Quantum criticality in a double quantum-dot system

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We discuss the realization of the quantum-critical non-Fermi liquid state, originally discovered within the two-impurity Kondo model, in double quantum-dot systems. Contrary to the common belief, the corresponding fixed point is robust against particle-hole and various other asymmetries, and is only unstable to charge transfer between the two dots. We propose an experimental set-up where such charge transfer processes are suppressed, allowing a controlled approach to the quantum critical state. We also discuss transport and scaling properties in the vicinity of the critical point.

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Quantum dots can be used to build single-electron transistors [1] and spin-based quantum bits [2], but equally interestingly, they serve as artificial atoms and allow to access correlated states of matter [3–5]. So far, most experiments focused on the study of Fermi-liquid states, with regular thermodynamic and transport properties at low temperatures [3, 5] and simple transitions or crossovers between them [4]. However, artificial molecules and mesoscopic structures can be used to realize and study non-Fermi liquids as well, characterized by singular properties and providing the simplest examples of quantum critical systems. However, due to their singular nature, these states are very elusive. In fact, only recently Oreg and Goldhaber-Gordon [6] proposed a controlled set-up to access the two-channel Kondo (2CK) fixed point [7, 8], being the paradigmatic example of non-Fermi liquid impurity system. Subsequently, this setup was successfully realized experimentally [9]. Dissipation has also been proposed to drive quantum phase transitions (QPT) in quantum dots [10, 11]. However, most dissipative QPT are of Kosterlitz-Thouless type, and therefore no true quantum-critical state is realized.

A non-Fermi-liquid state, similar to the one of the 2CK model, emerges in the two-impurity Kondo model (2IKM). This model, initially studied in the context of heavy-fermion QPT, consists of two impurity spins that are coupled to conduction electrons and, at the same time, interact with each other through an exchange interaction. Jones *et al.* [12] observed that in the 2IKM a quantum critical point (QCP) separates a "local-singlet" from a Kondo-screened phase. This QCP has been shown to be essentially equivalent to the 2CK fixed point [14], though its operator content and finite-size spectrum are different [13]. In fact, it has been observed that – unlike the 2CK fixed point – the QCP of the 2IKM is very sensitive to certain electron-hole symmetry-breaking processes, which can smooth the QPT into a cross-over [13, 15]. (A related non-Fermi liquid fixed point also appeared in a two-orbital Anderson model [16].)

The purpose of the present paper is to demonstrate that the QCP of the 2IKM can be realized and studied in a system of two quantum dots, shown in Fig. 1. Such a double-dot system has a number of interesting regimes [17], however, here we shall focus on a situation far from the charge degeneracy points, with one unpaired electron on each of the dots. Remarkably, the quantum critical state in this geometry is very robust against both the asymmetry of the device (parity) and electron-hole asymmetry, and a sharp phase transition appears as long as there is *no charge transfer* between the dots 1 and 2. We show that these charge transfer processes can be suppressed by inserting an artificial "antiferromagnetic insulator" between the two dots (see Fig. 1b).

Model. To start our analysis, let us assume that the charging energies $E_{C1,2}$ ($E_{C1} \approx E_{C2} \approx E_C$), associated with putting an extra electron to one of the two dots, are large compared to the level widths of the dots, Γ_γ ($\gamma = 1, 2$), and to the tunneling t between the two dots. Perturbatively integrating out virtual charge fluctuations

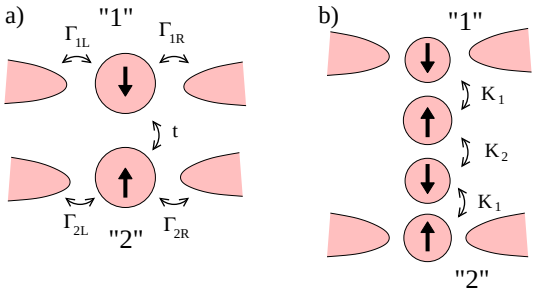

FIG. 1: a) System of two quantum dots studied in the paper. b) Modified set-up with suppressed charge transfer processes, with an even number of quantum dots inserted between the two main dots (1,2) attached to leads.

of the dots, we arrive at the following Hamiltonian:

$$H_{\rm int} = K\ \vec{S}_1\cdot\vec{S}_2 + \frac{1}{2}(J^{(1)}_{\gamma\gamma'}\vec{S}_1 + J^{(2)}_{\gamma\gamma'}\vec{S}_2)\psi^\dagger_\gamma\vec{\sigma}\psi_{\gamma'} \quad (1)$$
$$+ \ (V_{\gamma\gamma'} + Q_{\gamma\gamma'}\vec{S}_1\cdot\vec{S}_2)\psi^\dagger_\gamma\psi_{\gamma'} + \text{irrelevant terms}.$$

