PAMM · Proc. Appl. Math. Mech. 7, 4040029–4040030 (2007) / **DOI** 10.1002/pamm.200700473

A contribution to the modeling of metal foams on the mesoscale

Daniel Schwarzer[1,*], **Sebastien Gatouillat**[1], and **Carsten Proppe**[1]

[1] Institut für Technische Mechanik, Universität Karlsruhe (TH), 76128 Karlsruhe, Germany.

Due to their useful properties in lightweight construction and due to their excellent behavior in energy absorption for example in crash mechanics, metal foams became an interesting, often utilized and investigated material. For the determination of the mechanical properties of foams without the help of expensive experiments, a way for computing these properties is searched. The problem in doing so is that foams can be composed out of randomly distributed edges and faces with varying thickness and of other inhomogeneities on the mesoscale like imperfections. The goal in this paper is, to investigate the influence of these irregularities on the mechanical, linear elastic properties of a metal foam on the macroscale and to determine the size of a representative volume element, for which the irregularities on the mesoscale do not have a great influence on the linear elastic properties.

1 Starting point

This contribution starts with regular honeycomb structures, which were already introduced by Gibson and Ashby in [1]. In their book, these honeycomb structures are modeled as a network of beams based on different theories. In this work, the cell walls are assumed to be slender enough, so that Euler-Bernoulli's beam theory holds. The networks of beams are modeled in Abaqus with the help of the B23-Elements.
In order to compute upper and lower bounds of the elastic properties in the sense of Voigt and Reuss, different types of boundary conditions are applied: static uniform boundary conditions yielding in the lower Reuss bound and kinematic uniform boundary conditions giving the upper Voigt bound (see for example [3]). For each type of these boundary conditions four load cases are computed: two axial compression, a hydrostatic compression and a shear load case. From these load cases six in-plane elastic moduli can be calculated and it can be shown that the computed foam models are isotropic. Therefore only four of these six moduli are of interest: the Young's modulus, the Poisson's ratio, the bulk and the shear modulus. In this contribution only the Young's modulus is presented.
In a first step the structure of the honeycomb is randomized in order to represent the irregular structure of foams on the mesoscale (see fig. 1). In order to do that, so called Voronoï maps are computed, which are based on the grain growing from randomly distributed points. For each set of parameters of the honeycomb model and for each later introduced extensions of the model, 500 realizations (with eight load cases each) are computed in order to be sure that the mean value of the calculated moduli converges and the standard deviations are small enough.

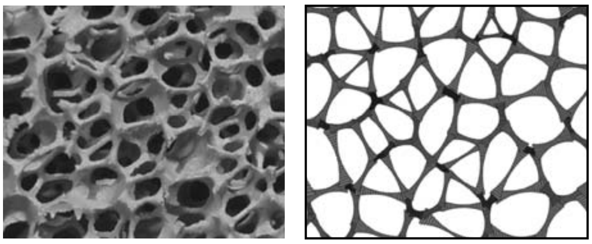

Fig. 1 Comparison of a real foam and the randomly generated model with varying thickness introduced later

2 Extensions to the model

Two different extensions are examined in this work: imperfections on the mesoscale and a varying thickness.

Imperfections on the mesoscale:
The imperfections on the mesoscale are modeled by randomly erasing nodes out of the generated Voronoï beam networks (see fig. 2). This procedure can have two possible consequences: an erased cell wall as well as an erased vertex, which means that three or more cell walls are influenced by this. The simulations were done with increasing number of erased nodes. In figure 3 and 4 the results are shown: The Young's modulus decreases rapidly by increasing the number of erased nodes, while the absolute value of the standard deviation nearly stays constant.

* Corresponding author E-mail: schwarzer@itm.uni-karlsruhe.de,

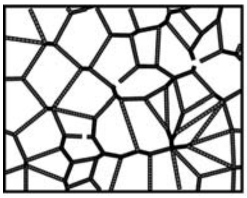

Fig. 2 Imperfections in the model: two possible consequences

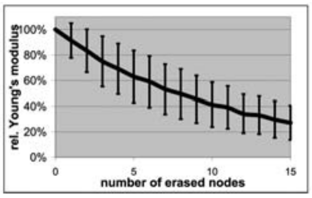

Fig. 3 Young's modulus versus number of erased nodes

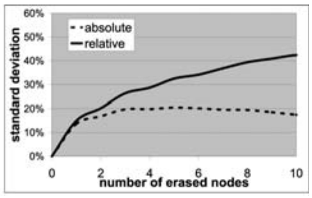

Fig. 4 stand. dev. of Young's mod. rel. to current and starting mean value

Varying thickness:

Another extension introduced to the model is a varying thickness. Principally there are two possible ways: One way is to calculate the stiffness of one cell wall by numerically integrating the differential equation for the displacement for a varying thickness function and evaluating the solution at the end of the beam. Another possibility is to approximate the thickness of a cell wall by the use of several beam elements with different but constant thicknesses. The second method needs about seven or eight elements to converge against the right solution which increases the computational effort but is easier to implement in a commercial finite element software.

