REPORT

No.: D3.3 – part 2

In-plane shear resistance of sandwich panels

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Symbols and notations

c	distance between fastenings at the transverse edge of a panel
d	nominal diameter of a fastener
d_1	minor diameter of the threaded part of a fastener
d_S	diameter of the unthreaded shank of a fastener
e	distance between fastenings
$f_{u,F1}$	tensile strength of the external face sheet
$f_{u,F2}$	tensile strength of the internal face sheet
h	height of a beam
i_p	polar radius of gyration
k	stiffness of a longitudinal spring
k_v	stiffness of a fastening
l	length of a beam or column
m	number of beams or purlins stabilised by a diaphragm
m_a	moment resulting from stabilisation
q_a	load resulting from stabilisation
q_T	shear force
q_w	wind load
t_{f1}	thickness of the external face sheet
t_{f2}	thickness of the internal face sheet
t_{sup}	thickness of substructure
v, Δv	displacement of a fastening, deflection of a beam
B	width of a panel
D	thickness of panel, here: thickness of panel at point of fastening
EI_w	warping stiffness
F_i	compression force of a stabilised component
GI_T	torsional stiffness
I	moment of inertia (of a diaphragm)
M	bending moment
M_{cr}	elastic critical moment
M^E	external moment acting on a diaphragm
M^I	internal moment
N	axial force
N_{cr}	elastic critical axial force
S	shear stiffness of a diaphragm

S_i	shear stiffness allotted to one beam
V	force at a fastener
V^F	force at a fastening, resulting from introduction of load
V^M	force at a fastening, resulting from an external moment
V^T	force at a fastening, resulting from shear force
V_{Rk}	load bearing capacity of a shear loaded fastening
α	amplification factor
γ	angle of shear [rad]

1 Preliminary remark

Sandwich panels are commonly used for enclosures of buildings. They are fixed to a substructure and they transfer transverse loads, for example snow and wind, to the substructure. When loaded by in-plane shear forces, sandwich panels have a high shear stiffness. Unlike for related building products such as trapezoidal sheeting or cassette profiles, the shear stiffness of sandwich panels is usually not taken into account for the design of the building. The high shear stiffness can be used for different stabilizing effects: Sandwich panels can restrain the lateral displacement of single components (e.g. beams, columns). Therefore flexural and lateral torsional buckling is prevented. By acting as diaphragm sandwich panels can also be used for global stabilisation of complete building structures and for transferring horizontal loads, e.g. wind loads.

To act as a diaphragm sandwich panels have to be connected to the substructure by direct fixings. If the in-plane shear stiffness of sandwich panels is taken into account for the design of the building the panels must not be modified during their life, e.g. no holes must be cut out. The design procedures presented in the report at hand cover predominantly static loadings, fatigue loading is not covered.

2 Sandwich panels and connections

If sandwich panels are used as diaphragms or for lateral restraint of single components, their special characteristics and especially the characteristics of the connections have to be taken into account.

Sandwich panels are screwed through the external face to the substructure. The head of the screw and the washer lie on the external face separated from the substructure by the relatively soft core layer, thus, contributing only slightly to the stiffness of the connection. As a result, the connections of sandwich panels have a higher flexibility compared to the connections of steel trapezoidal sheeting. This is aggravated by the fact that the faces consist of steel sheets whose thickness is half of the thickness used for steel trapezoidal sheeting.

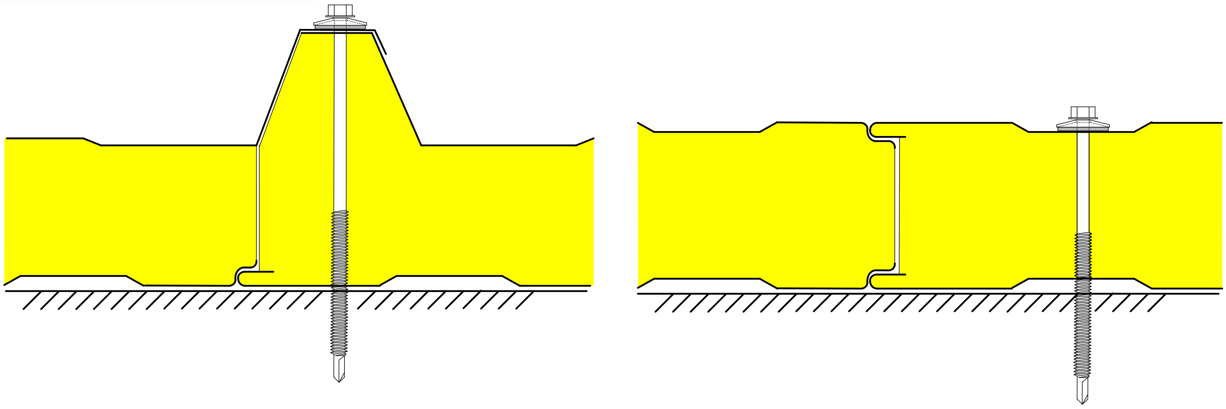

Fig. 2.1: Connections of sandwich panels to a substructure

Usually sandwich panels do not have any mechanical and thus, force transmitting connection of the longitudinal joints. The connection is mostly effected via a key and slot system. Only for roof panels the external faces are connected, where this connection mainly conduces to the water tightness of the longitudinal joint and the stabilization of the large free leg.

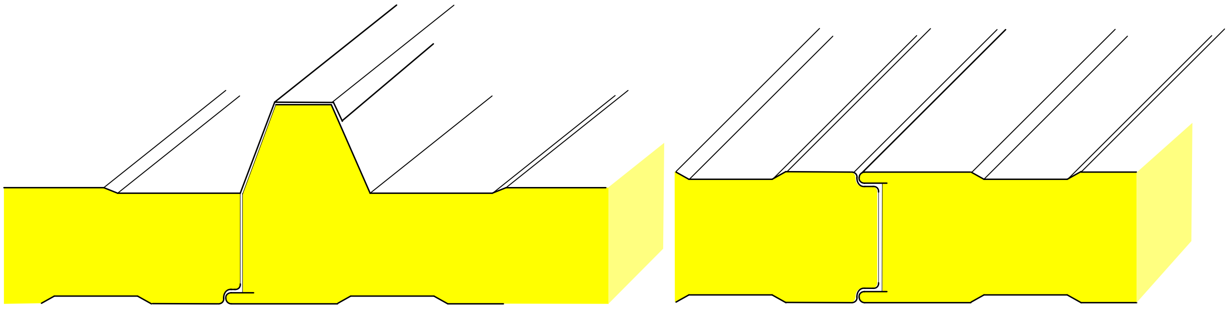

Fig. 2.2: Longitudinal joints

In common applications sandwich panels are used as single or multi span beams, only the transverse edges are fixed to the substructure. Because of that, the load-bearing behaviour of shear loaded sandwich panels does not correspond to the load-bearing behaviour of a real shear diaphragm as known from trapezoidal sheeting. Each panel is acting as an individual Vierendeel girder. A circumferential fixing on the substructure and a connection of the longitudinal joints is not common practice, even though feasible.

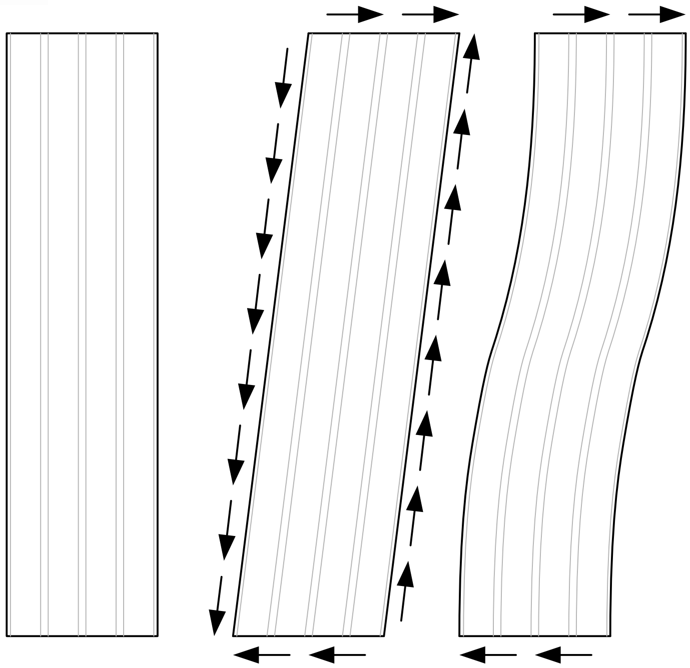

Fig. 2.3: Load bearing systems: shear diaphragm and Vierendeel girder

In different investigations it was shown, that not the panels but the connections are decisive for the stiffness and load-bearing capacity of shear diaphragms [1], [3], [8], [20]. In tests on shear diaphragms failure of the sandwich panels themselves could only be achieved after an additional reinforcement of the edges and connections. The in-plane shear stiffness of the panels is very much higher than the stiffness of the fastenings. Therefore, the panels can be regarded as approximately rigid in connection with flexible fastenings. For conventional applications a calculation model for determining the shear stiffness S and the load-bearing capacity of the diaphragm can be used, which only considers flexibilities of the fixings [1], [8].

3 Stiffness and load bearing capacity of fastenings

The stiffness k_v of connections of sandwich panels to a steel substructure can be determined according to the design model which was developed within the framework of the EASIE project [30] and is presented in [12]. The corresponding mechanical model is given in Fig. 3.1.

$$k_v = \frac{1}{\dfrac{x_F}{k_{F2}} + \dfrac{t_{\sup}^2 + 2\cdot(1-x_F)\cdot D\cdot t_{\sup}}{4\cdot C} + \dfrac{3\cdot(1-x_F)\cdot D\cdot t_{\sup}^2 + t_{\sup}^3}{24\cdot EI}} \tag{3.1}$$

with

$$x_F = 1 - \frac{\dfrac{1}{k_{F2}} - \dfrac{D\cdot t_{\sup}}{2\cdot C} - \dfrac{D\cdot t_{\sup}^2}{8\cdot EI}}{\dfrac{1}{k_{F2}} + \dfrac{D^2}{C} + \dfrac{D^2\cdot(2\cdot D + 3\cdot t_{\sup})}{6\cdot EI}} \tag{3.2}$$

The stiffness k_v consists of three single components.

Bending stiffness of the fastener

$$EI = 200.000 N/mm^2 \cdot \frac{\pi \cdot d_S^4}{64} \tag{3.3}$$

Rotational stiffness of clamping in the substructure

$$C = 2400 N/mm^2 \cdot \sqrt{t_{\sup} \cdot d_1^5} \tag{3.4}$$

Stiffness of internal face sheet (hole elongation)

$$k_{F2} = 6{,}93 \cdot \frac{f_{u,F2} \cdot \sqrt{t_{F2}^3 \cdot d_1}}{0{,}26mm + 0{,}8 \cdot t_{F2}} \qquad 0{,}40mm \le t_{F2} \le 0{,}70mm \tag{3.5}$$

$$k_{F2} = \frac{4{,}2 \cdot f_{u,F2} \cdot \sqrt{t_F^3 \cdot d_1}}{0{,}373mm} \qquad 0{,}70mm \le t_{F2} \le 1{,}00mm \tag{3.6}$$

t_{F2} thickness of internal face sheet

t_{sup} thickness of substructure

d_1 minor diameter of the threaded part of the fastener

d_S diameter of unthreaded shank

$f_{u,F2}$ tensile strength of internal face sheet

D thickness of panel at point of fastening

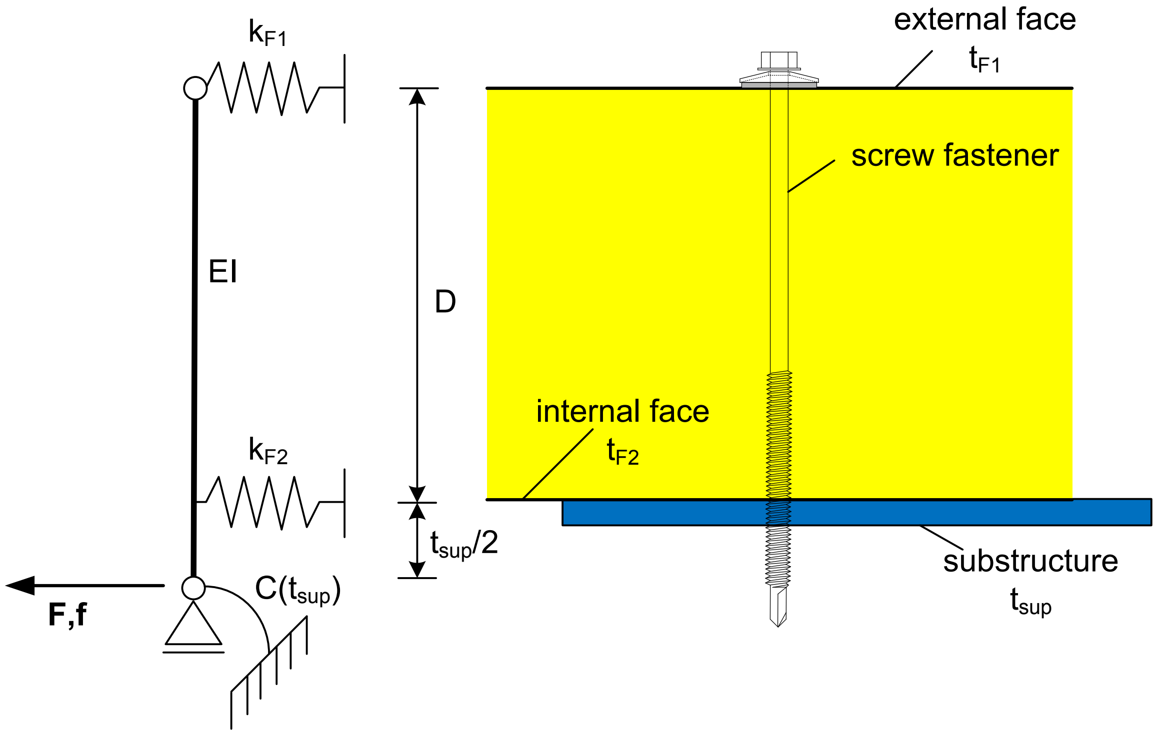

Fig. 3.1: Mechanical model of a fastening

The corresponding load-bearing capacity of a fastening is determined by testing and can be taken from (national) approvals, e.g. the German approval Z-14.4-407 [28] or from European technical approvals (ETA). It can also approximately be determined by the following formula which is also presented in [12].

$$V_{Rk} = 4{,}2 \cdot \sqrt{t_{F2}^{3} \cdot d_1} \cdot f_{u,F2} \tag{3.7}$$

The connections at longitudinal joints can also be taken into account for determination of the stiffness and load bearing capacity of shear diaphragms. According to [12] the stiffness of a fastening can be calculated with

$$k_v = 1900 \frac{N}{mm^3} \cdot t_{F1} \cdot d \tag{3.8}$$

The load-bearing capacity of a fastening at a longitudinal joint can be taken from (national) approval, e.g. [29] or from European technical approvals. Alternatively it can be determined by calculation according to EN 1993-1-3 [23].

$$V_{Rk} = 3{,}2 \cdot f_{u,F1} \cdot \sqrt{d \cdot t_{F1}^{3}} \tag{3.9}$$

with

d nominal diameter of fastener

t_{F1} thickness of external face sheet

$f_{u,F1}$ tensile strength of external face sheet

4 Shear stiffness of a diaphragm

The shear stiffness S [kN] of a shear diaphragm represents the resistance F of the diaphragm against a shear strain γ of 1 rad.

$$S = \frac{F}{\gamma} \tag{4.1}$$

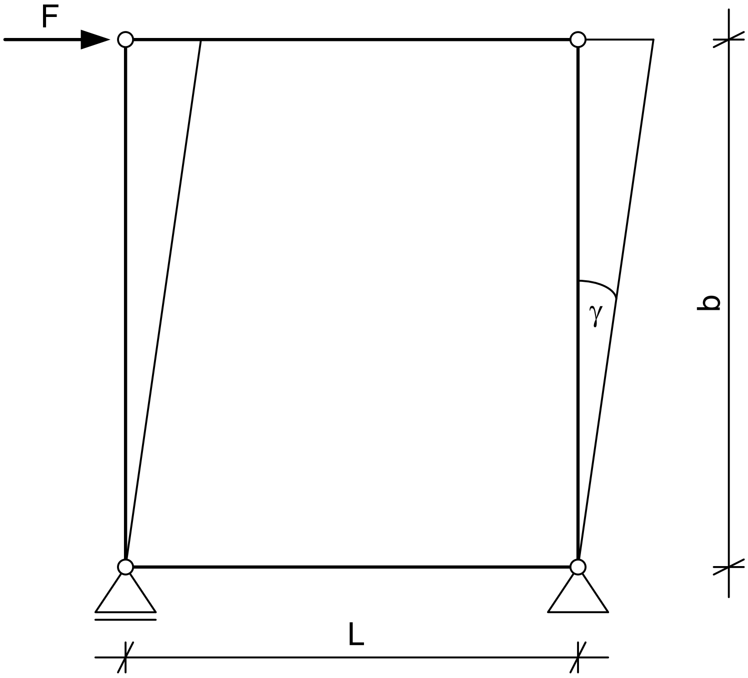

Fig. 4.1: Shear diaphragm

5 Stabilisation of beams and columns against buckling and lateral torsional buckling

5.1 Introduction

The constraint of lateral displacement of the flange of a beam or column connected to the sandwich panels of a diaphragm can be modeled by the approach of a longitudinal spring (Fig. 5.1). A partial or complete constraint of lateral displacement of the flange occurs in dependence on the spring stiffness k.

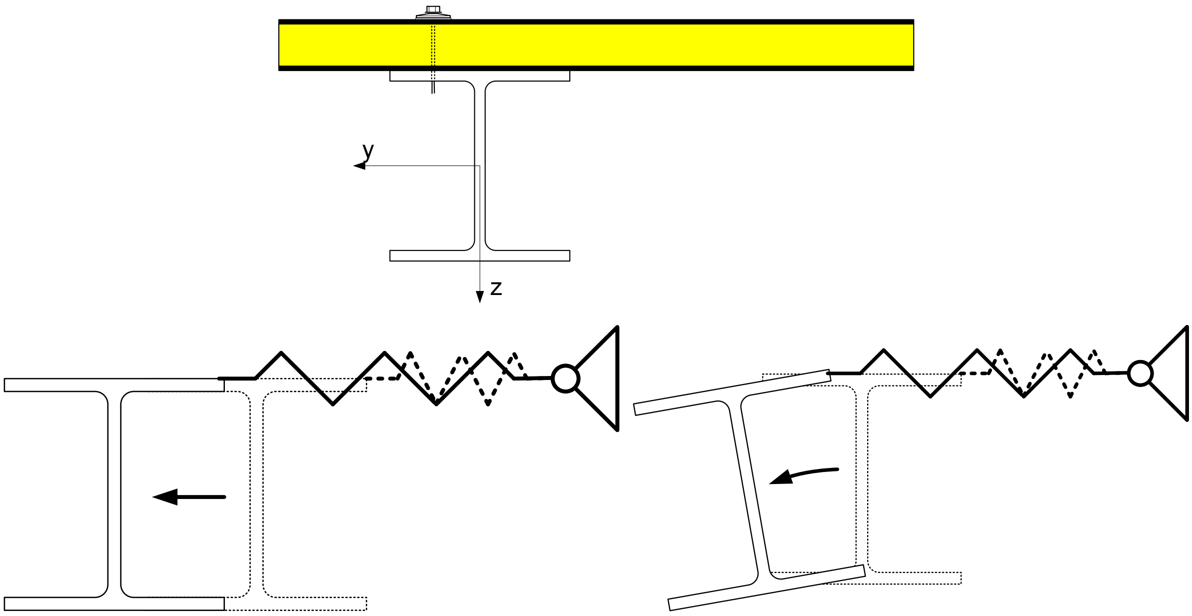

Fig. 5.1: Stabilization of beams by lateral restraint

The stiffness k of the longitudinal spring depends on the shear stiffness S_i. S_i is the shear stiffness of the diaphragm allotted to the investigated beam or column.

$$S_i = \frac{S}{m} \tag{5.1}$$

with

m number of beams or columns to be stabilized by the diaphragm

If a sinusoidal imperfection (deflection) is assumed the stiffness of the longitudinal spring can be determined by the following approach [5].

$$k = S_i \cdot \left(\frac{\pi}{l} \right)^2 \tag{5.2}$$

with

l length of stabilised beam

5.2 Complete restraint of lateral displacement

A complete constraint exists, if 90 % to 95 % (depending on the rules and standards) of M_{cr} (lateral torsional buckling) or $N_{cr,z}$ (flexural buckling) determined for a component with a rigid lateral bearing of the flange are reached. Then, the load-bearing capacity can be calculated assuming a rigid lateral support of the flange, which is connected to the sandwich panels. Either the plastic cross section capacity is reached or other stability proofs become decisive. If for a beam the flange subjected to compression is constraint, the plastic cross section capacity is reached. If the flange subjected to tension is constraint, lateral torsional buckling with a constraint torsional axis has to be investigated. For compression members buckling about the strong axis or (rarely) torsional buckling can become decisive.

