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Modelling the emergence of colloidal structures in Network Economics – The case of hinterland networks

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"That which is static and repetitive is boring.
That which is dynamic and random is confusing.
In between lies art."
John A. Locke

"It's like life.
It's never finished.
It just changes its forms.
I think consolidation is part of being healthy in an industry."

Ivan Seidenberg

Abstract

This paper develops a formal approach to explain how hinterland transport chains and corridors are formed. It is shown that the traditional discrete choice theory is not suited in situations with digressive cost functions and/or positive externalities which are quite typical in logistics businesses. Furthermore discrete choice theory does not allow for the modelling of the formation of new corridors or substructures.

The hinterland is modelled as several links with different characteristics. Two approaches have been investigated and developed. First, a dynamic approach is based on the Master Equation, mapping the emergence and the collapse of transport corridors. Results show the evolution on the formation of corridors and substructures applied to a theoretical prototypical case. Second, a static approach directly calculates the structure of the attractor-solutions (solutions to which the system could converge). These approaches together are based on the concept of the "Free economic energy" which includes the entropy and full costs. The correspondence between the dynamic and the static approach is proved mathematically. The new approach reconciles principles of dynamic modelling and of the standard econometric approaches for modelling freight.

Keywords: network effects, formation of clusters, destabilization of systems, emergent alternatives, entropy, Helmholtz free energy, dynamic freight modelling, hinterland, port

1 Introduction

The increased world wide trade flows and the trend towards containerisation create new challenges for macro-logistics systems such as ports, railway corridors or terminals. Ports have to manage the increasing freight flows and must offer an attractive hinterland network to remain competitive. Ports can profit from economies of scale and the positive externalities from container flows and between the hinterland and offshore transportation networks. From a policy perspective, the development of sustainable hinterland networks is crucial for a port because of the negative externalities of transportation and to improve the competitiveness of its economy.

Increased flows and intense competition lead to a hierarchy on transportation flows where the most loaded links become corridors. A freight corridor is a linear orientation of freight flows supported by an accumulation of transport infrastructures and activities serving these flows (Rodrigue, 2004). From this definition a question arises on how the corridors are formed. The answer to this question thus, may lie in the flows. They are serviced by a multi-stage freight distribution system but divided by mode and infrastructure servicing them. Furthermore, the evolution of corridors is governed by transportation demand where price, speed, reliability and safety play a major role.

Gateways (ports) are one of the most attracting infrastructures for flows. These infrastructures work on associative substructures by clusterisation of services, actors, commodities and effects are related to gateways. These structures distribute and collect all kind of goods to and from hinterlands. However, since the 1980ies the main flow of good has been the container. Its success comes from the standardised batch sizes it provides and from the flexibility offer for shipping commodities. They are ideally suited to amalgamate flows on ships and on hinterland corridors to achieve economies of scale (Friedrich et al., 2007). The container flows in hinterlands has grown by offering more services that are profitable when grouping commodities. This threats the classical role of ports' competition and gives the opportunity for new services and manufacturers to emerge in the hinterland.

Hinterlands and gateways present a kind of hen & egg problem, without the port there is no hinterland, and the hinterland is vital for the port. Therefore, if we focus on gateways, we should first look at their market area (hinterlands) where the drivers for shipping are located (e.g. Slack, 1985; Bird, 1988; Tongzon, 1995; Hesse and Rodrigue, 2004, Arduino et al., 2008). Moreover, the hinterlands are the core impact for attracting commodities to hub ports (Arduino et al., 2008) and their freight flows influence directly gateways (Notteboom and Rodrigue, 2004). To model the hinterlands structure we can focus to the dynamic development of transport corridors where two types of effects can be identified: effects which lead to self-stabilization and effects leading to competition and dynamics (Carrillo Murillo et al., 2008). Both effects cause irregular behaviour but they are generated for economical reasons. The self-stabilizing effects relate to (i) economies of scale through massification and recovery of fixed costs and to (ii) positive network externalities such as the Mohring effect, while effects leading to competition and dynamics describe positive interactions between sub-networks (hen & egg problem).

Both effects could strengthen the competitiveness of already performant systems but also cause destructive effects and they are the result from the continuous battles on markets and the wish of consumers to try something new. Moreover, changes in the preferences can also thread existing business models, consequently, the knowledge on the heterogeneity of actors and on their willingness to try new options is as important as the knowledge of the characteristics of the available services, i.e. their cost functions (Shy, 2001).

Freight substructures control hinterland transportation and they have been analysed in literature by characterising flows (e.g. van Klink and van den Berg, 1998), by the distribution of flows (e.g. Notteboom and Rodrigue, 2004), by assessing its accessibility and connectivity to other modes of transport (Carrillo Murillo et al., 2007), by surveys of stakeholders (e.g. De Langen and Chouly, 2004; De Langen 2007), by using discrete choice analysis (Malchow and Kanafani, 2004) and by using pseudo dynamic discrete choice models (de Jong and Ben-Akiva, 2007). Many studies conceptualise hinterlands, while little can be found for their modelling. Even if they are complementary for understanding interactions in hinterlands, the spread of these concepts had not been concentrated in a model.

Some attempts to analyse hinterland flows and its substructures have been carried on based on discrete choice models (Malchow and Kanafani, 2004 and de Jong and Ben-Akiva, 2007). Frome these experiences, we must consider discrete choice theory for modelling hinterlands.

However, one characteristic of discrete choice theory should already be anticipated: this theory assumes the options as given and fixed and are parallel to the entropy maximisation problem, but in cases of digressive cost functions, both approaches can get into struggle. Thus, we must go back to the assumptions behind these approaches in order to create a model capable to explain the irregular behaviour in the logistics and freight industry concerning hinterlands.

Before analysing the characteristics of a hinterland we present an alternative approach for modelling freight other than the classical models. The focus is on mapping network effects, instead of idealising a system (e.g. equal distribution of discrete choice theory), which is valid for real networks. In an environment where economies of scale becomes fundamental such as in the logistics industry, the recovery of fixed costs play an important role via the consolidation of flows which is the intuitive way for modelling dynamics. From this observation, three motivating approaches are combined in the development of the model proposed in this paper: the master equation in socio-dynamic systems (Weidlich, 2000), the entropy (Wilson, 1970) and individual responses to their environment (Helbing, 1997). The first two approaches play an important role in the mathematical deductions (follow section 2), while the third approach is implicitly added in the motivation of this paper.

In order to model behaviour of logistics and freight stakeholders in hinterlands, this paper is divided in six sections. In section 2 the theoretical framework is developed. It includes two different approaches for modelling decisions in hinterlands, namely, discrete choice theory and the introduction to the master equation in socio-dynamic systems. Once the model to proceed is selected, the development of a new conceptual approach to explain the process of the introduced model is shown in section 3. There, mathematical deductions for the new concepts are shown and the explanation of the theoretical case closes this section. Results are shown in section 4 and their interpretation is discussed in section 5. Finally, conclusions on the validation of the model are drawn in section 6.

2 Theoretical framework

2.1 Discrete choice theory

The theoretical economic and physiologic contribution from discrete choice theory is based on the random utility theory (e.g. Thurstone, 1929 and McFadden, 2001). It describes human selection of an option from a finite set of alternatives (Ortúzar and Willumsen, 1998). Moreover, discrete choice modelling is commonly undertaken using logit and probit models and in transportation is used as the major application to predict decisions in passenger transportation (e.g. Ben-Akiva and Lerman, 1985). Furthermore, after the success of discrete choice models to predict modal choice in passengers, this theory has been applied to freight transportation and logistics in several attempts. However, the transferability of these models does not fit for freight modelling since the real drivers goes beyond a static representation of networks. The problematic rises on the actors to model, when looking at the logistics industry, decisions are taken by several actors. Moreover, the number of operators is higher and all the decision plays on the recovery of fixed costs. Discrete choice theory take the alternatives as given and in real networks the number of actors is not fixed, since the service providers increase as the network evolutes.

