MANIPULATIONS

ANALYSIS OF TRADING BEHAVIOUR NOT CONFORMING WITH TRADING REGULATIONS

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universitätsverlag karlsruhe

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Jan Schröder

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Jan Schröder
Institut für Informationswirtschaft und -management
Universität Karlsruhe (TH)
Zirkel 2
76131 Karlsruhe, Germany
schroeder@iism.uni-karlsruhe.de

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Karlsruhe, December 2008 *Jan Schröder*

To my dear teachers.

Contents

1. Motivation

In comparison to social systems in former times where opinions of people were not accepted in or allowed for political decisions our western social life nowadays builds on valuations of individuals in a multitude of areas. For a democratic and plural society the voice of people is crucial to the continuity of the system definition itself. The valuations of small groups may decide about the flavour of politics i.e. are the decisive factor for a party joining the parliament or not. To derive the public opinion from individuals' beliefs all individuals can be asked to state their individual opinion which is done e.g. in ballots or polls [Noel 05]. Besides the correct detection of the public opinion itself, public interest also lies in having a foreknowledge of the result of an event if the formal aggregation is proceeded at a later date i.e. in an upcoming election. As conducting the full information sampling by raising all individual opinions (e.g. in an election) is in most cases very costly, this knowledge is usually built by information aggregation techniques which usually take only the opinions of a representative sample of the population responding with their true opinions [Bohm 99]. Usually, this is undertaken by surveys via various kinds of media. Recently a new method for predicting a full information aggregation's outcome has been devised. Following the efficient market hypothesis of Friedrich Hayek, all relevant information within a stock market is aggregated by the mechanism of a perfect market and is revealed and disseminated in the prices which is referred to as the price system [Haye 45, Muth 61]. As Eugene Fama states, the share prices within an efficient market continuously reflect the sum of all available and relevant information about future events [Fama 65]. Compared to a conventional survey, an information stock market system on the opinion poll's topic aggregates the traders' opinions while the prices reflect the aggregation of these opinions as the prediction of the market. Instead of asking a representative set of survey respondents for their own individual opinion on their own vote the information market asks its traders about their opinion on the electorate's votes, so which opinion the deciders will have. This concept was first described by Forsythe [Fors 92] and mechanises the idea of deriving a more adequate answer to a question by aggregating the suggestions of a set of people by means of an information market rather than by asking an expert or conducting a survey [Suro 04].

Within the literature such information systems are addressed by different terms like decision markets, information markets, idea futures, forecasting markets, idea stocks, information aggregation mechanisms or virtual stock markets [Tzir 07a, Tzir 07b].

Following the Hayek hypothesis of an efficient market the price system harnesses the selfish motivations of the buyers and sellers into a socially efficient outcome reaching a competitive allocation under neoliberal assumptions [Milg 92]. In a prediction market, *incentive compatibility* is defined as follows:

1. The traders expect more gains from the system by participating in the system than from not participating (they participate voluntarily) and

2. by revealing their true preference in the market the players earn more than by manipulation (they tell the truth).

If incentive compatibility holds in a prediction market, participating in the market and trading based on their honest expectations is a dominant strategy for rational traders in the market, because deviations from this strategy will result in monetary losses. Deficits within the incentive system of a stock market usually result in undesirable actions e.g. fraudulent actions like insider trading if traders have the chance to reach a higher return by following their fraudulent actions rather than by playing by the rules.

As in conventional stock markets, where the "estimation" for a company's value within its environment is traded, also prediction markets have to deal with disreputable traders that behave in a way that is illegal according to the actual market rules. First and foremost this is certainly the abuse of insider knowledge and the (fraudulent) buy and sell actions connected to it.

But in contrast to traditional stock markets, prediction markets may in many cases be conducted without a direct monetary incentive, i.e. there is no direct dependency between virtual stock market money and real money. The monetary incentive is usually substituted by some prizes to be won by the traders with the highest depot values after the market is finished and cleared. Within the market the traders then deal with virtual play money they are provided with when entering a new market (liquidity). As there is no monetary loss resulting from illegal decisions and actions in play money settings, the incentive compatibility of the market is strongly influenced, because cheating may lead to monetary gains in a play money market by e.g. increasing the chances of a player to win a price by using multiple accounts ("cheating" against the market rules) for money transfer.

This market setting generates other types of fraudulent actions not present in conventional stock markets. As the liquidity of the market is virtually generated by giving the play money to the traders for free, alliances among traders or multiple virtual identities of a single (human) trader provide an easy way to increase one's depot value besides regular market actions [Douc 02]. Here the motivation is egoistic, i.e. to win the prize and against the rules.

But prediction markets' problems are not only based on the virtuality of the traded money. Also the virtuality and the real world as its essential counterpart create serious problems for a prediction market's incentive system. Some traders in such a prediction market setting could have an interest in influencing the market in favour of the real event represented by some share (signalling). Basically, these traders assume that the decision makers of the real event recognise a share's price as the majority opinion and, e.g. the decision makers may follow that signal in case of price changes by changing their vote thus rallying to the majority opinion (Bandwagon Effect, see [Nade 93, Rhod 07]). Therefore, the manipulator attempts to influence the share's price by systematic buys or sells of large quantities of shares hoping to influence the decisions of traders in the real world. In contrast to the former manipulations, these manipulations do not always aim at personal profit. In fact, they have the rational intention to make collective profit in the real world by bearing costs within the virtual market i.e. costs of contributing to the own party reaching a certain percentage in an election vs. winning a price in the market system by effective (regular) trading.

Both the 'egoistic' and the 'signalling' behaviour emanate from perturbations in the incentive system. Manipulations result in growing dissatisfaction and abstinence of participants in the worst case. They may also discredit the medium prediction market as such by the disturbed confidence and reliability due to manipulated predictions. Manipulated predictions may also lead to high prediction errors. Trading systems that allow participants to carry out manipulative actions or whose fraudulent actions are simply not prevented from being carried out result in negative feedback loops by attracting even more malicious traders or they incentivise (frustrated) regular traders to become fraudulent thus maybe multiplying the effects.

As the numbers of users participating in such electronic markets are quite high, a manual supervision of the user accounts is simply impossible and algorithmic approaches are preferable. An idea of the ratio of fraudulent traders within systems without active fraud prevention can be extracted from the official consumer complaints within the *early days* of electronic auction markets in the United States where the Federal Trade Commission registered a total of 51,000 auction fraud complaints in 2002 [Comm 03] which in 2003 accounted for 48 % of Internet-related fraud complaints and in sum resulted in the presumed loss of USD 437 million in 2003 [Comm 04, Comm 08].

In general, an electronic market system acts as a managing instance organising the traders actions in matching buy and sell orders. The traders, therefore, interact with the market system and not with each other directly which provides anonymity among the traders. As most of the manipulation approaches by the participants violate this anonymity assumption for the networks derived from these anonymous transaction ties, this work focuses on the network connections seen as interconnections "among the traders". Thereby the status of a participant of a network is assumed to be related to the status of all other participants in the network. As this idea is the essential paradigm within the research area of social network analysis we analyse the market by means of a social network analysis technique. Regarded from this side, market manipulations can be seen as structures within a network that basically has an unstructured characteristic.

So in the end maintaining incentive compatibility of the market system is crucial to ensure an efficient market system and for prediction markets to provide reliable predictions. Due to aspects like market design e.g. a play money setting and the real world embedding of a prediction market (corresponding to a real world event which is to be predicted) this status of incentive compatibility cannot always be assured or it is even impossible i.e. if the incentive system of the real world cannot be changed. The detection of manipulations and the elimination of the manipulative traders can help to balance the system if incentive compatibility cannot fully be established by design of the system for the market rules and regulations. In addition, the detection of manipulations improves market efficiency.

1.1 Objectives

The main objective of this thesis is the description and detection of fraudulent trading patterns within prediction markets by means of a social network analysis technique, namely eigensystem analysis. Thereby basic fraudulent patterns within a prediction market are identified and classified with respect to the incentives of the traders and the prediction market system. The chosen analysis technique is adapted to the focus of this work and automatic procedures are described. These are necessary, because real-life electronic markets produce networks that are too large for manual analysis.

1.2 Outline

Following the motivation and this outline, Chapter 2 introduces prediction markets and gives an insight into the relevant research areas in the domain of prediction markets and the manipulations within such markets. Along with a detailed example the basic functionality of a prediction market is introduced in Section 2.1. Thereupon an outline of applications of prediction markets in its rather short history is given in Section 2.2, with the most prominent research and company internal markets mentioned. A literature overview in Section 2.3 categorises manipulations in traditional stock markets and describes types of manipulation specific to prediction markets. The work describing reasons for these manipulations to happen is given as well as the effects on the market caused by these manipulations. On the basis of the described related work the contribution of this work is described.

Chapter 3 describes the concept of incentive compatibility, the incentive structure of a prediction market system, and the manipulations as a result of the absence of incentive compatibility. Therefore, Section 3.1 describes incentive compatibility and its conditions for a prediction market system. The incentive systems of such markets are introduced in Section 3.2 with a focus on different scenarios a prediction market can be conducted within. Fraudulent actions and the connected trading patterns are derived from an incentive incompatible status of the market system in Section 3.3. After having given insight in the basic assumption of anonymity within a market network in Section 3.4 a short insight into fraud detection systems used within the domain is given in Section 3.5 along with a discussion of the contribution of the fraud detection techniques presented in this work.

As the main analysis technique used in this work stems from social network analysis, Chapter 4 introduces this field and related research areas that are important to understand the interpretation of the formal results from the analysis technique. First, two formal representations of a social network are introduced in Section 4.1 (matrix) and Section 4.2 (Graph). Section 4.3 describes basic measures on a graph used in the following work to describe structures within the network. Social network analysis techniques in the area of the used analysis technique is introduced in Section 4.4. In Section 4.5 topics relevant to these areas and the interpretation of the results of the applied analysis technique are introduced in brief.

For the analysis technique used Chapter 5 then gives the mathematical background. As the method, namely eigensystem analysis of complex Hermitian matrices, works in Hilbert space, the properties of Hermitian matrices are given in Section 5.1 with a special focus on metrics in Hilbert space. In Section 5.2 the representation of transaction networks as Hermitian matrices is described. The results from the eigensystem calculation is discussed in depth in Section 5.3. As eigensystems tend to appear quite complex in presentation a visualisation method is introduced. Structures within the eigensystem are described and a clustering with the eigensystem is introduced as a method of finding these patterns on the basis of the previously introduced metrics within Hilbert space.

The analysis in Chapter 6 brings together the prediction market transaction data, the analysis technique, and its capabilities of detecting certain structures within a network. In Section 6.1 several transaction networks are generated from the market's accounting system and its transaction records with respect to the pattern structures detectable by the introduced analysis method. Section 6.2 describes the introduced manipulation patterns in terms of these networks based on the patterns introduced in Section 3.3. Applying the analysis techniques from Section 5.3 to the networks described within this chapter, Section 6.3 gives structured procedures for the application and parametrisation.

Chapter 7 then applies these analysis ideas to market simulations and real world data. In Section 7.1 a regular market scenario is built which then is perturbed by defined transaction networks representing the introduced market manipulations. Along with several simulation runs on different perturbations the stability of the eigensystems can be shown. For the perturbations and their detection within the eigensystem stable boundaries of the spectrum-changes are exemplarily given. In the second part of this chapter the analysis methods are applied to transaction data from several prediction markets which were conducted between the years 2006 and 2007. After a short description of the basic parameters of the markets, Section 7.2 analyses the market data by means of the introduced techniques with each market having a special focus on one of each of the introduced manipulation patterns.

Closing remarks and an outlook to further work to be done within the addressed research areas are given in Chapter 8.

2. Related Work

2.1 Prediction Markets

To foresee the outcome of an uncertain future event or a hardly predictable development prediction markets can be used where a stock market is set up for the event on which the outcomes of the event are represented by the shares which are tied to them with a certain payoff (e.g. the proportional voting portion of a party in an election, the probability of the victory of a certain team or the sales figures of a product). In the moment before the first information on the real outcome of the event is made public, the market is closed. When the real outcome is known, the shares held by the traders are priced and bought back for the final value of the shares. The final value of a share can either be tied relative to the outcome of the election event e.g. x monetary units for the share that made x percent in the election (relative payoff), binary bound to the winning team at a sports event e.g. 100 monetary units for the winning team (binary payoff), zero for all other or tied to the absolute value of the event's value e.g. one monetary unit for each 1000 units sold of a product (absolute payoff). Table 2.1 lists the possible payoff functions for prediction markets.

In Figure 2.1 this functioning is illustrated by a trader having a certain estimation of the outcome of an election event (1). In the beginning the trader holds 20 shares of *Share1* which gives a total depot value of 220 Euro (2) at the current market price of 11 Euro for *Share1* he bought them for (1). Now in (3) the share prices change to 13 Euro for *Share1* and 29 Euro for *Share2*. As the trader assumes the share *Share2* to be underrated he decides to buy 30 shares of *Share2* (4). This gives a new total depot value of 1130 Euro. He also decides to sell all shares of *Share1* which gives him a profit of $2 * 20 = 40$ Euro as he bought the shares for 11 Euro per share and leaves the share portfolio's value at 870 Euro (5). In (6) the election day brought up the real outcome of the election with the party of *Share1* having 10 % and the party of *Share2* having 36 % of the total votes. This results in the system paying 10 Euro for every share of *Share1* and 36 Euro for every share of *Share2* and a total depot value after buyback of 1080 Euro (7) and a total speculative profit of 250 Euro for the trader.

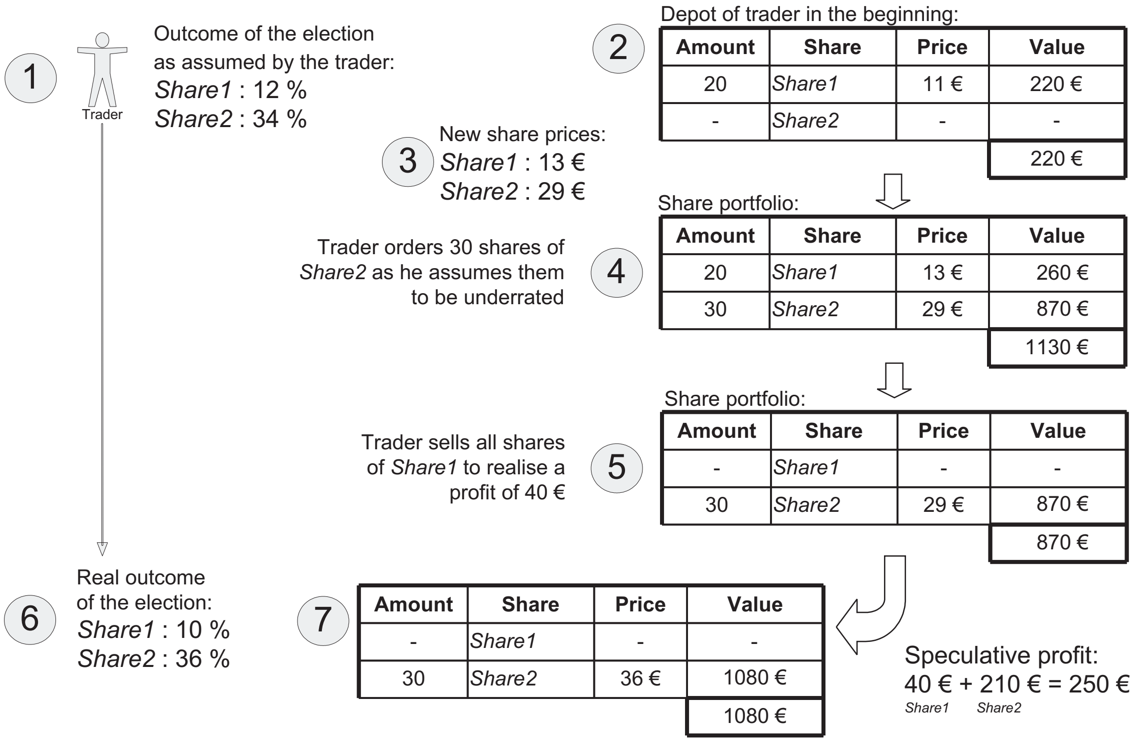

Figure 2.1: Trading in a prediction market system

Basis for the prediction market is the *primary market* (a fixed-price portfolio market) where traders can buy and sell a set of one share each (called a portfolio) for a fixed price without risk. E.g. in a political stock market where the result of an election is traded the portfolio consisting of one share for each party within the market costs 100 monetary units. In the *secondary market* (e.g. a continuous double auction market) the traders exchange these shares among each other with respect to their preferences. Within this setting the prediction market describes a zero-sum game with the given virtual money being redistributed among the traders. In play-money markets traders are usually not confronted with losses of real money, but with bankruptcy in the play money leading to the stop of all trading actions because of the lack of liquidity. The secondary market is usually designed as a continuous double auction matching buy and sell orders in the moment they overlap. Due to the low level of trading activities in the beginning of a market the initial phase of a market may be organised by a clearinghouse auction where the orders are collected until a certain point in time is reached. A market price is calculated and all queuing orders that overlap are matched and executed at a uniform price. This phase is then followed by the continuous double auction.

Hence the market price of a share at market closure reflects the aggregated perception of the traders for the probability of realising the event. This final forecast stands for the market's prediction itself and is often referred to as the market result which is compared to the outcome of the real life event. Also at each point in time during the trading, i.e. in the period where the shares' final values are unknown, the probability of occurence or expectations of the traders for that event to come true can be derived from the price of the corresponding share and this price reflects the continuous prediction of the traders about the outcome of the event. During the market, prices within the market and the perception of the outcome of the real event may influence each other [Schm 02].

The process resulting in the probabilities of the prediction market is conducted by rational, risk neutral traders selling shares which they estimate to be overvalued (i.e. the final price being lower than the current one) and buying shares which they estimate to be undervalued [Glos 85]. So the price of a share converges at a price level reflecting the aggregated preference of the traders for that share. Thereby the preferences of a trader correspond to his discounted expectations. In short term markets the discounting can be neglected [Span 03]. Several payoff functions can be used to generate different kinds of predictions where the payoff functions define how the final value of the shares are measured [Span 03, Wolf 04]. The most important payoff functions are listed in Table 2.1. The election market example above described a relative payoff for each share as seen in the bottom row of Table 2.1. The payoff is relative to the total opinion variation of 100% for all parties and directly bound to the percentage of the result the party achieves in the election. So with this payoff the system will buy back all shares held by traders into virtual money at the rate given by the parties' election percentage.

Predicting the outcome of a soccer tournament a relative payoff could be expressed by fixed virtual money values at which shares of certain teams are bought back in the case of reaching a certain place in the tournament i.e. 80 monetary units for each share reaching the first place, 50 for the second place and 20 for the third. But for the sake of clarity, the market design for such markets usually utilises a binary payoff described in the first row of Table 2.1 buying back the winning team's shares for 100 monetary units after market closure (end of final game). This type of markets are sometimes referred to as *winner-take-all* markets where the share of the realised event is denoted as *winner*. The middle row in Table 2.1 shows a payoff by absolute numbers which can be illustrated by a prediction market for a certain derivative and the market question asking for the price of the derivative at a certain due date. After market closure all shares are bought back for the actual course of the derivative in the real world. Within prediction market settings especially this type of payoff brings certain problems when used in a real money market where the number of traders and especially the real money they are able to buy shares for is not limited. The payoffs in this scenario are decoupled so depending on the price evolution of the derivative in the real world, the payoffs for the system in such a setting may become unlimited. Thus it could be safer for the market operator to use a binary payoff where the courses are classified into shares like "higher than value x at the due date t_d" and "lower than value x at the due date t_d" and the winning shares being bought back for a certain amount of money after market closure.

In the choice of the payoff function it should generally be paid attention to the traders' valuations for the share. Especially, in settings where a binary or absolute payoff is used together with long period markets, it may be difficult for the traders to value the initial price or prices at all as the measures are not known from everyday life and appear to be artificial (in comparison to an election event measured in percentual shares). Complex for the traders to evaluate are also long time spans,

Function of Disbursement	**Range of Values**	**Examples for the Market Design**
Binary payoff	0 - maximal value of payoff	- Winner-take-all markets - Market share of 30%, between 30-35% or above 35% at due date
Absolute numbers	0 - maximal possible value of the market condition	- Sales of a product in a certain period - Number of visitors of a movie
Relative numbers (Sum of prices of one share each result in a fixed portfolio value)	0 - 100	- Market share of different products in the same segment/competing products - Share of 100% for a party within an election

Table 2.1: Payoff functions for prediction markets (based on [Span 07b])

i.e. in a market setting which asks for the company's stock price one-year-ahead. The resulting volatility of the market may build a base for upcoming fraudulent actions as described in Section 6.2 if it is not based on high trading activity.

As most of the stock markets today are based on electronic communication within the market system also prediction markets are usually operated as electronic markets. Information technology is used by the market system to bring buyers and sellers together on a virtual market place. There the traders leave buy or sell offers (asks in case of the intention to buy and bids in case of the intention to sell shares). In most systems the traders are allowed to act from locations of personal choice as communication is conducted with internet connections. For some experimental scenarios the locations may be next to each other limited by the network connection. Normally in electronic markets traders interact with the market on screen without any direct physical connection to each other.

2.2 Application and Research Areas

Following betting markets at Wall Street on the outcome of presidential elections in the years after 1884 [Rhod 04] in 1988 the *Iowa Electronic Market* ran as the the first major electronic prediction market for the 1988 U.S. presidential elections [Fors 92] in the way described in Section 2.1. After several years of mainly political focus and predicting election outcomes the first business application of prediction markets have evolved with a market for completion dates of large scale software projects at Siemens in 1997 [Ortn 97, Ortn 98] and markets to predict the sales volume of printers at Hewlett-Packard from 1996 to 1999 [Plot 02]. Up to now several other application areas of prediction markets evolved like predicting the Oscar award winners [Lama 07] or the outcome of sports events like soccer matches and tournaments [Span 07a] which follow a long tradition of sports betting over the last century (i.e. sweepstakes, horsetrack race betting).

Since the beginning of prediction market research the main interest lies in the accuracy of the predictions. Starting with accuracy with respect to the real event's outcome and, therefore, the reliability of the idea of prediction markets itself, the focus nowadays changed to accuracy of the prediction in comparison to predictions of other forecasting media (i.e. surveys, betting markets) [Schm 02, Luck 07] or other auction types [Geye 07]. Of special interest is the influence of these parameters in the market's share prices and, therefore, the predictions [Schm 02]. Besides these parameters adjustable and fixed within the technical market system, a prediction market system essentially consists also of the (human) traders and a set of *market rules*. Market rules can be described as the totality of the legal and contractual rules and regulations that traders must observe in order to ensure that the market operates in an orderly fashion. The market's share prices and therefore the predictions are manipulated by traders not obeying these market rules.

2.3 Manipulation of Prediction Markets

In the literature markets are widely regarded as being more accurate for extracting diffuse information than other techniques like surveys or opinion polls [Fors 92, Chen 04, Hans 06a]. But this accuracy can be compromised by information traps (information existing in the market does not become revealed in prices, [Nth 99]), lack of liquidity, lack of meaningful equilibria or manipulation [Chen 04] which for the (human) trader's side results in behaviour like lying, manipulation, embezzlement, retribution, or system manipulation (sabotage) [Hans 06a].

Classification

In their empirical work on stock price manipulation Allen and Gale classify the field of manipulations in stock markets and give examples referred to in the following and based on the classification made by the United States Securities Act of 1934 [Alle 92]. In this classification manipulations fall into one or several of the categories

1. Action based manipulation.
2. Information based manipulation.
3. Trade based manipulation.

By *action based manipulation* traders try to change the perceived or actual value of assets. This may be achieved by a company communicating strategic decisions which it is aware of not taking them in the end (cp. 1863 Harlem Railway case. The New York City Council passed an ordinance allowing the Harlem Railway to build a streetcar system along Broadway which made the stock price rise from \$50 to \$75. Council members conspired and bought stock shortly before repealing the ordinance and trying to cover the short positions to their favour.), or achieved by even changing the companies' active environment in strategic manners with the plan of restoring it to the original setting after the effects are visible in the market (cp. 1901 American Steel and Wire case. The company's managers shortened the company's stocks and closed the steel mills. The stock price fell after announcing the closures but rose again when the company announced the reopening of the mills after the managers covered their positions.).

In *information based manipulation* the manipulation is based on releasing false information or spreading false rumours as done for example by journalists writing in favour of a certain trading pool or certain shares (cp. Levinson Curtis case. The pool manager Levinson conspired with the journalist Curtis who wrote a financial column in the New York Daily News in favour of the pool managed by Levinson making over $1 million per year).

Trade based manipulation is conducted by simply buying and selling shares without taking any publicly observable actions to alter the true value of the company or releasing false information to change the price. Allen and Gale state that simply "buying high" and "selling low" fits into this type of manipulation, although on the face of it, this behaviour does not fit into the basic assumption of manipulations being profitable - in fact these actions are just the reverse of what is required to make profit. But they also show that this type of manipulation can theoretically be profitable in an efficient market with rational traders if the traders have incomplete information about other traders and their intentions of trading (whether these intentions are manipulative or not).

The Securities Act aimed at the elimination of these manipulations in stock markets by e.g. making it illegal for directors and officers to sell short the securities of their own firm (action based manipulation) or requiring companies to issue information to the public on a regular basis so that the spreading of rumours would be more difficult (information based manipulations).

Occurrences

Manipulations described by Allen and Gale are traditionally defined as an attempt to profit from artificially changing stock prices. These manipulations focus on the goal of investor profits. In their recapitulation of manipulations within prediction markets of a *Century of Observational Data* Rhode and Strumpf especially focus on market prices as a general indicator for manipulations which reflect both, traditional market manipulations and those in prediction markets [Rhod 07]. In contrast to the main objective for manipulations in traditional financial markets (seeking profit) prediction markets have a richer set of motives for being manipulated. They argue that manipulators in prediction markets may be willing to accept losses if this has large and lasting effects on prices, thus hoping for real world deciders to reckon this prices as true prediction (see super-game in Section 3.2).

In their survey manipulations like large price jumps with initial price changes, appeared in all analysed prediction markets (1880-1944 Presidential Election Markets at Wall Street, 1988-present Iowa Electronic Market, and 2001-present TradeSports market), whether in markets where traders revealed their offers to publicity (public markets) or in markets where traders acted anonymously (anonymous markets). Other authors reported also successful price manipulations in a real life political stock market [Hans 04a] as well as unsuccessful ones in real life racetrack betting markets [Came 98], in real life prediction markets [Rhod 04], and in experimental prediction markets [Hans 06b].

For trade based manipulations Allen and Gale bring in a theory of manipulation via signalling. Due to their model a manipulative trader acts successful in imitating a well-informed insider within a surrounding of traders with rational expectations if the manipulative intentions of the manipulator are not revealed [Alle 92]. Bohm and Sonnegaard describe the vulnerability of prediction markets by coalitions performing circular trading [Bohm 99]. They describe that a coalition member A could offer a large number of shares to the market at a very low price and a coalition member B immediately afterwards purchases all standing offers. At a later date with higher prices in the same share, coalition member B offers to sell all shares at very high prices and coalition member B immediately accepts buying all standing shares. This procedure is particularly attractive when the queue of bids is short in the times of both actions. Hansen et al. [Hans 04a] describe an incentive for manipulation in a prediction market (i) which is covered by the media and (ii) in which a decisive vote illusion can be created. They present the behavioural model in Figure 2.2 as a basis for this type of manipulation. The figure shows the rationale of a political stock market without (left) and with (right) coverage by mass media. They argue that even if the probability of a single vote having a decisive influence in the outcome of the real world event is infinitesimal small (see e.g. Owen and Grofman [Owen 84]), a surrounding in which a vote illusion is present - i.e. the real world deciders believe in the influence of their vote - can occur. Together with a extensive mass media coverage this motivates the above mentioned acceptance for manipulators to take even losses for changing a share's price which Hansen et al. call the 'circle of influence'.

Consequences of manipulations

In terms of the long term change of prices which is traditionally assumed as being intended by stock price manipulation, Rhode and Strumpf as well as Wolfers and Zitzewitz find out that the effects of the manipulative actions are rather small [Rhod 07, Wolf 04]. Furthermore, Hansen et al. state that as the endowments of traders are finite the influence of trade based manipulations on market prices i.e. by traders buying shares to hold up the price can only partly be neutralised by just one rational trader [Hans 04a]. So effects of price manipulations are rather short-term [Wolf 04]. In historical data Rhode and Strumpf did not even find any effect of manipulating prices with trades [Rhod 04]. On the long term even positive effects on the accuracy of prices due to prediction market manipulations are reported by Hanson et al. [Hans 06b, Hans 06a, Hans 04b]. Besides as Rhode and Strumpf state, it is difficult and expensive to manipulate prediction markets beyond short periods [Rhod 07].

As Kyle, Spiegel and Subrahmanyam, and Hansen argue the market will be more accurate in its prediction as price manipulations result in a larger trading volume and more noise trading which compensates for the manipulative actions [Kyle 89, Spie 92, Hans 06a]. More behavioural aspects are introduced by Barzel and Silberberg who claim that a manipulated surrounding, that proper traders are aware of, results in an increase of the idea that the next vote is crucial and therefore

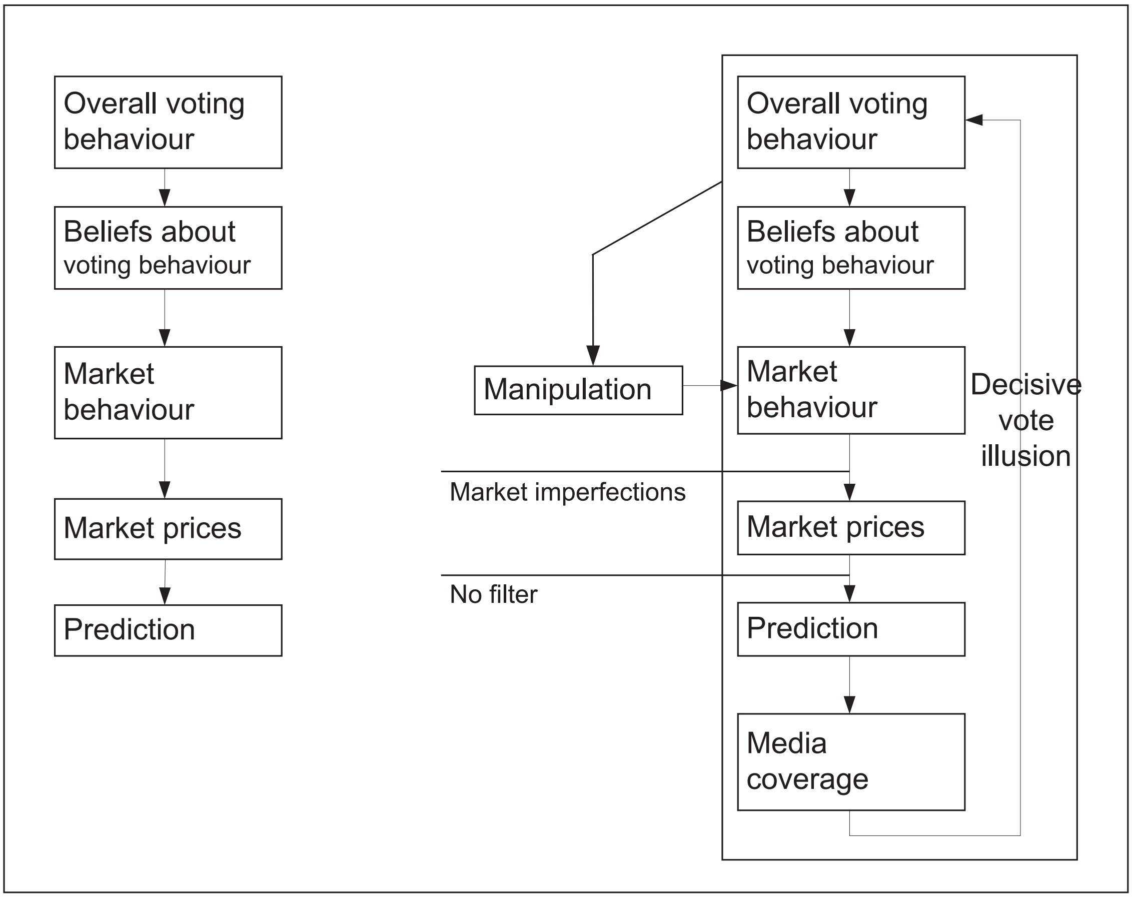

Figure 2.2: Rationale of a political stock market without (left) and with (right, 'circle of influence') coverage by mass media as depicted by Hansen et al. [Hans 04a]

the tendency to vote increases [Barz 73]. Following Hansen and Oprea also the opinions of traders seem more precise in an exceptional setting like a manipulated surrounding [Hans 04b]. In one of the first laboratory works on price manipulations in asset markets Hanson et al. report that manipulators submit higher bids than non-manipulators [Hans 06b]. In an experimental setting Hanson and Oprea find out that the mean of the manipulator's target price has no effect on prices whereas its variance increases the average price accuracy by raising the returns to informed trading and the incentives for traders to become informed [Hans 04b].

Due to Hanson et al. the overall information aggregation properties of prediction markets, the accuracy of the prices, and the correlation of the prices with the true, real world informational states are not harmed by the presence of manipulators [Hans 06b]. Manipulation seems indeed ineffective, if the presence of manipulation (price bias) is suspected and the direction is known. Non-manipulators in this case tend to accept contracts at lower prices as they seem not to do in a treatment where they don't suspect manipulative traders to exist. The bias from manipulation is therefore compensated by traders without manipulation incentives who have different thresholds at which they are willing to accept trades.

At least initial changes in prices due to manipulation are measurable in any manipulated setting [Rhod 07]. Manipulation tends to be a problem when all traders are very risk averse or when the harm from price errors correlates in unusual ways with manipulation errors [Hans 06a]. Also market characteristics like largeness and thickness, small number of possible outcomes or diversity of opinions seem to influence the ability of malicious traders to manipulate a market [Rhod 07]. As policy makers use information from prediction markets to motivate decisions, also Hanson et al. conclude that some individuals (manipulators) may be willing to take capital losses in these markets in order to distort the informativeness of prices and thereby indirectly control and influence policy [Hans 06b].

Summary

If the initially presented categories referred to manipulations in traditional stock markets, then they are nevertheless applicable to prediction markets. The manipulation types specific to prediction market are mainly manipulations that deal with illegal money transfer and price manipulation in order to change prices in the long term. These manipulations can be classified as an enhancement to the traditional types of manipulation and are subsumed into trade based manipulations. Almost all authors report their markets to be affected by manipulations which they assume to originate from profit-oriented motivations. Solely Hansen et al. present a behavioural model for the occurrence of one manipulation type typical for prediction markets. They describe the premise for a surrounding in which price manipulations are likely to appear by introducing two static requirements. Although some articles argue that the theoretical effects of manipulations in prediction markets are small or even have a positive effect on efficiency, they generally find these more or less positive effects of manipulation to be outperformed by the negative aspects. A manipulation detection and suppression is thus desirable, whereby so far no detailed literature exists for manipulation recognition in the very field of prediction markets. More information on the efforts in the field of manipulation detection by software systems is given in Section 3.5. However, in this work we focus on the manipulation types specific to prediction markets, namely illegal money transfer and price manipulation. We identify incentive compatibility as the essential characteristic of an electronic market which in the case of its absence accounts for the respective manipulations. In addition, a novel manipulation detection technique is introduced, that allows for an effective detection of those manipulations defined.

3. Fraud and the Breakdown of Incentive Compatibility

This chapter gives a short introduction to incentive compatibility and its breakdown in Section 3.1. Section 3.2 then discusses the incentive systems important to this work that participants of prediction markets are confronted with. Based on these incentives different motives for manipulation are described in Section 3.3. In Section 3.4 the character of anonymity within a transaction network evolved from market transactions is described. Concluding this chapter Section 3.5 introduces current ideas and frameworks that are dedicated to detect explicitly fraudulent actions within such markets.

3.1 Incentive Compatibility and its Breakdown

Basis to the idea of a prediction market and its information aggregation aspect is certainly the market mechanism that matches the expressed opinions encoded in buy and sell orders for shares tied to certain events and which results in a certain price taken as prediction. Underlying that functionality is the assumption of a market that not only provides the technical ability to match orders as described, but also incentivises its traders to systematically reveal their true opinion about the shares. If the market provides this characteristic the prices will instantly reflect all new available information in the price to all traders at any point in time and such a market is thus called an *efficient market* [Milg 92]. The condition for this to happen is called *incentive compatibility* which is reached if all participants reach their maximum outcome in the system by truthfully revealing any private information asked for by the market mechanism. This occurs, for example, if the market system reaches the most efficient system status if all participants' follow their plans of actions (which depict all affordable actions of the participant) and this results in a maximum personal profit (maximum profit plan). Formally, the individual trader's profit maximisation plans are separable and do not have external effects on other traders' plans. In a situation where a participant's maximum profit plan of action results in a system status that is less efficient than it would be without the participant following this plan the incentive compatibility of the market system is not given. In a market, incentive compatibility is not available, if traders can profit from fraud or cease to participate in the market because of fear from fraud. This is for example in a prediction market the case if the a trader gains more profit by cheating than by acting according to the rules. In a prediction market, failures of the incentive system and thus incentive incompatibility can appear by several means which are introduced in Section 3.2. As participants in a market are assumed to be profit maximising, these changes in the incentive system are usually exploited by the participants expecting a higher profit by doing so. These actions are usually forbidden by market rules. They are thus irregular and called fraudulent or are referred to as manipulations. Usually incentive compatibility is maintained by a system design that makes it more lucrative to play by the rules. This can be achieved by the traders having systematic higher rewards when playing conforming to the rules then when playing against them. This can be supported by guaranteed monetary incentives or systematic penalties (i.e. fines or system exclusion).

3.2 Incentive Systems of Prediction Markets

To induce participants within a system act conforming to the system rules, incentives are given. The ensemble of the arrangements in that direction is called the incentive system. In the following we introduce the incentive systems in the domain of prediction markets by describing a prediction market configuration in three different scenarios and the problems resulting from the different requirements for reaching incentive compatibility. The terms 'closed world' and 'open world' relate to the traded events of the prediction market. Where in a closed world prediction market neither the traders nor any other people have a personal tie to the traded events (i.e. in a laboratory experiment) the open world prediction market explicitly assumes this connections. Together with the personal ties, a market in an open world scenario describes a system visible to the public and especially to the deciders of the real world event. As introduced in Section 2.1 basic to any prediction market scenario is the payoff that takes place after the market is closed and the real world event's outcome is known. For the prediction market and its outcome in precise predictions the best traders are the ones that truthfully revealed their true opinion at any point in time during the market. So after market close the system should honour this truthful information revealing strategy by getting best rewards to players that did so.

1. Real money market in a closed world scenario

This scenario describes a prediction market where trader buy and sell shares within a market by using real world money. However, the prediction market is either isolated from the outside world or does not influence the events in the outside world (closed world assumption). So gains and losses are directly tied to the traders' wealth and if the closed world assumption holds, the traders' wealth is only a result of their trading behaviour. The gain G_w of traders realised from any interaction with the outside world w, therefore, is given by $G_w = 0$.

Consider a trader m who has created multiple accounts A and B and who uses transfers in order to improve the rank of account B. Does this behaviour increase his gain? For simplicity assume, he invests 1000 Euro per account (a total of 2000 Euro), he transfers 90% of his funds from account A to account B (fraudulent trade) and since he is clever, he succeeds in gaining 6 cent per Euro invested by regular trades. His gain without transfer is

$$G_R^m = G_A + G_B + G_w = 60 + 60 + 0 = 120 \text{ Euro},$$

where G_R^m denotes his total gain for the trader with a fraudulent behaviour which in this scenario is equal to the behaviour of a regular trader r. G_A denotes his gain from account A, and G_B his gain from account B. Next, for simplicity, we assume he first transfers the money from A to B and then executes his regular trades with the same cleverness as above. We clearly see, that the transfer from A to B only affects the money distribution between the accounts but not the gain: Account A has a balance of 100 Euro before he starts regular trading, account B has a balance of 1900 Euro which results in the total gain G_F as

$$G_R^m = 6 + 114 + 0 = 120 \text{ Euro}.$$

So, in this setting the money transfer from A to B does not lead to an extra gain from this type of fraudulent behaviour. However, since in a double auction market a fraudulent trader runs a certain risk, that some of his transfers are intercepted by other traders, there is a certain probability (5% in our example) that money will be lost in the transfer trades (5% of 900 Euro = 45 Euro) on the balance of the receivinig account. So in the first scenario the balance of his accounts after the transfer will be 100 Euro on account A, 1855 Euro on account B and his gains are

$$G_R^m = 6 + (-45 + 111,30) + 0 = 72,3 \text{ Euro}.$$

One immediately sees that the trader is better off without this type of fraudulent behaviour in this setting. Even, if he succeeds in executing fraud without losses, this does not earn him extra gains. A prediction market under the closed world assumption is incentive compatible. A rational trader will participate voluntarily because he expects to gain from his cleverness and he will not commit this type of fraud. In total this results in Inequation 3.1 with the rewards of regular trading return $Return(\cdot)_{\mathrm{RT}}$ being higher than the rewards of trading irregularly $Return(\cdot)_{\mathrm{IT}}$.

$$Return(\text{Money}_{\text{real}}, \text{World}_{\text{closed}})_{\mathrm{RT}} >> Return(\text{Money}_{\text{real}}, \text{World}_{\text{closed}})_{\mathrm{IT}} \approx 0 \quad (3.1)$$

2. Play money market in a closed world scenario

In this scenario the real money of the preceding scenario is replaced by play money without direct ties to the wealth of the traders. Each trader gets the play money by the system and has no losses of real wealth even in the case of going bankrupt in the market. In fact the only consequence of this is being not able to participate anymore in the case of a total loss.

As the monetary incentive of enhancing the own wealth is not given in this scenario, instead prizes are given to the best traders in the market. A trader is therefore better than another trader, if he traded better with respect to the trading of the other traders. In a market system where all traders get the same initial play money this is measured by the final depot values of the traders in the depot value ranking with the best players on top and the worst at bottom. So the main ambition of the traders can be described by "winning the game" by trading in a clever way to their own profit that conforms to the operator's goals.

But as loss and even bankruptcy does not harm the real world wealth of the participants, any means for reaching a high depot value in the end are available. So fraudulent actions like coalitions may appear where rings of traders agree on one trader earning the maximum profit thus transferring their initial play money to the agreed trader to let him have a better position in the depot value ranking. The won price is in the end divided by the ring. Usually such behaviour is regularised or forbidden by market rules.

If we again take G_R^m as the total return of the actions of trader m, his return of the two accounts he is in control of (A and B) is not any longer dependent on his own real wealth, thus not perceived as the sum of $G_A + G_B$ as in the previous scenario, but separately as $\max(G_A, G_B)$. In a play money setting this value is again connected with a certain prize whose value stands as the real world return of the market. The prize's value is expressed by the prize function $prize\,(r)$ for the play money result r with respect to the depot values of the other traders and the set of prize in connection to the depot value ranking. Taken the same numbers of initial investments for A and B of 1000 Euro each, and a gaining of 6 cent per Euro invested by regular trades, the gains of trader f would be

$$G_R^m = prize\,(\max(G_A, G_B)) + G_w = prize\,(\max(60, 60)) + 0 = prize\,(60)\ \text{Euro.}$$

Furthermore, as gains in each account can be seen separately and are not connected to each other by the wealth of the trader controlling them, also transfers from other sources than regular trading in the market (i.e. other accounts one is in control of) can be seen as gains to the benefited account. Adding the money transfer then extends the gains G_R^m to

$$G_R^m = prize\,(\max((-900 + 6), (900 + 114))) + 0 = prize\,(1014)\ \text{Euro.}$$

Now adding a risk of 5% for loosing money in a transaction results in

$$G_R^m = prize\,(\max((-900 + 6), (855 + 111, 30))) + 0 = prize\,(966, 30)\ \text{Euro.}$$

As can be seen, the risk of interception is lowering the gains, but, as also can be seen, this behaviour is still an advantage up to the risk of a total loss in the transaction due to an interception by other traders. The only limiting aspect is the amount of money available to transfer from an account in control of, as such a system usually does not allow for depot values below zero. The logical way to increase the gains of the account in favour of the transactions is to get in control of more than one account and arrange transfers from there.

So, in a closed world market setting with play money the system pays dividends for irregular behaviour which makes it more profitable for the trader to act irregular and receive the return $Return(\text{Money}_{\text{play}}, \text{World}_{\text{closed}})_{\text{IT}}$ than to act by the rules with the return $Return(\text{Money}_{\text{play}}, \text{World}_{\text{closed}})_{\text{RT}}$ which results in the incentive compatibility inequation in Inequation 3.2.

$$Return(\text{Money}_{\text{play}}, \text{World}_{\text{closed}})_{\text{RT}} << Return(\text{Money}_{\text{play}}, \text{World}_{\text{closed}})_{\text{IT}} \tag{3.2}$$

3. Real money market in an open world scenario

Transferring the closed world scenario with play money from above to an open world domain maintains the ties to the real loss of the trader in case that he does not trade compliant with the rules of the market which call for truthfully revealing the true opinion, but transfers the whole market system into a different surrounding. Now the traded events are tied to the real world with the traders or other people in the real world being interested in the outcome of the event. This brings up new incentives playing a decisive role by overlaying the market's inherent incentive system. If for example a trader is, by whatever means, personally tied to the outcome of the share's real world counterpart event i.e. he is a member of a certain party and interested in high market prices of this party (cp. signalling in Chapter 1, he could valuate the event coming true (victory of a certain party) higher than getting the gains or losses in the market (monetary losses below a certain amount). We refer to the traders' choice between the real world incentive system and the market incentive system as the *super game* e.g. a trader valuates gains for the party he is member of higher than his own profits reachable in the market. The term super game therefore addresses the choice of the trader to 'play' within the market's incentive system or within the real world's incentive system.

In this scenario a trader p starts operating with a single account G_C. Assume that the trader has again 1000 Euro invested and the gains by accomplishing clever regular trades on the market are 6%. As the returns of the real world G_w are not measurable in strict numbers we set these to x Euro. The total gains G_R^p of trader p are then described by

$G_R^p = G_C + G_w = 60 + x$ Euro.

Taken the fact that the trader's gains from the real world are ≥ 0 only if he influences the price, x will be zero in this case. Now as the trader tries to influence the market price (increase the price) he buys large amounts of stock without caring for clever trading. In fact we assume the trader to make losses of 6% which gives us the total return as

$G_R^p = -60 + x$ Euro

which is attractive to the trader if to him the price shift is worth 120 Euro (in real world money), so $x > 120$, as then the total gains exceed the previous situation where he does not influence the price.

So, the return in this scenario is not only a direct monetary one but also the one of being able to influence the real world event and the real world deciders, respectively, by signalling via the prices. The problem for the market operator in this scenario is that the external gains of the players are unknown to him. Therefore, he cannot regulate the market in such a way that incentive compatibility holds.

4. Play money market in an open world scenario

In this scenario the real money from the previous scenario is changed to play money. As in the second scenario, the effects to the traders are that the playing money is not directly tied to the wealth of the trader anymore. More precisely the total result G_R^p for trader p with account C trying to shift the prices of a share is given as

$G_R^p = prize\,(\max\,(G_C, 0)) + G_w$

which completely dissolves any boundaries by transaction costs or losses due to risk in the transaction as G_R^p will always be greater-to zero if any intention for signalling by prices is present ($G_R^p > 0$). The only limiting factors are the bankruptcy of the account due to the fact that a play money market usually does not allow for depot values below zero and the number of shares of the affected shares available.

To trader m trying to transfer money from one account A to another self controlled account B the situation in this scenario is the same as in the second scenario as the gains from the outside world G_w did not affect his strategy. Also in this scenario this type of money transfer among a self controlled coalition is more lucrative than trading regular with one trader.

Now the influence of the prediction market operator on both incentive systems depicted in Figure 3.1 is quite different. While the market incentive system in a play money market is fully controllable by the value and amount of the prizes, the real world incentive system can be out of reach as the possible gains of a trader outside the prediction market system are not observable neither controllable by the market operator. Furthermore the real world incentive system often deals with emotional bindings, i.e. in political stock markets the party *of* the trader, which may address psychological “reflexes” rather than rational behaviour.

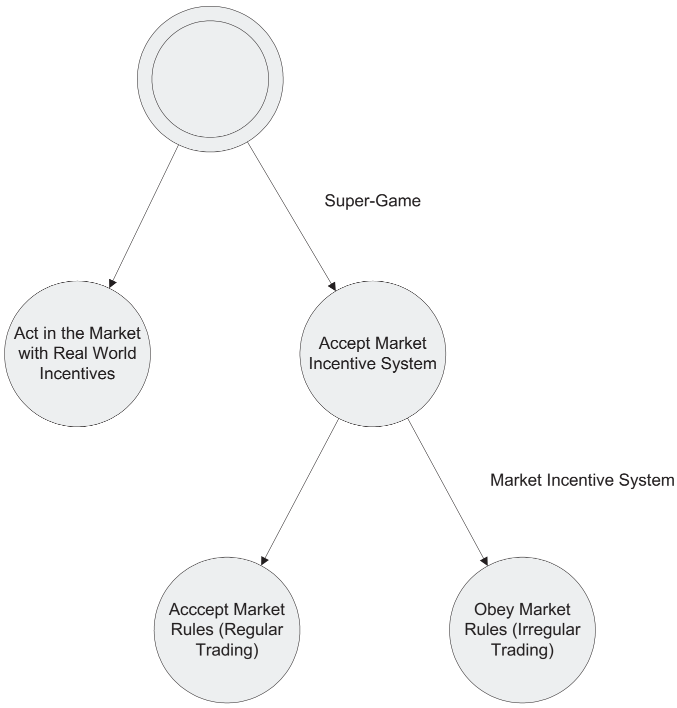

Figure 3.1: Incentive Systems of prediction markets

Both areas of conflict (incentive incompatible situations) are depicted next to each other in Figure 3.2. The boxes denote the incentive system where the depicted action plans lead to different gains with each denoting incentive incompatibility.

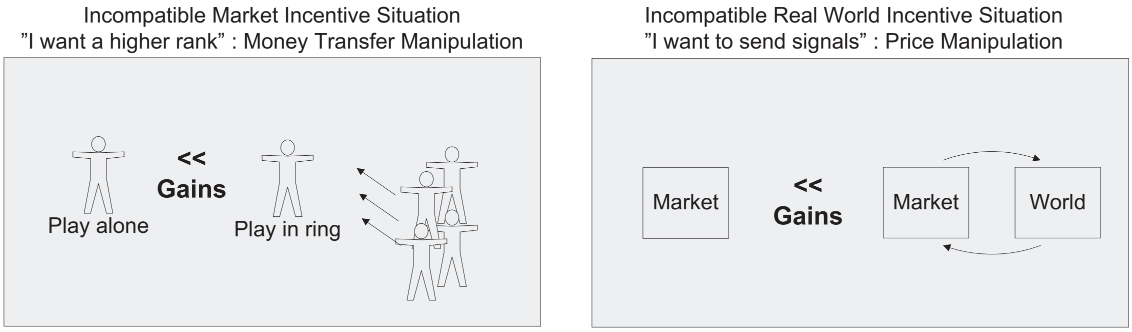

Figure 3.2: Incentive incompatible situations of prediction markets and action plans within

As mentioned in the beginning, the incentive systems have to be designed to provide incentive compatibility. As seen above this is by system design only given in the first scenario of real money used in a closed world scenario. Where the open world scenarios both offer incentive incompatibility only at the super-game level, the application of play money scenarios result in incentive incompatibilities on both the super-game and market incentive system level compared to Figure 3.1. In all cases the effects of a systematic incentive incompatibility are manipulation and fraudulent actions, as the return for these action plans are more profitable than for regular trading ones. To control the effects of this incompatibilities, usually rules that decrease the dividends for cheating thus increasing the costs for irregular behaviour, are introduced. This introduction of rules cause control costs that have to be paid by the system administrator but cause a reduction of attempts to commit fraud in the best case. The effect to control especially the incentive incompatibility on the level of the super-game, enhancing the incentive system itself may help. For example, to ensure a good quality of a play money market's incentives the prizes to be won could be enhanced so that the personal dividend in the market (i.e winning a prize) is higher than the personal dividend of the real world (i.e. my party wins by more people voting for it as the price signal of the prediction market made them to do so). But also this can end up in manipulation as with exclusiveness or too attractive prizes the incentive of "doing good" is enlarged to the negative "doing good by any cost or means" to win the price which also results in traders cheating. Finally some traders manipulate just for the incentive of having fun or as the end in itself by showing how clever they are.

3.3 Fraudulent Actions within Prediction Markets

To ensure a valid incentive system to all participants at any time a prediction market system needs the abidance of certain rules by the traders. Intentional renunciation of these rules is carried out for different motives which usually base on an incompatibility of incentives which makes it possible and even attractive to act fraudulent with rational thoughts as described in Section 3.1. The consequence of this environment is usually paired with the basic motivations that traders have. Whereas a "good minded" trader would not even act fraudulent if he is offered explicitly to do so, other traders may have a basic affinity to certain kinds of fraud and try to exploit any given possibility. The main motivations in a prediction market for doing so are described in the following sections.

3.3.1 Getting a Higher Rank

Within prediction markets egoism is the main incitement for the system to work in a proper manner and the predictions gain accuracy with the level of acting egoistically if incentive compatibility is given. But it is the egoism combined with playing by the rules that supports the system. As mentioned in Sections 2.1 and 3.2 prediction markets may be carried out in a play money setting with no direct connection between the money lost in the game and real assets of the participant. In such a system each participant gets a certain amount of initial play money to trade with. As the nature of electronic markets is virtual, traders especially in play money markets are usually able to apply for multiple accounts or join single (rule conforming) identities they are in reach of i.e. accounts from family members in terms of actions and effects. In both cases the traders try to augment the depot value of one selected account with the initial play money from the others. By these actions the central account the money is transferred to increases its depot value whereas the others decrease by the money transferred.

3.3.2 Signalling to the Outside World

As described in the motivation in Chapter 1 the second incitement for fraudulent actions is based on the shares' prices being regarded as the true prediction for the outcome of the event by the deciders of the real event. Thus the manipulator attempts to influence the shares' price by systematic buys or sells of large quantities of shares and consequently influence the decisions of traders in the real world. As with respect to the (manipulated and overvalued) market price the real world traders assume their private expectation to be undervalued, the manipulator hopes for the real world decider to decide in favour of this shares' outcome and thus a feedback effect of the affected market price. In contrast to a trader abiding to the market incentive system, for the trader trying to send signals in the super-game the incentives of the prediction market system are not relevant anymore. Even more they are completely replaced by the incentive of the real world i.e. the trader values the revenue of his contribution to the signal of the (over- or undervalued) prices onto the real world deciders (assumed higher votes in the real world) higher than the personal revenue to be reached within the market (winning a price in a play money setting or the money gain in a real money setting).

3.3.3 Consequences

The consequences of fraud are on the one side irregular price fluctuations which affect the prediction of the market. On the other side manipulations especially of the motivation of getting a better rank lead to increases of the depot value of the manipulator because of the manipulations. This usually leads to an increase of his ranking within the depot value ranking which could strike the incentive system of the market by non-manipulating traders supposing to be unsuccessful as their place in the ranking decreases. The options for dealing with detected fraud which are usually undertaken by the market administration are spelled out in the market regulations. Manipulating traders can be expelled from the market directly or disallowed to win a price depending on the market system. Despite the deactivation of manipulative traders the effects of the manipulation are still present in the market. As discussed in Section 2.3 prices will recover to the true rate but letting the manipulating traders stay in the market will surely end up in more dissatisfaction of other traders or market operators as the manipulating traders still show up in the top places of the depot value ranking. Empirical results in markets conducted at the Institute for Information Systems and Management in Karlsruhe show that the publication of traders supposed to be manipulating can normalise the negative effects of non-manipulating traders: In several markets the depot values of traders supposed to be manipulating were marked as being manipulated. In the effect the non-manipulating traders perceived the market as being fair again. However Section 2.3 also claims more precision for the market in the latter case by stronger arbitrage effects and higher activity of regular traders.

3.4 Anonymity in Trading Networks

Within an electronic market the traders interact with the *market system*. The market system manages the buy and sell orders and presents these to the traders usually as order books. In Figure 3.3 this market activity is depicted by traders who give their orders to the market system which matches the orders by a defined algorithm (auction mechanism). The orders are given by the traders in the following chronological sequence:

- Trader A: Buy 2 shares of *Share1* for a maximum of 6 monetary units
- Trader B: Sell 4 shares of *Share3* for a minimum of 5 monetary units
- Trader C: Sell 10 shares of*Share1* for a minimum of 5 monetary units

- Trader C: Buy 2 shares of *Share3* for a maximum of 6 monetary units
- Trader D: Buy 10 shares of *Share1* for a maximum of 5 monetary units.

In the order book only a certain number of sell and buy orders are displayed as can be seen in Figure 3.4. The number of orders shown is called the *depth of the order book*. With the displayed orders there is no information given about the identity of the selling or buying trader connected with the order. So the interactions of the traders are towards the market system rather than towards any other traders. With regard to the objective of a research based on the analysis of such a network this argument is essential especially where behavioural aspects like intended or unintended acts of traders towards other traders are relevant. In the case of certain manipulation patterns this is the case as for example traders trying to bypass this anonymity in the hope of transferring money from one self controlled account to the other.

