CALCULATION EXAMPLE

No.: D3.5

Design of frameless structures made of sandwich panels

Publisher:	Saskia Käpplein Thomas Misiek Karlsruher Institut für Technologie (KIT) Versuchsanstalt für Stahl, Holz und Steine

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Objects:	Global design of axially loaded sandwich panels Design of load application areas of axially loaded sandwich panels Design of frameless buildings for horizontal wind loads

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Versuchsanstalt für Stahl, Holz und Steine, Karlsruher Institut für Technologie (KIT) Tel.: +49 (0)721 608 42215
Abt. Stahl- und Leichtmetallbau, D-76128 Karlsruhe, Deutschland Fax: +49 (0)721 608 44078

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Saskia Käpplein, Thomas Misiek, Karlsruher Institut für Technologie (KIT), Versuchsanstalt für Stahl, Holz und Steine		
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Introduction

Sandwich panels are traditionally used as covering and isolating components, thus being secondary structural components of the building. The sandwich panels are mounted on a substructure and they transfer transverse loads as wind and snow to the substructure. The panels are subjected to bending moments and transverse forces only. A new application is to use sandwich panels with flat or lightly profiled faces in smaller buildings – such as cooling chambers, climatic chambers and clean rooms – without any load transferring substructure.

In this new type of application in addition to space enclosure, the sandwich panels have to transfer loads and to stabilise the building. The wall panels have to transfer normal forces arising from the superimposed load from overlying roof or ceiling panels. So the wall panels have to be designed for axial loads or a combination of axial and transverse loads. Furthermore horizontal wind loads have to be transferred to the foundation and the building has to be stabilised. Because of the lack of a substructure the sandwich panels have to transfer the horizontal loads. For this purpose the high in-plane shear stiffness and capacity of the sandwich panels is used.

Within the framework of work package 3 of the EASIE project, design methods for frameless buildings made of sandwich panels have been developed. The investigations are documented in Deliverable D3.3. In addition a Design guideline for frameless structures made of sandwich panels, which introduces the calculation procedures, was prepared (Deliverable D3.4 – part 1).

In the report at hand calculation examples for the design of frameless structures are given. In the examples the following design procedures are considered:

- *Calculation example No. 1:*
  Global design of axially loaded sandwich panels
- *Calculation example No. 2:*
  Design of load application areas of axially loaded sandwich panels
- *Calculation example No. 3:*
  Design of frameless buildings for horizontal wind loads

In the examples for the loads realistic values according to DIN 1055-4:2005 (wind loads) and DIN 1055-5:2005 (snow loads) have been used. The examples show that in small buildings without substructure axial loads as well as horizontal wind loads can be transferred by the sandwich panel without any problems.

CALCULATION EXAMPLE No. 1

Global design of a iall loaded sandwich panels

Table of contents

1 Preliminar remarks

In the following a calculation example for the design of axially loaded sandwich panels is presented. In the example the design of a wall panel of a small frameless building is shown (Fig. 1.1). The panel is loaded by self-weight and snow loads, which are introduced by the roof into the inner face of the wall panel. Also wind suction loads are considered. Both ends of the wall panel are regarded as hinged.

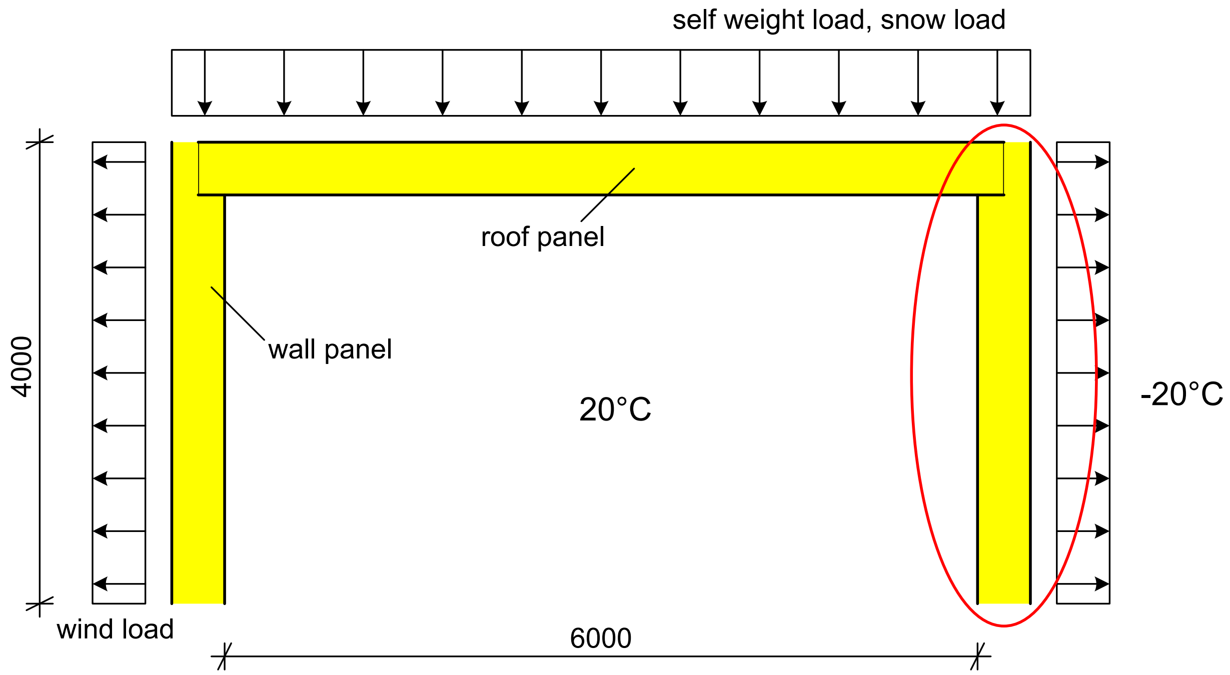

Fig. 1.1: S stem for calculation e ample

To demonstrate the effects of creeping of the core material more clearly the combination of the effects of action with snow as the dominant variable action is considered in the following example. In addition a wind suction load and a temperature difference (winter time) between inside and outside are taken into account. <u>For a complete design of the panel also other effects of action and combinations have to be considered and may be the relevant ones,</u> e.g. wind as dominant variable action and temperature difference in summer time.

The snow and wind loads, which are used in the example, have been determined according to the German standards DIN 1055-4 and DIN 1055-5.

2 Sandwich panel

The walls of the building consist of sandwich panels with lightly profiled steel faces and a polyurethane foam core. In the following section the geometry and the material properties of the panel are summarised. Usually these values are given on the CE-mark of the panel or in (national) approvals.

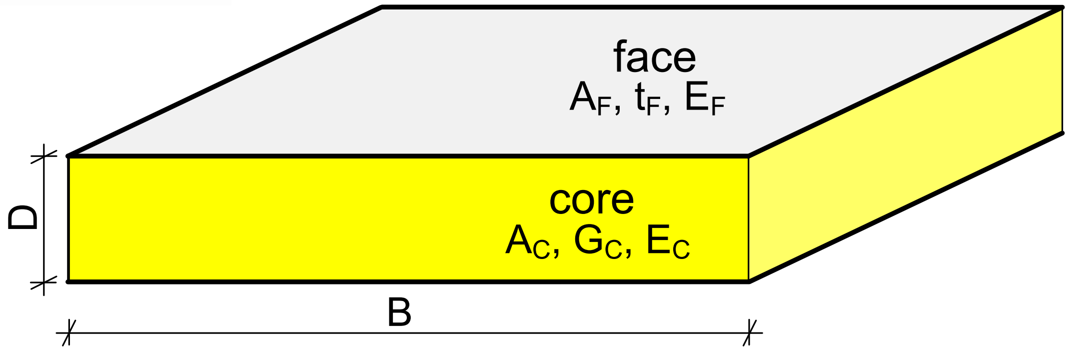

Fig. 1.2: **Cross section of a sandwich panel**

2.1 Geometry and material properties

Thickness of the panel	D = 100 mm
for lightly profiled faces D = e (distance e of centroids of faces)	
With of panel	B = 1000 mm
Thickness of faces	t_F = 0,75 mm
Elastic modulus of faces (steel)	E_F = 210.000 N/mm^2
Thermal expansion coefficient of faces	$\alpha_F = 12 \cdot 10^{-6}$/K
Wrinkling stress	σ_w = 70 N/mm^2
Yield stress of face	f_y = 280 N/mm^2
Shear modulus of core	G_C = 3,3 N/mm^2
Shear strength of core	f_{Cv} = 0,08 N/mm^2
Reduced long-term shear strength	$f_{Cv,t}$ = 0,03 N/mm^2
Creep coefficient 2000 h	φ_{2000} = 2,4
Creep coefficient 100.000 h	φ_{100000} = 7,0

With these values bending and shear stiffness of the panel can be calculated.

Bending stiffness (faces):

$$B_S = E_F \cdot \frac{A_{F1} \cdot A_{F2}}{A_{F1} + A_{F2}} \cdot e^2 = 787{,}5 \frac{kNm^2}{m}$$

Shear stiffness (core):

$$GA = G_C \cdot A_C = G_C \cdot B \cdot e = 330 \frac{kN}{m}$$

2.2 Material safety factors

For the following example the values given in EN 14509, Table E.9 are used.

Wrinkling failure of the face

γ_M = 1,25

Yielding of the face

$\gamma_M = 1{,}1$

Shear failure of the core

$\gamma_M = 1{,}50$

3 Actions and loads

3.1 Axial loads introduced by the roof panels

The self weight load of the roof panels is assumed to be

$g_k = 0{,}12$ kN/m^2

With the span of roof panels L = 6,0 m we get the axial load, which is introduced into the walls

$$N_{G,k} = g_k \cdot \frac{L}{2} = 0{,}36 \frac{kN}{m}$$

For the example at hand the snow load is assumed to be

$s_k = 0{,}68$ kN/m^2

The following axial load results from snow

$$N_{S,k} = s_k \cdot \frac{L}{2} = 2{,}04 \frac{kN}{m}$$

3.2 Moments introduced by the roof panels

The axial forces (self-weight, snow) are introduced into the internal face of the wall panel by contact. This results in additional moments M^N at the upper end of the wall panel (Fig. 1.3).

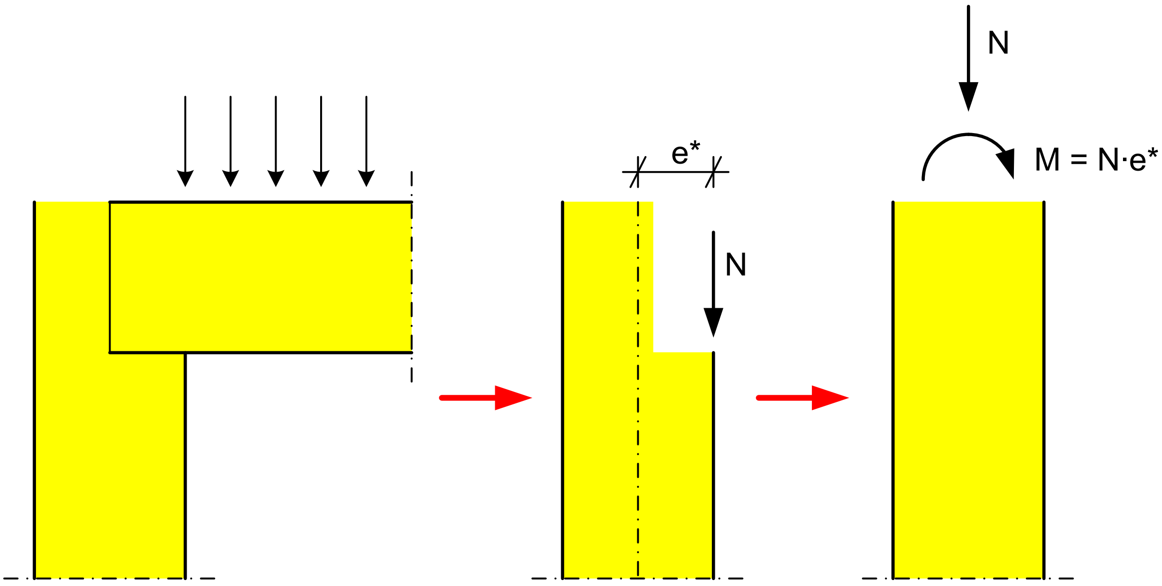

Fig. 1.3: Moment caused b a ial force

Because $t_{F1} = t_{F2}$ the eccentricity of the axial loads is

$$e^* = \frac{D}{2}$$

The end moments are

$$M^N_{G,k} = N_{G,k} \cdot e^* = 0{,}36\frac{kN}{m} \cdot \frac{100mm}{2} = 0{,}018\frac{kNm}{m}$$

$$M^N_{S,k} = N_{S,k} \cdot e^* = 2{,}04\frac{kN}{m} \cdot \frac{100mm}{2} = 0{,}102\frac{kNm}{m}$$

3.3 Transverse loads

For the example at hand the wind load (suction) is assumed to be
w_k = 0,78 kN/m^2

As global geometrical imperfection an initial deflection of the maximum allowable value for bowing according to EN 14509, D.2.9 is assumed.

$$e_0 = \frac{1}{500} \cdot l \leq 10mm$$

$$e_0 = \frac{1}{500} \cdot 4000mm = 8mm$$

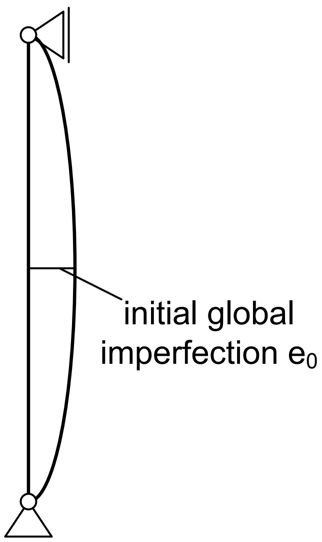

Fig. 1.4: Global imperfection

Together with the axial load the initial deflection causes bending moments and transverse forces. To be on the safe side for the imperfection the direction, which increases the moments caused by wind, snow and self-weight, is assumed. The global imperfection is considered by an equivalent transverse load q_{e0}.

$$q_{e0} = \frac{N \cdot 8 \cdot e_0}{L^2}$$

Global imperfection and self-weight:

$$q_{e0,G,k} = \frac{N_{G,k} \cdot 8 \cdot e_0}{L^2} = \frac{0{,}36 \cdot 8 \cdot 0{,}008}{4^2} = 0{,}00144 \frac{kN}{m^2}$$

Global imperfection and snow:

$$q_{e0,S,k} = \frac{N_{S,k} \cdot 8 \cdot e_0}{L^2} = \frac{2{,}04 \cdot 8 \cdot 0{,}008}{4^2} = 0{,}00816 \frac{kN}{m^2}$$

Temperature differences between inside and outside cause a deflection w_T of the panel.

$$w_T = \Delta T \cdot \frac{\alpha_F \cdot L^2}{e \cdot 8} = 40K \cdot \frac{12 \cdot 10^{-6} \cdot \frac{1}{K} \cdot 4000^2\, mm^2}{100mm \cdot 8} = 9{,}6mm$$

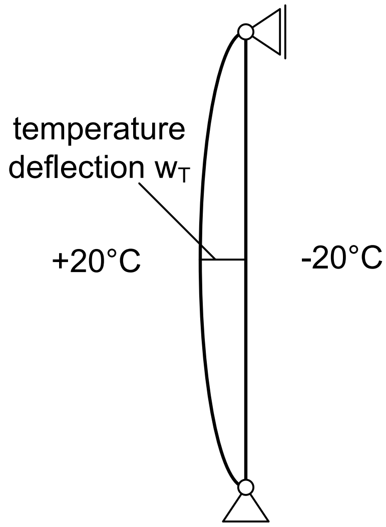

Fig. 1.5: Deflection caused b temperature differences

In conjunction with an axial load the deflection w_T causes bending moments and transverse forces.

In the considered example temperature differences between inside and outside cause a deflection in the opposite direction of the deflection caused by the wind suction load and the axial loads. So if the deflection w_T is considered, bending moment and transverse force decrease. Because of that in the example at hand temperature differences are neglected. That is on the safe side.

Remark:

To consider temperatures differences analogous to geometrical imperfections an equivalent transverse load q_{wT} can be used.

$$q_{wT} = \frac{N \cdot 8 \cdot w_T}{L^2} = \Delta T \cdot \frac{N \cdot \alpha_F}{e}$$

3.4 Summary of loads

A summary of the loads, which are considered in the calculation example at hand, is shown in Fig. 1.6. If sandwich panels are loaded by long-term loads (snow, self-weight) the shear deformation increases. For axially loaded panels the increase of deflections causes an increase of bending moments and transverse forces. Therefore creeping has not only to be considered in the determination of deflections but also in the determination of transverse forces and bending moments. In Fig. 1.6 the loads belonging to different periods of time are marked by different colours.