Here $\psi^\dagger_{\gamma\sigma} = \varrho_\gamma^{1/2}\sum_\epsilon c^\dagger_{\epsilon\gamma\sigma}$, with $c^\dagger_{\epsilon\gamma\sigma}$ being the creation operator of an electron state with spin σ and energy ϵ in the even combination of electrons in the leads attached to dot γ, and ϱ_γ their density of states at the Fermi energy. Apart from irrelevant terms, Eq. (1) is the most general Hamiltonian that describes the double-dot system in the regime where charge fluctuations are suppressed. The largest couplings are K, $J_\gamma \equiv J^{(\gamma)}_{\gamma\gamma}$, and $V_\gamma \equiv V_{\gamma\gamma}$, since these couplings are generated by second-order tunneling processes. They are typically of the size $J_1 \sim V_1 \sim \Gamma_1/E_C$, $J_2 \sim V_2 \sim \Gamma_2/E_C$, and $K \sim t^2/E_C$. The couplings V_1 and V_2 can be made small by tuning the dots close to the middle of their respective Coulomb blockade valleys. The second-largest couplings are associated with *charge transfer* between leads 1 and 2, and are all of order $V_{12} \sim Q_{12} \sim J^{(\gamma)}_{12} \sim (J_1J_2K/E_C)^{1/2}$. All other couplings are suppressed by further powers of t/E_C, Γ/E_C, and do not change the physics essentially.

Let us first study the Hamiltonian with the leading terms only, and no charge transfer between the two sides:

$$\tilde{H}_{\rm int} = K\ \vec{S}_1\vec{S}_2 + \frac{1}{2}(J_1\vec{S}_1\psi^\dagger_1\vec{\sigma}\psi_1 + J_2\vec{S}_2\psi^\dagger_2\vec{\sigma}\psi_2) \quad (2)$$
$$+ \ V_1\psi^\dagger_1\psi_1 + V_2\psi^\dagger_2\psi_2\ .$$

This Hamiltonian is characterized by three energy scales: Without the coupling K, the two spins on the two dots are screened independently at the Kondo temperatures $T_1 \approx \delta\epsilon\ e^{-1/J_1}$ and $T_2 \approx \delta\epsilon\ e^{-1/J_2}$, with $\delta\epsilon \ll E_C$ the typical level spacing on the dots [18]. These Kondo scales compete with K that tends to bind the two spins into an inter-impurity singlet.

Clearly, the terms in Eq. (2) may break both parity and electron-hole symmetry. Nevertheless, solving Eq. (2) using a numerical renormalization group (NRG) approach we find a sharp QPT upon variation of K for *any* value of the couplings J_γ and V_γ (in contrast to earlier statements). In all cases, the spectrum at the critical point can be described through a generalized version of the conformal field theory (CFT) of Affleck *et al.* [13], to be discussed below.

Asymmetric limit. Before diving into the CFT solution, let us give a simple and revealing physical picture of the physics in the limit $T_1 \gg K \gg T_2$. Here, the first spin is screened at a temperature $T \sim T_1$. Below that scale, a local Fermi-liquid description applies to the resulting Kondo-screened complex, and therefore, it acts as a *bath* which tries to screen the spin S_2 [19]. The effective dimensionless coupling between S_2 and the Kondo complex can be estimated as $\lambda_1 \sim K/T_1$. However, S_2 also couples to spin excitations in the leads attached to it, with a renormalized coupling $\lambda_2 \approx 1/\ln(T_1/T_2)$. Clearly, we end up with an effective 2CK model, which is known to display a QPT at $\lambda_1 = \lambda_2$, corresponding to the condition $T_2 \approx T_1\exp(-a\ T_1/K)$, with a a constant of the order of unity. The above argument is independent of the potential scattering terms. It shows that (i) The quantum-critical state is essentially identical to the two-channel Kondo state; (ii) Particle-hole or device (parity) symmetry are *not* required; (iii) The critical point is destroyed once there is charge transfer between channels 1 and 2. The phase diagram obtained from these simple arguments is shown in Fig. 2. A similar picture is obtained within a CFT approach [13].

