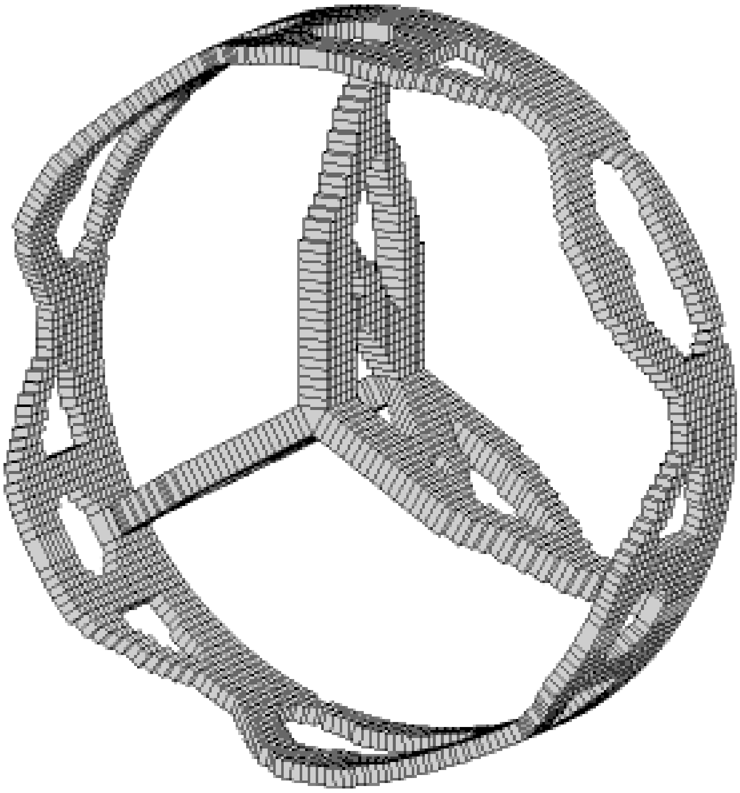
A New Approach for Sizing, Shape and Topology Optimization



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Abstract

This paper for the first time presents the new interface between CAOSS and MSC / NASTRAN. **CAOSS®** (**C**omputer **A**ided **O**ptimization **S**ystem **S**auter) is an optional finite element module for the efficient sizing, shape and topology optimization. The introduction gives a survey on the program itself and on its history. To begin with, the first chapter gives an overview of the optimization type (sizing, shape and topology) and the state-of-technology. In addition the principal differences between the mathematical programming methods and the methods based on optimum criteria are explained. A comprehensive chapter reveals the theoretical backgrounds of the optimum criteria for shape and topology optimization purposes as well as the especially developed controllers created by the authors. The interaction of CAOSS and MSC / NASTRAN and the optimization options new to the MSC / NASTRAN user in conjunction with CAOSS are emphasized. Finally industrial application examples give an impression of the capabilities.

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1. Introduction

In the tough international competition, companies can only survive if, besides highly innovative power they can provide strongly cost optimized products. Therefore in new procedures like the Simultaneous Engineering, the calculation engineer is already integrated into the concept phase of the product development process. Efficient methods of working require powerful optimization algorithms to be provided in addition to the discrete methods (FEM / BEM) proved worth while to support the calculation engineer in the draft and design phase.

In recent years many optimization approaches have been integrated into commercial FE programs. In industrial applications only few optimization methods are established partially. The problems to be solved in industry have to be abstracted dramatically. The further development of the optimization methods regarding application, integration and numerics is necessary and pushed ahead intensively.

In this context new optimization criteria and control strategies for sizing, shape and topology optimization [1, 57, 59, 61] were created at the University of

Karlsruhe, Germany. Based on these new strategies the computer program CAOSS was developed by engineers for engineers. Considering the hypothesis formulated in [57, 59] relating to the strain minimum, very fast and efficient control algorithms could be developed for sizing, shape and topology optimization based on FE models. CAOSS is a module extending FE programs for optimization purposes. Interfaces exist to various FE programs. In this paper the interface to MSC / NASTRAN is presented for the first time.

CAOSS was first presented to a wide public at the COMETT Optimization Conference of the European Union in May 1992. Both representatives of the industry and participating researchers from universities were impressed by the results. Since 1993 CAOSS has been distributed by CAE Partner (a company of the MSC Group) and has proved worth while in many companies. Many serial components have been optimized [17, 22, 31, 32].

In 1994 the program was awarded with the European Academic Software Award 1994 of the European Commission for the best academic software in the field of mechanics.

CAOSS has the following advantages:

- CAOSS is easy to use, flexible and requires only very little computing time.
- For optimization purposes, only an existing FE model is required (Bulk Data Deck), which need not be parametric thus old models can subsequently be optimized, too.
- The choice of design variables need not be taken into consideration to begin with. The number of iterations until a solution is found is very small and independent of the number of design variables.
- CAOSS is a module in addition to the already existing FE environment working largely in batch mode, thus ensuring a quick and easy lead-ing to optimization.

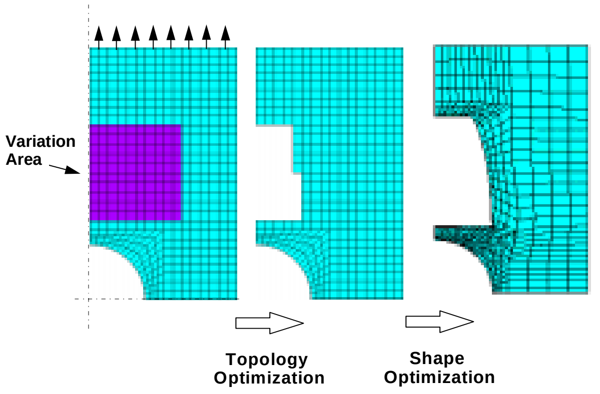

Figure 1: Integrated Design Concept using CAOSS

- The optimization procedure is clear and comprehensible. The approach corresponds to the way an engineer works.
- CAOSS can optimize structures of any size for various load cases at the same time.
- For shape optimization purposes a CAOSS internal mesh correction module adapts the mesh topology to the modified geometry. Thus remeshing with tetraeder elements is avoided.
- CAOSS runs under UNIX and DOS on workstations, mainframes and PC’s.

2. Optimization Types

The following gives an overview of the main differences among sizing, shape and topology optimization. Both the numerical and the user-specific characteristics are discussed shortly.

2.1. Sizing Optimization

For optimization purposes using „Sizing Variables“ i.e. cross sections and thicknesses of finite elements, many mathematical programming approaches have been tested and implemented into finite element programs (e.g. MSC / NASTRAN, PERMAS, COSMOS / M) and special optimization programs (e.g. MBB-LAGRANGE, STARS, ADS). During the optimization, the element properties are modified on the FE level. Due to the easy calculation of the sensitivities for sizing optimization purposes even realistic problems can be handled. Today these approaches can be considered as state-of-the-art.

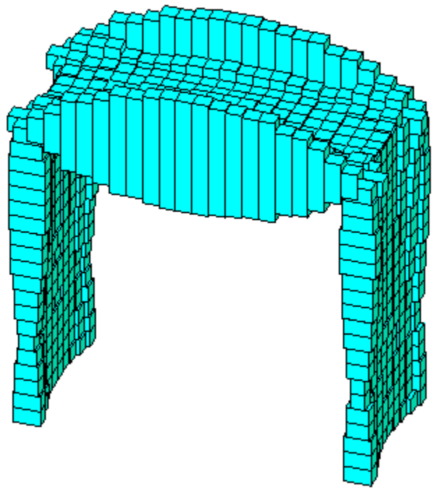

Figure 2: Sizing Optimization of a Shell Structure (For the Visualization, the Property ID’s are shown as Solids)

2.2. Shape Optimization

Compared with the sizing optimization the shape optimization is more complex. For the shape optimization two approaches are used:

a. Shape optimization based on FE models

The coordinates of the surface nodes are regarded as design variables which will be modified during the optimization. This usually leads to a large number of design variables which might cause considerable mathematical difficulties. Using suitable couplings of node displacements to define basis vectors, complex geometry changes can be described in the Solution 200 of MSC/NASTRAN with only few design variables [43, 50].