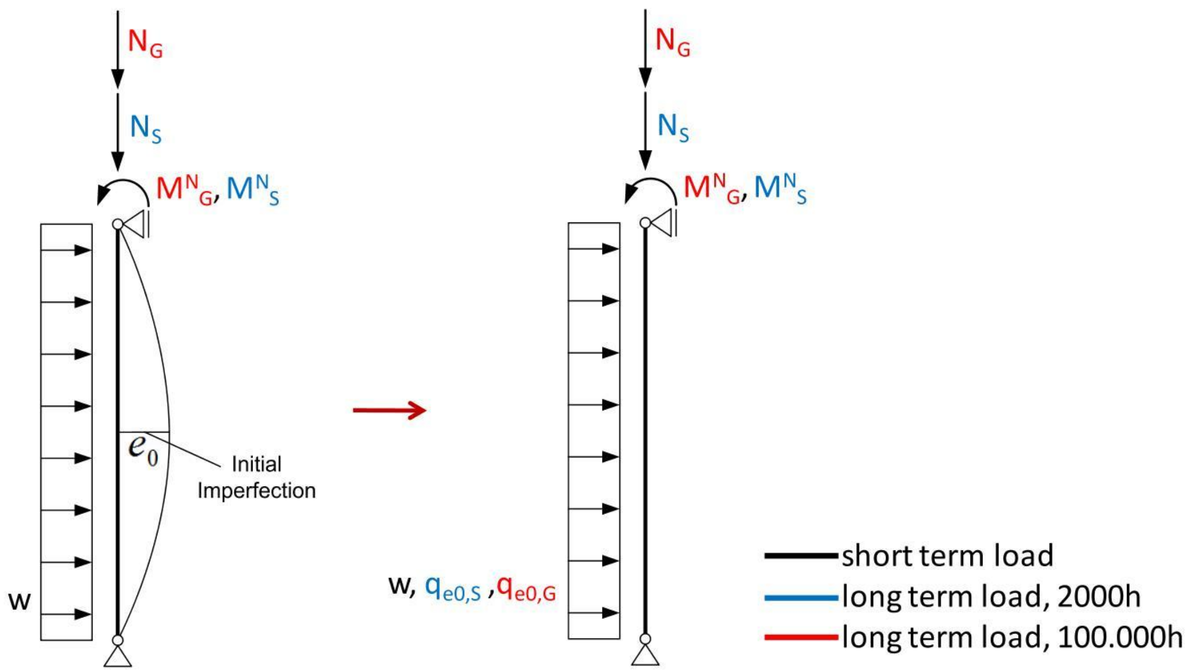

Fig. 1.6: Summary of loads

4 Combination rules, load factors and combination coefficients

The loads are combined according to the following combination rules.
Ultimate limit state (EN 14509, Table E.4)

$$S_d = \gamma_G \cdot G_k + \gamma_{Q1} \cdot Q_{k1} + \sum \gamma_{Qi} \cdot \Psi_{0i} \cdot Q_{ki}$$

Serviceability limit state, deflection (EN 14509, Table E.5)

$$S_d = G_k + \gamma_{11} \cdot Q_{k1} + \sum \Psi_{0i} \cdot \Psi_{1i} \cdot Q_{ki}$$

The load factors γ_F are given in Table E.8 of EN 14509.

Ultimate limit state:

Permanent action G (self weight): $\gamma_{F,G}$ = 1,35

Variable actions (snow, wind): $\gamma_{F,Q}$ = 1,50

Creeping: $\gamma_{F,C}$ = 1,00

Serviceability limit state:

γ_F = 1,00

For wind and snow the following combination coefficients are used (EN 14509, Table E.6).

Ψ_0 = 0,6

Ψ_1 = 0,75

5 Design loads

With the load factors and combination coefficients the design loads are determined.

Ultimate limit state

Permanent loads, self-weight

Axial load

$$N_{G,d} = \gamma_{FG} \cdot N_{G,k} = 1{,}35 \cdot 0{,}36 \frac{kN}{m} = 0{,}486 \frac{kN}{m}$$

Moment at the upper end

$$M_{G,d}^{N} = \gamma_{FG} \cdot M_{G,k}^{N} = 1{,}35 \cdot 0{,}018 \frac{kNm}{m} = 0{,}0243 \frac{kNm}{m}$$

Equivalent transverse load (initial deflection)

$$q_{e0,G,d} = \gamma_{FG} \cdot q_{e0,G,k} = 1{,}35 \cdot 0{,}00144 \frac{kN}{m^2} = 0{,}001944 \frac{kN}{m^2}$$

Variable loads, snow

Axial load:

$$N_{S,d} = \gamma_{FQ} \cdot N_{S,k} = 1{,}5 \cdot 2{,}04 \frac{kN}{m} = 3{,}06 \frac{kN}{m}$$

Moment at the upper end

$$M_{S,d}^{N} = \gamma_{FQ} \cdot M_{S,k}^{N} = 1{,}5 \cdot 0{,}102 \frac{kNm}{m} = 0{,}153 \frac{kNm}{m}$$

Equivalent transverse load (initial deflection)

$$q_{e0,S,d} = \gamma_{FG} \cdot q_{e0,S,k} = 1{,}5 \cdot 0{,}00816 \frac{kN}{m^2} = 0{,}01224 \frac{kN}{m^2}$$

Variable loads, wind

$$w_d = \gamma_{FQ} \cdot \Psi_0 \cdot w_k = 1{,}5 \cdot 0{,}6 \cdot 0{,}78 \frac{kN}{m} = 0{,}702 \frac{kN}{m}$$

<u>Serviceability limit state</u>

Permanent loads, self-weight

Axial load

$$N_{G,d} = N_{G,k} = 0{,}36 \frac{kN}{m}$$

Moment at the upper end

$$M_{G,d}^N = M_{G,k}^N = 0{,}018 \frac{kNm}{m}$$

Equivalent transverse load (initial deflection)

$$q_{e0,G,d} = q_{e0,G,k} = 0{,}00144 \frac{kN}{m^2} = 0{,}00144 \frac{kN}{m^2}$$

Variable loads, snow

Axial load

$$N_{S,d} = \Psi_{11} \cdot N_{S,k} = 0{,}75 \cdot 2{,}04 \frac{kN}{m} = 1{,}53 \frac{kN}{m}$$

Moment at the upper end

$$M_{S,d}^N = \Psi_{11} \cdot M_{S,k}^N = 0{,}75 \cdot 0{,}102 \frac{kNm}{m} = 0{,}0765 \frac{kNm}{m}$$

Equivalent transverse load (initial deflection)

$$q_{e0,S,d} = \Psi_{11} \cdot q_{e0,S,k} = 0{,}75 \cdot 0{,}00816 \frac{kN}{m^2} = 0{,}00612 \frac{kN}{m^2}$$

Variable loads, wind

$$w_d = \Psi_{02} \cdot \Psi_{12} \cdot w_k = 0{,}6 \cdot 0{,}75 \cdot 0{,}78 \frac{kN}{m} = 0{,}351 \frac{kN}{m}$$

6 Deflection, stress resultants and stresses according to 1st Order Theor , creep effects neglected

6.1 Deflection (serviceability limit state)

In the following the deflections caused by the different loads are given. Each deflection is divided into the bending part w_b and the shear part w_v.

Wind load:

$$w_W^I = w_{W,b} + w_{W,v} = \left(\frac{5}{384} \cdot \frac{w_d \cdot L^4}{B_S} \right) + \left(\frac{w_d \cdot L^2}{8 \cdot GA} \right)$$

$$w_W^I = \left(\frac{5}{384} \cdot \frac{0{,}351 \cdot 4^4}{787{,}5} \right) + \left(\frac{0{,}351 \cdot 4^2}{8 \cdot 330} \right) = 1{,}49 + 2{,}13 = 3{,}62mm$$

Snow load:

$$w_S^I = w_{S,b} + w_{S,v} = \left(\frac{M_{S,d}^N \cdot L^2}{16 \cdot B_S} + \frac{5}{384} \cdot \frac{q_{e0,S,d} \cdot L^4}{B_S} \right) + \left(\frac{q_{e0,S,d} \cdot L^2}{8 \cdot GA} \right)$$

$$w_S^I = \left(\frac{0{,}0765 \cdot 4^2}{16 \cdot 787{,}5} + \frac{5}{384} \cdot \frac{0{,}00612 \cdot 4^4}{787{,}5} \right) + \left(\frac{0{,}00612 \cdot 4^2}{8 \cdot 330} \right) = 0{,}12 + 0{,}04 = 0{,}16mm$$

Self-weight load:

$$w_G^I = w_{G,b} + w_{G,v} = \left(\frac{M_{G,d}^N \cdot L^2}{16 \cdot B_S} + \frac{5}{384} \cdot \frac{q_{e0,G,d} \cdot L^4}{B_S} \right) + \left(\frac{q_{e0,G,d} \cdot L^2}{8 \cdot GA} \right)$$

$$w_G^I = \left(\frac{0{,}018 \cdot 4^2}{16 \cdot 787{,}5} + \frac{5}{384} \cdot \frac{0{,}00144 \cdot 4^4}{787{,}5} \right) + \left(\frac{0{,}00144 \cdot 4^2}{8 \cdot 330} \right) = 0{,}029 + 0{,}009 = 0{,}038mm$$

So the deflection in mid-span is

$$w^I = w_W^I + w_S^I + w_G^I = 3{,}62 + 0{,}16 + 0{,}04 = 3{,}82mm$$

6.2 Stress resultants (ultimate limit state)

The design value of the normal force is

$$N_d = N_{G,d} + N_{S,d}$$

$$N_d = 0{,}486 + 3{,}06 = 3{,}546 \frac{kN}{m}$$

The distribution of bending moments caused by the different loads is given in Fig. 1.7.

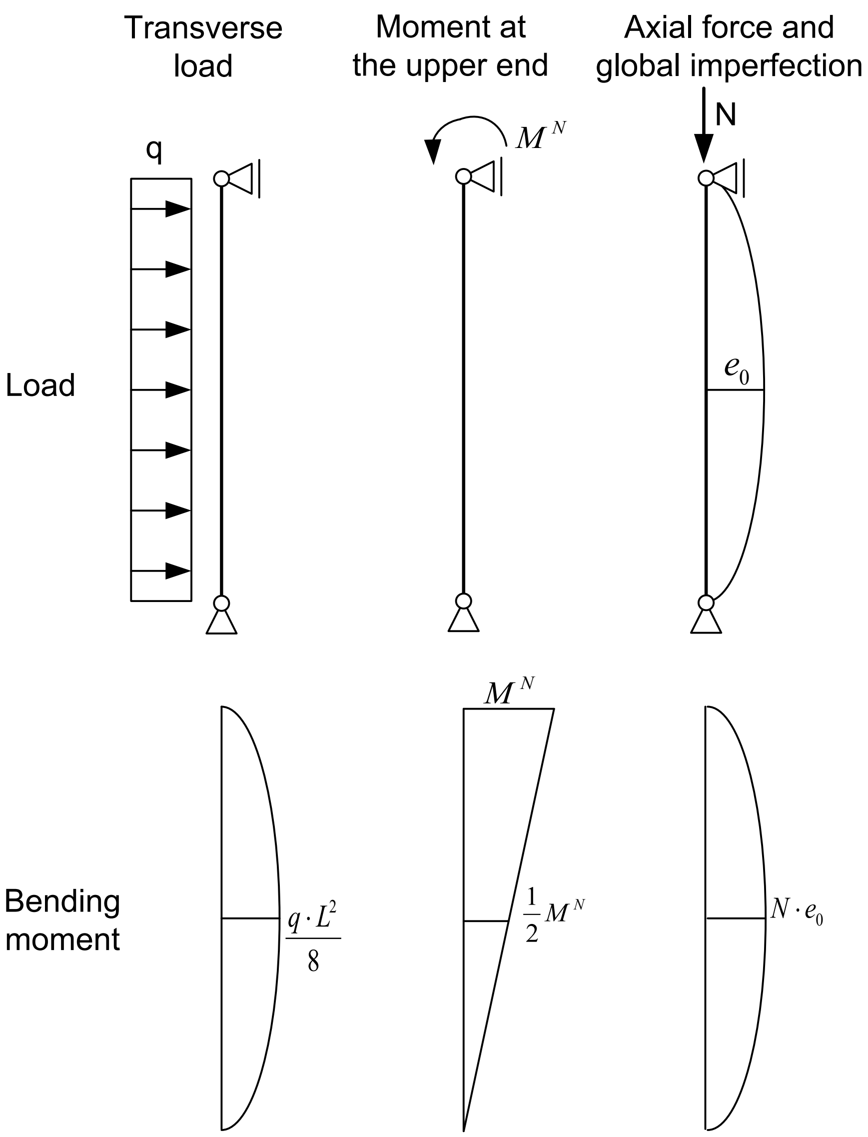

Fig. 1.7: Distribution of bending moments

The different loads cause the following bending moments in mid-span of the panel.

Wind load:

$$M_W^I = \frac{w_d \cdot L^2}{8} = \frac{0{,}702 \cdot 4^2}{8} = 1{,}404 \frac{kNm}{m}$$

Snow load:

$$M_S^I = \frac{1}{2} \cdot M_{S,d}^N + e_0 \cdot N_{S,d} = \frac{1}{2} \cdot 0{,}153 + 0{,}008 \cdot 3{,}06 = 0{,}101 \frac{kNm}{m}$$

Self-weight load:

$$M_G^I = \frac{1}{2} \cdot M_{G,d}^N + e_0 \cdot N_{G,d} = \frac{1}{2} \cdot 0{,}0243 + 0{,}008 \cdot 0{,}486 = 0{,}016 \frac{kNm}{m}$$

So in mid-span we have the bending moment

$$M_d^I = M_W^I + M_S^I + M_G^I = 1{,}404 + 0{,}101 + 0{,}016 = 1{,}521 \frac{kNm}{m}$$

The distribution of transverse forces caused by the different loads is given in Fig. 1.8.

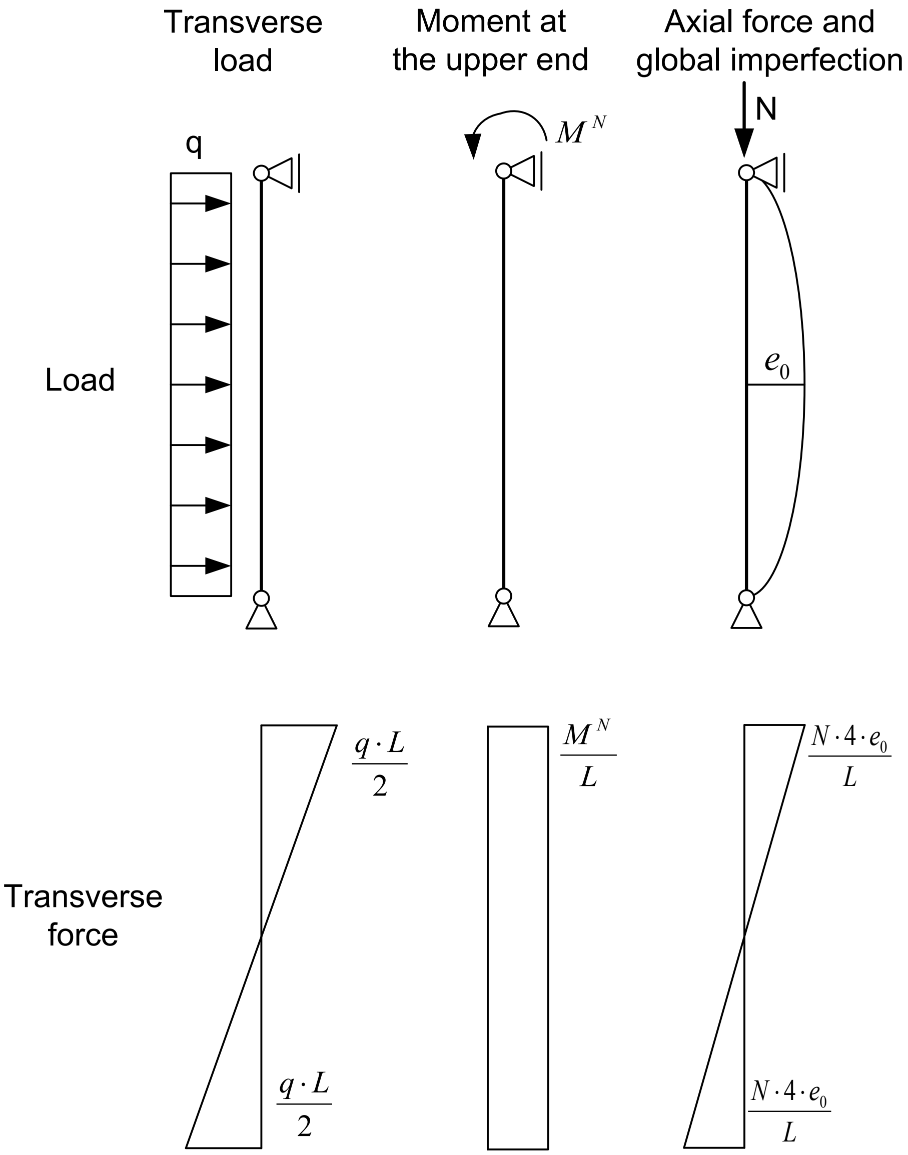

Fig. 1.8: Distribution of transverse forces

The different loads cause the following transverse forces at the supports of the panel.

Wind load:

$$V_{W,d}^I = \frac{w_d \cdot L}{2} = \frac{0{,}702 \cdot 4}{2} = 1{,}404 \frac{kN}{m}$$

Snow load:

$$V_{S,d}^{I} = \frac{M_{S,d}^{N}}{L} + \frac{4 \cdot e_0 \cdot N_{S,d}}{L} = \frac{0{,}153}{4} + \frac{4 \cdot 0{,}008 \cdot 3{,}06}{4} = 0{,}063 \frac{kN}{m}$$

Self-weight load:

$$V_{G,d}^{I} = \frac{M_{G,d}^{N}}{L} + \frac{4 \cdot e_0 \cdot N_{G,d}}{L} = \frac{0{,}0243}{4} + \frac{4 \cdot 0{,}008 \cdot 0{,}486}{4} = 0{,}01 \frac{kN}{m}$$

So at the supports we have the following transverse force

$$V_d^I = V_W^I + V_S^I + V_G^I = 1{,}404 + 0{,}063 + 0{,}01 = 1{,}477 \frac{kN}{m}$$

6.3 Stresses in face sheets and core

With the stress resultants the normal stresses in the face sheets and the shear stresses in the core are determined.