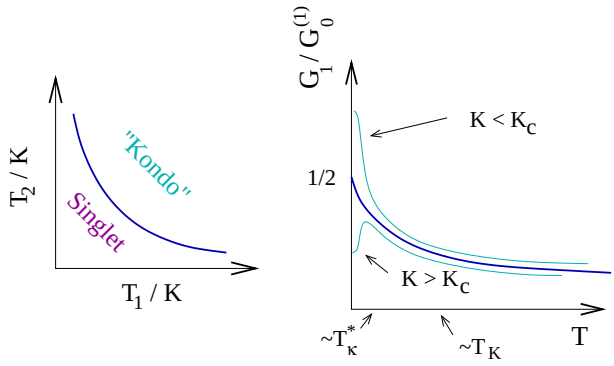

FIG. 2: Left: Phase diagram of the double-dot device in the absence of charge transfer. The two phases are separated by line of second order QPT, being very similar to the two-channel Kondo state. Right: Sketch of the temperature-dependent conductance through dot "1" for $T_1 \approx T_2 \approx T_K$, in the absence of charge transfer between the two sides.

Conformal field theory. Since we do not have electron-hole symmetry in any of the channels, we used only the symmetries $U_1(1)$ and $U_2(1)$ associated with charge conservation in the two channels and the global spin $SU(2)_2$ symmetry for the conformal field theory solution. In the corresponding coset construction, $U_1(1)\times U_2(1)\times SU(2)_2\times\mathcal{Z}_2$ [13], all primary states and primary fields are characterized by their two charges, Q_1 and Q_2, their spin j, and an Ising quantum number q (Id, σ, and ϵ).

At the critical point the entire finite-size spectrum can be characterized just by two phase shifts, $\delta_1, \delta_2 \in [0, \pi/2]$. Similar to Ref. 13, the finite size spectrum is obtained through fusion with the Ising field σ. The leading relevant operators at the fixed point are listed in Table I, where we also indicated the total charge, $Q = Q_1 + Q_2$ of every operator. Only operators with $Q = 0$ can occur in the Hamiltonian, and in the absence of magnetic field only spinless operators can appear, therefore there are only *two* possible relevant operators, ϕ and $\phi_\pm$ that can be present in the Hamiltonian. Therefore, in the vicinity of the QCP, the Hamiltonian can be written as

$$H = H^* + \kappa\ \phi + \delta\ \phi_+ + \delta^*\ \phi_- \ldots, \quad (3)$$

Q	(Q_1,Q_2)	j	Ising	x	operator
0	(0,0)	0	Id	0	$\sim H^*$
0	(0,0)	0	ϵ	$\frac{1}{2}$	$\phi \sim \delta K$
0	$\pm(1,-1)$	0	Id	$\frac{1}{2} \mp \frac{\delta_1-\delta_2}{\pi}$	$\phi_\pm \sim \psi_1^\dagger\psi_2$, $\psi_2^\dagger\psi_1$
0	(0,0)	1	Id	$\frac{1}{2}$	$\vec{\phi} \sim \vec{B}$
±1	$\pm(1,0)$	1/2	σ	$\frac{1}{2} \mp \frac{\delta_1}{\pi}$	$\sim \psi_1^\dagger, \psi_1$
±1	$\pm(0,1)$	1/2	σ	$\frac{1}{2} \mp \frac{\delta_2}{\pi}$	$\sim \psi_2^\dagger, \psi_2$
±2	$\pm(1,1)$	0	Id	$\frac{1}{2} \mp \frac{2(\delta_1+\delta_2)}{\pi}$	$\sim \psi_1^\dagger i\sigma_y\psi_2^\dagger$,$\psi_1 i\sigma_y\psi_2$

TABLE I: Operator content of the critical point.

where H^* denotes the fixed-point Hamiltonian. The coefficient $\kappa \approx \delta K/\sqrt{T_K} = (K-K_C)/\sqrt{T_K}$ measures the distance to the critical point, with $T_K \sim \min\{T_1, T_2\}$ being the Kondo scale associated with the formation of the non-Fermi liquid state. From the quantum numbers it is clear that the operators $\phi_\pm$ transfer exactly one charge from one side to the other, therefore the coefficient of δ is related to the amplitude of those operators in Eq. (1) that transfer charge between the two sides, and which have been neglected in Eq. (2). Both operators have scaling dimension 1/2 [20], and are thus relevant at the fixed point. However, κ can be tuned to zero, while δ always takes a finite value and generates a smooth cross-over at an energy scale $T_\delta^* \sim |\delta|^2$, even for $\kappa = 0$. As a result, a double-dot system never displays a true QPT. Nevertheless, as we shall see later, the parameter δ can be made small in a controlled way, such that the structure of the quantum critical point $\kappa = \delta = 0$ can be explored.

