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CENTRIFUGAL FILM DRAINAGE IN PACKED BEDS

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ABSTRACT

The model of centrifugal dewatering describes the course of measurements by any variation of the determining factors.

In comparison to models and measurements of literature also a equilibrium saturation can be calculated and has not to be measured for very long times in a centrifugal force field. The model is not only proofed for glass spheres but also for packed beds of irregular particles such as coal or quartz sand.

I. INTRODUCTION

Dewatering of packed beds in the centrifugal force field is in industry a mostly applied method for solid liquid separation: not finally because of the low costs compared to the costs of chemical dewatering.

In spite of variety of existing centrifuges and their manifold use in different products, there is no model in literature that allows a sure precalculation of the humidity of packed beds in the centrifugal-force field by any variation of the determining parameters.

The following model points out, that by use of two dimensionless parameters the moisture content, dependent on time, can be precalculated. In comparison to literature can be demonstrated too, that existing models have validity only in a certain sphere of parameter's variation.

2. PARTS OF FLUID IN PACKED BEDS

The whole fluid existing in a wetted packed bed will be subdivided into four different parts in order to precalculate them as shown in Figure 1.

Fig. 1. Parts of fluid in packed bed (schema)

The first part of fluid is bound in very fine pores in the particle itself. Because of the very high capillary forces this fluid only can be removed by chemical dewatering. For mechanical solid-liquid separation can be assumed that this part of fluid is a material constant value.

The caused loading (in the dimensions kg fluid/kg solid) and the saturation can be written as:

\[ \frac{N_s}{1} \text{ constant} \]  \hspace{1cm} (1a)

\[ N_s, S_{ls} \text{ constant} \]  \hspace{1cm} (1b)
The second part of fluid existing in a wetted packed bed is caused by a capillary rise (BATEL, SCHUBERT). According to equation 2 the height of the capillary rise \( h_\text{c} \) depends on the centrifugal acceleration factor \( C_m \), the surface tension \( \sigma \), the contact angle \( \theta \) and capillary radius. The capillary radius is determined by particle size and porosity and will be defined by equation 3.

\[
h_\text{c} = \frac{4 \cos \theta}{\sigma} \frac{1}{2} \frac{1}{r \rho \mu^2}
\]

\[
r_k = \frac{1}{3} \frac{1}{1 - c_k}
\]

As the height of the capillary rise in a packed bed is caused by the balance of forces (centrifugal force and capillary force) the quantity of fluid can be assumed as a function of centrifugal acceleration factor and \( \theta \) (equation 4a) at which \( M_2 \) means a quantity part of quantity \( M \) (equation 4) that contains all parameters that might influence the humidity of a packed bed.

\[
M = \{ h, S_\gamma, \sigma, \cos \theta, \varepsilon, \eta \} \quad (4)
\]

\[
M_{\text{hs}} = f_1(C_m, M_2) \quad (5a)
\]

\[
M_{\text{hs}}', S_{\text{hs}} = f_1(C_m, M_2, \varepsilon, \eta) \quad (5b)
\]

The loading and saturation caused by \( M_{\text{hs}} \) however depends to the height of the packed bed. With increasing mass of the solid material and with this, the height of the packed bed, the constant value of capillary fluid is less important. Therefore can be assumed for the function corresponding to equation (5b).

The third part of fluid is the so-called 'wedge fluid' at the contact points of the different particles. For the dependence of this fluid can be assumed a function (eq. (6a))

\[
S_{L_{2u}} = f_2(C_m, M_2) \quad (6a)
\]
respectively for loading and saturation

\[
S_{2u}, S_{2w} = f_2(C_m, M_2) \quad (6b)
\]

The last part of fluid is defined to be the only one that is dependent on time in the centrifugal force field. Generally can be taken a function

\[
S_{L_{p}} = f_3(C_m, M_3, t, \varepsilon, \eta, \sigma) \quad (7a)
\]

\[
S_{3p}(t), S_{3p}(t) = f_3(C_m, M_3, t, \varepsilon, \eta, \sigma) \quad (7b)
\]

