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Simulation of Hydroforming of Metal Sheets with an Efficient FE-Formulation Based on an Analytical Meshfree Description of a Compressible Fluid

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Hydroforming is very interesting for the automotive industry, as it offers a great variety to efficiently manufacture thin walled structures. In order to simulate such static large deformation processes under hydraulic or pneumatic pressure, an analytical meshfree description of a compressible heavy or weightless fluid is presented. Thus a fully nonlinear formulation of the fluid-structure-interaction only based on the surface of the wetted membrane structure and the constitutive equations for the fluid can be derived. Considering physically realistic boundary conditions this formulation finally leads for all combinations of fluids and gas to a symmetric tangential stiffness matrix including several dyadic rank updates, which can be cast into a very efficient solution procedure by sequential applications of the Sherman-Morrison formula. Furthermore other algorithms such as mesh refinements and structural contact can be restricted to the models for the work piece and the matrix.

1 Virtual Work Expression

We consider a compressible fluid volume v in the gravity field $\boldsymbol{g}$. The variation of its total potential energy $\delta\Pi^f$ is then given by the variation of the gravitational potential δV^f and the virtual work of the volume compression δW^f.

$$\delta\Pi^f = \delta V^f - \delta W^f = \delta\left(\rho\boldsymbol{g}\cdot\boldsymbol{s}\right) - p^k\delta v\ , \quad \text{with} \qquad p^k = -K\frac{v-v_0}{v_0}\ . \tag{1}$$

The gravitational potential V^f can be written in terms of the first order volume moment $\boldsymbol{s}$ and the fluid density ρ. Along with the bulk modulus K Hooke's law provides the connection between the volume compression and its internal pressure p^k. As already mentioned this approach aims at an analytical meshfree description of the heavy compressible fluid via the surrounding wetted surface. In the derived variation of the fluid potential (1) the controlling variables v and $\boldsymbol{s}$ can be represented by surface integrals, which yields along with the current normal vector $\boldsymbol{n}$ on the fluid boundary the following form:

$$\delta\Pi^f = \int_\eta\int_\xi (p^c - p^x - p^k)\boldsymbol{n}\cdot\delta\boldsymbol{u}\ d\xi d\eta = \int_\eta\int_\xi p^f\boldsymbol{n}\cdot\delta\boldsymbol{u}\ d\xi d\eta\ . \tag{2}$$

In (2) the mass conservation and the variation of density yield the pressure variable $p^c = \rho\boldsymbol{g}\cdot\boldsymbol{x}^c$ in the fluid's center of gravity $\boldsymbol{x}^c$; the position dependent pressure $p^x = \rho\boldsymbol{g}\cdot\boldsymbol{x}$ arises from the variation of $\boldsymbol{s}$. All three pressure parts p^c, p^x and p^k can be combined to the total pressure distribution p^f in the fluid.

2 Linearization and Solution

After linearizing the variational form of the fluid potential (2) and separate transformations a symmetric system of equations can be derived [1],[2],[4]. Using standard finite element isoparametric mapping with $\boldsymbol{N}$ denoting the matrix of shape functions of the wetted surface elements and $\boldsymbol{d}$ the vector of the corresponding discrete nodal displacements the so-called load stiffness matrix $\boldsymbol{K}^f_{elem}$ is obtained, which consists of the symmetric terms from the change of normal vector $\Delta\boldsymbol{n}$ and a change of the position dependent pressure Δp^x. In addition a dyadic rank two update of $\boldsymbol{K}^f_{elem}$ with the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, which couple the increments of the discrete nodal displacents $\Delta\boldsymbol{d}$ with the volume change Δv, is achieved (for further details see [3]). The residual at a time t yields the negative right hand side vector $\boldsymbol{f}^f_{elem}$. To simulate problems with more than one fluid filled chamber for each chamber i the corresponding fluid arrays have to be computed and summed up. Assembling the local vectors and matrices in their appropriate global arrays finally provides the fluid part to the linear set of equations describing the equilibrium condition. Therefore conventional algorithms such as structural contact, non-linear material models or mesh refinements for the adjoining shell elements can be easily applied.

