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Abstract

Production cost models are important analysis and decision support tools in the context of liberalised and competitive wholesale electricity markets. With the help of production cost models, it is possible to calculate the marginal cost of production of electricity which, by assuming a perfectly competitive market, should correspond to the spot price of electricity. Production cost models are based on a mathematical representation of the power system associated with an electricity market, and usually formulate the unit commitment and economic dispatch problems as an optimisation problem. Technical restrictions governing the operation of the system are integrated into the optimisation problem in the form of constraints. In this respect, three issues for the formulation and application of production cost models have been identified:

- Depending on the level of detail in the representation of the system, the formulation of the model and the solution of the optimisation problem might require significant data-gathering and computational effort.
- Owing to the integration of regional markets, in some cases it might be advantageous or necessary to model multiple interconnected regions. This, however, carries with it additional modelling effort.
- Traditional methodologies applied to determine the short-run marginal cost in mixed integral linear programming models, fail to incorporate the start-up cost of thermal power plants into the system marginal cost. This weakens the representativeness and validity of the results.

To deal with the issues of data availability and modelling effort, it is common practice to lower the level of detail of the model by simplifying the representation of the system. Such simplifications, however, could compromise the accuracy of the results and, ultimately, the usefulness of the model. Until now, no complete and detailed analysis of the effect of the level of detail on the performance of production cost models has been done. This work addresses the three issues mentioned above in an effort to make a contribution to the formulation of production cost models. To this end, a production cost model was developed which allows for a flexible representation of the power system. Variants of the model with different levels of detail were formulated and applied for the modelling of the power systems of three important European electricity markets. The performance of each model variant was evaluated by determining the accuracy of the results, measured as the difference between marginal costs and observed spot prices, and the computational effort. The results obtained with each model formulation show the tradeoffs between the modelling effort and the accuracy of the results. Additionally, a new methodology was developed to address the issue of the consideration of start-up cost of thermal power plants. The application of this methodology demonstrates the effect of the start-up costs on the modelled marginal cost and on the accuracy of the results.

Kurzfassung

Fundamentale Produktionskosten-Modelle (Production Cost Models) sind Werkzeuge zur Analyse der Preisbildung in liberalisierten Strommärkten. Mit Hilfe dieser Modelle ist es möglich, die Grenzkosten der Stromproduktion einzuschätzen, welche unter der Annahme eines perfekten Wettbewerbs im Markt dem Spotpreis für Strom entsprechen sollten. Strommarktmodelle basieren auf einer mathematischen Beschreibung des Strommarktes sowie des dahinter stehenden Stromversorgungssystems. Der Einsatz der Kraftwerke zur Deckung der Stromnachfrage wird in der Regel als Optimierungsproblem mit den technischen und wirtschaftlichen Restriktionen für den Systembetrieb als Nebenbedingungen formuliert. Bei der Formulierung und Implementierung von Strommarktmodellen spielen drei Faktoren eine wichtige Rolle:

- Abhängig vom Detaillierungsniveau der Systembeschreibung können die Modellformulierung und die Lösung des Optimierungsproblems erheblichen Datenwartungs- und Rechenaufwand erfordern.

- Wegen der Integration von regionalen Strommärkten ist es in manchen Fällen von Vorteil oder sogar notwendig, mehrere verbundene Regionen zu modellieren. Allerdings bringt die Modellierung mehrerer Regionen zusätzlichen Aufwand mit sich.

- Üblicherweise angewandte Verfahren zur Lösung von gemischt-ganzzahligen linearen Optimierungsproblemen lassen die Anfahrkosten von thermischen Kraftwerken in den Grenzkosten unberücksichtigt. Dies hat die systematische falsche Einschätzung der Strommarktpreise zur Folge.

In Anbetracht des Datenwartungs- und Rechenaufwands ist es üblich, die Systembeschreibung zu vereinfachen, um das Detaillierungsniveau des Modells zu reduzieren. Solche Vereinfachungen könnten aber der Genauigkeit der Ergebnisse und letztendlich der Brauchbarkeit des Modells schaden. Bisher wurden keine umfassenden Analysen über den Einfluss des Detaillierungsniveaus auf Modellierungsaufwand und Ergebnisgenauigkeit (Leistungsfähigkeit) von fundamentalen Strommarktmodellen durchgeführt. In der vorliegenden Arbeit werden diese Themen behandelt, um einen Beitrag zur Formulierung und Implementierung von Strommarktmodellen zu leisten. Hierfür wird ein Strommarktmodell entwickelt, welches eine flexible Systemdarstellung ermöglicht. Modellvarianten mit verschiedenen Detaillierungsniveaus werden formuliert und zur Modellierung von drei europäischen Strommärkten verwandt. Die Leistungsfähigkeit jeder Variante wird hinsichtlich der Genauigkeit der Ergebnisse und des Rechenaufwands gemessen. Zusätzlich wird eine neue Methodik entwickelt, um die Anfahrkosten thermischer Kraftwerksblöcke in den Grenzkosten zu berücksichtigen.

Chapter 1

Introduction

1.1 The Role of Market Models in Liberalised Electricity Markets

A global trend towards the liberalisation of the power industry has led to the creation of regional wholesale and retail markets for electricity. Wholesale markets are meeting places for major power producers and consumers to buy and sell electricity in bulk and in a competitive way. They are created with the objective of allowing the forces of supply and demand set a fair price for electrical energy.

In most wholesale markets electricity can be traded for physical delivery or financial settlement and for different time horizons. In the *spot market*, electricity is traded for the near future, a few hours ahead of the physical delivery. The *spot price* is the monetary value of a unit of electrical energy traded in the spot market. In a market with a uniform-pricing scheme, the spot price dictates the revenue of all power producers and the expenditure of all power consumers that participate in the spot market. It also serves as a reference for the price of power traded through bilateral contracts and the price for end customers in the retail market.

Electricity as a form of energy, possesses a series of attributes that separates it from most commodities traded in liberalised markets, and largely defines its price's behaviour. Because it cannot be stored efficiently and economically in large quantities, the supply and demand must constantly be in balance. In addition, the total demand for electricity of most power systems is practically irresponsive to the wholesale electricity price in the short-term. Therefore, the electricity price is determined by the multitude of factors that affect the costs of production and the levels of supply and demand in the short-term. Many of these factors are uncertain and stochastic in nature. For these reasons, electricity prices are very volatile and price spikes are common [75].

The inherent uncertainty of electricity prices exposes market participants to risks. They rely on tools and on the opinion of analysts to manage price and other kinds of risk. In this regard, *electricity market models* play a very important role. These are mathematical tools developed with the objective of directly or indirectly modelling electricity prices. They provide insight into the price building mechanism and a view of future electricity prices [72].

Several methodologies can be applied to model electricity markets. The selection of a particular methodology depends mainly on the objective of the modelling exercise. One of these modelling methodologies is the subject of analysis in this work. The approaches most commonly applied are introduced in the next section.

1.2 Methodological Approaches to Market Modelling

Based on the methodology, market models can be classified as follows:[1]

- Financial models
- Production cost models
- Market simulation models

Financial models apply techniques for the analysis of time series to identify behavioural patterns and trends in electricity prices, and to identify relationships with other variables. These observations can be used to formulate equations, or models, of electricity prices. By assuming that patterns, trends and relationships will repeat over time, financial models model can be used to forecast electricity prices directly.

Thanks to their computational efficiency, financial models are often applied in the stochastic modelling and analysis of electricity prices. This is useful in the valuation of financial products and physical assets.

The econometric techniques applied to formulate financial models require lengthy time series to build accurate models. Therefore, they are not adequate for the modelling of young markets or markets than have recently undergone structural changes. For this reason, financial models are mostly applied for the short-term forecast of electricity prices.

According to the microeconomic theory of pricing, in a competitive market where all participants act as price takers, the spot price of electricity should correspond to the short-run marginal cost of production. The so called ***production cost models*** are tools that aid in the calculation of the systems marginal cost. They are usually formulated as optimisation problems where the objective is the minimisation of the total operational costs. The result,

[1] Several classifications can be found in the literature. See for example [7, 12, 75]. Although Enzensberger [12] presents a more general classification of *energy system models* (Energiesystemmodelle), the categories can also by applied to a narrower classification of electricity market models.

is the optimal utilisation of the power generation units. From this result, the marginal cost of production can be derived.

Production cost models are based on a simplified mathematical representation of the technical and economical characteristics of the underlying power system. This property enables them to capture the kind of structural changes that econometric models are unable to, making them also adequate for the long-term modelling of electricity prices. On the other hand, it is sometimes difficult to access and gather the kind of technical data and parameters that production cost models require to make a bottom-up representation of the power system. Furthermore, depending on the level of detail, the modelling effort required to solve this kind of models can be significant.

The usefulness of production cost models in liberalised electricity markets has been the subject of discussion in recent years. Most of the criticism targets the assumption of perfect competition and the convergence of marginal costs and electricity prices. According to some observations, electricity prices in certain markets and at certain time periods seem not to correspond to marginal costs. For this reason the line of research has shifted towards the development of ***market simulation models***. Contrary to production cost models, market simulation models try to reproduce the strategic behaviour of the market participants [74]. Most market simulation models apply methodologies based on game theory, such as conjectural variations or agent based simulations, and are able to reproduce the outcome of markets with different levels of competition, from perfect to collusion.

Market simulation models require long time-series of detailed data in order to be calibrated, that is, to identify the behaviour of individual market players. In most electricity markets, the necessary information regarding the price of ask and bid offers and the cost of generation is not disclosed. Furthermore, for their application in the modelling of future electricity prices, modellers must make assumptions on the bidding strategies of market participants, which is a highly speculative exercise. For this reason, the results of market simulation models can be very uncertain.

According to some classifications, models based on the *system dynamics* methodology, belong to the market simulation category. System dynamics models attempt to represent complex dynamic relationships between market factors with the help of feedback loops and delayed reactions to stimuli. An application to the modelling of electricity markets can be found in [21]. These models are also difficult to calibrate and can display convergence difficulties.

Hybrid models that take advantage of different modelling methodologies can also be developed. For example, production cost modelling can be combined with financial methodologies to seize the ability of the former to generate electricity prices based on market fundamentals, and the ability of the latter to take into account stochastic factors [75].

1.3 The Case for Production Cost Models

In section 1.1, the role of market models as tools for risk management was mentioned. But market models serve several other purposes. For their versatility and sound theoretical basis, production cost models have found applications in various fields within the power industry, and have gained an important spot as one of the preferred analysis and decision support tools.

In the hand of savvy traders, they help identify business opportunities and support the market-making and speculative trading of futures, options and other physical and financial products. In this respect, they also play an important role for the creation of liquid markets.

Production cost models are also used by regulatory agencies for market monitoring. Thanks to their ability to reproduce the outcome of perfect competitive markets, they help assess the level of competitions and identify the exercise of market power [8, 23, 43, 69]. For the same reason, production cost models have been formulated prior to the liberalisation of regional power systems to simulate the possible outcome of a future spot market [2].

The applicability of production cost models goes well beyond the sole modelling of electricity prices. Since these models also reproduce the optimal scheduling of power generating units, many authors have applied production cost models to measure the effect of technical and economical factors on, for example, the utilisation of the installed capacity or the emission of greenhouse gases [28, 30, 33, 36, 47, 60].

As mentioned before, because of issues associate with the modelling effort, production cost models are mostly applied for the deterministic representation of electricity prices. Nevertheless, the backbone of many stochastic methods aimed at calculating the stochastic characteristics of the marginal cost, is based in some sort of production cost model [40, 73].

The above examples are evidence of the usefulness and value of production cost models. In spite of the criticism surrounding their application in liberalised electricity markets, production cost models clearly still have a strong case in their favour as fundamental tools. For this reason, they take the spotlight in this work.

1.4 Issues Associated with the Development and Application of Production Cost Models

Production cost models have been praised for their ability to make a detailed, bottom-up representation of power systems. This property suits them with the flexibility to model markets and systems with different configurations and operational frameworks. But in some circumstances, it is precisely this strength also their greatest pitfall. Next, three issues associated with the development and application of production cost models are discussed.

1.4.1 Data Availability, Modelling Effort and Level of Detail

Power plant operators in pursue of maximising their profits, optimise the usage of their units within the limits imposed by tight technical and safety restrictions. This is done with the help of modelling tools which rely on the mathematical representation of the operational characteristics of the power plants. Production cost models aimed at calculating the system marginal cost are formulated in a similar way to these tools.

The task of modelling an electricity market is, however, on a completely different scale. A production cost model must consider the whole power system participating in a market, and not the handful of power plants of a given operator.

In this respect, the formulation and application of these models presents some challenges. In the first place, the availability of complete and reliable data can be a serious issue [75]. The technical parameters of power plants, their operational costs and other essential data, is often regarded as being commercially sensitive and therefore is not made publicly available. Even data of public nature, such as the electricity consumption of nations or regions within interconnected power systems, is in some cases not gathered or published by the corresponding authorities. And even when it is published, the quality and consistency of the data can be doubtful.

The second challenge is related to the effort required to solve production cost models and obtain accurate results. Depending on their formulations, production cost models can consist of a large number of equations and variables. The resulting optimisation problem could require relatively high computing power and time.

Both the data availability and the modelling effort are closely related with level of detail of the model. By level of detail is understood the complexity of the mathematical representation of the system components and the time resolution of the input data.

Power plant operators make their best effort when optimising the commitment and dispatch of their units by considering as many of the relevant technical and economical operational constraints. Based on this premise, it follows that to reproduce the marginal cost of production accurately, a production cost model should also consider these constraints in detail. On the other hand, a higher level of detail carries with it the formulation of a higher number of equations and, in most cases, requires a larger amount of data and a higher modelling effort [59]. Therefore, the selection of the appropriate level of detail is of particular importance in the development of production cost models.

To deal with the issues of data availability and modelling effort, it is common practice to reduce the level of detail of models. This can be done by neglecting technical constraints or by decreasing the time resolution of the input data [75].

1.4.2 System Boundaries and the Consideration of the Power Exchange

Another issue to be reckoned with when developing production cost models, is the consideration of the power exchange between neighbouring power systems. This is especially relevant when modelling power systems spreading over a large geographic area where several regional markets might coexist.

Regional integration resulting from effective cross-border power trade can yield economic gains from cost reduction. It also increases the role that imports and exports play in the power balance and security of supply of the interconnected power systems [20, 21, 32, 43]. Depending on the level of integration, it could also have important effects on regional electricity prices [32, 43, 56].

In this regard, setting adequate system boundaries can be crucial for the success of any model. Boundaries determine which regions are modelled. From this depends which routes of power exchange are modelled endogenously and which are considered exogenous factors. If several regions are modelled, it is possible to take into account the influence of fundamental factors on the power exchange. However, there are the above-mentioned issues of data availability and modelling effort to consider. Even more, the uncertainty and inaccuracy in the modelling of individual regions is propagated throughout the model to all results.

This issues have been addressed in some models by relying on an uneven representation of the interconnected region. The idea is to make a greater effort in the modelling of the power systems of focus markets, while simplifying the representation of the power systems of less relevant regions, or of regions where the premises of the model do not fit. This strategy allows the endogenous calculation of the power exchange while in theory requiring a lower modelling effort. An uneven representation of the regional power systems, however, could also be a source of inaccuracy and uncertainty in the results. Different levels of detail are expected to yield different total costs for the same system, which could affect the power exchange, the regional power balance and, ultimately, the system marginal cost.

1.4.3 Incorporation of Start-up Costs into the System Marginal Cost

Depending on several technical and economical factors, the cost of starting up a power plant can be a significant part of the plant's operational costs. Start-up costs are therefore relevant for the scheduling decisions of power plants operators. Directly and indirectly, start-up costs play an important role in the total cost of generating electrical energy and thus in its market price.

It has been noticed by some authors that production cost models have a tendency to systematically underestimate spot prices during peak hours and to overestimate them during off-peak hours [23, 31, 61, 64]. This has been attributed to the fact that some methodologies applied to solve the optimisation problem, fail to incorporate the start-up costs into the system marginal cost.

If power plants operators would neglect the start-up from their operational costs and corresponding bids to the wholesale market, they would incur lower profit margins or even financial losses. For instance, a hypothetical peaking unit that must be started for a few hours during which it is the price setting unit, would only break-even by selling its output at a price equal to the average total cost of production including start-up costs.

During periods of low demand and low market prices it might be more economical for power plant operators to sell the produced energy at the market price even if this is lower than the average cost of production to guarantee that the unit stays in service during this period. This would prevent having to turn the plant off and subsequently on during the following period of high demand and carry the burden of the start-up costs. This temporary bidding strategy is what may cause spot prices to lay under the actual marginal cost of production.

The failure of production cost models to capture these realities, weakens the representativeness and validity of the system marginal cost.

1.5 Objectives

Production cost models are built with the objective of reproducing the marginal cost of production of electricity of power systems that, under certain assumptions, should approximate the spot price of electricity in a market operating in this system.

The usefulness of a model for this task is determined by its ability to produce accurate results with an acceptable modelling effort. The accuracy can be measured according to the deviation of the system marginal cost from the electricity price in the corresponding market [65, 75]. The modelling effort can be measured based on the time required to gather the necessary data (parameters, time series), keep the data up-to-date, and run the model (solve the optimisation problem). As mentioned before, both factors depend on the level of detail of the model.

The effect of the level of detail on the performance of production cost models has been the subject of little analysis until now. Any error introduced in the results by simplifications made in the formulation of the model is rarely quantified. Instead it is often the subject of conjecture, regarded as an inherent source of uncertainty or, at the most, qualitatively estimated with some sort of theoretical analysis.

Cumperayot [7], Kreuzberg [31] and Müsgens [44] are of the few to have quantified the effect that modelling certain constraints has on the modelled marginal cost of selected power systems. The analysis by Müsgens is limited to the effect of considering start-up costs and minimal load restriction in the operation of thermal power plants on the marginal cost of the German power system for one typical day of two months. However, the results are not judged against the corresponding market prices. Kreuzberg before him made a very similar analysis, but did compare the results of the base model with the spot price of the regional power exchange. The analysis by Cumperayot is much wider. But like Müsgens, he did not evaluate the accuracy of the results. Non of the aforementioned authors measured in any way the modelling effort.[2] And none has addressed the question of the effect of the level of detail on the modelling of the marginal cost of interconnected power systems using multiregional models.

The main objective of this work is to make a contribution to the formulation of production cost models by analysing the effect of the level of detail of production cost models on the accuracy of the results and on the modelling effort. In particular it will address the first two issues mentioned in the previous section, namely, the reduction of the level of detail in the formulation of the model as a way of managing the effort involved in the modelling of single and multiregional power systems. Additionally, a methodology to address the issue of the consideration of start-up costs in the system marginal costs should be developed and applied. These objectives will be achieved in the following steps:

1. The price building mechanism in wholesale electricity markets and the basis for the formulation and application of production cost models are briefly reviewed in chapter 2.

2. In chapter 3, the power system to be modelled is analysed and an adequate production cost model formulated. The most common modelling approaches and typical model simplifications are also discussed.

3. Variants of this model with different levels of detail are applied in the calculation of the marginal cost of the power systems associated with actual electricity markets based on historical data.

4. The accuracy of the results is evaluated by comparing the marginal cost with observed spot prices. The modelling effort of each variant is also measured. The results are summarised and analysed in chapter 4

[2] Only Kreuzberg provides a reference value of the computing time in [31, page 78].

Chapter 2

The Basis of Production Cost Modelling

A wholesale market for electricity can take many forms depending on the market design. This is the set of guidelines that determine, among other things, the terms of the trade and the rules market participants have to follow. The architecture of most markets is adapted to the particularities of the underlying power system. Nevertheless, most modern markets have learned form past experiences and now share several characteristics. The following is a superficial description of a generic wholesale market for electricity, of the sort that motivates and serves as basis for the formulation of production cost models. Detailed description of different wholesale markets can be found in the literature (e.g. [37, 68]) and in the documentation issued by local regulatory agencies and market operators.

2.1 The Wholesale Electricity Market

Wholesale electricity markets are meeting places for major power producers and consumers to buy and sell electricity in bulk and in a competitive way. They are created with the objective of allowing market forces set a fair price for electrical energy, and pursue the greater goal of maximising the system's welfare [76].

In most wholesale markets, electricity can be traded well in advance of and right until the time of delivery. The period of delivery might by as short as a quarter of an hour and as long as several years. Upon the due date, the deal is settled. Settlement could be financial or physical. In the former, only the amount of money corresponding to the value of the energy according to some index changes hands. In the latter, electrical energy is actually generated.

It is common to divide the wholesale market into smaller sub-markets based on the nature of the deals that take place in it. In the *forward market*, contracts with delivery date more than a few days in the future are traded. Usually the delivery time is equal or larger to one month and the settlement can be financial or physical. In the *spot market*, electricity is traded for physical delivery in the near future. Since the outcome of the spot market may affect the operating schedule of the power generation units, for technical and security reasons, financial settlement takes place ahead of delivery, usually one day in advance. For this reason, the spot market is also often referred to as the *day-ahead market*. The *spot price* is the monetary value of a unit of energy traded in the spot market. In some cases it is also possible to conduct continuous trading up to one hour ahead of delivery in a *real-time market*. According to some authors, not the day-ahead market but the real-time market should carry the title of spot-market, but this is only a matter of convention. Here, the initial definition (spot market=day-ahead market) will be used.

There are several ways in which electricity can be traded in the forward and spot markets. Figure 2.1 presents a broad classification of market designs according to the level of centralisation and form of arrangement. Many wholesale markets combine different modalities, for example facilitating a centralised and coordinated power exchange, while also allowing the bilateral trade between parties. In this respect, brokers, dealers, exchanges and pools play the most important roles.

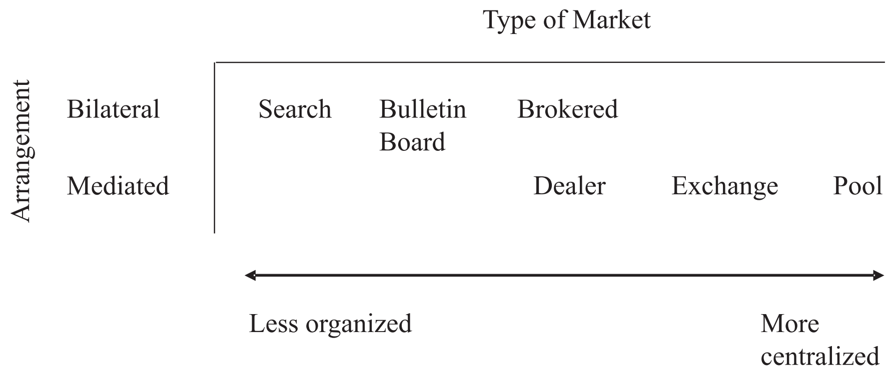


Figure 2.1: Market types according to the transaction arrangement and the level of coordination [68, page 86]

Power exchanges or power pools[1] are trading platforms which facilitate the trade of electricity in an efficient, transparent and competitive way. They profit mainly from fees charged for the participation in the exchange and for each transaction that takes place in them. Most power exchanges act as safe counterparts for all transactions, eliminating the counterparty risk present in bilateral trading. In most wholesale markets, regional authorities assign a power exchange the duty of operating a centralised and mediated spot market. The spot price determined in a power exchange is an important reference for the pricing of contracts traded in other sub-markets.

[1] Although there are structural differences between power exchanges and power pools, from here on, *power exchange* is used to refer to both.

Brokers and dealers are in the centre of the spectrum, creating decentralised but organised sub-markets. Brokers are entities that arrange the bilateral sale and purchase of electrical energy directly between buyers and sellers. They do not take own positions and profit from fees charged for the brokerage service. Dealers, in contrast, usually keep an inventory of contracts and profit from temporal price differences in the contracts (buying low and selling high). While in power exchanges mostly contracts with predefined and equal conditions for all market participants are traded, brokers offer greater flexibility, allowing and facilitating the trade of non-standard contracts.

The sub-markets created outside the auctions or continuous trading of the power exchanges and that are based on bilateral transactions, are commonly known as *over-the-counter* (OTC). Although OTC markets are usually associated with brokers, there is a growing trend towards power exchanges also facilitating and clearing bilateral deals and therefore taking part in the OTC market. This allows the trading of non-standard contracts without the counterparty risk of brokered bilateral trading.

According to the law of one price, in an efficient market all identical goods must have a single price [76]. Applied to the wholesale market for electricity, the price of a unit of energy for delivery at the same time and place must have the same price regardless of whether this unit is traded in the centralised or in the OTC markets. Any gap between the prices in these markets will be closed through arbitrage.

2.2 The Price Building Mechanism in a Centralised Auction-Based Spot Market

In centralised spot markets, trading can take place continuously – from the opening of the market for a specific contract until the time of delivery – or in periodic closed auctions. In the continuous trading, offers to buy or sell electricity are matched with other offers as they are received. For the closed auction, offers are accepted within a time period, at the end of which an auctioning algorithm determines a price. Auctions have the advantage of being transparent, economically efficient and flexible enough to accommodate different forms of bidding. In Europe, for instance, almost all power exchanges rely on some sort of closed auction to settle the spot market [37].

A typical closed auction for the spot market is organised and conducted on the trading day previous to the delivery day. The delivery day is divided into regular time intervals, commonly hours or half-hours. Those who wish to participate in the spot market submit a set of *bids* or offers to buy or sell electricity. Each bid represents a contract to deliver electricity during a time interval of the delivery day in a specific location, usually a control area of the system. The anatomy of a bid depends on the market design and is one of the main differences between centralised spot markets. Bids can be very simple, specifying only the price and volume of energy wished to be bought or sold. The traded volume can in some cases be expressed as a function of the price. Bids can also be associated with specific power generation units and include related operational or economical constraints. In most markets it is also possible to submit block bids, which are a series of consecutive and equal offers. In this case too, the volume could be price dependent or independent.

During the *market clearing* process, the market operator matches the individual bids for each time period for each control area in an auction to maximise the number of orders traded. Under the *uniform pricing* scheme, electricity is priced by aggregating the individual buy and sell orders for each period into demand and supply curves and intersecting them. If block bids were submitted, the aggregated supply and demand curves of each time period take into account the corresponding portion of the blocks. The price level in the intersection of the aggregated supply and demand curves is the *market clearing price* or *spot price* for electricity (see figure 2.2). All orders executed are priced at the level of the market clearing price. That is, all consumers pay the same price to all generators in the control area.[2] The algorithm used in the market clearing process must consider the interdependency over time of the aggregated supply and demand curves through the block orders, and must respect any other constraints imposed on the bids. Depending on the complexity of the bids, it might be necessary to iterate the market clearing process until a feasible solution to the spot market is found.

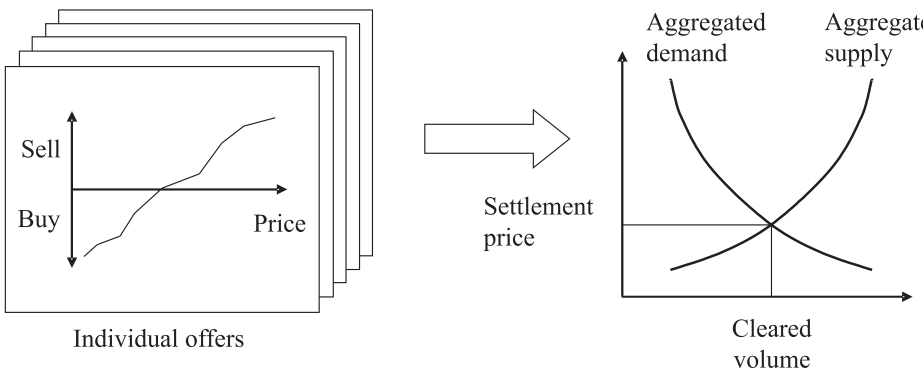


Figure 2.2: Schematic representation of the market clearing procedure and resulting price

[2]In markets with a zonal or nodal price schemes, the spot prices are determined for smaller portions of the system. This is usually done in the face of congestion of the transmission network, where the restricted flow of power leads to price differences between nodes (see section 3.1.5).

2.3 Modelling the Spot Price through the System Marginal Cost

Modelling the spot price for electricity in a wholesale market such as the one described before, can be a very challenging task. The main problem stems from the fact that producers and consumers have several option to participate in the spot market and in the wholesale market in general. Hence, modelling the portion of the supply and demand that is traded in each sub-market is difficult.

If the market design allows it, market participants often close long-term contracts for physical delivery or trade financial instruments in the forward market as means for managing risk. Later, they might try to leverage these positions by buying or selling in the spot market at better prices. As a result of this strategy, on the supply side of the spot market, mostly the capacity that has not yet been allocated to cover long-term obligations is offered. On the demand side, in addition to the purchase orders of actual consumers, the capacity that has already been sold forward might actually be submitted as purchase bids at a price lower than the one with which it was initially sold. Additionally, part of the available generating capacity is withheld from the market to fulfil the obligation of power plant operators to provide ancillary services, in particular capacity reserves.

But there is a way to work around these issues.

In a competitive market, each participant has only a small share of the total supply and is therefore unable to manipulate the market prices by adjusting its output. In this case, all participants act as price takers and must minimise their costs to maximise their profits. This is done by adjusting their output at a price equal to their average cost of production. Under these conditions, according to the general microeconomic theory of pricing, the spot price corresponds to the *marginal cost* or the incremental cost of producing an additional unit of output [76]. Since all market players minimise their individual production costs, the total supply cost will be also the minimal possible for the given level of demand. In the context of competitive electricity markets, such as the centralised spot market explained in the previous section, the marginal cost can be accordingly defined as the derivative with respect to the demand (volume) of the total cost of providing electricity at the level of the cleared demand.

Schweppe et al. [62] was of the first to suggest the use of *production cost models* to determine the marginal cost of production. Such models would be similar in formulation and implementation to the "*computerised economic dispatch logic*" used by power plant operators. These tools rely on the mathematical representation of the power plants and the constraints that govern their operation to optimise their utilisation.

Production cost models overcome the difficulties of modelling the aggregated supply and demand functions of a centralised market by modelling the complete power system.

In accordance to the premises of competitive markets, a production cost model determines the output of the power generation units to supply the total demand for electricity with minimal total cost.

The original idea of production cost modelling has been expanded in many directions to take into account the particularities of individual markets. In this respect, the level of complexity of models has an unlimited growth potential. It has also been contracted to the most elementary levels of simplification, neglecting all but a few fundamental constraints.

The simplest form of production cost model includes only two constraints:

1. The total output of all units must be equal to the instantaneous demand,

2. The total power generation of all units must be positive and lower than or equal to a generation capacity limit.

In this form of model, each power plant is treated as a price dependent selling bid, with zero volume before a price level, and a volume equal to its maximal output after the price level. The price level corresponds to the average variable cost of production of the unit when this operates at full capacity. When the individual bids are aggregated in increasing order according to price (the result of the auctioning process), the resulting supply curve corresponds to what is also known as the *merit order curve* of the system (see figure 2.3).

The spot or market clearing price, in accordance to the price building mechanism described in the previous section, is given by the average cost of the most expensive unit, or bid, needed to cover the demand. This unit is also called the *marginal unit.*

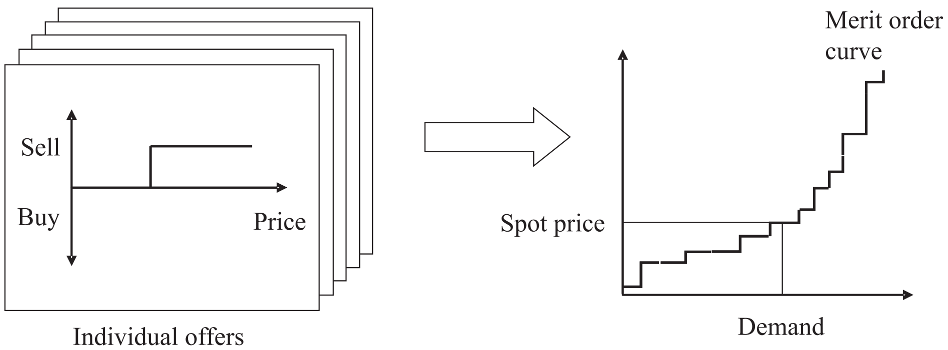


Figure 2.3: An example of a merit order curve

For its simplicity and transparency, this form of production cost model is widely used in the qualitative and quantitative analysis of spot prices. But its simplicity is also its major pitfall. Modelling the generation units as price dependent selling bids neglects important technical restrictions in their operation. This formulation assumes, for example, that all units are capable of a continuous output from zero to maximum, and that the average cost of production is constant in this range. Because of these approximations, the resulting marginal cost function (the merit order curve) is strictly increasing and smooth along the range of cumulated output. But as Schweppe [62] pointed out, due to the fuel consumption characteristics of the units and their start-up costs, in reality this curve might not be smooth or even monotonous. Even more, the merit order curve approach neglects important constraints in the operations of the system such as the provision of ancillary services. These constraints might also affect the scheduling of the units.

Chapter 3

Formulation of a Production Cost Model and Methodology for the Evaluation of its Performance

To fulfil the objectives proposed in his work, an adequate production cost model was developed. The following chapter presents the formulation of the model and the methodology applied for the analysis of the model's performance.

3.1 Model Formulation

As mentioned in previous sections, production cost models have a long tradition in the industry, and have been applied for the quantitative analysis of the generation scheduling and costs in many instances. For the development of this model special attention was paid to the formulation of preceding models. The following production cost models served as references:

- **A model developed by Becker** [2], that explored the marginal cost pricing of electricity with regard to the early market liberalization efforts in Europe.
- **The model E2M2[2] applied by Oeser** [47], actually a deterministic version of a stochastic model for generation scheduling under uncertainty, applied to investigate the thermal-generation-related environmental effects of wind power generation.
- **A model developed by Schwarz** [61] aiming to explain the rise in electricity prices in the German market and address allegations of market manipulation.
- **The model EUDIS developed and applied by Kreuzberg and Müsgens** [29, 30, 31, 32, 43], a multiregional model of the interconnected European power

system that applies a complex heuristic approach to model the generation scheduling with a relatively low computational effort.

- **A model developed by Cumperayot** [7] to simulate the marginal cost of the German power system, applied to answer questions similar to the ones posed in this work.

- **A model developed by Schröter** [60] to analyse the effect of the European emissions trading system on the generation scheduling in the German power system.

- **The model GEMM developed by Kramer** [28] to model marginal costs of electricity in some European countries.

This is only a selection of publicly accessible models, meant to cover the most common modelling methodologies and simplifications. Other models are developed by utilities or consulting firms for their own use, but these are normally not accessible to the general public.

Also a selection of generation scheduling tools were examined to have a view of the problem from the point of view of power plant operators.

3.1.1 Nomenclature

The system of equations that define an optimisation problem is composed of parameters and variables. While parameters have a constant value, the value of the variables is determined during the optimisation procedure. Both parameters and variables are defined for elements of sets. Sets have a discrete, finite and predefined total number of elements.

In the mathematical formulation of the objective function and constraints of the developed model, parameters and variables are represented by characters of the Roman or Greek alphabets. In some cases they are accompanied by superscripted letters indicative of a particular property that differentiates them from related parameters or variables with the same symbol. While P, for example, stands for *electric power*, P^{max} stands for *maximal* electric power. The elements for which a parameter or variable is defined are listed in subscript. Building on the previous example, the symbol $P^{max}_{(th,\,t)}$ corresponds to the maximal electric power for the elements th and t. If multiple elements of a set appear in an equation, they are distinguished by the prime (dash) symbol, as in t, t' and t''. Sets are represented by capital Roman letters, as in $th \in TH$. Variables are distinguished from parameters by a tilde underneath the corresponding symbol. $P^{max}_{(th,\,t)}$, for instance, is a parameters, and $\underset{\sim}{P}_{(th,\,t)}$ is a variable. Unless otherwise specified, variables are defined over $\mathbb{R}^{0+}$.