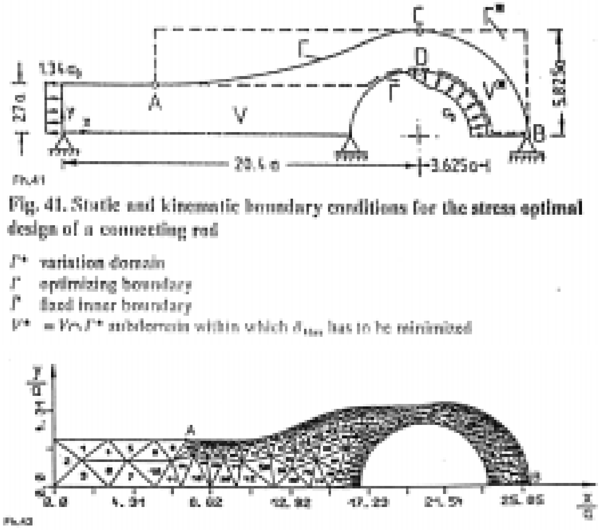

Figure 3: Shape Optimization based on a FE-Model (approx. 100 Design Variables) [67]

b.Shape optimization based on geometry models

Using the shape optimization method based on geometry models, the linkage of an FE model and a geometry model is maintained. As in this case the parameters of the geometry model are the design variables, the geometry model has to be fully parametric. Therefore the use of an efficient solid modeler is necessary. Each parameter modification of the geometry model also results in changes of the FE model. Within each optimization loop the entire FE model has to be set up anew according to the modifications of the geometry model parameters considering the boundary conditions. This method is used e.g. in the programs ANSYS, COSMOS/M and IDEAS. Many interfaces are put on the integration of an FE solver and a CAD system (e.g. ProEngineer). The selection of the design variables is left to the user. In general they differ due to his experiences and creativity. The results of the optimization essentially depend on the number and selection of the design variables. If free form surfaces are allowed, the selection of the design variables is very difficult. Using mathematical programming methods, the number of design variables must not be to large too, because this causes the numerical efforts to increase drastically.

The main difficulty with shape optimization is to transfer the surface changes to the FE mesh. Most programs avoid this transfer by an automatic remeshing in each optimization loop. Hence the original element topology (meshing) is destroyed and often models with only tetraeder elements are created. Only few programs are capable of such a transfer in a way that the mesh modification is analyzed starting from the modification of the

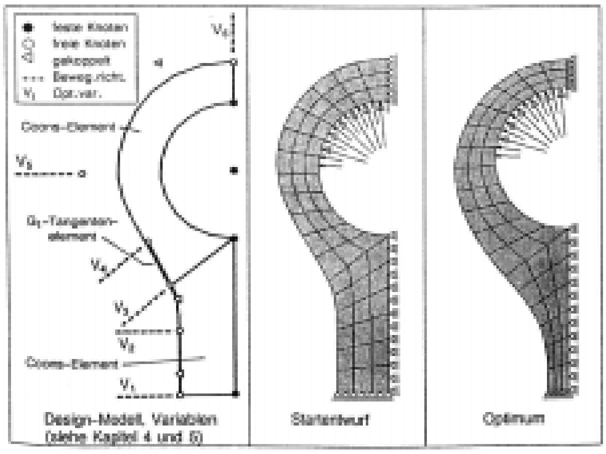

Figure 4: Shape Optimization based on a Geometry Model (6 Design Variables) [12]

surface while maintaining the element topology. To avoid the difficulties with remeshing some programs use p-elements for the shape optimization [49]. Still there is the problem of selecting suitable design variables. For large models the numerical efforts are extremely high.

2.3. Topology Optimization

Both for sizing and shape optimization a first design proposal, which is used as the start design, exists. The objective of general structural optimization methods is to compute even this first design proposal. Therefore an area (2D or 3D) with a homogeneous material distribution is used. Subsequently the functionally required boundary conditions (e.g. node constraints, nodal loads) are applied. The efforts for the modelling and preparation is extremely low. The optimum structural shape with the appropriate topology is issued as design proposal. The originally homogeneous material distribution becomes highly inhomogeneous. Areas arise with no more mass at all (openings and holes) and areas which contain high density mass (bars and struts).

Compared with the sizing and shape optimization the numerical efforts strongly increases. The number of design variables is typically between 5.000 and 100.000. Therefore large efforts have to be put into the sensitivity analylsis up to now. So far no method [8, 9, 10, 11, 33, 34, 40, 41, 54, 55] can be considered as a standard for calculating the optimum topology due to the above mentioned difficulties regarding the mathematical approaches. Essentially because of the expensive calculation efforts these approaches only can handle extremely simplified models. Commercially only few programs are available [2, 63]. Many FE developers work in this field [13, 69]. Because of the development of powerful iterative solvers and the more and more increasing computer capacities, topology optimization will be state-of-the-art very soon.

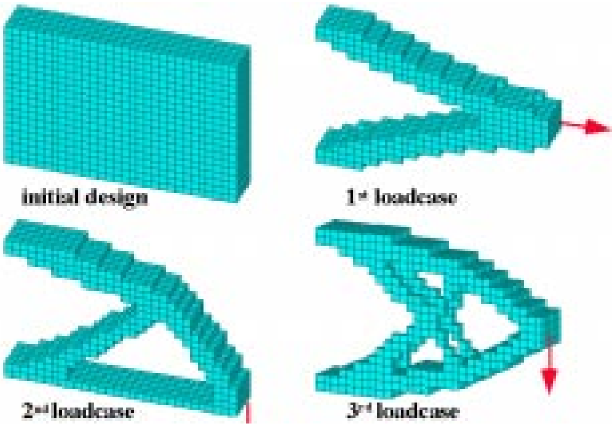

Figure 5: Topology of different Load Cases

3. Optimization strategies

Two approaches to the solution of a problem predominate over optimization. On the one hand these are the mathematical programming methods. They replace the mechanical model by a parametric mathematical substitute model which will then be studied with strictly mathematical methods. The other approach is based on the optimum criteria methods. For that reason requirements are formulated which are valid for the optimum design. If the optimum criteria can be applied to the certain task, the solution converges rapidly.

3.1. Mathematical Programming Methods

The structural optimization problem will be opened to the mathematical programming methods by formulating a substitute problem. The problem formulation [62] looks as follows.

$$min\ f(x)$$

$$g_i(x) = 0;\ j = 1, \dots, m_e$$

$$x \in IR^n;\ j = m_e + 1, \dots, m$$

$$x_i \le x \le x_u$$

In this case f(x) is the target function (e.g. structural weight, deformation, stress, expansion, eigenfrequencies, flutter / wobble rate, etc.). The function $g_j(x)$ represents mostly nonlinear boundary conditions, which constrain the solution.

The calculation of the solution is done in two steps, the calculation of the search direction d_ν and the increment γ. With this the vector γd_ν is determined, which leads from the current vector x_ν to the next solution $x_{\nu+1} = x_\nu + \gamma d_\nu$. For the calculation of the search direction and the increment many approaches exist, according to the various types of target functions and boundary conditions. Finally the solution produced by the analysis program must be verified. If the start values are „disadvantageous“, it might occur that the optimizer only identifies a local minimum as the solution. Therefore it is always recommendable to modify the start values. As the initial problem is transformed into a substitute model, the mathematical programming methods can generally be used. But they require an enormous amount of computing time which will even increase if the number of design variables and active restrictions is increased [62].

3.2. Optimum Criteria

In contrast to the mathematical programming methods, the optimum criteria methods take advantage of the knowledge on the physics and mechanics of the respective problem set. Theses will be postulated describing the optimum.

A well-known and ascertained physical law relating to structural mechanics is for instance the Fully Stressed Design which can actually only be applied to statically determined structures. An important mathematical optimum criterion is the Kuhn-Tucker condition which is for convex optimization purposes fulfilled necessarily and adequate in the optimum. The theses on stress homogenization and stress minimization are optimum criteria, too [59, 60].

Regarding the optimum criteria methods, these criteria and the response behaviour of modifications of the physical model are implemented into the algorithm. With suitable redesign rules, a convergence behaviour is achieved which cannot be attained with mathematical optimizers. Applying this particular physical and mechanical knowledge, the optimum criteria methods remain limited to the certain application areas. Applying this knowledge makes the individual optimization steps comprehensible. The remaining potential to the optimum, which is known from the optimum criteria, can easily and exactly be estimated.

The optimum criteria are particularly well proven for shape and topology optimization where a large number of design variables is required. The convergence speed is independent of the number of design variables.

4. A new „Control“ Method for Shape and Topology Optimization

4.1. Origin and Method Development

In 1991 Sauter of the „Institut für Maschinenkonstruktionslehre und Kraftfahrzeugbau“ the „Machine Design Institute“ of the University of Karlsruhe developed a theory [57] describing the optimum contour for components leading to minimum strain maxima. The theory is based on works of Baud [6, 7] and Neuber [47, 48] as well as extended statements of Schnack [64, 65, 66]. Objective is to minimize the maximum strain, modifying a given shape and given variation areas (areas where the surface is to be modified in whole or in part).

