On the Complexity of Scheduling with Power Control in Geometric SINR*

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Abstract

Although being a very fundamental problem in the field of wireless networks, the complexity of transmission scheduling with power control in the Geometric SINR model is still unknown. In this article, we show that the joint problem of finding transmission powers and scheduling the transmissions is NP-hard if the available powers are bounded, independent of the actual bounds. This also implies that scheduling with a finite number of power levels is NP-hard.

1 Introduction

One of the most fundamental problems in the field of wireless networks is the so-called scheduling problem. Given a set of transmission requests, the task is to distribute the wireless transmissions to time slots such that all transmission in the same slot can be active simultaneously without failures. Thus, one has to consider the interferences between the different transmissions. There have been many models proposed for modeling interference between concurrent transmissions. One of them is the Signal-To-Interference-Plus-Noise-Ratio model (SINR model). The SINR model is physically motivated and is believed to be reasonably realistic. A transmission is assumed to be successful if the ratio of the received signal strength and the sum of the interferences plus background noise is sufficiently high. Often, it is assumed that the signal strength is determined by the distance between sender and receiver. In this case, one speaks of the Geometric SINR (SINR_G) model.

The complexity of scheduling in the Geometric SINR model was first studied in [3]. The authors proved that the SCHEDULING problem and the ONE-SHOT SCHEDULING problem, the problem to fit as many transmissions as possible to the same time slot, are NP-hard if all senders use uniform transmission powers. Moreover, approximation algorithms for both problems were given. However, the complexity of scheduling with non-uniform transmission powers stayed unresolved and analyzing the complexity of the joint problem of power control and scheduling was proposed as an exciting research direction. In [5], the complexity of scheduling with power control was mentioned as one of five very essential problems to understanding sensor networks. Despite the attention that this problem recently received in the community, the complexity is still unresolved.

As main contribution, we prove the NP-hardness of scheduling with power control (SCHEDPC) and one-shot scheduling with power control (ONESHOTSCHEDPC), for the case that the available

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transmission powers are bounded. In particular, we show hardness for the case that among the available transmission powers there is a minimum power $P_{\min} > 0$ and a maximum power $P_{\max} < \infty$. This encloses all problems of practical relevance such as the problem where the transmission powers can be chosen from a finite set $\{P_{\min} = P_1, P_2, \ldots, P_j = P_{\max}\} \subset [P_{\min}, P_{\max}]$ of *power levels*. From a practical point of view, the limitation on arbitrary power bounds is not restricting, as wireless hardware usually has a maximum transmission power, and due to ambient noise every successful transmission has to exceed some minimum transmission power. Our proofs extend the NP-hardness proofs given in [3] for scheduling with uniform powers to the problem of scheduling with power control. The main idea is to construct the problem instances such that some senders are forced to transmit with minimum power $P_{\min}$, while the remaining senders are forced to use the maximum power $P_{\max}$. Thus, we know all transmission powers in advance and can use a construction similar to the one in [3].

2 Models and Notations

Our models and notations are similar to those used in [3] for the NP-hardness proof of scheduling without power control. The main distinction is that we allow arbitrary transmission powers for all senders.

In scheduling problems, we are given a set $L = \{l_1, \ldots, l_n\}$ of links, where each link $l_i = (s_i, r_i)$ represents a communication request from a sender s_i to a receiver r_i. The senders and receivers are distributed in the Euclidean plane and the distance between two nodes s_i, r_j is denoted by $d_{ij} = d(s_i, r_j)$. Thus, d_{ii} denotes the distance between a sender s_i and its corresponding receiver r_i.

Unlike [3], we do not assume that the senders use uniform transmission powers, but consider the choice of transmission powers P_{s_i} to be part of the addressed optimization problems.

