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## PIEZOELECTRIC CONTROL OF A MACHINE TOOL WITH PARALLEL KINEMATICS

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*Abstract:* In machine tools of parallel structure with two or three translatory degrees of freedom the rotatory degree of freedom is kinematically locked. Yet due to geometric faults, for example assembly errors or different geometries due to production tolerances, such machine tools exhibit an additional rotational behavior. Stresses within the structure occur leading to deflections of the tool center point, and thus, reducing the quality of the workpiece. For compensating these errors an adaptronic strut which can be implemented within such a machine tool has been developed. The strut comprises a piezoceramic sensor-actuator unit for controlled correction of those static and quasi-static deflections. Piezoceramic elements were chosen due to their high positioning accuracy and the small installation space required. The functional principle of a scale with a vibrating string is used for measuring the external load. Finally, an optimal control design for compensation is presented.

#### 1. Introduction

Fig. 1 shows a machine tool with parallel kinematics of three translatory degrees of freedom. Due to geometric errors such as assembly errors or differing geometries due to production tolerances stresses within the structure resulting in deflections of the tool center point (TCP) of the machine tool. Thus, the quality of the workpiece is reduced. An adaptronic strut as shown in Fig. 2 has been developed for compensating such errors. The strut, similar in shape to conventional struts in machine tools, is cut in two halves and a piezoelectric sensor-actuator unit is implemented in-between, giving the strut an additional degree of freedom.

The geometric deflections in focus of this contribution are mostly static or quasi-static, and thus, only inducing static or quasi-static signals on the piezoelectric sensor element. However, due to the internal leakage resistance of piezoceramic materials such signals are not measurable [1, 2]. By adapting the functional principle of a scale with a vibrating string a work-around for this problem was found. A string, which was mounted along the strut, as can be seen in Fig. 2, is excited by a solenoid. A dynamic signal is induced on the strut and onto the piezoelectric sensor. This signal can easily be

acquired and using frequency counters or phase-locked loops (PLL) its frequency course can be determined [5]. Equation (1) describes the relation

$$f_0 = \frac{1}{2l_s} \sqrt{\frac{T}{A\rho}} \tag{1}$$

between the eigenfrequency  $f_0$  of the string and pre-stress T on the string, with length  $l_{S_1}$  crosssectional area A and density  $\rho$  of the string. Thus, by measuring the frequency the external load on the strut can be determined. Further information about this functional principle can be found in [4, 5, 6].





Figure 1: Parallel kinematics machine tool with three translational degrees of freedom [3].

Figure 2: Adaptronic strut [4].

#### 2. Control concept for adaptronic strut

The simplest mechanical model of the adaptronic shown in Fig. 2 is a three-body oscillator as shown in Fig. 3. The upper and lower halves of the strut are lumped masses  $m_1$  and  $m_3$ , respectively, the piezoelectric element in-between is represented by  $m_2$ . The springs  $c_i$  and dampers  $d_i$  represent corresponding material properties, the force *F* represents external influences such as constraint forces on the strut.



Figure 3: Three-body oscillator as lumped mass approach for modeling the adaptronic strut.

The equations of motion for the system shown in Fig. 3 in state space form read

$$\frac{d}{dt}z = Az + bu + b_S u_S \tag{2}$$

with state space vector z representing positions and velocities of the lumped masses, actuator force  $u=F_r$  and disturbance force  $u_s=F$ .

Using the principle of Least Quadratic Regulator (LQR) the parameters for a state controller for this single variable system can be determined. The controller force  $F_r$  becomes

$$F_r = -r^T z \tag{3}$$

with state vector z. The control vector r is chosen such that the quadratic cost functional

$$J = \frac{1}{2} \int_{0}^{\infty} \left[ z^{T}(t) Q z(t) + \frac{1}{\kappa} F_{r}^{2} \right] dt$$

$$\tag{4}$$

is minimized. The scalar  $\kappa > 0$  is a value for the cost of the controller input whereas Q is a positive, semi-definite matrix weighting the system state. For more information on determining the controller parameters using LQR, see [7, 8, 9].