Normal stress in the face sheet subjected to compression:

$$\sigma_{F1,d} = \frac{N}{2 \cdot A_F} + \frac{M^{II}}{A_F \cdot D} = \frac{3546}{2 \cdot 750} + \frac{1.521.000}{750 \cdot 100} = 2{,}364 + 20{,}28 = 22{,}64 \frac{N}{mm^2}$$

Normal stress in the face sheet subjected to tension:

$$\sigma_{F2,d} = -\frac{N}{2 \cdot A_F} + \frac{M^{II}}{A_F \cdot D} = -\frac{3546}{2 \cdot 750} + \frac{1.521.000}{750 \cdot 100} = -2{,}364 + 20{,}28 = 17{,}92 \frac{N}{mm^2}$$

Shear stress in the core:

$$\tau_{C,d} = \frac{V}{A_C} = \frac{1477}{100.000} = 0{,}015 \frac{N}{mm^2}$$

7 Deflections, stress resultants and stresses according to 2nd Order Theor , creep effects neglected

7.1 Consideration of 2nd order theory - amplification factor

For axially loaded panels deflections and stress resultants have to be calculated according to 2nd Order Theory. Because of the axial load the deflections and the stress resultants increase. This is considered by the amplification factor α.

$$\alpha = \frac{1}{1 - \frac{N}{N_{cr}}} = \frac{1}{1 - \frac{3{,}546}{196{,}5}} = 1{,}02$$

Elastic buckling load of the faces (bending)

$$N_{ki} = \pi^2 \cdot \frac{B_S}{s_k^2} = \pi^2 \cdot \frac{787{,}5\,\frac{kNm^2}{m}}{(4m)^2} = 485{,}8\,\frac{kN}{m}$$

Elastic buckling load of the core (shear)

$$GA = 330kN$$

Elastic buckling load of the sandwich section

$$N_{cr} = \frac{N_{ki}}{1 + \frac{N_{ki}}{GA}} = \frac{485{,}8}{1 + \frac{485{,}8}{330}} = 196{,}5\,\frac{kN}{m}$$

7.2 Deflection and stress resultants

The deflections and stress resultants according to 2nd order theory are calculated by multiplying the values according to 1st order theory by the amplification factor α.

$$w^{II} = \alpha \cdot w^{I} = 1{,}02 \cdot 3{,}82 = 3{,}90mm$$

$$N^{II} = N^{I} = 3{,}546\,\frac{kN}{m}$$

$$M^{II} = \alpha \cdot M^{I} = 1{,}02 \cdot 1{,}521 = 1{,}551\,\frac{kNm}{m}$$

$$V^{II} = \alpha \cdot V^{I} = 1{,}02 \cdot 1{,}477 = 1{,}507\,\frac{kN}{m}$$

7.3 Stresses in face sheets and core

With the stress resultants the normal stresses in the face sheets and the shear stresses in the core are determined.

Normal stress in the face sheet subjected to compression:

$$\sigma_{F1,d} = \frac{N}{2 \cdot A_F} + \frac{M^{II}}{A_F \cdot D} = \frac{3546}{2 \cdot 750} + \frac{1.551.000}{750 \cdot 100} = 2{,}364 + 20{,}68 = 23{,}04\,\frac{N}{mm^2}$$

Normal stress in the face sheet subjected to tension:

$$\sigma_{F2,d} = -\frac{N}{2 \cdot A_F} + \frac{M^{II}}{A_F \cdot D} = -\frac{3546}{2 \cdot 750} + \frac{1.551.000}{750 \cdot 100} = -2{,}364 + 20{,}68 = 18{,}32\,\frac{N}{mm^2}$$

Shear stress in the core:

$$\tau_{C,d} = \frac{V^{II}}{A_C} = \frac{1507}{100.000} = 0{,}015\,\frac{N}{mm^2}$$

8 Deflections, stress resultants and stresses according to 2nd Order Theor , creep effects considered

8.1 Creep coefficients

If sandwich panel are loaded by long-term loads like snow and self-weight load, the shear deformations increase with constant loading. If the panels are loaded with axial loads an increase of deflections results also in an increase of bending moment and transverse force. To consider the creep effects the following creep coefficients φ_t are used.

Snow (2000h):

φ_{2000} = 2,4

Self-weight (100.000h):

$\varphi_{100.000}$ = 7,0

The increase of the deflection of the sandwich section of a panel is considered by the sandwich creep coefficient φ_{St}.

$$\varphi_{St} = \frac{k}{1+k} \cdot \varphi_t$$

The sandwich factor k represents the ratio between shear part and bending part of deflection

$$k = \frac{w_v}{w_b} = \frac{B_S}{GA} \cdot \frac{\int Q\overline{Q}dx}{\int M\overline{M}dx}$$

For the panel considered in the example at hand the sandwich factor k is

$$k = \frac{B_S}{GA \cdot L^2} \cdot \frac{48 \cdot e_0}{3 \cdot e^* + 5 \cdot e_0} = \frac{787{,}5}{330 \cdot 4^2} \cdot \frac{48 \cdot 8}{3 \cdot 50 + 5 \cdot 8} = 0{,}30$$

So we get the following sandwich creep factors.

Snow load (2000h):

$$\varphi_{S2000} = \frac{0{,}30}{1+0{,}30} \cdot 2{,}4 = 0{,}55$$

Self-weight load (100.000h):

$$\varphi_{S100.000} = \frac{0{,}30}{1+0{,}30} \cdot 7{,}0 = 1{,}62$$

8.2 Deflection and stress resultants

For the deflection creep effects are considered by multiplying the shear parts of the deflections due to long-term loads by the related creep coefficient φ_t.

$$w_t^{II} = \left[w_{W,b} + w_{W,v} + w_{S,b} + w_{S,v} \cdot (1+\varphi_{2000}) + w_{G,b} + w_{G,v} \cdot (1+\varphi_{100000})\right] \cdot \alpha$$

$$w_t^{II} = \left[1{,}49 + 2{,}13 + 0{,}12 + 0{,}04 \cdot (1+2{,}4) + 0{,}03 + 0{,}01 \cdot (1+7{,}0)\right] \cdot 1{,}02 = 4{,}07\,mm$$

For the stress resultants <u>bending moment</u> and <u>transverse force</u> the creep effects are considered by multiplying the long-term parts of the stress resultants by the related sandwich creep coefficient φ_{St}.

Axial force:

$$N_{d,t}^{II} = N_d^{I} = 3{,}546\,\frac{kN}{m}$$

Bending moment:

$$M_{d,t}^{II} = \left[M_W^{I} + M_S^{I} \cdot (1+\varphi_{S2000}) + M_G^{I}(1+\varphi_{S100000})\right] \cdot \alpha$$

$$M_{d,t}^{II} = \left[1{,}404 + 0{,}101 \cdot (1+0{,}55) + 0{,}016 \cdot (1+1{,}62)\right] \cdot 1{,}02 = 1{,}635\,\frac{kNm}{m}$$

Transverse force:

$$V_{d,t}^{II} = \left[V_W^{I} + V_S^{I} \cdot (1+\varphi_{S2000}) + V_G^{I}(1+\varphi_{S100000})\right] \cdot \alpha$$

$$V_{d,t}^{II} = \left[1{,}404 + 0{,}063 \cdot (1+0{,}55) + 0{,}01 \cdot (1+1{,}62)\right] \cdot 1{,}02 = 1{,}558\,\frac{kN}{m}$$

The shear strength of the core material is time-dependent. So for design purposes the transverse force is divided into a short-term and a long-term part.

$$V_d^{II} = V_{d,st}^{II} + V_{d,lt}^{II} = \left[1{,}404 \cdot 1{,}02\right] + \left[(0{,}063 \cdot 1{,}55 + 0{,}01 \cdot 2{,}62) \cdot 1{,}02\right] = 1{,}432\,\frac{kN}{m} + 0{,}126\,\frac{kN}{m}$$

8.3 Stresses in face sheets and core

With the stress resultants the normal stresses in the face sheets and the shear stresses in the core are determined. Effects of 2[nd] order theory as well as creep effects are considered in the following stresses.

Normal stress in the face sheet subjected to compression:

$$\sigma_{F1,d,t} = \frac{N}{2 \cdot A_F} + \frac{M_t^{II}}{A_F \cdot D} = \frac{3546}{2 \cdot 750} + \frac{1.635.000}{750 \cdot 100} = 2{,}364 + 21{,}8 = 24{,}16\,\frac{N}{mm^2}$$

Normal stress in the face sheet subjected to tension:

$$\sigma_{F2,d,t} = -\frac{N}{2 \cdot A_F} + \frac{M_t^{II}}{A_F \cdot D} = -\frac{3546}{2 \cdot 750} + \frac{1.635.000}{750 \cdot 100} = -2{,}364 + 21{,}8 = 19{,}44\,\frac{N}{mm^2}$$

Shear stress in the core:

$$\tau_{C,d,t} = \frac{V_t^{II}}{A_C}$$

$$\tau_{C,d} = \frac{1558}{100.000} = 0{,}016 \frac{N}{mm^2}$$

As the transverse force the shear stress is divided into a short-term part and a long-term part:

$$\tau_{C,d} = \tau_{Cd.st} + \tau_{Cd,lt} = \frac{1432}{100.000} + \frac{126}{100.000} = 0{,}014 \frac{N}{mm^2} + 0{,}00126 \frac{N}{mm^2}$$

9 Design calculations

To design the panel effects of 2nd order and creep effects have to be considered. So the deflections and stresses calculated in section 8 are used for the design calculations.

The deflection has to be less than the ultimate deflection w_{ult}. The ultimate deflection is chosen according to EN 14509, E.5.4.

$$w_t^{II} = 4{,}07mm \leq 40mm = \frac{L}{100}$$

The normal stress in the faces is compared to the yield strength and to the wrinkling stress, whereat in almost every case the wrinkling stress is decisive.

Yielding of the face in tension:

$$f_{y,d} = \frac{f_{y,k}}{\gamma_m} = \frac{280}{1{,}1} = 255 \frac{N}{mm^2}$$

$$\sigma_{F,d} = 19{,}44 N/mm^2 \leq f_{y,d} = 255 \frac{N}{mm^2}$$

Wrinkling of the face in compression:

$$\sigma_{w,d} = \frac{\sigma_{w,k}}{\gamma_M} = \frac{70}{1{,}25} = 56 \frac{N}{mm^2}$$

$$\sigma_{F,d} = 24{,}16 N/mm^2 \leq \sigma_{w,d} = 56 \frac{N}{mm^2}$$

The shear stress is compared to the shear strength of the core. In doing so it has to be considered, that the shear stress consists of a long-term and a short-term part and each part has to be regarded with the corresponding shear strength.

$$f_{Cv,d} = \frac{f_{Cv,k}}{\gamma_m} = \frac{0{,}08}{1{,}5} = 0{,}053 \frac{N}{mm^2}$$

$$f_{Cv,t,d} = \frac{f_{Cv,t,k}}{\gamma_m} = \frac{0,03}{1,5} = 0,02 \frac{N}{mm^2}$$

$$\frac{\tau_{C,d,st}}{f_{Cv,d}} + \frac{\tau_{C,d,lt}}{f_{Cv,t,d}} = \frac{0,014}{0,053} + \frac{0,00126}{0,02} = 0,26 + 0,063 \leq 1,0$$

10 Conclusions

In the sections above a calculation example for the design of an axially loaded wall panel of a frameless building is presented. For this purposes a combination of the effects of action has been chosen with snow as the dominant variable action. So the influence of creep effects and effects of 2nd order theory is demonstrated more clearly. However, the wind load has the main influence on the deflections, stress resultants and stresses. To design the panel additional combinations must be considered, e.g. wind as the dominant action etc.

Creeping as well as 2nd order effects has only a very small influence. In the above example the increase of the compression stress in the face is less than 10%, if creeping and 2nd order effects are considered. So in small buildings without substructure the axial loads can by transferred by the wall panels without any problems.

11 References

[1] EN 14509:2006: Self-supporting double skin metal faced insulating panels – Factory made products –Specifications.

[2] European recommendations for sandwich panels. ECCS/CIB-Report – Publication 257, ECCS TWG 7.9 and CIB W056, 2000.

[3] D3.3 – part 4: Axially loaded sandwich panels, Deliverable of EASIE project, June 2011.

[4] D3.4 part 1: Design guideline – Frameless buildings made of sandwich panels, Deliverable of EASIE project, October 2011.

[5] EN 1990:2002: Eurocode - Basis of structural design.

[6] DIN 1055-4:2005: Einwirkungen auf Tragwerke – Teil 4: Windlasten (Actions on structures – part 4: Wind loads).

[7] DIN 1055-5:2005: Einwirkungen auf Tragwerke – Teil 5: Schnee- und Eislasten (Actions on structures – part 5: Snowloads and ice loads).

CALCULATION EXAMPLE No. 2

Design of load application areas of a iall

loaded sandwich panels

Table of contents

1 Preliminar remarks

In the following a calculation example for the design of the load application area of an axially loaded sandwich panels is presented. In the example the connection between wall and roof is considered (Fig 2.1). The roof panels have a span of 6 m and they are loaded by self-weight and snow loads. These loads are introduced into the internal face of the wall panels as normal forces.

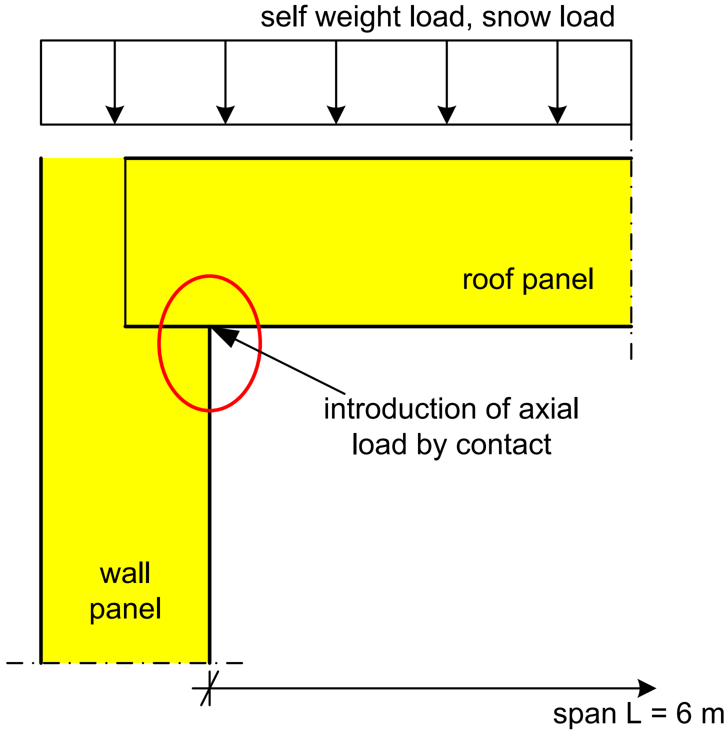

Fig 2.1: Load application detail of calculation e ample

The snow loads, which are considered in the example, have been determined according to the German standard DIN 1055-5 (snow load zone 2).

2 Sandwich panel

The walls of the building consist of sandwich panels with flat steel faces and a polyurethane foam core. In the following section all calculation requirements, which are needed to design the load application area, are summarised. Usually these values are given on the CE-mark of the panel or in (national) approvals.

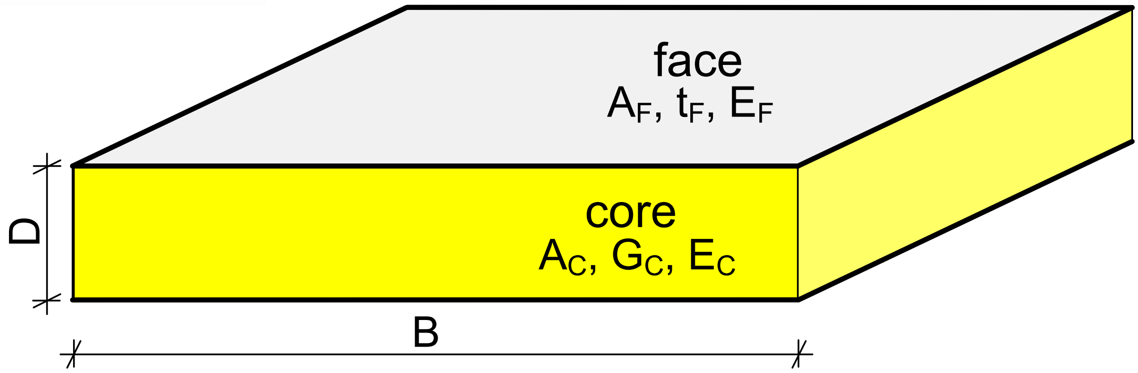

Fig 2.2: Cross section of a sandwich panel

2.1 Geometry and material properties

Thickness of faces	t_F = 0,50 mm
Elastic modulus of faces (steel)	E_F = 210.000 N/mm^2
Yield stress of face	$f_{y,F}$ = 280 N/mm^2
Wrinkling stress	σ_w = 70 N/mm^2
Shear modulus of core	G_C = 3,3 N/mm^2
Elastic modulus of core	E_C = 7,0 N/mm^2

The bending stiffness of a face sheet is

$$EI_F = E_F \cdot \frac{t_F^3}{12 \cdot (1-\nu_F^2)} = 210000 \frac{N}{mm^2} \cdot \frac{(0{,}50mm)^3}{12 \cdot (1-0{,}3^2)} = 2403 \frac{Nmm^2}{mm}$$

2.2 Material safety factor

For the following example the values given in EN 14509, Table E.9 are used. For crippling of the face at load application area the material factor for wrinkling of a face in mid-span can be used.

γ_M = 1,25 (ultimate limit state)

3 Actions and loads

To design the load application area we need the normal forces introduced from the roof into the wall panels. So self-weight of the roof and snow loads are considered.

The self-weight of the roof panels is assumed to be

g_k = 0,12 kN/m^2

With the span of roof panels L = 6,0 m we get the axial load, which is introduced into the walls

$$N_{G,k} = g_k \cdot \frac{L}{2} = 0{,}36 \frac{kN}{m}$$

For the example at hand the snow load is assumed to be

s_k = 0,68 kN/m^2

The following axial load results from snow.

$$N_{S,k} = s_k \cdot \frac{L}{2} = 2{,}04 \frac{kN}{m}$$

4 Combination rules and load factors

The load application area is designed for ultimate limit state. The following combination rule is used (EN 14509, Table E.4).

$$S_d = \gamma_G \cdot G_k + \gamma_{Q1} \cdot Q_{k1} + \sum \gamma_{Qi} \cdot \Psi_{0i} \cdot Q_{ki}$$

The load factors γ_F are given in Table E.8 of EN 14509.