The signal power $S_{r_j}(s_i)$ received at r_j from sender s_i depends on the transmission power P_{s_i} of s_i and the distance d_{ij} between nodes s_i and r_j. This article is based on the *Geometric Signal-to-Interference-plus-Noise-Ratio* model ($\mathrm{SINR_G}$ model). In the $\mathrm{SINR_G}$ model, it is assumed that the signal strength falls off with $d_{ij}^{-\alpha}$, i.e., $S_{r_j}(s_i) = P_{s_i}/d_{ij}^{\alpha}$. The *path-loss exponent* α defines how fast the signal strength decreases with increasing distance and depends on external conditions of the medium. As usual, it is assumed that $\alpha > 2$. Every sender s_j that sends concurrently with s_i causes an interference $I_{r_i}(s_j) = S_{r_i}(s_j)$ at the receiver r_i of link l_i. The notation $I_{r_i}(s_j)$ is used in order to highlight that we talk about interference and not about a useful signal. It is assumed that all interferences accumulate. The *total interference* I_{r_i} experienced by receiver r_i is given as the sum of all interferences caused by concurrently sending nodes, i.e., $I_{r_i} := \sum_{s_j \neq s_i} I_{r_i}(s_j)$. Furthermore, it is assumed that every receiver is exposed to an ambient noise with power N. In the $\mathrm{SINR_G}$ model, a transmission (s_i, r_i) is successful if and only if the ratio of the received signal strength $S_{r_i}(s_i)$ and the total interference I_{r_i} plus background noise N exceeds some minimum *SINR* β, i. e.,

$$SINR(r_i) = \frac{S_{r_i}(s_i)}{I_{r_i} + N} = \frac{\frac{P_{s_i}}{d_{ii}^{\alpha}}}{\sum_{j \neq i} \frac{P_{s_j}}{d_{ji}^{\alpha}} + N} \geq \beta, \tag{1}$$

with $\beta > 1$. In the following, we ignore the influence of background noise ($N = 0$) without loss of generality.

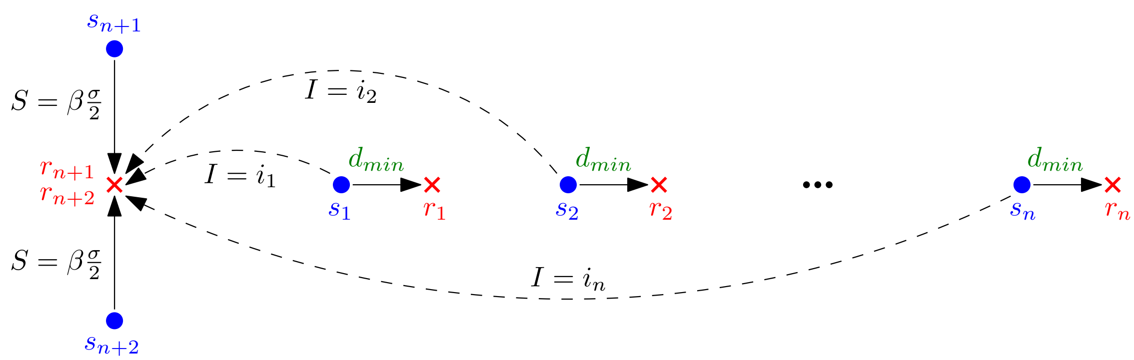

Figure 1: Reduction of Partition to SchedPC

3 Scheduling with Power Control (SchedPC)

In the problem of scheduling with power control (SchedPC), we are given a set $L = \{l_1, \ldots, l_n\}$ of transmission requests, as well as lower and upper bounds $P_{\min}$ and $P_{\max}$, $P_{\min} \neq P_{\max}$, on the available transmission powers. The aim is to assign every sender s_i a transmission power $P_{s_i} \in [P_{\min}, P_{\max}]$, and to distribute all requests of L to time slots such that all transmissions in the same slot can be executed simultaneously with the designated transmission powers. At this, a transmission $l_i = (s_i, r_i)$ is successful if and only if SINR inequality (1) is fulfilled. A *schedule* $\mathcal{S} = (\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_T)$ is a partition of L, where S_t denotes the set of links assigned to time slot t. T denotes the *length* of the schedule. Every sender is only active in the assigned time slot. A *power assignment* $\mathcal{P}$ is a function $\mathcal{P} : \{s_i | (s_i, r_i) \in L\} \to [P_{\min}, P_{\max}]$ that assigns to every sender s_i a valid transmission power $P_{s_i} := \mathcal{P}(s_i)$. A schedule $\mathcal{S}$ is said to be *valid* with respect to a power assignment $\mathcal{P}$ if all transmissions are successful in their corresponding time slots, using the designated powers. The SchedPC problem for $(L, P_{\min}, P_{\max})$ is to find a power assignment $\mathcal{P}$ and a valid schedule $\mathcal{S}$, such that $\mathcal{S}$ has minimal length among all valid power assignments and schedules.

