0–π Transitions in a Superconductor/Chiral magnet/Superconductor Junction

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We study the π phase in a superconductor-ferromagnet-superconductor Josephson junction, with a ferromagnet showing a cycloidal spiral spin modulation with in-plane propagation vector. Our results reveal a high sensitivity of the junction to the spiral order and indicate the presence of 0–π quantum phase transitions as function of the spiral wave vector. We find that the chiral magnetic order introduces chiral superconducting triplet pairs that strongly influence the physics in such Josephson junctions, with potential applications in nanoelectronics and spintronics.

It is by now well established that an equilibrium superconducting phase difference of π can be arranged between two singlet superconductors when separating them by a suitably chosen ferromagnetic material.1,2 Transitions between the π-state and the 0-state of such S-F-S Josephson junctions have been revealed in experiments through oscillations of the Josephson critical current with varying thickness of the ferromagnet3 or with varying temperature.4 The π Josephson junction is currently of considerable interest as an element complementary to the usual Josephson junction in the development of functional nanostructures,5 including superconducting electronics6 and quantum computing7.

Recently, there has been a rapid progress in the field of chiral magnetism8,9,10,11 that raises the expectations for applications of chiral magnets in spintronics. Chiral order occurs in inversion asymmetric magnetic materials9,11 that in the presence of spin-orbit coupling give rise to a Dzyaloshinskii-Moriya interaction \( D_{ij}(S_i \times S_j) \). This interaction favors a directionally non-collinear (spiral) spin structure of a specific chirality over the usual collinear arrangement favored by the Heisenberg exchange interaction \( J_{ij}(S_i \cdot S_j) \). A well-studied10,11 chiral magnet is the transition-metal compound MnSi, with the spiral wave length \( \Lambda \approx 180 \text{ Å} \). Nanoscale magnets or magnetic systems with reduced dimensionality that frequently lack inversion symmetry due to interfaces and surfaces are expected to exhibit chiral magnetism8. This has been confirmed by the recent observation9 of a spin spiral structure (with \( \Lambda \approx 12 \text{ nm} \)) in a single atomic layer of manganese on a tungsten substrate.

In this Letter, we combine chiral magnetism with superconductivity in a controllable Josephson nano-device where 0–π transitions can be induced by tuning the magnetic spiral wave vector \( Q \) (see Fig. 1). Whereas in bulk magnets \( Q \) can be manipulated e.g. by means of pressure, in nano-magnets alternative possibilities of control exist, as electric fields, geometry, or pinning layers. Such a Josephson device shows a surprisingly complex behavior with 0- to π-state transitions as function of spiral wave length \( \Lambda = 2\pi/Q \), that turn into zero temperature transitions for some critical wave vectors. However, below the threshold \( \Lambda_{th} = \pi \xi_j \), where \( \xi_j \) is the penetration depth of pairs into the chiral magnet (\( \xi_j \) depends on material constants), a qualitatively different behavior is found.

Within our model chiral magnetism and singlet superconductivity take place in mutually separated materials, and the magnetic spiral affects only the superconducting proximity amplitudes. This is in contrast to the case of coexisting superconducting and spiral magnetic order in the same material, e.g. in ferromagnetic superconductors12. We also contrast our model to the case of a helical spiral spin modulation with a propagation wave vector perpendicular to the S-F interface13, and the Josephson effect in S-F-S junctions with a Néel domain structure14. The physics studied in Refs.13,14 is dominated by the presence of long-range triplet components, that are absent in the present system15 (concerning the role of long-range triplet pairs in S-F-S hybrid structures see also16). In Refs.13,14, a strong dependence of the Josephson critical current \( I_c \) on the ferromagnet inhomogeneity is found due to these long-

FIG. 1: (Color online) S-CM-S Josephson junction where CM is a chiral ferromagnet with a cycloidal spiral spin modulation, i.e. the spins are confined to a plane (the \( x-y \)-plane) parallel to the spiral propagation direction (the \( y \)-axis).
range components. However, the related magnitude of $I_c$ is so small that the observation of such an effect is questionable. In this paper, we report a critical current with a much larger magnitude \cite{12}, which is essential for potential applications.

