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Computation of failure probabilities via local approximations

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For failure probability estimates of large structural systems, the numerical expensive evaluations of the limit state function have to be replaced by suitable approximations. Most of the methods proposed in the literature so far construct global approximations of the failure hypersurface. The global approximation of the failure hypersurface does not correspond to the local character of the most likely failure, which is often concentrated in one or several regions in the design space, and may therefore introduce a high approximation error for the probability of failure. Moreover, it is noted that global approximations are often constructed for parameter spaces that ignore constraints imposed by the physical nature of the problem.

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1 Introduction

In this study, a robust and efficient local approximation scheme of the limit state function for the estimation of failure probabilities is proposed. The major advantages of the proposed local approximation are that the limit state function is evaluated close to the region of most likely failure only and that it is not necessary to compute zeros of the limit state function. Moreover, an interaction between the importance sampling scheme and the limit state approximation scheme becomes possible.

The algorithm consists of four basic parts: (i) an initial optimization algorithm that directs the simulation to the region(s) of most likely failure; (ii) a local approximation algorithm for the limit state function that is based on the data points obtained in step 1; (iii) an importance sampling algorithm that makes use of the results obtained in step 1 and the local interpolation of the limit state function of step 2; (iv) an adaptation procedure in order to improve the importance sampling scheme and/or the quality of the interpolation result.

In the next sections, suitable implementations for these steps are introduced and discussed. After that, several examples are given in order to illustrate the efficiency of the proposed method.

2 Optimization algorithm

The problem addressed in this paper is the approximation of the probability of failure of a structure, which may be defined as

$$P_F = \int_F p(\boldsymbol{\theta})\,\mathrm{d}\,\boldsymbol{\theta}, \tag{1}$$

where $p(\boldsymbol{\theta})$ is the joint probability density function of the vector of random parameters $\boldsymbol{\theta}$, $F = \{\boldsymbol{\theta} | g(\boldsymbol{\theta}) \leq 0\}$ denotes the failure domain and $g(\boldsymbol{\theta})$ the limit state function. In engineering practice, the limit state function may be given in implicit form only, e.g. as a result of a stability analysis, or constructed as the union of several simple limit state functions, each representing a structural failure mode. For large finite element systems, the evaluation of the limit state function may be very time consuming and should be reduced to a minimum.

The probability of failure may be estimated from Monte Carlo simulation (MCS). Usually, the major contributions to the integral in equation (1) stem from regions around the design points, which are the points of maximum likelihood on the failure hypersurface $g(\boldsymbol{\theta}) = 0$. Efficient simulation techniques try to concentrate the samples on those regions. The computation of the design points is an optimization problem, that can be solved by nonlinear programming techniques or evolutionary methods.

3 Local approximation based on the moving least squares (MLS) method

The MLS approximation scheme has been frequently employed in conjunction with meshless methods. The MLS approximation has a reasonably high accuracy and can be easily extended to problems of arbitrary large dimension. Consider a neighborhood $\Omega_{\bar{\boldsymbol{\theta}}}$ of the sample $\bar{\boldsymbol{\theta}}$, for which the MLS approximation $\bar{g}(\boldsymbol{\theta})$ of the limit state function should be calculated. For $\boldsymbol{\theta} \in \Omega_{\bar{\boldsymbol{\theta}}}$, the MLS approximant is defined by $\bar{g}(\boldsymbol{\theta}) = \boldsymbol{p}^T(\boldsymbol{\theta})\boldsymbol{a}(\bar{\boldsymbol{\theta}})$, where $\boldsymbol{p}(\boldsymbol{\theta})$ is a complete monomial basis of order m and $\boldsymbol{a}(\bar{\boldsymbol{\theta}})$ contains the m interpolation coefficients, which are determined from the n given data points θ_i, $i = 1, \ldots, n$, by minimizing a weighted discrete L_2 norm.

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The weight functions have compact support. The support size r is an important geometrical parameter that determines the local character of the approximation. If r is too small, then the regularity of the minimization problem may not be assured. On the other hand, if r is too large, the local character of the interpolation scheme is violated. The quantities r may vary according to whether or not $\bar{\boldsymbol{\theta}}$ is close to the failure hypersurface. As long as there is a common scale factor, r can be used to introduce a quality measure of the approximation with respect to a given sample. From the fact that the local interpolation should be well defined for all sample points, one can obtain a minimal value for the scale factor. By comparing the scale factors of different samples, the reliability of the estimates of the probability of failure with respect to the underlying approximation can be judged.

4 Adaptivity

The adaptive procedure proposed here is based on a ranking of samples according to their quality with respect to the estimation of the failure probability. An important sample $\boldsymbol{\theta}_i$, for which the value of the limit state function can be predicted with relatively high uncertainty only is characterized by the following three properties: (i) the sample is close to the failure hypersurface, i.e. $|\bar{g}(\boldsymbol{\theta}_i)|$ is small; (ii) the probability $p(\boldsymbol{\theta}_i)$ of the sample is relatively large; (iii) the interpolation measure r_i is relatively large.

For the samples which introduce the highest amount of uncertainty, the exact value of the limit state function is calculated. These samples can be added to the base values of the interpolation algorithm. Moreover, the samples in the failure domain, but closest to the failure hypersurface with highest probability are used in the next iteration step as base samples for the importance sampling scheme.

5 Example

The increase in limit state function evaluations with the number n of random variables involved is investigated for the following example. For most approximation algorithms, the growth in the number of limit state function evaluations is superlinear. The following limit state function has been studied:

$$g(\boldsymbol{\theta}) = 4 - \theta_1 - \sum_{i=2}^{n} \frac{\theta_i^2}{8}. \tag{2}$$

An importance sampling density with unit variance has been introduced at the design point, which is given by $\theta_1 = 4$ and $\theta_i = 0$, $i = 2, ..., n$. By means of this density, 30000 samples were generated for each problem under consideration.

In the following, approximations of the failure probability were computed and further refined by introducing additional base points at which the limit state function is evaluated. From ten trials, the number of base points such that the approximation error for the failure probabilities falls below a certain limit during three succeeding adaptive refinement steps is recorded and averaged. The results are plotted in Figure 1 for a 5%-, 10%- and 15%-limit. It can be seen that the growth of the number of limit state function evaluations with dimension n of the problem is dependent on the error level: for an error of 15% the growth is sublinear, for 10% almost linear and for 5%, the number of limit state function evaluations grows superlinearly with dimension n.

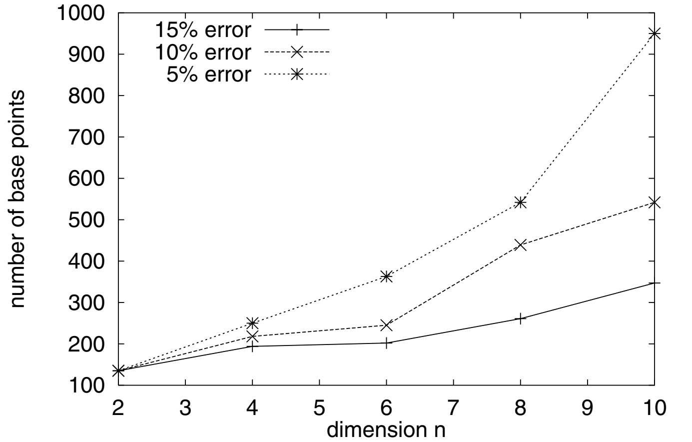

Fig. 1 Development of the number of limit state function evaluations to reach a prescribed precision with dimension n of the problem.

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