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Validation and Limits of Finite Inflatable Beam Elements

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Abstract:
Although nowadays inflatable tubular beams are often used in the field of civil engineering, by now there are only few publications dealing with finite deformation inflatable beam elements, see e.g. [1], [2] and [3]. All formulations of inflatable beams have several assumptions in common, as constant cross sections throughout the deformation, a constant internal gas pressure and the negligence of circumferential stresses. These assumptions have to be validated either by experiments or numerical analysis. In the current contribution beam-like structures are investigated using a finite element shell or membrane formulation and featuring a volume dependent gas loading, see e.g. [5] and [4]. In general the formulation substitutes the internal gas pressure by an energetically equivalent volume dependent loading and thus enables to check for potential gas pressure changes during the deformation process of the inflated beam as a consequence of volume changes. Further local deformations as occurring in the vicinity of supports or almost single loads can be considered. In this paper the focus will be only on the initial assumption of the beam theory that the biaxial stress state is neglected.

1 Mechanics of Inflatable Beams

The mechanics of the inflatable beam element are only briefly mentioned here. A more detailed description can be found in the literature, e.g. [3]. A virtual work approach is chosen for the description of a state of equilibrium using $\delta\mathcal{E}^{el}$ as the virtual elastic potential, $\delta\mathcal{W}^{g}$ as the virtual work of the internal gas pressure and $\delta\mathcal{W}^{ext}$ as the virtual work of the external forces.

$$\delta\mathcal{E} = \delta\mathcal{E}^{el} - \delta\mathcal{W}^{g} - \delta\mathcal{W}^{ext} = 0 \tag{1.1}$$

1.1 Kinematics

The beam element is supposed to feature Timoshenko kinematics, with rotation ϑ of the cross section. The position vector of an arbitrary point $\hat{\boldsymbol{P}}$ can be given in terms of the displacement $\boldsymbol{u}$ of a point $\boldsymbol{P}$ on the center line as follows:

$$\hat{\boldsymbol{x}} = \begin{pmatrix} X_1 & + u_1 & - \Delta\hat{X}_2 \sin\vartheta \\ X_2 & + u_2 & - \Delta\hat{X}_2 (1-\cos\vartheta) \\ X_3 & & + \hat{X}_3 \end{pmatrix} \tag{1.2}$$

The Green-Lagrange strain tensor $\boldsymbol{E}$ is set up using the deformation gradient $\boldsymbol{F}$.

$$\boldsymbol{E} = \frac{1}{2}\left(\boldsymbol{F}^T\boldsymbol{F} - \boldsymbol{I}\right) \ , \quad \text{with} \quad \boldsymbol{F} = \frac{\partial\hat{\boldsymbol{x}}}{\partial\hat{\boldsymbol{X}}} \tag{1.3}$$

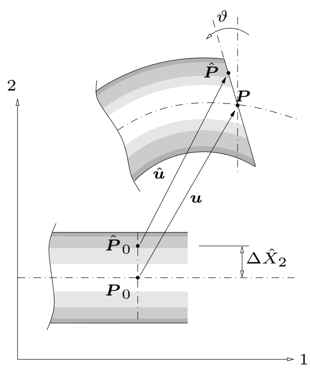

Bild 1.1: Kinematics of Timoshenko beam

Thus, assuming a hyperelastic material law and neglecting stresses in circumferential direction, the variation of the elastic potential is written as:

$$\delta \mathcal{E}^{el} = \int_{\mathcal{B}} \boldsymbol{S} : \delta \boldsymbol{E} dV \tag{1.4}$$

1.2 Virtual work of gas pressure

The virtual work of the internal gas pressure p can be described using the normal $\boldsymbol{n}$ on the surrounding wetted surface and a surface integral. It composes of a part $\delta\mathcal{W}^g_\circ$, which is associated to the curved tubular surface and a part $\delta\mathcal{W}^g_\exists$, associated to the end cap of the beam.

$$\delta\mathcal{W}^g = p \int_A \delta\hat{\boldsymbol{u}} \cdot \boldsymbol{n} dA = \delta\mathcal{W}^g_\circ + \delta\mathcal{W}^g_\exists \tag{1.5}$$

