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Large deformation frictional contact formulation based on a velocity description

Konyukhov Alexander[*1] and **Schweizerhof Karl**[**]

Germany, D-76131, Karlsruhe, Englerstrasse 2, University of Karlsruhe, Institute of Mechanics

The main idea of the velocity description is to describe kinematics of the contact in the specially defined local surface coordinate system which is related to the closest point projection procedure. All necessary operations for the description of the contact problems: kinematics, all differential operations etc. are considered from the geometrical point of view. While exploiting the differential geometry of the contact surfaces a consistent formulation for the contact problem, including friction is obtained.

1 Introduction

With frictional contact a specific interaction between bodies contacting each other along the surfaces of these bodies is described. Differential geometry provides a powerful mathematical tool to capture the modification of these surfaces due to deformation in the covariant form. Another essential feature to model frictional contact problems is the regularization of the contact conditions. For 2D frictional problems Wriggers et. al. [1] used an elasto-plastic analogy and a penalty regularization of the contact conditions. By then the return mapping algorithm developed for plasticity problems was linearized in the global coordinate system. Laursen and Simo [2], however, formulated the penalty based contact conditions and the return mapping algorithm via convective surface coordinates, but the following linearization performed in the global coordinate system led to an artificial non-symmetry of the contact tangent matrix e.g. in a FE discretization in the case of sticking. To overcome this Wriggers presented in [3] the regularization of the stick conditions based on a functional used in mesh tying procedures which consequently leads to a symmetric tangent matrix.

Despite the large amount of contributions a fully covariant description of contact is still not available in the literature. Therefore, we employ in this contribution the highly developed ”apparatus” of differential geometry to reconsider the frictional contact conditions in a specially defined spatial local coordinate system which corresponds to the well-known closest point procedure. The principle actions for non-frictional contact were developed by Konyukhov and Schweizerhof [4]. All differential operations necessary for kinematics and linearization are considered as covariant derivatives with a special attention of their values on the tangent plane. This leads to a simple geometrical structure of the tangent matrix. Each part of the full tangent matrix, such as the normal tangent matrix, the tangent matrix in the case of sticking and the tangent matrix in the case of sliding is subdivided into main, rotational and curvature parts. It is shown that the tangent matrix in the case of sticking directly preserves symmetry. Representative numerical examples show the robustness of the proposed approach.

2 Contact kinematics

A special local coordinate system related to the master surface at the penetration point C is defined as

$$\vec{r}(\xi^1,\xi^2,\xi^3) = \vec{\rho} + \vec{n}\,\xi^3. \tag{1}$$

By assuming the normal vector to be known, the projection procedure has already been taken into account into this consideration. The equilibrium equations for contact will now be formulated in the defined local coordinate system, however since contact is an interaction between surfaces, each necessary equation especially for the linearization can be considered on the tangent plane, i.e. at $\xi^3 = 0$. For this, we define all the geometrical and differential characteristics with special attention on their values on the tangent plane.

The value of the penetration g, essential for the formulation of the non-penetration conditions in contact mechanics, is exactly the third coordinate in our surface coordinate system:

$$\xi^3 = g = (\vec{r}_s - \vec{\rho}) \cdot \vec{n}. \tag{2}$$

The penalty method is used to enforce the non-smooth contact conditions. Thus, the normal contact force is taken as $N = \epsilon_N\,\xi^3$. The regularization of the tangent tractions T_i has to be considered on the tangent plane in the covariant

* Corresponding author: e-mail: Konyukhov@ifm.uni-karlsruhe.de, Phone: +49 721 608 3715, Fax: +49 721 608 7990
** e-mail: Karl.Schweizerhof@bau-verm.uni-karlsruhe.de

form. It leads to the following evolution equations:

$$\frac{dT_i}{dt} = (-\epsilon_T a_{ij} + \Gamma^k_{ij} T_k)\dot{\xi}^j - h^k_i T_k \dot{\xi}^3, \quad i = 1, 2, \tag{3}$$

where h^k_i are components of the curvature tensor and Γ^k_{ij} are Christoffel symbols for the master surface. The constitutive equations for the tangential tractions in the form of the evolution equation (3) is a key to construct the correct tangent matrix for sticking leading to the symmetric form. In order to derive the tangent matrix all differential operations have to be considered as covariant ones in a 3D spatial coordinate system and expressed then on the tangent plane. We represent here only the tangent matrix in the sticking case. The symmetry is easily verified:

