (31)$$

which should be substituted into Eq. (25) instead of $S_{\rm sc}$.

Now we turn to the case of weakly reflecting barriers, in which case the effective action is defined by Eqs. (22,23).

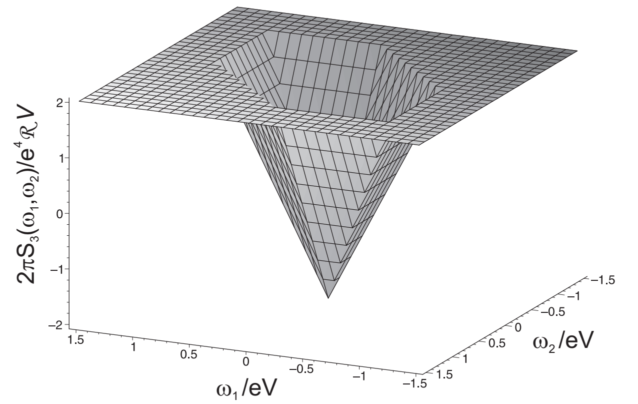

FIG. 1: Third cumulant of the current operator $\mathcal{S}_3$ at zero temperature as a function of frequencies ω_1 and ω_2. Note that $\mathcal{S}_3(\omega_1, \omega_2)$ changes its sign with increasing frequencies and flattens off at $|\omega_1|, |\omega_2| > eV$.

Combining this action with Eq. (25) for the odd ($m = 2l + 1 \geq 3$) current cumulants we obtain

$$\tilde{\mathcal{S}}_m = \frac{e^m \mathcal{R}}{2\pi} \delta(\omega_1 + \ldots + \omega_m) \Bigg[-2eV + \sum_{\mu_{1,2}=\pm 1} \sum_{\nu_j = \pm 1} (-1)^{\frac{\mu_1+\mu_2}{2}} \left(eV + \sum_{j=1}^m \frac{\nu_j \omega_j}{2} \right) \coth\left(\frac{eV}{2T} + \sum_{j=1}^m \frac{\nu_j \omega_j}{4T} \right) \times \prod_{j=1}^m \left(\frac{\mu_1 + \mu_2}{4} + \nu_j \left(\frac{\mu_1 - \mu_2}{4} - \coth\frac{\omega_j}{2T} \right) \right) \Bigg]. \quad (32)$$

Analogously for the even ($m = 2l \geq 4$) we find

$$\tilde{\mathcal{S}}_m = -\frac{e^m \mathcal{R}}{2\pi} \delta(\omega_1 + \ldots + \omega_m) \Bigg[2\sum_{j=1}^m \omega_j \coth\frac{\omega_j}{2T} + \sum_{\mu_{1,2}=\pm 1} \sum_{\nu_j=\pm 1} (-1)^{\frac{\mu_1+\mu_2}{2}} \times \left(eV + \sum_{j=1}^m \frac{\nu_j \omega_j}{2} \right) \coth\left(\frac{eV}{2T} + \sum_{j=1}^m \frac{\nu_j \omega_j}{4T} \right) \times \prod_{j=1}^m \left(\frac{\mu_1 + \mu_2}{4} + \nu_j \left(\frac{\mu_1 - \mu_2}{4} - \coth\frac{\omega_j}{2T} \right) \right) \Bigg]. \quad (33)$$

The delta-functions $\delta(\omega_1 + \ldots + \omega_m)$ in Eqs. (29,30,32,33) illustrate the cumulant invariance with respect to the time shifts $t_j \to t_j + \Delta t$. One can also consider "on-shell" cumulants $\mathcal{S}_m$ symbolically defined as

$$\mathcal{S}_m(\omega_1, \ldots, \omega_{m-1}) = = \frac{\tilde{\mathcal{S}}_m(\omega_1 + \ldots + \omega_{m-1}, -\omega_1, -\omega_2, \ldots, -\omega_{m-1})}{\delta(\omega_1 + \ldots + \omega_m)}. \quad (34)$$

This definition implies that one should first remove the $\delta-$function from Eqs. (29,30,32,33) and then replace the

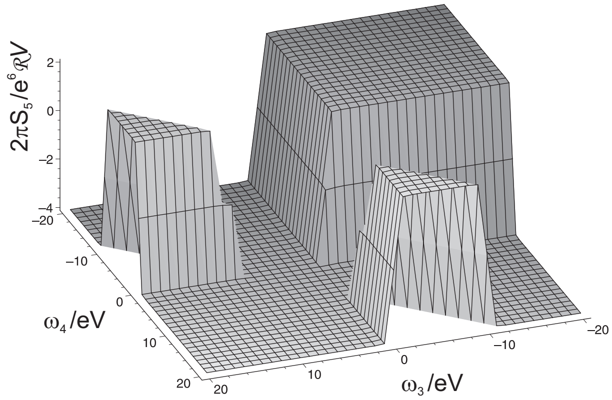

FIG. 2: Fifth current cumulant $\mathcal{S}_5$ at $T = 0$. Two frequencies are fixed, $\omega_1 = \omega_2 = -5eV$, while two others, ω_3 and ω_4 are varied.

frequencies as follows $\omega_1 \to \sum_{j=1}^{m-1} \omega_j$, $\omega_2 \to -\omega_1$, $\omega_3 \to -\omega_2$, ... $\omega_m \to -\omega_{m-1}$. The cumulants $\mathcal{S}_m$, defined in this way, depend on $m-1$ frequencies and in the limit $\omega_i \to 0$ reduce to the standard FCS expressions for the current cumulants.

The complete analytical expression for the third cumulant $\mathcal{S}_3(\omega_1, \omega_2)$ at arbitrary reflection probabilities has been previously derived in Ref. 8 by means of a somewhat different approach. In the limit of small $\mathcal{R}$ the result[8] reduces to that derived here.

The third current cumulant defined by Eqs. (32,34) with $m = 3$ is plotted in Fig. 1 in the limit $T = 0$. We note that the overall shape of the function $\mathcal{S}_3(\omega_1, \omega_2)$ remains the same at any transmission values T_n (cf., e.g., Fig. 2 of Ref. 21) except for the tunnel junction limit (all $T_n \ll 1$) in which case no frequency dispersion of $\mathcal{S}$ is observed. In a general case the cumulant $\mathcal{S}_3$ becomes dispersionless only at sufficiently high frequencies (or at low voltages) reaching a universal value[8,21] $\mathcal{S}_3 \to \beta e^4 gV/2\pi$.

Higher cumulants depend on larger number of frequencies and their behavior turns out to be more complicated. As an example in Fig. 2 we plot the fifth current cumulant (defined by Eqs. (32,34) with $m = 5$) at $T = 0$ as a function of two frequencies keeping two others fixed. Unlike the third cumulant, $\mathcal{S}_5$ does not tend to any universal frequency-independent value in the high frequency limit.

V. LOGARITHMIC CORRECTIONS AND RG

Let us turn on electron-electron interactions. In this section we assume the external circuit to be purely Ohmic, $Z_S(\omega) = R_S$. Further we adopt the standard set of approximations and assume that either the barrier dimensionless conductance $g = 2\sum_n T_n$, or that of the external circuit $g_S = 2\pi/e^2 R_S$ is large, $g \gg 1$, or $g_S \gg 1$. In this case in a wide range of parameters the effect of electron-electron interactions is to produce a negative correction to the system conductance. The latter interaction correction is proportional to $1/(g+g_S)$ and depends logarithmically[1,22] on voltage and temperature, thus becoming large at sufficiently small eV and T. This divergence demonstrates insufficiency of the first order perturbation theory in $1/(g+g_S)$ at low energies and makes it necessary to evaluate the higher order terms.

For the sake of definiteness below in this section we will assume $T \ll eV$ and define

$$L = \ln \frac{1}{eVR_0C},$$

where $R_0 = RR_S/(R+R_S)$ and $R = \pi/e^2 \sum_n T_n$ is the Landauer resistance of the barrier. The effective expansion parameter of the perturbation theory is then $L/(g+g_S)$.

