Decision Maps for Distributed Scenario-Based Multi-Criteria Decision Support

Zur Erlangung des akademischen Grades eines Doktors der Wirtschaftswissenschaften (Dr. rer. pol.)

von der Fakultät für Wirtschaftswissenschaften des Karlsruher Instituts für Technologie (KIT) genehmigte

D I S S E R T A T I O N

von

Dipl.-Math. Martina Comes

Referent:
Prof. Dr. O. Rentz
Korreferent:
Prof. Dr. S. Rachev

eingereicht am:	31.01.2011
Tag der mündlichen Prüfung:	04.07.2011

Prediction is very difficult,
especially if it's about the future.

Niels Bohr (1885–1965)

Vorwort

Diese Arbeit entstand im Rahmen meiner Tätigkeit am Institut für Industriebetriebslehre und Industrielle Produktion (IIP) des Karlsruher Instituts für Technologie (KIT). Die Grundlage der Arbeit lieferte unter anderem ein Forschungsprojekt zum industriellen Risikomanagement, das von der Europäischen Union im 7. Rahmenprogramm gefördert wurde.

Mit Freude nutze ich die Gelegenheit, den Menschen zu danken, die mich bei der Erstellung dieser Arbeit unterstützt haben. Besonderer Dank gilt meinem Doktorvater Prof. Dr. Otto Rentz für die Betreuung der Arbeit. Bei dem ehemaligen Leiter meiner Arbeitsgruppe „Technikbewertung und Risikomanagement" Dr. Michael Hiete möchte ich mich für zahlreiche Diskussionen, für die Freiheiten, die er mir einräumte, und für die stete Versorgung mit bester Rausch-Schokolade bedanken. Des weiteren gilt mein Dank Prof. Dr. Svetlozar Rachev für die Übernahme des Korreferates. Schließlich möchte ich Prof. Dr. Frank Schultmann für seine fachliche und persönliche Unterstützung danken.

Die gute Arbeitsatmosphäre am IIP hat sicher zum Gelingen der Arbeit beigetragen. Daher gilt mein Dank allen Kolleginnen und Kollegen des Instituts für die Zusammenarbeit auch über fachliche Grenzen hinweg. Besonderes möchte ich meinen ehemaligen und aktuellen Kollegen aus dem Bereich Risikomanagement, Dr. Valentin Bertsch, Marjorie Vannieuwenhuyse, Christian Trinks und Dr. Mirjam Merz, für die freundschaftliche Zusammenarbeit, die konstruktiven Diskussionen und die Unterstützung auch über diese Arbeit hinaus danken.

Während meiner Arbeit im Rahmen des Forschungsprojektes Diadem („Distributed Information Acquisition and Decision Making in Environmental Management") sind zahlreiche berufliche, aber auch persönliche Kontakte entstanden. Besonderer Dank gilt Dr. Claudine Conrado, die mit mir immer neue Fallbeispiele entwickelt hat, und Dr. Niek Wijngaards, der mich stets mit seiner Begeisterung und seinem Enthusiasmus für das Thema unterstützt und in unzähligen Skype-Sessions

zu fast jeder Tages- und Nachtzeit kleine und große Fragen und Probleme mit mir diskutiert hat.

Schließlich danke ich meiner Familie. Besonders danke ich meinem Bruder Christian für seinen organisatorischen und strategischen Rat sowie meinen Eltern Rita und Rudolf Comes, die mich stets vorbehaltlos unterstützt, an mich geglaubt, mich ermutigt und mir immer zur Seite gestanden haben.

Karlsruhe, im Juli 2011 Martina Comes

Contents

List of Figures

List of Tables

1. Introduction

> If we want things to stay as they are,
> things will have to change.
> (Giuseppe Tomasi di Lampedusa)

Behavioural studies have shown that, while we humans may be the best decision makers on the planet, we are not quite as good as we think we are. Particularly in strategic decision situations, decision-makers are subject to biases, inconsistencies and irrationalities. Strategic decision-making typically occurs in complex situations where the information upon which the decision is based is incomplete and error-prone, and its availability and quality changes over time [Rowe, 1994]. Decision theories, methods and tools are designed to assist decision-makers in maximising the benefit from their decisions' outcomes and minimise errors due to irrational behaviour [Guastello, 2006]. From a decision-analytical perspective, the two most important challenges in making strategic decisions are the presence of *fundamental uncertainty* and *complexity* [Montibeller and Franco, 2010].

Most decision-makers confronted with such problems attempt to use intuitive approaches to reduce the complexity until the problem seems manageable [Kiker et al., 2005]. In this unstructured process, however, important information may be lost, opposing points of view may be discarded and elements of uncertainty may be ignored. There are many reasons to expect that, on their own, decision-makers will experience difficulty making informed, rational choices in complex decision situations [McDaniels et al., 1999]. Therefore, the need arises for a well-structured and transparent decision support system facilitating information acquisition, sharing and processing, as well as the evaluation of decision alternatives, while avoiding information overload. This thesis develops a methodology to support strategic decision-making in complex and highly uncertain situations.

1.1. Strategic Decision-Making in Complex and Uncertain Situations

In daily life, everyone engages in decision-making. Some decisions have minor consequences and are made with little thought. Other decisions have much greater (potential) impact and justify the effort of reflecting and deliberating on possible decision alternatives before choosing one. Decision-making does not only involve assessing the consequences of the decision to be made, but also the context of the decision problem[1] and the skills and dispositions of the involved actors. The *context of the problem* involves the complexity of the questions to be answered, the presence of uncertainty, the number of available options as well as the time frame for both the decision-making and the time until the consequences will occur. The cognitive and social factors to be considered comprise the number and responsibilities of decision-makers and stakeholders, their respective skills, beliefs, attitudes towards risk, values and preferences.

This section provides a brief characterisation of typical decision problems arising in complex and highly uncertain situations. First, the decision-analytical framework for assessing the quality of a decision is specified. Second, a characterisation of decision-making under uncertainty is provided.

A decision is said to be *successful* or right if and only if its result is at least as good as every other possible result. It is said to be *rational* if and only if the decision-makers choose the alternative they have the best reason to select at the point in time at which the decision is made [Peterson, 2009]. *Instrumental rationality* assumes that the decision-makers have a set of aims and it is rational to do whatever they have the reason to assume will fulfil these aims best. Yet, a decision can be rational without being right and vice versa [Peterson, 2009]. This can happen when the actual result of a decision is successful, although there were no good reasons for anticipating success. Following this paradigm of instrumental rationality, decision-making under certainty equals choosing the alternative a_l from the set of feasible alternatives that has the best perfor-

[1] Following standard decision-analytical terminology, the term *decision problem* refers to all problems related to *making a decision,* from the structuring of the problem and the identification of feasible options to the assessment and ranking of these options and, ultimately, the decision-making itself [Belton and Stewart, 2002; Guitouni and Martel, 1998; French and Geldermann, 2005].

mance $R(a_l)$.[2] However, most decision problems are characterised by a lack of certainty.