The parameters and variables appearing in an equation are listed directly after its formulation. In the list is included the symbol, physical unit and a short description of the parameter or variable, as illustrated in the following example:

$$\underset{\sim}{P}_{(th,\,reg,\,t)} \leqslant P^{max}_{(th,\,t)}$$

for all $reg \in REG;\ th \in TH;\ t \in T$ with

$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg at time t
$P^{max}_{(th,\,t)}$	$\mathrm{MW_{el}}$	Maximal electric power output of unit th at time t

3.1.2 Model Functionality

As explained in section 2.3, the unit commitment problem proposed by a production cost model can be formulated as an optimisation problem. The objective is to minimise the total cost of production (see section 3.3.1). The optimisation variables are the output of the power generating units and the exchange of power between interconnected regions. Other auxiliary variables might also be required depending on the model formulation. Technical, economical, and environmental restrictions for the operation of the system are integrated into the optimisation problem in the form of constraints. These constraints form a system of linear or non-linear equations. Its solution is the optimal generation schedule. From this result, the marginal cost of production of electricity can be derived (see section 3.3.3). A schematic representation of the functionality of the model is shown in figure 3.1.

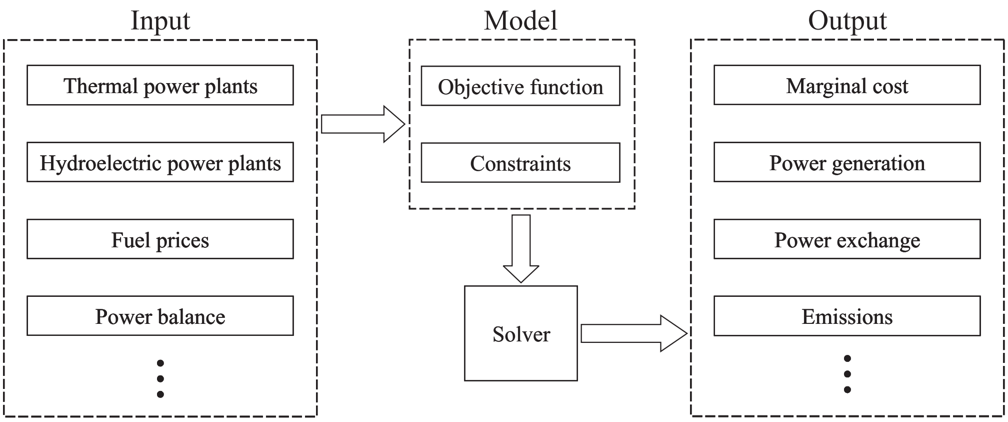


Figure 3.1: Production cost model functionality

3.1.3 System Boundaries

Production cost models overcome the difficulty of modelling the residual supply and demand participating in the spot market by modelling the complete power system behind the market instead (see section 2.3). For a model to make a valid representation of the market, it is important to correctly identify and set the boundaries of the corresponding power system. Since different methodologies are better suited to model thermal or hydro dominated power systems [19, 45, 46], the boundaries could also be crucial for the formulation of the model.

Nowadays it is common to find multiple electricity markets functioning within interconnected power systems. Electricity markets are usually delimited by the political boundaries of countries or of the states within. They might be further delimited by the boundaries of portions of the systems where electricity is allowed to flow freely through the transmission network and a single price can be calculated. The presence of congestion in the network produces different levels of scarcity across the system and leads to the formation of price differentials [30].[1] The boundary of a market can be determined by the points of the network (transformers, substations, lines) where bottlenecks are formed or represented. Usually these are also the points where the physical power exchange with the rest of the system is measured.

The electricity demand in a region is normally determined by measuring the output of power transformers serving a portion of the load. This allows counting the transmission losses as part of the total consumption. Depending on the voltage level of the network where power is measured, the load might be different. This is usually caused by the presence of net power injections in the sub-transmission or distribution network, such as the injection from distributed generation (e.g. small cogeneration units or wind turbines).

Figure 3.2 is a schematic representation of a power system and the parts being modelled. "Horizontally", the system is delimited by price regions. In the developed model, a regional market and the corresponding power system is identified with the index reg which belongs to the set of all modelled regions REG. "Vertically", the system is delimited by the points in the grid where demand is measured. The limits in both axes determine the portion of the system (number of power plants, level of consumption) to be modelled.

3.1.4 The Demand for Electricity and the Time Resolution

The demand for electricity of a power system determines the commitment and loading of power plants over time. It has therefore a direct effect on the cost of production and ultimately on the market prices.

Electricity as an energy source to drive appliances, lighting and machinery is hard to substitute in the short-term. In addition, residential, commercial and most industrial consumers negotiate with retailers long-term power delivery contracts at fixed prices. Therefore, the power consumption of most end consumers, and thus of the power system as a whole, is practically irresponsive in the short-term to variations in wholesale market prices.

[1] In regions where the level of congestion is not severe (not frequent), and this could be relieved or managed locally, it might be still possible to define a single price.

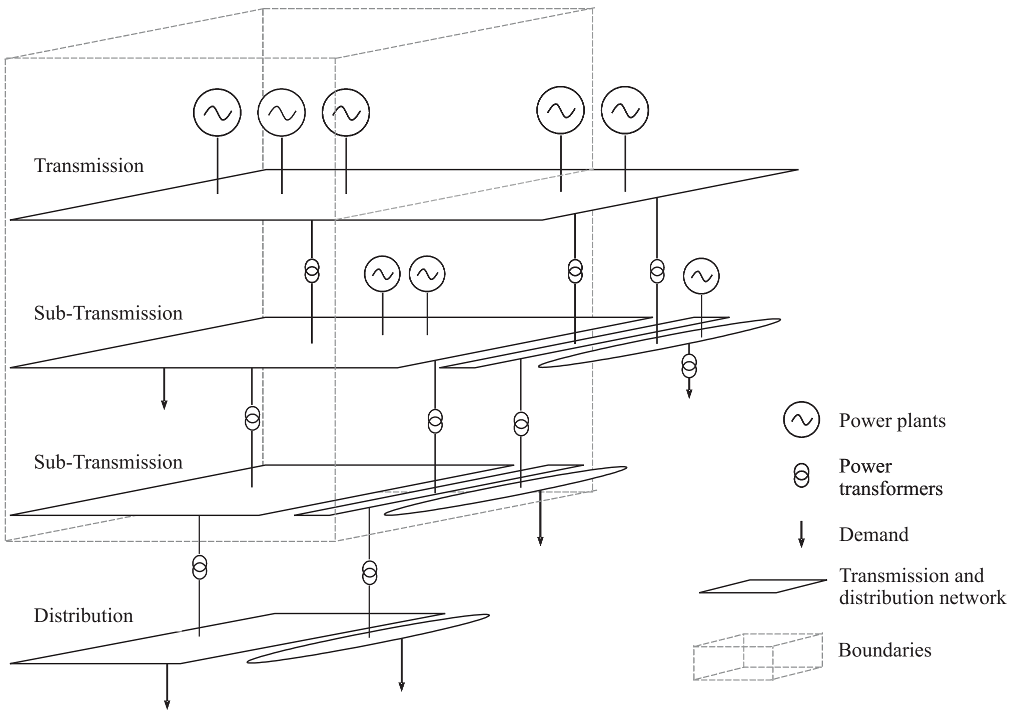


Figure 3.2: Schematic representation of the system boundaries

In the developed model, as in most production cost models, electricity is considered price irresponsive or *inelastic*.

The demand for electricity demand is determined by human activity and environmental factors. It is therefore characterised by various cycles over different time periods. The link to human activity can be easily recognised in weekly and daily cycles, consumption being higher on work days than on weekends, and during day than during night. Changes in temperature and solar radiation affect the consumption of electricity for room heating and illumination. Therefore, demand is cyclical over the seasons of the year, and daily consumption pattern are different during summer and winter months.

The representation of the demand and the formulation of production cost models go hand in hand. The modelling of certain constraints in the operation of thermal power plants requires a minimum of information on the order of occurrence and duration of load levels. For this reason, time series of the load is the preferred input data for production cost models. The resolution of the time series should be high enough to capture fluctuation over time of the load, wind power generation and other net power injections [36]. For the short term scheduling, a time resolution of 15 or 60 minutes is normally used [1, 18, 46, 67]. Since in most spot markets power is traded in hourly time intervals,[2] an hourly resolution allows the consideration of transactions within the spot market and are therefore preferred [46].

[2]By the time of this writing, only in the day-ahead markets of Australia and the United Kingdom, power is traded half-hourly

Power plant operators usually have first-hand knowledge of the load against which own generators are scheduled. But when it comes to modelling large power systems occupied by several utilities, the information necessary to model the load and the power balance is sometimes not easily accessible or is not at all available. Even more, optimising the commitment and dispatch of a big number of generators (in the order of hundreds or thousands) against hourly load values, especially for long time horizons, is unpractical due to the resulting computational effort. Modellers often deal with these issues by taking advantage of the cyclical nature of the demand, working with a few representative periods of the demand within the modelling horizon instead. If chronological data is available, *typical days* can be built by averaging the hourly values of the load of the corresponding real days. It is also common practice to average hourly values of the load of the typical days into larger *time segments* [39]. When using this approach, the model is formulated to optimise the output of the generators against the load of the time segments of the typical days. Of the models reviewed for this work, only Schwarz [61] uses hourly resolution.

The computational advantages of working with typical load periods can sometimes be outweighed by the loss of information on the chronology of the load and by the error introduced by the averaging of load levels. Both factors can affect the modelled schedule of the power plants and total costs of the system as demonstrated in [18, 38]. To be able to isolate the effects of the model simplifications on the results, the production cost model developed for this work optimises the commitment of power plants against chronological time series of the demand and a time resolution of one hour was chosen. The modelling time horizon is divided into T ordered time segments t such that $t = 1, \ldots, T$.

The condition of instantaneous balance between total supply and load in each region represents the first and most important constraint in the presented model:

$$L^{tot}_{(reg,t)} = P^{tot}_{(reg,t)} \tag{3.1}$$

for all $reg \in REG;\ t = 1, \ldots, T$ with

$L^{tot}_{(reg,t)}$	$\mathrm{MW_{el}}$	Total electricity demand of region reg at time t
$P^{tot}_{(reg,t)}$	$\mathrm{MW_{el}}$	Total net power injections into region reg at time t

To the total net power injections into a region belong the power generated by power plants in that region and the power exchange with neighbouring regions. Both sources can be endogenously modelled or given to the model as exogenous parameters. All sources represented in the model are discussed in the following sections.

3.1.5 Modelling the Transmission Network

In all modern power systems the electricity produced by power generators is transported over long distances and in bulk amounts over a transmission network to the point of consumption. The transmission network can affect the system marginal cost if the fees charged for its usage directly affect the production costs, or if constraints in its operation affect the scheduling of the power plants.

Transmission fees are charges paid for the right to be connected to, take power out of, or inject power into the network. These fees are meant to cover the costs of maintenance, ancillary services and transmission losses [24]. Additional fees might be charged if the limit of the transmission capacity of part of the network (one or more transmission lines) is reached. In such case, a *congestion fee* is set for the usage of the network. The presence of congestion in a transmission system is extremely important for many reasons. It signals that the capacity of the network is inadequate for the size of the market. Congestion can also force the creation of price zones, potentially reducing liquidity and hampering competition. Several methods can be applied to calculate and allocate transmission and congestion fees. The inclusion of the corresponding constraints and of cost components into the objective function should be tailored to the specific framework. An overview of methods for congestion management is presented by Shahidehpour et al. [65].

In most European power systems, transmission fees are paid largely or wholly by consumers. The portion paid by generators is normally negligible [16]. Fees arising from internal congestion (within the market) are determined by local transmission system operators based on the cost of relieving it by whichever method is used (e.g. counter-trading or re-dispatching) and are included in the transmission fees. This form of billing guarantees that all power traded in the market and to be delivered anywhere in the network has exactly the same price, independently of the distance or transmission path between producers and consumers. Internal transmission fees and the topology of the transmission network are therefore irrelevant for the modelling of marginal costs in these systems.

The calculation of fees for the usage of cross-border interconnections (with other markets) is a rather controversial issue. This is particularly true in the European power system due to the large number of interconnections and the vital role they play for the power exchange and security of supply in the region.

Cross-border interconnections are more prone to transmission congestion. For this reason, charges for their usage are dominated by congestion fees [51]. Congestion might occur either directly in the interconnection or somewhere else in the network but caused by the flow through the interconnection. In this case the congestion is usually represented on the interconnection itself.

Calculation of congestion fees and the methods for the distribution of the usable capacity is the subject of intense debate. While regulatory agencies traditionally push for market based, transparent and discrimination-free methods [9], transmission system operators would obviously favour methods that guarantee a source of revenue, and market participants find themselves divided between the increased risks and opportunities that come with different frameworks. In the last few years, many system operators and market players have given up to the pressure of regulators, relinquishing long-term contracts and changing existing methods in favour of explicit and implicit auctions [6, 27].

The idea of auctioning methods, just as in the auctioning of power in the spot market, is to let market forces set the price for the transmission capacity. If the capacity market works efficiently, the fair price for a congested interconnection should be equal to the difference of electricity prices between the two participating markets [6]. Higher or lower prices would imply that market participants are either loosing money form the cross-border trade or making extra money out of the capacity market. This in turn implies unjustified higher or lower revenues for the system operators. Assuming that transmission capacity markets work efficiently, and that the price for transmission capacity does, in fact, reflect the price gap between markets, the price paid by generators and power traders should not affect the scheduling of power plants. Under these premises, congestion fees for cross-border interconnections can also be neglected for the modelling of the marginal cost.

Further considerations made in the modelling of the power exchange between regions are discussed in section 3.1.8. They refer in particular to the paths and limits of the power flows.

3.1.6 Modelling the Electric Power Generation

The generation of electrical energy is achieved by transforming the energy contained in a primary source or fuel. A broad range of processes have been developed to utilise an equally broad range of fuels. The most widely used installations for power generation can be classified based on the fuel-process combination applied into thermal, hydro and wind power plants. Installations falling out of these three categories, such as photovoltaic panels, solar-thermal, wave and tidal power plants, play a less important role in modern power systems. The formulation of each kind in the model is done accordingly.

3.1.6.1 Thermal Power Plants

This category covers a wide variety of installations. All of them rely on thermodynamic processes to transform energy over many stages. In *steam turbine power plants*, the process begins with burning a fuel (oil, gas, coal, lignite, biomass, waste) in a furnace or combustion chamber, and using the heat released to generate steam at high temperature and pressure in a boiler. The steam then passes through and drives a set of turbines (hence the name) coupled to a generator. In *gas turbines power plants*, the mixture of hot gases resulting from the combustion is directly used to drive the turbine. Gas and steam turbines can also be combined into one cycle to make better use of the thermal energy resulting from the combustion. In *thermal nuclear power plants*, a nuclear reactor instead of a furnace generates the necessary heat to turn water into steam. All the processes mentioned above can be modelled with the same set of fundamental equations.[3]

Load Limits Thermal power plants are designed to operate safely, reliably, and economically within a range of power output. The upper limit is usually close to the nominal capacity of the unit. The lower limit depends on the technical characteristics of the unit and is related mainly to constraints in the operation of the steam generator [77]. At all times during the steady operation of a power plant, the power output must remain within these limits. In the model, if the power plant is not in service (not generating and injecting power into the system), the power output should be equal to zero. The operational states of the units can be represented with the help of the dimensionless binary variable $\underset{\sim}{U}_{(th,\,t)}$, where 1 represents *in-service*. The load limit constraints are formulated as follows:

$$\underset{\sim}{P}_{(th,\,reg,\,t)} \leqslant P^{max}_{(th,\,t)} \cdot \underset{\sim}{U}_{(th,\,t)} \tag{3.2}$$

$$\underset{\sim}{P}_{(th,\,reg,\,t)} \geqslant P^{min}_{(th,\,t)} \cdot \underset{\sim}{U}_{(th,\,t)} \tag{3.3}$$

with

$$\underset{\sim}{U}_{(th,\,t)} \in \{0;1\}$$

for all $reg \in REG;\ th \in TH;\ t = 1,\ldots,T$ with

$\underset{\sim}{P}_{(th,\,reg,\,t)}$	MW_{el}	Electric power injection of unit th into region reg at time t
$P^{max}_{(th,\,t)}$	MW_{el}	Maximal electric power output of unit th at time t
$P^{min}_{(th,\,t)}$	MW_{el}	Minimal electric power output of unit th at time t
$\underset{\sim}{U}_{(th,\,t)}$		State variable of unit th at time t

It is important to distinguish between the total or *gross* power output of the generator and the *net* power output made available to the system. In the latter, the own consumption of the plant to run pumps, fans and other auxiliary equipment is discounted. Only the net power output serves to cover the load and reserves. Hence, the values of P^{max} and P^{min} should be chosen accordingly.

Minimum up and down times Changes in the operational state of thermal power plants produce additional wear in its components due to the thermal stress. Therefore, a commitment with frequent state changes could reduce the technical lifetime of the unit significantly. To avoid this, power plant operators often impose restrictions in the minimum amount of time a power plant must remain in a certain state before changing it [18, 19,

[3] The dependency of the efficiency of gas turbines on the ambient temperature is hereby neglected. For this and other special considerations see [18, 63].

46, 67]. These restrictions are often neglected in production cost models as in [28, 47, 60]. Kreuzberg [31] argues that minimum times should only be considered when modelling imperfect competition, and imposes minimum up and down times of 2 time periods for all units in his model.

The model developed for this work is able to make full consideration of minimum up and down times. The impact of considering or neglecting these constraints in the modelling of the marginal cost will be subject to analysis in the following chapter.

For the commitment of the thermal units in the model, the following two conditions must be met:

$$\sum_{t'=t-t_{(th)}^{up,min}+1}^{t} \underset{\sim}{U}_{(th,t')} \geqslant t_{(th)}^{up,min} \cdot \left(\underset{\sim}{U}_{(th,t)} - \underset{\sim}{U}_{(th,t+1)}\right) \tag{3.4}$$

$$\sum_{t'=t-t_{(th)}^{down,min}}^{t-1} \left(1 - \underset{\sim}{U}_{(th,t')}\right) \geqslant t_{(th)}^{down,min} \cdot \left(\underset{\sim}{U}_{(th,t)} - \underset{\sim}{U}_{(th,t-1)}\right) \tag{3.5}$$

for all $th \in TH;\ t, t' = 1, \dots, T$ with

$t_{(th)}^{up,min}$	h	Minimum up-time of unit th
$t_{(th)}^{down,min}$	h	Minimum down-time of unit th
$\underset{\sim}{U}_{(th,t)}$		State variable of unit th at time t

The left hand side of equations (3.4) and (3.5) counts the number of periods that the unit has been in- or off-service prior to time t. The right hand side is not zero only if the value of the state variable changes between periods, i.e. when there is a change on the state of the unit.

Availability of Thermal Power Plants Power plants are complex systems with several interconnected and interacting components. If one of them fails to perform properly and the safe operation of the system is compromised, the power plant is forced out of service. Such events are stochastic in nature and future incidents can only be expressed in terms of probabilities. They can last from a few hours to several days, and might require the short-term commitment of reserves to cover the loss of capacity. To minimise the occurrence of unplanned outages and prolong the technical life of power plants, they undergo periodic revisions. During these revisions the generating capacity is totally or partially limited. Revisions can be scheduled well in advance to minimise their impact on the system. Both planned and unplanned outages affect the total capacity available in the system potentially affecting the outcome of the spot market.

This work focuses on the effect that the representation of deterministic factors might have on the modelling of the system marginal cost. Therefore, a deterministic approximation to the real availability of power plants is applied. An approach widely used in production cost modelling [28, 31, 47, 60] consists of reducing the nominal power output of power plants during a period of time to account for the average unavailable capacity during that period:

$$P^{max,avail}_{(th,\,t)} = P^{max}_{(th,\,t)} \cdot Avail_{(th,t)} \tag{3.6}$$

with

$$Avail_{(th,t)} \in [0, 1]$$

for all $th \in TH;\ t = 1, \ldots, T$ with

$P^{max}_{(th,\,t)}$	$\mathrm{MW_{el}}$	Maximal net electric power output of unit th
$P^{max,avail}_{(th,\,t)}$	$\mathrm{MW_{el}}$	Maximal available net electric power output of unit th at time t
$Avail_{(th,t)}$		Availability of unit th at time t

The upper limit of power output according to equation (3.2) is modified accordingly:

$$\underset{\sim}{P}_{(th,\,reg,\,t)} \leqslant P^{max,avail}_{(th,\,t)} \cdot \underset{\sim}{U}_{(th,\,t)} \tag{3.7}$$

for all $th \in TH;\ t, t' = 1, \ldots, T$ with

$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg at time t
$\underset{\sim}{U}_{(th,\,t)}$		State variable of unit th at time t
$P^{max,avail}_{(th,\,t)}$	$\mathrm{MW_{el}}$	Maximal available net electric power output of unit th at time t

This approximated representation of the real availability can influence the commitment of thermal units as shown by Flechner [19], affecting in particular the utilisation of peaking units. Cumperayot [7] demonstrated that with this approximation the system marginal cost tends to be underestimated, especially during peak hours.

Operational Costs The costs involved in the operation of a thermal power plant can be divided into *fixed* and *variable* costs. To the former belong those costs that incur at all times regardless of the operational state or the output of the plant, such as capital costs, insurance and taxes. They cannot be manipulated in the short-term and are therefore irrelevant for the unit commitment and economic dispatch decisions. Variable costs, on the other hand, depend on the operational state and on the output of the plant, and therefore must be considered when modelling marginal costs. Within the variable costs it is necessary to distinguish between those incurred during stationary operation (not necessarily at a fixed load level) and during start-up or shut-down procedures.

Costs in Stationary Operation They are made up of fuel cost, variable operation and maintenance (O&M) cost, and, if the market structure requires it, the value for emission allowances such as CO_2 or SO_x certificates.

The cost for fuel usually amounts for the biggest part of the operational costs. For each level of power output, the fuel cost can be calculated by multiplying the corresponding fuel input by the specific fuel price:

$$\underset{\sim}{C}^{fuel}_{(th,\,t)} = Q_{(th,\,t)} \cdot Pr_{fuel(th,\,t)}$$

for all $th \in TH;\ t = 1, \ldots, T$ with

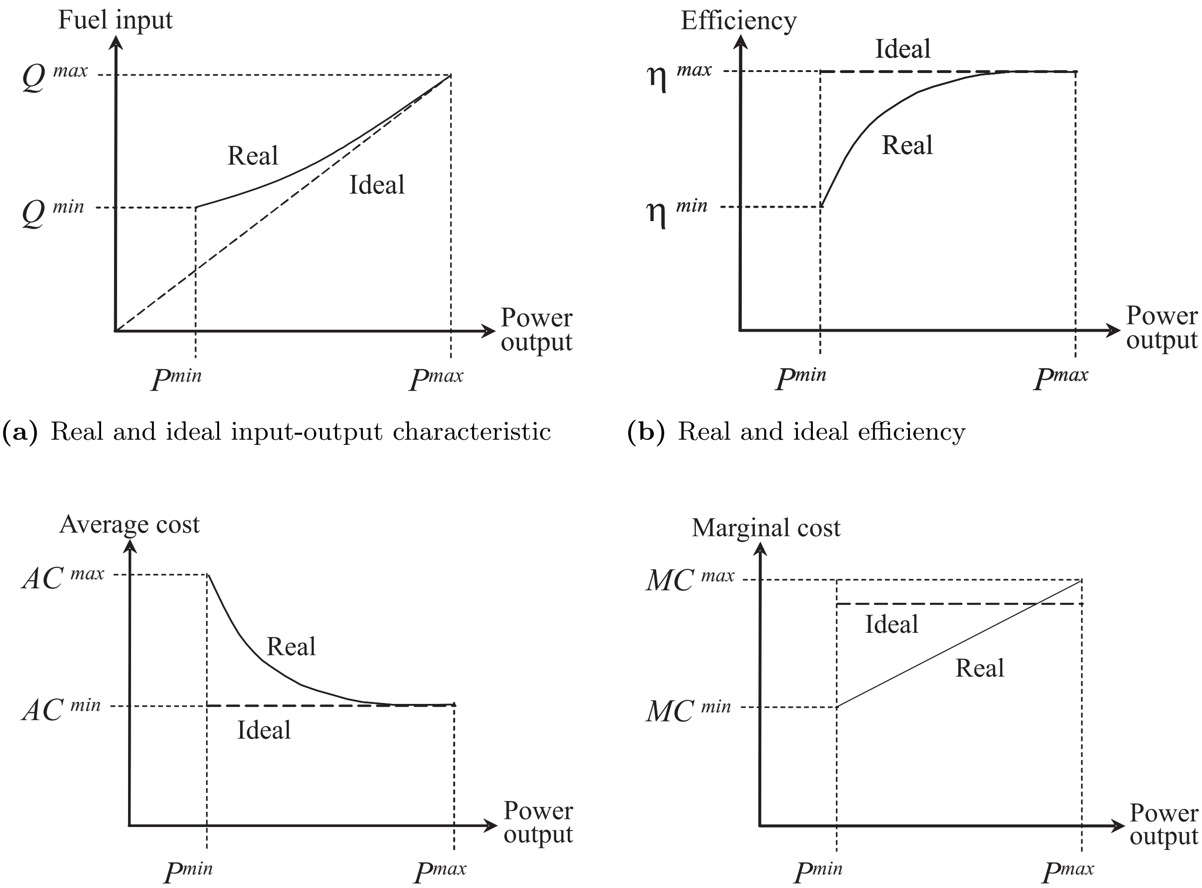
$\underset{\sim}{C}^{fuel}_{(th,\,t)}$	€	Fuel cost of unit th at time t
$Q_{(th,\,t)}$	$\mathrm{MW_{th}}$	Fuel input of unit th at time t
$Pr_{fuel(th,\,t)}$	$\frac{€}{\mathrm{MW_{th}}}$	Fuel price for unit th at time t

The fuel input is determined by the *input-output characteristic* of the thermal power plant.[4] Ideally, the fuel consumption would be zero for zero power output and increase linearly to the point of maximal output. In reality however, the input-output characteristic is a non-smooth function of the power output [3, 77]. The particular characteristic of a power plant is estimated empirically by measuring P and Q in discrete operation points. For modelling purposes, the characteristic is commonly fitted to a quadratic function defined between the load limits [59, 77] (see figure 3.3a). With this representation, the net total efficiency (the inverse of the heat rate) is also a non-linear function of the output, and is higher towards the nominal point of operation (see figure 3.3b). Accordingly, the average cost of production is maximal at P^{min} and decreases with the output (see figure 3.3c). For this reason it is advantageous to operate thermal units near their nominal capacity. The marginal cost, the first derivative of the total cost, increases linearly with the power output (see figure 3.3d).

In a unit commitment problem, the representation of the input-output characteristic as a quadratic function of the power output leads to a non-linear objective function. This representation is usually applied in models solved with the help of the *Lagrange Relaxation* technique [19, 46, 55, 67] (see section 3.3.2). A way to work around the non-linearity issue is to make a piecewise linear approximation of the input-output characteristic function (real or fitted) [18, 63]. However, this approach still requires knowledge of the precise shape of the curve, which generally is difficult to obtain.

An approximation commonly used in production cost models consists of assuming the ideal characteristic shown in figure 3.3. That is, assuming the efficiency to be constant and

[4] In plants with heat extraction (cogeneration power plants) the thermal input is a function of the total output and the rate between heat and power production (see [18]). The dispatch of all thermal units represented in this model is assumed to be power and not heat driven.



(a) Real and ideal input-output characteristic **(b)** Real and ideal efficiency

(c) Real and ideal average cost (AC) **(d)** Real and ideal marginal cost (MC)

Figure 3.3: Real and ideal operational characteristics of a thermal power plant

equal to its nominal value in the whole range of power output [28, 31, 33, 60, 61]. The fuel input can be easily determined based on the efficiency and the power output. The advantage of this approach is that the resulting objective function is linear and that the nominal efficiency is often given as a parameter to characterise the performance of a power plant.[5] As shown in figure 3.3c, the resulting average cost is constant for any output, and underestimates the real average cost. This could result in the units operating more often at partial load in the model. Because the ideal input-output characteristics passes through the origin (zero output for zero input), the ideal marginal cost is always equal or higher than the real (approximated) marginal cost.

A representation of the input-output characteristic which offers a compromise between the approximated and ideal characteristics discussed above, consists in neglecting the quadratic component of the approximated function, and defining a linear input-output function between the power output limits instead [18, 36, 59]. The approximated operational characteristics resulting from this representation are shown on figure 3.4.

[5]In this case, η includes the efficiency of the thermodynamic process (rate between the mechanical power output in the shaft of the turbine and the thermal power input in form of raw fuel), the efficiency of the generator and the plant's own power consumption.

The fuel cost in the points of operation at maximal and minimal output is calculated with the corresponding efficiencies:

$$\underset{\sim}{C}^{fuel}_{(th,t)}\Big|_{P^{max}_{(th,t)}} = C^{fuel,max}_{(th,t)} = \frac{P^{max}_{(th,t)}}{\eta^{max}_{(th,t)}} \cdot Pr^{fuel}_{(th,t)} \tag{3.8}$$

$$\underset{\sim}{C}^{fuel}_{(th,t)}\Big|_{P^{min}_{(th,t)}} = C^{fuel,min}_{(th,t)} = \frac{P^{min}_{(th,t)}}{\eta^{min}_{(th,t)}} \cdot Pr^{fuel}_{(th,t)} \tag{3.9}$$

for all $th \in TH;\ t = 1, \ldots, T$ with

$\underset{\sim}{C}^{fuel}_{(th,t)}$	€	Fuel cost of unit th at time t
$C^{fuel,max}_{(th,t)}$	€	Maximal fuel cost of unit th at time t
$C^{fuel,min}_{(th,t)}$	€	Minimal fuel cost of unit th at time t
$P^{max}_{(th,t)}$	$\mathrm{MW_{el}}$	Maximal electric power output of unit th at time t
$P^{min}_{(th,t)}$	$\mathrm{MW_{el}}$	Minimal electric power output of unit th at time t
$\eta^{max}_{(th,t)}$		Efficiency of unit th at maximal power output at time t
$\eta^{min}_{(th,t)}$		Efficiency of unit th at minimal power output at time t
$Pr^{fuel}_{(th,t)}$	$\frac{€}{\mathrm{MW_{el}}}$	Fuel price for unit th at time t

The efficiency at minimal output can be approximated from the nominal efficiency based on technology and size (nominal output) of the power plant. The fuel cost at any level of power output can be interpolated between the extreme values with the help of the optimisation variable $\underset{\sim}{\Delta}_{(th,t)}$:

$$\underset{\sim}{C}^{fuel}_{(th,t)} = C^{fuel,min}_{(th,t)} \cdot \underset{\sim}{U}_{(th,t)} + \underset{\sim}{\Delta}_{(th,t)} \cdot \left(C^{fuel,max}_{(th,t)} - C^{fuel,min}_{(th,t)}\right) \tag{3.10}$$

with

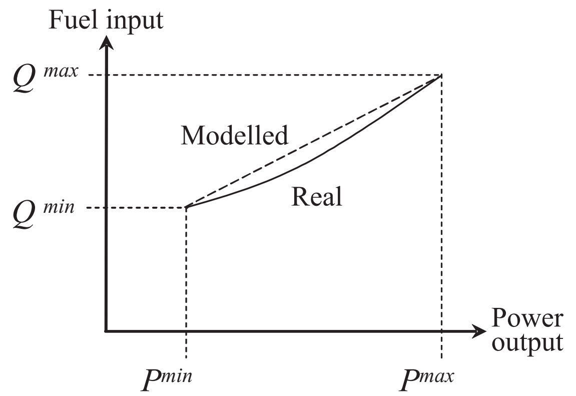
$$\underset{\sim}{\Delta}_{(th,t)} \in [0,1]$$

for all $th \in TH;\ t = 1, \ldots, T$ with

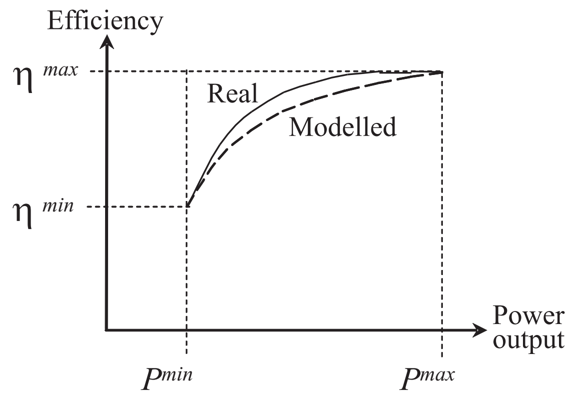
$\underset{\sim}{C}^{fuel}_{(th,t)}$	€	Fuel cost of unit th at time t
$C^{fuel,max}_{(th,t)}$	€	Maximal fuel cost of unit th at time t
$C^{fuel,min}_{(th,t)}$	€	Minimal fuel cost of unit th at time t
$\underset{\sim}{U}_{(th,t)}$		State variable of unit th at time t
$\underset{\sim}{\Delta}_{(th,t)}$		Interpolation variable of unit th at time t

The fuel cost and corresponding fuel input, can be mapped to the level of power output with the help of the same variable:

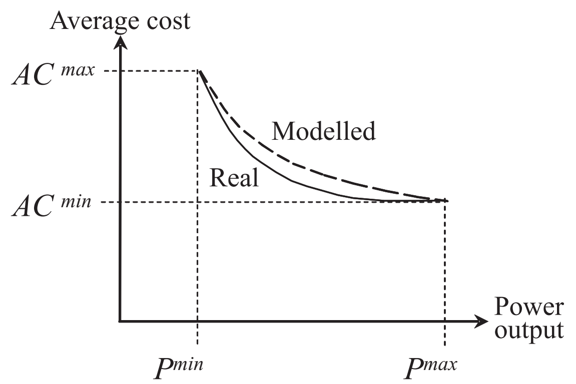
$$\underset{\sim}{P}_{(th,reg,t)} = P^{min}_{(th,t)} \cdot \underset{\sim}{U}_{(th,t)} + \underset{\sim}{\Delta}_{(th,t)} \cdot \left(P^{max}_{(th,t)} - P^{min}_{(th,t)}\right) \tag{3.11}$$



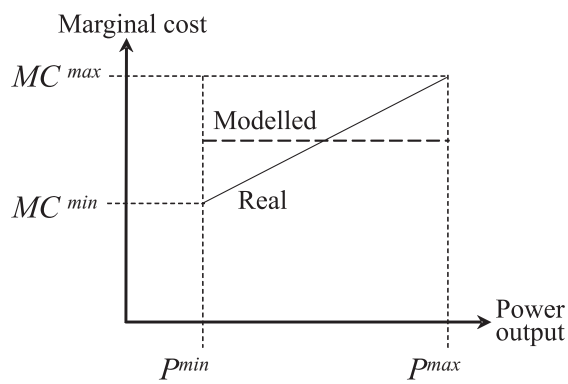
(a) Real and modelled input-output characteristic



(b) Real and modelled efficiency



(c) Real and modelled average cost (AC)



(d) Real and modelled marginal cost (MC)

Figure 3.4: Modelled operational characteristics of a thermal power plant

for all $reg \in REG;\ th \in TH;\ t = 1, \ldots, T$ with

$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg at time t
$P^{max}_{(th,\,t)}$	$\mathrm{MW_{el}}$	Maximal electric power output of unit th at time t
$P^{min}_{(th,\,t)}$	$\mathrm{MW_{el}}$	Minimal electric power output of unit th at time t
$\underset{\sim}{U}_{(th,\,t)}$		State variable of unit th at time t
$\underset{\sim}{\Delta}_{(th,\,t)}$		Interpolation variable of unit th at time t

According to equations (3.10) and (3.11), the fuel cost is greater than zero only when the unit is in the in-service state ($\underset{\sim}{U}_{(th,\,t)} > 0$), and takes a value between $C^{fuel,\,min}_{(th,\,t)}$ and $C^{fuel,\,max}_{(th,\,t)}$ depending on the level of $\underset{\sim}{\Delta}_{(th,\,t)}$. If the unit is not in service ($\underset{\sim}{U}_{(th,\,t)} = 0$), according to equations (3.3), (3.7) and (3.11), the interpolation variable, the electric power output, and the fuel cost is also zero.

