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Modeling of anisotropic surfaces within a covariant contact description

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Within covariant contact descriptions all necessary characteristics, such as kinematics of deformation and the weak form, as well as all necessary operations, such as linearization and derivation of the finite difference scheme, are described in the specially defined local surface coordinate system, which is related to the closest point projection procedure. This description allows to generalize all contact characteristics into an anisotropic domain for both adhesion and sliding behavior in a straightforward form. The model can be used to describe the average behavior of machined surfaces.

1 Introduction

Real properties of surfaces especially after some machining processes lead to the necessity to describe the contact friction conditions via equations, which are more general than the standard Coulomb friction law. A first generalization of sliding characteristics based on mechanics of a rigid block on an inclined plane was presented by Michalowski and Mroz [1]. Properties of the sliding anisotropy based on the "friction" tensor were theoretically investigated in He and Curnier [2] and e.g. Zmitrowicz [3]. On the other hand, a powerful numerical approach within the finite element method based on the regularization of the contact conditions became a standard in computational contact mechanics, see Wriggers [4]. The covariant description developed for frictional contact problems in Konyukhov and Schweizerhof [5] allows to generalize the contact interactions including both anisotropy for adhesion and anisotropy for friction and to build an effective numerical algorithm for arbitrary finite element implementation. The proposed approach can be easily applied to various anisotropic curved surfaces as e.g. an average model for machined surfaces.

2 Covariant description and its generalization for anisotropic surfaces

Within the covariant description all contact characteristics are described in a spatial local coordinate system defined on the master surface. This coordinate system is obtained at the projection point $\mathbf{C}$ based on the closest projection of the slave point $\mathbf{S}$:

$$\mathbf{r}_s(\xi^1, \xi^2, \xi^3) = \boldsymbol{\rho}(\xi^1, \xi^2) + \mathbf{n}\xi^3. \tag{1}$$

The first two convective coordinates ξ^1, ξ^2 define the surface point $\mathbf{C}$ and, therefore, are responsible for the tangential contact interaction. The third coordinate ξ^3 is the value of the penetration and is used to define the properties of the normal interaction. The complete contact traction vector $\mathbf{R}$ is defined in contravariant basis vectors $\boldsymbol{\rho}^i$ and $\mathbf{n}$ as follows:

$$\mathbf{R} = \mathbf{T} + \mathbf{N} = T_i\boldsymbol{\rho}^i + N\mathbf{n}. \tag{2}$$

Within the penalty regularization the normal traction is written in a closed form:

$$N = \epsilon_N \xi^3, \quad \text{if} \quad \xi^3 \leq 0; \tag{3}$$

while the tangent traction $\mathbf{T}$ is written in a rate form:

$$\frac{d\mathbf{T}}{dt} = -\epsilon_T \dot{\xi}^i \boldsymbol{\rho}_i, \tag{4}$$

where a full time derivative $\frac{d\mathbf{T}}{dt}$ is taken in covariant form in the spatial coordinate system on the tangent plane with $\xi^3 = 0$. A generalization for adhesion is obtained after the introduction of the adhesion tensor $\mathbf{B}$ in the evolution equation (4):

$$\frac{d\mathbf{T}}{dt} = \mathbf{B}(\mathbf{v}_s - \mathbf{v}), \tag{5}$$

where $\mathbf{v}_s - \mathbf{v} = \dot{\xi}^i \boldsymbol{\rho}_i$ is a relative velocity vector of the contact point $\mathbf{C}$. Introducing the friction tensor $\mathbf{F}$ defined in the surface metrics we obtain the generalization of the isotropic Coulomb friction law:

$$\Phi = \sqrt{f^{ij} T_i T_j} - N = \sqrt{\mathbf{T} \cdot \mathbf{F}\mathbf{T}} - N. \tag{6}$$

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The sliding criteria are written then as follows

$$if \;\; \Phi \leq 0 \rightarrow sticking, \quad if \;\; \Phi > 0 \rightarrow sliding. \tag{7}$$

The adhesion tensor $\mathbf{B}$ and the friction tensor $\mathbf{F}$ must fulfi ll some thermodynamical restrictions – for the theoretical developments concerning the friction tensor see [2] and [3]. In particular, the anisotropy can be inherited from the arbitrary coordinate system on the contact surface. Then e.g. the adhesion tensor for a plane with orthotropic properties has the following structure in Cartesian coordinates:

$$\mathbf{B} = -\frac{1}{x^2+y^2}\begin{bmatrix} \varepsilon_r x^2 + \varepsilon_\phi y^2 & (\varepsilon_r - \varepsilon_\phi)xy \\ (\varepsilon_r - \varepsilon_\phi)xy & \varepsilon_r y^2 + \varepsilon_\phi x^2 \end{bmatrix}, \tag{8}$$

where $\varepsilon_r, \varepsilon_\phi$ are the radial resp. the circumferential stiffnesses.