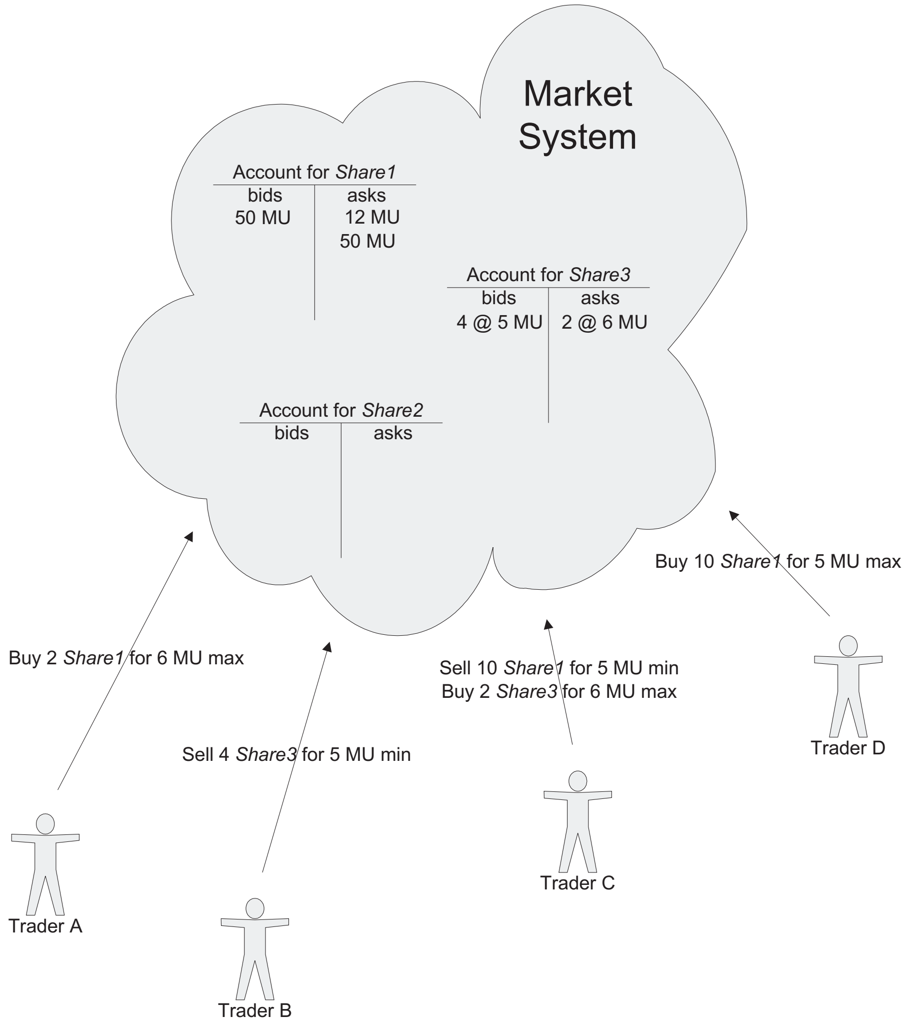

Figure 3.3: Trading towards the market system with MU as monetary units

So compared to a traditional market, where the traders deal with each other, a stock market and in particular an electronic market is based on the assumption that, depending on the market mechanism, the traders act anonymously. As the traders interact with the market system rather than with each other, the matching process depends only on the order parameters share, entry time of order, amount and due-date and not on the personal likes or dislikes of the traders regarding their counterpart in a transaction. So for building up transaction networks this context has to be kept in mind, especially when modelling networks that are going to be analysed in a social network context (i.e. seen as social networks [Fran 07]). This fact is utilised in analysing the traders' actions from a social network analysis point of view along with a method borrowed from this research area.

3.5 Fraud Detection Systems

To generate a smooth operation of the market, market operators try to enforce the market rules by using mechanisms to detect and subsequently prosecute irregularities. Given the sizes and propagation of electronic systems nowadays, building up efficient technical systems to detect and avoid irregularities becomes more important. The main area where fraud detecting systems are operated is in economic systems where fraud may result in severe consequences like monetary losses. Thus for economic systems within the domain of the internet the United States Department of Justice [Just 08] differenciate between the following fraud schemes:

- *Auction and Retail Schemes Online* with the most frequently reported form of internet fraud. Typically sellers pretend a good to be of a high value. After the payment is carried out, the delivered item is of a lower value than advertised (if sent anyway).
- *Business Opportunity/"Work-at-Home" Schemes Online* where customers are offered well paid job opportunities. For a certain fee the material is supposed to be delivered which after payment usually does not happen.
- *Identity Theft and Fraud* as the wrongful obtaining and using of someone else's personal data in some way that involves fraud or deception, typically for economic gain.
- *Investment Schemes Online* which consist of *Market Manipulation Schemes* and *Other Investment Schemes* with the first scheme describing the realisation of substantial profits by e.g. disseminating false and fraudulent information in an effort to cause illegal price shifts and buying and selling stocks before and after the action. The second scheme describes the combination of uses of the internet with traditional mass-marketing technology such as telemarketing to reach large numbers of potential victims.
- *Credit-Card Schemes* that describe credit card numbers theft and online usage.
- *Other Schemes* pool mainly false, misleading or legally inaccurate information propagation with the aim of people sending money to receive for certain service offers like cheap divorces which are not fulfilled in the end.

The fraudulent actions within this work emanate from prediction markets which are part of the virtual electronic markets domain. In the previous listing this is characterised by the mentioned *Investment Schemes Online* and especially the *Market Manipulation Schemes* in which several methods and techniques for fraud detection are used. But the use of enterprise internal systems is difficult to support by literature. Due to the protection of companies' interests and, therefore, missing data the existence of so called Anti Fraud Systems (AFS) or Fraud Detection System (FDS) within companies can only be guessed. However, references concerning this issue can be extracted from secondary reports as [Wire 00] where the results of effective FDS are reported, the information on promoted projects within research faculties or published number of companies, which work within this range. Approaches for the detection of fraud within the discussed domain emanate from different scientific areas. Basically all approaches try to determine deviations from regular behaviour with different solutions evaluating system data transformed to networks.

In a network analysis approach Bezerra and Wainer [Beze 08] examine the anomalies in transaction networks algorithmically in a graph built from event and task log file data automatically generated by a system. In a first step they derive the regular processing concept for the given data by mining regular patterns and interpret it as background emission. This is done by including every new input data into a graph and calculating an 'inclusion trace' by a certain proprietary metric based on a 'process mining' algorithm. The difference of a new trace to the existing ones is measured as 'inclusion costs' for the vertex. Comparing these costs, deviations and irregularities are detected by means of algorithmic comparison.

By means of visual network analysis Kobayashi and Ito [Koba 07] try to determine manipulations and to predict the future market process. They represent the actions of a participant in a market system graphically and guess the future actions based on so called explanation functions which provide some insight into the network and its configuration.

On the basis of Markov random fields Pandit et al. [Pand 07] present the fraud detection system NetProbe, which assists in determining abuse in an auction network by analysing the reputation system of the network. The reputation system queries the opinions of the counterpart of each transaction about each other which NetProbe utilises to build a network of the participants with honest and fraudulent links. By the use of a belief propagation method they model probabilities for a participant to be fraudulent and assign a status which is either fraud, accomplice or honest. The technology of NetProbe can be used with large networks and is exemplarily applied to a network with over 66,000 users and about 800,000 transactions.

With the purely graph based software package ADAGE McGlohon and Faloutsos [McGl 07] follow a direction similar to Kobayashi and Ito [Koba 07] where the network is represented in a visual context, whereby the focus here strongly lies on the temporal development of the network.

For the detection of manipulative actions within prediction markets the software "PSM" which in Section 7.2 of Chapter 7 is used to derive the real world market data offers a mechanism of cascaded structured queries to the transaction database which accomplishes a structured comparison of the extracted counterparts of each transaction finding overlaying transaction partners which correspond to certain manipulation behaviours.

Finally, all presented approaches examine the networks with the idea of finding out about certain developments which cannot be seen as rule conforming, thus as abnormal. Once determined, developments exceed a certain threshold value and are thereby classified. More general approaches like [Blum 06] consider rather the meta aspect, correlating several parameters of the system and thus try to take the context of the underlying system more into account. Here the results of different approaches (e.g. the ones specified above) are computed for the system separately. With the idea that certain combinations of results indicate a participant to be fraudulent to a certain probability the results are aggregated and a status of being fraudulent or not based on this probability is assigned to each participant.

The fraud detection approach presented in this thesis is applied to transaction data of prediction markets. As introduced in the objectives in Section 1.1 the presented eigensystem approach is adapted from the field of social network analysis which in the eigensystem methods focuses on the analysis of recursive relations among participants within a network. In the area of market manipulation detection approaches this enhances the analyses means with a new approach.

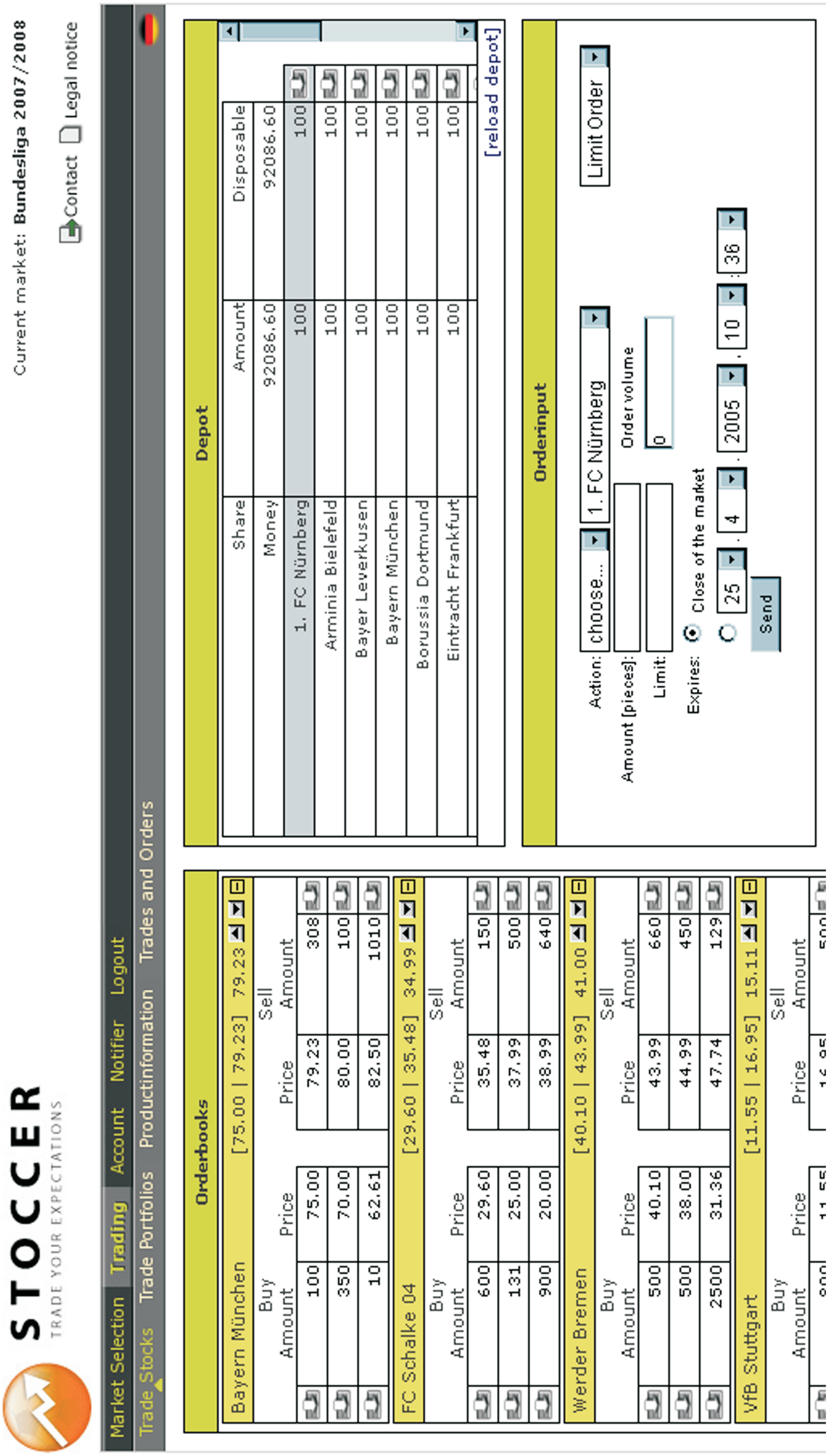

Figure 3.4: Trading screen of the prediction market system STOCCER with order books

4. Social Network Analysis

In principle social network analysis regards actions among participants in a network as dependent from each other. As this paradigm is not applicable for a basic rule-conforming prediction market it gains significance in the analysis of networks in which participants reckon the network as a social network and express that by their actions (trades). For electronic prediction markets this may be assumed for the case of certain manipulations detectable within the transaction records of the market, for instance in traders breaking the inherent market anonymity that will be described in Section 3.4 by trying to transfer money among two trading accounts they own themselves. Thus we focus on approaching the issue of detecting irregularities by means of social network analysis where these market irregularities are recognised as structure within a social network.

Social networks are usually formulated as participants in a network with possible connections among each other which depict the characteristic of the network. In the beginning of analyses on networks in the middle of the 20$^{\text{th}}$ century graphs and matrices were developed as formal ways to illustrate the networks. These still form the basis for the current techniques of visualisation and analysis within network analysis

4.1 Social Networks as Matrices

After Moreno startet to formalise relations between participants in a network [More 36], Forsythe and Katz [Fors 46] had the idea to illustrate social networks by means of a matrix A. A network of n participants is thereby represented by an $n \times n$ matrix. One row and one column is assigned to each participant with the matrix element a_{ij} depicting the edge weight of participant i to any other participant j in the network and a_{ji} depicting the edge weights of any other participants j in the network to i (see Table 4.1).

If the entries of the adjacency matrix are not binary, the underlying graph is called a *weighted graph*. In this case also the adjacency matrix may be called a *weighted adjacency matrix* [Dies 00]. If for any two participants within a network the edges to each other are of the same weight ($a_{ij} = a_{ji} \; \forall \; i, j$), the matrix can be called a *symmetric adjacency matrix* [Meye 00, P. 85]. As an example the matrix in Table 4.1 depicts a non-symmetric weighted adjacency matrix.

Participant	1	2	3	4	5
1	0	1	2	2	1
2	2	0	2	0	0
3	3	2	0	0	0
4	1	0	0	0	0
5	1	0	0	0	0

Table 4.1: Representation of a network as weighted non-symmetric adjacency matrix

4.2 Social Networks as Graphs

In his publications Lewin [Lewi 35, Lewi 36, Lewi 38] presented geometric interpretations and metrics for social networks by painting the graph structure with nodes and vertices. Bavelas [Bave 48, Bave 50] enhanced these ideas and published concrete ideas on formulating a network as a graph. The i^{th} and j^{th} participants are denoted as vertices v_i and v_j and the communication among participants v_i and v_j as edges e_{ij} of the graph. If the edges are weighted (e.g. with strength w_{ij} denoting the edge weight between vertices v_i and v_j) the graph is called a *weighted* graph, else an *unweighted*. In case of pure binary bindings (i.e. a connection between v_i and v_j exists or not) the edge weights w_{ij} are binary. At the same time the direction of an edge between two vertices can be expressed by an arrow. Such a graph is called *directed*, graphs without directions in edges as *undirected*. If in a graph vertices have no connection to any other vertices within the network they are called *isolates*, the graph is not connected.

So following [Jung 94] a weighted directed graph is denoted by

$$\mathcal{G} = \{V, E, \omega\} \tag{4.1}$$

with V as the set of vertices, E with $e \subseteq E^2$ as the set of edges and ω as a weighting function $E \mapsto \mathbb{N}$. A tuple $(v_i, v_j) \in V$ denotes an edge with $\omega(v_i, v_j) = w_{ij}$ describing the weight of the edge between vertices v_i and v_j.

A graph where the vertices can be split up into two disjunct subsets without any edges among the vertices of the same subset is called *bipartite*.

In Figure 4.1 a simple directed and weighted graph is given representing the weighted adjacency matrix A from Table 4.1 which can be calculated by

$$A = \begin{cases} w_{ij} \text{ if } v_{ij} \in V; i, j \in \{1, \ldots, n\} \\ 0 \text{ else.} \end{cases}$$

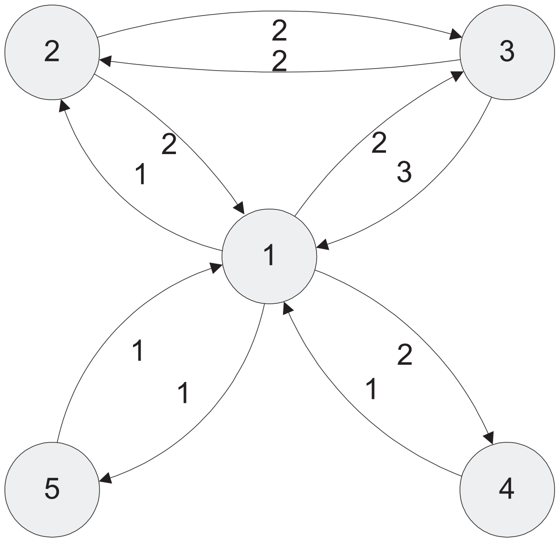

Figure 4.1: Directed weighted graph illustrating the weighted adjacency matrix from Table 4.1

4.3 Network Flows

Following Jungnickel in [Jung 94] within the graph $\mathcal{G}$

$$\delta_{\text{deg}}(v) \tag{4.2}$$

with $v \in V$ denotes the degree of a vertex (number of edges incident to the vertex) and can be split up to the indegree

$$\delta_{\text{deg}}^{in}(v) \tag{4.3}$$

and the outdegree

$$\delta_{\text{deg}}^{out}(v). \tag{4.4}$$

The weight of an edge from vertex v_i to vertex v_j is expressed by

$$\omega(v_i, v_j) = w_{ij} \tag{4.5}$$

with the outbound vertex flow of vertex v_i

$$\delta_{\text{flow}}^{out}(v_i) = \sum_{v_j \in V \setminus v_i} \omega(v_i, v_j) \tag{4.6}$$

and

$$\delta_{\text{flow}}^{in}(v_j) = \sum_{v_i \in V \setminus v_j} \omega(v_i, v_j) \tag{4.7}$$

the inbound vertex flow of vertex v_j.

From social network analysis we get the term *ego network* that describes the view on a focal vertex ("ego") and the vertices to whom ego is directly connected ("alters") plus the ties, if any, among the alters [Hann 05].

4.4 Social Network Analysis Methods

Within the research area of social network analysis vertices and edges between the vertices are the elemenary properties which are described in network structures. To analyse these as social networks, several methods were presented with different focuses in the analyses' results [Wass 99]. Some approaches try to detect cliques by clusters of individuals who are tightly connected to one another while others look for structural equivalence in sets of individuals by detecting similar relation patterns of individuals with respect to the rest of the network. For our purpose of analysing network structure evolved from stock markets we use a method which emphasises the character of detecting subgroup formation with regard to the manipulation patterns and the formation of traders involved in manipulation. In network analysis this is the main focus of a third group of methods that "don't 'look for' anything in particular - instead, they construct a continuous multidimensional representation of the network in which the coordinates of the individuals can be further analysed to obtain a variety of kinds of information about them and their relation to the rest of the network" [Rich 00], namely methods based on spectral decomposition.

The idea to analyse a graph's structure by means of spectral decomposition and the analysis of the graph's eigensystem reaches back to the 1950s. In 1953 Katz introduced a status index for a network participant which expressed its popularity within the network based on the status of all other participants of the network by using a recursive algorithm similar to the calculation of an eigensystem [Katz 53]. In the research area of network analysis measures this led to a set of centrality measures which understand the state of a network vertex as depending on other vertices [Wass 99].

Basically within the eigensystem based status measures the status index s_i of a vertex i is assumed to be dependent on the status indices of all other vertices within the network (popularity) [Bona 72, Cvet 97]. Assuming W as a symmetric adjacency matrix (with $w_{ij} \in \{0, \ldots, 1\}$), the recursive relation is depicted by

$$s_i = \sum_{j=1}^{n} w_{ij} s_j \text{ mit } j \in \{1, \ldots, n\} \tag{4.8}$$

which in matrix notation is

$$s = Ws \tag{4.9}$$

or

$$(W - I)s = 0, \tag{4.10}$$

respectively. Applying a multiplier λ to the left hand side which does "not violate the spirit of the model" [Bona 72] we get

$$\lambda s_i = (w_{i1}s_1 + w_{i2}s_2 + \cdots + w_{in}s_n) \tag{4.11}$$

which again in matrix notation is

$$\lambda \mathbf{s} = W\mathbf{s} \tag{4.12}$$

or

$$(W - \lambda I)\mathbf{s} = 0, \tag{4.13}$$

respectively. At the same time equation 4.13 denotes the eigenvalue equation with λ as eigenvalue and $\mathbf{s}$ the corresponding eigenvector [Bona 72].

In terms of traditional eigensystem analysis the original network and hence the connections between the vertices are seen as coordinates within a continuous multivariate space which a eigensystem analysis is conducted on. Thereby the variable space is rotated in order to maximise the variability of the first dimension while minimising the variance around the current variable by letting the variable axis "perpendicular to the first, point in the direction of the greatest remaining variability, and so on. This set of axes is a coordinate system that can be used to describe the relative positions of the set of points in the data. Most of the variability in the locations of points will be accounted for by the first few dimensions of this coordinate system. The coordinates of the points along each axis will be an eigenvector, and the length of the projection will be an eigenvalue." [Rich 00, Hill 07].

Within network analysis several methods utilising this basic decomposition were published like Correspondence Analysis, NEGOPY, CONCOR, CONVAR and Bonacich centrality with different approaches in the processing of the input network, and thus different interpretations of the results but all methods base on eigensystem calculation and analysis (for details see [Rich 00]).

4.5 Related Areas

Within this section several analysis techniques are introduced which are used in the field of finding structure within data by means of eigensystem analysis and aspects that are related to the eigensystem method used in the analysis in Chapter 6 and introduced in Section 5.1. Supporting the interpretation of the analysis' results in Chapter 6, Section 4.5.1 introduces the concept of principal components as an analysis technique closely related to the eigensystem analysis used here followed by a short introduction of the concept of Fourier decomposition in Section 4.5.2 as in the analysis stage denoted by Equation 5.14 in Section 5.1 a Fourier sum is depicted and interpreted.

4.5.1 Principal Components Analysis

Using eigensystem analysis as a tool to detect structure within a multidimensional sample of data without information on the dependencies in the sample, principal components analysis as a method of multivariate statistics tries to reduce the number of dimensions within the dataset. As explained in Section 4.4 structure within the dataset is detected by approximation of the relationships between the variables in linear combinations. This results in a set of principal components of which the relevant factors are being chosen. This choice is supported by using criteria like the algorithmical Kaiser measure where the optimal components are chosen by the relevance to the total variance of all factors. Thereby a percentage of covered variance decides about the relevant factors. If the sum of the most relevant factors' variance relative to the total variance exceeds this percentage, the optimal factors are found [Kais 60]. Another measure is depicted by the visual scree test which also utilises the covered variances of the single factors. Instead of a fixed share of the total variance the scree test detects the differences between the variances of the most relevant factors. If the difference in the two most relevant factors next to each other exceeds a certain limit, the optimal number of relevant factors is found [Catt 66]. Several other measures are described in [Tuck 69]. In principal components analysis the information content is measured by the explained variance in each dimension with the explained variance decreasing with any new dimension. So principal components analysis uses all variability within the dataset in the analysis.

4.5.2 Fourier Decomposition

In 1822 the book "Theórie analytique de la chaleur" (The Analytical Theory of Heat) by Jean Baptiste Fourier was published [Four 84]. In the context of heat distribution he introduced a univariate decomposition technique in which a discrete function over time could be approximated by the trigonometric series of a sum of unlimited sinusodial functions [Four 84]. As in the following Chapter 6 in Section 5.1 a network will be described by a Fourier sum in Equation 5.14, the original interpretation within the context of the decomposition of a univariate signal is given in the following.

Given information encoded in a continuous signal Fourier decomposition is used to transform this signal into a sum of discrete functions of sinus waves thus splitting the information into ordered subparts denoting the importance of each component to the main signal. More formally the arbitrary periodic function $f(x)$ with the wave count k can be described as a sum of sinusoidal and cosinusoidal oscillations, so harmonic functions $f(x)$ in Equation 4.14 with the Fourier coefficients a_k and b_k in Equations 4.15 and 4.16as and the wave counts of integer multiples $k = 2\pi/\lambda$.

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k \cos(kx) + b_k \sin(kx)) \tag{4.14}$$

$$a_k = \frac{1}{\pi} + \int_{-\pi}^{\pi} f(x)\cos(kx)\ dx,\ \ k \geq 0, \tag{4.15}$$

$$b_k = \frac{1}{\pi} + \int_{-\pi}^{\pi} f(x)\cos(kx)\ dx,\ \ k \geq 1 \tag{4.16}$$

The summands of Equation 4.14 represent the partial signal adding up to more detail with $k \to \infty$. Figure 4.2 shows four signals (black line) with the first four summands for each approximation (red, yellow, green, blue lines).

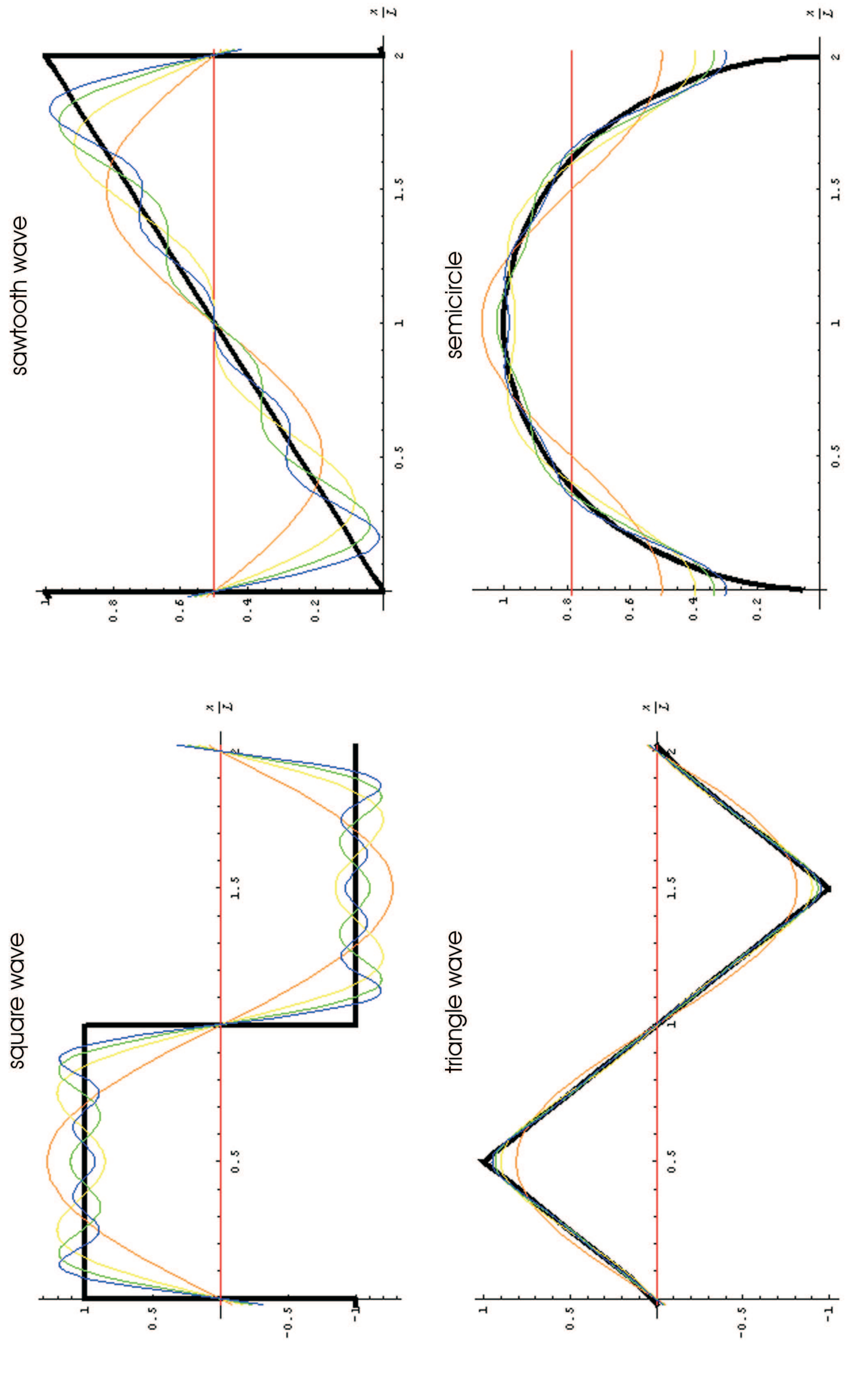

Figure 4.2: Fourier approximations of four signals

5. Eigensystems in Hilbert Space

This chapter describes the formal analysis method following up the social network analysis ideas of centrality measures based on eigensystem analyses in Section 4.4. Section 5.1 and 5.2 introduce the analysis method and the formulation of transaction networks as adequate input to the specific technique. Section 5.3 shows the encoding, eigensystem calculation and interpretation of certain graph patterns. The presented analysis accounts especially for the directional aspects of the analysed transaction networks. Thus an eigensystem analysis technique is utilised which is capable of analysing directed, weighted networks by transforming the network into a complex form obtaining a Hermitian matrix and thus operating in Hilbert space with the mathematical characteristics described in the following.

5.1 Hermitian Matrices

A complex number z can be represented in algebraic form or equivalently in exponential form as $z = a+ib = |z|e^{i\phi}$ with the real part of z being denoted as $Re(z) = a$, the imaginary part as $Im(z) = b$, the absolute value as $|z| = \sqrt{a^2 + b^2}$, and the phase as $0 \leq \phi = \arccos \frac{Re(z)}{|z|} \leq \pi$, with i the imaginary unit ($i^2 = -1$). $\overline{z} = a - ib$ denotes the complex conjugate of z. The angle or phase of a complex number z is denoted by $\varphi(z)$ and the absolute shift between two complex numbers z_1 and z_2 by $\varphi(z_1, z_2)$.

The outer product of two column vectors $\mathbf{x}$ and $\mathbf{y}$ with dimensions $n \times 1$ is defined as

$$\mathbf{x}\mathbf{y}^* = \begin{pmatrix} x_1\overline{y}_1 & \dots & x_1\overline{y}_n \\ \dots & \dots & \dots \\ x_n\overline{y}_1 & \dots & x_n\overline{y}_n \end{pmatrix} \tag{5.1}$$

with $\mathbf{y}^*$ representing the conjugate complex transpose of $\mathbf{y}$. The inner product of $\mathbf{x}, \mathbf{y} \in \mathbb{C}^n$ is defined as

$$\langle \mathbf{x} \mid \mathbf{y} \rangle = \mathbf{x}^* \mathbf{y} = \sum_{k=1}^{n} \overline{x}_k y_k. \tag{5.2}$$

The norm will be denoted $\| \mathbf{x} \|$ and defined as

$$\| \mathbf{x} \| = \sqrt{\langle \mathbf{x} \mid \mathbf{x} \rangle}. \tag{5.3}$$

A matrix H is called Hermitian, if and only if

$$H^* = H \tag{5.4}$$

This means that the matrix entries can be written as $h_{lk} = \overline{h}_{kl}$. Hermitian matrices are also normal

$$HH^* = H^* H. \tag{5.5}$$

Calculating the eigensystem of matrix H we get the set of real valued eigenvalues Λ where the trace of H equals the sum of the eigenvalues

$$tr(H) = \sum_{k=1}^{n} h_{kk} = \sum_{k=1}^{n} \lambda_k. \tag{5.6}$$

If, in addition, $h_{kk} = 0 \, \forall \, k$

$$tr(H) = 0 = \sum_{k=1}^{n} \lambda_k \tag{5.7}$$

thus the eigenvalues split up into the spectrum $\sigma(H)$ of H as a set of positive, zero and negative eigenvalues

$$\sigma(H) = \Lambda = \Lambda^+ \cup \Lambda^0 \cup \Lambda^- = \{\lambda_1, \lambda_2, \ldots, \lambda_n\} \in \mathbb{R} \tag{5.8}$$

and their corresponding set of orthogonal eigenvectors

$$\mathbf{X} = \mathbf{X}^+ \cup \mathbf{X}^0 \cup \mathbf{X}^- = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\} \text{ with } \mathbf{x}_i \in \mathbb{C}. \tag{5.9}$$

with $\mathbf{X}^{+}$ denoting the eigenvectors corresponding to positive eigenvalues, $\mathbf{X}^{-}$ the eigenvectors corresponding to negative eigenvalues and $\mathbf{X}^{0}$ the eigenvectors corresponding to zero eigenvalues.

An eigenvalue λ_i and its corresponding eigenvector $\mathbf{x}_i$ are referred to as "eigenpair".

The spectral radius of H is defined as

$$||H|| = \max\left(|\Lambda|\right), \tag{5.10}$$

thus a spectrum and each eigenvalue λ_i, respectively, may be normed by the spectral norm as

$$\lambda_{i'} = \frac{\lambda_i}{||H||}. \tag{5.11}$$

The eigenvectors of a Hermitian matrix H form a complete orthogonal system and can thus be transformed to an orthogonal basis. Since all eigenvalues of a Hermitian matrix are real the interpretation of the eigenvalues in the context of social network analysis does not pose any difficulty. In mathematical literature the eigenvalues are usually sorted by their value

$$\lambda_1 \geq \ldots 0 \geq \ldots \geq \lambda_n. \tag{5.12}$$

However, if not stated otherwise within this work we assume the eigenvalues to be sorted by their absolute values

$$|\lambda_1| \geq \ldots \geq |\lambda_n| \tag{5.13}$$

to help identifying the dominant substructures for interpretation.

The Hermitian matrix H can further be represented as the Fourier sum

$$H = \sum_{k=1}^{n} \lambda_k P_k \tag{5.14}$$

with

$$P_i = \mathbf{x}_i \mathbf{x}_i^* = \begin{pmatrix} x_{i1}\overline{x}_{i1} & \cdots & x_{i1}\overline{x}_{in} \\ \vdots & \ddots & \vdots \\ x_{in}\overline{x}_{i1} & \cdots & x_{in}\overline{x}_{in} \end{pmatrix} \tag{5.15}$$

denoting the orthogonal projector P_i of the eigenvector $\mathbf{x}_i$ corresponding to eigenvalue λ_i. Further

$$P_k^2 = P_k = P_k^* \tag{5.16}$$

and

$$\sum_{k=1}^{n} P_k = I. \tag{5.17}$$

The sum of the squares of all eigenvalues of the Hermitian matrix H

$$\| H \|^2 = \sum_{k=1}^{n} \lambda_k^2 \tag{5.18}$$

defines the total variation σ^2 contained in H.

The cumulated covered variance of the k^{th} subspace corresponding to the eigenvalues in decreasing order is described as

$$cumvar(\lambda_k) = \frac{\sum_{i=1}^{k} \lambda_i^2}{\sum_{i=1}^{N} \lambda_i^2} \tag{5.19}$$

with the eigenvalues sorted as noted in 5.13.

In the sense of centrality measures described in Section 4.4 the measures based on eigensystem analysis depict the most central vertex k in a graph by the largest absolute eigenvector component $x_{ik} = max_j\,|x_{ij}|$ of the eigenvector $\mathbf{x}_i$ corresponding to the largest absolute eigenvalue $\lambda_{\max} = max_l\,|\lambda_l|$. This also holds for the most central vertices in each substructure identified by the eigenvectors.

Invariant Properties

The measures used for describing the graphs' structures within the eigensystem are relations among the elements of the eigensystem namely distances between the subspaces denoted by the eigenvectors and their components. Thereby the properties of the complex numbers of the eigenvector components like absolute values and phase information describe structural properties of the original graph like direction of flows within a certain structure. Because of

$$\langle e^{i\phi_k}\mathbf{x}_k \mid e^{i\phi_l}\mathbf{x}_l\rangle \;= e^{-i\phi_k}e^{i\phi_l}\langle \mathbf{x}_k \mid \mathbf{x}_l\rangle \;= e^{i(\phi_k-\phi_l)}\langle \mathbf{x}_k \mid \mathbf{x}_l\rangle = e^{i(\phi_k-\phi_l)}\delta_{kl} \tag{5.20}$$

the eigenvectors are invariant to arbitrary rotation. Within this work we assume the eigenvectors $\mathbf{x}_i$ and thus their complex eigenvector components x_{ij} being rotated such that the phase of the eigenvector component corresponding to the maximum eigenvalue $\lambda_{\max}$ (cp. *anchor* $(\mathbf{x}_i)$ in Equation 5.30) has a phase of 0.

Metrics in Hilbert Space

Furthermore we work in Hilbert space which is a complete normed inner product space, the norm is defined by the scalar product $\|\mathbf{x}\|^2 = \langle \mathbf{x}|\mathbf{x}\rangle$ (see Equation 5.3). If vector $\mathbf{c}_y$ and $\mathbf{c}_z$ are normed to 1

$$\|\mathbf{c}_y\| = \|\mathbf{c}_z\| = 1 \tag{5.21}$$

we can calculate the distances of a vector $\mathbf{c}_y$ from a vector $\mathbf{c}_z$ by

$$\begin{aligned}
\|\mathbf{c}_y - \mathbf{c}_z\|^2 &= \langle \mathbf{c}_y - \mathbf{c}_z|\mathbf{c}_y - \mathbf{c}_z\rangle \\
&= \langle \mathbf{c}_y|\mathbf{c}_y\rangle + \langle \mathbf{c}_z|\mathbf{c}_z\rangle - \langle \mathbf{c}_y|\mathbf{c}_z\rangle - \langle \mathbf{c}_z|\mathbf{c}_y\rangle \\
&= \|\mathbf{c}_y\|^2 + \|\mathbf{c}_z\|^2 - \langle \mathbf{c}_y|\mathbf{c}_z\rangle - \overline{\langle \mathbf{c}_y|\mathbf{c}_z\rangle} \\
&= \|\mathbf{c}_y\|^2 + \|\mathbf{c}_z\|^2 - 2Re(\langle \mathbf{c}_y|\mathbf{c}_z\rangle) \\
&= 2 - 2Re(\langle \mathbf{c}_y|\mathbf{c}_z\rangle).
\end{aligned} \tag{5.22}$$

We identify the vector $\mathbf{c}_y$ with minimal distance to vector $\mathbf{c}_z$ by

$$\min_y \left(2 - 2Re(\langle \mathbf{c}_y|\mathbf{c}_z\rangle)\right). \tag{5.23}$$

Thereby the distance between $\mathbf{c}_y$ and $\mathbf{c}_z$ becomes minimal if the real part of a scalar product becomes minimal if

$$\max_y Re\left(\langle \mathbf{c}_y|\mathbf{c}_z\rangle\right) \leq 1. \tag{5.24}$$

This will be referred to in the spectral clustering method in Section 5.3.3.

5.2 Networks as Hermitian Matrices

Following Hoser and Geyer-Schulz [Hose 05] we take a network as a directed graph (digraph) in the representation of a real valued adjacency matrix A which is transformed into the Hermitian matrix H by first constructing the complex valued adjacency matrix

$$A_{\mathbb{C}} = A + i \cdot A^t. \tag{5.25}$$

Thereby the real part of a matrix entry a_{jk} can be seen as the outbound flow of vertex v_j to vertex v_k $(\omega(v_j, v_k))$ and the imaginary part as the inbound flow of vertex v_j from vertex v_k $(\omega(v_k, v_j))$.

$A_{\mathbb{C}}$ then is rotated to obtain the Hermitian adjacency matrix H as

$$H = A_{\mathbb{C}} \cdot e^{-i\frac{\pi}{4}}. \tag{5.26}$$

By transforming a graph's adjacency matrix into the complex Hermitian form described above and calculating the eigensystem of this matrix, all information of the graph structure and directional characteristic is retained within the eigensystem.

5.3 Analysis of the Eigensystem

Having introduced the analysis technique the calculated eigensystem of a graph without self incident vertices ($h_{kk} = 0 \;\forall\; k$) consists of the non-zero eigenvalues $\Lambda = \{\Lambda^+, \Lambda^-\} \in \mathbb{R}$ and the eigenvectors $\mathbf{X} = \{\mathbf{X}^+, \mathbf{X}^-\} \in \mathbb{C}$ (cp. Section 5.1) with certain structural characteristics. As introduced in Section 4.4 the eigensystem calculation of transaction networks describes the positions of the vertices towards each other in terms of centrality within the network which is used for the semantic interpretation of importance within the group. The following sections present the analysis of the eigensystem as a result of the preceeding calculations in terms of describing and finding the transaction patterns proposed in Section 6.2.

5.3.1 Visualisation of the Eigensystem

Complex numbers can be colour coded following the colour scheme given in Figure 5.1 which is derived from the colour circle. Thereby the absolute value $|x_j|$ of each component corresponds to the saturation of colour, while the phase $\phi(x_j)$ of the complex value encodes the colour itself. Thus a component with a low absolute value will be represented as a light saturated colour or even gray, while a component with a high absolute value will show a more or less saturated colour according to its phase. For example the value $\mathbf{x}_2^1 = 0.9e^{i\frac{\pi}{4}}$ will show an intensely saturated yellow, the value $\mathbf{x}_{22}^8 = 0.2e^{i\pi}$ will show a low saturated turquoise.

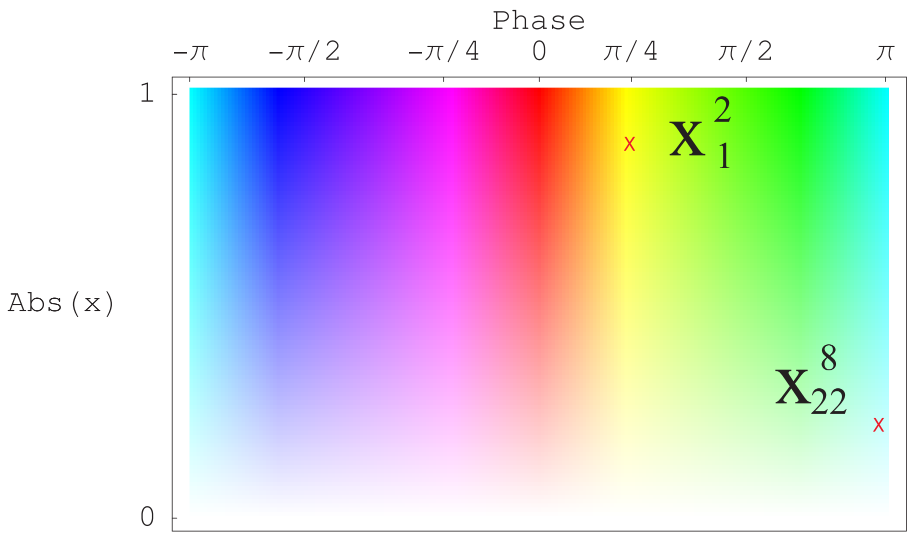

Figure 5.1: Colour coding of complex numbers

The visualisation of a graph with the weighted 4×4 adjacency matrix A_1

$$A_1 = \begin{pmatrix} 0 & 6 & 2 & 3 \\ 6 & 0 & 3 & 6 \\ 2 & 5 & 0 & 10 \\ 3 & 5 & 2 & 0 \end{pmatrix} \tag{5.27}$$

and the eigensystem shown in Table 5.1 is given in Figure 5.2 where the eigenvectors $\mathbf{x}_i$ are the rows of a graphical array $(\mathbf{x}_1, \ldots, \mathbf{x}_n)^T$. The eigenvectors are sorted by the absolute value of each eigenvector's corresponding eigenvalue in descending order. For the sake of clarity of the phase information the saturation is not applied in the visualisations throughout this work. A detailed description of this visualisation technique is given in [Hose 07c].

λ_k	19.85		-10.76		-9.17		0.084	
	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$
$\mathbf{x}_k$	0.41	-0.16	0.09	-0.15	0.60	-0.48	0.68	0
	0.54	-0.78	0.31	-2.97	0.71	2.40	0.33	0
	0.49	-3.07	0.59	0	0.35	2.55	0.54	-0.50
	0.55	0	0.74	-0.05	0.07	2.57	0.38	-2.71

Table 5.1: Eigensystem for A_1 (Equation 5.27) described in Table 5.1 with $z = |z|e^{i\phi}$

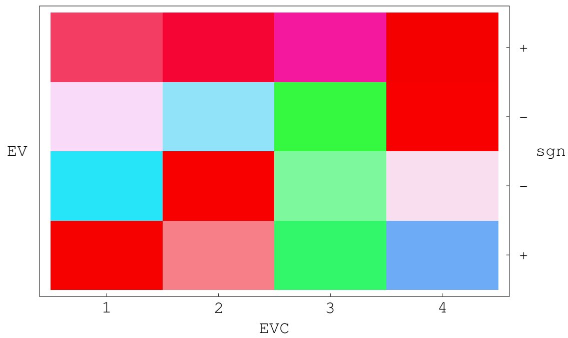

Figure 5.2: Visualisation of the eigensystem in Table 5.1

5.3.2 Structures represented by the Eigensystem

The structure of a network can be measured and described at several levels and by several means. Up to now network structures in terms of patterns were described by means of figurative language in Section 3.3 or graph theory in Section 6.2. After having processed transaction data from a network the proposed method results in the mentioned eigensystem. Now the input variables in form of networks hold all information on the network which is sustained within the eigensystem calculation. So the information is still contained in the resulting eigensystem but in a disaggregated way. To approach these results and what can be seen within the eigensystem we take a close look at the most extremal network structures in the sense of a trading network namely a strongly connected or complete structure and a star-like structure, respectively. In terms of graph structures the term *size* of the structure describes the number of vertices that are to be assigned to the structure and the term *strength* of the structure usually describes the amount of transactions within the structure, how visible the structure is in relation to the rest of the network.

In general the spectrum consists of the eigenvalues Λ representing the weighting factors of the subspaces given by the eigenvectors $\mathbf{X}$. Each of the eigenvalues describes a slice of the total variance within the data with the largest absolute values summing up to y % describing y % of the total structure of the network. If this main structures are covered by only a few eigenvalues, than the main structures can be extracted by analysing these eigenvalues and their corresponding eigenvectors.

To each of the eigenvalues λ_i one subspace is corresponding which is built up by the eigenvector $\mathbf{x}_i$. The structure of the eigenvector components i.e. the relations of the components to each other gives information on the structure within that subspace. The position j of the eigenvector component x_j within the eigenvector $\mathbf{x}$ indicates the original trader whose communication was encoded in row and column j in the original adjacency matrix.

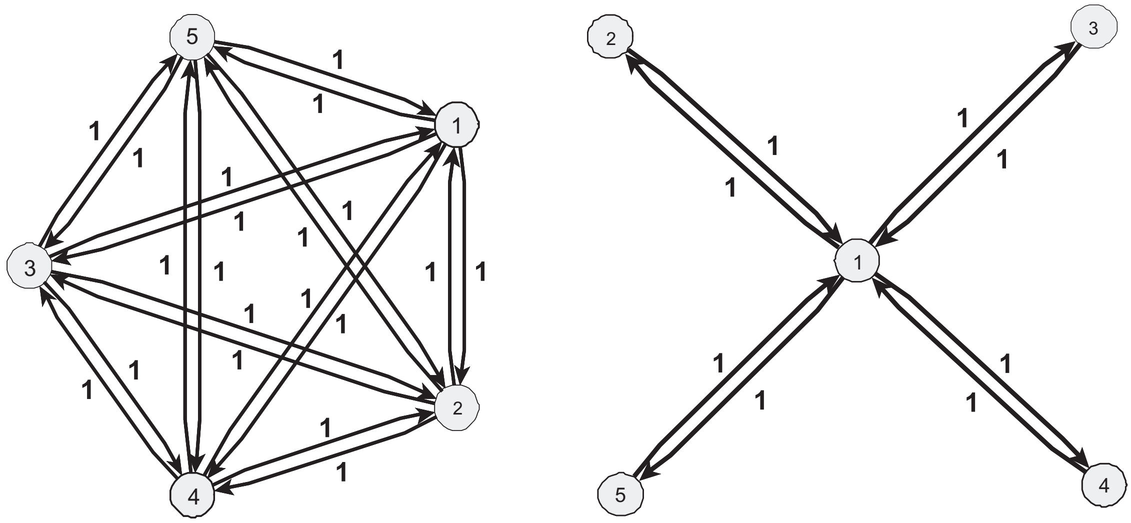

Figure 5.3: Complete and star graph structure

In Figure 5.3 the two most extremal types of graph structures for the research question at hand, a fully connected or complete structure and a star-like structure, respectively, are shown. From the eigensystem calculation for both graphs the eigensystem with eigenvalues Λ and eigenvectors $\mathbf{X}$ are obtained. In the following the arrangements of the eigenvalues and eigenvectors in case of these extremal structures is described.

Complete Structure

A complete network structure is depicted by a vertex set in which all vertices have connections to all other vertices with balanced connection weights. Traders in a network act anonymously towards the market system and therefore the orders are matched in a balanced way as described in Section 3.4. We assume that for a prediction market with rational traders and without manipulative actions the vertices within the different network types described in Section 6.1 tend to form complete structures. In terms of economic networks with the vertices as economic entities like companies this network structure would describe a market in perfect competition.

The eigensystem

In terms of acoustic signals a complete graph would be described as “ground noise”. That is because no specific pattern is emerging from the network formation but the arrangement of the vertices and flows is almost totally balanced. The traders in the respect of not contributing a certain structure to the network structure are referred to as “noise traders”. If the network consists of complete and uniformly distributed ties among the vertices as shown in the adjacency matrix in Table 5.28 and depicted in Figure 5.4 this structure is expressed by a maximum number of minimal valued negative eigenvalues, limited by Equation 5.6 with one eigenvalue being positive compensating for the low valued eigenvalues.

$$A_2 = \begin{pmatrix} 0 & 30 & 30 & 30 & 30 \\ 30 & 0 & 30 & 30 & 30 \\ 30 & 30 & 0 & 30 & 30 \\ 30 & 30 & 30 & 0 & 30 \\ 30 & 30 & 30 & 30 & 0 \end{pmatrix} \tag{5.28}$$

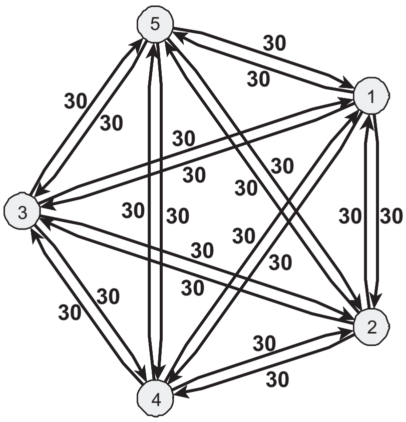

Figure 5.4: Analysed complete graph structure

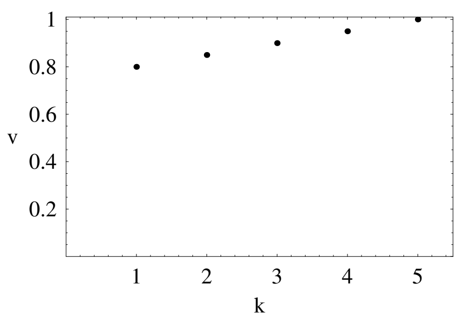

Figure 5.5: Cumulated variance of the analysed complete graph structure of Figure 5.4

The cumulated variance of the data depicted in Figure 5.5 is covered almost completely by the first subspace denoted by the maximum absolute eigenvalue. As the underlying graph depicts no structure in particular the corresponding eigenvector's structure reflects this situation in holding information of the completeness of the graph in the components. Following Section 5.3.1 the visualisation of the eigensystem is given in Figure 5.6. The eigenvectors are depicted by the rows with the eigenvector components (EVC) in columns. The right edge of the figure denotes the sign (sgn) of the eigenvalue corresponding to each of the eigenvectors. The figure shows the explained structure of the complete graph in the first eigenvector corresponding to a positive eigenvalue without any other eigenvector with alternated algebraic sign corresponding to the first one in regard to the phase of an eigenvector component.

λ_k	169.71		-42.43		-42.43		-42.43		-42.43	
	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\phi(z)$	$\mid z \mid$
	0.45	0	0.08	0	0.03	0	0.25	0	0.89	0
	0.45	0	0.39	0	0.15	0	0.77	3.14	0.22	3.14
$\mathbf{x}_k$	0.45	3,14	0.56	3.14	0.22	3.14	0.34	3.14	0.22	0
	0.45	0	0.51	0	0.45	3.14	0.34	3.14	0.22	3.14
	0.45	0	0.51	3.14	0.85	3.14	0.34	3.14	0.22	3.14

Table 5.2: Eigensystem for A_2 described in Table 5.28 with $z = |z|e^{i\phi}$

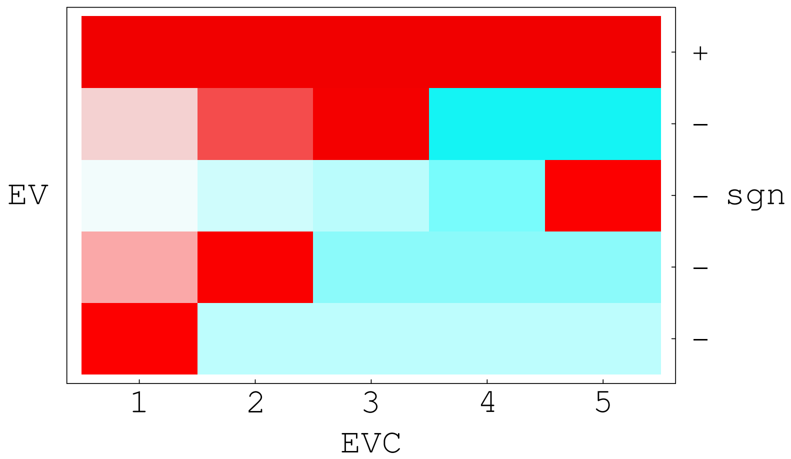

Figure 5.6: Visualisation of the eigensystem in Table 5.2

Structure of a Star Graph

Having structure in the network, the eigenvalues as weighting factors of the subspaces start to vary in their values up to the most centralised structure where all vertices are connected to only one central vertex. Modelling the economic network structure of a pure monopoly results in the most central structure with no connections among the vertices connected to the monopolist which depicts the centre of the network. This structure in terms of graph structures is described as star-like structure or simply as a star graph.

The eigensystem

Let H be a matrix

$$H = \begin{pmatrix} 0 & B \\ B^* & 0 \end{pmatrix}. \tag{5.29}$$

As the non-zero eigenvalues are the positive and negative square roots of the singular values of the matrix B the spectrum of a pure (undisturbed) star graph shows a symmetry along the x-axis when the eigenvalues are sorted by value. As a consequence the eigenvectors corresponding to the two eigenvalues with the same absolute value but opposite sign show the following characteristics: The absolute values for each component are the same in both eigenvectors but the phase shift between the eigenvectors' components depend on the position of the vertex in the graph. If the eigenvectors are rotated as described in Section 5.1 the eigenvector components are equal in phase if the corresponding vertex is the centre of the star structure. They have a phase shift of π if the corresponding vertices are alters [Meye 00].

The k^{th} vertex in the original graph (which by convention holds the k^{th} row and column within the adjacency matrix) is represented by the k^{th} component of each eigenvector denoting the subspaces. The k^{th} vertex in the original graph can be seen as the most relevant within a substructure i if the absolute value of the k^{th} component of the i^{th} eigenvector is maximal for all components of this eigenvector

$$anchor(\mathbf{x}_i) = Abs(x_{ik}) = max_j Abs(x_{ij}). \tag{5.30}$$

In the case of Equation 5.30 k is called the *anchor* of the i^{th} eigenvector $\mathbf{x}_i$.

If there exists another eigenvector l with the anchor on the same eigenvector component position which additionally has no phase shift in the argument to the phase of the anchor of the i^{th} eigenvector

$$\varphi(x_{ik}, x_{lk}) \approx 0 \text{ with } anchor(\mathbf{x}_l) = Abs(x_{lk}) = max_j Abs(\mathbf{x}^l_j) \tag{5.31}$$

a star-like or central structure is present. All other components of the l^{th} subspace that are connected to the anchor and thus to this structure have a phase shift of approximately π

$$\varphi\left(\mathbf{x}_{mk}, \mathbf{x}_{ik}\right) \approx \pi, m \neq i. \tag{5.32}$$

The extent to which the Equations 5.31 and 5.32 approach equality shows the extent of the 'purity' of the structure which can be expressed in certain thresholds for each approximation. $\varphi(\mathbf{x}_{mk}, \mathbf{x}_{ik})$ additionally holds information about the encoded network flow of the pattern. Applying the inverse rotation to the obtaining of the Hermitian adjacency matrix in Equation 5.26 gives hints on the network flow within the found structure which can be described as 'towards' the centre vertex (inbound) for $|\varphi(\mathbf{x}_{mk}, \mathbf{x}_{ik}) - \pi| < 0$ and 'from' the centre vertex (outbound) for $|\varphi(\mathbf{x}_{mk}, \mathbf{x}_{ik}) - \pi| > 0$.

$$A_3 = \begin{pmatrix} 0 & 30 & 30 & 30 & 30 \\ 30 & 0 & 0 & 0 & 0 \\ 30 & 0 & 0 & 0 & 0 \\ 30 & 0 & 0 & 0 & 0 \\ 30 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{5.33}$$

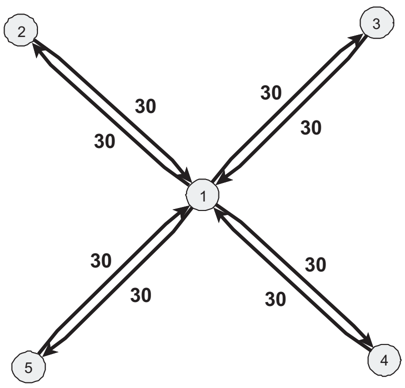

Figure 5.7: Analysed equally weighted star graph structure

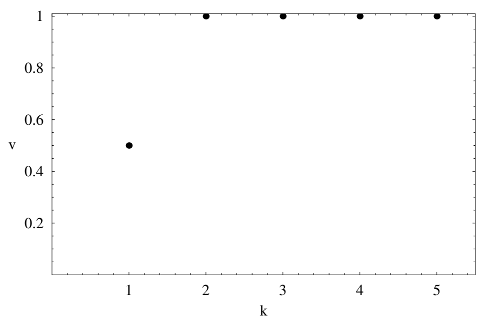

Figure 5.8: Cumulated variance of the analysed star graph structure of Figure 5.7

In the weighted adjacency matrix in Table 5.33 which is depicted by Figure 5.7 a star graph structure is given with its eigensystem in Table 5.3 and the visualisation of the eigenvectors following the descriptions in Section 5.3.1. It can be clearly seen that the structure is 'pure' which results in a phase shift of π from the first to the second eigenvector in the most central vertex 1 denoted by the first eigenvector component. The other vertices i belonging to the structure have a phase shift of zero in their corresponding eigenvector component x_i. The cumulated variance of the eigenspectrum given in Figure 5.8 clearly shows the two corresponding eigenvalues which have alternating algebraic signs in their eigenvalues. The visualisation of the eigensystem in Figure 5.9 with the same formal arrangements as Figure 5.6 shows a clear phase shift of π of all eigenvector components of the corresponding eigenvectors. The second eigenvector component in contrast has a phase shift of zero (red box in both rows) which indicates the centre of the star graph. The third to fifth row are blank as the eigenvalues are zero.