The difference between clear and vague probabilities was first discussed by Knight [1921], who distinguished *risk,* which can be represented by precise probabilities, and (unmeasurable) *uncertainty,* which cannot. Keynes [1921] introduced a distinction between *judged probability* representing the balance of evidence in favour of a particular proposition and the *weight of evidence* representing the quantity of evidence supporting that balance. To describe and operationalise a lack of knowledge, this thesis adopts the terminology introduced by Knight and Keynes, which is still commonly used in today's decision theory [e.g., Clemen and Reilly, 1999; Morgan and Henrion, 1990; Peterson, 2009]. Here, the terms risk, ignorance and uncertainty have the following precise meaning.

- In decisions under *risk* the decision-makers know the probability of possible results.
- In decisions under *ignorance* these probabilities are unknown or non-existent.
- *Uncertainty* can be used as a synonym for ignorance or as a broader term referring to both risk and ignorance. In this thesis, the latter definition is used. More precisely, uncertainty implies that in a specific situation the decision-makers do not have access to (all) information that quantitatively and qualitatively is appropriate to describe or predict a system, its behaviour or other characteristics deterministically [Zimmermann, 2000]. Uncertainty is prevalent in most types of information and knowledge, particularly in model-based decision support systems. It arises from incompleteness of information or knowledge, linguistic imprecision, measurement errors, approximations and simplifications [Peterson, 2009].

The assessment of uncertainties is particularly important in *high-consequence systems* [Oberkampf et al., 2004], which are open systems whose behaviour has a significant effect on the world outside the system itself (e.g., power plants, aviation or medical systems). Failure in high consequence systems can result in significant threats for society, economy and environment [Mc Carthy et al., 1997]. Furthermore, explicit consideration of uncertainty is important when the decision-makers' *attitude towards risk* is relevant [Morgan and Henrion, 1990].

[2] As there is no uncertainty, both the impact of a_l on the attributes' scores and the preferences can be determined uniquely.

Often, national and international legislation, industry standards and company guidelines require that a quantitative evaluation of the present uncertainties should be included in the analysis of results [van der Keur et al., 2010; Swart et al., 2009].

1.2. Decision Support in Strategic Emergency Management

This thesis has been motivated by the decision problems typically arising in strategic risk and emergency management for the chemical industry. Generally, managing risks to and emerging from today's complex and globally-interlaced production systems has become increasingly important in the light of a rising number of extreme events and man-made emergencies [Hiete and Merz, 2009]. The chemical industry (particularly all activities related to processing, and transporting hazardous chemicals) plays an exceptional role in technological emergencies, since the release of hazardous substances may result in severe consequences [Hiete and Merz, 2009; Khan and Abbasi, 2002]. Examples of recent major chemical incidents in Europe include the Seveso disaster, where hazardous compounds were released from a reactor producing trichlorophenol in 1976 [Khan and Abbasi, 1999], the explosion of a fireworks store in Enschede in 2000 [Wybo and Lonka, 2003], the explosion of a warehouse in Toulouse, where mainly ammonium nitrate and ammonium nitrate-based fertilisers were stored, in 2001 [Dechy et al., 2004], and the release of caustic sludge from an aluminium plant's waste reservoir in Kolontar in 2010 [Enserink, 2010].

Although the methods and approaches presented throughout this thesis are applicable in different fields sharing some common features, here the particular characteristics and constraints of strategic decision-making in emergency management are addressed. In emergency management, the concept of *risk* usually encompasses three aspects [Kaplan and Garrick, 1981; Kaplan, 1997]:

- a *scenario* allowing for an analysis of the situation and its development,
- the probability or *likelihood* of the scenario, and
- the *consequences* of the scenario, usually represented as a multidimensional vector, as there are typically different types of losses.

Risk management comprises identification, assessment, analysis and mitigation of risks for population, society and environment [Aven, 2004; French et al., 2009].

In emergency management, decision-makers need to identify and evaluate alternatives taking into account multiple, at least partly conflicting objectives. Health and safety aspects, the environmental impact, technical and organisational feasibility, costs of mitigation measures, as well as economic and social consequences have to be considered [Geldermann et al., 2009]. The aim is to select an alternative with the smallest risk to people, society and environment under different possible evolutions of the situation. In other words, a *robust alternative* is to be identified, where robustness refers to the ability of a solution to cope with uncertain or non-anticipated developments [Wallenius et al., 2008]. Finally, making decisions in strategic emergency management requires the integration of multi-faceted input from stakeholders and experts with different values and objectives [Bertsch, 2008].

In summary, decisions in strategic emergency management typically feature the following characteristics [Bertsch, 2008; Geldermann et al., 2006; Mustajoki et al., 2007; Wright and Goodwin, 2009]:

- a finite set of feasible alternatives $A = \{a_1, \ldots, a_k\}$ to choose from,
- multiple goals, which are often conflicting,
- the involvement of multiple, frequently locally-dispersed experts, each of whom has different knowledge, skills, competences and preferences,
- the need to consider information of heterogeneous origin, type and quality,
- constrained time to make a decision and bounded availability of experts, but no ad-hoc or real-time decision making,
- the need for transparency, comprehensibility and documentation to enhance acceptance and compliance and to account for the decisions made if necessary.

Therefore, in strategic emergency management the need for distributed, timely, coherent and effective decision support arises.

1.3. Objectives and Structure of the Thesis

In complex strategic decision-making situations, decisions must often be made among a finite set of feasible alternatives with respect to multiple objectives [Belton and Stewart, 2002]. *Multi-Criteria Decision Analysis* (MCDA) supports decision-makers in these situations, as MCDA allows for a transparent evaluation of alternatives [von Winterfeldt and Edwards, 1986]. Still, the use of

MCDA can be problematic when uncertainties are significant [Durbach and Stewart, 2003]. Uncertainties, however, play an important role in most strategic decision-making situations, as information is often imprecise, uncertain or lacking [Morgan and Henrion, 1990; Montibeller and Franco, 2010].