Significant changes were made w.r.t. the control of the surface compared with Schnacks’ gradientless shape optimization [30, 64, 65, 67] and the related optimization strategy according to the rules of nature by Mattheck [36, 38]. Thus the convergence speed and the handling were highly improved. An essential extension was made for areas not to be modified so that now even contact problems can be optimized. The incorporation of user-optimized functionalities was of special importance.

In 1992 the same research team expanded the optimum criteria w.r.t sizing and topology optimization purposes of 2D and 3D structures, loaded by one or several load cases.

The entire description of the theory of these approaches would go beyond this paper. The authors published detailed papers on shape optimization [56, 57, 59]. Therefore only the first two theses are explained. For further details, please refer to [60].

4.2. The Shape Optimization Theory

The formulation of the shape optimization problem is:

An area $\mathbf{V} \subset \mathbf{IR}^{\mathbf{q}}$**(q=2, 3)** with the edge δV is given, whereby V resp. δV are defined by the component resp. its edge.

The maximum load stress B_{max} resulting from a prescribed component load shall be minimized by an „optimum edge“ between two given edge points A and B (A, B $\in$ δV).

The optimum edge ($\Gamma \subset \delta V$) is required lying within a specific variation area x ($\Gamma \subset \Gamma^*$) defined by the design boundary conditions so that the load stress maximum is minimized in V.

The maximum load stress derives from one or more load cases.

Preconditions are that:

1. the law of stress decay is in effect,.
2. there is a maximum load stress on the component edge δV.

If the maximum load stress exists on the edge δV within the variation area Γ^*, the following theses apply to a load case:

Thesis 1

a. The load stress on an edge Γ between two given points A and B is minimal, if the load stress on the edge Γ is constant.

b. A constant load stress on the edge Γ exists, if the edge Γ between the limiting points A and B does not adjoin to the border of the variation area Γ^*.

Thesis 2

a. If the edge Γ adjoins to the border of the variation area, the load stress on the portion Γ_f between the transition points A' and B' is constant and the load stress on the edges Γ_{ga} (AA') and Γ_{gb} (BB') is smaller than on Γ_f.

b. The longer the edge Γ_f is, the smaller the maximum load stress is on Γ_f. If Γ_f is equal to Γ, the load stress is minimal (if the variation limits A and B are fixed) and Thesis 1 is applicable.

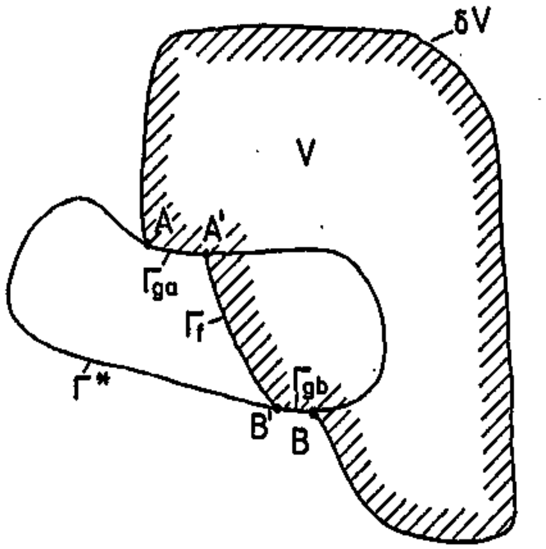

Figure 6: Load Stress Minimum in Case Edge Γ adjoins to the Borders of the Variation Area Γ^*

Example relating to Thesis 1 and 2 (shaft shoulder):

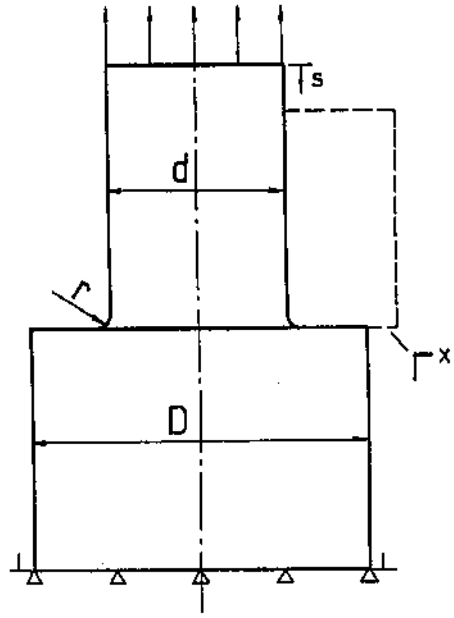

Figure 7: Shaft Shoulder with Variation Area Γ^*

The example is a shaft shoulder loaded with tensile stress. The axial cross-section shows the contour of the start design and the variation area Γ^*. The vertical longitudinal axis is the rotational axis, the force acts in vertical direction. The complete load stress homogenization results from an increase in material at the transition between shaft and radius. The variation area has been defined in a way that no surface displacement is allowed into the component. In 1934 Baud [6] found a similar contour for flat bars loaded with tensile stress.

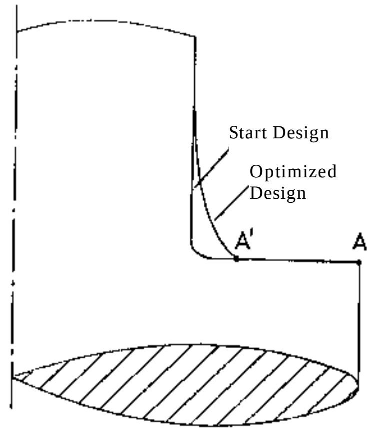

Figure 8: Profile of the Shaft Shoulder loaded with Tensile Stress before and after the Optimization

The reference stress curve according to von Mises along the surface (being the running coordinate along the surface) shows the complete load stress homogenization in the area where the edge to be optimized is not limited by the variation area. The maintained constancy in this profile can vividly be described by illustrating that the edge effect for all cross-sections is exactly compensated by its cross-sectional increase. This directly causes the curvature to constantly grow. The arc of the edge AA' lies on the edge of the variation area. In this area, the load stress is not constant, but smaller than on the edge lying within the variation area. This example proves Thesis 2a. If the variation area is reduced, the constant load stress level of the free edge will increase.

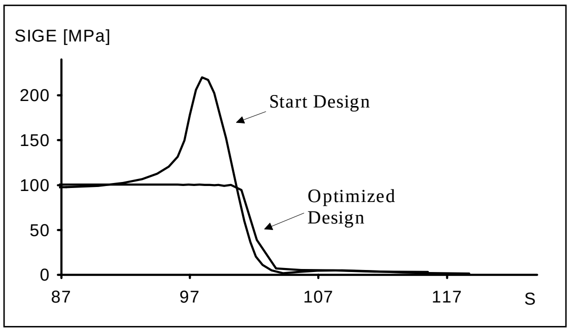

Figure 9: Reference Stress along the Surface of the Shaft Shoulder loaded with Tensile Stress

4.3. The Topology Optimization Theory

Actually this is an applied energy equation. The energy equation says that for elastic systems the outer work W_a is preserved without loss as shape change energy in the deformed system. Therefore the displacement depends on the accumulated strain energy. According to the energy equation the shape change energy U yields from the identity with the external shape change energy W_a which results from the n applied generalized forces F_i.

$$U = W_a = \frac{1}{2}\sum_{i=1}^{n} F_i \cdot w_i$$

For maximizing the stiffness, the minimum of the strain energy has to be found:

$$min(U) = min\left(F^T H F\right)$$

This corresponds to the equation of Menabrea, i.e. a statically undetermined system, which is stressless without external loads, has a stationary minimal shape change energy value. In accordance with the hypothesis of Beltrami this also leads to a strain minimum.

For the optimization, the shape change energy U in the variation area is homogenized and minimized by specific changes of the compliance matrix. Introducing a fictive porosity and with a defined mass limit (mean porosity) the stiffness variation can be transformed into a density variation. The similarity to the homogenization approach of Bendsoe and Kikuchi and the method of Mlejnek is obvious [9, 10, 11, 34, 40].

5. A New „Control“ Method

The use of optimum criteria requires the formulation of redesign rules which, in dependence on the state variables (load resp. energy), moves rapidly towards an optimum component via feedback, thus avoiding the problem of the highly computer-bound sensitivity analysis. Using this approach, the important task is actually to find an appropriate controller. A controller can be designed the more precisely, the more exactly the system behaviour is known.