However, a pseudo-dynamic model based on discrete choice theory has been applied to the Norwegian and Swedish transportation network (de Jong and Ben-Akiva, 2007). This logistic model focuses on the individual sender and receiver decisions at a firm level (micro level) for determining the optimal size unit and the possible transport chains for every flow of goods. By optimizing the total logistic costs, the solution raises after an iterative process. Here the use of economies of scale is applied, since a higher consolidation of transport units results in lower transport costs per tonne. In this study, three plausible contributions have been suggested for freight modelling, namely, the inclusion of the size of goods, the use of distribution centres (consolidation) for transportation chains and the updating of cost functions during the iterations. The consideration of updating the production costs goes beyond static modelling. However, this model does not fit for freight since it is not able to represent dynamic behaviour due to its tendency to spread decisions (follow section 5). In other words, this model is not able to map the heterogeneity of actors which is remarkable in freight transportation (Park, 1995) for the formation of transport chains and corridors.

2.2 Master equation

As a more flexible approach to model hinterlands is the Master equation which is able to map dynamic behaviour of actors. Moreover, the master equation is even able to show the same logit-like solutions than discrete choice theory solutions. An interesting feature is its mesoscopic final solutions suited for descriptive freight modelling.

In classical and quantum mechanics, the master equation is a set of differential equations to describe the time evolution of a system. This is done by the probability of the system to reach a discrete set of states (Kampen, 2007). In social dynamics this concept has been applied to analyse the change of behaviour of individuals by means of self motivation (Weidlich, 2000). The research of Weidlich results in a generalization of the master equation to describe each kind of system based on four elements, three variables and a set of parameters. They are collective material, extensive personal, intensive personal and trend and control parameters, respectively. In table 3.1, examples of elements of his approach were compiled. As one can observe, this approach can be applied at the distribution and modal choice stages of the classical four step model (Ortúzar and Willumsen, 1998).

Table 3.1 - Examples of macrovariables

Collective material variables (m)	Extensive personal variables (n)	Intensive Personal variables (v)	Trend and control parameters (k)
Stock and flow variables	Distribution of attitudes over the population analyzed	Extrovert and introvert properties of individuals	Constant coefficients from statistics
Capital stock			
total inventory of the industry			
total number of buildings in a city			
production			
investment			
capacity	Or individual Attitudes (partial opinions about the systems of individuals)		
gross national product			
gross income			
Price	Choice set variable of individuals		
quality of commodities			
productivity (production per worker)			
capacity utilization rate			
income per worker			

Source: Author's own representation based on Weidlich, 2000.

This more generalized version of the master equation contains too much probabilities and low empirical data based on statistics. Therefore, there must be the intention to derive from the master equation simpler equations of motion for simpler quantities to be comparable to the evolution of a single real social system (Weidlich, 2000).

The construction method of stationary solutions (states) during the evolution of a system is based on the migratory master equation of accessible states (possibility of migration) $n = \{n_1, ..., n_i, ..., n_L\}$ with integers $n_i \geq 0$ is finite and the master equation must possess a stationary solution (Weidlich, 2000). Formally expressed:

$$w_{ji}(n_{ji}, n) = p_{ji}(n) \cdot n_i = \mu_{ji} \cdot n_i \cdot \exp[\alpha \cdot (u_j(n_{ji}) - u_i(n))]$$

$$\text{with} \qquad \mu_{ji} = \mu_{ij} \tag{2.1}$$

where:

w_{ji}	pair of motions from i to j and from j to i
$u_j(n_{ji})$	utility function of a possible state j
$u_i(n)$	utility function of a given state i
$p_{ji}(n)$	probability of state migration
μ_{ji}	flexibility or mobility of migration
n_i	initial distribution
α	sensitivity on cost differences

for the non vanishing transition rates from n to n_{ji}. The utility functions depend on the initial configuration n and the final configuration n_{ji}. This generalisation leads to transition rates which are nonlinear functions of the configuration variables, and this statement changes the dynamics of the system fundamentally. However, the behaviour of the present model by following the random distribution allows individuals to react to the market. Furthermore, a property of the model is that migration is allowed to integer pair of movements w_{ji} adding dynamics over the network.

An important advantage when applying this approach relates to the non biased solutions by following a Markov process by means of lack of memory or non influence of the system from past states. Hence, emergence of new alternatives without a preference for the predetermined maximum is possible. Maximums can be illustrated as major providers of logistics services, clusters or corridors for instances.

To model the formation of colloidal structures such as corridors, we must enable the emergence of new strong alternatives in the transition phase (e.g. by a destabilization of the most probable path). In this case a so called bifurcation takes place. An original unimodal probability distribution evolves to a bimodal or multimodal distribution over the space with two or more relative maxima. Deviations of these maxima thus, obtain higher relative weight at phase transitions. There the macro variables undergo critical fluctuations until they practically stabilize around one of the new arising maxima. Indeed, the system follows an imminent destabilization ensuing phase transition. Besides, migratory transitions are favoured (or disfavoured), if the utility of the destination alternative exceeds (or falls short of) the utility of the initial alternative.

3 Development of a new conceptual approach

An alternative modelling for hinterlands must be directed on the logistics industry. Several aspects can drive freight businesses as the effects for trying new alternatives. However, the most important aspect in the logistics industry is the capability of reducing cost by consolidating flows. Indeed, dynamic modelling is implicitly directed on recovery of fixed costs by means of economies of scale. Therefore, we seek a model capable to show the dynamic interactions for searching the best (and new) solutions for shipping commodities.

Supported by the theoretical framework, two methods have been considered for modelling hinterlands. Thus, we make a comparison between the discrete choice theory and the master equation. Since both methods are used for simulation, we propose to test them by using the same initial distribution and condition to compare both results.

3.1 Methods Comparison

The comparison of both approaches was made by using the same parameters, in this case $\alpha = 0{,}01$, and the same digressive cost function. The α parameter in discrete choice theory expresses the lack of information and heterogeneity of actors, while in master equation relates to the sensitivity of cost differences. For more information on the cost function follow section 3.5.

The hinterland concept is based on nine different alternatives with a cost function based on the number of shipments per alternative. The more shipments on the same alternative, the less the unitary costs per alternative. The first test consist in evaluate the results of both approaches by using a homogeneous distribution. This means that each alternative has 5 shipments and the total fixed amount of shipments on the whole hinterland equals to 45. Figures 3.1 to 3.3 show the test applied for both approaches, the “x” axis represents the alternatives available for the same task (in this case paths), while the “y” axis shows two scales: the left side relates to the unitary costs and the right side represents the number of shipments per alternative. The graphic must be read by one of the “x” axis for a better understanding of the graph, then switching to the other component. First, we test the models by using a spread initial distribution with no distinctions for any alternative at both approaches. Second, we represent all possible distributions identically (imitating an initial distribution) to test the critical mass for keeping or eliminating one alternative. Third, we add a shipment to the distribution in order to test the breakpoint of the critical mass already tested. Note that initial states and their characteristics are marked with small signs while final states (solutions) are marked with the same sign but bigger. Only the first test is aggregated in one graph (due to the static representation for the logit model), while figs. 3.2 and 3.3 contain two graphs: the left-side graphs represent the test applied to the discrete choice model and the right-side graphs show the test applied to the master equation model.

