PAMM · Proc. Appl. Math. Mech. **6**, 331–332 (2006) / **DOI** 10.1002/pamm.200610147

Bifurcation Behavior of a 1DOF Sliding Friction Oscillator

Daniel Schwarzer*[1], **Hartmut Hetzler**[1], and **Wolfgang Seemann**[1]

[1] Institut für Technische Mechanik, Universität Karlsruhe (TH), 76128 Karlsruhe

This article deals with different types of friction models and their influence on the behavior of a simple 1 degree-of-freedom (DOF) sliding friction oscillator which is in literature commonly referred to as "mass-on-a-belt"-oscillator. The examined friction characteristics are assumed to be proportional to the applied normal force and only dependend on the relative velocity between the mass and the belt. For an exponential and a generalized cubic friction characteristic, the linear stability of the steady-state and the bifurcation behavior in the sliding domain are examined. It is shown that the resulting phase plots of the observed system are strongly dependent on the chosen friction characteristic.

1 Introduction and mechanical model

Motivated by investigations on the phenomenon of noisy brakes, a simple model to reflect the occuring oscillations corresponding to groan is searched. Experiments have shown that the measured frequencies of groan are in case of disc-brakes independent on the driving speed. Therefore self-excitation suggests itself as the basic mechanism for this phenomenon.

In the literature, the simplest mechanical model for investigating oscillations due to friction is the so-called "mass-on-a-belt"-oscillator: A lumped mass m connected to the inertial system by a linear elasticity (coefficient c) and a dashpod (damping coefficient d), is pressed by a force F_B onto a belt moving with the constant velocity v_0. Between the belt and the mass, the normal force N and the friction force R act, of which the latter is in the sense of Coulomb only proportional to that normal force. The factor of proportion μ is assumed to be at least linearly dependend on the relative velocity $v_{rel} = v_0 - \dot{x}$ between the mass (coordinate x) and the belt (see fig 1).

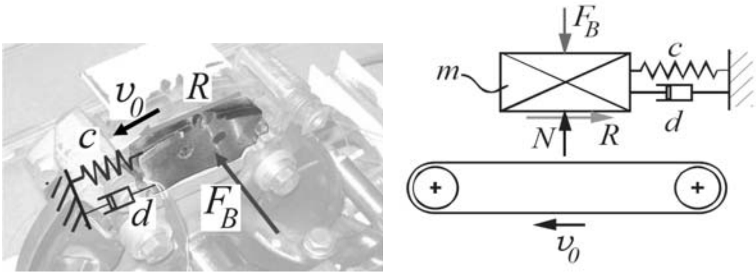

Fig. 1 Disc-brake for experiments and mechanical model

In the observed sliding domain ($v_{rel} > 0$), after translation of the parameters into the existing steady-state, the equation of motion reads

$$\ddot{z}+2D\omega_0\dot{z}+\omega_0^2 z = \frac{N}{m}(\mu(v_0-\dot{z})-\mu(v_0)), \tag{1}$$

with the natural frequency $\omega_0 = \sqrt{\frac{c}{m}}$ and the dimensionless damping measure D.

2 Non-linear investigations

In addition to a linear analysis of the steady-state (see [4]), further investigations are carried out by means of a first-order averaging method: First, the coordinates are transformed to polar coordinates (amplitude $A(t) > 0$ and phase $\psi(t)$) and then projected to a slower time by averaging over one period T ($\frac{1}{T}\int_0^T f(t)dt$). Therefore this method is often referred to as *method of slowly changing phase and amplitude* [2]. Applying this method to eq. (1) yields two differential equations, one amplitude equation in the form $\dot{A} = A \cdot f(A)$ and another equation for the phase. The latter yields $\dot{\psi} = 0$, indicating that the phase ψ stays constant over all time.

The search for steady-states (i.e. $\dot{A} \stackrel{!}{=} 0$) and the analysis of their stability show for friction characteristics, which are at least cubical in the relative velocity, two main results:

(I) A steady-state with the amplitude $A_1 = 0$ always exists. This result can also be found by a linear analysis.

(II) The system undergoes a Hopf-bifurcation: On the one hand, there exists only one steady-state. By changing the sign of the bifurcation parameter, this steady-state changes its stability and will be sourrounded by a limit cycle. Whether the bifurcation is super- or subcritical and with it the stability of the steady-state and of the limit cycle resp., is decided by the shape of the used friction characteristic (see chapter 3).

* Corresponding author: e-mail: schwarzer@itm.uni-karlsruhe.de

3 Results for different friction characteristics

Motivated by the measured friction characteristic shown in fig. 3, an exponential ansatz, $\mu(v_{rel}) = \mu_\infty + \Delta\mu e^{-a v_{rel}}$, is investigated first. The occuring Hopf-bifurcation then is subcritical. Therefore an unstable limit cycle exists in the neighborhood of the stable steady-state (cf. fig. 2). For small pertubations e.g., this can cause instabilities although linear analysis does not predict them.

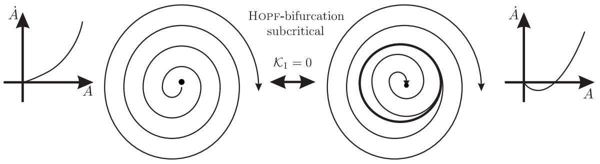

Fig. 2 Subcritical Hopf-bifurcation

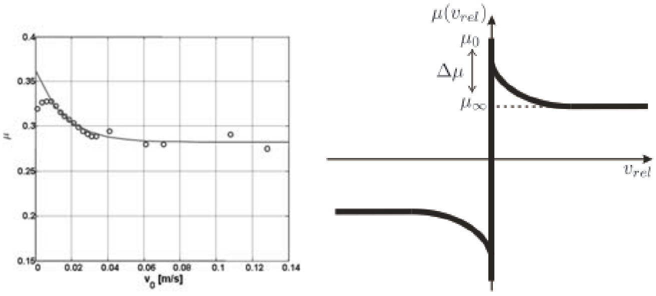

Fig. 3 Left: experimentally measured friction characteristic (dots) and exponential approximation (line). Right: parameters of used ansatz

The results of the linear and non-linear analysis and the occuring phase plots are evaluated with respect to two parameters (the dimensionless damping coefficient D and the parameter a of the friction characteristic) and plotted in fig. 4. More details are discussed in [3].