For mechanically loaded components the authors derived redesign rules for sizing, shape and topology optimization purposes from the optimum criteria, discussing the principal static and not the dynamical damper and memo characteristics nor the adaptivity and „learning capabilities“ of the controllers. For practical use the integration of restrictions and functionalities and various particular cases have to be obeyed.

The description assumes that the optimization is based on finite element models.

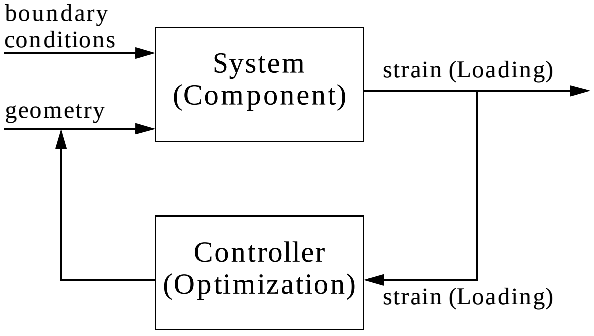

Figure 10: Optimization based on the Optimum Criteria

5.1. Sizing Optimization

The input parameters for the controller are the element-specific characteristics (Property-ID's, Shell thicknesses, cross sections, etc.) and the local element loads, the output parameters are the modification of the element-specific characteristics. Dependent on the strain level the thicknesses resp. cross sections are increased or reduced so that the strains seek homogeneous values. Basically this complies with the Fully Stressed Design rules.

5.2. Shape Optimization

The input parameters for the controller are the local node coordinates and the local node strains. The output parameters are the local modifications of the node coordinates. The controller reduces the surface curvature by applying mass at points with high strains. For low strains the surface curvature is increased by removing mass at these points.

5.3. Topology Optimization

For topology optimization the assignment of the FE data to the control parameters is not as clear as for sizing and shape optimization. The input parameters are the material distribution (Property-ID's, Element Property-ID's) and the local element strains. The redesign rule says how the required resp. allowed material has to be distributed regarding the boundary conditions and the set target mass so that the remaining mass will than be equally loaded considering all load cases. Starting from a homogeneous material distribution, mass will be compressed in areas of high energy density and diluted in areas of low energy density. To simulate the inhomogeneous material distribution for shell structures, the thickness of the shells (Property-ID of the finite element) resp. Young's modulus are well suited. For solid structures the inhomogeneous material distribution is easily simulated by using Young's modulus. Hence follows that the output parameters are new assignments of the element properties including material modifications.

6. CAOSS and MSC/NASTRAN

6.1. Optimization with MSC/NASTRAN

For many years MSC/NASTRAN has comprised powerful sizing optimization solutions which are based on mature mathematical algorithms [43]. In addition MSC/NASTRAN provides sensitivities for many areas. With the Solution 200, which is based on a mathematical approach, an efficient shape optimization module has been implemented in Version 68 [50, 52]. Hereby complex restrictions can be defined and the combination of different analysis types is possible. The proper selection and definition of the basis vectors mainly depends on the experiences and creativity of the user. The efforts the user must take is very much influenced by the used Pre- and Postprocessor.

6.2. Optimization using CAOSS and MSC/NASTRAN

With the integration of CAOSS into MSC/NASTRAN further opportunities are opened to the user. In a very easy way shape optimizations can be performed. The more flexible the surfaces are the more advantageous are the optimization results. If the optimum design has to fulfill complex restrictions which are not included in the optimum criteria, preoptimizations can be performed with CAOSS very easily providing the basis vectors for the Solution 200. With the CAOSS topology optimization capabilities the MSC/NASTRAN functionalities are considerably improved.

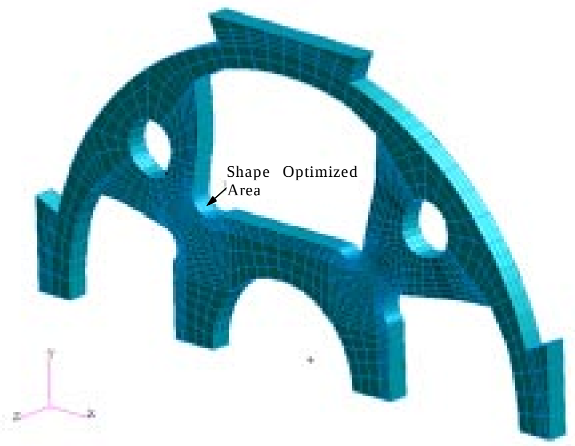

Figure 11: Optimization of a Flyweel Clutch using MSC/NASTRAN, MSC/PATRAN and CAOSS

6.3. Communication between CAOSS and MSC/NASTRAN

The CAOSS concept ensures that the MSC/NASTRAN user may continue working in his well known FE environment. He need not learn further pre- or postprocessors and may use the MSC/NASTRAN solver. Both the FE model for the FE calculation (FE bulk data deck) and the optimization model for the optimization (optimization bulk data deck) are created using e.g. MSC/PATRAN or MSC for Windows. The FE bulk data deck contains the input for the solver containing all FE-specific model data and boundary conditions. Besides the node and element information, the optimization bulk data deck contains further optimization boundary conditions for the optimization run. These might be simple node restraints or e.g. shell structures can be defined used as displacement limits during the shape optimization. This is a way to define complex geometric optimization boundary conditions very easily.

The interactive CAOSS preprocessor (CAOSS_PREP) reads the optimization bulk data deck and generates a file controlling the optimization. Additionally required resp. possible options are set for the optimization (e.g. definition of design variables, optimization type, load stress hypothesis, variation and restricted areas, node restraints, stop criteria) which are not contained in the optimization model directly.

The optimization loop can be started as soon as the FE model, the optimization model and the control file have been created. In case of an identical FE model and optimization model (e.g. for the topology optimization) all information (CAOSS control file included) can be written to a MSC/NASTRAN input file. In this case all CAOSS-specific commands begin with $CAOSS. The CAOSS optimization itself runs in batch mode. In the first iteration the original FE bulk data deck is calculated with MSC/NASTRAN. The optimization module (CAOSS_OPT) directly accesses the XDB data base of MSC/NASTRAN to read the analysis results. For the postprocessing of the results with MSC/PATRAN, a DMAP sequence creates an OUTPUT2 for each iteration. The load stress quantities of the structure are evaluated automatically by CAOSS_OPT to determine the modifications of the FE model obeying the defined optimization boundary conditions. CAOSS_OPT modifies the model in the FE bulk data deck which is the input for the solver during the next iteration. In detail these are property and element modifications for the sizing optimization and grid location modifications for the shape optimization. For the topology optimization the material cards, property ID's and elements are modified. The new results deriving from the MSC/NASTRAN analysis are checked by CAOSS_OPT for the stop criterion.

For the shape optimization an adaptive mesh correction algorithm is implemented in CAOSS which runs without an additional FE analysis. The original mesh topology is maintained thus not destroying the elaborate mesh. This mesh adaption preserves the quality of the mesh. Large element distortions only appear for extremely large shape modifications.

The interaction between the CAOSS modules on the one hand and MSC/NASTRAN on the other hand is controlled by a control file on the operating system level (UNIX-Shell). Modifications for special prob-