For beams without normal force a minimum value of the shear stiffness S_i is given in [22] and [25].

$$S_i \geq \left(EI_w \cdot \frac{\pi^2}{l^2} + GI_t + EI_z \cdot \frac{\pi^2}{l^2} \cdot \frac{h^2}{4} \right) \cdot \frac{70}{h^2} \tag{5.3}$$

with

h height of cross section

GI_T torsional stiffness

EI_w warping stiffness

i_p polar radius of gyration

If this condition is fulfilled, at least 95 % of M_{cr} determined for a component with a rigid lateral bearing of the flange are reached. So no proof of lateral torsional buckling is necessary. Formula (5.3) is connected to the requirement that the sandwich panels must be mechanically connected to each other in the longitudinal joints and the shear diaphragm must be circumferential supported.

5.3 Partial constraint of lateral displacement

If the diaphragm can provide a partial restraint only, the proof of buckling about the weak axis or of lateral torsional buckling must be done. In doing so, the stiffness of the diaphragm can be included in the calculation of the critical axial force N_{cr} or the critical moment M_{cr}. For the determination of M_{cr} and N_{cr} a longitudinal spring with the stiffness k (section 5.1, formula (5.2)) is taken into account.

This results in an increased critical buckling force [8]

$$N_{cr} = \frac{k\frac{l^2}{\pi^2}\cdot(i_p^2+b^2)+N_{cr,z}\cdot i_p^2+GI_T+EI_w\frac{\pi^2}{l^2}}{2\cdot i_p^2} \pm \sqrt{\left(\frac{k\frac{l^2}{\pi^2}\cdot(i_p^2+b^2)+N_{cr,z}\cdot i_p^2+GI_T+EI_w\frac{\pi^2}{l^2}}{2\cdot i_p^2}\right)^2 - \frac{\left(N_{cr,z}+k\frac{l^2}{\pi^2}\right)\cdot\left(EI_w\frac{\pi^2}{l^2}+GI_T+k\frac{l^2}{\pi^2}\cdot b^2\right)}{i_p^2}+\frac{\left(k\frac{l^2}{\pi^2}\right)^2\cdot b^2}{i_p^2}} \tag{5.4}$$

or an increased critical moment [8].

$$M_{cr} = k\frac{l^2}{\pi^2}b \pm \sqrt{\left(N_{cr,z}+k\frac{l^2}{\pi^2}\right)\cdot\left(EI_w\frac{\pi^2}{l^2}+GI_T+k\frac{l^2}{\pi^2}b^2\right)} \tag{5.5}$$

with

$$N_{cr,z} = \frac{\pi^2\cdot EI_z}{l^2} \quad \text{(buckling load of unconstraint column)} \tag{5.6}$$

b distance of brace point from centre of gravity, usually $b=-\frac{h}{2}$ (compressed flange supported)

GI_T torsional stiffness

EI_w warping stiffness

Insertion of the shear stiffness S_i leads to the following formulae

$$N_{cr} = \frac{S_i\cdot(i_p^2+b^2)+N_{cr,z}\cdot i_p^2+GI_T+EI_w\frac{\pi^2}{l^2}}{2\cdot i_p^2} \pm \sqrt{\left(\frac{S_i\cdot(i_p^2+b^2)+N_{cr,z}\cdot i_p^2+GI_T+EI_w\frac{\pi^2}{l^2}}{2\cdot i_p^2}\right)^2 - \frac{\left(N_{cr,z}+S_i\right)\cdot\left(EI_w\frac{\pi^2}{l^2}+GI_T+S_i\cdot b^2\right)}{i_p^2}+\frac{S_i^2\cdot b^2}{i_p^2}} \tag{5.7}$$

$$M_{cr} = S_i \cdot b \pm \sqrt{\left(N_{cr,z} + S_i\right) \cdot \left(EI_w \frac{\pi^2}{l^2} + GI_T + S_i \cdot b^2\right)} \tag{5.8}$$

For the critical elastic lateral torsional buckling moment there are also different approximation formulae available [17], e.g.

$$M_{cr} = M_{cr}^0 + 0{,}5 \cdot S_i \cdot h \tag{5.9}$$

with

$$M_{cr}^0 = C_1 \cdot N_{cr,z} \cdot \left(\sqrt{\frac{I_w + 0{,}039 \cdot l^2 \cdot I_T}{I_z} + 0{,}25 \cdot z_p^2} + 0{,}5 \cdot z_p\right) \tag{5.10}$$

z_p distance of point of load application to center of gravity of the profile

C_1 factor for consideration of distribution of moment, e.g. according to [25] table10

With this increased values the proofs against lateral torsional buckling and weak axis buckling are provided, for example according to [22].

In the case of partial restraint, a connection of the longitudinal joints and a circumferential fixing of the diaphragm are not necessary according to the investigation on the stabilisation of compression members documented in [8]. The common unidirectional spanning application with fixings at the transverse edges only is sufficient.

Based on the requirement that the plastic cross section capacity of the profile shall be utilized and thus, lateral torsional buckling is not decisive, the following requirement on the shear stiffness applies according to [9].

$$S_i \geq 10{,}18 \cdot \frac{M_{pl,y}}{h} - 4{,}31 \cdot \frac{EI_z}{l^2} \cdot \left(-1 + \sqrt{1 + 1{,}86 \cdot \frac{\pi^2 \cdot EI_w + GI_t \cdot l^2}{EI_z \cdot h^2}}\right) \tag{5.11}$$

On the safe side (5.11) can be simplified to

$$S_i \geq 10{,}18 \cdot \frac{M_{pl,y}}{h} \tag{5.12}$$

This requirement is based on the concept of partial constraint, i.e. a lateral torsional buckling proof is done and the shear stiffness is included in the calculation. The requirement given in (5.11) was developed for a single span beam with lateral restraint of the compressed flange. In [9] it is shown, that the requirement given above can also be used for multi-span beams with a non-dimensional slenderness $\overline{\lambda}_M < 1$.

6 Determination of shear stiffness S

6.1 Preliminary remarks

A model to determine the shear stiffness S of diaphragms made of sandwich panels was developed in [1]. In addition to the shear stiffness also the load bearing capacity of a diaphragm can be determined using this model. The model can be used for unidirectional spanning sandwich panels with fixing at the transverse edges only, as well as for panels which are also connected at the longitudinal edges and joints. If panels have only connections at the transverse edges each panel acts as an individual element. If the panels are connected at the longitudinal joints and edges, they influence each other. This results in an increased shear stiffness S and in an increased load bearing capacity.

The in-plane shear stiffness of a sandwich panel is very much higher than the stiffness of its fixings. Therefore the fixings are decisive for the stiffness and load bearing capacity of a shear diaphragm. This assumption was confirmed by different investigations [1], [8]. In the design model the panels are assumed to be rigid and the fastenings are represented by longitudinal springs with the stiffness k_v.

The determination of the stiffness S of the shear diaphragm is based on the assumption that displacements of the sandwich panels only occur parallel to their longitudinal edges. A rotation of the diaphragm about an axis orthogonal to the plane of the panel is neglected. This assumption is also confirmed by the results of the investigations of [8]. Therefore at all fastenings only displacements and forces in longitudinal direction are considered.

6.2 Unidirectional spanning panels

Sandwich panels are often connected to the substructure at the transverse edges only. They do not have connections at the longitudinal joints. This is common practice, especially for wall panels. Each panel of a diaphragm acts as an individual element. When loaded by in plane shear forces each panel rotates around a reference point P, which is located in the centre of the panel (centre of gravity of the fasteners). The panels remain parallel to the longitudinal edges of the diaphragm and they are parallel to each other (Fig. 6.1, Fig. 6.2).

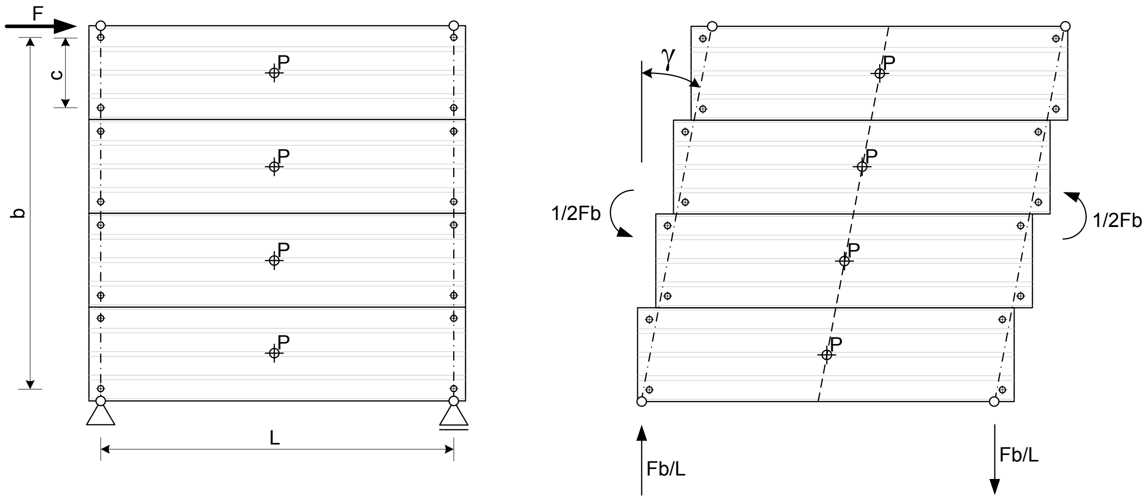

Fig. 6.1: Shear diaphragm with unidirectional spanning sandwich panels

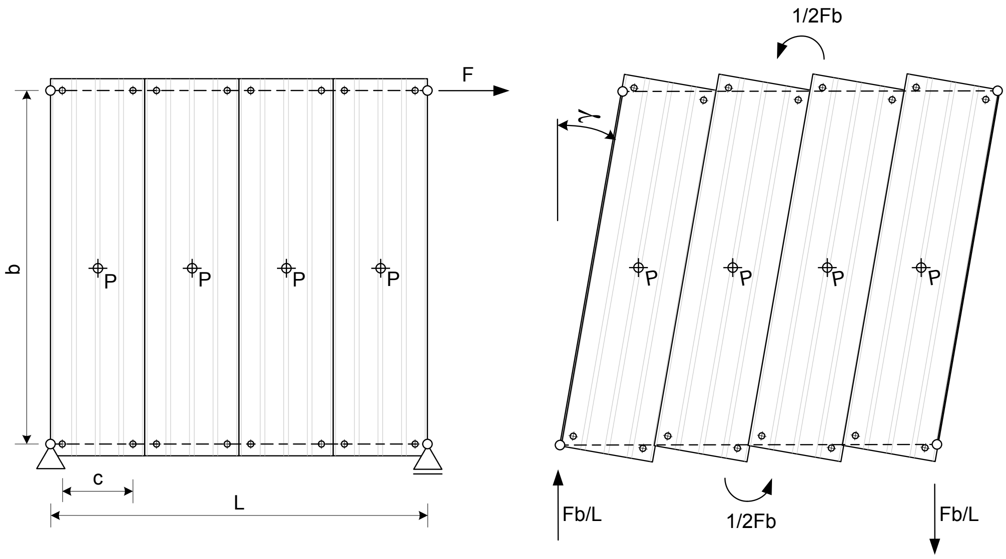

Fig. 6.2: Shear diaphragm with unidirectional spanning sandwich panels – continuation

At the fastenings forces and displacements in direction of the longitudinal edges occur. The relative displacement Δv of a fastening can be defined by the angle of shear γ and the distance x to the reference point.

$$\Delta v = \gamma \cdot x \tag{6.1}$$

With the stiffness k_v of the fastening the force V acting at a fastening can be determined.

$$V = k_v \cdot \Delta v = k_v \cdot \gamma \cdot x \tag{6.2}$$

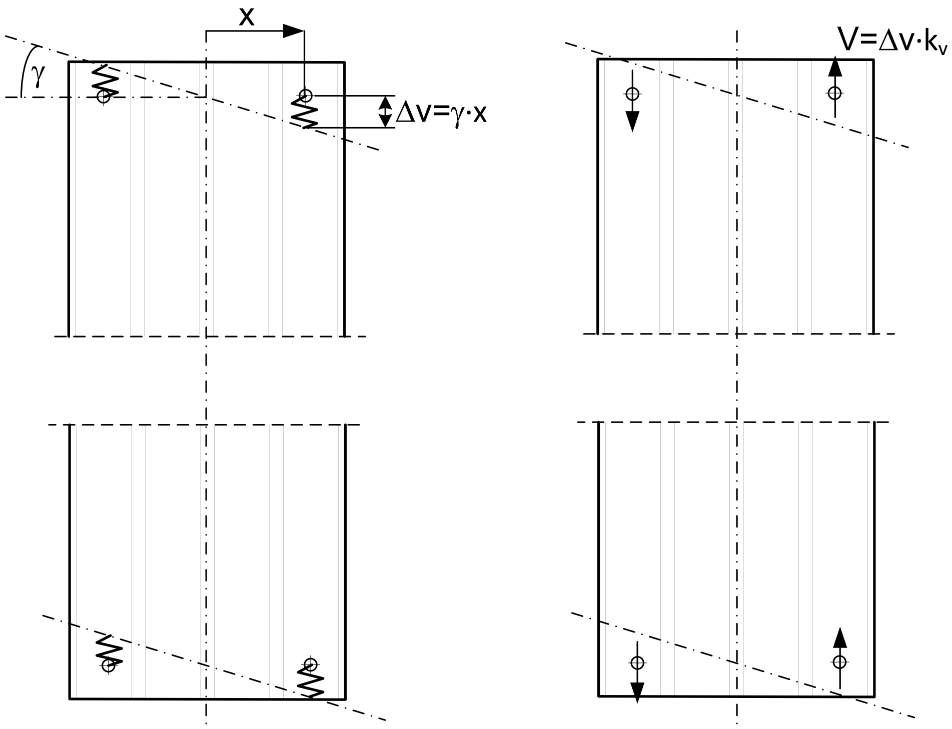

Fig. 6.3: Displacement and forces of fastenings

With the forces of the connections the internal moment of the shear diaphragm is determined by addition over all n sandwich panels and their n_i connections.

$$M^I = \sum_i \sum_k \left(F \cdot x_{i,k}\right) = \sum_i \sum_k \left(k_v \cdot \Delta v_{i,k} \cdot x_{i,k}\right) = \sum_i \sum_k \left(k_v \cdot \gamma \cdot x_{i,k}^2\right) \tag{6.3}$$

with

i summation over panels

k summation over fastenings of a panel

If c is the distance between two fasteners, which are symmetrical to the reference point, the distance x to the reference point is

$$x_{i,k} = \frac{c_{i,k}}{2} \tag{6.4}$$

The contribution of one pair of fasteners to the internal moment is

$$M_{i,k}^I = k_v \cdot \gamma \cdot \frac{c_{i,k}^2}{2} \tag{6.5}$$

So the internal moment can be written as

$$M^I = \sum_i \sum_k \left(k_v \cdot \gamma \cdot \frac{c_{i,k}^2}{2}\right) \tag{6.6}$$

The internal moment has to counteract the external moment.

$$M^E = F \cdot b = S \cdot \gamma \cdot b \tag{6.7}$$

Equalization of internal and external moment provides the shear stiffness S of the diaphragm.

$$S = \frac{k_v}{2 \cdot b} \cdot \sum_i \sum_k c_{i,k}^2 = \frac{I}{b} \tag{6.8}$$

I is the moment of inertia of the diaphragm.

$$I = \underbrace{\sum_i \sum_k \left(\frac{k_v}{2} \cdot c_{i,k}^2 \right)}_{I_i} = \sum_i I_i \tag{6.9}$$

For simplification the width B of one sandwich panel can be set to the distance c_1 of the outer fasteners at the transverse edges for some applications (e.g. stabilization of beams and columns).

With

$$B = c_1 \tag{6.10}$$

and

$$b = n_i \cdot B = n_i \cdot c_1 \tag{6.11}$$

n_i number of sandwich panel

formula (6.8) can be simplified to

$$S = \frac{k_v}{2 \cdot n_i \cdot c_1} \cdot n_i \cdot \sum_k c_{i,k}^2 = \frac{1}{2} k_v \cdot \sum_k \frac{c_k^2}{c_1} \tag{6.12}$$

Similar approaches are presented in [5] for panels with two fasteners at each transverse edge and in [11] for diaphragms made of trapezoidal sheeting.

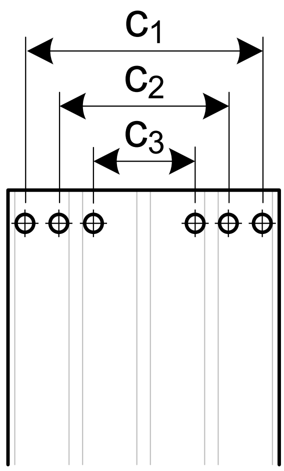

Fig. 6.4: Consideration of several fasteners

6.3 Panels with connections at the longitudinal edges and joints

Connections of the longitudinal joints of the panels and connections to the substructure at the longitudinal edges can also be considered using the design model given in [1]. These connections increase the shear stiffness and also the load bearing capacity of diaphragms. The panels do not act as individual members, they influence each other. Therefore the reference point P_i of a panel s not located in the centre of the panel. It depends on the stiffness of the fastenings.

