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On the dynamics of a nonlinear rotor-floating ring bearing system

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Today, in high speed applications the rotors are commonly supported by hydrodynamic journal bearings. One typical configuration of journal bearings incorporated in automotive turbochargers is the floating ring bearing. Rotors supported by floating ring bearings have many advantages, regarding costs and power consumption for example. However, they might become unstable with increasing speed of rotation. At the onset of instability both the perfectly balanced and unbalanced rotor undergo self-excited vibrations which could cause the mechanical breakdown of the system. The "oil whip"-phenomenon, very well known from the investigations of the plain journal bearing occurs here in a different form. At the stability limit the rotor begins either oscillating with about the half of the ring speed or the half of the ring speed plus the half of the journal speed depending on the system parameters.

For this reason a rotor-floating ring bearing model is presented showing the mentioned characteristics. By applying the nonlinear equations of motion the limit cycles of the system are determined and its loss of stability is investigated.

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1 Introduction and mechanical model

Nowadays in automotive turbochargers the application of floating ring bearings is widely-used especially because of their low costs and reduced power losses. The main problem of rotors supported by floating ring bearings is that they might become unstable with increasing speed of rotation. Thereby self-excited vibrations are the cause of the loss of stability.

The simplest mechanical model for investigating oscillations due to nonlinear bearing forces is made up of a rigid symmetrical perfectly balanced rotor of mass $2m_1$ which is supported by two identical floating ring bearings with a ring mass of m_2. The

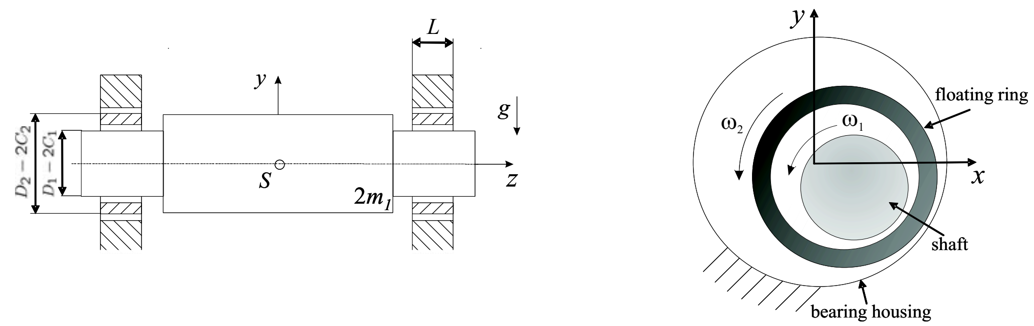

Fig. 1 Mechanical model of a rotor supported by floating ring bearings

floating ring bearing is a kind of journal bearing in which there is a loose ring between the shaft and the bearing housing. In this way the fluid film is separated into an inner film and an outer film whereas the diameter of the shaft is $D_1 - 2C_1$ and the outer diameter of the floating ring is $D_2 - 2C_2$. The radial clearances of the inner and outer film are denoted by C_1 and C_2 respectively. While the shaft rotates with constant angular velocity ω_1, the difference in the friction torque due to the inner film and the outer film forces the ring to rotate with an angular velocity of ω_2. In the following investigation the torque balance is neglected because the rotational speed of the ring is assumed to be constant. Further assumptions are made by using same bearing lengths L as well as same lubricant viscosities η for both films. Referring to figure 1 the equations of motion for the rotor-floating ring bearing model can be written as

$$\begin{aligned} m_1\ddot{x}_1 &= F_{x1}(x_1, \dot{x}_1, y_1, \dot{y}_1, x_2, \dot{x}_2, y_2, \dot{y}_2) \\ m_1\ddot{y}_1 &= F_{y1}(x_1, \dot{x}_1, y_1, \dot{y}_1, x_2, \dot{x}_2, y_2, \dot{y}_2) - W_1 \\ m_2\ddot{x}_2 &= F_{x2}(x_2, \dot{x}_2, y_2, \dot{y}_2) - F_{x1}(x_1, \dot{x}_1, y_1, \dot{y}_1, x_2, \dot{x}_2, y_2, \dot{y}_2) \\ m_2\ddot{y}_2 &= F_{y2}(x_2, \dot{x}_2, y_2, \dot{y}_2) - F_{y1}(x_1, \dot{x}_1, y_1, \dot{y}_1, x_2, \dot{x}_2, y_2, \dot{y}_2) - W_2. \end{aligned} \tag{1}$$

The loads in the equations of motion (1) are on the one hand the weight $2W_1$ of the rotor and on the other hand the weight W_2 of one ring. By applying the π film Ocvirk short bearing approximation to solve the Reynolds equation, an analytical