Due to the dyadic rank-two update of the global load stiffness matrix $\boldsymbol{K}^f$ for each fluid filled chamber $i = 1..n$ its characteristic band structure, which is necessary for an efficient solution, is lost. Therefore a proper solution algorithm for this kind

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of problems can be found in the Sherman-Morrison formula, which provides the inverse of a dyadic updated matrix without increasing the band width. In the case of a $2n$ rank update for the corresponding n fluid filled chambers this algorithm needs to be applied sequentially. Thus the solution vector $\Delta\boldsymbol{d}$ and the hybrid variables Δp_i^c, Δp_i^k and $\Delta\rho_i$ of chamber i can be computed using this specific structure.

3 Numerical Example

As a typical example for fluid loaded shell structures a combined procedure of deep drawing with subsequent hydroforming is chosen. For additional this example illustrates the versatility of the derived equations, because they can be applied after previous loadsteps and combined with other conventional algorithms like structural contact in this example. Fig. 1 [a] and [b] show the deep drawing process. The hydroforming process is initialized by a displacement controlled piston (Fig. 1 [c]), which compresses the compressible fluid below the metal sheet and thus steadily increases the pressure on its lower side until the final deformation (Fig. 1 [d]) is reached.

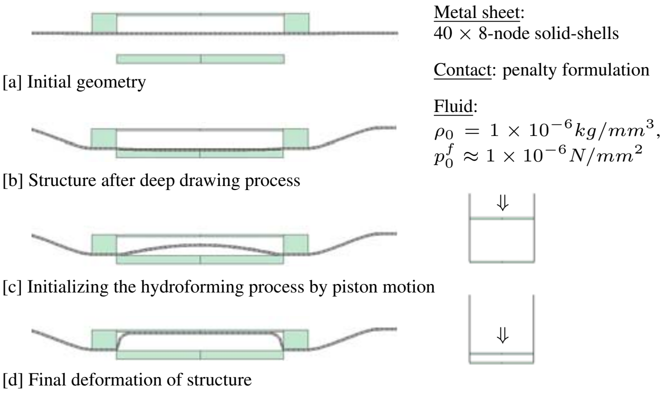

Fig. 1 States of the hydroforming process of a thin metal sheet

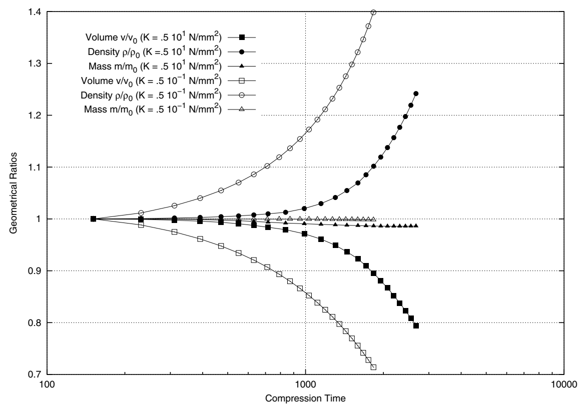

Fig. 2 Volume ratios vs. density ratios for two different bulk moduli K

Fig. 2 displays the change of the state variables for a very low compressible fluid with a bulk modulus of $K = 5.0N/mm^2$ and a more compressible one with $K = .05N/mm^2$. It can be seen that the more compressible fluid directly reacts to the the pistion motion with an increase of density and the associated decrease of volume. Therefore with this fluid the desired deformation cannot be achieved. Whereas the less compressible fluid almost keeps its initial volume and thus enables the deformation of the metal sheet. Only from a time $t = 1000$ on, when contact between the sheet and the punch is established, the fluid changes its volume and density. Further on the ratios of current fluid masses to the initial masses are close to 1.0. Thus mass conservation is preserved, which was an essential assumption in the governing equations.

4 Conclusions

With the derived equations a meshless simulation of a heavy compressible fluid was presented, with the fluid pressure distribution p^f as the interface between the fluid boundary and the adjoining finite elements. An efficient solution algorithm was given with the sequential application of the Sherman-Morrison formula. In future works it is intended to simulate chambers with fluid and gas filling in arbitrary combination. Our aim is also on the possible separation of fluid chambers during the deformation process.

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