We study the S-CM-S junction shown in Fig. 11 within the framework of the quasiclassical theory of superconductivity and consider the diffusive limit. Furthermore, we shall assume that the pair correlations induced in the chiral magnet, quantified by the anomalous Green function $f$, are small. This is fulfilled for temperatures close to the superconducting critical temperature $T_c$, but also for much smaller temperatures $T$ provided that the S-CM interface transparency is small. We decompose the $2 \times 2$ spin-matrix $f$ as $f = f_s i \sigma_y + i (f_t \gamma \sigma_y)$, where $f_s$ is the singlet component and $f_t$ is the triplet vector (here $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ is a vector of Pauli matrices). These components obey \cite{15} in the magnet system a set of linearized Usadel equations

\begin{equation}
(D \nabla^2 - 2 \varepsilon_n) f_s = 2iJ \cdot f_t, \tag{1}
\end{equation}

\begin{equation}
(D \nabla^2 - 2 \varepsilon_n) f_t = 2iJ f_s, \tag{2}
\end{equation}

where $\varepsilon_n = \pi T (2n+1)$ is the Matsubara frequency with $n$ a positive integer. Quantities for negative frequencies are obtained through symmetry relations \cite{15}, the components $f_s$ and $f_t$ being respectively even and odd in $\varepsilon_n$. The $z$-axis is perpendicular to the interfaces, and the CM region is delimited by $|z| < d_f/2$, where $d_f$ is the thickness of the CM layer. The exchange field $J$ is nonzero in the CM region, while the singlet superconducting order parameter $\Delta_s$ is nonzero only in the S regions. The S and CM parts can have different diffusion constants, $D_s$ and $D_f$, and also different superconducting coherence lengths $\xi_s, \xi_f = \sqrt{D_s, f/2\pi T_c}$. For simplicity, we assume that the two S regions, and also the two S/CM interfaces, are characterized by identical parameters. Another important length scale is the magnetic length $\xi = \sqrt{D_f/\|J\|}$.

The exchange field $J$ rotates within the $x-y$ plane in the CM film with a spiral wave vector $Q_y$, $J(y) = J (\cos Q y, \sin Q y, 0)$. \tag{3}

As a result $f$ depends on both spatial coordinates $x$ and $y$. The triplet vector $f_t$ is found to be parallel to $J$ everywhere \cite{12}. It is convenient to introduce chiral triplet components $f_\pm = (f_{tx} \pm i f_{ty}) e^{\pm i Q y}$. In the CM layer, the singlet component $f_s$ and the two chiral triplet components $f_\pm$ are then given by

\begin{equation}
f_t (z) = \sum_{\epsilon = \pm 1} \varphi_{l,\epsilon} \left[ a_{\epsilon} \cosh (k_\epsilon z) + b_{\epsilon} \sinh (k_\epsilon z) \right], \tag{4}
\end{equation}

where $l = s$ or $\pm$, $\varphi_{s,\epsilon} = \eta_s, \varphi_{\pm,\epsilon} = \epsilon, \varphi_{s,\epsilon} = -\epsilon$ and

\begin{equation}
k_\epsilon = \sqrt{2(\varepsilon_n + i \epsilon \eta_{-\epsilon})/D_f}, \tag{5}
\end{equation}

where $\eta_s = \sqrt{1 - \eta^2 + i \eta}$ for $\eta \leq 1, \eta > 1$, \tag{6}

\begin{equation}
\gamma_s \eta_s \partial_z f_t (z) = \xi_s \partial_z f_t (z), \tag{7}
\end{equation}

\begin{equation}
\gamma_\eta \delta \xi \partial_z f_t (z) = \pm [f_t (z) - f_t (z)], \tag{8}
\end{equation}

for the triplet ($l = \pm$) and singlet ($l = s$) amplitudes. The parameters $\gamma_s$ and $\gamma_\eta$ are related to the conductivity mismatch between the two sides ($\gamma_s / \xi_s = \sigma_f / \sigma_s$ with the bulk conductivities $\sigma_f$ in CM and $\sigma_s$ in S) and the boundary resistance, respectively. The signs $\pm$ in Eq. (8) refer to the interfaces at $z = \pm d_f/2$, respectively. In the following, we define the short-hand notation $\delta (\pm) = f_s (\pm d_f/2)$ for the singlet amplitudes at the interfaces.