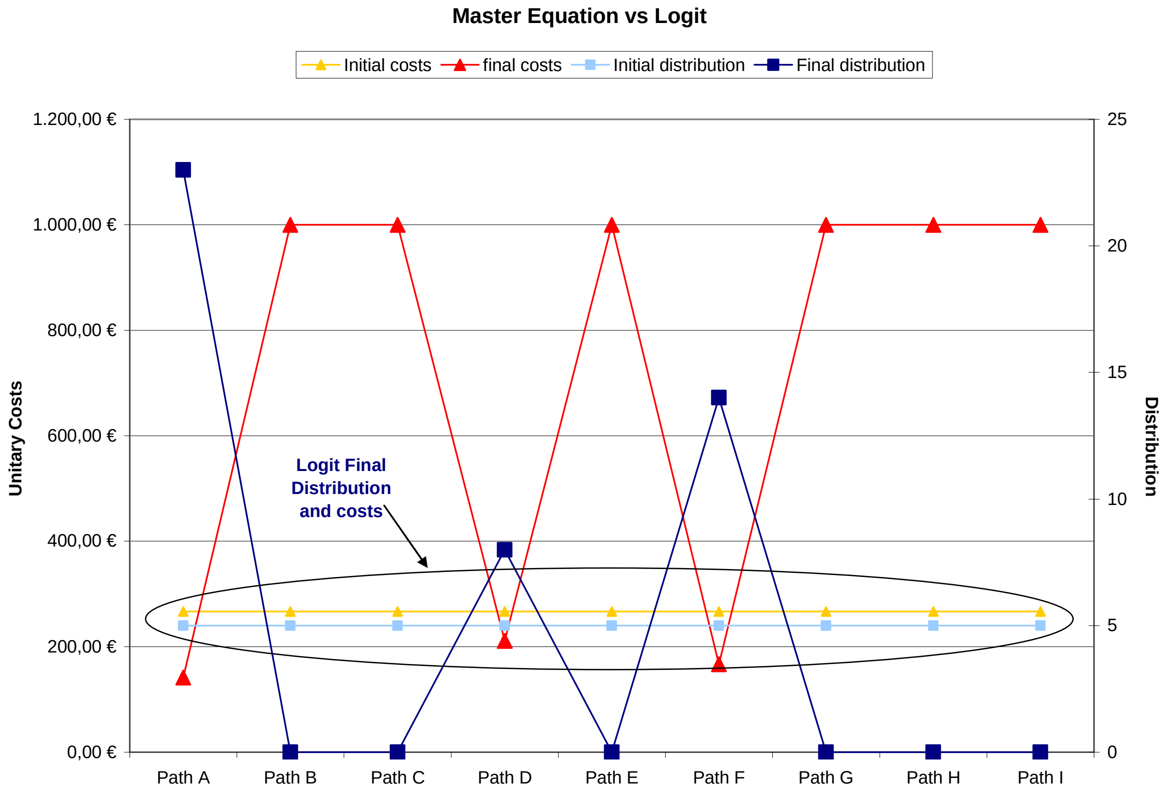

Figure 3.1 Equal spread initial distribution test

On one hand, the first test for the logit model shows an immediate equal distribution since the first iteration. This feature states the deterministic behaviour over the data, since it does not allow flows among the different alternatives. In other words, the final distribution remains the same for the logit model (see the distribution and costs in the circle in fig. 3.1). On the other hand, the emergent master equation approach presents a dynamic behaviour. It allows the emergence of consolidation for three clusters with different hierarchy on its results (mesostructures). The path A being the most remarkable with 23 grouped shipments, followed by the path F with 14 shipments and the third remaining alternative is the path D with 8 shipments. The rest of paths are eliminated when using the master equation and it bring us to test why some paths are eliminated. Even if the logit model presents an equal probability distribution we consider it for an analysis on the number of elements an alternative to be self-sustaining. When applying this test to the logit model we can obtain a better understanding on the same feature for the master equation. For this test we used an initial situation similar to the normal distribution (fig. 3.2), then, we added a shipment on one alternative to follow the criteria for keeping an alternative (fig. 3.3).

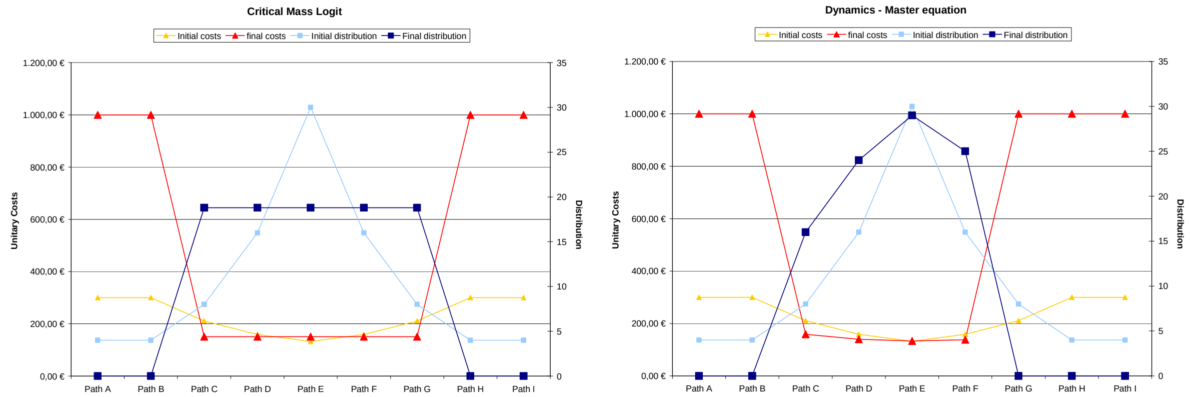

Figure 3.2 Self-sustainability test

At first sight we can observe self-sustainability effect biased by the initial distribution on the logit model. In other words, it makes a selection based on the alternatives with the most number of shipments assigned in the initial distribution. However, once the alternatives were selected, it distributes with equal probability the fixed number of shipments in all the alternatives. The master equation shows an unbiased behaviour since it allows the elimination of one of the members kept by the logit model. The results are almost similar for the number of remaining alternatives. However, the distribution of shipments among them is more dynamic allowing the emergence of new alternatives on the hierarchy of the hinterland. With these results, we proceed to apply the breakpoint test with the same distribution to know whether both models are able to sustain their alternatives.

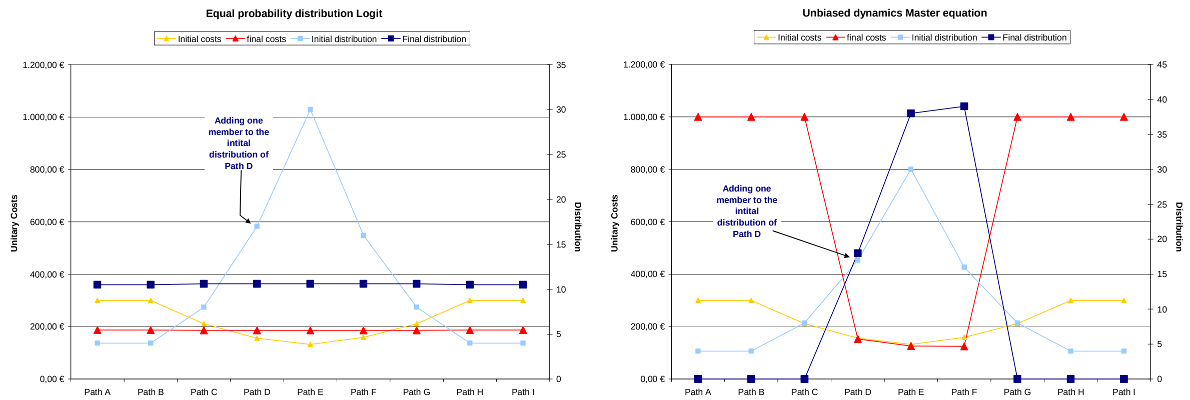

Figure 3.3 Breakpoint of self-sustainable solutions test

The breakpoint test relates to the addition of one shipment on path D to know whether the results of self-sustaining are broken. It is important to underline however, the drastic change of the logit results when adding one user to the system in one alternative and keeping every alternative for the final distribution. There, small changes at the initial distribution eliminate the breakpoint allowing equal probability of shipments to be distributed over the entire system. The results of the master equation show also a change on the system by consolidating more the shipments to reduce the number of remaining alternatives. Therefore, the number of total shipments on the hinterland plays an important role for defining the final number of remaining alternatives and we confirm the hypothesis for model freight with the master equation approach. More interesting is to observe the possibility of an emergent cluster on the top of the hierarchy in the hinterland. Note that path E at the right-side graph of fig 3.3 not only has the capability to remain until the final distribution (and consequently to reduce full costs) but allows an alternative with 7% of the demand to emerge as the first cluster. Therefore, we propose a deeper analysis with different initial distribution to test the remaining number of alternatives at both models. The results shown for this analysis however, are shown in figures A-6 and A-7 in annex.