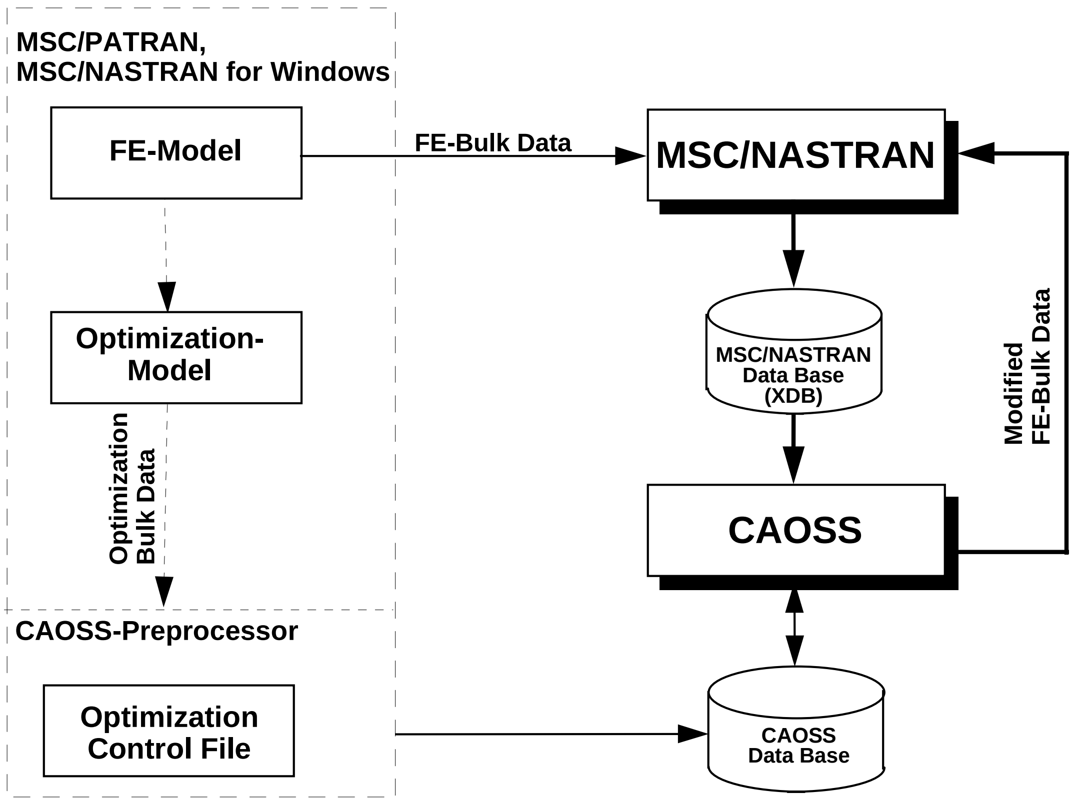

Figure 12: Schematic Flow of the Optimization using CAOSS and MSC/NASTRAN

lems or company-specific peculiarities can be performed easily.

6.4. Sample-Listing

The following shows an MSC/NASTRAN input file for topology optimization purposes. Both the Bulk Data Section from the FE model and the Bulk Data Section of the optimization model are identically for this problem. CAOSS_PREP reads the entire MSC/NASTRAN input file including the special CAOSS commands ($CAOSS ...). MSC/NASTRAN interprets these lines as comments. The shown CAOSS commands are sufficient for the topology optimization of a small problem. Totally, CAOSS provides approx. 20 different commands.

```
________________________________________________
$                    FILE MANAGEMENT SECTION (FMS)
ASSIGN output2='onehole.op2' status=unknown unit=12
$
$_______________________________________________
$                 EXECUTIVE CONTROL SECTION (ECS)
SOL SESTATICS
TIME 10000
$ DMAP to create two data bases: OUTPUT2 and XDB
INCLUDE 'XDBAOUT.DAT'
CEND
$_______________________________________________
$                      CASE CONTROL SECTION (CCS)
ECHO = NONE
DISPLACEMENT = ALL
SPCFORCE = ALL
OLOAD = ALL
FORCE = ALL
STRESS = ALL
SPC = 1
STRFIELD = ALL
SUBCASE 1
 LOAD = 1
SUBCASE 2
 LOAD = 2
SETS DEFINITIONS
SET 1 = ALL
SURFACE 1 SET 1 SYSTEM BASIC NORMAL X3
VOLUME 1 SET 1 SYSTEM BASIC
$
$_______________________________________________
$                        BULK DATA SECTION (BDS)
BEGIN BULK
PARAM,DBDICT,2
PARAM,POST,0
INCLUDE 'MODELL.BDF'
```