Due to the leakage of pair correlations into the central CM region, the amplitudes $\delta (\pm)$ are expected to be reduced compared with the bulk value in S. This inverse proximity effect can be important in hybrid structures involving ferromagnets (see e.g. Ref. \cite{16}). However, the spatial dependences of $n_s$ as well as of the triplet components can be disregarded in S when $\gamma \ll 1 + \gamma_\eta d_f / \xi_f$, and the rigid boundary conditions hold (see, e.g., Ref. \cite{17}), with $\delta (\pm) \approx \sqrt{\Delta s} e^{\pm i \phi / 2} / \sqrt{\varepsilon_n^2 + \Delta_s^2}$, where $\phi$ is the phase difference between the two superconductors. Using Eqs. (7)-(8) within this assumption, we express $a_s$ and $b_s$ as functions of $\delta (\pm)$

\begin{equation}
a_s = \frac{\delta (\pm) + \delta (-)}{2} \frac{1}{(\eta_s + \eta_{-s}) A_s}, \tag{9}
\end{equation}

\begin{equation}
b_s = \frac{\delta (\pm) - \delta (-)}{2} \frac{1}{(\eta_s + \eta_{-s}) B_s}, \tag{10}
\end{equation}

where $A_s = \cosh (x_s) + \gamma_\eta k_s \eta_s \sinh (x_s), B_s = \sinh (x_s) + \gamma_\eta k_s \xi \cosh (x_s), x_s = k_s d_f/2$.

The current flowing through the S-CM-S junction is

\begin{equation}
I = 2e \frac{D_{f}}{\pi} N_f ST \sum_n \text{Im} [ f_{\eta_{\pm}} \partial_{\eta_{\pm}} f_{\eta_{\pm}} - f_{\eta_{\pm}} \partial_\eta f_{\eta_{\pm}} - f_{\eta_{\pm}} \partial_{\eta_{\pm}} f_{\eta_{\pm}}] \tag{11}
\end{equation}

where $N_f$ is the Fermi-level density of states per spin in CM and $S$ is the cross-section area. We insert $f_{\eta_{\pm}} \partial_{\eta_{\pm}} f_{\eta_{\pm}} = (f_{\eta_{\pm}} \partial_{\eta_{\pm}} f_{\eta_{\pm}}^* + f_{\eta_{\pm}} \partial_{\eta_{\pm}} f_{\eta_{\pm}}^*) / 2$ and Eq. (11), and express $I$ as a function of $a_{\eta}$ and $b_{\eta}$ as

\begin{equation}
I = 4e \frac{D_{f}}{\pi} N_f ST \sum_{n \geq 0, \eta_{\pm}, \eta_{\pm}} \text{Im} [(\eta_{\eta_{\pm}}^\eta_{\pm} - \eta_{\eta_{\eta}} \epsilon^\epsilon a_{\eta_{\eta_{\eta}}} b_{\eta_{\eta_{\eta}}}] \tag{12}
\end{equation}

For $\eta < 1$ only the terms with $\epsilon \neq \epsilon'$ contribute, while for $\eta > 1$ only the terms with $\epsilon = \epsilon'$ contribute (the case
indeed is applicable in the parameter range we consider. The more general expression (12) to verify that Eq. (14) is of Eq. (14). For small thicknesses ($Q$, temperature dependence), resistance, $\sigma$, of Eq. (14), such as $\Delta$ terms in Eq. (14), such as $\Delta^2 \xi / 4$ for $\varepsilon_n = \pi T_c$.

$\eta = 1$ is defined via the corresponding limit in Eq. (12). In agreement with current conservation, the dependence on $z$ vanishes.

It then follows from Eqs. (9), (10), and (12) that

$$I_c R_N = 4 V_0 \left( \frac{d_f}{\xi_f} + 2 \gamma_b \right) \sum_{n \geq 0} \frac{T_c}{\varepsilon_n^2} \frac{k_c \xi_f \eta_c}{A_c B_c (\eta_c + \eta_{-c})},$$

where $R_N = (d_f + 2 \gamma_b \xi_f) / \sigma_f S$ is the normal state resistance, $\sigma_f = 2 e^2 N_f D_f$ is the conductivity of the CM layer, and $V_0 = \pi \Delta^2 / 4 e^2 T_c$. On the other hand, for low barrier transparencies ($\gamma_b \gg 1$), we have $A_c \approx \gamma_b k_c \xi_f \sinh(k_c d_f / 2)$ and $B_c \approx \gamma_b k_c \xi_f \cosh(k_c d_f / 2)$, which lead to

$$I_c R_N = \frac{4 \pi T_c}{\gamma_b} \sum_{n \geq 0} \sum_{\varepsilon = \pm 1} \frac{\Delta^2}{\varepsilon_n^2 + \Delta^2} \frac{\eta_c (\eta_c + \eta_{-c})}{k_c \xi_f \sinh(k_c d_f / 2)}.$$

In the absence of inhomogeneity ($Q = 0$), we then recover expressions for the critical current in the literature. Note that the temperature $T$ appears through several terms in Eq. (14), such as $\Delta_z$ (here we assume the BCS temperature dependence), $\varepsilon_n$ and $k_c$.