In the following, we extend the NP-hardness proof for scheduling without power control, given in [3], to the more general SchedPC problem. In particular, we show that SchedPC is NP-hard for arbitrary $P_{\min}$, $P_{\max}$.

To this extent, we will give a polynomial time reduction of an NP-complete problem to SchedPC, the Partition problem, which has been shown to be NP-complete in [4]: Given a set $\mathcal{I} = \{i_1, \ldots, i_n\}$ of integers, find $\mathcal{I}_1, \mathcal{I}_2 \subset \mathcal{I}$ such that $\mathcal{I}_1 \cap \mathcal{I}_2 = \emptyset$ and

$$\sum_{i_j \in \mathcal{I}_1} i_j = \sum_{i_j \in \mathcal{I}_2} i_j = \frac{1}{2} \sum_{i_j \in \mathcal{I}} i_j \ .$$

Let $\mathcal{I} = \{i_1, \ldots, i_n\}$ be an instance of Partition. Without loss of generality, we assume that all elements are distinct and positive and we set $\sum_{j=1}^{n} i_j = \sigma$. In order to solve the Partition problem for $\mathcal{I}$, we construct an instance $L_{\mathcal{I}} = \{l_1, \ldots, l_{n+2}\}$ of SchedPC with $n+2$ links for arbitrary $P_{\min}$ and $P_{\max}$ as follows. We will prove that there exists a schedule of length 2 if and only if the Partition instance $\mathcal{I}$ has a solution.

What makes it hard to find an NP-hardness proof for SchedPC is that every sender can arbitrarily choose its transmission power from an interval of possible powers. In order to handle this, we construct our SchedPC instance such that some of the senders have to use transmission power $P_{\max}$, and the remaining senders have to use power $P_{\min}$. Thus, we know all the transmission powers in advance. This construction is shown in Figure 1.

For every integer $i_j \in \mathcal{I}$ we introduce a link $l_j = (s_j, r_j)$. Every sender s_j is placed at position

$$\mathrm{pos}(s_j) = \left(\left(\frac{P_{\min}}{i_j} \right)^{1/\alpha}, 0 \right), \quad \forall 1 \le j \le n \ .$$

The position is chosen such that the interference caused at the origin $(0, 0)$ of the coordinate system equals i_j when s_j sends with power $P_{\min}$. Next, we place the receivers such that every transmission l_j can be executed successfully, even if s_j sends with power $P_{\min}$ and every other sender sends with power $P_{\max}$. For this, every sender-receiver-pair has to be placed sufficiently close together. As we will show later, a distance

$$d_{\min} = P_{\min}^{1/\alpha} \cdot \frac{\frac{1}{(i_{\max}-1)^{1/\alpha}} - \frac{1}{i_{\max}^{1/\alpha}}}{1 + (\frac{P_{\max}}{P_{\min}} n\beta)^{\frac{1}{\alpha}}} \ ,$$

where $i_{\max}$ is the maximum value in $\mathcal{I}$, is sufficient. Thus, we place every receiver $r_i, 1 \le i \le n$, at position

$$pos(r_i) = pos(s_i) + (d_{\min}, 0) \ .$$

Finally, we have to place l_{n+1} and l_{n+2}. We positioned the senders $s_1, \ldots, s_n$ such that the interference which they cause at the origin is proportional to $i_1, \ldots, i_n$. In order to take advantage of this property, we place r_{n+1} and r_{n+2} at the origin.