For realizing the state controller described above the system state must be known. However, since not all system states are measured an additional element must be introduced. The so-called Luenberger observer estimates the system states according to

$$\frac{d}{dt}\hat{z} = A\hat{z} + bu + l(\hat{y} - y).$$
<sup>(5)</sup>

The observer matrix l is chosen such that the eigenvalues of the observed system are further on the left of the imaginary axis than the eigenvalues of the controlled system. This guarantees that the observer is faster than the controller. That can easily be achieved using the pole placement procedure according to Ackermann since according to Föllinger [8] the two sets of eigenvalues can be set independently.

#### 3. Simulation results

To test the efficiency of the developed state controller the three-body oscillator was modeled within the commercial multi-body software program MSC.Adams. The co-simulation interface to MATLAB/ SIMULINK was used for realizing the controller and the observer within the system. The exchange interval between these two programs was set to  $\Delta t = 0.1$  ms which equals the maximum step size of the BDF integrator used.

As initial conditions for all simulations presented in the following all system states, i.e. both, positions and velocities of the bodies, are set to zero. The system experiences the influence of an external disturbance force

$$F = 2kN \sigma(t - 0.05s), \tag{6}$$

the tip deflection of the strut, i.e. the position  $x_3$  of body  $m_3$ , is the control variable.







Figure 4: LQR state controlled system under influence of external disturbance force F

Figure 5: Closed-loop system with position feedback

Fig. 4 depicts the reaction of the controlled system on the disturbance force F (6). A steady state control error is remaining. Thus, the presented control concept is not sufficient and has to be enhanced. Adding an additional feedback path as shown in Fig. 5, an improvement of the control can be achieved. The required control elements in detail are a pre-filter set to F = 1, an integrator H = 1/s in the feedback path, the LQR controlled system G and a compensation element C. This compensator is set using the root locus procedure [10, 11] such that an optimal system behavior and a large stability margin are obtained. A simple proportional element proved to be sufficient.

As depicted in Fig. 6 the steady state error vanishes if the enhanced control concept is used. The influence of the external disturbance force is compensated quickly. The actuator force required for this control operation can be seen in Fig. 7.



Figure 6: LQR-PI controlled system under influence of external disturbance force *F* 

Figure 7: Actuator force

In the following the presented control concept derived by the simple lumped mass approach is used for more advanced models of the strut. In a first enhancement the lumped masses are substituted by flexible bodies. Fig. 8 depicts the strut comprising upper and lower halves with the piezoelectric element in-between. With the tip deflection of the strut as control variable and using the same parameters for the control concept presented the efficiency of the designed control configuration under the same influence of external force F(6) is shown in Fig. 9.



Figure 8: Simple model of adaptronic strut with flexible bodies

Figure 9: LQR-PI controlled system under the influence of external disturbance force *F* 

Using the CAD data for accurate modeling of the geometry of the strut the model depicted in Fig. 10 arises. Again the tip deflection of the strut is used as control variable and the same settings for the control parameters are used. Fig. 11 shows the response of the controlled system to an external disturbance force  $F' = \frac{1}{2} F$ . At the reduced magnitude of the applied external force it can already be seen that the simple control concept reaches its limits when used with the accurate model. However, since mainly static and quasi-static loads are in focus of the application, higher loads might still be an issue of compensation if they are slowly changing. Therefore, further studies on this subject must be done.





Figure 10: Accurate model of adaptronic strut by use of CAD data

Figure 11: LQR-PI controlled system under influence of external force F'=1/2 F

#### 4. Conclusions

An adaptronic strut for compensating static and quasi-static errors in machine tools with parallel kinematics has been presented. The functional principle of a scale with vibrating string was used for measuring the occurring loads with piezoelectric elements. The developed control concept for the strut was tested on three different models of the strut: a lumped mass model, a flexible body model and finally a flexible body model by CAD data. Future studies include the integration of one or more controlled struts in a model of a machine tool and further investigations of the behavior of the enhanced machine tool during machining processes like turning or milling.

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