First published in:

QUATERNION-BASED DELAY TIME DETERMINATION FOR KINEMATIC OPTICAL MEASURING SYSTEMS

Claudia Depenthal, *Geodetic Institute, University Karlsruhe, Germany*

Abstract: **By using kinematic optical measuring systems in spatiotemporal positioning necessarily all involved sensors of the measuring systems have to be synchronized. Otherwise existing delay times in a measuring system will lead to deviations in space-time position. These delay times will be determinated with a developed time-referenced 4D calibration system, which is qualified for tracking optical measuring systems of any kind. The base of this calibration system is built up by a tiltable rotating arm driven by a rotary direct drive. The rotating arm is supplemented by a further rotary direct drive mounted on a freely movable tripod. The developed modeling for determinability of the delay times is based on the theory of quaternions. The fundamental idea of modeling is equivalent to the fact that every measurand of the test item, which is measured at a particular time, could be assigned to an explicit position of the rotating arm.**

Index Terms**: kinematic measurements, calibration system, time-referenced, rotating arm.**

I. INTRODUCTION

Kinematic optical measuring systems such as lasertracker, robotic-tacheometer or iGPS are employed for the space-time position determination of moving object points. These kind of measuring systems are multi-sensor systems and for a spatiotemporal positioning all involved sensors have to be synchronized. But the existing of dead time and latency will lead to deviations in space-time position. To determinate these delay times a time-referenced 4D calibration system and an adequate modeling was developed. The 4D calibration system consists of a tiltable rotating arm with a rotary direct drive and for polar measuring systems it is supplemented by a further rotary direct drive, which is mounted on a movable tripod under the measuring system. The main item of the calibration system are the direct drives and the control system with the real- time multi-axis servo motion controller PMAC (Programmable Multi-Axes Controller), which is used for the position and velocity control of the direct drives. The calibration system delivers a trajectory with a high accuracy for discrete space-time positions. The details of the 4D calibration system are described in [1] and [2]. The emphasis of this paper lies in the time referencing and modeling for the delay time determination, which will be exemplary shown for a polar measurement system with two rotations. Further modelings are described in [1].

II. DELAY TIME

A kinematic measurement process has to assign an accurate spatiotemporal position to a moving object. In fact there is a difference between the measured spatiotemporal position and the theoretical position. The dimension depends on the measurement system and especially on the time response of the measurand. In this context the terms *dead time* and *latency* where often used, however there are exist different definitions for theses terms. For this reason and in combination with the time-referenced calibration system these terms are not applicable and the term *delay time* will be used instead.

The delay time is defined as a time difference between a reference point and the development time for every measurement value of a measurement system, e.g. a distance. A clearly defined reference point is the time of the measurement request t_{rq}, because this is the time at which a complete measurement is expected. Fig. 1 shows an example for a time scale with angle and distance measurement at discrete positions. The 4D calibration system delivers for every measurement request the reference time $t_{rq,i}$. In this example (Fig. 1) a certain time later the angle and distance will be measured and the delay time is now defined in relation to the measurement request, as $t_{h,i}$ for the angle measurement and $t_{d,i}$ for the distance.

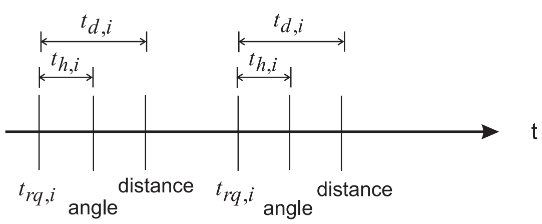

Fig.1 – Time scale with time of measurement request $t_{rq,i}$, delay time for angle measurement $t_{h,i}$ and delay time for distance $t_{d,i}$.

III. TIME REFERENCING

The meaning of time referencing is, that specific procedures have to be kept at the same point of time on a given time scale. For time referencing only real-time systems can be used. A system is said to be real-time if the total correctness of the result of a real-time data processing depends not only upon its logical correctness, but also upon time in which it is performed [3]. A real-time system also has to be

978-1-4244-3861-7/09/$25.00 ©2009 IEEE

guarantied a temporal deterministical behavior [4]. In relation to the calibration system and a test item – a measurement system – the time referencing is used with two different procedures, an external trigger or a serial interface. In this way a calibration system assigns a position – time and location – to a clearly defined reference point.