$$pos(r_{n+1}) = pos(r_{n+2}) = (0, 0) \ ,$$

Last, we place their senders s_{n+1} and s_{n+2} perpendicular to the x-axis at distance $(2P_{\max}/\beta\sigma)^{1/\alpha}$, i.e.,

$$pos(s_{n+1}) = \left(0, \left(\frac{2P_{\max}}{\beta \cdot \sigma} \right)^{1/\alpha} \right) \ ,$$

$$pos(s_{n+2}) = \left(0, -\left(\frac{2P_{\max}}{\beta \cdot \sigma} \right)^{1/\alpha} \right) \ .$$

In the following, we show that

- in every 2-slot solution of the SchedPC problem, the senders $s_1, \ldots, s_n$ have to use transmission power $P_{\min}$,
- in every 2-slot solution of the SchedPC problem, the senders s_{n+1} and s_{n+2} have to use transmission power $P_{\max}$,
- there exists a 2-slot solution to the constructed SchedPC instance if and only if the Partition problem has a solution, and
- every 2-slot solution of the SchedPC instance implies a solution to the corresponding Partition problem.

Let us start with some observations: As r_{n+1} and r_{n+2} share the same position, l_{n+1} and l_{n+2} cannot be scheduled simultaneously. Thus, every schedule needs at least two slots. Moreover, s_{n+1} and s_{n+2} have the same distance to every receiver.

Lemma 1 *Every transmission l_i, $1 \leq i \leq n$, is successful with transmission power $P_{\min}$, no matter how many other links are active concurrently and no matter which transmission powers they use.*

Proof. Obviously, the worst thing that can happen is that all senders s_j, $1 \leq j \leq n \wedge j \neq i$, and one of the senders s_{n+1}, s_{n+2}, transmit concurrently with power $P_{\max}$. Let $L_i = \{l_j | 1 \leq j \leq n{+}1, i \neq j\}$. Since the positions of the sender nodes $s_1, \ldots, s_n$ depend on the values of $i_1, \ldots, i_n$, we can determine the minimum distance between two senders s_j and s_k, $j \neq k$,

$$\begin{aligned} d(s_j, s_k) &= |d(s_j, r_{n+1}) - d(s_k, r_{n+1})| && (2)\\ &= \left| \left(\frac{P_{\min}}{i_j}\right)^{\frac{1}{\alpha}} - \left(\frac{P_{\min}}{i_k}\right)^{\frac{1}{\alpha}} \right| && (3)\\ &\geq P_{\min}^{\frac{1}{\alpha}} \left(\frac{1}{(i_{\max} - 1)^{1/\alpha}} - \frac{1}{i_{\max}^{1/\alpha}} \right) && (4) \end{aligned}$$

Thus, the sender s_j closest to r_i, $i \neq j$, is located at least at distance $d(s_j, s_i) - d_{\min}$ from r_i. All other senders (including s_{n+1} and s_{n+2}) are farther away. Now, we can show a lower bound for $SINR(r_i)$:

$$\begin{aligned} SINR(r_i) &= \frac{\frac{P_{s_i}}{d_{ii}^{\alpha}}}{\sum_{l_j \in L_i} \frac{P_{s_j}}{d_{ji}^{\alpha}}} && (5)\\ &\geq \frac{\frac{P_{\min}}{d_{ii}^{\alpha}}}{\sum_{l_j \in L_i} \frac{P_{\max}}{d_{ji}^{\alpha}}} && (6)\\ &\geq \frac{\frac{P_{\min}}{d_{\min}^{\alpha}}}{\frac{nP_{\max}}{(d(s_j,s_i)-d_{\min})^{\alpha}}} && (7)\\ &\geq \frac{1}{n}\frac{P_{\min}}{P_{\max}} \left(\left(1 + \left(\frac{P_{\max}}{P_{\min}} n\beta\right)^{\frac{1}{\alpha}}\right) - 1 \right)^{\alpha} && (8)\\ &= \beta && (9) \end{aligned}$$