Scenarios offer a possibility to deal with uncertainty as they explore fundamentally different descriptions of a situation and its possible developments [Schoemaker, 1993]. Being plausible, consistent and coherent [Schnaars, 1987], scenarios appeal to decision-makers and help overcoming cognitive biases such as overconfidence or misjudgement of likelihoods [Wright and Goodwin, 2009]. To construct scenarios describing *large and complex* decision problems, knowledge and expertise from various domains has to be brought together [Morgan and Henrion, 1990; Shaw and Fox, 1993]. Although recently a small number of approaches evaluating scenarios with respect to multiple goals have been developed [Diakoulaki and Karangelis, 2007; Goodwin and Wright, 2001; Hites et al., 2006; Montibeller and Belton, 2006], none of these approaches systematically integrates scenario construction and evaluation of alternatives. While usually both SBR and MCDA are used exclusively for long term (not time critical) problems, the approach developed in this thesis facilitates using *Scenario-Based Multi-Criteria Decision Analysis* for complex time critical (but not ad-hoc or real-time) decision support.

The approach presented in this thesis is targeted at situations where the complexity of the decision problem can be reduced by dividing the overall goal into a number of (possibly conflicting) criteria that can be operationalised by means of measurable attributes. Similarly, it is assumed that the decision problem itself can be divided into a number of sub-problems that can be analysed and solved by experts with a particular domain of knowledge and skills. This distributed approach enables adapting the reasoning principles flexibly to the types and qualities of information available. When information is uncertain, imprecise or not available, there are several principles to handle this lack of knowledge, e.g., *probabilistic* and *Bayesian techniques*, *fuzzy logic* or the reduction of the complete domain to sets of possible states. Each of these principles is targeted at dealing with a certain type and quality of information. When these types and qualities of information vary for different parts of the problem at hand, the use of one unique principle capturing all arising uncertainties becomes problematic. Therefore, this thesis proposes dealing with the uncertainties in a distributed manner allowing each expert to choose the most appropriate principle for the part of the problem he has to handle.

This thesis aims at developing a methodology providing *robust decision support for complex strategic decision problems.* The thesis shows that *Scenario-Based Reasoning is a means to achieve robust Multi-Criteria Decision Support.* Robustness is a concept related to the stability of results. It addresses the question how flawed or defective the models and data can be without jeopardising the results' quality [Ben-Haim, 2000; Regan et al., 2005; Roy, 2010]. A highly immune alternative is preferred over an alternative that is vulnerable to errors. To achieve the aim of robust Multi-Criteria Decision Support, the following objectives must be achieved.

O.1 The *different types of information* collected must be combined and processed into meaningful scenarios. Different ways of capturing the uncertainty of the information must be taken into account in a well-structured and transparent manner.

O.2 The scenarios must be *purposeful.* They need to answer the question(s) relevant to the problem at hand and must be tailored to the recipients' information needs.

O.3 The scenarios must be *acceptable* and *credible* for their recipients. Therefore, the recipients' quality requirements (e.g., in terms of correctness, plausibility, consistency and coherence) must be met.

O.4 A distributed system taking into account information from various sources to *evaluate each alternative* for varying scenarios must be implemented.

O.5 Approaches facilitating the analysis of the *robustness* of and *risks* associated with each alternative with respect to several objectives must be developed and implemented.

O.6 The scenario building and evaluation processes must be *manageable* and respect *constraints* in terms of time available for the decision-making process, bounded availability of experts, limited resources and capacities for information processing.

To show how these objectives can be achieved by the presented methodology, this thesis is structured as follows.

Chapter 2 describes the background and positions the methodology developed in this thesis. Firstly, it briefly explains the purpose and use of various techniques for *Multi-Criteria Decision Support.* Secondly, it addresses the problem of *uncertainty* in decision support systems. Finally, it introduces the use of *scenarios* for decision support under fundamental uncertainty. It reviews diverse scenario-based techniques and summarises the requirements for scenario-based decision support.

Chapter 3 provides the newly developed *formalisations of scenarios and sets of scenarios.* These formalisations provide a flexible structure that clarifies the concepts used and facilitates the implementation of scenario-based techniques in distributed settings. The guiding principle is that, although scenarios for complex situations cannot be computed in an automated way and experts must perform many tasks manually, there are structural aspects that can be used as basis for automated support. Using tools and algorithms to do tedious, exacting or computationally intense tasks helps to eliminate human errors and reduces information overload. Consequently, human experts can focus on work for which their particular skills and domain of knowledge are essential and most valuable. Chapter 3 first defines *single scenarios* in a manner that allow tools and techniques from graph theory and network models from the field of Artificial Intelligence to be applied. The second part of this chapter formalises the concept of *sets of scenarios* and focuses on capturing the *relationships* between individual scenarios and sets of scenarios. The formalisation provides the basis for the definition of two types of scenario construction processes, which make sure that *objective O.1* is achieved.

Chapter 4 specifies in a precise and strict way the *requirements* that single scenarios and sets of scenarios must meet to be acceptable for their recipients. To this end, both the structure and the contents of the scenarios are considered. The chapter provides a characterisation of sets of scenarios and types of Scenario-Based Reasoning, allowing for a definition of requirements for Scenario-Based Reasoning ensuring that objective O.2 is met. Furthermore, Chapter 4 presents an operationalisation of scenario quality requirements and provides the basis for achieving objective O.3.

Chapter 5 presents the key results of this thesis. *Decision Maps,* a new framework facilitating scenario construction and assessment with MCDA techniques, combine Directed Acyclic Graphs (DAGs) managing information distribution, processing and filtering, with MCDA evaluation techniques. This formal approach enables applying powerful methods and tools from set and graph theory as well as approaches from the Artificial Intelligence community. Via DAGs, co-operations between the best timely available experts (both human experts and automated systems) are established. These experts build assessments of the situation and prognoses of how it might unfold into the future (i.e., scenarios) that are the basis for the decision. It is shown that this integrated approach allows for a detailed evaluation of the scenarios' results with respect to multiple criteria. Thus objective O.4 is achieved. The evaluation of scenarios enables an analysis of the *robustness* of the alternative. This analysis is supported by an aggregation of the scenario results according to the decision-makers' (risk) preferences. Additionally, complementary methods to investigate the risks associated with each alternative are presented. Hence, objective O.5 is achieved. While the use of scenarios enhances the robustness of MCDA results, the use of MCDA provides a rationale for constructing *relevant* scenarios. In this manner, the problem structuring techniques of MAVT are used as a means to structure and manage information processing. This approach enables information overload of the experts and decision-makers to be reduced and objective O.6 to be achieved.