From the tests shown above, the master equation represents better the distribution on hinterlands and its logistics environments than the deterministic logit model. This statement is fundamental since it allows the possibility to distribute shipments with non biased distributions. Moreover, this feature of the master equation is able to represent one of the physical attitudes (if not the most important) of logistics companies which are willing to change its decision on shipments in the short term (e.g. De Langen and Chouly, 2004; De Langen 2007). A clear example of skilled strategies practiced in the logistics industry is due to asymmetries of information. Besides, dynamic behaviour at the final results of master equation are represented by higher consolidating flows and reducing more the overall costs. Furthermore, it does not idealize the system by grouping all shipments in one alternative, but it has the possibility to create mesostructures at final solutions (e.g. environments with quasi min cost but not the min values similar to real cases due to network effects[1]). However a question arise on which is the process that determines the optimal number of final solutions (mesostructures), since it showed an irregular number of remaining alternatives or paths. A step to follow is to know the structure between the configuration of mesostructures and macrovariables leading to the final distributions.

It is possible to go beyond the master equation if we want information about the structures of its results. Due to the stochastic processes of the master equation, the information on the solutions presents a possibility for calibrating the choice model. For this purpose, three deductions are formulated to end with the concept of the free economic energy (FEE).

3.2 Master equation with optimal number of alternatives

We aim to find the optimal number of clusters were a stable state is reached. A stable state in the master equation is an environment in which interactions are minimised in the distribution, since the optimal number of clusters has the same number of individuals in each alternative. Consequently, the differential in costs is inexistent and every alternative reaches a stable state. First, we start from equation 2.1 to analyse the stabilization process for calculating the optimal maxima of the master equation.

$$\sum_{i \neq 1} n_1 \exp(\alpha(c_1 - c_i)) = \sum_{i \neq 1} n_i \exp(\alpha(c_i - c_1)) \tag{3.1}$$

Extracting and grouping the terms in both sides, we have

$$n_1 \exp(2 \cdot \alpha \cdot c_1) = \sum_{i \neq 1} n_i \exp(2 \cdot \alpha \cdot c_i) \tag{3.2}$$

The left hand side term can be interpreted as the potentials of alternative 1 to loose or gain a member, while the right hand side term relates to the same potentials of the whole system. For a stable state, therefore, it can be assumed that both potentials are minimised. This is the classical competition approach that can be expressed for the master equation. Since we aim to know the optimal number of clusters we can use any of the terms of equation 3.2. Generalising the deduction we aim to

[1] For more information concerning network effects in transportation follow Carrillo Murillo et al., 2008.

$$\min\left(n_i \cdot \exp\left(2 \cdot \alpha \cdot c_i\right)\right) \tag{3.3}$$

for obtaining the optimal number of members for every alternative i. The result giving the critical mass (number of shipments of every alternative for a stable state of the whole system) alternative i is

$$n_i{}^* = -\frac{1}{2 \cdot \alpha \cdot \frac{\partial c_i\left(n_i\right)}{\partial n_i}}$$

(3.4)

and this solution plays an important role for determining the behaviour of the model.

However, when applying the latter case to every pair of movements the problem turns out to be more complex since we should include some potential costs as an incentive. Therefore, the migration from an alternative n (outbond) to an alternative m (inbound) and/or from a macrostate $i = 1$ to the new configuration $i \neq 1$ (e.g. $i = 2$), we have:

$$(m+1) \cdot \exp(\alpha \cdot (c_1 - (c_2 + \Delta c)) < (n-1) \cdot \exp(\alpha \cdot (c_2 + \Delta c - c_1)) \tag{3.5}$$

with

$$\Delta c = c_1 - co_2 \tag{3.6}$$

where

co_i potential costs of the i alternative

The result minimising the equation 3.5 is

$$\underbrace{ln(m+1) - \ln(n-1)}_{S*} < 2\alpha \cdot \Delta c \tag{3.7}$$

It is important to notice in equation 3.7 that the coefficient with value 2, in the right side of the equation, represents both movements as in equation 3.4. Therefore, we must take into consideration a factor 2 in all the calculations for representing both, the inbound and outbound flows. On the other hand, the right side of the equation 3.7 (indicated as S*) is similar to the entropy change for the similar case of the pair of movements explained in the following section.

3.3 Cohesion between Master equation and Entropy

The concept of entropy was introduced by Wilson (1970) for urban and regional modelling. In transportation it has been largely used for measuring all the possible states of distribution in a system. The method of Wilson for measuring entropy is:

$$w = \ln\left(\frac{N!}{n_1! \cdot n_2! \cdot n_3! \ldots \cdot n_n!}\right) \tag{3.8}$$

Applying the equation 3.8 for the pair of movements, we model for every state of the process the entropy respectively by eliminating the constant T! for a fixed number of shipments.

For state $i = 1$

$$S_1 = \ln\left(\frac{1}{m!}\right) \cdot \ln\left(\frac{1}{n!}\right) \tag{3.9}$$

And for state $i \neq 1$ (e.g. 2)

$$S_2 = \ln\left(\frac{1}{(m+1)!}\right) \cdot \ln\left(\frac{1}{(n-1)!}\right) \tag{3.10}$$

Formally writing the process from equation 3.9 to equation 3.10 we have

$$S_{1-2} = \underbrace{\ln(m+1) - \ln(n)}_{S^*} + \underbrace{\ln\left(\frac{1}{(m+1)!}\right) + \ln\left(\frac{1}{(n-1)!}\right)}_{S2} \tag{3.11}$$

The last two terms of the equation 3.11 correspond with the entropy of the state after a forced exchange of alternative from one cluster to another, while the first two terms correspond to the exchange triggered by the change of entropy between states.

For large number of individuals in the system $m + n$, the entropy changes S^* from inequality 3.7 and the entropy changes S^* from equation 3.11 are almost identical and show a link between the master equation and the entropy maximizing concept.

Therefore, the change of state is represented by the ΔS during the time which represents Δc as well and these costs must be considered for obtaining information of the evolution of the system. Therefore, from equations 3.7 and 3.11, we obtain:

$$\Delta c > \frac{\Delta S}{2 \cdot \alpha} \tag{3.12}$$

3.4 Free Economic Energy optimisation

From the physics of energy, an interesting context related to dynamics in a system is the free energy. There are several variants of this equation, but we focus in the so-called Helmholtz Energy. Basically, the thermodynamic potential is derived from the fundamental equation of thermodynamics. The free energy consists on two terms, namely, the internal energy U and the entropy S. The latter is influenced by the temperature changes in relation to the rest of the system. The whole concept stands for the necessary energy to generate a thermal equilibrium with its surroundings in the system. Formally, we apply the Helmholtz free energy as the free economic energy of an alternative as:

$$FEE = H = U - T \cdot S \tag{3.13}$$

By transferring this equation to transportation, the internal energy U is interpreted as the cost for the overall system, the entropy S is equal to the entropy of the equation 3.8 for large number of actors and the temperature $T = \frac{1}{2 \cdot \alpha}$.

The value of the temperature follows three explanations. In the original model of Helmholtz the temperature forces the change of entropy which is done by (i) α in the equation 3.1, the entropy of Wilson is the (ii) *inverse* of the number of members of every alternative (e.g. equation 3.8) and the parameter (iii) 2 relates to the inbound and outbound flows as deduced in equations 3.4 and 3.7.