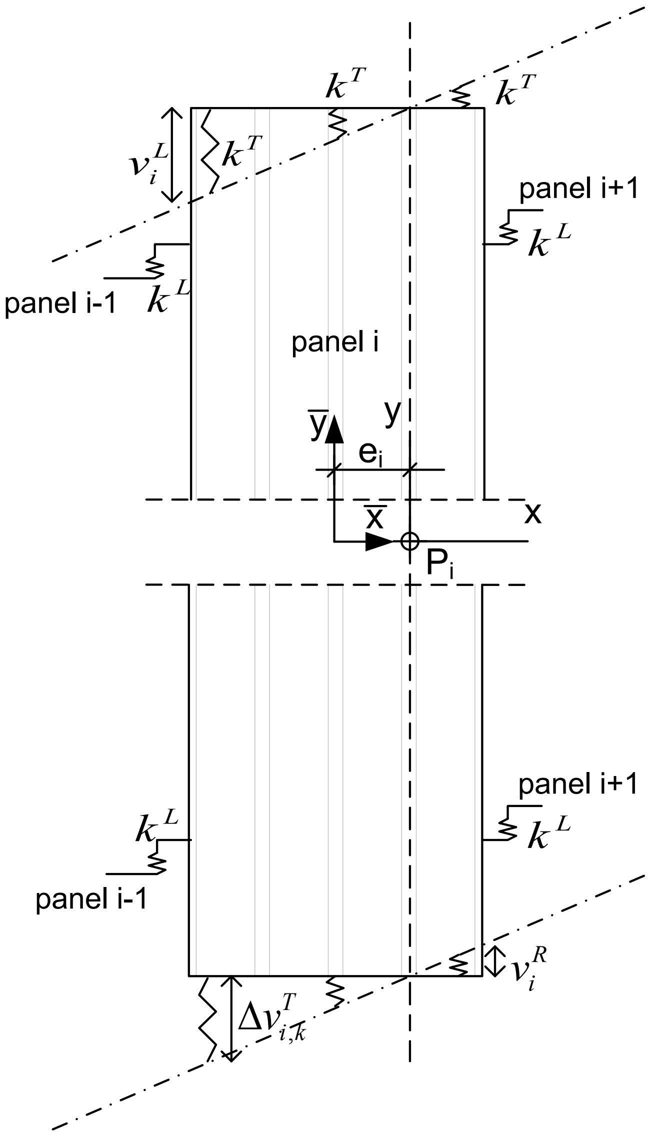

Fig. 6.5: **Model for determination of reference points**

At the transverse edges the panels have connections to the substructure with the stiffness k_v^T and at the longitudinal joints they have connections with the stiffness k_v^L. For each panel a local coordinate system ($\bar{x}/\bar{y}$) is chosen. At the connections of panel i the following relative displacements Δv to the substructure or to adjacent panels occur.

Relative displacement of connection k at the transverse edge:

$$\Delta v_{i,k}^{T} = \gamma \cdot x_{i,k} = \gamma \cdot (\bar{x}_{i,k}^{T} - e_i) \tag{6.13}$$

with

$x_{i,k}$ distance of connection i,k to the reference point

e_i distance of reference point from point of origin of the local coordinate system ($\bar{x}/\bar{y}$)

Relative displacement of connections at the left longitudinal edge:

$$\Delta v_i^l = \gamma \cdot (x_i^l - x_{i-1}^r) = \gamma \cdot (\bar{x}_i^l - e_i - \bar{x}_{i-1}^r + e_{i-1}) \tag{6.14}$$

Relative displacement of connections at the right longitudinal edge:

$$\Delta v_i^r = \gamma \cdot (x_i^r - x_{i+1}^l) = \gamma \cdot (\bar{x}_i^r - e_i - \bar{x}_{i+1}^l + e_{i+1}) \tag{6.15}$$

with

x_i^l distance of the left longitudinal edge to the reference point

x_i^r distance of the right longitudinal edge to the reference point

The force of a connection results from the stiffness k_v of the connection multiplied by the corresponding relative displacement Δv.

Force of connection k of the transverse edge:

$$V_{i,k}^T = k_v^T \cdot \Delta v_{i,k}^T \tag{6.16}$$

Force of connections at the left longitudinal edge:

$$V_i^l = k_v^L \cdot \Delta v_i^l \tag{6.17}$$

Force of connections at the right longitudinal edge:

$$V_i^r = k_v^L \cdot \Delta v_i^r \tag{6.18}$$

For panel i equilibrium of forces in longitudinal direction is established as follows.

$$n_i^l \cdot V_i^l + n_i^r \cdot V_i^r + \sum_k V_{i,k}^T = 0 \tag{6.19}$$

$$n_i^l \cdot \gamma \cdot (\bar{x}_i^l - e_i - \bar{x}_{i-1}^r + e_{i-1}) \cdot k_v^L + n_i^r \cdot \gamma \cdot (\bar{x}_i^r - e_i - \bar{x}_{i+1}^l + e_{i+1}) \cdot k_v^L + \gamma \cdot k_v^T \sum_k \bar{x}_{i,k}^T - n_i^T \cdot \gamma \cdot e_i \cdot k_v^T = 0$$

with

n_i^l number of fastenings at the left longitudinal joint

n_i^r number of fastenings at the right longitudinal joint

n_i^T number of fastenings at the transverse edge

Rearranging formula (6.19) results in

$$A_i \cdot e_{i-1} - B_i \cdot e_i + C_i \cdot e_{i+1} = A_i \cdot \left(\bar{x}_{i-1}^r - \bar{x}_i^l\right) + C_i \cdot \left(\bar{x}_{i+1}^l - \bar{x}_i^r\right) - D_i \tag{6.20}$$

With

$$A_i = n_i^l \cdot k_v^L \tag{6.21}$$

$$B_i = n_i^l \cdot k_v^L + n_i^T \cdot k_v^T + n_i^r \cdot k_v^L \tag{6.22}$$

$$C_i = n_i^r \cdot k_v^L \tag{6.23}$$

$$D_i = k_v^T \cdot \sum_k \bar{x}_{i,k}^T \tag{6.24}$$

So, for n panels, an equation system with n equations and n unknown distances e_i of the reference points occurs. Solving the equation system leads to the locations of the reference points.

For panel i the internal moment can be calculated with

$$M_i^I = n_i^l \cdot V_i^l \cdot x_i^l + n_i^r \cdot V_i^r \cdot x_i^r + \sum_k V_{i,k}^T \cdot x_{i,k}^T \tag{6.25}$$

$$M_i^I = n_i^l \cdot k_v^L \cdot \gamma \cdot (x_i^l - x_{i-1}^r) \cdot x_i^l + n_i^r \cdot k_v^L \cdot \gamma \cdot (x_i^r - x_{i+1}^l) \cdot x_i^r + \sum_k k_v^T \cdot \gamma \cdot x_{i,k}^T \cdot x_{i,k}^T \tag{6.26}$$

$$M_i^I = \gamma \cdot \left(n_i^l \cdot k_v^L \cdot (x_i^l - x_{i-1}^r) \cdot x_i^l + n_i^r \cdot k_v^L \cdot (x_i^r - x_{i+1}^l) \cdot x_i^r + k_v^T \cdot \sum_k \left(x_{i,k}^T \right)^2 \right) \tag{6.27}$$

For a shear diaphragm consisting of n panels the internal moment M^I is

$$M^I = \underbrace{\gamma \cdot \sum_i \underbrace{\sum_k n_i^l \cdot k_v^L \cdot (x_i^l - x_{i-1}^r) \cdot x_i^l + n_i^r \cdot k_v^L \cdot (x_i^r - x_{i+1}^l) \cdot x_i^r + k_v^T \cdot \sum_k \left(x_{i,k}^T \right)^2}_{I_i}}_{I} \tag{6.28}$$

I is the moment of inertia of the shear diaphragm. The contribution I_i of panel i to the moment of inertia is

$$I_i = \sum_k k_v^T \cdot \left(x_{i,k} \right)^2 + n_i^l \cdot k_v^L \left(x_i^l - x_{i-1}^r \right) \cdot x_i^l + n_i^r \cdot k_v^L \cdot \left(x_i^r - x_{i+1}^l \right) \cdot x_i^r \tag{6.29}$$

The contribution $I_{i,k}$ of a single fastening to the moment of inertia is

$$I_{i,k} = \begin{cases} k_v^T \cdot \left(x_{i,k} \right)^2 & \text{transverse edge} \\ k_v^L \cdot \left(x_i^l - x_{i-1}^r \right) \cdot x_i^l & \text{left joint/edge} \\ k_v^L \cdot \left(x_i^r - x_{i+1}^l \right) \cdot x_i^r & \text{right joint/edge} \end{cases} \tag{6.30}$$

The internal moment has to counteract the external moment.

$$M^E = F \cdot b = S \cdot \gamma \cdot b \tag{6.31}$$

Equalization of the internal and the external moment provides the shear stiffness S of the diaphragm.

$$S = \frac{M^I}{\gamma \cdot b} = \frac{I}{b} \tag{6.32}$$

If there are also connections at the longitudinal edges of the diaphragm, for the fictive panels 0 and n+1 the following coordinates have to be used for determination of the reference points:

$$\bar{x}_0^r = x_0^r = 0 \tag{6.33}$$

$$\bar{x}_{n+1}^l = x_{n+1}^l = 0 \tag{6.34}$$

Remark:

At the connections of the panels to the substructure the loads are transferred almost completely by the internal face. In common applications at the longitudinal joints only the external faces are connected. If the panels act as diaphragm and both kinds of connections are taken into account for load transfer, forces have to be transferred through the panel, i.e. through the core material, from one face to the other. According to the experimental investigations on shear diaphragms presented in [1] and [8] for common applications this is not a problem.

In [8] longitudinal joints of wall panels were connected with blind rivets. Based on the geometry of the joint with an overlapping lip one rivet was clinched through four sheets. So the connection of the longitudinal joints is very stiff and consequently comparatively high forces occur. In [8] this kind of connection is regarded even as stiff enough to assume a rigid shear wall without any slip in the longitudinal joints. Also in this case the transfer of loads through the panels was not a problem.

6.4 Verification

The calculation procedure for the stiffness of the fastenings, which was developed within the EASIE project [30], was verified in conjunction with the model for shear diaphragms, given above. For this purpose tests on diaphragms presented in [1] were recalculated. The tested diaphragm and all relevant dimensions and parameters are presented in Fig. 6.6.

In [1] two tests were performed and the force F needed to reach an angle γ = 1/375 rad was measured.

no.	force F at γ = 1/375
01	3,90 kN
02	4,44 kN

Tab. 6.1: Measured forces

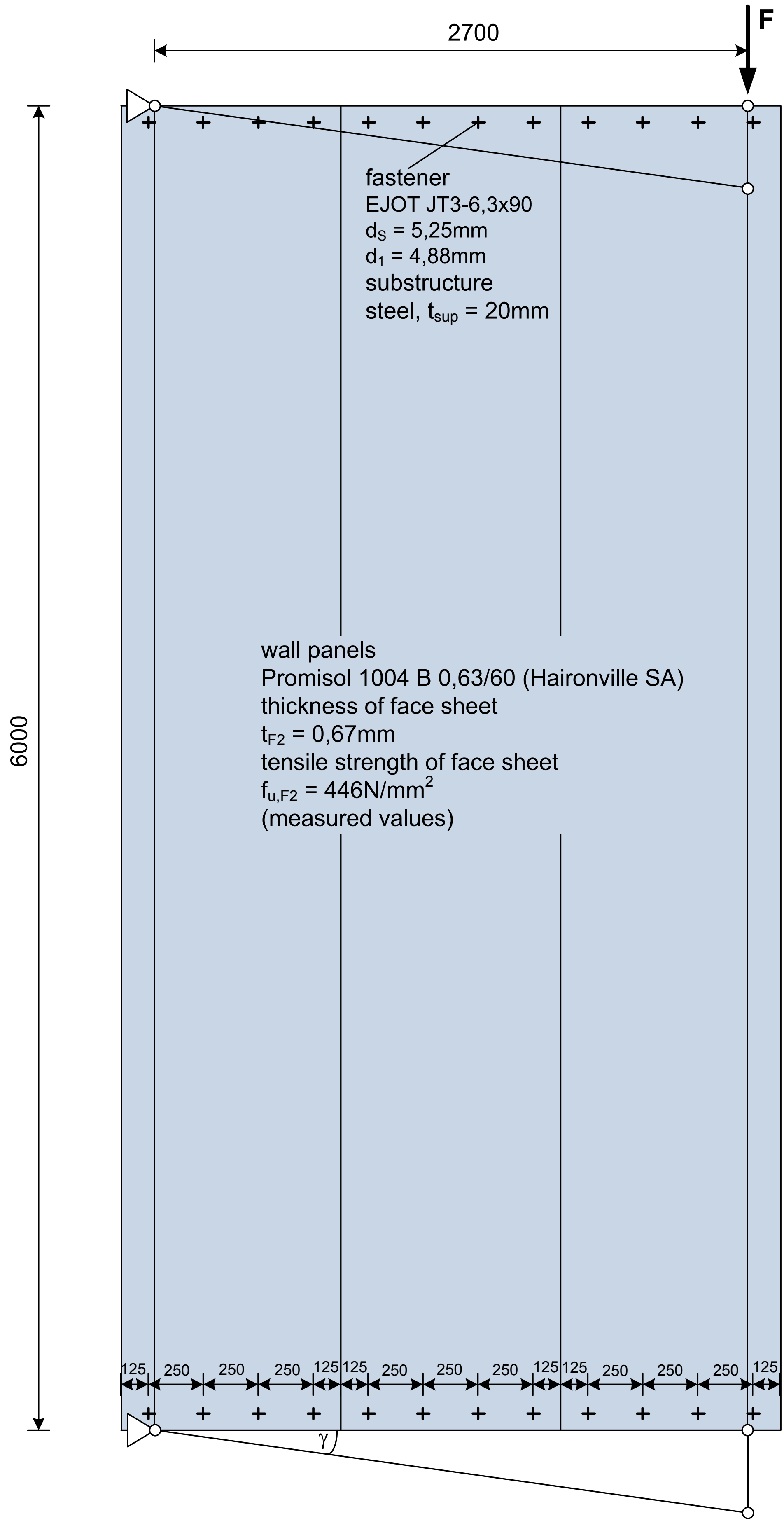

Fig. 6.6: **Test arrangement in [1]**

With the forces F measured in the tests the angle γ is recalculated using the model for shear diaphragms and the model for the stiffness of fastenings presented above.
According to the formulae given in section 3 the fastenings have the stiffness

$$k_v = 2{,}47\frac{kN}{mm} \tag{6.35}$$

With the stiffness k_v the moment of inertia of the diaphragm is:

$$I = \sum_i \sum_k \frac{k_v}{2} \cdot c_{i,k}^2 = 3 \cdot 2 \cdot \frac{2{,}47kN/mm}{2} \cdot \left((250mm)^2 + (750mm)^2\right) = 4631{,}25kNm \tag{6.36}$$

The external moments are

$$M^E = F \cdot 2700mm \tag{6.37}$$

no.	external moment M^E
01	10,53 kNm
02	11,99 kNm

Tab. 6.2: External moments

The calculated values of the angle γ are given in the following table. In this table also the derivation from the test results is shown.

$$\gamma = \frac{M^E}{I} \tag{6.38}$$

no.	angle γ	deivation
01	0,0023 rad	13,7 %
02	0,0026 rad	2,5 %

Tab. 6.3: Calculated angle γ and deviation from measured value

7 Loads on diaphragms

7.1 Transfer of horizontal loads

If a diaphragm is used to transfer load, e.g. wind loads, these loads are applied continuously or by the beams or purlins which support the panels. The kind of load application depends on the direction of the load in relation to the direction of span of the panels. If the load acts in longitudinal direction of the panels the forces are applied continuously (Fig. 7.1). If the direction of the load is orthogonal to the span of the panels the load is applied by the supporting beams (Fig. 7.2).

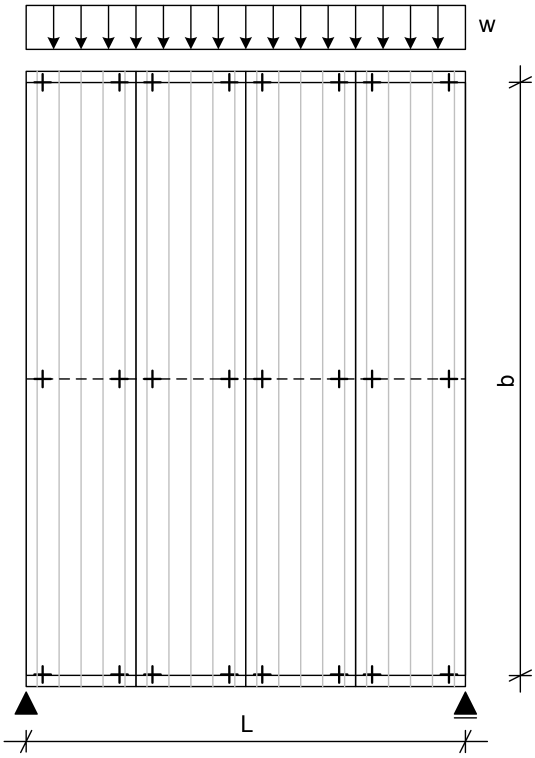

Fig. 7.1: Continuous application of loads

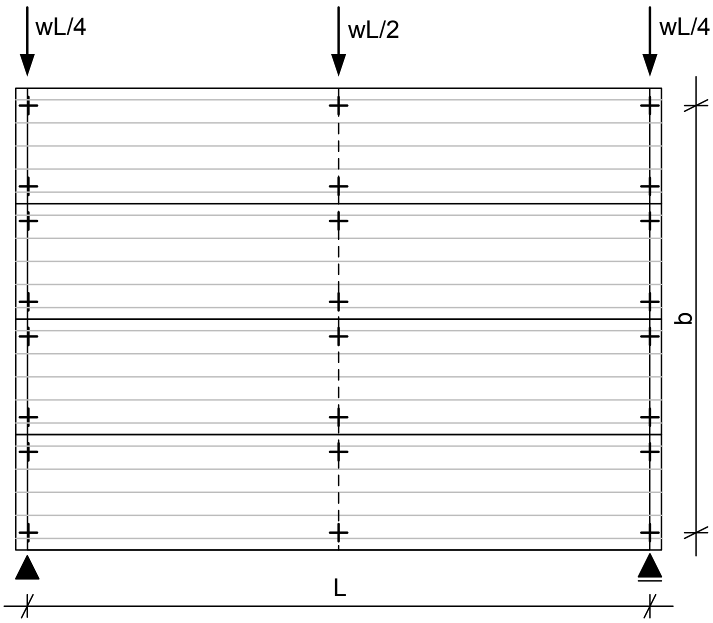

Fig. 7.2: Application of loads by supporting beams

7.2 Stabilisation of beams and columns

An imperfection of the beam in form of a sinusoidal half-wave is assumed.

$$v(x) = v_0 \cdot \sin\left(\frac{\pi \cdot x}{l}\right) \tag{7.1}$$

The imperfection v_0 at x = l/2 can be determined according to [22] or [25].

$$v_0 = \frac{l}{500} \cdot \sqrt{0{,}5 \cdot \left(1 + \frac{1}{m}\right)} \qquad \text{(EN 1993-1-1)} \tag{7.2}$$

$$v_0 = \frac{l}{400} \cdot \sqrt{\frac{10}{l}} \cdot 0{,}5 \cdot \left(1 + \frac{1}{m}\right) \qquad \text{(DIN 18800-2)} \tag{7.3}$$

with

m number of components to be stabilized

In some cases v_0 has to be increased by the deflection $v_{w,0}$ from transverse loads (e.g. wind loads). This is necessary, if the diaphragm is used to transfer wind loads and to stabilise beams. The deflection $v_{w,0}$ caused by the wind load is also increased be the effects of the compression force.

$$v_{w,0} = \frac{1}{S} \cdot \frac{w \cdot l^2}{8} \tag{7.4}$$

Further the compression force F_i in the component to be stabilized is assumed to be constant in longitudinal direction of the beam. If a beam without axial loading is considered, the normal force resulting from the bending moment $M_{S,d}$ in the flange subjected to compression is

$$F_i = \frac{M_d}{h - t} \tag{7.5}$$

For the stabilisation of a compression member subjected to axial pressure (7.5) has to be modified to

$$F_i = N_d \tag{7.6}$$

and for the stabilization of a beam-column with axial force and bending moment

$$F_i = N_d + \frac{M_d}{h - t} \tag{7.7}$$

or

$$F_i = \frac{N_d}{2} + \frac{M_d}{h - t} \tag{7.8}$$

depending on whether flexural buckling (equation (7.7)) or lateral torsional buckling (equation (7.8)) is concerned.

