## Characteristic bending strength of beech glulam

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# Abstract

This project was carried out to derive the characteristic bending strength of beech glulam. 47 full size glulam beams with combined symmetrical lay-up were produced. For this purpose mechanical grading was used to classify the lamellae according to the dynamic MOE. The beams were tested according to EN 408. The bending strength exceeds 44,5 N/mm<sup>2</sup>. The structural properties of beech boards were determined as input for a finite-element-based computer model. It was specifically developed to predict the bending strength of beech glulam. Comparisons between the experimental data and the analytical results from the computer model show a good agreement. Depending on the bending strength of finger joints and the tensile strength of boards design proposals were numerically derived taking into account visual and/or mechanical grading.

# Résumé

L'article présente une méthode de calcul pour la résistance à la flexion du bois lamellé collé en hêtre. On a produit 47 poutres expérimentales qui avaient la dimension d'éléments de construction et une structure symmétrique et combinée. On a classé les planches de bois suivant leur résistance en déterminant leur rigidité axiale. Les poutres ont été testées en flexion à quatre points selon l'EN 408. Les résultats montrent une résistance à la flexion de plus de 44,5 N/mm<sup>2</sup>. Les propriétés structurelles d'une quantité de planches de bois de hêtre ont été déterminées. Ces données sont utilisées dans un modèle de calcul qui se base sur la méthode des éléments finis. Ce modèle permet de prédire la résistance à la flexion des poutres en bois lamellé collé en hêtre. Un bon accord a été observé entre les résultats expérimentaux et les résultats théoriques. Avec le modèle de calcul on donne quelques propositions pour la détermination des valeurs caractéristiques qui prennent pour base la résistance à la flexion d'aboutages à entures et la résistance à la traction de planches. Dans ces propositions on tient compte du classement visuel ou/et mécanique des bois suivant leur résistance.

## 1 Introduction

The bending strength of glulam depends on the tensile strength of the lamellae and of the finger joints which may correlate. If the correlation is known, it is possible to determine the characteristic bending strength of glulam depending only on the characteristic tensile strength of the lamellae. In the case of softwood this leads to the calculation model in EN 1194, where a linear relation between the two values is given. The bending strength of the glulam and the tensile strength of the lamellae are determined based on test methods defined in EN 408. The results of these test methods lead to the so called laminating effect. This means, that the bending strength of glulam is generally higher than the tensile strength of the lamellae. The high tensile strength of beech (fagus silvatica) raises the question, whether the common

relation according to EN 1194 is also valid for a characteristic tensile strength exceeding 26 N/mm<sup>2</sup> or if a different relation more accurately describes the laminating effect for beech glulam. The aim of the present study is to answer this question and to provide design models for beech glulam. An overview of the conducted work and research is given below: Regression equations were derived to predict the mechanical properties of 150 mm long board sections and 150 mm long finger joints. The structural properties of 1888 beech lamellae graded according to the dynamic MOE from longitudinal vibration (=  $E_{dyn}$ ) are described. A new calculation model consisting of a simulation programme and a finite element programme was developed. This model is appropriate to numerically reproduce 4 point bending tests according to EN 408. Full size beams with combined lay-up having mechanical properties of beech glulam can be analysed. In an experimental investigation 4 point bending tests on full size beech glulam incorporating the 1888 boards were carried out. A comparison between the results from the bending tests and the numerical results is given to verify the calculation model. 4 point bending tests on finger joints taking into account visual and mechanical grading were conducted. The results clarify the influence of the grading method on the strength of finger joints. Different visual and mechanical grading procedures partly suitable for practical application were developed for use in the calculation model. The required input data were generated by computer-aided grading using the database of the 1888 boards. The numerical determination of bending strength of 600 mm high beams considering the grading proposals provides a database making it possible to describe the laminating effect and to work out a design model for beech glulam.

### 2 Modelling lamellae

The bending strength and the MOE of a glulam beam modelled in the simulation programme is mainly affected by the ratios of values tensile strength (=  $f_t$ ) / MOE (=  $E_t$ ) and compression strength (=  $f_c$ ) / MOE (=  $E_c$ ). These values vary in longitudinal direction of the lamellae. The following empirical equations were developed to determine the mechanical properties (in N/mm<sup>2</sup>) of lamellae discretised in 150 mm steps. In the following equations, an additional index j denotes finger joints. The extensive database describing the tension and compression tests on 150 mm board sections and finger joints was provided by Glos et al. [2]. More background information concerning the property variation and the effects of autocorrelation can be found in [3] and general concepts of simulating glulam beams in [4].

