PROCEEDINGS Part II

22 - 25 April 1986
Ostend, Belgium

Editors: R. Vanbrabant
J. Hermia
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AN EXPANDED MATHEMATICAL MODEL DESCRIBING THE CONVEYING OF PASTY MATERIAL IN DECANTER CENTRIFUGES

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ABSTRACT

When applying solid-bowl centrifuging procedures to slurries with very fine particle size distribution, the conveying of the settled solids inside the bowl can cause specific problems. The sludge can flow back down the cone, counter-current to the movement of the scroll conveyor, first through the clearance between the top of the conveyor blades and the inner surface of the bowl and secondly along the helical canal formed by the conveyor blades and the cone.

A mathematical model is developed describing the conveyance or the backflow of the sludge by variation of the machine geometry, the operating conditions, and the rheological behaviour of the pasty material. Using this model, data achieved by test runs with pilot machines can be applied to an industrial scale decanter, i.e. to set guidelines for the geometric properties and the operating conditions.

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I. INTRODUCTION

When processing suspensions with very fine particle size distributions (<50 μm in centrifugal decanters), problems arising from the limited separation efficiency are accompanied by special problems concerning the discharge of the solids.

Whereas granular solids which are easy to filter and dewater can generally be transported without difficulty by the screw from the cylindrical section through the conical section of the bowl from where they are discharged (Fig. 1), backflow occurs with fine-grained sludges which impairs the clarifying efficiency of the decanter.

This backflow also leads to higher torque as well as increased wear on the screw (or worn). There are two paths by which the paste can achieve backflow. The sediment can either travel back down the conical section by flowing in the clearance between the blades of the screw and bowl (or drum), or it can spiral back on the screw thus achieving helical backflow [3].

Each of these effects (actually it is usually a combination of both effects) means that the forward conveying section of the screw is opposed by a backflow which leads to a situation where the height of the solid layer accumulating in the cylinder rises significantly despite the continuous conveyance of the solid up the conical section.

Although the conveyance process of granular dewatered solids has been examined, but, as of yet, remains largely unpublished, there exists today no method by which to predict the "conveyance properties" of sludges through using the different rheological properties, different
geometries or operating parameters.

One must be content with an empirical approach, whereby few specific conclusions for different instances of applications regarding machine and process can be drawn. Thus, it can happen, for example, that the functioning of a decanter which is geometrically enlarged, can come to a total halt, although the centrifuge functioned perfectly on a scale model.

II. MATHEMATICAL EVALUATION OF THE CONVEYANCE PROCESS OF THE PASTE MATERIAL IN A DECANTER

In the following, a mathematical model will be introduced through which it is possible to quantitatively evaluate the conveyance process, or, more precisely, the backflow in the decanter with respect to the geometry of the machines, operating parameters and the rheological properties of the sediment to give reliable conclusions in respect to the "scale up".

A. Mass Balance of the Solid

The total mass balance of the solid material in a decanter is as follows:

\[ \dot{m}_{\text{feed}} = \dot{m}_{\text{DS}} + \dot{m}_{\text{DC}} \]  

(1)

where \( \dot{m}_{\text{DS}} \) = discharge of solid from the cone

\( \dot{m}_{\text{DC}} \) = discharge of solid in centrate

The mass balance of parts for a single continuous thread screw is:

\[ \dot{m}_{\text{screw},0} - \dot{m}_{\text{screw},0} \cdot \dot{m}_{\text{screw},0} = \dot{m}_{\text{screw},1} + \dot{m}_{\text{screw},1} \cdot \dot{m}_{\text{screw},1} + \cdots \]  

(2)

Assuming that at the point of discharge from the cone \( \dot{m}_{\text{screw},0} \) and \( \dot{m}_{\text{gap},0} \) are equal to zero, and that \( \dot{m}_{\text{screw},0} = \dot{m}_{\text{DS}} \), we can simplify Eq. (2) to:

\[ \dot{m}_{\text{DS}} = \dot{m}_{\text{screw},n} + \dot{m}_{\text{screw},n} \cdot \dot{m}_{\text{gap},n} \]  

(3)

Substituting Eq. (3) into Eq. (1), we get:

\[ \dot{m}_{\text{feed}} = \dot{m}_{\text{screw},n} + \dot{m}_{\text{screw},n} \cdot \dot{m}_{\text{gap},n} + \dot{m}_{\text{DC}} \]  

(4)

Fig. 2. Mass balance of parts (solids) in decanter

In Eq. (4), \( \dot{m}_{\text{screw},n} \), \( \dot{m}_{\text{spiral},n} \), and \( \dot{m}_{\text{gap},n} \) appear as unknowns.