□

Lemma 2 *There exists a solution to a* Partition *problem $\mathcal{I}$ if and only if there exists a 2-slot schedule for $L_{\mathcal{I}}$. In the corresponding schedule, the senders $s_1, \ldots, s_n$ have to use transmission power $P_{\min}$ and the senders s_{n+1} and s_{n+2} have to use transmission power $P_{\max}$.*

Proof. We start with showing that every solution to the Partition problem implies a valid 2-slot schedule with corresponding power assignment $\mathcal{P}$. Let us assume that we know two subsets $\mathcal{I}_1, \mathcal{I}_2 \subset \mathcal{I}$, whose elements sum up to $\sigma/2$. For every $i_j \in \mathcal{I}_1$, we assign the link l_j to the first time slot. Moreover, we assign l_{n+1} to the first time slot. The remaining links are assigned to the second time slot. We set $P_{s_1} = P_{s_2} = \cdots = P_{s_n} = P_{\min}$ and $P_{s_{n+1}} = P_{s_{n+2}} = P_{\max}$. We know from Lemma 1 that transmissions $l_1, \ldots, l_n$ are always successful, so let us focus on the receivers r_{n+1} and r_{n+2}. The situation is the same for both receivers, so it suffices to examine r_{n+1}. The signal power r_{n+1} receives from its sender node s_{n+1} is

$$S_{r_{n+1}}(s_{n+1}) = \frac{P_{\max}}{\left(\left(\frac{2P_{\max}}{\beta\sigma}\right)^{\frac{1}{\alpha}}\right)^{\alpha}} = \frac{\beta\sigma}{2}.$$

Besides, r_{n+1} experiences from each sender s_j the interference

$$I_{r_{n+1}}(s_j) = \frac{P_{\min}}{\left(\left(\frac{P_{\min}}{i_j}\right)^{\frac{1}{\alpha}}\right)^{\alpha}} = i_j.$$

This results in a total interference of

$$I_{r_{n+1}} = \sum_{i_j \in \mathcal{I}_1} i_j = \frac{\sigma}{2}.$$

For the *SINR* at r_{n+1} we get

$$SINR(r_{n+1}) = \frac{S_{r_{n+1}}(s_{n+1})}{I_{r_{n+1}}} = \frac{\beta\sigma/2}{\sigma/2} = \beta.$$

Altogether, the constructed 2-slot schedule is valid for the given power assignment $\mathcal{P}$. It is easy to see that $\mathcal{P}$ is the only possible power assignment for the given schedule. If s_{n+1} or s_{n+2} would send with less power than $P_{\max}$, the corresponding *SINR* would fall below β. The same thing would happen if one of the other senders would use a transmission power above $P_{\min}$.

Finally, we have to show that we cannot find a 2-slot schedule for $L_{\mathcal{I}}$ if there does not exist a solution to the PARTITION problem. No solution to the PARTITION problem implies that for every partition of $\mathcal{I}$ into two subsets, the sum of one set is greater than $\sigma/2$. This means that, even if all senders other than s_{n+1} and s_{n+2} use $P_{\min}$, the minimum transmission power possible, the interference at r_{n+1} in slot 1 or the interference at r_{n+2} in slot 2 exceeds $\sigma/2$. Even if s_{n+1} and s_{n+2} send with the maximum transmission power $P_{\max}$, the signal only arrives at the receivers with power $\beta\sigma/2$. Thus, the *SINR* of either l_{n+1} or l_{n+2} is below β and the transmission fails. □

From the above construction and observations, we can conclude the main theorem of this section:

Theorem 1 *The* SCHEDPC *problem in* SINR_G *is NP-hard.*

4 One-Shot Scheduling with Power Control (OneShotSchedPC)

Instead of asking for a shortest schedule for a given set of links, one can ask for a maximum number of transmissions to be carried out in a single time slot. This problem is named ONESHOTSCHEDPC problem, and like in the SCHEDPC problem, we are given a set $L = \{l_1, \ldots l_n\}$ of links and upper and lower bounds $P_{\max}$ and $P_{\min}$ on the available transmission powers. Additionally, all links are