## 2.1 Mechanical properties of board sections

The regression equations (1) to (4) predict the mechanical properties of 150 mm long board sections. The MOE is closely correlated with the strength. Hence the MOE is modelled first and appears as independent variable when modelling the strength values. The DEB-value (0,05-0,85) is a knot ratio quantifying single knots in accordance with DIN 4074.  $\rho_0$  (575-820 kg/m<sup>3</sup>) is the oven-dry density of beech and u (+/-12%) is the moisture content.

$$\ln(E_{c}) = -3,46 + 3,91 \cdot 10^{-2} \cdot \rho_{0} - 7,44 \cdot 10^{-2} \cdot u - 1,92 \cdot DEB - 2,75 \cdot 10^{-5} \cdot \rho_{0}^{2}$$
(1)

$$\ln(f_{c}) = 2,88 + 1,13 \cdot 10^{-4} \cdot E_{c} - 2,71 \cdot 10^{-9} \cdot E_{c}^{2}$$
<sup>(2)</sup>

$$\ln(E_{t}) = 3,36 \cdot 10^{-1} + 2,64 \cdot 10^{-2} \cdot \rho_{0} - 1,56 \cdot DEB - 1,87 \cdot 10^{-5} \cdot \rho_{0}^{2}$$
(3)

$$\ln(f_t) = 3,09 + 9,76 \cdot 10^{-5} \cdot E_t - 1,54 \cdot 10^{-4} \cdot E_t \cdot DEB$$
(4)

### 2.2 Mechanical properties of finger joints

The regression equations (5) and (6) predict the mechanical properties of finger joints in the compression zone of the beam. Aicher et al. [5] carried out tensile tests on finger joints. They found a correlation with  $r^2 = 0.45$  between the tensile strength and the lower longitudinal MOE of the connected boards. The MOE of both boards was measured in the range 350 mm in length. The distance between the finger joint and the beginning of the range was 58 mm. The correlation indicates the use of the minimum dynamic MOE =  $E_{dyn,min}$  (9700-20600 N/mm<sup>2</sup>) as independent variable when predicting the mechanical properties in the tensile zone of the beam. The equations (7) and (8) are used when modelling mechanical grading by measuring the dynamic MOE. In this case the MOE difference between the connected boards is small. Visual grading does not consider the MOE as grading parameter of the boards. Hence connections between boards with a low and high MOE are possible. For this case the equations (9) and (10) applies.  $\rho_{0,min}$  and  $\rho_{0,max}$  (kg/m<sup>3</sup>) indicate the smallest and highest density, respectively, of the joined boards.

$$E_{c,j} = 1,01 \cdot 10^{5} - 1,55 \cdot 10^{4} \cdot u + 6,44 \cdot 10^{2} \cdot u^{2} + 9,57 \cdot 10^{-3} \cdot \rho_{0,max}^{2}$$
(5)

$$\mathbf{f}_{c,j} = -2,10 \cdot 10^2 + 40, 4 \cdot \mathbf{u} - 1,74 \cdot \mathbf{u}^2 + 2,73 \cdot 10^{-6} \cdot \mathbf{E}_{c,j} \cdot \boldsymbol{\rho}_{0,\min}$$
(6)

$$E_{t,j} = 3,20 \cdot 10^3 + 0,823 \cdot E_{dyn,min}$$
<sup>(7)</sup>

$$f_{t,j} = 63, 2 - 8, 27 \cdot 10^{-5} \cdot \rho_{0,\min}^{2} + 1, 82 \cdot 10^{-7} \cdot E_{dyn,\min} \cdot E_{t,j}$$
(8)

$$E_{t,j} = 7,67 \cdot 10^3 + 0,538 \cdot E_{dyn,min}$$
(9)

$$f_{t,j} = 54, 5 - 5, 04 \cdot 10^{-5} \cdot \rho_{0,\min}^{2} + 1, 60 \cdot 10^{-7} \cdot E_{dyn,\min} \cdot E_{t,j}$$
(10)

### 2.3 Structural properties of beech

### 2.3.1 Material, methods and results

The mechanical properties calculated from the equations (1) to (10) depend on the structural properties density, knot ratio and moisture content. 1888 boards were examined to determine these properties. Three sawmills located in Germany each delivered about one third of the testing material, see Table 1. The gross density (= air dry mass/volume) and the dynamic MOE as grading parameter of each board were measured. The measurement is described in [1]. The boards were graded according to the system shown in Table 2. Fig. 1 depicts the yield. About one third of the boards is in the highest grades 4 and 5. This allowed a combined lay-up with lamellae of high stiffness in the outer zone of the test beams.