Chapters 6, 7 and 8 provide the formalisations and methods for scenario management allowing O.6 to be achieved while respecting the requirements of O.3. Scenario management provides mechanisms to control the number of scenarios. Additionally, it facilitates the handling of novel and/or outdated information. Chapter 6 is dedicated to scenario information management. It shows how information on the progress and status of the construction of (multiple) scenarios can be captured. Furthermore, an abstract information model is described to capture Scenario-Based Reasoning processes with sufficient annotations to support scenario management. Chapter 7 introduces useful operations for comparing scenarios and defines new concepts such as similarity and equivalence of scenarios. These concepts facilitate *scenario management,* presented in Chapter 8. Scenario management entails the characterisation and selection of the most relevant scenarios, the pruning of scenarios (by defining degrees of acceptability according to the users' preferences) and the updating of scenarios.

Chapter 9 illustrates the method developed by means of an *example* from strategic emergency management. Reactive chemical hazards have been a significant concern for facilities that process, handle, transport or store reactive chemicals (such as chlorine), as these incidents have led to numerous losses in the process industries and have affected the environment and the public [Wei et al., 2004]. Hence, in Chapter 9, a set of precautionary decision alternatives is investigated for a chemical incident involving the potential release of a large amount of chlorine.

Chapter 10 concludes this thesis with a *summary* of the main results and a *discussion* that highlights the advantages of the approaches developed and evinces some limitations. Furthermore, a number of aspects are identified for future research and possibilities how these open issues could be solved using the approaches elaborated in this thesis are developed and discussed.

2. Decision Support in Complex and Uncertain Situations

Everyone complains of his memory,
no one of his judgement.
(François de la Rochefoucauld)

Strategic decision-making requires the analysis of complex and uncertain situations to assess the impact of feasible alternatives. Different techniques to handle the prevailing complexity and uncertainty have been developed. Each of these techniques is tailored to a particular class of problems sharing certain characteristic features and properties. This chapter reviews the most important techniques for decision support under uncertainty and provides the background for the novel methodology that is developed within this thesis.

Section 2.1 provides an overview of *Multi-Criteria Decision Support* techniques integrating the decision makers' preferences to model trade-offs between different goals. In Section 2.2, different types of quantifiable uncertainty and methods for handling them are explored. *Probabilistic* and *fuzzy techniques* for decision support are reviewed and their respective strengths and drawbacks are discussed. Section 2.3 introduces *scenario-based methods* as a means to deal with non-quantifiable or severe uncertainty. The most common methods of scenario planning and analysis as well as network techniques facilitating scenario construction are briefly reviewed.

2.1. Multi-Criteria Decision Support Systems

Numerous Multi-Criteria Decision Support Systems have been developed [Belton and Stewart, 2002]. In the field of Multi-Criteria Decision Analysis (MCDA), all systems and methods deal with the problem that comparisons within a set of alternatives must be made comprehensively with respect to multiple objectives.

To this end, abstract and vague higher-level goals are expressed in terms of a number of relatively precise but generally conflicting criteria [Stewart, 1992]. A view shared by many MCDA practitioners is that one of the principal benefits from the use of this well-structured approach to decision-making is the learning about the problem itself as well as about the value judgements and priorities of all involved parties contributing to an increased respective understanding [Belton and Hodgkin, 1999].

2.1.1. Multi-Criteria Decision Analysis

Plenty of MCDA methods have been developed, applied and described. Below is a characterisation of MCDA methods based on the following features: the number of decision makers, the availability of support, the uncertainty in the data and information, and a characterisation of the set of alternatives.

Number of decision makers: a decision can be made by an individual or a group [Luce and Raiffa, 1989]. In the first context, *unitary* value judgements (from a single individual or a homogeneous group) need to be taken into account, and the methods applied can be relatively informal [Stewart, 1992].

Contrarily, when the interests of a larger group of decision makers with different priorities and/or objectives need to be respected, these *conflicting interests* must be resolved to build a compromise [Kiker et al., 2005]. In this case, systems facilitating the communication between all negotiating parties gain in importance [Korhonen et al., 1992]. As it is essential to demonstrate that each interest has been considered, decisions must be made transparently, and the need for documenting the rationale for the choices made increases [Luce and Raiffa, 1989]. This requirement necessitates the use of rather formal methods implying that all factors which must be taken into account in arriving at an evaluation of alternatives are prescribed [Stewart, 1992; Williams and Steele, 2002].

In higher-level strategic decision-making, decision makers need to come to a decision on behalf of a much larger group or community [Kiker et al., 2005; Stewart, 1992]. Examples are strategic managerial decisions in large corporations, decisions by public authorities or decisions on environmental issues that involve shared resources. Therefore, the system developed in this thesis is targeted at situations when there is a *diversity of interests* that must be respected.

Availability of support: usually, decision support systems are designed with a specific type of user in mind [Belton and Hodgkin, 1999]. Yet, the systems are often made available to a diverse set of users with a wide range of skills and experience including expert analysts and lay decision makers [Eden and Ackermann, 1996]. A common classification is the distinction between a *facilitated decision process,* where a moderator leads the decision makers through the process and the *"Do-it-yourself"* user [Belton and Hodgkin, 1999; Hodgkin et al., 2005]. The role of the facilitator is to guide and steer the discussions. Particularly, the group dynamics should be managed and ultimately, a closure or a consensus should be reached including a commitment to action [Eden, 1992b; Montibeller and Franco, 2010].

Uncertainty in underlying data and information: although decision-making processes are prone to multiple types of uncertainty stemming from different sources, for the classification of MCDA methods, one mostly refers to the (un-)certainty of the consequences $x(a)$ of implementing an alternative a [Guitouni and Martel, 1998; Luce and Raiffa, 1989]. A decision is said to be made

- *under certainty,* when the implementation of alternative a leads invariably to a specific outcome $x(a)$ [Luce and Raiffa, 1989].
- *under risk,* when the implementation of alternative a leads to a possible set of outcomes $X(a)$ where each $x(a) \in X(a)$ occurs with a known probability $p\big(x(a)\big) \in [0,1]$ [Luce and Raiffa, 1989].
- *under fuzziness,* when the implementation of alternative a leads to a possible set of outcomes $X(a)$ where the membership of each $x(a) \in X(a)$ to a set $\widetilde{X}(a) \subseteq X(a)$ is characterised by a function $\mu\big(x(a)\big) \in [0,1]$ [Bellman and Zadeh, 1970].
- *under severe uncertainty,* when the implementation of alternative a leads to a possible set of outcomes $X(a)$ where the likelihood of each $x(a) \in X(a)$ is unknown [Luce and Raiffa, 1989]. In these situations one distinguishes decisions under *ignorance* referring to situations without any information quantifying the likelihood of events [Hogarth and Kunreuther, 1995] and decisions under *ambiguity* describing situations when uncertainties about the quantification of likelihood are created by missing information that is relevant and could be known [Camerer and Weber, 1992].