Let us first evaluate the second order contribution to the interaction correction $\propto L^2/(g+g_S)^2$. For this purpose it is sufficient to expand the effective action to the third order in φ^-. One finds[2]

$$\begin{aligned} iS = iS_S - \frac{i}{e^2}\int_0^t dt'\, \varphi^- \left[C\ddot{\varphi}^+ + \frac{1}{R}\dot{\varphi}^+\right] \\ -\frac{1}{2e^2R}\int_0^t dt_1 dt_2 \alpha(t_1-t_2)\varphi^-(t_1)\varphi^-(t_2) \\ \times \left\{1-\beta+\beta\cos\left[\varphi^+(t_1)-\varphi^+(t_2)\right]\right\} \\ +\frac{i\beta}{6e^2R}\int_0^t d\tau(\varphi^-(\tau))^3\dot{\varphi}^+(\tau) \\ -\frac{2\pi i\gamma}{3e^2R}\int_0^t d\tau_1 d\tau_2 d\tau_3 \varphi^-(\tau_1)\varphi^-(\tau_2)\varphi^-(\tau_3) \\ \times f(\tau_2,\tau_1)f(\tau_3,\tau_2)f(\tau_1,\tau_3), \end{aligned} \quad (35)$$

where

$$\beta = \frac{\sum_n T_n(1-T_n)}{\sum_n T_n}, \quad \gamma = \frac{\sum_n T_n^2(1-T_n)}{\sum_n T_n} \quad (36)$$

and

$$f(\tau_2,\tau_1) = \frac{T\sin\left[(\varphi^+(\tau_2)-\varphi^+(\tau_1))/2\right]}{\sinh\left[\pi T(\tau_2-\tau_1)\right]}. \quad (37)$$

The current is expressed via the path integral

$$I(t) = \frac{-e\int \mathcal{D}\varphi^\pm \frac{\delta S_{\rm sc}[\varphi^\pm]}{\delta\varphi^-(t)} e^{iS[\varphi^\pm]}}{\int \mathcal{D}\varphi^\pm e^{iS[\varphi^\pm]}}. \quad (38)$$

This formula can be derived, e.g., from Eq. (5) making use of the definition for the current operator

$$\hat{I}(t) = \frac{e}{2}\frac{d(\hat{N}_L(t)-\hat{N}_R(t))}{dt}, \quad (39)$$

where $\hat{N}_{L,R}(t) = \sum_\sigma \int_{L,R} d^3\boldsymbol{r}\, \hat{\Psi}^\dagger_\sigma(t,\boldsymbol{r})\hat{\Psi}_\sigma(t,\boldsymbol{r})$ is the total number of electrons on the left (right) side of the barrier.

Evaluating this integral with the approximate action (35) and assuming $eVR_0C \lesssim 1$ we get

$$I = \frac{V}{R}\left[1-\frac{2\beta L}{g+g_S}+\frac{2L^2}{(g+g_S)^2}\left(\beta-\frac{\beta^2 R_S}{R+R_S}-2\gamma\right)\right. \left. +\mathcal{O}\left(\frac{L^3}{(g+g_S)^3}\right)\right]. \quad (40)$$

In order to find the higher order terms of the perturbation theory in $L/(g+g_S)$ one needs to retain the contributions to the effective action of order $\sim(\varphi^-)^4$ and higher. An alternative way is to treat the effective action by means of the RG approach[3,4] which allows to recover the leading logarithmic contributions to all orders.

Previously, the RG equations for this problem were formulated either in the limit[3] $g_S \gg 1, g$ or in the opposite limit[4] $g_S \to 0$ and $g \gg 1$. In a general case the RG equations for the channel transmissions read:

$$\frac{d\tilde{T}_n}{dL} = -\frac{\tilde{T}_n(1-\tilde{T}_n)}{\sum_k \tilde{T}_k + (g_S/2)}. \quad (41)$$

The implicit solution for these equations can be obtained in the form

$$\tilde{T}_n(L) = \frac{T_n(1-z)}{1-T_n z}$$
$$L = -\sum_k \ln(1-T_k z) - \frac{g_S}{2}\ln(1-z), \quad (42)$$

where the parameter z changes from 0 to 1.

Eqs. (42) demonstrate that as the voltage decreases the channels are "turned off" by interactions one by one depending on their transmission values. Most transparent channels $R_n \ll 1$ remain open down to lowest voltages. Resolving Eqs. (42) one can explicitly determine the renormalized (voltage dependent) transmissions $\tilde{T}_n$ and, substituting $\tilde{T}_n$ into the Landauer formula, derive the expression for the $I-V$ curve. Unfortunately it remains unclear whether this approach is sufficient down to the lowest energies/voltages in which case instanton effects[6,23] need to be taken into account. The analysis of this problem is beyond the scope of the present paper.

VI. CURRENT IN THE LIMIT $\mathcal{R} \ll 1$

Let us now evaluate the interaction correction to the current for a somewhat different physical limit. In contrast to the previous section, here we will make no assumptions about both dimensionless conductances g and g_S, i.e. the conductances of both the barrier and the external circuit can no longer be large. At the same time we focus our attention on almost transparent scatterers assuming $\mathcal{R} \ll 1$.

We again make use of Eq. (38) combining it with the effective action (22). In the path integral we perform a shift $\varphi^+ \to \varphi^+ + eVt$, where $V = V_x/(1+e^2N_{\rm ch}R_S/\pi)$ is the voltage drop at the barrier in the absence of interactions. This shift helps to eliminate the linear in V_x term from the action (11), but simultaneously introduces the voltage V in the last term of Eq. (23). The variational derivative of the action takes the form:

$$-e\frac{\delta S_{\rm sc}[\varphi^\pm]}{\delta\varphi^-(t)} = \frac{e\dot{\varphi}^+(t)}{\pi}(N_{\rm ch} - \mathcal{R}\cos\varphi^-(t))$$
$$-\frac{2ie\mathcal{R}}{\pi}\int dx\alpha(t-x)\sin\frac{\varphi^-(x)}{2}\cos\frac{\varphi^-(t)}{2}$$
$$\times\cos\left[eV(t-x)+\varphi^+(t)-\varphi^+(y)\right.$$
$$\left.+i\int dz\frac{T\sinh\pi T(t-y)\varphi^-(z)}{\sinh\pi T(t-z)\sinh\pi T(y-z)}\right]+\dots \quad (43)$$

Here ... stands for the terms which give no contribution to the current.

Evaluating the path integral (38) we can put $\mathcal{R}=0$ in the action $S[\varphi^\pm]$ in Eq. (38) and keep the terms $\propto \mathcal{R}$ only in $\delta S_{\rm sc}/\delta\varphi^-$. It is possible because the terms coming from first order in $\mathcal{R}$ correction to the action vanish. The integral over φ^+ gives $\delta-$function which fixes φ^-. Since the latter phase turns out to be small, $|\varphi^-| < \pi$, we are allowed to use the action in the form (21). The path integral becomes Gaussian and can be evaluated exactly. The result reads

$$I = \frac{e^2}{\pi}(N_{\rm ch}-\mathcal{R})V$$
$$+\frac{2e\mathcal{R}}{\pi}\int_0^\infty dt\,\alpha(t)\,e^{F(t)}\sin\left[\frac{K(t)}{2}\right]\sin[eVt]. \quad (44)$$

Here we have defined the functions

$$K(t) = \int\frac{d\omega}{2\pi}\frac{ie^2\,{\rm e}^{-i\omega t}}{(\omega+i0)\left(-i\omega C+\frac{e^2N_{\rm ch}}{\pi}+\frac{1}{Z_S(\omega)}\right)}, \quad (45)$$

$$F(t) = \int\frac{d\omega}{2\pi}\frac{e^2\omega\coth\frac{\omega}{2T}}{-i\omega C+\frac{e^2N_{\rm ch}}{\pi}+\frac{1}{Z_S(\omega)}}\frac{1-\cos\omega t}{\omega^2}. \quad (46)$$

Let us point out that the form of Eq. (44) resembles to a certain extent that of the result for the $I-V$ curve derived perturbatively in $T_n \ll 1$ for externally shunted tunnel barriers within the so-called $P(E)$-theory[15]. Here, in contrast, the interaction term in Eq. (44) was derived perturbatively in $\mathcal{R} = \sum_n(1-T_n)$. Another important feature of our result is the presence of the term $e^{+F(t)}$ under the time integral in Eq. (44). This exponent becomes large in the long time limit and should be contrasted with the decaying exponent $e^{-F(t)}$ in the corresponding expression[15] derived in the limit $T_n \ll 1$.

In the limit $V \to \infty$ the $I-V$ dependence (44) tends to the following simple form

$$I = \frac{e^2}{\pi}(N_{\rm ch}-\mathcal{R})V - \frac{e^2}{\pi}\mathcal{R}\frac{e}{2C}, \quad (47)$$

i.e. the $I-V$ curve has the offset. This result formally holds for any $Z_S(\omega)$ and at any temperature. In practice, the offset might be difficult to observe at high conductances and temperature.