The imperfection v_0 and the normal force F_i result in the moment M(x).

$$M(x) = F_i \cdot v(x) = F_i \cdot v_0 \cdot \sin\left(\frac{\pi \cdot x}{l}\right) \tag{7.9}$$

Because of the axial compression force the moment must be amplified according to effects of 2nd order theory. These effects are usually considered by an amplification factor. For a beam without shear flexibility the amplification factor is

$$\alpha = \frac{1}{1 - \frac{N}{N_{cr}}} \tag{7.10}$$

If only the shear stiffness of a beam is taken into account we get according to [19]

$$\alpha = \frac{1}{1 - \frac{F_i}{S_i}} \tag{7.11}$$

The amplification results in the following moment according to 2nd order theory.

$$M^{II}(x) = F_i \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \cdot \sin\left(\frac{\pi \cdot x}{l}\right) \tag{7.12}$$

With this moment a restraining load q_a acting on the diaphragm can be determined.

$$q_a(x) = -\left(M^{II}(x)\right)'' \tag{7.13}$$

So the restraining load resulting from the stabilization of a beam is

$$q_a(x) = F_i \cdot \left(\frac{\pi}{l}\right)^2 \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \cdot \sin\left(\frac{\pi \cdot x}{l}\right) \tag{7.14}$$

The load $q_a(x)$ is the restraining load, which avoids a transverse displacement of the beam or of the compressed flange.

Instead of avoiding transverse displacement a beam can also be restraint by avoiding rotation about the z-axis. To avoid rotation a moment $m_a(x)$ is assumed. The moment $m_a(x)$ represents a restraining moment per unit length [kNm/m] along the longitudinal axis of the beam [10].

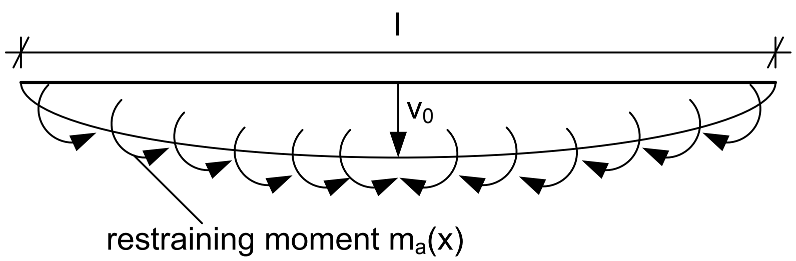

Fig. 7.3: **Restraining moment**

The restraining moment can be calculated with

$$m_a(x) = -\left(M^{II}(x)\right)' \tag{7.15}$$

$$m_a(x) = F_i \cdot \left(\frac{\pi}{l}\right) \cdot v_0 \cdot \frac{1}{1-\frac{F_i}{S_i}} \cdot \cos\left(\frac{\pi \cdot x}{l}\right) \tag{7.16}$$

Remark:

An alternative derivation of the restraining moment $m_a(x)$ can be found in [8]. If a beam is restraint by sandwich panels, the fasteners introduce a force V in the panel. E.g. we have two fasteners at a transverse edge and they have the distance c; we get the moment

$$M = V \cdot c \tag{7.17}$$

This moment M we get for each panel. The forces V and therefore also the moments M depend on the rotation v´ of the beam. So the highest moment acts at the ends of the beam and M = 0 in the middle of the beam. If the moments M are smeared over the length of the beam, we get a moment m(x) per unit length.

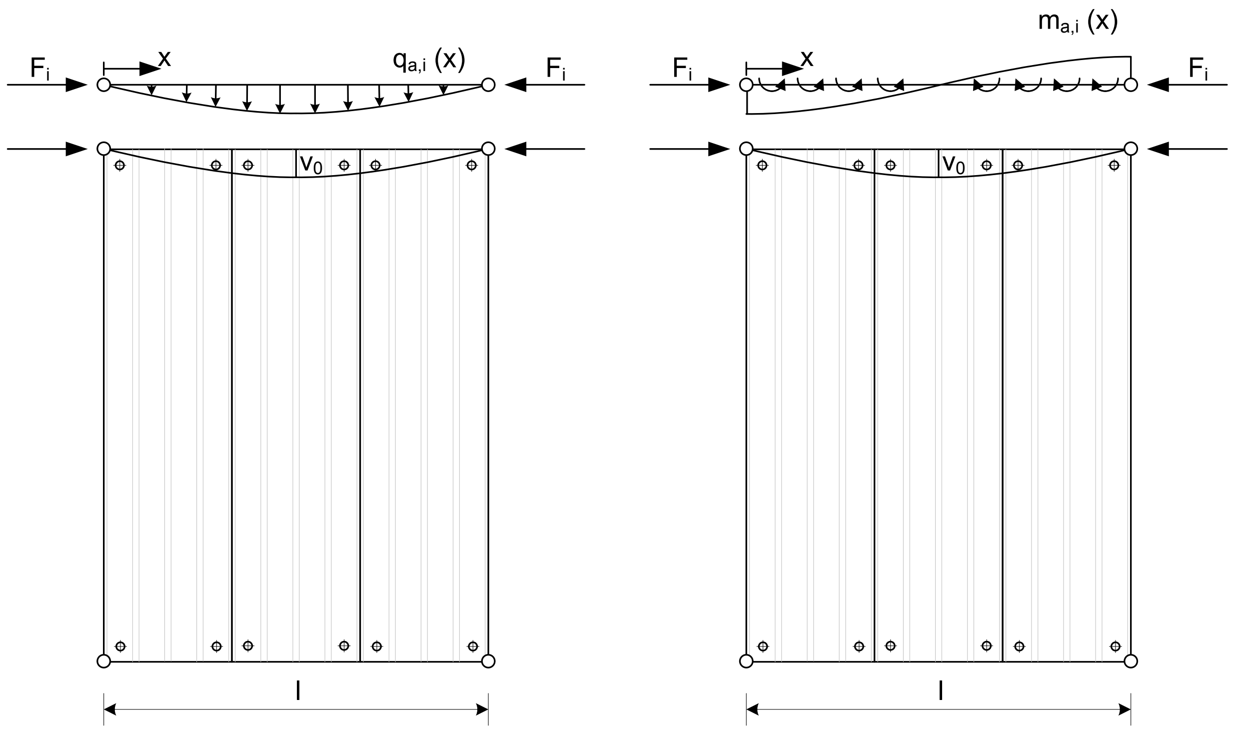

Fig. 7.4: Load resulting from stabilization of a single component

Remark:

In the formulae given above the bending stiffness of the stabilized component (e.g. the compressed flange of the beam) is neglected. The bending stiffness can be considered by modification of the amplification factor. Further the normal force F_i was assumed to be constant in longitudinal direction of the beam. Both assumptions are on the safe side. To consider an in-constant normal force - resulting from a bending moment on a single-span beam - in [19] an adjustment of the amplification factor with the factor 1/2 is suggested.

$$\alpha = \frac{1}{1 - \frac{1}{2} \cdot \frac{F_i}{S_i + EI \cdot \left(\frac{\pi}{l}\right)^2}} \tag{7.18}$$

8 Design of fastenings

8.1 Introduction

If sandwich panels are used to transfer loads in the plane of the panels or for stabilisation of single components, for the design of the fastenings additional forces have to be taken into account. This applies for fastenings of the sandwich panels to the substructure as well as for fastenings at the longitudinal joints.

8.2 Wind loads applied by the supporting beams

If sandwich panels, e.g. roof panels, are used to transfer wind loads, the loads often act in transverse direction to the span of the panels. In this case the load is applied by the supporting beams, e.g. as given in the following figure.

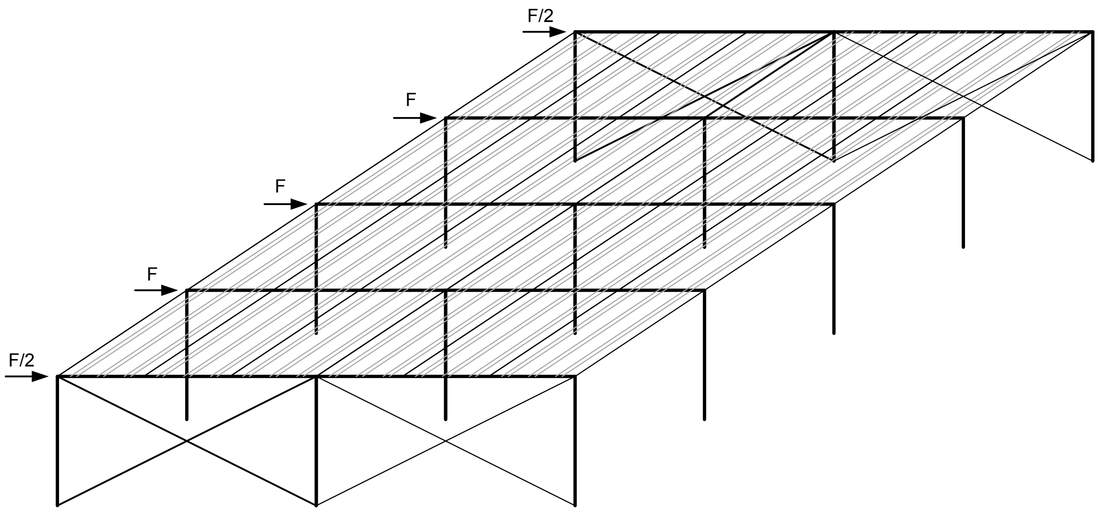

Fig. 8.1: Transfer of horizontal loads

Fig. 8.2 shows the diaphragm, which is used to transfer the horizontal load to the exterior braced walls of the buildung. The horizontal loads induce an external Moment M^E. For the example given in Fig. 8.2, the external moment is calculated to

$$M^E = \frac{F}{2} \cdot L + F \cdot \frac{L}{2} = F \cdot L \tag{8.1}$$

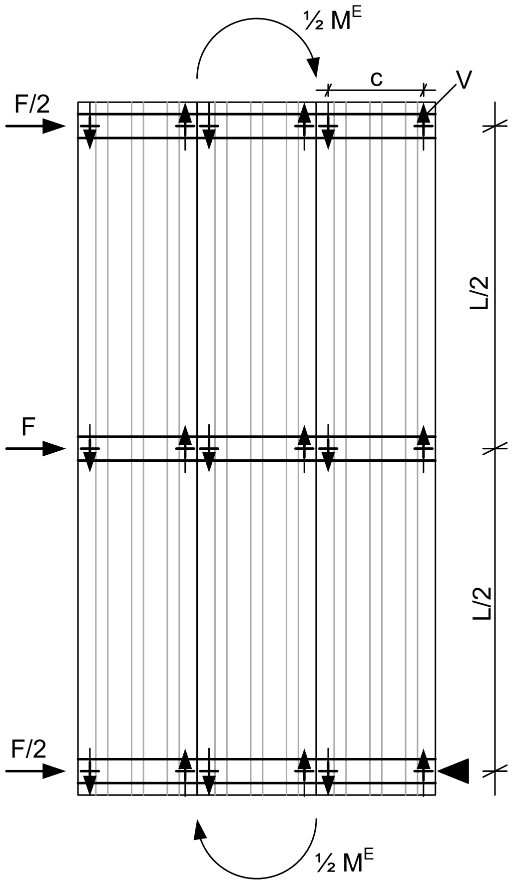

Fig. 8.2: Shear diaphragm used to transfer horizontal loads

The external moment results in forces V^M at the fastenings. The forces V^M act in longitudinal direction of the panels. Determination of these forces is presented in the following sections (8.2.1 and 8.2.2).

In addition to the forces V^M resulting from the external moment, forces V^F resulting directly from the introduction of the horizontal load in the panels have to be considered. The load introduced by one beam can be distributed uniformly on the fasteners at this beam.

$$V^F = \frac{F}{n} \tag{8.2}$$

n number of fasteners at the beam

To determine the resulting force V acting on one fastening, the forces V^M and V^F are added vectorially.

$$V = \sqrt{\left(V^M\right)^2 + \left(V^F\right)^2} \tag{8.3}$$

8.2.1 Panels without connections at the longitudinal joints

An internal moment M^I has to withstand the external moment M^E. The internal moment results in forces of the fastenings. For a diaphragm without connections at the longitudinal joints and two fastenings at each transverse edge the internal moment is

$$M^I = n \cdot V^M \cdot c \tag{8.4}$$

with

n number of pairs of fastenings to the substructure (one pair = two fasteners)

c distance between the fasteners

The force V^M acting on a fastening can be determined by equalisation of internal and external moment.

$$V^M = \frac{M^E}{n \cdot c} \tag{8.5}$$

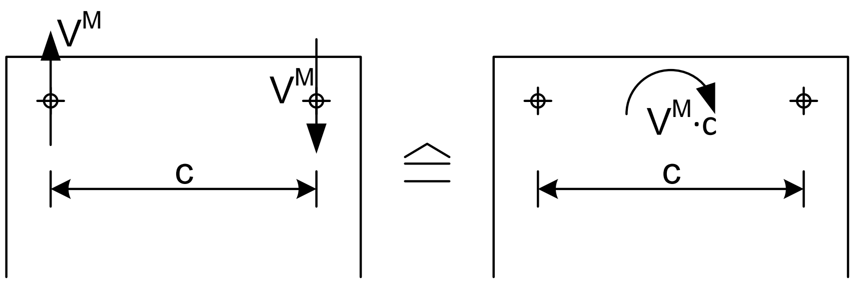

Fig. 8.3: Internal moment

More than two fastenings at a transverse edge can be considered by the displacement figure.

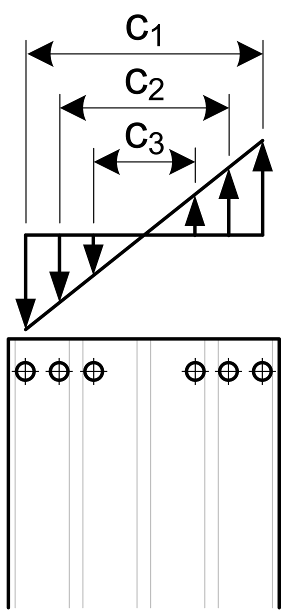

Fig. 8.4: Consideration of additional fasteners

In this case (8.4) is transformed to

$$M^I = n \cdot V^M \cdot \sum_k \left(\frac{c_{i,k}}{c_1} \right) \tag{8.6}$$

The maximum force V^M acting on a fastening is determined by equalization of the internal and the external moment.

$$V^M = \frac{M^E}{n \cdot \sum_k \left(\frac{c_{i,k}}{c_1} \right)} \tag{8.7}$$

8.2.2 Panels with connections at the longitudinal joints

An internal moment M^I has to withstand the external moment M^E. The internal moment results in forces V^M of the fastenings. For a diaphragm with connections at the longitudinal joints the internal moment is

$$M^I = \gamma \cdot I \tag{8.8}$$

with

I moment of inertia as given in section 6.3

With the external moment M^E the internal moment $M^I_{i,k}$ acting on a fastening can be calculated to

$$M_{i,k} = \frac{M^E}{I} \cdot I_{i,k} \tag{8.9}$$

with

$I_{i,k}$ contribution of fastening k of panel i to the moment of inertia (see also section 6.3)

The force acting on a fastening can be determined by the following formulae

$$V^M_{i,k} = \frac{M^I_{i,k}}{x_{i,k}} = \begin{cases} \frac{M^E}{I} \cdot k_v^T \cdot x_{i,k} & \text{transverse edge} \\ \frac{M^E}{I} \cdot k_v^L \cdot \left(x_i^l - x_{i-1}^r \right) & \text{left edge/joint} \\ \frac{M^E}{I} \cdot k_v^L \cdot \left(x_i^r - x_{i+1}^l \right) & \text{right edge/joint} \end{cases} \tag{8.10}$$

with

$x_{i,k}$ distance of fastening from reference point

Remark:

The connections at the longitudinal joints have a wide influence on the stiffness and the load-bearing capacity of a diaphragm. This influence becomes apparent, if the examples presented in annex 1 and 2 are compared. In addition the connections at the longitudinal joints have usually a high stiffness compared to the connections at the transverse edges. Therefore the following procedure is recommended for the design of a diaphragm.

- Design of fastenings at longitudinal joints:
  To determine the forces in the fastenings at longitudinal joints for all fastenings the real stiffness k_v is used.
- Design of fastenings at the transverse edges and determination of the shear stiffness S of the diaphragm:
  To be on the safe side for the determination of the forces in the fastenings at the transverse edges and of the stiffness S of the diaphragm it is recommended to use a reduced stiffness for the fastenings of the longitudinal joints.

8.3 Loads in longitudinal direction of the panels

In longitudinal direction of the panels wind loads as well as loads resulting from stabilisation of single components can act. Wind loads are usually constant, whereas stabilisation loads are assumed to have a sinusoidal form.