Therefore, FEE would be an indicator for the stability of the system, meaning that an optimal consolidation leads to a great stable condition. For N number of actors of the system where n is the average number of actors in each cluster, N/n determines the number of clusters for the next stable state of a system. The entropy of a cluster, therefore, corresponds to the Wilson (1970) approach with the Stirling's approximation, being

$$S = -\frac{N}{n} \cdot n \cdot \ln n = -N \cdot \ln n \tag{3.14}$$

and the equation of internal energy or cost is

$$U = N \cdot AC(n) \tag{3.15}$$

where

$AC(n)$ average cost of cluster n

Combining equations 3.14 and 3.15 in 3.13, we have

$$FEE = N \cdot AC(n) + \frac{1}{2 \cdot \alpha} \cdot N \cdot \ln(n) \tag{3.16}$$

Since the aim is to consolidate the decisions, minimising the equation 3.16 we have

$$\frac{d(FEE)}{dn} = \frac{dAC(n)}{dn} + \frac{1}{2 \cdot \alpha} \cdot \frac{1}{n} = 0 \tag{3.17}$$

Giving a stable size of cluster $n*$

$$n* = -\frac{1}{2 \cdot \alpha} \cdot \frac{1}{\frac{dAC(n)}{dn}} \tag{3.18}$$

This is exactly the equation 3.4 that we find from the approach of the master function. Under the consideration of substitution in equation 3.18, we obtain the connection between the master equation and the concept of free economic energy (or Helmholtz energy).

3.5 Theoretical case

In order to test both approaches, discrete choice and master equation (follow section 3), we may use a theoretical case for the purpose of the present study. The same cost function is used for the following results aim of this study. Since our objective is to propose an alternative approach for modelling decisions on hinterlands, we may illustrate the cost function with a practical case. A case on transportation can be a competitive network based on nine different alternatives (e.g. paths or links to reach the same origin and destination) to perform a shipment. The distribution of shipments is fixed via the initial distribution, on which the cost function is depending on the number of individuals using those alternatives during the entire time. Therefore, the cost function will decrease as the number of users increase using economics of scale. In this case, the heterogeneity of users is implemented and the total result of the final distribution over the alternatives is the cumulated distribution of the microstates (e.g. local forwarders, three party logistics providers and shippers) at every alternative. A more illustrative way to view the example case is the evolution of hinterland chains (or corridors) and its hierarchy along the time. There, the cost function plays an important role for the final decision making. If we want to model inland terminals, the investments are the main target and we focus on the recovery of fixed costs. However, if we want to focus an analysis on the operators, for instance a rented train, we must target on the frequency as the main aim of the analysis.

Now let the cost function depends on two variables, fixed cost C_f and convergence cost C_c (the cost that converges to the value of the asymptote) for every n number of users. Using a digressive function according to economics of scale for C_f, we have:

$$f(c) = c_i(C_F, C_C, n) = \frac{C_F}{n_i} + C_C$$

for

$$n > 0 \qquad (3.20)$$

A maximum cost value is fixed according to the cost calculation to avoid the steepness of the curve. In annex, a graph of the cost function is shown (table A-5).

4 Results

Table 3.6 Initial distribution for the extended model

	Path A	Path B	Path C	Path D	Path E	Path F	Path G	Path H	Path I
Actor 1	1	1	1	1	1	1	1	1	1
Actor 2	1	1	1	1	1	1	1	1	1
Actor 3	1	1	1	1	1	1	1	1	1
Actor 4	1	1	1	1	1	1	1	1	1
Actor 5	1	1	1	1	1	1	1	1	1
Sum	5	5	5	5	5	5	5	5	5

The starting point for testing the improvements is a matrix with nine alternatives and five market actors. The initial occupancy of this matrix is equally distributed with a shipping task of exactly one transportation unit (see table 3.6). The employed cost function is the same presented in section 4. For this example the parameter values are (mobility factor) $\mu = 0{,}001$ and $\alpha = 0{,}01$. A possible result after 100 iterations is given in table 3.7 and graphically displayed in figure 3.1. Both show a concentration of units of transport on a small number of connections. This feature shows economies of scale in a stable state of the system.

Table 3.7 Final distribution

	Path A	Path B	Path C	Path D	Path E	Path F	Path G	Path H	Path I
Actor 1	2	0	0	0	4	2	0	0	1
Actor 2	1	0	0	0	5	0	0	0	3
Actor 3	3	0	0	0	2	1	0	0	3
Actor 4	4	0	0	0	2	1	0	0	2
Actor 5	4	0	0	0	1	1	0	0	3
Sum	14	0	0	0	14	5	0	0	12

By applying the cost function to the results issues from section 3.3 or equation 3.18 for the cost function 3.20 the $C_f = 1000$ euros (see figure A-4 in annex) which seems suitable for a practical example of shipping a container unit on the hinterland leg as a maximum cost. The derivation of the cost function is:

$$\frac{dAC(n)}{dn} = \frac{d\left(\frac{1000}{n}\right)}{dn} = -\frac{1000}{n^2} \qquad (4.1)$$

Integrating equation 4.1 in equation 3.18 we end up with:

$$n* = 20$$

For a better understanding of the concept of optimal number of clusters, let us show the evolution process of the master equation for the theoretical case in fig. 3.1. Since the exact number is 45/20 = 2,25 clusters, the entire system reacts by energy (cost) efficiency and for 100 iterations, the final distribution results in 4 clusters with a weak number of consignments in one cluster.

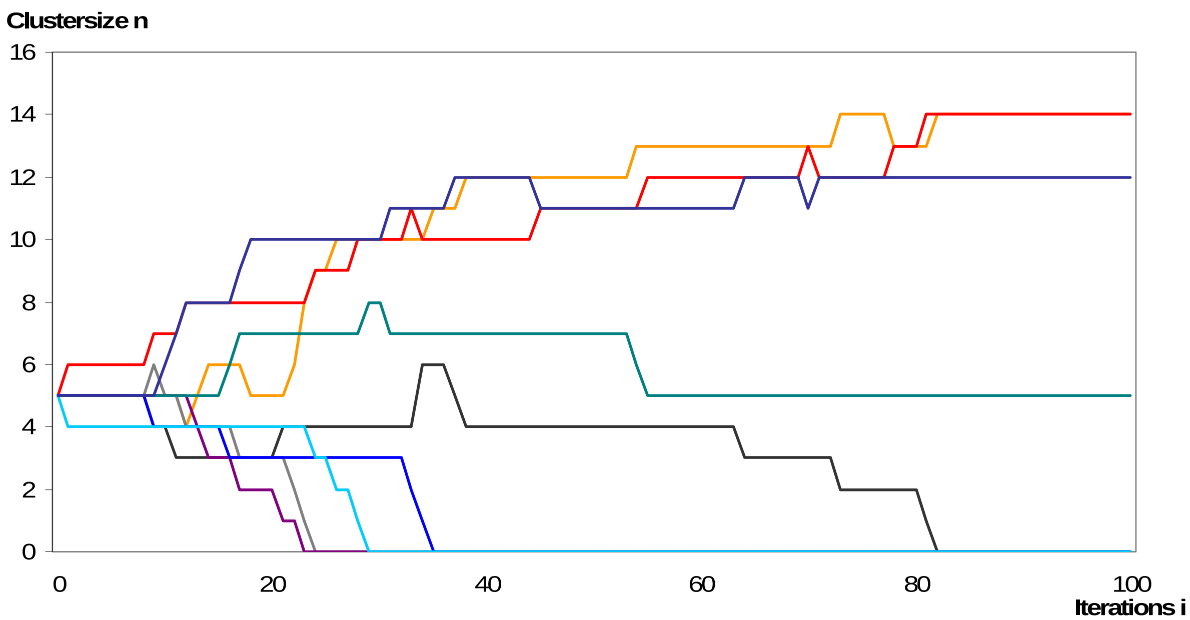

Figure 3.1Possible distribution of the cluster formation

It seems that the number of clusters goes in that direction since the top remaining clusters show an important advantage over the fourth remaining alternative. This statement is open to discussion since more tests are needed to prove the accuracy of the model.