In the following we study the influence of an exchange field with chiral order on the Josephson effect on the basis of Eq. (14). For small thicknesses $d_f$ we have used the more general expression (12) to verify that Eq. (14) indeed is applicable in the parameter range we consider. As we show in Fig. 2 the chiral magnetic order introduces a surprisingly rich behavior: the magnitude of $I_c$ as function of increasing wave vector $Q$ presents initial oscillations and suppression, followed by increase and final saturation. Depending on the thickness of the CM layer, there can be one or several $0 - \pi$ and $\pi - 0$ transitions as function of the spiral order wave vector $Q$. Above a certain value of $Q$ ($Q \xi_f = 2$) is indicated by the vertical line in the figure $I_c$ is positive independently of other model parameters, meaning that the junction phase difference is stabilized at zero. Physically, this can be understood as an averaging out of the exchange field within one magnetic length $\xi_f$. Technically, this critical value of $Q$ separates a region with complex eigenvalues $k_c$ ($\eta < 1$, oscillating $I_c$) from a region with real $k_c$ ($\eta > 1$, monotonously increasing $I_c$), see the inset of Fig. 2. For $\eta < 1$, the complex $k_c$ leads to a non-monotonic dependence of $I_c$ as function of $Q$. In the large-$Q$ limit, the Josephson critical current for a junction with a normal metal is recovered.

In Fig. 3(a) we study in more detail the critical current within the region $0 \leq Q \xi_f \leq 2$ supporting oscillations. For an intermediate thick magnetic film (here $d_f = 2.7 \xi_f$) it is possible to see both $0 - \pi$ and $\pi - 0$ transitions as function of $Q$, with a reasonably large critical current. The phase transitions shift to lower values of $Q$ with increasing temperature. As seen in Fig. 3(b), the spiral order can also induce $0 - \pi$ transitions as function of temperature for certain parameter ranges.

Phase-diagrams of the $\pi - 0$ transitions are presented in Fig. 4. We see that in the low-$T$ region [panel (a)] the phase transition line $T_{\pi - 0}(Q)$ develops a very steep slope. This insensitivity to temperature variations can be of importance for device applications. Although at ultra-low temperatures a more sophisticated theory than the mean field approach presented here should be used, our results in Fig. 4 give a strong indication of a $\pi - 0$ transition as a function of $Q$ also at zero temperature. Thus, the system of a chiral magnet sandwiched between two superconductors is of potential interest for the study.
of critical behavior near a quantum critical point.

In the right panel of Fig. 4 the very different behaviors for $Q\xi J < 2$ and $> 2$ are also seen. For $Q\xi J < 2$ the spiral order shifts the transition lines towards thicker magnetic films, but the transition line never disappears from the phase diagram. Only in the region $Q\xi J > 2$ is the averaging of the exchange field over the magnetic length so effective as to prevent $0 - \pi$ transitions.

In summary, we have studied the Josephson effect in an S-CM-S junction in the presence of an in-plane cycloidal spin spiral structure in the magnet. We have found that the presence of a spin spiral can change the ground state of the Josephson junction, and lead to a transition between a $\pi$-junction and a $0$-junction for a critical spiral wave vector. The dependences of the Josephson effect on magnet thickness and on temperature depend sensitively on the wave vector of the chiral order in the magnet. We predict that a quantum-critical point should exist in the phase-diagram for suitably chosen sample parameters. We expect that these effects will have potential applications for new types of functional nanoscale structures. The ultimate goal for the future is to tune Josephson junctions with one or more chiral magnets by controlling the phases or magnitudes of the spiral magnetic wave vectors.

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Note added. - After submission, we became aware of work by Crouzy et al. [21], who study in-plane magnetic Néel domain walls. Their model is markedly different from ours, but leads to similar findings about the periodicity of $0$ to $\pi$ transitions with the magnetic inhomogeneity.

[17] The magnitude of $L$, is in our case [see Eqs. (13), (14) below] not characterized by a small prefactor $\propto \xi J/\xi f$ or $\xi J^{3}/\xi f$ in strong contrast to formulas derived in Refs. [13] and [14], respectively.
[20] We assume that the following inequalities hold $(\gamma_0 k_\xi J)^{-1} \ll \text{tanh}(x_c) \ll \gamma_0 k_\xi J$.