Table 1Sample size and cross-sectional dimensions (mm)

source	sample size	height / width (mm)
Spessart	670	41/121
Nordhessen	659	40/116
Schönbuch	559	44/115

grade	range of dynamic MOE (N/mm <sup>2</sup> )
1	$E_{dyn} \le 13000$
2	$13000 < E_{dyn} \le 14000$
3	$14000 < E_{dyn} \le 15000$
4	$15000 < E_{dyn} \le 16000$
5	$16000 < E_{dyn}$

Table 2Mechanical grading according to dynamic MOE



Fig. 1 Absolute yield in the 5 grades

The knots were determined according to DIN 4074 considering only the single knot with the DEB-value. All the knots appearing in the boards were taken into account in order to reproducing their appearance while simulating the lamellae. A typical feature of beech is the high amount of boards being free from knots, see Fig. 2.



Fig. 2 Fraction of boards being free from knots



Fig. 3 Dynamic MOE depending on maximum DEB-value

Fig. 3 shows the relation between the dynamic MOE and the maximum DEB-value. It is evident, that the dynamic MOE decreases with increasing maximum DEB-value. The trend is independent of the source of the boards. The linear relation is superposed by a strong residual scattering. Hence the following proposals for mechanical grading in section 7.1 additionally consider the maximum DEB-value as a second grading parameter. The moisture content of the boards is given in Table 3.

source	mean	std deviation	range
Spessart Nordhessen	11,3 9 95	0,595 0,538	8,15-13,4 8,85-11,9
Schönbuch	10,5	0,597	9,14-11,7

Table 3Moisture content statistics (%)

### **2.3.2** Density functions of the structural properties

The simulation of the structural properties of the lamellae is based on random number generation taken from density functions. These were fitted to the empirical data. The fit was carried out for each of the grades in Table 2 and each structural property. The advantage of this approach is a very exact simulation of the structural properties within a grade. The lognormal and beta density function are suitable to describe the structural properties. Table 4 provides the gross density statistics for each grade. The grading influence on the statistics and on the shape of the density functions is evident. Fig. 4 exemplifies this for grades 1, 3 and 5.

Table 4Gross density statistics (kg/m³)

grade	n	mean	std deviation
1	336	657	32,5
2	428	662	30,9
3	444	674	28,6
4	307	685	27,0
5	373	706	30,2



Fig. 4 Fitted beta density for gross density

The relation between the oven-dry density (in kg/m<sup>3</sup>) of beech and the gross density (= $\rho_{gross}$  in kg/m<sup>3</sup>) at about 10 % moisture content is given by equation (11).

$$\rho_0 = 22,7 + 0,952 \cdot \rho_{\text{gross}} \tag{11}$$

Table 5 shows the maximum DEB-value statistics. Only boards with knots were taken into account. It is evident that grading according to the dynamic MOE is also an efficient method to detect knots. The fraction of boards with knots decreases with higher grades. Mean and std deviation show a similar trend. There are as shown in Fig. 5 significant differences between the shape of the fitted beta density functions. Further DEB-values being smaller than the maximum DEB-value appearing along the board are simulated following the method developed by Görlacher [1]. In the current case of beech the method additionally takes into account the number of sections with knots. The statistics of this feature shows Table 6. Fig. 6 displays the moderate influence of the grading technique according to Table 2 on the number of sections with knots. The fitted density curves are quite similar.