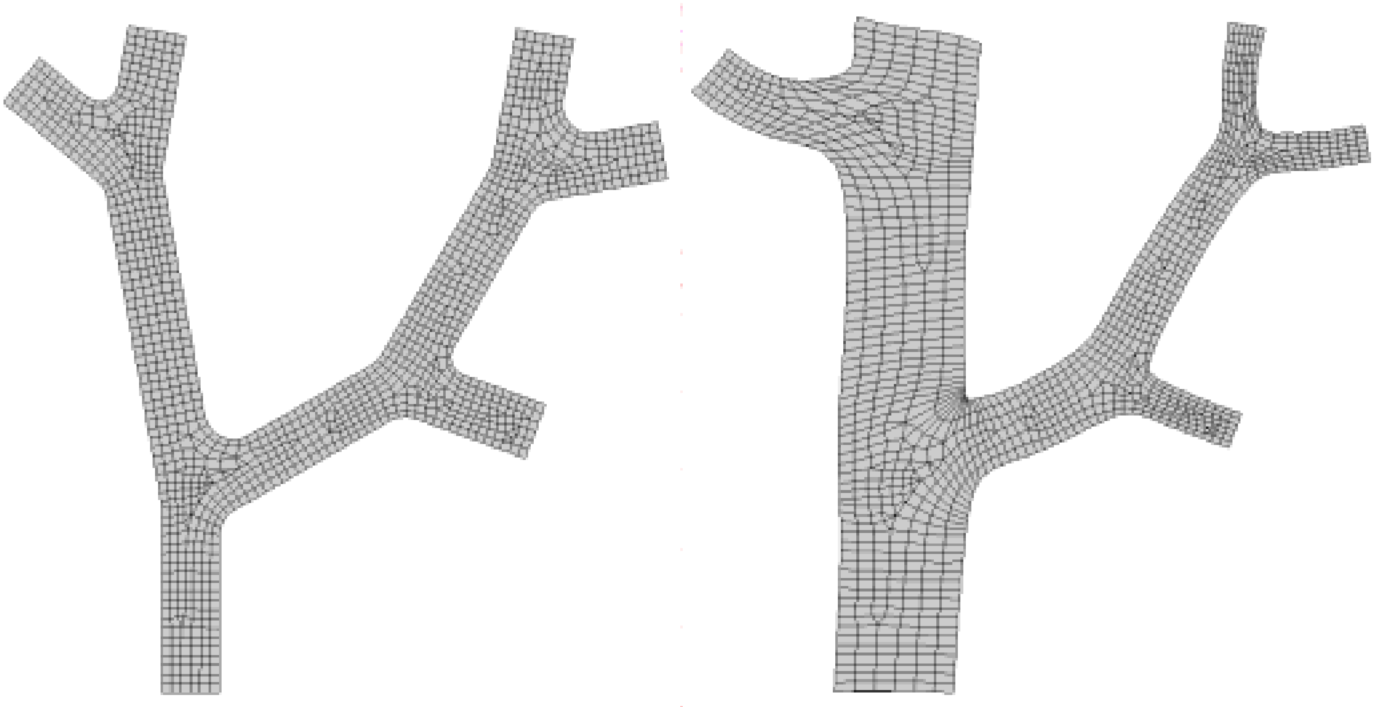

Figure 13: Comparison of the Mesh Topology before and after Optimization

```
ENDDATA
$ ______________________________________________
$                CAOSS CONTROL DATA SECTION (CCDS)
$CAOSS $                       This is a short example for
                               CAOSS_PREP input for topology
$CAOSS SEL,NODE, S, LOC_Y, 39, 100
$CAOSS SEL,NODE, R, LOC_X, -1,  50
$CAOSS SEL,ELEM, S, ND_ALL
$CAOSS GROUP,ELEM, TOPO_ELGR
$CAOSS DV_DEF, TOPO, TOPO_ELGR, 30
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$CAOSS OPT_CONT, TOPO_ELGR, SPEED_TOPO, FAST
$CAOSS SAVE                       $ create CAOSS data base
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7. Examples

In the following, typical structures for sizing, shape and topology optimizations are shown which have been all computed with CAOSS.

7.1. Sizing Optimization

The sizing optimization will exemplarily be shown optimizing a small structure. An axisymmetric bottle is modelled over 120° (a third segment). In the start design all 1.500 shell elements have the same thickness. The bottle is loaded with a vertical load distribution at the top. The bottle ground is fixed ring-like in axial direction. 1.500 design variables (one for each element) are defined altogether. The shell elements were allowed to change in discrete steps, with 40 different thicknesses defined. The shell thicknesses of the bottle top elements were coupled. Further optimization boundary conditions were defined with minimum and maximum shell thicknesses for certain element groups. The optimized model, which needed 3 iterations to be computed, shows an extensive area with a shell thickness distribution according to a homogeneous strain distribution. For the visualization, the Property-ID’s of the shell thicknesses are shown as solids. The coupling restrictions of the shell thicknesses of the bottle top elements is apparent. Obviously the shell thicknesses of elements of certain element groups meet the maximum or minimum limit, e.g. at the transition from the bottle ground to the bulb or from the bulb to the neck.

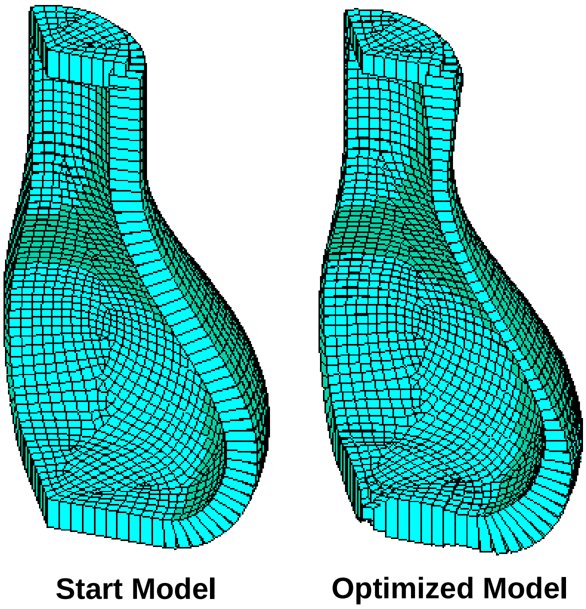


Figure 14: FE-Model of the Bottle

This example was calculated within approx. 5 min on a PC. It shows that in particular cases sizing optimization based on the optimum criteria can be used efficiently. The problem can be calculated with the Solution 200 of MSC/NASTRAN, too. If buckling and stability criteria or dynamical characteristics have to be considered for the optimization, CAOSS can quickly compute a preoptimized model, which provides the start design for the following optimization with Solution 200.

7.2. Shape Optimization

Example 1 - Primary Flywheel

This is the shape optimization of a complete assembly of a primary side of a damped flywheel clutch by the car supplier LuK, Bühl (Germany) [22, 31]. The entire 3D-model has approx. 30.000 Degrees of Freedom (DOF) and considers both the geometrical nonlinear structural behaviour and several load cases.

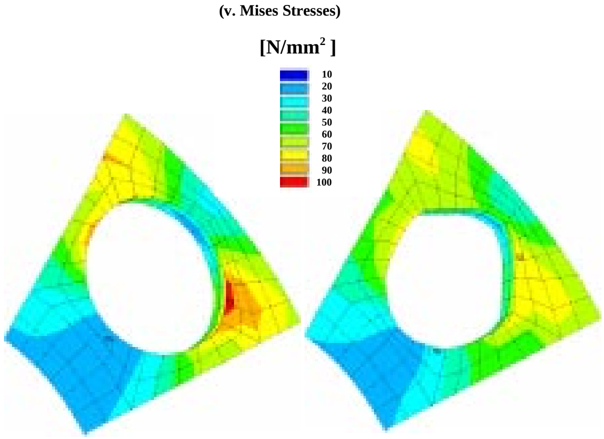


Figure 15: Comparison of the Stresses in the Ventilation Hole

The assembly is welded from three parts. The primary side and the cover are deep drawing parts, the gear rim is machined. The primary flywheel is loaded alternately. The rotation of the component leads to centrifugal forces due to both the mass of the assembly itself and secondary parts within the assembly. Furthermore a hydrostatic pressure is caused through the fluid between the primary side and the cover. In addition these forces are overlaid by an alternating torque. As the maximum alternating load appears in the ventilation holes, their form is to be optimized. For functional reasons a bore hole as large as possible is desired. With regard to the production and function the following restrictions have to be obeyed:

- The primary side must be produced as a deep drawing part.
- The ventilation bore holes must have the same geometry.
- The outer contour of the primary side must be maintained.

The objective of this optimization was the modification of the bore hole shape to minimize the alternating strains caused by the two load cases. During the optimization, all boundaries of the ventilation holes were allowed to move. The demand for a symmetric geometry of the ventilation hole was

