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GENERALIZED CLOSEST POINT PROJECTION PROCEDURES FOR CONTACT ANALYSES: ON EXISTENCE AND UNIQUENESS FOR ARBITRARY CONTACT SURFACES.

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Abstract.   *The uniqueness and existence of the closest point projection procedure (CPP) widely used in contact mechanics as well as in other fields of computational mechanics, e.g. in plasticity, are analyzed in for various contact situations. Starting from C^2-continuous surfaces "a proximity domain" allowing a unique projection onto a surface is created based on the geometrical properties of a given surface. It is shown that in order to construct a continuous projection domain for arbitrary globally C^1, or C^0–continuous surfaces, the projection algorithm should be generalized and also include a projection onto a curved edge and onto corner points.*

1 INTRODUCTION

The closest point projection procedure is often introduced as a numerical scheme to compute convective coordinates of a point projected onto a surface. It appeared in early publications for finite element models either as a result of the linearization of non-penetration conditions, see Kikuchi and Oden [1], or as a result of the split of a displacement vector into normal and tangential direction, see Hallquist et.al. [2], the closest point has become the first necessary step in almost all applied methods in computational contact mechanics, e.g. such as Penalty, Mortar and Augmented Lagrangian Methods. Despite the enormous number of publications on contact mechanics, there are only a few publications covering to some completeness the problem of uniqueness and existence of the closest point procedure for arbitrary approximations of the contact surfaces as well as describing effective numerical algorithms to overcome non-trivial problems.

In the current contribution, we provide analytical tools allowing to create, a-priori, *proximity domains of contact surfaces* from which a given contact point is always **uniquely projected**. This approach is based on the geometrical properties of contact surfaces exploiting the covariant description for contact problems developed in Konyukhov and Schweizerhof [3], [4].

2 Formulation of the closest point projection procedure in geometrical terms

The closest point projection problem is usually formulated as an extremal problem

$$||\mathbf{r} - \boldsymbol{\rho}(\xi^1, \xi^2)|| \to \min, \quad \longrightarrow (\mathbf{r} - \boldsymbol{\rho}) \cdot (\mathbf{r} - \boldsymbol{\rho}) \to \min, \tag{1}$$

where $\boldsymbol{\rho} = \boldsymbol{\rho}(\xi^1, \xi^2)$ is an arbitrary parameterization of a surface which can be given either by e.g. a finite element, or by a spline, or by a NURBS approximation. Problem (1) is then solved mostly numerically. However, then a fundamental problem arises: Does the solution of (1) exist? And, if it exists, then, is it unique for any arbitrary surface approximation? *The initial and strict answer is fully covered by the application of the famous theorem from the convex analysis to problem (1).*

If the function

$$\mathbf{F}(\xi^1, \xi^2) = \frac{1}{2}(\mathbf{r} - \boldsymbol{\rho}) \cdot (\mathbf{r} - \boldsymbol{\rho}) \tag{2}$$

is convex in a domain $(\xi^1, \xi^2) \in D$, *then the solution of problem (1) exists and is unique in this domain.*

Using this criterion we can focus on the geometrical properties of the surfaces. Assuming first C^2-continuous surfaces, we can analyze the second derivative of the function $\mathbf{F}$

$$\mathbf{F}'' = \left[\begin{array}{cc} \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_1 - \boldsymbol{\rho}_{11} \cdot (\mathbf{r} - \boldsymbol{\rho}) & \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_2 - \boldsymbol{\rho}_{12} \cdot (\mathbf{r} - \boldsymbol{\rho}) \\ \boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_1 - \boldsymbol{\rho}_{21} \cdot (\mathbf{r} - \boldsymbol{\rho}) & \boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_2 - \boldsymbol{\rho}_{22} \cdot (\mathbf{r} - \boldsymbol{\rho}) \end{array} \right], \tag{3}$$
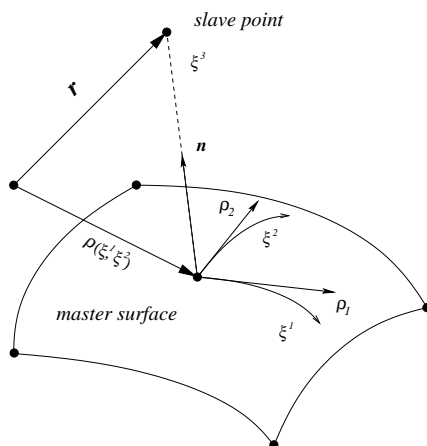
Figure 1: Closest point procedure and definition of the spatial coordinate system.

where a short notation for the partial derivatives has been introduced as

$$\boldsymbol{\rho}_i = \frac{\partial \boldsymbol{\rho}}{\partial \xi^i}, \quad \boldsymbol{\rho}_{ij} = \frac{\partial^2 \boldsymbol{\rho}}{\partial \xi^i \partial \xi^j}. \tag{4}$$

Having introduced a 3D spatial coordinate system related to the surface coordinate system, see Fig. 1, as follows:

$$\mathbf{r}(\xi^1, \xi^2, \xi^3) = \boldsymbol{\rho} + \mathbf{n}\xi^3, \tag{5}$$

eqn. (3) is transformed as

$$\mathbf{F}'' = \begin{bmatrix} a_{11} - \xi^3 h_{11} & a_{12} - \xi^3 h_{12} \\ a_{21} - \xi^3 h_{21} & a_{22} - \xi^3 h_{22} \end{bmatrix}, \tag{6}$$

where a_{ij} and h_{ij} are covariant components of the metric resp. curvature tensors.