For the same example we applied the three alternative concepts discussed in section 3.3, namely, the energy, the entropy and the Helmholtz free energy. These deductions are displayed graphically in figures 3.2, 3.3 and 3.4 respectively.

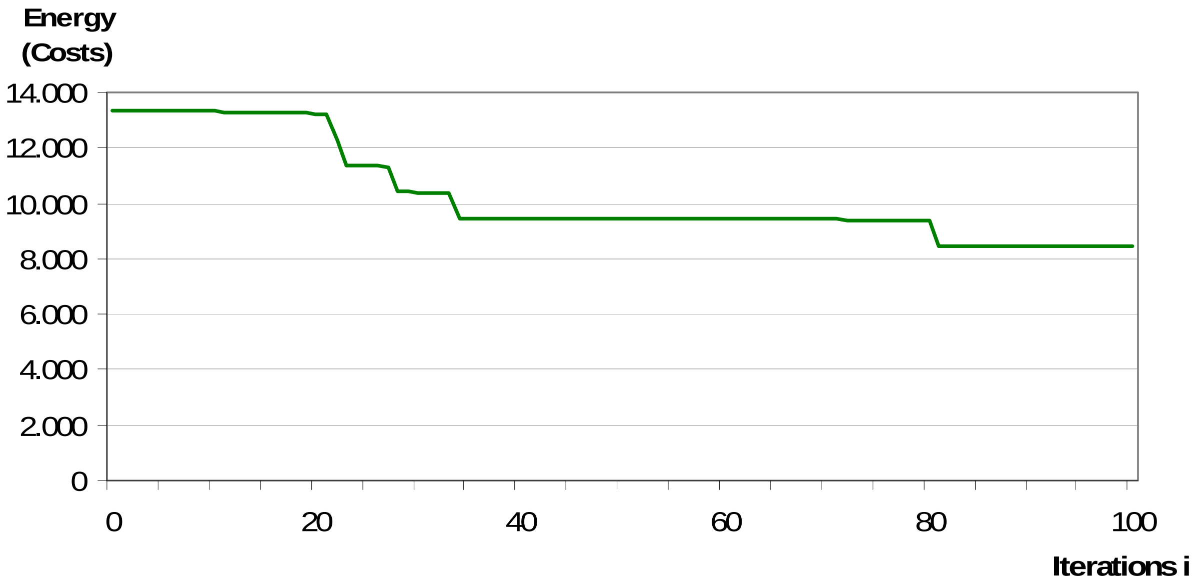

Figure 3.2 Energy progress

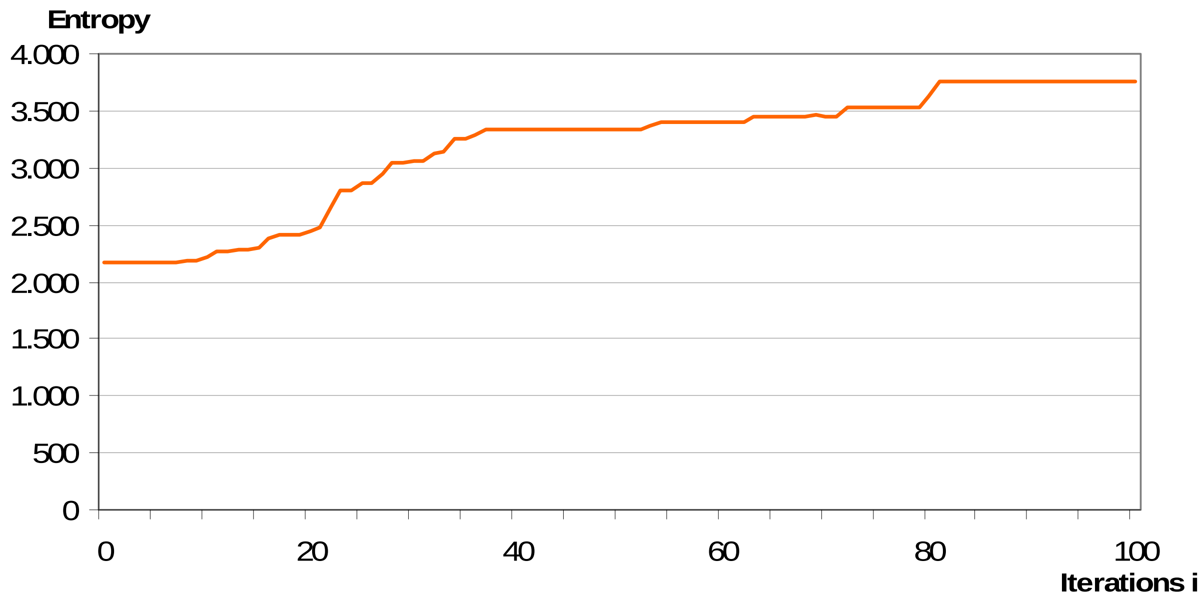

Figure 3.3 Entropy evolution

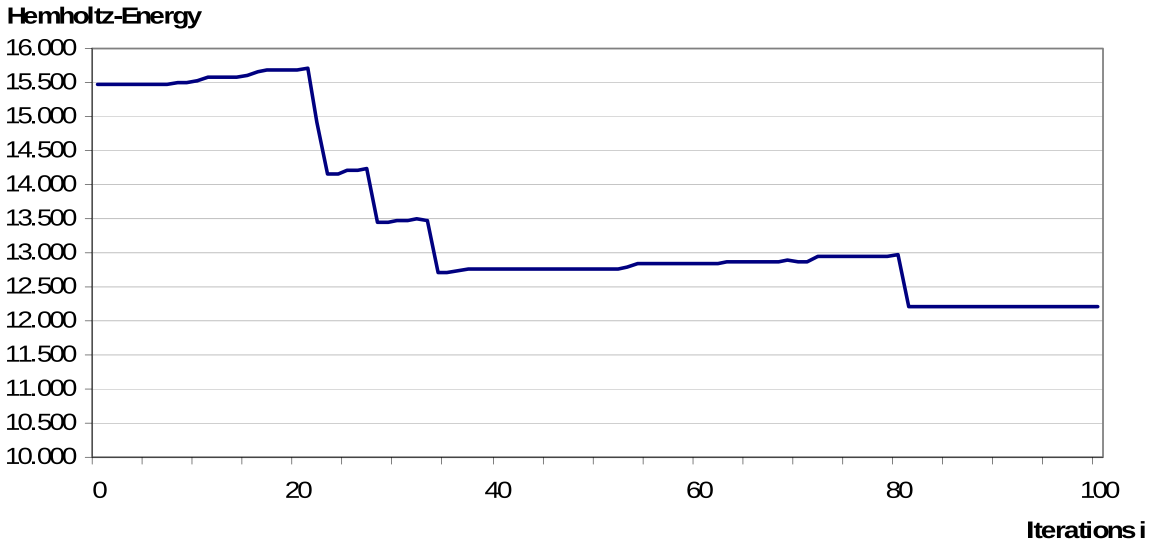

Figure 3.4 Free Economic Energy development

5 Interpretation of results

First, the transition rates flow between all alternatives allowing the possibility of elimination and feeding some of them without a progressive change but variable (see yellow alternative in table 3.1). In the first 20 iterations the interactions among the alternatives for remaining on the game is dynamic, then, in between the 20th and the 40th iterations four alternatives are swiped out, and it is after the 80th iteration that a fifth cluster is eliminated by the model. This characteristic allow the actors to play in a game were several interactions are possible before the selection of the remaining alternatives.

The energy (cost) of the overall model present results as expected. Due to economies of scale the overall costs decrease with the evolution of the system (cluster formation). However, the entropy costs represent an obstacle for the interchange of decisions as the system evolutes. This can be illustrated as the increased competitiveness with small number of alternatives. There, the entropy emerges as a constraint due to the increased competition at the remaining alternatives. For illustration, this is similar to the competition in game theory of small number of actors where the difference in alternatives is relatively small.