Table 5Maximum DEB-value statistics

grade	n (with knots)	fraction	mean	std deviation
1	289	0,86	0,358	0,183
2	316	0,74	0,261	0,164
3	295	0,66	0,222	0,141
4	200	0,65	0,183	0,119
5	207	0,55	0,153	0,110



Fig. 5 Fitted beta density for maximum DEB-value

grade	n (with knots)	mean	std deviation
1	289	2,85	1,92
2	316	2,77	1,81
3	295	2,43	1,56
4	200	2,55	1,67
5	207	2,24	1,50



Fig. 6 Fitted lognormal density for number of sections with knots

The histogram and fitted lognormal density for the board length is shown in Fig. 7. The distribution of the empirical data is irregular. This is caused by the different length of the basic material. The original board length is chiefly 3 m and the rest 4 m and 5 m. The required preparation of the board ends with regard to finger jointing causes a reduction up to 1,5 m. Hence a maximum range of about 3,2 m can be observed.



Fig. 7 Histogram and fitted lognormal density of the board length

### **3** Calculation model

### 3.1 Simulation programme

The simulation programme is comparable to the real glulam production. A continuous lamella is generated consisting of simulated boards and finger joints. The mechanical properties are determined in steps 150 mm in length. Considering that each board is an individual item, the structural properties and their variation are determined individually for each board. Here, the effect of autocorrelation is taken into account. The result are boards of low up to high quality. The activation of different density functions enables the simulation of a grading process according to the method in Table 2 as well as the grading proposals in Table 13 with regard to practical application. In general beams with combined lay-ups are simulated taking into account the economical use of the higher grade boards (Fig. 8). There is as a minimum 2 lamellae in the outer zone.



Fig. 8 Lay-up of a combined glulam beam

The dynamic MOE is a dependent variable and the leading mechanically determined grading parameter. In the simulation programme he is calculated from the stiffness properties of a simulated board with formula (12): the common rule of serial connection of springs having different stiffness. The factor of 1,05 considers the influence of the dynamic testing method.  $E_{stat}$  is the mean MOE in terms of static load.  $E_i$  quantifies the variable MOE of a single section and i is the number of sections along the discretised lamella. If the value of the dynamic MOE - calculated in a loop - is within the limits of the desired grade, the board is included in the simulated beam. All the mechanical properties are stored in a two-dimensional array for later use in the finite element programme. Additionally, the simulation programme can provide for a characteristic tensile strength of finger joints.

$$E_{dyn} \approx 1,05 \cdot E_{stat} = 1,05 \cdot \frac{N}{\sum_{i=1}^{N} \frac{1}{E_i}}$$
 (12)

### **3.2** Finite element programme

The beam bending strength and MOE are calculated using the finite element programme ANSYS Version 5.7. Fig. 9 shows the mechanical model. Instead of a load a stepwise displacement  $\Delta u$  is applied in the middle of the loading equipment. The load in the vertical compression members (F) is stored after each step for later determination of the maximum load (F<sub>max</sub>). The bending strength (f<sub>m</sub>) is calculated using formula (13) with the section modulus (W).

$$f_{\rm m} = \frac{F_{\rm max} \cdot \ell/3}{W} \tag{13}$$



Fig. 9 Finite element model

The material is orthotropic. In the compressive zone ideal elastoplasticity and in the tensile zone ideal elasticity until failure is assumed. A failure in the outermost lamination generally stops the calculation. Failure occur if the tension stress in the centre of an element lies in between a range +/-0.5% of the tensile strength of the board section. Element failure in the tensile zone outside the outermost lamination do not constitute beam failure and hence is allowed during the calculation. Those elements were deactivated during the calculation by multiplying their stiffness by a severe reduction factor.

#### 4 **Bending tests on beech glulam beams**

47 test beams were produced and tested according to EN 408. Differing from Table 2 the boards coming from Nordhessen and Schönbuch were graded in up to 7 grades, see Table 7. The range of variation concerning the MOE of the outer laminations of the combined test beams was increased. Hence 2 test beams were produced with lamellae in zone 1 belonging to grade 7. The beams are divided into 5 series differing in terms of beam height and grade of lamellae.

Grade	range of dynamic MOE (N/mm <sup>2</sup> )
$5 \rightarrow \begin{cases} 5^* \\ 6 \end{cases}$	$\begin{array}{l} 16000 < E_{dyn} \leq 17000 \\ 17000 < E_{dyn} \leq 18000 \end{array}$
2	$18000 < E_{dyn}$

Table 7 Subdivision of grade 5 boards coming from Nordhessen and Schönbuch

#### 4.1 Beam lay-up

3 strength classes and 2 beam heights were realised, see Table 8. Fig. 8 and Table 9 give details of the beam lay-up. The total amount of 1888 boards was used to produce the beams. This confirms the economical aspect of the proposed grading system in Table 2.