Wind loads:

$$q_w(x) = w_0 \tag{8.11}$$

or alternatively

$$m_w(x) = \int q_w(x) = w_0 \cdot \left(x - \frac{l}{2} \right) \tag{8.12}$$

Loads resulting from stabilisation (see also section 7.2):

$$q_a(x) = F_i \cdot \left(\frac{\pi}{l} \right)^2 \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \cdot \sin\left(\frac{\pi \cdot x}{l} \right) \tag{8.13}$$

or alternatively

$$m_a(x) = F_i \cdot \left(\frac{\pi}{l} \right) \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \cdot \cos\left(\frac{\pi \cdot x}{l} \right) \tag{8.14}$$

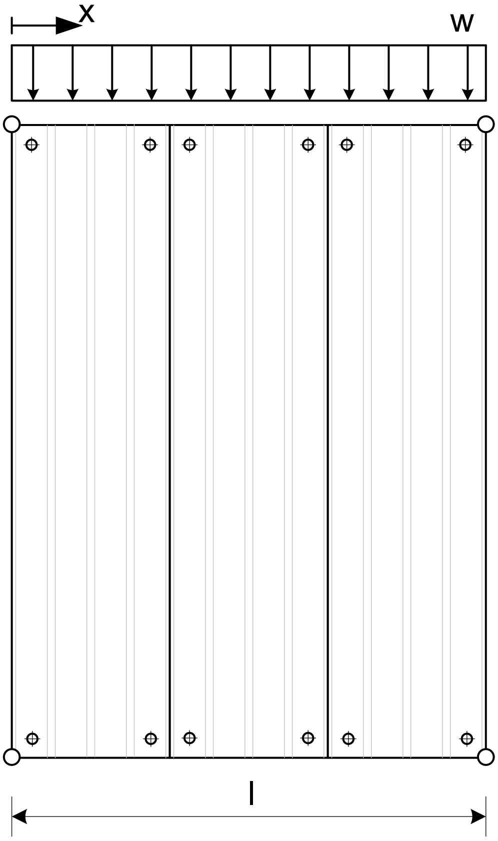

Fig. 8.5: Wind load acting in longitudinal direction of the panel

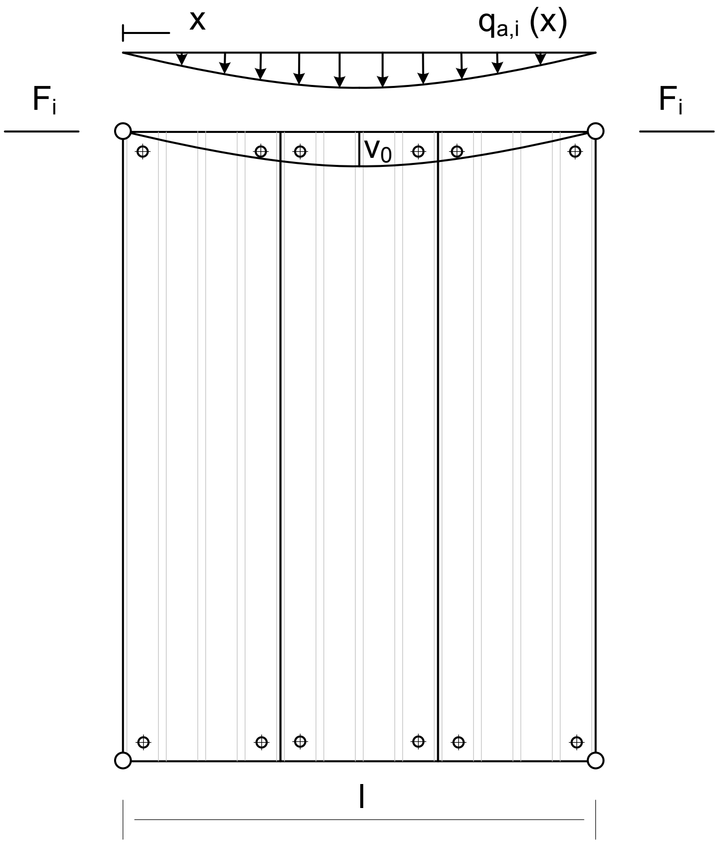

Fig. 8.6: Load resulting from stabilization of a beam

8.3.1 Unidirectional spanning panels

The moment m(x) has its maximum m_0 at the ends of the beam (x=0, x=l).
Wind loads:

$$m_{w,0} = \frac{l \cdot w_0}{2} \tag{8.15}$$

Loads resulting from stabilisation:

$$m_{a,0} = F_i \cdot \left(\frac{\pi}{l}\right) \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \tag{8.16}$$

The forces of the fastenings of the panel have to withstand the moment m(x). So the moment m_0 (per unit length) is converted to the moment M_0 acting on one panel. The moment M_0 is approximately

$$M_0 = m_0 \cdot B \tag{8.17}$$

with
B width of the panel
The moment M_0 leads to forces V^M in the fastenings. These forces act in longitudinal direction of the panel. For a panel connected to the substructure with two fasteners at each transverse edge the force V^M in the highest stressed connections (panels at x=0 and x = l) is approximately

$$V^M = \frac{M_0}{c} \tag{8.18}$$

If further connections to the substructure exist, the forces in the fastenings decrease. Further fastenings can be considered via the displacement figure (Fig. 8.4).

$$M_0 = V^M \cdot \sum_k \frac{c_{i,k}^2}{c_1} \tag{8.19}$$

The force in the highest stressed fastening is

$$V^M = \frac{M_0}{\sum \frac{c_{i,k}^2}{c_1}} \tag{8.20}$$

On the transverse edges of the diaphragm also shear forces occur. The shear forces V^0 can be determined by equilibrium on the panel.

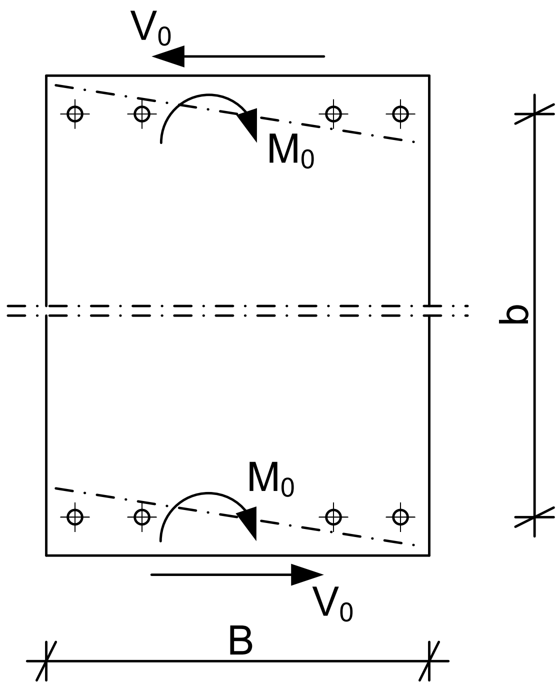

Fig. 8.7: **Equilibrium on a panel**

$$V_0 = \frac{m \cdot M_0}{b} \tag{8.21}$$

m number of stabilized beams

If a constant distribution on the fastenings of the transverse edge is assumed, for one fastening the force V^T in transverse direction is

$$V^T = \frac{V_0}{n} \tag{8.22}$$

with

n number of fastenings at a transverse edge

The resulting force of one fastening is determined by vectorial summation of the forces V^M and V^T.

$$V = \sqrt{\left(V^M\right)^2 + \left(V^T\right)^2} \tag{8.23}$$

8.3.2 Panels with connections at the longitudinal edges and joints

If the panels have connections at the longitudinal edges and joints, the load-bearing behaviour corresponds to that of a real shear diaphragm, like for example diaphragms made of trapezoidal sheeting. Therefore on the transverse edge of the diaphragm the following shear force acts in dependence of the applied load.

Wind load:

$$q_{w,T}(x) = \frac{w_0}{b} \cdot \left(\frac{l}{2} - x\right) \tag{8.24}$$

Loads resulting from stabilisation:

$$q_{a,T}(x) = \frac{\sum F_i}{b} \cdot \frac{\pi}{l} \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \cdot \cos\left(\frac{\pi \cdot x}{l}\right) \tag{8.25}$$

with

ΣF_i summation of normal forces of the beams, which are stabilized by the diaphragm

In the following figure the shear forces in diaphragms loaded by wind loads and stabilisation loads are shown.

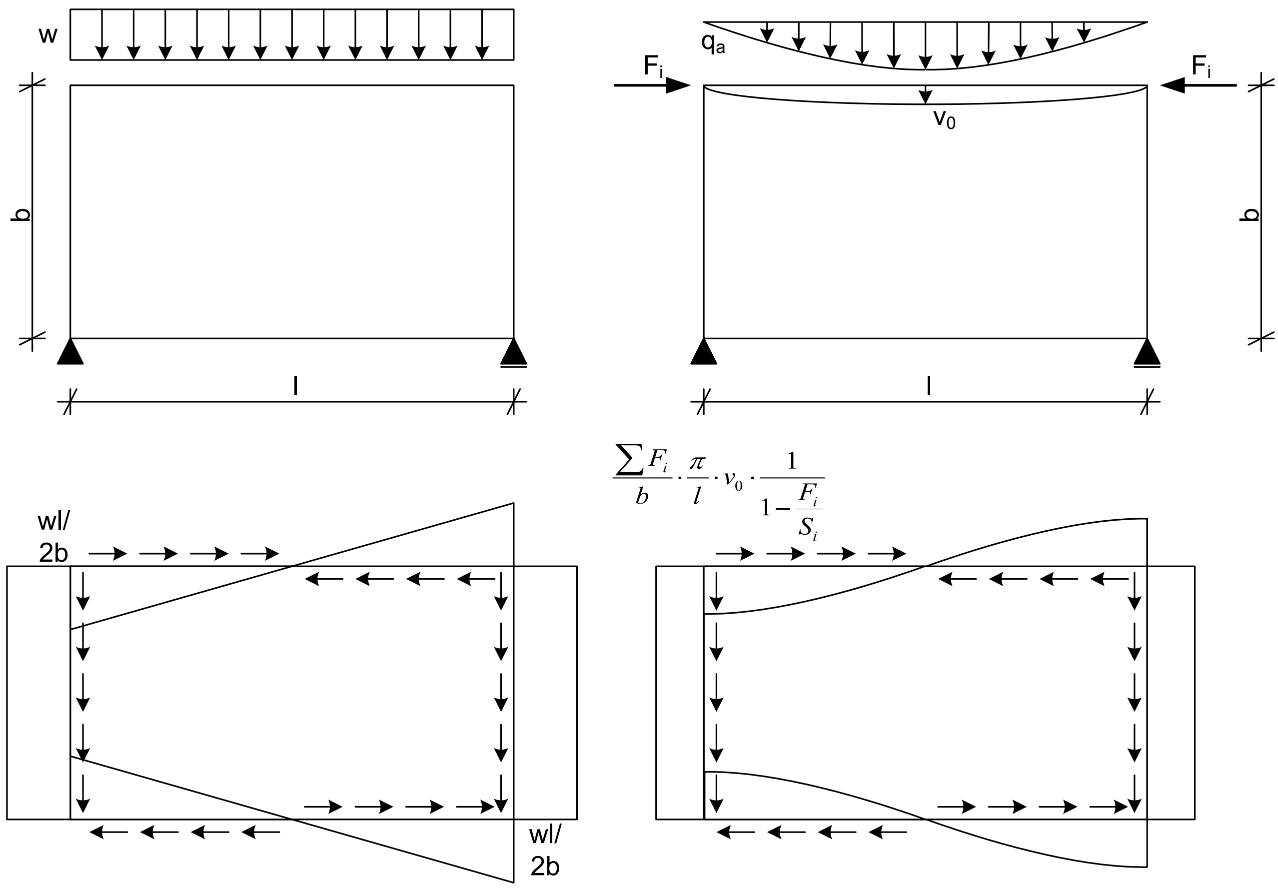

Fig. 8.8: **Shear forces of diaphragms**

At the transverse edges of a diaphragm two loads are acting – the load q_w and q_a resulting from introduction of loads in the diaphragm and the load q_T resulting from the shear force. For determination of the resulting force it is important to note, that both forces have different directions and therefore have to be added vectorially.

The force acting on one fastener of the transverse edge can be determined by multiplying the load q and q_T by the corresponding width, i.e. by the distance of the fastenings.

For wind loads the following forces occur.

From introduction of load in the diaphragm:

$$V^w = w_0 \cdot e \quad \text{(constant for all fasteners of the beam)} \tag{8.26}$$

From shear force:

$$V^T = \frac{l \cdot w_0}{2 \cdot b} \cdot e \quad \text{(highest force at the end of the beams)} \tag{8.27}$$

with

e distance between fastenings

Resulting force:

$$V = \sqrt{\left(V^w\right)^2 + \left(V^T\right)^2} \quad \text{(highest force at the end of the beams)} \tag{8.28}$$

For loads resulting from stabilization the following forces occur.

From introduction of load in the diaphragm:

$$V^a = F_i \cdot \left(\frac{\pi}{l}\right)^2 \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \cdot e \quad \text{(highest force at x = l/2)} \tag{8.29}$$

From shear force:

$$V^T = \frac{\sum F_i}{b} \cdot \frac{\pi}{l} \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \cdot e \quad \text{(highest force at x = 0, x = l)} \tag{8.30}$$

The forces at the longitudinal edges resulting from the load q(x) are determined by integration of q(x) over the length of the beam and distribution to both longitudinal edges of the shear diaphragm.

Wind loads:

$$V_w = \frac{1}{2} \cdot \int_0^l q_w dx = \frac{w_0 \cdot l}{2} \tag{8.31}$$

Loads resulting from stabilisation:

$$V_{a,i} = \frac{1}{2} \cdot \int_0^l q_{a,i} dx = F_i \cdot \left(\frac{\pi}{l}\right) \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \cdot \tag{8.32}$$

For stabilising forces the forces V_i have to be added up over the number of beams to be stabilised. The resulting force is distributed uniformly to the fasteners of a longitudinal edge.

Wind loads:

$$V = \frac{V_w}{n} \tag{8.33}$$

Loads resulting from stabilisation:

$$V = \frac{m \cdot V_{a,i}}{n} \tag{8.34}$$

with

m number of beams to be stabilized

n number of fastenings at a longitudinal edge

To determine the forces at the longitudinal joints the equations (8.31) and (8.32) have to be combined with the linear approach (for wind loads) or with the cosine approach (for loads resulting from stabilization) of equation (8.24) and (8.25). Also at the longitudinal joints the forces V_i have to be added up over the number of beams to be stabilized and are distributed uniformly to the fasteners of a joint.

Wind loads:

$$V_w = \frac{w_0 \cdot l}{2} \cdot \left(1 - \frac{2 \cdot x}{l}\right) \tag{8.35}$$

Loads resulting from stabilisation:

$$V_{a,i} = F_i \cdot \left(\frac{\pi}{l}\right) \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \cdot \cos\left(\frac{\pi \cdot x}{l}\right) \tag{8.36}$$

For simplification and on the safe side it is recommended to use the forces determined for the longitudinal edges according to (8.31) and (8.32) also to design the fastenings of longitudinal joints.

Remark:

In the calculation procedure given above a uniform distribution of forces to the fasteners of a longitudinal edge is assumed. This requires a beam-like behaviour of the diaphragm. This is only valid for diaphragms with dimensions, which allow staying in the beam theory (approximately l/b > 5). For diaphragms with other dimension the forces are distributed discontinuously to the fasteners of a beam.

9 Determination of shear angle γ

For serviceability limit state the global deformation of the diaphragm should be limited. Therefore it has to be shown that the shear angle γ caused by an external moment is less than an ultimate angle γ_{max}. The angle γ can be determined by equalisation of the internal and the external moment.

$$M^E = M^I = \gamma \cdot I \tag{9.1}$$

$$\gamma = \frac{M^E}{I} \tag{9.2}$$

with

I moment of inertia of the diaphragm

At present for diaphragms made of sandwich panels there are not any values for γ_{max} available. According to DIN 18807 [26], [27] the ultimate shear angle for diaphragms made of trapezoidal sheeting is

$$\gamma_{\max} \leq \frac{1}{750} \tag{9.3}$$

If for a building no other value is given, this value could also be used for diaphragms made of sandwich panels.

10 Partial safety factors

To design diaphragms two design calculations have to be done [26], [27]. The global deformation of the building, i.e. the shear angle γ, has to be limited to a limit shear angle γ_{max}. This is done for the serviceability limit state. So the loads have to be determined using the load factors γ_F for serviceability limit state.

As a second step the fasteners have to be designed. This has to be done for ultimate limit state. The loads have to be determined using the load factors γ_F for ultimate limit state. The design value of the load-bearing capacity of the fastenings is determined by dividing the characteristic load bearing capacity by the material safety factor γ_M.

The partial safety factors γ_F and γ_M are given by national specifications. They can be found in EN 1990 [24] and the related national annex.

11 Summery

Because of their high in-plane shear stiffness sandwich panels can be used to transfer horizontal loads and to stabilise beams and columns against buckling and lateral torsional buckling. In report D3.3 – part 2 the model for determination of the stiffness of diaphragms made of sandwich panels, which was developed in [1], is presented and explicated. Also procedures for the determination of the forces at fastenings, which are decisive for the load bearing behaviour and capacity of diaphragms, are given. In addition some calculation examples are given in the annex of the report at hand.

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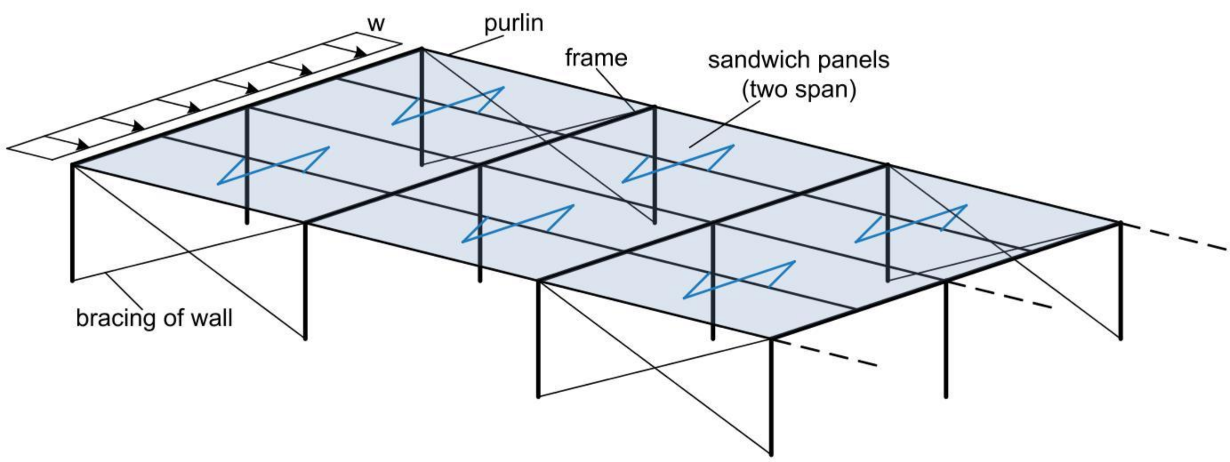
Calculation example No. 1:

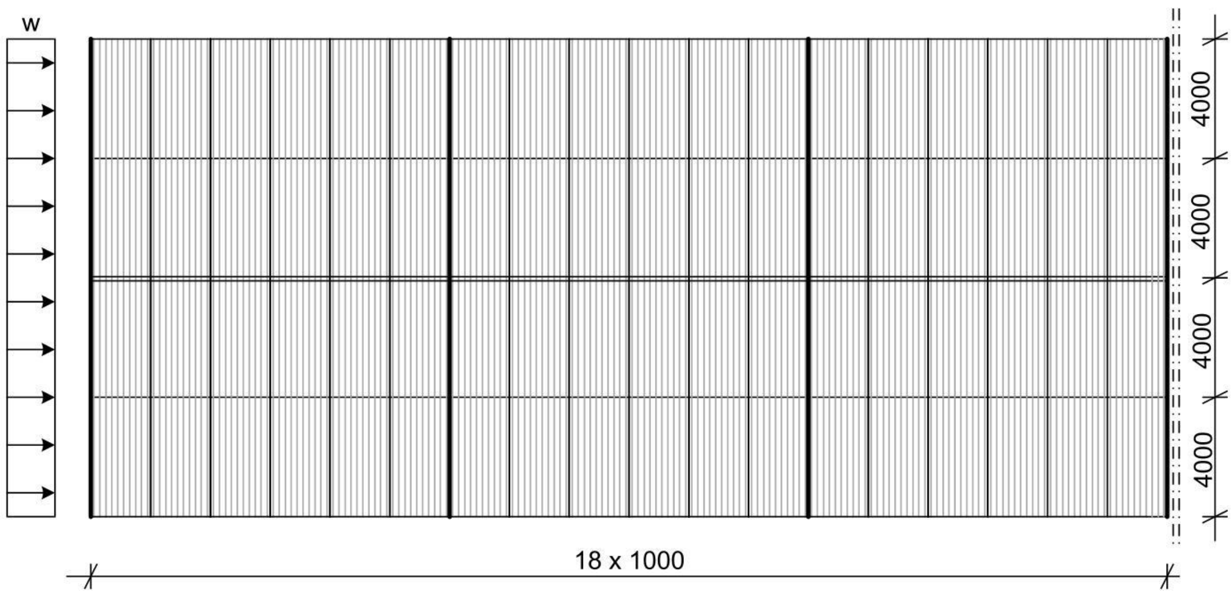
Transfer of wind loads - Panels without connections at the longitudinal edges and joints

To stabilise the building in the following figure the wind load w has to be transferred to the external braced walls. For transferring the load a diaphragm consisting of the sandwich panels at the roof of the building is used.