5.3.3 Clustering with the Eigensystem

The results of the analysis can be analysed by utilising the metrics in Hilbert space introduced in Section 5.1. By making use of the eigenvector information, distances

λ_k	-84.85		84.85		0		0		0	
	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\phi(z)$	$\mid z \mid$
	0.71	0	0.71	3.14	0	0	0	0	0	0
	0.35	0	0.35	0	0	0	0	0	0	0
$\mathbf{x}_k$	0.35	0	0.35	3.14	0	0	0	0	0	0
	0.35	0	0.35	3.14	0	0	0	0	0	0
	0.35	0	0.35	3.14	0	0	0	0	0	0

Table 5.3: Eigensystem for A_3 described in Table 5.33 with $z = |z|e^{i\phi}$

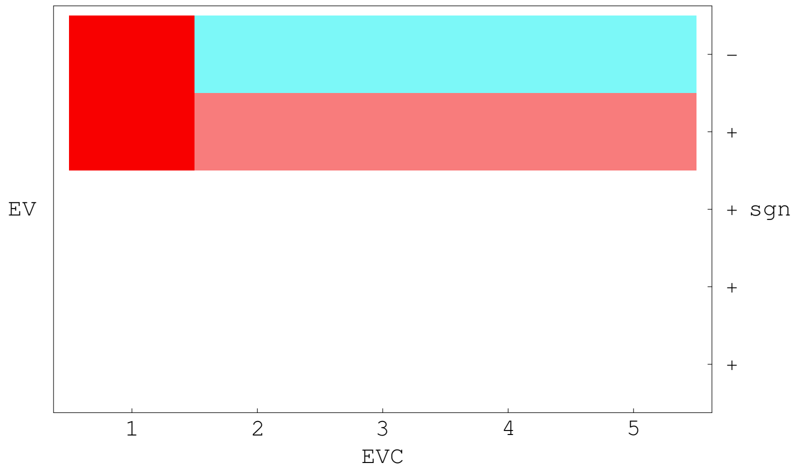

Figure 5.9: Visualisation of the eigensystem in Table 5.3

between the subspaces of the eigensystem can be calculated. This groups and orders the patterns by their dominant and associate subspaces of their corresponding vertices for further interpretation and possible detection of subgroups of manipulative traders. As the traders' trading actions are encoded within the subspaces, a clustering of the traders with regard to centralised structures can be accomplished.

Taken the ideas of Section 5.1 and given the eigensystem of H we take the set of positive eigen values Λ^+ with $\lambda_1^+, \lambda_2^+, \ldots, \lambda_l^+$ and their corresponding eigenvectors $\mathbf{X}^+$ with $\mathbf{x}_1^+, \mathbf{x}_2^+, \ldots, \mathbf{x}_l^+$ and build the matrix R as

$$R_{n\times l} = \left(\lambda_1^+\mathbf{x}_1^+|\lambda_2^+\mathbf{x}_2^+|\ldots|\lambda_l^+\mathbf{x}_l^+\right). \tag{5.34}$$

As we work in Hilbert space, distances are defined by scalar products. So with this matrix R and its complex conjugate transpose we build the scalar product matrix S with $s \in \mathbb{C}$, $S = S^*$ and

$$S_{n\times n} = RR^*. \tag{5.35}$$

S represents thus the distance between points in an l-dimensional subspace where

$$s_{ij} = \sum_k r_{ik}\overline{r}_{jk} = \sum_l \lambda_l^2 x_{li}\overline{x}_{lj}. \tag{5.36}$$

Following Equation 5.24 the distance between the vectors is minimal if $Re\left(\langle \mathbf{r}_i | \mathbf{r}_j \rangle\right)$ is maximal for $\|\mathbf{r}_i\| = \|\mathbf{r}_j\| = 1$ or $Re\left(\langle \mathbf{r}_i | \mathbf{r}_j \rangle\right) \leq 1$ where $\langle \mathbf{r}_i | \mathbf{r}_j \rangle$ denotes the distance of two points in an l-dimensional subspace.

Section 5.1 mentions that all information of the underlying network is maintained in the combination of projectors P_i and eigenvalues Λ, so the scalar product matrix of equation 5.35 holds all structural information contained in the eigenvectors $\mathbf{X}^+$ corresponding to the positive eigenvalues Λ^+ which is shown by

$$P_m = \mathbf{x}_m \mathbf{x}_m^* \tag{5.37}$$

$$\Rightarrow p_{jk}^m = x_{mj} \overline{x}_{mk} \tag{5.38}$$

$$\Rightarrow s_{jk} = \sum_m \lambda_m^2 p_{jk}^m \tag{5.39}$$

with s_{jk} denoting the sum of the components of the respective projector weighted by the square of the respective positive eigenvalue λ_m.

Following [Hose 07b] we find clusters q_k within the scalar product matrix S by assigning the vertices of the network to the cluster with cluster centre $\mathbf{x}_j^+$ such that a vertex i belongs to a cluster q_k if $Re(s_{iq_k}) = max_j Re(s_{ij})$. As to Equation 5.7 at least one of the eigenvalues of $\mathbf{X}$ has to be different to 0, l and therefore the minimum number of clusters is at least one, at most $n-1$ for the analysed network.

6. Analysis

Having described transaction networks in general in Chapter 4 with their representation as graphs (cp. Section 4.2) or adjacency matrices (cp. Section 4.1), this chapter describes the relevant network structures of prediction markets built by the basic entities inherent to such a market in Section 6.1 by means of formal graph theory. In Section 6.2 abstract graph structures are shown that describe several types of trading-behaviour within a market. The closing remarks of the analysis chapter in Section 6.3 present procedures for fraud detection by means of the introduced analysis technique. The procedures make use of the structural characteristics of the eigensystem in relation to the underlying graph structure as described in Section 5.3.

6.1 Network Definition and Formation

As in a stock market *traders* in a prediction market exchange *shares* and *money*. The exchange is usually accomplished by having a *market mechanism* conducting the transactions either for all shares or each share type separately but without the traders being informed about their counterpart in any transaction. This anonymity in trading networks and trading towards the system is described in detail in Section 3.4 and is depicted by Figure 3.3. Shares, money and the market mechanism are called the entities of the market. To achieve communication networks with direct communication between the traders, we acquire direct connections among the traders from the transaction data of the market (matched trading activities of the traders towards the market) that are available within the market system as depicted by Figure 6.1. These direct connections are then organised into several networks.

Following [Fran 06] a stock market and its moneyflows can be modelled as an accounting system in which a simple transaction record has the form

RECORD <debit account>, <credit account>, <amount>.

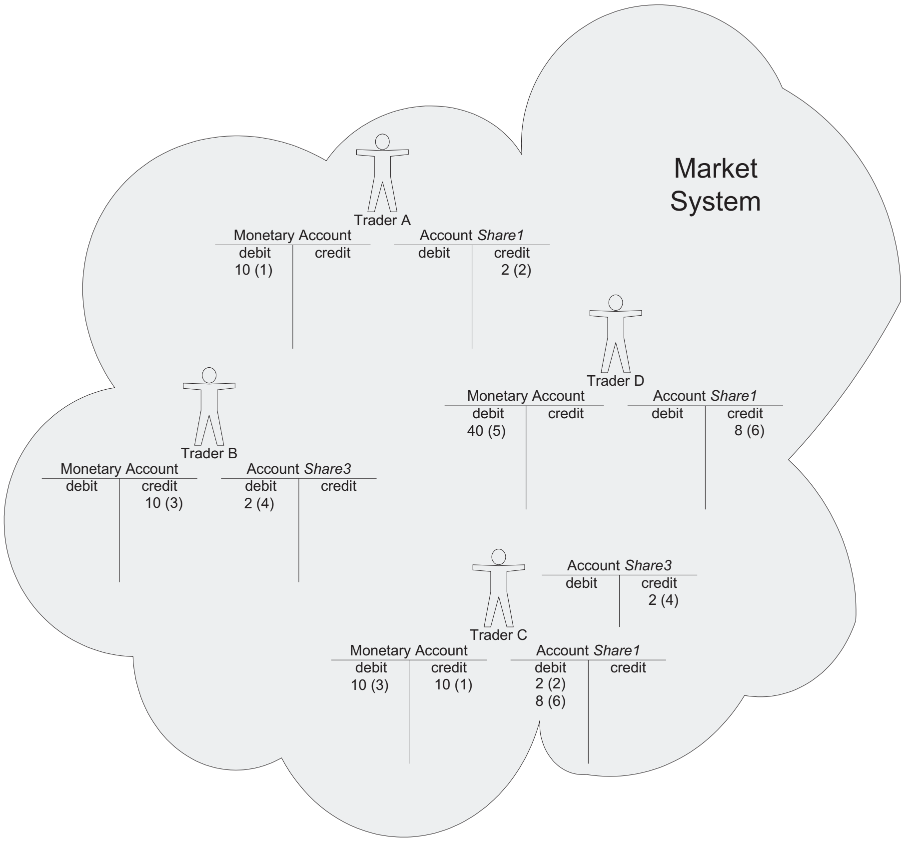

Figure 6.1: Accounts within a market system with transactions of Figure 3.3

This describes the transaction flows of an <amount> from the <debit account> to the <credit account>. Expressing additionally also the stock market's full information on traded shares in the same manner requires a small transformation. Without loss of information all stock market actions can be expressed in an modified accounting system solely by simple transaction records like

RECORD <debit share account>, <credit share account>, <x>

with <x> denoting the quantity of a share to be exchanged between a <debitshare account> and a <credit share account> and

RECORD <debit monetary account>, <credit monetary account>, <p $\times$ x>

with the monetary units as price p multiplied by the quantity x flowing from the <debit monetary account> to the <credit monetary account>. For the transaction data given in Figure 3.3 the matched orders result after elimination of the matching mechanism of the share in the accounting records with the transaction number as prefix:

- (1) RECORD <Trader A Monetary>, <Trader C Monetary>, <10>

- (2) RECORD <Trader C Share1>, <Trader A Share1>, <2>
- (3) RECORD <Trader C Monetary>, <Trader B Monetary>, <10>
- (4) RECORD <Trader B Share3>, <Trader C Share3>, <2>
- (5) RECORD <Trader D Monetary>, <Trader C Monetary>, <40>
- (6) RECORD <Trader C Share1>, <Trader D Share1>, <8>.

These transaction records are illustrated with 9 accounts in Figure 6.1 with the transaction records grouped by traders and the attached transaction number.

As the objective here is the analysis of trading patterns we are primarily interested in the analysis of the transaction networks namely the networks of moneyflow and shareflow. For this reason we focus on the cumulated flow quantities money and shares separately and split up the total accounting system's network into subnetworks of total moneyflow, total shareflow, sharewise moneyflow and sharewise shareflow. The generation of these submarkets is described by the following subsets. As not all information is traceable by the basic accounting system as described above, the market system has to be expanded to the tracking of these parameters. Especially the accounts have to be splitted up into a debit and a credit share account for each trader. The full market network derived from the market accounting system in Figure 6.1 is given in Figure 6.2. Within the figures the monetary units are described as *MU*. A moneyflow is for example expressed by “2 * 2 MU = 4 MU” which describes the flow of two times two monetary units which results in four monetary units in total. A shareflow is given by “2 *Share1* @ 2 MU” which describes the flow of two shares of *Share1* at the price of two monetary units.

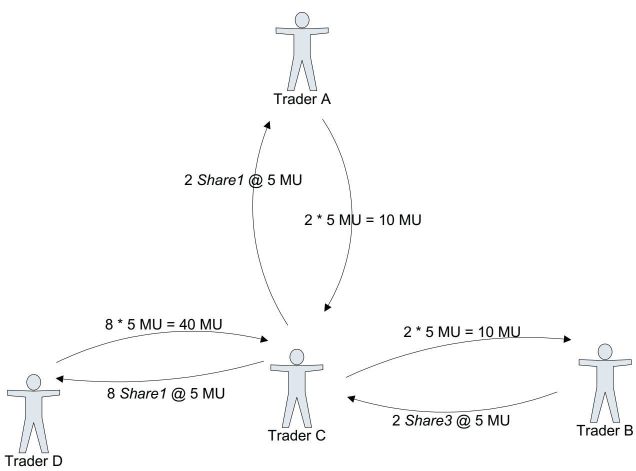

Figure 6.2: Full market network derived from market accounting system in Figure 6.1

6.1.1 Global Moneyflow Network

Considering only the flow of monetary units (MU) between the traders in the market, the network is described by the graph $\mathcal{G}^M = \left\{V^T, E^M, \omega^M\right\}$ with the traders V^T as the vertices, the flows of money E^M as edges and the weight of the edge as the amount of money $w^M_{i,j}$ exchanged by the two traders i and j represented by the two connected vertices v_i and v_j. The monetary flow for a transaction thereby is defined as *share price times order volume* which results in the network depicted by Figure 6.3. The information taken from the total accounting system are the records conforming to

RECORD <debit account>, <credit account>, <amount>

or from the modified accounting system according to

RECORD <debit monetary account>, <credit monetary account>, <p $\times$ x>.

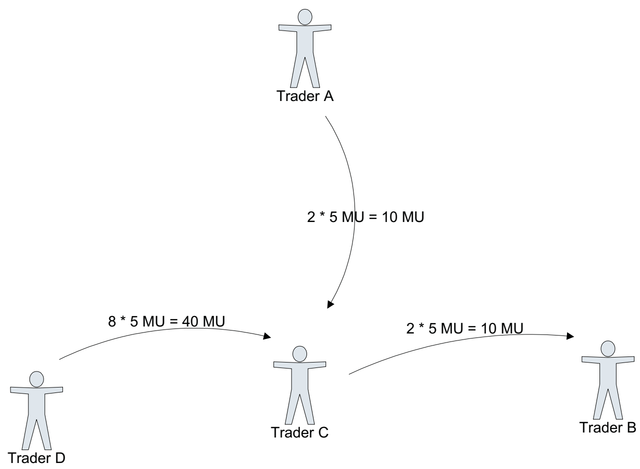

Figure 6.3: Global moneyflow network as subnetwork of the full market network in Figure 6.2

6.1.2 Global Shareflow Network

The shares exchanged within the system can be modeled as the shareflow network which is expressed by the graph $\mathcal{G}^S = \left\{V^T, E^Q, \omega^Q\right\}$ with Q designating the flow of shares. This subnetwork is depicted by Figure 6.4 whose information is taken from the modified accounting system records conforming to

RECORD <debit share account>, <credit share account>, <x>

with the share accounts of all traders for that share.

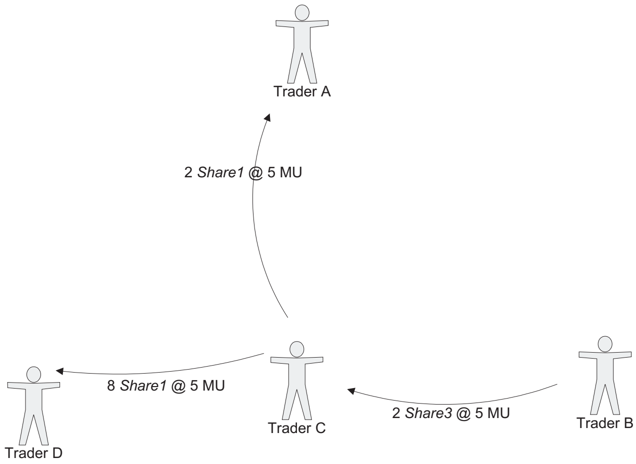

Figure 6.4: Global shareflow network as subnetwork of the full market network in Figure 6.2

6.1.3 Sharewise Moneyflow Network

Restricting the previously built networks to the money flows of only one type of shares results in $\mathcal{G}_s^M = \left\{V^T, E^{M^s}, \omega^{M^s}\right\}$ with $s \in S = \{s_1, \ldots, s_n\}$ denoting one of the n types of shares within the market. $\mathcal{G}_s^M$ describes one network for each share within the market and is illustrated by Figure 6.5. Within the total accounting system this network represents the records conforming to

RECORD <debit account>, <credit account>, <amount$_s$>

or in the the modified accounting system according to

RECORD <debit monetary account$_s$>, <credit monetary account$_s$>, <p × x>

with <debit monetary account$_s$> denoting the accounting system entries within the monetary accounts of traders that were involved in transactions of a certain share s.

6.1.4 Sharewise Shareflow Network

The second flow quantity within a stock market is the number of shares exchanged among the traders. Taken as a separate network, this results in the graph representation $\mathcal{G}_s^S = \left\{V^T, E^{Q^s}, \omega^{Q^s}\right\}$ with again the traders V^T as vertices, but this time with the shareflows E^Q of share s as edges and the sum of shares ω^{Q^s} of share s exchanged between the two vertices connected to the edge as the weight of this edge. The modified accounting system's records information used to build up this network are records conforming to

RECORD <debit share account$_s$>, <credit share account$_s$>, <x>

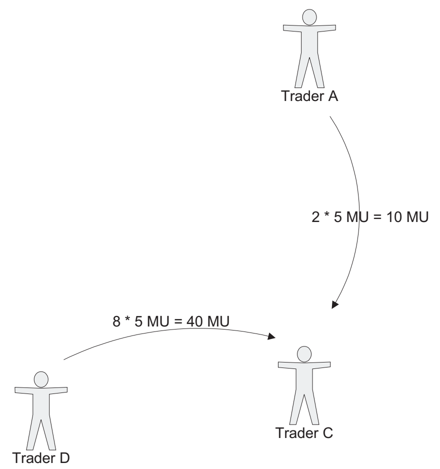

Figure 6.5: Sharewise moneyflow network as subnetwork of the full market network in Figure 6.2

which results in the graph depicted by Figure 6.6.

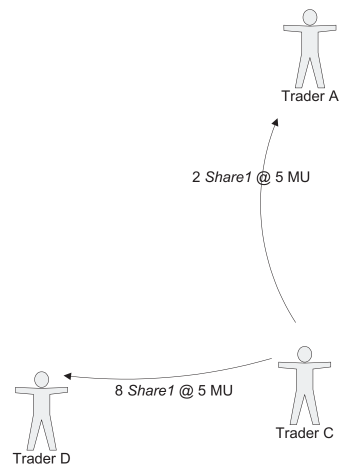

Figure 6.6: Sharewise shareflow network for *Share1* as subnetwork of the full market network in Figure 6.2

6.1.5 Summary

After having extracted direct transaction networks among the traders we are now able to describe the fraudulent actions from Section 3.3 in terms of transaction patterns. The acquired transaction networks provide an appropriate basis to describe the patterns with regard to their selective appearance.

6.2 Manipulation Patterns

As described in Section 3.3 the patterns looked for are motivated by incentive incompatibility as described in Section 3.2. We distinguish the following two types of fraudulent traders:

1. Money transfer manipulator (“I want a higher rank”)
2. Price manipulator (“I want to signal”)

To differentiate between these types we describe the actions taken by a regular trader, a trader trying to rank himself better and a signalling trader by means of the notations introduced in Section 4.3. Behaviour in a market is expressed by the trading of shares among the traders which results in money and shareflows with the relevant networks introduced in Section 6.1. All traders within a prediction market system leave their marks in certain transaction patterns. All patterns define structures within transaction networks derivable from the transaction records of the accounting system underlying a market as mentioned in Section 6.1. In the case of traders misbehaving in a way described in Section 3.3, their actions are based on the patterns described in the following.

As mentioned in Section 3.4 traders in markets considered here usually act anonymously. The anonymity is linked to the amount of liquidity of the market. The less order activity there is, the less anonymity is present. In a liquid market, a market with high order activity, the market reacts on all new bid and ask offers that lie beyond the current price for each case in a very short time period (seconds). In such a situation the traders are barely aware of their counterpart within a transaction even if the depth of the order book is high (see Section 3.4). So, almost none of the traders of can tell its counterpart in a transaction and the set of their transactions describes a quite balanced network structure. Figure 6.7 illustrates this liquid and illiquid times of a market by the spread which can be narrow (high activity) or wide (low activity).

In the following we describe the patterns using the measures vertex degree and vertex flow from Section 4.3 in the context of the networks described in Section 6.1. Thus we limit the measures to shares i.e. the edges being bound to a certain share s with respect of money or quantity denoted by superscription with S, s or M respectively i.e. $\delta^{s,in}_{\text{flow}}(v)$ for the inbound shareflow of share s for vertex v or $\delta^{S|M,out}_{\text{deg}}(v)$ for the outbound vertex degrees with respect to the total money or share flow.

The fact from Section 5.3.1 that the results of the eigen analysis contains all information that was put into the calculation enables the description of the manipulative patterns in terms of the eigensystem. The main patterns describing the network are usually found by analysing the eigenvectors corresponding to the largest absolute eigenvalues that cover a sufficient percentage of total variance. The patterns are

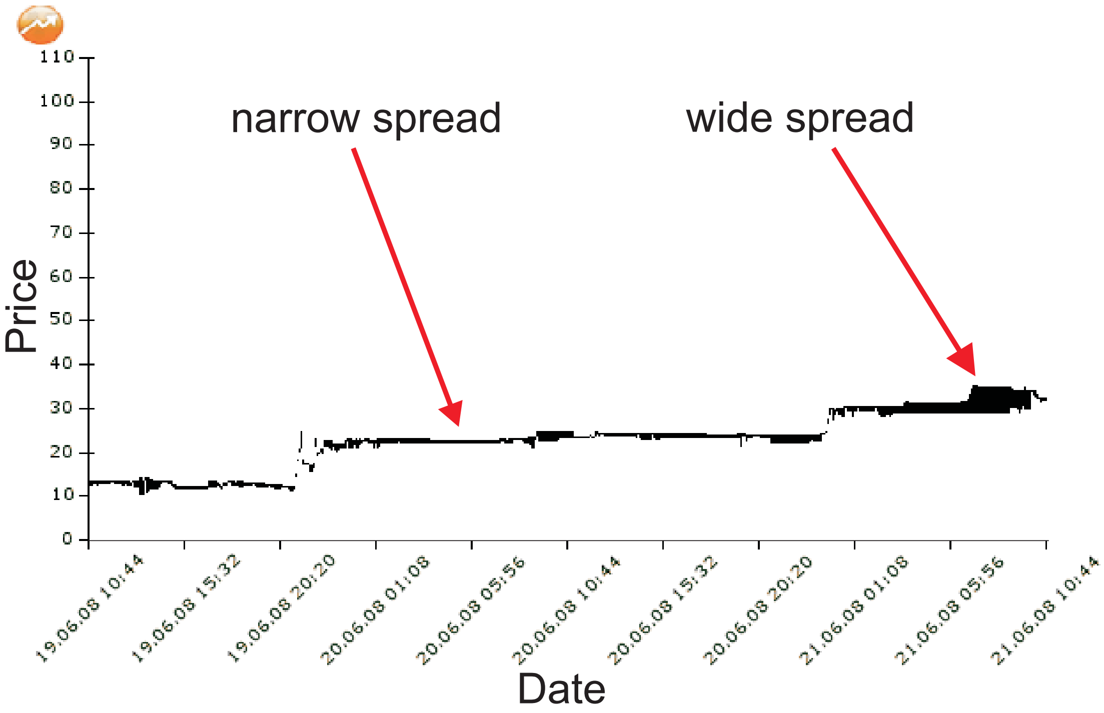

Figure 6.7: Spread of the share for Germany in the Euro-08 championships

thereby depicted by centralised structures that contain one or more vertices depending on the actions of the manipulator towards the network which are given at the end of each pattern description.

6.2.1 Regular Trading

In a market with regular trading behaviour of all traders the anonymity of all trading partners is fully maintained (even in non-liquid periods) and the transaction network has approximately balanced trades among the active traders. The trading partners are chosen by the system with respect to the open orders within the relevant order book. So generating the networks in Section 6.1 for an active market with a sufficient trading history evenness can be assumed and roughly approximated by a transaction graph of the active traders showing a quite complete structures i.e. strongly connected with balanced connections for each of all vertices

$$\delta_{\text{deg}}^{M|S,in}(v_i) \approx \delta_{\text{deg}}^{M|S,out}(v_i) \tag{6.1}$$

as well as within the share and money flow counts for the vertices

$$\delta_{\text{flow}}^{M|S,out}(u) \approx \delta_{\text{flow}}^{M|S,in}(u) \tag{6.2}$$

see Section 4.3). This structure is generated by regular traders that act heterogeneously (arbitrage traders, fundamental traders) and itself may produce strong deviations from this description due to the amount of heterogeneity. However, the structure will change in the moment the market is penetrated by traders with malicious intentions.

6.2.2 Money Transfer Pattern

Following Section 3.3 the pattern of malicious intentions emanates the market internal incentive system where in the case of play money markets the cost of trading according to the market rules can be significantly higher than the cost of manipulating. Here the trader tries to augment the monetary depot value of his own trading account by trying to transfer money from other accounts controlled by him (multiple identity accounts, accounts of other traders collaborating with him). In terms of patterns we refer to this pattern as the *money transfer pattern*. Within the transaction networks from Section 6.1 this pattern manifests itself in vertices with strong asymmetric flow counts in the global moneyflow network (high inbound money flow count for the vertex in favour of the manipulation, $\delta_{\text{flow}}^{M,in}(u) \gg \delta_{\text{flow}}^{M,out}(u)$), which is usually bound to few other selfcontrolled or allied traders as money sources in the global moneyflow network in Section 6.1.1 ($\delta_{\text{deg}}^{M,in}(u) \gg \delta_{\text{deg}}^{M,out}(u)$, $\delta_{\text{deg}}^{M,in}(u) \rightarrow |\,\{\text{accounts controlled by } u\}\,|$). These traders try to bypass the market inherent anonymity (cp. Section 3.4) to point out the counterpart of a certain order placed to the order book and thereby controlling the transaction flow. Low liquidity and trading volume in certain shares strongly supports this fraudulent activity by opening space for breaking the described market and order book inherent anonymity. This is possible in markets during times of low activity i.e. during nighttime - with respect to the time zone of a locally operated market. The transactions of the center of the structure to the other traders is dependent to the extent this trader tries to 'cover' his actions among regular behaviour, but is systematically lower than the ties with his associates.

In Figure 6.8 the graph of such a network is given exemplarily with a group of three fraudulent traders where one is favoured by the fraudulent actions. The fraudulent trader favoured by the manipulation here has balanced moneyflows to other traders but high moneyflows from accomplices. With respect to the center vertex the group appears as a strong centralised inbound structure with strong inbound moneyflows from a small number of other vertices (accomplices). During runtime of a market these actions result in high positions of these traders in the depot value ranking. Figure 6.9 shows a ranking with the assumed fraudulent traders marked "(RV)" for Regelverletzung (german for rule breaching) on top 20 positions as described in Section 3.3 (the total ranking had 2099 depot value entries).

The pattern within the eigensystem

The intention of a behaviour that tries to get a better rank lies in gathering as much money as possible and thus, in a play money market, increasing the own depot value

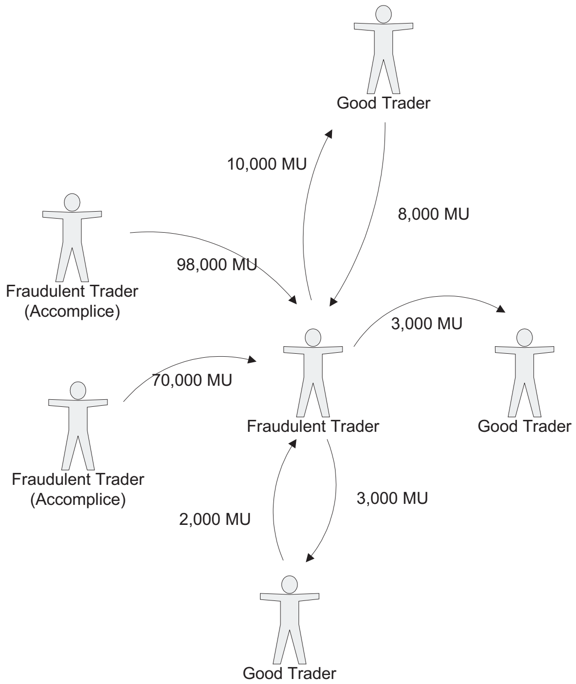

Figure 6.8: Example of traders acting conforming to a money transfer pattern

to have a good position in the ranking as the higher the position there, the better the reward. This can be detected by analysing the global moneyflow network $\mathcal{G}^M$ described in Section 6.1.1. Here the vertex showing a manipulative behaviour of getting a better rank depicts the center of a star-like structure. The trader depicted by the central vertex is thereby hand in glove with vertices he is directly connected with. The direction of the moneyflow is inbound to the manipulating vertex in favour of the manipulation, so the centre of the star-like structure.

6.2.3 Price Manipulation Pattern

Compared with the motivation to get a higher rank, the signalling manipulator acts not for his own account (or account of a certain group), but is even willing to give up personal assets for the real world's incentive i.e. the favoured party winning. The intention of acting is to influence the market value of a certain share, lowering or heightening the share's price to a level different from the price that would be reached without the manipulation. This is achieved by large buy or sell offers of the given share. In terms of patterns we refer to this pattern as the *price manipulation pattern*. While the share is being manipulated it is called overvalued or undervalued, repectively, while the wrong rating is created by the efforts of a single trader or a (small) group of traders.

Price manipulation shows up in the sharewise shareflow network structures described in Section 6.1.4. Vertices of traders acting in the described way buy (overrating, heightening the share's price) or sell (underrating, lowering the share's price) large

Platz	Benutzer		Ergebnis
1.	GB-star		2201725.35
2.	derfla66		2142852.51
3.	emheld		2060216.96
4.	spanier	(RV)	1255686.69
5.	rolfsuter		1232933.80
6.	siggy		1192518.15
7.	blaeuer		1191727.58
8.	Elite Force		1104088.42
9.	ergond	(RV)	1102194.84
10.	Mr. CPS	(RV)	1097212.83
11.	mwuo1		1085579.00
12.	olily330		1053340.05
13.	HJS		1052376.84
14.	cruz	(RV)	1025229.06
15.	elpibo		987625.61
16.	wilmots24	(RV)	981431.08
17.	klatschi		968268.23
18.	boix		870572.97
19.	mindli44		864468.22
20.	wolfso		853926.77

Figure 6.9: User ranking of the prediction market for the 2007 elections for the national parliament in Switzerland with traders assumed to manipulate being marked (RV) (cp. Section 3.3)

amounts of shares from or to arbitrary other traders. The type of the network thus reflects the transactions of the share the manipulative actions are accomplished in.

In Figure 6.10 a shareflow network for share *Share3* is described as it could be extracted from a market system following Section 6.1.4. If the depicted connections to the central trader (Fraudulent Trader) are all connections of this trader with respect to share *Share3* a price manipulation pattern can be assumed. Here the fraudulent trader tries to act in only one direction namely either outbound or inbound. The pattern structure can be described as centralised with regular amounts of connections to other traders but notably stronger outbound than inbound shareflows in case of lowering the price or stronger inbound than outbound shareflows in case of increasing the price (cp. Section 3.3.2). During runtime of a market an external indication for this pattern can be found in a notable difference between the price of a certain share in the market and the course of the same share within other forecasting systems like polls.

The pattern within the eigensystem

Transactions based on signalling intentions differ from the ones motivated by increasing the rank as the goal of the manipulation. Here the actions target the shares or more exactly one specific share with the manipulative actions aiming at the overvaluation or undervaluation of the share's price. So the patterns are to be found in the results of the sharewise shareflow networks $\mathcal{G}_s^S$ described in Section 6.1.4. The depot of the manipulative trader thereby depicts strong inbound or outbound flows in case of overvaluation or undervaluation, respectively.

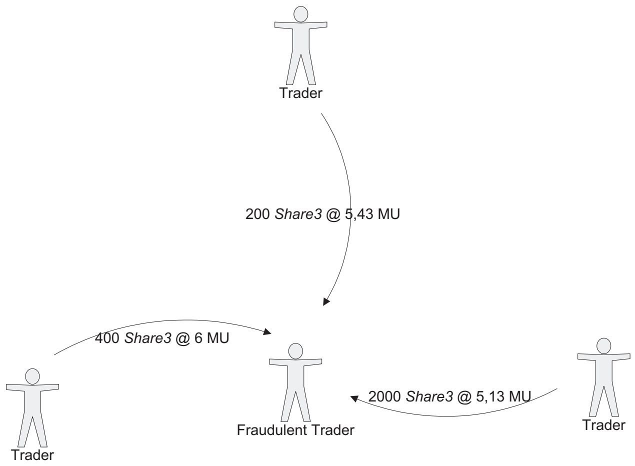

Figure 6.10: Example for a trader acting conforming to a price manipulation pattern

6.3 Procedures

On the basis of the proposed patterns and their characteristics in the eigensystem procedures are described that serve the purpose of finding these strucures within the eigensystem. First, the patterns are sought with threshold variables introduced for defining the accuracy of the search with respect to the 'pureness' of the patterns. In the second part the eigensystem is analysed by the clustering methods described in Section 5.3.3. By metrics that give information about vertices belonging to certain structures relations among the vertices and information about the transaction flow within that structure are determined.

6.3.1 Pattern Search

Having in mind the pattern descriptions of Section 6.2 structured procedures for analysing the eigensystem of a transaction network and its meanings are described. The procedure in a first step detects basic structures introduced in Section 6.2 by the analysis of the spectrum of the eigensystem. In a second step details about the found structures are discovered by analysing the eigenvectors corresponding to the relevant eigenvalues found in the first step. Thereby the vertices of the network relevant for the structure are found by a systematic comparison of the eigenvectors' components. Also information of the direction of flows is unveiled by the components' complex phase information which hints at the source of actions. So which trader originally affects the structure most, is identified. In the following listing the items describe the steps of the procedure.

The required configuration parameters for the described procedures are:

- x depicting the cumulated variance percentage the relevant eigenvalues should cover,

- δ_1 depicting a threshold for accuracy in selecting corresponding eigenvalues,
- δ_2 depicting a threshold for accuracy in selecting corresponding anchors,
- δ_3 depicting a threshold for accuracy in selecting corresponding members.

1. Find patterns: Analyse the spectrum
   (a) Sort subspaces according to their importance
      - Order eigenvalues by absolute value (see Equation 5.13).

   (b) Decide about relevant patterns
      - Select the set of eigenvalues that cover x percent of the data variance (see Equation 5.19).

   (c) Find patterns with a certain accuracy δ
      - Find corresponding eigenvalues $i \in \mathbb{R}^+$ and $j \in \mathbb{R}^-$ with $|\lambda_i| \approx |\lambda_j|$ or $||\lambda_i| - |\lambda_j|| \leq \delta_1$, respectively, with δ_1 depicting the given threshold for accuracy.

2. Find out details about the patterns: Analyse eigenvectors corresponding to the matched eigenvalues λ_i and λ_j
   (a) Find the central trader in the structure (anchor)
      - Check phase information of the largest eigenvector component x_{im} with $Abs(x_{im}) = \max_k Abs(x_{ik})$ of $\mathbf{x}_i$ against phase information of the same eigenvector component x_{jm} within other corresponding eigenvectors $\mathbf{x}_j$. If the phase information is less than a given threshold δ_2, $\varphi(x_{im}, x_{jm}) \leq \delta_2$, then they are corresponding and thus m is the central trader or anchor within this structure denoted by the subspace.

   (b) Find traders that belong to the structure (members)
      - Check phase information of the other eigenvector components x_{in} with $n \neq m$ of $\mathbf{x}_i$ against the phase information of the other eigenvector components x_{jn} with $n \neq m$ of $\mathbf{x}_j$. If the absolute difference of phase information difference $\varphi(x_{in}, x_{jn})$ and π is less than a given threshold δ_3, $|\pi - \varphi(x_{in}, x_{jn})| \leq \delta_3$, then they are corresponding and thus belong to the same structure as members.

   (c) Discover details about the direction of communication within the pattern
      - For a member n the two corresponding eigenvector components x_{in} and x_{jn} the direction of communication flow towards or from the central trader can be detected with
         - $\pi - \varphi(x_{io}, x_{jo}) < 0$ for flows towards the centre node (inbound with respect to the centre node).
         - $\pi - \varphi(x_{io}, x_{jo}) > 0$ for flows from the centre node (outbound with respect to the centre node).

6.3.2 Clustering

To enhance the basic procedure of finding the described structures within the eigensystem we apply the clustering technique introduced in Section 5.3.3.

Taken the descriptions of manipulative trading patterns within an eigensystem in Section 6.2 the search for these patterns can be described as procedures for systematically analysing the eigensystem. The procedures make use of the characteristics like strengths of eigenvalues and direction of eigenvectors within the eigensystem to identify the two fraud patterns investigated. Following Section 5.1 Euclidian distances among the subspaces denoted by the eigenvectors $\mathbf{X}$ are calculated which provide a fast solution to the complex task of a structured comparison described in the Section 6.3.1, resulting in assignments of vertices to their subnetworks. The clustering technique reorders the eigensystem regarding the structural most central vertices and calculates the members of the subnetworks of these centres as described in Section 5.1.

A structured procedure for analysing the eigensystem and thus the network according to Section 5.3.3 is depicted by the following steps. Again the items describe the steps of the procedure.

1. Calculate the scalar product matrix
    - Build the scalar product matrix S as depicted by Equation 5.35.
2. Discover structural groups by relevant relations between vertices.
    - Find the most prominent vertices and assign counterparts by least distances within the scalar product matrix S and the procedure described at the bottom of Section 5.3.3.

6.4 Summary

As we have seen in this chapter the market system by default does not provide the direct trader-to-trader transaction data needed for the analysis of transaction networks proposed in Chapter 4. We introduce the network definitions needed for the proposed analysis in Section 6.1 and their derivation from an abstract accounting system. The manipulation patterns based on the fraudulent actions introduced in Section 3.3 are then described with regard to their appearance within these networks and in the context and appearance in the resulting eigensystems in Section 6.2. All characteristics of these appearances are then summarised in the procedures for finding patterns within the eigensystem of the proposed networks in Section 6.3.

7. Application in Prediction Markets

Within this chapter the ideas of the last chapter are applied to simulation market data in Section 7.1 and data from real world markets in Section 7.2. Both, simulation and real world markets result in transaction networks that are analysed and discussed by the techniques and interpretations introduced in Chapter 6. The simulation was conducted using a software framework written in JAVA[1] and MATHEMATICA[2]. For the real world data a market system written in PHP[3] was utilised.

7.1 Simulation Data

For the purpose of analysing simulated market data comparable to the transaction network evolving in real world markets we first assume the simulated market data to be free of the described manipulative actions (cp. Section 6.2.1). We then introduce perturbations on this regular market structure and display the changes in the eigensystem. As an intended perturbation we introduce the money transfer pattern described in Section 6.2.2 resulting mainly from the market's inherent incentive system (see Section 3.3.1). Additionally a rough approximation of the boundaries of changes in the spectrum due to such a perturbation is given in Section 7.1.2. Finally the price manipulation pattern described in Section 6.2.3 is added to the regular trading pattern which mainly results from incentive incompatibility in the super-game described in Section 3.3.2.

For each of the simulations the original simulated adjacency matrix showing the transaction network of the matrix is given. This matrix is then transformed into

[1] http://java.sun.com/
[2] http://www.wolfram.com/
[3] http://www.php.net/

a Hermitian adajacency matrix following the description in Section 5.2. For this matrix the eigensystem is calculated resulting in a set of eigenvalues and their corresponding eigenvectors which for the simulated graphs is given in total as a tabular listing ordered by the descending absolute eigenvalues. Following the result of the eigensystem calculation the real and absolute values distribution of the eigenvalues is given. As the relevance of the eigenvalues in terms of their information content can be described by means of the variance the cumulated variance is given to each of the eigensystems resulting from the simulations. To retain clarity the simulated markets are built with 20 traders which results in presentable data. Each simulation is shown in two exemplary runs with varied parameters followed by the results of several simulation runs to demonstrate the stability of the eigensystem. For simulation purposes we take only parts of a market, thus accepting slight violations of the accounting system i.e. the balance sheet total has to be zero. As we only intend to show the impact of perturbation on markets this can be considered as an acceptable approximation.

The parts of this section consist of the following simulations:

1. Regular Market Activity (Section 7.1.1)
   - Analysis of a network of a regular market environment (pages 72 to 81)
     - Adjacency matrix, eigensystem, spectrum, cumulated variance, and eigensystem visualisation in Figures 7.1 to 7.5
     - Spectra and cumulated variance of multiple simulations in Figures 7.6 and 7.7
   - Analysis of a network of a regular market environment with an increased standard deviation of the connection weights (pages 77 to 81)
     - Adjacency matrix, eigensystem, spectrum, cumulated variance, and eigensystem visualisation in Figures 7.8 to 7.12
     - Spectra and cumulated variance of multiple simulations in Figures 7.13 and 7.14
2. Money Transfer Pattern (Section 7.1.2)
   - Analysis of a regular market with money transfer perturbation (pages 83 to 87)
     - Adjacency matrix, eigensystem, spectrum, cumulated variance, and eigensystem visualisation in Figures 7.15 to 7.19
     - Spectra and cumulated variance of multiple simulations in Figures 7.38 and 7.39
   - Analysis of a regular market with an increased standard deviation of the connection weights and a money transfer perturbation (pages 88 to 94)
     - Adjacency matrix, eigensystem, spectrum, cumulated variance, and eigensystem visualisation in Figures 7.22 to 7.26

        - Spectra and cumulated variance of multiple simulations in Figures 7.45 and 7.46
    - Simulations for lower and upper bounds of structural change in spectrum
        - Spectra and cumulated variance of multiple simulations of a lower and upper bound for the regular market with money transfer pattern in Figures 7.29 and 7.30
        - Spectra and cumulated variance of multiple simulations of a lower and upper bound for the regular market with an increased standard deviation of the connection weights with money transfer pattern in Figures 7.31 and 7.32

3. Price Manipulation Pattern (Section 7.1.3)

    - Analysis of a regular market with price manipulation perturbation (pages 96 to 100)
        - Adjacency matrix, eigensystem, spectrum, cumulated variance, and eigensystem visualisation in Figures 7.33 to 7.37
        - Spectra and cumulated variance of multiple simulations in Figures 7.38 and 7.39
    - Analysis of a regular market with an increased standard deviation of the connection weights and a price manipulation perturbation (pages 101 to 107)
        - Adjacency matrix, eigensystem, spectrum, cumulated variance, and eigensystem visualisation in Figures 7.40 to 7.44
        - Spectra and cumulated variance of multiple simulations in Figures 7.45 and 7.46
    - Simulations for lower and upper bounds of structural change in spectrum
        - Spectra and cumulated variance of multiple simulations of a lower and upper bound for the regular market with price manipulation pattern in Figures 7.47 and 7.48
        - Spectra and cumulated variance of multiple simulations of a lower and upper bound for the regular market with an increased standard deviation of the connection weights with price manipulation pattern in Figures 7.49 and 7.50

4. Multiple Patterns (Section 7.1.4)

    - Analysis of a regular market with a perturbation by multiple patterns (pages 109 to 113)
        - Adjacency matrix, eigensystem, spectrum, cumulated variance, and eigensystem visualisation in Figures 7.51 to 7.55
        - Spectra and cumulated variance of multiple simulations in Figures 7.56 and 7.57
    - Analysis of a regular market with an increased standard deviation of the connection weights and a perturbation by multiple patterns (pages 114 to 118)

- Adjacency matrix, eigensystem, spectrum, cumulated variance, and eigensystem visualisation in Figures 7.58 to 7.62
- Spectra and cumulated variance of multiple simulations in Figures 7.63 and 7.64.

7.1.1 Regular Market Activity

As shown in Section 3.4 the traders within a market of interest act anonymous towards the market system and therefore do regularly not know any of the matched trading partners. As the matching algorithm within such a market does only match by formal rules and not semantic likes or dislikes of certain traders or aspects of traders, the connections among the traders within the trader-trader network are arbitrary. As a transaction network evolves over time these arbitrary set connections add up to a joint connection value. Thus for the simulation networks the connection strengths are assumed to be identically normally distributed with an average strength depending on the trading activity of the individual trader. Thus for a regular market, a market with no fraudulent or manipulation activity, the transaction network results in a complete graph structure.

Simulating the introduced transaction setting we obtain e.g. the graph structure listed in the adjacency matrices in Figures 7.1 and 7.8. The transaction matrices were built by means of connections chosen randomly from a normal distribution with different standard deviations σ.

Following the simulation of the market's transaction data we calculate the eigensystem of the adjacency matrices which before were transformed to the Hermitian form in the way described in Chapter 6. This results in the eigenvalues and their corresponding eigenvectors given in Tables 7.1 and 7.2 with the eigenvalues in descending absolute value order. The real value distribution of the normed eigenvalues sorted by absolute size is given in Figures 7.2 and 7.9. In Figures 7.3 and 7.10 this distributions are given additionally in absolute values. In the cumulated variance plots in Figures 7.4 and 7.11 it can be clearly seen that for such a market structure almost 90 % of the total variance is described by the first eigenvalue. Regarding the cumulated variance the three largest absolute eigenvalues sum up to around 95 % of the total variance. The eigensystems in Tables 7.1 and 7.2 depict a complete network graph. Characteristic for this structure is the lack of corresponding eigenvalues in the sense of the procedures in Section 6.3. As can be seen the most dominant elements of the eigenspaces corresponding to the eigenvalues with alternating algebraic sign are not the same. For the eigensystems in Tables 7.1 and 7.2 this can be seen in the eigenvectors corresponding to the second and third eigenvalue (alternating algebraic sign) and their corresponding eigenvectors 2 and 3 (third to sixth column in upper row) where the main elements are in different components (component 8 in eigenvector 2 and component 5 in eigenvector 3 as indicated by a phase of zero as the eigenspaces are rotated as described in Section 5.1). As the rest of the eigenvalues' contributions to the total variances is low and adds no significant structures to the main structures, the whole spectra have no significant patterns but the one of slightly perturbed complete structures (see patterns in the eigensystem in Section 5.3.2). Thus the procedures in Section 6.3 do not match eigenvalue pairs with alternating algebraic signs whose eigenvector components have the required phase information structure (see procedures in Section 5.3.2).

As introduced in Section 5.3.1 the visualisation of the eigensystems of Figures 7.1 and 7.2 are given in Figures 7.5 and 7.12. Here the complete structures of the underlying graphs can be clearly assessed by the first eigenvectors (lying in the first row) and their phases that do not vary much for all eigenvector components. Figures 7.6 and 7.13 give the result for 10 simulation runs with the same distribution parameters for each network. They show a stability in the spectrum in the distribution of the major eigenvalues due to the cumulated variances that are also depicted in a superimposed way in Figures 7.7 and 7.14. To be able to compare distributions from the different simulations, the spectra of the simulation runs are superimposed in Figures 7.6 and 7.13 normed by the spectral norm of Equation 5.11. Variations in the eigenvalues are almost invisible in Figures 7.6 and 7.7. Even when increasing the standard deviation considerably, the spectra remain quite stable, although the variations have increased (Figures 7.13 and 7.14).

As can be seen in the two different network simulation runs in this section, the spectrum reacts on changes within the input transaction data. The spectrum of the homogeneous complete graph structure in Figure 7.2 appears quite perfectly according to the pattern description in Section 5.3.2 whereas the complete network with an increasing standard deviation results in a spectrum which appears more structured as can be seen in the low eigenvalues in Figure 7.9. Now the question arises in how far structure induced by random trading can be discerned from structure induced by manipulative trading. To investigate this we induce perturbation similar to the manipulative trading patterns introduced in Section 6.2. Since manipulative trading needs to have an impact on either the upper ranks in the depot value ranking (money transfer pattern) or the market prices (price manipulation pattern) the perturbation can be assumed to be rather high. The simulations with these two patterns are presented in the following sections.

$$\begin{pmatrix}
0 & 744 & 683 & 750 & 688 & 718 & 709 & 795 & 660 & 707 & 725 & 708 & 658 & 676 & 699 & 702 & 715 & 658 & 714 & 642 \\
730 & 0 & 672 & 785 & 655 & 652 & 719 & 740 & 737 & 730 & 678 & 628 & 767 & 697 & 689 & 679 & 749 & 720 & 689 & 692 \\
725 & 612 & 0 & 674 & 709 & 640 & 753 & 729 & 683 & 678 & 657 & 623 & 757 & 688 & 635 & 686 & 686 & 752 & 592 & 681 \\
748 & 607 & 783 & 0 & 710 & 727 & 760 & 701 & 709 & 671 & 749 & 730 & 699 & 702 & 797 & 735 & 786 & 793 & 733 & 708 \\
745 & 677 & 703 & 652 & 0 & 768 & 656 & 749 & 693 & 783 & 677 & 759 & 714 & 755 & 665 & 686 & 721 & 668 & 684 & 601 \\
811 & 681 & 698 & 656 & 640 & 0 & 605 & 734 & 748 & 661 & 713 & 773 & 675 & 733 & 715 & 712 & 730 & 646 & 641 & 686 \\
663 & 735 & 700 & 737 & 641 & 665 & 0 & 663 & 695 & 653 & 705 & 682 & 677 & 721 & 634 & 670 & 710 & 766 & 764 & 667 \\
687 & 784 & 762 & 685 & 625 & 706 & 774 & 0 & 698 & 736 & 698 & 772 & 639 & 660 & 687 & 619 & 818 & 710 & 762 & 732 \\
720 & 786 & 634 & 727 & 813 & 689 & 682 & 714 & 0 & 679 & 738 & 767 & 704 & 709 & 740 & 615 & 757 & 613 & 691 & 682 \\
719 & 751 & 743 & 755 & 685 & 662 & 598 & 797 & 663 & 0 & 563 & 733 & 758 & 716 & 739 & 779 & 740 & 700 & 568 & 692 \\
688 & 693 & 717 & 726 & 715 & 694 & 676 & 670 & 668 & 787 & 0 & 734 & 657 & 682 & 776 & 695 & 723 & 780 & 656 & 735 \\
661 & 655 & 680 & 656 & 733 & 731 & 730 & 705 & 738 & 689 & 667 & 0 & 740 & 863 & 617 & 694 & 735 & 715 & 727 & 748 \\
699 & 729 & 615 & 670 & 673 & 673 & 750 & 746 & 739 & 722 & 724 & 699 & 0 & 695 & 676 & 722 & 663 & 602 & 844 & 676 \\
770 & 616 & 710 & 620 & 680 & 671 & 698 & 639 & 643 & 773 & 658 & 679 & 687 & 0 & 672 & 816 & 766 & 659 & 715 & 776 \\
682 & 587 & 652 & 700 & 722 & 716 & 647 & 703 & 649 & 725 & 681 & 735 & 713 & 647 & 0 & 718 & 686 & 704 & 674 & 658 \\
679 & 758 & 549 & 645 & 649 & 681 & 712 & 645 & 609 & 673 & 742 & 677 & 644 & 710 & 757 & 0 & 666 & 608 & 667 & 705 \\
783 & 701 & 684 & 744 & 750 & 592 & 702 & 649 & 716 & 757 & 666 & 727 & 697 & 773 & 631 & 731 & 0 & 666 & 674 & 663 \\
732 & 664 & 729 & 713 & 655 & 741 & 694 & 661 & 723 & 696 & 651 & 628 & 666 & 727 & 708 & 660 & 729 & 0 & 695 & 656 \\
680 & 669 & 680 & 705 & 699 & 635 & 656 & 707 & 791 & 677 & 733 & 755 & 749 & 660 & 679 & 698 & 744 & 678 & 0 & 693 \\
804 & 698 & 671 & 679 & 766 & 658 & 665 & 692 & 651 & 693 & 698 & 661 & 706 & 769 & 657 & 642 & 673 & 704 & 661 & 0
\end{pmatrix}$$

Figure 7.1: Simulated transaction network of a regular market environment with $\mu = 700$ and $\sigma = 50$

λ_k	18804.8		-1488.39		-1406.26		-1357.22		-1276.1		-1227.95		-1171.59		-1146.94		-1107.83		-1042.54	
	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$
$\mathbf{x}_k$	0.2275	0.0326	0.2233	-0.1864	0.2191	1.8697	0.2279	0.2365	0.2234	-0.9526	0.2211	0.3467	0.2215	-2.9863	0.2268	-2.8451	0.2243	-0.1517	0.2257	1.7947
	0.1879	0.0103	0.1846	2.863	0.3035	0	0.0747	-2.6258	0.1669	0.5485	0.1933	-0.6594	0.2126	2.2263	0.3458	2.9267	0.1543	2.7523	0.3039	-2.0777
	0.1744	0.0236	0.4122	1.7632	0.0881	-2.8675	0.269	-2.9375	0.241	-0.8985	0.1814	-0.5787	0.1057	2.0228	0.1056	-2.4806	0.3529	1.7936	0.1029	-0.676
	0.2377	0	0.1368	0.2801	0.1743	-0.9169	0.3006	-1.5267	0.2032	-0.7714	0.0988	-3.0913	0.2618	-2.8218	0.3453	0	0.1427	1.4537	0.4334	1.703
	0.3383	0.0143	0.1963	-0.6272	0.104	-0.1034	0.0687	0.9684	0.0971	-2.4678	0.3641	2.4036	0.1828	1.6975	0.1291	-1.4457	0.1925	2.8613	0.1847	-1.3018
	0.2092	0.0109	0.1579	-3.056	0.1772	-2.1832	0.1964	0.111	0.3644	1.6419	0.234	1.9671	0.1166	1.303	0.1393	1.676	0.3476	0.7438	0.1924	-0.8007
	0.2098	0.021	0.1216	-1.9994	0.2893	1.4651	0.1045	1.2317	0.1931	-3.1053	0.1516	-2.8834	0.0054	3.0272	0.1844	1.6386	0.1053	0	0.2275	-2.5457
	0.2857	0.0156	0.1003	-1.07	0.1307	-2.2987	0.4838	-1.8759	0.2577	-2.6056	0.1856	1.6279	0.122	-0.1709	0.1444	1.0737	0.0699	-2.1515	0.0649	-2.9522
	0.1152	0.0104	0.1863	0.2001	0.0663	2.4431	0.3503	-1.0283	0.2542	0.8508	0.2241	-1.8752	0.4886	-0.7475	0.1963	-2.338	0.0796	-1.3247	0.2103	1.7568
	0.2254	0.0244	0.3035	2.7585	0.3122	2.4229	0.2509	0	0.1765	-2.6473	0.2076	-2.4909	0.0658	-1.1388	0.2593	-2.8671	0.2508	0.8743	0.0897	0.5847
	0.2847	0.0072	0.2195	-2.2563	0.2249	1.148	0.1087	1.4238	0.21	-1.0106	0.1062	-0.8838	0.2628	0	0.2718	-0.6322	0.2704	-2.277	0.0186	0
	0.3027	0.0193	0.1245	1.7425	0.0674	0.3786	0.2121	-2.6638	0.0336	-2.5873	0.1519	0	0.3192	-2.2151	0.3614	-1.0156	0.2531	0.5351	0.1747	1.5211
	0.1157	0.0191	0.2202	0	0.2296	-1.937	0.0758	1.4498	0.156	1.7459	0.1937	1.1484	0.1182	-1.2453	0.1763	1.6383	0.2938	1.1205	0.0804	-0.2351
	0.0468	0.032	0.2603	-1.3981	0.4232	-1.3876	0.1144	-1.6744	0.2905	0	0.1252	-1.7296	0.2723	-1.3986	0.1365	-2.0001	0.2225	-1.6171	0.1997	-0.7518
	0.4321	0.0258	0.2241	2.684	0.139	0.3937	0.157	2.8559	0.0764	-0.8149	0.4368	2.5292	0.1668	-2.8025	0.0675	2.6771	0.1325	-0.6569	0.4094	-2.8316
	0.0849	0.0375	0.2853	-0.0004	0.2123	2.5776	0.1157	-1.7583	0.4131	2.9815	0.3891	-0.4873	0.17	2.0683	0.1972	-1.0573	0.1062	2.7338	0.1766	-1.8447
	0.1571	0.0368	0.2429	2.1659	0.1403	2.6946	0.1849	2.3816	0.2078	1.1958	0.0451	1.2743	0.1694	-0.863	0.1216	1.3399	0.3286	-2.7329	0.2132	0.0363
	0.1678	0.0201	0.0754	0.3406	0.1514	-1.7011	0.127	1.3239	0.1179	2.5462	0.2567	0.2376	0.2281	-1.3957	0.3256	-2.2274	0.1289	-2.5059	0.096	2.3754
	0.0685	0.0147	0.1583	-3.0377	0.4185	-2.655	0.25	0.2245	0.1212	0.4598	0.0742	-1.3861	0.1945	2.0597	0.1552	-1.3047	0.3006	-2.3769	0.2842	3.006
	0.1959	0.0176	0.3332	1.5661	0.088	1.8208	0.2817	2.2598	0.281	2.0142	0.1728	-3.0005	0.2943	-2.8611	0.2437	-0.181	0.1456	-0.351	0.2597	2.8038