A. External Trigger

An external trigger-signal is realized e.g. with a function generator using the rising edge of a rectangular signal as trigger. The quality of the time referencing is depending on the edge steepness and the gate delay. Fig. 2 shows a TTL (transistor-transistor logic) circuit for a rising edge with a steepness of 1 µs. Within the high level both, the measurement system (t_1) and calibration system (t_2), detect the trigger, but not at the same place. A time lag Δt within the referencing arise from the gate delay of both systems and can be calculated by the difference (1) of the trigger point, which are shifted by the gate delay

$$\Delta t = t_4 - t_3 \quad . \qquad (1)$$

The gate delay of a measurement system is rarely known. The gate delay of the calibration system is less than 100 ns. The reference point for the delay time determination is the trigger point detected by the calibration system.

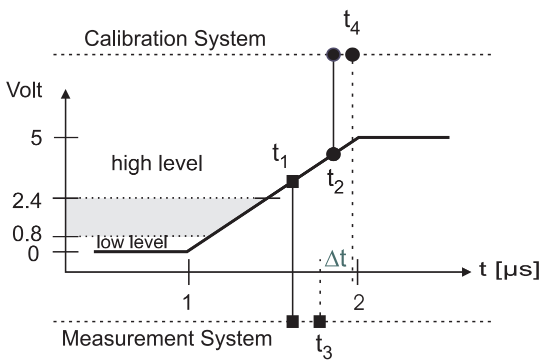

Fig.2 – Time referencing with external trigger: rising edge of a rectangular signal.

B. Serial Interface

The other method of communication between a calibration system and a test item is a serial interface. The communication is made up of request and reply in terms of the data item. The trailing edge of the start bit of the data item will be captured and at the same time a trigger signal must be send to PMAC. To realize this in real-time a FPGA-modul (Field Programmable Gate Array) with a resolution of 25 ns is used. Fig. 3 shows the time referencing for an assuming data transfer rate of 19200 baud. The period between two trailing edges constitutes 103 µs, so that the trailing edge of the start bit must be captured within this period. The same procedure can be done with the reply of the test item. In this way for every start bit of the request and reply a spatiotemporal position is known. The reference point for the delay time determination is the start bit of the request under considering of the known length of the data item.

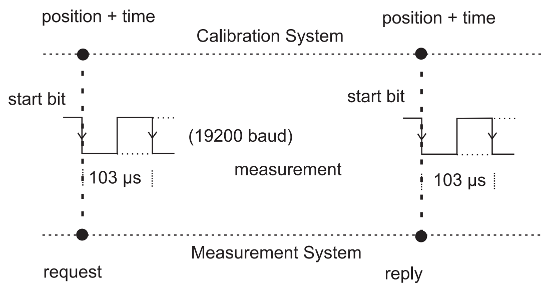

Fig.3 – Time referencing with serial line: capture the trailing edge of a data item.

IV. MODELLING

The aim of modeling is the determination of the delay time for every measurand of a test item. Because kinematic measurements are characterized by no repeated measurements the model must bear as unique unknown the delay time for every measurand at a discrete measurement point. Therefore the measurand itself must be expressed as a function of the delay time. To reach this aim the modeling is developed on quaternion-based rotations.

A. Background of Quaternions

A quaternion may be regarded as 4-tupel of real numbers, that is, as an element of $\Re^4$ [5]. A quaternion is defined as the sum

$$q = q_0 + \mathbf{q} = q_0 + iq_x + jq_y + kq_z \quad . \qquad (2)$$

In this sum q_0 is called the scalar part and **q** the vector part of the quaternion. The scalars q_0, q_x, q_y and q_z are called the components of the quaternion. Quaternions have there own algebra and the non-commutative multiplication of two quaternions p and q are given in (3).

$$pq = p_0 q_0 - \mathbf{p} \cdot \mathbf{q} + p_0 \mathbf{q} + q_0 \mathbf{p} + \mathbf{p} \times \mathbf{q} \qquad (3)$$

For a rotation in $\Re^3$ the quaternion has to be a unit quaternion. The triple product (4) with the quaternion q, the complex conjugate quaternion q^* and the pure quaternion p (p=(0,**p**), $\mathbf{p} \in \Re^3$) delivers as result a pure quaternion w

$$w = qpq^* \quad . \qquad (4)$$

(4) can summarized in the matrix Q (6) – with the quaternion terms of q – and the multiplication with vector $\mathbf{p} \in \Re^3$ results directly the vector $\mathbf{w} \in \Re^3$.