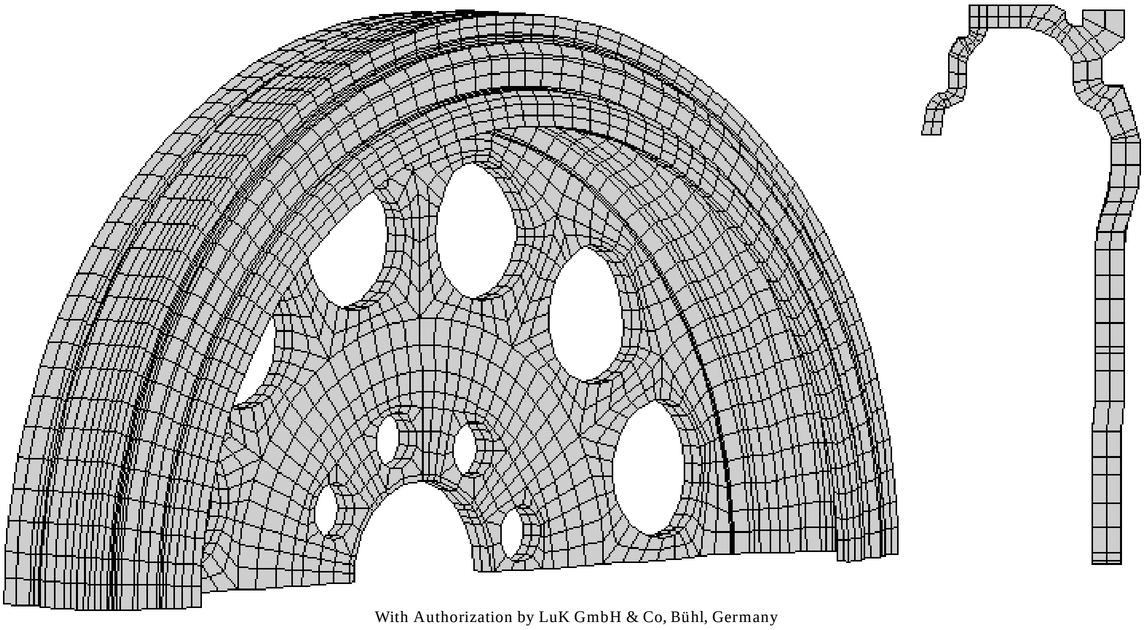


Figure 16: FE-Model of the Flywheel

easily met bycoupling the related nodes at the resprective holes. Furthermore the production requirement „constant component thickness“ could be fulfilled by coupling the node displacement over the thickness.

In 4 iterations the maximum stress was reduced by approx. 22%. Although the analysis along the rim of the bore hole shows the reduction of the stresses of the original structure, an entire homogenization has not yet been achieved. Especially the area close to the axle is lowloaded

.

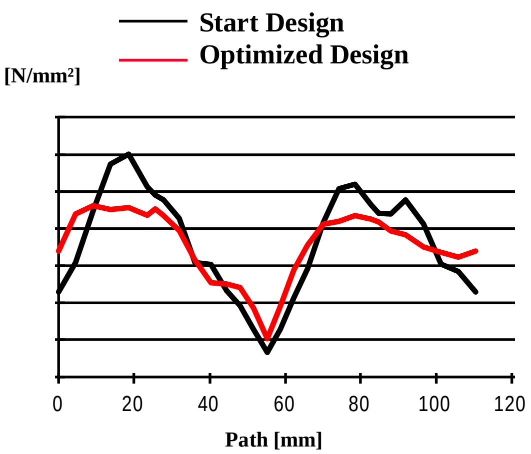


Figure 17: Stress Curves of the Bore before and after the Optimization

Example 2 - Rocker Arm

CAOSS was used for the shape optimization of an exhaust valve rocker arm by MTU Friedrichshafen (Germany), manufacturer of high performance diesel engines [17]. Take advantage of the symmetries, the 3D model has approx. 15.000 DOF. Main loads result from the superposition of a bending moment, caused by tappet forces, and a force fit.

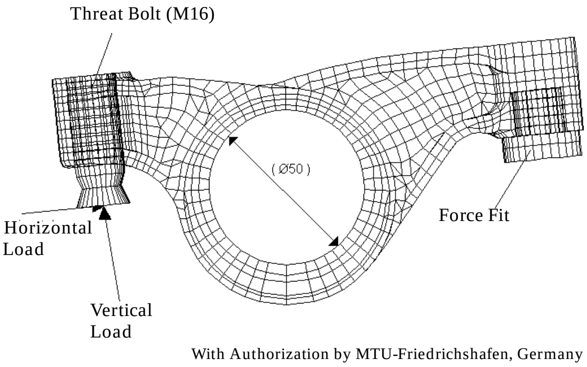


Figure 18: FE-Model of the Rocker Arm

For the shape optimization of the rocker arm the following production and functional restrictions had to be obeyed:

- For the holding fixture, two planes parallel to each other must lie at the upper belts. Their position is optional.

- To further ensure forging these components, no relievings in the outer and inner contours are allowed.
- For an easy machining of the face plane of the force fit, a run-out and a relieving for the cutter have to be ensured.
- The inner contour of the bores and the adjacent planes must not be changed

.

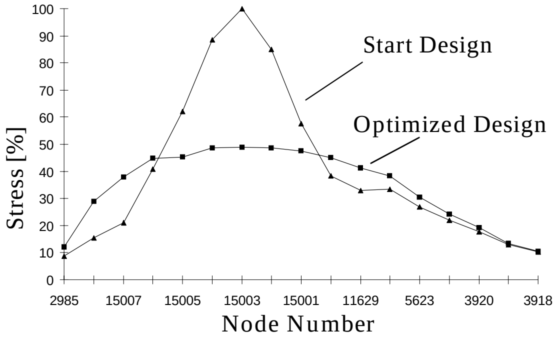


Figure 19: Stress Curves at the critical Location before and after Optimization

The objective was to reduce the local strain peaks meeting the geometric boundary conditions. The stiffness of the component should be maintained or even increased. Coupling the certain node displacements, the requirements „parallelity“ and „forgeability“ were fulfilled. With constrained areas (areas where the component must not move into) the production restrictions were fulfilled, too

After 5 iterations the maximum strain of the component was reduced approx. by 35%, increasing the stiffness approx. by 10%. The reference stress in the critical area clearly shows a more homogeneous curve.

Example 3 - Brake Carrier

The brake carrier of a car brake by the car supplier Automotive Lucas, Koblenz (Germany) [32] has more than 30.000 DOF, too. The maximum strain could be reduced about 30% due to shape modifications at the critical location by only very few iterations.

Example 4 - Contact Surfaces

In the next example the pressure-loaded contact surface between cylinders with a symmetry axis in common is shape optimized.

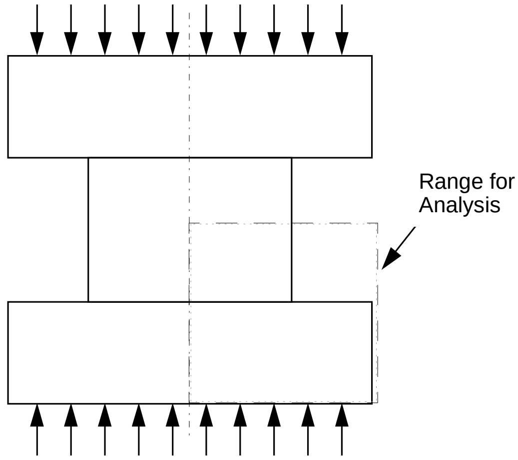


Figure 21: FE-Model (Three Cylinders)

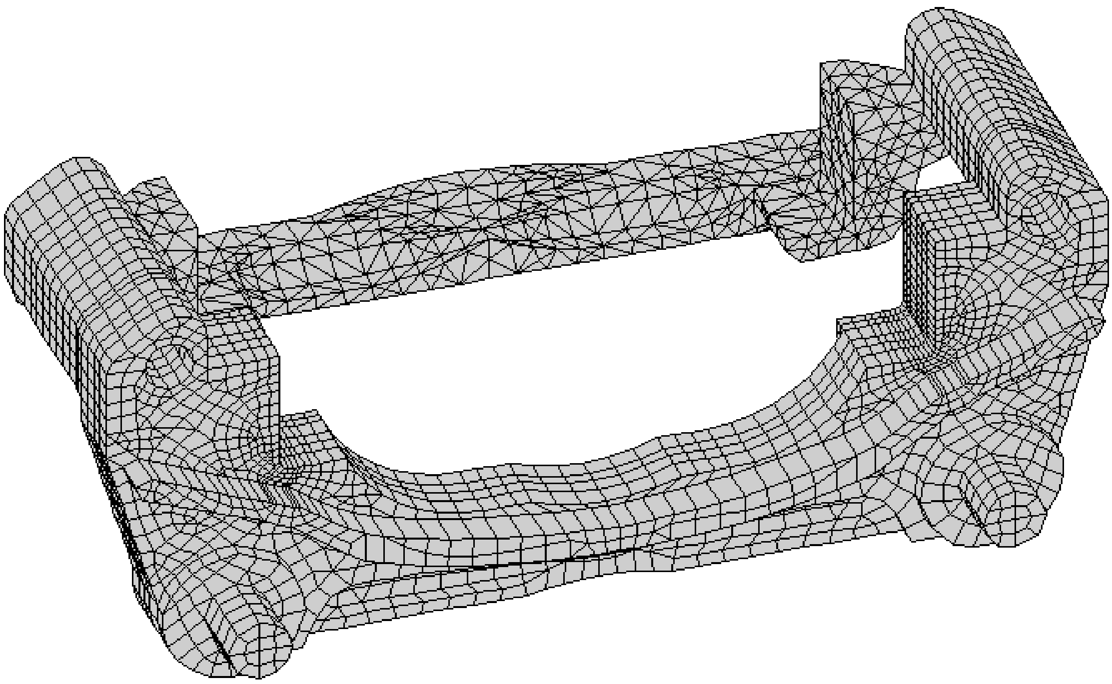


Figure 20: FE-Model of the Brake Carrier

The cylinder with the smaller diameter is compressed between the larger cylinders. In the initial state the different diameters lead to a stress peak at the edge of the common contact surface. The nominal stress in the contact surface is 100 MPa. Due to the symmetry, only a quarter of the cross section has to be modelled. The compression curve in the contact surface increases with rising radii. Singularity problems were encountered at the outer edge of the cylinder so that the compression level cannot be determined exactly (approx. 240 MPa

To analyze compression-loaded contact surfaces, special algorithms have been developed and implemented into CAOSS. Material increase at locations with high contact stresses would not lead to proper results.).

In the following two possible solutions are discussed.

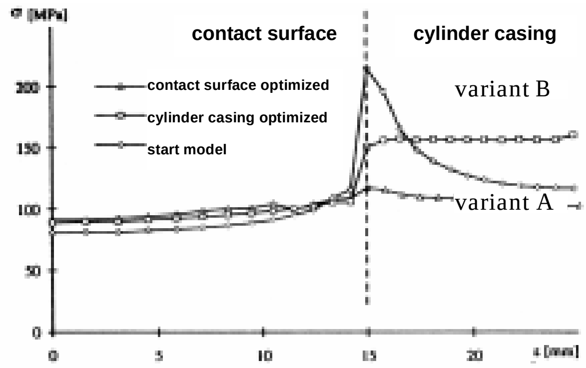


Figure 22: Compression Curve in the Contact Surface

The first possibility is to make the end face of the smaller cylinder convex (Variant A). This corresponds to the principle of the outer contouring of rolling elements. Secondly an appropriate relieving can make the smaller cylinder pliable above the edge (Variant B). This corresponds to the principle of the inner contouring of rolling elements.