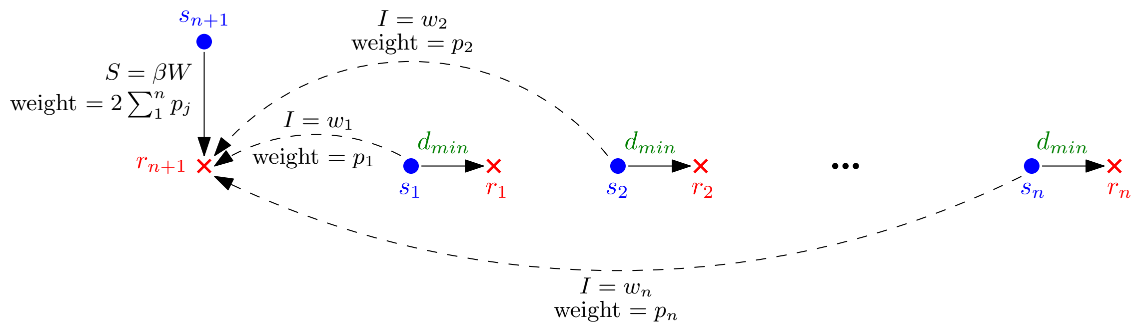

Figure 2: Reduction of Knapsack to OneShotSchedPC

weighted, i. e., we are given a weight w_i for every link l_i. Now, in the OneShotSchedPC problem, we try to fill one single slot as good as possible. Thus, the objective is to find a subset $\mathcal{S} \subseteq L$ as well as a power assignment $\mathcal{P}$ such that all links in $\mathcal{S}$ can be scheduled concurrently and $\mathcal{S}$ maximizes the total weight $\sum_{l_j \in \mathcal{S}} w_j$.

The One-Shot Scheduling problem without power control was proved to be NP-hard in the SINR_G model in [3]. In the following, we extend this proof to the decision problem of One-Shot Scheduling with Power Control (OneShotSchedPC).

Again, we give a polynomial time reduction from an NP-complete problem. As in [3], we give a reduction from the well-known Knapsack problem [2], where we are given a set of items, $X = \{x_1, \ldots, x_n\}$, with *values* p_j and *weights* w_j. The goal is to pick the most valuable set of items that does not exceed a given overall weight W. Formally, we aim at finding a set $X' \subseteq X$, with $\sum_{x_i \in X'} w_i \leq W$, that maximizes $\sum_{x_i \in X'} p_i$

We give a polynomial time reduction from Knapsack to OneShotSchedPC similar to the one from Partition to SchedPC. It is depicted in Figure 2. Given an instance $\mathcal{I}$ of Knapsack, we use $n+1$ links $L_{\mathcal{I}} = \{l_1, \ldots, l_{n+1}\}$ and arbitrary but fixed $P_{\min}$ and $P_{\max}$. The first n links $l_1, \ldots, l_n$ represent the items of the Knapsack problem. Link l_{n+1} enforces the connection between optimal solutions of OneShotSchedPC and optimal solutions of Knapsack. Without loss of generality, we assume that all items have distinct integer weights. This time, we place the senders $s_1, \ldots, s_n$ such that the received power from s_i at the origin $(0, 0)$ equals w_i if s_i transmits with power $P_{\min}$, i.e.,

$$pos(s_i) = \left(\left(\frac{P_{\min}}{w_i}\right)^{1/\alpha}, 0\right), \quad \forall 1 \leq j \leq n \ .$$