Permanent action: $\gamma_{F,G}$ = 1,35

Variable actions: $\gamma_{F,Q}$ = 1,50

5 Design load and effects of action

With the load factors and combination coefficients the design value of the axial load is determined.

$$N_d = \gamma_{FG} \cdot N_{G,k} + \gamma_{FQ} \cdot N_{S,k} = 1{,}35 \cdot 0{,}36 \frac{kN}{m} + 1{,}50 \cdot 2{,}04 \frac{kN}{m} = 3{,}546 \frac{kN}{m}$$

The following design stress is introduced into the inner face of the wall panels:

$$\sigma_d = \frac{N_d}{t_F} = \frac{3546 \frac{N}{mm}}{0{,}50 mm} = 7{,}1 \frac{N}{mm^2}$$

6 Resistance value

The resistance value needed for the design of the load application area is the crippling stress $\sigma_{c.}$. The crippling stress of the free edge is determined based on the wrinkling stress σ_w in mid-span.

At first for the considered panel the imperfection factor α is determined.

$$\alpha = \frac{1+\chi_w \cdot \lambda_w^2 \cdot (\chi_w - 1) - \chi_w}{\chi_w \cdot (\lambda_w - \lambda_0)} \geq 0{,}21$$

$$\alpha = \frac{1+0{,}250 \cdot 1{,}419^2 \cdot (0{,}250 - 1) - 0{,}250}{0{,}250 \cdot (1{,}419 - 0{,}7)} = 2{,}07$$

with

reduction factor for wrinkling stress:

$$\chi_w = \frac{\sigma_w}{f_{y,F}} = \frac{70}{280} = 0{,}250$$

elastic buckling stress (wrinkling):

$$\sigma_{cr,w} = \frac{3}{A_F} \cdot \sqrt[3]{\frac{2}{9} \cdot EI_F \cdot G_C \cdot E_C} = \frac{3}{0{,}50} \cdot \sqrt[3]{\frac{2}{9} \cdot 2403 \cdot 3{,}3 \cdot 7{,}0} = 139 \frac{N}{mm^2}$$

slenderness for wrinkling:

$$\lambda_w = \sqrt{\frac{f_{y,F}}{\sigma_{cr,w}}} = \sqrt{\frac{280}{139}} = 1{,}419$$

$$\lambda_0 = 0{,}7$$

If the imperfection factor α is known, the crippling stress σ_c of the free edge can be calculated.

$$\sigma_{c,k} = 0{,}54 \cdot \chi_c \cdot f_{y,F} = 0{,}54 \cdot 0{,}139 \cdot 280 = 21{,}0 \frac{N}{mm^2}$$

with

slenderness for crippling:

$$\lambda_c = \sqrt{2} \cdot \lambda_w = \sqrt{2} \cdot 1{,}419 = 2{,}007$$

reduction factor for crippling stress:

$$\phi = \frac{1}{2} \cdot \left(1 + \alpha \cdot (\lambda_c - \lambda_0) + \lambda_c^2\right) = \frac{1}{2} \cdot \left(1 + 2{,}07 \cdot (2{,}007 - 0{,}7) + 2{,}007^2\right) = 3{,}867$$

$$\chi_c = \frac{1}{\phi + \sqrt{\phi^2 - \lambda_c^2}} = \frac{1}{3{,}867 + \sqrt{3{,}867^2 - 2{,}007^2}} = 0{,}139$$

7 Design calculation

To design the load application area the crippling stress has to be compared to the normal stress σ_d introduced into the face of the panel.

$$\sigma_{c,d} = \frac{\sigma_{c,k}}{\gamma_M} = \frac{21{,}0}{1{,}25} = 16{,}8 \frac{N}{mm^2}$$

$$\sigma_d = 7{,}1 \frac{N}{mm^2} \leq \sigma_{c,d} = 16{,}8 \frac{N}{mm^2}$$

8 Conclusions

In the sections above a calculation example for the design of the load application area of an axially loaded sandwich panel is presented. As loading self-weight and snow loads, which are introduced from the roof into the internal face of the wall panel, are considered. The example shows that for small buildings without substructure the normal forces can be introduced from the roof into the face of the wall panels without any problems.

9 References

[1] D3.3 – part 5: Introduction of loads into axially loaded sandwich panels, Deliverable of EASIE project, October 2011.

[2] D3.4 – part 1: Design guideline – Frameless buildings made of sandwich panels, Deliverable of EASIE project, October 2011.

[3] EN 14509:2006: Self-supporting double skin metal faced insulating panels – Factory made products –Specifications.

[4] European recommendations for sandwich panels. ECCS/CIB-Report – Publication 257, ECCS TWG 7.9 and CIB W056, 2000.

[5] EN 1990:2002: Eurocode - Basis of structural design.

[6] DIN 1055-5:2005: Einwirkungen auf Tragwerke – Teil 5: Schnee- und Eislasten (Actions on structures – part 5: Snowloads and ice loads).

CALCULATION EXAMPLE No. 3

Design of frameless buildings for hori ontal wind loads

Table of contents

1 Preliminar remarks

In the following some calculation examples for the design of sandwich panels, which transfer horizontal loads by in-plane shear resistance, are presented. Because of the high in-plane shear stiffness and capacity of sandwich panels only the fastenings have to be designed for in-plane shear loads. In the examples the determination of the forces the fastenings have to be designed for is demonstrated. Also displacements of the building are determined. For all examples determination of forces and displacements by numerical calculation is shown. For some simple applications also determination by analytical calculation is presented.

2 Building and load

2.1 Building and fastenings

The following examples deal with the building shown in Fig. 3.1.

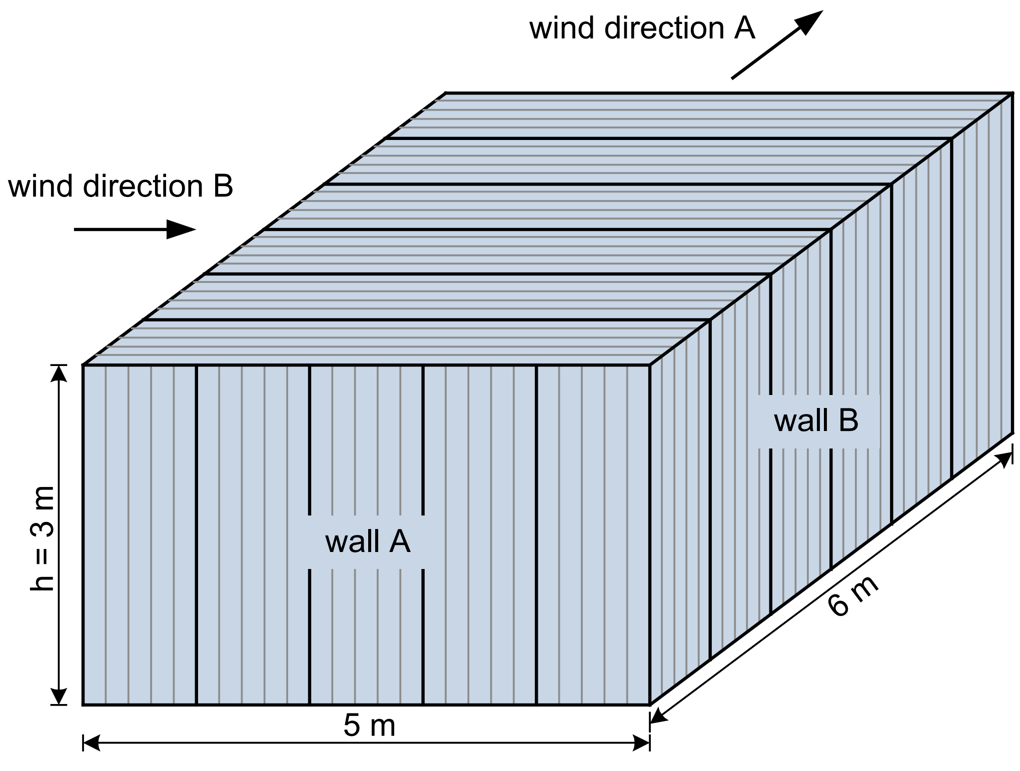

Fig. 3.1: Frameless building of calculation e amples

At the lower end the wall panels are fixed to the foundation by steel angles, which are screwed to the panels (Fig. 3.2). The distance between the connections is 200 mm. So each panel is fixed by five connections. Both faces are connected to an angle, so each connection consists of two fastenings. The stiffness of a fastening is 6,5 kN/mm.

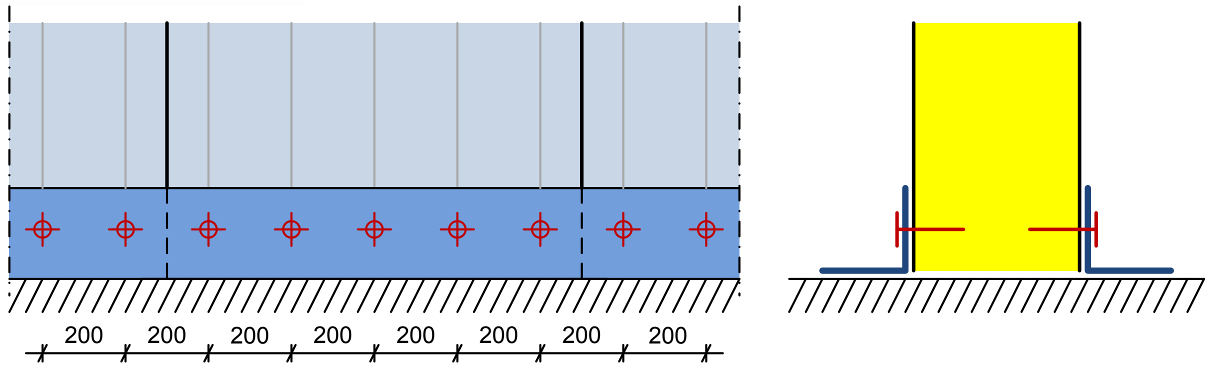

Fig. 3.2: **Connections between wall and foundation**

Fig. 3.3 shows the connections between wall and roof. Steel angles and self-drilling screws are used. The connections have the distance 333 mm. The internal as well as the external face sheet are connected. Altogether one connection consists of four fastenings. The stiffness of a fastening is 6,5 kN/mm.

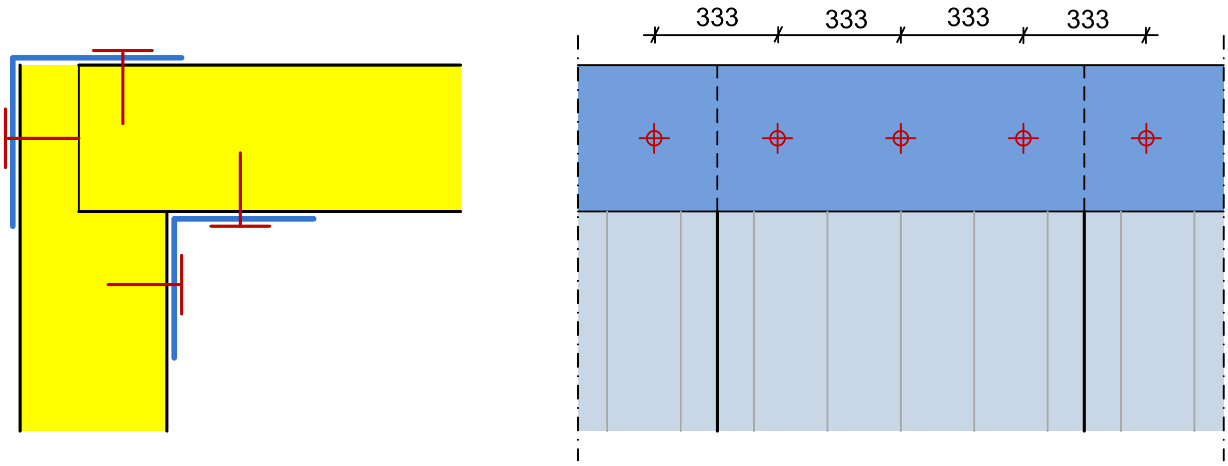

Fig. 3.3: **Connection between wall and roof**

The longitudinal joints of the roof panels are connected by blind rivets. Both face sheets are connected. So one rivet is mounted from the internal and one rivet is mounted from the external side. There are 10 connections at each joint (distance 500 mm). The stiffness of a fastening is 12 kN/mm.

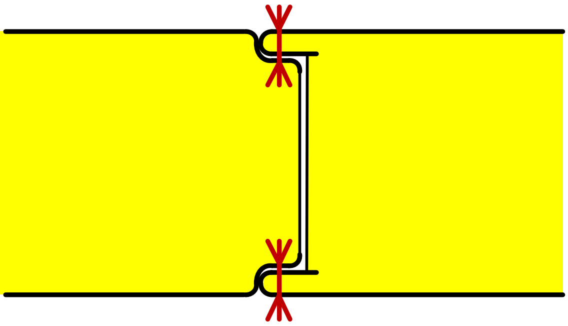

Fig. 3.4: Connection of longitudinal joints

Connections of the joints of the wall panels are not necessarily required. But connecting the joints increases the stiffness and the load bearing capacity of the building. In the following an example for a wall without connections of the joints (section 6) as well as an example for a wall with connections of the joints (section 7) is presented. It is assumed that the same fasteners are used as for the joints of the roof panels (stiffness of a fastening 12 kN/mm). The distance between the connections is 500 mm, i.e. there are 6 connections at each joint.

2.2 Actions and loads

The wind load has to be determined according to national standards or according EN 1991-1-4 and the related national annex. For the examples presented in the report at hand the following design loads are assumed.

Wind suction load: $w^*_{Sd} = 0{,}52\ kN/m^2$

Wind compression load: $w^*_{Cd} = 0{,}78\ kN/m^2$

The walls of the building are directly loaded by the loads given above. From the wall the load is transferred to the foundation and to the roof. The following line loads are introduced into the roof.

Wind suction load:

$$w_{Sd} = w^*_{Sd} \cdot \frac{L}{2} = 0{,}52 \frac{kN}{m^2} \cdot \frac{3m}{2} = 0{,}78 \frac{kN}{m}$$

Wind compression load:

$$w_{Cd} = w^*_{Cd} \cdot \frac{L}{2} = 0{,}78 \frac{kN}{m^2} \cdot \frac{3m}{2} = 1{,}17 \frac{kN}{m}$$

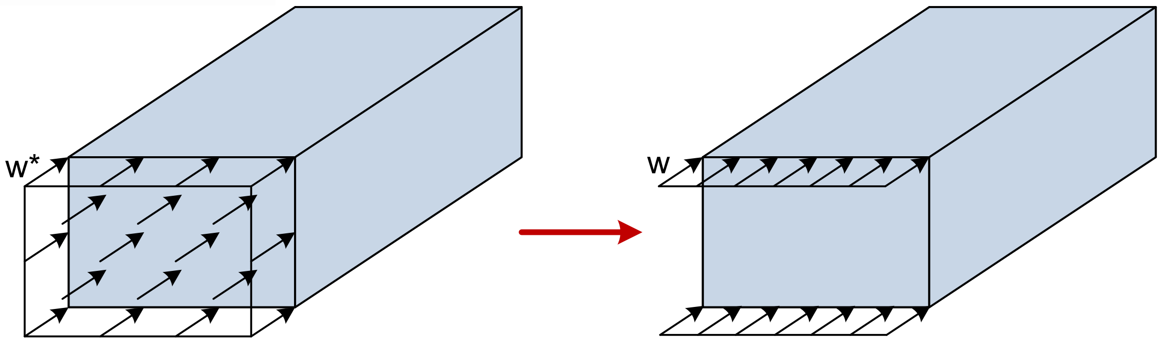

Fig. 3.5: **Hori ontal loads introduced into the roof**

3 Stiffness of connections

Based on the stiffness of the fastenings the stiffness of the connections is determined. This stiffness is used for the FE-model of the numerical calculation.

At the connection between wall and foundation one connection consists of two fastenings, which are arranged parallel. So the stiffness of both fastenings has to be added to determine the stiffness of the connection.

$$k_C = 2 \cdot k_v = 2 \cdot 6{,}5 \frac{kN}{mm} = 13{,}0 \frac{kN}{mm}$$

At the connection between wall and roof the force is transferred from the roof panel to the angle and subsequently from the angle to the wall panel. Two fastenings are arranged in series. So the stiffness of one fastening has to be divided by two. Because there are two angles with this kind of connection both stiffness's have to be added.

$$k_C = 2 \cdot \frac{k_v}{2} = 2 \cdot \frac{6{,}5}{2} \frac{kN}{mm} = 6{,}5 \frac{kN}{mm}$$

At the connections of the longitudinal joints (roof and wall) two fastenings are arranged parallel. So the stiffness of both fastenings is added.

$$k_C = 2 \cdot k_v = 12{,}0 \frac{kN}{mm} = 24{,}0 \frac{kN}{mm}$$

If the forces of the fastenings and the displacements of the building are determined analytically, for simplification the forces of the connections are smeared over the width and length of the panels. The resulting stiffness per unit length is given in the following.

Connection between wall and foundation:

$$k_W = \frac{k_C}{B/n} = \frac{13{,}0}{200} = 0{,}065 \frac{kN}{mm^2}$$

Connection between transverse edges of roof and wall:

$$k_{Wt} = \frac{k_C}{B/n} = \frac{6{,}5}{333} = 0{,}0195 \frac{kN}{mm^2} = 19500 \frac{kN}{m^2}$$

Connection between longitudinal edges of roof and wall:

$$k_{Wl} = \frac{k_C}{L/n} = \frac{6{,}5}{333} = 0{,}0195 \frac{kN}{mm^2} = 19500 \frac{kN}{m^2}$$

Connection of longitudinal joints:

$$k_J = \frac{k_C}{L/n} = \frac{24}{500} = 0{,}048 \frac{kN}{mm^2} = 48000 \frac{kN}{m^2}$$

4 Design of roof - wind direction A

4.1 System and loads

The system and the loads are given in Fig. 3.6. The roof is supported on the walls. Only in-plane shear forces are introduced into the walls. The connections to the wall have the stiffness 6,5 kN/mm. At the longitudinal joints the panels are connected with 10 connections per joint (distance 500 mm). The connections have the stiffness 24 kN/mm. The longitudinal edges of the outer panels are loaded by a wind suction or compression load.