It is important to mention that with this representation of the input-output characteristic, the marginal cost of the thermal units is always lower than the ideal marginal cost, and lays somewhere in between the minimal and maximal real marginal cost. This can be appreciated in figure 3.4d.

With the introduction of the so-called Emission Trading Scheme (ETS) in the European Union [17], power generators are given allowances for the emissions of greenhouse gases, and a cap is imposed on the total emissions of several energy-intensive industrial sectors. Like any other scarce commodity traded in an open market, CO_2 certificates could acquire a commercial value. Generators have an incentive to save certificates by emitting below their allocated allowances and selling the surplus of allowances in the market. The profit they forgo by keeping and using their certificates (opportunity cost) instead of selling them is considered part of the operational costs. It is calculated based on the fuel used by the power plant, the plant's specific CO_2 emissions, and the price for CO_2 certificates in the market:

$$\underset{\sim}{C}^{CO_2}_{(th,\,t)} = \frac{c^{CO_2}_{(th)}}{\eta_{(th,\,t)} \cdot H_{(th)}} \cdot Pr^{CO_2}_{(reg,\,t)} \cdot \underset{\sim}{P}_{(th,\,reg,\,t)} \tag{3.12}$$

for all $reg \in REG;\ th \in TH;\ t = 1, \ldots, T$ with

$\underset{\sim}{C}^{CO_2}_{(th,\,t)}$	€	CO_2 cost of unit th at time t
$c^{CO_2}_{(th)}$	$\frac{\mathrm{t}_{CO_2}}{kg}$	Specific emissions of the fuel used by unit th
$\eta_{(th,\,t)}$	$\frac{\mathrm{MW_{el}}}{\mathrm{MW_{th}}}$	Efficiency of unit th at time t
$H_{(th)}$	$\frac{\mathrm{MW_{th}}}{kg}$	Calorific value of the fuel of unit th
$Pr^{CO_2}_{(reg,\,t)}$	$\frac{€}{\mathrm{t}_{CO_2}}$	Price for a metric ton of CO_2 equivalent to an emissions certificate in region reg at time t
$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg at time t

Since the CO_2 emissions and related operational costs depend on the efficiency of the power plant, they must be calculated for each operational point. This can be done by calculating the maximal and minimal CO_2 cost and interpolating the same way as for the fuel cost for any level of electric power output:

$$C^{CO_2,\,max}_{(th,\,t)} = \frac{E^{CO_2}_{(th)}}{\eta^{max}_{(th,\,t)} \cdot H_{(th)}} \cdot Pr^{CO_2}_{(reg,\,t)} \cdot P^{max}_{(th,\,t)} \tag{3.13}$$

$$C^{CO_2,\,min}_{(th,\,t)} = \frac{E^{CO_2}_{(th)}}{\eta^{min}_{(th,\,t)} \cdot H_{(th)}} \cdot Pr^{CO_2}_{(reg,\,t)} \cdot P^{min}_{(th,\,t)} \tag{3.14}$$

$$\underset{\sim}{C}^{CO_2}_{(th,\,t)} = C^{CO_2,\,min}_{(th,\,t)} \cdot \underset{\sim}{U}_{(th,\,t)} + \underset{\sim}{\Delta}_{(th,\,t)} \cdot \left(C^{CO_2,\,max}_{(th,\,t)} - C^{CO_2,\,min}_{(th,\,t)} \right) \tag{3.15}$$

for all $reg \in REG;\ th \in TH;\ t = 1, \ldots, T$ with

$\underset{\sim}{C}^{CO_2}_{(th,\,t)}$	€	CO_2 cost of unit th at time t
$C^{CO_2,\,max}_{(th,\,t)}$	€	Maximal CO_2 cost of unit th at time t
$C^{CO_2,\,min}_{(th,\,t)}$	€	Minimal CO_2 cost of unit th at time t

$P^{max}_{(th,\,t)}$	$\mathrm{MW_{el}}$	Maximal electric power output of unit th at time t
$P^{min}_{(th,\,t)}$	$\mathrm{MW_{el}}$	Minimal electric power output of unit th at time t
$\eta^{max}_{(th,\,t)}$	$\frac{\mathrm{MW_{el}}}{\mathrm{MW_{th}}}$	Efficiency of unit th at maximal power output at time t
$\eta^{min}_{(th,\,t)}$	$\frac{\mathrm{MW_{el}}}{\mathrm{MW_{th}}}$	Efficiency of unit th at minimal power output at time t
$E^{CO_2}_{(th)}$	$\frac{\mathrm{t_{CO_2}}}{kg}$	Specific emissions of the fuel used by unit th
$H_{(th)}$	$\frac{\mathrm{MW_{th}}}{kg}$	Calorific value of the fuel of unit th
$Pr^{CO_2}_{(reg,\,t)}$	$\frac{€}{\mathrm{t_{CO_2}}}$	Price for a metric ton of CO_2 equivalent to an emissions certificate in region reg at time t
$\underset{\sim}{U}_{(th,\,t)}$		State variable of unit th at time t
$\underset{\sim}{\Delta}_{(th,\,t)}$		Interpolation variable for the costs of unit th at time t

In addition to fuel and CO_2 related costs, operational costs also arise, for example, from the operation of ancillary services, waste disposal (waste waters, ashes, chemicals) and the operation of flue-gas cleaning installations for the removal of sulphur and nitrous oxides. In the model formulated here, these costs are aggregated into a single component proportional to the power output:

$$\underset{\sim}{C}^{add}_{(th,\,t)} = c^{add}_{(th,\,t)} \cdot \underset{\sim}{P}_{(th,\,reg,\,t)} \tag{3.16}$$

for all $reg \in REG;\ th \in TH;\ t = 1, \ldots, T$ with

$\underset{\sim}{C}^{add}_{(th,\,t)}$	€	Additional operational costs of unit th at time t
$c^{add}_{(th,\,t)}$	$\frac{€}{\mathrm{MW_{el}}}$	Specific additional operational costs of unit th at time t
$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg in time t

The total operational cost of a thermal power plant in stationary operation results from adding up the three individual cost components calculated with equations (3.10), (3.15) and (3.16):

$$\underset{\sim}{C}^{op}_{(th,\,t)} = \underset{\sim}{C}^{fuel}_{(th,\,t)} + \underset{\sim}{C}^{CO_2}_{(th,\,t)} + \underset{\sim}{C}^{add}_{(th,\,t)} \tag{3.17}$$

for all $th \in TH;\ t = 1, \ldots, T$ with

$\underset{\sim}{C}^{op}_{(th,\,t)}$	€	Total operational costs in stationary operation of unit th at time t
$\underset{\sim}{C}^{fuel}_{(th,\,t)}$	€	Fuel cost of unit th at time t
$\underset{\sim}{C}^{CO_2}_{(th,\,t)}$	€	CO_2 cost of unit th at time t
$\underset{\sim}{C}^{add}_{(th,\,t)}$	€	Additional operational costs of unit th at time t

Start-up cost This portion of the operational costs arises each time a plant goes into the in-service state and starts producing and injecting power into the grid. Depending on plant specific factors such as generating technology and fuel, start-up costs can amount to several equivalent hours of operation at full load and play an important role in the profitability of a unit. Therefore, this cost component can also have important effects on the unit commitment decisions of power plant operators [65, 75], which in turn affect the marginal cost of production [30, 44].

Start-up costs stem mainly from the additional wear of the plant's components due to the thermal stress, and from the own power consumption of the plant while operating below P^{min} (where, in addition, the plant's efficiency is relatively low), not yet synchronised to the grid, and therefore unable to sell its output. Other plant specific costs, such as the energy required for fuel preconditioning (fuel oil preheating, for instance) or the additional personnel required to perform the start-up, are also taken into account. Costs are also associated to shut-down procedures. These costs may be included in the start-up costs [59, 62].

The time and energy required for each start-up depends on the temperature of the combustion chamber and boiler at the beginning of the start-up procedure. Therefore, a portion of the cost is variable. The cooling-down characteristic of thermal power plants can be approximated by an exponential function of the time the installation has been in standby (down-time) [2, 19, 59, 65, 67]. Accordingly, the variable portion of the cost increases with the accumulated down-time before the start-up procedure begins, and reaches a maximal value when the unit is cooled-down. The total cost can be calculated as:

$$\underset{\sim}{C}^{start}_{(th,\,t)} = C^{start,\,min}_{(th,\,t)} + \left(C^{start,\,max}_{(th,\,t)} - C^{start,\,min}_{(th,\,t)}\right) \cdot \left(1 - e^{-\frac{t^{down}_{(th,\,t)}}{\tau_{(th)}}}\right) \tag{3.18}$$

for all $th \in TH;\ t = 1, \ldots, T$ with

$\underset{\sim}{C}^{start}_{(th,\,t)}$	€	Start-up costs of unit th at time t
$C^{start,\,min}_{(th,\,t)}$	€	Minimal start-up costs of unit th at time t
$C^{start,\,max}_{(th,\,t)}$	€	Maximal start-up costs of unit th at time t
$t^{down}_{(th,\,t)}$	h	Accumulated down time of unit th at time t
$\tau_{(th)}$	h	Time constant of the cooling-down characteristic of unit th

After a down-time of $2{,}3$ times τ, circa 90% of $\underset{\sim}{C}^{start}_{(th,\,t)}\Big|_{t\to\infty}$ is reached. The value of τ varies depending on the fuel, technology and nominal output of the power plant.

Several approaches have been proposed to take into account start-up costs in unit commitment models. Some of them directly apply cost functions like (3.18) [19, 46, 55, 59, 67]. This however brings a non-linear factor into a model, and requires knowledge of several plant-specific parameters. Other approaches avoid the non-linearity issue by utilising a piecewise linear approximation of the variable cost component [11, 18]. And in some models, just a constant (down-time independent) charge per start-up is considered [36].

As pointed out by Weber [75], modelling start-up costs is a difficult task when dealing with national or regional systems due to the computational cost involved and data availability issues. Nevertheless, since there is plenty of evidence demonstrating the importance of start-up costs for the unit commitment of thermal power plants [18, 19], an effort is usually made to integrate start-up costs into the formulation of production cost models. Of the models examined, those by Becker [2] and Cumperayot [7] consider variable start-up costs as represented by (3.18). In the models EUDIS [31], E2M2s [47] and in the model applied by Schwarz [61], start-up costs are assumed to be constant. An intermediate approach was applied by Schröter [60] who, based on the methodology developed by Krämer [33], defined "hot", "warm" and "cold" starts to take into account a degree of dependency on the down-time.

Dubois [11] and Filter [18] showed that assuming constant start-up costs, although it tends to underestimate total operational costs, produces the same unit commitment as the piecewise-linear-approximation approach, and with considerably lower computational effort. Therefore, in this model, start-up costs are assumed to be constant and are calculated as in [33] according to the relation:

$$\underset{\sim}{C}^{start}_{(th,\,t)} = \varphi \cdot C^{start}_{(th,\,t)}\Big|_{t^{down}_{(th,\,t)} \to \infty} = \varphi \cdot C^{start,max}_{(th,\,t)} \tag{3.19}$$

for all $th \in TH;\ t = 1, \ldots, T$ with

$\underset{\sim}{C}^{start}_{(th,\,t)}$	€	Start-up costs of unit th at time t
$C^{start,\,max}_{(th,\,t)}$	€	Maximal start-up costs of unit th at time t
$t^{down}_{(th,\,t)}$	h	Accumulated down time of unit th at time t
φ		Start-up factor

The start-up factor φ is assigned a value between 0 and 1 to represent the current temperature of the plant and the corresponding level of the start-up cost. In the developed model, the start-up factor is chosen as 0,5 to represent a warm start.

The maximal value of the start-up costs is determined based on the nominal power of the unit and its operational costs in stationary operation:

$$C^{start,max}_{(th,\,t)} = \underset{\sim}{C}^{op}_{(th,\,t)}\Big|_{P^{max}_{(th,\,t)}} \cdot CF^{fuel}_{(th)} \cdot t^{start}_{(th)} \cdot \left(\frac{1}{1 - CF^{add}_{(th)}}\right) \tag{3.20}$$

for all $th \in TH;\ t = 1, \ldots, T$ with

$C^{start,\,max}_{(th,\,t)}$	€	Maximal start-up costs of unit th at time t
$\underset{\sim}{C}^{op}_{(th,\,t)}$	€	Total operational costs in stationary operation of thermal unit th at time t
$P^{max}_{(th,\,t)}$	MW_{el}	Maximal electric power output of unit th at time t

$CF^{fuel}_{(th)}$		Cost factor for increased fuel cost of unit th during a start-up procedure
$CF^{add}_{(th)}$		Cost factor for increased additional costs of unit th during a start-up procedure
$t^{start}_{(th)}$	h	Duration of a cold start of unit th

The factors for the increased operational costs $CF^{fuel}_{(th)}$ and $CF^{add}_{(th)}$, depend on the fuel and technology employed (for exemplary values, see [33, 60]).

In reality, start-up costs are accumulated over the period of time it takes the power plant to go from standby to the in-service state and into stationary operation. For thermal conventional blocks, the start-up time is in the range of 2 to 5 hours, depending on technology and fuel [11, 67]. For nuclear power plants the start-up time can be of up to 25 hours. In the presented model, start-up costs are incurred in the very moment a unit is set into service. A start-up is signalled by a change of the state-variable $\underset{\sim}{U}_{(th,\,t)}$ between time segments.

$$\underset{\sim}{C}^{\,start}_{(th,\,t)} = \varphi \cdot C^{start,max}_{(th,\,t)} \cdot \underset{\sim}{Y}_{(th,\,t)} \tag{3.21}$$

where

$$\underset{\sim}{Y}_{(th,\,t)} \geqslant \underset{\sim}{U}_{(th,\,t)} - \underset{\sim}{U}_{(th,\,t-1)} \tag{3.22}$$

for all $th \in TH;\ t = 1, \ldots, T$ with

$\underset{\sim}{C}^{\,start}_{(th,\,t)}$	€	Start-up costs of unit th at time t
$C^{start,\,max}_{(th,\,t)}$	€	Maximal start-up costs of unit th at time t
$\underset{\sim}{Y}_{(th,\,t)}$		Start-up variable of unit th at time t
$\underset{\sim}{U}_{(th,\,t)}$		State variable of unit th at time t
φ		Start-up factor

3.1.6.2 Hydroelectric Power Plants

Hydroelectric power plants (*hydro* power plants for short) transform the kinetic energy of flowing water into electrical energy by letting water drive a turbine coupled to a generator. In general, a hydro unit can be thought of as system composed by an upper and a lower water reservoir connected through a waterway (pipe or channel) and a turbine in between, as presented schematically in figure 3.5.

The potential energy caused by the height difference will tend to move the water from the upper to the lower reservoir, driving the turbine and generator. In the developed model,

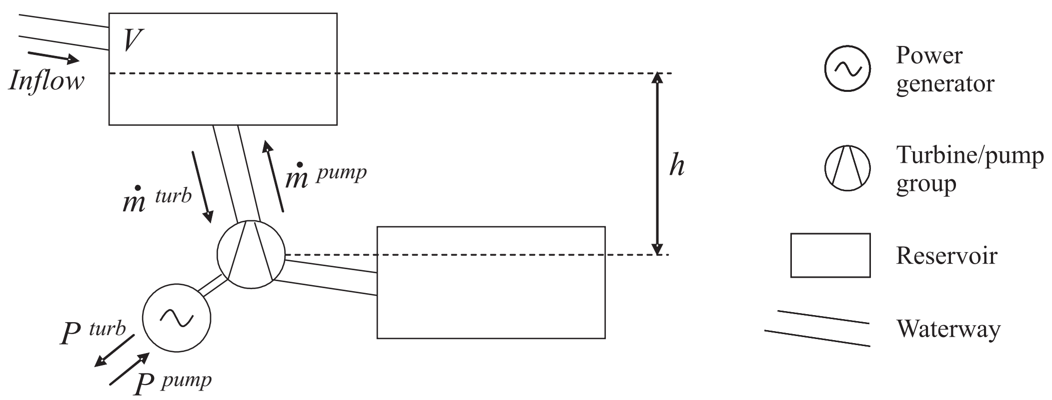


Figure 3.5: Schematic representation of a hydroelectric power plant

the net electric power generated by a hydro power plant is proportional to the head of the reservoirs and to the instantaneous flow of water trough the turbine:

$$\underset{\sim}{P}^{turb}_{(hy,\,reg,\,t)} = \underset{\sim}{\dot{m}}^{turb}_{(hy,\,t)} \cdot g \cdot h_{(hy)} \cdot \eta^{turb}_{(hy)} \tag{3.23}$$

for all $reg \in REG;\ hy \in HY\ ;\ t = 1, \ldots, T$ with

$\underset{\sim}{P}^{turb}_{(hy,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit hy into region reg at time t
$\underset{\sim}{\dot{m}}^{turb}_{(hy,\,t)}$	$\frac{\mathrm{kg}}{\mathrm{s}}$	Mass flow of water through the turbine of unit hy at time t
g	$\frac{\mathrm{m}}{\mathrm{s}^2}$	Constant acceleration due to gravity
$h_{(hy)}$	m	Head of the reservoirs of unit hy
$\eta^{turb}_{(hy)}$		Total efficiency of unit hy

The total efficiency of the unit includes that of the turbine and the generator. The efficiency also takes into account losses due to friction along the waterway. In reality, the total efficiency is a function of the water flow, and both the efficiency and the maximal water flow (and consequently the maximal power output) are functions of the water head. The head at the same time depends on the volume of water stored. Based on the results of the analysis by Flechner [19], the effect of assuming constant efficiency and maximal flow on the scheduling on the power plants are here neglected.

The power output of a hydro power plant is limited by the maximal mass of water that can flow through the turbine and by the volume of water stored in the reservoirs:

$$\underset{\sim}{P}^{turb}_{(hy,\,reg,\,t)} \leqslant P^{turb,\,max}_{(hy,\,t)} \tag{3.24}$$

$$P^{turb,\,max}_{(hy,\,t)} \leqslant \sum_{res\,\in RESHY} \underset{\sim}{V}_{(res,t)} \cdot \rho \cdot g \cdot h_{(hy)} \cdot \eta^{turb}_{(hy)} \cdot \frac{1}{\Delta t_{(t)}} \tag{3.25}$$

$$P^{turb,\,max}_{(hy,\,t)} \leqslant \dot{m}^{turb,\,max}_{(hy,\,t)} \cdot g \cdot h_{(hy)} \cdot \eta^{turb}_{(hy)} \tag{3.26}$$

for all $hy \in HY\,;\ res \in RES\,;\ t = 1, \ldots, T$ with

$RESHY := \{\ (res, hy) \subset RES \times HY \mid$ Hydro unit hy is situated below reservoir $res\}$

$P^{turb,\,max}_{(hy,t)}$	$\mathrm{MW_{el}}$	Maximal electric power output of unit hy at time t
$\underset{\sim}{P}^{turb}_{(hy,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit hy into region reg at time t
$\dot{m}^{turb,\,max}_{(hy,\,t)}$	$\frac{\mathrm{kg}}{\mathrm{s}}$	Maximal mass flow of water through the turbine of unit hy at time t
$\eta^{turb}_{(hy)}$		Total efficiency of unit hy in turbine-mode
$\underset{\sim}{V}_{(res,t)}$	m^3	Volume of water stored in reservoir res at time t
$h_{(hy)}$	m	Head of unit hy
ρ	$\frac{\mathrm{kg}}{\mathrm{m}^3}$	Density of water
g	$\frac{\mathrm{m}}{\mathrm{s}^2}$	Constant acceleration due to gravity
$\Delta\, t_{(t)}$	s	Duration of the time interval t

Although it is generally desirable to operate water turbines with a minimal water flow to avoid cavitation, the minimal electric output is relatively low in comparison to the nominal output of the unit. Furthermore, hydro units have the ability to change their load levels rapidly and without incurring any additional operational cost. With this in mind, and taking into account the hourly time resolution used here, any minimum load requirements can be neglected.

Hydroelectric units can be classified according to the head and technology used (kind of turbine employed) into *run-of-river* and *storage* power plants. The former are built across large and continuous natural streams of water. They are unable to accumulate large volumes of water and rely on relatively low heads but big flows of water to generate power. The power output of run-of-river plants depends therefore mainly on the instantaneous water flow on the river beds. With no fuel cost and very low operational charges, they make very economical power sources, so their power output, when available, is always injected into the grid. Hence, they are considered in most models as exogenous time series of their power output.

Storage power plants on the other hand are able to accumulate large volumes of water, which the plant operator is able to dispose of at will. Storage power plants are built with a larger height difference between reservoirs than run-of-river power plants. This enables the conversion of the potential energy of the water stored in the upper reservoir into the kinetic energy necessary to drive the turbine. Thanks to their flexibility and efficiency, storage power plants are very valuable assets for utilities, providing power and reserve capacity (see section 3.1.7) on demand. They can have important effects on the spot prices by replacing the otherwise expensive units required to cover the load in peak hours [32, 43].

Some storage plants additionally have the capability of pumping water from a lower to an upper reservoir. These so called *pumped-storage* units are currently the most efficient way to store large quantities of energy. They take advantage of the daily and weekly cycles of the power demand by consuming pumping-power in the valley hours when power prices are low, and turbining the stored water in the peak hours when power prices are high. Besides taking advantage of the peak/off-peak price spreads, pumped-storage units may also have the effect of reducing operational costs by preventing mid-load power plants from shutting down during off-peak hours.

The electric power consumed by the pumps is calculated similarly to the electric power generated by the turbines in (3.23):

$$\underset{\sim}{P}^{pump}_{(hy,\,reg,\,t)} = \underset{\sim}{\dot{m}}^{pump}_{(hy,\,t)} \cdot g \cdot h_{(hy)} \cdot \frac{1}{\eta^{pump}_{(hy)}} \tag{3.27}$$

for all $reg \in REG;\ hy \in HY;\ t = 1, \ldots, T$ with

$\underset{\sim}{P}^{pump}_{(hy,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power consumption of unit hy from region reg at time t
$\underset{\sim}{\dot{m}}^{pump}_{(hy,\,t)}$	$\frac{\mathrm{kg}}{\mathrm{s}}$	Mass flow of water through the pump of unit hy at time t
g	$\frac{\mathrm{m}}{\mathrm{s}^2}$	Constant acceleration due to gravity
$h_{(hy)}$	m	Head of unit hy
$\eta^{pump}_{(hy)}$		Total efficiency of unit hy in pump-mode

The power consumption of a hydro power plant is limited by the maximal mass of water that can flow through the pump and by the volume of water stored in the reservoirs:

$$\underset{\sim}{P}^{pump}_{(hy,\,reg,\,t)} \leqslant P^{pump,\,max}_{(hy,\,t)} \tag{3.28}$$

$$\underset{\sim}{P}^{pump}_{(hy,\,reg,\,t)} \leqslant \sum_{res\,\in HYRES} \underset{\sim}{V}_{(res,t)} \cdot \rho \cdot g \cdot h_{(hy)} \cdot \frac{1}{\eta^{pump}_{(hy)}} \cdot \frac{1}{\Delta t_{(t)}} \tag{3.29}$$

$$\underset{\sim}{P}^{pump}_{(hy,\,reg,\,t)} \leqslant \dot{m}^{pump,\,max}_{(hy,\,t)} \cdot g \cdot h_{(hy)} \cdot \frac{1}{\eta^{pump}_{(hy)}} \tag{3.30}$$

for all $hy \in HY;\ res \in RES;\ t = 1, \ldots, T$ with

$HYRES := \{\ (hy, res) \subset HY \times RES \mid$ Hydro unit hy is situated above reservoir $res\}$

$P^{pump,\,max}_{(hy,t)}$	$\mathrm{MW_{el}}$	Maximal electric power consumption of unit hy at time t
$P^{pump}_{(hy,t)}$	$\mathrm{MW_{el}}$	Electric power consumption of unit hy from region reg at time t
$\dot{m}^{pump,\,max}_{(hy,\,t)}$	$\frac{\mathrm{kg}}{\mathrm{s}}$	Maximal mass flow of water through the pump of unit hy at time t
$\eta^{pump}_{(hy)}$		Total efficiency of unit hy in pump-mode
$\underset{\sim}{V}_{(res,t)}$	m^3	Volume of water stored in reservoir res at time t
$h_{(hy)}$	m	Head of unit hy
ρ	$\frac{\mathrm{kg}}{\mathrm{m}^3}$	Density of water
g	$\frac{\mathrm{m}}{\mathrm{s}^2}$	Constant acceleration due to gravity
$\Delta\, t_{(t)}$	s	Duration of the time interval t

The pumps of most pumped-storage power plants are driven by the same synchronous machines used to generate electricity, which must operate at constant speed when either generating or pumping. This implies that flow of most pumps and the electrical power that the machines consume, cannot be regulated. Most pumps have therefore only binary modes of operation. Assuming that the time required to switch modes is small compared to the time resolution of the model, it is save to neglect this restriction. Should the pumped-storage power plants in a system be aggregated into a single equivalent unit or a few equivalent units for modelling purposes, it is only permissible to assume continuous point of operation, since several different pumps can achieve several different levels of power. Even more, there is a growing tendency to the application of adjustable synchronous machines in pump storage power plants [19].

The volumes of water stored in the reservoirs are balanced in the model by keeping account of all positive and negative flows. It is in general possible for several reservoirs to be connected to the same waterway, and for several waterways to part from or to converge into the same reservoir. By mapping reservoirs to waterways it is possible to reproduce the topology of the system. In the model, all flows are added to the volumes according to this topology:

$$\begin{aligned}
\underset{\sim}{V}_{(res,t)} = & \underset{\sim}{V}_{(res,t-1)} \\
& + \mathit{Inflow}_{(res,t)} \\
& - \sum_{hy \in RESHY} \underset{\sim}{\dot{m}}^{turb}_{(hy,\,t)} \cdot \frac{1}{\rho} \cdot \Delta t_{(t)} \\
& + \sum_{hy \in RESHY} \underset{\sim}{\dot{m}}^{pump}_{(hy,\,t)} \cdot \frac{1}{\rho} \cdot \Delta t_{(t)} \\
& + \sum_{hy \in HYRES} \underset{\sim}{\dot{m}}^{turb}_{(hy,\,t)} \cdot \frac{1}{\rho} \cdot \Delta t_{(t)} \\
& - \sum_{hy \in HYRES} \underset{\sim}{\dot{m}}^{pump}_{(hy,\,t)} \cdot \frac{1}{\rho} \cdot \Delta t_{(t)}
\end{aligned} \tag{3.31}$$

for all $res \in RES\,;\ hy \in HY\,;\ t = 1, \ldots, T$ with

$HYRES := \{\ (hy, res) \subset HY \times RES \mid \text{Hydro unit } hy \text{ is situated above reservoir } res\ \}$
$RESHY := \{\ (res, hy) \subset RES \times HY \mid \text{Hydro unit } hy \text{ is situated below reservoir } res\ \}$

$\underset{\sim}{V}_{(res,t)}$	m^3	Volume of water stored in reservoir res at time t
$Inflow_{(res,t)}$	$\frac{\mathrm{m}^3}{\mathrm{s}}$	Net natural inflow into reservoir res at time t
$\underset{\sim}{\dot{m}}^{turb}_{(hy,\,t)}$	$\frac{\mathrm{kg}}{\mathrm{s}}$	Mass flow of water through the turbine of unit hy at time t
$\underset{\sim}{\dot{m}}^{pump}_{(hy,\,t)}$	$\frac{\mathrm{kg}}{\mathrm{s}}$	Mass flow of water through the pump of unit hy at time t
ρ	$\frac{\mathrm{kg}}{\mathrm{m}^3}$	Density of water
$\Delta\, t_{(t)}$	s	Duration of the time interval t

To account for restrictions on the amount of water allowed in the reservoirs, (3.31) is complemented with the following constraint:

$$\underset{\sim}{V}_{(res,t)} \leqslant V^{max}_{(res,t)} \tag{3.32}$$

for all $res \in RES\,;\ t = 1, \ldots, T$ with

$\underset{\sim}{V}_{(res,t)}$	m^3	Volume of water stored in reservoir res at time t
$V^{max}_{(res,t)}$	m^3	Maximal volume of water that can be stored in reservoir res at time t

Although storage power plants have the flexibility that run-of-river units lack, they still rely on a scarce primary energy source. The administration of the hydrological resources is done in most cases based on the value of the water in the reservoirs. The value of water is determined by current spot prices and by the expectation of power producers regarding future spot prices and water inflows [19, 55, 66]. Since the amount of water in riverbeds is associated with precipitation and snowmelt, most models that optimise the dispatch of storage power plants consider the inflows as a stochastic variable. This kind of problem is then more appropriately solved using stochastic optimisation techniques. In the developed model, the power generation from storage power plants with natural inflows is therefore considered as an exogenous time series. Pump storage power plants without natural inflows are immune to random environmental variables. They are modelled according to equations (3.23) to (3.31).

Hydro power plants are less prone to unplanned outages, and planned outages last considerably shorter than that of their thermal counterparts. The unavailability of the hydro power plants is neglected in the model.

3.1.6.3 Other Power Sources

To give an accurate representation of the power balance in the system, it is necessary to account for the power generation from power plants not modelled according to the parameters and constraints defined in sections 3.1.6.1 and 3.1.6.2. This might even include thermal and hydro power plants, if it is determined that their commitment and dispatch is not done optimally against the system's short-term power demand. Examples of these power sources are:

- The power generation from renewable sources, such as wind, biomass and solar energy, which, as the output of run-of-river power plants, does depend mainly on the availability of the primary energy sources.
- The electric power output of cogeneration units driven by the system's heat demand.
- The output of power plants owned by industrial power consumers, driven by the industry's own power demand.

These sources of power are taken into account in the power balance in the form of exogenous time series. The power generation is added to the right hand side of equation (3.1) together with the power generation from modelled thermal and hydro units:

$$
\begin{aligned}
L^{tot}_{(reg,t)} = & \underset{\sim}{P}_{(th,\,reg,\,t)} + \underset{\sim}{P}^{turb}_{(hy,\,reg,\,t)} - \underset{\sim}{P}^{pump}_{(hy,\,reg,\,t)} \\
& + P^{misc}_{(reg,\,t)}
\end{aligned}
\tag{3.33}
$$

for all $reg \in REG;\ th \in TH;\ hy \in HY\ ;\ t = 1, \ldots, T$ with

$L^{tot}_{(reg,\,t)}$	$\mathrm{MW_{el}}$	Total electricity demand of region reg at time t
$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg at time t
$\underset{\sim}{P}^{turb}_{(hy,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit hy into region reg at time t
$\underset{\sim}{P}^{pump}_{(hy,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power consumption of unit hy in region reg at time t
$P^{misc}_{(reg,\,t)}$	$\mathrm{MW_{el}}$	Miscellaneous net power injections into region reg at time t

3.1.7 Modelling the System Reserves

The operation of a power system is only possible if there is an instantaneous and continuous balance between the supply and demand of real power. Any sudden difference between the projected and real supply or demand, would produce a deviation of the system frequency away from its nominal value. If after such an event equilibrium is not restored on time, and the deviation from the nominal frequency is big enough, the system could become unstable and may be rendered inoperable. System reserves are meant to guarantee the safe and reliable operation of the system by timely compensating for these imbalances.

The most common causes of deviations from the schedule are differences between expected and real demand (load forecast error) and unplanned outages of generating units and network components.

Loss of supply or demand in the system would cause the frequency to decrease or increase respectively. The *positive reserves* would provide extra generating capacity in the first case[6] and *negative reserves* would do the opposite in the second case. Additionally, both system reserves are further subdivided according to access (activation) and service time into *primary*, *secondary* and *tertiary* .

Transmission system operators and other overseeing agencies have issued guidelines regarding the implementation of the system reserves. Yet almost every system in the world has its own guidelines. Nonetheless, they are all based on the same basic guidelines accepted by the engineering community. Rebours [54] presents a summary of the different

[6] The so called "interruptible load" could in theory also provide positive reserve by rapidly disconnecting localised loads. For technical reasons, this practice is not widely spread in today's power systems and, in some cases, it has not been yet authorised by the regulatory authorities. Therefore, it is normally neglected in production cost models.

definitions and specifications of system reserves of some major power systems. In all of them, however, the distinction between primary, secondary and tertiary is done in some form. The following description of the systems reserves is based on the guidelines issued by the *Union for the Co-ordination of Transmission of Electricity* (UCTE) in Europe [70].

The *primary reserve* is the first line of defence against disturbances and it is meant to stabilise the frequency of the system. It is automatically and locally activated by the frequency control of the generators. The primary reserve is required to act immediately after an event and to be able to deliver its full capacity within 30 seconds and for at least 15 minutes.

The *secondary reserve* will replace the primary reserve if the imbalance persists after 30 seconds, and will bring back the system frequency and the exchange schedule between control areas to their target values. The secondary reserve must be able to deliver its full capacity within 15 minutes and remain online for as long as required. It is also activated automatically but by a centralised *load-frequency* controller.

According to the UCTE rules, in the portion of the interconnected European power system monitored by the UCTE, the primary reserve must be able to cope with a load imbalance of up to 3000 MW [70].[7] The contribution of each control area within the UCTE to the primary reserve pool is proportional to the share of power generation of the control area (eq. (3.34)). The amount of secondary reserve is also calculated individually for each control area based on its peak demand (eq. (3.35)).

$$Res^{prim}_{(reg)} = \frac{E_{(reg)}}{E_{(UCTE)}} \cdot 3000 \tag{3.34}$$

$$Res^{sec}_{(reg)} = \sqrt{a \cdot L^{max}_{(reg)} + b^2} - b \tag{3.35}$$

with

$$a = 10 \text{ MW}$$
$$b = 150 \text{ MW}$$

for all $reg \in REG;\ t = 1, \ldots, T$ with

$Res^{prim}_{(reg)}$	MW_{el}	Positive and negative primary reserve requirement of region reg at time t
$Res^{sec}_{(reg)}$	MW_{el}	Positive and negative secondary reserve requirement of region reg at time t
$E_{(reg)}$	MWh_{el}	Energy generation of region reg
$E_{(UCTE)}$	MWh_{el}	Energy generation of the UCTE system
$L^{max}_{(reg)}$	MW_{el}	Peak demand

[7] Reference scenario as of 2003.

The UCTE also defines a *tertiary reserve* capacity with the objective of taking over and replacing the secondary reserve if necessary. The recommendations of the UCTE regarding this reserve are vague, and a big variety of other definitions can be found. But in most cases the tertiary reserve is required to be in line and at full capacity up to between 15 and 30 minutes after the primary reserve has been activated, and should remain fully available for as long as required. The amount of tertiary reserve in the UCTE synchronous area must be greater or equal to the capacity of the largest generating unit operating in the system or the amount of secondary reserve, whichever is bigger.

The requirements for activation time and ramp-rate, limit the number of units that are able to provide each part of the reserves. Primary reserve can exclusively be provided by partially loaded operating, or *spinning*, units in the form of a "reserve band" of free capacity, which allows them to steer in both directions. Secondary reserves is also covered by the free spinning capacity, as well as by storage and pump storage power plants. Tertiary reserves can additionally be covered by rapid starting gas turbines, even if they are in the off state.

The requirement for spinning capacity to cover the reserves affect the economic dispatch of the units by not allowing those ones assigned to cover the reserves to operate at full load (for positive reserve) or at minimum load (for negative reserve) [46, 55, 67], even when this implies incurring higher operational cots. Therefore, system reserves should be taken into account in the formulation of production cost models. There is some disagreement, however, on how and to which extent this should be done.