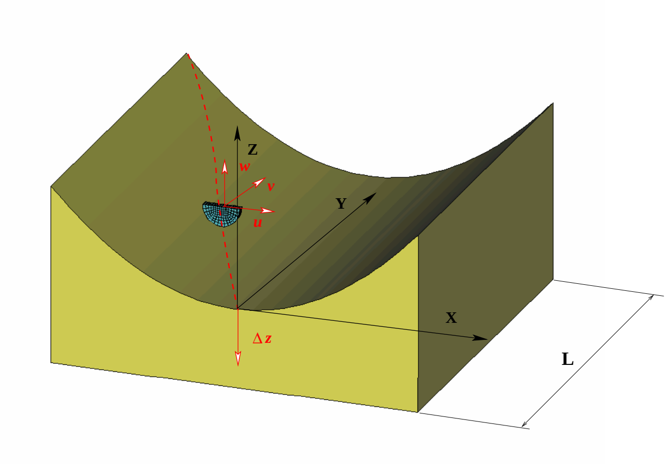
$$D_v(\delta W^T_c) = -\varepsilon_T \int_s (\delta\vec{r}_s - \delta\vec{\rho}) a^{ij} \vec{\rho}_i \otimes \vec{\rho}_j (\vec{v}_s - \vec{v}) ds - \int_s T_i \left[(\delta\vec{r}_s - \delta\vec{\rho})\, a^{il} a^{jk}\, \vec{\rho}_k \otimes \vec{\rho}_l\, \vec{v}_j + \right. \tag{4}$$

$$\left. \delta\vec{\rho}_{,j}\, a^{ik} a^{jl}\, \vec{\rho}_k \otimes \vec{\rho}_l\, (\vec{v}_s - \vec{v}) \right] ds + \int_s T_i h^{ij} (\delta\vec{r}_s - \delta\vec{\rho}) \cdot (\vec{\rho}_j \otimes \vec{n} + \vec{n} \otimes \vec{\rho}_j)(\vec{v}_s - \vec{v}) ds.$$

Similar to the normal tangent matrix, see [4], the tangential tangent matrix for friction can be subdivided into a main, a rotational and a pure curvature part.

3 Numerical examples

For the numerical solution, the frictional problems can be subdivided into two types depending on the integration step for the evolution eqn. (3) within the return-mapping scheme. The first type involves the consideration of the sticking-sliding zone and requires small integrations steps as the change of states has to be found. For this problem, the full tangent matrix can be reduced up to the main part with negligible loss of efficiency. The second type involves only full sliding. Then load steps a-priori larger than a threshold value are applicable for this, however, for the computation it appears necessary to keep the rotational part together with the main part of the tangent matrix (4) to obtain reasonable convergence rates. The curvature part can be still omitted without loss of efficiency. In order to show this effect for a large sliding problem we consider a spiral motion of a semi-circular cylinder on the surface of a parabolical cylinder, see Fig. 1. The corresponding table shows the comparison between computations with the main matrix only and the main matrix plus the rotational matrix for the first 20 iterations. Obviously, it is definitely advantageous to use the tangent matrix without curvature parts (case 1), but keeping the rotational parts in this problem.



Case 1			Case 2		
No. l.s.	No. it./l.s.	Cum. No. it.	No. l.s.	No. it./l.s.	Cum. No. it.
1	9	9	1	10	10
2	8	17	2-18	13	231
3	5	22	19	12	243
4-20	4	90	20	11	254

Fig. 1 Spiral motion of a deformable circular semi-cylinder on a rigid parabolical cylinder. Segment-to-analytical surface approach. Case 1: excluding only curvature matrix; case 2: only main matrix. Comparison of no. of iterations for the fist 20 load steps (l.s.)

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