Renormalization group. To obtain an estimate for the (dangerous) coupling δ in Eq. (3) we need to compute the renormalization of the various processes that correspond to charge transfer in Eq. (1) [21]. To this purpose, let us assume that $T_1 \approx T_2 \approx T_K$ and construct the perturbative scaling equations for the couplings in Eq. (1). In leading logarithmic order they read

$$\frac{d\underline{J}^{(\gamma)}}{dl} = \left(\underline{J}^{(\gamma)}\right)^2 , \quad \frac{d\underline{Q}^{(\gamma)}}{dl} = \frac{d\underline{V}^{(\gamma)}}{dl} = 0 . \qquad (4)$$

Here $l = \ln(\delta\epsilon/\Lambda)$ denotes the logarithmic energy scale, and we introduced a matrix notation in the lead indices, $Q_{\gamma\gamma'} \to \underline{Q}, \ldots$. From these equations we readily see that the most dangerous operators are the off-diagonal parts of the $J^{(\gamma)}$ which increase along the RG flow. However, in the perturbative regime the ratios $J_{12}^{(1)}/J_1$ and $J_{12}^{(2)}/J_2$ remain approximately constant. At the scale T_K we have $J_1 \sim J_2 \sim 1$, from which we immediately obtain an estimate for the parameter δ: $\delta \sim \sqrt{T_K}(K/E_C)^{1/2}$. Thus, for a double-dot system we find: $T_{\delta,DD}^* \sim T_K K/E_C$. For typical semiconductor quantum-dot parameters, $E_C \sim$ 20 K, and $K \sim T_K \sim 0.5$ K, this gives a cross-over scale $T_{\delta,DD}^* \sim 12$ mK, which, while not very large, might be enough to spoil an observation of the non-Fermi liquid behavior.

Suppressing charge transfer. T_δ^* can be suppressed by creating an artificial antiferromagnetic insulator to mediate the exchange interaction between the two main dots 1,2. The simplest arrangement is shown in Fig. 1b, where we connect the two dots with two additional quantum dots with one electron on each of them. For simplicity, let us assume that the charging energies of all dots are similar, but the tunneling-generated exchange coupling K_2 between the two central dots is somewhat larger than the one between the outer dots and their neighbors, $E_C > K_2 > K_1$ (see Fig. 1). In this limit, at energy scales below K_2 the spins on the central dots are bound to a singlet, and their role is essentially restricted to mediate an antiferromagnetic interaction $K \sim K_1^2/K_2$ between the two main dots. With parameters $K_2 \approx 3$ K and $K_1 \approx 1.5$ K this gives a coupling in the range of $K \sim 1\text{K} \sim T_K$. On the other hand, $J_{12}^{(\gamma)} \sim (J_1 J_2 K_2 K_1^2/E_C^3)^{1/2}$, and therefore T_δ^* is reduced to

$$T_{\delta,4D}^* \sim T_K \left(\frac{K_1}{E_C}\right)^2 \frac{K_2}{E_C} . \qquad (5)$$

With the above parameters we find $T_{\delta,4D}^* \approx 10^{-3}\, T_K \approx$ 0.5 mK. This value can readily be decreased even further by inserting more quantum dots in the middle.

Transport. In the remainder of the paper we thus assume that T_δ^* is smaller than the experimentally relevant temperature scales, i.e., we set $\delta = 0$. Let us furthermore concentrate on $T_1 \approx T_2 \approx T_K$. CFT allows to predict various observables in the regime close to the QCP, $\kappa \approx 0$. We first note that in the absence of charge transfer, the linear conductance through dot γ is simply related to the T-matrix $T^{(\gamma)}$ of the conduction electrons in the corresponding electrodes as $G_1 = G_0^{(1)}\, \mathrm{Im}\{T^{(1)}/2\}$ with $G_0^{(1)} = \frac{2e^2}{h}\, 4\Gamma_{L1}\Gamma_{R1}/(\Gamma_{L1}+\Gamma_{R1})^2$ (see Fig. 1). At the fixed point (i.e., zero temperature), $T^{(1)} = i(1-S^{(1)})$, with $S^{(\gamma)}$ the S-matrix of the electrons in lead γ [22]. Similar to the analysis of [22] we find that $S^{(1)} = S^{(2)} = 0$ at the QCP, and thus the conductance is $G_1(T=0) = G_0^{(1)}/2$ for $K = K_C$. The approach to this value is determined by the leading irrelevant operator, which, similar to the electron-hole symmetrical case, is ϕ', the derived field from ϕ [23]. At $K = K_C$, the finite-temperature corrections to $G_0^{(1)}$ can be computed by perturbation theory in ϕ', with the result $G_{1,QCP}(T) = G_0^{(1)}\left(1 - \alpha_1\sqrt{T/T_K} + \ldots\right)$. Here α_1 is a non-universal constant of order unity that depends on the asymmetry of the device and on the phase shifts. At finite source-drain voltages, V, the deviation $\delta G_1 \equiv G_0^{(1)} - G_1(T)$ will display scaling properties, similar to those of the 2CK model [24, 25]