Below we will consider an important limit of purely Ohmic external environment $Z_S(\omega) = R_S$. If both temperature and voltage are sufficiently small, $T, eV \ll (R+R_S)/RR_SC$, the integral in Eq. (44) can be evaluated analytically. We obtain

$$I = \frac{e^2 N_{\rm ch}}{\pi} V - \left(\frac{{\rm e}^{\gamma_0}(R+R_S)}{2\pi T R R_S C}\right)^{\frac{2}{g+g_S}} \times \frac{4\pi e T \mathcal{R}}{\Gamma\left(2-\frac{2}{g+g_S}\right)\left|\Gamma\left(\frac{1}{g+g_S}+i\frac{eV}{2\pi T}\right)\right|^2} \times \frac{\sinh\frac{eV}{2T}}{\cosh\frac{eV}{T} - \cos\frac{2\pi}{g+g_S}}, \quad (48)$$

where $\Gamma(x)$ is the gamma-function and $\gamma_0 \simeq 0.577$ is the Euler constant. At low temperatures, $T \ll eV$, Eq. (48) yields the differential conductance

$$\frac{dI}{dV} = \frac{e^2 N_{\rm ch}}{\pi} - \frac{e^2 \mathcal{R}}{\pi\Gamma\left(1-\frac{2}{g+g_s}\right)}\left(\frac{{\rm e}^{\gamma_0}(R+R_S)}{e|V|RR_SC}\right)^{\frac{2}{g+g_s}}, \quad (49)$$

while in the zero bias limit $eV \ll T$ one recovers the linear conductance

$$G = \frac{e^2 N_{\rm ch}}{\pi} - \frac{e^2\mathcal{R}}{2\sqrt{\pi}}\frac{\Gamma\left(1-\frac{1}{g+g_s}\right)}{\Gamma\left(\frac{3}{2}-\frac{1}{g+g_s}\right)}\left(\frac{{\rm e}^{\gamma_0}(R+R_S)}{\pi T R R_S C}\right)^{\frac{2}{g+g_s}}. \quad (50)$$

We note that Eqs. (44-50), being perturbative in $\mathcal{R}$, become inapplicable at voltages/temperatures below the energy scale E^*, which can be estimated as

$$E^* = \frac{R+R_S}{RR_SC}\left(\frac{\mathcal{R}}{N_{\rm ch}}\right)^{\frac{g+g_s}{2}}. \quad (51)$$

At the same time the above results are non-perturbative in both $1/g$ and $1/g_S$ and, hence, provide complementary information to the results obtained from the RG analysis discussed in the previous section.

VII. CURRENT NOISE

We now turn to the current noise and again consider the system with almost perfectly transmitting channels assuming $\mathcal{R} \ll 1$. The current noise spectrum is defined in a standard manner:

$$\mathcal{S}_I(\omega) = \int dt\, \mathcal{S}(t)\, {\rm e}^{i\omega t}, \quad (52)$$

where

$$\mathcal{S}(t) = \langle \hat{I}(t)\hat{I}(0) + \hat{I}(0)\hat{I}(t)\rangle - 2\langle I(0)\rangle^2. \quad (53)$$

In the absence of interactions and in the limit $\mathcal{R} \ll 1$ the low frequency noise spectrum scales as[10] $\mathcal{S}_I(0) \propto \mathcal{R}$. Below we will demonstrate that electron-electron interactions yield an additional contribution to $\mathcal{S}_I(0)$ which is also proportional to $\mathcal{R}$ but can be much larger than the non-interacting result. In other words, a dramatic *increase* of the current shot noise by electron-electron interactions is expected provided both conductances g and g_S are not very large.

The noise spectrum will be evaluated with the aid of the path integral

$$\mathcal{S}_I(\omega) = -2ie^2\int dt\, {\rm e}^{i\omega t}\frac{\int\mathcal{D}\varphi^\pm\, \frac{\delta^2 S_{\rm sc}[\varphi^\pm]}{\delta\varphi^-(t)\delta\varphi^-(0)}\, {\rm e}^{iS[\varphi^\pm]}}{\int\mathcal{D}\varphi^\pm\, {\rm e}^{iS[\varphi^\pm]}}. \quad (54)$$

This expression follows directly from Eqs. (5,39).

The variational derivative of the effective action $S_{\rm sc}$ can be evaluated in a straightforward manner. We obtain

$$-2ie^2\frac{\delta^2 S_{\rm sc}[\varphi^\pm]}{\delta\varphi^-(t)\delta\varphi^-(0)} = \frac{2ie^2\mathcal{R}}{\pi}\,\dot\varphi^+(t)\sin[\varphi^-(0)]\delta(t) + \frac{2e^2N_{\rm ch}}{\pi}\alpha(t) - \frac{2e^2\mathcal{R}}{\pi}\alpha(t)\big(\cos[\varphi^-(t)]+\cos[\varphi^-(0)]\big)$$
$$+\frac{2e^2\mathcal{R}}{\pi}\,\alpha(t)\cos\left[\varphi^+(t)-\varphi^+(0)-i\int dz\big(T\coth[\pi T(t-z)]+T\coth[\pi Tz]\big)\varphi^-(z)\right]\cos\frac{\varphi^-(t)}{2}\cos\frac{\varphi^-(0)}{2}$$
$$+\frac{4ie^2\mathcal{R}}{\pi}\int dx\,\alpha(t-x)\sin\left[\varphi^+(t)-\varphi^+(x)-i\int dz\big(T\coth[\pi T(t-z)]+T\coth[\pi T(z-x)]\big)\varphi^-(z)\right]$$
$$\times\big(T\coth[\pi Tt]-T\coth[\pi Tx]\big)\cos\frac{\varphi^-(t)}{2}\sin\frac{\varphi^-(x)}{2}$$
$$+\frac{4ie^2\mathcal{R}}{\pi}\int dx\,\alpha(-x)\sin\left[\varphi^+(x)-\varphi^+(0)-i\int dz\big(T\coth[\pi T(x-z)]+T\coth[\pi Tz]\big)\varphi^-(z)\right]$$
$$\times\big(T\coth[\pi T(x-t)]-T\coth[\pi Tt]\big)\cos\frac{\varphi^-(0)}{2}\sin\frac{\varphi^-(x)}{2}+\ldots. \quad (55)$$

Here we again omit terms which contribution to the path integral vanishes.

Since we are going to evaluate linear in $\mathcal{R}$ contributions to the noise spectrum, it suffices to set $\mathcal{R} = 0$ in the expression for S in the exponent of Eq. (54). As in the previous section integrating first over φ^+ one can verify that only the values $|\varphi^-| < \pi$ give a non-vanishing contribution. Under this approximation the path integral becomes Gaussian and we find

$$\mathcal{S}_I(t) = \frac{e^4\mathcal{R}}{\pi C}\delta(t) + \frac{2e^2(N_{\rm ch} - 2\mathcal{R})}{\pi}\alpha(t) + \frac{2e^2\mathcal{R}}{\pi}\alpha(t)\,{\rm e}^{F(t)}\cos\left[\frac{K(t)}{2}\right]\cos[eVt] + \frac{4e^2\mathcal{R}}{\pi}\int dx\,\alpha(x)\,{\rm e}^{F(x)}\sin\left[\frac{K(x)}{2}\right]\cos[eVx] \times \left(T\coth[\pi T(x-t)] + T\coth[\pi T(x+t)]\right). \quad (56)$$

Let us define the function

$$\Phi(t) = F(t) + \frac{i}{2}K(|t|){\rm sign}t = e^2\int\frac{d\omega}{2\pi}\frac{(1-\cos\omega t)\coth\frac{\omega}{2T} + i\sin\omega t}{\omega\left(-i\omega C + \frac{e^2N_{\rm ch}}{\pi} + \frac{1}{Z_S(\omega)}\right)}, \quad (57)$$

and express the noise power spectrum as follows:

$$\mathcal{S}_I(\omega) = \frac{e^4\mathcal{R}}{\pi C} + \frac{2e^2}{\pi}(N_{\rm ch} - 2\mathcal{R})\omega\coth\frac{\omega}{2T} + \frac{2e^2\mathcal{R}}{\pi}\int dx\,\alpha(x)\,{\rm e}^{\Phi(x)}\cos[eVx]\cos\omega x - \frac{4ie^2\mathcal{R}\coth\frac{\omega}{2T}}{\pi}\int dx\,\alpha(x)\,{\rm e}^{\Phi(x)}\cos[eVx]\sin\omega x. \quad (58)$$

Making use of the property $\Phi(t - i/T) = \Phi(-t)$ one can prove the following identity:

$$\int dx\,\alpha(x)\,{\rm e}^{\Phi(x)}\cos\omega x = -\frac{e^2}{2C} + \omega\coth\frac{\omega}{2T} + i\coth\frac{\omega}{2T}\int dx\,\alpha(x)\,{\rm e}^{\Phi(x)}\sin\omega x. \quad (59)$$