The compression curve in the contact surface shows significant strain reductions for both possibilities. The modification of the outer contour corresponds to the solution, known from the bearing technology [16]. The contour of Variant A considerably depends on Young's Modulus of the applied materials. To visualize the convexity, Young's Modulus of the material is set to only 2.100 MPa. For the inner-contouring the stress singularity disappears for an edge angle of 80°.

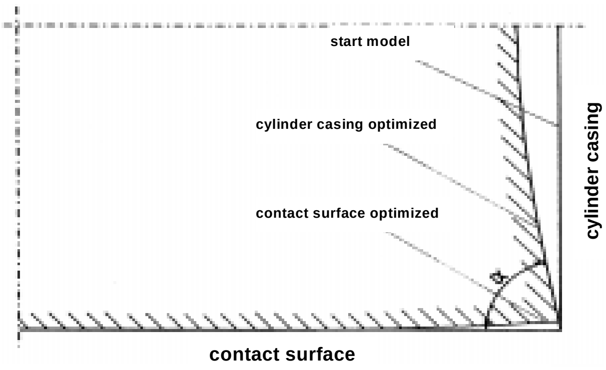
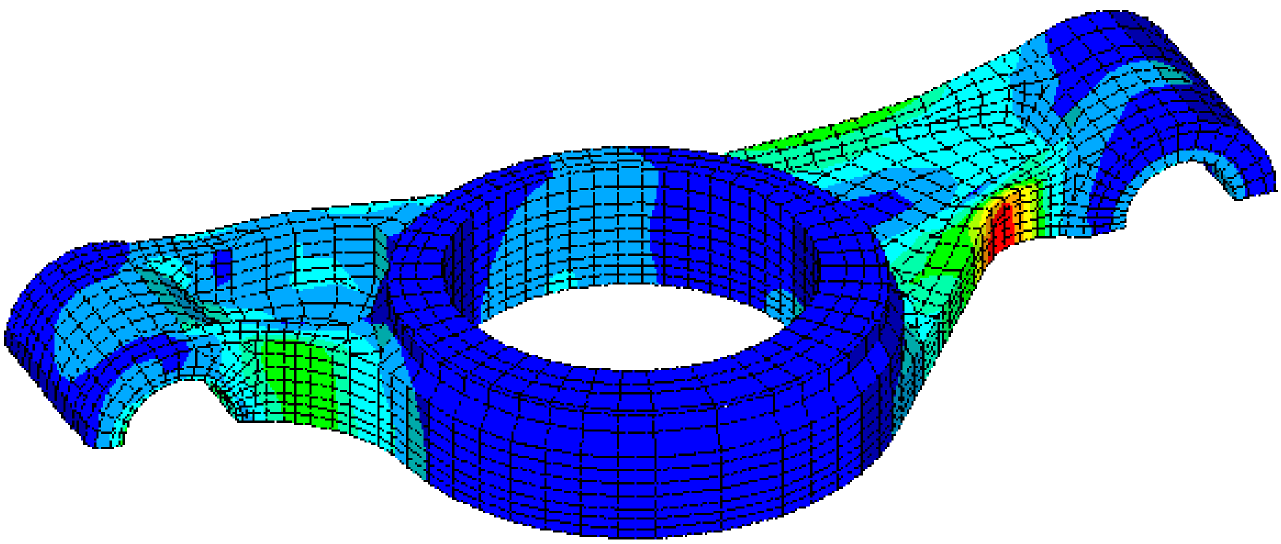


Figure 23: Contour of both Possibilities



With Authorization by MTU-Friedrichshafen, Germany

Figure 24: Stress Plot of the Rocker Arm

Example 1 - Railway Bridge

The applied loads and boundary conditions of the first example correspond to a simple railway bridge. A rectangular solid with the left surface fixed and two translational DOF at the right is used as the start design. This arrangement is required from a technical point of view, to compensate length expansions of the bridge. A further demand is to define a lane at the bottom of the solid, achieved by a thin horizontal rated area (area which will be formed as solid material in every case). A constant surface load is applied to the top side. The shown topology results from a target mass of 42% of the initial volume (i.e. 42% of the variation area are formed as solid material at the end of the optimization) and is formed as one solid bend with attached tensile struts. The attachment of the struts to the bend and the lane show slightly the formation of radii. The high symmetry of the resulting structure, even with the unsymmetric boundary conditions, is of special interest.

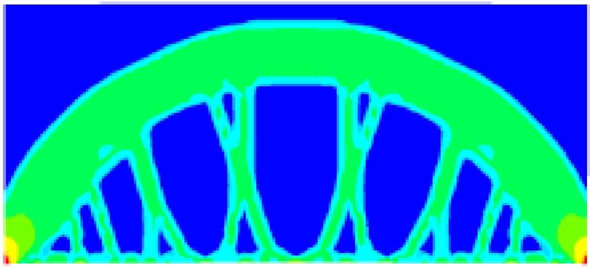


Figure 25: Topology Optimized Model

Example 2 - Car Body

The second example is a car body which undergoes a topology optimization with regard to six different load cases [24]. To show explicitly that the optimization of 3D structures causes no further efforts, the car body is designed with approx. 5.000 3D solid elements. Hence follow 30.000 DOF for the FE calculation and 5.000 design variables for the optimization. For the car front, forces in several directions were assumed, resulting e.g. from crash situations. At the rear end only an axial force is assumed. Element rows are „frozen“ at the front and rear end. The shown structure comes up with a target mass of 43%. Decisive solution parameters are bending moments, caused by the lateral effective loads. Slender frameworks were formed at the front end, due to the immediate affect of the loads applied from various directions.

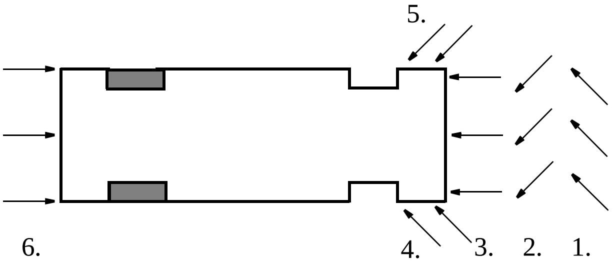


Figure 26: Geometry and Loads (6 Load Cases)

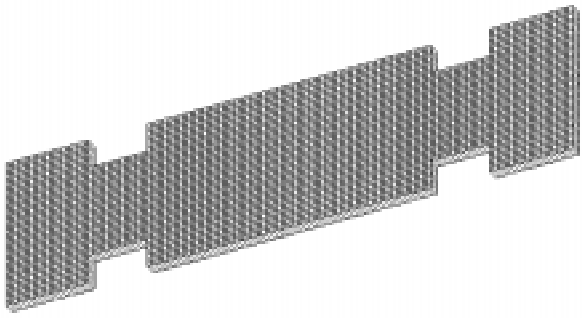


Figure 27: FE-Model of the Start Design

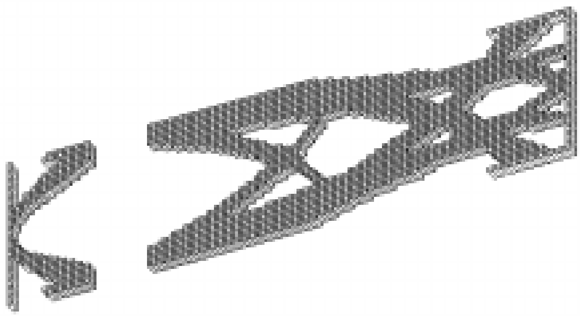


Figure 28: Topology Optimized Model

Example 3 - Star

The initial design consists of a ring, stiffened with 120° symmetric struts. The model is designed with 7.000 3D solid elements and has approx. 45.000 DOF. In the center of the struts, the star is loaded with an axial force. The model is fixed axially on the ring between the struts. For the struts the load results mainly in bending moments and for the ring in a superposition of bending and torsional moments. The optimization with a defined target mass of 50% leads to the shown structure.

Remarks on the Topology Optimization

The computer simulation results are very schematic and simplified, but show that the results often lead to unconventional or unexpected designs, which normally are not taken into consideration. This way, the topology optimization supports the engineers creativity in the draft and design process.

For low target masses, mostly lean structures are appear. If these structures are interpreted as frameworks with tensile and compression struts no stability conditions (bending, buckling) are considered [29].

For industrial applications, the structures often are so complicated that they require equations with 20.000 to 200.000 DOF and even more which have to be solved. For this iterative solvers would lead to lower computation efforts of which the preconditioning is derived from each previous iteration [25].

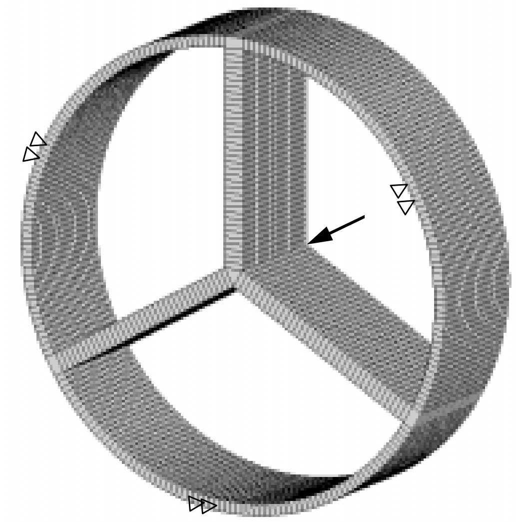


Figure 29: FE-Model with Boundary Conditions

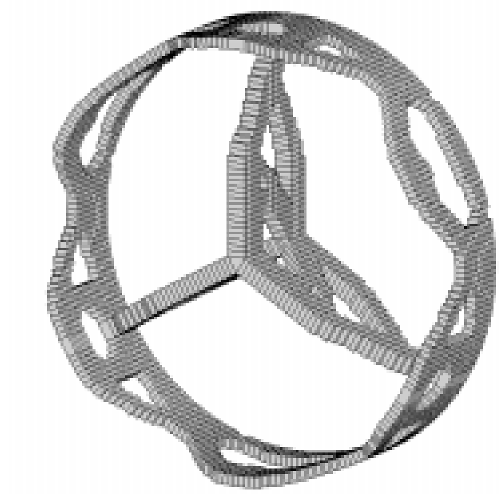


Figure 30: Topology Optimized Model

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