$w_k = 1{,}59 kN/m$ (serviceability limit state)

$w_d = 2{,}39 kN/m$ (ultimate limit state)





The panels are connected to the purlins as given in the figure below. There are not any connections at the longitudinal joints of the panels (or these connections are neglected).

The fastenings have the stiffness

$k_v^T = 2{,}34 kN/mm$

Remark:

The stiffness of the fastening has been determined according to section 3 for the following values:

Self-drilling screw with nominal diameter 5,5 mm:

Minor diameter of thread d_1 = 4,19 mm

Diameter of shank d_S = 4,55 mm

Sandwich panels:

Thickness D = 60 mm

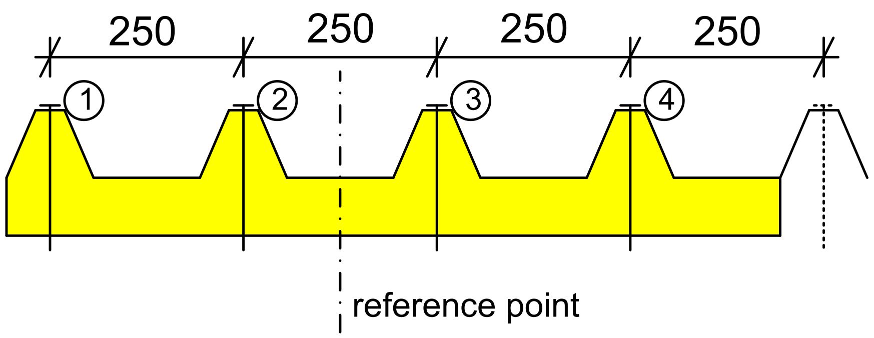
Thickness of face sheets t_F = 0,6mm

Tensile strength of face sheets f_{uF} = 360 N/mm^2

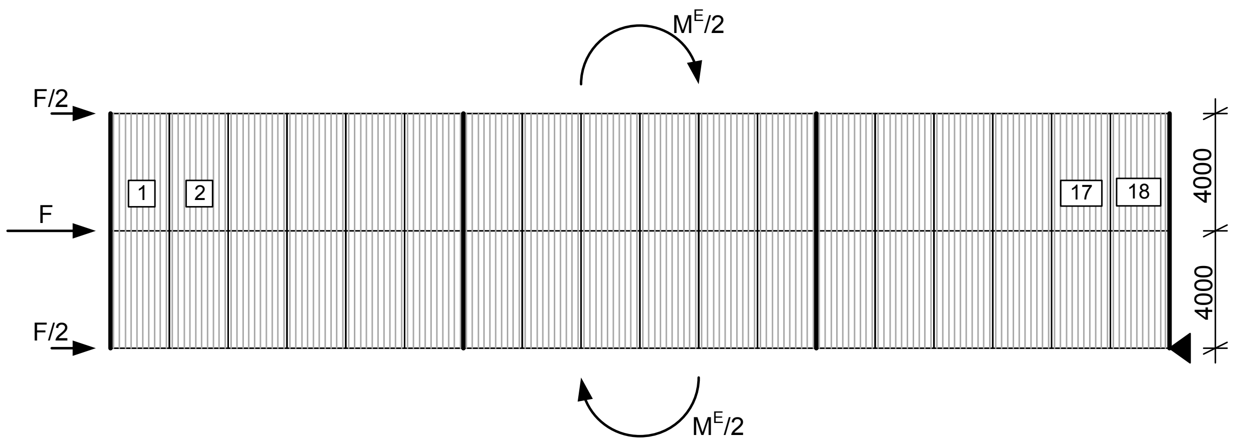
Substructure:

Steel

Thickness of substructure t_{sup} = 12 mm



The diaphragm, which is used to transfer the horizontal load w, is given in the following figure.



Determination of shear stiffness S

For panels without connections at the longitudinal joints the reference point P is located in the centre of gravity of the fasteners. So the shear stiffness S of the diaphragm given above is

$$S = \frac{I}{b} = \frac{\sum_i \sum_k \left(k_v \cdot \frac{c_{i,k}^2}{2} \right)}{b} = \frac{3 \cdot 18 \cdot 2{,}34 kN/mm \cdot \left(\frac{(0{,}75m)^2}{2} + \frac{(0{,}25m)^2}{2} \right)}{8m} = \frac{39487 kNm}{8m} = 4936 kN$$

Determination of forces in the fastenings (for ultimate limit state)

The wind load w is applied to the panels by the purlins. The concentrated loads applied to the purlins are

$$F = \frac{16m \cdot 2{,}39 kN/m}{4} = 9{,}56 kN$$

$$\frac{F}{2} = 4{,}78 kN$$

The loads, which are introduced by the purlins, lead to an external moment.

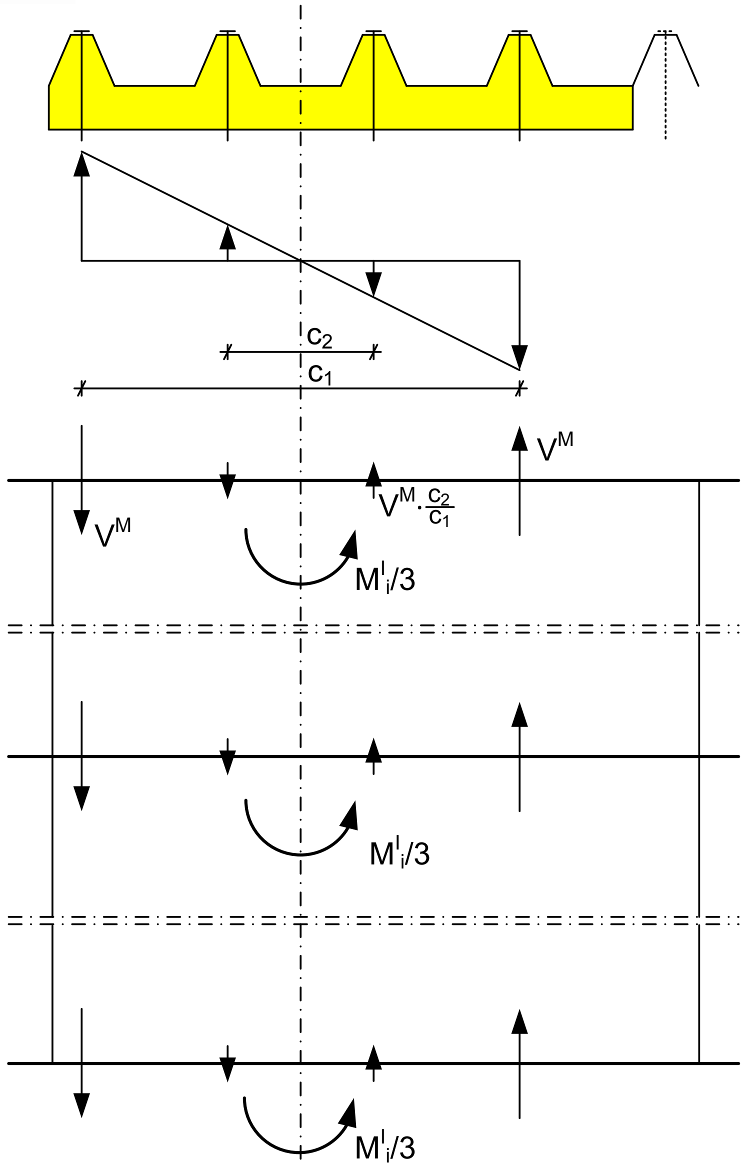
$$M^E = \frac{F}{2} \cdot 8m + F \cdot 4m = 76{,}48 kNm$$

An internal moment M^I has to withstand the external moment M^E. The contribution M^I_i of one panel to the internal moment is

$$M_i^I = n \cdot V^M \cdot \sum \frac{c_{i,k}^2}{c_1} = n \cdot V^M \cdot \left(c_1 + \frac{c_2^2}{c_1} \right) = 3 \cdot V^M \cdot \left(0{,}75m + \frac{(0{,}25m)^2}{0{,}75m} \right) = 2{,}5m \cdot V^M$$

The internal moment of the diaphragm (18 panels) is

$$M^I = 18 \cdot M_i^I = 45m \cdot V^M$$



The force in the most stressed fastening is

$$V^{M} = \frac{M^{E}}{45m} = 1{,}70kN$$

Introducing the load in the panels causes an additional force V^F in the fastenings. To determine the force V^F a constant distribution over the fastenings of a beam is assumed.

$$V^{F} = \frac{F}{4 \cdot 18} = \frac{9{,}56kN}{4 \cdot 18} = 0{,}13kN$$

The forces caused by the external moment and by introduction of the load have to be added. It has to be noted, that the forces V^F and V^M do not act in the same direction. The resulting force V of a fastening is

$$V = \sqrt{\left(V^{M}\right)^{2} + \left(V^{F}\right)^{2}} = 1{,}70kN$$

The force of a fastener given above has to less than the design value of the load bearing capacity.

$$V_{Rd} = \frac{V_{Rk}}{\gamma_m}$$

γ_m has to be chosen according to national specifications.

V_{Rk} can be determined by calculation according to section 3 or be testing (values given in approvals).

Determination of shear angle (serviceability limit state)

Load applied by the beams:

$$F = \frac{16m \cdot 1{,}59kN/m}{4} = 6{,}36kN$$

$$\frac{F}{2} = 3{,}18kN$$

External moment:

$$M^E = \frac{F}{2} \cdot 8m + F \cdot 4m = 50{,}88kNm$$

Shear angle:

$$\gamma = \frac{M^E}{I} = \frac{50{,}88kNm}{39487kNm} = 0{,}0013rad$$

This angle has to be less than γ_{max} (usually 1/750 rad)

Calculation example No. 2:

Transfer of wind loads - Panels with connections at the longitudinal edges and joints

The diaphragm given in example No. 1 is designed, but also connections at the longitudinal joints are considered.

The connections at the transverse edges have the stiffness

$$k_v^T = 2{,}34 kN/mm$$

The connections at the longitudinal joints have the stiffness

$$k_v^L = 7{,}00 kN/mm$$

At the longitudinal joints the fastenings have a distance of 400 mm. So there are 20 fastenings at each joint.

Determination of reference points

For each panel the factors A_i to D_i are determined and the respective equation is formed.

Panel 1:

$$A_i = n_i^l \cdot k_v^L = 0$$

$$B_i = n_i^l \cdot k_v^L + n_i^T \cdot k_v^T + n_i^r \cdot k_v^L = 0 + 12 \cdot 2{,}34 kN/mm + 20 \cdot 7{,}00 kN/mm = 168{,}08 kN/mm$$

$$C_i = n_i^r \cdot k_v^L = 20 \cdot 7{,}00 kN/mm = 140{,}00 kN/mm$$

$$D_i = k_v^T \cdot \sum_{k=1}^{n_i^T} \overline{x}_{i,k}^T = k_v^T \cdot (-375 - 125 + 125 + 375) mm = 0$$

$$0 - 168{,}08 \cdot e_i + 140{,}00 \cdot e_{i+1} = 0 + 140{,}00 \cdot (-375 - 625) - 0$$

$$0 - 168{,}08 \cdot e_i + 140{,}00 \cdot e_{i+1} = -140 kN$$

Panel 2-17:

$$A_i = n_i^l \cdot k_v^L = 20 \cdot 7{,}00 = 140{,}00 kN/mm$$

$$B_i = n_i^l \cdot k_v^L + n_i^T \cdot k_v^T + n_i^r \cdot k_v^L = 20 \cdot 7{,}00 + 12 \cdot 2{,}34 + 20 \cdot 7{,}00 = 308{,}08 kN/mm$$

$$C_i = n_i^r \cdot k_v^L = 20 \cdot 7{,}00 kN/mm = 140{,}00 kN/mm$$

$$D_i = k_v^T \cdot \sum_{k=1}^{n_i^T} \overline{x}_{i,k}^T = k_v^T \cdot (-375 - 125 + 125 + 375) mm = 0$$

$$140{,}00 \cdot e_{i-1} - 308{,}08 \cdot e_i + 140{,}00 \cdot e_{i+1} = 140{,}00 \cdot (625 - (-375)) + 140{,}00 \cdot (-375 - 625) - 0$$
$$140{,}00 \cdot e_{i-1} - 308{,}08 \cdot e_i + 140{,}00 \cdot e_{i+1} = 0$$

Panel 18:

$$A_i = n_i^l \cdot k_v^L = 20 \cdot 7{,}00 kN/mm = 140{,}00 kN/mm$$

$$B_i = n_i^l \cdot k_v^L + n_i^T \cdot k_v^T + n_i^r \cdot k_v^L = 20 \cdot 7{,}00 kN/mm + 12 \cdot 2{,}34 kN/mm + 0 = 168{,}08 kN/mm$$

$$C_i = n_i^r \cdot k_v^L = 0$$

$$D_i = k_v^T \cdot \sum_{k=1}^{n_i^T} \overline{x}_{i,k}^T = k_v^T \cdot (-375 - 125 + 125 + 375)mm = 0$$

$$140{,}00 \cdot e_{i-1} - 168{,}08 \cdot e_i + 0 = 140{,}00 \cdot (625 - (-375)) + 0 - 0$$

$$140{,}00 \cdot e_{i-1} - 168{,}08 \cdot e_i + 0 = 140 kN$$

So we get the following equation system.

$$[A] \cdot [e] = [b]$$

with

$$A = \begin{bmatrix}
-168 & 140 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
140 & -308 & 140 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 140 & -308 & 140 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 140 & -308 & 140 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 140 & -308 & 140 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 140 & -308 & 140 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 140 & -308 & 140 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 140 & -308 & 140 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 140 & -308 & 140 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 140 & -308 & 140 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 140 & -308 & 140 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 140 & -308 & 140 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 140 & -308 & 140 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 140 & -308 & 140 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 140 & -308 & 140 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 140 & -308 & 140 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 140 & -308 & 140 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 140 & -168
\end{bmatrix}$$

$$b = \begin{bmatrix}
-140000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 140000
\end{bmatrix}$$

Solving the equation system results in the locations of the reference points e_i of the panels.

$$e = \begin{bmatrix} 1787 \\ 1145 \\ 733 \\ 468 \\ 297 \\ 185 \\ 111 \\ 59 \\ 18 \\ -18 \\ -59 \\ -111 \\ -185 \\ -297 \\ -468 \\ -733 \\ -1145 \\ -1787 \end{bmatrix}$$

The distance $x_{i,k}$ of a fastening to the reference point is given in the following table.

$$x_{i,k} = \bar{x}_{i,k} - e_i$$

reference point		x: distance of the fastening from reference point					
		left edge	right edge	transverse edge			
panel	e_i			1 (left)	2	3	4 (right)
	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]
1	1787	-	-1162	-2162	-1912	-1662	-1412
2	1145	-1520	-520	-1520	-1270	-1020	-770
3	733	-1108	-108	-1108	-858	-608	-358
4	468	-843	157	-843	-593	-343	-93
5	297	-672	328	-672	-422	-172	78
6	185	-560	440	-560	-310	-60	190
7	111	-486	514	-486	-236	14	264
8	59	-434	566	-434	-184	66	316
9	18	-393	607	-393	-143	107	357
10	-18	-357	643	-357	-107	143	393
11	-59	-316	684	-316	-66	184	434
12	-111	-264	736	-264	-14	236	486
13	-185	-190	810	-190	60	310	560
14	-297	-78	922	-78	172	422	672
15	-468	93	1093	93	343	593	843
16	-733	358	1358	358	608	858	1108
17	-1145	770	1770	770	1020	1270	1520
18	-1787	1412	-	1412	1662	1912	2162

Determination of shear stiffness S

For each fastening the moment of inertia $I_{i,k}$ is determined.

$$I_{i,k} = \begin{cases} k_v^T \cdot \left(x_{i,k}\right)^2 & \text{transverse edge} \\ k_v^L \cdot \left(x_i^l - x_{i-1}^r\right) \cdot x_i^l & \text{left edge} \\ k_v^L \cdot \left(x_i^r - x_{i+1}^l\right) \cdot x_i^r & \text{right edge} \end{cases}$$

The moments of inertia of the fastenings are added up to get the moment of inertia I of the diaphragm.

panel	$I_{i,k}$: contribution of a fastening to the moment of inertia						I_i
	left edge	right edge	transverse edge				
			1 (left)	2	3	4 (right)	
	[kNmm]	[kNmm]	[kNmm]	[kNmm]	[kNmm]	[kNmm]	[kNmm]
1	-	-2913889	10934044	8551180	6460816	4662953	33549204
2	3812782	-2140249	5406219	3774088	2434458	1387327	72456937
3	4560449	-555581	2872644	1722554	864964	299875	97377459
4	4337305	910958	1662907	822850	275294	20237	113309113
5	3898351	2040585	1056324	416476	69129	14281	123447335
6	3484527	2848853	734623	225316	8510	84203	129825561
7	3148208	3410991	552467	130216	465	163215	133723081
8	2877765	3803786	440214	78995	10275	234056	135921628
9	2642265	4090683	362057	48086	26615	297644	136862165
10	2404912	4321595	297644	26615	48086	362057	136733340
11	2124456	4536478	234056	10275	78995	440214	135509312
12	1752278	4767997	163215	465	130216	552467	132944576
13	1229065	5039274	84203	8510	225316	734623	128524721
14	485838	5348893	14281	69129	416476	1056324	121363249
15	-539584	5623580	20237	275294	822850	1662907	110023774
16	-1841857	5589447	299875	864964	1722554	2872644	92231921
17	-3169247	4439891	1387327	2434458	3774088	5406219	64419153
18	-3540998	-	4662953	6460816	8551180	10934044	21007022
						$I = \Sigma I_i$	1919229551

The shear stiffness S of the diaphragm is

$$S = \frac{I}{b} = \frac{1919230 kNm}{8m} = 239904 kN$$

Forces at the fastenings (ultimate limit state)

The force V^M at a fastening resulting from the external moment is

$$V_{i,k}^{M} = \begin{cases} \dfrac{M^E}{I} \cdot k_v^T \cdot x_{i,k} & \text{transverse edge} \\ \dfrac{M^E}{I} \cdot k_v^L \cdot \left(x_i^l - x_{i-1}^r\right) & \text{left edge} \\ \dfrac{M^E}{I} \cdot k_v^L \cdot \left(x_i^r - x_{i+1}^l\right) & \text{right edge} \end{cases}$$

panel	V^M: force at a fastening caused by external moment					
	left edge	right edge	transverse edge			
			1 (left)	2	3	4 (right)
	[kN]	[kN]	[kN]	[kN]	[kN]	[kN]
1	-	0,100	-0,202	-0,178	-0,155	-0,132
2	-0,100	0,164	-0,142	-0,118	-0,095	-0,072
3	-0,164	0,205	-0,103	-0,080	-0,057	-0,033
4	-0,205	0,231	-0,079	-0,055	-0,032	-0,009
5	-0,231	0,248	-0,063	-0,039	-0,016	0,007
6	-0,248	0,258	-0,052	-0,029	-0,006	0,018
7	-0,258	0,264	-0,045	-0,022	0,001	0,025
8	-0,264	0,268	-0,040	-0,017	0,006	0,029
9	-0,268	0,269	-0,037	-0,013	0,010	0,033
10	-0,269	0,268	-0,033	-0,010	0,013	0,037
11	-0,268	0,264	-0,029	-0,006	0,017	0,040
12	-0,264	0,258	-0,025	-0,001	0,022	0,045
13	-0,258	0,248	-0,018	0,006	0,029	0,052
14	-0,248	0,231	-0,007	0,016	0,039	0,063
15	-0,231	0,205	0,009	0,032	0,055	0,079
16	-0,205	0,164	0,033	0,057	0,080	0,103
17	-0,164	0,100	0,072	0,095	0,118	0,142
18	-0,100	-	0,132	0,155	0,178	0,202

The introduction of the load in the panels causes an additional force V^F in the fasteners

$$V^F = \frac{F}{4 \cdot 18} = \frac{9{,}56kN}{4 \cdot 18} = 0{,}13kN$$

For the fasteners at the transverse edges the force V^F has to be added to the force V^M.

$$V = \sqrt{\left(V^M\right)^2 + \left(V^F\right)^2}$$

panel	V: force at a fastening					
	left edge	right edge	transverse edge			
			1 (left)	2	3	4 (right)
	[kN]	[kN]	[kN]	[kN]	[kN]	[kN]
1	-	0,100	0,240	0,221	0,202	0,185
2	-0,100	0,164	0,192	0,176	0,161	0,149
3	-0,164	0,205	0,166	0,153	0,142	0,134
4	-0,205	0,231	0,152	0,141	0,134	0,130
5	-0,231	0,248	0,144	0,136	0,131	0,130
6	-0,248	0,258	0,140	0,133	0,130	0,131
7	-0,258	0,264	0,138	0,132	0,130	0,132
8	-0,264	0,268	0,136	0,131	0,130	0,133
9	-0,268	0,269	0,135	0,131	0,130	0,134
10	-0,269	0,268	0,134	0,130	0,131	0,135
11	-0,268	0,264	0,133	0,130	0,131	0,136
12	-0,264	0,258	0,132	0,130	0,132	0,138
13	-0,258	0,248	0,131	0,130	0,133	0,140
14	-0,248	0,231	0,130	0,131	0,136	0,144
15	-0,231	0,205	0,130	0,134	0,141	0,152
16	-0,205	0,164	0,134	0,142	0,153	0,166
17	-0,164	0,100	0,149	0,161	0,176	0,192
18	-0,100	-	0,185	0,202	0,221	0,240

The force of a fastener given above has to less than the design value of the load bearing capacity.

$$V_{Rd} = \frac{V_{Rk}}{\gamma_m}$$

γ_m has to be chosen according to national specifications.