Because spinning and non-spinning capacities play different roles in providing system reserves, if reserves are to be included in the model, it is important to make full consideration of the operating states of the power plants [19]. This implies modelling single generating units, and at least taking into account their minimal load and start up costs. If these constraints are neglected, all units, independently of their operating state, would be able to provide spinning reserve, automatically making the constraint irrelevant for the unit commitment and load dispatch.

In the model applied by Cumperayot [7], only secondary and *minutes reserve* (the "fast" portion of the tertiary reserve in the German system) are represented. Even though the author agrees that generators provide a "primary reserve band" of free capacity, he chose to neglected it.[8] The *hourly reserve* (the "slow" or cold portion of the tertiary reserve) is also neglected, since the required activation time makes it irrelevant for the generation scheduling. The negative portion of reserves did not get any attention in this model or in the analysis.

In the E2M2^{s} model applied by Oeser [47] reserves are aggregated and the positive portion is divided into spinning and non-spinning components.

[8] If it was considered in the maximal net power output of the units, this was not explicitly mentioned.

Kramer [28] is forced to aggregate the thermal units by fuel and vintage into equivalent blocks to be able to apply the GEMM model to the power systems of 10 European countries. Since minimal load and start-up costs are neglected, also are the system reserves.

The heuristic approach applied in the model EUDIS allows primary and secondary reserves to be modelled, even though power plants are aggregated and load duration curves instead of chronological time series are used. Tertiary reserves are neglected on the basis that they are provided exclusively by cold capacity and that notice for tertiary reserve can be considered long enough "*not to affect the dispatching of the units*". This however assumes that free spinning capacity and storage and pump storage power plants, the latter being endogenously modelled in EUDIS, do not contribute to the tertiary reserves. It might be true that storage and pumped-storage power plants are deemed too valuable in practice to serve as peak-shaving units, and their capacity is in fact not withheld to cover the reserves. But this is a bold assumption not supported by the theory.

The model applied by Schröter [60] also resorts to a heuristic approach [36] to solve the unit commitment and load dispatch problem. Here portions of the "base-capacity", "mid-capacity" and "peak-capacity" are blocked to cover the reserves.

The model applied by Schwarz and Lang [61], even though it considers single units,[9] system reserves are "considered in a rather simple way" and "with less detail" than in the model EUDIS .

In accordance with the restrictions imposed by the activation time and ramp-rates, the models EUDIS and E2M2^s impose due restrictions on the units able to serve the reserves (the latter only does it for the tertiary reserves). The model by Schröter does it in a very coarse way by assigning portions of the load to portions of the capacity. In the models by Cumperayot and Lang there is no mention regarding qualifying units.

There seems to be no agreement on the relevance of reserves for the formulation of production cost models and on how the reserves should be modelled. Furthermore, their effect on the modelling of the system marginal costs has been subject to little analysis. Some attention is dedicated to this matter in this work. For this purpose the proposed model is able to consider all three kinds of reserves in a flexible way. Primary and secondary reserves are aggregated into single spinning reserve requirements in each direction (positive and negative). The reserve requirements should be covered by the free spinning capacity of the qualifying thermal and hydro units according to the following equations:

$$\begin{aligned} Res_{(reg,t)}^{spin,pos} \leqslant & \sum_{th} \left(P_{(th,t)}^{max,avail} \cdot \underset{\sim}{U}_{(th,t)} - \underset{\sim}{P}_{(th,reg,t)} \right) \\ & + \sum_{hy} \left(P_{(hy,t)}^{turb,\,max} - \underset{\sim}{P}_{(hy,reg,t)}^{turb} + \underset{\sim}{P}_{(hy,reg,t)}^{pump} \right) \end{aligned} \tag{3.36}$$

[9] "conventional power plants" with more than 50 MW capacity.

for all $reg \in REG;\ th \in THSPPOS;\ hy \in HYSPPOS;\ t = 1, \dots, T$

$$\begin{aligned} Res^{spin,neg}_{(reg,t)} \leqslant & \sum_{th} \left(\underset{\sim}{P}_{(th,reg,t)} - P^{min}_{(th,t)} \cdot \underset{\sim}{U}_{(th,t)} \right) \\ & + \sum_{hy} \left(P^{pump,\,max}_{(hy,t)} - \underset{\sim}{P}^{pump}_{(hy,reg,t)} + \underset{\sim}{P}^{turb}_{(hy,reg,t)} \right) \end{aligned} \tag{3.37}$$

for all $reg \in REG;\ th \in THSPNEG;\ hy \in HYSPNEG;\ t = 1, \dots, T$ with

$THSPPOS := \{\ th \in TH \mid$ Thermal unit contributes to the positive spinning reserve$\}$

$THSPNEG := \{\ th \in TH \mid$ Thermal unit contributes to the negative spinning reserve$\}$

$HYSPPOS := \{\ hy \in HY \mid$ Hydro unit contributes to the positive spinning reserve$\}$

$HYSPNEG := \{\ hy \in HY \mid$ Hydro unit contributes to the negative spinning reserve$\}$

$Res^{spin,pos}_{(reg,t)}$	$\mathrm{MW_{el}}$	Positive primary and secondary reserve requirements of region reg at time t
$Res^{spin,neg}_{(reg,t)}$	$\mathrm{MW_{el}}$	Negative primary and secondary reserve requirements of region reg at time t
$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg at time t
$\underset{\sim}{P}^{turb}_{(hy,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit hy into region reg at time t
$\underset{\sim}{P}^{pump}_{(hy,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power consumption of unit hy in region reg at time t
$P^{max,avail}_{(th,t)}$	$\mathrm{MW_{el}}$	Maximal available net electric power output of unit th at time t
$P^{turb,\,max}_{(hy,t)}$	$\mathrm{MW_{el}}$	Maximal net electric power output of unit hy at time t
$P^{pump,\,max}_{(hy,t)}$	$\mathrm{MW_{el}}$	Maximal net electric power consumption of unit hy at time t
$\underset{\sim}{U}_{(th,\,t)}$		State variable of unit th at time t

Positive and negative tertiary reserves are also allocated to the free spinning capacity of selected power plants, as well as to the installed capacity of an even narrower selection of thermal units regardless of their operational state. To guarantee that the same capacity is not allocated to serve primary, secondary and tertiary reserve simultaneously, the selected capacity should cover the totality of the reserves.

$$\begin{aligned} Res^{spin,pos}_{(reg,t)} + Res^{tert,\,pos}_{(reg,t)} \leqslant & \sum_{th} \left(P^{max,\,avail}_{(th,t)} \cdot \underset{\sim}{U}_{(th,\,t)} - \underset{\sim}{P}_{(th,\,reg,\,t)} \right) \\ & + \sum_{gt} \left(P^{max,\,avail}_{(gt,\,t)} - \underset{\sim}{P}_{(gt,\,reg,\,t)} \right) \\ & + \sum_{hy} \left(P^{turb,\,max}_{(hy,\,t)} - \underset{\sim}{P}^{turb}_{(hy,\,reg,\,t)} + \underset{\sim}{P}^{pump}_{(hy,\,reg,\,t)} \right) \end{aligned} \tag{3.38}$$

for all $reg \in REG;\ th \in THTERPOS;\ hy \in HYTERPOS;\ gt \in GTTERPOS;\ t = 1, \dots, T$

$$\begin{aligned} Res^{spin,neg}_{(reg,t)} + Res^{tert,\,neg}_{(reg,t)} \leqslant & \sum_{th} \left(\underset{\sim}{P}_{(th,\,reg,\,t)} - P^{min}_{(th,\,t)} \cdot \underset{\sim}{U}_{(th,\,t)} \right) \\ & + \sum_{hy} \left(P^{pump,\,max}_{(hy,\,t)} - \underset{\sim}{P}^{pump}_{(hy,\,reg,\,t)} + \underset{\sim}{P}^{turb}_{(hy,\,reg,\,t)} \right) \end{aligned} \tag{3.39}$$

for all $reg \in REG;\ th \in THTERNEG;\ hy \in HYTERNEG;\ t = 1, \ldots, T$ with

$THTERPOS := \{\ th \in TH \mid$ Selection of thermal units contributing to the positive tertiary reserve with their free spinning capacity$\}$

$THTERNEG := \{\ th \in TH \mid$ Selection of thermal units contributing to the negative tertiary reserve with their free spinning capacity$\}$

$GTTERPOS := \{\ th \in TH \mid$ Selection of thermal units contributing to the positive tertiary reserve with their available capacity regardless of their operational state$\}$

$HYTERPOS := \{\ hy \in HY \mid$ Selection of hydro units contributing to the positive tertiary reserve$\}$

$HYTERNEG := \{\ hy \in HY \mid$ Selection of hydro units contributing to the negative tertiary reserve$\}$

$Res^{tert,pos}_{(reg,t)}$	$\mathrm{MW_{el}}$	Positive tertiary reserve of region reg at time t
$Res^{tert,neg}_{(reg,t)}$	$\mathrm{MW_{el}}$	Negative tertiary reserve of region reg at time t
$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg at time t
$\underset{\sim}{P}^{turb}_{(hy,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit hy into region reg at time t
$\underset{\sim}{P}^{pump}_{(hy,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power consumption of unit hy in region reg at time t
$P^{max,avail}_{(th,t)}$	$\mathrm{MW_{el}}$	Maximal available net electric power output of unit th at time t
$P^{min}_{(th,\,t)}$	$\mathrm{MW_{el}}$	Minimal electric power output of unit th at time t
$P^{turb,\,max}_{(hy,t)}$	$\mathrm{MW_{el}}$	Maximal net electric power output of unit hy at time t
$P^{pump,\,max}_{(hy,t)}$	$\mathrm{MW_{el}}$	Maximal net electric power consumption of unit hy at time t
$\underset{\sim}{U}_{(th,\,t)}$		State variable of unit th at time t

3.1.8 Modelling the Power Exchange Between Systems

When power is traded across interconnected regions, the schedule of the local generators will reflect this transaction, resulting in a surplus of power (generation above local demand) in the exporting region and vice versa. This is known as the *scheduled power exchange.* The actual flow of power through the network, however, is governed by the physical properties of the transmission lines, namely their impedance. Power may not flow from the point of injection to the point of withdrawal through the shortest transmission line(s) joining them. Instead, it will distribute itself across all available connections between both regions according to the shortest electrical path between the points, the one offering the least opposition to the flow. The result, is the *physical power exchange.*

In the developed model, the exchange of power between modelled and non-modelled regions is considered in the form of exogenous time series. The time series corresponding to each region are aggregated into a single net-import time series. The power exchange between modelled regions is determined endogenously. Its representation is simplified, first, by aggregating the flows through individual interconnection single flows, and second, by modelling scheduled instead of physical exchanges.

The amount of power that a transmission line and its connecting substations are able to carry is limited by the thermal rating of the wires and maximum output of the power transformers. Additional constraints are also set to ensure the safe and reliable operation of the system in the case of an eventuality and to account for errors in the forecasted load. In the model, aggregated ties are characterised by their capacity for transmission of real power taken into account these constraints. This capacity might be different for power flows in each direction.

To prevent power flowing through the same link in both directions simultaneously, a single optimisation variable per interconnection is assigned. It may take any real value within the maximal transmission capacity in each direction:

$$-P^{ex,\,max}_{(reg',\,reg,\,t)} \leqslant \underset{\sim}{P}^{ex}_{(reg,\,reg',\,t)} \leqslant P^{ex,\,max}_{(reg,\,reg',\,t)} \tag{3.40}$$

for all $reg,\ reg' \in REG;\ t = 1, \ldots, T$ with

$\underset{\sim}{P}^{ex}_{(reg,\,reg',\,t)}$	$\mathrm{MW_{el}}$	Electric power flow from region reg to reg'
$P^{ex,\,max}_{(reg,\,reg',\,t)}$	$\mathrm{MW_{el}}$	Maximal electric power flow from region reg to reg'

The power flowing in or out of a region is added to the power supply of that region with the appropriate algebraic sign. The power balance equation (3.33) is modified accordingly:

$$\begin{aligned} L^{tot}_{(reg,t)} = & \underset{\sim}{P}_{(th,\,reg,\,t)} + \underset{\sim}{P}^{turb}_{(hy,\,reg,\,t)} - \underset{\sim}{P}^{pump}_{(hy,\,reg,\,t)} \\ & + P^{misc}_{(reg,\,t)} \\ & + \sum_{reg'} \underset{\sim}{P}^{ex}_{(reg,\,reg',\,t)} \\ & + P^{ex}_{(reg,\,t)} \end{aligned} \tag{3.41}$$

for all $reg,\ reg' \in REG;\ th \in TH;\ hy \in HY;\ t = 1, \ldots, T$ with

$L^{tot}_{(reg,\,t)}$	$\mathrm{MW_{el}}$	Total electricity demand of region reg at time t
$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg at time t
$\underset{\sim}{P}^{turb}_{(hy,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit hy into region reg at time t
$\underset{\sim}{P}^{pump}_{(hy,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power consumption of unit hy in region reg at time t
$P^{misc}_{(reg,\,t)}$	$\mathrm{MW_{el}}$	Miscellaneous net power injections into region reg at time t
$\underset{\sim}{P}^{ex}_{(reg,\,reg',\,t)}$	$\mathrm{MW_{el}}$	Electric power flow from region reg to reg'
$P^{ex}_{(reg,\,t)}$	$\mathrm{MW_{el}}$	Net power import from non-modelled regions to region reg at time t

By assuming the application of market based mechanisms for the allocation of scarce transmission capacity between regional markets, it is possible to neglect the fees paid for the usage of the interconnections (see section 3.1.5).

3.2 Model Variants

To analyse the effect of model simplification on the system marginal costs, several model variants based on the full model presented on the previous sections were formulated. Different levels of detail are achieved in each variant by including or neglecting constraints.

The inclusion of certain constraints in a model variant depends on the consideration of other constraints. For example, taking into account the system reserves only makes sense if minimal load requirements are also considered. Otherwise, according to equations (3.36) and (3.37), all power plants, regardless of their state (in-service or not), would be able to cover the reserves. This contradicts the definition of spinning reserve and undermines the effect of the reserve requirements on the scheduling of the units. Therefore, not all possible combinations of constraints make sense.

Six model variants, named *Level 1* to *Level 6*, were formulated. They reflect increasing levels of complexity on the representation of the system. The reference model is the simplest of all variants (Level 1). This allows to asses gains (or losses) in performance (accuracy vs. computational effort) by raising the complexity of the model. Common to the formulation of all six versions is:

- the load balance constraint, given by equation (3.41),
- the upper limit of the power output of the thermal units, equal to the nominal available capacity according to equation (3.7),
- the representation of hydro power plants according to the set of equations (3.23) to (3.31),
- the limits imposed on the power exchange according to equation (3.40).

In the following, a detailed description of the six model variants is presented.

Level 1
Merit-order-curve model
Only the fundamental constraints of load balance and maximum power output of power plants are considered. The state and start-up variables of the thermal units are neglected. The total net efficiency of the thermal units is assumed to be constant and equal to the unit's nominal efficiency.

Level 2
Minimal load constraints
In addition to the constraints of Level 1, the operational states, minimal load requirements (eq. (3.3)) and start-up costs of the thermal units (eq. (3.19)) are considered.

Level 3
System reserves
From this level on, the system reserves requirements are taken into account. It is possible to customise the kind of system reserves that are considered (spinning and/or tertiary) by including or excluding the relevant equations (eq. (3.36), (3.37), (3.38) and (3.39)). Tertiary reserve requirements can only be considered in combination with the spinning reserve requirements.

Level 4
Variable efficiency
The input-output characteristic of the thermal units is modelled as a linearised function of the real characteristic.

Level 5
Minimum times
Minimum up-down time requirements are imposed in the commitment of the power plants (equations (3.4) and (3.5)) are included. The efficiency of the thermal units is assumed constant and equal to the unit's nominal efficiency.

Level 6
Variable efficiency and minimum times
Minimum up-down time requirements are imposed in the commitment of the power plants. Efficiency of the thermal units is considered a function of the power output. The input-output characteristic is assumed a linear function (Levels 4 and 5 combined).

As mentioned above, in model variants Level 3 to Level 6 the requirements for system reserves are taken into account. To examine the effect of the extent to which reserves are considered, four additional variations of the main six model variants were also formulated:

Level X.1 Only positive spinning reserve is considered and can be supplied by all (spinning) units.

Level X.2 Positive and negative spinning reserve is considered and can be supplied by all (spinning) units.

Level X.3 Positive and negative spinning reserve is considered and can only be supplied by the qualifying units.

Level X.4 Positive and negative spinning and tertiary reserves are considered and can only be supplied by the qualifying units.

3.3 Solving the Optimisation Problem

3.3.1 The Objective Function

As first mentioned in section 2.3 and then in section 3.1.2, for the modelling of the marginal cost, the unit commitment problem is formulated as an optimisation problem. The goal is to minimise the total costs of production while covering the demand for electricity and respecting all other constraints, namely:

$$C^{tot} = \sum_{t=1}^{T} \sum_{th \in TH} \left(\underset{\sim}{C}^{op}_{(th,\,t)} + \underset{\sim}{C}^{start}_{(th,\,t)} \right) \xrightarrow{!} min \tag{3.42}$$

with

C^{tot}	€	Total operational costs
$\underset{\sim}{C}^{op}_{(th,\,t)}$	€	Total operational costs in stationary operation of thermal unit th at time t
$\underset{\sim}{C}^{start}_{(th,\,t)}$	€	Start-up costs of unit th at time t

3.3.2 Optimisation Technique

The technique applied to solve an optimisation problem depends on the characteristics and scale of the problem and on the precision requirements of the solution. In particular the following properties of a problem should be considered:

- the linearity (or non-linearity) of the objective function and constraints,
- the presence of stochastic parameters,
- the presence of discrete optimisation variables.

An overview and a concise comparison of commonly applied optimisation techniques is offered in [18, 59].

Unit commitment models often include non-linear, non-convex constraints and discrete variables in their formulation. This narrows the selection of optimisation techniques that can be applied to solve them. The so called *system decomposition* technique appears to be the preferred approach [7, 19, 46, 55, 65]. It consists of dividing the original problem into smaller, more manageable subproblems. These are coupled to each other and with overlapping constraints via *Lagrange multipliers* by a coordinator. Among the technique's advantages stand out its computational efficiency for large optimisation problems. This allows the consideration of stochastic variables and the application of the technique in *stochastic programming* [1, 67]. On the down side, the algorithm must be tailored to each optimisation problem, and is less robust than other techniques in terms of finding feasible solutions. This makes it unsuitable for the present analysis.

The optimisation problem resulting from the system modelling presented in this work is mainly characterised by:

- the objective function and all constraints being linear,
- all parameters being equal to their expected values,
- the presence of continuous and discrete (binary) variables.

A suitable alternative to the system decomposition, is the *Mixed-Integer Linear Programming* (MILP) technique [11, 18]. Its main advantages are its sound theoretical foundation and the availability of several commercial-and non-commercial solvers (optimisation programs) working with standardised data formats. These solvers are therefore applicable to diverse model formulations. Computationally, however, the MILP technique is less efficient than the Lagrange relaxation technique, and is therefore less often applied to solve unit commitment problems [22, 34, 55].

Most solvers take a two-step approach to solving a MILP problem. First, the level of the integer variables that offer the best feasible solution is found. In the second step, the integer variables remain fixed and a final *linear programming* LP problem is solved. The result is the value of the continuous variables, namely the electric output of the power plants. This 2-step approach is adequate for calculating the system marginal cost in spite of the presence of integer variables in the model as demonstrated in [48].

The *Branch-and-Bound* group of algorithms are often applied to the combinatorial optimisation problem. They combine techniques for the partial enumeration and search of possible integers solutions with a relaxation strategy, which is efficient for large problems. It often relies on a series of heuristic techniques that can be customised to particular problems. Both the relaxation part of the combinatorial optimisation as well as the final LP problem can be solved by applying the well known Simplex algorithm or the interior point method [35, 52].

3.3.3 Calculation of the System Marginal Cost

The derivative of the objective function of an optimisation problem with respect to any constraint (the marginal cost of that constraint), can be obtained directly from the solution of the *dual* problem of the original (or *primal*) problem [25, 52, 53]. Commercial MILP solvers usually solve LP problems in their dual form, since this is often easier when the solution is bounded by a large number of constraints. Together with the optimal solution and the corresponding value of the optimisation variables, a sensitivity report of the problem is provided. This report includes the marginal costs of all constraints.

In the developed model, the system marginal cost is obtained from the sensitivity report of the solution of the final LP problem (after the solution of the combinatorial problem) as the marginal cost of the load balance constraint.

In theory, the rate of change of the objective function with respect to the load, might differ whether the load balance constraint is either tightened or relaxed. This is especially true for problems with non-convex or discontinuous objective functions. Even more, the marginal cost has often been wrongly defined as the change in costs by producing one *more* **or** *less* unit of output. These facts have given rise to concepts such as "right-hand-side" and "left-hand-side" marginal costs [68]. However, the *duality theorems* state that for the solution of the dual (primal) problem there is only one solution of the corresponding primal (dual) problem. This implies that the marginal cost of the constraints of the primal are, as the solution of the dual, unique [52].

3.3.4 System Marginal Cost with Specific Start-up Costs

The incomplete consideration of the start-up costs in the marginal cost of production has been identified as one of the issues affecting the accuracy of production cost models (see section 1.4). In line with the objectives of this work, the exact nature of this issue was analysed and a methodology was developed that could in theory correct it and render a more accurate representation of the spot price.

As mentioned in the previous section, to solve the final LP problem, from which the marginal cost is obtained, all integer variables have been fixed. The marginal cost of this problem is only defined in the subspace of solutions allowed by the current value of the integer variables. This implies that start-up costs, the part of the variable costs depending on the state-variables, is irrelevant for the marginal cost.

Schwarz et al. [61] and Sensfuß et al. [64] suggested the calculation of "*specific start-up costs*" as a way of integrating the start-up costs into the variable costs in stationary operation. *Positive specific start-up cost* would reflect the cost of the units which are started during the optimisation period, and *negative specific start-up cost* would reflect the bidding strategy of power plant operators during periods of low market prices.

The methodology of the specific start-up costs relies on a optimisation procedure with two successive stages. In the first stage, the optimisation program is solved and the marginal cost is calculated as described in sections 3.3.2 and 3.3.3. Based on the resulting unit commitment and occurrence of start-ups, the specific start-up costs are calculated. In the second stage, the specific costs are added to the operational costs of the units and the problem is solved once again.

3.3.4.1 Positive Specific Start-Up Cost

In the methodology applied by Schwarz et al., the positive specific start-up cost of each unit is calculated for each day by assessing the number of hours during the day that a power plant is in operation, and dividing the start-up cost by the total power output during that day. Sensfuß et al. takes an alternative approach. The generating period is calculated continuously and not only for one day; and the start-up costs are divided by the maximal output of the unit.

There certain inconsistencies in both methodologies. In the case of Schwarz et al., if the specific start-up cost is calculated for each day based on the incurred start-up cost during that day, it is not clear how units that are started once and remain operational for several consecutive days (for example during the working days of a week) should be treated. In the case of Sensfuß et al., since the start-up cost is divided by the maximal output of the unit, any unit could recover the total start-up cost only by operating at full load during the complete generating period.

In this work the two original ideas have been combined and slightly modified. Positive specific start-up costs are calculated individually for each hour according to equation (3.43), by dividing the total start-up cost at the beginning of each generating period through the total power output during that period. A generating period is identified by a change in the start-up variable $\underset{\sim}{Y}_{(th,\,t)}$ and begins at time t_0. Its length is determined by counting the consecutive number of time segments afterwards when the unit is online ($\underset{\sim}{U}_{(th,\,t)} = 1$):

$$\underset{\sim}{c}^{start,\,pos}_{(th,\,t)} = \frac{\underset{\sim}{C}^{\,start}_{(th,\,t_0)}}{\sum\limits_{\substack{t' \in \\ TON_{(th,\,t_0)}}} \underset{\sim}{P}_{(th,\,reg,\,t')} \cdot \Delta\, t_{(t')}} \tag{3.43}$$

for all $reg \in REG;\ th \in TH$ with

$TON_{(th,\,t_0)} := \{\ t \in T \mid t \geqslant t_0 \cap t < min\{t'' \in T \mid \underset{\sim}{U}_{(th,\,t'')} = 0\ ,\ t'' \geqslant t_0\}\ \}$

$\underset{\sim}{c}^{start,\,pos}_{(th,\,t)}$	$\frac{€}{\mathrm{MWh_{el}}}$	Positive specific start-up costs of unit th at time t
$\underset{\sim}{C}^{\,start}_{(th,\,t)}$	€	Start-up costs of unit th at time t
$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg at time t
$\Delta\, t_{(t)}$	h	Duration of the time interval t

By calculating the positive specific start-up cost in this way, a power plant would break even (cover its total costs including start-up costs) by generating an amount of energy equal to the output during $TON_{(th,\,t_0)}$.

3.3.4.2 Negative Specific Start-Up Cost

During periods of low demand and low market prices, many peak- and mid-load units might become uneconomical based on their variable operational costs and expected spot prices. It would be natural for these units to go out of service during this period to avoid losses. In the following period of high demand and high market prices, if these same units become profitable, they could be started again. However, this carries with it the corresponding start-up costs. If the start-up cost surpasses the losses of selling their energy at market prices during a given period, generators have the incentive of bidding below their operational costs to guarantee that they remain in service during that period. This bidding strategy could produce market prices that lay below the actual marginal cost of the system.

This logic can be reproduced with the help of negative specific start-up costs, which would be subtracted from the variable costs in stationary operation to reflect the price discount in the bids. Negative specific start-up costs would be calculated for a closed time period $TOFF$ beginning at the time t_0 when the average variable cost of the unit in stationary operation is higher than the system marginal cost. The negative profit margin of a unit during that time period is given by:

$$\sum_{\substack{t \in \\ TOFF_{(th,\,t_0)}}} \underset{\sim}{P}_{(th,\,reg,\,t)} \cdot \Delta\, t_{(t)} \cdot \left(\frac{\underset{\sim}{C}^{op}_{(th,\,t)}}{\underset{\sim}{P}_{(th,\,reg,\,t)} \cdot \Delta\, t_{(t)}} - MC_{(reg,\,t)} \right) \tag{3.44}$$

for all $reg \in REG;\ th \in TH$ with

$$TOFF_{(th,\,t_0)} := \{\ t \in T \mid t \geqslant t_0 \cap t < min\{t' \in T \mid \frac{\underset{\sim}{C}^{op}_{(th,\,t')}}{\underset{\sim}{P}_{(th,\,reg,\,t')} \cdot \Delta\, t_{(t')}} \leqslant MC_{(reg,\,t')}\ ,\ t' \geqslant t_0\}\ \}$$

$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg at time t
$MC_{(reg,\,t)}$	$\frac{€}{\mathrm{MWh_{el}}}$	Marginal cost of region reg at time t
$\underset{\sim}{C}^{op}_{(th,\,t)}$	€	Total operational costs in stationary operation of unit th at time t
$\Delta\, t_{(t)}$	h	Duration of the time interval t

Power plant operators would in theory be willing to lower the price of their offers to the level where the loss equals the start-up costs. The negative specific start-up cost $c^{start,\,neg}_{(th,\,t)}$ in each hour of $TOFF_{(th,\,t_0)}$ should be such that:

$$\sum_{\substack{t \in \\ TOFF_{(th,\,t_0)}}} \underset{\sim}{P}_{(th,\,reg,\,t)} \cdot \Delta\, t_{(t)} \cdot \left(\frac{\underset{\sim}{C}^{op}_{(th,\,t)}}{\underset{\sim}{P}_{(th,\,reg,\,t)} \cdot \Delta\, t_{(t)}} - \underset{\sim}{c}^{start,\,neg}_{(th,\,t)} - MC_{(reg,\,t)} \right) = \varphi \cdot C^{start,\,max}_{(th,\,t_0)} \tag{3.45}$$

for all $reg \in REG;\ th \in TH$ with

$$TOFF_{(th,\,t_0)} := \{\ t \in T \mid t \geqslant t_0 \cap t < min\{t' \in T \mid \frac{\underset{\sim}{C}^{op}_{(th,\,t')}}{\underset{\sim}{P}_{(th,\,reg,\,t')} \cdot \Delta\, t_{(t')}} \leqslant MC_{(reg,\,t')}\ ,\ t' \geqslant t_0\}\ \}$$

$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg at time t
$MC_{(reg,\,t)}$	$\frac{€}{\mathrm{MWh_{el}}}$	Marginal cost of region reg at time t
$\underset{\sim}{C}^{op}_{(th,\,t)}$	€	Total operational costs in stationary operation of unit th at time t
$\underset{\sim}{c}^{start,\,neg}_{(th,\,t)}$	$\frac{€}{\mathrm{MWh_{el}}}$	Negative specific start-up costs of unit th at time t

$C^{start,\,max}_{(th,\,t)}$	€	Maximal start-up costs of unit th at time t
$\Delta\, t_{(t)}$	h	Duration of the time interval t
φ		Start-up factor

Assuming that the negative specific start-up cost of each unit is constant within the considered time period and solving for $\underset{\sim}{c}^{start,\,neg}_{(th,\,t)}$:

$$\underset{\sim}{c}^{start,\,neg}_{(th,\,t)} = \frac{\sum\limits_{\substack{t\in\\ TOFF_{(th,\,t_0)}}} \underset{\sim}{P}_{(th,\,reg,\,t)} \cdot \Delta\, t_{(t)} \cdot \left(\frac{\underset{\sim}{C}^{op}_{(th,\,t)}}{\underset{\sim}{P}_{(th,\,reg,\,t)} \cdot \Delta\, t_{(t)}} - MC_{(reg,\,t)} \right) - \varphi \cdot C^{start,\,max}_{(th,\,t_0)}}{\sum\limits_{\substack{t\in\\ TOFF_{(th,\,t_0)}}} \underset{\sim}{P}_{(th,\,reg,\,t)} \cdot \Delta\, t_{(t)}} \tag{3.46}$$

for all $reg \in REG;\; th \in TH$ with

$TOFF_{(th,\,t_0)} := \{\; t \in T \mid t \geqslant t_0 \cap t < min\{t' \in T \mid \frac{\underset{\sim}{C}^{op}_{(th,\,t')}}{\underset{\sim}{P}_{(th,\,reg,\,t')} \cdot \Delta\, t_{(t')}} \leqslant MC_{(reg,\,t')}\;,\; t' \geqslant t_0\}\;\}$

$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg at time t
$MC_{(reg,\,t)}$	$\frac{€}{\mathrm{MWh_{el}}}$	Marginal cost of region reg at time t
$\underset{\sim}{C}^{op}_{(th,\,t)}$	€	Total operational costs in stationary operation of unit th at time t
$C^{start,\,max}_{(th,\,t)}$	€	Maximal start-up costs of unit th at time t
$\underset{\sim}{c}^{start,\,neg}_{(th,\,t)}$	$\frac{€}{\mathrm{MWh_{el}}}$	Negative specific start-up costs of unit th at time t
$\Delta\, t_{(t)}$	h	Duration of the time interval t
φ		Start-up factor

The negative specific start-up cost of each unit is calculated only in those time periods $TOFF_{(th,\,t_0)}$ where the negative profit margin is higher than the start-up cost. That is:

$$\underset{\sim}{c}^{start,\,neg}_{(th,\,t)} \geqslant 0 \Leftrightarrow \sum\limits_{\substack{t\in\\ TOFF_{(th,\,t_0)}}} \underset{\sim}{P}_{(th,\,reg,\,t)} \cdot \Delta\, t_{(t)} \cdot \left(\frac{\underset{\sim}{C}^{op}_{(th,\,t)}}{\underset{\sim}{P}_{(th,\,reg,\,t)} \cdot \Delta\, t_{(t)}} - MC_{(reg,\,t)} \right) \geqslant \varphi \cdot C^{start,\,max}_{(th,\,t)} \tag{3.47}$$

Otherwise the specific start-up cost would be negative and have the undesired effect of rising the operational cost. This can be deduced from (3.46).

Sensfuß et al. also applied a methodology to calculate negative specific start-up costs. In that case they are calculated by simply dividing the total start-up cost of a unit by the equivalent energy output at full load during a given period, and not the actual output. Moreover, this is done for those units and during those time periods where the modelled market price lays below the unit's operational costs, regardless of the size of the profit margin. This might cause the problems mentioned above, of negative start-up costs being lower than zero.[10]

3.3.4.3 Calculation of the Modified System Marginal Cost

After calculated as explained above, the positive and negative specific start-up costs are added to the operational costs as a power-output-dependent component:

$$\begin{aligned} \underset{\sim}{C}^{op}_{(th,\,t)} = \underset{\sim}{C}^{fuel}_{(th,\,t)} + \underset{\sim}{C}^{CO_2}_{(th,\,t)} + \underset{\sim}{C}^{add}_{(th,\,t)} \\ + \left(\underset{\sim}{c}^{start,\,pos}_{(th,\,t)} - \underset{\sim}{c}^{start,\,neg}_{(th,\,t)}\right) \cdot \underset{\sim}{P}_{(th,\,reg,\,t)} \cdot \Delta\, t_{(t)} \end{aligned} \tag{3.48}$$

for all $reg \in REG;\ th \in TH;\ t = 1, \ldots, T$ with

$\underset{\sim}{C}^{op}_{(th,\,t)}$	€	Total operational costs in stationary operation of unit th at time t
$\underset{\sim}{C}^{fuel}_{(th,\,t)}$	€	Fuel cost of unit th at time t
$\underset{\sim}{C}^{CO_2}_{(th,\,t)}$	€	CO_2 cost of unit th at time t
$\underset{\sim}{C}^{add}_{(th,\,t)}$	€	Additional operational costs of unit th at time t
$\underset{\sim}{c}^{start,\,pos}_{(th,\,t)}$	$\frac{€}{\mathrm{MWh_{el}}}$	Positive specific start-up costs of unit th at time t
$\underset{\sim}{c}^{start,\,neg}_{(th,\,t)}$	$\frac{€}{\mathrm{MWh_{el}}}$	Negative specific start-up costs of unit th at time t
$\underset{\sim}{P}_{(th,\,reg,\,t)}$	$\mathrm{MW_{el}}$	Electric power injection of unit th into region reg at time t
$\Delta\, t_{(t)}$	h	Duration of the time interval t

Prior to the second optimisation, the state variables must be fixed at their current value. Otherwise, the optimisation could produce a unit commitment that disagrees with the one on which the calculation of the specific start-up costs was based. In addition, all start-up variables are set to zero, so that start-up costs do not appear twice in the total cost. Finally, the resulting problem is solved once more. The new marginal costs will reflect the increase or decrease in the operational cost of the marginal unit due to the specific start-up costs.

The addition of the specific start-up cost might change the relative cost of each units and produce a slightly different economic dispatch after the second optimisation. Taking this into account would result in an iterative process. It is here assumed that the difference in the overall dispatch is too small to justify such a process.

In the methodology by Schwarz, in the second step the dispatch of the thermal units is optimised for each hour separately. But since in the developed model pumped-storage units can also be modelled and due to the constraint for the balance of the reservoirs, the optimisation problem must be solved in closed form over the complete modelling period.

[10] This might explain why in their application of the methodology to a simulation of the German spot market, the effect of the specific start-up cost on the simulated prices is relatively small [64, page 11].

3.4 Performance Evaluation of the Model Variants

Performance is defined here as the ability of a model to produce accurate results with an acceptable modelling effort. Although accuracy and effort can be measured objectively, there is no absolute function to measure the performance. The main issues here are the scaling of each factor and their relative weights. Depending on their importance, which itself depends on the application, a model that yields accurate results but with a relatively high computational effort might, or might not, be deemed as "useful" as one that produces inaccurate results more rapidly. To facilitate an individual judgment of performance, separate measurements of accuracy and of modelling effort are provided here.