The acceptance of time-constant parts of fluid according to equation 5a and 5b only contradicts apparently to what one can see if watching the variation of fluid at the different contact points (figure 2).
the following partial differential equation will be got

\[
\frac{\partial z}{\partial t} = -g / \nu \left( \frac{1}{2} \frac{z^2}{r_k} - \frac{1}{2} \left( r_k - s \right)^2 \right) - \left( r_k - s \right)^2 \ln \left( \frac{r_k}{r_k - s} \right) \frac{r_k}{r_k - s} \frac{\partial z}{\partial r}
\]

(10)

A particular solution of equation (10) expressed as saturation for the case of a capillary is given in equation (11)

\[
S_f(t) = \frac{4}{3} \lambda_H - \frac{1}{2}
\]

(11)

whereas

\[
\lambda_H = \frac{4 a \eta}{c_m \rho H}
\]

(12)

Equation (12) describes the time-dependent course of saturation in a packed bed.

3. EXPERIMENTAL DATA

Looking at typical measurements with glass spheres which were got in a special centrifuge (tumbler centrifuge) and compared to the theoretical course expressed by equation (6) there is similar to the results of BIERNANKI a shifting found out. In figure (5) this is demonstrated for three different values of the centrifugal acceleration factor $c_m$.

Fig. 5. Comparison of theoretical and experimental course of loading for three different values of acceleration factor $c_m$.
Subtracting the theoretical values of saturation or loading at certain times from a lot of experimental values the time-constant differences shown in figure (6) are found out.

![Figure 6](image)

**Figure 6.** Time-constant differences of saturation for different values of \( C_p \) (glass spheres, \( x = 2.7 \) mm)

Dependent to the equilibrium between centrifugal force and capillary force in a packed bed, which is known in literature as Bondnumber (HERZOGH)\(^1\)

\[
\frac{F_c}{F_p} = \frac{C_m}{C_m} = \text{Bondnumber}
\]

(13)

for all varied porosities, centrifugal acceleration factors, heights of packed beds, particle sizes and particle size distributions all points of measurements can be sized by one course only (figure 7).

In this figure the third and fourth field of the Bond-diagram is presented which can schematically be explained in figure 8.

![Figure 7](image)

**Figure 7.** Equilibrium saturation \( s \) versus Bond-number \( B_0 \) (glass spheres, different particle sizes)

![Figure 8](image)

**Figure 8.** The four fields of the Bond-diagram

In field one the capillary force is higher than the centrifugal force and the capillary rise (also for \( C_p > 1 \)) is higher than any height of a packed bed that can be realized in a centrifuge. The saturation \( S \) still remains at value one.

In the second field the capillary rise has a value greater zero but less than the maximum height of a packed bed. That means that for height \( H \), a saturation \( S = 1 \) is possible but for the same acceleration factor \( C_m \) and height \( H \), or \( H_j \) the saturation is less than one.
Many authors found out that the equilibrium saturation depends on a power $-0.25$ to the centrifugal acceleration factor $C$. They have done experiments in all fields of the Bond-diagram and the experimental data are studied by regression in a fair good manner by this power.

In fact the power of $C_0$ changes from 0 or $-1$. Therefore, the shows figure 9, dependent to the field in the Bond-diagram, there can be a very good correspondence to the experimental data. In cases in which the power of $C_0$ is not in the region of $-0.25$, there is a considerable deviation between theory and experimental data.

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SYMBOLS

3c 2  = Bond-number  
C   = Centrifugal acceleration  
factor  
F   = Force  
H   = height  
S   = saturation  
S0   = specific surface  
V'   = volume  
dc   = diameter of capillary  
h0, hSC  = head of capillary rise  
m   = mass  
r0   = radius capillary  
t   = time  
x   = particle size  
δ   = thickness of film  
\cos\delta   = cos of contact angle  
r   = porosity  
\eta   = viscosity  
M   = parameter  
\rho   = density  
\sigma   = surface tension