V_{Rk} can be determined by calculation according to section 3 or be testing (values given in approvals).

Determination of shear angle (serviceability limit state)

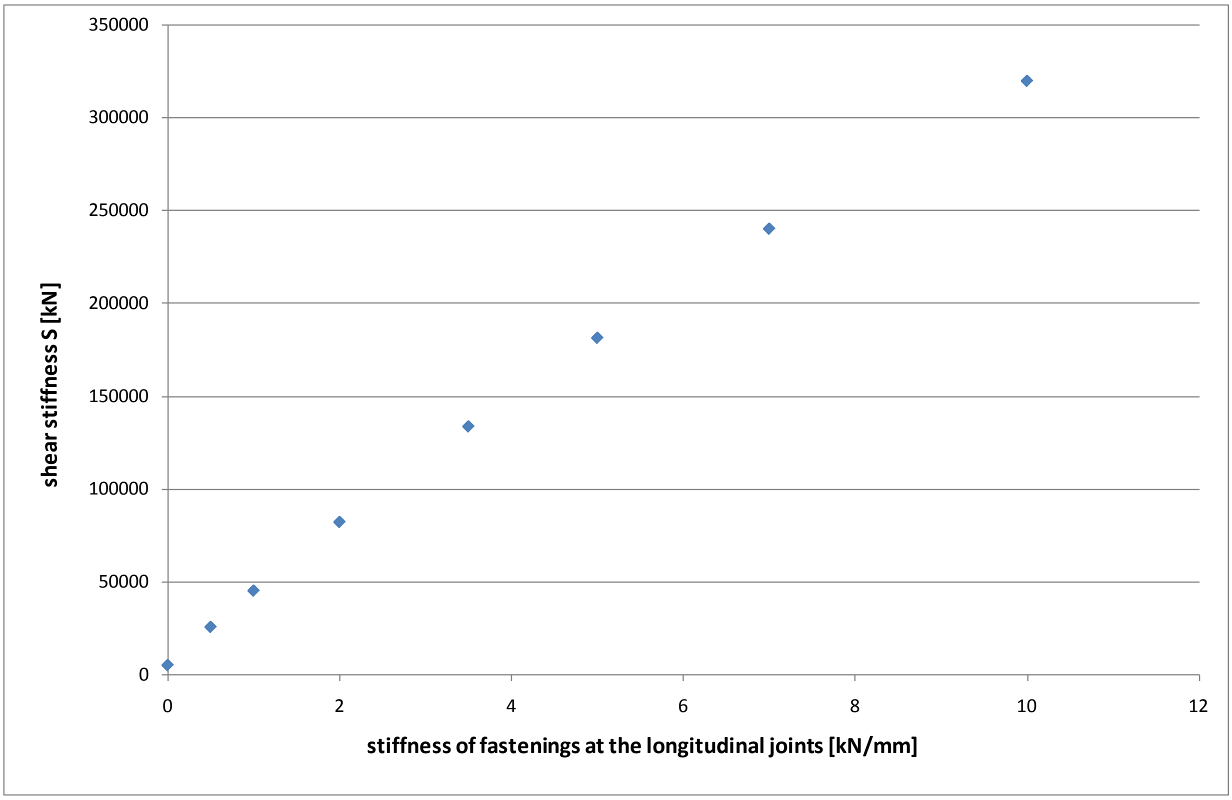
$$\gamma = \frac{M^E}{I} = \frac{50{,}88 kNm}{1919230 kNm} = 0{,}00003 rad$$

This angle has to be less than γ_{max} (usually 1/750 rad)

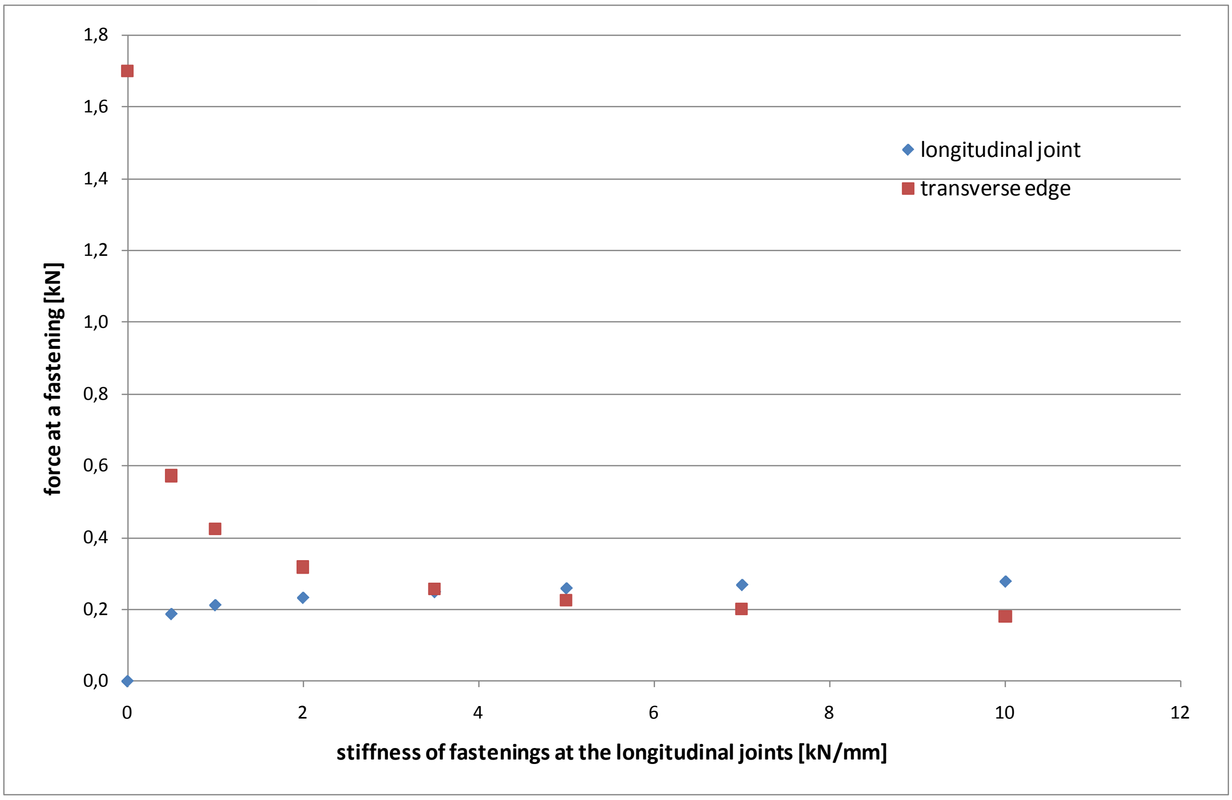
Remarks

If the examples No. 1 and No. 2 are compared, it is obvious that the fastenings at the longitudinal joints have a wide influence on the stiffness and also on the load bearing capacity of the diaphragm. For the diaphragm without fastenings at the longitudinal joints (example no. 1) the shear stiffness is S = 4936 kN, whereas for the diaphragm with connections at the longitudinal joints (example no. 2) the shear stiffness is S = 239904 kN.

The shear stiffness S increases with increasing stiffness k_v^L of the fastenings at the longitudinal joints. In the following diagram the shear stiffness S resulting from a stiffness of the fastenings between 0 und 10 kN/mm is shown.

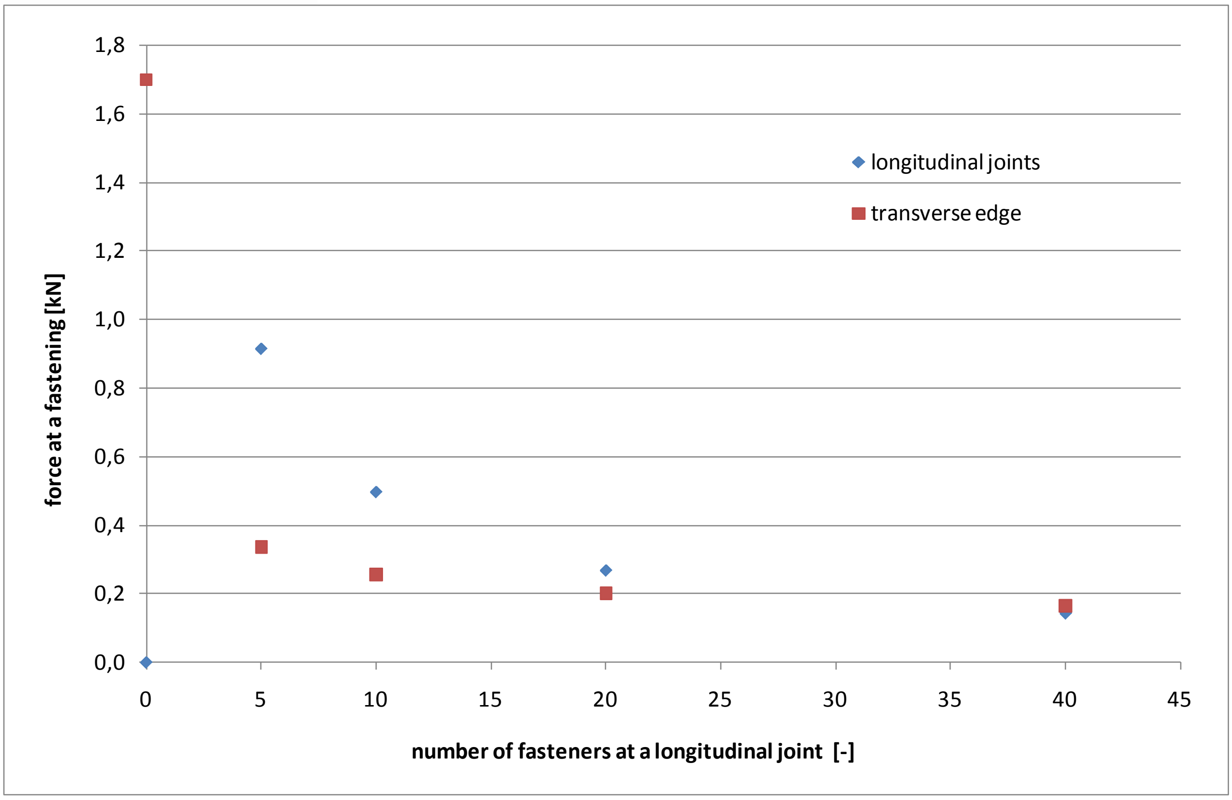


With increasing stiffness of the fastenings at the longitudinal joints the forces at the fastenings increase, whereas the forces at the fastenings at the transverse edges decrease. In the following diagram the forces at the most stressed fastening of the transverse edges and the forces at the most stressed fastening of the longitudinal joints are given for different stiffness of the fastenings at the longitudinal joints.



Concerning the forces of the fastenings there is a wide difference between diaphragms with and diaphragms without connections at the longitudinal joints. Even if the fastenings at the longitudinal joints have only a very low (not realistic) stiffness, the forces of the fastenings at the transverse edge decrease significantly. A further variation of the stiffness results in a comparatively slight modification of the forces of the fastenings.

Also an increase of the number of fastenings at the longitudinal joints increases the stiffness and load bearing capacity of the diaphragm. With an increasing number of fastenings the forces at a fastener at a longitudinal joint as well as at the transverse edges decrease. This is shown in the following diagram, where the forces of the most stressed fastenings are given for different numbers of fasteners at the longitudinal joints.



If a fictive diaphragm with stiff connections at the longitudinal joints is considered, the panels of the diaphragm act together as a single rigid element. In this case the reference points of all panels are located in the center of the diaphragm. For a stiffness $k_v^L = 10^5$ kN/mm, which corresponds approximately to a stiff connection, the shear stiffness of the diaphragm is S = 1704755 kN. In the most stressed fastenings the following forces occur:

Longitudinal joints:

$$V = 0{,}319 kN$$

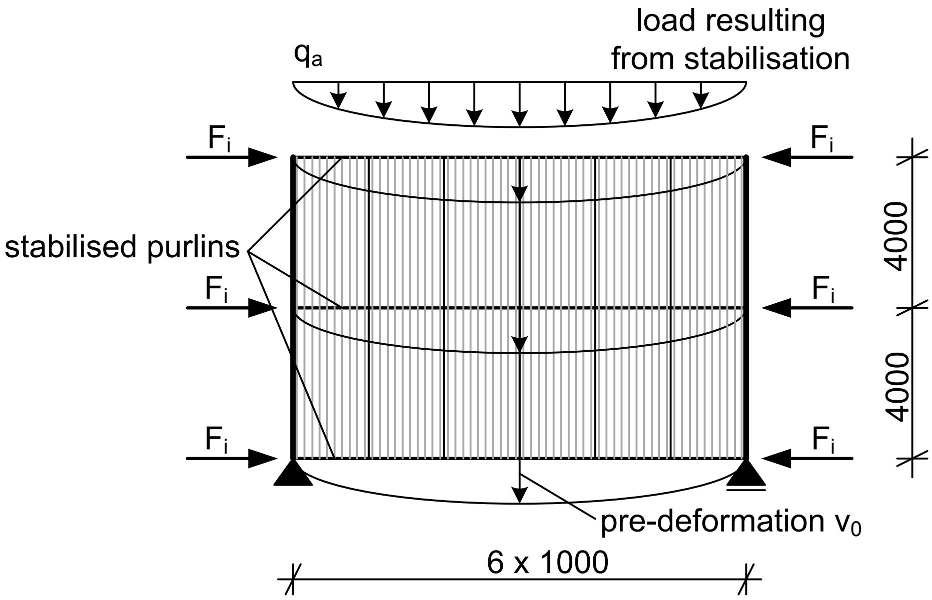
Transverse edges:

$$V = 0{,}116 kN$$

Calculation example No. 3:

Stabilization of single components - Panels without connections at the longitudinal edges and joints

The diaphragm bellow is used to stabilize the purlins, which support the sandwich panels. The purlins are loaded by a bending moment. So they have to be stabilised against lateral torsional buckling.



Imperfection:

$$v_0 = \frac{l}{500} \cdot \sqrt{0{,}5 \cdot \left(1 + \frac{1}{m}\right)} = \frac{6000mm}{500} \cdot \sqrt{0{,}5 \cdot \left(1 + \frac{1}{3}\right)} = 9{,}8mm$$

Compression force in the upper flange:

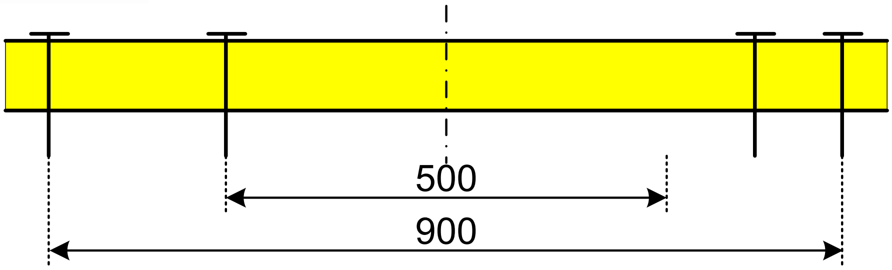
$F_i = 150kN$ (ultimate limit state)

The compression force F_i results from a bending moment.

At the transverse edges the panels are connected to the substructure as given below. The fastenings have the stiffness

$k_v = 2{,}34kN / mm$

There are not any connections at the longitudinal edges and joints.