Then, combining Eqs. (44), (58) and (59) we obtain

$$\mathcal{S}_I(\omega) = \frac{2e^2(N_{\rm ch} - 2\mathcal{R})}{\pi}\omega\coth\frac{\omega}{2T} + \frac{e^2\mathcal{R}}{\pi}\sum_\pm(\omega \pm eV)\coth\frac{\omega \pm eV}{2T} + e\sum_\pm\left[2\,\delta I\left(\frac{\omega}{e} \pm V\right)\coth\frac{\omega}{2T} - \delta I\left(\frac{\omega}{e} \pm V\right)\coth\frac{\omega \pm eV}{2T}\right], \quad (60)$$

where $\delta I(V) = I(V) - \frac{e^2(N_{\rm ch} - \mathcal{R})}{\pi}V$ is the interaction correction to the current, $I(V)$ is defined in Eq. (44). The frequency dependence of the noise spectrum is illustrated in Fig. 3. One can see that the noise is enhanced due to interaction at low frequencies $|\omega| \lesssim eV$, and reduced at $|\omega| \gtrsim eV$.

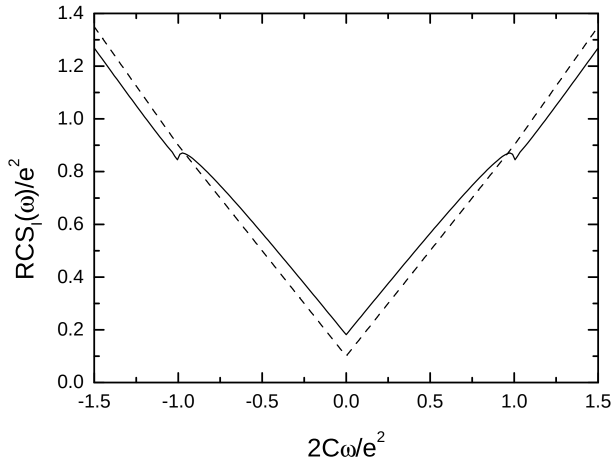

FIG. 3: Frequency dependence of the noise power spectrum (solid line) at $T = 0$ and in the presence of interactions. For comparison the noise spectrum in the absence of interactions is shown by the dashed line. Both curves are plotted for $N_{\rm ch} = 1$, $\mathcal{R} = 0.1$, $g_S = 2$ and $V = e/2C$.

It is easy to see that Eq. (60) satisfies the fluctuation-dissipation theorem (FDT). Indeed, applying an ac bias, $V(t) = V_0\cos\omega t$, and repeating the analysis of the previous section one can verify that the real part of the zero-bias conductance is related to the current in the following way:

$$\operatorname{Re} G(\omega) = \frac{e}{\omega}I\left(\frac{\omega}{e}\right). \quad (61)$$

Making use of this identity and setting $V = 0$ in Eq. (60), one finds

$$\mathcal{S}_I(\omega, V = 0) = 2\left[\operatorname{Re} G(\omega)\right]\omega\coth\frac{\omega}{2T} \quad (62)$$

in agreement with FDT.

In the non-interacting limit Eq. (60) reduces to the Khlus formula[10] expanded in $\mathcal{R}$ to the first order. We also note that the result for the noise spectrum for highly transmitting barriers (60) is to some extent similar to that for the case of tunnel junctions. The latter can be expressed in terms of the $I - V$ dependence in the following way:

$$S_I^{\rm tun}(\omega) = e\sum_\pm I\left(\frac{\omega}{e} \pm eV\right)\coth\frac{\omega \pm eV}{2T}. \quad (63)$$

This is a general result which holds in the non-interacting limit[24] as well as in the presence of an arbitrary external impedance[25]. We observe that in both Eqs. (60) and (63) the effect of electron-electron interactions is fully described by the interaction term contained in the $I - V$ curve.

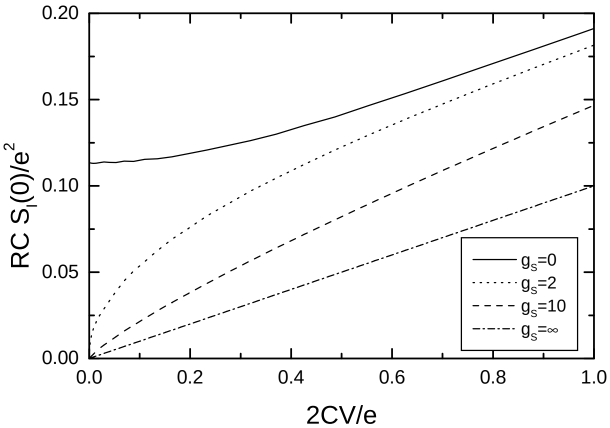

FIG. 4: The zero frequency shot noise power spectrum $\mathcal{S}_I(0)$ as a function of voltage V at $T=0$ for $N_{\rm ch}=1$, $R=\pi/e^2$ and $\mathcal{R}=0.1$. The conductance g_S effectively controls the interaction strength, i.e. at small g_S the interaction is strong, while it tends to zero in the limit $g_S\to\infty$. The interaction-induced excess noise is clearly observed even at rather large values of g_S.

Consider now the low frequency limit $\omega\to 0$. From the above results we obtain

$$\mathcal{S}_I(0) = \left(\frac{e^2}{\pi}(N_{\rm ch}-2\mathcal{R}) + 2\frac{d}{dV}\delta I(V)\right)4T + 2e\left(\frac{e^2\mathcal{R}}{\pi}V - \delta I(V)\right)\coth\frac{eV}{2T}. \tag{64}$$

In the shot noise regime $T\ll eV$ this formula reduces to

$$\mathcal{S}_I(0) = 2e\left(\frac{e^2\mathcal{R}}{\pi}V - \delta I(V)\right). \tag{65}$$

Since the interaction correction is negative, $\delta I(V)<0$, we arrive at the conclusion (cf. Ref. 2) that for highly transmitting barriers the shot noise is enhanced by electron-electron interactions. At high voltages the noise spectrum takes the form

$$\mathcal{S}_I(0) = \frac{2e^3\mathcal{R}}{\pi}\left(V + \frac{e}{2C}\right), \tag{66}$$

i.e. in this limit interactions induce voltage-independent excess noise which magnitude scales linearly with the Coulomb gap $e/2C$.

In the limit $eVRR_SC/(R+R_S)\ll 1$ from Eqs. (65) and (48) one obtains the result

$$\mathcal{S}_I(0) = \frac{2e^2\mathcal{R}(R+R_S)e^{\frac{2\gamma_0}{g+g_s}}}{\pi\Gamma\left(2-\frac{2}{g+g_s}\right)RR_SC}\left(\frac{e|V|RR_SC}{R+R_S}\right)^{1-\frac{2}{g+g_s}} \tag{67}$$

This expression is valid provided $e|V|>E^*$, where E^* is defined in Eq. (51). In the limit $g+g_S\to\infty$ Eq. (67) reduces to the non-interacting result. For $g+g_S\gg 1$ the interaction effects remain weak and Eq. (67) agrees with the perturbative results[2]. In the limit $g+g_S\gg 1$ the same expressions can also be reproduced with the aid of the RG analysis (41,42) combined with the formula $S_I = \frac{2e^2}{\pi}eV\sum_n\tilde T_n(1-\tilde T_n)$. For smaller conductance values the excess noise becomes larger and for $g+g_S$ of order one the shot noise is strongly enhanced by electron-electron interactions. The effect of interactions on the shot noise spectrum $\mathcal{S}_I(0)$ of highly transmitting coherent scatterers is clearly observed in Fig. 4.

VIII. QUANTUM DOTS

So far we have considered a single scatterer embedded in the electromagnetic environment with the impedance $Z_S(\omega)$. Another important physical situation is that of so-called quantum dots. A quantum dot can be modeled by a system of two scatterers connected in series via a sufficiently small island. In this section we will demonstrate that some of the results derived above for a single scatterer can be directly generalized to the case of quantum dots.

Let us denote the number of channels in the left and right barriers N_l and N_r. As before, we will also assume that all these channels are almost open and $\mathcal{R}_l,\mathcal{R}_r\ll 1$. Throughout our analysis we will stick to a simplified description which amounts to treating the effect of the left barrier on the right one (and vice versa) as that of an Ohmic resistor. This approximation is justified if the electron distribution function inside the dot remains in equilibrium, i.e. equal to the Fermi function with the temperature of the leads. This is the case either in the linear in voltage regime or, else, provided inelastic relaxation of electrons inside the dot is sufficiently strong. The latter condition may apply for large quantum dots and/or at high enough temperatures. In addition, below we will ignore the gate modulation of the current flowing across the quantum dot.