λ_k	-1028.23		-936.1		-886.24		-865.74		-776.8		-759.74		-673.48		-582.36		-567.85		-503.43	
	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$	$\lvert z\rvert$	$\phi(z)$
$\mathbf{x}_k$	0.2236	-2.416	0.2265	-0.8193	0.2238	-0.5882	0.2256	-2.7794	0.2203	0.7854	0.2192	1.4408	0.2278	-2.6803	0.221	-0.5602	0.2224	-1.0991	0.2208	-1.1452
	0.2248	-1.8011	0.1533	-1.5275	0.3746	2.0914	0.1567	-2.3229	0.0913	-1.5853	0.2343	-2.4954	0.1632	-2.5443	0.1387	-2.5285	0.3466	-1.9467	0.1364	0
	0.0862	0.0529	0.2823	-0.7136	0.1665	2.9159	0.3762	1.5526	0.1204	2.3534	0.317	0.988	0.1611	2.739	0.1429	-2.8809	0.138	0	0.1854	-2.9658
	0.0182	-2.652	0.1902	-2.3983	0.1032	0.8474	0.0894	0.5315	0.2558	0.6802	0.1804	-0.6811	0.3288	-0.8282	0.0872	-0.8663	0.1512	-0.2229	0.2555	1.3999
	0.2242	1.0652	0.2795	1.7964	0.0113	0.3634	0.3974	-2.6598	0.2498	2.6695	0.2352	0	0.1434	-2.6915	0.3317	0.5378	0.0349	-2.6564	0.1797	-2.7601
	0.1272	-1.6229	0.4525	1.5767	0.187	1.9362	0.2263	-1.0081	0.1313	0	0.0873	2.0841	0.2577	0.9136	0.0792	0.6835	0.0979	2.2475	0.2622	-2.5664
	0.5147	0.1779	0.0954	-1.7023	0.1899	0.3562	0.1663	-2.3269	0.1748	3.051	0.2146	1.1976	0.1427	1.1934	0.2653	-2.8795	0.421	0.9063	0.0845	0.6025
	0.1475	1.2537	0.0521	0	0.2505	-1.8681	0.0666	1.3607	0.3229	-1.7784	0.1325	1.9636	0.3449	-2.6241	0.3341	1.5408	0.1101	-0.8011	0.2353	-0.1464
	0.1259	0.3998	0.1863	2.8167	0.0751	1.7659	0.1849	2.2753	0.2291	-0.3367	0.2006	-1.5073	0.1833	-2.7202	0.3583	1.4971	0.1779	2.7165	0.0743	1.4949
	0.3451	0.1683	0.2302	0.5085	0.2454	-0.9086	0.0548	0.3241	0.238	-2.2886	0.2255	-2.2248	0.1951	-1.8635	0.2115	-0.6052	0.1342	-1.191	0.1902	-1.9973
	0.1998	-2.6913	0.0626	1.9379	0.2701	-3.0469	0.3321	3.0868	0.3121	2.1839	0.166	2.3733	0.2441	0.8117	0.1686	-0.6718	0.2321	-2.2506	0.2065	1.7586
	0.0643	-1.0732	0.3354	1.2926	0.1696	0.8584	0.0947	-1.645	0.3445	-2.1375	0.1201	-0.2528	0.2159	0.2438	0.2155	1.9757	0.0596	2.6102	0.3287	-2.9899
	0.1053	-2.117	0.194	3.0338	0.4193	-2.7854	0.1112	-0.1647	0.166	-2.551	0.2561	-0.2813	0.4314	-1.8601	0.0708	0.1602	0.1452	2.0743	0.3712	-0.5923
	0.196	0	0.1164	0.1916	0.097	-1.1372	0.274	-0.8105	0.0439	0.2764	0.1196	-1.3505	0.1078	0.4651	0.1872	-2.6285	0.4395	2.6145	0.2511	-2.3322
	0.3093	1.5371	0.0354	2.7137	0.2684	-2.4336	0.1315	0.4933	0.0831	2.5739	0.1001	-2.2416	0.1129	-0.1803	0.1322	0	0.1694	-0.8456	0.2074	2.47
	0.2237	-0.846	0.1229	-2.5453	0.2607	-0.0843	0.0906	2.3926	0.1394	-2.5498	0.2122	-3.1148	0.2911	0	0.2163	-1.115	0.1777	2.6973	0.2275	-1.305
	0.1674	2.18	0.2023	-1.6594	0.0971	0	0.2014	0.7788	0.2487	1.2431	0.4603	-1.9455	0.1853	-2.129	0.236	3.0488	0.0991	-0.0334	0.3574	3.1228
	0.1346	-2.8324	0.4052	3.0187	0.0271	1.0573	0.3452	-1.1002	0.4244	-1.6619	0.2743	2.1385	0.2177	2.086	0.1351	-1.7121	0.0643	0.1141	0.1902	2.1111
	0.2456	2.9894	0.136	1.1856	0.2938	-1.1417	0.0857	0	0.0833	2.763	0.1765	0.1274	0.0211	2.1492	0.3671	1.9774	0.3499	2.4556	0.1061	0.8583
	0.2406	-2.9235	0.1393	0.4689	0.2193	-2.4242	0.3121	2.6622	0.1379	-0.2623	0.2264	-1.4778	0.0761	1.5152	0.211	-2.3506	0.2653	-2.2179	0.0796	-2.5213

Table 7.1: Eigensystem for the simulated regular market activity described in Figure 7.1 with $z = |z|e^{i\phi}$

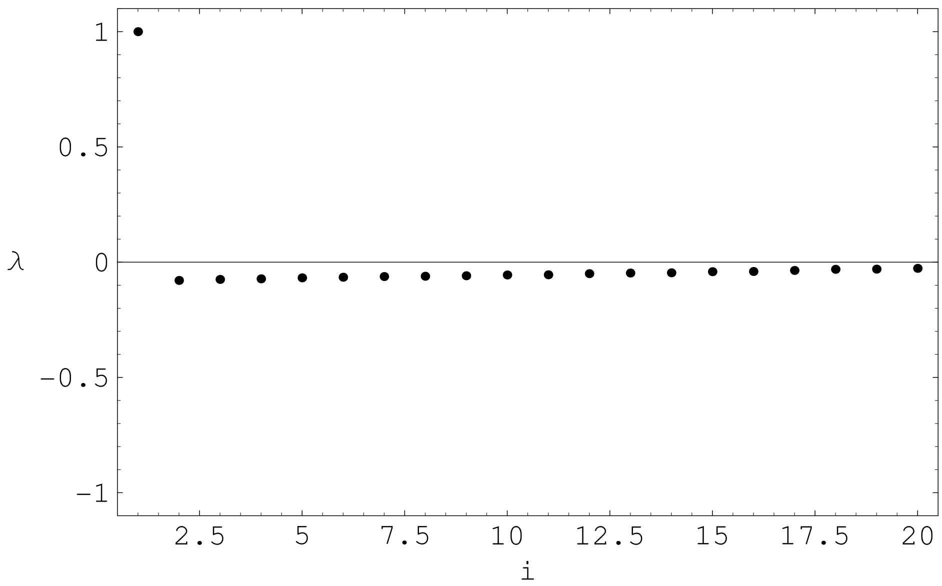

Figure 7.2: Normed eigenvalues of the simulated regular market described in Figure 7.1

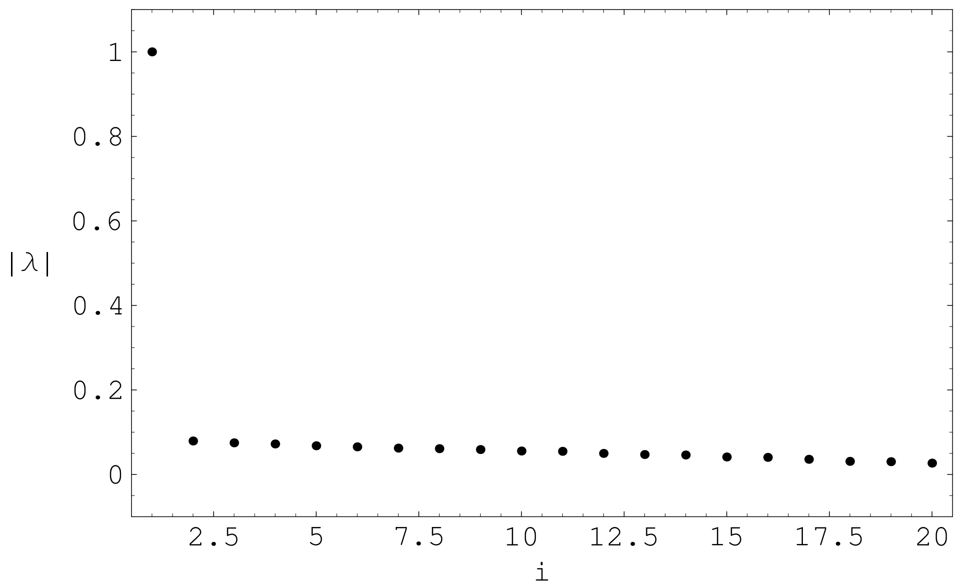

Figure 7.3: Normed eigenvalues of the simulated regular market described in Figure 7.1 with absolute values

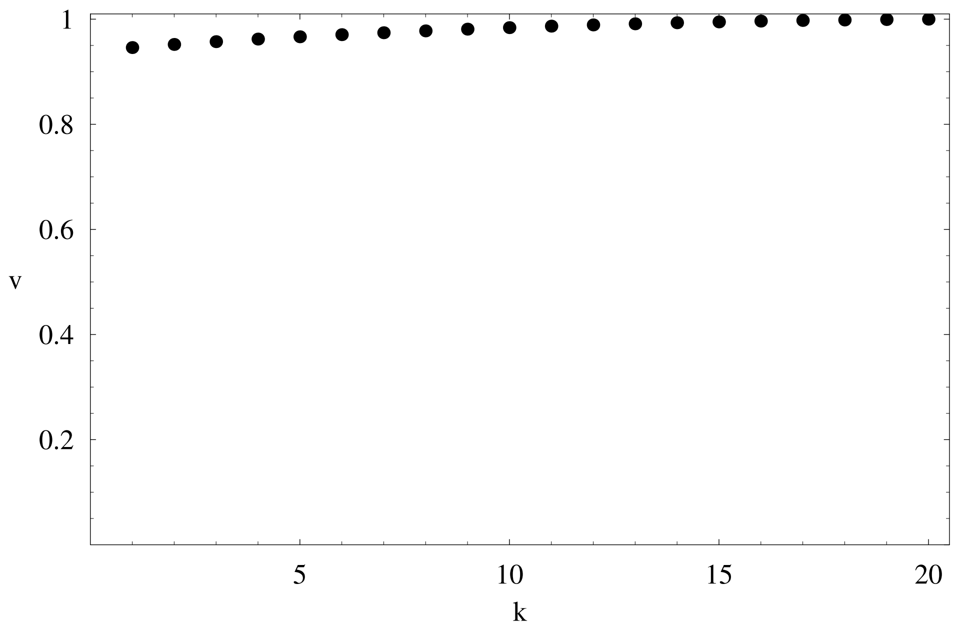

Figure 7.4: Cumulated variance of the 20 first largest absolute eigenvalues of the simulated regular market described in Figure 7.1

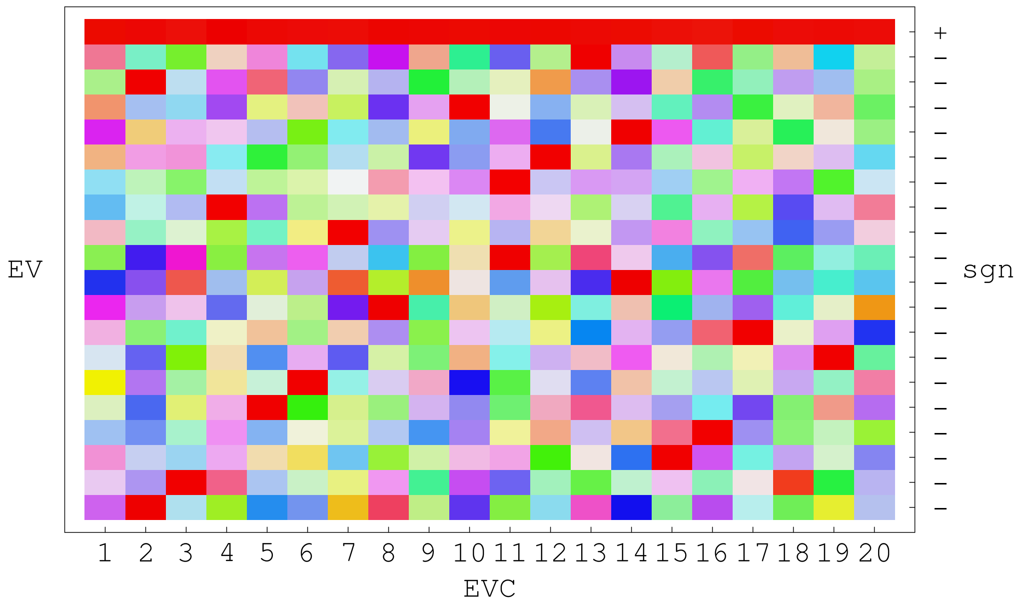

Figure 7.5: Eigensystem of the simulated regular market described in Figure 7.1

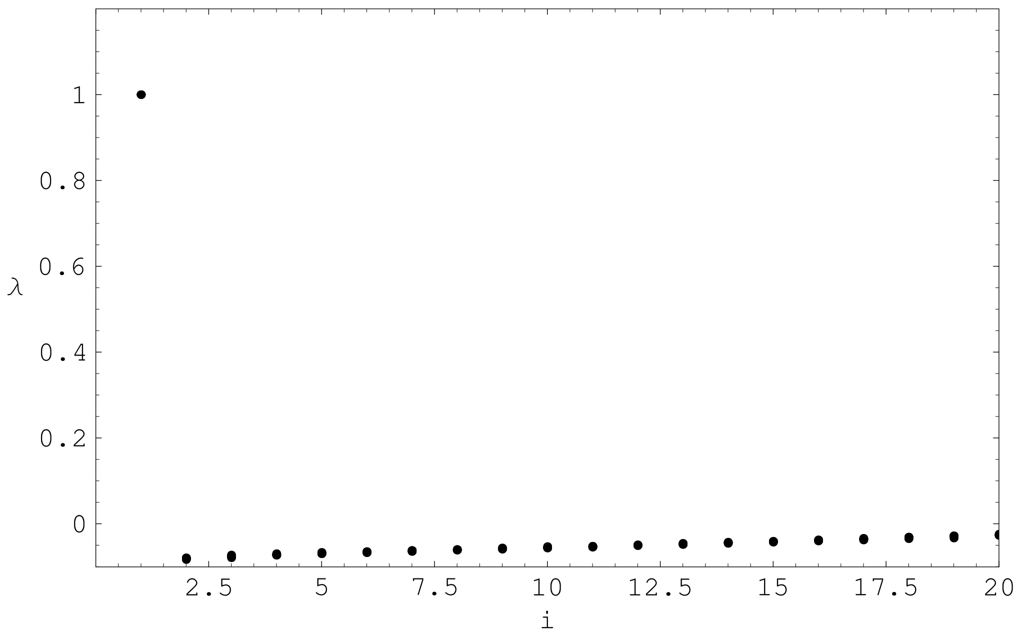

Figure 7.6: Superimposed normed eigenvalues of the simulated regular market described in Figure 7.1 for 10 simulation runs

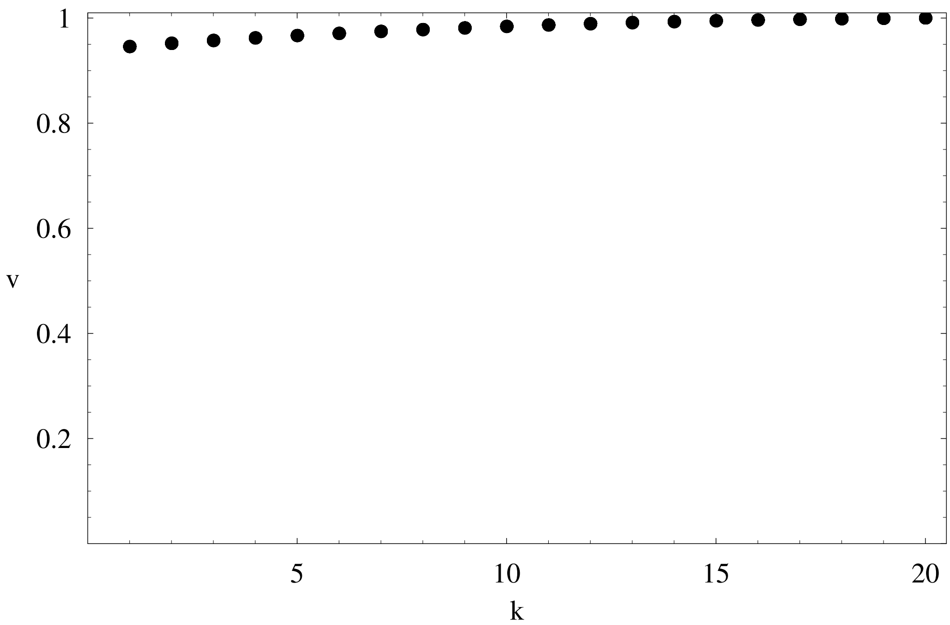

Figure 7.7: Superimposed cumulated variances of the 20 first largest absolute eigenvalues of the simulated regular market described in Figure 7.1 for 10 simulation runs

$$\begin{pmatrix}
0 & 849 & 705 & 776 & 556 & 584 & 929 & 764 & 767 & 699 & 591 & 341 & 835 & 629 & 893 & 718 & 588 & 593 & 557 & 579 \\
595 & 0 & 707 & 739 & 732 & 795 & 397 & 742 & 625 & 685 & 653 & 530 & 900 & 886 & 842 & 883 & 813 & 748 & 584 & 561 \\
607 & 749 & 0 & 666 & 620 & 514 & 546 & 687 & 679 & 527 & 802 & 844 & 743 & 605 & 656 & 824 & 706 & 559 & 790 & 598 \\
532 & 926 & 654 & 0 & 829 & 566 & 646 & 723 & 652 & 655 & 900 & 679 & 673 & 698 & 654 & 859 & 671 & 629 & 912 & 541 \\
635 & 949 & 663 & 587 & 0 & 507 & 783 & 768 & 854 & 720 & 582 & 764 & 954 & 705 & 877 & 613 & 539 & 892 & 657 & 790 \\
762 & 829 & 511 & 772 & 440 & 0 & 990 & 501 & 587 & 762 & 667 & 787 & 817 & 685 & 743 & 760 & 535 & 597 & 489 & 952 \\
867 & 756 & 860 & 1039 & 685 & 648 & 0 & 507 & 832 & 741 & 512 & 552 & 444 & 577 & 494 & 474 & 495 & 826 & 591 & 655 \\
720 & 678 & 815 & 743 & 439 & 629 & 768 & 0 & 823 & 653 & 962 & 902 & 572 & 703 & 592 & 840 & 663 & 500 & 893 & 572 \\
689 & 546 & 731 & 638 & 401 & 600 & 710 & 516 & 0 & 661 & 764 & 571 & 444 & 632 & 710 & 817 & 838 & 553 & 638 & 698 \\
683 & 381 & 629 & 902 & 834 & 682 & 748 & 636 & 752 & 0 & 711 & 334 & 986 & 725 & 445 & 763 & 617 & 742 & 537 & 577 \\
803 & 538 & 694 & 875 & 745 & 755 & 564 & 861 & 745 & 804 & 0 & 551 & 443 & 612 & 833 & 778 & 980 & 900 & 844 & 607 \\
733 & 618 & 763 & 943 & 605 & 505 & 685 & 639 & 673 & 692 & 590 & 0 & 844 & 620 & 614 & 769 & 514 & 591 & 761 & 793 \\
447 & 719 & 714 & 575 & 443 & 655 & 774 & 770 & 714 & 627 & 957 & 619 & 0 & 666 & 405 & 743 & 714 & 737 & 363 & 656 \\
670 & 749 & 636 & 816 & 833 & 589 & 539 & 535 & 562 & 528 & 819 & 766 & 737 & 0 & 784 & 782 & 1016 & 701 & 492 & 837 \\
872 & 739 & 609 & 1001 & 651 & 775 & 779 & 754 & 846 & 713 & 619 & 917 & 696 & 689 & 0 & 638 & 631 & 828 & 538 & 816 \\
556 & 610 & 597 & 838 & 696 & 595 & 995 & 883 & 597 & 784 & 744 & 899 & 492 & 683 & 718 & 0 & 600 & 966 & 960 & 1057 \\
811 & 629 & 741 & 727 & 848 & 583 & 627 & 469 & 556 & 294 & 462 & 658 & 649 & 847 & 816 & 810 & 0 & 789 & 700 & 434 \\
841 & 744 & 470 & 707 & 955 & 569 & 730 & 603 & 935 & 732 & 627 & 733 & 752 & 629 & 542 & 833 & 473 & 0 & 637 & 638 \\
836 & 770 & 744 & 770 & 845 & 425 & 661 & 857 & 492 & 449 & 854 & 705 & 749 & 918 & 1024 & 726 & 864 & 581 & 0 & 565 \\
402 & 940 & 759 & 818 & 658 & 805 & 670 & 782 & 452 & 896 & 814 & 651 & 580 & 717 & 567 & 694 & 577 & 691 & 908 & 0
\end{pmatrix}$$

Figure 7.8: Simulated transaction network of a regular market environment with $\mu = 700$ and $\sigma = 150$

λ_k	18716		-2452.53		-2182.28		-2030.71		-1937.06		-1651.44		-1546.46		-1469.45		-1282.09		-1109.11	
	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$
	0.2202	0.0025	0.2291	2.9046	0.2175	1.6255	0.2391	-0.9814	0.2254	-2.1723	0.212	2.8711	0.2209	-0.1434	0.2242	1.0984	0.2143	1.0815	0.2147	0.4764
	0.2123	0.0091	0.1805	-0.7289	0.1378	0	0.2026	-2.039	0.276	1.5666	0.2314	1.5993	0.3714	-1.274	0.042	-0.6498	0.0886	2.344	0.2725	2.8857
	0.0949	0.0087	0.4193	-2.5352	0.0414	2.7514	0.2873	2.5463	0.131	-1.3339	0.0725	2.6614	0.0664	2.5907	0.2102	0	0.2496	-1.3266	0.2237	0.7467
	0.4218	0.0488	0.0752	-2.7636	0.1781	1.9755	0.2794	-2.4116	0.0464	-1.2378	0.3218	-2.2193	0.0633	1.8311	0.3211	-1.0101	0.0216	-1.7675	0.0917	-2.7846
$\mathbf{x}_k$	0.1237	-0.0349	0.1001	2.5037	0.0947	-3.1005	0.2396	0.1282	0.2977	-2.7935	0.1687	2.3432	0.1981	-1.3751	0.1218	2.7323	0.1265	-1.2635	0.2543	2.1308
	0.088	-0.0535	0.2973	3.0738	0.2027	-3.1218	0.0939	1.4013	0.2934	3.0685	0.0731	-1.0958	0.3123	2.588	0.268	1.9584	0.3342	-2.1614	0.2234	-0.4454
	0.0271	0.0352	0.1712	0	0.2672	-2.9604	0.0886	0.2775	0.256	0.7939	0.1381	-2.4813	0.3095	-1.4625	0.2112	-2.7152	0.3413	-1.294	0.1494	-1.9664
	0.1263	-0.0164	0.129	-0.1057	0.3882	2.4556	0.1209	2.4884	0.3095	1.3588	0.2037	0.5025	0.0425	-0.4798	0.3111	1.9314	0.2583	0.0471	0.1479	0.3576
	0.3198	0.0364	0.0875	1.9953	0.1059	0.808	0.1279	1.8107	0.267	-0.6582	0.0428	0	0.1588	-2.9943	0.2966	2.5825	0.1059	2.8063	0.2741	-2.7975
	0.288	-0.0016	0.1889	-1.6854	0.0825	-0.8155	0.2088	-1.3284	0.2542	1.4016	0.197	-0.0739	0.3098	0.5745	0.1115	-1.1999	0.2658	-0.6413	0.1421	1.9366
	0.0305	-0.0112	0.2706	-0.0821	0.1072	-0.717	0.372	0.5362	0.1202	-1.6124	0.3805	2.6563	0.0767	-1.2021	0.2222	-2.3465	0.1281	-3.0084	0.1618	0
	0.1196	-0.006	0.2704	-0.868	0.4666	1.0274	0.5215	0	0.0938	1.8814	0.1232	0.2973	0.1275	-0.693	0.0809	-1.2329	0.1245	-2.935	0.35	0.9551
	0.2063	0.0356	0.2622	1.6325	0.3587	-3.0672	0.1815	-2.4265	0.1131	0.1707	0.3208	-0.339	0.0658	-0.5063	0.0255	-2.2446	0.0943	3.1076	0.0486	-0.0943
	0.1661	-0.0078	0.0477	2.0755	0.1628	-1.3742	0.147	1.2297	0.2193	0	0.2234	3.0731	0.2328	2.784	0.1601	1.7928	0.1431	0	0.2697	0.214
	0.364	-0.0316	0.2037	0.6525	0.0551	-2.3316	0.0673	2.8918	0.0822	1.3539	0.1434	1.4339	0.2952	1.1101	0.3502	-0.2784	0.4086	-2.9155	0.2392	-2.9903
	0.2304	0	0.3173	1.622	0.2064	-1.7132	0.1418	1.3311	0.0894	-2.4353	0.1178	-0.9058	0.1956	-2.3614	0.2916	-0.99	0.0533	0.9877	0.2663	-2.9811
	0.2106	0.013	0.0805	-1.45	0.3324	-1.9484	0.2142	2.6936	0.2168	-2.4789	0.4494	-0.9811	0.1995	0	0.0678	1.4076	0.216	-2.6918	0.2035	-2.6454
	0.1673	0.0104	0.3714	-1.7904	0.1248	2.5667	0.1904	1.9149	0.2855	1.3139	0.1287	2.2805	0.3672	1.5707	0.202	-0.9856	0.1462	2.138	0.1295	-0.6511
	0.3689	-0.0356	0.1634	-2.8953	0.1303	-0.0681	0.1263	1.3292	0.1366	1.688	0.1183	-2.235	0.2551	1.9708	0.1567	-2.8951	0.4231	-3.0566	0.2568	-1.9775
	0.1231	-0.0193	0.1227	-0.0086	0.1925	1.8685	0.0702	-1.716	0.3684	-2.4093	0.2794	-1.4048	0.0591	1.4214	0.3354	1.5988	0.1123	1.0542	0.3017	1.9265

λ_k	-1017.84		-860.11		-734.07		618.66		-609.1		-483.31		-343.05		270.3		115.32		-11.69	
	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$
	0.2323	0.1423	0.2186	2.4989	0.2168	-1.2125	0.225	1.6589	0.231	2.0169	0.2405	3.0009	0.2143	1.9757	0.2248	-2.1745	0.2261	-0.1265	0.2226	2.8727
	0.2782	2.9431	0.1333	1.8021	0.3516	-2.2887	0.0811	0.787	0.1665	-1.4179	0.2966	2.8361	0.1987	-0.2955	0.2378	-0.6693	0.152	2.3069	0.2251	-2.4818
	0.2681	-0.8837	0.126	-2.7022	0.0451	2.6373	0.0777	-1.1584	0.3624	-1.885	0.2613	0.6223	0.2109	2.465	0.274	0.1068	0.2571	1.5181	0.2577	2.3094
	0.1268	-2.4629	0.4627	0	0.2541	1.3027	0.1629	-1.6169	0.0523	1.989	0.0606	-2.525	0.0254	2.6383	0.1788	-2.1695	0.068	2.4266	0.3463	-1.1348
	0.3277	-0.5251	0.0693	-2.4911	0.2621	1.5679	0.3736	1.1521	0.2175	-1.458	0.2922	0.3176	0.2149	-1.6702	0.0912	-1.2591	0.366	-2.1741	0.122	0.4563
$\mathbf{x}_k$	0.1966	-1.4998	0.1255	0.3802	0.1168	-2.9108	0.1498	2.6025	0.2494	-1.8356	0.1971	2.5202	0.3165	0	0.1024	2.282	0.3175	1.2497	0.169	-2.0736
	0.1232	-2.8214	0.2041	-3.1236	0.1008	0.1051	0.2558	3.0047	0.1312	-2.1588	0.2116	1.2606	0.3711	1.8313	0.2247	-2.8508	0.2751	-0.1968	0.2619	-0.2269
	0.1873	2.2356	0.0779	1.2134	0.1288	0.5527	0.0508	-1.6353	0.2622	-2.7963	0.371	-1.7174	0.282	2.1144	0.1261	1.188	0.323	-1.8014	0.1058	2.2062
	0.1455	0.1435	0.0339	2.0807	0.3643	1.3706	0.4041	-2.6321	0.1071	0	0.2636	-1.5074	0.2042	-2.3675	0.3266	1.3207	0.1964	0	0.0607	1.9252
	0.3862	-2.0349	0.2429	-2.1289	0.099	1.9131	0.1629	-2.8953	0.2367	1.3937	0.1826	-1.2654	0.1576	-1.4785	0.3404	-2.0352	0.1553	-1.4889	0.1723	-1.4109
	0.1445	-0.2371	0.0365	-0.3636	0.1272	1.5577	0.1647	-1.0394	0.3874	2.3949	0.1119	-3.0773	0.2054	-2.0341	0.3581	1.4707	0.1365	-1.5471	0.3188	-1.9289
	0.1178	-1.9047	0.2575	-0.1207	0.1151	-2.319	0.0379	2.0448	0.2442	-0.1548	0.1343	0	0.134	1.9513	0.0532	2.3783	0.0373	2.9408	0.2081	2.0867
	0.3315	0.0282	0.1221	-0.6438	0.1378	-1.7738	0.4774	2.8447	0.1838	2.7732	0.2665	0.4862	0.0504	1.5388	0.1694	0	0.2049	-1.8091	0.2131	-1.7108
	0.2645	2.6453	0.0883	0.6298	0.2151	0	0.1911	0.0314	0.3059	-0.1391	0.0382	0.5995	0.4029	-0.0139	0.1756	-0.1913	0.3891	1.8705	0.2255	-1.3701
	0.0535	0	0.3301	-0.7715	0.2507	0.3886	0.1701	1.181	0.1113	-0.1645	0.161	-2.5718	0.1972	0.8868	0.1954	2.5892	0.1914	0.993	0.0393	2.6237
	0.2475	-2.6781	0.3767	2.2497	0.3601	-1.4433	0.2012	-2.1278	0.3378	2.8474	0.1102	-1.1993	0.1605	-2.1275	0.0455	1.987	0.0638	2.5289	0.1818	0.1564
	0.151	2.4208	0.2456	2.3749	0.062	1.9607	0.1945	0	0.1494	1.1294	0.2211	0.5454	0.1856	1.0781	0.2483	0.8288	0.019	0.6322	0.3448	-0.8409
	0.2274	1.8778	0.1615	-0.7424	0.4272	-2.3219	0.1744	1.9599	0.0378	-0.6328	0.33	2.8964	0.135	-2.1068	0.0709	-2.5805	0.153	-1.4872	0.151	0
	0.2075	-0.2614	0.2655	2.9206	0.1265	-1.2737	0.1572	-0.5569	0.1106	2.4436	0.2208	-1.5598	0.2053	-1.3709	0.1514	-2.418	0.1446	-2.8016	0.3685	1.9552
	0.1335	0.7903	0.2792	2.6048	0.1471	1.3429	0.1872	-2.7326	0.149	-1.8328	0.1584	1.7642	0.234	-0.6274	0.397	-3.1304	0.2807	2.6379	0.0586	3.0785

Table 7.2: Eigensystem for the simulated regular market activity described in Figure 7.8 with $z = |z|e^{i\phi}$

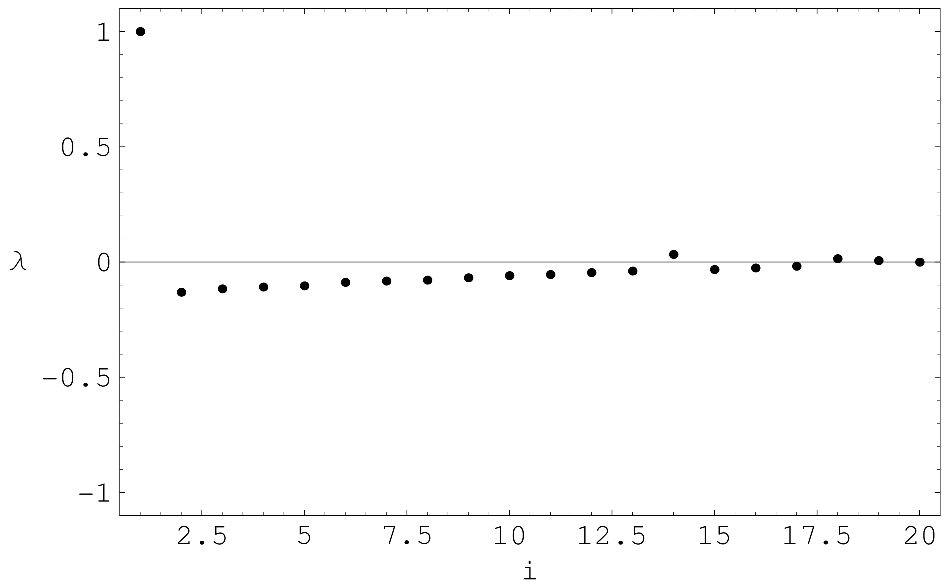

Figure 7.9: Normed eigenvalues of the simulated regular market described in Figure 7.8

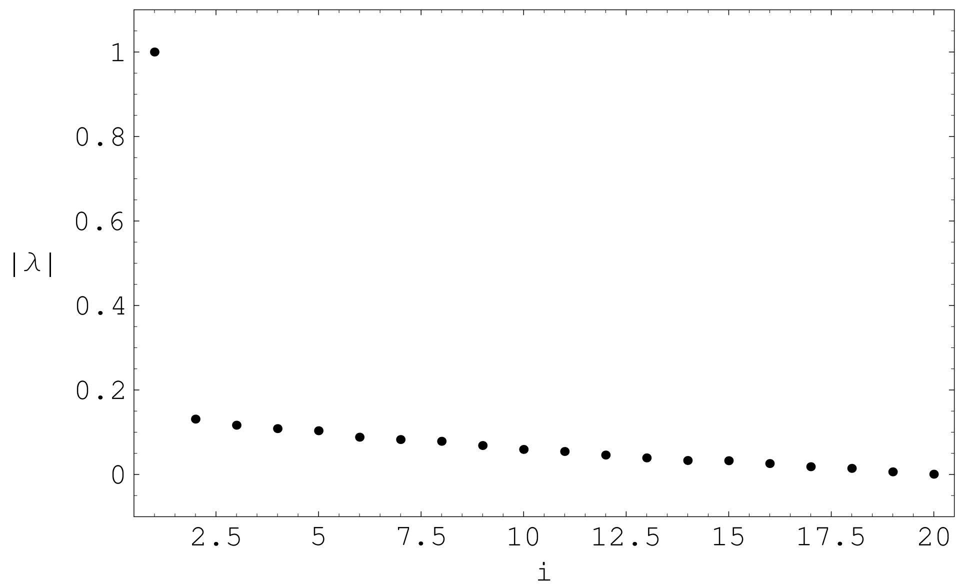

Figure 7.10: Normed eigenvalues of the simulated regular market described in Figure 7.8 with absolute values

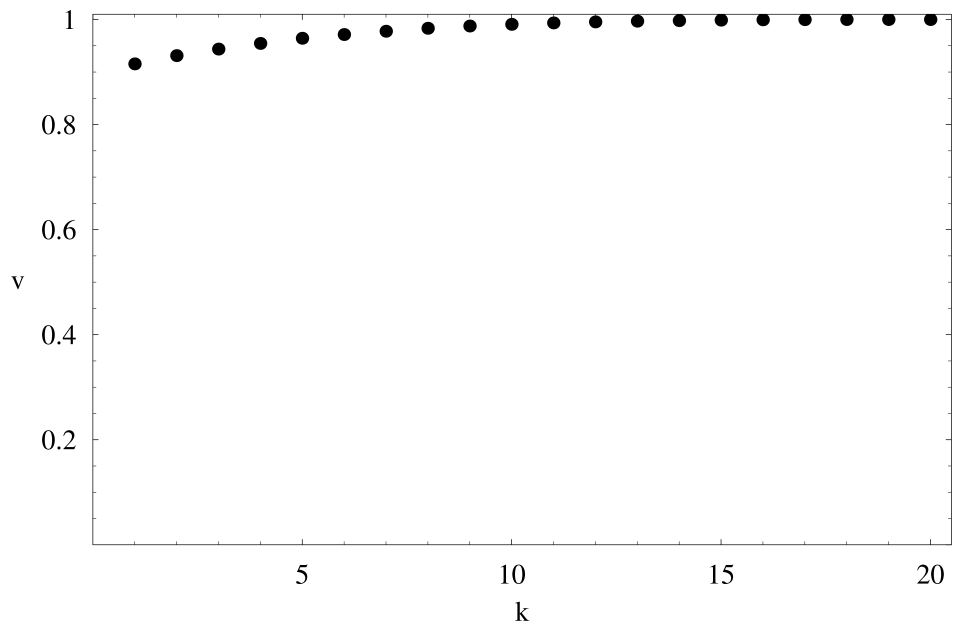

Figure 7.11: Cumulated variance of the 20 first largest absolute eigenvalues of the simulated regular market described in Figure 7.8

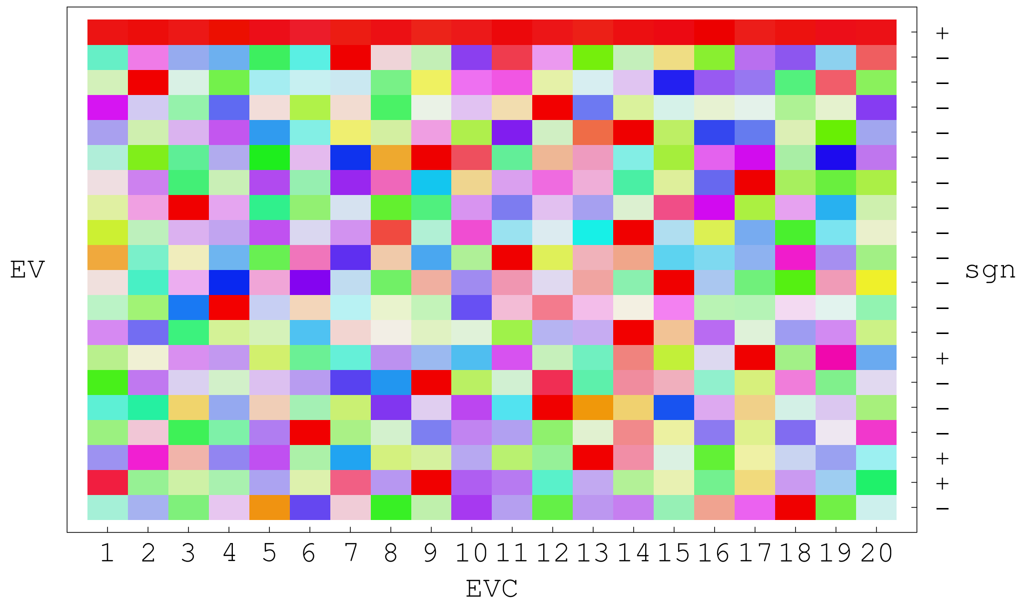

Figure 7.12: Eigensystem of the simulated regular market described in Figure 7.8

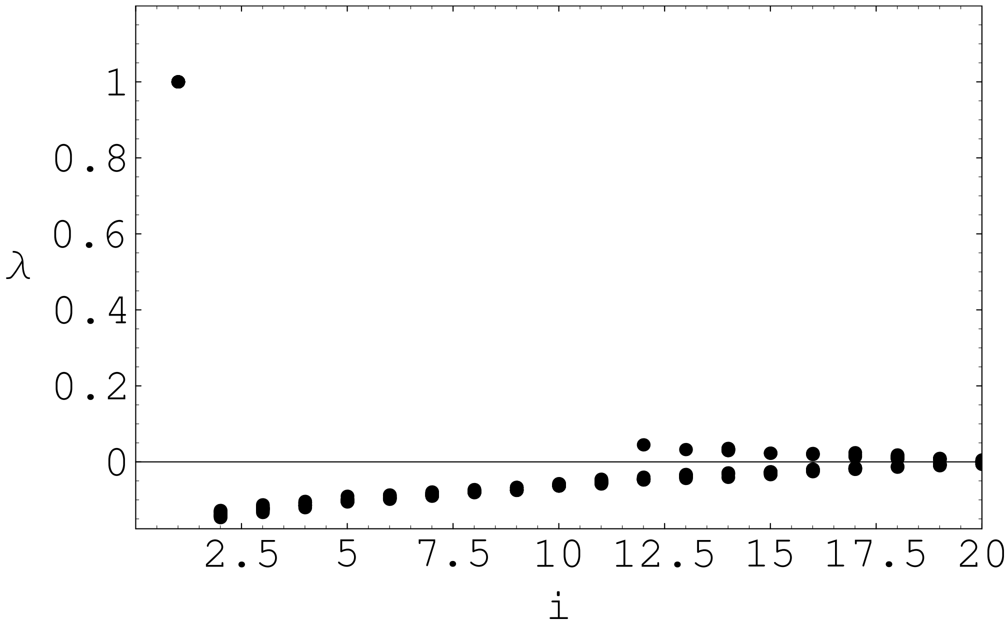

Figure 7.13: Superimposed normed eigenvalues of the simulated regular market described in Figure 7.8 for 10 simulation runs

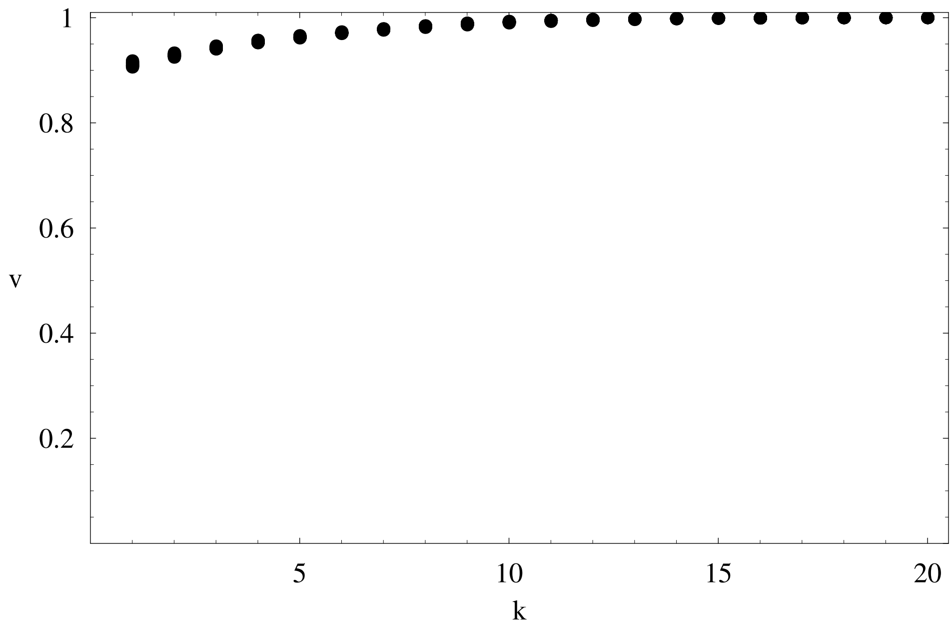

Figure 7.14: Superimposed cumulated variances of the 20 first largest absolute eigenvalues of the simulated regular market described in Figure 7.8 for 10 simulation runs

7.1.2 Money Transfer Pattern

In Figures 7.15 and 7.22 the transaction networks of a regular market from the previous section is perturbed by a strong transaction flow from trader 11 to trader 12 and a low one from trader 12 to trader 11 (depicted by rows and columns 11 and 12, respectively). Again the results of the eigensystem calculation of the Hermitian adjacency matrix is given in Tables 7.3 and 7.4 and the spectrum distribution in Figures 7.16, 7.23, 7.17, and 7.24. In comparison to the eigensystem of the undisturbed complete communication structures in the previous section the presence of the added structure results in the changing of the cumulated variance in Figures 7.18 and 7.25. Still the complete structure is present in the first eigenvector corresponding to the first eigenvalue describing about 70 % of the cumulated variance. But this time two corresponding eigenvalues follow on positions 2 and 3 in the ordered absolute eigenvalues. Following the procedures in Section 6.3 this is detected by the alternation of the algebraic sign between the 2$^{\text{nd}}$ and 3$^{\text{rd}}$ eigenvalue, the quite same phase for the centre node of the structure and the phase shift of π of the vertices directly involved in the pattern. Scaled to several runs the eigensystem is stable as depicted for 10 simulation runs for each network in the cumulated variances in Figures 7.20 and 7.27 and the spectra in Figures 7.21 and 7.28. In the visualisations in Figure 7.19 and 7.26 this can be seen in a vivid way by the colour of the eigenvector components 11 and 12 in the 2$^{\text{nd}}$ and 3$^{\text{rd}}$ rows.

The impact of a perturbation on the eigensystem for Hermitian matrices can be shown to follow the following rough boundaries $\lambda_i + \epsilon_1 \geq \beta_i \geq \lambda_i + \epsilon_n \forall i$. This holds for perturbations of linear form for a Hermitian unperturbed matrix A and a Hermitian perturbation matrix E. The corresponding eigenvalues are $\lambda_i \in \sigma(A), \epsilon_i \in \sigma(E)$. The resulting perturbed matrix $B = A + E$ has the eigenvalues β_i [Meye 00, p.551].

Thus the perturbation of a Hermitian matrix A depends on the norm of the perturbation matrix E since the norm is equivalent to the maximum eigenvalue of the Hermitian matrix E. Following this approach it can be shown that a manipulative trading behaviour becomes visible already by a small perturbation in the input adjacency matrix relative to the transactions of the other traders with a small variance. If on the other hand the market shows comparably large standard deviation, the manipulative trading behaviour has to be stronger to result in a visible impact. This is shown in the following examples depicted by Figures 7.29 and 7.30 for the transaction network described in Figure 7.15 and in Figures 7.31 and 7.32 for the transaction network described in Figure 7.22. In the examples for each standard deviation a perturbation with no visible impact on the spectrum (Figures 7.29 and 7.31) and a perturbation with a visible impact on the spectrum (Figures 7.30 and 7.32) are shown. Whether the visible perturbation is a manipulative pattern as described can only be seen by using the eigenvector information as proposed in Section 6.3.

$$
\begin{pmatrix}
0 & 602 & 754 & 719 & 743 & 791 & 706 & 671 & 641 & 771 & 618 & 721 & 638 & 656 & 671 & 664 & 647 & 647 & 691 & 723 \\
684 & 0 & 711 & 703 & 772 & 667 & 668 & 812 & 732 & 643 & 689 & 661 & 682 & 683 & 738 & 698 & 833 & 675 & 666 & 698 \\
700 & 701 & 0 & 687 & 590 & 699 & 794 & 716 & 632 & 720 & 712 & 708 & 717 & 640 & 825 & 699 & 688 & 629 & 676 & 638 \\
737 & 646 & 727 & 0 & 727 & 745 & 681 & 775 & 670 & 630 & 638 & 646 & 693 & 733 & 680 & 722 & 577 & 756 & 786 & 751 \\
657 & 691 & 698 & 722 & 0 & 660 & 767 & 702 & 705 & 703 & 685 & 615 & 733 & 698 & 659 & 791 & 653 & 641 & 648 & 725 \\
731 & 651 & 742 & 717 & 707 & 0 & 658 & 640 & 643 & 733 & 755 & 673 & 781 & 671 & 652 & 630 & 600 & 719 & 766 & 689 \\
672 & 629 & 669 & 700 & 677 & 699 & 0 & 701 & 671 & 660 & 628 & 661 & 615 & 792 & 772 & 639 & 655 & 759 & 729 & 687 \\
611 & 735 & 573 & 663 & 731 & 660 & 742 & 0 & 724 & 618 & 733 & 726 & 732 & 776 & 859 & 643 & 619 & 683 & 732 & 682 \\
659 & 710 & 637 & 643 & 663 & 679 & 665 & 756 & 0 & 741 & 688 & 750 & 649 & 659 & 648 & 775 & 717 & 636 & 727 & 738 \\
774 & 729 & 791 & 637 & 710 & 722 & 701 & 710 & 723 & 0 & 734 & 693 & 685 & 650 & 630 & 717 & 718 & 648 & 817 & 750 \\
691 & 698 & 700 & 843 & 714 & 639 & 642 & 661 & 687 & 633 & 0 & \underline{3700} & 696 & 752 & 611 & 768 & 614 & 746 & 737 & 634 \\
630 & 745 & 731 & 698 & 739 & 626 & 694 & 699 & 723 & 652 & \underline{0} & 0 & 759 & 705 & 720 & 704 & 701 & 686 & 657 & 712 \\
660 & 649 & 729 & 735 & 666 & 750 & 738 & 706 & 738 & 632 & 773 & 720 & 0 & 691 & 774 & 718 & 792 & 651 & 756 & 672 \\
774 & 637 & 692 & 659 & 724 & 701 & 753 & 731 & 729 & 623 & 685 & 648 & 820 & 0 & 612 & 669 & 761 & 704 & 706 & 702 \\
700 & 760 & 681 & 727 & 636 & 708 & 704 & 689 & 712 & 719 & 716 & 737 & 693 & 712 & 0 & 755 & 734 & 644 & 635 & 719 \\
780 & 718 & 695 & 815 & 689 & 696 & 708 & 716 & 679 & 735 & 711 & 756 & 672 & 670 & 663 & 0 & 667 & 753 & 668 & 716 \\
751 & 591 & 627 & 710 & 704 & 706 & 727 & 675 & 569 & 574 & 767 & 670 & 786 & 789 & 634 & 664 & 0 & 726 & 693 & 685 \\
627 & 733 & 785 & 743 & 647 & 760 & 736 & 624 & 724 & 648 & 673 & 662 & 742 & 654 & 761 & 667 & 648 & 0 & 628 & 617 \\
803 & 665 & 647 & 739 & 642 & 854 & 742 & 740 & 681 & 613 & 680 & 654 & 748 & 633 & 683 & 667 & 700 & 688 & 0 & 682 \\
748 & 692 & 690 & 683 & 682 & 613 & 725 & 792 & 708 & 701 & 727 & 802 & 641 & 693 & 646 & 756 & 779 & 636 & 723 & 0
\end{pmatrix}
$$

Figure 7.15: Simulated transaction network of a market environment following the network in Figure 7.1 with $\mu = 700$ and $\sigma = 50$ and a money transfer perturbation of 3000 from trader 11 to 12 and -700 from trader 12 to 11

λ_k	18970.9		-4038.93		1854.76		-1464.8		-1423.73		-1366.88		-1294.44		-1197.36		-1145.65		-1110.19	
	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$
	0.2205	-0.1072	0.2199	0.2225	0.2204	-2.537	0.2238	0.753	0.2192	2.045	0.2208	-1.9224	0.2211	-1.2113	0.2228	-1.771	0.2187	-3.0572	0.219	-0.192
	0.01	-0.1335	0.041	2.324	0.0438	-2.9415	0.039	1.8381	0.0472	-1.3492	0.0335	-1.8349	0.0227	0.1854	0.0388	-2.5022	0.0366	-0.3393	0.0334	-1.0285
	0.0811	-0.1147	0.0807	2.0136	0.0678	-2.6588	0.0656	1.5776	0.0849	2.4451	0.1027	1.8124	0.103	2.5894	0.078	2.1665	0.0575	-0.5743	0.0872	0.8089
	0.1114	-0.1102	0.1785	-2.8882	0.2044	-3.0201	0.2073	-0.8125	0.1748	-1.3744	0.2526	-2.5764	0.2609	-3.0605	0.3345	0	0.1769	-1.8842	0.1733	0.9329
	0.2784	-0.1179	0.1749	2.4939	0.1921	-2.8591	0.2696	-2.8969	0.1767	1.482	0.1148	1.7964	0.1876	-2.4233	0.4072	0.7617	0.1437	-1.8307	0.2689	-3.1225
$\mathbf{x}_k$	0.3199	-0.1118	0.2815	1.9755	0.3134	-2.2256	0.2125	-0.8005	0.2211	-0.7546	0.1239	-0.6026	0.1698	-2.5641	0.0779	-2.531	0.1884	0	0.1091	-1.6937
	0.2597	-0.0988	0.2264	1.3948	0.2643	-2.669	0.0329	-0.0249	0.2772	0.1097	0.338	-1.8979	0.2913	0.9939	0.1732	-1.5183	0.1879	-1.2783	0.2398	2.9178
	0.0656	-0.1091	0.1379	1.7	0.0233	-2.4333	0.4208	-0.5754	0.1274	2.3248	0.3526	1.9203	0.2727	1.854	0.0839	2.0338	0.1243	1.3722	0.392	-0.4328
	0.3044	-0.1201	0.2844	1.6376	0.2934	-2.7045	0.1744	1.201	0.1562	-0.4786	0.4185	2.3868	0.1053	-2.1576	0.2304	-0.8065	0.2439	-1.9814	0.3621	-0.1099
	0.1001	-0.1474	0.1369	2.0689	0.3528	-2.492	0.319	-2.0531	0.3618	-1.3558	0.2094	-2.3335	0.2152	1.5932	0.234	0.8974	0.1628	1.8609	0.2208	-1.2366
	0.3325	-0.2348	0.187	0	0.2411	0	0.1531	-2.6171	0.0841	3.1385	0.1264	-2.4906	0.0385	-1.6223	0.1715	0.3819	0.3632	-0.5006	0.1438	3.042
	0.2338	0	0.485	-2.1629	0.06	1.0638	0.0287	-0.0024	0.2051	0.8772	0.1201	0.288	0.0124	1.8681	0.1215	2.8741	0.4229	-0.3494	0.1105	-2.3323
	0.1912	-0.1209	0.0852	1.9667	0.1155	-2.5261	0.2223	2.2514	0.2557	1.0534	0.0806	-2.1212	0.5027	1.676	0.0405	-0.9997	0.3217	-3.0068	0.1897	-2.4926
	0.2258	-0.1207	0.2831	2.8072	0.1169	-2.7799	0.132	-2.7628	0.3929	-1.8139	0.2261	-0.0223	0.1965	-0.939	0.3491	2.991	0.1026	-0.7372	0.1653	0
	0.3318	-0.1232	0.2333	1.4725	0.1934	-2.54	0.3916	-2.4965	0.1156	0	0.0993	0.8568	0.3946	-0.9705	0.2126	1.7512	0.1996	-2.721	0.2405	-1.1782
	0.1264	-0.1243	0.2505	2.1381	0.1539	-2.7304	0.2468	0.5492	0.3056	-2.1144	0.1041	0	0.0914	1.2497	0.1274	-2.849	0.2696	1.723	0.1389	2.7837
	0.0748	-0.1172	0.131	1.8447	0.3488	-2.3898	0.3268	0	0.2342	-2.4086	0.29	1.0923	0.2727	-1.5571	0.2571	0.8191	0.179	-0.2154	0.0632	-2.7568
	0.1521	-0.1201	0.1041	2.7253	0.3536	-2.7864	0.1283	-2.7434	0.3888	2.0217	0.2829	-2.0549	0.0675	-1.4644	0.2717	1.9244	0.2101	1.9008	0.1586	1.7418
	0.3358	-0.1122	0.0572	2.9023	0.1237	-2.5525	0.0045	2.6594	0.0405	3.0065	0.0666	0.9738	0.2272	0	0.31	0.2017	0.2366	-3.1246	0.4506	2.7726
	0.2404	-0.1261	0.3547	1.3218	0.2746	-2.4182	0.2105	2.8159	0.0158	-1.6806	0.3352	3.1065	0.0603	2.7595	0.2214	-2.7121	0.232	2.0417	0.18	2.3219

λ_k	-1059.49		-1032.01		-953.48		-871.38		-805.7		-745.12		-712.83		-656.16		-526.08		-421.38	
	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$
	0.246	-2.6155	0.246	0.5509	0.2249	2.445	0.2214	-1.3486	0.2215	-0.2447	0.2237	2.0415	0.2179	2.7545	0.2176	-1.6148	0.2224	0.1219	0.222	1.7703
	0.6979	1.6483	0.6968	0	0.0568	-0.7265	0.0363	-1.6667	0.0479	2.7303	0.0232	1.8944	0.0527	-1.4243	0.0315	2.017	0.0259	-3.0436	0.051	-1.5342
	0.6674	2.0787	0.6662	-2.9368	0.0687	2.4087	0.0768	-1.4153	0.0836	-0.2557	0.0451	-1.9086	0.091	0	0.0659	2.8615	0.0847	1.3678	0.0638	-3.1002
	0.0321	0.5376	0.0225	2.7732	0.2258	2.1858	0.3452	1.5819	0.1458	-2.9712	0.1656	2.1634	0.3838	1.4253	0.2439	-0.6781	0.2571	-0.2983	0.2052	1.9102
	0.0036	1.9466	0.0181	-0.835	0.2179	0.5952	0.2193	0	0.4172	2.3806	0.0948	-2.8956	0.2068	-1.1398	0.2935	0	0.1699	-0.3243	0.0853	-0.2831
$\mathbf{x}_k$	0.011	-0.7295	0.0126	3.011	0.3577	-1.1963	0.1573	0.5187	0.2588	3.0103	0.367	1.7502	0.1958	-2.6996	0.0761	-2.6175	0.3492	0.0105	0.0883	2.0987
	0.0109	2.3661	0.0194	-0.6096	0.2858	0	0.1125	1.7534	0.2887	0	0.1602	-1.0268	0.1306	1.2667	0.1363	0.521	0.3618	2.8575	0.2031	1.5203
	0.0227	-0.9531	0.0088	-2.2721	0.1513	-2.8688	0.0927	-1.0033	0.1143	-2.0355	0.2548	-0.1902	0.3486	2.4108	0.1943	0.2217	0.0485	2.6223	0.3692	-1.3308
	0.023	-2.7374	0.0179	2.2871	0.2083	-1.015	0.0992	3.0812	0.1966	2.9204	0.1863	-0.6173	0.0824	-2.6194	0.3404	0.6876	0.0368	-1.1264	0.0642	-1.6606
	0.0074	0.5883	0.0375	-1.9603	0.1642	-0.8483	0.3881	0.1013	0.1178	0.6817	0.2062	-1.3781	0.2586	-2.7991	0.2041	2.4587	0.1651	0	0.1508	-2.4847
	0.0226	1.7414	0.038	-0.7662	0.1249	-0.3291	0.1786	-0.2954	0.2163	-3.0357	0.3375	-2.6935	0.1239	1.1818	0.0674	2.6354	0.3101	-2.3603	0.4901	0.4055
	0.0281	-0.8682	0.0311	1.9696	0.1979	2.256	0.3554	-0.6925	0.0601	0.1802	0.3362	-1.0717	0.1504	-2.5002	0.2141	-3.0314	0.1026	2.2247	0.2837	-1.4436
	0.027	-0.8507	0.0149	-1.8649	0.3444	-2.7279	0.0827	1.3098	0.2632	-2.853	0.2395	-1.2613	0.1144	-1.2597	0.2918	-2.4454	0.1055	-3.0394	0.2557	1.2536
	0.0162	-2.9428	0.0134	-1.6855	0.1167	1.3464	0.2694	2.9317	0.2017	0.1338	0.1458	-2.3101	0.2954	-3.1062	0.3001	-0.7172	0.1369	-2.8774	0.2989	0.2983
	0.0166	-0.6208	0.0435	2.4844	0.2225	2.6946	0.1737	-0.0725	0.2378	-0.0698	0.1787	1.1311	0.1142	0.8581	0.2381	2.906	0.2292	-2.4281	0.1738	-1.9854
	0.0149	-2.9293	0.021	-2.0537	0.3174	-3.0359	0.3823	1.8862	0.2328	2.5921	0.2292	1.4096	0.0491	0.3659	0.2697	0.801	0.4268	-0.4234	0.0656	-0.3199
	0.0189	-0.8257	0.0293	1.3901	0.295	-2.2737	0.2074	-2.9008	0.1637	0.6475	0.3104	-2.8568	0.2678	-0.8246	0.0418	-2.1608	0.3241	2.1262	0.0189	0
	0.0184	-0.1734	0.0233	1.6779	0.2228	-0.8773	0.1439	2.8771	0.1817	2.0602	0.2488	2.2601	0.273	-1.3255	0.3066	1.916	0.1808	3.1393	0.2801	2.1545
	0.0097	1.7106	0.0057	0.5779	0.2371	0.8542	0.1783	1.0227	0.3805	-2.3613	0.2419	0	0.0975	2.9907	0.2553	-2.2936	0.1664	1.1611	0.2488	1.3618
	0.0343	0	0.021	1.6853	0.0997	2.0708	0.2462	2.9948	0.2338	-0.8826	0.0416	-1.0055	0.4374	1.4117	0.2571	-0.1922	0.1652	0.0954	0.1785	-1.2932