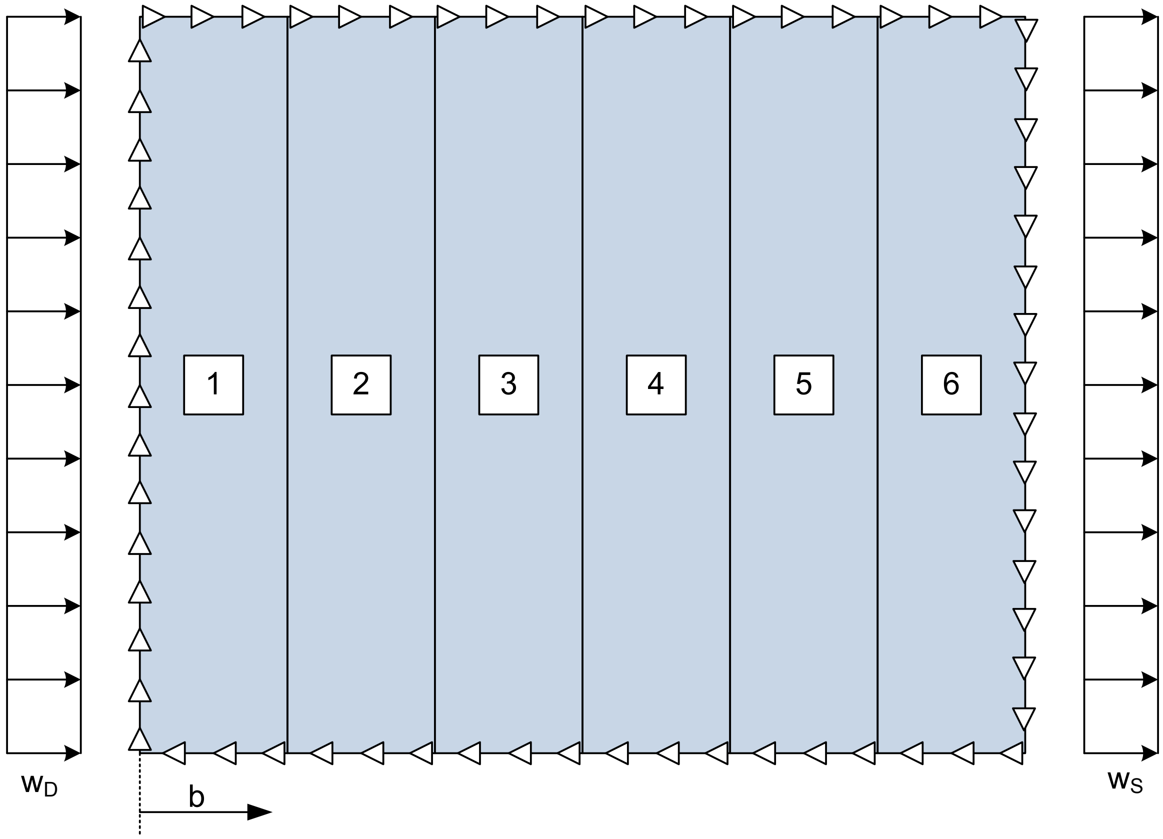

Fig. 3.6: S stem of roof

4.2 Determination of forces and displacements by numerical calculation

In the FE-model the connections are represented by longitudinal springs. The stiffness of the springs corresponds to the stiffness of the connection. The connections between roof and wall are represented by one spring. The direction of this spring is parallel to the wall. Because in the joints forces in longitudinal as well as in transverse direction can be transferred, the connections are represented by two springs which are orthogonal to each other.

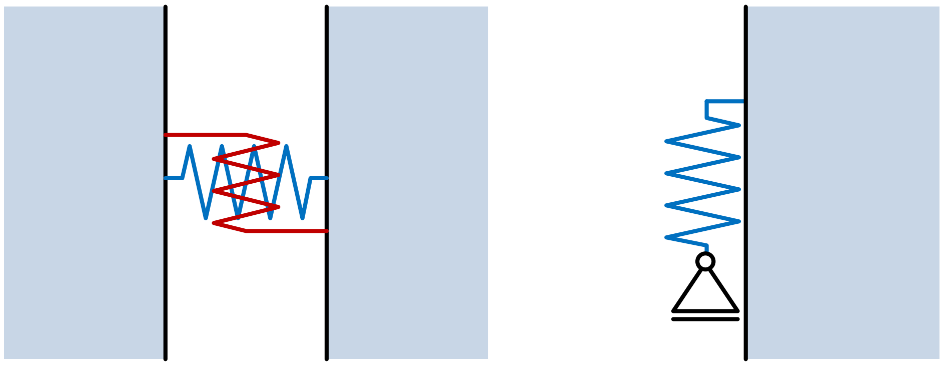

Fig. 3.7: Connections in the FE-model (longitudinal joint and connection between wall and roof)

In the following figure the FE-model used for the numerical calculations is shown.

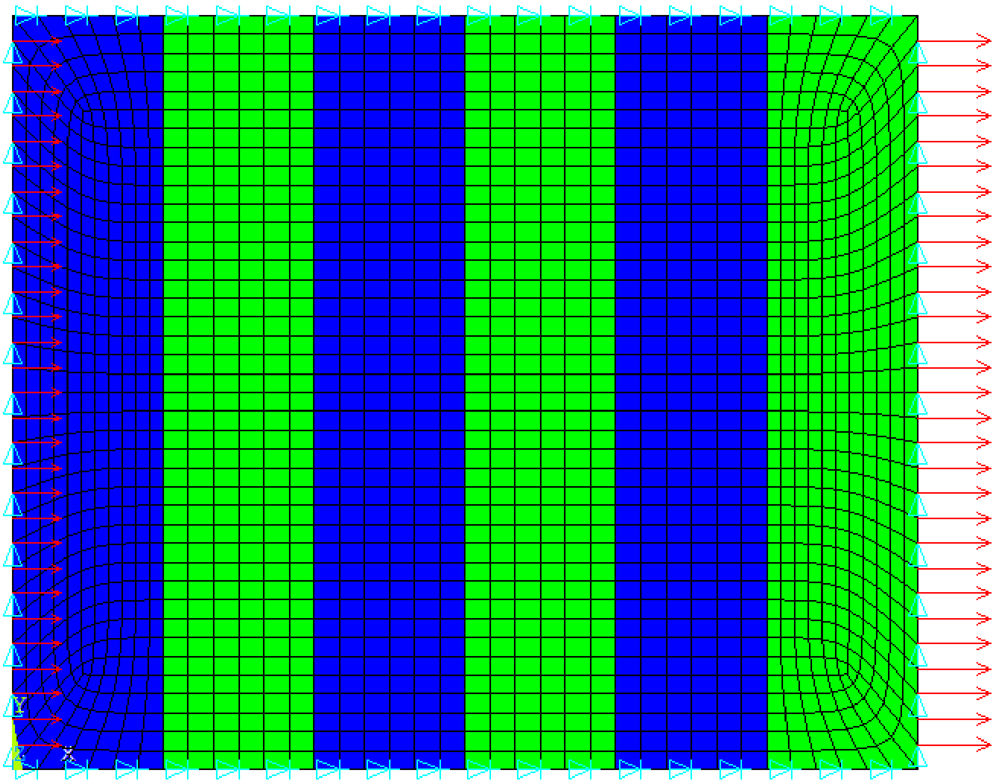

Fig. 3.8: FE-model of the roof

By the numerical calculation the displacements of the panels are determined. The highest displacement (0,059 mm) occurs at panel 6.

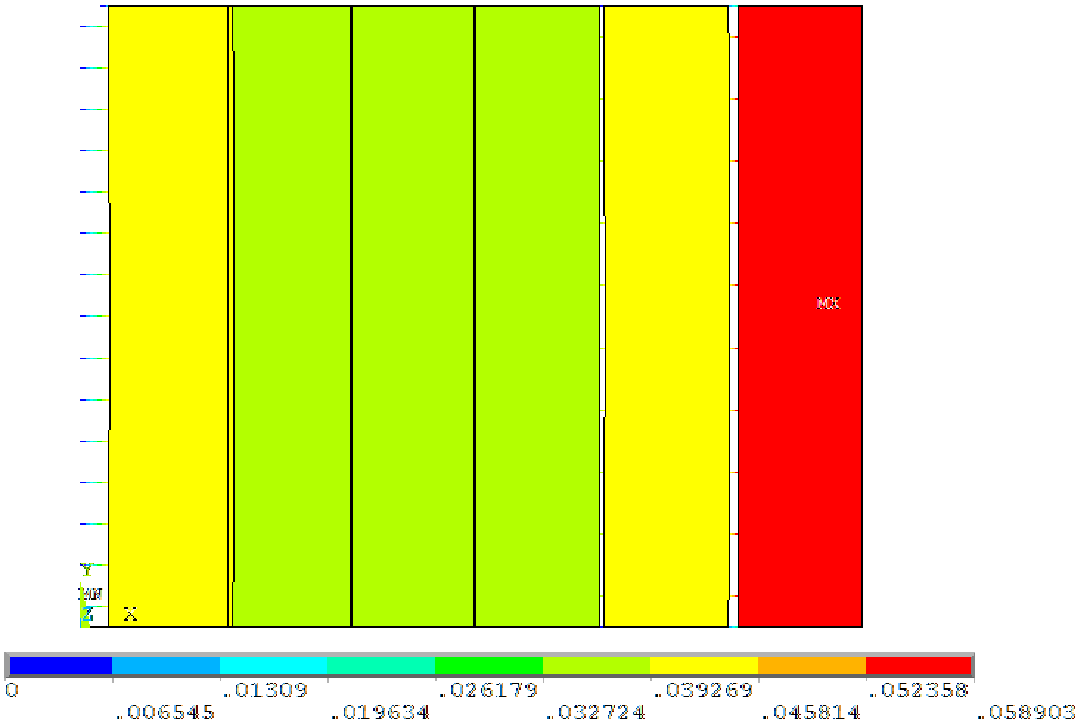

Fig. 3.9: Displacement of sandwich panels [mm]

Also the forces of the connections are determined by the numerical calculation. Fig. 3.10 shows the forces of the connections at the transverse edges of the panels. The highest forces (380 N) occur at the panel 6, where the wind suction loads are introduced. Because internal and external face sheet are connected, the force of the highest stressed fastenings is

$$V_d = \frac{376N}{2} = 188N$$

For this force the fastenings have to be designed.

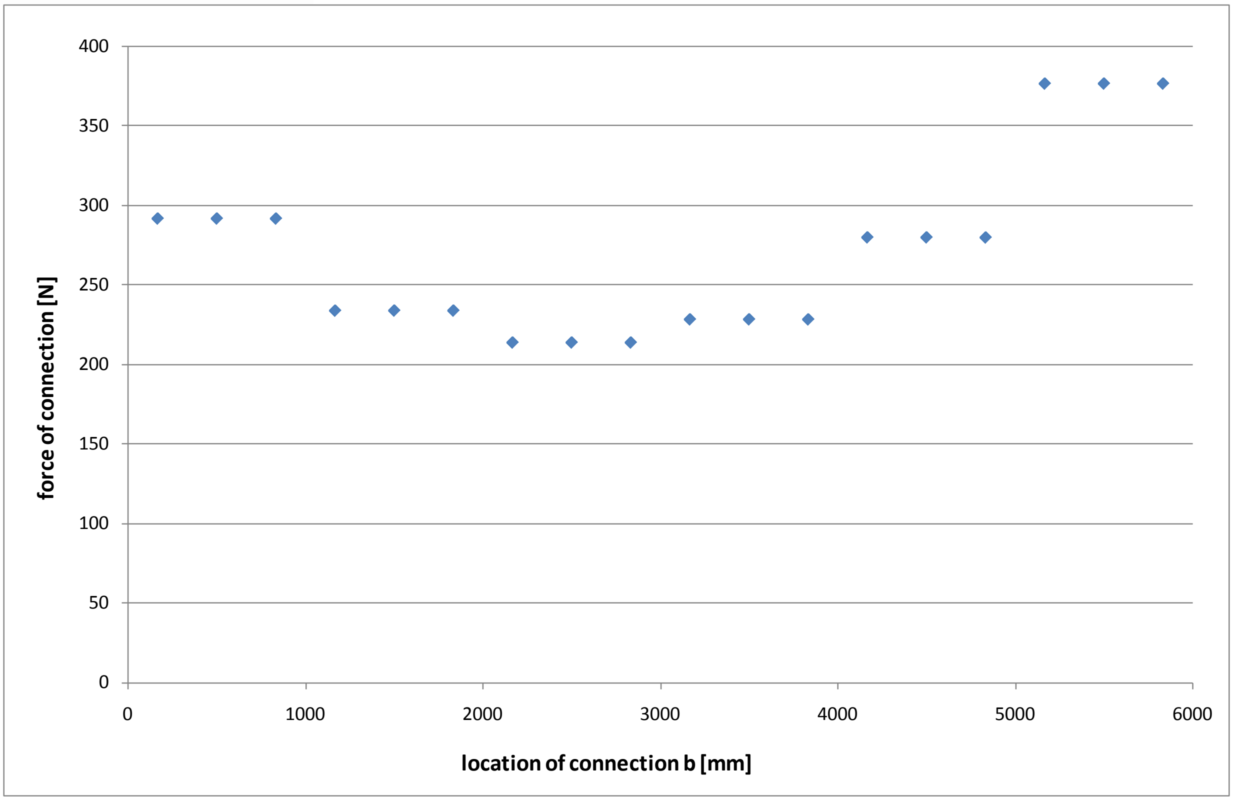

Fig. 3.10: Forces of connections determined b numerical calculation

At the longitudinal joints only forces in transverse direction of the panels occur. All connections of a joint are loaded by the same force. The forces of the connections of the longitudinal joints are given in Tab 3.1. Because a connection consists of two fastenings, the force of a fastening is half the force of the connection.

joint between panel	Force of connection [N]	Force of fastening [N]
1 and 2	215	107 (compression)
2 and 3	74	37 (compression)
3 and 4	54	27 (tension)
4 and 5	191	96 (tension)
5 and 6	359	180 (tension)

Tab 3.1: Forces of connections at joints determined b numerical calculation

4.3 Determination of forces and displacements by analytical calculation

Alternatively to the numerical calculation shown above the forces and displacements can be determined by an analytical calculation. For each panel the factors A_i, B_i, C_i and F_i are determined and the respective equation is formed.

Panel 1:

$$A_1 = k_J^l \cdot L = 0$$

$$B_1 = -\left(k_J^l \cdot L + k_J^r \cdot L + 2 \cdot k_{Wt} \cdot B\right) = -\left(0 + 48000 \cdot 5 + 2 \cdot 19500 \cdot 1\right) = -279000 \frac{kN}{m}$$

$$C_1 = k_J^r \cdot L = 48000 \cdot 5 = 240000 \frac{kN}{m}$$

$$F_1 = w_1 \cdot L = 0{,}78 \cdot 5 = 3{,}9 kN$$

$$0 - 279000 \cdot v_{x,1} + 240000 \cdot v_{x,2} + 3{,}9 = 0$$

Panel 2 to 5:

$$A_i = k_J^l \cdot L = 48000 \cdot 5 = 240000 \frac{kN}{m}$$

$$B_i = -\left(k_J^l \cdot L + k_J^r \cdot L + 2 \cdot k_{Wt} \cdot B\right) = -\left(48000 \cdot 5 + 48000 \cdot 5 + 2 \cdot 19500 \cdot 1\right) = -519000 \frac{kN}{m}$$

$$C_i = k_J^r \cdot L = 48000 \cdot 5 = 240000 \frac{kN}{m}$$

$$F_i = 0$$

$$240000 \cdot v_{x,i-1} - 519000 \cdot v_{x,i} + 240000 \cdot v_{x,i+1} + 0 = 0$$

Panel 6:

$$A_6 = k_J^l \cdot L = 48000 \cdot 5 = 240000 \frac{kN}{m}$$

$$B_6 = -\left(k_J^l \cdot L + k_J^r \cdot L + 2 \cdot k_{Wt} \cdot B\right) = -\left(48000 \cdot 5 + 0 + 2 \cdot 19500 \cdot 1\right) = -279000 \frac{kN}{m}$$

$$C_6 = k_J^r \cdot L = 0$$

$$F_6 = w_6 \cdot L = 1{,}17 \cdot 5 = 5{,}85 kN$$

$$240000 \cdot v_{x,5} - 279000 \cdot v_{x,6} + 0 + 5{,}85 = 0$$

So we get the following equation system.

$$\begin{bmatrix} -279000 & 240000 & 0 & 0 & 0 & 0 \\ 240000 & -519000 & 240000 & 0 & 0 & 0 \\ 0 & 240000 & -519000 & 240000 & 0 & 0 \\ 0 & 0 & 240000 & -519000 & 240000 & 0 \\ 0 & 0 & 0 & 240000 & -519000 & 240000 \\ 0 & 0 & 0 & 0 & 240000 & -249000 \end{bmatrix} \cdot \left[v_x\right] = \begin{bmatrix} -3{,}90 \\ 0 \\ 0 \\ 0 \\ 0 \\ -5{,}85 \end{bmatrix}$$

Solving the equation system results in the displacement $v_{x,i}$ of the panels.

$$[v_x] = \begin{bmatrix} 0{,}045mm \\ 0{,}036mm \\ 0{,}033mm \\ 0{,}035mm \\ 0{,}043mm \\ 0{,}058mm \end{bmatrix}$$

If the displacements are known the forces of the connections can be determined. At the transverse edges of the panels the force of the highest stressed connections (panel 6) is

$$V = v_{x,6} \cdot k_{Wt} \cdot \frac{B}{n} = 0{,}058 \cdot 19500 \cdot 0{,}333 = 0{,}377kN$$

For the force of one fastening we get

$$V_d = \frac{377N}{2} = 189N$$

For the connections of the joints the forces are determined by the following formula. In Tab 3.2 the forces are given for all joints.

$$V_d = \frac{\Delta v_{x,i} \cdot k_J \cdot L}{n} = \Delta v_{x,i} \cdot 48000 \cdot 0{,}333$$

$$\Delta v_{x,i} = \begin{cases} v_{x,i-1} - v_{x,i} \\ v_{x,i+1} - v_{x,i} \end{cases}$$

joint between panel	Δv_x [mm]	force of connection [kN]	force of fastening [N]
1 and 2	0,009	0,216	108
2 and 3	0,003	0,072	36
3 and 4	0,002	0,048	24
4 and 5	0,008	0,192	96
5 and 6	0,015	0,360	180

Tab 3.2: Forces of connections at joints determined b anal tical calculation

5 Design of roof - wind direction B

5.1 System and loads

For the numerical calculation the system given in the section above is used. But the transverse edges of the panels are loaded by the wind loads. The system and the loads are given in Fig. 3.11.