3 Proximity criteria for different surfaces

"A proximity domain" is created as a domain where the second derivative is positive $\mathbf{F}'' > 0$. Surprisingly, the analysis of positivity in eqn. (6) has a geometrical interpretation leading to the Sylvester criteria enforcing the positivity in eqn (6) which in due course can be written in terms of principal curvatures k_1, k_2 as:

$$\left(\frac{1}{\xi^3} - k_1 \right) > 0$$

$$\left(\frac{1}{\xi^3} - k_1 \right) \left(\frac{1}{\xi^3} - k_2 \right) > 0. \tag{7}$$

3

Within the last transformation, it is assumed that the coordinate ξ^3 is a positive value within the normal vector direction $\mathbf{n}$, as chosen in eqn. (5). Thus, zones with positive and negative coordinates ξ^3 should be distinguished for various geometrical structures of surface points. The domains are different for an elliptic point with $k_1 \cdot k_2 > 0$, for a
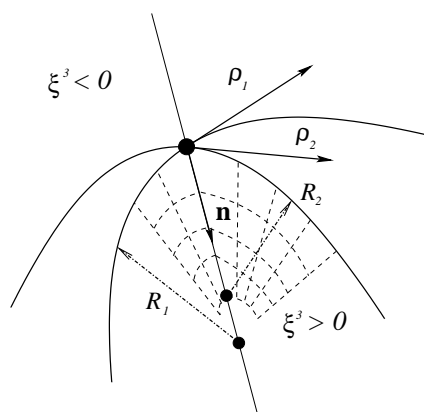

Figure 2: Elliptic point. Structure of the surface and projection domain assuming C^2-continuity.

hyperbolic point with $k_1 \cdot k_2 < 0$, a parabolic $k_1 = 0, k_2 \neq 0$, or for a flat point $k_1 = k_2 = 0$. Exemplarily we show the structure for an elliptic point for the convex part, keeping in mind eqn. (7),

$$\Omega(\xi^1, \xi^2, \xi^3) := \{\mathbf{r} = \boldsymbol{\rho} + \xi^3\mathbf{n}, \ \ where \ \ 0 < \xi^3 < \min\{\frac{1}{\xi_1^3}, \frac{1}{\xi_2^3}\}, \tag{8}$$

see Fig.2. For the concave part with $\xi^3 < 0$ the criteria leads to an infinite domain:

$$\Omega(\xi^1, \xi^2, \xi^3) := \{\mathbf{r} = \boldsymbol{\rho} + \xi^3\mathbf{n}, \ \ with \ \ -\infty < \xi^3 < 0\}. \tag{9}$$

Domains for hyperbolic with $k_1 \cdot k_2 < 0$, and parabolic points $k_1 = 0, k_2 \neq 0$, with are constructed in the similar fashion. A flat point (besides the case $k_1 = k_2 = 0$ globally on the surface) requires the analysis of higher derivatives.

It is necessary to define additional projection procedures in order to create a proximity domain for the 3D space allowing a continuous mapping of any path laying inside. These projections include a projection onto an edge and onto a corner. The idea is illustrated in Fig. 3 for the contact surface of a hexaeder focusing on the surfaces containing the corner point O.

4 CONCLUSIONS

1. The fundamental problems of existence and uniqueness of the closest point projection procedure are investigated.

2. The consideration of the differential properties allows to create "projection domains" from which a projection of any point is uniquely defined.
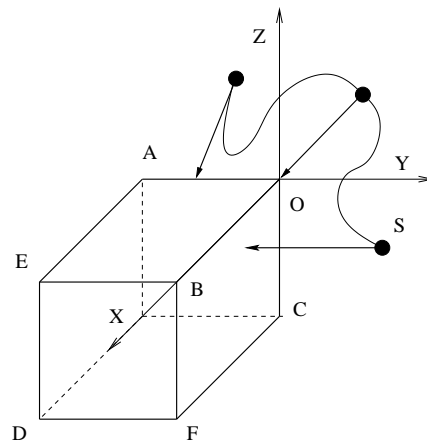
Figure 3: Hexaedral contact surface in the vicinity of O. A continuous proximity domain consists of domains for sides, edges OA, OB, OC and, finally, of a domain for the corner point O.

3. For arbitrary C^0 continuous surfaces a projection routine should be generalized to include projections onto objects of lower geometrical dimensions, such as curved lines and points.

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