These types of uncertainty and state-of-the-art methods dealing with each of them are discussed in-depth in Section 2.2.

The **set of alternatives** A can be continuous or discrete and infinite or finite. In the continuous case, A is usually characterised by a set of boundary conditions [Korhonen et al., 1992], while for the discrete and finite case A can be written as a set $A = \{a_1, \dots, a_l\}$, $l \in \mathbb{N}$. According to the properties of A, one distinguishes the following classes of MCDA methods:

Multi-Objective Decision Making (MODM) methods are used when A is a continuous set [Hwang et al., 1980]. MODM problems are typically written in the form

$$\begin{array}{ll} \max\limits_{a} & f(a) \\ \text{subject to} & a \in A, \end{array}$$

where $A \subseteq \mathbb{R}^n$ represents the set of feasible alternatives and the objective function $f = (f_1, \dots, f_k)^T : A \to \mathbb{R}^k$ has individual real-valued objectives $f_i : A \to \mathbb{R}$ for $i = 1, \dots, k$ [Klamroth and Miettinen, 2008; Shin and Ravindran, 1991]. Usually, A is implicitly defined by a set of the constraints [Hwang et al., 1980; Shin and Ravindran, 1991], i.e., there are functions $g_j : A \to \mathbb{R}$ $(j = 1 \dots, m)$ allowing for defining A by requiring $g_j(a) \leq 0 \ \forall \ a \in A$ $(j = 1, \dots, m)$.

Due to the conflicting nature of the objectives, there is usually not one optimal solution simultaneously maximizing all criteria, but several mathematically equally good solutions exist, called *efficient, non-dominated, non-inferior* or *Pareto-optimal solutions* [Klamroth and Miettinen, 2008; Shin and Ravindran, 1991]. These share the property that no improvement in any objective is possible without sacrificing on one or more of the other objectives. *Preferential information* is necessary to identify the best among the efficient solutions. Methods solving MODM problems can be classified according to the role of the decision makers and the time in the decision-making process when their preferences are elicited [Klamroth and Miettinen, 2008; Zitzler et al., 2000]:

- *no articulation* of preference information is needed,
- *a priori articulation:* preferences are elicited at the beginning of the search process (e.g., by assigning weights to each objective),
- *progressive* or *interactive articulation:* the decision makers actively take part in an iterative solution process and specify the preferential information gradually,
- *a posteriori articulation:* first, the set of efficient solutions is generated, then the decision makers are supposed to select the most satisfactory solution.

In **Multi-Attribute Decision-Making** (MADM) the decisions consists in selecting one alternative out of a small (discrete and finite) set of feasible, mutually exclusive alternatives. As this thesis uses MADM, the next section is dedicated to a description of different MADM techniques. Subsequently, *Multi-Attribute Value Theory* (MAVT), a deterministic MADM technique, which serves as an exemplary technique throughout this thesis, is explained in detail.

2.1.2. Multi-Attribute Decision-Making

Multi-attribute decision support systems include models and methods that aid the decision makers to choose one alternative out of a list of feasible options respecting multiple criteria. That means, an alternative a_k from the set $A = \{a_1, \ldots, a_l\}$, $l \in \mathbb{N}$ must be selected. To this end, the alternatives are ranked based on preferential information. Generally, one distinguishes *compensatory* and *non-compensatory approaches.* Compensatory approaches allow for balancing a poor performance in one criterion by a good performance in another, while this is impossible in non-compensatory approaches [Guitouni and Martel, 1998].

Compensatory approaches can be divided into Value System and Disaggregation-Aggregation approaches. *Value System approaches* (e.g., Multi-Attribute Value and Utility Theory) aim at the construction of a value system that aggregates the decision makers' preferences on the criteria based on strict assumptions on the preference relations. They require complete and transitive preference relations as well as the commensurability of criteria [Keeney et al., 1979; French, 1986; Siskos and Spyridakos, 1999; von Winterfeldt and Edwards, 1986]. The elicited preferences are used as a basis to construct a unique (value or utility) function aggregating the partial preferences and performances of an alternative on multiple criteria [Siskos and Spyridakos, 1999]. The trade-offs between the criteria are modelled by weights that are used as factors in the aggregation approach (e.g., in the Simple Additive Weighing (SAW) method) [Belton and Stewart, 2002]. As this thesis exploits Multi-Attribute Value Theory (MAVT) approaches, Section 2.1.3 is dedicated to MAVT.
Disaggregation-Aggregation (D-A) approaches such as the Utility Additive (*UTA*) method aim at analysing the decision makers' behaviour and cognitive style [Jacquet-Lagréze and Siskos, 1982, 2001]. The underlying decision support procedures can be divided into two phases. Firstly, in the disaggregation phase a preference model is constructed from decision makers' judgements on a limited set of reference alternatives that are familiar to them. Secondly, the aggregation phase exploits the information elicited to construct value or utility functions as in the Value System approaches presented above [Siskos et al., 1999]. The D-A

approaches use special linear programming techniques (such as the UTASTAR algorithm [Jacquet-Lagréze and Siskos, 2001]) to assess these functions so that the rankings obtained are as consistent as possible with the elicited preferences.

The most common **non-compensatory approaches** are *outranking approaches* (e.g., *PROMETHEE* [Behzadian et al., 2010; Brans and Vincke, 1985] or *ELECTRE* [Roy, 1991]). Outranking approaches aim at the construction of outranking relations that model the *incomparability* among alternatives. Outranking relations are binary relations between pairs of alternatives $\left(a_{k_i}, a_{k_j}\right)$ $(k_i, k_j \in \{1, \ldots, l\})$. The outranking relation models the strength of arguments supporting the statement that a_{k_i} is at least as good as a_{k_j} *(concordance)* and the strength of the arguments against this statement *(discordance)* [Roy, 1991]. In most approaches, the relation is determined on the basis of the comparison of alternatives to some reference profiles (fictitious alternatives a^*) [Doumpos and Zopounidis, 2004].

2.1.3. Multi-Attribute Value Theory

Due to its success in strategic decision-making [Bertsch et al., 2006; Chang and Yeh, 2001; French, 1996; Geldermann et al., 2009], the MCDA approach used in this thesis is *Multi-Attribute Value Theory* (MAVT).