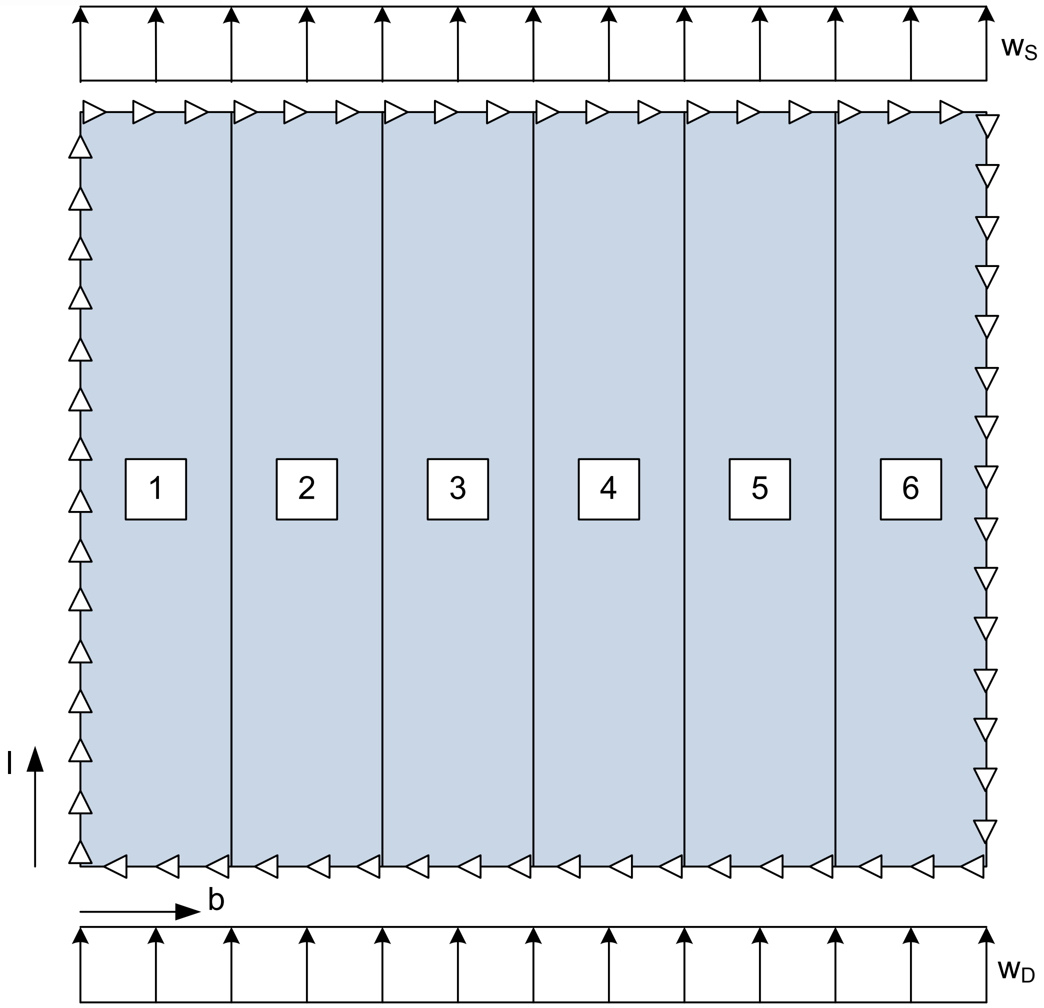

Fig. 3.11: **S stem of roof**

In the following figure the FE-model is shown.

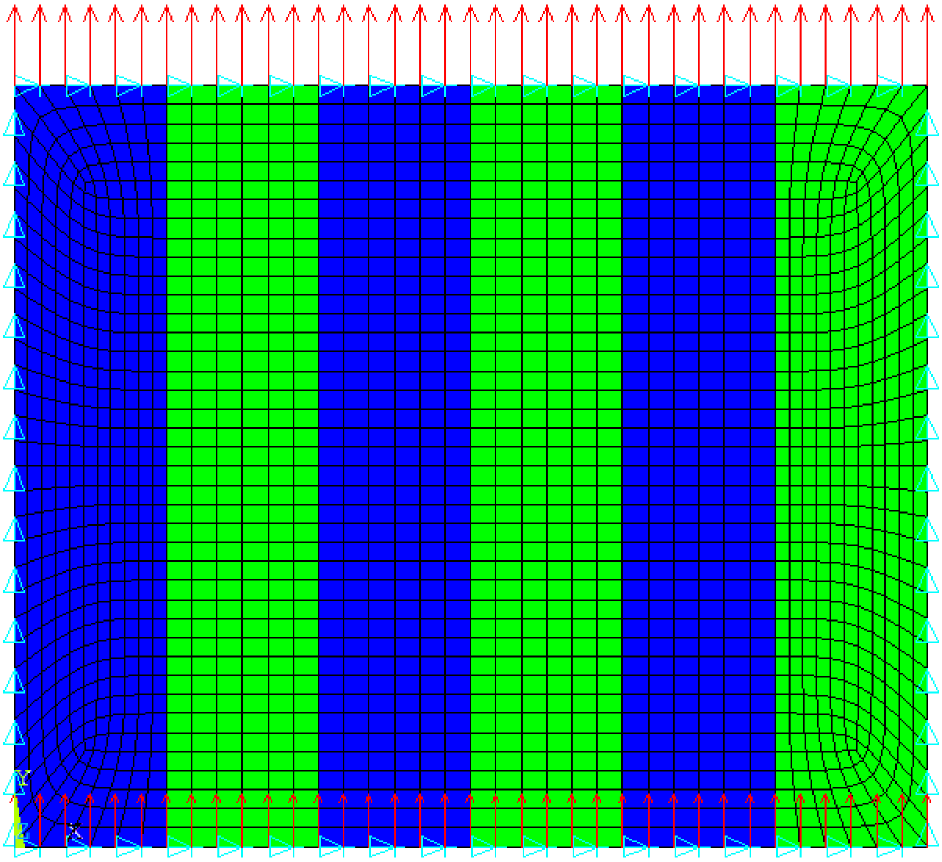

Fig. 3.12: **FE-model of the roof**

The displacement and the rotation of the panels are given in the following figures.

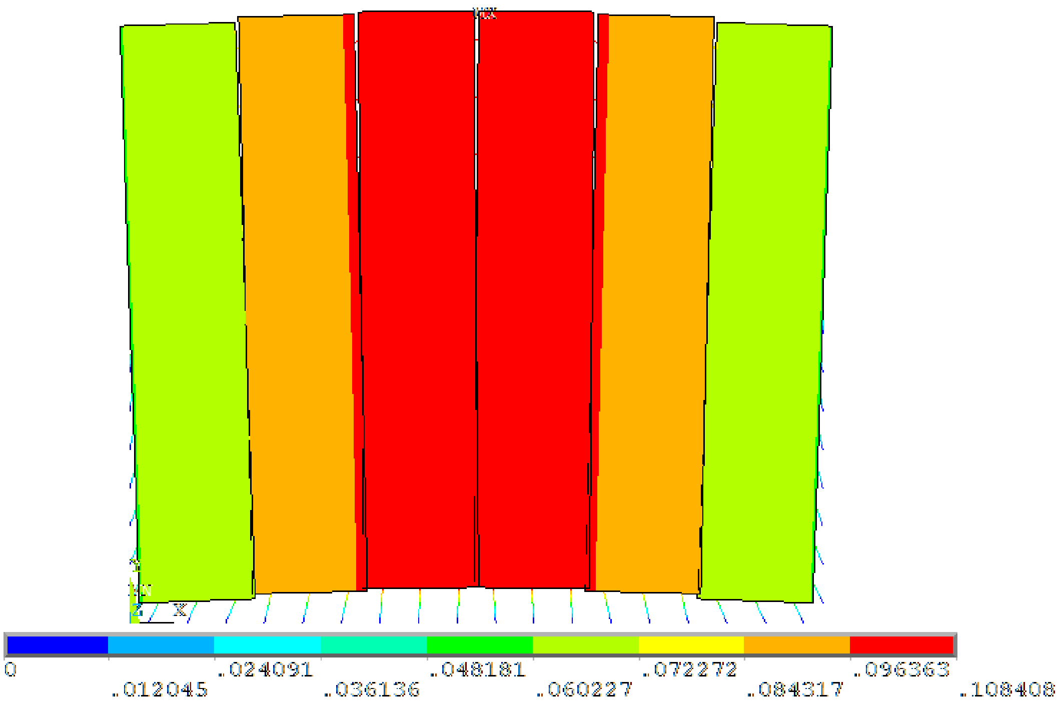

Fig. 3.13: **Displacement of panels [mm]**

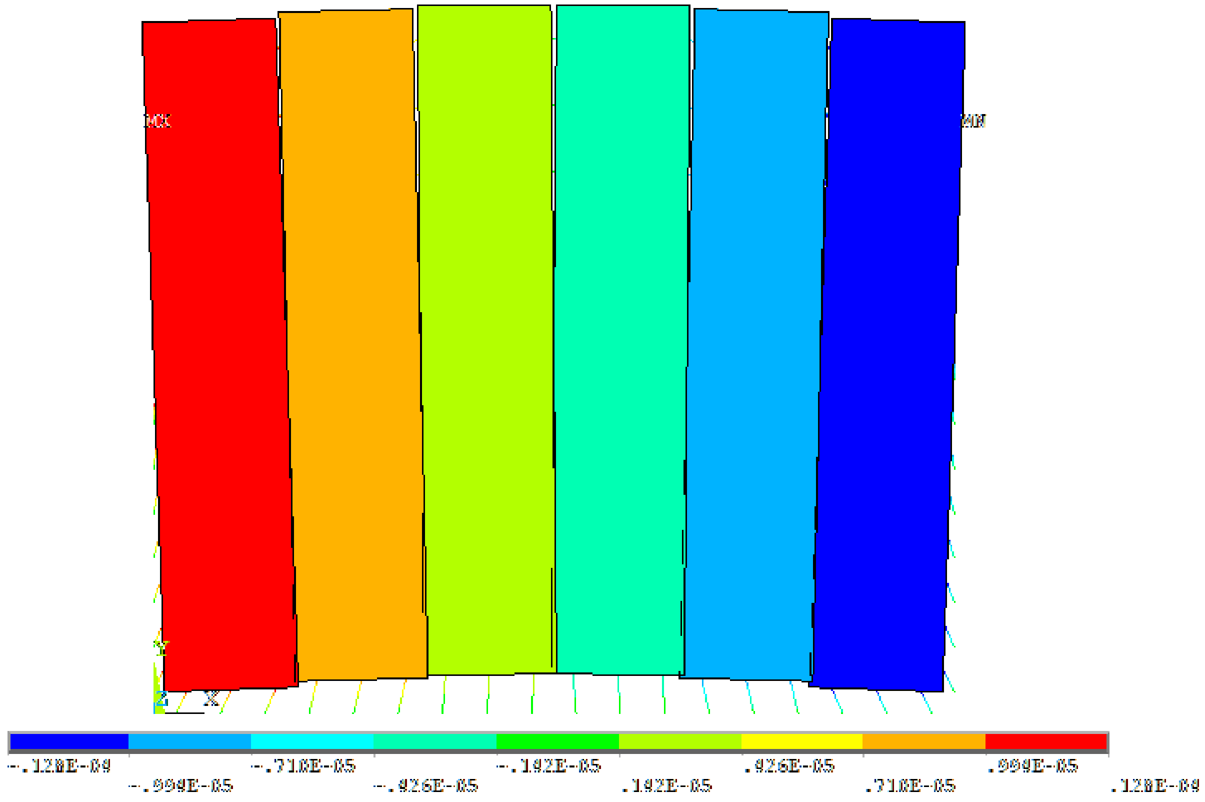

Fig. 3.14: **Rotation of panels [rad]**

Also the forces of the connections are determined by the numerical calculation. Fig. 3.15 shows the forces of the connections at the transverse edges of the panels (connection between wall and roof). The highest forces (200 N) occur at panel 1 and 6, where also the highest rotation occurs.

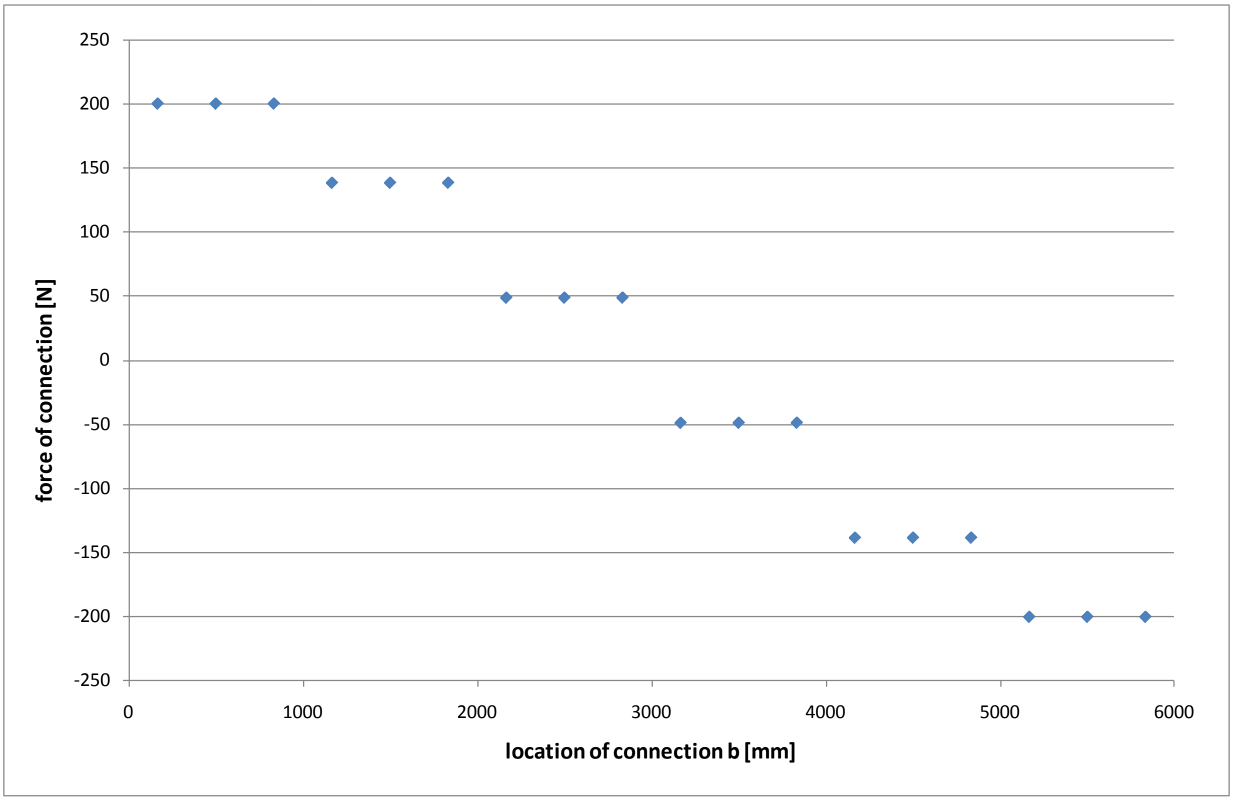

Fig. 3.15: Forces of connection at transverse edges

At the connection between the longitudinal edges of panel 1 or 6 and the wall all connections are loaded by the same force.

$$V = 390N$$

So the fastenings of the connections between wall and roof have to be designed for the following force

$$V_d = \frac{390N}{2} = 195N$$

The connections of the joints between the roof panels are loaded by transverse as well as by longitudinal forces. The forces in longitudinal direction are constant for all connections of a joint.

joint between panel	force of a connection [N]
1 and 2	390
2 and 3	195
3 and 4	0
4 and 5	195
5 and 6	390

Tab 3.3: Forces of connection of joints (force in longitudinal direction)

The forces in transverse direction are linearly distributed over the length of the joint. The highest force occurs at the outer connections of a joint. The forces are given in the following table.

		joint between panel				
		1 and 2	2 and 3	3 and 4	4 and 5	5 and 6
location of connection l [mm]	250	-205	-297	-324	-297	-205
	750	-159	-231	-252	-231	-159
	1250	-113	-165	-180	-165	-113
	1750	-68	-99	-108	-99	-68
	2250	-23	-33	-36	-33	-23
	2750	23	33	36	33	23
	3250	68	99	108	99	68
	3750	113	165	180	165	113
	4250	159	231	252	231	159
	4750	205	297	324	297	205

Tab 3.4: Forces of connection at joints (force in transverse direction)

With the longitudinal and the transverse forces the resulting forces of the connections are determined.

$$V = \sqrt{V_x^2 + V_y^2}$$

The resulting forces are summarised in Tab 3.5.

		joint between panel				
		1 and 2	2 and 3	3 and 4	4 and 5	5 and 6
location of connection l [mm]	250	440	355	324	355	440
	750	421	302	252	302	421
	1250	406	255	180	255	406
	1750	396	219	108	219	396
	2250	391	198	36	198	391
	2750	391	198	36	198	391
	3250	396	219	108	219	396
	3750	406	255	180	255	406
	4250	421	302	252	302	421
	4750	440	355	324	355	440