The differential area vector $\boldsymbol{n}dA$ on the tubular surface is given by the vector cross product of the two tangential vectors $\hat{\boldsymbol{x}}_{,1}$ and $\hat{\boldsymbol{x}}_{,2}$ on the curvilinear geometry of the beam (see figure 1.2). Assuming a cross section A_0 with constant radius R_0 throughout the deformation, an a priori integration in circumferential direction ξ_2 can be performed, using $\partial/\partial\xi_2 = \frac{1}{R_0}(\cdot)_{,\varphi}$. With $(\cdot)_{,1}$ denoting the derivation to the convective coordinate ξ_1, the virtual work of gas pressure for the tubular domain part yields

$$\begin{aligned} \delta\mathcal{W}^g_\circ &= pA_0 \int_l \sin\vartheta\vartheta_{,1}\delta u_1 - \cos\vartheta\vartheta_{,1}\delta u_2 \\ &\quad + (\cos\vartheta u_{2,1} - \sin\vartheta(1+u_{1,1}))\,\delta\vartheta dX_1 \ . \end{aligned} \tag{1.6}$$

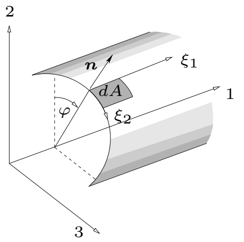

Bild 1.2: Curvilinear coordinates ξ_1, ξ_2 on tubular domain part $\circ$

The normal vector $\boldsymbol{n}$ on the end cap follows from figure 1.3 to

$$\boldsymbol{n} = [\cos\vartheta, \sin\vartheta, 0]^T \tag{1.7}$$

and thus the virtual work of gas pressure for the end cap part:

$$\delta\mathcal{W}^g_\exists = pA_0 \left(\delta u_1 \cos\vartheta + \delta u_2 \sin\vartheta\right)\big|_\exists \tag{1.8}$$

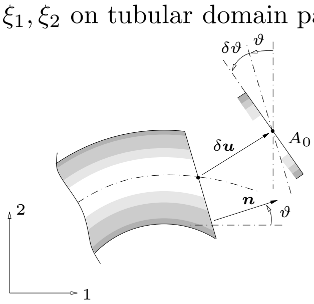

Bild 1.3: Kinematics on end cap $\exists$

Subsequent linearization and discretization of the center line deflections $\boldsymbol{u}$ and the cross section rotations ϑ yield a finite element formulation for a finite inflatable beam element, which subsequently will be compared with a 3D membrane formulation featuring volume dependent gas loading (see also [4]).

2 Numerical Examples

For the verification of the beam model an inflated cantilever (initial length $L_0 = 1000mm$, initial radius $R_0 = 30mm$, thickness $t = 1mm$ and Young's modulus $E = 100MPa$) has been chosen.

To investigate the influence of the biaxial stress state in the membrane the computation was performed with Poisson ratios $\nu = 0.0$ and $\nu = 0.4$. As initial values for the beam radius, the current radius R of the 3D membrane model in the deformed configuration has been taken. Comparing the horizontal tip displacement after the inflation process (see table 2), we find that in case of $\nu = 0.4$ the beam solution with $\bar{u} = 171.6mm$ is far from the 3D membrane solution with $\bar{u} = 75.6mm$. Neglecting the lateral contraction by using $\nu = 0.0$ both tip displacements are in adequate agreement. Further, it could be observed that the load deflection behavior for a subsequent transversal tip loading of $F = 2N$ leads in the case for $\nu = 0.0$ to a poor approximation, which is due to the fact that the finite beam element behaves too stiff. The surprisingly good agreement between the load deflection curves in the case of $\nu = 0.4$ is only by accident, because the overestimation of the bending moment due to a too large $\bar{u} = 171.6mm$ is compensated by the higher stiffness of the beam model. Hence it can be stated that the negligence of the biaxial stress state is the major source of error in the simplified beam model and must be overcome by e.g. an additional energy term in the virtual work approach. But, as also shown in the literature, [3] the beam model is at least applicable for small deflections.

	gas pressure	membrane solution	beam solution	load deflection curve
$\nu = 0.4$	0.1061 MPa	75.6 mm	171.6 mm	good
$\nu = 0.0$	0.1165 MPa	167.1 mm	190.2 mm	poor

Tabelle 2.1: Horizontal tip displacements $\bar{u}$ after inflation of beam for membrane and beam model

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