In MAVT, the decision process starts by *structuring the problem* taking it from an initial intuitive understanding to a description that facilitates quantitative or numerical analysis [von Winterfeldt and Edwards, 1986]. The problem structuring phase results in an *attribute tree* hierarchically ordering the decision makers' aims at different abstraction levels, cf. Figure 2.1. The tree shows how the overall objective is divided first into criteria (possibly sub-criteria etc.), until finally the level of attributes is reached. It is assumed that each higher level criterion can be operationalised by a set of attributes that allows for measuring (or quantitatively estimating) the consequences arising from the implementation of any alternative [Stewart, 1992]. That means, for each alternative $a_k \in A$, each attribute j $(j = 1, \ldots, n)$ is assigned a score $x_{kj} \in \mathbb{R}$.

In the next step of MAVT, *preferential information* is elicited [von Winterfeldt and Edwards, 1986]. The attribute scores x_{kj} are normalised to ensure the com-

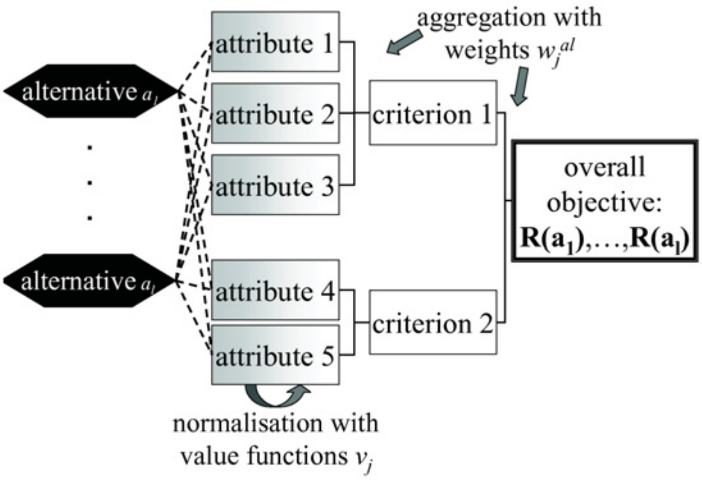


Figure 2.1.: *Structuring The Decision Problem with an Attribute Tree. Attributes are depicted in gradient boxes, criteria and overall goal in white boxes, alternatives in diamonds.*

parability of attributes that are measured on different scales. To this purpose, for each attribute j ($j = 1, \ldots, n$) a *value function*

$$\begin{aligned} v_j : \mathbb{R} &\rightarrow [0,1] \\ x_{kj} &\mapsto v_j(x_{kj}) \end{aligned}$$

mapping the attribute's score to a number in $[0, 1]$ is defined. v_j expresses how important it is to attain a performance close to the optimal possible performance in attribute j. Each value function v_j maps the best score ($\max_{a_k \in A} \{x_{kj}\}$ for strictly increasing and $\min_{a_k \in A} \{x_{kj}\}$ for strictly decreasing preferences) to 1, whereas the worst score is mapped to 0 [Keeney et al., 1979].

The trade-offs between different criteria are captured in *weight vectors* $w^{al} \in [0,1]^{n(al)}$ for each abstraction level[3] al ($al = 1, \ldots, m$). Each weight w_j^{al} ($j = 1, \ldots, n(al)$) describes for each criterion j its relative importance with respect to the other criteria at the same abstraction level. More precisely, the weights indicate the relative importance of changing the level of performance on the respective criteria from their worst to their best levels [Raiffa, 2006]. Besides requiring $0 \leq w_j^{al} \leq 1$ for all $j = 1, \ldots, n(al)$ and $al = 1, \ldots, m$, it is demanded that $\sum_{j=1}^{n(al)} w_j^{al} = 1$ for each abstraction level al.

[3] The abstraction levels correspond to the levels of aggregation presented in the attribute tree, cf. Figure 2.1.

The last step in MAVT is the *aggregation* of values $v_j(x_{kj})$ to the result with respect to the overall goal, $R(a_k)$. To this end, an aggregation operator $agg_D = agg(w^1, \ldots, w^m)$:

$$\begin{aligned} agg_D : [0,1]^n &\rightarrow [0,1] \\ v(x_k) &\mapsto agg_D(v(x_k)) = R(a_k), \end{aligned}$$

where $v(x_k) \in [0,1]^n$ is the vector of attribute values assuming that alternative a_k is implemented. As the space $[0,1]$ with the relation $\leq$ is totally ordered, a comparison of results $R(a_k) = agg_D(v(x_k))$ is enabled. In this manner, a ranking of alternatives $a_k \in A$ is achieved.

MAVT results should not be understood as an imperative prescription but as support and guidance for the decision makers [Belton and Stewart, 2002]. Most of the time, the perceptions of the decision makers will change during the decision support process [French et al., 2009]. Therefore, it is vital that the modelling process is of dynamic, cyclic nature, until a requisite decision model, whose form and content are sufficient to solve the problem, is reached [Phillips, 1984].

MAVT allows for resolving complexity in the decision makers' value judgements and goals by introducing the problem structuring phase, which models the problem by an attribute tree. The handling of uncertain information is, however, hardly addressed, as MAVT assumes that all attribute scores are known with certainty and well-defined [Fenton and Neil, 2001].

An approach allowing for the integration of uncertainty in MAVT is *sensitivity analysis* [Bertsch et al., 2007; Hiete et al., 2010; Ríos-Insua and French, 1991]. Sensitivity analyses are usually applied ex post by varying the input parameters used for the generation of an initial result (e.g., via simulation) [Saltelli et al., 2008]. That means, sensitivity analyses are rather targeted at testing the robustness of results to perturbations of the input and model parameters than exploring fundamentally different developments or paths. While standard sensitivity analysis focuses on variations of one parameter, there are more general approaches analysing the effect of *simultaneous* variations of multiple parameters using sampling techniques (e.g., Monte-Carlo methods [Bertsch, 2008; Butler et al., 1997; Geldermann et al., 2006; Hiete et al., 2010]). Although these methods have linear complexity, simulations can become computationally expensive when the models themselves become complex [Morgan and Henrion, 1990]. As Monte-Carlo techniques require a considerable number of simulations, their application can be problematic when time is limited. Lastly, the modelling of impact factors as independent random variables (as usually done in Monte-Carlo simulations) can lead to contradictory descriptions of the situation and thus to results hard to explain to the users [Ferson, 1996].

2.2. Classification and Treatment of Quantifiable Uncertainty in Strategic Decision-Making

This section focuses on approaches for handling quantifiable uncertainties in MADM. The approaches presented assume that it is possible to quantify for any event the likelihood of its occurrence. It has already been emphasised that strategic decisions are prone to a number of different types of uncertainty (cf. Section 1.1) that need to be treated in different ways [Bedford and Cooke, 2001; French, 1995; Helton, 1994; Morgan and Henrion, 1990]. This section first provides an outline of types of uncertainty that occur typically in strategic decision-making situations. Subsequently, the most important approaches to handle quantifiable uncertainty (namely, probabilistic and fuzzy approaches) and their application in the field of MADM are presented.