The thickness of each cell wall is modeled with the help of the simple quadratic function $t(x) = t_0 \left[3(1 - t_{\text{rel}}) \left(\frac{2x}{\ell} - 1\right)^2 + t_{\text{rel}}\right]$, where the parameter t_{rel} is a measure for the curvature being concave for $0 < t_{\text{rel}} < 1$, and the parameter t_0 as a measure for the equivalent thickness of a constant cell wall [2]. In this work, the parameters are not chosen randomly: t_{rel} is assumed to be linear dependent on the beam length (see figure 7) and t_0 is assumed to be dependent on the circumference and the area of the adjoining pores in that sense, that a cell wall with small adjoining pores gets relatively more material than a cell wall with bigger pores as neighbors (see figure 6).

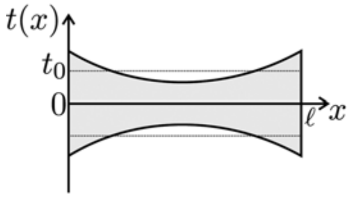

Fig. 5 Function of varying thickness [2]

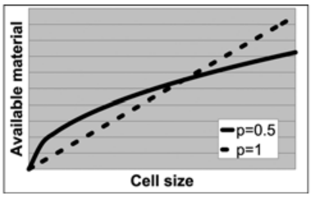

Fig. 6 Choice of parameter t_0

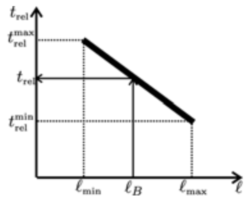

Fig. 7 Choice of parameter t_{rel}

3 Results - all extensions in one model

All mentioned extensions are introduced into the model. The deformation of one model can be seen in figure 8. In figure 9 the Young's modulus is shown for an increasing sample size. It is observable, that it converges and the standard deviation decreases as expected. For finding the size of a representative volume element, two possible criteria can be evaluated: a statistical one in the sense of mean value and standard deviation and the discrepancy $D = \frac{P_{\text{KUBC}} - P_{\text{SUBC}}}{\frac{1}{2}(P_{\text{KUBC}} + P_{\text{SUBC}})}$ measuring the relative error between the upper and the lower bound [4]. The difference of these two criteria can be seen in figure 10.

With the help of this work, it is now possible to determine the size of a volume element, which can be interpretated as representative.

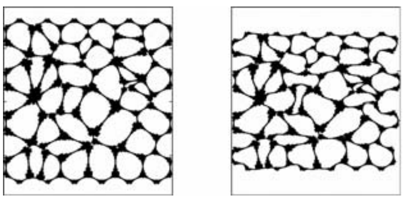

Fig. 8 Undeformed and deformed model

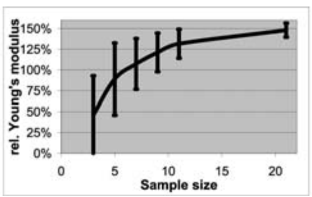

Fig. 9 Young's modulus

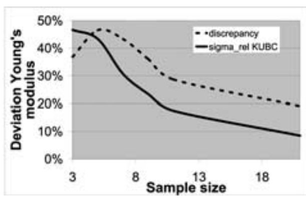

Fig. 10 Error of Young's modulus

References

[1] L.J. Gibson, and M.F. Ashby, Cellular solids - Structure and properties, 2nd edition (University Press, Cambridge, 1997).
[2] H. Harders, Ermüdung von Aluminiumschaum, Dissertation an der Technischen Universität Braunschweig, 2005.
[3] T. Kanit, S. Forest, I. Galliet, V. Mounoury and D. Jeulin, Int. J. of Solids and Structures **40**, 3647-3679 (2003).
[4] Z.F. Khisaeva, M. Ostoja-Starzewski, J. of Elasticity **85**, 153-173 (2006).