Let us denote the voltage drop across the left/right barrier as $V_{l,r}$. Obviously, $V_l+V_r=V$, where V is the total bias voltage. From the current conservation conditions at both barriers one finds

$$I = \frac{e^2(N_l-\mathcal{R}_l)}{\pi}V_l + \delta I_l(V_l),$$
$$I = \frac{e^2(N_r-\mathcal{R}_r)}{\pi}V_r + \delta I_r(V_r). \tag{68}$$

Here δI_l is the interaction correction to the current in the left barrier given by the last term in Eq. (44) and Eqs. (45,46) with the following replacements: $N_{\rm ch}\to N_l$, $1/Z_S(\omega)\to(\frac{e^2}{\pi})N_r$, $C\to C_\Sigma$, where C_Σ is the total capacitance of the quantum dot. The interaction correction δI_r is defined analogously.

Resolving Eqs. (68) and keeping the first order in $\mathcal{R}_{l,r}$, one finds

$$I = \frac{e^2}{\pi}\frac{N_lN_r}{N}V - \frac{e^2}{\pi}\frac{N_r^2\mathcal{R}_l + N_l^2\mathcal{R}_r}{N^2}V$$

$$+\frac{N_r}{N}\delta I_l\left(\frac{N_r}{N}V\right)+\frac{N_l}{N}\delta I_r\left(\frac{N_l}{N}V\right), \quad (69)$$

where $N = N_l + N_r$. Assuming now $T \ll eV$ and $eVR_lR_rC_\Sigma/(R_l+R_r) \ll 1$ we obtain

$$I = \frac{e^2}{\pi}\frac{N_lN_r}{N}V - \frac{2e^{\gamma_0/N}eE_C(N_r^{2-1/N}\mathcal{R}_l + N_l^{2-1/N}\mathcal{R}_r)}{\pi^2N^{2-2/N}\Gamma\left(2-\frac{1}{N}\right)}\left(\frac{\pi eV}{2E_C}\right)^{1-1/N} \quad (70)$$

and

$$\frac{dI}{dV} = \frac{e^2}{\pi}\frac{N_lN_r}{N} - \frac{e^2}{\pi}\frac{N_r^{2-1/N}\mathcal{R}_l + N_l^{2-1/N}\mathcal{R}_r}{N^{2-2/N}\Gamma\left(2-\frac{1}{N}\right)}\left(\frac{2\,e^{\gamma_0}E_C}{\pi eV}\right)^{1/N}, \quad (71)$$

where $E_C = e^2/2C_\Sigma$ is the charging energy. In the opposite limit $eV \ll T \ll (R_l+R_r)/R_lR_rC_\Sigma$ we arrive at the expression for the linear conductance

$$G = \frac{e^2}{\pi}\frac{N_lN_r}{N} - \frac{e^2}{2\sqrt{\pi}}\frac{\Gamma\left(1-\frac{1}{2N}\right)(N_r^2\mathcal{R}_l + N_l^2\mathcal{R}_r)}{\Gamma\left(\frac{3}{2}-\frac{1}{2N}\right)N^{2-1/N}}\left(\frac{2\,e^{\gamma_0}E_C}{\pi^2T}\right)^{1/N} \quad (72)$$

In the limit $N_l = N_r = 1$ this result reduces to that derived in Ref. 17 by means of the bosonization technique. Eq. (72) is also consistent with the corresponding result of Ref. 18 in the case of arbitrary number of channels $N_{l,r}$.

Under the assumption of strong inelastic relaxation inside the dot current fluctuations across two barriers can be considered uncorrelated and one can also evaluate the noise spectrum of the system. Combining Eqs. $\delta I = \delta V_l/R_l + \xi_l$, $\delta I = \delta V_r/R_r + \xi_r$ and $\delta V_l + \delta V_r = 0$ we arrive at the relation between the current fluctuations in the system δI and fluctuations across each of the barriers $\xi_{l,r}$:

$$\delta I = \frac{N_r\xi_l + N_l\xi_r}{N_l+N_r}. \quad (73)$$

Accordingly, the noise power spectrum $\mathcal{S}_I \propto \langle\delta I^2\rangle$ takes the form

$$\mathcal{S}_I = \frac{N_r^2}{N^2}\mathcal{S}_l + \frac{N_l^2}{N^2}\mathcal{S}_r, \quad (74)$$

where the noise spectra $\mathcal{S}_{l,r}$ are defined by Eq. (67) with the corresponding parameters. Here we consider only the shot noise regime $T \ll eV$ and assume $eV \ll NE_C$, in which case one finds

$$\mathcal{S}_I = \frac{4\,e^{\gamma_0/N}(N_r^2\mathcal{R}_l + N_l^2\mathcal{R}_r)}{\pi^2N^{2-1/N}\Gamma\left(2-\frac{1}{N}\right)}e^2E_C\left(\frac{\pi eV}{2E_C}\right)^{1-1/N}. \quad (75)$$

As before, we observe strong enhancement of the shot noise by electron-electron interactions provided the number of channels in the dot N is not large.

Further work is needed in order to relax the assumption about strong inelastic relaxation inside the dot. In the metallic limit $g \gg 1$ the corresponding analysis has been developed in Ref. 9.

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APPENDIX A: TRANSFORMATION OF THE ACTION

Let us demonstrate the equivalence of the actions (3) and (7). First we evaluate the Green function $\check{G}_V$ which satisfies the equation

$$\check{G}_V^{-1}(t_1,\boldsymbol{r}_1)\check{G}_V(t_1,t_2;\boldsymbol{r}_1,\boldsymbol{r}_2) = \delta(t_1-t_2)\delta(\boldsymbol{r}_1-\boldsymbol{r}_2), \quad (A1)$$

where $\check{G}_V^{-1}(t,\boldsymbol{r})$ is defined in Eq. (4). The general solution of Eq. (A1) can be written in the form

$$\check{G}_V = \begin{pmatrix} -i\theta(t_1-t_2)\mathcal{U}_1(t_1,t_2) & 0 \\ 0 & i\theta(t_2-t_1)\mathcal{U}_2(t_1,t_2)\end{pmatrix} + i\begin{pmatrix}\mathcal{U}_1(t_1,t) & 0 \\ 0 & \mathcal{U}_2(t_1,t)\end{pmatrix}\begin{pmatrix}\hat\rho_V(t) & -\hat\rho_V(t) \\ \hat\rho_V(t)-1 & -\hat\rho_V(t)\end{pmatrix} \times \begin{pmatrix}\mathcal{U}_1(t,t_2) & 0 \\ 0 & \mathcal{U}_2(t,t_2)\end{pmatrix}. \quad (A2)$$

Here $\hat\rho_V(t)$ is an arbitrary operator. Below we will use the convention according to which the product of operators involves only the coordinate integration, i.e.: $\mathcal{U}^2(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r_2}) = \int d^3\boldsymbol{r}'\mathcal{U}(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}')\mathcal{U}(t_1,t_2,\boldsymbol{r}',\boldsymbol{r}_2)$. It is also important to keep in mind that $0 < t_1, t_2 < t$, and the Green function $\check{G}_V$ implicitly depends on the final time t.