Tab 3.5: Forces of connection at joints (resulting forces)

The highest force of a connection is 440 N. So the fastenings of the joints should be designed for the following force.

$$V_d = \frac{440N}{2} = 220N$$

5.2 Determination of forces and displacements by analytical calculation

Alternatively to the numerical calculation shown above the forces and displacements can be determined by an analytical calculation.
Wind suction and compression load are added to the wind load w

$$w = w_S + w_C = 0{,}78 + 1{,}17 = 1{,}95 \frac{kN}{m}$$

The forces introduced into the walls being parallel to the direction of the load (wall A) are

$$V_1^l = -V_n^r = \frac{n_{SW}}{2} \cdot w \cdot B = \frac{6}{2} \cdot 1{,}95 \cdot 1 = 5{,}85kN$$

At the joints the following forces in longitudinal direction occur.
Joint between panel 1 and 2:

$$V_1^r = V_2^l = V_1^l - w \cdot B = 5{,}85 - 1{,}95 \cdot 1 = 3{,}90kN$$

Joint between panel 2 and 3:

$$V_2^r = V_3^l = V_2^l - w \cdot B = 3{,}90 - 1{,}95 \cdot 1 = 1{,}95kN$$

Joint between panel 3 and 4:

$$V_3^r = V_4^l = V_3^l - w \cdot B = 1{,}95 - 1{,}95 \cdot 1 = 0$$

Joint between panel 4 and 5:

$$V_4^r = V_5^l = V_4^l - w \cdot B = 0 - 1{,}95 \cdot 1 = -1{,}95kN$$

Joint between panel 5 and 6:

$$V_5^r = V_6^l = V_5^l - w \cdot B = 1{,}95 - 1{,}95 \cdot 1 = -3{,}90kN$$

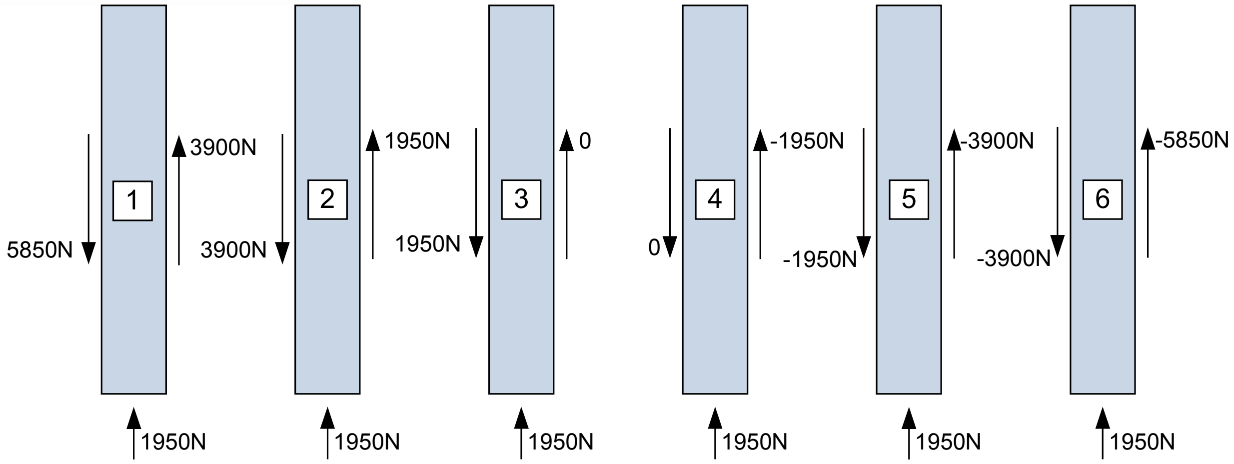

Fig. 3.16: Forces in longitudinal direction

To determine the rotations of the panels the factors A_i to D_i are determined for each panel and the corresponding equation is formed.

Panel 1:

$$A_1 = -\frac{k_J^l \cdot L^3}{12} = 0$$

$$B_1 = \frac{k_J^l \cdot L^3}{12} + \frac{k_{Wt} \cdot B \cdot L^2}{2} + \frac{k_J^r \cdot L^3}{12} = 0 + \frac{19500 \cdot 1 \cdot 5^2}{2} + \frac{48000 \cdot 5^3}{12} = 743750 kNm$$

$$C_1 = -\frac{k_J^r \cdot L^3}{12} = -\frac{48000 \cdot 5^3}{12} = -500000 kNm$$

$$D_1 = \left(V_1^l + V_1^r\right) \cdot \frac{B}{2} = (5{,}85 + 3{,}90) \cdot \frac{1}{2} = 4{,}875 kNm$$

Panel 2 to 5:

$$A_i = -\frac{k_J^l \cdot L^3}{12} = -\frac{48000 \cdot 5^3}{12} = -500000 kNm$$

$$B_i = \frac{k_J^l \cdot L^3}{12} + \frac{k_{Wt} \cdot B \cdot L^2}{2} + \frac{k_J^r \cdot L^3}{12} = \frac{48000 \cdot 5^3}{12} + \frac{19500 \cdot 1 \cdot 5^2}{2} + \frac{48000 \cdot 5^3}{12} = 1243750 kNm$$

$$C_i = -\frac{k_J^r \cdot L^3}{12} = -\frac{48000 \cdot 5^3}{12} = -500000 kNm$$

$$D_2 = \left(V_2^l + V_2^r\right) \cdot \frac{B}{2} = (3{,}90 + 1{,}95) \cdot \frac{1}{2} = 2{,}925 kNm$$

$$D_3 = \left(V_3^l + V_3^r\right) \cdot \frac{B}{2} = (1{,}95 + 0) \cdot \frac{1}{2} = 0{,}975 kNm$$

$$D_4 = \left(V_4^l + V_4^r\right) \cdot \frac{B}{2} = (0 - 1{,}95) \cdot \frac{1}{2} = -0{,}975 kNm$$

$$D_5 = \left(V_5^l + V_5^r\right) \cdot \frac{B}{2} = (-1{,}95 - 3{,}90) \cdot \frac{1}{2} = -2{,}925 kNm$$

Panel 6:

$$A_6 = -\frac{k_J^l \cdot L^3}{12} = -\frac{48000 \cdot 5^3}{12} = -500000kNm$$

$$B_6 = \frac{k_J^l \cdot L^3}{12} + \frac{k_{Wt} \cdot B \cdot L^2}{2} + \frac{k_J^r \cdot L^3}{12} = \frac{48000 \cdot 5^3}{12} + \frac{19500 \cdot 1 \cdot 5^2}{2} + 0 = 743750kNm$$

$$C_6 = -\frac{k_J^r \cdot L^3}{12} = 0$$

$$D_6 = \left(V_1^l + V_1^r\right) \cdot \frac{B}{2} = (-3{,}90 - 5{,}85) \cdot \frac{1}{2} = -4{,}875kNm$$

So we get the following equation system.

$$\begin{bmatrix} 743750 & -500000 & 0 & 0 & 0 & 0 \\ -500000 & 1243750 & -500000 & 0 & 0 & 0 \\ 0 & -500000 & 1243750 & -500000 & 0 & 0 \\ 0 & 0 & -500000 & 1243750 & -500000 & 0 \\ 0 & 0 & 0 & -500000 & 1243750 & -500000 \\ 0 & 0 & 0 & 0 & -500000 & 743750 \end{bmatrix} \cdot [\varphi] = \begin{bmatrix} 4{,}875 \\ 2{,}925 \\ 0{,}975 \\ -0{,}975 \\ -2{,}925 \\ -4{,}875 \end{bmatrix}$$

Solving the equation system results on the rotations φ_i of the panels.

$$[\varphi] = \begin{bmatrix} 1{,}23 \cdot 10^{-5} rad \\ 8{,}48 \cdot 10^{-6} rad \\ 2{,}99 \cdot 10^{-6} rad \\ -2{,}99 \cdot 10^{-6} rad \\ -8{,}48 \cdot 10^{-6} rad \\ -1{,}23 \cdot 10^{-5} rad \end{bmatrix}$$

If the rotations are known the forces of the connections can be determined. For the connections of the longitudinal edges of panel 1 and 6 (connection between wall and roof) we get the following forces.

$$V = \frac{V_1^l}{n} = \frac{5{,}85}{15} = 0{,}39kN$$

For the connections at the transverse edges the highest forces occur at the panels with the highest rotation, i.e. at panel 1 and 6. The force of a connection is

$$V = k_{Wt} \cdot \varphi_i \cdot \frac{L}{2} \cdot \frac{B}{n} = 19500 \cdot 1{,}23 \cdot 10^{-5} \cdot \frac{5}{2} \cdot 0{,}333 = 0{,}20kN$$

So the fastenings at the connections between roof and wall have to be designed for the following force.

$$V_d = \frac{390N}{2} = 195N$$

At the joints of the panels forces in transverse and in longitudinal direction are transferred. The force in longitudinal direction is determined by dividing the force V_i^l or V_i^r by the number of connections (n = 10).

joint between panel	force of joint [kN]	force of a connection [N]
1 and 2	0,39	390
2 and 3	1,95	195
3 and 4	0	0
4 and 5	1,95	195
5 and 6	3,90	390

Tab 3.6: Forces of connection at joints (force in longitudinal direction)

The force in transverse direction is not constant for the connections of a joint. In the following for each joint the force of the connection with the highest transverse load is determined.

Joint between panel 1 and 2:

$$V_x = k_J \cdot (\varphi_1 - \varphi_2) \cdot \frac{L}{2} \cdot \frac{L}{n} = 48000 \cdot \left(1{,}23 \cdot 10^{-5} - 8{,}48 \cdot 10^{-6}\right) \cdot \frac{5}{2} \cdot 0{,}50 = 0{,}223kN$$

Joint between panel 2 and 3:

$$V_x = k_J \cdot (\varphi_2 - \varphi_3) \cdot \frac{L}{2} \cdot \frac{L}{n} = 48000 \cdot \left(8{,}48 \cdot 10^{-6} - 2{,}99 \cdot 10^{-6}\right) \cdot \frac{5}{2} \cdot 0{,}50 = 0{,}330kN$$

Joint between panel 3 and 4:

$$V_x = k_J \cdot (\varphi_3 - \varphi_4) \cdot \frac{L}{2} \cdot \frac{L}{n} = 48000 \cdot \left(2{,}99 \cdot 10^{-6} + 2{,}99 \cdot 10^{-6}\right) \cdot \frac{5}{2} \cdot 0{,}50 = 0{,}359kN$$

Joint between panel 4 and 5:

$$V_x = k_J \cdot (\varphi_4 - \varphi_5) \cdot \frac{L}{2} \cdot \frac{L}{n} = 48000 \cdot \left(-2{,}99 \cdot 10^{-6} + 8{,}48 \cdot 10^{-6}\right) \cdot \frac{5}{2} \cdot 0{,}50 = 0{,}330kN$$

Joint between panel 5 and 6:

$$V_x = k_J \cdot (\varphi_5 - \varphi_6) \cdot \frac{L}{2} \cdot \frac{L}{n} = 48000 \cdot \left(-8{,}48 \cdot 10^{-6} + 1{,}23 \cdot 10^{-5}\right) \cdot \frac{5}{2} \cdot 0{,}50 = 0{,}223kN$$

The resulting force is determined by vectorial summation of longitudinal and transverse force.

$$V = \sqrt{V_x^2 + V_y^2}$$

joint between panel	force in longitudinal direction [N]	force in transverse direction [N]	resulting force [N]
1 and 2	390	223	449
2 and 3	195	330	383
3 and 4	0	359	359
4 and 5	195	330	383
5 and 6	390	223	449

Tab 3.7: Forces of connection at joints (ma imum value of a joint)

So the fastenings of the joints have to be designed for the following force.

$$V_d = \frac{449N}{2} = 225N$$

Also the global displacement of the panels can be determined analytically. One part of the displacement is caused by the force in longitudinal direction. The displacement $v_{y,1,v}$ of the outer panel of the roof is

$$v_{y,1,v} = \frac{V_1^l}{L \cdot k_W} = \frac{5{,}85}{5 \cdot 19500} = 6{,}00 \cdot 10^{-5}\,m = 0{,}0600mm$$

The displacement of panel 2 relative to panel 1 is

$$v_{y,2/1,v} = \frac{V_2^l}{L \cdot k_J} = \frac{3{,}90}{5 \cdot 48000} = 1{,}63 \cdot 10^{-5}\,m = 0{,}0163mm$$

The displacement of panel 3 relative to panel 2 is

$$v_{y,3/2,v} = \frac{V_3^l}{L \cdot k_J} = \frac{1{,}95}{5 \cdot 48000} = 8{,}13 \cdot 10^{-6}\,m = 0{,}00813mm$$

Also the rotations cause a displacement of the panels in longitudinal direction. In the following this part of the displacement is determined for panel 1 to 3.

$$v_{y,1,\varphi} = B \cdot \varphi_1 = 1000 \cdot 1{,}23 \cdot 10^{-5} = 0{,}0123mm$$

$$v_{y,2,\varphi} = B \cdot \varphi_2 = 1000 \cdot 8{,}48 \cdot 10^{-6} = 0{,}00848mm$$

$$v_{y,2,\varphi} = B \cdot \varphi_2 = 1000 \cdot 2{,}99 \cdot 10^{-6} = 0{,}00299mm$$

The maximum displacement occurs at the joint between panel 3 and 4. It is determined by summation of the displacements $v_{y,i,v}$ and $v_{y,i,\varphi}$ over the panels, starting at panel 1.

$$v_{y,3} = \frac{V_1^l}{L \cdot k_W} + \frac{V_2^l}{L \cdot k_J} + \frac{V_3^l}{L \cdot k_J} + B \cdot \varphi_1 + B \cdot \varphi_2 + B \cdot \varphi_3$$

$$v_{y,3} = 0{,}06 + 0{,}0163 + 0{,}00813 + 0{,}0123 + 0{,}00848 + 0{,}00299 = 0{,}108mm$$

6 Design of wall without connections at joints

From the roof the in-plane shear forces are introduced into the walls. In the following the forces of the fastenings and the displacements of the panels of wall A are determined. For the present example it is assumed that there are not any connections of the joints of the wall panels. The wall A is loaded by in-plane shear forces, if the wind load has the direction B. The load introduced into one panel is (cf. section 5.2, Fig. 3.16)

$$F_d = \frac{V_1^l}{5} = \frac{5{,}85}{5} = 1{,}17kN$$

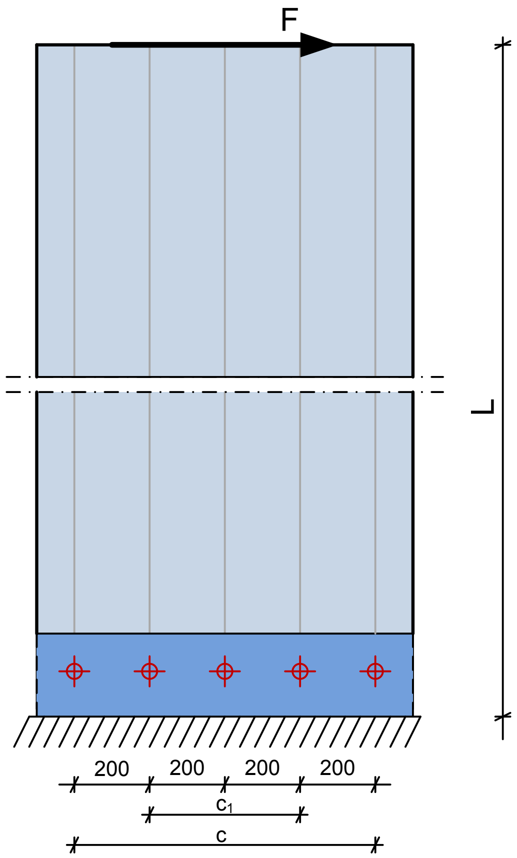

Fig. 3.17: S stem of wall panel

The connections between wall and foundation transfer forces in transverse and in longitudinal direction.

Force in transverse direction:

$$V_x = \frac{F}{n} = \frac{1{,}17}{5} = 0{,}234kN$$

Force in longitudinal direction (outer connections):

$$V_y = \frac{F \cdot L}{c + \frac{c_1^2}{c}} = \frac{1{,}17 \cdot 3}{0{,}80 + \frac{0{,}40^2}{0{,}80}} = 3{,}51kN$$

Resulting force:

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{0{,}234^2 + 3{,}510^2} = 3{,}518kN$$

A connection consists of two fastenings. So the fastenings have to be designed for the following force.

$$V_d = \frac{3518N}{2} = 1759N$$

The moment causes the following rotation.

$$\varphi = \frac{12 \cdot F \cdot L}{B^3 \cdot k_W} = \frac{12 \cdot 1{,}17 \cdot 3}{1^3 \cdot 65000} = 6{,}48 \cdot 10^{-4} rad$$

The transverse displacement of the panel is (upper end of panel)

$$v_x = \frac{F}{B \cdot k_W} + \varphi \cdot L = \frac{1{,}17}{1 \cdot 65000} + 6{,}48 \cdot 10^{-4} \cdot 3 = 0{,}00196m = 1{,}96mm$$

7 Design of wall with connections of joints

The wall of the previous section is also considered in this section. But now the connections of the longitudinal joints of the wall panels are taken into account. So not only a single panel, but the complete wall has to be considered. The system and the loads are given in Fig. 3.18. Each connection between wall and roof (three connections per panel) is loaded by 390 N (1170 N per panel).