B. Calculation of the helical backflow

The flow of high concentrated suspensions as in sludge or, in this case, pastes, can exhibit large differences in flow profiles depending upon the roughness of the walls of the drum.

Fig. 3. Velocity profile

a) for the shear flow with yield value \( \tau_0 \)
b) for the slip- and shear flow

This means the helical backflow can not simply be described by the shear stress
function of the medium
\[ \tau = \tau_0 + K \cdot \gamma \]  
where adhesion to the wall is assumed (especially where there exists no compact ground layer). Instead, one must also take into account the slip function
\[ \tau = C' \cdot \gamma \cdot g_R + \tau_1 \]  
(6)

thus taking into respect the effect of the roughness of the surface area of the wall when estimating the helical backflow (Fig. 3) [2].

The helical backflow along the worm spierral can be calculated as follows [3]:
\[ \delta_{\text{helical}} = \beta \cdot (1 - R) \cdot \left[ (G \cdot \cos \theta) \cdot \left\{ \left[ \frac{2}{h^2} \cdot \frac{1}{h} \right] - (G \cdot \cos \theta) \right\} \right] \]

where
\[ \beta = \arcsin \left( \frac{h}{h + t} \right) \]  
and
\[ \delta = \arcsin \left( \frac{h}{h + t} \right) \]  

(7)

C. Calculation of backflow between the blades and drum (\( \delta_{\text{blade}} \)).

In calculating the backflow between the blade and drum, \( \delta_{\text{blade}} \) must be calculated for shearing and slip stresses into account. Just as we did for the calculation of the helical backflow, \( \delta_{\text{blade}} \) is made using several assumptions, one of which is that the screw blades are in the form of annular disks corresponding to a screw with a pitch of nought.

The solid mass flow restricted by the shear function is formulated as follows:
\[ \delta_{\text{solid}} = \beta \cdot (1 - R) \cdot \gamma \cdot \left[ \frac{(G \cdot \cos \theta) \cdot \left\{ \left[ \frac{2}{h^2} \cdot \frac{1}{h} \right] - (G \cdot \cos \theta) \right\}}{\left( \frac{2}{h^2} \cdot \frac{1}{h} \right) - (G \cdot \cos \theta)} \right] \]

(8)

Eq. (10) is applicable in the case that the shear gradient in the direction of the periphery is much larger than the shear gradient in the direction of the bowl wall (\( y > x \)).

Eq. (11) is applicable in the case that the shear gradient in the direction of the bowl wall is much larger than the shear gradient in the direction of the periphery (\( y < x \)).

\[ \delta_{\text{solid}} = \beta \cdot (1 - R) \cdot \gamma \cdot \left[ \frac{(G \cdot \cos \theta) \cdot \left\{ \left[ \frac{2}{h^2} \cdot \frac{1}{h} \right] - (G \cdot \cos \theta) \right\}}{\left( \frac{2}{h^2} \cdot \frac{1}{h} \right) - (G \cdot \cos \theta)} \right] \]

(9)

The zone in the clearance where the shear stress is not large enough to have an effect is given by \( \tau_0 \) where
\[ \tau_0 = \left( \frac{G \cdot \cos \theta \cdot \left\{ \left[ \frac{2}{h^2} \cdot \frac{1}{h} \right] - (G \cdot \cos \theta) \right\}}{\left( \frac{2}{h^2} \cdot \frac{1}{h} \right) - (G \cdot \cos \theta)} \right) \]

The mass flow of solids caused by sliding on the metal surface can be calculated as follows:
\[ \delta_{\text{slide}} = \beta \cdot (1 - R) \cdot \gamma \cdot \left[ \frac{(G \cdot \cos \theta) \cdot \left\{ \left[ \frac{2}{h^2} \cdot \frac{1}{h} \right] - (G \cdot \cos \theta) \right\}}{\left( \frac{2}{h^2} \cdot \frac{1}{h} \right) - (G \cdot \cos \theta)} \right] \]

By experimental analysis on the backflow through the clearance using a centrifuge built specifically for this purpose, it can be determined that the influence of the different rates of rotation between the worm and the drum have little effect on the rate of backflow through the clearance. Thus, for practical purposes, Eq. (11) will be used to calculate the
Fig. 4. Calculated backflow profile

volume of solid and fluid as measured from the drum to the surface and V87 stands for the volume of the sludge, which displaces this fluid.