Table 8	Acronym for strength class of the 5 series	

height h (mm)	340	600
strength class		
very high high low	VH-34 H-34 L-34	VH-60 H-60
source span ℓ (m)	Spessart 5,10	Nordhessen Schönbuch 9,00

Table 9Strength class and combined beam lay-up

strength class	grade of boards in zone 1	grade of boards in zone 2
very high	5	3
high	4	2
low	1	1

# 4.2 Results

Fig. 10 shows the relation between bending strength and MOE. The statistics of these values are given in Table 10. The following conclusions can be drawn:

- The grade of the lamellae obviously affects the bending strength and the MOE of the tested beams.
- The strength depends on the height.
- The strength values belonging to the strength classes "high" and "very high" exceed the lower limit of 44,5 N/mm<sup>2</sup>. The 5<sup>th</sup> percentile strength value of beech glulam manufactured from mechanically graded boards is in the order of this value.

		height h (mm)	340	600	
/mm²)	very high	n mean std dev. min	11* 63,7 7,51 50,4	10 55,9 5,78 46,9	
g strength (N	high	n mean std dev. min	12 57,8 9,32 46,6	8 50,5 5,21 44,5	
bending	low	n mean std dev. min	5 43,3 7,25 35,0	- - -	
(1	very high	n mean std dev. min	12 15500 490 14700	10 16000 799 15100	
loe (N/mm <sup>2</sup>	high	n mean std dev. min	12 14400 383 13700	8 14400 265 14000	
X	low	n mean std dev. min	5 12300 403 11800	- - - -	
* A poorl caused a s	* A poorly manufactured finger joint in the outermost lamella considering the adhesive application caused a strength value of 32.7 N/mm <sup>2</sup> . This value is disregarded.				

Table 10Bending strength and MOE statistics



Fig. 10 Bending strength depending on MOE; beam height h = 34 cm



Fig. 10 (Continuation) Bending strength depending on MOE; beam height h = 60 cm

### 5 Bending strength of finger joints

108 bending tests on finger joints manufactured from visually graded boards were performed, see Table 11. A further 259 tests were carried out to study the influence of mechanical grading on the bending strength of finger joints, see Table 12. These specimens were manufactured in the laboratory from the undamaged parts of tested beams. It was possible to assign the specimens to the grades and to the source of the connected boards. All the bending tests were conducted flatways according to EN 408 with a span of 15 times the height. The flexural MOE obtained by vibration methods is the reference parameter, see Fig. 11 and [6].

 Table 11
 Sample size and cross-sectional dimensions

source	Spessart	Nordhessen	Schönbuch
Ν	31	56	21
width/height (mm)	110/34	100/30	105/36

source grade	Spessart	Nordhessen	Schönbuch
2	21	20	22
4	18	22	22
5 Σ	$\frac{24}{88}$	$\frac{19}{83}$	$\frac{22}{88}$
width/height (mm)	110/33	100/29	105/34

 Table 12
 Sample size and cross-sectional dimensions



Fig. 11 Flatways flexural vibration; the finger joint connection is in the middle of the specimen and  $\ell$  is the specimen length

### 5.1 Visual grading of boards

The relation between bending strength and flexural MOE is shown in Fig. 12. The 3 regression lines confirm the influence of stiffness on the bending strength. The  $5^{\text{th}}$  percentile is 56 N/mm<sup>2</sup> in case of visual grading.



Fig. 12 Bending strength depending on flexural MOE

## 5.2 Mechanical grading of boards

Fig. 13 shows the relation between bending strength and flexural MOE and Fig. 14 the mean and  $5^{th}$  percentile value of bending strength comparing the different grades. It is remarkable,

that no increase of bending strength between grades 4 and 5 can be proved. The  $5^{\text{th}}$  percentile value of the 127 specimens belonging to grades 4 and 5 both amount to 68,8 N/mm<sup>2</sup>. In terms of technical feasibility mechanical grading of grades 4 and 5 can lead to a  $5^{\text{th}}$  percentile value exceeding 70 N/mm<sup>2</sup>.