Table 7.3: Eigensystem for the simulated market with money transfer pattern described in Figure 7.15 with $z = |z|e^{i\phi}$

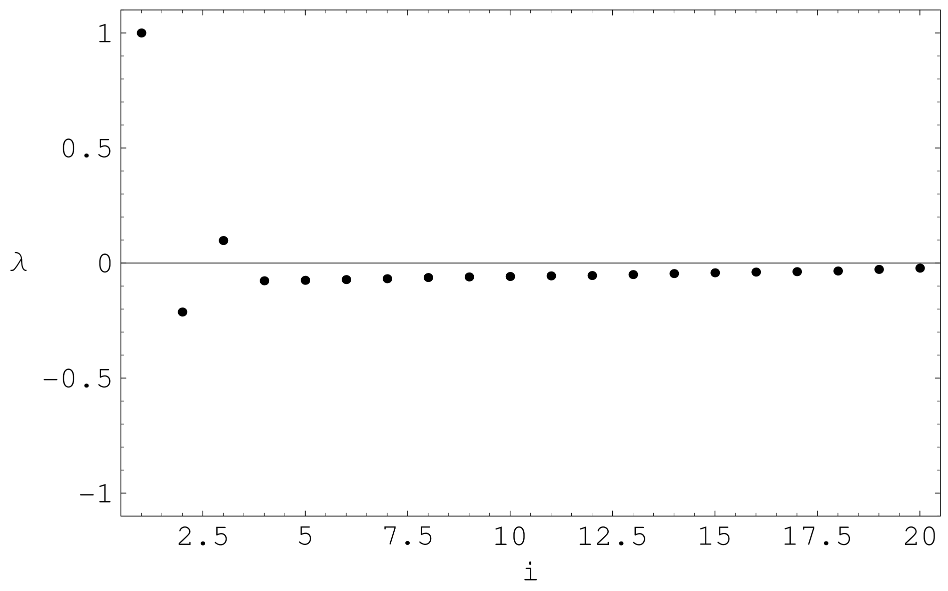

Figure 7.16: Normed eigenvalues of the simulated market with money transfer pattern described in Figure 7.15

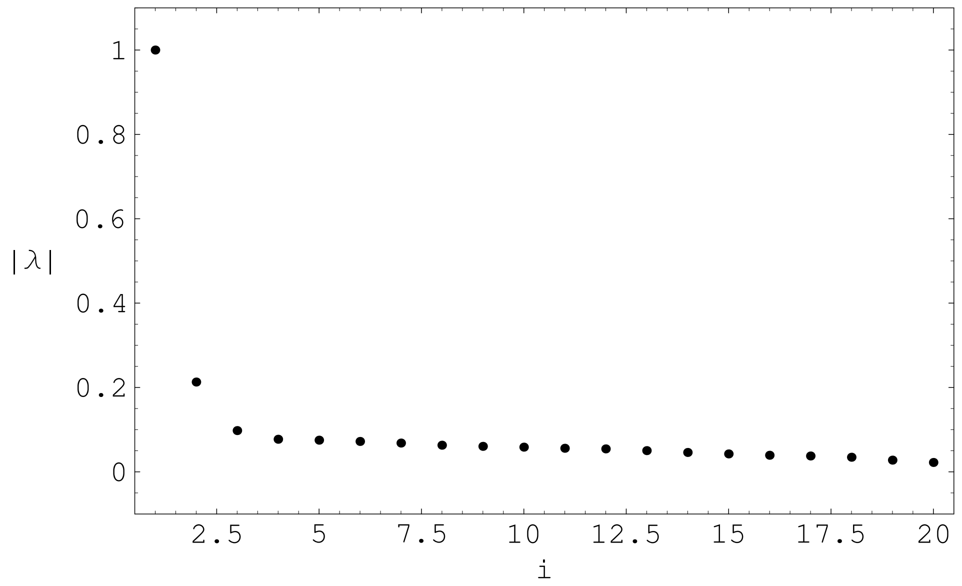

Figure 7.17: Normed eigenvalues of the simulated market with money transfer pattern described in Figure 7.15 with absolute values

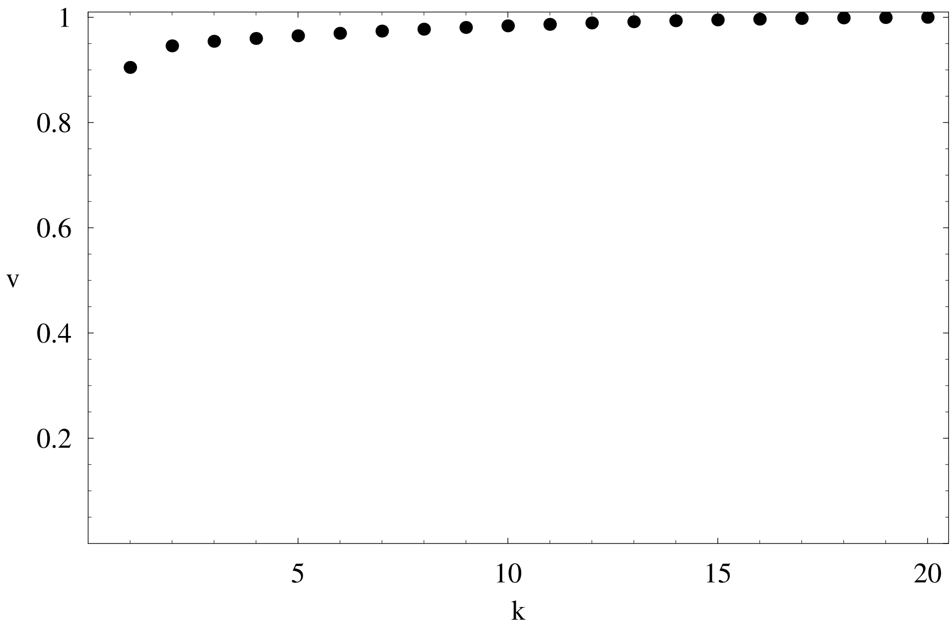

Figure 7.18: Cumulated variance of the 20 first largest absolute eigenvalues of the simulated market with money transfer pattern described in Figure 7.15

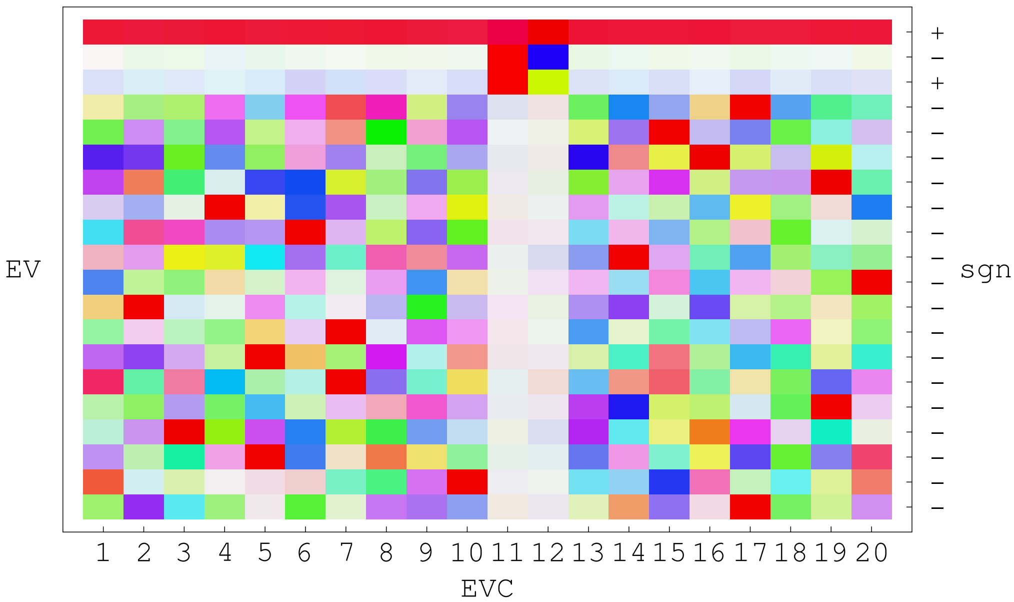

Figure 7.19: Eigensystem of the simulated market with money transfer pattern described in Figure 7.15

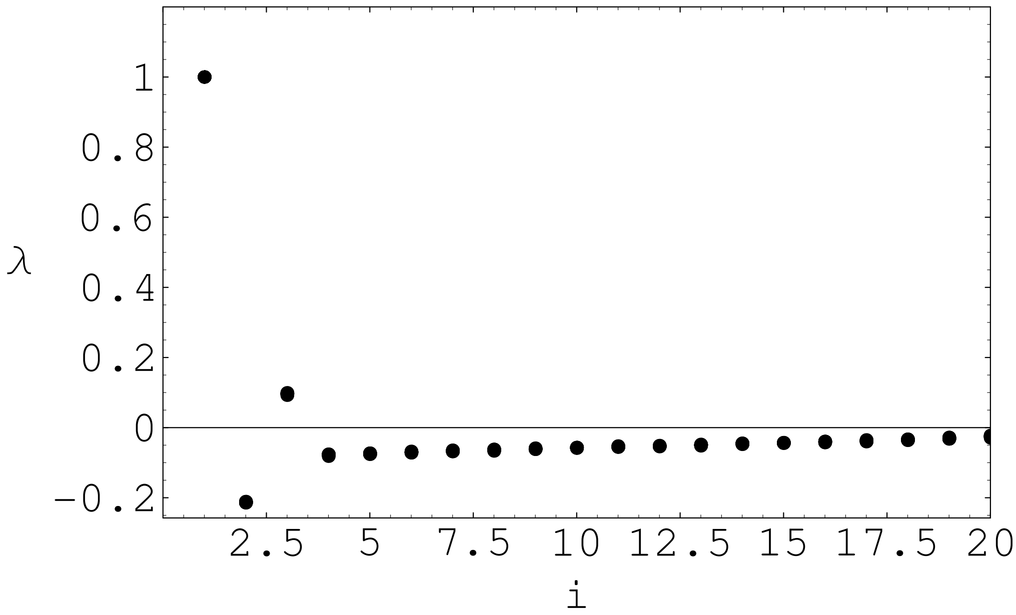

Figure 7.20: Superimposed normed eigenvalues of the simulated market with money transfer pattern described in Figure 7.15 for 10 simulation runs

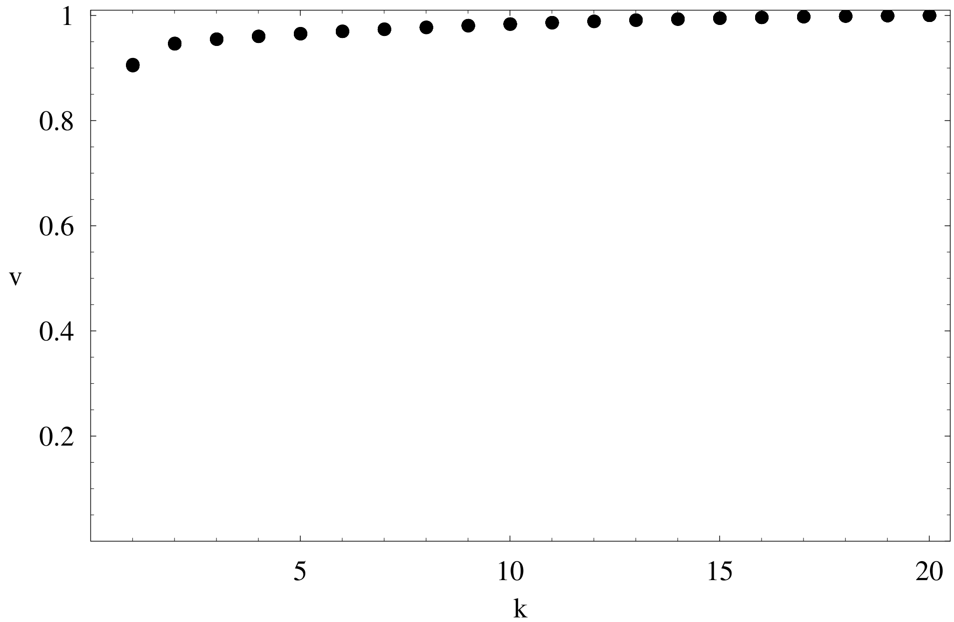

Figure 7.21: Superimposed cumulated variances of the 20 first largest absolute eigenvalues of the simulated market with money transfer pattern described in Figure 7.15 for 10 simulation runs

$$\begin{pmatrix}
0 & 583 & 781 & 506 & 921 & 694 & 812 & 814 & 585 & 832 & 464 & 618 & 852 & 675 & 764 & 580 & 758 & 519 & 665 & 452 \\
733 & 0 & 574 & 1043 & 537 & 752 & 651 & 646 & 625 & 555 & 691 & 670 & 704 & 693 & 612 & 668 & 665 & 539 & 692 & 860 \\
644 & 275 & 0 & 635 & 425 & 626 & 904 & 676 & 676 & 642 & 579 & 765 & 830 & 423 & 498 & 754 & 802 & 592 & 753 & 663 \\
643 & 749 & 839 & 0 & 390 & 787 & 757 & 593 & 764 & 751 & 715 & 928 & 717 & 712 & 730 & 646 & 626 & 384 & 654 & 799 \\
643 & 806 & 547 & 906 & 0 & 637 & 968 & 589 & 446 & 795 & 628 & 834 & 693 & 868 & 701 & 621 & 753 & 579 & 758 & 860 \\
593 & 819 & 500 & 653 & 663 & 0 & 552 & 841 & 744 & 825 & 566 & 585 & 511 & 808 & 563 & 332 & 850 & 607 & 692 & 686 \\
566 & 580 & 827 & 419 & 388 & 643 & 0 & 851 & 687 & 743 & 887 & 848 & 803 & 695 & 776 & 545 & 613 & 902 & 864 & 777 \\
707 & 742 & 809 & 700 & 669 & 1058 & 882 & 0 & 569 & 648 & 732 & 613 & 702 & 610 & 836 & 460 & 640 & 773 & 641 & 932 \\
1004 & 546 & 607 & 707 & 640 & 499 & 721 & 1027 & 0 & 955 & 634 & 747 & 863 & 597 & 705 & 813 & 1101 & 823 & 600 & 545 \\
660 & 656 & 515 & 744 & 649 & 522 & 697 & 659 & 695 & 0 & 847 & 739 & 908 & 532 & 709 & 806 & 1007 & 594 & 565 & 527 \\
556 & 880 & 1031 & 503 & 845 & 686 & 658 & 736 & 1026 & 655 & 0 & 4600 & 407 & 406 & 583 & 690 & 873 & 698 & 715 & 766 \\
771 & 797 & 699 & 929 & 727 & 654 & 489 & 439 & 627 & 755 & 0 & 0 & 724 & 527 & 755 & 416 & 753 & 523 & 613 & 728 \\
814 & 646 & 546 & 745 & 840 & 733 & 628 & 853 & 569 & 548 & 844 & 856 & 0 & 803 & 620 & 429 & 806 & 591 & 865 & 791 \\
782 & 588 & 469 & 700 & 588 & 570 & 695 & 738 & 878 & 459 & 726 & 624 & 914 & 0 & 735 & 690 & 876 & 609 & 604 & 601 \\
784 & 609 & 524 & 597 & 747 & 448 & 598 & 874 & 525 & 587 & 367 & 593 & 890 & 688 & 0 & 704 & 497 & 771 & 1044 & 692 \\
741 & 464 & 493 & 561 & 591 & 522 & 528 & 654 & 842 & 720 & 684 & 724 & 537 & 875 & 976 & 0 & 672 & 605 & 847 & 794 \\
721 & 890 & 634 & 735 & 885 & 359 & 798 & 630 & 619 & 660 & 700 & 456 & 541 & 586 & 833 & 775 & 0 & 812 & 522 & 725 \\
873 & 639 & 655 & 871 & 709 & 654 & 720 & 470 & 593 & 670 & 837 & 390 & 818 & 507 & 673 & 633 & 867 & 0 & 748 & 640 \\
564 & 647 & 724 & 590 & 816 & 624 & 559 & 707 & 812 & 821 & 791 & 550 & 625 & 813 & 622 & 615 & 594 & 1093 & 0 & 536 \\
601 & 629 & 565 & 656 & 709 & 666 & 304 & 826 & 955 & 768 & 778 & 883 & 690 & 683 & 841 & 439 & 1178 & 773 & 686 & 0
\end{pmatrix}$$

Figure 7.22: Simulated transaction network of a market environment following the network in Figure 7.8 with $\mu = 700$ and $\sigma = 150$ and a money transfer perturbation of 3900 from trader 11 to 12 and -700 from trader 12 to 11

λ_k	18869.8		-5033.7		2813.6		-2464.07		-2210.02		-1889.31		-1731.92		-1583.44		-1491.95		-1198.08	
	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$
	0.2194	0.1708	0.2138	1.0244	0.2072	-2.964	0.2205	0.5656	0.2213	-0.1322	0.2062	1.9584	0.2206	-0.327	0.228	0	0.23	-2.4607	0.2214	2.9734
	0.0431	0.1379	0.0593	2.7042	0.0361	2.7657	0.1223	2.7292	0.0204	-2.8502	0.0091	-1.2489	0.0352	1.9671	0.0424	1.1208	0.0556	2.671	0.0715	1.9023
	0.1424	0.1569	0.0461	-2.7401	0.0313	1.7058	0.0471	-2.8602	0.0519	2.985	0.0798	-0.0687	0.1001	0.666	0.1182	-1.4384	0.0563	2.6082	0.0672	-0.1981
	0.076	0.152	0.1268	1.1717	0.071	-3.052	0.2441	1.7097	0.2539	-0.5294	0.2266	1.8347	0.359	-1.0087	0.261	0.8712	0.2582	0.3354	0.2019	-0.4344
$\mathbf{x}_k$	0.1318	0.1207	0.236	2.6141	0.3516	-3.0252	0.0998	2.7234	0.2044	-1.9695	0.0878	-0.4263	0.0049	0	0.2765	-2.3329	0.1716	0.228	0.2176	1.4483
	0.1978	0.138	0.2328	2.9126	0.1315	-2.9783	0.182	0.0517	0.153	1.0201	0.2942	0.6953	0.224	1.0354	0.3597	-2.5262	0.2016	-1.4641	0.1006	-1.6764
	0.2136	0.1267	0.1819	1.2083	0.0945	-1.8458	0.2652	0.9082	0.4051	-1.7979	0.1848	-0.8055	0.3418	-2.2578	0.1591	2.0784	0.2677	-0.0393	0.3179	2.4326
	0.533	0.1423	0.101	-2.6061	0.1948	-2.3026	0.2525	-3.1231	0.3106	-0.8236	0.3037	-2.9887	0.0725	-2.4593	0.062	-1.6832	0.0949	-1.9097	0.1515	0.717
	0.1049	0.1159	0.1886	-2.2773	0.2569	2.8277	0.1382	-1.666	0.0925	2.1117	0.1376	-0.5114	0.2775	0.3069	0.0547	0.9816	0.5349	0	0.2718	1.0568
	0.2826	0.1592	0.3513	1.9177	0.2319	-2.4693	0.1881	-1.6953	0.1392	3.0102	0.375	-2.9136	0.1624	-1.9223	0.3	-2.2634	0.1304	-2.7772	0.1462	-2.9128
	0.2906	0	0.4336	0	0.2434	0	0.3581	-2.5555	0.1981	-1.5573	0.1213	2.0248	0.0756	1.5998	0.1764	1.6503	0.3216	-2.2426	0.1079	-1.7265
	0.1212	0.3122	0.2135	-2.221	0.3116	1.006	0.4082	0.8791	0.1818	0.5593	0.0936	2.3011	0.26	-3.0748	0.4127	1.5679	0.0848	-2.186	0.1653	-3.1351
	0.1253	0.1533	0.2565	1.1839	0.2833	-2.2426	0.1113	2.1746	0.257	1.0055	0.02	-1.718	0.2778	2.1677	0.1016	-1.9717	0.1591	-1.9481	0.4263	2.2754
	0.2503	0.1305	0.2163	1.4372	0.1721	-2.3875	0.2108	-1.3838	0.1513	-3.0025	0.2657	2.957	0.1809	-1.7715	0.1308	2.2378	0.0701	2.2772	0.3453	-2.1418
	0.0816	0.1879	0.0709	0.4042	0.1812	-2.9964	0.0278	-1.5456	0.1108	2.8535	0.3683	0	0.1658	2.167	0.2312	1.3175	0.0914	-1.9697	0.3622	0
	0.234	0.0987	0.3094	2.2543	0.1081	-2.5312	0.2809	1.4289	0.2804	0.6155	0.3765	-2.5227	0.0385	2.2458	0.1739	-2.199	0.3872	-0.3919	0.1673	2.7953
	0.3186	0.2168	0.2484	2.9561	0.061	-2.8355	0.2361	-2.5749	0.409	0	0.2016	0.753	0.2776	-2.4987	0.2353	2.3389	0.1439	2.1932	0.1279	-1.3619
	0.0856	0.1393	0.301	2.8955	0.3222	-2.3282	0.3306	-2.8489	0.3071	-3.0033	0.2024	2.8498	0.3512	1.5447	0.1936	1.7707	0.1427	-1.2888	0.1751	-2.2277
	0.1175	0.1593	0.0813	2.8726	0.3073	-2.3822	0.2005	-0.5395	0.0951	-0.438	0.2126	-1.9729	0.0724	-1.3571	0.2676	-0.2812	0.106	1.8427	0.2211	-0.679
	0.2922	0.1379	0.121	1.1602	0.3599	-2.8413	0.0796	0	0.0831	2.5955	0.1441	-3.097	0.3432	0.5472	0.2431	0.207	0.2626	1.335	0.1697	2.49

λ_k	-1137.68		-1022.66		-797.71		-709.73		-588.2		379.25		-368.77		278.88		-114.83		0.56	
	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$	$\vert z \vert$	$\phi(z)$
	0.2607	-1.1898	0.2553	-1.0392	0.2273	-1.4091	0.212	-2.1335	0.2179	1.06	0.2058	1.4448	0.2323	-1.2137	0.2153	1.024	0.2217	1.5805	0.227	2.847
	0.6906	0	0.6811	-2.7912	0.0932	0.6007	0.0361	-2.812	0.0247	-1.4586	0.0008	-2.3199	0.0633	-3.024	0.0916	0.8569	0.0376	-2.0294	0.0378	0.2024
	0.644	-0.3842	0.6435	1.0224	0.1205	2.5854	0.1492	0.3211	0.1567	0.9765	0.0722	-0.5947	0.0941	-2.4871	0.126	2.9921	0.114	0.8029	0.0243	-2.3795
	0.0783	2.5753	0.0554	2.1892	0.0911	0.1185	0.1938	0.3408	0.0896	1.7418	0.2993	-1.8677	0.214	-2.7067	0.2024	1.3932	0.2585	1.6281	0.4325	1.2907
	0.0428	2.3457	0.0491	-1.7347	0.3743	-3.0077	0.2499	-0.5638	0.104	2.2651	0.1367	2.8092	0.4357	0	0.1995	0.3097	0.3275	-0.8279	0.1319	-1.2762
$\mathbf{x}_k$	0.0544	-1.2826	0.0997	2.3365	0.1074	2.009	0.1717	3.0329	0.4296	-2.74	0.1428	-2.2247	0.2873	-1.7922	0.3157	-1.6242	0.1672	-3.1193	0.2167	-3.11
	0.0532	-2.1829	0.067	-2.1783	0.3076	-2.1758	0.1815	2.9946	0.1637	-1.4391	0.2015	2.3285	0.2519	1.6074	0.2306	2.7453	0.0635	0.3913	0.0771	-1.9493
	0.0545	2.0834	0.0575	0	0.2691	1.7867	0.2627	-2.6647	0.2583	-2.9251	0.2629	2.8524	0.1949	2.9889	0.1155	-2.647	0.1783	-2.417	0.0876	3.105
	0.0435	0.9755	0.025	0.2599	0.1206	-0.5176	0.2781	2.589	0.2634	3.0462	0.1147	0	0.1211	1.812	0.2665	-1.3791	0.3596	2.0223	0.095	2.095
	0.026	-2.7944	0.0211	-1.4382	0.0224	0	0.0959	0.3581	0.3802	-2.1532	0.2434	0.0818	0.087	0.4787	0.2924	-0.257	0.2585	1.7159	0.1352	-1.937
	0.0255	-0.9826	0.0549	-1.6625	0.2625	1.3507	0.0571	2.4611	0.2839	2.7378	0.1438	0.4931	0.2711	-0.3582	0.2219	2.4905	0.1749	-1.4368	0.0607	1.2549
	0.0497	3.1087	0.0416	2.3349	0.0421	-0.9624	0.3013	-0.0244	0.1061	-0.7916	0.3287	2.5678	0.0806	-0.1359	0.1867	2.7139	0.19	-1.555	0.2613	1.6357
	0.0419	1.6057	0.0748	-2.4232	0.2106	-0.9743	0.3284	1.1576	0.1495	0	0.2671	2.4339	0.186	-1.4858	0.1586	-2.3784	0.2682	2.4933	0.2828	2.0113
	0.0905	-2.6669	0.0213	1.0271	0.2242	-2.2743	0.3118	-2.9469	0.2024	-0.4829	0.2145	-1.3137	0.1529	0.3188	0.1585	1.6112	0.1634	3.0408	0.4804	0.9869
	0.0192	-2.416	0.0843	-0.4651	0.537	-0.7911	0.2714	-0.9151	0.0871	-2.0395	0.2971	1.5555	0.2526	-1.9655	0.1282	2.587	0.0863	-1.3869	0.1823	0.2017
	0.0557	-1.8385	0.0788	1.2838	0.2112	1.2274	0.1035	-1.6634	0.2515	2.4527	0.3083	0.4906	0.2215	0.2924	0.0779	2.263	0.086	1.8792	0.2084	0
	0.0412	-0.7405	0.0766	2.6038	0.1828	2.6464	0.2915	1.2406	0.1365	1.1596	0.135	0.131	0.197	1.8627	0.3067	0	0.2526	-2.2121	0.2011	-1.9856
	0.0146	-3.0457	0.0315	-2.8129	0.0786	2.4886	0.0455	-3.0491	0.1443	0.1256	0.1303	1.3528	0.3671	2.6684	0.2476	-1.0687	0.2566	0	0.1808	-1.174
	0.0508	1.0614	0.0205	-2.5268	0.2014	0.7235	0.1692	2.2899	0.2811	2.3823	0.1345	-3.0605	0.1788	-0.0494	0.4588	-2.3391	0.392	-0.1707	0.2972	0.2889
	0.0097	-1.4286	0.0344	-1.3554	0.0683	3.0919	0.3383	0	0.2684	1.4929	0.3987	-1.5574	0.1871	3.0749	0.0566	-1.5473	0.1973	-2.2988	0.1824	0.457

Table 7.4: Eigensystem for the simulated market with money transfer pattern described in Figure 7.22 with $z = |z|e^{i\phi}$

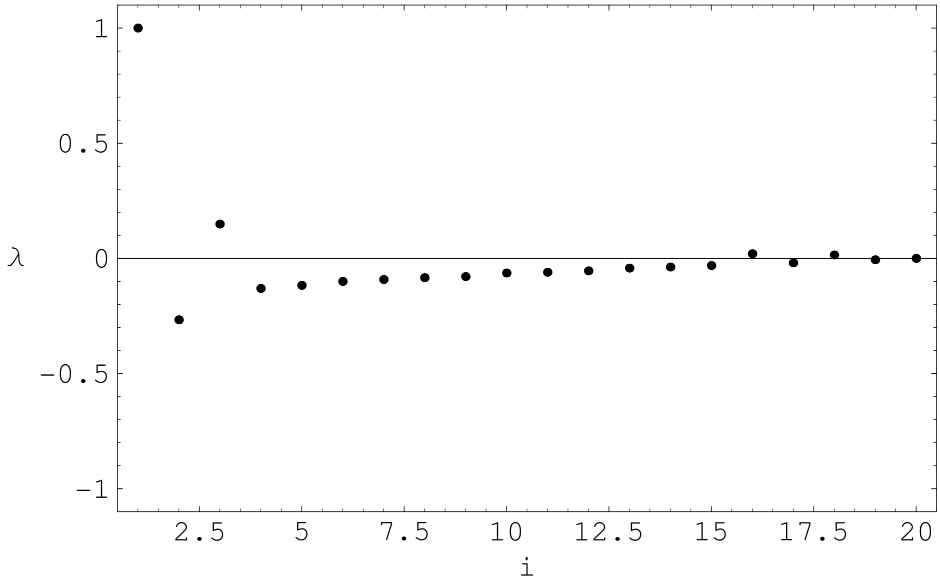

Figure 7.23: Normed eigenvalues of the simulated market with money transfer pattern described in Figure 7.22

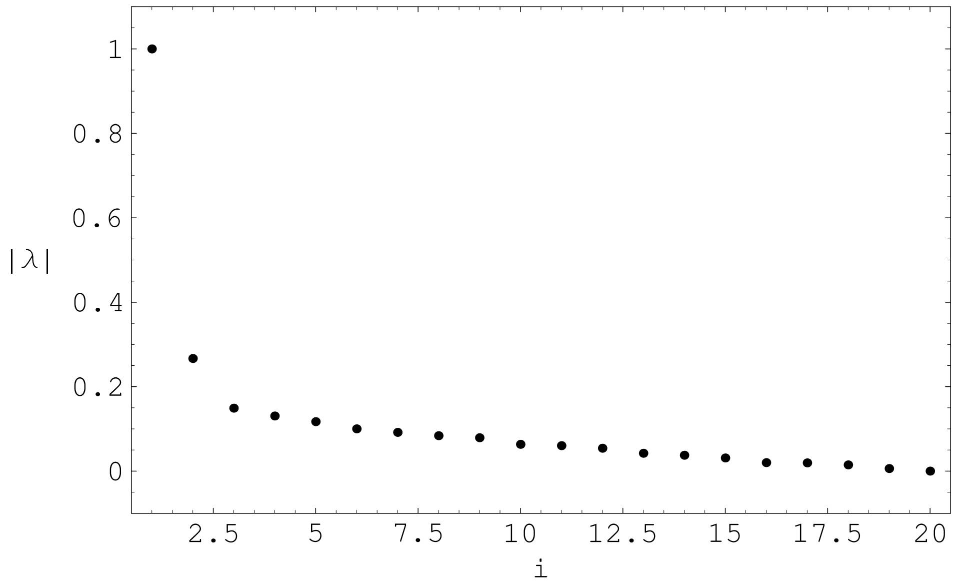

Figure 7.24: Normed eigenvalues of the simulated market with money transfer pattern described in Figure 7.22 with absolute values

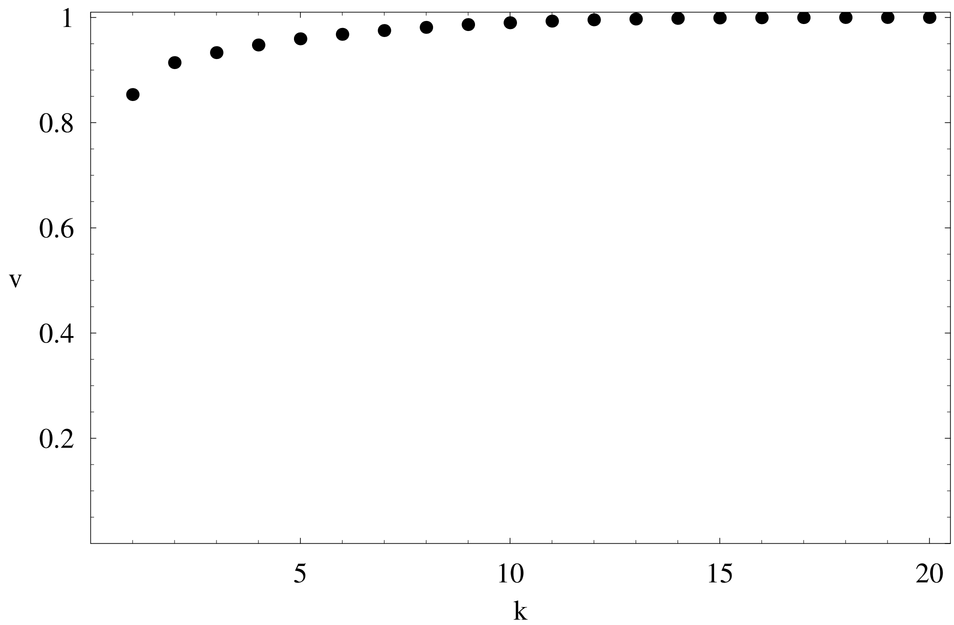

Figure 7.25: Cumulated variance of the 20 first largest absolute eigenvalues of the simulated market with money transfer pattern described in Figure 7.22

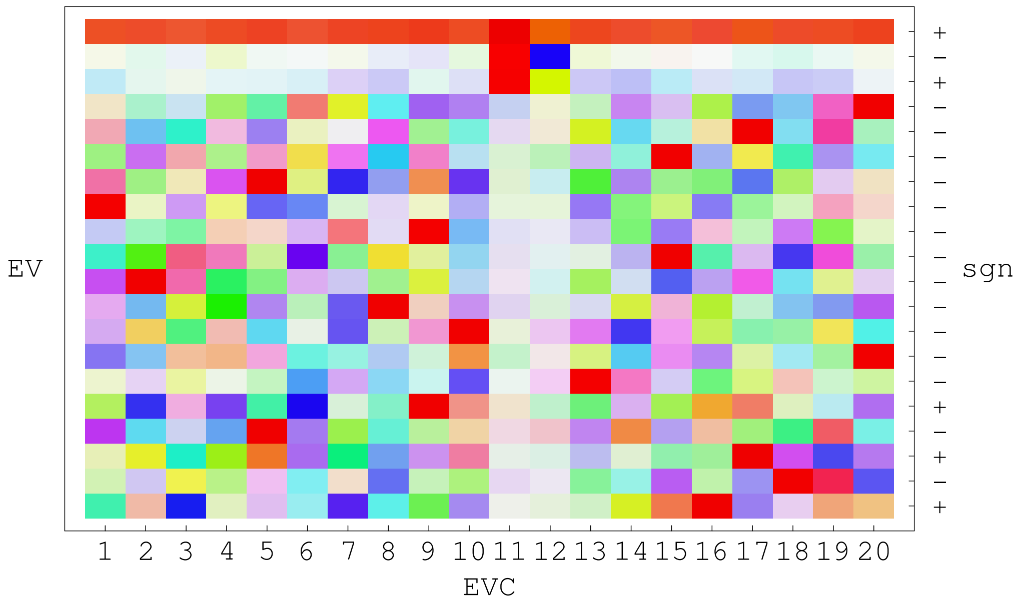

Figure 7.26: Eigensystem of a simulated market with money transfer pattern described in Figure 7.22

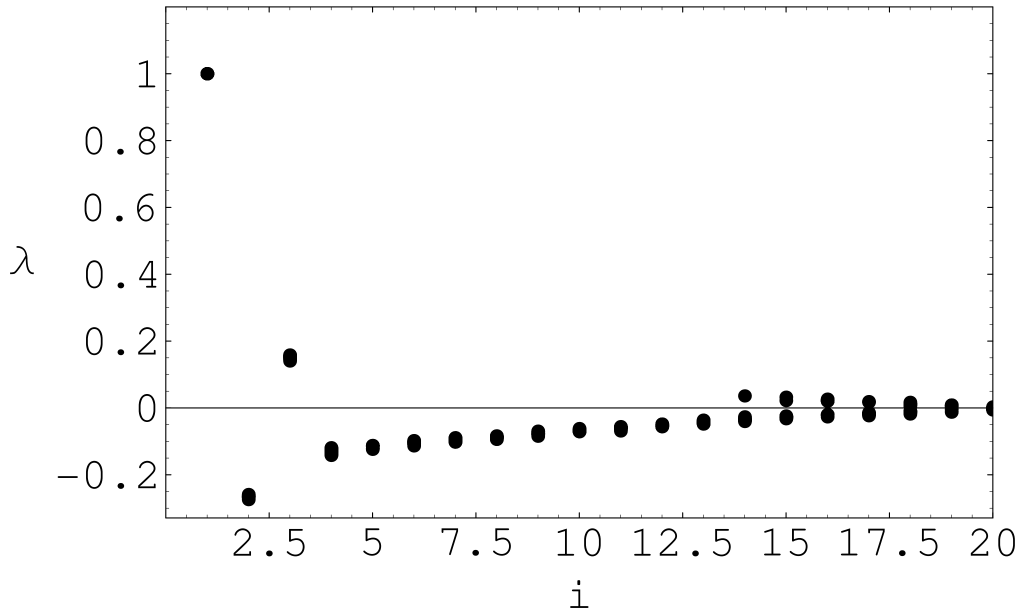

Figure 7.27: Superimposed normed eigenvalues of the simulated market with money transfer pattern described in Figure 7.22 for 10 simulation runs

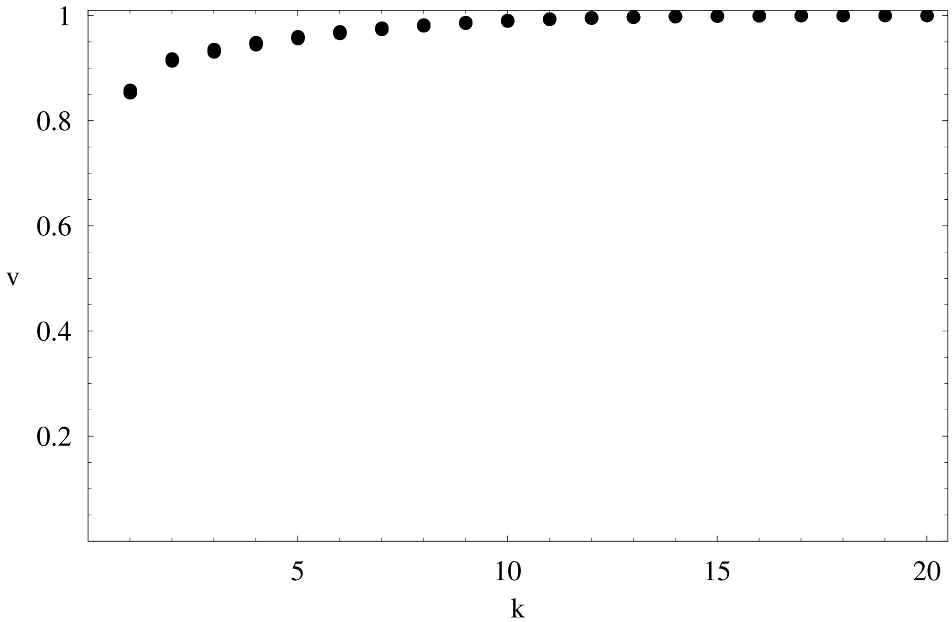

Figure 7.28: Superimposed cumulated variances of the 20 first largest absolute eigenvalues of the simulated market with money transfer pattern described in Figure 7.22 for 10 simulation runs

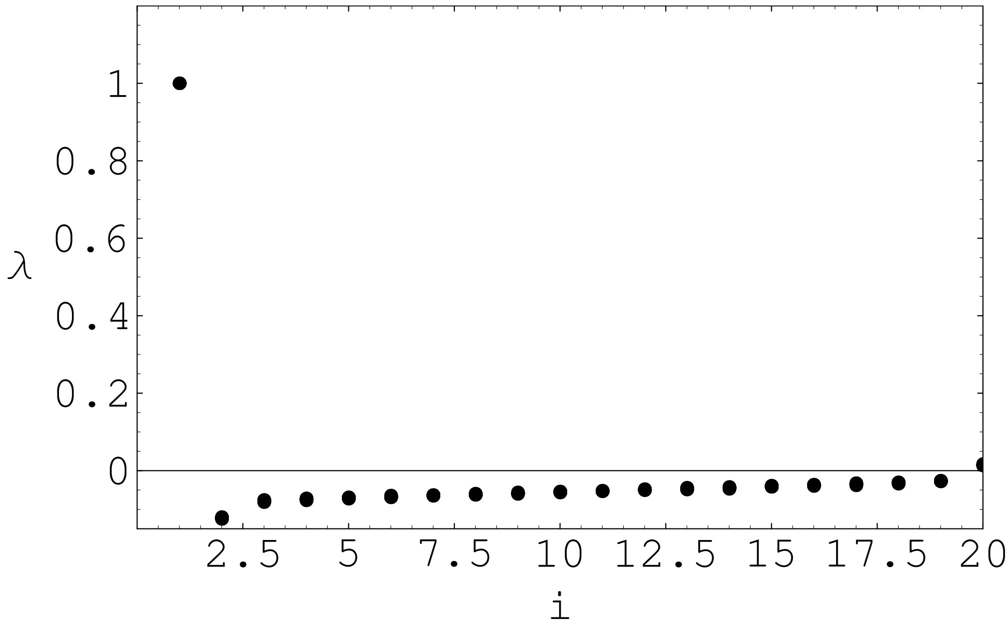

Figure 7.29: Superimposed normed eigenvalues of the simulated regular market described in Figure 7.1 for 10 simulation runs with a money transfer manipulation perturbation of 1100 from trader 11 to 12 and −700 from trader 12 to 11

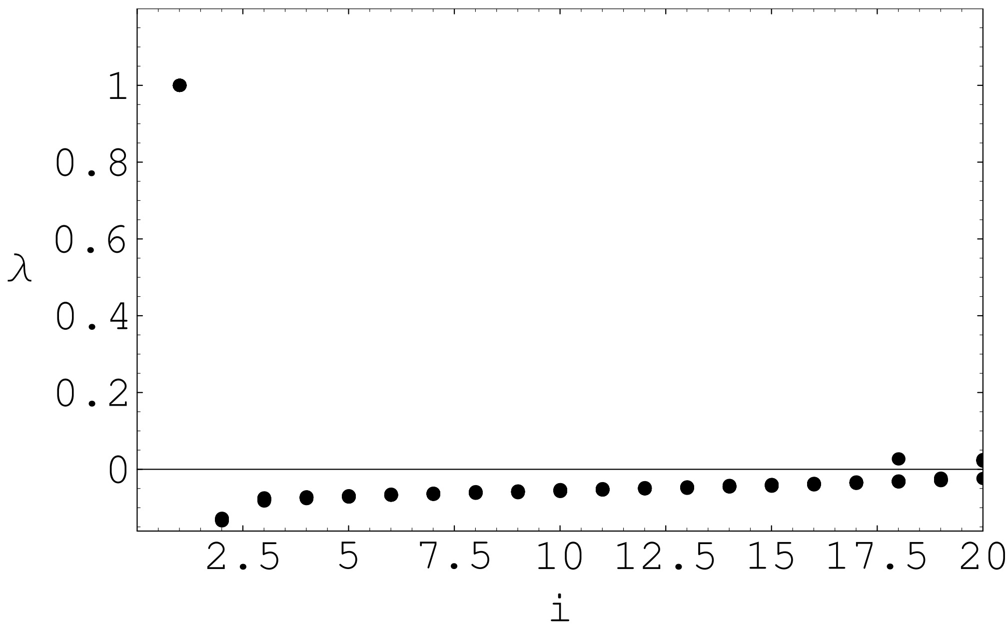

Figure 7.30: Superimposed normed eigenvalues of the simulated regular market described in Figure 7.1 for 10 simulation runs with a money transfer manipulation perturbation of 1300 from trader 11 to 12 and −700 from trader 12 to 11

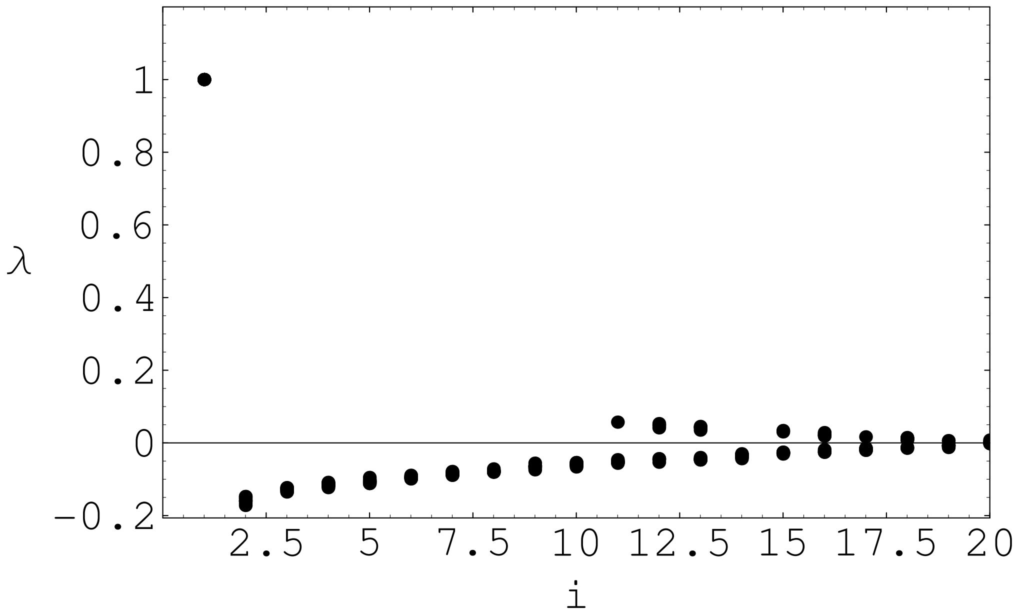

Figure 7.31: Superimposed normed eigenvalues of the simulated regular market described in Figure 7.8 for 10 simulation runs with a money transfer manipulation perturbation of 1300 from trader 11 to 12 and −700 from trader 12 to 11

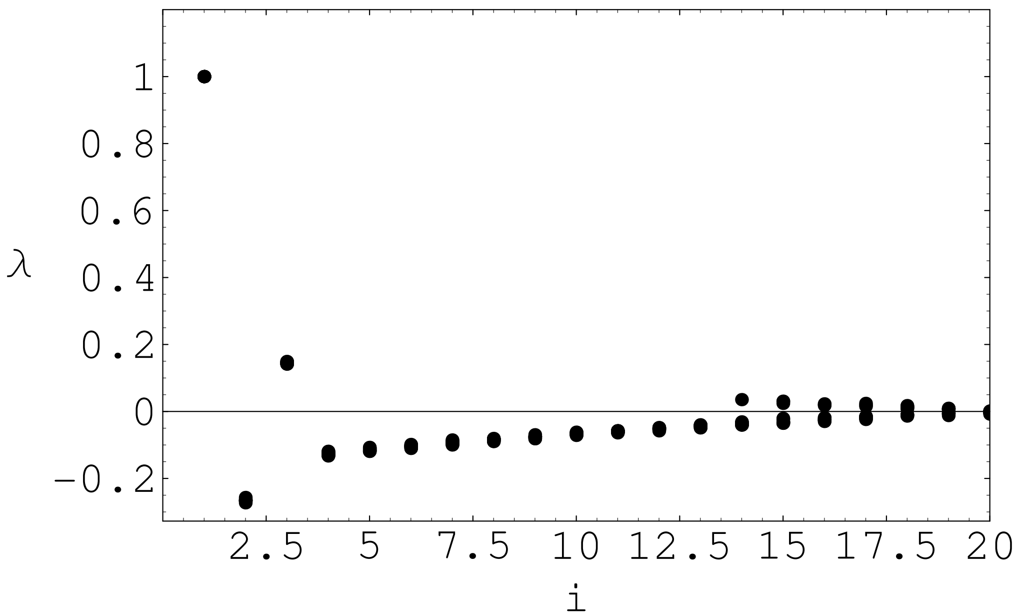

Figure 7.32: Superimposed normed eigenvalues of the simulated regular market described in Figure 7.8 for 10 simulation runs with a money transfer manipulation perturbation of 3900 from trader 11 to 12 and −700 from trader 12 to 11

7.1.3 Price Manipulation Pattern

For the third simulation we add a price manipulation pattern to the transaction network activities of Section 7.1.1. The added structures are built by inbound transactions to trader 2 from traders $1, 3, 4, \ldots, 20$ added to the networks in Figures 7.1 and 7.8 with the strengths given in the captions of the Figures of the perturbed networks in Figures 7.33 and 7.40. The eigensystems to these networks are given in Tables 7.5 and 7.5. Compared to the regular market activity and the regular market with the money transaction perturbation the strengths of the added pattern is the dominant structure within the spectra depicted by Figures 7.34 and 7.35. This is stated by the cumulated variances depicted in Figures 7.36 and 7.43 where the first eigenvalue holds about 85 % of all variance in the data. As in the regular market's eigensystems the spectra here have alternating algebraic signs from the 1^{st} to the 2^{nd} eigenvalue, but here the eigenvector components on positions 2 have the same phase and the ones of the components $1, 3, 4, \ldots, 20$ are rotated by π which can be seen in the visualisation of the eigensystem in Figures 7.37 and 7.37 in the first row (first eigenvector) compared to the second (second eigenvector). Also in this case the spectra give the same information after the simulation is carried out multiple times (see Figures 7.38 and 7.45 and Figures 7.39 and 7.39). Since only the norm of the perturbation matrix is relevant in the case of a star pattern in the price manipulation pattern the strength of the trading between ego and alters in the star can be smaller for each pair than within the money transfer pattern. Figures 7.47 and 7.48 and Figures 7.49 and 7.50 give examples for rough lower and upper bounds of the visibility of the perturbations in the case of a price manipulation pattern with respect to the configuration of the basic structure from the regular market activities of Figures 7.1 and 7.8 in Section 7.1.1.

$$
\left(\begin{array}{cccccccccccccccccccc}
0 & 3942 & 695 & 582 & 634 & 667 & 627 & 663 & 704 & 744 & 702 & 611 & 658 & 733 & 703 & 665 & 699 & 802 & 766 & 679 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
815 & 3965 & 0 & 721 & 741 & 723 & 773 & 728 & 797 & 730 & 625 & 742 & 650 & 733 & 714 & 758 & 690 & 700 & 695 & 754 \\
719 & 4056 & 654 & 0 & 818 & 726 & 686 & 725 & 720 & 649 & 804 & 702 & 712 & 741 & 728 & 669 & 670 & 682 & 674 & 626 \\
719 & 4039 & 713 & 703 & 0 & 672 & 697 & 639 & 653 & 787 & 751 & 618 & 618 & 676 & 643 & 716 & 699 & 690 & 720 & 719 \\
675 & 3992 & 603 & 714 & 726 & 0 & 676 & 639 & 693 & 751 & 676 & 737 & 586 & 713 & 732 & 671 & 688 & 605 & 803 & 707 \\
604 & 4051 & 776 & 812 & 614 & 766 & 0 & 686 & 661 & 721 & 771 & 730 & 737 & 664 & 701 & 637 & 709 & 786 & 689 & 655 \\
771 & 3981 & 841 & 680 & 734 & 701 & 663 & 0 & 704 & 659 & 719 & 705 & 717 & 713 & 636 & 717 & 703 & 645 & 674 & 685 \\
705 & 3997 & 787 & 724 & 736 & 782 & 689 & 688 & 0 & 637 & 651 & 759 & 693 & 686 & 617 & 670 & 678 & 626 & 629 & 743 \\
678 & 3907 & 682 & 676 & 638 & 711 & 733 & 675 & 726 & 0 & 706 & 652 & 749 & 706 & 637 & 664 & 745 & 759 & 748 & 703 \\
670 & 4036 & 742 & 686 & 726 & 678 & 651 & 789 & 660 & 742 & 0 & 589 & 705 & 703 & 680 & 760 & 710 & 743 & 587 & 678 \\
680 & 3997 & 624 & 719 & 681 & 724 & 716 & 740 & 711 & 727 & 681 & 0 & 655 & 701 & 727 & 705 & 669 & 705 & 699 & 805 \\
725 & 4114 & 716 & 615 & 670 & 704 & 661 & 630 & 700 & 662 & 780 & 701 & 0 & 743 & 714 & 728 & 730 & 759 & 682 & 646 \\
746 & 3978 & 695 & 661 & 672 & 571 & 693 & 668 & 641 & 636 & 748 & 778 & 702 & 0 & 621 & 726 & 724 & 688 & 676 & 665 \\
785 & 4016 & 667 & 769 & 671 & 764 & 749 & 758 & 745 & 659 & 691 & 787 & 678 & 724 & 0 & 712 & 699 & 719 & 720 & 730 \\
737 & 4075 & 715 & 730 & 698 & 640 & 601 & 661 & 681 & 643 & 750 & 684 & 767 & 726 & 733 & 0 & 650 & 644 & 719 & 744 \\
682 & 3851 & 711 & 649 & 636 & 619 & 672 & 656 & 763 & 773 & 740 & 718 & 745 & 658 & 780 & 705 & 0 & 685 & 681 & 803 \\
690 & 4040 & 698 & 728 & 777 & 718 & 632 & 742 & 741 & 680 & 610 & 686 & 752 & 724 & 718 & 723 & 725 & 0 & 713 & 640 \\
781 & 3996 & 663 & 723 & 681 & 715 & 727 & 740 & 722 & 794 & 717 & 664 & 733 & 690 & 603 & 675 & 744 & 621 & 0 & 749 \\
667 & 4032 & 642 & 707 & 724 & 655 & 649 & 750 & 689 & 739 & 680 & 803 & 736 & 761 & 618 & 682 & 640 & 644 & 793 & 0
\end{array}\right)
$$

Figure 7.33: Simulated transaction network of a market environment following the network in Figure 7.1 with $\mu = 700$ and $\sigma = 50$ and a price manipulation perturbation of 3300 to trader 2 from traders $1, 3, 4, \ldots, 20$