The task at hand is to evaluate the operator $\hat\rho_V(t)$. For this purpose let us make use of the Dyson equation

$$\check{G}_V(t_1,t_2) = \check{G}_0(t_1,t_2) - \int_0^t dt'\check{G}_0(t_1,t')\,e\check{V}(t')\,\check{G}_V(t',t_2), \quad (A3)$$

where

$$\check{V}(t) = \begin{pmatrix} V_1(t) & 0 \\ 0 & V_2(t)\end{pmatrix}, \quad (A4)$$

and $\check{G}_0$ is the Keldysh-Green function of noninteracting electrons:

$$\check{G}_0 = \begin{pmatrix} -i\theta(t_1-t_2)\mathcal{U}_0(t_1,t_2) & 0 \\ 0 & i\theta(t_2-t_1)\mathcal{U}_0(t_1,t_2)\end{pmatrix}$$

$$+ i \begin{pmatrix} \mathcal{U}_0(t_1,0) & 0 \\ 0 & \mathcal{U}_0(t_1,0) \end{pmatrix} \begin{pmatrix} \hat\rho_0 & -\hat\rho_0 \\ \hat\rho_0 - 1 & -\hat\rho_0 \end{pmatrix}$$
$$\times \begin{pmatrix} \mathcal{U}_0(0,t_2) & 0 \\ 0 & \mathcal{U}_0(0,t_2) \end{pmatrix}. \tag{A5}$$

Here $\mathcal{U}_0$ is the evolution operator for non-interacting electrons defined by eq. (8) with $V_{1,2} = 0$ and $\hat\rho_0$ is the initial reduced single-electron density matrix operator defined as $\langle \boldsymbol{r}|\hat\rho_0|\boldsymbol{r}'\rangle = \mathrm{tr}\left(\hat\Psi^\dagger_\uparrow(\boldsymbol{r}')\hat\Psi_\uparrow(\boldsymbol{r})\hat{\boldsymbol{\rho}}_0\right)$, where $\hat{\boldsymbol{\rho}}_0$ is the initial many-particle density matrix. Substituting the general solution (A2) into Eq. (A3) and making use of the Dyson equation for the evolution operators

$$\mathcal{U}_{1,2}(t,t') = \mathcal{U}_0(t,t') + i\int_{t'}^t ds\, \mathcal{U}_0(t,s) e V_{1,2}(s)\mathcal{U}_{1,2}(s,t'),$$

we find[26]:

$$\hat\rho_V(t) = \mathcal{U}_1(t,0)\hat\rho_0$$
$$\times\left[1 + (\mathcal{U}_2(0,t)\mathcal{U}_1(t,0) - 1)\hat\rho_0\right]^{-1}\mathcal{U}_2(0,t). \tag{A6}$$

As a next step, let us fulfill the following replacements $eV_1 \to eV^+ + \lambda V^-/2$, $eV_2 \to eV^+ - \lambda V^-/2$ and evaluate the derivative $i\partial S_{el}/\partial\lambda$, where $iS_{el} = 2\mathrm{Tr}\ln \check G_V^{-1}$. With the aid of Eq. (4) we obtain

$$i\frac{\partial S_{el}}{\partial \lambda} = \int_0^t ds \int d^3\boldsymbol{r}\, \big(G^{11}_{V,\lambda}(s,s,\boldsymbol{r},\boldsymbol{r})$$
$$- G^{22}_{V,\lambda}(s,s,\boldsymbol{r},\boldsymbol{r})\big)\, eV^-(s,\boldsymbol{r}). \tag{A7}$$

Employing Eqs. (A2), (A6) and using the properties of the trace of the product of operators, we find

$$i\frac{\partial S_{el}}{\partial\lambda} = i\int_0^t ds\, \mathrm{tr}\Big[\big(\mathcal{U}_{1,\lambda}(s,t)\hat\rho_{V,\lambda}(t)\mathcal{U}_{1,\lambda}(t,s)$$
$$+\mathcal{U}_{2,\lambda}(s,t)\hat\rho_{V,\lambda}(t)\mathcal{U}_{2,\lambda}(t,s)\big)\, e\hat V^-(s)\Big]$$
$$= i\int_0^t ds$$
$$\times\, \mathrm{tr}\Big[\big(\mathcal{U}_{2,\lambda}(0,t)\mathcal{U}_{1,\lambda}(t,s)e\hat V^-(s)\mathcal{U}_{1,\lambda}(s,0)$$
$$+\mathcal{U}_{2,\lambda}(0,s)e\hat V^-(s)\mathcal{U}_{2,\lambda}(s,t)\mathcal{U}_{1,\lambda}(t,0)\big)$$
$$\times\ \hat\rho_0[1 + (\hat{\mathcal{U}}_{2,\lambda}(0,t)\hat{\mathcal{U}}_{1,\lambda}(t,0) - 1)\hat\rho_0]^{-1}\Big]. \tag{A8}$$

What remains is to integrate Eq. (A8) over λ from 0 to 1. This task is accomplished with the aid of the identity

$$\frac{\partial}{\partial\lambda}\left(\mathcal{U}_{2,\lambda}(0,t)\mathcal{U}_{1,\lambda}(t,0)\right) =$$
$$\frac{i}{2}\int_0^t ds\big(\mathcal{U}_{2,\lambda}(0,t)\mathcal{U}_{1,\lambda}(t,s)e\hat V^-(s)\mathcal{U}_{1,\lambda}(s,0)$$
$$+\mathcal{U}_{2,\lambda}(0,s)e\hat V^-(s)\mathcal{U}_{2,\lambda}(s,t)\mathcal{U}_{1,\lambda}(t,0)\big). \tag{A9}$$

Since the action S equals to zero at $\lambda = 0$, we arrive at the final result

$$2\mathrm{Tr}\ln\check G_V^{-1} = 2\mathrm{tr}\ln\left[1 + (\mathcal{U}_2(0,t)\mathcal{U}_1(t,0) - 1)\hat\rho_0\right], \tag{A10}$$

which proves the equivalence of Eqs. (3) and (7).

APPENDIX B: FULLY TRANSMITTING BARRIERS

Let us demonstrate that the action S_0 (18) can be exactly transformed to the form (19). For this purpose we first expand the action (18) in the following series

$$iS_0[a,\varphi^+] = 2\mathrm{Tr}\left\{a(x)\rho_0(y-x)\mathrm{e}^{i\frac{\varphi^+(y)-\varphi^+(x)}{2}}\right\}$$
$$-2\sum_{N=2}^\infty \frac{(-1)^N J_N}{N}, \tag{B1}$$

where

$$J_N = \int dx_1\ldots dx_N\, \rho_0(x_1-x_2)a(x_2)\rho_0(x_2-x_3)\ldots$$
$$\times\,\rho_0(x_{N-1}-x_N)a(x_N)\rho_0(x_N-x_1)a(x_N), \tag{B2}$$

and rewrite J_N in the form

$$J_N = \int dx \int_{-E_{\min}}^\infty \frac{dE}{2\pi}\int\frac{d\omega_1\ldots d\omega_N}{(2\pi)^N}\, n(E)$$
$$\times\, n(E+\omega_1)n(E+\omega_1+\omega_2)\ldots n(E+\omega_1+\ldots+\omega_{N-1})$$
$$\times\,\tilde a(\omega_1)\tilde a(\omega_2)\ldots\tilde a(\omega_N)\,\mathrm{e}^{i(\omega_1+\ldots+\omega_N)x}, \tag{B3}$$

where $\tilde a(\omega) = \int dx\, \mathrm{e}^{i\omega x}a(x)$ is the Fourier transform of $a(x)$, and we have also introduced the cutoff at a large negative energy $-E_{\min}$.

We will use the following property of the Fermi function

$$n(E)n(E+\omega) = \frac{1}{2}\left(1 - \coth\frac{\omega}{2T}\right)n(E)$$
$$+\frac{1}{2}\left(1+\coth\frac{\omega}{2T}\right)n(E+\omega). \tag{B4}$$

This equation is analogous to the Ward identity for the electron Matsubara Green function: $\mathcal{G}(\omega+\Omega,p+q)\mathcal{G}(\omega,p) = \mathcal{G}(\Omega,q)[\mathcal{G}(\omega,p) - \mathcal{G}(\omega+\Omega,p+q)]$. The latter identity ensures that all higher order symmetrized polarization bubbles for the one-dimensional electron gas vanish thus turning the RPA approximation into an exact procedure[20]. Analogously, the identity (B4) helps to simplify the integrals (B3).