Fig. 13 Bending strength depending on flexural MOE



Fig. 14 Mean (top) and 5<sup>th</sup> percentile (bottom) bending strength value over grade of connected boards

## 6 Verifying the calculation model

The beams of the 5 series were modelled with the calculation model taking into account the lay-up and the distribution of the structural properties of the laminated boards. 500 simulations were conducted per series. The 5<sup>th</sup> percentile value of the tensile strength of finger joints was predicted using equation (8). The values are 53 N/mm<sup>2</sup> (grade 5), 48 N/mm<sup>2</sup> (grade 4) and 36 N/mm<sup>2</sup> (grade 1). The moderate increase of tensile strength of 53/48 = 1,10 between grades 4 with 5 is not confirmed by the bending tests on finger joints. Considering the small sample size of test specimens in grades 4 and 5 it is still plausible to assume higher values in grade 5. Fig. 15 compares the test results and the simulations. The test results are situated

mainly in the range mean value +/- std deviation of the simulation. The dependence of the strength on the height and the influence of board grade on the bending strength is reproduced correctly. Hence the calculation model is suitable to predict the bending strength of beech glulam.



Fig. 15 Test results in comparison with simulation results

# 7 Design proposals for beech glulam

## 7.1 Grading methods

The design proposals were determined numerically. For that 7 grading methods as shown in Table 13 were developed having different influence on the tensile strength of the boards. The data of the 1888 boards were used to determine the appropriate density functions of the structural properties. These density functions were integrated into the calculation model to numerically reproduce the grading methods. The glulam bending strength was calculated for each of the models to study the influence of the boards tensile strength on the glulam bending strength. Thereby the characteristic tensile strength of the finger joints varied from 20 N/mm<sup>2</sup> to 60 N/mm<sup>2</sup> in steps of 5 N/mm<sup>2</sup>. In this way 900 calculations were performed per step within a single grading method. The simulated beams have 20 laminations, a height of 600 mm and a span of 10,80 m. The following 5<sup>th</sup> percentile values were determined by the non-parametric method.

grade	knots	MOE (N/mm <sup>2</sup> )	
А	$0,33 < \text{DEB} \le 0,80$	-	
В	$0,20 < \text{DEB} \le 0,33$	-	
С	$DEB \le 0.33$	-	
D	$DEB \le 0,20$	-	
E	$DEB \le 0,042$	-	
F	$DEB \le 0,20$	$15000 < E_{dyn}$	
G	$DEB \le 0,042$	$15000 < E_{dyn}$	

Table 13Grading methods

## 7.2 Laminating effect

### 7.2.1 Laminating effect in terms of simulated strength values of 150 mm long sections

The curves in Fig. 16 point out the relation between the characteristic glulam bending strength (=  $f_{m,g,k}$ ) and the variable characteristic tensile strength of finger joints (=  $f_{t,j,k,sim}$ ). The maximum characteristic bending strength (=  $f_{m,g,k,max}$ ) for each of the grades is clearly visible. The dashed line in Fig. 16 represents a linear relation between the glulam bending strength and the finger joint tensile strength, see equation (14). The gradient of this line is independent of the grading method or the tensile strength of boards, respectively, and applies until the trend becomes non-linear. The unit of strength values in the equations (14) - (21) is N/mm<sup>2</sup>.



Fig. 16 Characteristic bending strength of glulam depending on simulated characteristic finger joint tensile strength

$$f_{m,g,k} = -3 + 1,04 \cdot f_{t,j,k,sim}$$
(14)

The computation of the laminating effect in Table 14 shows the decrease of  $\lambda$  with increasing characteristic tensile strength of the boards (=  $f_{t,l,k,sim}$ ). Here, the characteristic tensile strength of the boards was determined using the calculation model. This is as expected and caused by the more homogeneous material properties in higher grades. Hence the laminating effect disappears as reported by Falk and Colling [7] for the case of softwood.

grade	$f_{t,l,k,sim}$ (N/mm <sup>2</sup> )	$f_{m,g,k,max}$ (N/mm <sup>2</sup> )	$\lambda = \frac{f_{\text{m,g,k,max}}}{f_{\text{t,l,k,sim}}}$
А	17	21	1,24
В	23	27,5	1,20
С	29	33	1,14
D	34	36,5	1,07
E	38	39,5	1,04
F	46	47,3	1,03
G	54	55,5	1,03