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solution for the hydrodynamic forces is obtained. Hence, the forces generated on the shaft and the ring by the inner film are nonlinear functions of the relative position and relative velocity of the shaft center to the center of the floating ring, while the forces generated on the ring by the outer film depend solely on the position and velocity of the ring center. After introducing the dimensionless time $\tau = \omega_1 t$ and the following dimensionless parameters

$$\sigma = \frac{1}{4}\frac{D_1 L^3 \eta}{C_1^2 \sqrt{m_1 C W_1}}, \quad \bar{\omega} = \sqrt{\frac{m_1 C_1}{W_1}} C_1, \quad \gamma = \frac{C_2}{C_1}, \quad \delta = \frac{D_2}{D_1}, \quad \mu = \frac{m_2}{m_1}, \quad \Omega = \frac{\omega_2}{\omega_1} = \frac{1}{1 + \frac{\delta^3}{\gamma}} \tag{2}$$

the equations of motion (1) can be written by means of a coordinate transformation in nondimensional state space form

$$\mathbf{X}' = \mathbf{F}(\mathbf{X}, \sigma, \bar{\omega}, \gamma, \delta, \mu) \tag{3}$$

whereas σ is a kind of reciprocal load parameter and $\bar{\omega}$ a dimensionless angular velocity.

2 Stability analysis

As a result of a linear stability analysis of the static equilibrium position the stability chart in dependence of σ and $\bar{\omega}$ is shown in figure 2a) for a given rotor-bearing system. At the stability limit the system undergoes a Hopf bifurcation. Here, two curves correspond to a loss of stability at purely imaginary eigenvalues where a so-called symmetric Hopf bifurcation occurs at their intersection point. In addition to the linear stability analysis further investigations are carried out by applying the center manifold theory after a linear transformation of coordinates in (3). In the case of a Hopf bifurcation with one pair of purely imaginary eigenvalues the center manifold $\mathbf{y}_s = \mathbf{h}(\mathbf{y}_c)$ is the appropriate method of reducing the 8-dimensional system to a 2-dimensional bifurcation system

$$\mathbf{y}'_c = \mathbf{J}_c \mathbf{y}_c + \mathbf{g}_c(\mathbf{y}_c, \mathbf{h}(\mathbf{y}_c)) \tag{4}$$

which depends only on the critical variables $\mathbf{y}_c$. Thereby $\mathbf{y}_s$ are the noncritical variables and all eigenvalues of the Jordan matrix $\mathbf{J}_c$ have a zero real part. In general an exact solution of the bifurcation equation (4) is not possible. Therefore, an approximation $\mathbf{y}_s = \mathbf{H}(\mathbf{y}_c)$ for the center manifold is calculated in form of a series in the critical variables. Inserting the approximation into (4) the bifurcation equations can be transformed into normal form (see [3])

$$r' = ar^3 + O(|r|^5), \quad \phi' = \Omega + br^2 + O(|r|^4). \tag{5}$$

Thus, the Hopf bifurcation is for $a < 0$ supercritical and for $a > 0$ subcritical, respectively. Corresponding to the stability chart the parameter a is plotted in dependence of the load parameter σ in figure 2a). By using the numerical continuation software AUTO [4] the results obtained for the parameter a can be verified e.g. for $\sigma_1 = 0{,}4$ and $\sigma_2 = 0{,}5$ (cf. figure 2b)). In the bifurcation diagrams, the dimensionless eccentricities of the shaft and the floating ring are defined with respect to the radial clearance C_1 of the inner film.

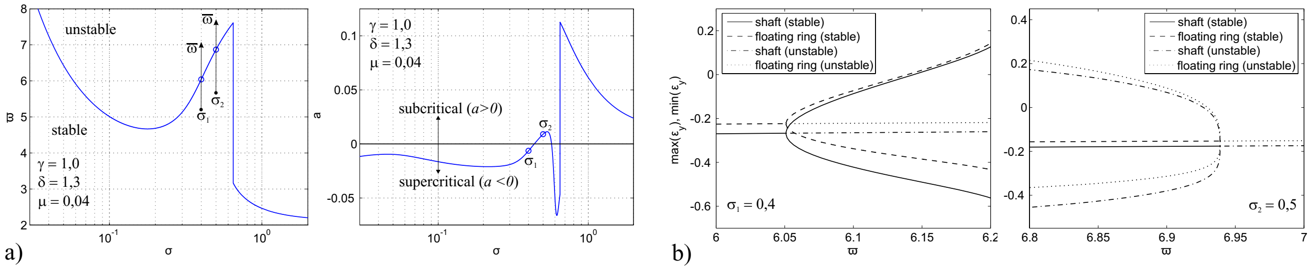

Fig. 2 a) Stability chart and corresponding area of sub- or supercritical bifurcation b) Bifurcation diagrams for $\sigma_1 = 0{,}4$ and $\sigma_2 = 0{,}5$ (maximal/minimal dimensionless displacements in vertical direction of shaft and floating ring dependent on $\bar{\omega}$).

3 Conclusion

In this study an analytical method of the bifurcation theory is applied to investigate the stability of the static equilibrium position of a rotor supported by floating ring bearings. Depending on floating ring bearing design parameters the existing Hopf bifurcation at the stability limit is either sub- or supercritical.

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