b) Degree of Fullness, F%: This is a factor which allows one to judge the volume of sludge in the decanter in comparison to the ideal situation where there is no backflow. This is a measurement standard for the actual duration of the sediment in the machine and is defined as the relation of the total volume of sludge with backflow (V87) to the total volume of sludge without backflow (V85). (20)

\[
F\% = \text{V87/V85}
\]

In Fig. 4, R21 is the residual moisture of the discharge, R22 is at the point of transition from cone to cylinder, and R23 is the residual moisture at the end of the cylinder, where the suspension is added in the case of a co-current flow decanter. In the previous example for calculation of the backflow profile, it was seen that only 17.5% of the total volume of sludge was composed of clear liquid. In this example, the firm material would stay in the centrifuge 4.7 times longer than if the machine functions without any backflow. When the drum revolves 5 times slower from 3000 min⁻¹ to 750 min⁻¹ (Fig. 5), it can be observed that the "degree of effectiveness" increases from 17.5% to 70.14%.

On the other hand, (Fig. 6) one can observe the degree of effectiveness by holding the rate of rotation of the drum constant and raising the differential speed (the difference of the rates of rotation between the worn and drum).

Fig. 5. Backflow profile after slowing the rotation of the drum

Fig. 6. Backflow profile with higher differential speed

When considering these possibilities, one should naturally take into consideration other undesirable effects which could occur, for example, disrupting the clarification through turbulence caused by the increasing of the differential speed, or making sedimentation more difficult and/or increasing the residual moisture by slowing the speed of rotation of the drum. The above-mentioned examples are simple operational conditions. Fig. 7 shows the dependence of MC on mass flow \(\beta\) and on variations of centrifuging factor \(r\) (g-factor).

III. CONSIDERATIONS FOR THE "SCALE-UP"

The transferring and applying of results from the pilot plant to the large machi-
When the pilot machine (Fig.5) is enlarged by a factor of four (x4), the RG - degree of effectiveness - of the enlarged machine is 0.1 (Fig.8).

Fig. 8. Backflow profile of the "scale-up".

In Fig.8 it can also be seen that the height of the backflow is higher than the fluid level, or niveau. This means that the solid is also being carried out in the clarified-liquid discharge before the sediment can reach the solid discharge level. This means that the decanter cannot produce the desired solid material, the clarification process is fully disrupted, and the unseparated suspension flow through the fluid outlet.

Fig. 9 shows that it would be thoroughly worthless to install a large machine when the centrifuging factor (g-value) is held constant by the "scale-up". The large machine would only function without backflow within a very small margin.
Lockflow in the clearance.

To calculate the driving potential \( \Delta H = \frac{2}{3} \Delta P R_c \), one must consider two cases:

a) The backflow of mass is not large enough to fully cover the total width between two blades to come into contact with the reverse side of the blade. In this case, calculations of the driving potential yield:

\[
\begin{align*}
\Delta z &= \frac{2}{2} R_c \Delta \alpha - (h_j^2 - 2R_j h_j - 2b R_j \sin 3) \\
\Delta H &= \frac{2}{3} \Delta \alpha R_c (h_j^2 - 2R_j h_j - 2b R_j \sin 3) \\
\end{align*}
\] (14)

b) The backflow of mass is high enough that the blockage extends all the way to the reverse side of the preceding blade. One can calculate the driving potential in this case only when one knows the height of the backflow on the neighboring blade.

\[
\begin{align*}
\Delta z &= \frac{2}{2} R_c \Delta \alpha - (h_j^2 - 2R_j h_j - 2b R_j \sin 3) + 2R_j (h_j - 1 - G \sin 3) \\
\Delta H &= \frac{2}{3} \Delta \alpha R_c (h_j^2 - 2R_j h_j - 2b R_j \sin 3) + 2R_j (h_j - 1 - G \sin 3) \\
\end{align*}
\] (15)

The total mass backflow through the clearance can be described by combining Eq. (11) and Eq. (13):

\[\Delta \text{gap} = \Delta \text{shear} + \Delta \text{slide} \] (16)

D. Formulation of the mass flow due to the screw (screw)

The mass flow which is advanced by the screw, \( \Delta \text{screw} \), yields, through the difference of the rotation rates of the screw and the drum, the following:

\[\Delta \text{screw} = \frac{2}{3} R_c \Delta \alpha R_h (G - b) \Delta \text{diff} \cos^2 \] (17)

E. Overall transfer equation

One can now insert Eq. (17), Eq. (16) and Eq. (17) into the Balance Equation, Eq. (4). The resulting mass transfer equation is:

\[\Delta \text{diff} = \frac{2}{3} R_c \Delta \alpha R_h (G - b) \Delta \text{diff} \cos^2 \] (18)

Eq. (18) deals with an ordinary differential equation of a higher order: an equation which can now be analytically solved. The total height of the backflow at any given point, or, more specifically, the varying of the height with the changing of the measuring coordinates, appears in the equation as an unknown variable. Thus, to solve Eq. (18), one must employ "numerical methods". For the case where the centrifugal has a firm ground layer between the blades and the drum \( (G \neq 0) \), then Eq. (18) can be solved with relative ease. One uses the point of solid discharge as a starting place and calculates the height of the backflow along the channel of the worn out through iteration methods.