In what follows we will employ the procedure similar to that applied within the imaginary time formalism[27]. Let us interchange $\{\omega_1,\ldots,\omega_N\}$ $N-1$ times in the following way. We first put ω_N in front of ω_1, then we place it in-between ω_1 and ω_2, and so on. This set of changes is summarized in the table:

$$\begin{array}{ll} & \{\omega_1\ \ \omega_2\ \ \omega_3\ \ \omega_4\ \ \ldots\ \ \omega_N\} \\ & \qquad\qquad\quad\ \downarrow \\ (1) & \{\omega_N\ \ \omega_1\ \ \omega_2\ \ \omega_3\ \ \ldots\ \ \omega_{N-1}\} \\ (2) & \{\omega_1\ \ \omega_N\ \ \omega_2\ \ \omega_3\ \ \ldots\ \ \omega_{N-1}\} \\ (3) & \{\omega_1\ \ \omega_2\ \ \omega_N\ \ \omega_3\ \ \ldots\ \ \omega_{N-1}\} \\ \ldots & \\ (N-1) & \{\omega_1\ \ \omega_2\ \ \ldots\ \ \omega_{N-2}\ \ \omega_N\ \ \omega_{N-1}\}. \end{array} \tag{B5}$$

Having made these changes in the integral (B3) we express J_N as a sum of $N-1$ corresponding integrals divided by $N-1$. Afterwards we apply the identity (B4) in each of these integrals excluding ω_N from the arguments of the Fermi functions. Specifically, in the term corresponding to the sequence (1) in the table (B5) we split the product of the first two Fermi functions $n(E)n(E+\omega_N)$, in the term (2) we split the product of the second and the third Fermi functions, and so on. Then we get

$$\begin{aligned}
&J_N = \frac{1}{N-1}\int dx \int_{-E_{\min}}^{\infty}\frac{dE}{2\pi}\int\frac{d\omega_1\ldots d\omega_N}{(2\pi)^N}\\
&\times[\tilde f(-\omega_N)n(E)+\tilde f(\omega_N)n(E+\omega_N)]\\
&\times n(E+\omega_N+\omega_1)\ldots n(E+\omega_N+\omega_1+\ldots+\omega_{N-2})\\
&\times \tilde a(\omega_1)\tilde a(\omega_2)\ldots\tilde a(\omega_N)\,\mathrm{e}^{i(\omega_1+\ldots+\omega_N)x}\\
&+\frac{1}{N-1}\sum_{k=2}^{N-1}\int dx\int_{-E_{\min}}^{\infty}\frac{dE}{2\pi}\int\frac{d\omega_1\ldots d\omega_N}{(2\pi)^N}\\
&n(E)n(E+\omega_1)\ldots\times[\tilde f(-\omega_N)n(E+\omega_1+\ldots+\omega_{k-1})\\
&\tilde f(\omega_N)n(E+\omega_1+\ldots+\omega_{k-1}+\omega_N)]\\
&\times n(E+\omega_1+\ldots+\omega_{k-1}+\omega_N+\omega_k)\times\ldots\\
&\times n(E+\omega_1+\ldots+\omega_{k-1}+\omega_N+\omega_k+\ldots+\omega_{N-2})\\
&\times \tilde a(\omega_1)\tilde a(\omega_2)\ldots\tilde a(\omega_N)\,\mathrm{e}^{i(\omega_1+\ldots+\omega_N)x}, \qquad (B6)
\end{aligned}$$

where we have defined $\tilde f(\omega)=\frac{1}{2}\left(1+\coth\frac{\omega}{2T}\right)$.

Now let us perform the frequency shifts $\omega_j\to\omega_j+\omega_N$ in all the terms as well as the energy shift $E\to E+\omega_N$ in the second term of the first integral in order to eliminate ω_N from the arguments of the Fermi functions. As a result we arrive at the following expression

$$\begin{aligned}
&J_N = \frac{1}{N-1}\int dx\int_{-E_{\min}}^{\infty}\frac{dE}{2\pi}\int\frac{d\omega_1\ldots d\omega_N}{(2\pi)^N}\\
&\times[\tilde f(-\omega_N)\tilde a(\omega_1-\omega_N)+\tilde f(\omega_N)\tilde a(\omega_1)\mathrm{e}^{i\omega_N x}]\\
&\times n(E)n(E+\omega_1)\ldots n(E+\omega_1+\ldots+\omega_{N-2})\\
&\times\tilde a(\omega_2)\ldots\tilde a(\omega_N)\,\mathrm{e}^{i(\omega_1+\ldots+\omega_{N-1})x}\\
&+\frac{1}{N-1}\sum_{k=2}^{N-1}\int dx\int_{-E_{\min}}^{\infty}\frac{dE}{2\pi}\int\frac{d\omega_1\ldots d\omega_N}{(2\pi)^N}\\
&\times[\tilde f(-\omega_N)\tilde a(\omega_{k-1})\tilde a(\omega_k-\omega_N)\\
&+\tilde f(\omega_N)\tilde a(\omega_{k-1}-\omega_N)\tilde a(\omega_k)]\\
&n(E)n(E+\omega_1)\ldots n(E+\omega_1+\ldots+\omega_{N-2})\\
&\times\tilde a(\omega_1)\tilde a(\omega_2)\ldots\tilde a(\omega_{k-2})\tilde a(\omega_{k+1})\ldots\tilde a(\omega_N)\\
&\times\,\mathrm{e}^{i(\omega_1+\ldots+\omega_{N-1})x}\\
&+\frac{1}{N-1}\int dx\int_{-E_{\min}+\omega_N}^{-E_{\min}}\frac{dE}{2\pi}\int\frac{d\omega_1\ldots d\omega_N}{(2\pi)^N}\\
&\times n(E)n(E+\omega_1)\ldots n(E+\omega_1+\ldots+\omega_{N-2})\\
&\times\tilde f(\omega_N)\tilde a(\omega_1)\tilde a(\omega_2)\ldots\tilde a(\omega_N)\,\mathrm{e}^{i(\omega_1+\ldots+\omega_N)x}. \quad (B7)
\end{aligned}$$

We will assume that $|\omega_1|,\ldots,|\omega_N|\ll E_{\min}$ which implies that $n(E)n(E+\omega_1)\ldots n(E+\omega_1+\ldots+\omega_{N-2})=1$ in the interval $-E_{\min}<E<-E_{\min}+\omega_N$. This fact allows us to evaluate the integral over E in the last term. In addition, one observes that $\tilde f(\omega_N)+\tilde f(-\omega_N)=1$ and, hence, the function $\tilde f_n$ should drop out from all the other terms. We obtain

$$\begin{aligned}
&J_N=\frac{1}{N-1}\int dx\int_{-E_{\min}}^{\infty}\frac{dE}{2\pi}\int\frac{d\omega_1\ldots d\omega_N}{(2\pi)^N}\\
&\times\tilde a(\omega_1)\ldots\tilde a(\omega_N)\,\mathrm{e}^{i(\omega_1+\ldots+\omega_{N-1})x}\sum_{j=1}^{N-1}\frac{\tilde a(\omega_j-\omega_N)}{\tilde a(\omega_j)}\\
&\times n(E)n(E+\omega_1)\ldots n(E+\omega_1+\ldots+\omega_{N-2})\\
&-\frac{1}{N-1}\int dx\int\frac{d\omega_1\ldots d\omega_N}{(2\pi)^{N+1}}\,\omega_N\tilde f(\omega_N)\\
&\times\tilde a(\omega_1)\tilde a(\omega_2)\ldots\tilde a(\omega_N)\,\mathrm{e}^{i(\omega_1+\ldots+\omega_N)x}. \qquad (B8)
\end{aligned}$$

Returning to the coordinate representation and assuming $a(x)\to 0$ for $|x|\to\infty$, we find

$$\begin{aligned}
&J_N=\frac{1}{N-1}\int dx_1\ldots dx_{N-1}\,\rho_0(x_1-x_2)\\
&\times\rho_0(x_2-x_3)\ldots\rho_0(x_{N-2}-x_{N-1})\rho_0(x_{N-1}-x_1)\\
&\times\prod_{j=1}^{N-1}a(x_j)\left(\sum_{j=1}^{N-1}a(x_j)\right) \qquad (B9)\\
&-\int\frac{dxdy}{2\pi}\,\alpha(x-y)\frac{a^{N-1}(x)a(y)+a(x)a^{N-1}(y)}{4(N-1)}.
\end{aligned}$$

To summarize, making use of Eq. (B4) we have reduced the number of the functions ρ_0 under the integral by one. One can employ this procedure further and, again applying Eq. (B4), finally reduce the number of the functions ρ_0 down to two. As a result, J_N takes the form

$$\begin{aligned}
J_N&=\int dxdy\rho_0(x-y)\rho_0(y-x)\sum_{k=1}^{N-1}\gamma_{kN}a^k(x)a^{N-k}(y)\\
&+\int\frac{dxdy}{2\pi}\alpha(x-y)\sum_{k=1}^{N-1}\beta_{kN}a^k(x)a^{N-k}(y). \quad (B10)
\end{aligned}$$

Now let us apply the identity

$$\rho_0(x-y)\rho_0(y-x)=\rho_0(0)\delta(x-y)-\frac{\alpha(x-y)}{4\pi}, \quad (B11)$$

which follows directly from Eq. (B4), and regroup the terms in Eq. (B10). Then we obtain

$$\begin{aligned}
J_N&=\int dx\rho_0(0)u_Na^N(x)\\
&+\int\frac{dxdy}{2\pi}\,\alpha(x-y)\sum_{k=1}^{N-1}\alpha_{kN}a^k(x)a^{N-k}(y). \quad (B12)
\end{aligned}$$

