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A note on the nonlinear modeling of piezoelectric rods

Piezoceramic materials are usually calculated using linear constitutive equations if the exciting electric field is small. If the structure is excited near a resonance frequency the assumption of a linear material becomes invalid. Therefore nonlinear constitutive equations have to be used if the super harmonics in the measured velocity signals should be modeled. In this paper nonlinear constitutive equations are derived from the electric enthalpy density to describe the longitudinal oscillations of a piezoceramic rod. The kinetic energy considers the inertia effects in transversal direction so the model is not limited to the slender rod theory. With Hamilton's principle a variational principle is derived which is approximated using a Rayleigh Ritz ansatz with the first eigenfunction of the linearized system. The resulting nonlinear ODE is solved by application of the harmonic balance method. The amplitudes of displacement of the fundamental, the second and the third super harmonic oscillations are solved numerically. In a first step the results of the theoretical model are compared with measurements. The material parameters of the nonlinear constitutive equations are calculated using a parametric identification method. As a result it is shown that the theoretical model can describe the Duffing type nonlinearities found in measurements.

1. Theoretical Model

The model of the piezoceramic rod is shown in figure 1. A coordinate system is introduced such that the 3-axis is aligned with the poling direction. The left and right end of the rod are stress-free (mechanical boundary conditions) and the rod is excited by a harmonic electric voltage $\varphi(t)$ (electrical boundary condition). To derive a variational principle for the piezoceramic, it is convenient to use Hamilton's principle for dielectric media

$$\delta \int_{t_0}^{t_1} L\,dt + \int_{t_0}^{t_1} \delta W\,dt = 0, \quad L = T - H \tag{1}$$

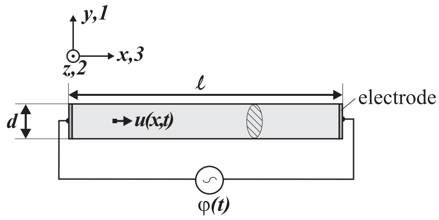

Figure 1: Model of the piezoceramic rod

in which L is a modified Lagrangefunction and δW the virtual work [3]. From measurements it is known, that the longitudinal oscillations of a piezoelectric rod show nonlinear behavior if the rod is excited near a resonance frequency even at low electric fields. To model the observed jump phenomenon and the superharmonics an uniaxial state of stresses is assumed. Furthermore the electric displacement in transversal direction will be neglected. Therefore the electric enthalphy can be written as

$$H = \int_V \frac{1}{4}c_2 S_3^4 + (\frac{1}{3}c_1 - \frac{1}{3}e_2 E_3)S_3^3 + (\frac{1}{2}\overline{c}_{33} - \frac{1}{2}e_1 E_3)S_3^2 - \overline{e}_{33}E_3 S_3 - \frac{1}{2}\overline{\epsilon}_{33}E_3^2 \mathrm{dV}. \tag{2}$$

In this equation the independent variables are the electric field E_3 and the strain S_3, where the index $()_3$ denotes the polarization axis of the rod. Furthermore the following abbreviations are used

$$\overline{c}_{33} = c_{33}^E - \frac{2(c_{13}^E)^2}{c_{11}^E + c_{12}^E}, \quad \overline{e}_{33} = e_{33} - \frac{2c_{13}^E e_{31}}{c_{11}^E + c_{12}^E}, \quad \overline{\epsilon}_{33} = \epsilon_{33}^S + \frac{2e_{31}^2}{c_{11}^E + c_{12}^E}, \tag{3}$$

in which c_{ij} denotes the stiffness tensor components, e_{ij} the piezoelectric tensor components and ϵ_{ij} the permittivity tensor components, in case the Voigt notation is used. By assuming a linear velocity in transversal direction, the kinetic energy can be written as

$$T = \int_V \frac{1}{2}\rho\dot{u}^2 + \frac{1}{2}\rho[(y\dot{S}_1)^2 + (z\dot{S}_2)^2]\mathrm{dV}. \tag{4}$$

A detailed derivation of the above potential functions can be found in [1] and [2]. Nevertheless the nonlinear

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constitutive equations, which are based on equation (2) are given by

$$T_3 = \frac{\partial H}{\partial S_3} = c_2 S_3^3 + (c_1 - e_2 E_3) S_3^2 + (\overline{c}_{33} - e_1 E_3) S_3 - \overline{e}_{33} E_3 \tag{5}$$

$$D_3 = -\frac{\partial H}{\partial E_3} = \frac{1}{3} e_2 S_3^3 + \frac{1}{2} e_1 S_3^2 + \overline{e}_{33} S_3 + \overline{\epsilon}_{33} E_3, \tag{6}$$

with the stress T_3 and the electric displacement D_3 in poling direction.