$$\delta G_1/G_0 = \sqrt{T/T_K}\, F(V/T) , \qquad (6)$$

where the (non-universal) function F has the properties $F(x \ll 1) \approx$ const and $F(x \gg 1) \propto \sqrt{x}$.

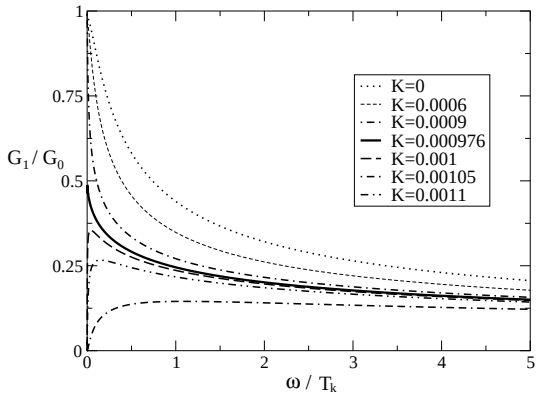

FIG. 3: NRG results for $G_1(\omega)$ (in unit of G_0) for different RKKY couplings. The parameters used here correspond to Kondo couplings $J_1 \approx 0.1$, $J_2 \approx 0.2$, the potential scattering terms $V_1 \approx 0.003$, $V_2 \approx 0.02$, where the energy unit is the half bandwidth of the conduction electrons. The critical RKKY coupling is $K_c \approx 0.000976$. The frequency ω in the plot is in units of T_K where T_K is defined as the half-width of $G_1(\omega)$ for $K = K_c$.

For small but finite κ, another crossover occurs at an energy scale $T_\kappa^* = \kappa^2 \approx (K - K_C)^2/T_K$: For $\kappa > 0$ a inter-impurity singlet state is formed, while for $\kappa < 0$ a Kondo state is recovered. At these fixed points the S-matrices are given by $S^{(\gamma)} = e^{2i\delta_\gamma}$ $(K > K_c)$ and $S^{(\gamma)} = -e^{2i\,\delta_\gamma}$ $(K < K_C)$, with both of these fixed points are of Fermi-liquid type, and therefore the conductance at them scales as $G_{1,\mathrm{singlet}} = G_0\,\left(\sin^2(\delta_1) + \beta_1 (T/T_\kappa^*)^2 + \ldots\right)$ and $G_{1,\mathrm{screened}} = G_0\ (\cos^2(\delta_1) - \gamma_1 (T/T_\kappa^*)^2 + \ldots)$, respectively, with β_1 and γ_1 again non-universal constants of order of unity [26]. The properties of $G_1(T)$ are summarized in Fig. 2.

A numerical computation of the finite-temperature scaling functions in the vicinity of the QCP is notoriously difficult. However, we can compute the AC conductance $G_1(\omega)$ [27] by applying the NRG approach to the Anderson Hamiltonian corresponding to Eq. (2). The results of this calculation for a generic situation without particle-hole and parity symmetries are shown in Fig. 3. The various crossovers can be clearly observed in $G_1(\omega)$ as a function of frequency, which displays a behavior qualitatively similar to $G_1(T)$.

Summary. We have demonstrated that the quantum phase transition of the two-impurity Kondo model can be experimentally accessed using double quantum-dot devices. The non-Fermi liquid state is robust against particle-hole and device asymmetries; it is destroyed by charge transfer between the two main dots, which, however, can be effectively suppressed with additional quantum dots in the set-up. Using a combination of analytical and numerical methods we have made predictions for relevant energy scales and transport quantities.

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