λ_k	28522		-10684.7		-1470.39		-1414.6		-1348.48		-1308.81		-1257.76		-1191.22		-1119.54		-1056.31	
	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$
	0.1944	-0.7729	0.5219	2.3161	0.1973	2.3176	0.197	-0.0106	0.1953	-3.0176	0.1941	-1.2791	0.1955	-2.7429	0.1954	2.732	0.1955	-0.1507	0.1944	-0.8637
	0.115	0	0.8527	0	0.1121	-1.1041	0.1236	0.5735	0.1247	2.9419	0.1214	-0.5482	0.1259	-1.0987	0.1177	-2.699	0.1195	1.4608	0.1105	1.538
	0.3532	-0.7964	0.0055	2.3923	0.0615	1.1403	0.1656	-2.8268	0.2957	-1.217	0.06	0.4476	0.1605	-0.7259	0.3051	-0.4275	0.1667	2.5303	0.1663	2.6501
	0.1757	-0.7879	0.0046	2.3633	0.4087	-1.631	0.167	-1.6957	0.1991	-3.1359	0.3067	0	0.4475	-2.8694	0.184	-0.9523	0.2514	-1.2986	0.1765	-2.0992
	0.2067	-0.7818	0.0023	2.3447	0.2698	-3.1383	0.1736	1.9133	0.0286	-3.1286	0.2867	-2.8633	0.0825	0.9477	0.0798	1.6284	0.2161	0.8028	0.1774	-2.9397
$\mathbf{x}_k$	0.2638	-0.782	0.0049	2.3441	0.1972	-2.5512	0.3933	-2.3226	0.1818	-2.2983	0.2987	-2.7838	0.1775	-1.2976	0.1481	-2.6421	0.044	-0.5453	0.1177	-0.109
	0.2151	-0.7957	0.0028	2.3855	0.3747	2.1414	0.2933	0	0.2267	2.1975	0.1156	2.7921	0.1102	-2.0334	0.2667	2.2985	0.182	0.8679	0.1422	2.2474
	0.0605	-0.7875	0.0046	2.3623	0.144	-1.6257	0.2094	-0.1525	0.2802	-1.5797	0.1289	-2.6959	0.1556	1.689	0.1721	0.4919	0.2769	-1.9681	0.5646	2.3937
	0.0518	-0.7804	0.0097	2.3419	0.1161	-1.0909	0.2376	0.5256	0.2629	0.5125	0.1502	-1.7088	0.1238	0.969	0.0979	-2.6937	0.1443	3.0588	0.2766	0
	0.0602	-0.7819	0.0089	2.3461	0.1182	0.111	0.129	0.8869	0.1915	2.6618	0.2532	-0.6082	0.1384	0.5374	0.2336	0	0.454	-2.6739	0.2489	-1.0707
	0.2888	-0.7781	0.0042	2.3335	0.2927	1.1719	0.0871	-2.5835	0.1089	0	0.27	-0.8245	0.3078	2.1408	0.5084	2.7571	0.1665	-2.0499	0.0464	-0.8369
	0.1985	-0.7855	0.0033	2.3563	0.1056	2.4103	0.2319	2.553	0.294	-0.3576	0.2318	0.0388	0.2972	3.1149	0.1642	0.4459	0.3331	1.8103	0.0661	-0.9004
	0.2434	-0.7847	0.0043	2.3543	0.2303	-2.6354	0.3874	-1.8364	0.0521	1.575	0.2943	-0.8792	0.0451	0.5979	0.0876	0.2977	0.1986	0.959	0.0979	2.0602
	0.2765	-0.7737	0.0068	2.3203	0.0433	-1.9158	0.1342	2.2228	0.3036	1.4677	0.2145	3.0642	0.1782	-0.9902	0.2472	-1.9538	0.2183	-1.8389	0.1535	-1.5769
	0.3052	-0.8027	0.0048	2.4089	0.0906	-0.1485	0.1983	1.9766	0.2789	1.4759	0.1668	-0.2015	0.206	0	0.1206	1.7668	0.112	0.9158	0.2522	2.9217
	0.3127	-0.784	0.0078	2.353	0.1685	-0.1582	0.1445	0.256	0.2825	-2.7535	0.0755	1.9614	0.2541	2.4111	0.222	-1.0441	0.0982	1.8437	0.1903	0.0179
	0.1663	-0.788	0.0051	2.3646	0.3896	2.534	0.0247	-2.6311	0.1875	-2.142	0.0986	1.941	0.0904	2.239	0.3263	-2.2584	0.3646	-1.9468	0.1488	2.6565
	0.0618	-0.79	0.0066	2.3702	0.189	0	0.2347	-1.9927	0.1816	-0.4951	0.3682	-3.1193	0.4132	-2.9676	0.205	-2.0945	0.1254	1.9114	0.1814	2.0321
	0.3718	-0.787	0.005	2.3619	0.1237	-0.7261	0.3654	2.1558	0.3404	0.4508	0.0276	1.361	0.3244	2.9504	0.2317	-1.8028	0.1048	0	0.3731	2.1437
	0.0753	-0.7818	0.0034	2.3476	0.2939	0.0545	0.1547	-0.7515	0.1226	3.0267	0.3506	-0.4478	0.0516	-1.229	0.0874	2.5987	0.2521	-0.7518	0.1865	2.7533

λ_k	-978.41		-943.08		-904.34		-865.44		-807.05		-777.05		-714.68		-659.22		-554.07		-466.87	
	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$	$\mid z \mid$	$\phi(z)$
	0.1966	-1.2257	0.1961	0.8504	0.1973	3.1124	0.1947	-2.0715	0.1964	-0.5048	0.1964	-0.1851	0.193	2.8088	0.1962	-1.6914	0.1964	-1.2144	0.1966	0.5078
	0.1218	2.1174	0.1182	-2.9478	0.1301	0.5456	0.1186	0.96	0.1201	-1.7407	0.127	2.7781	0.1061	3.0894	0.1231	0.0394	0.1176	-3.1327	0.1217	1.0799
	0.329	-2.5462	0.2731	2.8314	0.2509	2.7736	0.1965	1.662	0.1907	0.3446	0.0287	-1.4402	0.1739	0	0.3956	-1.2097	0.0997	-1.7592	0.2458	1.7926
	0.0794	-2.2104	0.1705	-0.8564	0.2333	-1.7255	0.1052	1.2852	0.277	-1.4139	0.1576	2.3137	0.0748	0.0406	0.1777	0.2065	0.2308	2.653	0.1382	-1.773
	0.4084	-2.924	0.2216	-0.1261	0.0926	-1.2605	0.2014	-0.308	0.2217	2.8687	0.2359	2.1362	0.2122	-0.6121	0.1402	-0.1584	0.4004	-2.048	0.2919	-1.6723
$\mathbf{x}_k$	0.1577	0.7814	0.1423	-2.7686	0.2992	1.1503	0.2618	-2.0902	0.0825	-1.6815	0.3177	1.6849	0.3128	-1.0266	0.2038	0.8976	0.2714	2.696	0.1462	-2.8171
	0.0637	2.8674	0.3096	2.335	0.1943	2.9454	0.1741	1.4019	0.4001	0.3671	0.0993	-2.3097	0.1997	0.2044	0.208	0	0.1219	1.1865	0.3001	-1.0784
	0.2094	0	0.2123	2.8771	0.12	2.1968	0.0453	1.5811	0.045	0.1219	0.1657	-1.1926	0.2322	-0.968	0.1356	-2.2123	0.0827	-2.8848	0.4163	0.3541
	0.1802	-2.6564	0.3545	0	0.1606	-0.6552	0.3791	-2.6999	0.0674	-0.0958	0.296	0.7967	0.333	-0.5947	0.1851	-2.3891	0.3848	1.685	0.0196	2.535
	0.1864	2.4596	0.1919	-1.3378	0.2383	0.8346	0.3055	-1.1862	0.1521	3.0153	0.335	-1.3776	0.3048	-1.1619	0.1646	0.954	0.1147	0	0.1816	-0.7325
	0.1943	-0.3738	0.1509	2.9381	0.2146	-1.4843	0.1807	-1.5865	0.3391	1.9918	0.1234	3.0054	0.0945	-2.293	0.0404	0.087	0.2278	-3.1181	0.1536	0
	0.293	0.3577	0.2114	-0.8203	0.1545	1.0109	0.1117	1.5988	0.2623	1.0672	0.2896	-2.1347	0.2989	2.2985	0.2005	-2.3268	0.2317	1.7306	0.1486	-2.7727
	0.0917	2.3704	0.2674	2.3266	0.4562	0	0.2016	-3.1135	0.2699	-2.7487	0.342	0.7511	0.1061	2.1572	0.0772	-2.527	0.1046	1.5766	0.207	0.4877
	0.2701	2.8772	0.1753	-2.4706	0.3067	0.9829	0.3299	0.305	0.0689	0	0.062	1.1996	0.2829	1.7704	0.3923	-2.5805	0.2033	-2.0344	0.1043	-0.4332
	0.1362	0.9653	0.253	0.7448	0.1165	-2.8708	0.3663	-2.4324	0.2845	-1.0757	0.3002	-2.5074	0.262	2.8063	0.1582	1.6278	0.2245	2.1937	0.2742	2.4766
	0.3521	1.8832	0.1805	2.9222	0.2371	-2.6464	0.1985	1.4931	0.209	-2.9328	0.2053	1.6925	0.2186	1.7352	0.4012	-2.8125	0.1766	-2.4908	0.1725	1.0413
	0.0747	1.7166	0.2291	-0.3063	0.2268	2.5474	0.2717	3.0062	0.3424	1.6596	0.2695	2.8179	0.0152	2.086	0.2507	2.2087	0.2582	0.0742	0.0497	0.7316
	0.0491	-1.641	0.1679	0.0898	0.079	-0.3515	0.235	0	0.1512	2.0643	0.2509	0	0.2831	-2.9152	0.229	0.7877	0.0805	2.0401	0.3981	1.1396
	0.182	-2.1057	0.1835	-2.848	0.0965	2.3635	0.0691	2.3503	0.1167	-1.9409	0.1678	0.4438	0.2482	-1.9752	0.0794	2.5756	0.2771	-0.3429	0.1408	-1.7305
	0.3696	-1.0736	0.301	-1.9997	0.2916	-0.5835	0.1339	-0.3794	0.2203	2.6729	0.0997	-1.8514	0.1652	1.2685	0.2546	-2.6984	0.2795	0.2166	0.2803	-2.3428

Table 7.5: Eigensystem for the price manipulation pattern described in Figure 7.33 with $z = |z|e^{i\phi}$

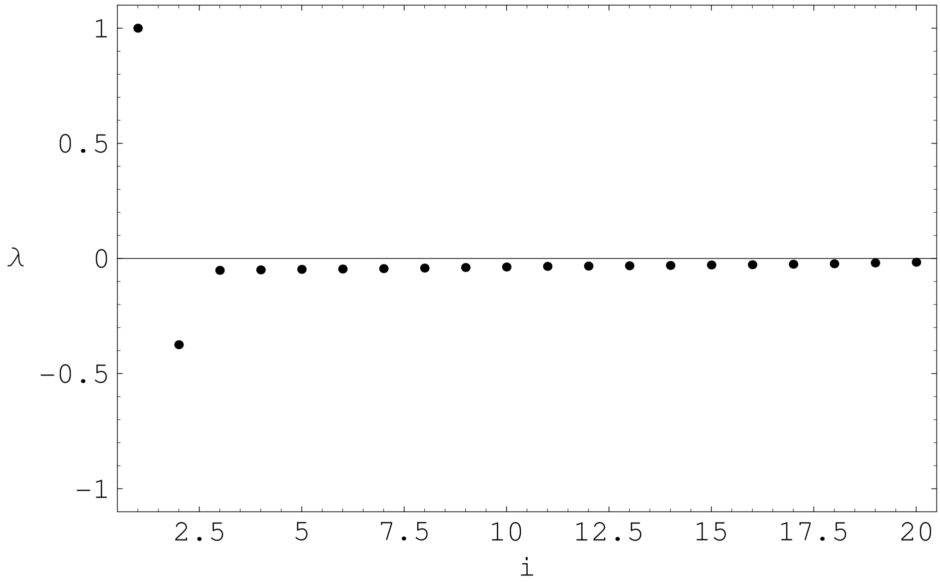

Figure 7.34: Normed eigenvalues of a simulated market described in Figure 7.33 with price manipulation

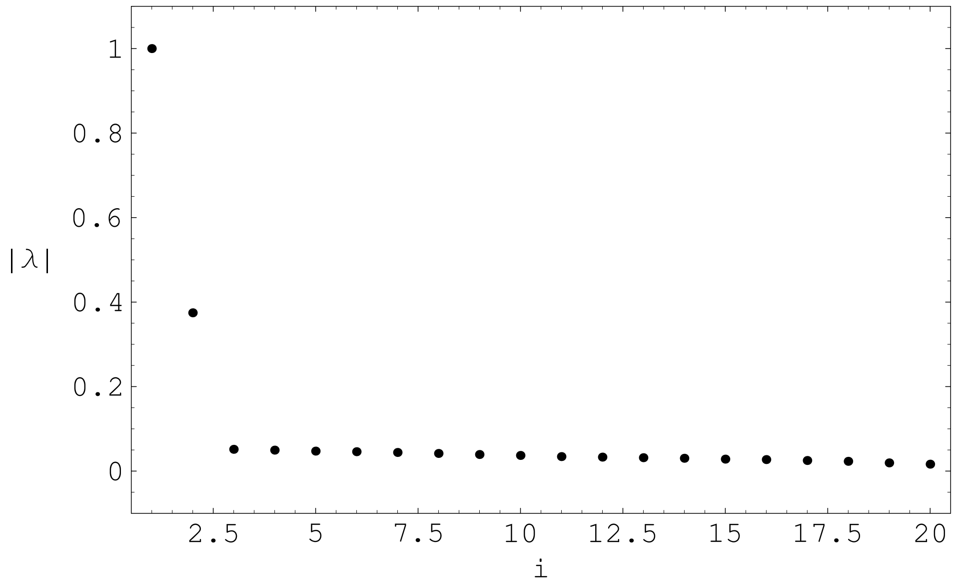

Figure 7.35: Normed eigenvalues of the simulated market described in Figure 7.33 with price manipulation with absolute values

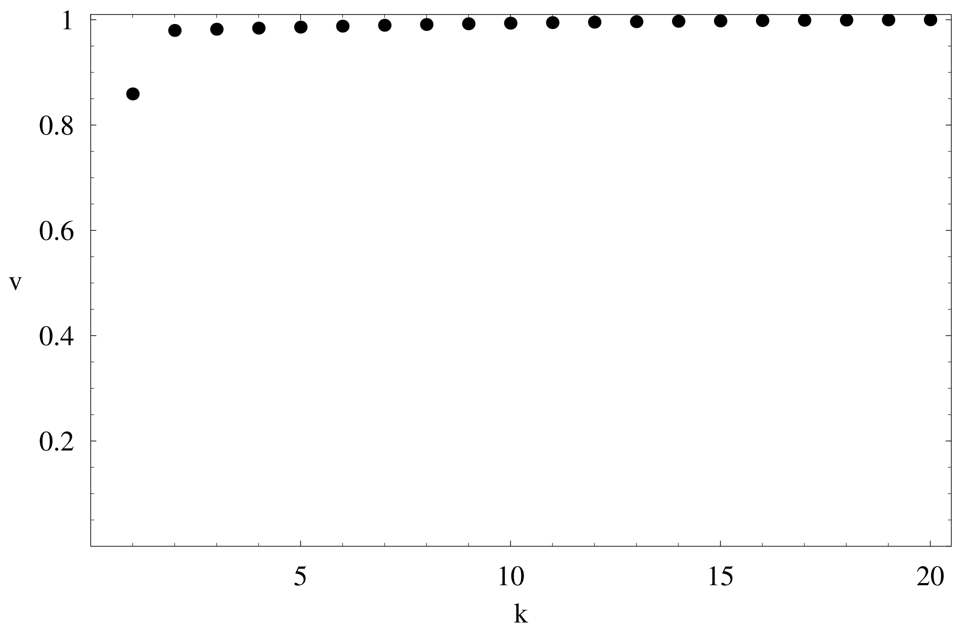

Figure 7.36: Cumulated variance of the 20 first largest absolute eigenvalues of the simulated market described in Figure 7.33 with price manipulation pattern

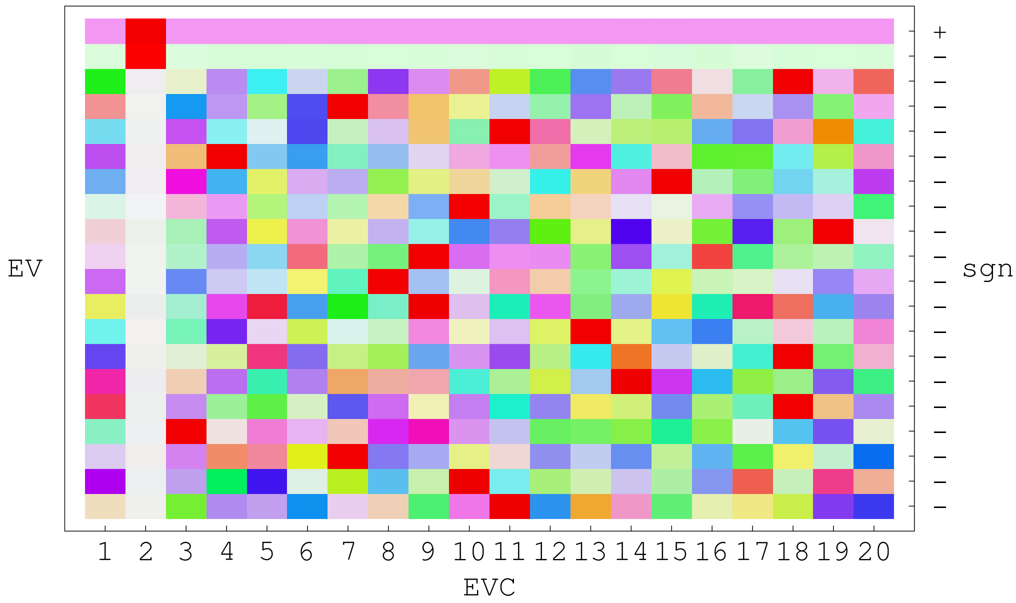

Figure 7.37: Eigensystem of the simulated market described in Figure 7.33 with price manipulation pattern

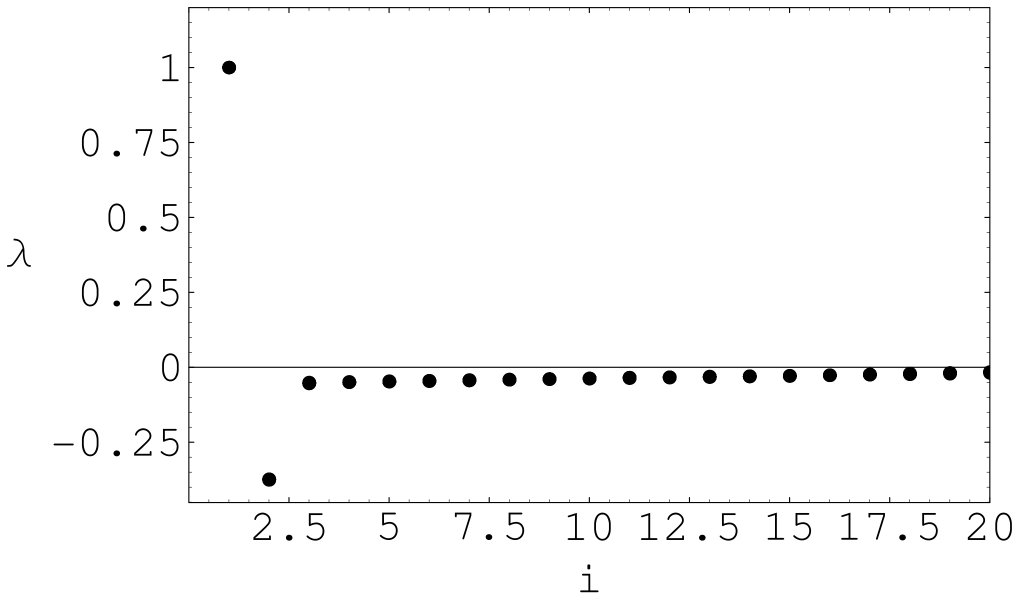

Figure 7.38: Superimposed normed eigenvalues of the simulated market described in Figure 7.33 for 10 simulation runs with price manipulation pattern

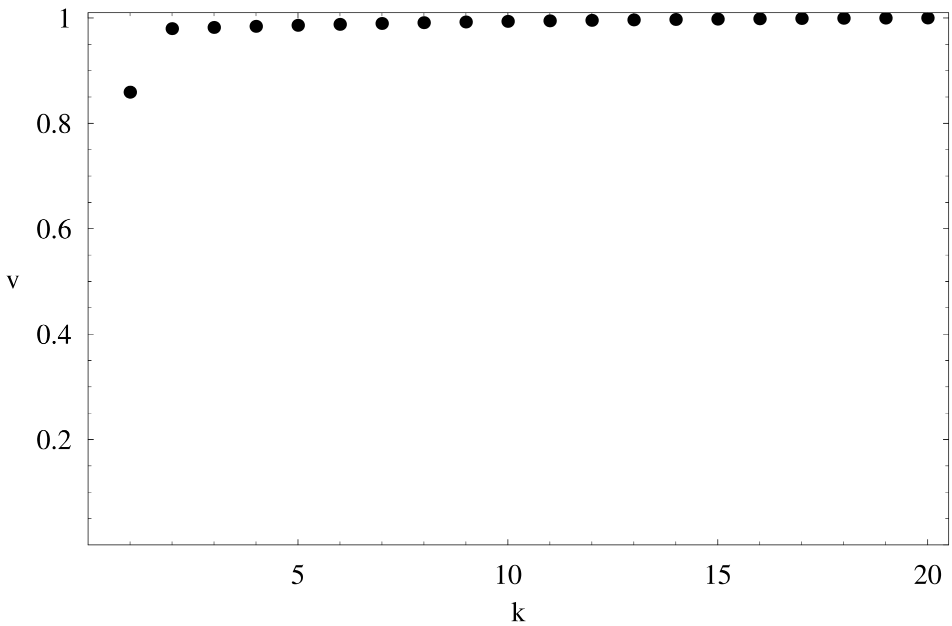

Figure 7.39: Superimposed cumulated variances of the 20 first largest absolute eigenvalues of the simulated market described in Figure 7.33 for 10 simulation runs with price manipulation pattern

$$
\begin{pmatrix}
0 & 2957 & 731 & 654 & 764 & 644 & 715 & 794 & 686 & 698 & 676 & 592 & 697 & 755 & 718 & 670 & 720 & 725 & 654 & 741 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
711 & 3126 & 0 & 666 & 632 & 648 & 723 & 695 & 692 & 754 & 641 & 754 & 681 & 788 & 675 & 667 & 796 & 774 & 744 & 658 \\
651 & 2913 & 726 & 0 & 634 & 626 & 694 & 700 & 657 & 685 & 628 & 672 & 691 & 803 & 711 & 749 & 642 & 606 & 737 & 669 \\
595 & 3072 & 790 & 696 & 0 & 630 & 686 & 710 & 640 & 651 & 774 & 704 & 654 & 694 & 699 & 730 & 720 & 683 & 687 & 650 \\
753 & 3056 & 789 & 749 & 678 & 0 & 717 & 612 & 665 & 638 & 738 & 765 & 601 & 726 & 679 & 740 & 631 & 708 & 677 & 702 \\
759 & 3053 & 738 & 731 & 651 & 721 & 0 & 733 & 743 & 640 & 756 & 697 & 738 & 704 & 717 & 705 & 778 & 700 & 724 & 650 \\
639 & 3030 & 681 & 742 & 654 & 669 & 661 & 0 & 661 & 641 & 638 & 676 & 776 & 717 & 738 & 627 & 770 & 748 & 647 & 674 \\
750 & 3069 & 679 & 693 & 674 & 732 & 725 & 715 & 0 & 702 & 730 & 683 & 674 & 767 & 646 & 621 & 807 & 754 & 737 & 757 \\
729 & 2946 & 657 & 716 & 764 & 727 & 623 & 672 & 600 & 0 & 616 & 642 & 696 & 672 & 740 & 730 & 719 & 749 & 721 & 604 \\
612 & 3145 & 795 & 649 & 753 & 736 & 754 & 822 & 730 & 709 & 0 & 705 & 702 & 682 & 715 & 659 & 700 & 693 & 676 & 736 \\
717 & 2980 & 761 & 634 & 700 & 719 & 629 & 689 & 724 & 639 & 783 & 0 & 691 & 692 & 838 & 674 & 650 & 650 & 693 & 786 \\
677 & 3005 & 753 & 728 & 754 & 711 & 771 & 628 & 655 & 730 & 748 & 743 & 0 & 656 & 683 & 627 & 689 & 620 & 705 & 692 \\
696 & 2970 & 615 & 678 & 639 & 707 & 703 & 647 & 712 & 707 & 652 & 721 & 717 & 0 & 655 & 716 & 632 & 648 & 669 & 661 \\
697 & 2975 & 642 & 781 & 650 & 718 & 733 & 741 & 612 & 645 & 734 & 735 & 682 & 686 & 0 & 758 & 669 & 674 & 738 & 711 \\
733 & 2962 & 704 & 671 & 700 & 717 & 665 & 726 & 740 & 707 & 710 & 618 & 691 & 715 & 720 & 0 & 619 & 744 & 679 & 721 \\
736 & 3032 & 692 & 719 & 706 & 686 & 725 & 604 & 660 & 654 & 708 & 669 & 714 & 752 & 702 & 718 & 0 & 782 & 648 & 745 \\
729 & 2969 & 665 & 628 & 697 & 691 & 716 & 617 & 739 & 694 & 688 & 668 & 766 & 697 & 673 & 673 & 722 & 0 & 756 & 712 \\
627 & 3075 & 761 & 704 & 710 & 681 & 728 & 694 & 670 & 717 & 671 & 712 & 754 & 737 & 750 & 677 & 735 & 653 & 0 & 725 \\
659 & 2976 & 704 & 698 & 727 & 689 & 774 & 777 & 650 & 704 & 770 & 687 & 662 & 673 & 656 & 679 & 705 & 759 & 692 & 0
\end{pmatrix}
$$

Figure 7.40: Simulated transaction network of a market environment following the network in Figure 7.8 with $\mu = 700$ and $\sigma = 150$ and a price manipulation perturbation of 2300 to trader 2 from traders $1, 3, 4, \ldots, 20$

λ_k	24771.7		-6987.76		-1459.04		-1397.63		-1328.57		-1269.83		-1212.63		-1181.15		-1109.65		-1038.76	
	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$
$\mathbf{x}_k$	0.2013	-0.7899	0.4689	2.3819	0.207	0	0.1988	-1.5913	0.2021	-2.902	0.2026	-1.692	0.2059	0.8146	0.2017	-3.0614	0.2032	2.8074	0.1987	2.1624
	0.0998	0	0.8827	0	0.1184	-0.9881	0.0969	0.3324	0.1191	-2.9301	0.1152	-0.7045	0.1066	-1.2769	0.1129	-0.1621	0.1165	1.7291	0.1032	-1.582
	0.3691	-0.7803	0.0116	2.3304	0.3069	-1.8623	0.2423	-2.4337	0.0706	1.1448	0.1862	0.3864	0.1723	-2.9762	0.303	-1.5049	0.1666	-1.8204	0.2205	-2.8822
	0.0197	-0.7781	0.0018	2.323	0.1763	0.8403	0.2489	-2.2394	0.2127	1.668	0.3151	-1.7874	0.3064	-0.6708	0.3084	-2.155	0.1748	-0.5632	0.1423	0
	0.1763	-0.783	0.0172	2.3488	0.3322	-2.0733	0.1944	-1.2943	0.2642	2.7361	0.2463	1.915	0.248	-0.5877	0.2452	2.1207	0.1289	2.3925	0.3196	0.9177
	0.2751	-0.7886	0.005	2.3679	0.3289	2.2357	0.0362	-0.5595	0.2596	3.0329	0.1279	2.8869	0.2879	-1.4706	0.1572	-0.9126	0.4315	-1.8536	0.0489	-2.3767
	0.0675	-0.7892	0.0066	2.3736	0.1095	2.7613	0.1209	-3.0202	0.2188	-2.0258	0.1202	1.4868	0.0689	0.9365	0.0953	1.8728	0.1667	1.9256	0.2923	-2.5782
	0.131	-0.7793	0.0042	2.3291	0.1934	-2.6777	0.2722	1.8335	0.3219	2.4491	0.1728	-3.0808	0.1893	-0.9531	0.2236	-0.808	0.1312	2.4734	0.1889	-0.4953
	0.1539	-0.802	0.0029	2.423	0.1494	-1.7722	0.2046	0.4136	0.1564	-1.8667	0.1377	0	0.087	-1.7816	0.2155	-2.1761	0.1021	-2.4204	0.2974	0.5768
	0.3705	-0.7869	0.0066	2.3633	0.2637	-2.8954	0.4336	2.3215	0.1394	-2.037	0.3642	-0.5189	0.085	1.8841	0.0669	-2.9665	0.2747	-0.3599	0.0743	-1.217
	0.2327	-0.79	0.0088	2.376	0.2462	-0.3877	0.1236	-2.9069	0.3219	-0.9307	0.1851	-1.8749	0.2029	0.7042	0.098	-0.4241	0.2233	0	0.1488	-3.1143
	0.1583	-0.7916	0.0038	2.3841	0.0454	1.1188	0.1375	2.3556	0.1906	-1.9426	0.1315	1.4522	0.4608	2.6967	0.1858	-3.0432	0.0733	2.4616	0.0556	0.5453
	0.2492	-0.785	0.0048	2.3585	0.1973	0.7996	0.2066	0	0.1331	-0.8417	0.2273	1.9909	0.2886	-1.6269	0.0877	-0.8623	0.4826	1.5853	0.1682	-0.9912
	0.2297	-0.7649	0.004	2.2607	0.204	-2.7397	0.1193	2.4494	0.1744	0	0.3633	-2.9883	0.3208	1.3308	0.368	1.5884	0.1708	0.9845	0.2316	-3.1135
	0.3382	-0.7824	0.0057	2.341	0.0767	-2.3145	0.2188	-0.3375	0.2311	0.4666	0.1852	-0.0953	0.3239	0.319	0.1479	1.4967	0.0171	-0.9958	0.2374	2.1058
	0.0847	-0.7898	0.0031	2.3804	0.2992	0.3706	0.1287	2.2368	0.1111	-0.3045	0.3951	1.1092	0.0892	2.4996	0.3082	0	0.1134	-2.8114	0.0339	-1.056
	0.0319	-0.7829	0.0082	2.3392	0.2615	2.0984	0.312	0.6314	0.3974	1.6009	0.0577	-2.1452	0.1875	2.8883	0.1256	1.1305	0.19	-0.6581	0.4984	-0.1562
	0.2416	-0.7816	0.0114	2.3355	0.1678	1.7345	0.3148	2.331	0.133	0.4655	0.1991	-2.9181	0.1653	0	0.3495	-2.2888	0.3	1.3662	0.2855	-0.3382
	0.3626	-0.7887	0.0056	2.3715	0.2499	-0.0396	0.3311	1.9953	0.3232	-2.777	0.137	-2.23	0.0783	-1.8342	0.2127	1.5955	0.2129	2.5292	0.0683	-3.1076
	0.1305	-0.7874	0.0065	2.3666	0.2458	-1.5816	0.1057	-1.172	0.1684	-1.1075	0.2267	-2.6551	0.0417	1.9635	0.2785	1.7242	0.2263	2.6971	0.2493	-0.4887

λ_k	-994.83		-950.58		-912.5		-847.5		-809.43		-747.25		-736.65		-664.88		-582.24		-540.78	
	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$
$\mathbf{x}_k$	0.2068	-0.7428	0.2017	-0.0679	0.2024	-2.2031	0.2014	0.341	0.2028	0	0.2007	1.0215	0.2039	1.2333	0.202	-1.6013	0.2044	0	0.2025	2.8854
	0.1219	-2.7774	0.102	1.93	0.1055	0.6256	0.1019	-0.7383	0.0996	-3.05	0.1015	2.0996	0.1077	1.4114	0.0996	2.8062	0.1146	2.8861	0.1002	1.4288
	0.3075	-0.6488	0.3425	-1.3343	0.0411	-2.9696	0.1089	2.3863	0.1628	1.1538	0.2002	1.715	0.2771	0.6238	0.2023	2.6642	0.1263	-2.6185	0.2181	1.9313
	0.1866	-0.4764	0.2771	1.4801	0.374	0.7527	0.0362	1.7894	0.3036	0.0822	0.1947	1.2864	0.1945	-1.8779	0.0827	-0.2889	0.1347	-2.3895	0.2768	-1.2285
	0.0228	-3.0865	0.1204	-0.0083	0.2537	-0.141	0.4087	2.8625	0.1769	1.3129	0.0974	0.2232	0.1958	0.7026	0.2947	-0.8416	0.0964	1.9732	0.167	-0.2339
	0.2741	1.4022	0.1809	-0.4101	0.1423	1.548	0.2016	0.4578	0.2769	-2.0786	0.1632	0	0.2072	-2.4439	0.066	-3.0933	0.3356	-1.563	0.0541	-0.1536
	0.0774	1.2636	0.2542	2.001	0.2045	-3.0742	0.0816	-1.3623	0.2033	-0.1401	0.163	0.3351	0.4207	-2.8871	0.498	2.075	0.3249	2.5828	0.2486	2.005
	0.3013	1.1595	0.2292	-0.921	0.2023	2.9491	0.2506	0	0.3188	2.1324	0.3936	-2.7093	0.1326	-2.3673	0.2359	0	0.0993	2.1827	0.1115	2.2219
	0.55	2.7063	0.0834	1.5003	0.118	0	0.1382	-0.7005	0.2549	2.6288	0.3806	1.5836	0.2052	3.0607	0.1471	2.3904	0.1909	-0.0638	0.2517	2.4281
	0.0686	0.0086	0.1966	-1.4254	0.1486	0.0102	0.2425	-1.222	0.2488	-2.4253	0.1155	-1.9686	0.0865	0	0.1023	-1.4775	0.2806	-1.6445	0.2535	-2.3519
	0.1278	2.9984	0.1282	-2.9162	0.4031	-1.2959	0.1468	-1.6522	0.0606	2.2437	0.2887	-0.7184	0.259	-2.7015	0.1238	2.7529	0.157	1.7545	0.4453	0.726
	0.0286	2.0479	0.2139	-0.9195	0.1516	-2.4429	0.4919	1.5652	0.1964	-3.01	0.2013	3.0169	0.1395	2.8634	0.2049	2.8851	0.4189	-1.8709	0.1323	0
	0.0634	-1.4	0.2238	0.1437	0.3325	-1.9438	0.1408	-1.7693	0.1802	-1.1315	0.0486	-2.5807	0.1691	-0.9572	0.1678	1.6308	0.2544	2.0838	0.3065	-1.5456
	0.1755	2.1413	0.2572	0	0.1189	0.7085	0.1512	-2.2652	0.1384	1.8192	0.1973	1.8037	0.1342	-2.3013	0.1697	0.8479	0.2874	-1.226	0.2939	-2.1586
	0.1915	2.0174	0.2617	2.8108	0.2007	-2.088	0.2489	0.8743	0.2372	-1.9711	0.1971	-2.4028	0.3054	3.097	0.2846	-0.7425	0.281	-2.3771	0.0711	-0.2789
	0.0885	-2.341	0.3034	3.1275	0.1011	-1.4273	0.1788	-1.9203	0.3196	2.5159	0.268	-0.1209	0.2796	0.1673	0.2529	-0.5254	0.3002	-0.8992	0.2302	0.201
	0.0618	-2.4314	0.0733	2.6427	0.2334	2.0449	0.2258	1.8518	0.2001	0.6127	0.0688	-1.6419	0.2726	1.8254	0.2498	2.8561	0.0943	-2.7746	0.1775	-2.8171
	0.1534	1.4892	0.2714	-2.8399	0.2931	-2.5877	0.0682	2.4076	0.2998	-2.7013	0.3246	-0.2332	0.1712	1.6705	0.0848	1.7547	0.092	0.2727	0.0897	-2.982
	0.3162	1.8794	0.0945	-2.8604	0.1844	0.6016	0.3057	3.14	0.1255	-1.3999	0.2973	2.3979	0.0526	-0.6654	0.2064	0.9596	0.1069	-3.078	0.2731	-1.632
	0.3311	0	0.3433	-1.9836	0.2606	1.458	0.1781	2.8147	0.2336	2.1753	0.1461	-1.596	0.3095	-2.4445	0.3212	-2.6303	0.1148	0.9546	0.1541	1.1435

Table 7.6: Eigensystem for the price manipulation pattern described in Figure 7.40 with $z = |z|e^{i\phi}$

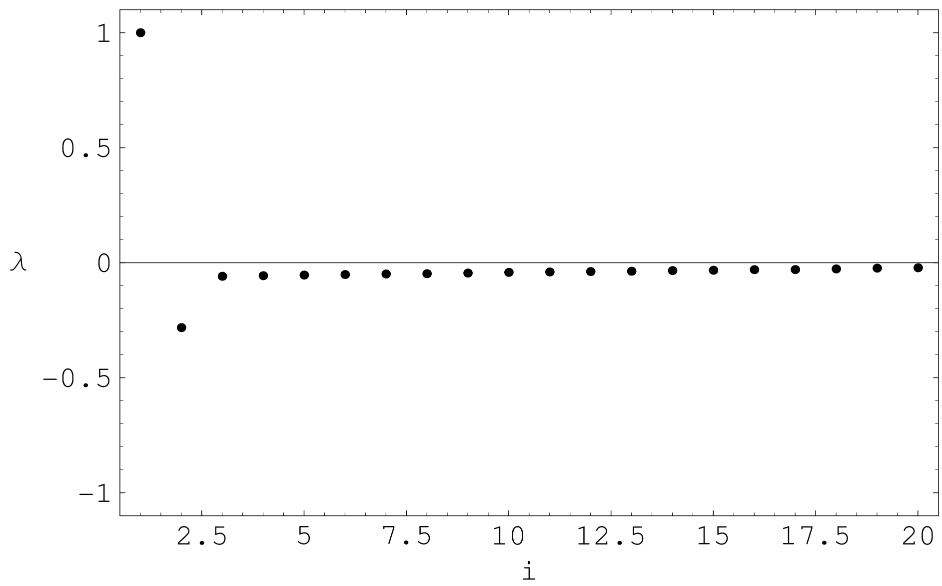

Figure 7.41: Normed eigenvalues of the simulated market described in Figure 7.40 with price manipulation

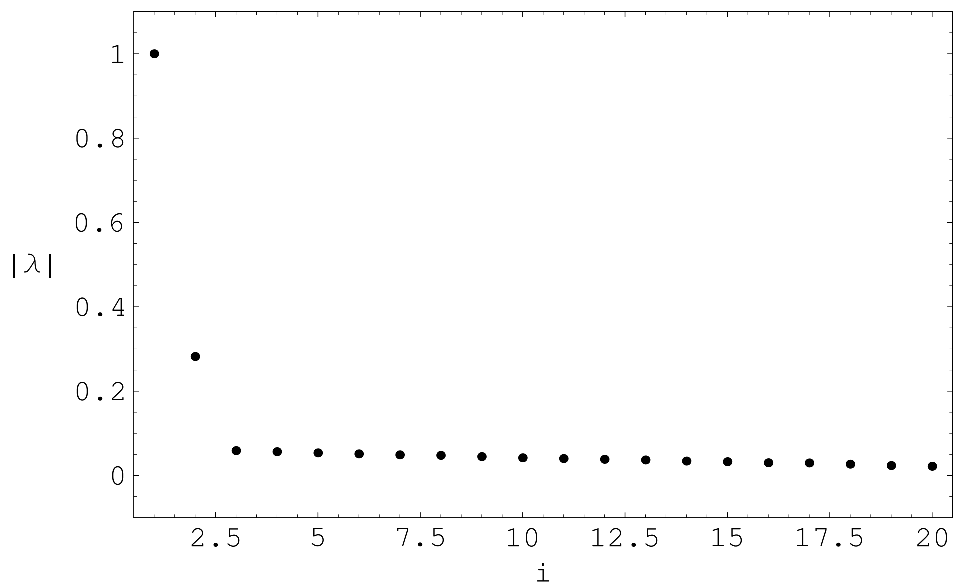

Figure 7.42: Normed eigenvalues of the simulated market described in Figure 7.40 with price manipulation with absolute values

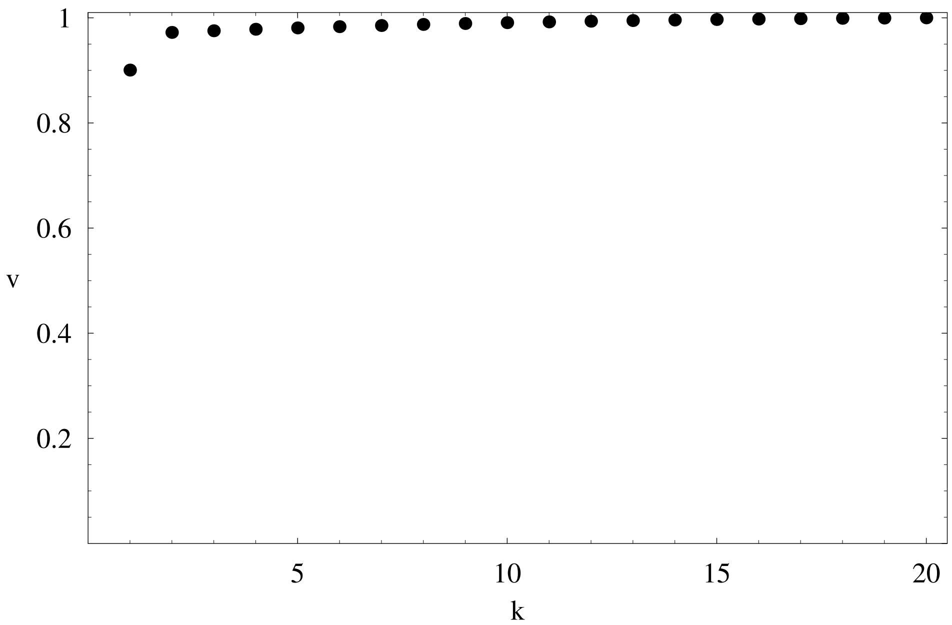

Figure 7.43: Cumulated variance of the 20 first largest absolute eigenvalues of the simulated market described in Figure 7.40 with price manipulation pattern

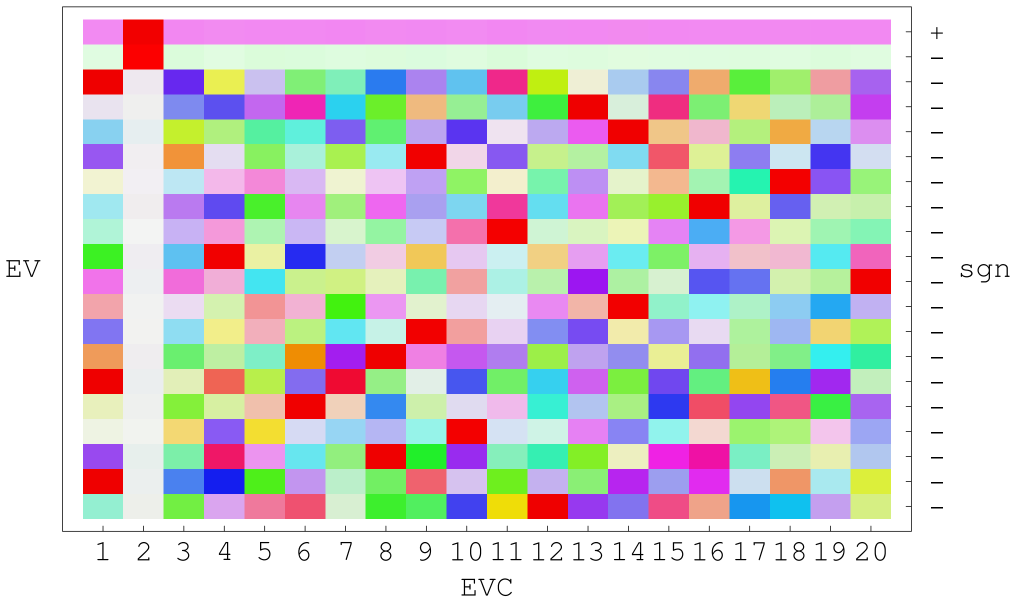

Figure 7.44: Eigensystem of the simulated market described in Figure 7.40 with price manipulation pattern

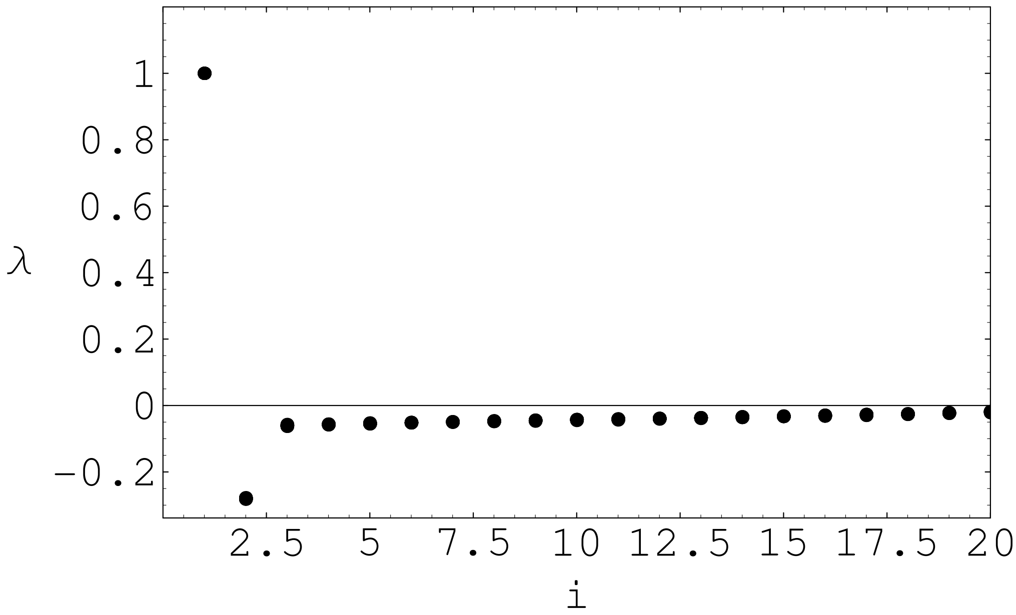

Figure 7.45: Superimposed normed eigenvalues of the simulated market described in Figure 7.40 for 10 simulation runs with price manipulation pattern

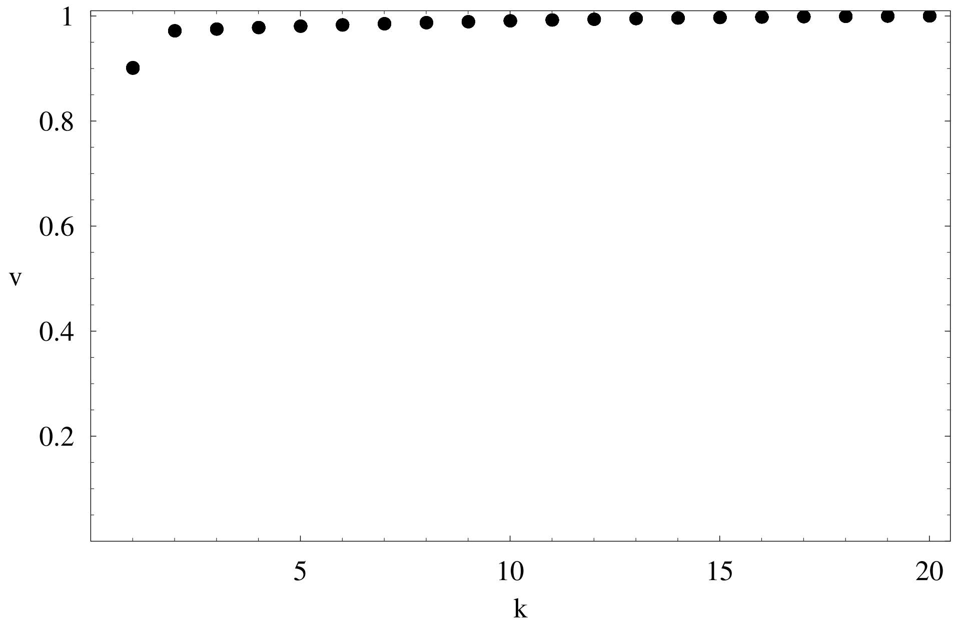

Figure 7.46: Superimposed cumulated variances of the 20 first largest absolute eigenvalues of the simulated market described in Figure 7.40 for 10 simulation runs with price manipulation pattern

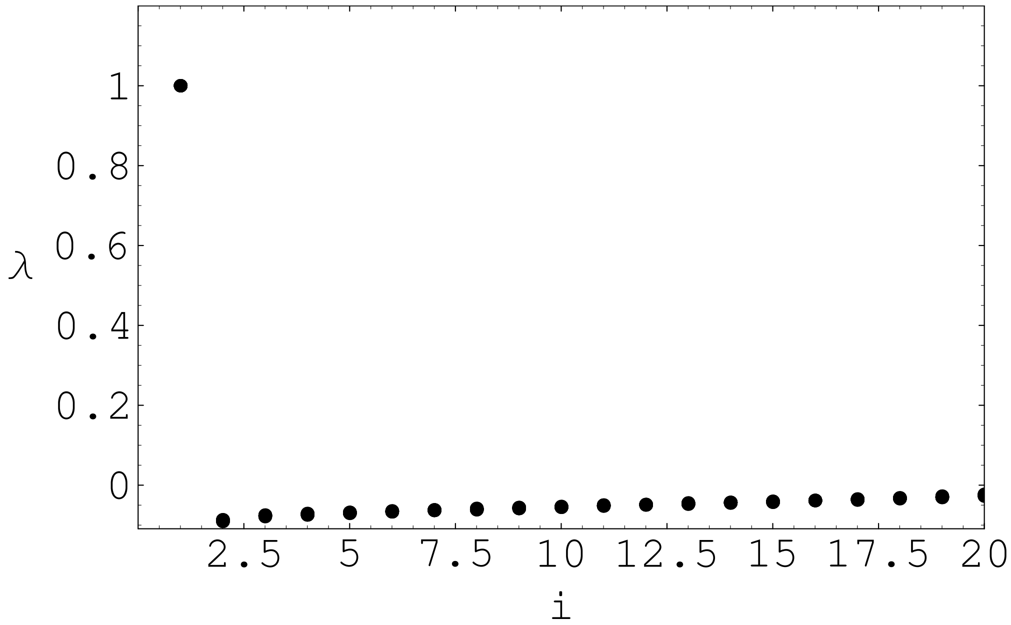

Figure 7.47: Superimposed normed eigenvalues of the simulated regular market described in Figure 7.1 for 10 simulation runs with a price manipulation perturbation of 600 to trader 2 from traders $1, 3, 4, \ldots, 20$

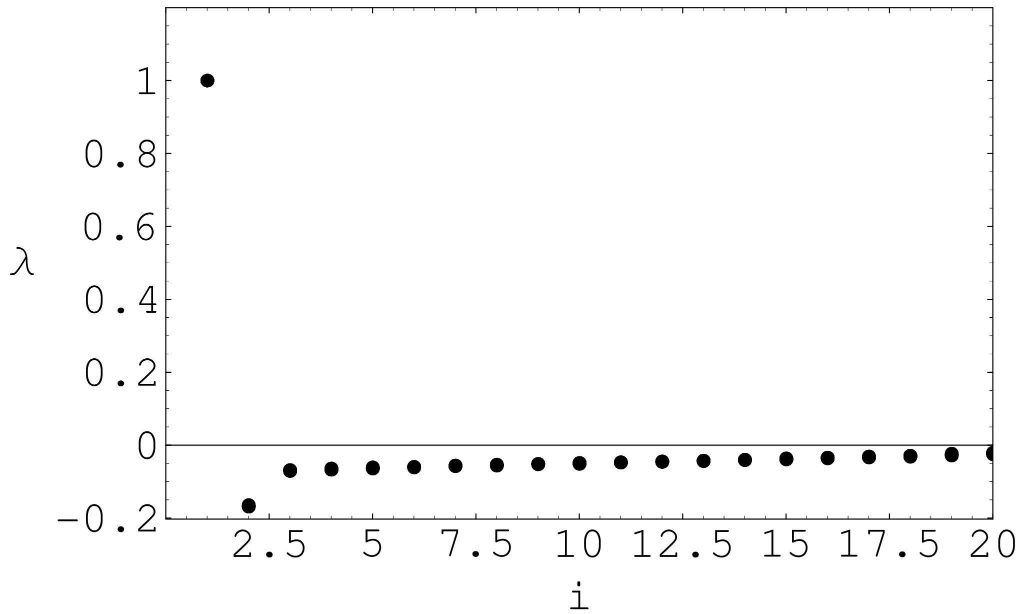

Figure 7.48: Superimposed normed eigenvalues of the simulated regular market described in Figure 7.1 for 10 simulation runs with a price manipulation perturbation of 1300 to trader 2 from traders $1, 3, 4, \ldots, 20$

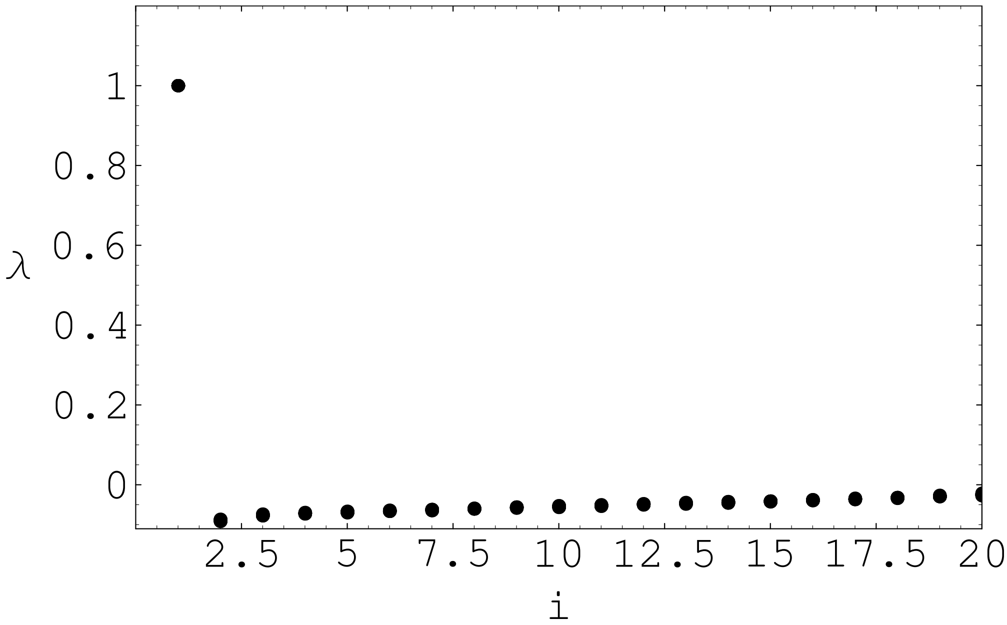

Figure 7.49: Superimposed normed eigenvalues of the simulated regular market described in Figure 7.8 for 10 simulation runs with a price manipulation perturbation of 600 to trader 2 from traders $1, 3, 4, \ldots, 20$

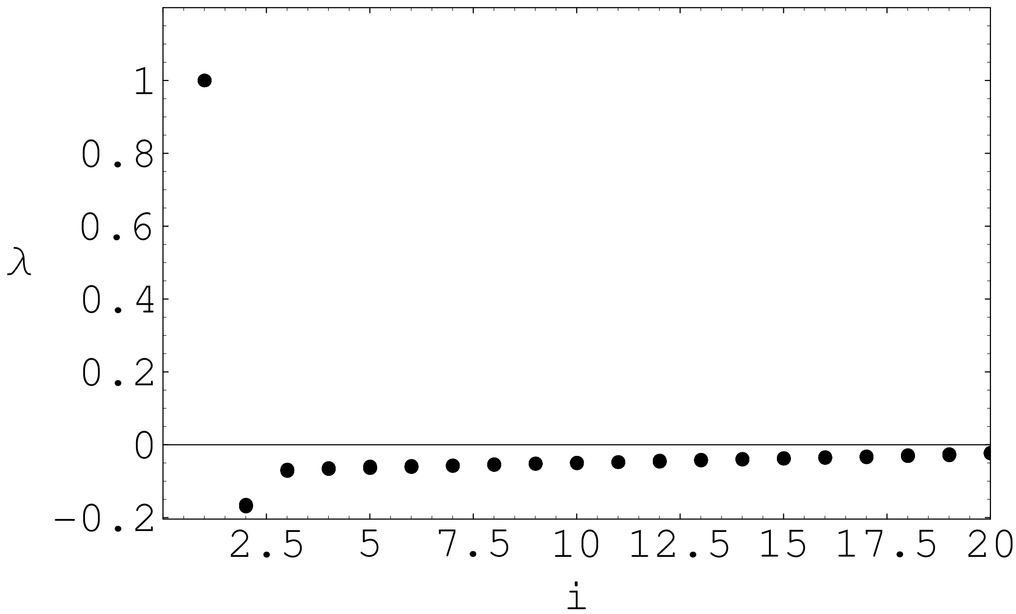

Figure 7.50: Superimposed normed eigenvalues of the simulated regular market described in Figure 7.1 for 10 simulation runs with a price manipulation perturbation of 1300 to trader 2 from traders $1, 3, 4, \ldots, 20$

7.1.4 Multiple Patterns

Finally, as in real markets the discussed patterns are usually not occuring separately, this simulation focuses on the occurrence of multiple patterns in one set of transaction data. For simulation reasons we apply the pattern structures of the money transfer pattern and the price manipulation pattern as introduced before. Both are added as a perturbation to the basic structure of the regular market activities from Section 7.1.1. The building rules for both patterns correspond to the ones used in Sections 7.1.2 and 7.1.3. As result we again obtain the set of the adjacency matrices (Figures 7.51 and 7.58), the eigensystem listings (Figures 7.7 and 7.8), the spectra distributions (Figures 7.52 and 7.59, 7.53, and 7.60), the cumulated variance plots (Figure 7.54 and 7.61), and the visualisations in Figures 7.55 and 7.62. We can clearly see that the single structures described in the previous sections are still present. The plots of the cumulated variances in Figures 7.54 and 7.61 show that the eigenvalues representing the patterns have the highest values and are given in decreasing order according to their share of the information within the total covered variances. For multiple runs of the simulation the results are depicted by the eigenvalues in Figures 7.56 and 7.63 and the variances in Figures 7.57 and 7.64 where the structural variation is quite small and the structural characteristics of the major eigenvalues like strength and algebraic signs are corresponding in all of the superimposed spectra. Additionally with the application onto the different basic characteristics in the networks of the regular market structure in Figures 7.1 and 7.8 we varied the strengths of the added patterns by increasing the structure of the money transfer pattern and increasing the structure of the price manipulation pattern. This can be seen in the spectra of 10 simulation runs in Figures 7.56 and 7.63 in the first four eigenvalues. In the visualisations in Figures 7.55 and 7.62 the properties of both of the patterns are present. These are the star like structure from traders $1, 3, 4, \ldots, 20$ towards trader 2 forming a price manipulation pattern (lines 1 and 2) and the money transfer pattern between traders 11 and 12 (lines 3 and 4).