2.2.1. Risk, Uncertainty, Ambiguity and Ignorance

For large and complex engineered or natural systems the task of assessing the consequences of a decision is often beyond the capabilities of human decision makers and experts [Pavlin et al., 2009b]. The available information or knowledge base typically consists of a mixture of (partial) knowledge, assumptions and ignorance [Sluijs et al., 2005]. Therefore, decision makers must consider uncertainties of various types and sources. Yet, model-based assessments of complex problems are usually limited to one type of uncertainty [Walker et al., 2003]. As different types of uncertainty have different characteristics, relative magnitudes and adequate means for quantifying them, it is necessary to handle them in different ways [Bedford and Cooke, 2001; Helton, 1994; Morgan and Henrion, 1990; Walker et al., 2003].

One of the most widely recognised classifications is the distinction between *aleatory* and *epistemic* uncertainties [Bae et al., 2004; Bedford and Cooke, 2001; Helton, 1994; Hora, 1996; Kiureghian and Ditlevsen, 2009; Paté-Cornell, 2002]:

Aleatory uncertainties arise due to the inherent variability of the (physical) system or the environment under consideration. The approach commonly used to deal with aleatory uncertainty is the use of *(conditional) probability distributions* [Akter and Simonovic, 2005; Rowe, 1994]. When substantial statistics are available, the use of probability distributions is uncontested [Helton, 1994; Akter and Simonovic, 2005; Ferson and Hajagos, 2004; Morgan and Henrion, 1990].

Epistemic uncertainties result from a from a certain level of ignorance on the system. They forestall the precise assessment of a particular value of interest due to the limitation in the available information or knowledge. In principle, these uncertainties can be quantified by experts, but they cannot be measured. Particularly, they cannot be represented appropriately by probabilistic techniques, since the characterisation of epistemic uncertainties via probability distributions imposes a large amount of unjustified information [Jakeman et al., 2010]. For instance, limited understanding or misrepresentation of the modelled process can lead to a misjudgement of the influence of input on a model's predictions. Epistemic uncertainties are reducible and can therefore be interpreted as a measure for how much *could* be controlled if required [Bedford and Cooke, 2001].

Epistemic uncertainties are widespread in the area of strategic decision-making. Often, goals, constraints and the consequences of the alternatives are not known precisely [Bedford and Cooke, 2001; Helton, 1994]. Epistemic uncertainties present in this domain include the subjectivity and imprecision in the decision-making process itself (e.g., uncertainties in the preferences). Frequently, techniques from fuzzy logic are used to represent these uncertainties [Akter and Simonovic, 2005; Gong and Zhang, 2008; Yu and Tzeng, 2006].

Alternative classifications characterise uncertainty according to

- the *sources of uncertainty* within the modelling and decision-making process [Bertsch, 2008; Funtowicz and Ravetz, 1990; Morgan and Henrion, 1990; Rowe, 1994; Walker et al., 2003],
- the *level of uncertainty.* The uncertainty manifests itself on the scale between deterministic knowledge and total ignorance [Camerer and Weber, 1992; Tannert et al., 2007; Walker et al., 2003].

When founding the classification of uncertainty on its **sources** (i.e., on the location of uncertainty within the model-based decision framework), the classification refers to the following issues.

- The *framing* or domain of the problem sets the boundaries of the system modelled (e.g., the time span considered). It is crucial in any model-based system, as it clarifies the issues to be addressed and the outcomes of interest to be estimated by the model [Hertwich et al., 2000; Walker et al., 2003]. This includes, e.g., economic, environmental, political, social and technological factors that form the context of the problem [Refsgaard et al., 2007].
- *Data uncertainty* (or uncertainty of model input) refers to the description of the system under scrutiny and the variables that drive changes in this system [Walker et al., 2003].
- *Parameter uncertainty* is associated to the values chosen for the model's parameters [Bertsch et al., 2007].
- *Model uncertainty* is a general concept associated with how the model input translates into results [Draper, 1995]. This type of uncertainty comprises both the conceptual model (i.e., the variables and inference mechanisms that are chosen to describe the system) and the implemented (computational) model [French, 1995; Stainforth et al., 2007; van Asselt and Rotmans, 2002; Walker et al., 2003]. Model uncertainty can therefore be further divided into two parts: model *structure* uncertainty, which is uncertainty about the form of the model itself (including inter alia the simplifications and abstractions made, alternative model formulations and model assumptions) and model *technical* uncertainty, which is uncertainty arising from the implementation of the model.
- *Model outcome uncertainty* denotes the accumulated uncertainty associated with the results and their interpretation [French, 1995].

2.2.2. Principles for Handling Quantifiable Uncertainty

This section focuses on techniques to quantify uncertainties by means of probabilistic and fuzzy approaches. It explains both the prerequisites and constraints for using the respective approaches as well as the MADM methods that support decision makers in situations which fall below the respective paradigm. Section 2.3 develops a framework for handling information on events for which the likelihood of an outcome cannot be quantified.

2.2.2.1. Probability Theory and Bayesian Models: Principles

The oldest and most widely used technique to quantify uncertainty is *probability theory,* which measures uncertainty as a real number $P \in [0,1]$, where an event A with $P(A) = 0$ is impossible, whereas an event B with $P(B) = 1$ is sure [Peterson, 2009]. In the *frequentist* view the probability measure P of an event A represents the proportion with which A would occur if the process was repeated an infinite number of times (e.g., the probability of head or tail when throwing an ideal coin) [French et al., 2009]. In the *propensity* view, the probability is interpreted as a property of the system under scrutiny, e.g., the symmetry of an ideal coin results in equal probabilities for having a head" or a tail [Walley and Fine, 1982]. In the *subjectivist* or *Bayesian* view, the probability of A measures a person's degree of belief in A given the person's current knowledge and information [Morgan and Henrion, 1990].

Probabilistic techniques share the advantage that they fulfil some properties (or axioms) which have been designed to guarantee the *coherence of beliefs*. A violation of these axioms leads to inconsistency and exposes a person to the risk of a combination of bets that will lead to a sure loss (e.g., the "Dutch book" [Shafer, 1985]). This violation illustrates not that it is impossible to not follow the suggestions made by probability theory, but that applying them can protect decision makers from some kind of irrationality.