$$\mathbf{w} = Q\mathbf{p} \tag{5}$$

$$Q = \begin{pmatrix} 2q_0^2 - 1 + 2q_x^2 & 2q_x q_y - 2q_0 q_z & 2q_x q_z + 2q_0 q_y \\ 2q_x q_y + 2q_0 q_z & 2q_0^2 - 1 + 2q_y^2 & 2q_y q_z - 2q_0 q_x \\ 2q_x q_z - 2q_0 q_y & 2q_y q_z + 2q_0 q_x & 2q_0^2 - 1 + 2q_z^2 \end{pmatrix} \tag{6}$$

For any unit quaternion (2) the components can be expressed by

$$q_0 = \cos\left(\frac{\theta}{2}\right), \quad \mathbf{q} = \mathbf{u}\sin\left(\frac{\theta}{2}\right) \tag{7}$$

with θ as rotation angle and **u** as unit vector in the direction of **q**. The triple product (4) may be interpreted geometrically as a rotation of the vector **p** through an angle θ, about **q** as the rotation axis, to the new position of the vector **w** (Fig. 4).

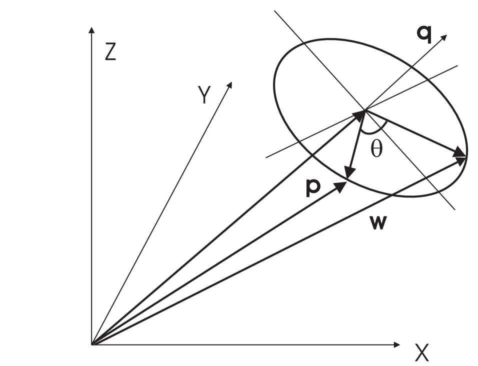

Fig. 4 – Quaternion rotation: rotation axis vector **q**, rotation angle θ.

B. Polar model with two rotations

For a polar measurement system a trajectory which consists of two rotations can be used. The advantage is that the horizontal angle range of the measurement system will be used completely. The first rotation is the movement of the rotating arm and the second one is the rotation of the direct drive under the test item. If the test item, e.g. a robotic-tacheometer, has locked a prism on the rotating arm and the rotating arm starts rotation, the test item follows the prism. The starting point of the rotating arm system is the so called *homepoint* $\mathbf{p_{D,1}}=(r,0,0)^T$, where r is the known radius of the rotating arm (Fig. 5). For the rotation of the point $\mathbf{p_{D,1}}$ to the position $\mathbf{p_{D,2}}$ the triple product (8) will be used. For the quaternion q_1 in (8) the trigonometric form will be inserted. The rotation axis of (9) corresponds to the z-axis of the rotating arm system. The rotation angle which is used in (4) will be replaced by the relation (10). Through the control system of the calibration system the angular velocity ω_D of the rotating arm is known.

$$p_{D,2} = q_1 p_{D,1} q_1^* \tag{8}$$

$$q_1 = \left(\cos\left(\frac{t\omega_D}{2}\right), \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \sin\left(\frac{t\omega_D}{2}\right) \right) \tag{9}$$

$$\theta = t\omega_D \tag{10}$$

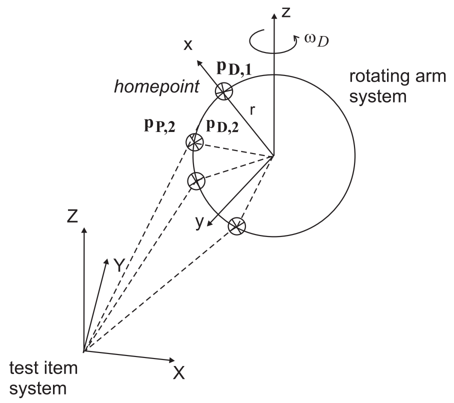

Fig.5 – First rotation by the rotating arm, with homepoint and test item system.

The next step is to transform (8) from the rotating arm system to the test item system. In [6] a co-ordinate transformation is described using unit quaternions. In a static reference measurement the quaternions q_R and p_{tr} are determined for the rotation and translation between both co-ordinate systems. The result of the co-ordinate transformation for (8) is the new pure quaternion (11)

$$p_{P,2} = q_R p_{D,2} q_R^* + p_{tr} = q_R \left(q_1 p_{D,1} q_1^* \right) q_1^* + p_{tr}, \tag{11}$$

which can be directly assigned as a vector $\mathbf{p_{P,2}}$ in $\Re^3$. In this way every position on the rotating arm can be expressed by (11) and with the known angular velocity in (9) the vector $\mathbf{p_{P,2}}$ depends only on the time.