$$
\begin{pmatrix}
0 & 3903 & 654 & 697 & 721 & 692 & 759 & 682 & 816 & 671 & 727 & 617 & 765 & 746 & 641 & 684 & 725 & 732 & 760 & 643 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 599 & 673 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
623 & 4002 & 0 & 696 & 723 & 743 & 701 & 617 & 726 & 665 & 681 & 723 & 693 & 642 & 574 & 675 & 669 & 643 & 761 & 757 \\
715 & 3961 & 723 & 0 & 635 & 654 & 728 & 761 & 652 & 666 & 668 & 707 & 775 & 702 & 667 & 706 & 711 & 722 & 705 & 745 \\
743 & 3975 & 722 & 761 & 0 & 746 & 714 & 671 & 666 & 689 & 685 & 695 & 694 & 705 & 695 & 786 & 704 & 758 & 626 & 656 \\
723 & 3989 & 738 & 703 & 669 & 0 & 723 & 793 & 735 & 750 & 681 & 709 & 662 & 643 & 761 & 658 & 752 & 732 & 718 & 778 \\
716 & 3997 & 744 & 712 & 670 & 700 & 0 & 669 & 696 & 612 & 588 & 669 & 648 & 711 & 727 & 761 & 684 & 652 & 789 & 781 \\
663 & 3987 & 711 & 793 & 638 & 744 & 713 & 0 & 713 & 721 & 715 & 714 & 603 & 633 & 759 & 692 & 776 & 667 & 694 & 711 \\
582 & 4010 & 711 & 643 & 695 & 713 & 655 & 744 & 0 & 633 & 615 & 742 & 684 & 690 & 659 & 652 & 771 & 714 & 735 & 718 \\
793 & 3997 & 726 & 630 & 660 & 709 & 817 & 697 & 742 & 0 & 745 & 709 & 719 & 759 & 615 & 678 & 725 & 709 & 784 & 719 \\
677 & 687 & 707 & 635 & 689 & 684 & 686 & 639 & 685 & 669 & 0 & 3700 & 671 & 636 & 728 & 664 & 678 & 686 & 748 & 797 \\
635 & 669 & 700 & 647 & 678 & 749 & 672 & 650 & 698 & 714 & 0 & 0 & 729 & 617 & 747 & 696 & 703 & 638 & 755 & 745 \\
681 & 3975 & 562 & 678 & 785 & 640 & 755 & 690 & 652 & 724 & 675 & 686 & 0 & 760 & 702 & 629 & 697 & 696 & 675 & 695 \\
669 & 4065 & 658 & 647 & 668 & 559 & 697 & 729 & 616 & 691 & 644 & 635 & 674 & 0 & 678 & 793 & 721 & 638 & 682 & 753 \\
676 & 3922 & 811 & 653 & 673 & 712 & 713 & 701 & 701 & 675 & 720 & 792 & 674 & 695 & 0 & 672 & 674 & 710 & 675 & 780 \\
730 & 3965 & 614 & 618 & 633 & 774 & 632 & 661 & 724 & 700 & 658 & 683 & 688 & 682 & 733 & 0 & 671 & 702 & 640 & 737 \\
769 & 4091 & 724 & 652 & 713 & 698 & 723 & 684 & 698 & 775 & 719 & 710 & 726 & 721 & 694 & 696 & 0 & 628 & 664 & 651 \\
755 & 3955 & 751 & 656 & 674 & 707 & 620 & 710 & 750 & 655 & 764 & 734 & 622 & 682 & 685 & 766 & 794 & 0 & 711 & 684 \\
728 & 3989 & 571 & 708 & 682 & 677 & 852 & 742 & 751 & 756 & 643 & 645 & 650 & 704 & 770 & 695 & 646 & 753 & 0 & 644 \\
724 & 3975 & 734 & 684 & 677 & 757 & 667 & 793 & 734 & 662 & 734 & 801 & 703 & 683 & 642 & 752 & 728 & 844 & 556 & 0
\end{pmatrix}
$$

Figure 7.51: Simulated transaction network of a market environment following the network in Figure 7.1 with $\mu = 700$ and $\sigma = 50$ and a price manipulation perturbation of 3300 to trader 2 from traders $1, 3, 4, \ldots, 20$ and money transfer perturbation of 3000 from trader 11 to 12 and -700 from trader 12 to 11

λ_k	27358.4		-10792.9		-3721.44		3017.91		-1470.02		-1382.6		-1331.03		-1255.68		-1180.62		-1140.46	
	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$
	0.2015	-0.7761	0.5088	2.3427	0.2008	-3.1225	0.2002	-2.7582	0.201	2.4249	0.2041	0	0.2033	-1.5897	0.2028	-2.2948	0.2022	0	0.2031	1.7031
	0.1214	0	0.8145	0	0.1364	2.9438	0.1302	-1.7803	0.1319	-0.1814	0.1294	-1.6009	0.128	1.3583	0.1293	2.1838	0.1334	-2.1699	0.1321	0.3513
	0.0213	-0.7664	0.1505	2.3145	0.0108	-0.5429	0.0208	2.856	0.0057	1.6068	0.0153	0.5284	0.0303	-1.5402	0.0113	-2.183	0.0344	-1.7918	0.0198	-1.6868
	0.036	-0.783	0.2332	2.3514	0.0177	-0.1985	0.0323	-2.8122	0.0263	-2.8515	0.0282	0.9859	0.0452	1.6203	0.0302	1.0205	0.0286	-1.772	0.0223	-2.8943
	0.1564	-0.7835	0.0021	2.3613	0.4139	-0.633	0.1072	-3.1128	0.2632	-1.2197	0.071	2.8969	0.2762	-1.6663	0.0842	2.4138	0.0902	0.1268	0.2483	-1.411
$\mathbf{x}_k$	0.3585	-0.7794	0.003	2.3553	0.1191	0.959	0.1512	2.7267	0.1589	-0.1171	0.1607	-1.6469	0.2396	2.2056	0.2456	0	0.2867	0.5863	0.2419	-2.4858
	0.1871	-0.7661	0.0119	2.3268	0.2458	-0.9107	0.3223	3.0009	0.0557	-2.9754	0.335	-2.4634	0.0892	2.9658	0.4087	2.9375	0.2283	2.3536	0.1661	2.3554
	0.154	-0.7726	0.002	2.3144	0.124	0.1843	0.2928	-2.5471	0.3665	-2.3462	0.3808	-3.0345	0.2068	-1.7592	0.2805	-1.9504	0.1823	-1.402	0.2353	0.7933
	0.4524	-0.7627	0.0046	2.3081	0.0883	-0.3689	0.2685	2.8746	0.0951	2.1906	0.3233	-1.2618	0.3057	2.1688	0.114	2.2472	0.2416	-1.5514	0.2853	0.8384
	0.1952	-0.7847	0.0029	2.3557	0.309	1.3386	0.0893	-2.9871	0.2251	-2.4442	0.2428	0.3565	0.3398	1.2306	0.0786	-2.5941	0.3438	-3.068	0.2644	-2.2406
	0.1317	-0.779	0.0022	-0.7847	0.0991	0	0.1497	0	0.2683	0.9851	0.1435	2.3178	0.3278	-2.6082	0.3494	-0.1748	0.281	-0.5433	0.1157	2.8871
	0.1872	-0.5909	0.004	-1.4218	0.1801	-2.3444	0.1593	0.8675	0.1633	3.081	0.1677	-1.1722	0.4241	-2.7686	0.2575	-1.5524	0.3031	-1.6232	0.3424	-2.0483
	0.0453	-0.773	0.0072	2.3353	0.0642	0.8975	0.1889	2.9744	0.1873	0.2637	0.3745	-1.3156	0.129	-2.1792	0.0605	-0.9705	0.0563	2.9472	0.335	0.4955
	0.3258	-0.7666	0.0099	2.3183	0.1609	-1.1396	0.5193	-2.76	0.2189	2.0693	0.2347	-2.3501	0.0615	1.201	0.2065	-0.2662	0.2537	-0.164	0.2607	-2.138
	0.2945	-0.7776	0.0071	2.3363	0.1726	0.8475	0.2288	1.3835	0.4502	-2.6364	0.169	1.0094	0.0741	1.3476	0.2604	-2.0613	0.2185	-1.1334	0.2337	0
	0.0592	-0.7634	0.0072	2.3071	0.2154	-0.0321	0.1309	-3.0852	0.1889	1.2284	0.1809	1.3446	0.0982	-2.4875	0.1961	0.3639	0.0423	-2.9313	0.1615	2.1317
	0.2703	-0.7679	0.0114	2.3135	0.3767	0.5095	0.3889	-2.8565	0.389	0.4216	0.2367	2.0739	0.0695	-0.4548	0.3321	0.1851	0.219	2.5271	0.1859	-0.1654
	0.2649	-0.774	0.0056	2.315	0.3002	0.473	0.135	-2.0618	0.1513	-2.8086	0.1281	-2.4438	0.2072	-1.2117	0.23	-0.0642	0.4445	2.2545	0.2823	-0.1177
	0.228	-0.7725	0.0039	2.357	0.3708	-2.9668	0.2129	2.6074	0.2278	0	0.2175	1.9577	0.231	-0.4884	0.2744	1.2	0.2075	0.5843	0.1064	-2.6854
	0.2056	-0.7693	0.0063	2.318	0.241	2.2771	0.01	1.1182	0.0614	-0.4424	0.2529	0.6283	0.3492	0	0.1715	2.28	0.1027	1.2503	0.2626	1.4209

λ_k	-1102.35		-1024.25		-950.36		-857.04		-836.93		-793.3		-738.4		-689.18		-629.03		-480.7	
	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$	$\|z\|$	$\phi(z)$
	0.152	1.3291	0.1538	2.9244	0.1997	2.1743	0.2007	-0.6422	0.2007	0.3882	0.2005	-2.7435	0.2054	-1.8354	0.2023	-1.0467	0.2026	2.955	0.205	2.9468
	0.1415	-1.4139	0.149	1.2639	0.1347	0.9586	0.1395	2.3504	0.1301	-1.7547	0.131	1.0275	0.1383	0.7534	0.1282	3.1387	0.1321	0.2572	0.1297	1.9188
	0.698	1.9574	0.6962	0.8828	0.0133	0.1358	0.0251	1.2763	0.021	2.6121	0.0082	-0.9703	0.0071	0.026	0.0015	-2.0078	0.013	0	0.0015	1.1307
	0.6816	0.4409	0.6815	-0.5877	0.0309	1.9561	0.0534	0	0.0131	0.9422	0.0317	-2.5369	0.0285	1.8688	0.0188	1.8184	0.0417	0.5176	0.0173	2.9848
	0.0276	-2.368	0.0242	-2.328	0.2261	2.1247	0.0761	-3.1334	0.1859	0	0.1107	-2.2251	0.086	0	0.3014	2.0948	0.5261	1.7044	0.2917	-2.3049
$\mathbf{x}_k$	0.0181	2.3046	0.0195	2.4516	0.267	-2.3635	0.302	3.106	0.2446	0.003	0.3024	1.2205	0.3161	1.5169	0.2825	-1.2072	0.1062	-1.6626	0.1396	0.7329
	0.0148	-1.1885	0.0275	1.7926	0.1576	-2.052	0.1782	-0.0845	0.2606	-1.1724	0.2725	-1.368	0.0623	2.416	0.1262	-1.8572	0.109	-0.4138	0.4462	-2.0639
	0.011	2.8562	0.0068	-2.6919	0.3206	2.3833	0.114	-0.2521	0.2397	-2.1014	0.2309	-2.8401	0.1543	2.1205	0.158	1.7213	0.0905	-1.5967	0.3196	0.2557
	0.0096	-1.4949	0.0128	-1.6275	0.2871	-2.8617	0.349	1.792	0.0642	3.1197	0.2372	-0.6854	0.1288	-0.6679	0.2059	0	0.081	-1.5249	0.1588	0.7137
	0.018	2.2798	0.02	-1.2599	0.0933	2.8222	0.2912	-2.524	0.3495	-2.8818	0.2596	1.8485	0.3019	-2.5081	0.0269	-2.3869	0.0754	-2.246	0.2403	2.8102
	0.0061	-2.1389	0.0274	2.8629	0.0721	1.4958	0.1213	-1.83	0.0648	0.9239	0.2784	-2.9313	0.5389	-2.4039	0.3122	-0.5552	0.1752	1.7778	0.1225	-3.0905
	0.0141	-0.0931	0.0206	-2.0186	0.2128	2.5928	0.0695	1.1231	0.1405	-1.6907	0.1746	1.0396	0.2938	2.2009	0.4409	1.8724	0.025	1.9139	0.1214	-0.9231
	0.0056	2.9327	0.008	0.9298	0.059	2.3402	0.313	1.3043	0.4278	0.9878	0.1259	1.9978	0.2689	-0.5033	0.1471	1.5712	0.4252	1.9054	0.2682	-3.0726
	0.0325	-1.7823	0.0042	-2.0616	0.3155	-0.3323	0.2636	-1.3905	0.2208	2.986	0.0677	0.1777	0.038	-1.303	0.197	2.0594	0.2096	1.9142	0.0943	-2.1817
	0.0064	2.9102	0.0102	1.5931	0.134	0	0.4299	2.6897	0.2905	0.6935	0.1232	1.1467	0.1578	-2.6034	0.1198	2.6037	0.1553	-1.0808	0.2414	-0.6027
	0.0157	-2.3921	0.0117	-1.0213	0.4714	-0.6018	0.0759	-1.6072	0.1828	-0.9374	0.5101	0	0.1509	-2.9453	0.3915	-2.6941	0.1178	2.4824	0.277	-0.4066
	0.0112	0	0.0135	2.9202	0.1056	-2.6628	0.1604	-0.3164	0.1857	-1.8817	0.0584	-0.9811	0.2278	-1.5773	0.1415	2.0376	0.2856	-2.4341	0.01	1.9347
	0.0136	1.0511	0.0246	0	0.2208	0.3185	0.2848	-2.7859	0.24	0.9125	0.0838	-2.101	0.2739	-2.3434	0.2138	-0.1513	0.2961	2.7251	0.074	0.4158
	0.0117	0.6769	0.0119	-0.6085	0.264	0.9228	0.2055	2.743	0.2867	-1.8938	0.3582	-0.8274	0.0825	2.4011	0.2211	-0.6872	0.1925	-1.3799	0.2598	-2.6064
	0.0357	2.0699	0.0114	0.934	0.2765	-1.8103	0.2486	-0.964	0.2201	-2.7746	0.2099	2.2048	0.2514	2.0323	0.2305	-1.41	0.345	1.1877	0.3653	0

Table 7.7: Eigensystem for the multiple pattern structure described in Figure 7.51 with $z = |z|e^{i\phi}$

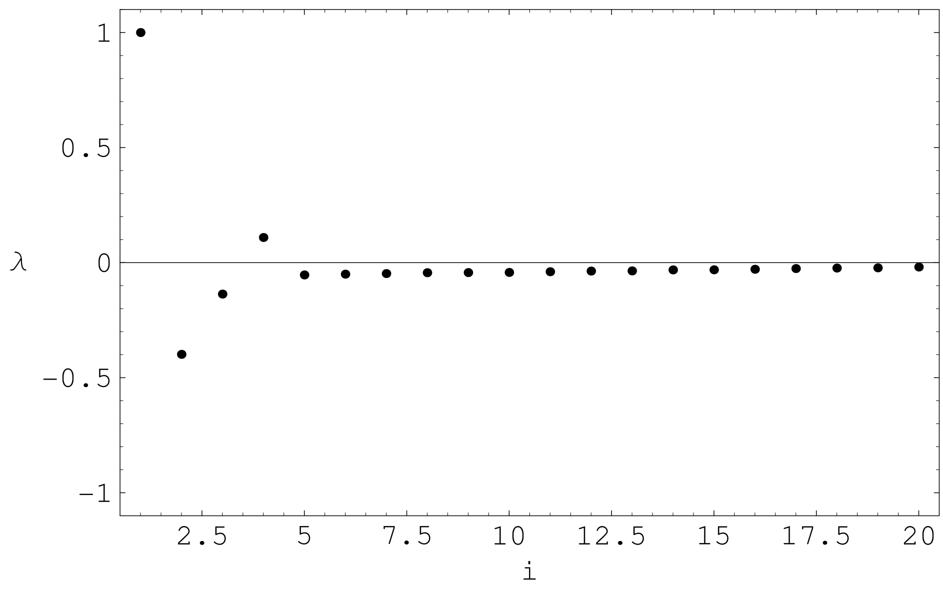

Figure 7.52: Normed eigenvalues of the simulated market described in Figure 7.51 with price manipulation and money transfer pattern

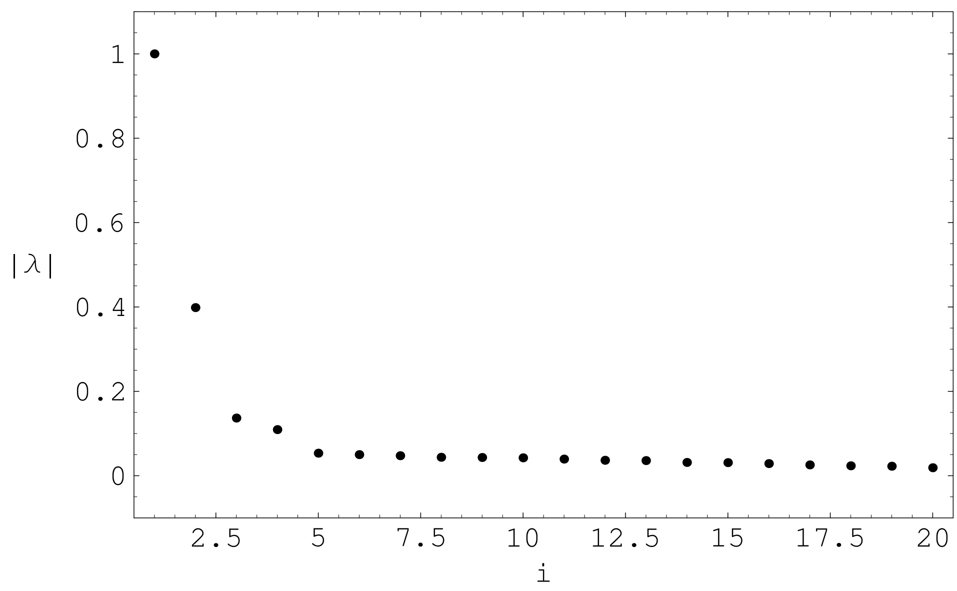

Figure 7.53: Normed eigenvalues of the simulated market described in Figure 7.51 with price manipulation and money transfer pattern with absolute values

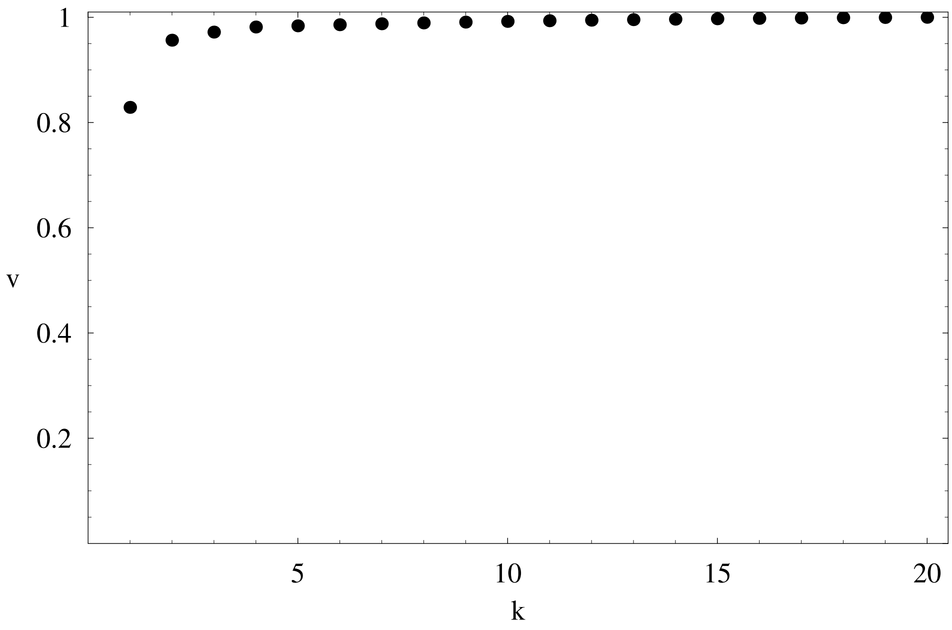

Figure 7.54: Cumulated variance of the 20 first largest absolute eigenvalues of the market described in Figure 7.51 with price manipulation money transfer pattern

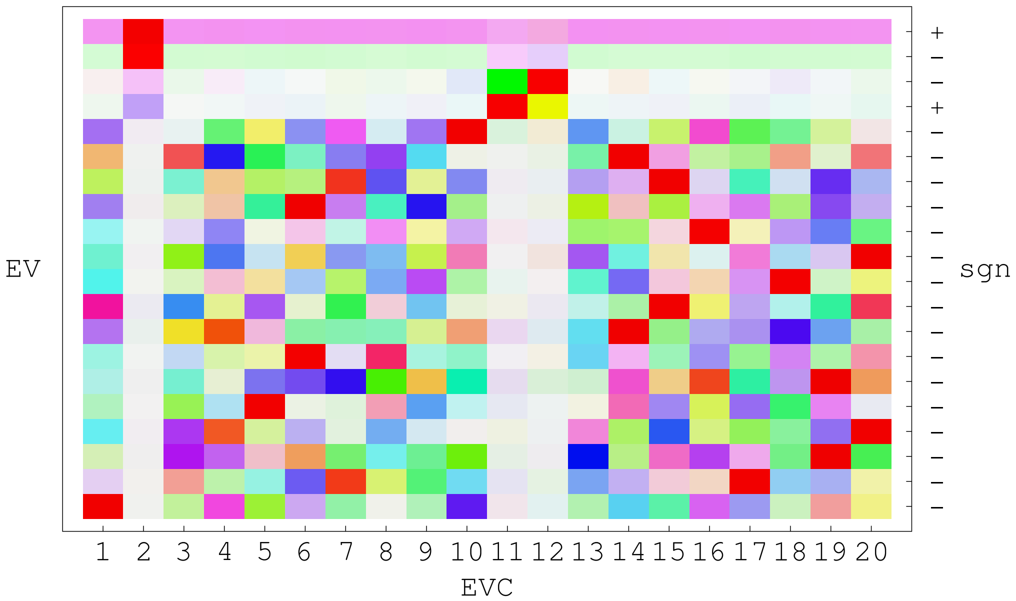

Figure 7.55: Eigensystem of the simulated market described in Figure 7.51 with price manipulation and money transfer pattern

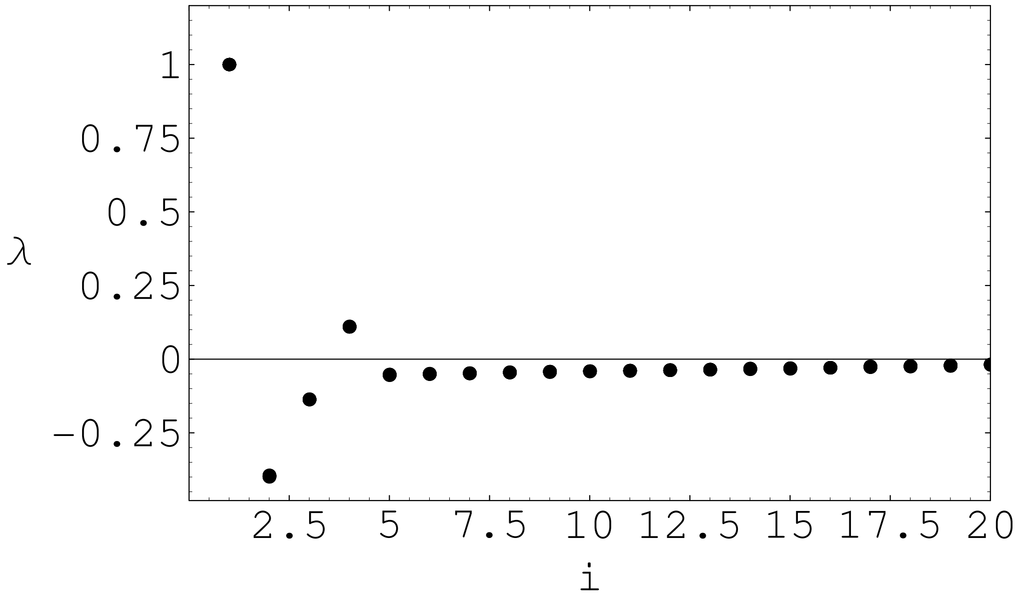

Figure 7.56: Superimposed normed eigenvalues of the simulated market described in Figure 7.51 with price manipulation and money transfer pattern for 10 simulation runs

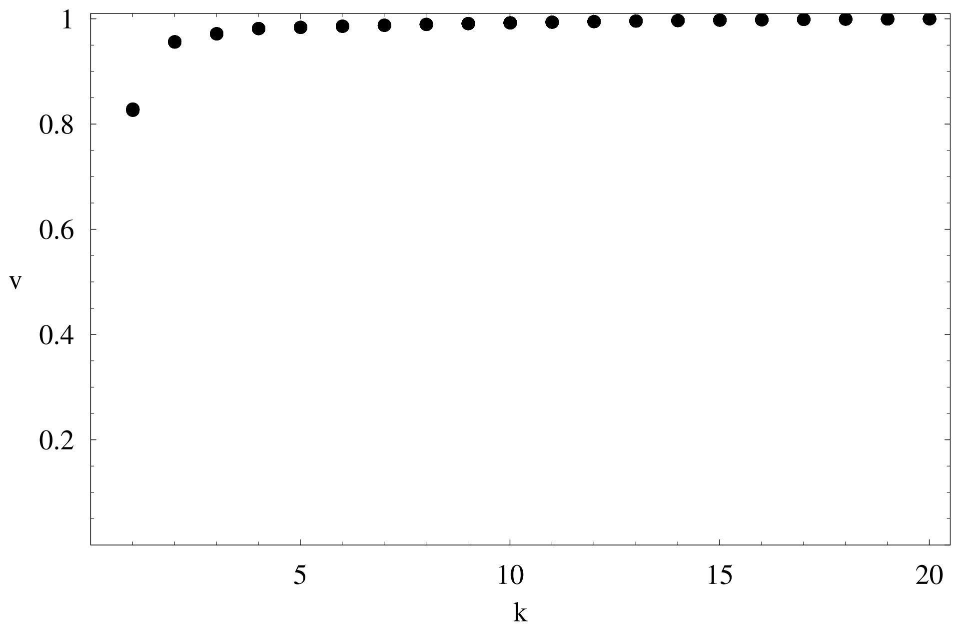

Figure 7.57: Superimposed cumulated variances of the 20 first largest absolute eigenvalues of the market described in Figure 7.51 with price manipulation money transfer pattern for 10 simulation runs

$$\begin{pmatrix}
0 & 3047 & 806 & 704 & 708 & 778 & 723 & 670 & 667 & 701 & 782 & 647 & 704 & 693 & 697 & 708 & 756 & 638 & 738 & 691 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 756 & 424 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
662 & 2980 & 0 & 692 & 683 & 684 & 618 & 734 & 710 & 691 & 670 & 748 & 786 & 691 & 638 & 554 & 654 & 710 & 821 & 727 \\
745 & 3039 & 700 & 0 & 748 & 725 & 763 & 551 & 670 & 699 & 880 & 647 & 730 & 662 & 776 & 685 & 752 & 707 & 713 & 692 \\
677 & 3011 & 655 & 754 & 0 & 690 & 670 & 670 & 761 & 724 & 616 & 573 & 632 & 718 & 683 & 608 & 677 & 776 & 787 & 698 \\
729 & 2926 & 760 & 686 & 589 & 0 & 626 & 718 & 741 & 701 & 752 & 593 & 660 & 703 & 736 & 729 & 612 & 649 & 715 & 702 \\
678 & 2952 & 661 & 701 & 705 & 723 & 0 & 624 & 742 & 797 & 751 & 730 & 626 & 652 & 742 & 713 & 698 & 684 & 696 & 699 \\
734 & 3013 & 660 & 715 & 764 & 626 & 738 & 0 & 642 & 669 & 559 & 530 & 656 & 734 & 605 & 712 & 628 & 757 & 719 & 734 \\
697 & 2904 & 675 & 768 & 682 & 789 & 682 & 749 & 0 & 782 & 559 & 453 & 730 & 707 & 716 & 611 & 694 & 695 & 710 & 644 \\
679 & 2929 & 622 & 558 & 697 & 663 & 663 & 700 & 714 & 0 & 652 & 922 & 736 & 677 & 631 & 739 & 768 & 623 & 641 & 755 \\
867 & 554 & 818 & 770 & 613 & 460 & 689 & 1017 & 616 & 647 & 0 & 4600 & 662 & 842 & 710 & 741 & 868 & 599 & 516 & 829 \\
462 & 617 & 841 & 590 & 746 & 496 & 781 & 474 & 912 & 711 & 0 & 0 & 907 & 765 & 605 & 930 & 821 & 750 & 462 & 774 \\
658 & 3020 & 699 & 626 & 641 & 701 & 742 & 716 & 693 & 634 & 1004 & 880 & 0 & 663 & 641 & 679 & 783 & 751 & 688 & 767 \\
740 & 3032 & 707 & 687 & 640 & 737 & 721 & 708 & 709 & 729 & 627 & 466 & 640 & 0 & 727 & 655 & 762 & 724 & 737 & 666 \\
712 & 2980 & 668 & 773 & 675 & 709 & 705 & 730 & 700 & 799 & 497 & 644 & 604 & 739 & 0 & 701 & 750 & 681 & 723 & 696 \\
801 & 3082 & 705 & 729 & 641 & 629 & 672 & 686 & 607 & 701 & 833 & 672 & 709 & 735 & 722 & 0 & 749 & 727 & 666 & 638 \\
775 & 3079 & 715 & 706 & 731 & 627 & 736 & 687 & 720 & 674 & 661 & 784 & 732 & 764 & 685 & 661 & 0 & 689 & 738 & 627 \\
616 & 3013 & 757 & 693 & 702 & 625 & 673 & 686 & 684 & 720 & 784 & 901 & 722 & 639 & 743 & 738 & 673 & 0 & 694 & 695 \\
753 & 2959 & 734 & 755 & 722 & 747 & 699 & 762 & 635 & 635 & 706 & 708 & 729 & 682 & 738 & 639 & 680 & 660 & 0 & 741 \\
662 & 2964 & 769 & 644 & 681 & 646 & 729 & 699 & 648 & 825 & 685 & 653 & 609 & 700 & 694 & 690 & 768 & 634 & 748 & 0
\end{pmatrix}$$

Figure 7.58: Simulated transaction network of a market environment following the network in Figure 7.8 with $\mu = 700$ and $\sigma = 150$ and a price manipulation perturbation of 2300 to trader 2 from traders $1, 3, 4, \ldots, 20$ and money transfer perturbation of 3900 from trader 11 to 12 and -700 from trader 12 to 11

λ_k	24058.9		-7417.37		-4712		3824.42		-1432.14		-1350.12		-1287.9		-1268.1		-1173.72		-1137.17	
	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$
	0.2101	-0.7729	0.4481	2.3607	0.2082	0.6399	0.2093	-2.9317	0.205	-2.9249	0.2021	1.7235	0.2068	1.122	0.2055	2.625	0.2038	2.5813	0.2067	-2.8613
	0.1191	0	0.796	0	0.1224	-0.7078	0.1266	-2.636	0.1204	-0.1808	0.1024	3.0056	0.1176	-0.2241	0.1204	-1.3745	0.1096	-1.5227	0.1064	-0.8953
	0.0616	-0.7539	0.2937	2.2762	0.0277	2.3969	0.0201	1.3095	0.0278	0	0.0324	-0.6475	0.0325	-0.8112	0.1105	2.4272	0.0326	-1.211	0.0773	-1.5616
	0.0386	-0.7762	0.2715	2.3131	0.0313	0.0392	0.0419	-2.9471	0.0698	-1.9474	0.0879	1.1098	0.0244	-2.2642	0.0378	-2.1066	0.0946	0	0.0098	0.1058
	0.2153	-0.7679	0.0281	2.3282	0.3448	2.5998	0.3443	2.2682	0.1378	1.5674	0.2653	-2.0547	0.1456	-1.6219	0.1567	2.2194	0.2813	-3.0887	0.2973	0.8434
$\mathbf{x}_k$	0.2302	-0.7771	0.0112	2.2373	0.2655	2.2917	0.1899	3.074	0.3394	2.0929	0.1119	-0.2274	0.0957	-2.3096	0.1013	-0.6525	0.1869	2.1145	0.407	0
	0.2597	-0.7664	0.0247	2.25	0.1794	3.0512	0.07	2.1734	0.1117	1.9361	0.2991	1.1904	0.1304	-2.1091	0.2701	2.5476	0.3172	3.0356	0.1468	2.087
	0.2612	-0.7589	0.0132	2.4794	0.3086	0.6238	0.2115	2.166	0.2968	-1.378	0.1856	-2.1978	0.2859	-1.5506	0.3501	-1.6599	0.232	1.3136	0.2149	-2.9946
	0.1181	-0.7623	0.0051	2.3808	0.0431	-2.6473	0.3831	1.9618	0.0754	0.0245	0.3336	2.8731	0.228	1.4263	0.223	-0.1315	0.0948	-1.7009	0.2383	2.6535
	0.2813	-0.7541	0.015	2.1315	0.1169	2.2947	0.2392	2.3972	0.3026	-1.7905	0.4031	0	0.3001	-2.2916	0.2411	0.8281	0.276	-0.126	0.1319	-1.2815
	0.2961	-0.8106	0.01	-0.659	0.2042	2.4084	0.08	-0.8842	0.2116	1.6913	0.1603	-2.4662	0.4687	1.7272	0.1351	-2.7189	0.2199	0.0425	0.0428	1.7373
	0.3968	-0.608	0.0039	-1.8825	0.3857	0	0.2776	0	0.2075	1.9753	0.0437	-0.0282	0.1992	1.9666	0.1201	0.8899	0.161	-2.8324	0.33	0.0004
	0.2219	-0.7761	0.0085	2.1856	0.1432	-2.7752	0.0106	-1.2838	0.2373	-2.7913	0.1794	-2.0377	0.1215	0.1181	0.2778	1.0665	0.2955	1.2055	0.0857	-1.9824
	0.0868	-0.7589	0.0227	2.4081	0.0683	0.0367	0.3764	1.9602	0.3745	-1.7644	0.1063	2.767	0.232	1.5438	0.1707	2.0311	0.357	-1.2677	0.1771	-0.0981
	0.2897	-0.7687	0.0195	2.3837	0.2502	1.5868	0.2621	2.255	0.1531	0.7536	0.2846	-1.7843	0.3028	0	0.2681	1.4888	0.2985	1.7144	0.0666	-1.9201
	0.1954	-0.7714	0.0227	2.3071	0.2857	-2.6053	0.1685	1.701	0.3299	0.4705	0.0924	-1.6857	0.3505	1.6991	0.224	-1.3362	0.0887	-1.0271	0.1161	0.9386
	0.0109	-0.7569	0.0253	2.2991	0.1638	1.902	0.2439	1.1625	0.2063	0.9657	0.2269	-0.2115	0.1427	-2.01	0.1899	-1.3576	0.2815	-2.4469	0.2307	-1.9675
	0.2832	-0.7735	0.0318	2.1792	0.1092	2.6893	0.0478	-2.9224	0.2718	2.8525	0.3777	1.4567	0.0592	2.6688	0.2461	1.1194	0.2144	2.0625	0.2966	-2.1072
	0.2464	-0.7726	0.0195	2.1979	0.305	1.9768	0.3199	3.1334	0.1552	-1.5069	0.3142	0.3652	0.1921	2.2292	0.4177	0	0.2523	-2.5256	0.1951	2.0126
	0.208	-0.7598	0.0079	2.3356	0.3405	1.3224	0.2082	1.5962	0.2205	1.8286	0.0769	2.4804	0.2597	0.8599	0.2435	-1.1924	0.0889	-0.2541	0.4442	1.2846

λ_k	-1091.69		-1050.14		-931.1		-886.05		-835.75		-785.26		-720.11		-669.9		-583.68		-551.16	
	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$	$\lvert z \rvert$	$\phi(z)$
	0.1874	-2.4423	0.1841	-1.887	0.2093	-1.9781	0.2076	2.7663	0.2062	2.909	0.208	-0.9106	0.2123	2.0548	0.2076	2.7452	0.2074	2.36	0.2071	1.744
	0.2655	-1.412	0.2256	-1.5268	0.1392	-1.7173	0.1275	-0.0175	0.1043	-1.3789	0.1466	1.2746	0.1289	-0.0808	0.1231	-0.5316	0.0949	1.5257	0.12	2.3522
	0.6471	-0.7748	0.6677	2.8771	0.0722	2.0344	0.0254	0.87	0.0665	-1.4445	0.0434	-2.0777	0.041	-0.7501	0.0717	-2.0283	0.0555	-1.9832	0.0096	2.6723
	0.6631	-0.8478	0.6645	2	0.0194	-0.6408	0.067	-3.0039	0.0588	0.6474	0.025	-1.0336	0.0342	-2.1884	0.0159	2.6976	0.0765	2.3138	0.031	-1.7559
	0.0378	-3.0499	0.0489	-1.24	0.3434	-1.388	0.1053	-1.9861	0.1382	0.8362	0.2868	2.8168	0.1882	1.6465	0.1505	-0.836	0.19	2.2277	0.2906	-2.3664
$\mathbf{x}_k$	0.0301	2.6021	0.0194	0.945	0.336	-1.6325	0.2571	1.7072	0.1153	-2.045	0.2115	-1.0851	0.1314	2.8216	0.371	2.0714	0.1099	-2.0159	0.2956	-0.3024
	0.0206	0	0.0302	1.4717	0.249	1.8741	0.0439	2.1769	0.3883	-2.8461	0.1959	1.1039	0.3405	0.8001	0.2829	-0.2005	0.3273	2.0195	0.1524	-1.1539
	0.042	-0.2309	0.0583	1.3423	0.0929	-2.2891	0.1879	0.2148	0.142	1.8361	0.1666	2.5663	0.3429	2.227	0.0361	-2.9514	0.3597	0	0.1397	2.9375
	0.0561	-1.8572	0.0383	2.1583	0.0703	-1.0896	0.1935	1.822	0.2448	0.1455	0.2814	1.3155	0.3736	2.2402	0.2259	0.5609	0.2686	-1.9997	0.3227	-0.8898
	0.0368	-0.1158	0.0502	-1.3601	0.1182	-0.9405	0.2345	1.6518	0.3206	2.4659	0.2635	2.5843	0.1878	0.872	0.1387	-1.3321	0.2315	-1.1125	0.0299	0
	0.0156	0.4599	0.0149	-2.9914	0.2385	-0.7634	0.417	2.0523	0.0098	0.5468	0.159	2.7419	0.2711	1.3669	0.1694	1.7266	0.2975	1.1398	0.2413	2.4123
	0.0291	1.437	0.0337	0.1783	0.1172	2.3204	0.1368	2.3744	0.4115	3.0109	0.1731	-1.8447	0.0814	-3.1215	0.1351	-0.558	0.2355	-2.2877	0.2623	-0.6679
	0.0263	1.6118	0.0316	0.8514	0.1013	-2.0141	0.4247	2.8266	0.2662	2.7684	0.3043	0	0.2604	-1.4841	0.2253	0	0.3529	-1.1046	0.237	2.3654
	0.0148	1.0642	0.0277	-2.9207	0.0721	0	0.4919	0	0.0766	0.8985	0.1472	-0.4781	0.1178	1.6231	0.3337	2.4896	0.2038	1.5097	0.0994	1.5781
	0.058	2.1251	0.0067	0	0.1402	1.8892	0.143	-1.1805	0.3735	0	0.2317	1.2024	0.1972	-0.8532	0.1578	2.7223	0.0755	2.5852	0.3416	-0.7415
	0.0417	-0.5497	0.0326	-1.2551	0.3587	-2.8497	0.1077	-0.6722	0.0753	-1.7743	0.078	0.7623	0.3153	-2.0626	0.3353	2.0929	0.3379	0.9729	0.2422	0.6547
	0.0505	-2.793	0.0778	2.7689	0.2711	0.9281	0.1513	-2.0417	0.0833	1.0049	0.3734	0.0732	0.2124	0.5733	0.3967	-0.2626	0.1198	1.7618	0.4022	1.3724
	0.1237	-2.8619	0.0618	-2.8782	0.4435	1.6847	0.1997	-0.1974	0.3048	-0.6287	0.0442	2.2359	0.1523	-2.1662	0.3084	-0.59	0.0921	-0.0681	0.1127	-2.2728
	0.0273	1.154	0.0401	-0.3188	0.2685	2.1441	0.1068	-2.319	0.124	2.2461	0.3133	-1.9938	0.2026	1.1371	0.0473	2.2424	0.0603	-1.7545	0.2477	-3.0335
	0.0358	-2.8006	0.0329	1.2859	0.185	0.5068	0.0868	-2.4169	0.257	-2.6442	0.3496	-2.6266	0.2549	0	0.1528	-2.3929	0.2689	-0.4347	0.1336	-0.9237

Table 7.8: Eigensystem for the multiple pattern structure described in Figure 7.58 with $z = |z|e^{i\phi}$

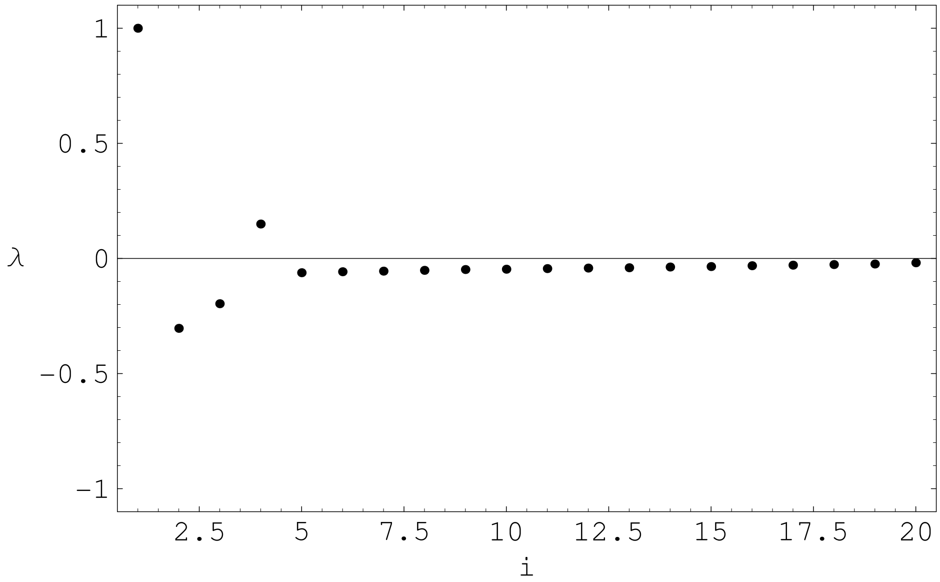

Figure 7.59: Normed eigenvalues of the simulated market described in Figure 7.58 with price manipulation and money transfer pattern

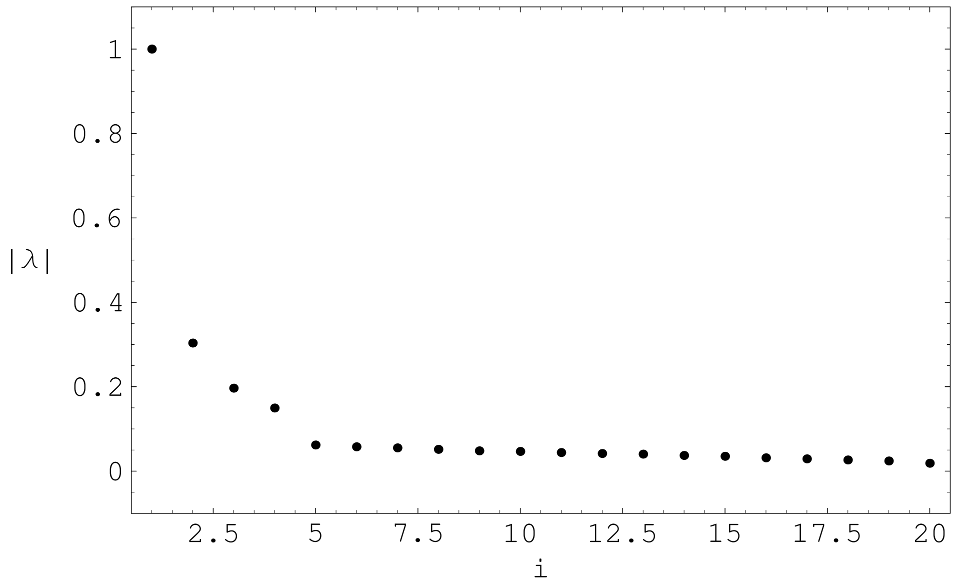

Figure 7.60: Normed eigenvalues of the simulated market described in Figure 7.58 with price manipulation and money transfer pattern with absolute values

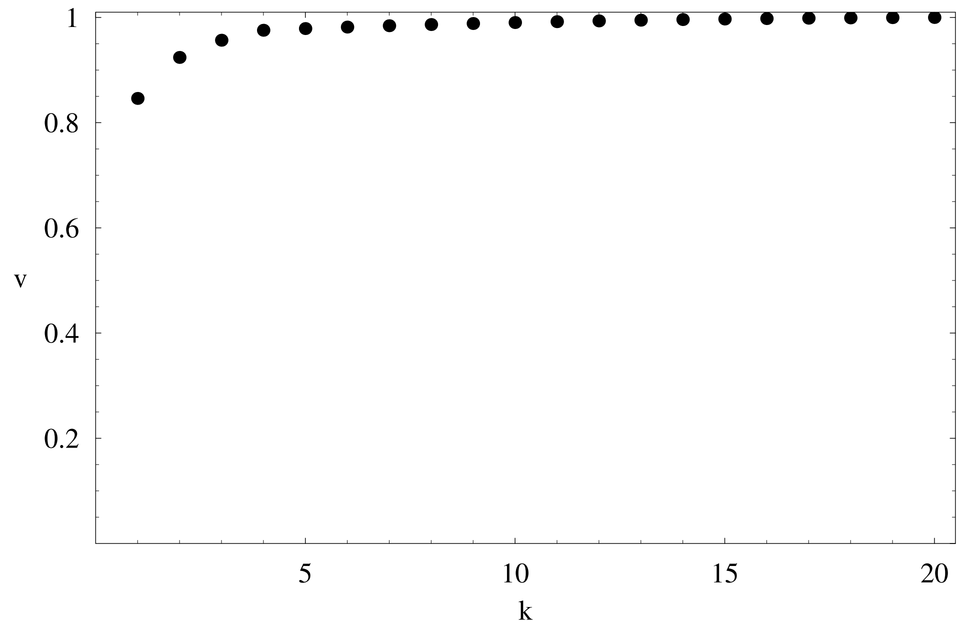

Figure 7.61: Cumulated variance of the 20 first largest absolute eigenvalues of the market described in Figure 7.58 with price manipulation money transfer pattern

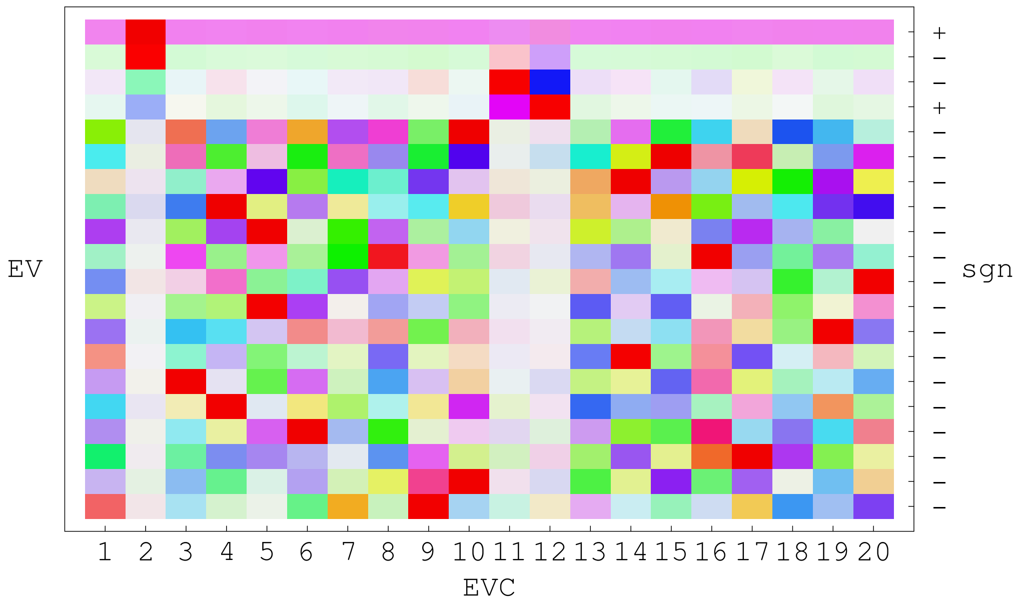

Figure 7.62: Eigensystem of the simulated market described in Figure 7.58 with price manipulation and money transfer pattern

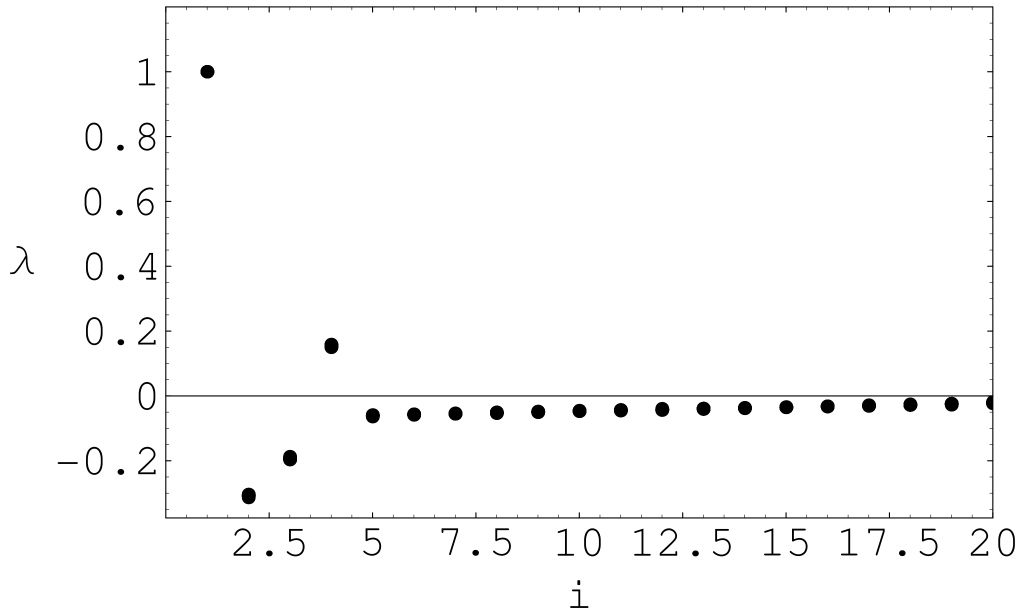

Figure 7.63: Superimposed normed eigenvalues of the simulated market described in Figure 7.58 with price manipulation and money transfer pattern for 10 simulation runs

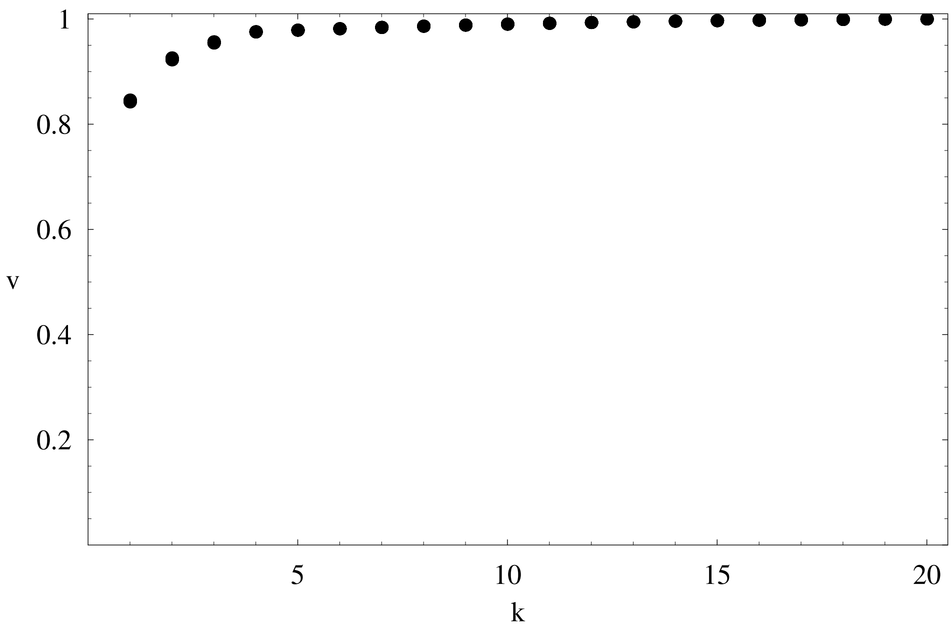

Figure 7.64: Superimposed cumulated variances of the 20 first largest absolute eigenvalues of the market described in Figure 7.58 with price manipulation money transfer pattern for 10 simulation runs

7.1.5 Summary

The preceding chapters of this work introduced the transaction patterns resulting from certain trader behaviours within a prediction market. Within this section these transaction patterns were simulated separately as perturbations to a basic network structure and approached by the proposed analysis method. For each of these scenarios the results gave a clear identification of the defined patterns and their stability within the analysis results i.e. the patterns within the spectra. The gap of structural change within the spectrum due to perturbations in the analysed networks is approached exemplarily by analysing simulation settings for networks below and above that gap. Therefore, simulation runs for networks with certain perturbations (simulating manipulative actions) of the basic structure of a regular market network that do not effect the spectrum structure of a regular market network without perturbations, and perturbations that do effect the spectrum structure towards the spectrum structure described for the certain manipulation patterns are given.

The eigensystem of the regular market's transaction behaviour in Section 7.1.1 described a quite complete network. Adding the transaction patterns for irregular money transfer and price manipulation to this basic network in Sections 7.1.2 and 7.1.3 result in eigensystems whose analysis result gives clear diagnoses of these added structures following the required eigensystem structures of Section 6.2 in case of containing these patterns. Section 7.1.4 finally shows how the structures add up jointly within the transaction structure as well as in the analysis result. Also here the patterns are differentiable from the basic regular market behaviour.

With these simulation results at hand the analysis approach is in the following section applied to real world market data focusing one pattern in each of the markets.

7.2 Real World Market Data

The data from real world markets was collected during the operation of the software framework "PSM" (acronym for *Political Stock Market*) of the Institute for Information Systems and Management at Universität Karlsruhe (TH) in Germany.

Data from the following markets was analysed in this section:

1. Elections for the Baden-Württemberg state parliament in Germany 2006
2. Elections for the national parliament in Switzerland 2007

The system was operated in cooperation with several media partners. The markets were communicated as operated by a research unit, but there was no experimental design disturbing the character of an honest prediction market. The participants traded in markets with an incentive system where attractive prices could be won by traders with the highest final depot values. Because of the system design of the software PSM, full datasets with all trading information document each action of traders within the market.

Below we describe and analyse the market designs in detail. In the first market for the state parliament in Baden-Württemberg the focus was on the money transfer pattern, in the market for the national parliament in Switzerland on price manipulation. As abbreviation for 'monetary unit' MU is used instead.

7.2.1 Elections for the Baden-Württemberg State Parliament in Germany 2006

The market for the elections for the Baden-Württemberg state parliament in Germany was conducted with four partners coming from the newspaper business, namely *Badische Zeitung* in Freiburg, *Badische Neueste Nachrichten* in Karlsruhe, *Stuttgarter Zeitung* in Stuttgart and *Heilbronner Stimme* in Heilbronn. The participants were mainly customers of these media partners, so mainly readers of rather serious and politically balanced newspapers all over the election region. In addition the trader group contained some participants from previous markets. The market ran from January, 31st 2006 until election day on March, 26th 2006 for about twelve weeks and was stopped with the closing time of the polling stations at 18:00 CET when the first official information on the voter's decision was allowed to be released. More detailed data about the market is given in Table 7.9.

Within the area of market manipulations this market was not interrupted by market surveillance during runtime. Participants blamed for having manipulated the market were excluded from the lottery without being informed during runtime. All participants were informed on these efforts to find out about manipulators and the consequent exclusion by the terms and conditions of the market.

Market opening	2006-01-12 16:23:44
Market close	2006-03-26 18:00:00
Number of traders (at least one sell or buy transaction)	306 traders
Number of traders (at least one sell transaction)	190 traders
Number of traders (at least one buy transaction)	291 traders
Number of transactions	10786 transactions
Number of shares	7 shares
Average volume per trade	214.6 shares
Average moneyflow per trade	2,462.1 MU
Moneyflow in total	26,556,378 MU
Shareflow in total	2,314,197 shares

Table 7.9: Statistical Data on the 2006 state parliament elections in Baden-Württemberg in Germany

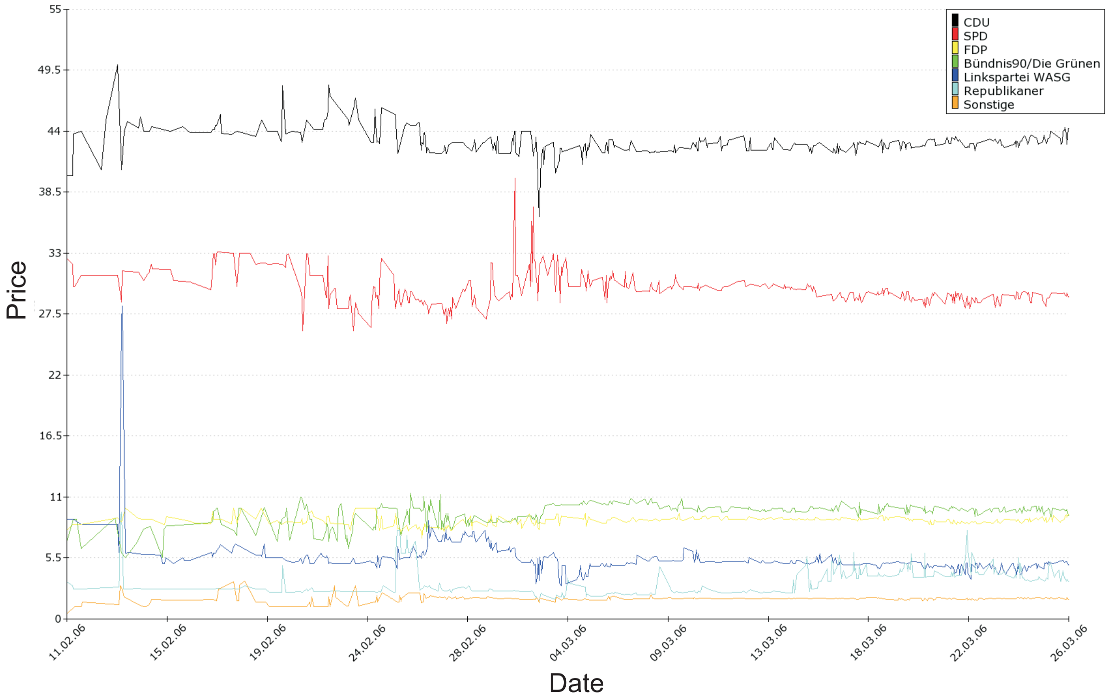

Figure 7.65: Prices of the market for the 2006 state parliament elections in Baden-Württemberg in Germany

7.2.1.1 Analysis

As the subject of the analysis in this section is the money transfer pattern introduced in Section 6.2.2, the global moneyflow network of the market (see Section 6.1.1) is extracted from the market's transaction data. For this market the existence of a money transfer manipulation could be assumed as the depot values of the most prominent positions in the highscore, especially the one of the trader with nickname "elfriede", were exceeding a value which was plausible with regard to the other trader's depot values (cp. Table 7.10).

