PAMM · Proc. Appl. Math. Mech. **6**, 433–434 (2006) / **DOI** 10.1002/pamm.200610197

Proposing a Pretwisted Bimorph Actuator

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Though only small strains are available in piezoceramic materials, bending actuators provide reasonable deflections. Due to beam kinematics bending actuators usually are slender beams having flat cross-sections. This feature allows for maximum deflection in one direction. However, the axis orthogonal to it usually is not actuated. Instead of combining two straight bending actuators to overcome this problem we propose a bending actuator which is pretwisted. Controlling the pretwisted actuator segment-wise provides bending in several independent directions as well.

Investigated is a pretwisted bimorph, similar to a helicoid. The active elements along the beam axis are subdivided and controlled separately, hence allowing independent control of the curvature of each section. Herein we give a first characterization of the pretwisted bimorph actuator.

1 Modeling the Actuator

The actuator is a slender beam of length L, made of two piezoceramic layers, driven in bender configuration - a so-called bimorph. Its rectangular cross-section is rotated by a pretwist angle $\varphi(z) = \Phi z$ similar to a helicoid. At the end of the bimorph the pretwist angle is $\varphi(L) = \varphi_e$ yielding the pretwist angle rate $\Phi = \varphi_e/L$. As shown in figure 1 the bimorph is subdivided into N_{seg} active elements of equal length $\Delta L = L/N_{seg}$.

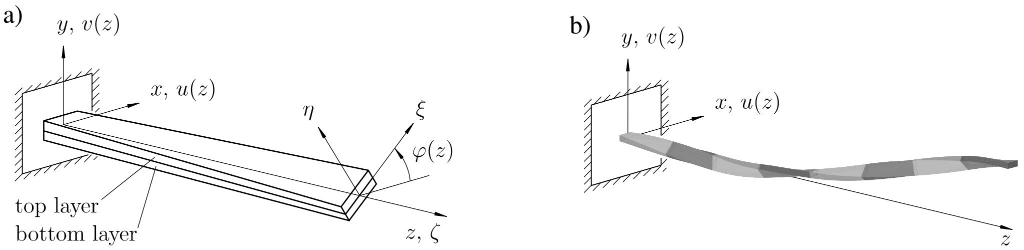

Fig. 1 clamped-free pretwisted bimorph actuator: a) global x, y, z-coordinate system and local ξ, η, ζ-coordinate system, coinciding with the principal axes of each cross-section b) actuator consisting of $N_{seg} = 8$ independent active elements, deformation for an arbitrary electric load is shown

As the bimorph is pretwisted an appropriate beam model is needed. Beams being pretwisted have attracted much interest in the past because they are well in use yet complex to model. In [1] ROSEN gives a detailed survey of approaches and results on this topic. We apply the rather simple Euler-Bernoulli beam theory and linear piezoelectricity [2] to allow for a first straightforward understanding of the bimorph's performance. Because of this assumption, Φ is confined to small values. However restrictions on $\varphi(z)$ are not needed.

For simplification we assume herein that an electric load applied to the i-th element bends the neutral axis only within the η-ζ-plane and gives a curvature K_i. I.e. K_i abbreviates the electric load, geometric parameters and material parameters. Thus the deflection of the beam including all active elements yields

$$\begin{bmatrix} u(z) \\ v(z) \end{bmatrix} = \int_0^z \int_0^{z^*} \sum_{i=1}^{N_{seg}} K_i \left[\sigma(\overline{z} - \{i-1\}\Delta L) - \sigma(\overline{z} - i\Delta L)\right] \begin{bmatrix} \sin \Phi\overline{z} \\ \cos \Phi\overline{z} \end{bmatrix} d\overline{z}\, dz^* \tag{1}$$

where $\sigma(z)$ is the *Heaviside*-function. Hence we restrict discussion on the deflection u_L and v_L at the end of the beam, since this leads to the work space of the actuator. Using equation (1) we get

$$\boldsymbol{u}_L = \begin{bmatrix} u_L \\ v_L \end{bmatrix} = L^2\, \boldsymbol{G}\, [K_1, \ldots, K_{N_{seg}}] \tag{2}$$

where $\boldsymbol{G}$ denotes a $2 \times N_{seg}$ transfer matrix. The matrix depends only on the geometric parameters φ_e and N_{seg} and is characteristic for a certain pretwisted bimorph.