Table 14 Laminating effect  $\lambda$ 

The simulation results as described in section 7 were merged into a single database and a multiple nonlinear regression analysis was performed to derive a general design proposal resulting in equation (15). It should be noted that the strength values of the independent variables refer to 150 mm long sections. The coefficient of correlation amounts to 0,99.

$$f_{m,g,k} = -2,09 + 0,913 \cdot f_{t,j,k,sim} - 0,0202 \cdot f_{t,j,k,sim}^2 - 0,0128 \cdot f_{t,l,k,sim}^2 + 0,0344 \cdot f_{t,j,k,sim} \cdot f_{t,l,k,sim}$$
(15)

### 7.2.2 Laminating effect in terms of strength values derived from standard test methods

Equation (15) will be transformed with the intention to replace the independent variables with strength values derived from tests according to EN 408. First a relation between tensile strength and bending strength of finger joints has to be established. Blaß et al. [3] performed multiple tensile tests on 150 mm long finger joints and bending tests on finger joints according to EN 408. They proposed the relation in equation (16). Here,  $f_{m,j,k}$  is the characteristic bending strength of finger joints. Colling et al. [8] proposed a quite similar factor of 1/0,7 = 1,43 for softwood. Transforming the characteristic tensile strength of 150 mm board sections and the characteristic tensile strength of boards according to EN 408 (=  $f_{t,l,k}$ ), it is assumed that the test method affects more the measured strength values in case of lower lamination quality than in case of higher quality. A linear relation was derived as equation (17). The intercept and the gradient were determined fulfilling the following conditions: The relevant characteristic tensile strength according to EN 408 should amount to 70% (89%) of the characteristic tensile strength of 150 mm long board sections, if the latter value is 24 N/mm<sup>2</sup> (54 N/mm<sup>2</sup>). This coincides with values found by Colling and Falk [9], proposing a range from 71% up to 83%, see also Falk and Colling [7].

Inserting equations (16) and (17) in equation (15) leads to the equation describing the characteristic bending strength of beech glulam in terms of strength values determined by standard test methods, see (18).

$$\mathbf{f}_{\mathrm{m,j,k}} \approx 1,40 \cdot \mathbf{f}_{\mathrm{t,j,k,sim}} \tag{16}$$

$$f_{t,l,k} \approx -8,088 + 1,037 \cdot f_{t,l,k,sim}$$
(17)

$$f_{m,g,k} = -2,87 + 0,844 \cdot f_{m,j,k} - 0,0103 \cdot f_{m,j,k}^{2} - 0,192 \cdot f_{t,l,k} - 0,0119 \cdot f_{t,l,k}^{2} + 0,0237 \cdot f_{m,j,k} \cdot f_{t,l,k}$$
(18)

The finger joint bending strength resulted in a characteristic value of 56 N/mm<sup>2</sup> in case of visual grading and 70 N/mm<sup>2</sup> in case of mechanical grading. Considering these values in equation (18), two design proposals can be derived for beech glulam made of visually graded boards, see equation (19), and mechanically graded boards, see equation (20). The beech glulam design proposals in comparison with the current model in EN 1194 referring to softwood, see equation (21), are shown in Fig. 17. There, the model according to equation (20) seems to be an adequate continuation of the model according to equation (21).

$$\mathbf{f}_{m,g,k} = 12,0+1,13 \cdot \mathbf{f}_{t,l,k} - 0,0119 \cdot \mathbf{f}_{t,l,k}^{2}$$
(19)

$$f_{m,g,k} = 5,66 + 1,47 \cdot f_{t,l,k} - 0,0119 \cdot f_{t,l,k}^{2}$$
<sup>(20)</sup>

$$f_{m,g,k} = 7 + 1,15 \cdot f_{t,l,k}$$
(21)



Fig. 17 Equations (19) and (20) in comparison with equation (21)

# 8 Conclusions

An extensive numerical and experimental investigation was carried out to determine the characteristic bending strength of beech glulam. Bending tests on full size beams composed of mechanically graded boards confirm a characteristic bending strength of at least 44,5 N/mm<sup>2</sup>. A numerical approach was used to derive design proposals for beams with a height of 600 mm. These proposals are valid for visual and mechanical grading of beech lamellae. Visual grading considering knots allows a characteristic bending strength up to 36 N/mm<sup>2</sup>. Mechanical grading using the dynamic MOE from longitudinal vibration is a precondition to achieve characteristic bending strength values up to 48 N/mm<sup>2</sup>.

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