For a more general view, the friction characteristic now is assumed to be a cubic polynom of the form $\mu(v_{rel}) = \mu_0 + k_1 v_{rel} + k_2 v_{rel}^2 + k_3 v_{rel}^3$, representing an approximation for the chosen exponential ansatz as well as other in literature commonly used characteristics (cf. [1]). With the abbreviations $\mathcal{K}_1 = 2D\omega_0 + \frac{N}{m}(k_1 + 2v_0 k_2 + 3v_0^2 k_3)$, $\mathcal{K}_2 = -\frac{N}{m}(k_2 + 3v_0 k_3)$ and $\mathcal{K}_3 = \frac{N}{m}(k_3)$, eq. 1 then yields $\ddot{z} + \omega_0 z = -\mathcal{K}_1 \dot{z} - \mathcal{K}_2 \dot{z}^2 - \mathcal{K}_3 \dot{z}^3$.

After "averaging" this equation, two steady-states can be calculated: $A_1 = 0$ and $A_2 = \sqrt{-\frac{4\mathcal{K}_1}{3\omega_0^2 \mathcal{K}_3}}$. The stability and existence of these steady-states are dependend on the friction parameters k_1, k_2 and k_3. This dependence is shown in fig. 5 (as an example for the section $k_2 = 0$). More detailed results can be found in [5].

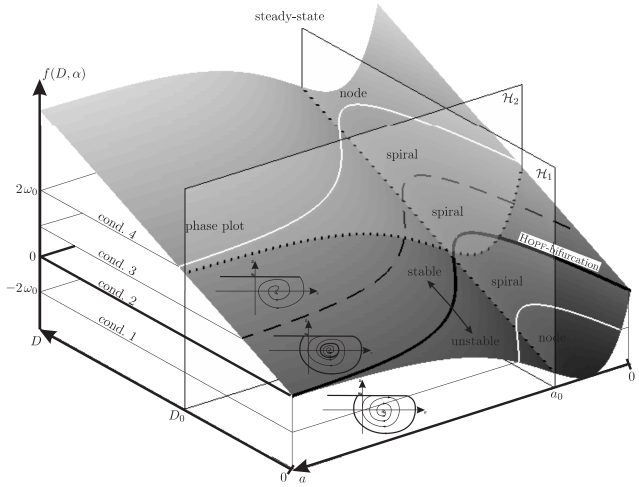

Fig. 4 Results of investigations dependend on the dimensionless damping coefficient D and the parameter a

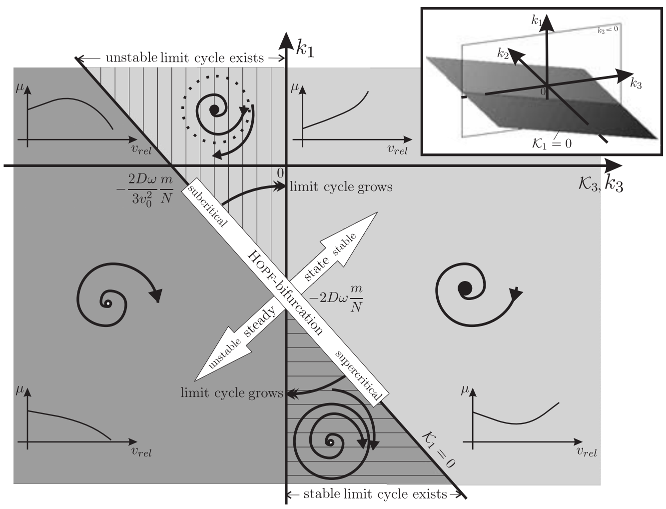

Fig. 5 Influence of the parameters k_1, k_2, k_3 of a cubic friction characteristic on the phase plot (e.g. section $k_2 = 0$)

Concluding this paper it can be stated that the occuring phase plots of the "mass-on-a-belt"-oscillator are strongly dependent on the chosen friction characteristisc.

References

[1] A. Fidlin, Nonlinear oscillations in Mechanical Engineering, Springer Verlag, 2006.
[2] P. Hagedorn, Non-linear oscillations, Clarendon press, Oxford, 1988.
[3] H. Hetzler, W. Seemann and D.Schwarzer, Analytical investigations of Hopf-bifurcations occuring in a 1DOF sliding friction oscillator with application to disc-brake vibrations, proceedings to IDETC'05, DETC2005-84312, ASME 2005.
[4] H. Hetzler, D. Schwarzer and W. Seemann, Analytical investigation of steady-state stability and local bifurcations occuring in sliding friction oscillators with application to low-frequency disc brake noise, Communications in Nonlinear Science and Numerical Simulations (CNSNS), Elsevier, 2006, in press.
[5] H. Hetzler, D. Schwarzer and W. Seemann, Steady-state stability and bifurcations of friction oscillators due to velocity dependent friction characteristics, Proceedings of the Inst. of Mech. Engineers, Part K, Journal of Multi-body Dynamics, 2006, in process.