More important to observe is the FEE (Helmholtz free energy) which allow changes, but before a drastic change (e.g. the elimination of one alternative) the cost increase gradually. Before an alternative is eliminated, the individuals optimise their own utility creating an external effect for the whole system. However, after the effect is spread over the network, this effect is transmitted to the network and it disappears causing a global minimum. This is graphically shown by the increases before every drastic decreasing (the elimination of one alternative) step in fig. 3-4.

6 Conclusions

Before the background of growing freight flows and the trend of grouping intermodal transport networks, new approaches to model the emergence of transport corridors are required. They could be helpful in marketing (to support the strategies of transport operators in a competition environment) and for economic transport policy. Furthermore, the user’s optimisation is mandatory from the point of view of the overall system.

Two approaches were introduced for modelling hinterlands and its inner-interactions. As a first approach we tested them and we selected the Master equation given its superiority for modelling consolidation of shipments and its performance for creating mesostructures. The master equation maps the evolution of markets over time and allows transport corridor emergence. Since its functioning depends on the cost function, it allows the cluster formation (consolidation) for modelling corridors, or mapping hierarchy between decision-makers. An alternative assumption is the consideration of asymmetrical information as in real networks, since an individual can not own all the information of a system or network.

Given the stochastic characteristics of the Master equation we modelled its development for gathering information about the structures of its results. This situation guided us to introduce concepts from physics. The first motivation to go beyond the master equation was the understanding of the optimal number of remaining clusters. Three concepts were developed by mathematical deductions issued from the master equation and the Helmholtz energy or Free Economic Energy.

When studying the rules governing the evolution of dynamic hinterland systems, it has been found out that the model seek to maximise the Free Economic Energy. The FEE concept is formed by two components, namely, the full costs and the entropy. In order to point solutions, the FEE acts as an emergent concept able to map all intermediate solutions (mesostructures) considering the whole system. Furthermore, with the FEE critical states (e.g. before an alternative is shut down) can be mapped and its results can be applied for policy as well as for transport forecasting in hinterlands. Critical states are linked with real situations since logistics companies in the hinterland game play with strategic decisions by trying different alternatives and allowing new alternatives to emerge as principal options. Besides, all of these phenomena can be mapped via real drivers (full costs) and characteristics on the distribution of the system (entropy).

On the other hand, the optimal number of clusters is based on the minimisation of interactions in between alternatives. This is the nearest stable state where the final distribution is changing of form (by following a random distribution) but with the same number of remaining alternatives. In the case shown in section 4, due to the limitation of the iterations, the deduction for the optimal number of final clusters out of the concepts developed in section 3 seems to converge. However, more tests are needed to prove the accuracy of the optimal number of clusters.

We have developed a formal mathematical tool to model the emergence of hinterland networks. Its use can be descriptive for policy makers, but also for decision support aiming to companies. Moreover, the FEE shows the opportunities of the market for both, experimental and statistical regression models. This model represents a clear aid for better understanding self-organised system such as hinterlands.

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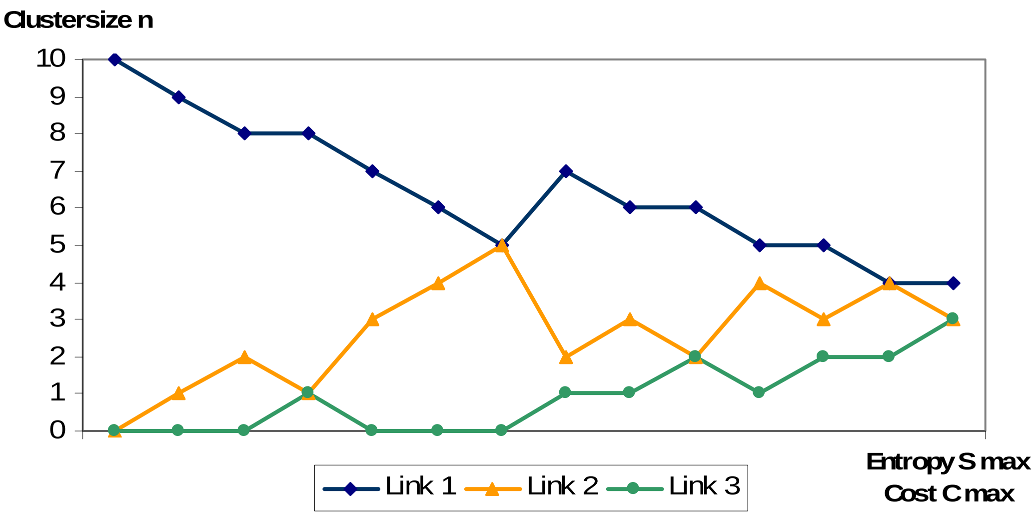
Annex

Paths			Costs			Total Costs	Entropy			Total Entropy
n_1	n_2	n_3	C_1	C_2	C_3	C	S_1	S_2	S_3	S
10	**0**	**0**	85,00	0,00	0,00	85,00	-15,10	0,00	0,00	-15,10
9	**1**	**0**	84,13	41,13	0,00	125,25	-12,80	0,00	0,00	-12,80
8	**2**	**0**	84,00	57,00	0,00	141,00	-10,60	-0,69	0,00	-11,30
8	**1**	**1**	84,00	41,13	41,13	166,25	-10,60	0,00	0,00	-10,60
7	**3**	**0**	83,88	68,38	0,00	152,25	-8,53	-1,79	0,00	-10,32
6	**4**	**0**	83,00	76,00	0,00	159,00	-6,58	-3,18	0,00	-9,76
5	**5**	**0**	80,63	80,63	0,00	161,25	-4,79	-4,79	0,00	-9,57
7	**2**	**1**	83,88	57,00	41,13	182,00	-8,53	-0,69	0,00	-9,22
6	**3**	**1**	83,00	68,38	41,13	192,50	-6,58	-1,79	0,00	-8,37
6	**2**	**2**	83,00	57,00	57,00	197,00	-6,58	-0,69	-0,69	-7,97
5	**4**	**1**	80,63	76,00	41,13	197,75	-4,79	-3,18	0,00	-7,97
5	**3**	**2**	80,63	68,38	57,00	206,00	-4,79	-1,79	-0,69	-7,27
4	**4**	**2**	76,00	76,00	57,00	209,00	-3,18	-3,18	-0,69	-7,05
4	**3**	**3**	76,00	68,38	68,38	212,75	-3,18	-1,79	-1,79	-6,76

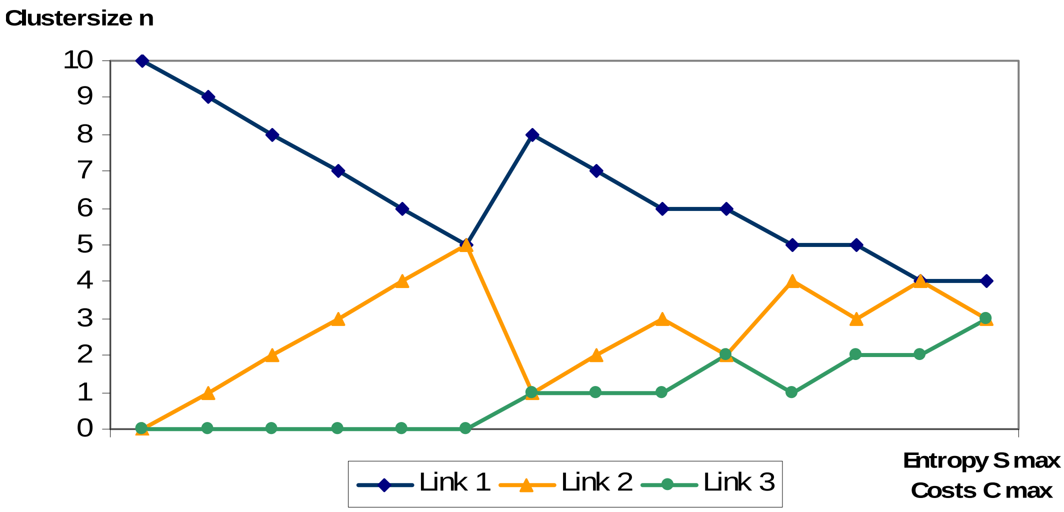
A-1 Example of Entropy maximisation per Entropy order