The accuracy of the result of the different model variants is tested by comparing the system marginal cost with historical market prices. A modified definition of the *mean absolute percentage error* (MAPE) proposed by Shahidehpour et al. [65] was used to measure how well the profile of the hourly marginal costs approximates the profile of the spot price. In this modified version, the percentage deviation of the hourly marginal cost is measured against the average spot price during the modelling period instead of the hourly spot price. This prevents the percentage error from taking extreme values when the spot price is very small. The MAPE is defined as:

$$\mathrm{MAPE}_{(reg)} = \frac{1}{T} \cdot \sum_{t} \left| \frac{\left(MC_{(reg,t)} - Pr^{spot}_{(reg,t)} \right) \cdot 100}{\overline{Pr}^{spot}_{(reg)}} \right| \tag{3.49}$$

with

$$\overline{Pr}^{spot}_{(reg)} = \frac{1}{T} \cdot \sum_{t} Pr^{spot}_{(reg,t)} \tag{3.50}$$

for all $reg \in REG, t = 1, \ldots, T$ with

$\mathrm{MAPE}_{(reg)}$		Mean absolute percentage error of the hourly marginal cost of region reg in the modelled time horizon
$MC_{(reg,t)}$	$\frac{€}{\mathrm{MWh_{el}}}$	Marginal cost of region reg at time t
$Pr^{spot}_{(reg,t)}$	$\frac{€}{\mathrm{MWh_{el}}}$	Spot price of the electricity in the market of region reg at time t
$\overline{Pr}^{spot}_{(reg)}$	$\frac{€}{\mathrm{MWh_{el}}}$	Average spot price of electricity in the market of region reg in the modelled time horizon

The deviation of the average marginal cost from the average spot price during *base*, *peak* and *off-peak* hours is also measured. This measure serves to assess whether a model is systematically under or overestimating spot prices in either period. The exact definition of the peak period often differs between markets, as it depends on the power consumption patterns of the system. Here peak hours are defined for all working days, namely from Monday to Friday except holidays, from the 8th to the 20th hour of the day, from 8:00 to 19:59 that is. The remaining hours of the week are considered *off-peak* hours.

To measure the modelling effort, the computation time was used. This includes only the time it takes to solve the system of equations and produce an optimal solution. The compilation time, the time required by the solver to build the system of equations, was not taken into account.[11]

The computation time depends greatly on the model implementation, everything from how "well" the model is coded, to the particular solver used, its version and settings. For the model developed here it was noticed, for example, that some model variants preformed better if $\underset{\sim}{U}_{th,\,t} = 1 \; \forall(th,\, t)$ was provided as starting value, while others performed worse. To be able to compare "apples with apples", in all cases, the model variants were applied to the various test cases without providing starting values and using most of the standard settings of the solver. Only the required tolerance of the solution to the integer problem, defined as the relative gap between the integer solution at any given iteration and the best possible integer solution [26, 41], was lowered from the standard 10 % to 1 %. The model was developed using GAMS® V22.3 and solved with CPLEX® V10.0 running on a Linux-based system with a clock speed of 2,2 GHz and 4 GB of RAM memory.

[11] The compilation time could be eliminated by "prefabricating" the system of equations, and updating the coefficient of the variables with each new application.

Chapter 4

Effect of the Level of Detail of the Production Cost Model on the Determination of the System Marginal Cost

The production cost model described in the previous chapter was applied to model the power systems of three regions within the European interconnected power system. The effect of the level of detail in the model formulation on the model's performance was analysed.

4.1 Models of the French, German and Spanish Power Systems

As subjects for the analysis, models of the French, German and Spanish power systems were developed. Each was formulated as an independent but interconnected region. Their situation in the European continent is illustrated in figure 4.1.

These three systems were selected for a multitude of reasons. Together, they house almost half of the installed generating capacity in the interconnected power system of the European Union (EU), and amount for almost the same portion of the power generation and consumption (see table 4.1). Differences in their power generation mix, consumption patterns and overall power balance, allow to make a better generalisation of the results. In these regions function three of the most important and liquid power exchanges in Europe: Powernext in France, the EEX in Germany and OMEL in Spain. Each system and the corresponding markets are bounded by the national borders of the these countries, which eases the delimitation task.[1] The three power systems and their corresponding markets exist and operate under the legal framework of the European Union and therefore share common sets of rules. Even though several regional differences persist, it is possible to apply the same modelling and analysis methodology for all three system.

A special consideration was made for Germany regarding the amount of operating reserves. Two mayor German utilities, EnBW and RWE, have participation in hydro power plants located outside the national borders. These units were not taken into account in the installed capacity of the country. However, the power generated by the units is directly injected into the German control areas. The actual power generation, which is not explicitly known, is accounted for in the net power exchange between the German region and the countries where the units are located. It was assumed that both companies allocate considerable portions of their reserve requirements to these units. Their contribution to the secondary and tertiary reserves was assumed and discounted from the theoretical totals calculated according to equation (3.35) and from the assumed amount of tertiary reserve.

The necessary historical data to model the years 2005 and 2006 was gathered. The information on the installed capacity was obtained from a commercial power plants database. In total, 1275 single thermal units and 33 hydro-storage units were modelled. Information regarding the power balance was largely extracted from public information providers such as national and regional statistics offices and industry associations. Gaps in the information were filled with modelled data based on own assumptions.

The fact that France, Germany and Spain are neighbouring countries and their power systems are interconnected, also allows to investigate the effect of the level of detail in the modelling of the marginal cost of multiregional power systems. In this regard, it must be noted that the methods applied for the allocation of the transmission capacity between the corresponding markets suffered several changes during the years 2005 and 2006. Figure 4.2 presents the time-line of the allocations methods in place during this time. Before June 2006, non-market-based and discriminatory allocation methods such as *pro-rata* or *priority lists*[2] were still in place, and portions of the capacity were still withheld by long-

[1] The integration of the electricity markets of Spain and Portugal, will lead to the creation of the Iberian Market of Electrical Energy (MIBEL). The integration process began in late 2001 and should conclude with the launch of the single Iberian Electricity Market Operator (OMI). During the integration process, and prior to the creation of OMI, a transitory integrated spot and intra-day market operator (OMIE) and a transitory integrated futures market operator (OMIP) should be created [42]. OMIP began its activities in July 2006 [50]. The incumbent spot market operator in Spain (OMEL), assumed the role of OMIE and began operating the Iberian spot and day-ahead market in July 2007 [49]. The accurate modelling of the spot market of MIBEL from this date on, would require the consideration of both the Spanish and Portuguese power systems. In this work, in all time periods the Spanish power system is modelled as a single region, the power exchange with the Portuguese power system is considered an exogenous time series, and the marginal cost is assessed against the historical spot price in MIBEL.

[2] For definitions on these and other allocation methods see for example [6, 27].

Figure 4.1: Location and geopolitical boundaries of the French, German and Spanish power systems

	Maximal Generating Capacity [MW_{el}]	Portion of EU's [%]	Total Net Electricity Production [TWh_{el}]	Portion of EU's [%]	Total Electricity Demand [TWh_{el}]	Portion of EU's [%]
EU 25	725.498		3.039		3.014	
France	115.500	15,9	549	18,1	482	16,0
Germany	132.265	18,2	581	19,1	564	18,7
Spain	75.953	10,5	280	9,2	272	9,0
Total	323.718	44,6	1.410	46,4	1.318	43,7

Table 4.1: Key figures of the French, German and Spanish power systems as of December 2005 [71]

term contracts. This is important for the assessment of the results, since in the model an efficient market-based mechanism is assumed (see section 3.1.8).

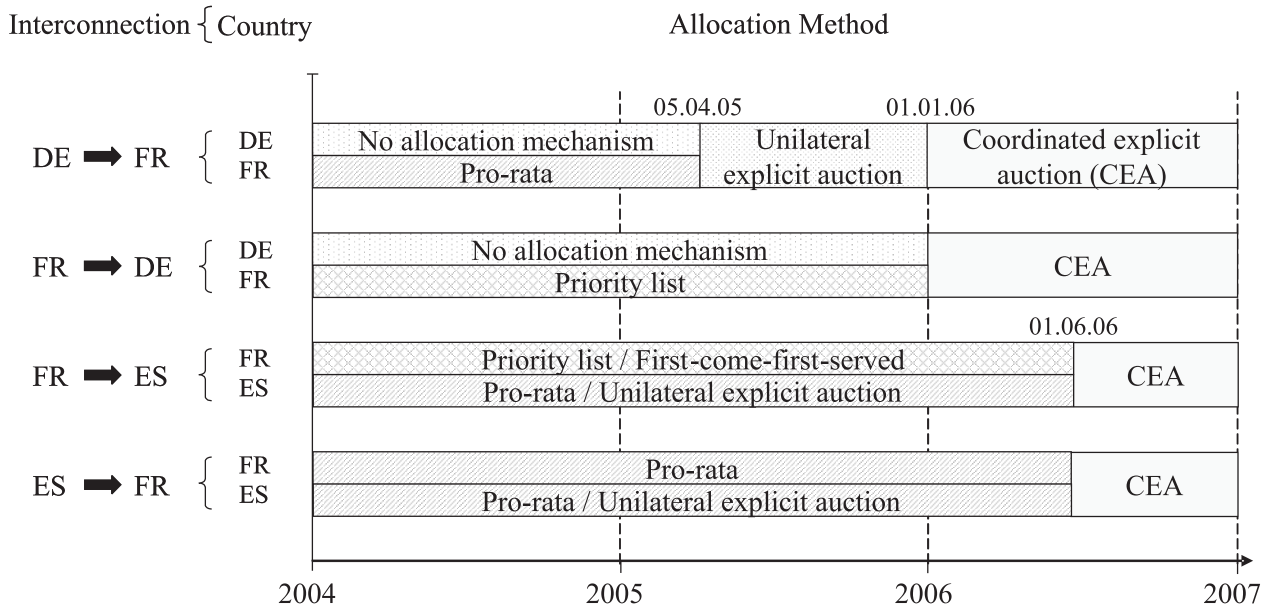


Figure 4.2: Time line of the implementation of methods for the day-ahead allocation of the transmission capacity of the interconnections between France, Germany and Spain [10, 14, 15, 57, 58].

4.2 Effect on the Modelling of the Marginal Cost of Systems with Exogenous Exchange of Electricity

For the first analysis, the exchange of power between each modelled region and its neighbouring regions was taken into account in the form of exogenous time series, and was integrated into the total domestic supply according to equation (3.41).

As explained in section 3.1.6, pumped-storage power plants take advantage of the cycles in power demand and prices to store power in water reservoirs. Most pumped-storage units without natural inflows have characteristic daily or weekly cycles of operation depending on the size of the reservoirs. To capture this property and achieve a realistic representation of the commitment of pumped-storage units, a week (any set of seven consecutive days) was chosen as the shortest modelling time horizon. This approach was also suggested by Müsgens [43] and Schweppe et al. [62]. To account for the difference in the typical profile of the power demand according to the seasons of the year (see section 3.1.4), the first week of February and August of the years 2005 and 2006 were modelled as representative winter and summer weeks.

Next, the German system is used as case study. The results for the first week of August of 2005 are presented and analysed in detail. Figure 4.3 presents the error of the hourly marginal cost. Figure 4.4 shows the relative error of the average marginal cost during base and peak hours for model variants[3] Level 1.4 to Level 6.4 in relation to the spot price in the power exchange.

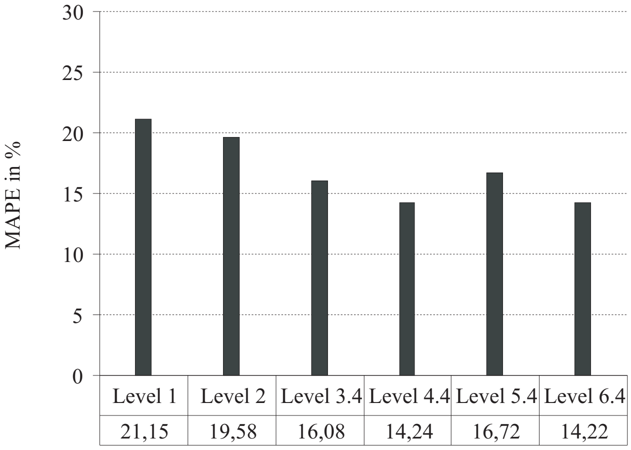


Figure 4.3: Error of the hourly marginal cost of the German power system during the first week of August 2005

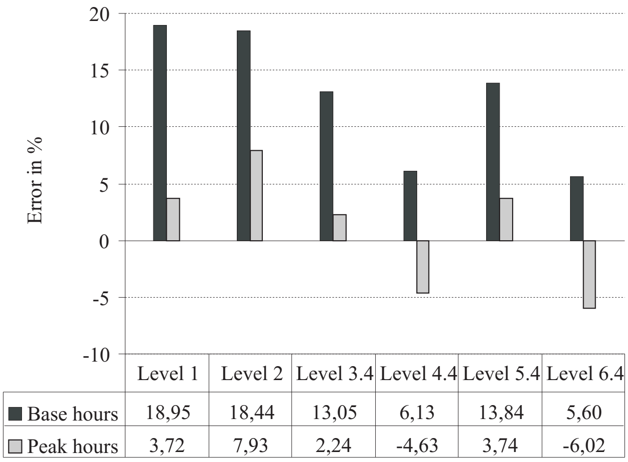


Figure 4.4: Relative error of the average marginal cost of the German power system during the base and peak hours of the first week of August 2005

The error of the marginal cost is highest for Level 1, the simplest of all model variants, where most constraints in the operation of the power plants and all system reserves are neglected. Figure 4.5 shows the utilisation of the capacity of each thermal unit and the merit order curve of the system. As expected for this case, units are dispatched in the order given by their operational costs.

When minimum load requirements and start-up costs are also considered in variant Level 2, the marginal costs during peak and off-peak hours are affected in different ways: peaks are pulled higher while valleys are pushed lower. Owing to this effect, the profile of the marginal cost comes closer to the spot price as indicated by the reduction in the MAPE. In average, the marginal cost is also lower than for Level 1. This is better appreciated by looking at the hourly values of the marginal cost and of the spot price in figure 4.6.

The explanation for the change in the marginal cost can be found in the change in the commitment of the thermal power plants. In figure 4.7 it is observed that in Level 2 units

[3]See section 3.2 in page 49 for the description of the model variants.

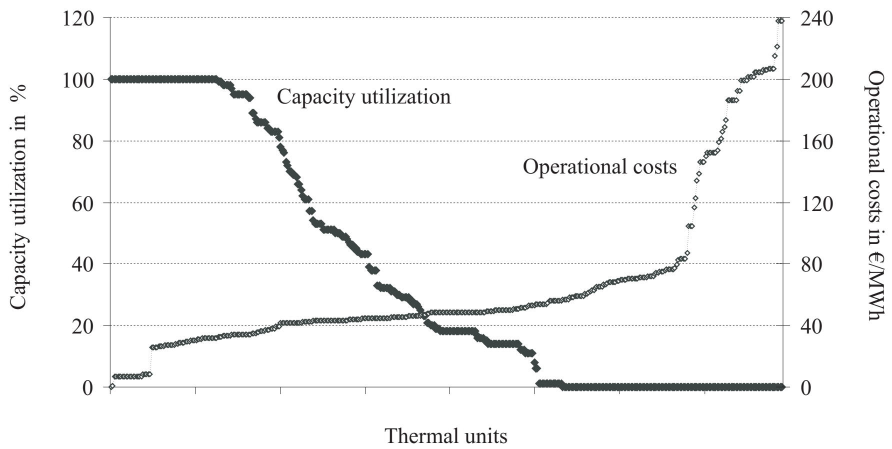


Figure 4.5: Capacity utilisation and merit-order curve for model variant Level 1 of the German power system for the first week of August 2005

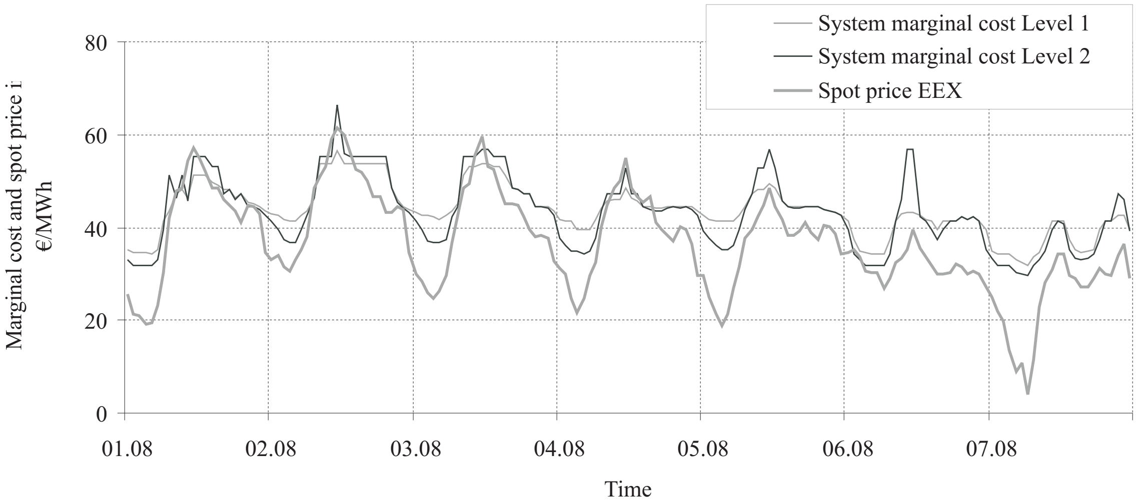


Figure 4.6: Hourly spot price and marginal costs of model variants Level 1 and Level 2 of the German power system during the first week of August 2005

are not anymore dispatched according to the merit-order curve ranking. The gray bars referenced to the right ordinate of the diagram, show the change in the utilisation of the capacity between model variants Level 2 and Level 1. A shift in generation from the most economical units to the mid-load units is noticed. This effect was also observed by Müsgens in [43]. However, here a different explanation is offered. When minimal load requirements and start-up costs are taken into account, its is more economical for mid-load units to operate at their minimal allowed output during the hours of low demand than shutting-down and starting-up again the following period. To keep with the load balance, base-load units must reduce their output. As a result, during the off-peak hours the most expensive unit in operation does not set the marginal cost. Instead, the slope of the objective function correspond to the marginal cost of the most economical unit in operation, which is usually operating at minimal load. This is illustrated in figure 4.8. It is also important to notice that the hours where the system marginal cost is higher than the operational cost of the most expensive thermal unit, the marginal cost is set by the pumped storage units.

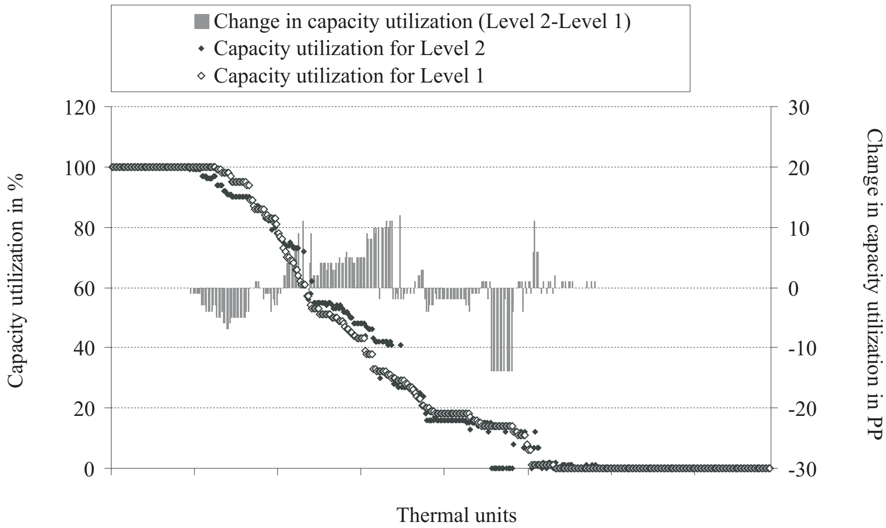


Figure 4.7: Change in the capacity utilisation between model variants Level 1 and Level 2 of the German power system during the first week of August 2005

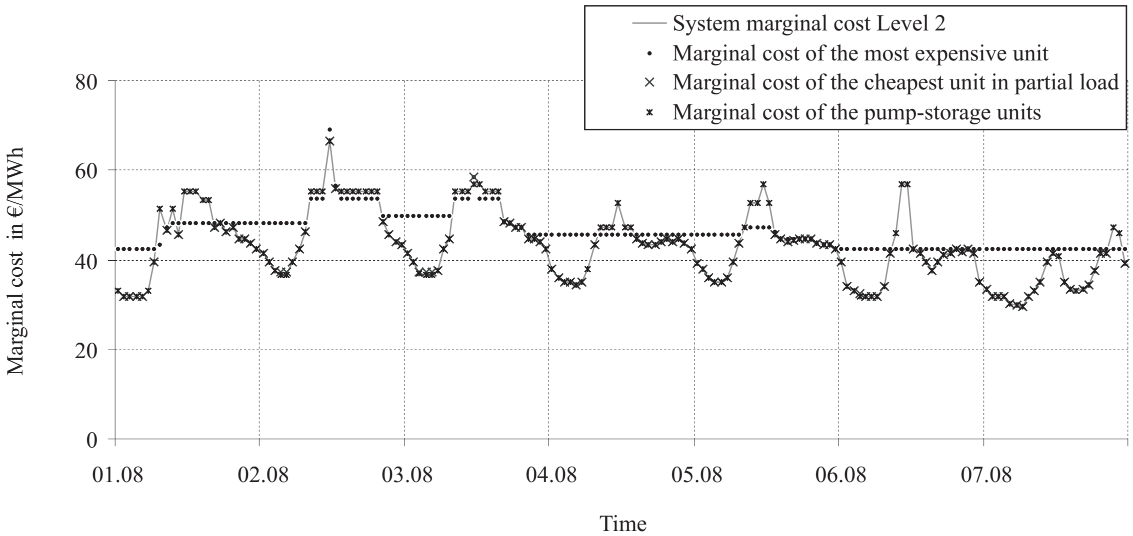


Figure 4.8: Decomposition of the marginal cost for model variant Level 2 of the German power system during the first week of August 2005

When system reserves are taken into account in model variant Level 3.4, the MAPE decreases further (see figure 4.3). This time, the average marginal cost is lower than for Level 2 during both base and peak hours. The utilisation of the thermal units is once more examined and is presented in figure 4.9. Again there is a reduction in the output of base-load units in favour of more expensive ones. As mentioned in section 3.1.7, the requirement for free spinning capacity to supply positive primary and secondary reserves, forces more units to be online that what otherwise be necessary if all units would be allowed to operate at their maximal capacity. On the other hand, the requirement for free spinning capacity to supply negative reserves, forces the qualifying units to operate above their minimal output during hours of low demand. The result is the marginal cost being determined by the cheapest unit operating at partial load and not the most expensive unit required to cover the load. This is shown in figure 4.10.

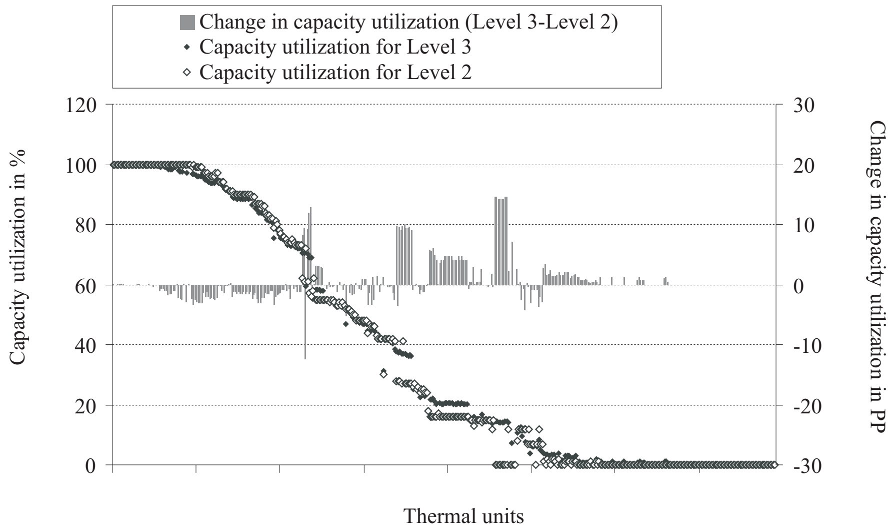


Figure 4.9: Change in the capacity utilisation between model variants Level 2 and Level 3.4 of the German power system during the first week of August 2005

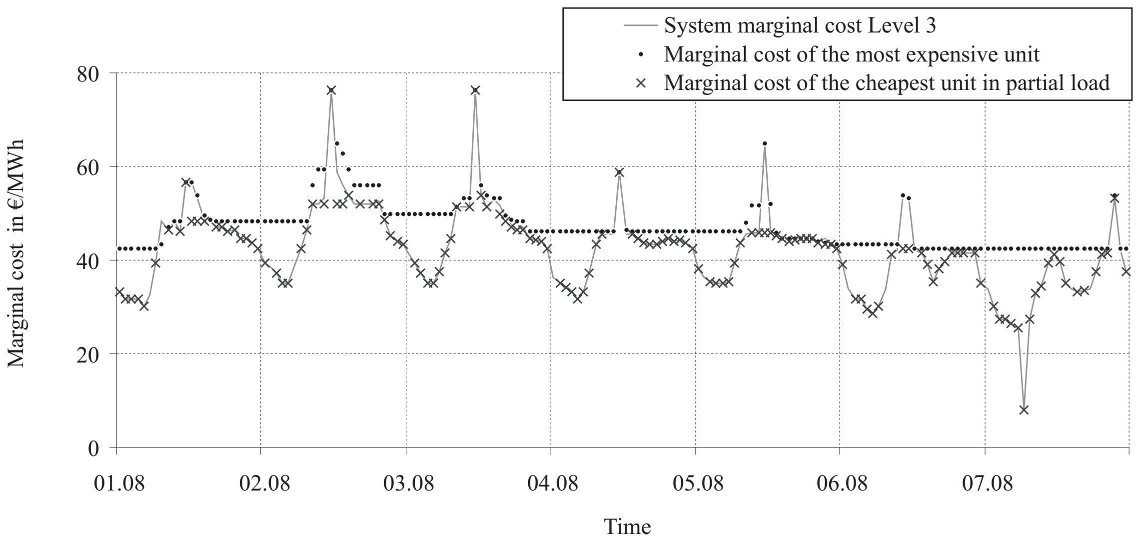


Figure 4.10: Decomposition of the marginal cost for model variant Level 3.4 of the German power system during the first week of August 2005

The error of the hourly marginal cost reaches a minimum with model variant Level 4.4, when the efficiency is not considered constant but a function of the output based on the linearised input-output characteristic instead. Figure 4.11 again shows a shift in the utilisation of the thermal power plants, with an increase in the output from base-load units. As explained in section 3.1.6.1 and illustrated in figure 3.4, when the input-output function is approximated according to equation (3.10), the average costs are higher at partial load than when the efficiency is assumed constant. As a consequence, it is more economical to operate certain units closer to their nominal output. The marginal cost of the units, on the other hand, is constant along the complete range of power output, but is lower than the ideal marginal cost. In consequence, the system marginal cost is lower than for model variant Level 3.4, as shown in the diagrams of figures 4.4 and 4.12.

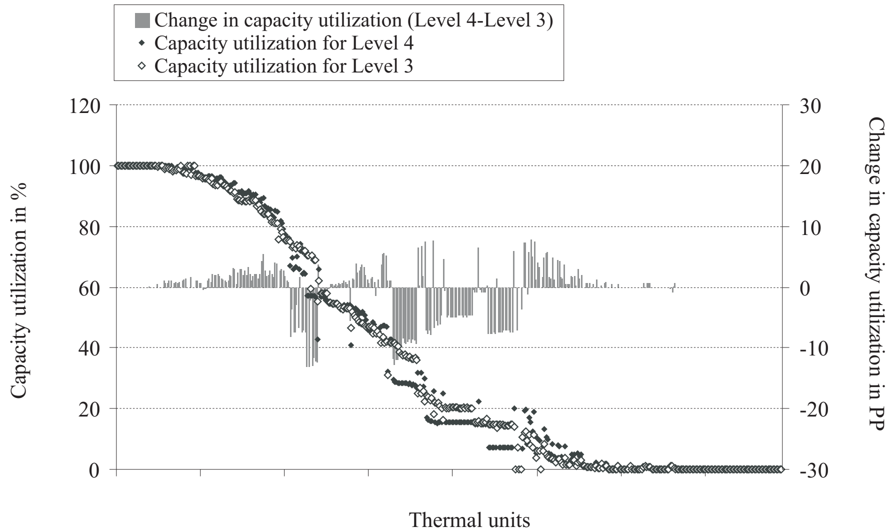


Figure 4.11: Change in the capacity utilisation between model variants Level 3.4 and Level 4.4 of the German power system during the first week of August 2005

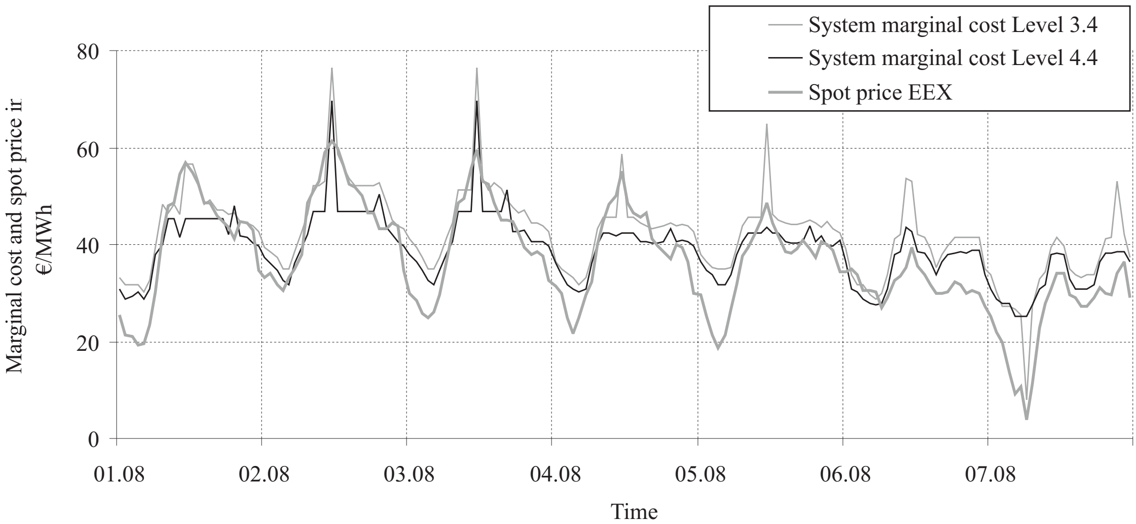
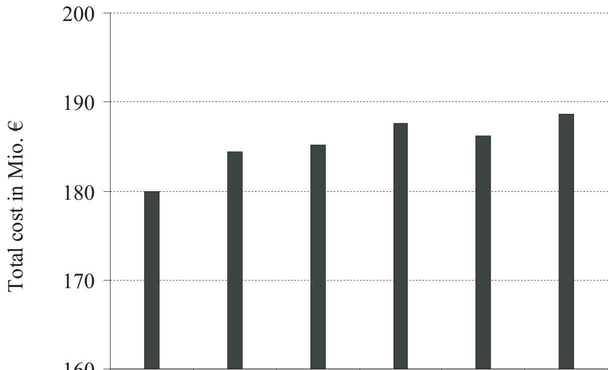


Figure 4.12: Hourly spot price and marginal costs of model variants Level 3.4 and Level 4.4 of the German power system during the first week of August 2005

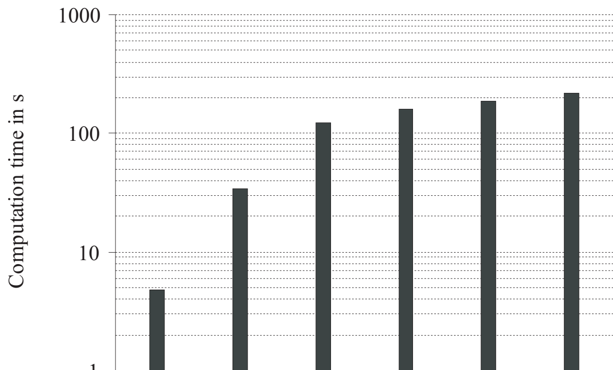
When the efficiency is assumed constant and minimum up and down time requirements are taken into account in model variant Level 5.4, the MAPE is considerably higher than when the time constraints are neglected in variant Level 4.4. It is important to notice that when all constraints are included in variant Level 6.4, the MAPE is lower than when the efficiency is assumed constant (Level 5.4), but not lower than when minimum time requirements are neglected but the efficiency is not considered to be constant (Level 4.4). The same can be said about the error of the base and peak marginal cost. This hints to the fact that imposing constraints in the minimal up and down times has little impact on the unit commitment and on the marginal cost. This suspicion is confirmed by looking at the value of the objective function (total operational cost) in figure 4.13. As expected, increasing the level of detail of the model makes the value of the solution worse (higher). But with the inclusion of minimum up and down time constraints (Levels 5.4 and 6.4), the value of the objective function in the point of optimality changes only slightly in comparison to the solution of the problem when these constraints are neglected (Levels 3.4 and 4.4 respectively).



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.4	179,92	184,47	185,21	187,61	186,18	188,65

Figure 4.13: Total cost of the German power system during the first week of August 2005

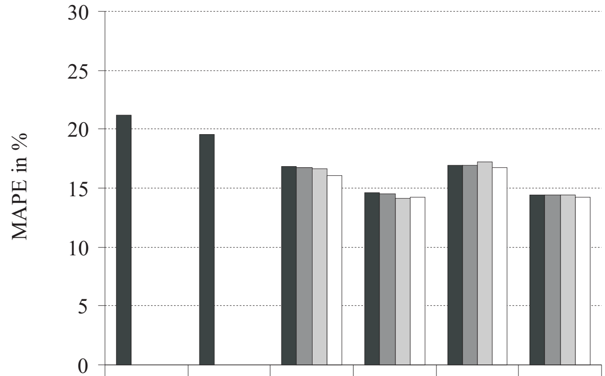
Regarding the modelling effort, the computation time is presented in figure 4.14. A direct relation with the level of detail can be easily appreciated. In this case, it takes 210 more seconds to solve the model formulated according to model variant Level 6.4 than according to Level 1, an increase of over 4000 %. Is important to notice how for Levels 5.4 and 6.4 the computation times are 52 % and 36 % higher than for Levels 3.4 and 4.4 respectively, while at the same time the accuracy of the results does not improves considerably.



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.4	5	34	124	160	188	216

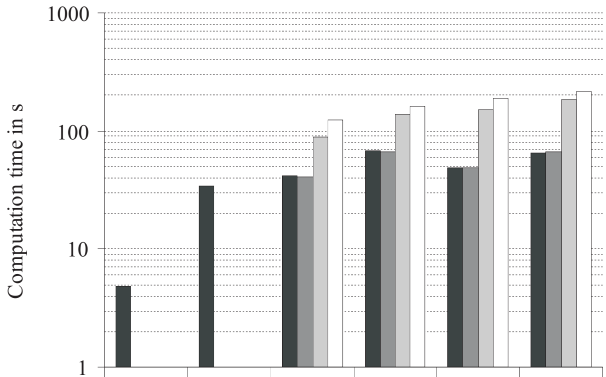
Figure 4.14: Computation time for model variants of the German power system during the first week of August 2005

The effect of the representation of the system reserves on the model performance is summarised in the MAPE and the computation times in figures 4.15 and 4.16. Simplifications in the modelling are in this case related to a slight increase in the error of the hourly marginal cost. The modelling effort, on the other hand, is much lower when no restriction on the units able to provide primary and secondary reserves is imposed (Level X.2). The effect of the negative portion of the reserves (Level X.1) is negligible for both the marginal cost and the modelling effort. This indicates that in most hours, even in the periods of low demand when many units run at their minimal load, the margin between the total power output and its lower bound is still enough to cover the negative reserves.