Fig. 4 shows a calculated backflow profile for a machine of specific geometry and operating parameters.

Two concepts must now be introduced in order to be able to judge the functioning capability of the decanter better.

a) Degree of Effectivity, \( NG \); this is a measurement or standard which measures the volume of the clear liquid in the overall volume of liquid and solid. \( NG \) is defined as:

\[NG = \left(1 - \frac{\text{NIV}}{\text{VIV}}\right) \times 100\% \] (19)

in which \( \text{NIV} \) stands for the Niveau Volume (or the level of the total volume inside) and \( \text{VIV} \) stands for the Volume Inside.
Comparing Fig. (9) to Fig. (7), it can be recognized that the large machines, in respect to the backflow, would exhibit the same behaviour as when the parameter velocity of the drum, not the centrifugal factor, is held constant. This can also be shown using Table I with Eq. (18). For this reason, the centrifugal factor for the large machine is:  

\[ C_G = C_T / 1.1 \]  

Now, when the solid mass flow \( \dot{m}_s \) (gpm) is not enlarged by \( 1.1^2 \) as in the drag force law [4], but instead is enlarged by \( 1.1^2 \) using the theory of equivalent shear rates[5], then it becomes obvious that by constant parameter velocity, the large machines can function even more favorably than the pilot machine (Fig. 20).

![Diagram](image)

Fig. 10. Operational field due to "scale=4x" (9Hg, 9Hg, 14"

IV. INFLUENCE OF THE GAP WIDTH ON THE BACKFLOW

The former examples have been calculated under the assumption that there is a firm ground layer. For the case where the sediment can flow back through the gap between wall and blade as well as flow in a spiral form along the blades, the backflow profile becomes even more unfavorable. This means that the backflow mass is greater than when the gap width is zero. At this point it should be mentioned, that the addition of the backflow through the gap to the calculation seriously overestimates the entire calculating process.

Fig. 11 demonstrates the influence of the gap width on the degree of effectiveness.

For \( s = 0 \text{ mm} \), the degree of effectiveness is identical to that in the example in Fig. 6. It can also be seen in Fig. 11 that enlarging the gap width from 0 mm to 0.35 mm is more than enough to decrease the degree of effectiveness from 48.9 % to 0 %.

At first the influence of the gap width upon the solid backflow appears great. In practice, it can be seen that the gap closes itself after a certain amount of time in operation by building up a firm ground layer. This means that, for most cases, the influence of the gap width can be ignored.

Certain assumptions are made for the derivation of the mathematical models, or, in this case, for the calculation of the backflow profile, where quantitative statements can be made with caution. With this mathematical model, one is much better equipped to make precise relative statements, especially when one wishes to apply the experimental results from a pilot machine to an industrial-size machine. One must also look critically at the fact that an extremely large number of experiments are necessary for the determination of the rheological data (\( \tau_0, K, n, \gamma, \gamma' \)) since they are continuously coupled with the residual moisture of the sediment. Since the change in the residual moisture of the sludge during the conveying by the screw is not known, it will be estimated as being linear. One can also assume that the residual moisture in the conical section of the decanter is hardly, if at all, changed by the lessening of the centrifugal acceleration. Thus, the value for BF1 to BF2 can be considered to be constant.

The described mathematical model can be checked through experimental data from an industrial test. For this test, the
behaviour of a starch suspension in
two genetically different decanters
are employed. Thereby it was determined
that the larger machine did not dis-
charge any solid. After investigating
the theoretical data of the product,
these experimental findings could be
mathematically proven.

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SYMBOLS

b blade thickness
C C-bowl, g-factor
C° slip-factor
C° degree of fullness
g acceleration due to gravity
h pitch
h height of cake, level of accu-
mulated sludge
i e-mailing factor, scale-up
factor
K shear-factor
m slip-exponent
n solid-mass flow
n shear-exponent
differential speed
d rotational speed of drum
e degree of effectiveness
p pressure
r radius
s residual moisture
g gap width, clearance
v perimeter velocity of the drum
v, s slide velocity
v volume flow
w mean radius of particles