Again, we make sure that $l_1, \ldots, l_n$ can be scheduled with power $P_{\min}$, regardless of all other links. Thus, we have to put r_i close enough to s_i. As in Section 3, the sender-receiver distance

$$d_{\min} = P_{\min}^{\frac{1}{\alpha}} \cdot \frac{\frac{1}{(w_{\max}-1)^{1/\alpha}} - \frac{1}{w_{\max}^{1/\alpha}}}{1 + (\frac{P_{\max}}{P_{\min}} n\beta)^{\frac{1}{\alpha}}} \ ,$$

where $w_{\max}$ is the largest weight in the Knapsack instance, is sufficient. This gives

$$pos(r_i) = pos(s_i) + (d_{\min}, 0) \ .$$

Thereafter, we have to place the additional link l_{n+1}. The receiver r_{n+1} is placed at $(0, 0)$

$$pos(r_{n+1}) = (0, 0)$$

and we place the sender s_{n+1} such that the received power at $(0,0)$ is βW if s_{n+1} sends with maximum power P_{max}:

$$pos(s_{n+1}) = \left(0, \left(\frac{P_{\max}}{\beta W}\right)^{1/\alpha}\right)$$

Finally, we have to assign the links appropriate weights. The links $l_1, \ldots, l_n$ are assigned the value of the corresponding item:

$$weight(l_i) = p_i, \quad \forall 1 \le i \le n \tag{10}$$

The weight of our special link l_{n+1} is set to twice the value of all items

$$weight(l_{n+1}) = 2 \cdot \sum_{j=1}^{n} p_j,$$

in order to make sure that l_{n+1} is part of every optimal solution of the OneShotSchedPC instance.

Lemma 3 *Let* $(\mathcal{S}_{\text{OPT}}, \mathcal{P}_{\text{OPT}})$ *with schedule* $\mathcal{S}_{\text{OPT}}$ *and power assignment* $\mathcal{P}_{\text{OPT}}$ *be an optimum solution of* OneShotSchedPC *instance* $(L_{\mathcal{I}}, P_{\min}, P_{\max})$*. Then,* $(\mathcal{S}_{\text{OPT}}, \mathcal{P}^*)$ *with* $P_{s_1} = P_{s_2} = \cdots = P_{s_n} = P_{\min}$ *and* $P_{n+1} = P_{\max}$ *is also an optimum solution of* $\mathcal{I}$*.*

Proof. If we set the transmission powers of $s_1, \ldots, s_n$ to $P_{\min}$ then we do not lose anything, as we defined $d_{\min}$ such that $l_1, \ldots, l_n$ are successful with power $P_{\min}$, no matter which other senders are active simultaneously and no matter which transmission powers they use. Moreover, we also can set $P_{n+1} = P_{\max}$, as this does not influence the links $l_1, \ldots, l_n$ and it only increases $SINR(r_{n+1})$. So the schedule $\mathcal{S}_{\text{OPT}}$ is also valid with respect to $\mathcal{P}^*$.

This means that we can assume without loss of generality that every optimum schedule has to be valid with power assignment $\mathcal{P}^*$. Now we use this property to show that every optimum schedule implies a valid solution to the Knapsack problem. In particular, we have to show that

$$\sum_{l_j \in \mathcal{S}_{\text{OPT}}} w_j \le W.$$

This follows from the SINR constraint of l_{n+1}:

$$\begin{aligned} SINR(r_{n+1}) &= \frac{S_{r_{n+1}}(s_{n+1})}{I_{r_{n+1}}} && (11) \\ &= \frac{P_{\max} / \left(\left(\frac{P_{\max}}{\beta W}\right)^{\frac{1}{\alpha}}\right)^{\alpha}}{\sum_{l_j \in \mathcal{S}_{OPT}} P_{\min} / \left(\left(\frac{P_{\min}}{w_j}\right)^{\frac{1}{\alpha}}\right)^{\alpha}} && (12) \\ &= \beta \cdot \frac{W}{\sum_{l_j \in \mathcal{S}_{OPT}} w_j} && (13) \end{aligned}$$