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	21,15	19,58	16,86	14,63	16,96	14,43
Levels X.2			16,78	14,55	16,94	14,46
Levels X.3			16,61	14,13	17,24	14,46
Levels X.4			16,08	14,24	16,72	14,22

Figure 4.15: Error of the hourly marginal cost of the German power system during the first week of August 2005



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	5	34	42	68	49	65
Levels X.2			41	66	49	66
Levels X.3			89	139	153	185
Levels X.4			124	160	188	216

Figure 4.16: Computation time for the model variants of the German power system during the first week of August 2005

The remaining results are presented in Appendix A in the form of diagrams of the MAPE, the relative error of the marginal cost during base and peak hours, the total cost and the computational effort. In absolute terms, there are significant differences in the

5 magnitudes used to assess the model's performance, both for the same region and time period across model variants, and for the same model variant across modelled region and time period. The modelling effort for the most complex model variant (Level 6.4), for instance, varies between 15 and over 900 seconds across all 12 test cases (three regions by four weeks). Similarly, the MAPE varies between 10 and 24 %, and the error of the marginal cost in base hours between 9 and -6 %. This shows the diversity achieved with the three modelled regions and four time periods considered. Quantitatively, however, and in spite of the individual characteristics of each system, there is no significant difference in the effect of the level of detail on the performance of the model.

Overall, an average reduction in the MAPE of 5,25 percentage points was achieved between the least and most accurate model variant. The model variants with the highest and lowest MAPE are not the same in all cases. The corresponding model variants were identified and are discussed in the next subsection.

The models of the French and German power systems during the first weeks of February, show a behaviour not fitting the pattern observed in the other cases. These two cases were singled out for a more detailed analysis. In particular, the sharp reduction of the MAPE and relative error of the marginal cost for Levels 3 and 5 when the representation of the reserves is simplified (Level X.2 vs. Level X.3) stands out (see figures A.1.1, A.3.1, A.5.1 and A.7.1 in Appendix A). The explanation for this phenomenon was found in particularities of the power demand of both systems during this time period. A peak in the load occurs every working day of the week at 18:00. To cover the demand, it is necessary to commit additional capacity during this and the two following hours. Due to the restrictions imposed on the units able to provide the reserves in Levels 3.3 and 3.4, the units committed in these cases have considerably higher operational costs than the ones committed in Levels 3.1 and 3.2. As a result, the marginal cost in Levels 3.3 and 3.4 reaches much higher values (see for example figures A.5.3 and A.7.3 in Appendix A). A possible explanation is that the units qualifying to provide primary and secondary reserves in these markets were wrongly selected in the model. However, the fact that the error of the hourly marginal cost decreases for model variants Level 4, indicates that the error is actually associated with the representation of the efficiency of the thermal units. In the case of the model for Germany, the discrepancy of the information provided by the MAPE and the error of the average marginal cost during base and peak hours, in particular for model variant Level 4, stands out. As can be seen in figure 4.17, even though the relative deviation is much higher for Level 4.3 than for Level 4.2, the profile of the marginal cost of Level 4.3 resembles much better that of the spot price. The relative error of the average marginal cost is in this case a misleading measurement of the quality of the results.

Also worth examining closer, is the apparent decrease in the value of the objective function associated with an increase in the level of detail of the model in some cases,

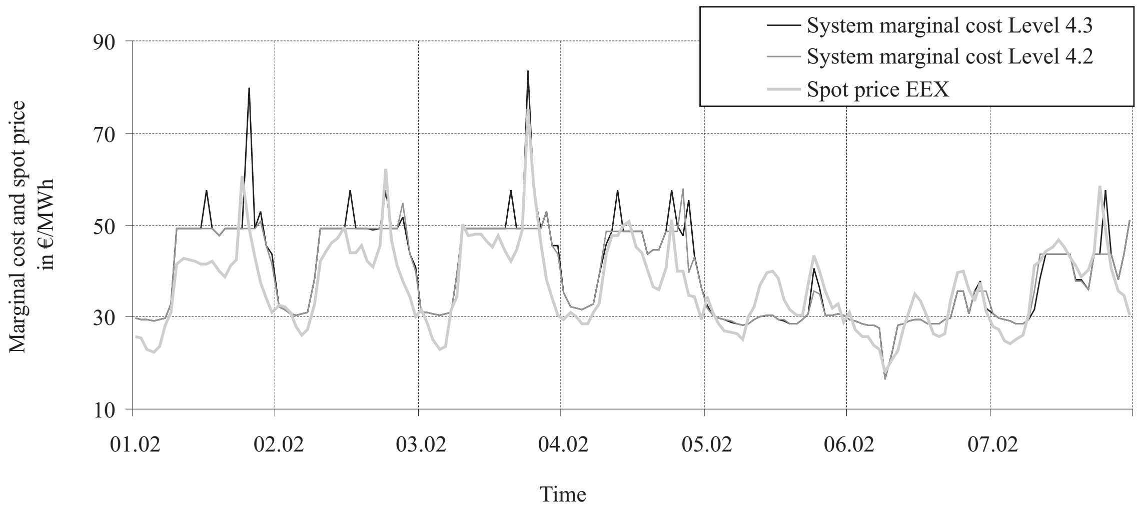


Figure 4.17: Hourly spot price and marginal costs of model variants Level 4.2 and Level 4.3 of the German power system during the first week of August 2005

something that contradicts the theory. In the model of the French power system during the first week of August 2005, for example, the total cost for model variant Level 4.3 is around 135 thousand Euros or 0,11 % lower than for Level 4.2 (see figure A.2.5 in Appendix A). The complete solving procedure was examined, and it was found that the problem lies on the accuracy of the solver. In the first case (Level 4.3) the combinatorial optimisation process stops when the current solution has a relative gap of 0,0146 %, well below the required tolerance (see section 3.4), and an optimality value of 124,28 Mio.€. In the second case (Level 4.2), the relative gap and the optimality value are, at 0,1806 % and 124,42 Mio.€ respectively, much higher. Also in this last case, the gap is lower than the minimum required, but clearly the solution is not as accurate as in the first case. When the minimal tolerance is reset at 0,01 % (near the gap of the solution of variant Level 4.3), the new solution obtained for Level 4.2 has a gap of exactly 0,0100 %, and the objective function a value of 124,22 Mio.€. This solution is only slightly "better" than the solution of model variant Level 4.3, but 0,16 % lower than the first solution. This result actually agrees with the theory. The computation time is however 22 times higher than before (705 s vs. 35 s), and the MAPE improved marginally from 26,90 % to 26,64 %. In other words, in the first attempt the solver found relatively fast a feasible solution (a local minimum) which satisfied the precision requirements of the solver, and settled with it.

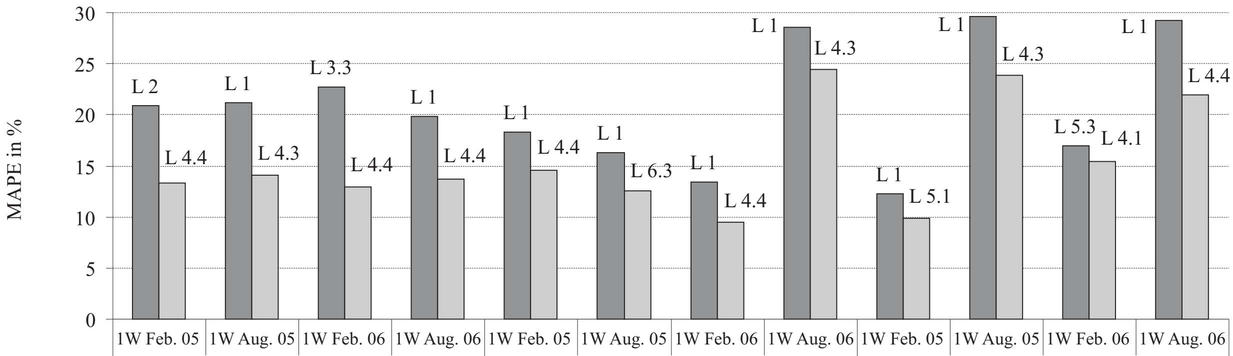
It should be kept in mind that commercial solver such as CPLEX often rely, at least partially, on heuristic approaches in their branch-and-bound or branch-and-cut algorithms to find the solution of combinatorial problems. For that reason, finding a feasible solution is not always a systematic process. It could actually happen "by chance" and with different relative effort for various applications of the same model formulation.

4.2.1 Identification of the Optimal Level of Detail

As mentioned in section 3.4, nothing definitive can be said about the optimal level of detail of a model in terms of performance, without knowing the relative importance of the accuracy and the modelling effort for the particular model application. Nevertheless, an attempt is made to identify the model variant offering a fair compromise between these factors. The assessment is based solely on the error of the hourly marginal cost and on the computation time.

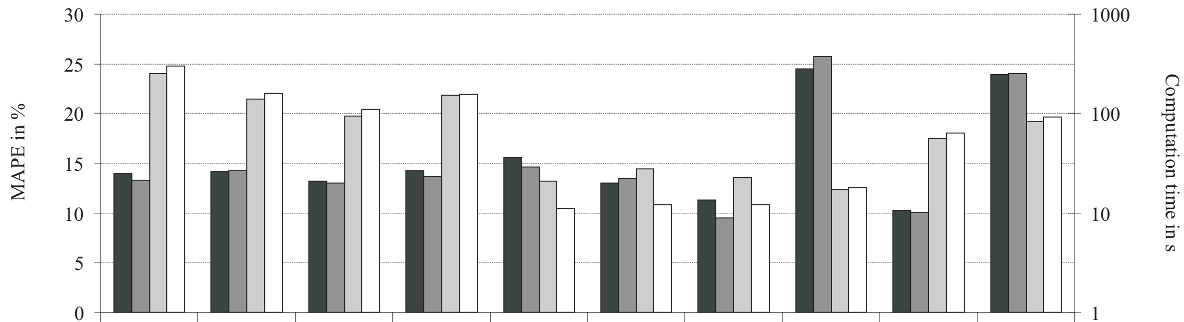
The diagram in figure 4.18 presents the selection of the least and most accurate model variants for each case. The labels on top of the horizontal bars give the corresponding level of detail. Clearly there is no absolute winner, but variants Level 4.4 and 4.3 seem good candidates to share the title of the model formulation with the highest accuracy. In the cases where these two variants were not the ones with the lowest MAPE, the corresponding values for Levels 4.4 and 4.3 are only tenths of percentage points higher (see figures A.1.1, A.3.1 and A.10.1 in Appendix A). In comparison with more detailed model variants (Level 5.X and Level 6.X, when minimum up and down times are considered), the MAPE of Levels 4.3 and 4.4 is always similar or lower. The modelling effort, on the other hand, is in all cases considerable lower.

To discern between variants Level 4.3 and Level 4.4, we look at the performance of these two model formulations in figure 4.19. It would be safe to say that the extra effort required to solve model variant Level 4.4 is relatively small in comparison to the potential gains in accuracy. Even more, for the model of the Spanish region, the effort of solving the model when tertiary reserves are taken into account (Level 4.4) is considerable lower than when they are neglected (Level 4.3). Therefore, Level 4.4 is here identified as the best model variant.



	Germany				Spain				France			
	1W Feb. 05	1W Aug. 05	1W Feb. 06	1W Aug. 06	1W Feb. 05	1W Aug. 05	1W Feb. 06	1W Aug. 06	1W Feb. 05	1W Aug. 05	1W Feb. 06	1W Aug. 06
max(MAPE)	20,86	21,15	22,75	19,85	18,27	16,32	13,39	28,56	12,30	29,59	17,00	29,22
min(MAPE)	13,28	14,13	12,97	13,68	14,59	12,58	9,48	24,48	9,88	23,89	15,43	21,92

Figure 4.18: Variants of the models for French, German and Spanish power systems during the first week of February and August 2005 and 2006 with the highest and lowest mean average percentage error of the marginal cost



	1W Feb. 05	1W Aug. 05	1W Feb. 06	1W Aug. 06	1W Feb. 05	1W Aug. 05	1W Feb. 06	1W Aug. 06	1W Feb. 05	1W Aug. 05
	Germany				Spain				France	
MAPE(Level 4.3)	13,94	14,13	13,19	14,25	15,53	12,96	11,26	24,48	10,26	23,89
MAPE(Level 4.4)	13,28	14,24	12,97	13,68	14,59	13,48	9,48	25,75	10,11	23,99
Time(Level 4.3)	252	139	94	152	21	28	23	17	56	83
Time(Level 4.4)	298	160	110	156	11	12	12	18	63	92

Figure 4.19: Error of the hourly marginal cost and computation times for variants of the models of the German, French and Spanish power systems during the first week of February and August 2005 and 2006

The reduction in the MAPE achieved with model variant Level 4.4 with respect to the reference model formulation (Level 1), is in average of 4,55 percentage points and varies in between 1,10 and 7,30 percentage points across the 12 modelled test cases.

4.2.2 Conclusions and Recommendations

With the previous analysis, the relationship between the level of detail and the performance of variants of the production cost model developed was established. In general, model variants with a higher level of detail in the representation of the system yield more accurate results than simpler variants.

The inclusion of technical and economical restrictions in the representation of the system in the form of constraints, increases the effort required to solve the unit commitment and load dispatch problems. In particular, it was found that the modelling effort is somewhat sensitive to the modelling of system reserves. The effect is significant when tertiary reserves are neglected (Level X.4 vs. Level X.3) in some cases, and in most cases when there is no discrimination on the units able to supply primary and secondary reserves (Level X.3 vs. Level X.2).

The consideration of minimal up and down time requirements in the commitment of thermal units often produces a significant increase in the modelling effort but does not affect the model accuracy considerably.

Also worth noticing is the tendency of the marginal cost to decrease as the number of constraints in the model increases. Special cases where identified where a higher level of detail in the representation of the system reserves can also produce higher marginal costs during peak hours.

It is important to mention that the observations made here in some cases does not support those made by previous authors. Worth noticing is the high sensitivity of the model performance to the representation of the system reserves. This contradicts the observation made by Cumperayot [7, page 77], who found that neglecting secondary and tertiary reserves has a "*...negligible impact on the determination of the system marginal cost...*"[4]. In another case, Kreuzberg [31, page 66] alleges (in a footnote) that adding variable efficiencies to the representation of thermal units "*...would strongly increase computing requirements without adding much to the realism of the model ...*". While the remark on the computational effort seems correct, considering variable efficiencies does seems to render a better (perhaps more "realistic") representation of the system.

In conclusion, the influence of the level of detail on the accuracy of the results and modelling effort should not be assumed or neglected without first assessing it, quantitatively at best. Only then, it is possible to have a clear understanding of the results and the inherent error and uncertainties of the modelled marginal cost.

4.3 Effect on the Modelling of the Marginal Cost of Multiregional Systems

The following analysis examines the effect of the level of detail of the production cost model on the marginal cost of multiregional power systems. It is motivated by the attempt to reduce the modelling effort by simplifying the representation of regions adjacent to one or more main regions of interest. Examples of this approach can be found in [28, 31].

Two systems with two regions each where modelled: Germany-France (DE-FR) and France-Spain (FR-ES). The modelled time periods were the same as in the previous analysis: the first week of February and August of 2005 and 2006. The power exchange between each pair of regions was represented through an optimisation variable according to equation (3.40), and integrated into the power balance according to equation (3.41).

Based on the results presented and discussed in the previous section, a narrower combination of three model variants was tried. In all cases one of the regions was modelled according to model variant Level 4.4, the model formulation identified as the best performer. The adjacent region was modelled according to either:

- **Level 4.4**, for the same reasons given above,
- **Level 1**, for being the simplest model representation and requiring the least modelling effort.

[4] For a model of the German power system during the year 2001.

The results from each variant were compared with the results obtained for the first analysis, that is, with each region modelled according to variant Level 4.4 and taking into account the scheduled power exchange with all adjacent regions as exogenous time series.

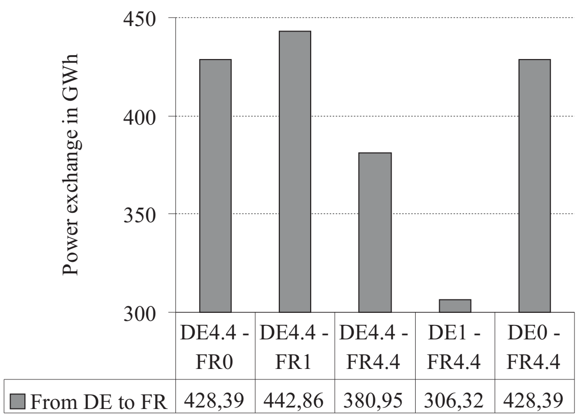
For simplicity reasons, the short form of the name of the regions and the level of detail used for their representation are used to refer to a particular test case. The expression DE4.4-FR1, for example, makes reference to the model of the German and French power systems where the first is modelled according to Level 4.4 and the latter according to Level 1. The expression FR4.4-ES0 corresponds to the model of the French system according to Level 4.4 where the scheduled power exchange with Spain is included into the power balance of the French system as an exogenous time series.

Before looking at the results, it is important to mention that congestion is rather common in these two interconnections. All throughout the year 2005 and on an hourly basis, 57 % of the time the scheduled exchange between France and Spain was 95 % or higher than the allowed net transfer capacity (NTC) in either direction [57]. In contrast, the level of congestion in the interconnection between France and Germany in the same period was at 9 % much lower. But for the first week of February and August of that year, the proportions are considerably higher. The exact values for the first week of February and August of 2005 and 2006 are shown in table 4.2.

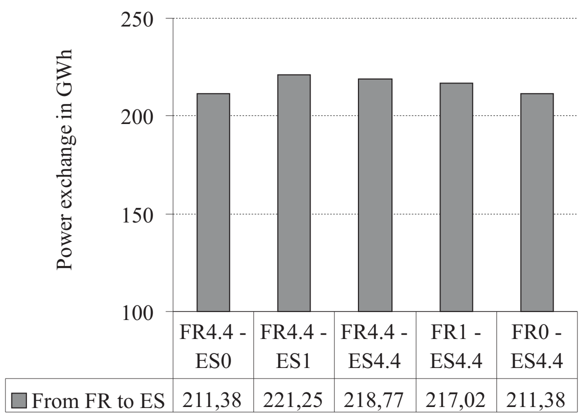
	Proportion of hours with congestion [%]	
	Germany-France	France-Spain
1st week February, 2005	0	73
1st week August, 2005	76	96
All through 2005	9	58
1st week February, 2006	0	59
1st week August, 2006	40	38
All through 2006	16	44

Table 4.2: Congestion in the interconnections between the French, German and Spanish electricity markets

First, the effect that the level of detail in the modelling of each region has on the power exchange is analysed, as this will help understand and explain the effect on the marginal cost. The results show that the effect on the power exchange is related to the level of utilisation of the interconnection. This is illustrated in the diagrams in figure 4.20, which show the scheduled and modelled power exchange between pair of regions for the first week of February 2005 (the remaining diagrams are presented in Appendix A). For the three cases with congestion of over 70 % the effect is rather limited (see figures A.13.2, A.14.1 and A.14.2). This implies that the regional costs are not affected in a level that would alter the cost disparity between regions significantly. The deviation of the modelled from the scheduled power exchange is in these cases, and for all model variants, relatively small. Only in the interconnection between Germany and France during the first week of August there is a considerable difference between the schedule and the model. The historical data shows that the scheduled exchange was actually higher than the notified NTC in 71 % of the hours by up to 35 %. This explains why the power flow from France to Germany is severely underestimated by all model variants (see figure A.14.1 in Appendix A).



(a) From the German to the French region



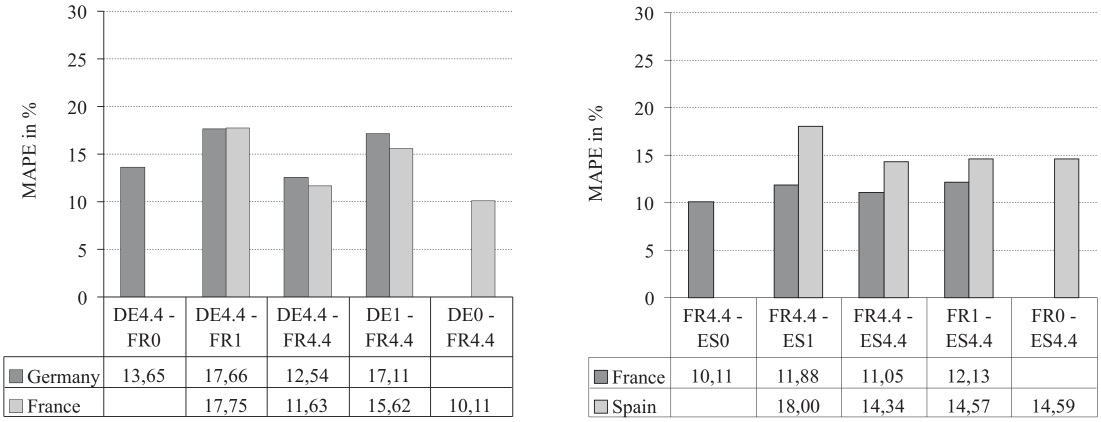
(b) From the French to the Spanish region

Figure 4.20: Power exchange during the first week of February 2005 for different model variants

In the cases with a lower level of congestion, not only is the effect of the level of detail on the modelled power exchange much more significant, also there is a clear correlation between the representation of the importing and exporting regions and the power exchange: the higher the relative level of detail of the exporting region, the higher the level of exports from that region (see figures A.13.1, A.15.1 and A.15.2 in Appendix A).

Regardless of the level of congestion and the level of detail of the model, differences between the scheduled and modelled power exchange should be judged carefully. As mentioned in section 4.1, contrary to the model's assumptions, during some of the modelled time periods the transmission capacity of both interconnections was not allocated according to market-based mechanisms. Instead, pro-rata and priority lists were used, and some of the capacity was withheld on the basis of long-term contracts. The evidence presented in [4, 5, 13] shows that, in spite of the implementation of explicit auctions, there was still little or no correlation between the scheduled power exchange and the price spread between markets. Power was often exported from Germany into France or from France into Spain, even if the spread in the spot prices in those directions was negative.

Regarding the effect of the level of detail of the models on the marginal cost, as expected based on previous observations, simplifications made in the representation of the modelled regions reduce the accuracy of the results. It is important to notice that simplifications made in either one of the two interconnected regions lowers the accuracy of the results for both regions in all tested cases. This is illustrated in figure 4.21 as diagrams of the MAPE for the same regions and time period as presented above.



(a) French and German power systems **(b)** French and Spanish power systems

Figure 4.21: Error of the hourly marginal cost during the first week of February 2005 for different model variants

No relation between the accuracy in the modelling of the marginal cost and the accuracy in the modelling of the power exchange can be established. What is more, the accuracy in the modelling of the power exchange is in no way related to the level of congestion.

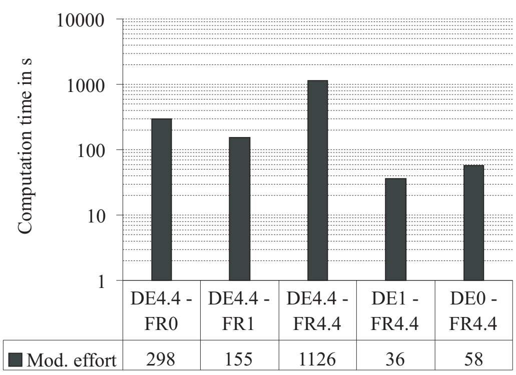
It is remarkable that in six out of the eight modelled cases, the added MAPE of both regions is the lowest when the power exchange is considered exogenously. Only in some cases the MAPE of the marginal cost for one of the regions is lower when both regions are coupled and modelled according to variant Level 4.4 than for the reference model (when the exchange is not determined endogenously). But in no case and for no combination of model variants is the MAPE of both regions lower than in the reference case simultaneously.

Regarding the modelling effort, the computation times for the cases cited above are presented in figure 4.22. The models of the interconnected regions according to variant Level 4.4 are in all cases the most difficult to solve. The computation time for these cases is significantly higher than when both regions are modelled separately.

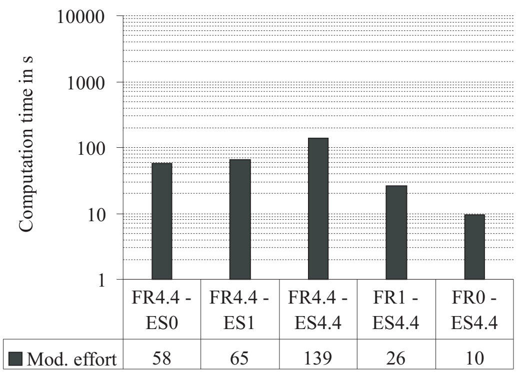
4.3.1 Conclusions and Recommendations

The effect of the level of detail on the performance of multiregional production cost models was examined. This was done with the help of two systems, each comprising two interconnected regions modelled during four different weekly time periods.

The results show that an even and detailed representation of each region yields the most accurate results. However, the resolution of such modelled requires a higher modelling effort than when the interconnected regions are modelled separately.



(a) French and German power systems



(b) French and Spanish power systems

Figure 4.22: Computation times for different model variants during the first week of February 2005

It was also demonstrated that better results and a lower modelling effort can be achieved by considering the power exchange between regions as an exogenous factor, rather than by determining it endogenously. Inaccuracies in the representation of each regions may propagate in the model, leading to errors in the representation of the power exchange and in the representation of the marginal cost of each region. Also, the comparison of the modelled and scheduled power exchange showed that is in general difficult to reproduce the latter. Independent analyses of the markets for electricity of the three modelled regions and of the historical power exchange between the markets, revealed that the cross-border power trade does not always obey to market fundamentals such as price spreads. This could be blamed on the inefficiency or absence of markets for transmission capacities. The opposite condition, namely the allocation of the transmission capacity via (efficient) market-based mechanisms is a primal assumption of the production cost model developed.

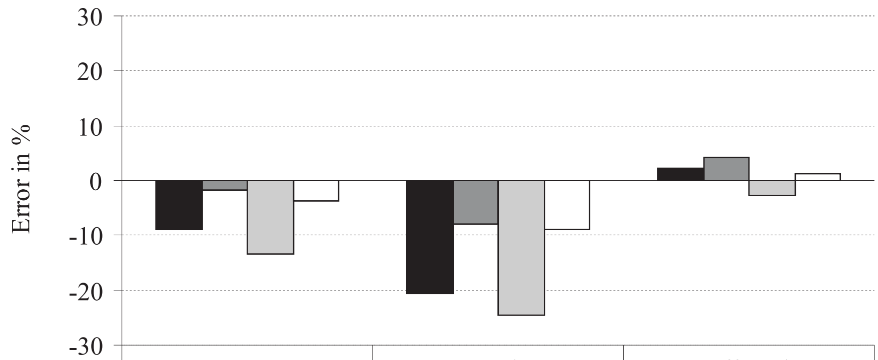
In this sense, the endogenous modelling of the power exchange presents no significant advantage for the modelling of the marginal cost of individual regions. This is of course based on the premise that the scheduled power exchange is known. When modelling future marginal costs, for instance, this is clearly not the case. The decision whether to model interconnected power systems as such, or to consider the power exchange between them an exogenous factor, should be taken based on the perceived (un)certainty of the model of each region, and of the power exchange forecast at hand.

4.4 Marginal Cost with Specific Start-up Costs

As mentioned in section 3.1.6.1 and demonstrated in section 4.2, start-up costs play and important role in the commitment of power generating units and thus in the modelling of marginal costs. In section 3.3.4 it was also discussed that the closed MILP approach applied to solve the unit commitment and load dispatch problem, fails to take into account start-up costs on the system's marginal cost. The methodology presented in that section was applied in the modelling of the marginal cost of the three modelled regions in an effort to render a more realistic representation of the supply cost function.

The results from the analysis done in section 4.2 do not precisely support the thesis that production cost models always underestimate spot prices in peak hours and overestimates then in off-peak hours. It was shown that the marginal cost may as well be higher than the spot prices depending as much on the level of detail of the model as on the system and time period modelled. Therefore, for the following analysis the modelling period was extended to the calendar years 2005 and 2006, still in weekly intervals. The model variant used to calculate the reference marginal cost was Level 4.4, the one offering the best performance.

Positive and negative specific start-up costs are supposed to have different effects on the marginal cost. While the former is expected to correct the systematic underestimation of the spot prices in peak hours, the latter should target the overestimation of the prices during off-peak hours. It should be remembered that it is possible for both positive and negative specific start-up costs to be greater than zero for the same generating unit at the same time. To be able to appreciate the effect of each specific start-up cost on the marginal cost, the diagram in figure 4.23 presents the result of applying the methodology to the model of the German power system during the year 2005, first with positive and negative start-up costs separately and then combined. The bars represent the error of the average reference (normal) and modified marginal costs during base, peak and off-peak hours relative to the market spot price.



	Base	Peak	Off-peak
Normal	-9,01	-20,62	2,12
With positive start-up costs	-1,77	-8,04	4,25
With negative start-up costs	-13,42	-24,56	-2,73
With pos. and neg. start-up costs	-3,71	-8,94	1,32

Figure 4.23: Relative error of the average marginal cost of the German power system for the year 2005 in base and peak hours including specific start-up costs

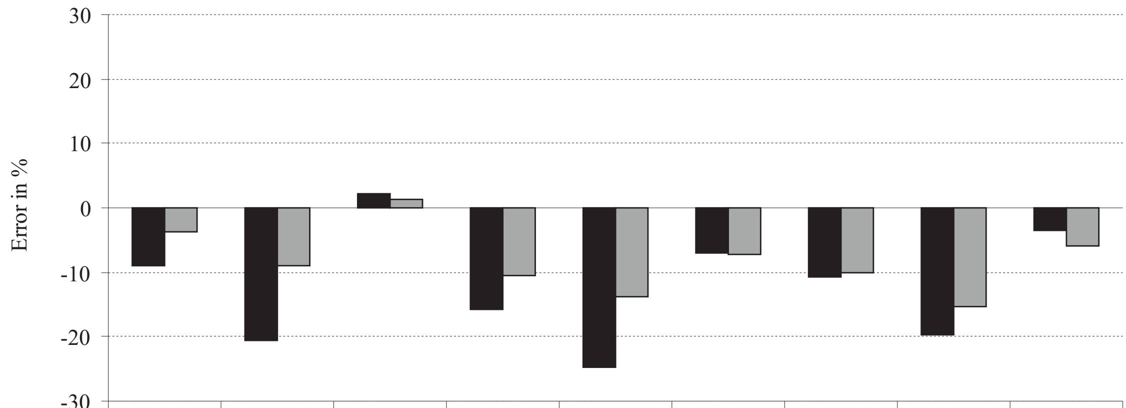
As expected, positive specific start-up costs raise the marginal costs specially during peak hours. It also has the undesired effect of rising the marginal cost in the off-peak hours.

This can be attributed to the starting-up of power plants for periods of high demand on the weekends and holidays, when all the hours are classified as off-peak (see section 3.4). Negative specific start-up costs, on their part, have the expected effect of lowering the marginal cost in particular in off-peak hours. The effect of both positive and negative start-up costs combined is lesser than the effect of each one individually. Nevertheless, the error of the modified marginal cost in base, peak and off-peak hours is lower. In this case, with the help of the positive and negative specific start-up costs, the error of the marginal cost was reduced over 50 % in base and peak hours and almost 40 % in off-peak hours. However, peak prices remain underestimated and off-peak prices remain overestimated.

The results of applying the specific start-up costs methodology to the three modelled regions during both years are presented in figures 4.24 and 4.25. In all cases, the reference marginal cost in base and peak periods is lower than the spot prices. In off-peak periods, however, the results are mixed. The error of the modified marginal costs is lower than the error of the reference marginal cost in base and peak hours in all cases. But in those cases where the model already underestimated the market prices in off-peak hours (France in both years and Spain in 2005), the application of the specific start-up costs methodology worsened the accuracy of the results.

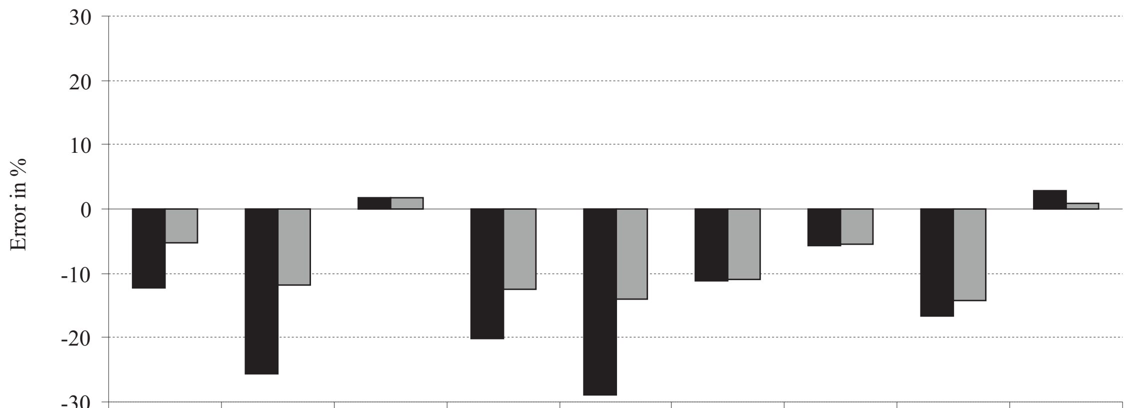
It is also important to notice that the impact of the specific start-up costs is different for the three regions. In this respect, stands out the little effect of the negative specific start-up costs on the French region during off-peak hours. This can be attributed to the relative large number of nuclear power plants operating in the French system. In the model, these units have very similar operational costs and the lowest operational costs of all power plants. This implies that during off-peak hours, if one of these units sets the price, the operational cost of the other units is unlikely to be lower than the marginal cost of the system. In the case of the Spanish region, the effect of the specific start-up costs is almost negligible. A detailed analysis of the unit commitment in this system revealed that the number of start-ups is relatively small. This can be explained by the relatively "flat" load profile and a large portion of storage and pumped-storage power plants present in the system. The latter helps the system follow the load profile without the necessity of changing the state of the thermal units.

To assess the effect of the specific start-up costs on the hourly marginal cost, the error of the hourly marginal cost of the 104 weeks modelled was calculated. The results are presented in figure 4.26. For all three regions, the average MAPE of the modified marginal cost is higher than that of the reference marginal cost. From this results and from the error of the average marginal cost, it can be inferred that with the application of the specific start-up costs methodology the modified marginal cost reaches more extreme values. When averaged over a longer period of time, the modified marginal cost is closer to market prices. But extreme values do not occur at the same time as in the market. Thus the higher MAPE.