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2 Work Space and Reduced Set of Inputs

Due to constraints on the electric field and consequently on the curvature, assuming $-\hat{K} \leq K_i \leq \hat{K}$, following equation (2) the deflection $\boldsymbol{u}_L$ cannot exceed a certain hull ∂P. This gives the work space P, depending only on $\boldsymbol{G}$, L and $\hat{K}$. Let us consider the curvatures K_i of each element the inputs of the actuator. In the following we assume an actuator having only N_{red} independent inputs. Now we attempt to find a pattern which connects the independent inputs to all N_{seg} inputs while maximizing the hull ∂P. Basically it means that we optimize a pattern

$$\boldsymbol{r} = [r_1,\, r_2,\, \ldots,\, r_{N_{seg}}] \tag{3}$$

where $r_j = \{1, 2, \ldots, N_{red}\}$ denotes the input to be loaded on the j-th element, leading to a reduced transfer matrix

$$\boldsymbol{G}_{red} = \boldsymbol{R}(\boldsymbol{r})\,\boldsymbol{G}. \tag{4}$$

The $N_{seg} \times N_{red}$ index matrix $\boldsymbol{R}$ decodes $\boldsymbol{r}$ accordingly. To formulate a scalar objective function we compute the area of the work space $A_{hull} = f(P)$. This approach of course neglects the shape of the work space. Hence the discrete optimization problem can be formulated as

$$\begin{aligned} A_{hull} &= f(P\{L, \boldsymbol{r}, \boldsymbol{G}, \hat{K}\}) = \max!, \\ \boldsymbol{r} &= [r_1, r_2, \ldots, r_{N_{seg}}]. \end{aligned} \tag{5}$$

Herein we did not focus on a sophisticated optimization algorithm instead via combinatorics all patterns $\boldsymbol{r}$ were determined and tested. This approach is a priori limited to small numbers of N_{seg} and N_{red}, though the algorithm reliably identifies the pattern for the global maximum of A_{hull}. Figure 2 illustrates some results of the optimization. Apparently an actuator of only 2 or 3 independent inputs allows for a hull close to that of an actuator having $N_{seg} = 11$ independent inputs, provided a well-chosen pattern and pretwist angle is selected.

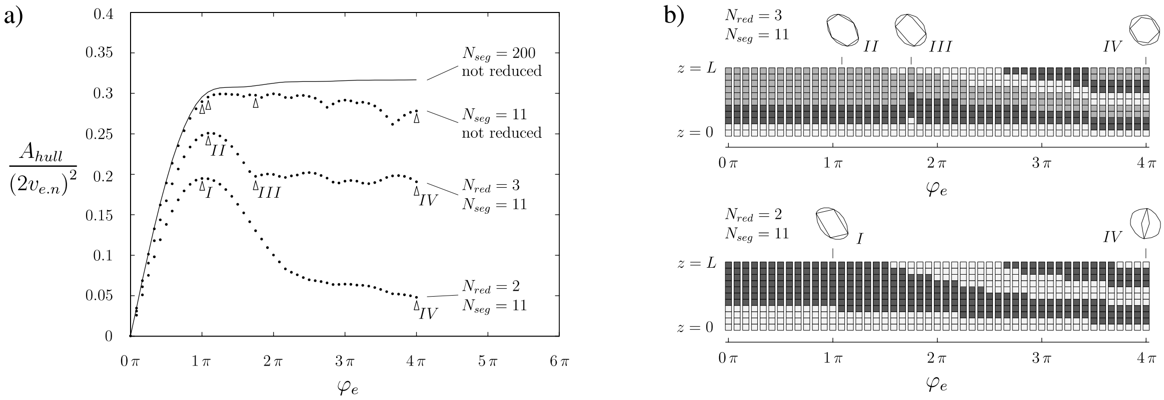

Fig. 2 Optimizing the pattern $\boldsymbol{r}$ for maximum A_{hull}; actuators with full sets of independent inputs and actuators with reduced number of inputs are compared: a) A_{hull} vs. φ_e and b) pattern $\boldsymbol{r}$ vs. φ_e are shown. In addition designated hulls are plotted, comparing ∂P for full set and reduced set of inputs. The value $v_{e.n}$ denotes the maximum deflection v_L of a straight actuator.

3 Conclusion

Herein the concept of a pretwisted bimorph actuator was presented and a first model was developed. The work space of the actuator was introduced and an optimization problem was solved to reduce the number of independent inputs while maximizing the area of the work space. The conclusion can be drawn that an optimized pattern allows for a sound work space even for very few inputs compared to the number of elements.

Since we focused on a first insight of a pretwisted bimorph actuator the models and approaches applied were kept simple. For many aspects given herein more refined approaches exist, were published by several groups and ready to adopt. From our point of view the concept proposed is worth the effort to combine and apply these approaches. In particular kinematics of pretwisted beams including dynamics, modeling of curved piezoceramics and control of the actuator shall be implemented.

References

[1] A. Rosen, Structural and dynamic behaviour of pretwisted rods and beams, Appl. Mech. Rev. **44**, 483-515 (1991)
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