Rank	Nickname	Virtual Depot Money
1	elfriede	226688.71
2	herrrtie	176265.35
3	gruener	172738.39
4	MarcEichler	166769.19
5	potato joe	164285.52
6	Maio_Shan	156898.10
7	Ritvars	155599.08
8	henning	147850.64

Table 7.10: First 8 highest ranks in the highscore for the market for the 2006 state parliament elections in Baden-Württemberg in Germany

Following Section 6.2.2 an irregular trading pattern is given by a strong asymmetric total moneyflow among two traders which themselves have little or below other connections to other traders of the network. Now analysing this market transaction data in the manner of the simulations in Section 7.1, the eigensystem results in 306 eigenvalues out of which the 21 largest absolute values are displayed with their real values in Table 7.11. The spectrum in total is given in Figure 7.67 (ordered by the descending absolute values) and in Figure 7.68 with absolute values.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
1058206.58	-1031266.15	524487.49	-372072.73	258457.61	-253073.92
λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}
243497.00	-228817.40	201615.16	-201527.61	-191045.04	182719.01
λ_{13}	λ_{14}	λ_{15}	λ_{16}	λ_{17}	λ_{18}
-176745.81	163198.67	156091.17	-150107.99	141420.50	-140327.93
λ_{19}	λ_{20}	λ_{21}			
133133.33	-129003.39	127808.00			

Table 7.11: First 21 largest absolute eigenvalues of the market for the 2006 state parliament elections in Baden-Württemberg in Germany

These eigenvalues have 306 corresponding eigenvectors with each having 306 eigenvector components which again split up in absolute value and phase information. As to the page boundaries of this work the full eigensystem of dimensions 306×306 cannot displayed properly, the visualisation introduced in Section 5.3.1 can help to get an insight into the structure of the eigensystem. The eigenvectors $\mathbf{x}_i$ can be found in the rows with each column j corresponding to the eigenvector component x_{ij}. The eigenvectors are thereby ordered top down by their corresponding absolute eigenvalues λ_i in descending order. As the k^{th} eigenvalue λ_k is assumed to be a weighting factor for the subspace build by the k^{th} eigenvector $\mathbf{x}_k$ and thus as an indicator of importance of this subspace, the eigenvectors lying on top represent the most interesting subspaces holding the main structural information of the graph's structure. As can be seen in the plot of the cumulated variance in Figure 7.69 already the eight largest absolute eigenvalues cover about 80 % of the total variance. As the eigenvalues of these are alternating in the algebraic sign, a first indicator for strong star-like structures is given. Following the pattern search procedures of Section 6.3 we find quite clear phase shifts in the corresponding eigenvectors which support the star-like structure and additionally identify the direction of each structure.

Visually this result can easily be verified if the eigensystem is processed by the cluster algorithm as introduced in Section 5.3.3 and applied in Section 6.3. The subspaces' components are thereby reordered according to information of 'closeness' of a variable within all subspaces. Thus the central vertices of the found patterns are ordered by their relevance with the related vertices in the found pattern lying next to them. The reassembled eigenvectors are shown in Figure 7.70 where the eigenvectors still build the rows, but with the eigenvector components rearranged in the described manner. In comparison to Figure 7.66 the corresponding eigenvectors' corresponding components are gathered next to each other. As in the case of the sought structures their phases have a shift of 0 if they are centre and a shift of π if they are alters to the centre, the resulting colour change information can be taken from the colour table in Figure 5.1. A phase of 0 is indicated by red, a phase of π by cyan.

As an example for the found patterns within the total moneyflow network Figures 7.71 and 7.72 show the total network for the traders 1922 ('elfriede') and 1816 ('henning'). Both traders are supposed to manipulate by the money transfer pattern with trader 1922 collaborating with 3 other traders (traders 1898, 1858, and 1775) and trader 1816 collaborating with 2 (traders 1826 and 1831). The actions towards the regular market of traders 1922 and 1816 differs as trader 1922 trades only little with traders not collaborating with him but trader 1816 behaves as a trader with balanced trading volumes to the rest of the market beside his allies.

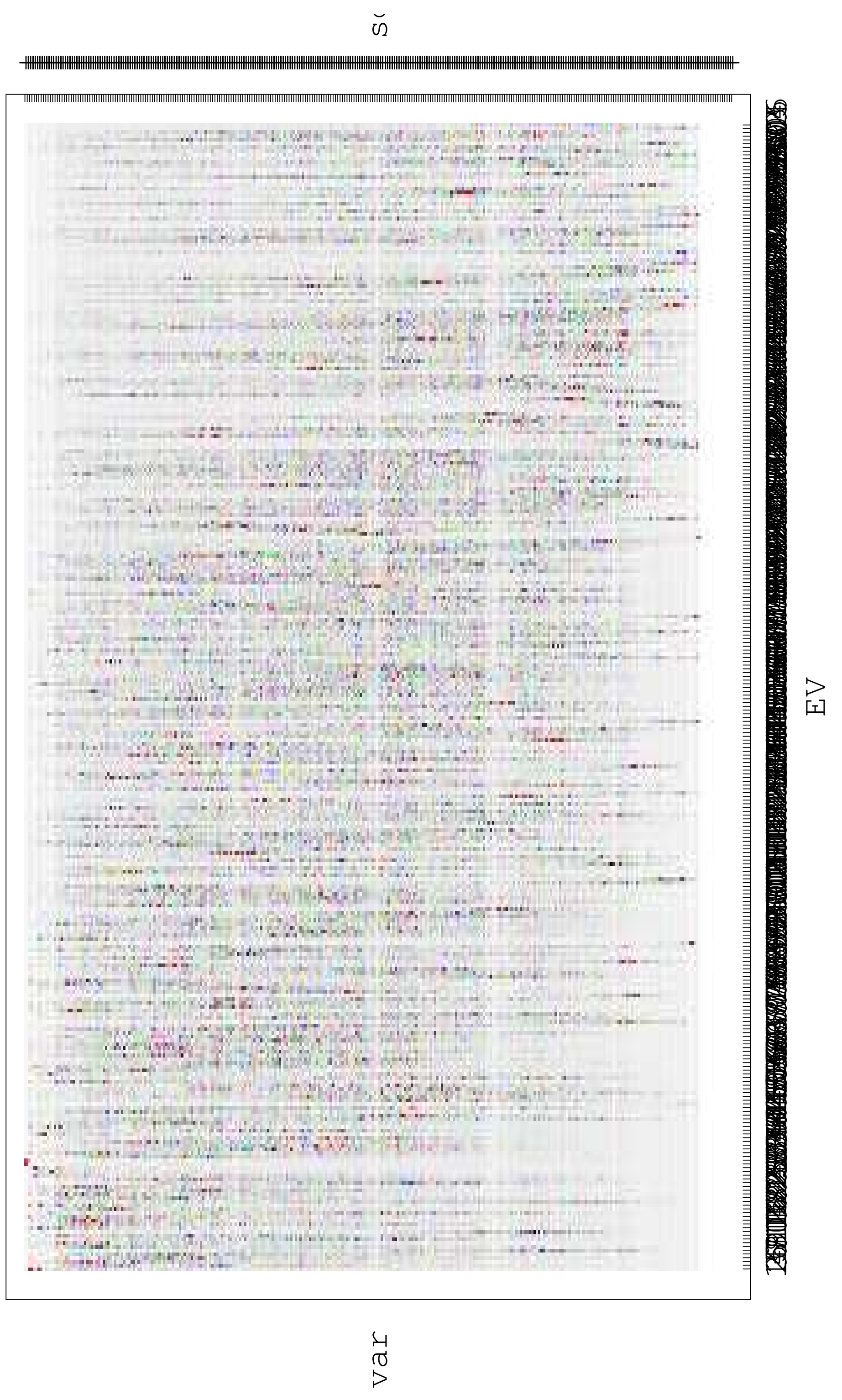

Figure 7.66: Visualisation of the eigensystem of the global moneyflow network of the market for the 2006 state parliament elections in Baden-Württemberg in Germany following Section 5.3.1

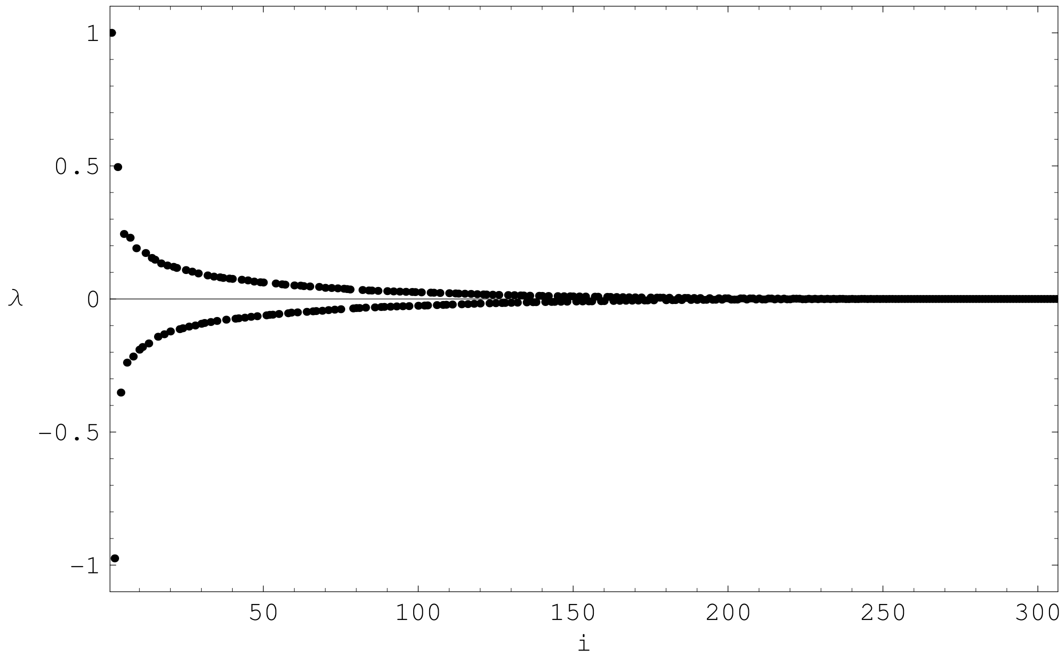

Figure 7.67: Normed eigenvalues of the global moneyflow network of the market for the 2006 state parliament elections in Baden-Württemberg in Germany

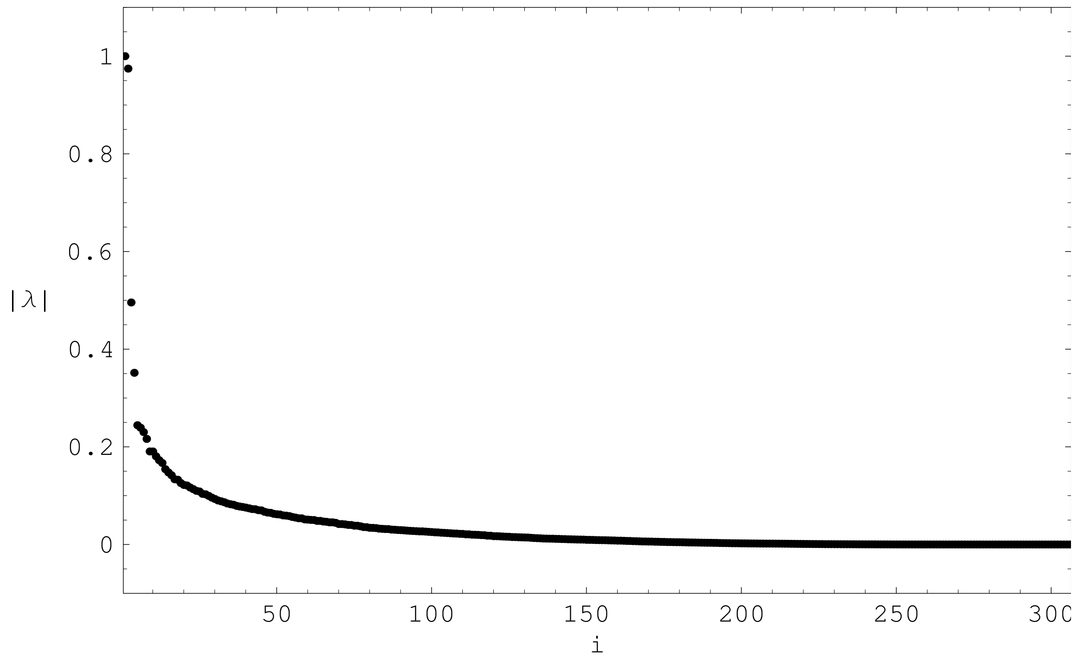

Figure 7.68: Normed eigenvalues of the global moneyflow network of the market for the 2006 state parliament elections in Baden-Württemberg in Germany with absolute values

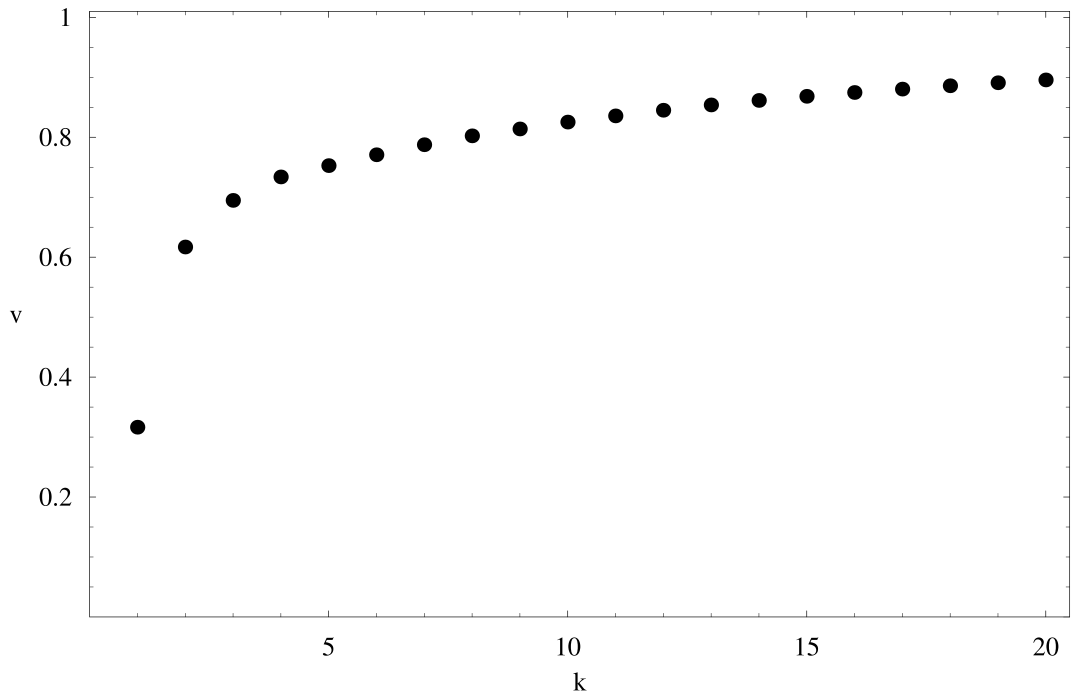

Figure 7.69: Cumulated variance of the global moneyflow network of the 20 first largest absolute eigenvalues of the market for the 2006 state parliament elections in Baden-Württemberg in Germany

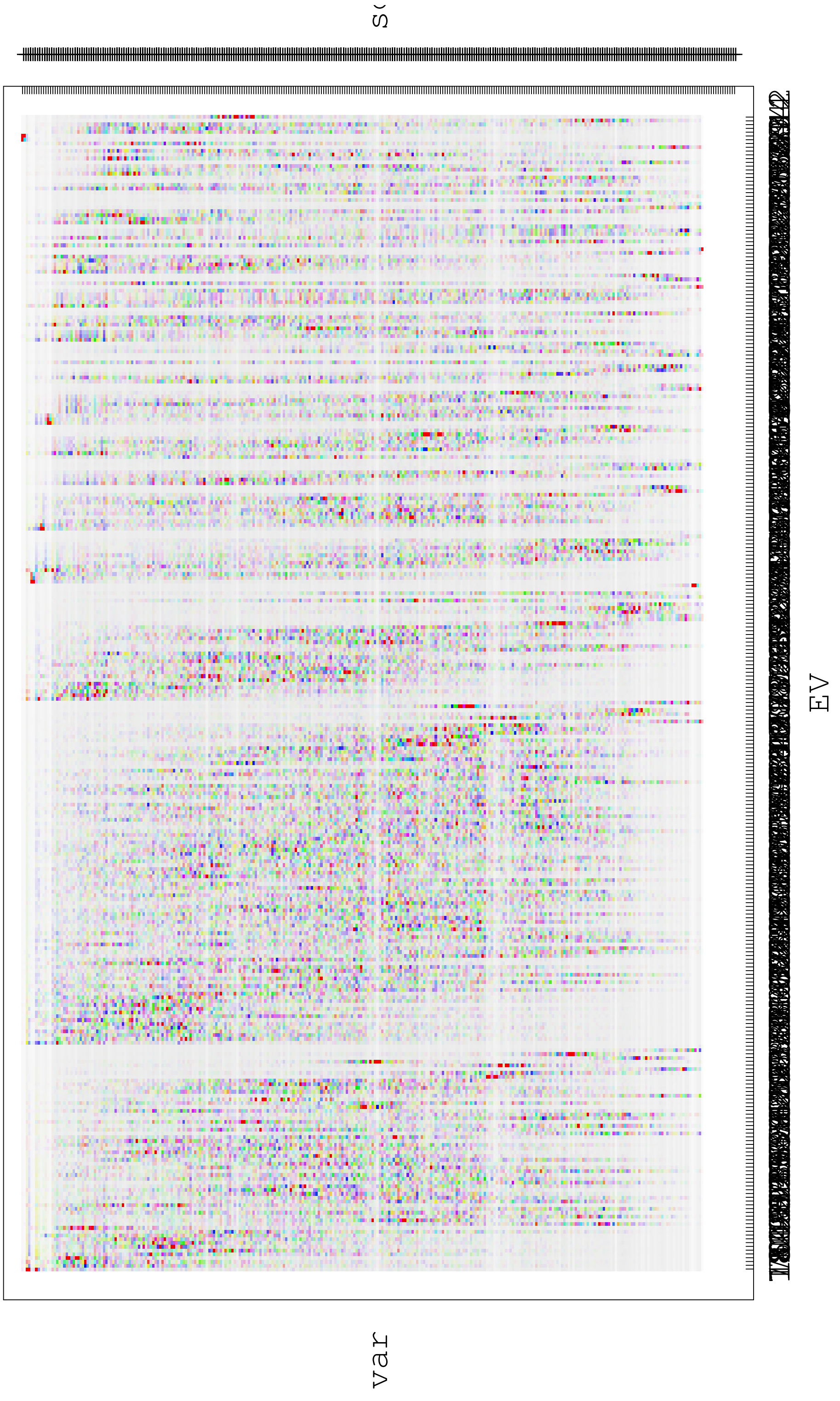

Figure 7.70: Visualisation of the global moneyflow network of the eigensystem of the market for the 2006 state parliament elections in Baden-Württemberg in Germany following Section 5.3.1 (clustering applied)

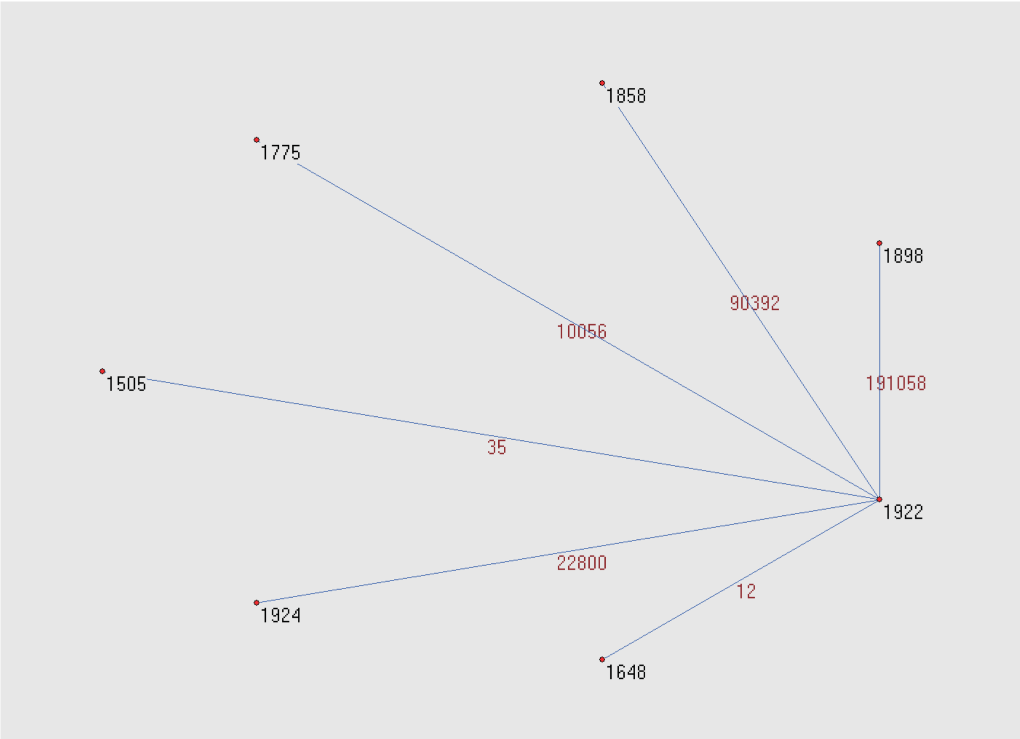

Figure 7.71: Total moneyflow network for user 1922 (username 'elfriede') with the strength of moneyflows visualised by numbers showing the difference of inbound and outbound flow

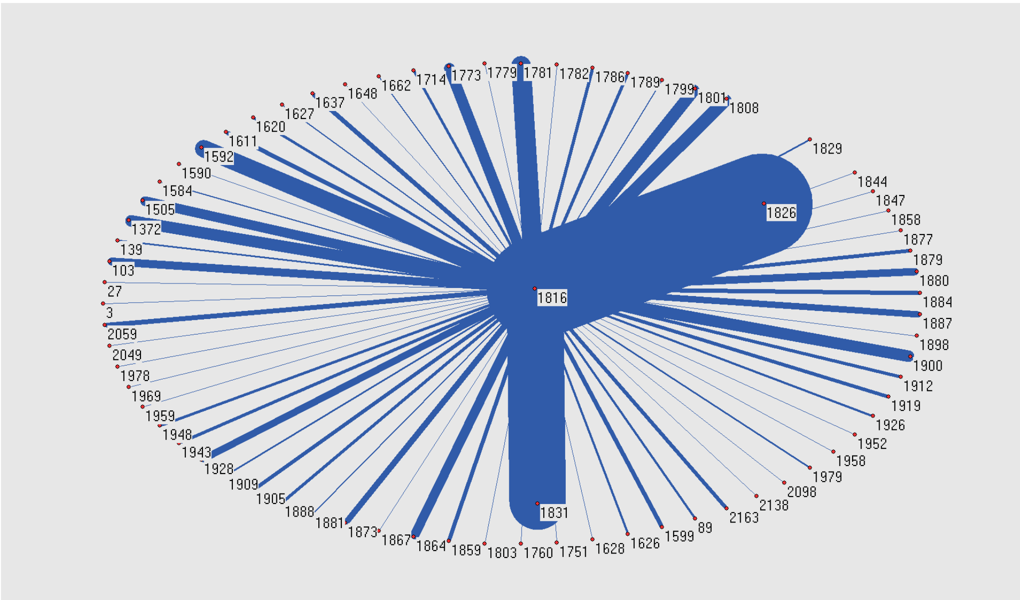

Figure 7.72: Total moneyflow network for user 1816 (username 'henning') with the strength of the difference of moneyflows visualised by strength of ties

7.2.2 Elections for the National Parliament in Switzerland 2007

Like the previous market, the market for the national parliament elections (Nationalrat) in Switzerland in 2007 was not disturbed during runtime by the system operators. This is noteworthy as massive and continuous price manipulations took place which put the market and the results into a wrong perspective. The liberal stance on market surveillance was supported by the liberal character of the media partner (*Neue Zürcher Zeitung Online*) who chose to handle the manipulation by journalistic means. In the areas of online and print, the media partner reported on the manipulations within the market and the fact that manipulating traders would be excluded from the lottery, as it was also stated in the terms and conditions for the market system. As in the market described before, manipulating traders were not blocked from trading during runtime and were not informed about their exclusion. The participants were mainly customers of the media partner.

Market opening	2007-09-11 10:00:00
Market close	2007-10-21 12:00:00
Number of traders (at least one sell or buy transaction)	511 traders
Number of traders (at least one sell transaction)	310 traders
Number of traders (at least one buy transaction)	487 traders
Number of transactions	16,421 transactions
Number of shares	8 shares
Average volume per trade	350.7 shares
Average moneyflow per trade	3,781.4 MU
Moneyflow in total	62,093,811 MU
Shareflow in total	5,759,656 shares

Table 7.12: Statistical Data of the 2007 national parliament elections in Switzerland

7.2.2.1 Analysis

Finding information on price manipulation in prediction markets we follow the pattern description in Section 6.2.3 which proposes to analyse the shareflow network (cp. Section 6.1.4) of the share that is supposed to be manipulated. In the resulting eigensystem analysis of this transaction data the patterns show up in the way introduced in Section 6.2 and in the simulation in Section 7.1.3. The eigensystem analysis results are illustrated by parts of the eigensystem in Table 7.13, the eigenvectors in colour table visualisation in Figure 7.74, the normed real and absolute eigenvalues distribution in Figures 7.75 and 7.76, the cumulated variance plot in Figure 7.77 and the eigenvectors with the cluster algorithm introduced in Section 5.3.3 applied. As for the network data gathered in the described prediction market for the Swiss national parliament it was obvious during the active period of the market that especially the price of the smallest party in the market (Grünliberale Partei GLP in Basel) was quite different from the values of the polls and, last but not least, the final result of the election where the final market price was 5.76 MU compared to the final share price of 1.40

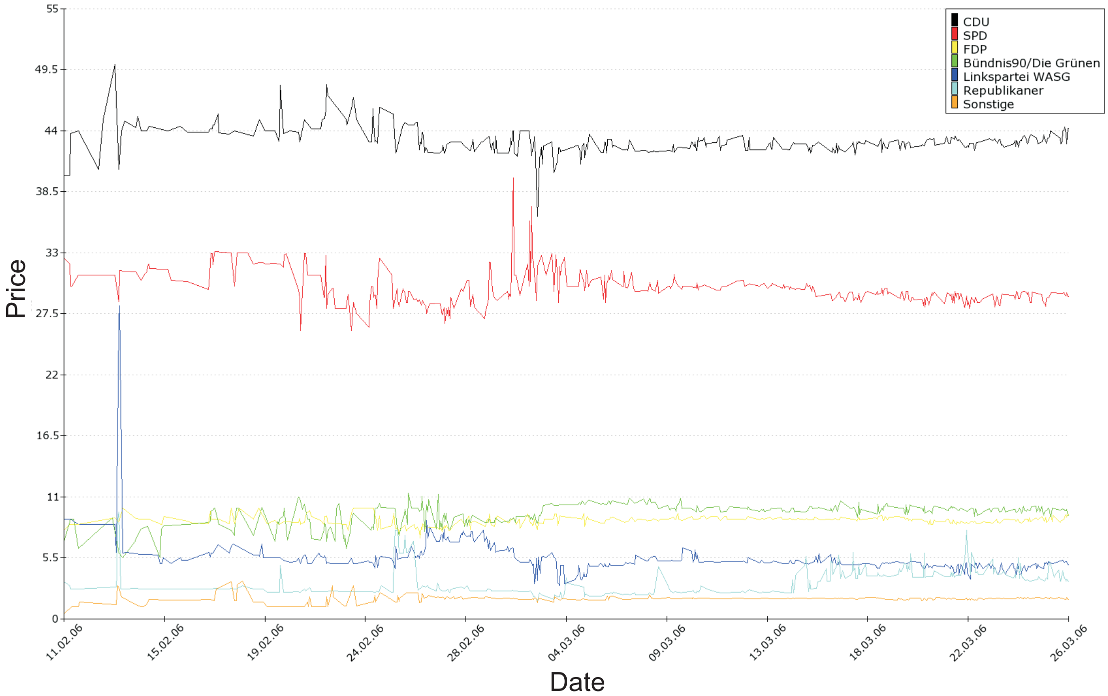

Figure 7.73: Prices of the market for the 2007 national parliament elections in Switzerland

In the analysis results the second eigenvalue pair showed a strong inbound pattern on trader 3224 whose inbound and outbound connections are given in Figures 7.79 and 7.80. In the clustering of the eigensystem this trader appears in the second cluster as central vertex which is visualised in Figure 7.78 and can be found in the upper left, second red box, as the eigenvector components are ordered by the clusters and their structure within as described in Section 5.3.3. This central structure of trader 3224 within the shareflow network hints at a price manipulation which is even more probable by the comparison of the inbound and outbound networks shown in Figures 7.79 and 7.80.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
80036.10	-80032.60	43524.80	-41968.30	36415.00	-30059.00	29495.70
λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}	λ_{13}	λ_{14}
-25525.40	19535.40	-18481.50	18440.80	14301.90	-13074.50	-12623.80
λ_{15}	λ_{16}	λ_{17}	λ_{18}	λ_{19}	λ_{20}	λ_{21}
-9061.48	8626.6	-7473.57	7202.77	-6590.15	6556.27	6270.35

Table 7.13: First 21 largest absolute eigenvalues

Figure 7.74: Visualisation of the eigensystem of the shareflow network for the GLP-share of the 2007 national parliament elections in Switzerland following Section 5.3.1

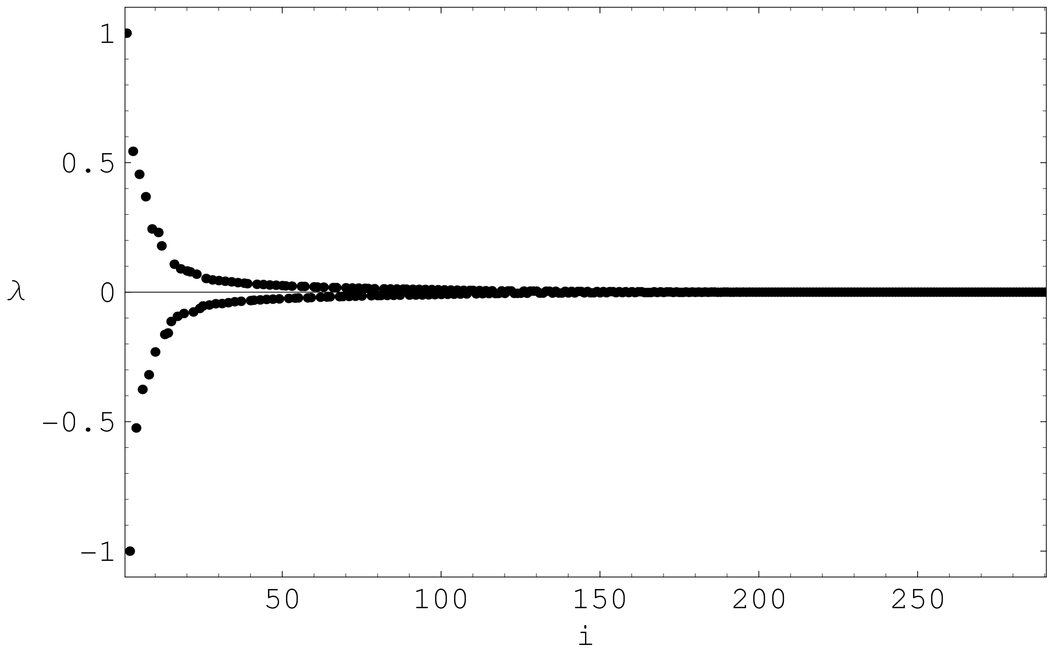

Figure 7.75: Normed eigenvalues of the shareflow network for the GLP-share of the 2007 national parliament elections in Switzerland

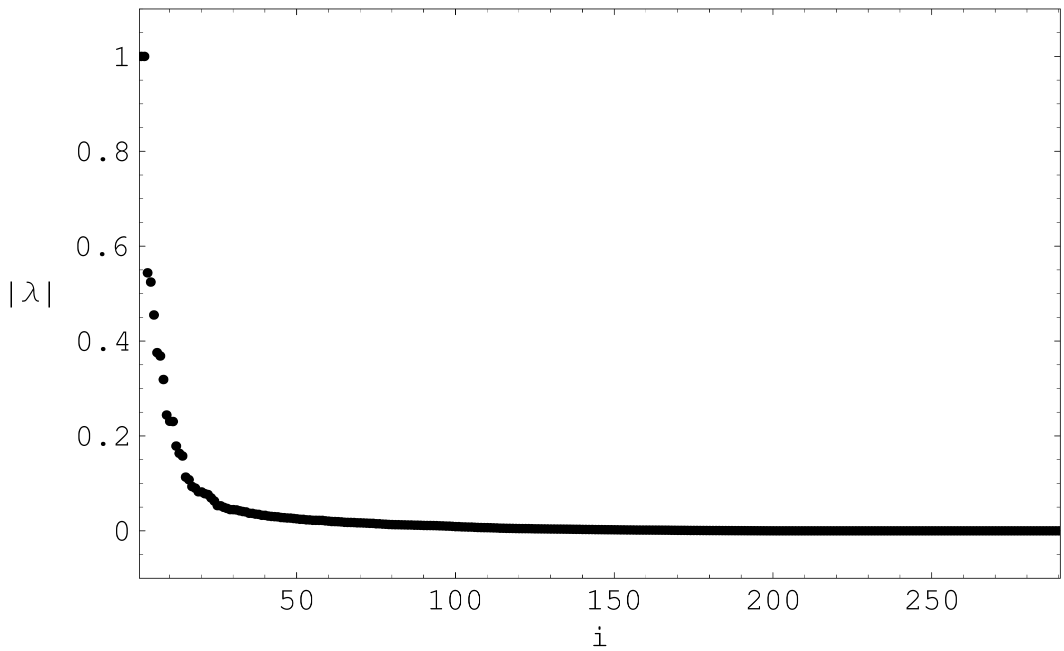

Figure 7.76: Normed eigenvalues of the shareflow network for the GLP-share of the 2007 national parliament elections in Switzerland with absolute values

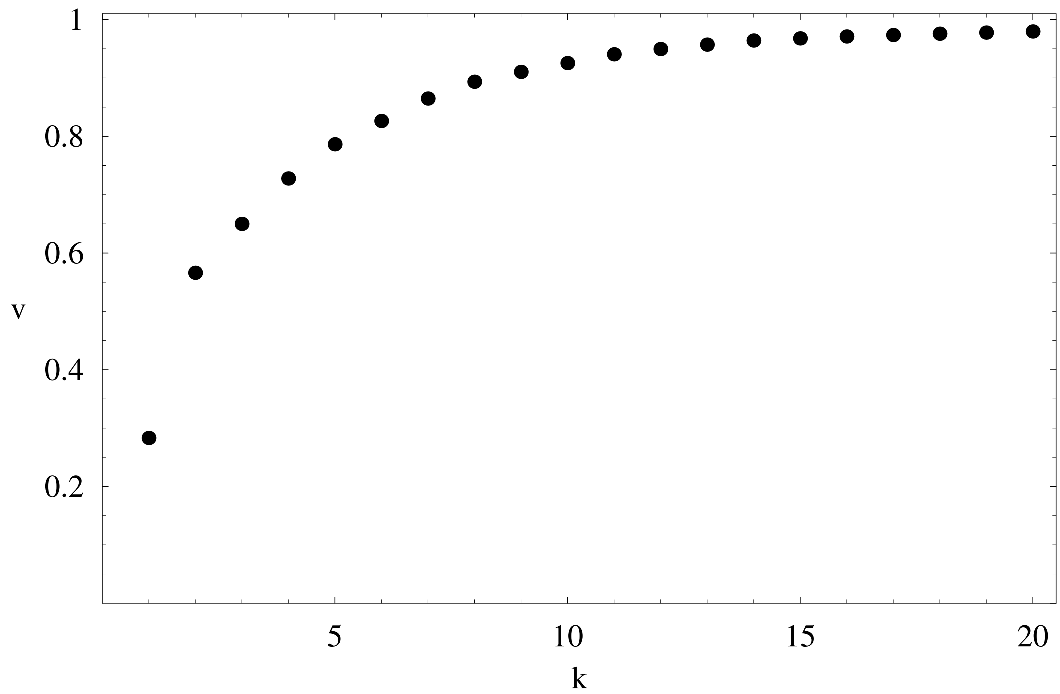

Figure 7.77: Cumulated variance of the shareflow network for the GLP-share of the 20 first largest absolute eigenvalues of the 2007 national parliament elections in Switzerland

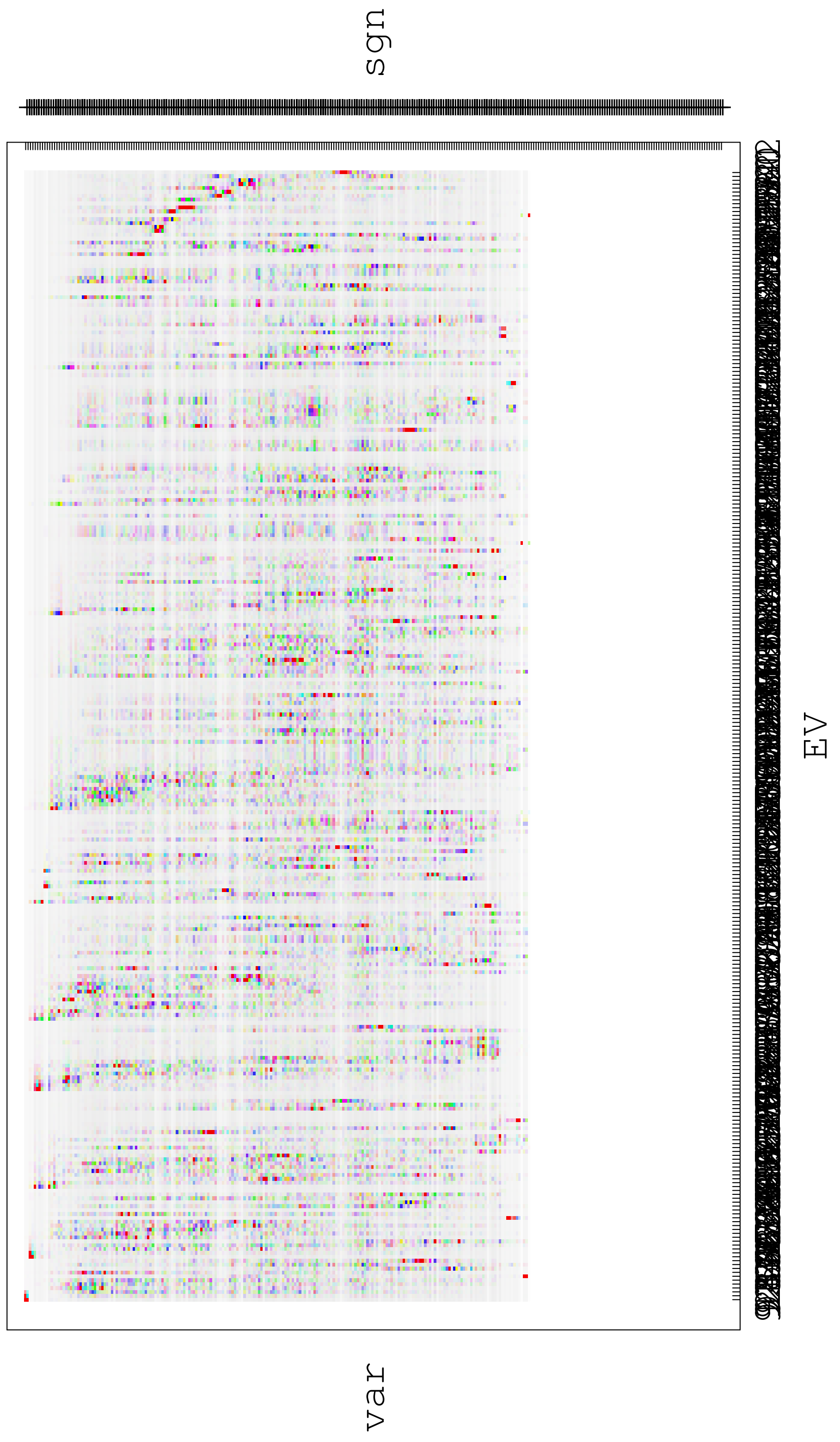

Figure 7.78: Visualisation of the shareflow network for the GLP-share of the eigensystem of the 2007 national parliament elections in Switzerland following Section 5.3.1 (clustering applied)

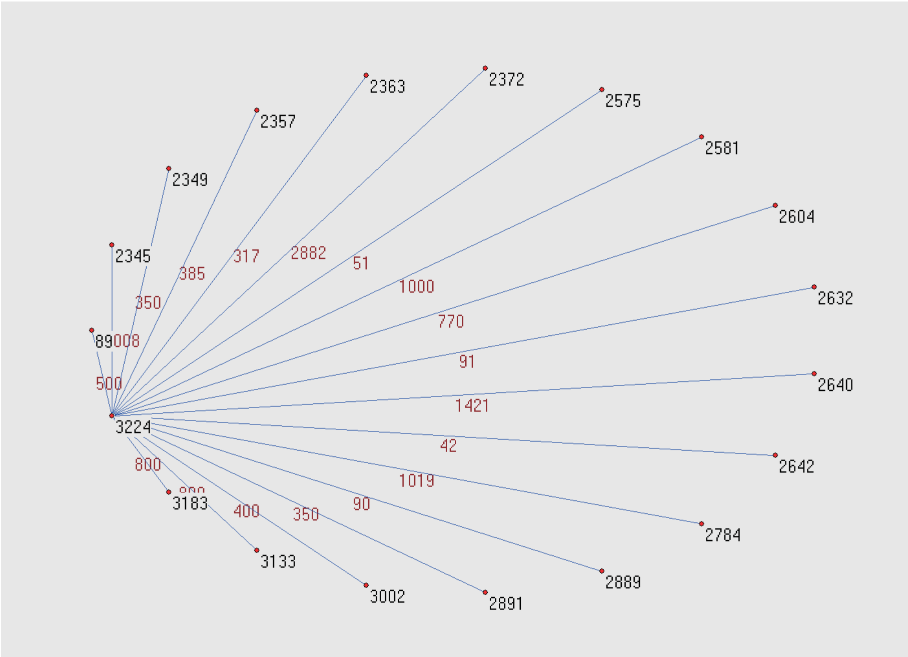

Figure 7.79: Inbound shareflow network of share 'GLP' for user 3224 with the strength of shareflow visualised by numbers showing the cumulated shareflow

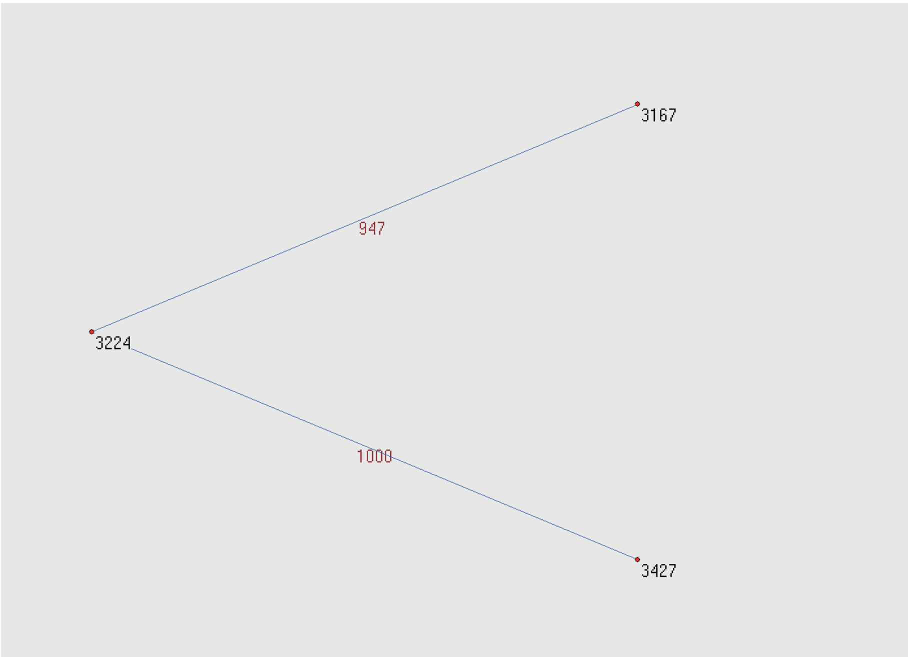

Figure 7.80: Outbound shareflow network of share 'GLP' for user 3224 with the strength of shareflow visualised by numbers showing the cumulated shareflow

7.3 Discussion

By the analyses accomplished within this chapter the effectiveness of the analysis method from Chapter 6 could be demonstrated. Market transaction data with the specific patterns of Section 6.2 introduced separately were generated and analysed. In the results of these analyses the pattern structure was clearly discoverable by the procedures introduced in 6.3 based upon the properties within the resulting data eigensystem described in Section 6.2.

For identifying the specific patterns within real market transaction data, the networks of interest were built from the market system data and analysed in a similar manner to the simulated data. As a result the network data put into the analysis was split with the single patterns being extracted. In conclusion to the real data put into the analysis, the separate extraction of traders with a manipulator's characteristic directly from the network flows stored in the database came up with the similar traders and trader groups, respectively, assuring the assumed suspicion of the trader acting irregular.

As all analysis techniques within this work were presented in a research context, it is useful to give some remarks on certain aspects of a proper application in other settings or domains. The remarks are arranged in their assignment to the aspects *software and programming*, *network characteristics* denoting primarily the characteristics of transaction networks built from prediction markets, but transferable to other network definitions, and *systematic failures* in the domain of prediction markets but also transferable to other market domains.

- Software and Programming
    - As the numerical precision of the common software calculation packages may be very different and usually not well documented, analyses have to be carried out carefully with being aware about the precision of the respective package used e.g. the amount of digits up to which two eigenvalues are distinguishable. This holds also for the internal calculation procedures in the software where one has to be aware of i.e. different norming in certain stages of the eigensystem calculation.
    - One major aspect in the choice of the software framework used for the analysis should be the aspect of real time mining of the data. The analysis of this work was conducted with static data after the end of a market. In a second step this could be taken closer to the real time mining by applying the static calculations to time slices of the transaction network data, but true real time mining besides a good hardware configuration makes high demands on the software used and especially the efficiency and stability of the algorithms of the mathematical solvers used.

- Network Characteristics
  - As transaction networks evolve over time the assumed state of a complete graph structure for a well functioning market is reached only after a certain time of market activity. In the case of traders allowed to participate in the market after market start this is even more difficult to achieve. In the case the networks are not well defined the supposed analysis technique may leave manipulations undiscovered as the regular trading itself would appear like manipulative actions thus similar to the real manipulations. Depending on the configuration of the market, complete structures within the observed network can be achieved by only analysing certain time slices of the network which hold only partial information of the network, but enough data to discover the proposed manipulation structures. Furthermore, a good extract of partial data can also result in the manipulative structure to show up in a quite clearer way as the manipulative transactions are not 'covered' by regular trades from other time slices of the transaction network.
  - Manipulative traders may cover their irregular actions by regular ones. This is especially the case for regular traders that only perform a small amount of manipulations. Also here the analysis of parts of the transaction networks may help to discover these transaction structures.
  - The basic assumption in Section 7.1.3 that a market is quite even in its degree and flow distribution can be touched not only by manipulative traders but also fundamental or arbitrage traders. Their actions may overlap with the patterns of manipulative traders introduced within this work in certain network types. Thus along with a semantic verification of the plausibility of the result this fact should be considered.
- Systematic failures (usually common to all manipulation detection systems)
  - If a market system is confronted especially with price manipulations in a large scale, trading may get stuck, because none of the target shares is available on the market anymore. In such a situation analyses of transaction networks give results only up to the point in time trading was disrupted. In this case an analysis of networks which also considers not executed orders could be helpful.
  - Wrong analysis results could appear if in transactions belonging to a money transfer pattern the wrong 'recipient' is matched to the starting offer of the manipulator. This may happen by chance if a regular trader acts with an adequate offer at the same time as the manipulator does.
  - Generally, the final decision about a trader being a price manipulator is rather difficult as the trader's intentions can only be assumed.

8. Conclusion and Outlook

This work describes the incentive systems of a prediction market, incentive compatibility and the reasons, effects and detection of irregular trading behaviour that follows the absence of incentive compatibility. This behaviour is analysed and classified by the the different transaction patterns of traders within such a system with respect to the incentives given by the system. To balance incentive incompatibility these traders can then be excluded from active trading or in a market with play money from participation in the lottery in the end depending on the market operator's strategy.

To solve the problem of detecting the traders acting according to the introduced manipulation patterns this work focuses on the trading characteristics of irregular traders in different transaction networks that can be extracted from a prediction market system. As the used eigensystem analysis approach, emanating from the area of social network analysis, focuses on analysing networks in terms of central structures the transaction data relevant to a certain trading pattern is described in central structures. This is achieved by first disaggregating the total market transaction data into subnetworks of certain entities like money and shares and then describing the manipulative actions as centralised structures. Procedures for finding the patterns and, therefore, the relevant traders are given as well as a method for clustering the resulting eigensystem by inherent metric information. The proposed analysis technique is applied to simulated network data providing clear transaction patterns in the network definition. Finally the transaction data of two public prediction markets that were conducted within the scope of the research question are analysed. The results of this analyses give a clear insight into the trader behaviour and separate the 'good' ones from the relevant 'bad' ones that disturb the incentive compatibility. Especially by the application of the clustering technique the cliques associated to the pattern and its initiating trader are discovered.

As a new tool in the forecasting business, prediction markets operate in areas nowadays forecasting technologies can hardly compete with. Even if carried out online traditional opinion polls do not offer aspects like the real time operation a continuous market system can provide. Additionally, prediction markets provide a high involvement of the trader which is usually not present or even not achievable by traditional prediction tools. But as prediction markets are implemented in open world scenarios with play money they may suffer from a gambling character where the system and results could be considered as systematically being not serious. This impression is even strengthened if the market predictions are wrong. So as prediction markets achieve more public interest and gain attractiveness it is essential that they provide clear predictions and, therefore, are recognised as a valid and serious forecasting tool. The essential aspect to provide an attractive and well functioning prediction market system is a working incentive system where incentive incompatibility and its systematic result (fraud and manipulation) is balanced by sufficient arrangements to increase the cost of accomplishing these actions for the manipulator like prosecution and penalisation of these irregular actions. This work gives a new way of detecting irregular trading behaviour by means of an eigensystem analysis technique.

In the context of this work the presented methods for fraud detection were set up in a research domain. Continuing the proposed ideas it would be useful to implement the presented methods in a robust system which serves the application domain adequately. Crucial in implementing is the efficient data procession in the case of the eigensystem calculations which requires a good eigensolver package at low algorithmical costs. For the methods analysing the eigensystems the main challenge is surely the level of detail the structures can be detected. Especially in a real time mining environment the differentiation between noise (regular) trading patterns and the irregular ones is hard to distinguish as the upcoming network is unknown in each point in time before the market closes. An efficient usage of subsequent time windows to calculate eigensystems on the static states of the network could be a starting point to this analysis. By an elegant comparison among these time slices even a continuous picture of the evolving network could be drawn. This could be enhanced by overlapping time windows. Within the domain of markets the unmatched orders could be used as another source for information that tracks back to the full set of users actions in the market system and could give a deeper insight into the trading behaviour and manipulative acting. As this work only uses cumulated numbers of shares or moneyflows it could also give more insight into the actions of a trader to use information like shareflow and moneyflow with respect to the number of trades or price of a share in the analysis.

Regarding the incentive compatibility and the incentive systems of prediction markets in real world applications it is necessary to get a deeper insight into the behavioural mechanisms like the impact of fraud onto regular traders of the markets. This is essential to provide good quality solutions in the control of incentive incompatible status of systems and thus good prediction quality and error measuring as the results of such markets. As introduced in the motivation to this work in Chapter 1 prediction markets provide a brilliant combination of social sciences, economic research and information technology that makes it possible to experience certain domains (i.e. the political one) from a different but modern point of view. Thus a solid formal basis for dealing with the mentioned issues is crucial to the public perception of prediction markets as a reliable prediction instrument.

A. Abbreviations and Symbols

Within this work the following conventions are maintained if not stated otherwise.

Basic graph entities and sets	
$S = \{s_1, \ldots, s_n\}$	Set of shares
$V = \{v_1, v_2, \ldots, v_m\}$	Abstract set of vertices
V^T	Set of traders
$E = \{e_1, e_2, \ldots, e_o\}$	Abstract set of edges
E^M	Set of moneyflow edges
E^{M^s}	Set of moneyflow edges for a certain share $s \in S$
E^Q	Set of shareflow edges
$\omega(v_i, v_j) = w_{ij}$ with $i, j \in V$	Applied weighting function and weight of edge between vertex v_i and vertex v_j
Graphs and measures	
$\mathcal{G} = \{V, E, \omega\}$	Basic Graph as described in Equation 4.1
$\mathcal{G}^M = \left\{V^T, E^M, \omega^M\right\}$	Graph of moneyflow network as described in Section 6.1.1
$\mathcal{G}^S = \left\{V^T, E^Q, \omega^Q\right\}$	Graph of shareflow network as described in Section 6.1.2
$\mathcal{G}^M_s = \left\{V^T, E^{M^s}, \omega^{M^s}\right\}$	Graph of sharewise shareflow network for share $s \in S$ as described in Section 6.1.4
$\delta_{\deg}(v)$	Degree of a vertex v (all edges incident to vertex v as described in Equation 4.2
$\delta^{in}_{\deg}(v)$	Degree (inbound) of a vertex v (all edges incident to vertex v and directed to vertex v) as described in Equation 4.3
$\delta^{out}_{\deg}(v)$	Degree (outbound) of a vertex v (all edges incident to vertex v and directed away from vertex v) as described in Equation 4.4

$\delta_{\text{flow}}^{in}(v_j) = \sum_{v_i\ \in\ V \setminus\ v_j} \omega(v_i, v_j)$	Abstract inbound flow of a vertex(sum of all flows of edges incident to vertex v_i and directed to vertex v_i) as described in Equation 4.7
$\delta_{\text{flow}}^{out}(v_i) = \sum_{v_j\ \in\ V \setminus\ v_i} \omega(v_i, v_j)$	Abstract outbound flow of a vertex (sum of all flows of edges incident to vertex v_i and directed away from vertex v_i) as described in Equation 4.6
General Mathematical Notation	
$\varphi(z_1)$	The phase information of $z_1 \in \mathbb{C}$
$\varphi(z_1, z_2)$	The angle between z_1 and z_2 with z_1, $z_2 \in \mathbb{C}$
$\mathbf{a}$	Vector a
$\mathbf{a}^*$	Complex conjugate transpose of vector a
a_i	i^{th} element of vector a
$\overline{a}_i$	Complex conjugate of the i^{th} element of vector a
$\mathbf{B}$	Set of vectors
$\mathbf{b}_i$	i^{th} vector within the set of vectors $\mathbf{B}$
b_{ij}	j^{th} element of the i^{th} vector within the set of vectors $\mathbf{B}$
C	Matrix C
C^t	Transpose of matrix C
C^*	Complex conjugate transpose of matrix C
Mathematical Notation	
Λ	Set of all eigenvalues (Spectrum)
λ_i	Eigenvalue i
$\mathbf{X}$	Set of all eigenvectors
$\mathbf{x}_i$	Eigenvector i
x_{ij}	j^{th} eigenvector component of eigenvector i
Abbreviations and Textual Concerns	
MU	Monetary Unit
moneyflow	Flow of money
shareflow	Flow of shares
e.g.	exempli gratia; for example
cp.	compare to
i.e.	id est; that is
et al.	et alii; and others

List of Figures

List of Tables

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As a new tool in the forecasting business, prediction markets operate in areas current forecasting technologies can hardly compete with. Even traditional opinion polls carried out online do not offer aspects like the real time operation a continuous market system can provide. Additionally, prediction markets provide a high involvement of the trader, usually not present or even achievable by traditional prediction tools. But as prediction markets are implemented with play money in open world scenarios, they may suffer from a gambling character where the system and results could be considered as not being systematically serious. This impression is strengthened further if the market predictions are wrong. So as prediction markets achieve more public interest and gain attractiveness, it is essential that they provide clear predictions and, are therefore recognised as a valid and serious forecasting tool. The essential aspect in providing an attractive and well functioning prediction market system is incentive compatibility. Fraud and manipulation are the systematic results of incentive incompatibility, which, if present, have to be detected and balanced. This can be accomplished by sufficient arrangements to increase the cost of carrying out these manipulative actions.

This work gives a new way of detecting irregular trading behaviour by means of a generalised eigensystem analysis technique, used mainly in the area of the social network analysis. Derived from incentive incompatible situations, the motivations for manipulations are described on a formal level. The manipulations are expressed as abstract trading patterns which can be detected in certain types of transaction networks within a prediction market system.

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