Determination of shear stiffness S

The shear stiffness of the diaphragm is

$$S = \frac{k_v \cdot \sum\sum \frac{c_{i,k}^2}{2}}{b} = 3 \cdot 6 \cdot \frac{2{,}34kN/mm \cdot \left(\frac{(0{,}9m)^2}{2} + \frac{(0{,}5m)^2}{2} \right)}{6m} = 3720kN$$

The shear stiffness available for one beam is

$$S_i = \frac{3720}{3} = 1240kN$$

Forces at the fastenings (ultimate limit state)

The maximum bracing moment (x = 0 and x = l) is

$$m_0 = F_i \cdot \left(\frac{\pi}{l} \right) \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} = 150kN \cdot \frac{\pi}{6000mm} \cdot 9{,}8mm \cdot \frac{1}{1 - \frac{150}{1240}} = 0{,}88kNm/m$$

The moment acting on one panel is

$$M_0 = m_0 \cdot B = 0{,}88kNm/m \cdot 1m = 0{,}88kNm$$

The moment M_0 results in forces V^M at the fastenings. The force at the outer fasteners of the panel is

$$V^M = \frac{M_0}{\sum \frac{c_k^2}{c_1}} = \frac{0{,}88kNm}{0{,}9m + \frac{(0{,}5m)^2}{0{,}9m}} = 0{,}75kN$$

At the transverse edges of the diaphragm also a force V_0 in transverse direction occurs.

$$V_0 = \frac{3 \cdot M_0}{L} = \frac{3 \cdot 0{,}88kNm}{8m} = 0{,}33kN$$

The force V_0 is continuously distributed to the 4 fasteners of a transverse edge.

$$V^T = \frac{0{,}33kN}{4} = 0{,}08kN$$

For the fasteners at the transverse edges the force V^T has to be added to the force V^M.

$$V = \sqrt{\left(V^M\right)^2 + \left(V^T\right)^2} = \sqrt{0{,}75^2 + 0{,}08^2} = 0{,}75kN$$

The force of a fastener given above has to less than the design value of the load bearing capacity.

$$V_{Rd} = \frac{V_{Rk}}{\gamma_m}$$

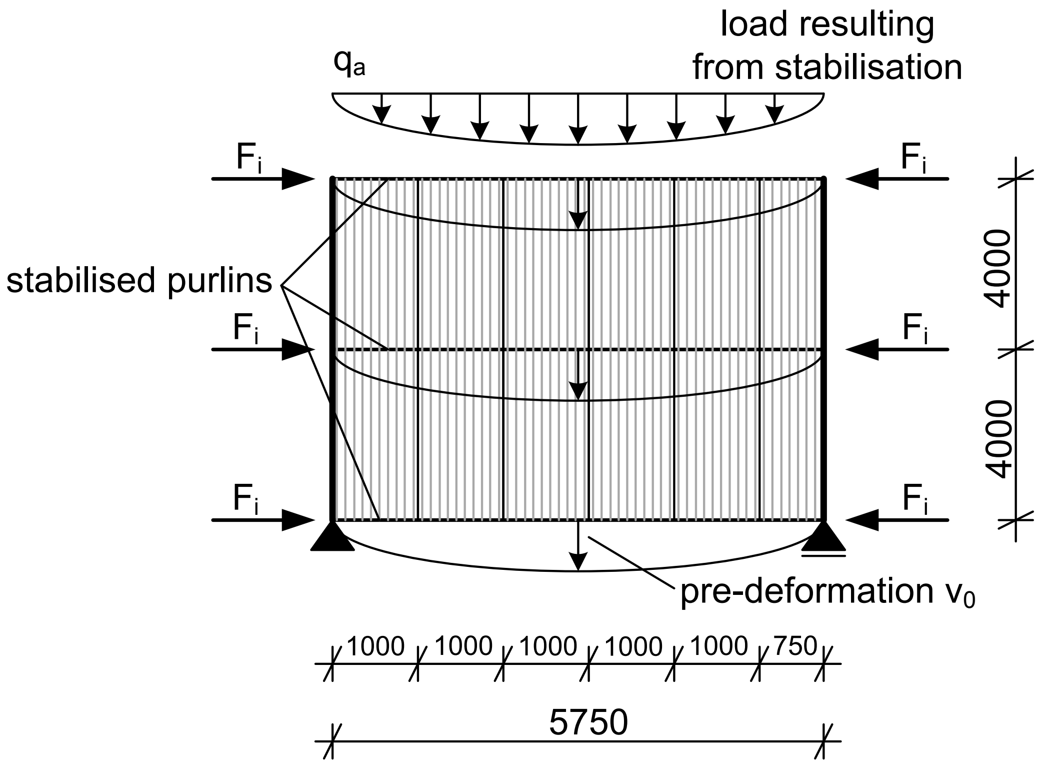
γ_m has to be chosen according to national specifications.

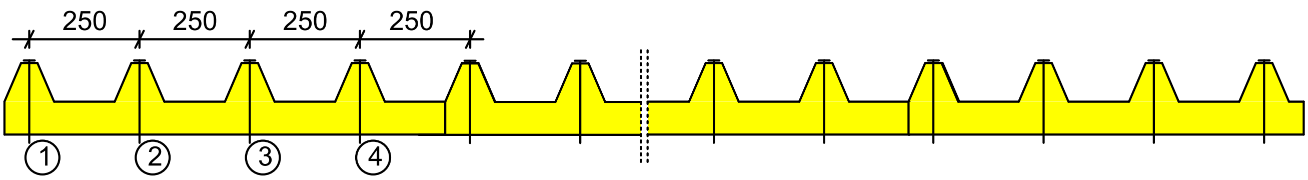
V_{Rk} can be determined by calculation according to section 3 or be testing (values given in approvals).

Calculation example No. 4:

Stabilization of single components - Panels with connections at the longitudinal edges and joints

The diaphragm bellow is used to stabilize the beams, which support the sandwich panels. The beams are loaded by a bending moment. So the beams have to be stabilized against lateral torsional buckling.





The fastenings at the transverse and at the longitudinal edges have the stiffness

$$k_v^T = 2{,}34 kN/mm$$

The fastenings at the longitudinal joints have the stiffness

$$k_v^L = 7{,}00 kN/mm$$

At the longitudinal joints the panels are connected with a distance of 400 mm between the fasteners. So there are 20 fastenings at each longitudinal edge and joint.

Pre-deformation v_0 of the beams:

$$v_0 = \frac{l}{500} \cdot \sqrt{0{,}5 \cdot \left(1 + \frac{1}{m}\right)} = \frac{5750mm}{500} \cdot \sqrt{0{,}5 \cdot \left(1 + \frac{1}{3}\right)} = 9{,}4mm$$

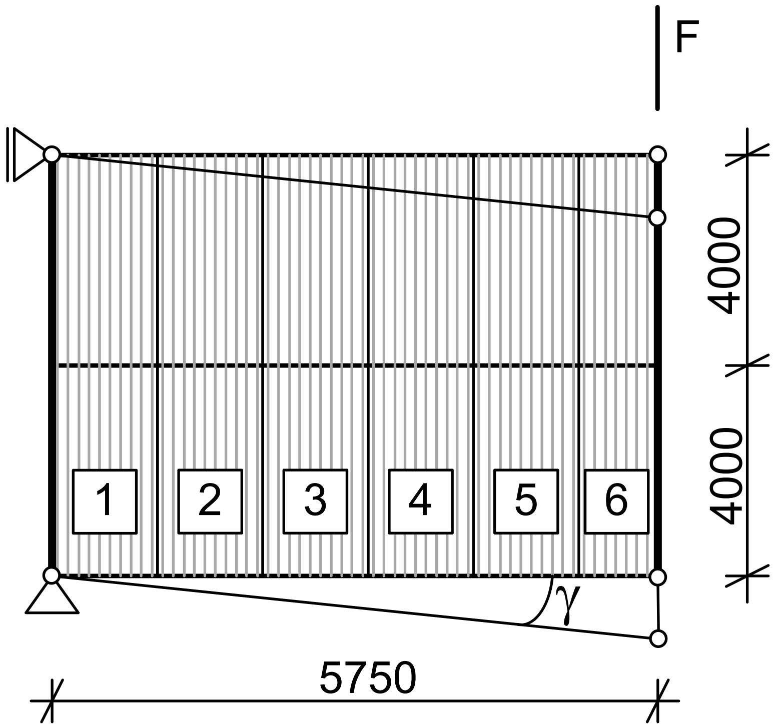
Compression force in the upper flange of the beams:

$F_i = 250kN$ (ultimate limit state)

The compression force F_i results from a bending moment.

<u>Determination of shear stiffness S of the diaphragm</u>

To determine the shear stiffness S the diaphragm given bellow is considered.



For each panel the factors A_i to D_i are determined and the respective equation is formed.

Panel 1:

$$A_i = n_i^l \cdot k_v^T = 20 \cdot 2{,}34kN/mm = 46{,}8kN/mm$$

$$B_i = n_i^l \cdot k_v^T + n_i^T \cdot k_v^T + n_i^r \cdot k_v^L = 20 \cdot 2{,}34 + 12 \cdot 2{,}34 + 20 \cdot 7{,}00 = 214{,}88kN/mm$$

$$C_i = n_i^r \cdot k_v^L = 20 \cdot 7{,}00kN/mm = 140{,}00kN/mm$$

$$D_i = k_v^T \cdot \sum_{k=1}^{n_i^T} \bar{x}_{i,k}^T = k_v^T \cdot (-375 - 125 + 125 + 375)mm = 0$$

$$0 - 214{,}88 \cdot e_i + 140{,}00 \cdot e_{i+1} = 46{,}8 \cdot (0 - (-375)) + 140{,}00 \cdot (-375 - 625) - 0$$

$$0 - 214{,}88 \cdot e_i + 140 \cdot e_{i+1} = -122450kN$$

Panel 2-5:

$$A_i = n_i^l \cdot k_v^L = 20 \cdot 7{,}00 = 140{,}00 kN/mm$$

$$B_i = n_i^l \cdot k_v^L + n_i^T \cdot k_v^T + n_i^r \cdot k_v^L = 20 \cdot 7{,}00 + 12 \cdot 2{,}34 + 20 \cdot 7{,}00 = 308{,}08 kN/mm$$

$$C_i = n_i^r \cdot k_v^L = 20 \cdot 7{,}00 kN/mm = 140{,}00 kN/mm$$

$$D_i = k_v^T \cdot \sum_{k=1}^{n_i^T} \overline{x}_{i,k}^T = k_v^T \cdot (-375 - 125 + 125 + 375) mm = 0$$

$$140{,}00 \cdot e_{i-1} - 308{,}08 \cdot e_i + 140{,}00 \cdot e_{i+1} = 140{,}00 \cdot (625 - (-375)) + 140{,}00 \cdot (-375 - 625) - 0$$
$$140{,}00 \cdot e_{i-1} - 308{,}08 \cdot e_i + 140{,}00 \cdot e_{i+1} = 0$$

Panel 6:

$$A_i = n_i^l \cdot k_v^L = 20 \cdot 7{,}00 kN/mm = 140{,}00 kN/mm$$

$$B_i = n_i^l \cdot k_v^L + n_i^T \cdot k_v^T + n_i^r \cdot k_v^T = 20 \cdot 7{,}00 + 12 \cdot 2{,}34 + 20 \cdot 2{,}34 = 214{,}88 kN/mm$$

$$C_i = n_i^r \cdot k_v^T = 20 \cdot 2{,}34 kN/mm = 46{,}8 kN/mm$$

$$D_i = k_v^T \cdot \sum_{k=1}^{n_i^T} \overline{x}_{i,k}^T = k_v^T \cdot (-375 - 125 + 125 + 375) mm = 0$$

$$140{,}00 \cdot e_{i-1} - 214{,}88 \cdot e_i + 0 = 140{,}00 \cdot (375 - (-375)) + 46{,}8 \cdot (0 - 375) - 0$$

$$140{,}00 \cdot e_{i-1} - 214{,}88 \cdot e_i + 0 = 87450 kN$$

So we get the following equation system.

$$\begin{bmatrix} -214 & 140 & 0 & 0 & 0 & 0 \\ 140 & -308 & 140 & 0 & 0 & 0 \\ 0 & 140 & -308 & 140 & 0 & 0 \\ 0 & 0 & 140 & -308 & 140 & 0 \\ 0 & 0 & 0 & 140 & -308 & 140 \\ 0 & 0 & 0 & 0 & 140 & -214 \end{bmatrix} \cdot \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{bmatrix} = \begin{bmatrix} -122450 \\ 0 \\ 0 \\ 0 \\ 0 \\ 87450 \end{bmatrix}$$

Solving the equation system results in the locations of the reference points e_i.

$$e = \begin{bmatrix} 901 \\ 509 \\ 218 \\ -29 \\ -281 \\ -590 \end{bmatrix}$$

The distances of the fastenings from the corresponding reference point are given in the following table.

$$x_{i,k} = \bar{x}_{i,k} - e_i$$

reference point		x: distance of the fasteners from reference point					
		left edge	right edge	transverse edge			
panel	e			1 (left)	2	3	4 (right)
	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]
1	901	-1276	-276	-1276	-1026	-776	-526
2	509	-884	116	-884	-634	-384	-134
3	218	-593	407	-593	-343	-93	157
4	-29	-346	654	-346	-96	154	404
5	-281	-94	906	-94	156	406	656
6	-590	215	965	215	465	715	965

For each fastening the contribution $I_{i,k}$ to the moment of inertia I is determined.

$$I_{i,k} = \begin{cases} k_v^T \cdot \left(x_{i,k}\right)^2 & \text{transverse edge} \\ k_v^L \cdot \left(x_i^l - x_{i-1}^r\right) \cdot x_{i,k} & \text{left edge} \\ k_v^L \cdot \left(x_i^r - x_{i+1}^l\right) \cdot x_{i,k} & \text{right edge} \end{cases}$$

The moments of inertia of the fastenings are added up to get the moment of inertia I of the diaphragm.

	$I_{i,k}$: contribution of a fastener to the moment of inertia						I_i
	left edge	right edge	transverse edge				
panel			1 (left)	2	3	4 (right)	
	[kNmm]	[kNmm]	[kNmm]	[kNmm]	[kNmm]	[kNmm]	[kNmm]
1	3811593	-1174703	3811593	2464598	1410102	648107	77741001
2	3757350	577604	1827326	939659	344491	41823	96158977
3	2945542	2145151	823223	275510	20296	57582	105343693
4	1825935	3420090	280659	21711	55263	381314	107137340
5	490578	4383670	20572	57119	386166	1007713	101899674
6	-1040992	2179990	108372	506411	1196950	2179990	34755122
						$I = \Sigma I_i$	523035808

The shear stiffness S of the diaphragm is

$$S = \frac{I}{b} = \frac{523035808kNmm}{5750mm} = 90962kN$$

The shear stiffness S_i, which stabilizes one beam, is

$$S_i = \frac{S}{m} = \frac{93962kN}{3} = 30320kN$$

Load acting on the diaphragm

Stabilisation of a beam results in the following load.

$$q_a(x) = F_i \cdot \left(\frac{\pi}{l}\right)^2 \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \cdot \sin\left(\frac{\pi \cdot x}{l}\right)$$

In addition the shear force given bellow acts at the transverse edges of the diaphragm.

$$q_T(x) = \frac{\sum F_i}{b} \cdot \frac{\pi}{l} \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \cdot \cos\left(\frac{\pi \cdot x}{l}\right)$$

Forces at the fastenings (ultimate limit state)

At the transverse edge the following forces act on the fastenings.

From introduction of q_a:

$$V^a = F_i \cdot \left(\frac{\pi}{l}\right)^2 \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \cdot e = 250kN \cdot \left(\frac{\pi}{5750mm}\right)^2 \cdot 9{,}4mm \cdot \frac{1}{1 - \frac{250kN}{30320kN}} \cdot 250mm = 0{,}18kN$$

(max force at x = l/2)

From q_T:

$$V^T = \frac{\sum F_i}{b} \cdot \frac{\pi}{l} \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} \cdot e = \frac{3 \cdot 250kN}{8000mm} \cdot \frac{\pi}{5750mm} \cdot 9{,}4mm \cdot \frac{1}{1 - \frac{250kN}{30320kN}} \cdot 250mm = 0{,}12kN$$

(max force at x = 0 and x = l)

All fastenings of the transverse edge should be designed for the force V = 0,18 kN.

Stabilisation of one beam results in the following forces at the longitudinal edges:

$$V_i = F_i \cdot \left(\frac{\pi}{l}\right) \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}} = 250kN \cdot \frac{\pi}{5750mm} \cdot 9{,}4mm \cdot \frac{1}{1 - \frac{250kN}{30320kN}} = 1{,}30kN$$

The forces V_i have to be added for all stabilised beams and are distributed uniformly to the 20 fasteners of a longitudinal edge.

$$V = \frac{3 \cdot V_i}{20} = 0{,}20 KN$$

On the safe side the fastenings at the longitudinal joints should also be designed for the force

$$V = 0{,}20 kN$$

The force of a fastener given above has to less than the design value of the load bearing capacity.

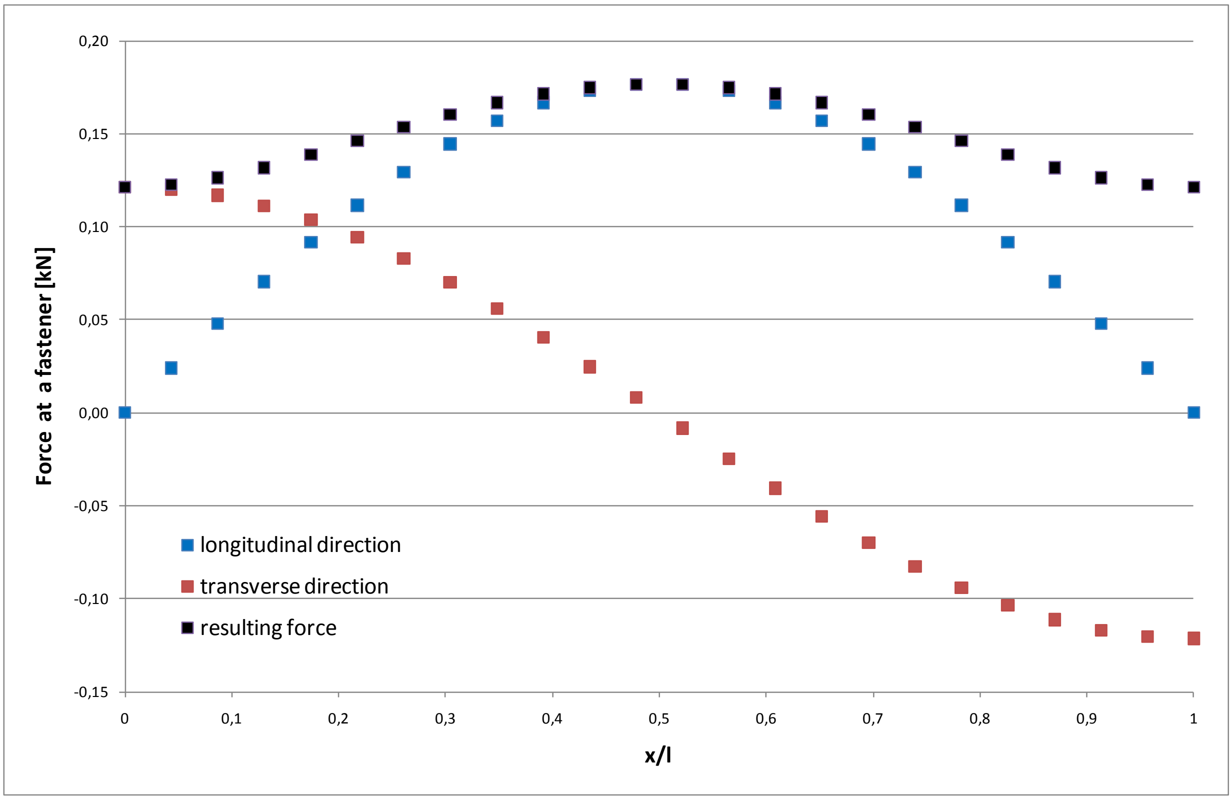
$$V_{Rd} = \frac{V_{Rk}}{\gamma_m}$$

γ_m has to be chosen according to national specifications.

V_{Rk} can be determined by calculation according to section 3 or be testing (values given in approvals).

Remarks:

To design all fasteners of the transverse edge for the same force is a simplification, but it is on the safe side. The exact forces of each fastener are given in the following diagram.



A further simplification is to calculate all forces with the width e = 250 mm. For the fasteners at the corners of the diaphragm (x = 0 and x = l) the exact width is e/2. These fasteners are also loaded by the shear forces acting on the longitudinal edges. Also for this force the exact with is only e/2. These forces have to be added. But it is on the safe side to determine both forces with the width e and to design the fastening for the major of both forces.

Introduction of the stabilising load $q_a(x)$ in the panels results in an additional compression force, which is introduced in the internal face sheets of the panels. This results in an additional moment in the panel.

$$N = \int_0^l q_a(x) = 2 \cdot F_i \cdot \left(\frac{\pi}{l}\right) \cdot v_0 \cdot \frac{1}{1 - \frac{F_i}{S_i}}$$

For the above example this compression force is

$$N = 2 \cdot 250kN \cdot \frac{\pi}{5750mm} \cdot 9{,}4mm \cdot \frac{1}{1 - \frac{250kN}{30320kN}} = 2{,}59kN$$

The force N = 2,59 kN acts on the 6 panels of the diaphragm. This comparatively low force (as well as the moment resulting from this force) is negligible.