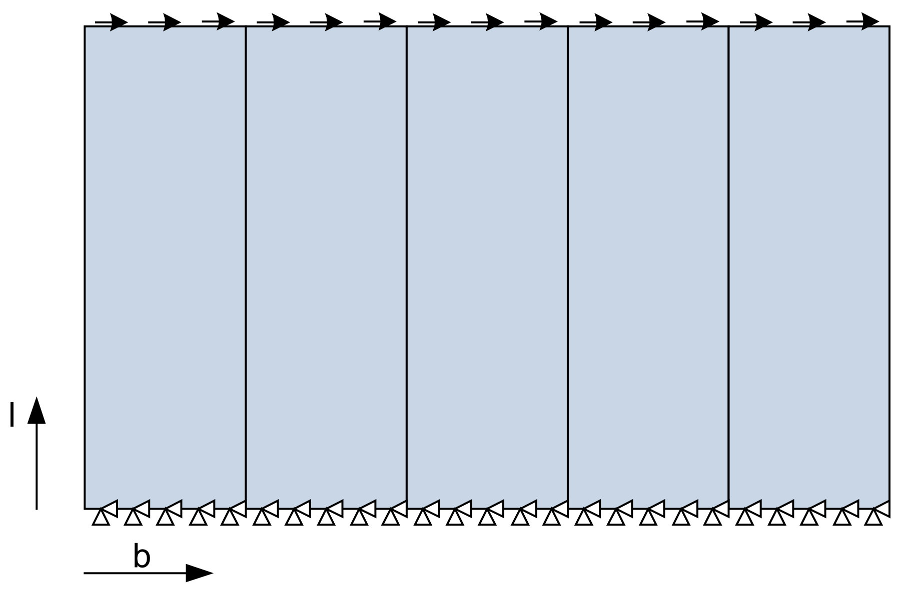

Fig. 3.18: S stem of the wall

The following figure shows the FE-model used for the calculations.

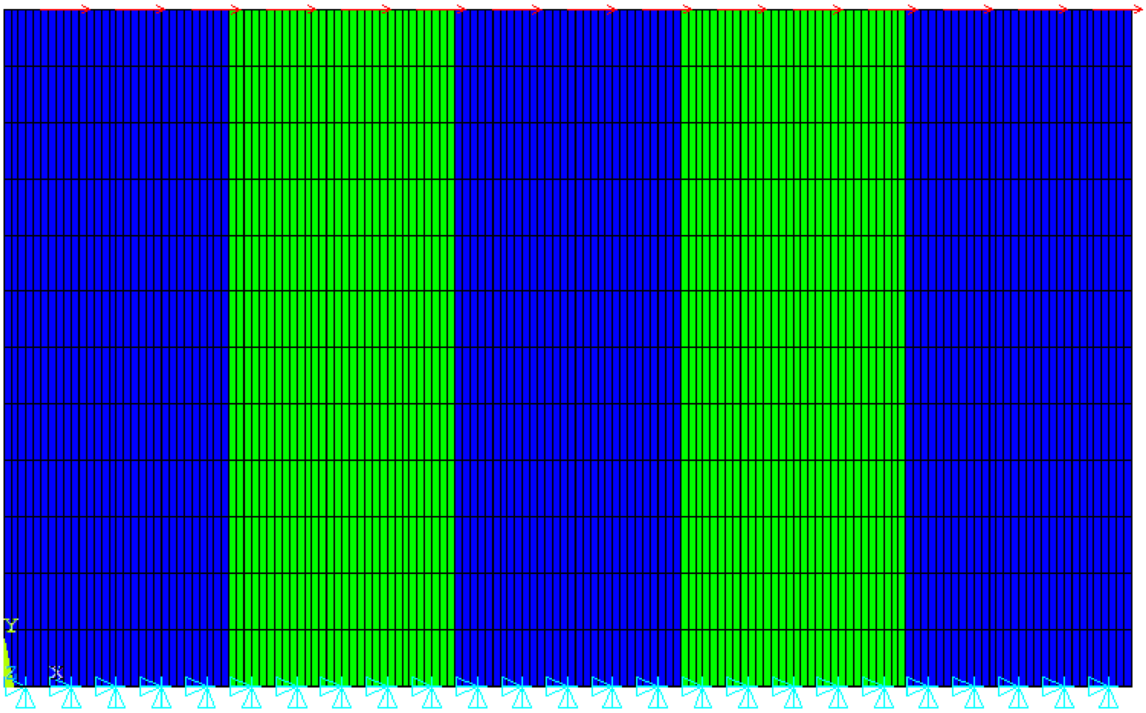

Fig. 3.19: FE-model of the wall

The displacement and the rotation of the panels are given in the following figures.

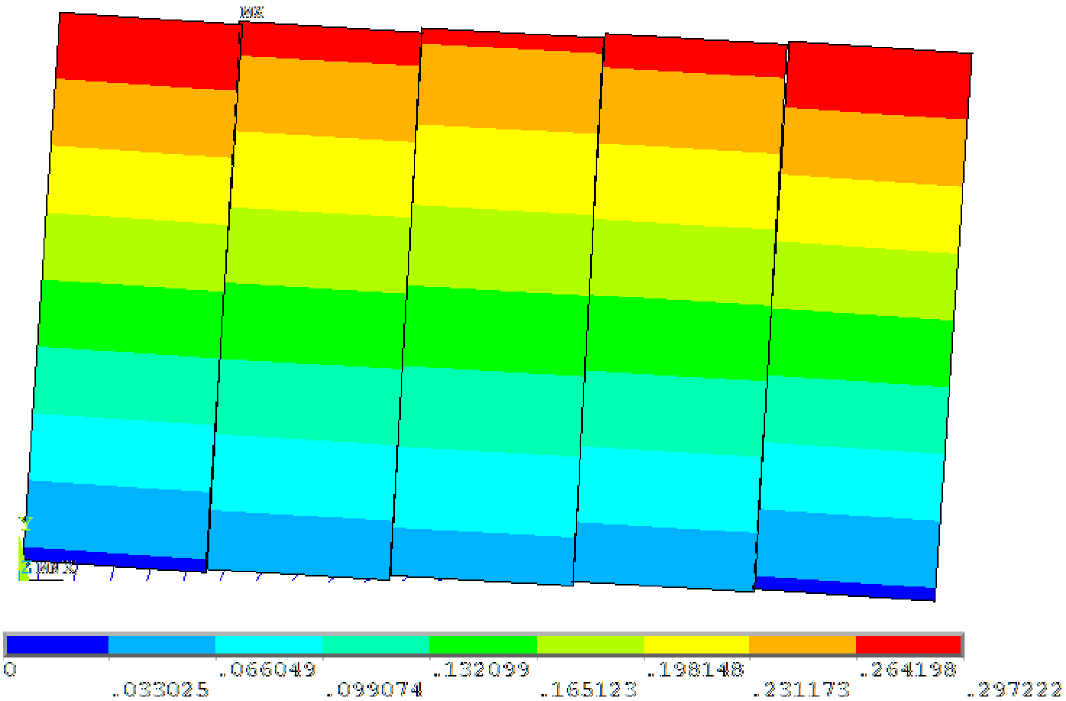

Fig. 3.20: Displacement of panels (transverse direction), [mm]

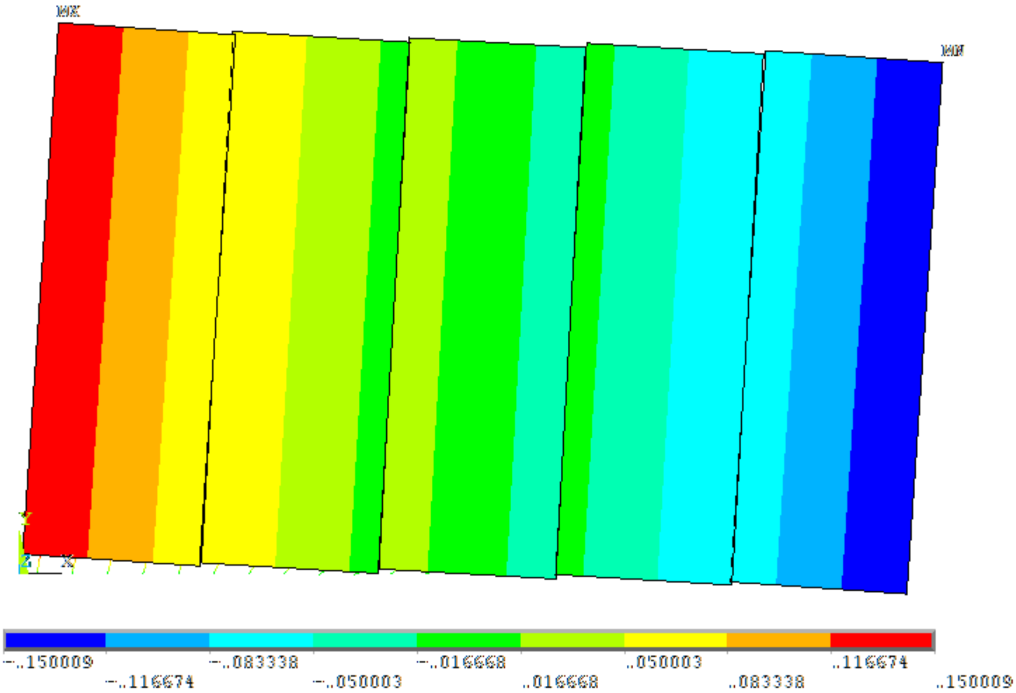

Fig. 3.21: **Displacement of panels (longitudinal direction), [mm]**

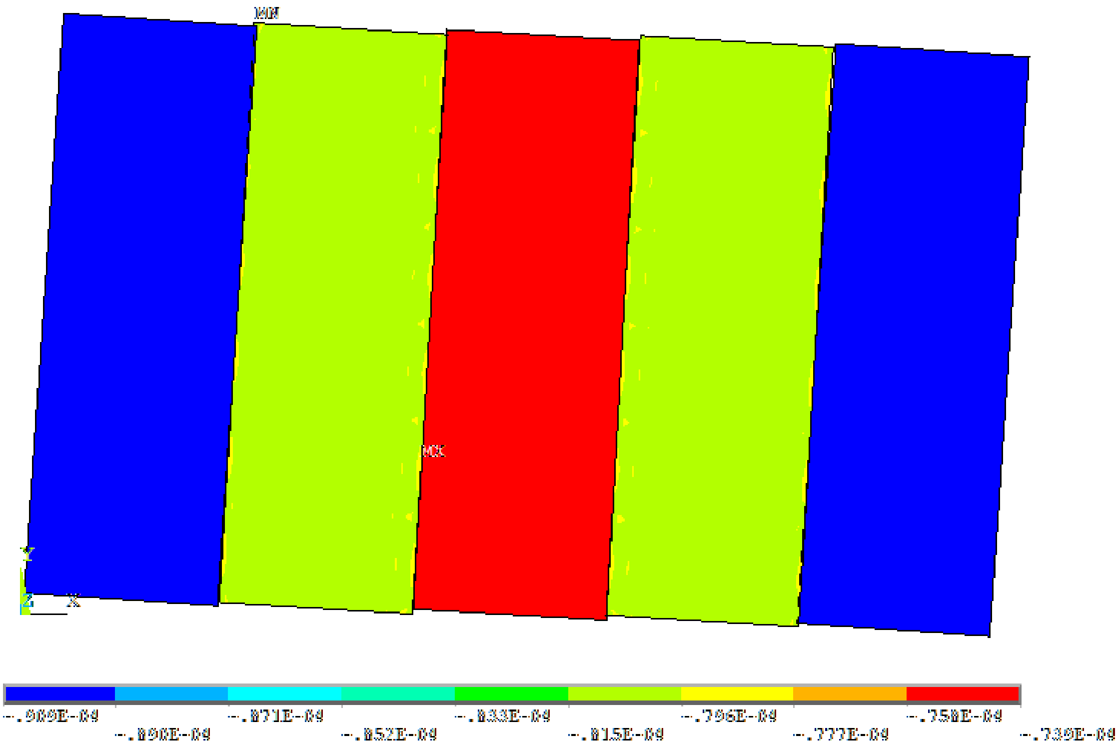

Fig. 3.22: **Rotation of panels, [rad]**

Also the forces of the connections are determined by the numerical calculation. Fig. 3.23 shows the forces of the connections between panel and foundation. The highest forces (932 N) occur at the outer connections. Because internal and external face sheet are connected, the force of the highest loaded fastenings is

$$V_d = \frac{932N}{2} = 466N$$

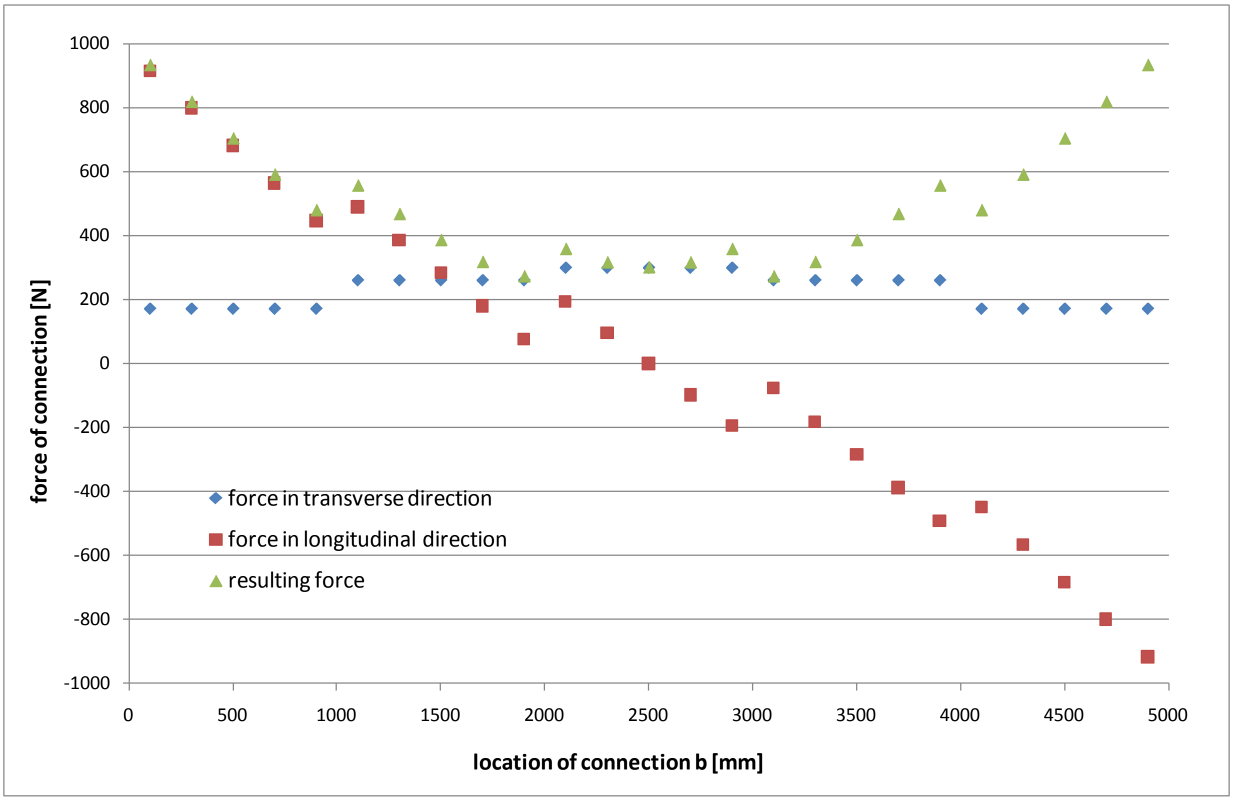

Fig. 3.23: Forces of connections between panels and foundation

The connections of the joints between the roof panels are loaded by transverse as well as by longitudinal forces. The forces in longitudinal direction are constant for all connections of a joint.

joint between panel	force of a connection [N]
1 and 2	568
2 and 3	805
3 and 4	805
4 and 5	865

Tab 3.8: Forces of connection at joints (force in longitudinal direction)

The forces in transverse direction are linearly distributed over the length of the joint. The forces are given in the following table.

		joint between panel			
		1 and 2	2 and 3	3 and 4	4 and 5
location of connec-tion l [mm]	250	266	116	-116	-266
	750	139	58	-58	-139
	1250	12	1	-1	-12
	1750	-115	-57	57	115
	2250	-242	-114	114	242

Tab 3.9: Forces of connection at joints (force in transverse direction) [N]

Based on the longitudinal and the transverse forces determined by the numerical calculation the resulting forces are determined.

$$V = \sqrt{V_x^2 + V_y^2}$$

The resulting forces are summarised in Tab 3.10.

		joint between panel			
		1 and 2	2 and 3	3 and 4	4 and 5
location of connec-tion l [mm]	250	627	813	813	627
	750	585	807	807	585
	1250	568	805	805	568
	1750	580	807	807	580
	2250	617	813	813	617

Tab 3.10: Forces of connection at joints (resulting forces) [N]

The highest force of a connection is 813 N. So the fastenings of the joints should be designed for the following force.

$$V_d = \frac{813N}{2} = 407N$$

Remarks:

If the joints of wall panels are connected displacements and rotation as well as the centre of rotation are unknown. To determine them no set of linear equations is available. So also for simple configurations an analytical calculation is not reasonable.

In section 6 and 7 the same wall is designed, whereat only in section 7 the connections of the longitudinal joints are considered. If the forces and displacements of both examples are compared, the influence of the connection becomes obvious.

If there are no connections of the joints the fastenings of the connections between wall and foundation have to be designed for 1759 N. With the additional connections given in the example this force decreases to 466 N. Anyhow the force of a fastening of the joint is comparatively small (407 N).

For the wall without connections of the joints the highest transverse displacement of a panel is 1,96 mm. If the panels are connected to the adjacent panels, the displacement decreases to 0,30 mm.

8 Conclusions

In the sections above some calculation examples for the design of frameless buildings loaded by horizontal wind loads are presented. In the examples the determination of the forces of fastenings and the determination of displacements of panels is demonstrated. Numerical calculations are an effective method to determine forces and displacements. For some simple applications forces and displacement can also be determined by analytical calculation procedures.

With the given examples it could be shown, that the displacements, which are caused by horizontal wind loads are quite small. Also the forces the fastenings have to be designed for are as small that they can be transferred by conventional mechanical fasteners.

9 References

[1] EN 14509:2006: Self-supporting double skin metal faced insulating panels – Factory made products –Specifications.

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[3] EN 1990:2002: Eurocode - Basis of structural design.

[4] EN 1991-1-4:2005: Eurocode 1: Actions on structures – Part 1-4: General actions, Wind actions.