The second rotation results from the direct drive under the test item. If the direct drive starts with rotation the test item must countersteer to keep the prism. This second rotation means for a test item – a polar measurement system – that the horizontal angle is enlarged. If only the second rotation were used the measuring result of the test item is a circle, with the measured distance as radius and the test item is then the center of the circle. Fig. 6 illustrates such a situation. If the prism changes the position on the rotating arm, a new circle with the measured distance is resulted. For this rotation a further quaternion q_2 (12) will be used. The rotating axis corresponds to the Z-axis of the test item co-ordinate system and the rotation angle is build by the relation between the angular velocity ω_P and the time. Through the control system of the calibration system the angular velocity ω_P of the second direct drive is known again.

$$q_2 = \left(\cos\left(\frac{t\omega_P}{2}\right), \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \sin\left(\frac{t\omega_P}{2}\right) \right) \qquad (12)$$

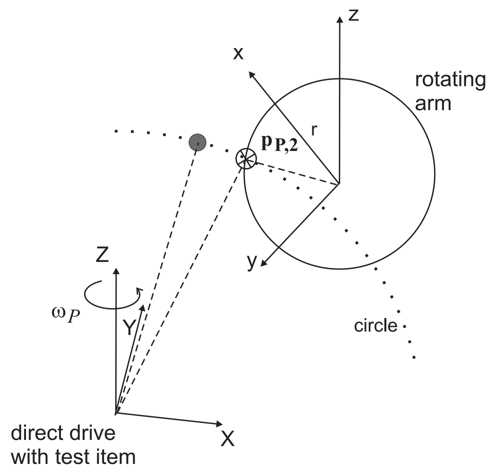

Fig.6 – Second rotation: rotating arm and direct drive with test item

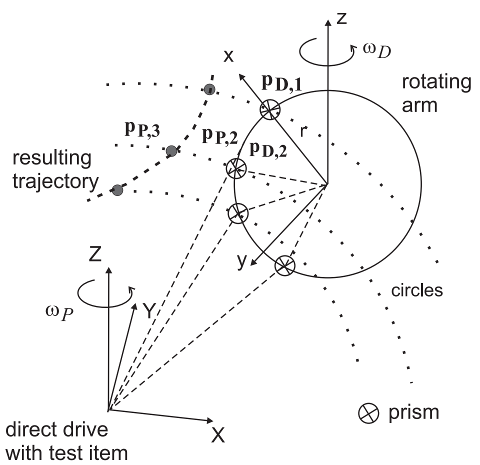

Fig.7 – Two rotations: rotating arm and direct drive with test item, resulting trajectory

If both – rotating arm and the second direct drive – are rotating, a new trajectory is resulted. Fig. 7 shows, in principle, the formation of a new trajectory, which is neither a circle of the rotating arm nor a circle with the test item as center. The starting point is the homepoint $\mathbf{p}_{P,1}$. Through the rotation of the rotating arm the prism change the position to $\mathbf{p}_{P,2}$ and at the same time the rotated test item must countersteer and therefore the horizontal angle will be changed and the result is position $\mathbf{p}_{P,3}$. The complete resulting trajectory primarily depends on the angular velocity of both direct drives and the arrangement of the tiltable rotating arm. Fig. 8 shows an ideal trajectory which results from an angular velocity of 30°/s for the rotating arm arranged in a vertical position and 50°/s for the direct drive under the test item. Another example shows Fig. 9 for a the rotating arm arranged in a horizontal position with an angular velocity of 35°/s and 10°/s for the second direct drive.