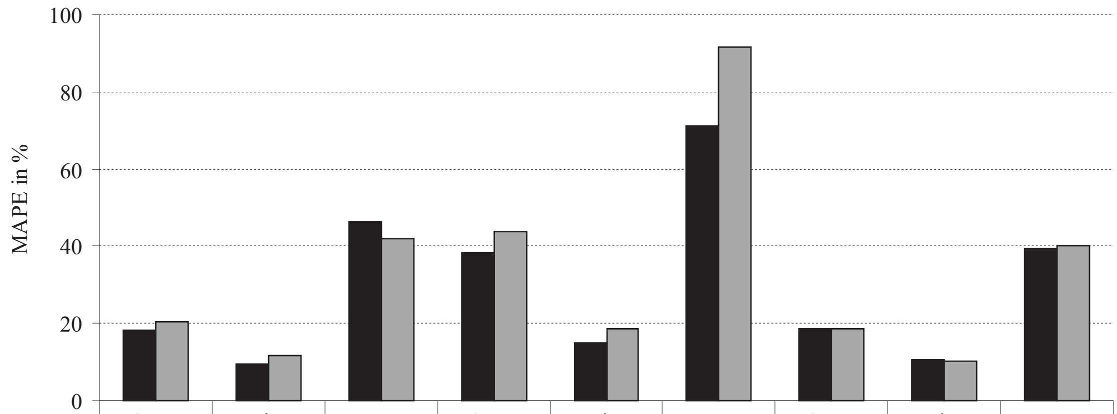
	Germany			France			Spain		
	Base	Peak	Off-peak	Base	Peak	Off-peak	Base	Peak	Off-peak
Normal	-9,01	-20,62	2,12	-15,68	-24,65	-7,06	-10,70	-19,73	-3,54
Modified	-3,71	-8,94	1,32	-10,41	-13,74	-7,24	-9,99	-15,28	-5,81

Figure 4.24: Relative error of the average marginal cost of the French, German and Spanish power systems during the year 2005



	Germany			France			Spain		
	Base	Peak	Off-peak	Base	Peak	Off-peak	Base	Peak	Off-peak
Normal	-12,25	-25,56	1,84	-20,10	-28,90	-11,27	-5,62	-16,69	2,74
Modified	-5,24	-11,86	1,79	-12,41	-13,93	-10,92	-5,57	-14,26	0,98

Figure 4.25: Relative error of the average marginal cost of the French, German and Spanish power systems during the year 2006



	Germany			France			Spain		
	Avg	Min	Max	Avg	Min	Max	Avg	Min	Max
Normal	18,07	9,45	46,50	38,37	15,14	71,01	18,70	10,54	39,50
Modified	20,38	11,74	42,11	43,71	18,45	91,79	18,77	10,29	40,32

Figure 4.26: Error of the hourly marginal cost of the of the French, German and Spanish power systems during the years 2005 and 2006

4.4.1 Conclusions and Recommendations

The methodology developed to integrate start-up costs into the system marginal cost, proved to be beneficial for the modelling of the market prices in the modelled regions. When specific-start up costs are included into the variable operational costs of selected thermal power plants, the average of the modified marginal cost is closer to the market prices. But in most cases, i.e. for most weeks, the hourly profile of the modified marginal cost, differs more form the profile of the spot price. From this observation, the conclusion can be drawn that the methodology is more useful for correcting the average marginal cost produced by the model.

It should be taken into account that both positive and negative specific start-up costs are calculated based on the total start-up cost of the thermal units. Therefore, the effect of the methodology depends on the estimation of the total start-up costs. For instance, higher start-up costs would produce higher positive specific start-up costs and lower negative specific start-up costs. In the model applied here, total start-up costs are assumed constant and equivalent to a warm start (see section 3.1.6.1). Different results should be obtained if the start-up costs are modelled according to a warm start or as a function of the down-time of the units.

The specific-start up costs methodology is based on the assumption that the imperfect consideration of the cost components of the power plants is alone responsible for the systematic misestimation of the spot prices. However, the marginal cost depends as much on the modelling methodology as on input factors such as load and other costs. As pointed out by Schwartz [61, page 3], different assumptions on components of the load balance can lead to different conclusions regarding the efficiency of the market or the accuracy of the model. Furthermore, the methodology also assumes that production cost models systematically underestimate peak prices and underestimating off-peak prices. However, if this hypothesis does not hold for a particular system, the methodology could by unhelpful in improving the accuracy of the results.

Is recommended the application of this methodology preceded by an exhaustive analysis, preferably over a long time period, of the ability of a production cost model to reproduce spot prices, and of the influence of input factors such as the load and costs on the marginal cost.

Chapter 5

Summary

Production cost models are important analysis and decision support tools in the context of liberalised and competitive wholesale electricity markets. With the help of production cost models, it is possible to calculate the marginal cost of production of electricity. Under the assumption that the wholesale market is perfectly competitive, the marginal cost should correspond to the spot price of electricity.

Production cost models are based on a mathematical representation of the technical and economical characteristics of the power system associated with an electricity market, and usually formulate the unit commitment and economic dispatch of the power generation resources as an optimisation problem. Technical restrictions governing the operation of the system, are integrated into the optimisation problem in the form of constraints. In this respect, three issues for the formulation and application of production cost models have been identified:

1. Depending on the level of detail in the representation of the system, the formulation of the model and the solution of the optimisation problem might require significant data-gathering and computational effort.

2. Owing to the integration of regional markets, in some cases it might be advantageous or necessary to model multiple interconnected regions. This, however, carries with it additional modelling effort.

3. Traditional methodologies applied to solve the optimisation problem, fail to incorporate the start-up cost of thermal power plants into the system marginal cost. It has been observed that this leads to a systematic misestimation of market prices in particular in times of extreme high or low power demand.

To deal with the issues of data availability and modelling effort, it is common practice to lower the level of detail of the model by simplifying the representation of the system. Such simplifications, however, could compromise the accuracy of the results, and ultimately, the

usefulness of the model. Until now, no comprehensive analysis of the effect of the level of detail on the performance of production cost models has been done.

The present work was developed with the objective of making a contribution to the formulation of production cost models, specially analysing the effect of the level of detail of the models on the accuracy of their results and on the associated computational effort. A further objective was to develop and apply a methodology to address the issue of the consideration of start-up costs in the calculation of the system marginal cost.

To fulfil these objectives, a production cost model was developed, which allows for a flexible representation of the power system. Variants of the model, 18 in total, with different levels of detail were formulated. The model variant with the lowest level of detail served as reference for the evaluation of more detailed variants. The model variants were applied for the modelling of the power systems of three important European electricity markets during exemplary weeks of the years 2005 and 2006. The fact that the three systems have adjoining borders and their power systems are interconnected, also allowed the construction of multiregional models. The performance of each model variant applied to each system was evaluated based on the accuracy of the results and the computational effort. The accuracy was determined by comparing the marginal costs of each region with observed spot prices.

For the first analysis, the power exchange between regions was taken into account as an exogenous factor. The results show the tradeoffs between the modelling effort and the accuracy of the results.

In general, a direct relationship between the level of detail of the model and the accuracy of the results was observed. For the cases modelled, the average difference in the mean average percentage error (MAPE) of the hourly marginal cost of the least and the most detailed model variants is 5,25 percentage points. Increasing the level of detail, in general also increases the modelling effort required to solve the model. The computation time ranges in between 1 and 3 seconds for the reference model and between 15 and over 900 seconds for the most complex model variant.

Based on the results of this first analysis, one model variant was singled out as the one offering the best balance between accuracy and modelling effort. The model in question takes into account the following constraints in the operation of thermal power plants:

- Minimal and maximal power output requirements,
- Charges for the start-up as part of their variable costs,
- A linearised representation of the fuel-input versus power-output characteristic

The minimum up and down time requirements of the units are hereby neglected. The model variant also takes into account the requirements for primary, secondary and tertiary system capacity reserves. The average difference between the MAPE of the marginal cost resulting from this and the least detailed model variant is 4,55 percentage points.

It was observed that increasing the level of detail in the model has the effect of decreasing the average marginal cost. The requirements for system reserves and the lower efficiency of thermal units when operating at partial load were identified as the main causes of this effect.

From the modelling of multiple interconnected regions, it was observed that the level of detail can have important effects on the modelled power exchange and thus on the regional power balance. This, in turn, has an influence in the system marginal cost of the systems. Modelling the regions with an uneven level of detail offers a lower modelling effort than when all regions are modelled with the same level of detail. The accuracy of the results, however, also decreases. Furthermore, in all cases the performance of the multiregional models was worse than the performance of the models of the individual regions taking into account the power exchange as an exogenous input parameter.

To address the issue of the consideration of the start-up cost of thermal power plants, a methodology was developed to incorporate this cost component into the system marginal cost. The methodology was applied in the modelling of the three power systems during two whole calendar years. The addition of positive and negative specific start-up costs to the variable costs in stationary operation of thermal power plants, does modify the system marginal cost as expected, raising it during peak hours and lowering it during off-peak hours. The magnitude of the effect, however, is different across the modelled power systems. The error of the average marginal cost decreases for all three systems in both years, which proved the benefits of the developed methodology.

To summarise, the level of detail of production cost models and the delimitation of the modelled system can have important effects on the performance of the model. The magnitude of these effects, however, depends on the modelled system. To maximise the usefulness of production cost models it is important to pay attention to both the level of detail and the system delimitation during the development and implementation phases of the model.

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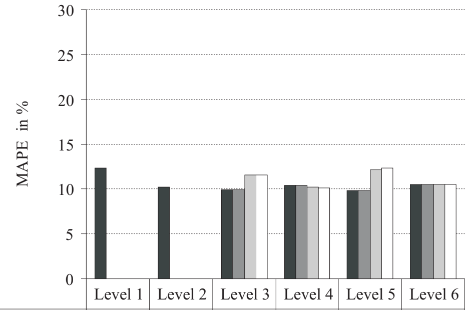
Appendix A

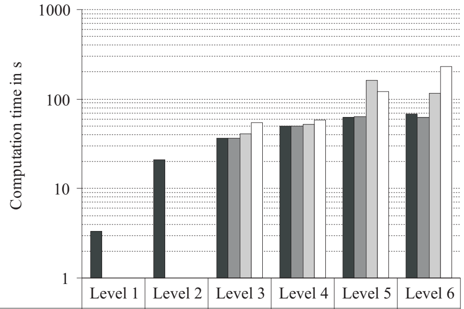
Results

A.1 Model of the French Power System for the First Week of February 2005



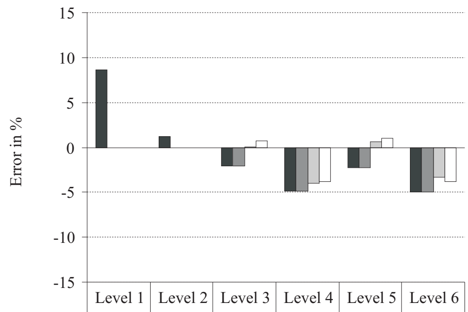
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	12,36	10,18	9,92	10,44	9,88	10,55
Levels X.2			9,92	10,44	9,88	10,55
Levels X.3			11,61	10,26	12,18	10,55
Levels X.4			11,55	10,11	12,30	10,51

Figure A.1.1: Error of the hourly marginal cost



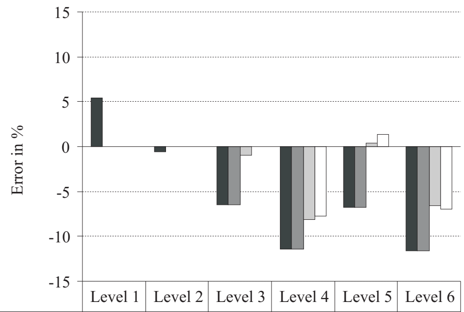
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	3	21	36	50	62	67
Levels X.2			37	50	63	62
Levels X.3			41	52	161	117
Levels X.4			54	58	121	230

Figure A.1.2: Computation time



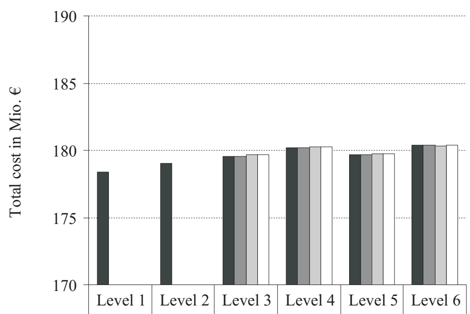
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	8,61	1,17	-2,10	-4,82	-2,24	-4,93
Levels X.2			-2,10	-4,82	-2,24	-4,93
Levels X.3			0,05	-3,99	0,61	-3,36
Levels X.4			0,75	-3,83	1,02	-3,78

Figure A.1.3: Error of the average marginal cost during base hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	5,42	-0,60	-6,51	-11,41	-6,80	-11,62
Levels X.2			-6,51	-11,41	-6,79	-11,62
Levels X.3			-0,96	-8,11	0,36	-6,59
Levels X.4			-0,02	-7,77	1,31	-6,94

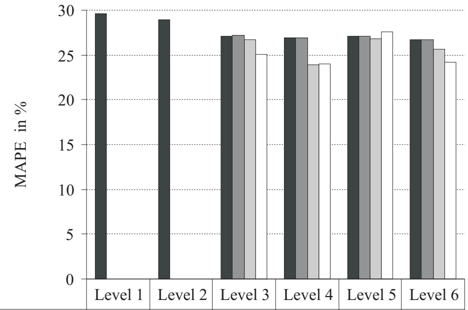
Figure A.1.4: Error of the average marginal cost during peak hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	178,37	179,03	179,53	180,19	179,66	180,39
Levels X.2			179,53	180,19	179,65	180,38
Levels X.3			179,66	180,24	179,76	180,35
Levels X.4			179,67	180,25	179,75	180,37

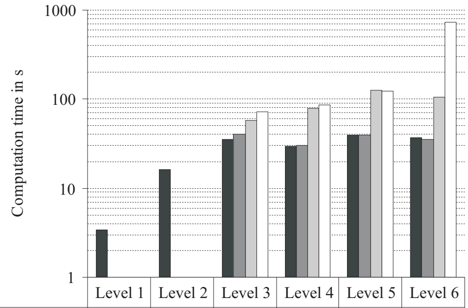
Figure A.1.5: Total cost

A.2 Model of the French Power System for the First Week of August 2005



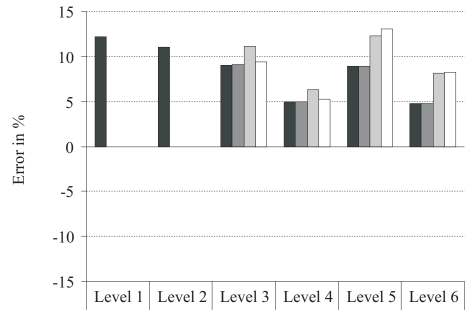
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	29,59	28,98	27,13	26,90	27,11	26,74
Levels X.2			27,15	26,90	27,11	26,74
Levels X.3			26,71	23,89	26,77	25,65
Levels X.4			25,03	23,99	27,56	24,16

Figure A.2.1: Error of the hourly marginal cost



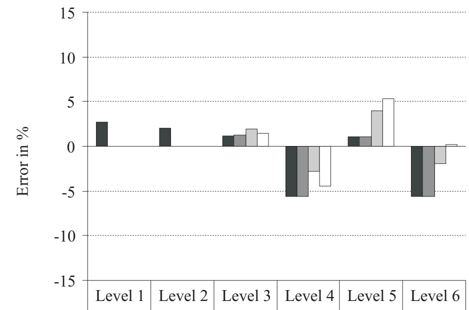
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	3	16	35	30	40	37
Levels X.2			40	30	40	35
Levels X.3			57	80	127	106
Levels X.4			72	87	122	729

Figure A.2.2: Computation time



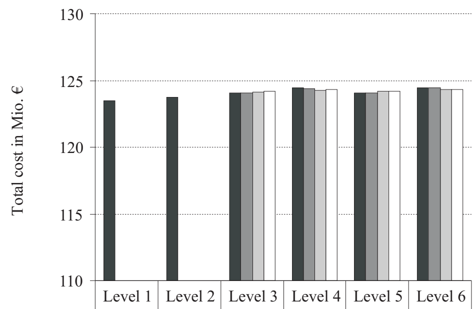
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	12,20	11,02	8,99	4,94	8,94	4,80
Levels X.2			9,08	4,94	8,94	4,80
Levels X.3			11,17	6,28	12,31	8,18
Levels X.4			9,38	5,25	13,11	8,23

Figure A.2.3: Error of the average marginal cost during base hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	2,74	2,01	1,16	-5,63	1,06	-5,61
Levels X.2			1,28	-5,63	1,06	-5,61
Levels X.3			1,98	-2,84	3,97	-1,92
Levels X.4			1,50	-4,43	5,35	0,16

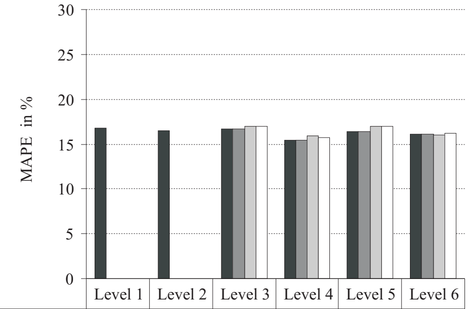
Figure A.2.4: Error of the average marginal cost during peak hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	123,47	123,76	124,09	124,42	124,09	124,44
Levels X.2			124,09	124,42	124,09	124,45
Levels X.3			124,11	124,28	124,18	124,33
Levels X.4			124,17	124,30	124,17	124,32

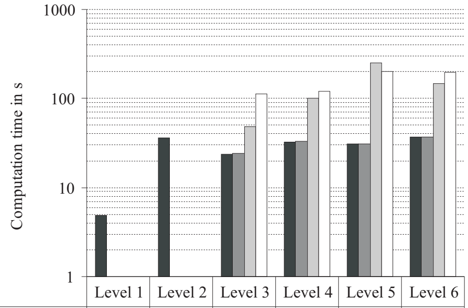
Figure A.2.5: Total cost

A.3 Model of the French Power System for the First Week of February 2006



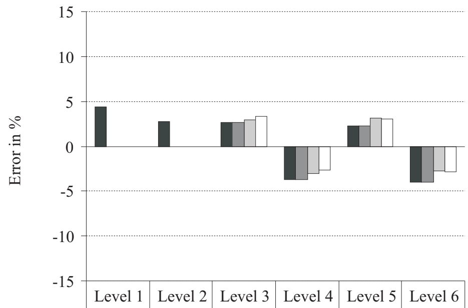
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	16,79	16,46	16,69	15,43	16,39	16,08
Levels X.2			16,69	15,43	16,39	16,08
Levels X.3			16,98	15,88	17,00	15,99
Levels X.4			17,00	15,69	16,98	16,16

Figure A.3.1: Error of the hourly marginal cost



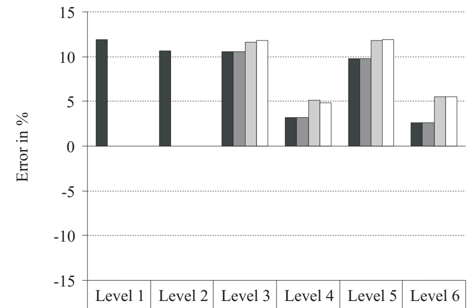
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	5	36	24	33	31	37
Levels X.2			24	33	31	37
Levels X.3			49	101	251	149
Levels X.4			112	119	200	198

Figure A.3.2: Computation time



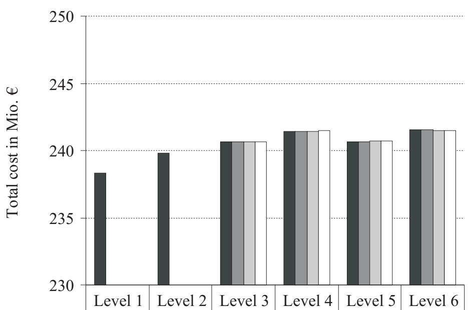
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	4,35	2,77	2,68	-3,72	2,25	-4,00
Levels X.2			2,68	-3,72	2,25	-4,00
Levels X.3			2,93	-3,07	3,12	-2,77
Levels X.4			3,31	-2,63	3,04	-2,87

Figure A.3.3: Error of the average marginal cost during base hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	11,93	10,69	10,57	3,23	9,82	2,62
Levels X.2			10,57	3,23	9,82	2,62
Levels X.3			11,60	5,09	11,78	5,55
Levels X.4			11,76	4,84	11,88	5,52

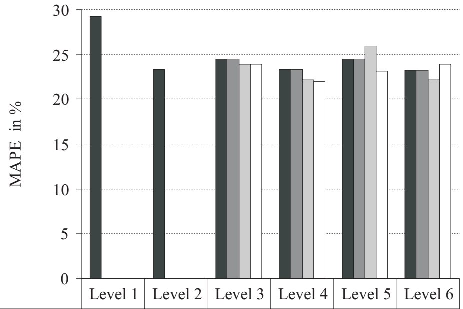
Figure A.3.4: Error of the average marginal cost during peak hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	238,30	239,79	240,64	241,44	240,66	241,53
Levels X.2			240,64	241,44	240,66	241,53
Levels X.3			240,67	241,45	240,72	241,49
Levels X.4			240,66	241,47	240,69	241,47

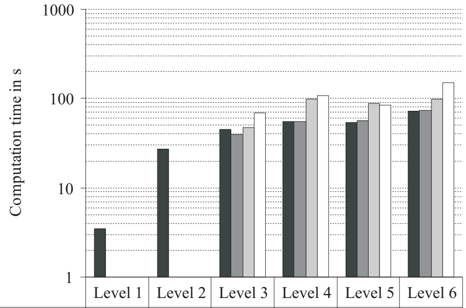
Figure A.3.5: Total cost

A.4 Model of the French Power System for the First Week of August 2006



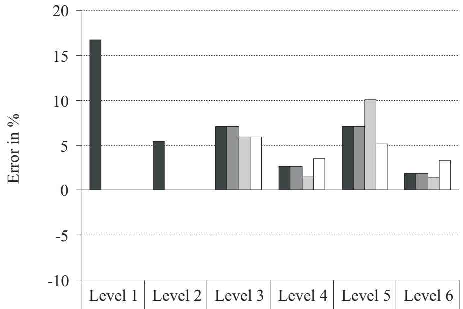
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	29,22	23,32	24,52	23,32	24,52	23,25
Levels X.2			24,52	23,32	24,52	23,26
Levels X.3			23,90	22,12	25,94	22,13
Levels X.4			23,90	21,92	23,18	23,92

Figure A.4.1: Error of the hourly marginal cost



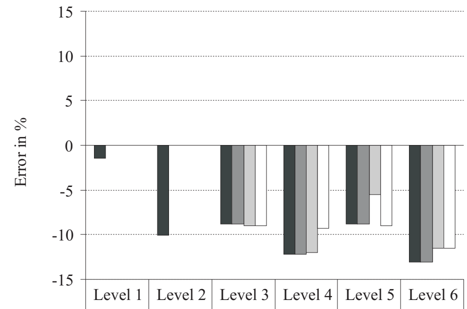
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	3	27	46	55	54	71
Levels X.2			40	55	57	74
Levels X.3			48	98	88	99
Levels X.4			69	107	85	150

Figure A.4.2: Computation time



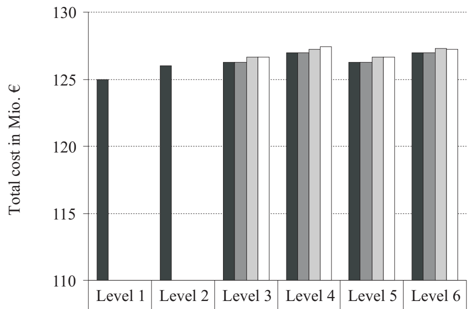
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	16,73	5,48	7,08	2,59	7,08	1,84
Levels X.2			7,08	2,59	7,08	1,84
Levels X.3			5,87	1,49	10,08	1,39
Levels X.4			5,87	3,48	5,15	3,29

Figure A.4.3: Error of the average marginal cost during base hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	-1,47	-10,07	-8,80	-12,17	-8,80	-13,06
Levels X.2			-8,80	-12,17	-8,80	-13,06
Levels X.3			-9,03	-11,99	-5,50	-11,49
Levels X.4			-9,03	-9,25	-9,03	-11,49

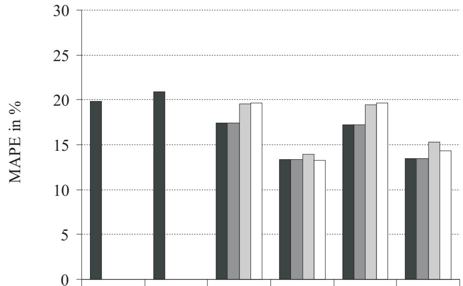
Figure A.4.4: Error of the average marginal cost during peak hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	124,94	126,00	126,29	126,98	126,29	126,99
Levels X.2			126,29	126,98	126,29	126,96
Levels X.3			126,63	127,25	126,63	127,27
Levels X.4	0,00	0,00	126,63	127,41	126,63	127,23

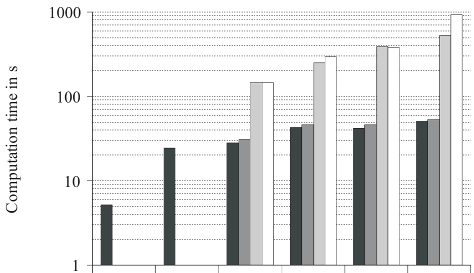
Figure A.4.5: Total cost

A.5 Model of the German Power System for the First Week of February 2005



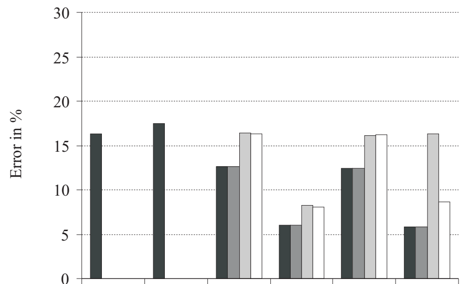
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	19,88	20,86	17,42	13,32	17,26	13,47
Levels X.2			17,39	13,32	17,26	13,48
Levels X.3			19,57	13,94	19,44	15,26
Levels X.4			19,65	13,28	19,67	14,33

Figure A.5.1: Error of the hourly marginal cost



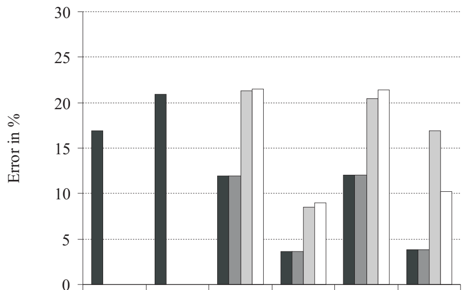
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	5	24	28	43	42	51
Levels X.2			31	46	46	53
Levels X.3			146	252	393	531
Levels X.4			147	298	381	942

Figure A.5.2: Computation time



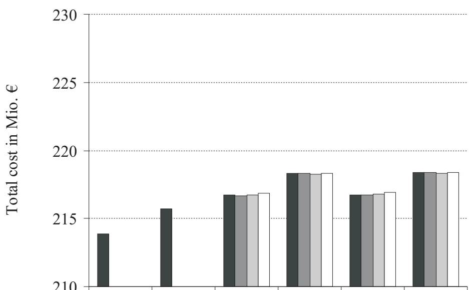
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	16,27	17,49	12,60	5,98	12,46	5,82
Levels X.2			12,58	5,98	12,46	5,81
Levels X.3			16,44	8,25	16,16	16,27
Levels X.4			16,32	8,07	16,22	8,61

Figure A.5.3: Error of the average marginal cost during base hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	16,88	20,96	11,97	3,64	12,08	3,78
Levels X.2			11,97	3,64	12,08	3,78
Levels X.3			21,27	8,53	20,47	16,88
Levels X.4			21,47	9,00	21,37	10,21

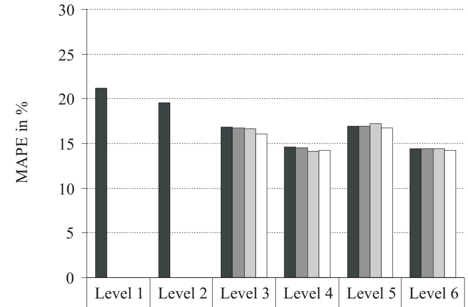
Figure A.5.4: Error of the average marginal cost during peak hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	213,87	215,71	216,70	218,31	216,72	218,39
Levels X.2			216,70	218,31	216,72	218,39
Levels X.3			216,71	218,22	216,77	218,29
Levels X.4			216,83	218,30	216,90	218,39

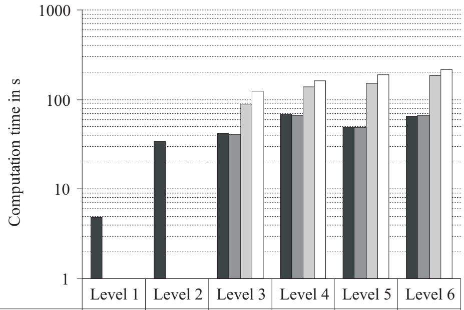
Figure A.5.5: Total cost

A.6 Model of the German Power System for the First Week of August 2005



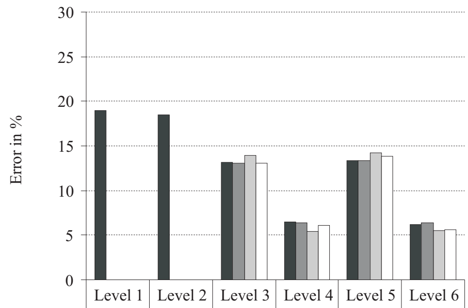
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	21,15	19,58	16,86	14,63	16,96	14,43
Levels X.2			16,78	14,55	16,94	14,46
Levels X.3			16,61	14,13	17,24	14,46
Levels X.4			16,08	14,24	16,72	14,22

Figure A.6.1: Error of the hourly marginal cost



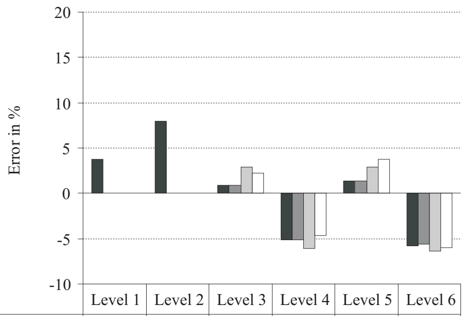
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	5	34	42	68	49	65
Levels X.2			41	66	49	66
Levels X.3			89	139	153	185
Levels X.4			124	160	188	216

Figure A.6.2: Computation time



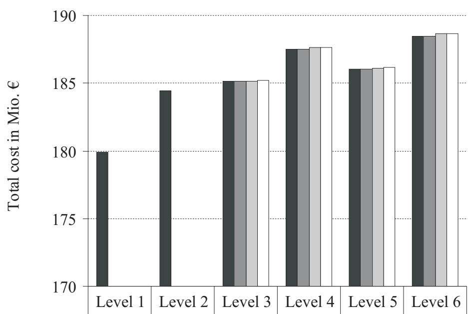
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	18,95	18,44	13,14	6,48	13,37	6,23
Levels X.2			13,05	6,39	13,36	6,34
Levels X.3			13,95	5,45	14,19	5,52
Levels X.4			13,05	6,13	13,84	5,60

Figure A.6.3: Error of the average marginal cost during base hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	3,72	7,93	0,93	-5,12	1,38	-5,79
Levels X.2			0,93	-5,13	1,38	-5,61
Levels X.3			2,92	-6,04	2,93	-6,33
Levels X.4			2,24	-4,63	3,74	-6,02

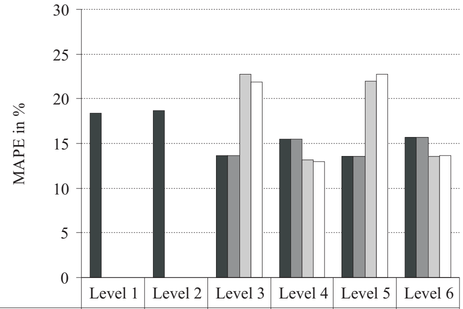
Figure A.6.4: Error of the average marginal cost during peak hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	179,92	184,47	185,12	187,52	186,05	188,47
Levels X.2			185,12	187,52	186,05	188,47
Levels X.3			185,15	187,62	186,12	188,63
Levels X.4			185,21	187,61	186,18	188,65

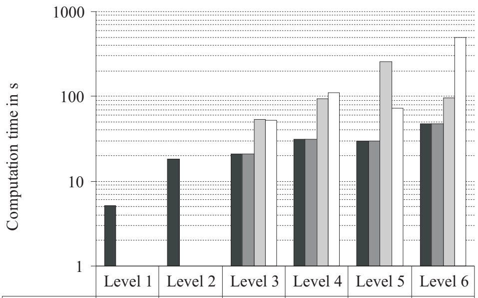
Figure A.6.5: Total cost

A.7 Model of the German Power System for the First Week of February 2006



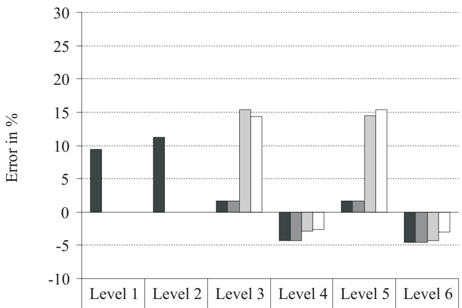
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	18,39	18,70	13,63	15,44	13,53	15,67
Levels X.2			13,63	15,44	13,53	15,67
Levels X.3			22,75	13,19	21,93	13,53
Levels X.4			21,84	12,97	22,73	13,64

Figure A.7.1: Error of the hourly marginal cost



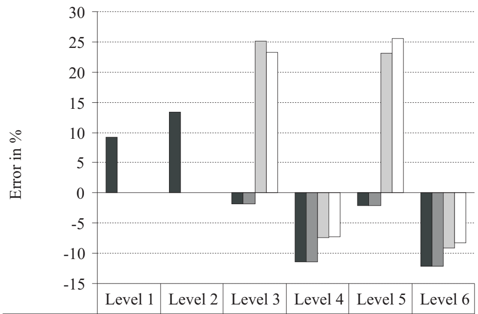
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	5	18	21	32	30	48
Levels X.2			21	32	30	48
Levels X.3			54	94	259	96
Levels X.4			52	110	73	490

Figure A.7.2: Computation time



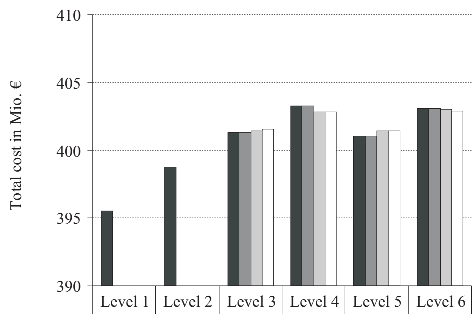
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	9,37	11,19	1,66	-4,29	1,64	-4,62
Levels X.2			1,66	-4,29	1,64	-4,62
Levels X.3			15,35	-2,87	14,41	-4,35
Levels X.4			14,30	-2,61	15,35	-2,97

Figure A.7.3: Error of the average marginal cost during base hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	9,28	13,33	-1,85	-11,36	-2,11	-12,12
Levels X.2			-1,85	-11,36	-2,11	-12,12
Levels X.3			25,17	-7,44	23,19	-9,07
Levels X.4			23,27	-7,20	25,55	-8,27

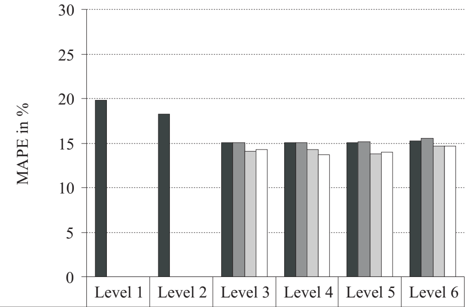
Figure A.7.4: Error of the average marginal cost during peak hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	395,54	398,78	401,29	403,27	401,07	403,07
Levels X.2			401,29	403,27	401,07	403,07
Levels X.3			401,41	402,82	401,43	402,99
Levels X.4			401,53	402,82	401,42	402,90

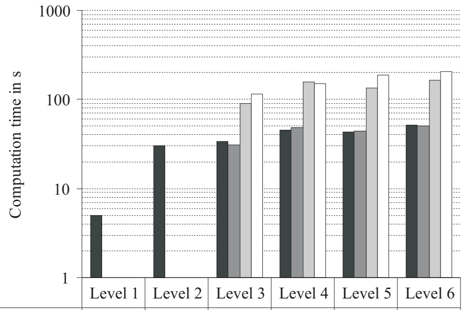
Figure A.7.5: Total cost

A.8 Model of the German Power System for the First Week of August 2006



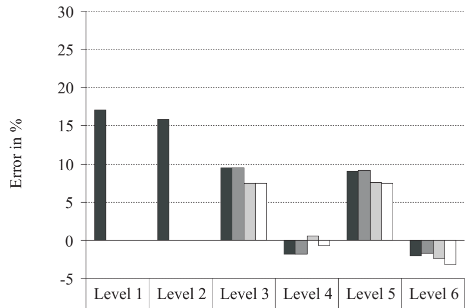
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	19,85	18,23	15,07	15,04	15,09	15,21
Levels X.2			15,07	15,03	15,12	15,50
Levels X.3			14,07	14,25	13,79	14,65
Levels X.4			14,22	13,68	13,99	14,65