Paths			Costs			Total Costs	Entropy			Total Entropy
n_1	n_2	n_3	C_1	C_2	C_3	C	S_1	S_2	S_3	S
10	**0**	**0**	85,00	0,00	0,00	85,00	-15,10	0,00	0,00	-15,10
9	**1**	**0**	84,13	41,13	0,00	125,25	-12,80	0,00	0,00	-12,80
8	**2**	**0**	84,00	57,00	0,00	141,00	-10,60	-0,69	0,00	-11,30
7	**3**	**0**	83,88	68,38	0,00	152,25	-8,53	-1,79	0,00	-10,32
6	**4**	**0**	83,00	76,00	0,00	159,00	-6,58	-3,18	0,00	-9,76
5	**5**	**0**	80,63	80,63	0,00	161,25	-4,79	-4,79	0,00	-9,57
8	**1**	**1**	84,00	41,13	41,13	166,25	-10,60	0,00	0,00	-10,60
7	**2**	**1**	83,88	57,00	41,13	182,00	-8,53	-0,69	0,00	-9,22
6	**3**	**1**	83,00	68,38	41,13	192,50	-6,58	-1,79	0,00	-8,37
6	**2**	**2**	83,00	57,00	57,00	197,00	-6,58	-0,69	-0,69	-7,97
5	**4**	**1**	80,63	76,00	41,13	197,75	-4,79	-3,18	0,00	-7,97
5	**3**	**2**	80,63	68,38	57,00	206,00	-4,79	-1,79	-0,69	-7,27
4	**4**	**2**	76,00	76,00	57,00	209,00	-3,18	-3,18	-0,69	-7,05
4	**3**	**3**	76,00	68,38	68,38	212,75	-3,18	-1,79	-1,79	-6,76

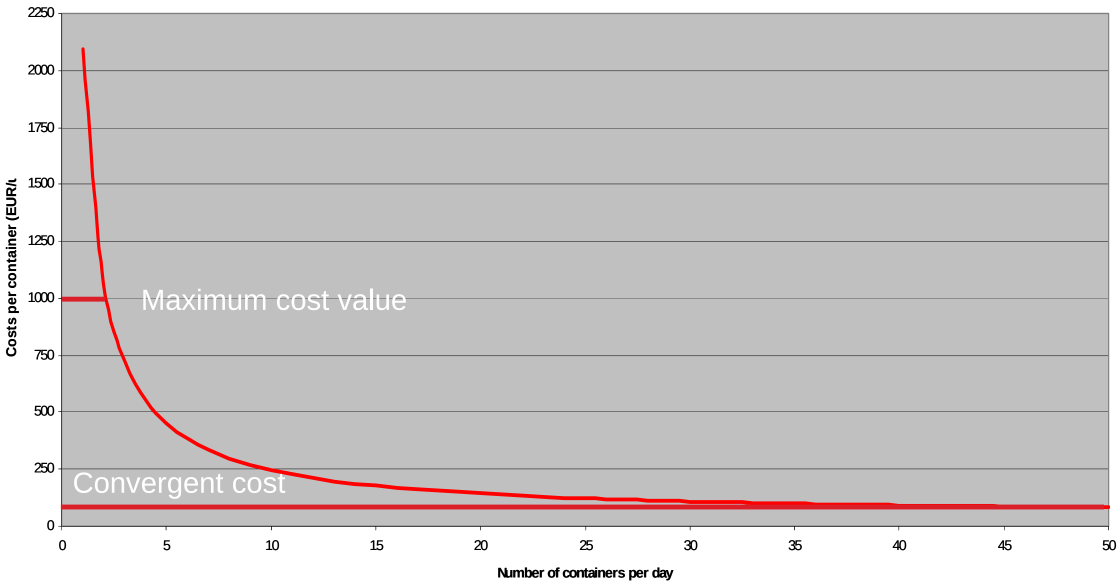
A-2 Example of Cost minimisation per Cost order



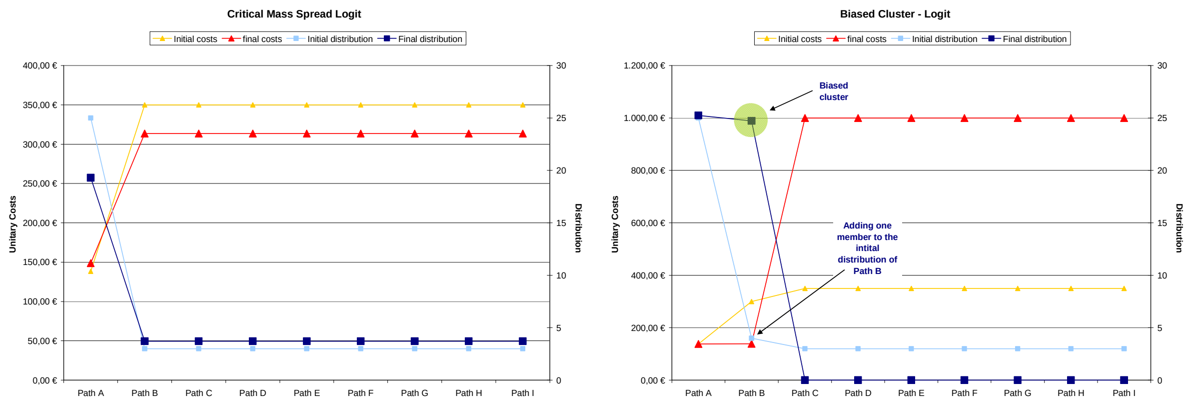
A-3 Evolution of Entropy maximisation



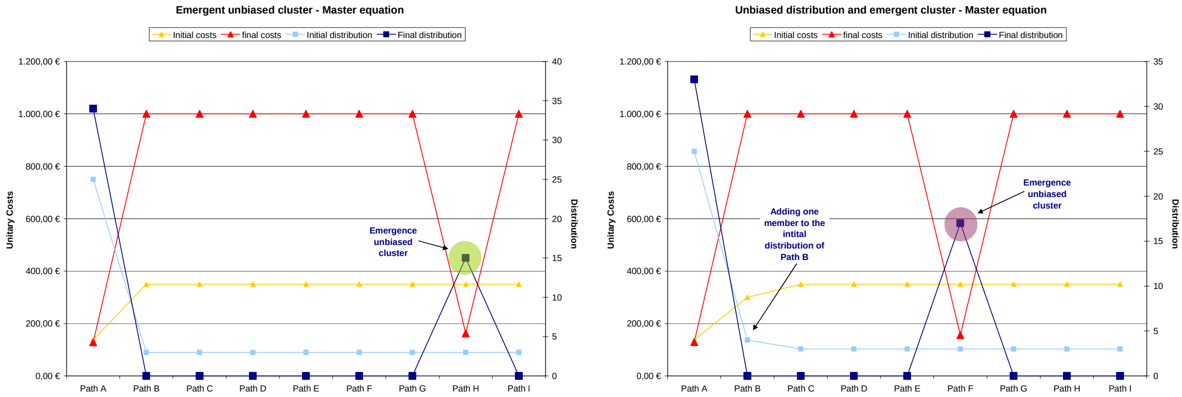
A-4 Evolution of Cost minimisation



A-5 Graphical Cost Function



A-6 Critical mass spread and Biased cluster emergence - Logit



A-7 Cluster emergence and unbiased distribution – Master equation