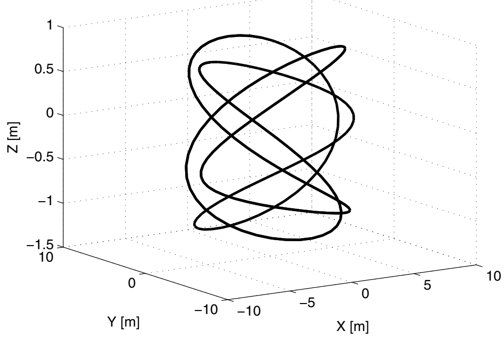

Fig.8 – resulting ideal trajectory: angular velocity 30°/s for the rotating arm arranged in a vertical position and 50°/s for the second direct drive

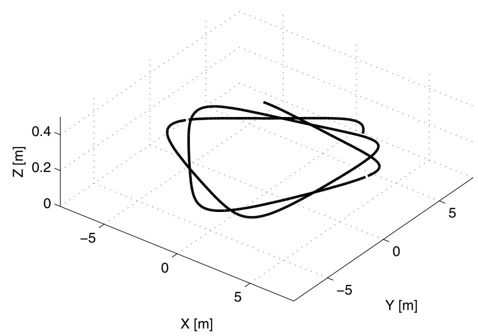

Fig.9 – resulting ideal trajectory: angular velocity 35°/s for the rotating arm arranged in a horizontal position and 10°/s for the second direct drive

The position $\mathbf{p}_{P,3}$ of the new trajectory (Fig. 7) is developed with the quaternion q_2 (12). The triple product (13) consist of the quaternion (12) and the pure quaternion (8)

$$\begin{aligned} p_{p,3} &= q_2 p_{P,2} q_2^* \\ &= (q_1 q_R q_2) p_{D,1} (q_1 q_R q_2)^* + q_1 q_{tr} q_1^* \end{aligned} \qquad (13)$$

The expanded result include the quaternion (9), the quaternion q_R and q_{tr} and shows the efficient concatenation of multiple rotations. The result of (13) is again a pure quaternion and therefore the vector $\mathbf{p_{P,3}}$ is assigned directly. In this way every discrete position of the trajectory can be expressed by the triple product (13) respectively through the vector $\mathbf{p_{P,3}}$. Every measurand of a polar measurement system can be determined now as a function of the time t, which is, as a delay time, the only unknown in this function. The last step is now the determination of this delay time.

C. Determination of the delay time

The delay time is defined in section II. The time referencing of the 4D calibration system delivers the time for every measurement request of the test item. From vector $\mathbf{p_{P,3}}$ every polar measurand can be calculated, e.g. the distance

$$s\left(t_{d,i}\right)=p_{P,3,X}^{2}+p_{P,3,Y}^{2}+p_{P,3,Z}^{2} \tag{14}$$

In this case the measured distance s_i is known, while the delay time $t_{d,i}$ is unknown. The non-linear equation cannot be solved analytical, but must be solved numerically by Newton's method. The iterative solution follows from the definition of recursion of Newton Iteration [7]

$$\left(t_{d,i}\right)_{k+1}=\left(t_{d,i}\right)_{k}-\frac{f\left(\left(t_{d,i}\right)_{k}\right)}{\dot{f}\left(\left(t_{d,i}\right)_{k}\right)} \tag{15}$$

$$\dot{f}\left(\left(t_{d,i}\right)_{k}\right)\neq 0 \qquad k=0,1,2,...$$

For the example of the distance measurement the function in (15) has the following form

$$f\left(\left(t_{d,i}\right)_{k}\right)=p_{P,3,X}^{2}+p_{P,3,Y}^{2}+p_{P,3,Z}^{2}-s_{i}^{2} \tag{16}$$

The initial value for the delay time is the time of the measurement request. This value will be close to the unknown value of the delay time and a solution will be found after a few iteration steps. This determination process will be done for every discrete measuring position and can be used for all polar measurands.

V. EXEMPLARY MEASURING RESULT

In this section an exemplary result is presented for the delay time determination based on the polar model with two rotations. The result shall be shown the effectiveness of the quaternion-based modeling. More detailed results and explanations are presented in [1].

A Leica robotic-tacheometer TCRA1201 is used as test item. The time referencing is realized about a serial interface (cf. section III B). In this case a reference time for the measuring request and reply will be received by the time-referenced calibration system. The rotating arm is arranged in a horizontal position. The kinematic measuring is executed by an angular velocity of 35°/s for the rotating arm and 10°/s for the direct drive under the test item. The ideal trajectory for seven circulations of the rotating arm shows Fig. 9. The robotic-tacheometer follows the prism while it will be rotated through the direct drive. In this way the robotic-tacheometer use its complete horizontal angle array. In this constellation between test item and rotating arm the delay time of the horizontal angle and the distance is determined. For every discrete measuring position the delay time is calculated in relation to the time of measuring request.