So, in order for l_{n+1} to be transmitted successfully, which means that $SINR(r_{n+1}) \ge \beta$, it must hold that $\sum_{l_j \in \mathcal{S}_{\text{OPT}}} w_j \le W$. Moreover, it is easy to verify that every solution X' of the Knapsack problem with value V implies a solution $(\mathcal{S}, \mathcal{P}^*)$ of the corresponding OneShotSchedPC problem with value $V' = V + 2 \cdot \sum_{j=1}^{n} p_j$. Thus, thanks to the choice of weights in (10), every solution that maximizes the overall weight of our OneShotSchedPC instance at the same time maximizes the value of the corresponding solution to the underlying KNAPSACK problem. Altogether, we have shown that KNAPSACK is polynomial time reducible to ONESHOTSCHEDPC. □

Again, we conclude with the main theorem:

Theorem 2 ONESHOTSCHEDPC *in* SINR_G *is NP-hard.*

5 Power Control with a Finite Set of Power Levels

In modern hardware, the transmitters usually can choose their transmission power from a finite set of powers. Thus, the complexity of scheduling in this scenario has high practical relevance. The NP-hardness proofs given in the previous sections do not use the fact that available transmission powers formed a continous interval $[P_{\min}, P_{\max}]$ and easily extend to any case where the available transmission powers have a minimum $P_{\min} > 0$ and a maximum $P_{\max}$. This includes the case of a finite set $\{P_{\min} = P_1, \ldots, P_k = P_{\max}\}$ of available power levels.

6 Proving NP-completeness

In [3], the authors claim to prove the problems SCHEDULING and ONESHOTSCHEDULING to be in NP. They argue that a solution to these problems—which did not include the problem of finding power levels—, can be checked in polynomial time by comparing all signal-to-noise ratios to the given threshold β. What might hold true for a model of computation featuring real-valued arithmetics, links to a long-standing open problem in complexity theory: Even in the presumably easy case of $\alpha = 3$ and $\beta \in \mathbb{N}$ and all s_i and r_i placed on an integer grid, comparing signal-to-noise ratios to the threshold β boils down to the comparison of a sum of square roots of integers to an integer. Unfortunately, it is an open problem whether or not this can be done in polynomial time on a Turing machine, which is necessary to prove the above problems to be in NP [1]. At the time being, even the decision problem whether a set of nodes on the integer grid have a euclidean minimum spanning tree with a length bounded by a given integer cannot be claimed to be in P.

Since the problems considered in this paper are generalizations of the problems in [3], proving NP-completeness for SCHEDPC and ONESHOTSCHEDPC also calls for this gap in complexity theory to be closed.

7 Conclusion

We have shown that *scheduling with power control* and *one-shot scheduling with power control* are NP-hard, as long as we are given a maximum and minimum on the possible transmission powers. This also encloses the practically relevant cases where the senders are allowed to choose their power from a finite set of powers. The complexity of the more general problem with completely arbitrary transmission powers is still unresolved. However, this problem has no real practical relevance as the hardware usually sets an upper limit on the transmission power and the omnipresent background noise defines a lower limit on any reasonable transmission power.

References

[1] Q. Cheng. On Comparing Sums of Square Roots of Small Integers. In *Mathematical Foundations of Computer Science 06*, volume 4162 of *LNCS*, pages 250–255. Springer, 2006.

[2] M. R. Garey and D. S. Johnson. *Computers and Intractability : A Guide to the Theory of NP-Completeness (Series of Books in the Mathematical Sciences)*. W. H. Freeman, January 1979.

[3] O. Goussevskaia, Y. A. Oswald, and R. Wattenhofer. Complexity in geometric SINR. In *MobiHoc '07: Proceedings of the 8th ACM international symposium on Mobile ad hoc networking and computing*, pages 100–109, New York, NY, USA, 2007. ACM.

[4] R. M. Karp. Reducibility among combinatorial problems. In R. E. Miller and J. W. Thatcher, editors, *Complexity of Computer Computations*, pages 85–103. Plenum Press, 1972.

[5] T. Locher, P. von Rickenbach, and R. Wattenhofer. Sensor Networks Continue to Puzzle: Selected Open Problems. In *9th International Conference on Distributed Computing and Networking (ICDCN)*, Kolkata, India, Jan. 2008.