Figure A.8.1: Error of the hourly marginal cost



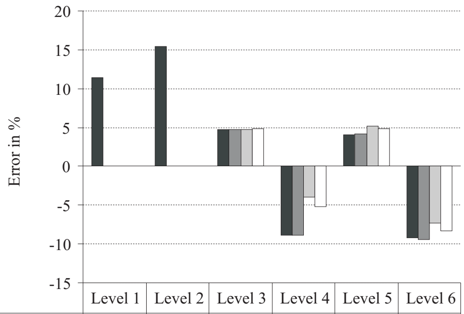
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	5	30	34	45	43	52
Levels X.2			31	48	44	50
Levels X.3			89	156	135	165
Levels X.4			116	152	187	207

Figure A.8.2: Computation time



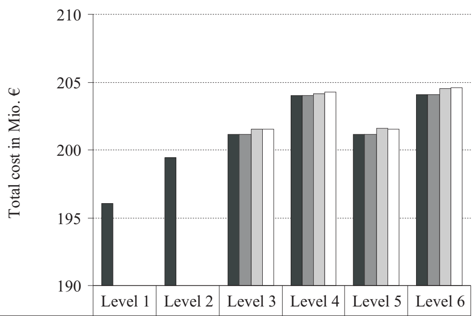
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	17,06	15,85	9,45	-1,85	9,08	-2,03
Levels X.2			9,45	-1,86	9,11	-1,72
Levels X.3			7,43	0,55	7,57	-2,38
Levels X.4			7,41	-0,67	7,48	-3,14

Figure A.8.3: Error of the average marginal cost during base hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	11,43	15,46	4,73	-8,91	4,11	-9,21
Levels X.2			4,73	-8,90	4,14	-9,41
Levels X.3			4,73	-4,01	5,21	-7,36
Levels X.4			4,83	-5,22	4,90	-8,28

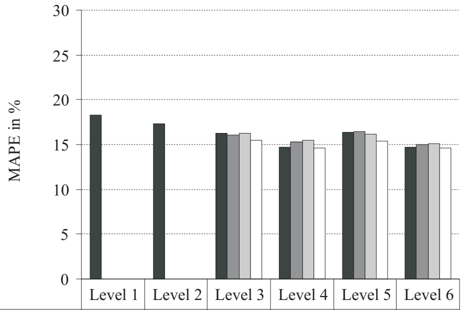
Figure A.8.4: Error of the average marginal cost during peak hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	196,06	199,43	201,14	204,00	201,15	204,09
Levels X.2			201,14	204,02	201,16	204,10
Levels X.3			201,53	204,17	201,56	204,51
Levels X.4			201,53	204,28	201,55	204,60

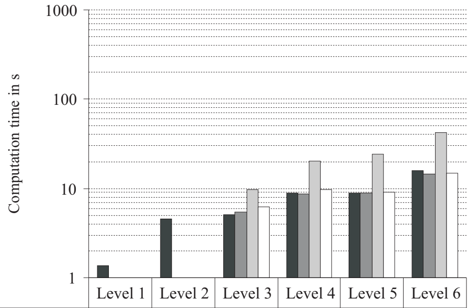
Figure A.8.5: Total cost

A.9 Model of the Spanish Power System for the First Week of February 2005



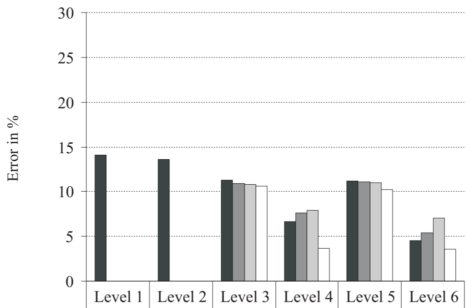
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	18,27	17,32	16,26	14,75	16,36	14,74
Levels X.2			16,06	15,31	16,45	14,96
Levels X.3			16,21	15,53	16,14	15,11
Levels X.4			15,53	14,59	15,42	14,64

Figure A.9.1: Error of the hourly marginal cost



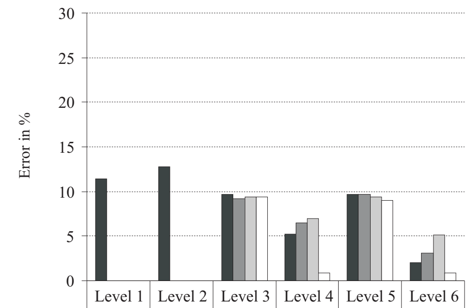
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	1	5	5	9	9	16
Levels X.2			5	9	9	15
Levels X.3			10	20	24	42
Levels X.4			6	10	9	15

Figure A.9.2: Computation time



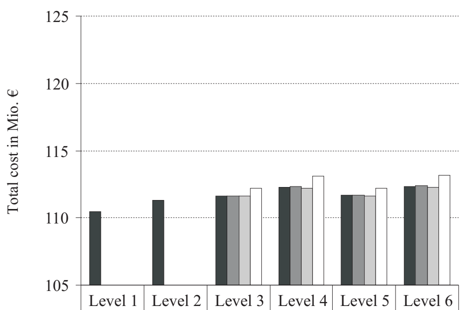
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	14,09	13,61	11,24	6,68	11,19	4,58
Levels X.2			10,87	7,63	11,06	5,36
Levels X.3			10,85	7,91	10,96	7,00
Levels X.4			10,59	3,66	10,21	3,57

Figure A.9.3: Error of the average marginal cost during base hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	11,44	12,81	9,65	5,25	9,65	2,03
Levels X.2			9,19	6,45	9,65	3,13
Levels X.3			9,37	6,99	9,37	5,14
Levels X.4			9,41	0,90	9,01	0,87

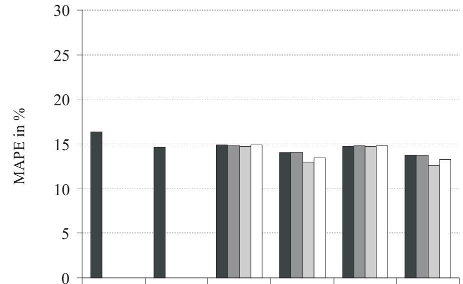
Figure A.9.4: Error of the average marginal cost during peak hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	110,45	111,32	111,65	112,28	111,67	112,35
Levels X.2			111,65	112,32	111,66	112,38
Levels X.3			111,64	112,22	111,62	112,26
Levels X.4			112,20	113,12	112,17	113,15

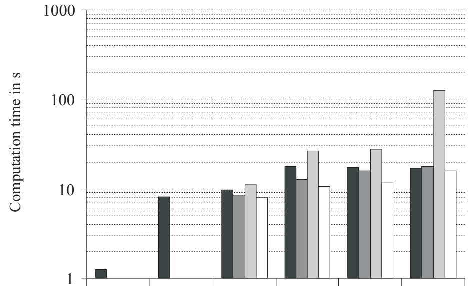
Figure A.9.5: Total cost

A.10 Model of the Spanish Power System for the First Week of August 2005



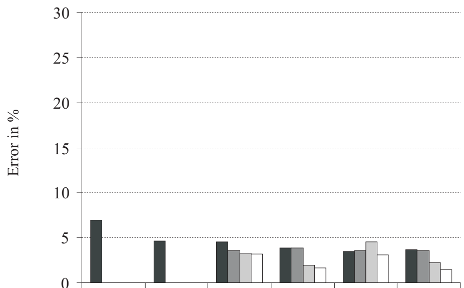
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	16,32	14,60	14,86	14,02	14,74	13,74
Levels X.2			14,84	14,01	14,76	13,74
Levels X.3			14,71	12,96	14,69	12,58
Levels X.4			14,89	13,48	14,76	13,29

Figure A.10.1: Error of the hourly marginal cost



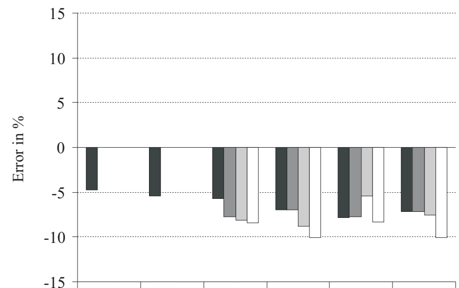
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	1	8	10	18	17	17
Levels X.2			9	13	16	18
Levels X.3			11	26	28	127
Levels X.4			8	11	12	16

Figure A.10.2: Computation time



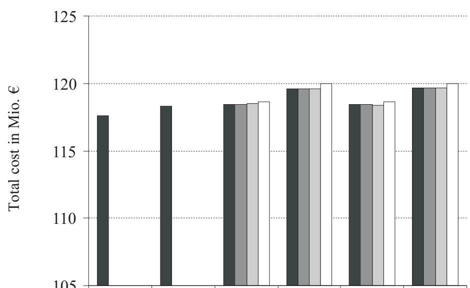
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	6,93	4,60	4,50	3,89	3,50	3,62
Levels X.2			3,54	3,89	3,55	3,62
Levels X.3			3,27	1,96	4,54	2,23
Levels X.4			3,20	1,68	3,13	1,48

Figure A.10.3: Error of the average marginal cost during base hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	-4,74	-5,44	-5,70	-6,94	-7,83	-7,20
Levels X.2			-7,77	-6,94	-7,73	-7,20
Levels X.3			-8,13	-8,84	-5,38	-7,54
Levels X.4			-8,41	-10,03	-8,30	-10,05

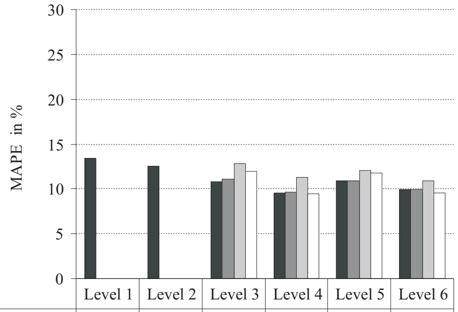
Figure A.10.4: Error of the average marginal cost during peak hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	117,60	118,29	118,43	119,62	118,46	119,67
Levels X.2			118,42	119,62	118,44	119,67
Levels X.3			118,47	119,60	118,41	119,63
Levels X.4			118,65	119,99	118,63	120,01

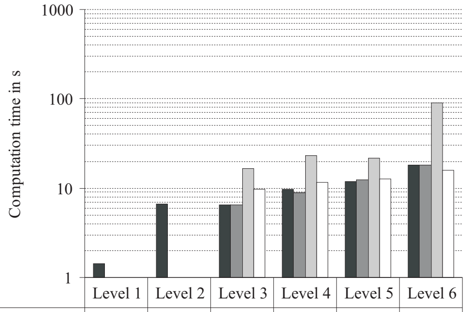
Figure A.10.5: Total cost

A.11 Model of the Spanish Power System for the First Week of February 2006



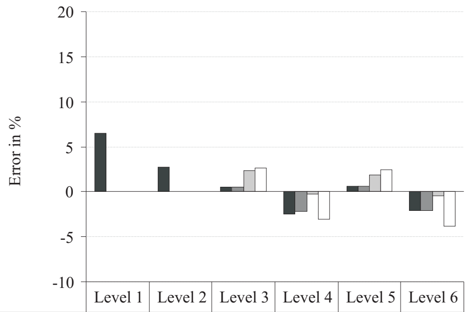
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	13,39	12,58	10,83	9,54	10,89	9,94
Levels X.2			11,11	9,63	10,89	9,94
Levels X.3			12,87	11,26	12,06	10,86
Levels X.4			11,95	9,48	11,77	9,56

Figure A.11.1: Error of the hourly marginal cost



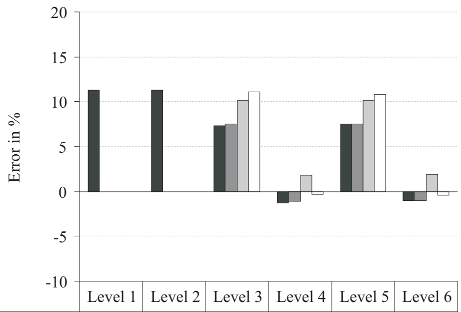
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	1	7	7	10	12	18
Levels X.2			6	9	12	18
Levels X.3			16	23	22	89
Levels X.4			10	12	13	16

Figure A.11.2: Computation time



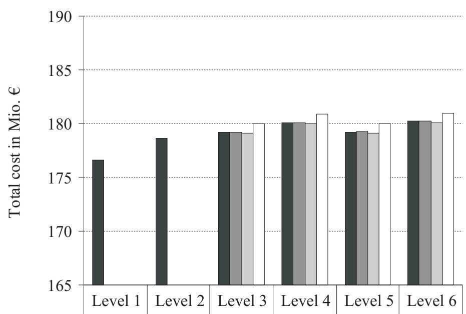
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	6,52	2,74	0,53	-2,49	0,61	-2,04
Levels X.2			0,53	-2,20	0,61	-2,04
Levels X.3			2,34	-0,28	1,90	-0,46
Levels X.4			2,60	-3,01	2,49	-3,80

Figure A.11.3: Error of the average marginal cost during base hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	11,32	11,25	7,32	-1,26	7,52	-0,95
Levels X.2			7,53	-1,10	7,52	-0,95
Levels X.3			10,17	1,77	10,17	1,92
Levels X.4			11,06	-0,29	10,83	-0,45

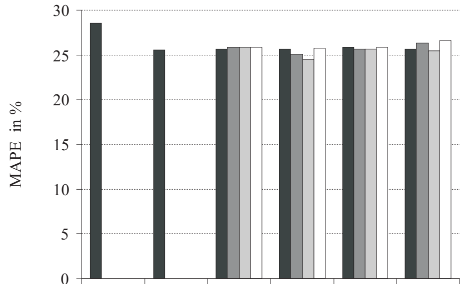
Figure A.11.4: Error of the average marginal cost during peak hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	176,59	178,61	179,23	180,11	179,23	180,22
Levels X.2			179,21	180,11	179,26	180,22
Levels X.3			179,12	180,01	179,14	180,10
Levels X.4			179,98	180,86	179,97	180,98

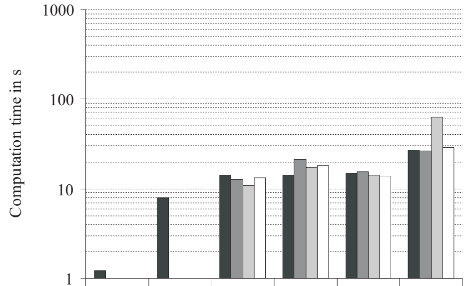
Figure A.11.5: Total cost

A.12 Model of the Spanish Power System for the First Week of August 2006



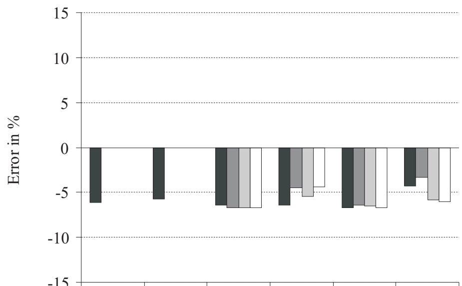
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	28,56	25,58	25,66	25,66	25,87	25,66
Levels X.2			25,85	25,09	25,66	26,34
Levels X.3			25,86	24,48	25,67	25,45
Levels X.4			25,85	25,75	25,83	26,66

Figure A.12.1: Error of the hourly marginal cost



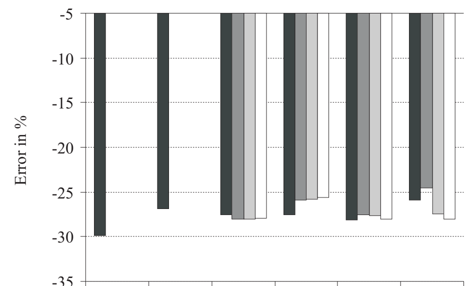
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	1	8	14	14	15	27
Levels X.2			13	21	15	26
Levels X.3			11	17	14	63
Levels X.4			13	18	14	29

Figure A.12.2: Computation time



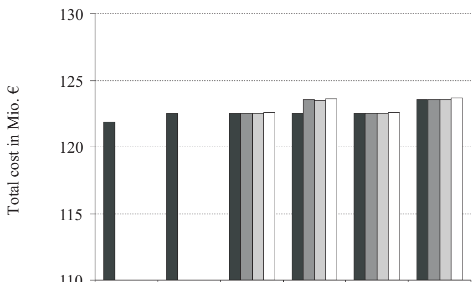
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	-6,17	-5,77	-6,46	-6,46	-6,72	-4,32
Levels X.2			-6,70	-4,44	-6,45	-3,34
Levels X.3			-6,71	-5,44	-6,47	-5,81
Levels X.4			-6,68	-4,43	-6,68	-6,06

Figure A.12.3: Error of the average marginal cost during base hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	-29,90	-26,88	-27,58	-27,58	-28,08	-25,93
Levels X.2			-28,05	-25,89	-27,58	-24,54
Levels X.3			-28,07	-25,81	-27,60	-27,46
Levels X.4			-27,96	-25,64	-27,99	-28,02

Figure A.12.4: Error of the average marginal cost during peak hours



	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Levels X.1	121,87	122,51	122,52	122,52	122,52	123,53
Levels X.2			122,52	123,53	122,52	123,54
Levels X.3			122,52	123,51	122,52	123,52
Levels X.4	0,00	0,00	122,61	123,64	122,61	123,68

Figure A.12.5: Total cost

A.13 Multiregional Models for the First Week of February 2005

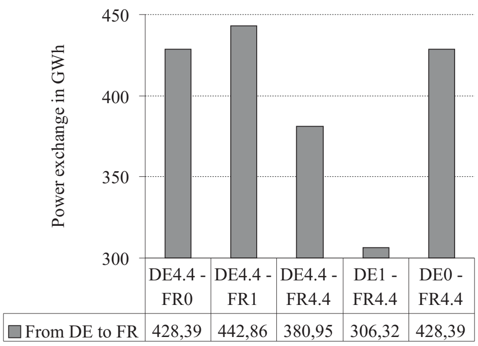


Figure A.13.1: Power exchange from the German to the French region

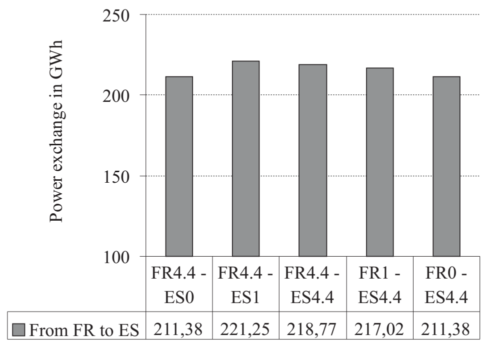


Figure A.13.2: Power exchange from the French to the Spanish region

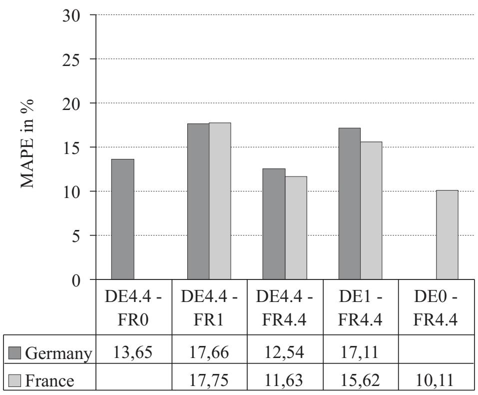


Figure A.13.3: Error of the hourly marginal cost of the French and German power systems

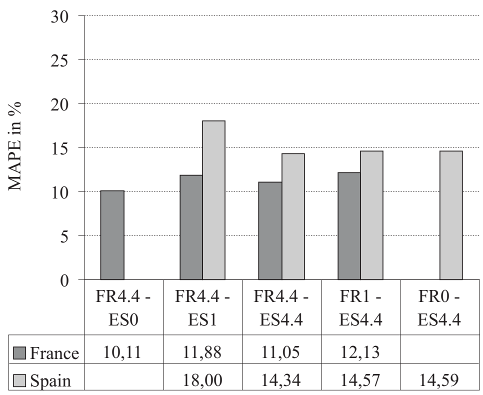


Figure A.13.4: Error of the hourly marginal cost of the French and Spanish power systems

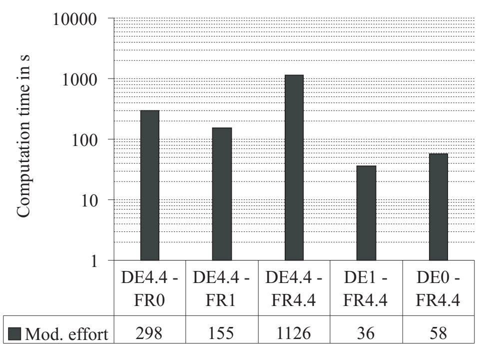


Figure A.13.5: Computation time for the French–German power system

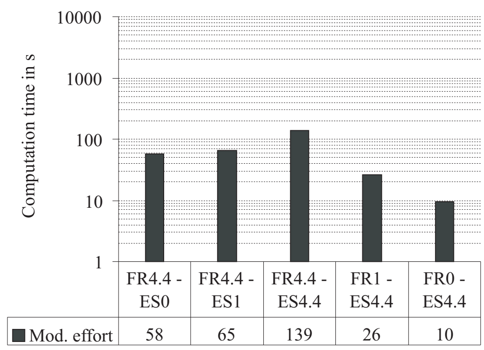
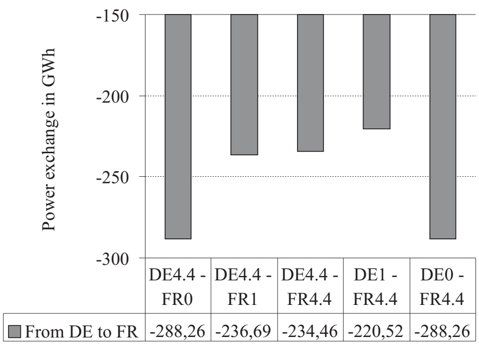


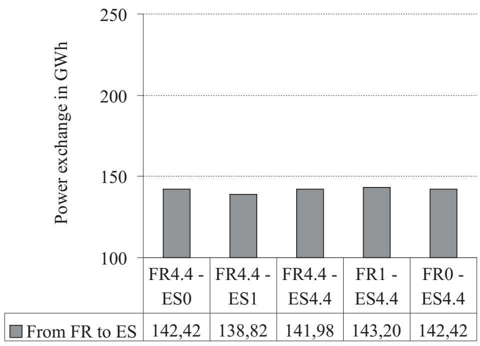
Figure A.13.6: Computation time for the French–Spanish power system

A.14 Multiregional Models for the First Week of August 2005



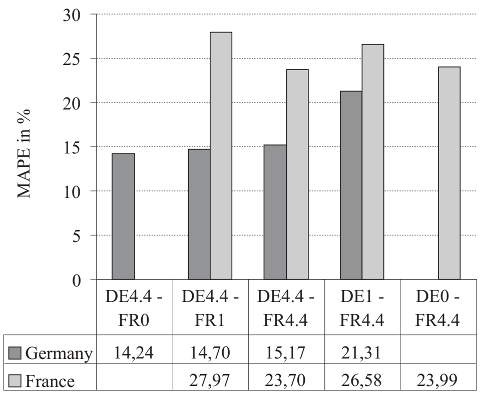
	DE4.4 - FR0	DE4.4 - FR1	DE4.4 - FR4.4	DE1 - FR4.4	DE0 - FR4.4
From DE to FR	-288,26	-236,69	-234,46	-220,52	-288,26

Figure A.14.1: Power exchange from the German to the French region



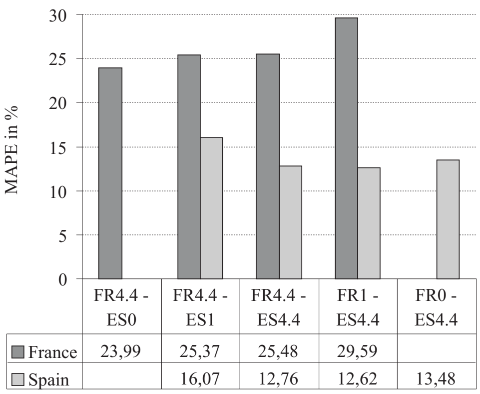
	FR4.4 - ES0	FR4.4 - ES1	FR4.4 - ES4.4	FR1 - ES4.4	FR0 - ES4.4
From FR to ES	142,42	138,82	141,98	143,20	142,42

Figure A.14.2: Power exchange from the French to the Spanish region



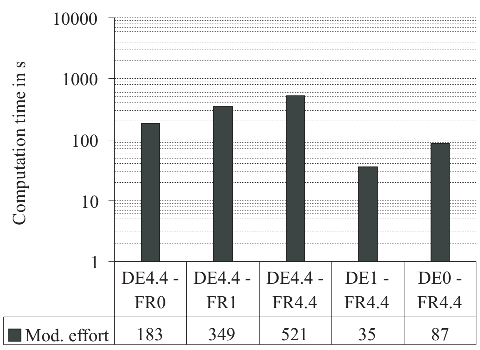
	DE4.4 - FR0	DE4.4 - FR1	DE4.4 - FR4.4	DE1 - FR4.4	DE0 - FR4.4
Germany	14,24	14,70	15,17	21,31	
France		27,97	23,70	26,58	23,99

Figure A.14.3: Error of the hourly marginal cost of the French and German power systems



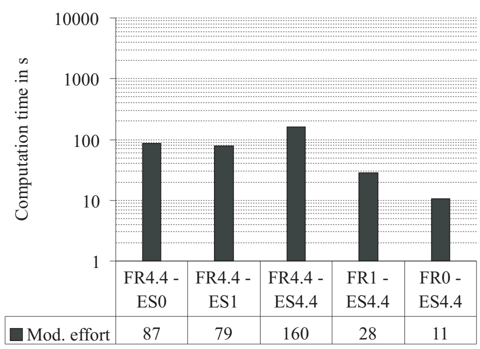
	FR4.4 - ES0	FR4.4 - ES1	FR4.4 - ES4.4	FR1 - ES4.4	FR0 - ES4.4
France	23,99	25,37	25,48	29,59	
Spain		16,07	12,76	12,62	13,48

Figure A.14.4: Error of the hourly marginal cost of the French and Spanish power systems



	DE4.4 - FR0	DE4.4 - FR1	DE4.4 - FR4.4	DE1 - FR4.4	DE0 - FR4.4
Mod. effort	183	349	521	35	87

Figure A.14.5: Computation time for the French–German power system



	FR4.4 - ES0	FR4.4 - ES1	FR4.4 - ES4.4	FR1 - ES4.4	FR0 - ES4.4
Mod. effort	87	79	160	28	11

Figure A.14.6: Computation time for the French–Spanish power system

A.15 Multiregional Models for the First Week of February 2006

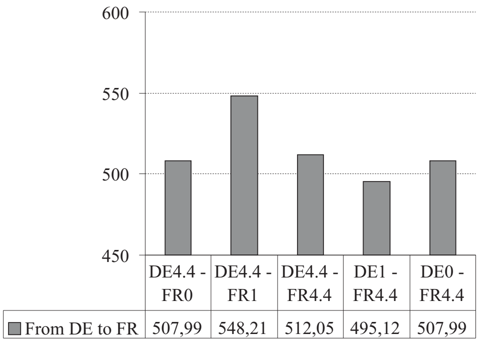


Figure A.15.1: Power exchange from the German to the French region

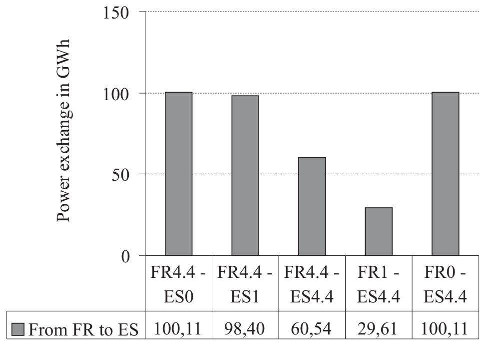


Figure A.15.2: Power exchange from the French to the Spanish region

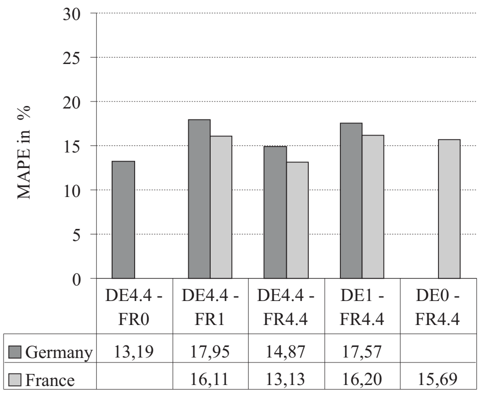


Figure A.15.3: Error of the hourly marginal cost of the French and German power systems

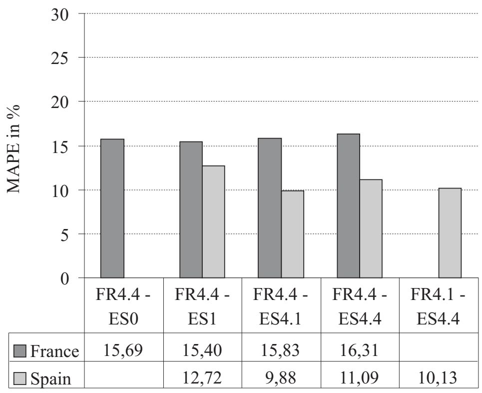


Figure A.15.4: Error of the hourly marginal cost of the French and Spanish power systems

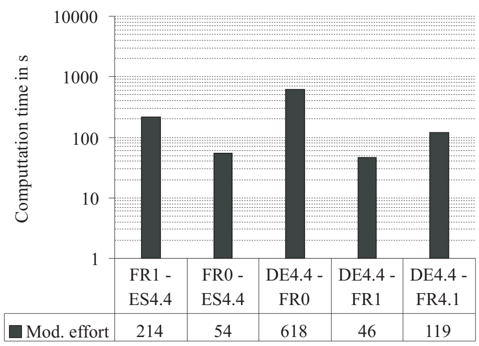


Figure A.15.5: Computation time for the French–German power system

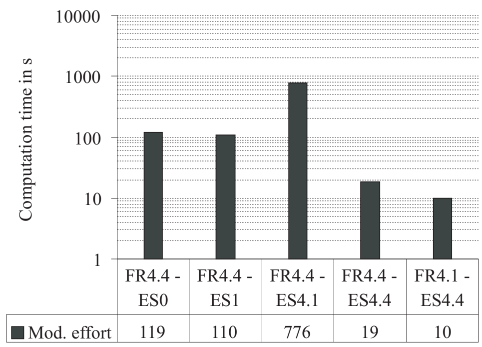
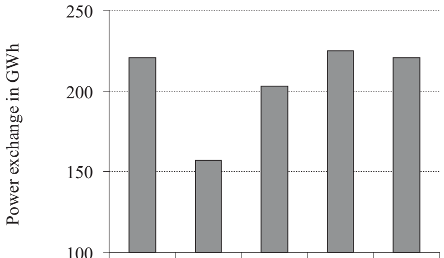


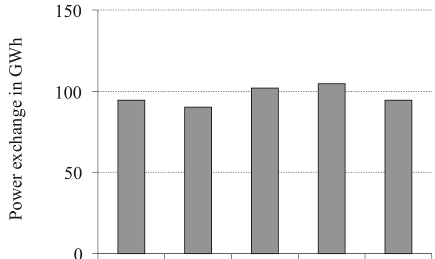
Figure A.15.6: Computation time for the French–Spanish power system

A.16 Multiregional Models for the First Week of August 2006



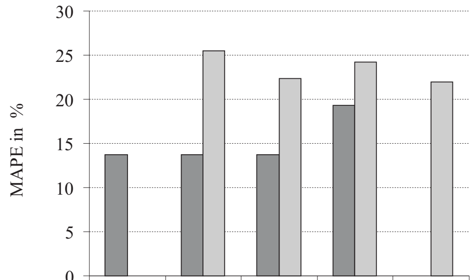
	DE4.4 - FR0	DE4.4 - FR1	DE4.4 - FR4.4	DE1 - FR4.4	DE0 - FR4.4
From DE to FR	220,80	157,15	202,82	224,77	220,80

Figure A.16.1: Power exchange from the German to the French region



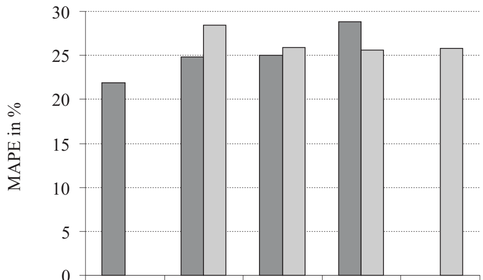
	FR4.4 - ES0	FR4.4 - ES1	FR4.4 - ES4.4	FR1 - ES4.4	FR0 - ES4.4
From FR to ES	94,30	90,09	101,77	104,47	94,30

Figure A.16.2: Power exchange from the French to the Spanish region



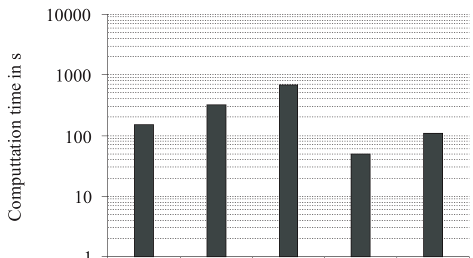
	DE4.4 - FR0	DE4.4 - FR1	DE4.4 - FR4.4	DE1 - FR4.4	DE0 - FR4.4
Germany	13,68	13,68	13,69	19,31	
France		25,46	22,32	24,25	21,92

Figure A.16.3: Error of the hourly marginal cost of the French and German power systems



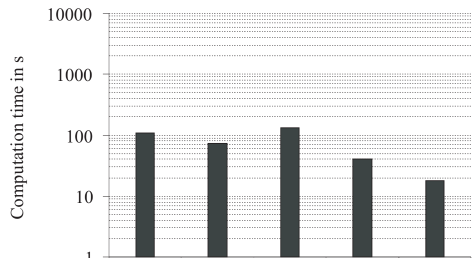
	FR4.4 - ES0	FR4.4 - ES1	FR4.4 - ES4.4	FR1 - ES4.4	FR0 - ES4.4
France	21,92	24,86	25,02	28,84	
Spain		28,47	25,91	25,65	25,75

Figure A.16.4: Error of the hourly marginal cost of the French and Spanish power systems



	DE4.4 - FR0	DE4.4 - FR1	DE4.4 - FR4.4	DE1 - FR4.4	DE0 - FR4.4
Mod. effort	152	319	683	49	107

Figure A.16.5: Computation time for the French–German power system



	FR4.4 - ES0	FR4.4 - ES1	FR4.4 - ES4.4	FR1 - ES4.4	FR0 - ES4.4
Mod. effort	107	73	134	41	18

Figure A.16.6: Computation time for the French–Spanish power system

Production cost models are among the preferred tools for the analysis of electricity markets based on fundamental factors. Models of this kind typically make a representation of electricity markets and of the underlying power system by formulating the unit commitment and economic dispatch problem as an optimisation program.

The insight production cost models provide into the operation of the system and the price building mechanism can be used to support production, planning and trading activities in the context liberalised wholesale electricity markets.

This book makes a contribution to the formulation and implementation of production cost models by addressing issues associated with the level of detail in the representation of the system, the accuracy of the results and the modelling effort. To this end a production cost model was formulated and applied to estimate the short-run marginal cost of the power systems of three important European electricity markets.

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