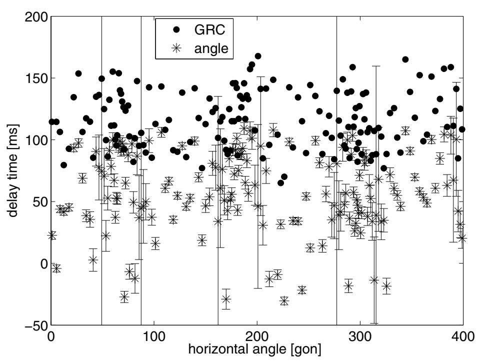

Fig.10 – delay time of the horizontal angle in relation to the time of measuring request. GRC is the time of measuring reply.

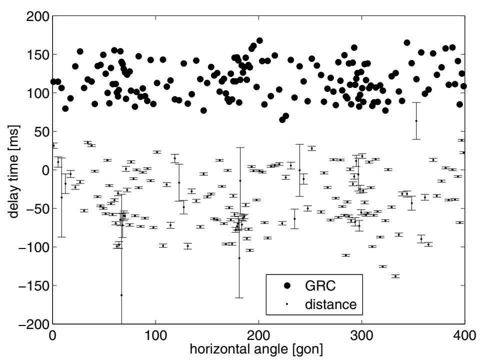

Fig. 11 – delay time of the distance in relation to the time of measuring request. GRC is the time of measuring reply.

Fig. 10 shows the result for the delay times of the horizontal angle about one complete rotation of the robotic-tacheometer. The delay times scatter between -30 ms and 100 ms. The time of measuring reply

(GRC) shows the variation in every single measurement period.

Fig. 11 shows the result for the delay times of the distance. The delay times scatter between -125 ms and 35 ms. The predominant values are negative and that means that the measurement values result before the actual measurement request. Such a result can be achieved only with a time-referencing, because the time of measurement request represents the reference time, which is known by the calibration system, and the delay time is than calculated in relation to this reference.

VI. CONCLUSION

By using kinematic optical measuring systems delay times will lead to deviations in space-time position determination of moving object point. A time-referenced 4D calibration system was developed for the determination of delay times. The rotating arm of the calibration system bases on a rotary direct drive and a real-time control system for position and time. Especially for polar measurement systems the calibration system is expanded by a second direct drive under the test item. The modeling for the delay time determination based on the theory of quaternions. The aim of the modeling was to obtain a function for every measurand of a measurement system, which depends only on the delay time. In this paper the polar model for two rotations, the rotating arm and the second direct drive, has been presented. The time-referencing with trigger or serial interface can assign a time to every measurement request of the test item. This time can also be used as start value for the Newton Iteration for the calculation of the delay time. First measuring results have shown the successful modeling based on the theory of quaternions.

REFERENCES

[1]. Depenthal C. Entwicklung eines zeitreferenzierten 4-D Kalibriersystems für optisch kinematische Messsysteme. Dissertation Universität Karlsruhe (in print) 2008.

[2]. Depenthal C. A Time-referenced 4D Calibration System for Kinematic Optical Measuring Systems. Proceedings of 1st International Conference on Machine Control & Guidance, 24-26 June 2008, Zurich, Switzerland

[3]. Wörn H., Bringschulte U. Echtzeitsysteme. Springer-Verlag Berlin Heidelberg, 2005.

[4]. Mächtel M. Entstehung von Latenzzeiten in Betriebssystemen und Methoden zur Messtechnischen Erfassung. Fortschritt-Berichte VDI Reihe 8 Nr. 808, VDI-Verlag Düsseldorf, 2000.

[5]. Kuipers J. B. Quaternions and Rotation Sequences - A Primer with Applications to Orbits, Aerospace, and Virtual Reality. Princeton University Press, 1999.

[6]. Horn B. K. P. Closed-form solution of absolute orientation using unit quaternions. Journal of the Optical Society of America A, Vol.4, No.4 p. 629-642, 1987.

[7]. Bronstein I. N., Semendjajew K. A. Taschenbuch der Mathematik. 25. durchgesehene Auflage, Grosche G (Hrsg.), Ziegler V (Hrsg.), B G Teubner Verlagsgesellschaft, 1991

Dipl.-Ing. Claudia Depenthal
Geodetic Institute
University Karlsruhe, Germany
E-mail: depenthal@gik.uka.de