Introducing the potential functions into the variational principle (1) and using GAUSS law for dielectric media $\mathrm{div} D = 0$ and the field equation $E = -\mathrm{grad}\,\varphi$, an equation is obtained, which depends only on the displacement $u(x,t)$ and the electric potential $\varphi(t)$ and various derivatives of $u(x,t)$ and $\varphi(t)$ with respect to the time t and the coordinate x. The variational principle can not be solved analytically. Therefore a first order RAYLEIGH-RITZ ansatz is introduced by using the first eigenfunction of the linearized system

$$u(x,t) = U(x)p(t). \tag{7}$$

Obtained is a nonlinear ordinary differential equation of the form

$$\begin{aligned} C_1\,\ddot{p}(t) + C_2\,\ddot{p}(t)p(t) + C_3\,\ddot{p}(t)p(t)^2 + C_4\,\ddot{p}(t)p(t)^3 + C_5\,\ddot{p}(t)p(t)^4 + C_6\,\dot{p}(t) + C_7\,\dot{p}(t)^2 p(t) &\; + \\ C_8\,\dot{p}(t)^2 p(t)^2 + C_9\,\dot{p}(t)^2 p(t)^3 + C_{10}\,\dot{p}(t)^2 + C_{11}\,p(t)^5 + C_{12}\,p(t)^4 + C_{13}\,p(t)^3 + C_{14}\,p(t)^2 &\; + \\ C_{15}\,p(t)^2\varphi(t) + \underbrace{C_{16}}_{=0}\,p(t)^2\ddot{\varphi}(t) + C_{17}\,p(t) + C_{18}\,p(t)\varphi(t) + \underbrace{C_{19}}_{=0}\,p(t)\ddot{\varphi}(t) + C_{20}\,\ddot{\varphi}(t) &= \varphi(t). \end{aligned} \tag{8}$$

It can be seen, that a parameter excitation by the voltage $\varphi(t)$ occurs. The harmonic balance method is used to solve the nonlinear ODE. The fundamental, the first and second higher harmonics are taken into account. The resulting equations are solved numerically for the amplitudes of the displacement functions.

2. Results

The comparison of the fundamental harmonic oscillation with a measurement for a low voltage excitation is shown in figure 2. The measurements are carried out at a piezoceramic rod (diameter 25mm, length 20mm, material PIC141 PI-Ceramic, Germany). The material parameters used for the simulation are determined by using the NELDER-MEAD simplex algorithm where the objective function is the L_2-norm of the difference between measurement and simulation data. As one can see it is possible to describe the DUFFING-type nonlinearities with the presented model. The figure shows the sweep up mode, which means that the frequency is increased, while the amplitudes of velocity are measured with a gain-phase analyzer. It is well known, that the frequency at which the jump occurs depends on the sweep mode. Furthermore it can be seen, that the amplitude function inclines to the left, which is the so called softening effect. The material parameters used for the above simulation are in SI-units: $c_1 = 1.2e-4, c_2 = -9e14, e_1 = 1e-4, e_2 = -1e8$. The main parameters, which are responsible for the nonlinear behavior are the piezoelectric parameter e_2 and the stiffness parameter c_2. Both values lead the curve incline to left in case of negative values. A detailed discussion of the parameter identification will be published soon. Future work will focus on the improvement of the parameter identification and the application of the model to a piezoelectric transformer [1].

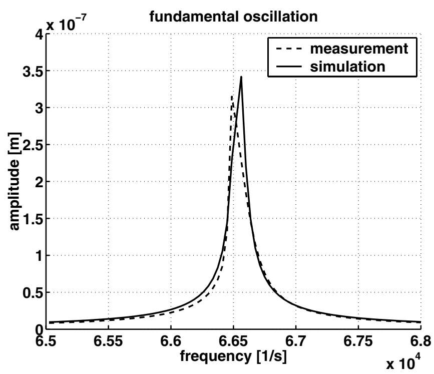

Figure 2: Comparison of the fundamental amplitudes (simulation / measurement)

3. References

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