In order to find the coefficients u_N and α_{kN} let us rewrite Eq. (B9) in the form

$$\begin{aligned}
&J_N[a(x)]=-i\frac{d}{d\lambda}\frac{J_{N-1}[a(x)\mathrm{e}^{i\lambda a(x)}]}{N-1}\bigg|_{\lambda=0}\\
&-\int\frac{dxdy}{2\pi}\,\alpha(x-y)\frac{a^{N-1}(x)a(y)+a(x)a^{N-1}(y)}{4(N-1)},
\end{aligned}$$

and derive the following equations:

$$\alpha_{1,N} = \frac{N-2}{N-1}\alpha_{1,N-1} - \frac{1}{4(N-1)},$$
$$\alpha_{N-1,N} = \frac{N-2}{N-1}\alpha_{N-2,N-1} - \frac{1}{4(N-1)},$$
$$\alpha_{k,N} = \frac{k-1}{N-1}\alpha_{k-1,N-1} - \frac{N-1-k}{N-1}\alpha_{k,N-1},$$
$$u_N = u_{N-1} \qquad \text{(B13)}$$

together with the boundary conditions $u_1 = 1$ and $\alpha_{1,2} = -1/2$. These equations can be solved with the result

$$u_n = 1, \quad \alpha_{kN} = -\frac{N}{4k(N-k)}. \qquad \text{(B14)}$$

Let us now recall that the function $\rho_0(y-x)$ in Eq. (18) is multiplied by $\mathrm{e}^{i\frac{\varphi^+(y)-\varphi^+(x)}{2}}$. Hence, in Eq. (B12) $\rho_0(0)$ has to be replaced by

$$\lim_{x\to y}\rho_0(y-x)e^{i\frac{\varphi^+(y)-\varphi^+(x)}{2}} = \rho_0(0) + \frac{\dot\varphi^+}{4\pi}.$$

This observation brings us to the following expression for J_N:

$$J_N = \int dx\,\left(\rho_0(0) + \frac{\dot\varphi^+(x)}{4\pi}\right)a^N(x)$$
$$- \int dxdy\,\alpha(x-y)\sum_{k=1}^{N-1}\frac{Na^k(x)a^{N-k}(y)}{8\pi k(N-k)}. \qquad \text{(B15)}$$

Substituting J_N into Eq. (B1), we finally arrive at Eq. (19).

APPENDIX C: WEAK REFLECTION LIMIT

Let us derive the effective action of a coherent scatterer in the weak reflection limit making use of the perturbation theory in R_n. In order to proceed we define the operator

$$D_a(x,y) = \langle x|\left[1+\hat\rho_0\hat a\right]^{-1}\hat\rho_0|y\rangle, \qquad \text{(C1)}$$

where $\langle x|\hat\rho_0|y\rangle = \rho_0(x-y)$ and $\langle x|\hat\rho\hat a|y\rangle = \rho_0(x-y)a(y)$. Expanding Eq. (13) in $\hat r'^\dagger\hat r'$ one finds

$$iS^{(1)}_{\rm sc} = -2\mathcal{R}\int dx$$
$$\times\left[\lim_{x\to y}\left(D_{a_1}(x,y)\mathrm{e}^{-i\frac{\varphi^+(x)-\varphi^+(y)}{2}}\right)a_1(x)\right.$$
$$\left. + \lim_{x\to y}\left(D_{a_2}(x,y)\mathrm{e}^{i\frac{\varphi^+(x)-\varphi^+(y)}{2}}\right)a_2(x)\right]$$
$$+8\mathcal{R}\int dxdy\,D_{a_2}(x,y)D_{a_1}(y,x)\,\mathrm{e}^{i[\varphi^+(x)-\varphi^+(y)]}$$
$$\times\sin\frac{\varphi^-(x)}{2}\sin\frac{\varphi^-(y)}{2}. \qquad \text{(C2)}$$

Here we have defined $a_{1,2}(x) = \theta(t-x)\theta(x)\big(\mathrm{e}^{\pm i\varphi^-(x)}-1\big)$, and $D_a(x,y) = \langle x|\hat D_a|y\rangle$.

The function $D_a(x,y)$ can be found in the following way. According to Eq. (18) one has

$$iS_0[a+\delta a,0] = iS_0[a,0] + 2\int dx\,D(x,x)\delta a(x)$$
$$-\int dxdy\,D(x,y)\delta a(y)D(y,x)\delta a(x) + \mathcal{O}(\delta a^3(x)). \qquad \text{(C3)}$$

On the other hand, from Eq. (19) we find

$$iS_0[a+\delta a,0] = iS_0[a,0] - \int dx\frac{\rho_0(0)}{(1+a(x))^2}\delta a^2(x)$$
$$+\int dx\frac{2\rho_0(0)+\frac{1}{2\pi}\int dy\alpha(x-y)\ln[1+a(y)]}{1+a(x)}\delta a(x)$$
$$+\int\frac{dxdy}{4\pi}\alpha(x-y)\left[\frac{\delta a(x)\delta a(y)}{(1+a(x))(1+a(y))}\right.$$
$$\left.-\frac{\delta a^2(x)\ln[1+a(y)]}{2(1+a(x))^2} - \frac{\delta a^2(y)\ln[1+a(x)]}{2(1+a(y))^2}\right]$$
$$+\mathcal{O}(\delta a^3(x)). \qquad \text{(C4)}$$

Comparing Eqs. (C3) and (C4) we obtain

$$D_a(x,x) = \frac{\rho_0(0)+\frac{1}{4\pi}\int dy\alpha(x-y)\ln[1+a(y)]}{1+a(x)}, \qquad \text{(C5)}$$

$$D_a(x,y)D_a(y,x) =$$
$$= \left[\rho_0(0)+\int\frac{dy}{4\pi}\alpha(x-y)\ln[1+a(y)]\right]\frac{\delta(x-y)}{(1+a(x))^2}$$
$$-\frac{\alpha(x-y)}{4\pi(1+a(x))(1+a(y))}. \qquad \text{(C6)}$$

These relations are consistent with the following form of $D_a(x,y)$:

$$D_a(x,y) \;=\; \frac{\mathrm{e}^{\frac{iT}{2}\int dz\left(\coth\pi T(x-z)-\coth\pi T(y-z)\right)\ln[1+a(z)]}}{\sqrt{(1+a(x))(1+a(y))}}$$
$$\times\,\rho_0(x-y)\,. \qquad \text{(C7)}$$

Since Eqs. (C5,C6) do not uniquely determine the function $D_a(x,y)$, Eq. (C7) has to be additionally checked. To this end we make a shift $a(x)\to a(x)+\delta a(x)$ and find the linear in $\delta a(x)$ correction to $D_a(x,y)$. From Eq. (C1) we obtain

$$\delta D_a(x,y) = -\int dz\,D_a(x,z)\delta a(z)D_a(z,y). \qquad \text{(C8)}$$

Defining the function $f(x) = \frac{1}{2}\delta(x) - \frac{i}{2}T\coth\pi Tx$, and making use of the identity

$$\rho_0(x-y)\rho_0(y-z) = [f(x-y)+f(y-z)]\rho_0(x-z), \qquad \text{(C9)}$$

which is a direct consequence of Eq. (B4), one can verify that the property (C8) also holds for the function (C7).

We then conclude that the two functions (C1) and (C7) may differ only by a shift of the argument $a(x)$. The latter shift is zero since in both cases at $a(x) = 0$ one gets $D_{a=0}(x, y) = \rho_0(x - y)$. Thus, we conclude that Eq. (C7) is indeed correct.

What remains is to substitute this result into Eq. (C2) and get

$$
\begin{aligned}
iS^{(1)}_{\rm sc} \;=\; & -4\mathcal{R}\int_0^t dx\rho_0(0)[1-\cos\varphi^-(x)] \\
& + \frac{i\mathcal{R}}{\pi}\int_0^t dx\dot{\varphi}^+(x)\sin\varphi^-(x) \\
& + \frac{\mathcal{R}}{\pi}\int_0^t dxdy\alpha(x-y)\varphi^-(x)\sin\varphi^-(y) \\
& + 8\mathcal{R}\int_0^t dxdy\rho_0(x-y)\rho_0(y-x)\mathrm{e}^{i[\varphi^+(x)-\varphi^+(y)]} \\
& \times \mathrm{e}^{\int_0^t dz[T\coth\pi T(x-z)-T\coth\pi T(y-z)]\varphi^-(z)} \\
& \times \sin\frac{\varphi^-(x)}{2}\sin\frac{\varphi^-(y)}{2}. \qquad \text{(C10)}
\end{aligned}
$$

In the last term of this equation we again apply the identity (B11) and arrive at the final expression for the effective action (23).

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