The principle of maximum dissipation is applied to formulate the anisotropic friction problem in a continuous form. Afterwards, the return-mapping algorithm with regard to inequalities (7) within the backward Euler scheme for the evolution equation (5) is applied to obtain all characteristics for sliding, such as a sliding force $\mathbf{T}^{sl}$ and a sliding displacement vector $\boldsymbol{\xi}^{sl}$. These variables can be viewed as a pair of conjugate variables for the energy dissipation function, which allows to defi ne them separately, e.g. the sliding force $\mathbf{T}^{sl}$ is computed as follows:

$$\mathbf{T}^{sl} = -\frac{\hat{\mathbf{T}}}{\sqrt{\hat{\mathbf{T}} \cdot \mathbf{F}\hat{\mathbf{T}}}}|N| \quad \text{with} \quad \hat{\mathbf{T}} = \mathbf{BFT}^{tr}, \tag{9}$$

where the trial force $\mathbf{T}^{tr}$ is computed via the backward Euler scheme for the evolution equation (5). The linearization procedure for a Newton's type iterative solution of the global system of equations is performed as covariant differential operations on the tangent plane in the spatial coordinate system.

3 Numerical example

A plane with orthotropic properties in a polar coordinate system is chosen here to show the possibility to model prescribed machined properties within the proposed approach. A block is positioned on the orthotropic plate, see Fig. 1. The block is pressed down with prescribed vertical displacement **w**, then the horizontal displacements **v** are incrementally applied. Controlling the stiffness parameters $\varepsilon_r, \varepsilon_\phi$, one can enforce various motions e.g. also a motion on a circular path.

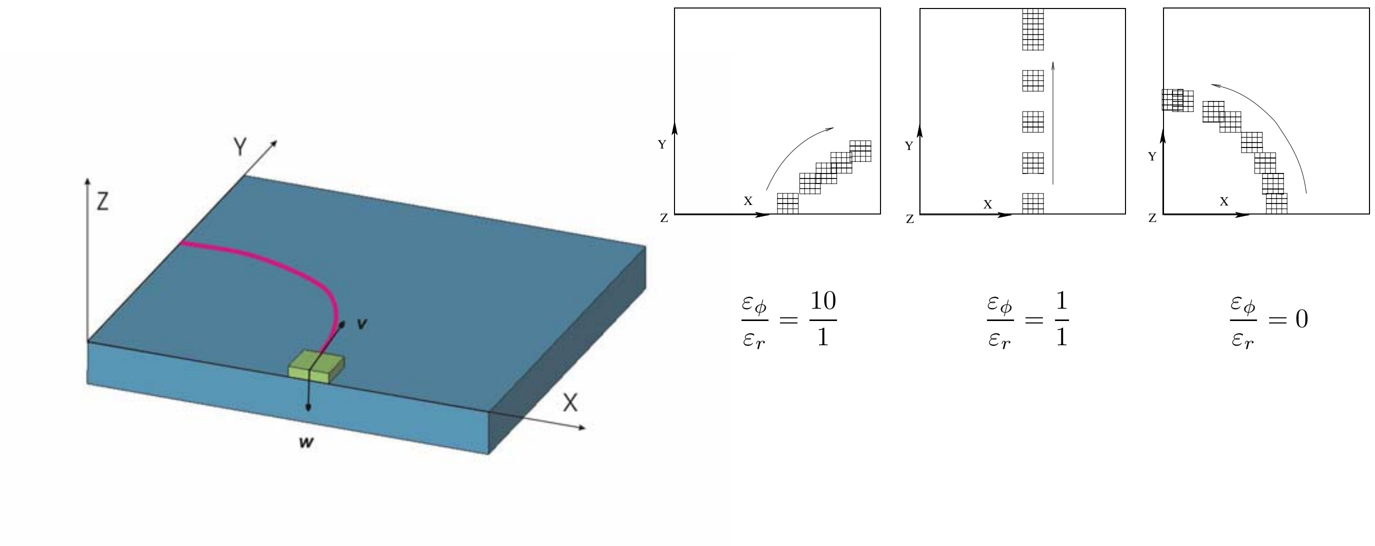

Fig. 1 Motion of a block on the plane with polar orthotropy. Variation of parameters leading to different motions.

References

[1] R. Michalowski, Z. Mroz, Associated and non-associated sliding rules in contact friction problems. Arch. Mech. **30**, 259–276 (1978).
[2] Q.-C. He, A. Curnier, Anisotropic dry friction between two orthotropic surfaces undergoing large displacements, Eur. J. Mech. A-Solid. **12**, 631–666 (1993).
[3] A. Zmitrowicz, A theoretical model of anisotropic dry friction, Wear. **73**, 9–39 (1981).
[4] P. Wriggers, Computational Contact Mechanics. (John Wiley & Sons, 2002).
[5] A. Konyukhov, K. Schweizerhof, Covariant description for frictional contact problems, Comput. Mech. **35**, 190–213 (2005).