The classical theory of decision under risk combines the principle of mathematical expectation with the assumption of decreasing marginal utility, which jointly imply risk aversion [Von Neumann and Morgenstern, 1953]. Yet, a number of behavioural phenomena reflecting varying *risk attitudes* have challenged the validity of the classical theory [Tversky and Wakker, 1995]. Particularly, there is considerable evidence that preferences between risky prospects are not linear in the probabilities. The certainty effect, demonstrated by Allais, is the best-known example of this phenomenon [French et al., 2009]. Finally, risk preferences depend not only on the degree of uncertainty but also on its source. For instance, people sometimes prefer to bet on known rather than unknown probabilities [Tversky and Wakker, 1995].

2.2.2.2. Multi-Attribute Decision Support under Risk

While intra-criteria preference functions under certainty have been referred to as value functions (cf. Section 2.1.3), preference functions under risk are called

utility functions [Belton and Stewart, 2002]. To define utility functions, it is first required that the following preference axioms for any preference ordering $\prec$ (and the derived preference function) are fulfilled. Let P, Q and R be three random variables, on which $\prec$ is defined. It is required that the following properties hold:

$$P \prec Q \Rightarrow \lambda P + (1-\lambda)R \prec \lambda Q + (1-\lambda)R \ \forall 0 < \lambda \leq 1$$
$$P \prec Q \prec R \Rightarrow \exists \alpha, \beta \in (0,1): \ \alpha P + (1-\alpha)R \prec Q \prec \beta P + (1-\beta)R$$

That means, $\prec$ must be a weak ordering.

According to the von Neumann-Morgenstern utility theory [Von Neumann and Morgenstern, 1953], the above preference function axioms hold if and only if there exists a real-valued *utility function* u such that for all random variables P, Q in a given probability space Ω, all possible outcomes X and for $P \prec Q$

$$\int_{x \in X} P(x) \cdot u(x) \ \mathrm{dx} \leq \int_{x \in X} Q(x) \cdot u(x) \ \mathrm{dx}.$$

In this case, $\int_{x \in X} P(x)u(x) \ \mathrm{dx}$ and $\int_{x \in X} Q(x)u(x) \ \mathrm{dx}$ are the *expected utilities* of P and Q respectively (in contrast to the expected value, which is $\int_{x \in X} P(x)x \ \mathrm{dx}$ and $\int_{x \in X} Q(x)x \ \mathrm{dx}$). Moreover, the utility function u is unique up to a positive linear transformation [French, 1986]. If the set of consequences X is discrete, above equation is equivalent to

$$\sum_{x \in X} P(x) \cdot u(x) \leq \sum_{x \in X} Q(x) \cdot u(x),$$

i.e., the expected utility of a decision is calculated as the sum of utilities of its consequences weighted by their probabilities.

Expected utility theory states that a decision can be made based on the valuation of possible consequences of implementing an alternative $a_k \in A$, $x_i \in X(a_k)$, by the function u: if an appropriate utility $u(x_i)$ is assigned to the possible consequences of a_k and the expected utility

$$E\big(u(a_k)\big) = \int_{x_i \in X(a_k)} P(x_i)\, u(x_i) \,\mathrm{dx}$$

of each alternative a_k is calculated, then it is rational to choose the alternative a_k with the highest expected utility [Fishburn, 1968; Friedman and Savage, 1952; Savage, 1972].

In Multi-Attribute Utility Theory (MAUT), $x_k = x_{kj}$ $(j = 1, \dots, n)$ is considered as a function of uncertain random factors with known density function $P(x_k)$. The utility function u serves as a characterisation of the decision makers' inclination towards risk [Pratt, 1964]. Keeney [1971] provided a theoretical framework and a set of assumptions on the preferences that allow for utility functions to be decomposed. This decomposition facilitates the preference elicitation and the aggregation of results [Fishburn, 1968]. Particularly, *preferential* and *utility independence* are required.

Preferential independence implies that the preference order for l performances $x_{1j}, \dots, x_{lj}$ of a given attribute j (for alternatives $a_1, \dots, a_l$) does *not* depend on the performances $x_{1k}, \dots, x_{lk}$ of any other attribute $k \neq j$. Preferential independence concerns only preferences for consequences and neglects probabilistic considerations. The latter are also referred to as *lotteries* because frequently, the model of lotteries is used to elicit utilities [Fishburn, 1968].

Utility independence implies that the decision makers' risk attitude for a given subset of attributes does not depend on the performances (and the probabilities of these performances) of the other attributes. For instance, if an attribute j is utility independent of all other attributes, the preferences for various scores of j, $x_{1j}, \dots, x_{lj}$ (and lotteries over these scores) keeping all other attributes' scores constant do not depend on the particular scores of the other attributes [Ananda and Herath, 2005]. Additionally, it is required that this independence does not only hold for single attributes, but also for sets of attributes. Utility independence is not a reflexive relation: if j is utility independent from k the reverse does not necessarily follow [Keeney and Raiffa, 1976]. Furthermore, utility independence implies preferential independence, but the opposite is not true [Ananda and Herath, 2005]. Approaches to test if a set of attributes Att fulfils the utility independence conditions have been described by Keeney and Raiffa [1976].

If each attribute $j \in Att$ is independent of all attributes $k \in Att, k \neq j$, the utility function can be represented as an additive or a multiplicative function [Belton and Stewart, 2002; Fry et al., 1996; Keeney, 1977]:

$$u(x_k) = u(x_{k1}, \dots, x_{kn}) = \sum_{j=1}^{n} w_j u_j(x_{kj}),$$

$$1 + W u(x_k) = 1 + W u(x_{k1}, \dots, x_{kn}) = \prod_{j=1}^{n} \left(1 + W w_j u_j(x_{kj})\right),$$

where u_j is the utility function on attribute j's scores with values in $[0,1]$ and $0 \leq w_j \leq 1$. If $\sum_{j=1}^{N} w_j \neq 1$, a scaling constant $W > -1$ must be determined [Belton and Stewart, 2002].

MAUT has been criticised for being based on unrealistic assumptions. The believes and preferences of the decision makers have been considered too complex to be represented by the (quantitative) concepts of utility theory [French, 1986; Gass, 2005]. MAUT requires the decision makers to give judgements about preferences among imaginary bets. Therefore, Edwards [1977] argued that untutored decision makers may either reject the whole process or accept answers suggested by the sequence of questions rather than by their own preferences. To elicit utility functions nevertheless, several preference elicitation techniques have been developed [Edwards, 1977; Ananda and Herath, 2005].

Another problem of using MAUT is the fact that the deduction of adequate probability distributions can be problematic, particularly if not only aleatory but also epistemic uncertainties are present [Belton and Stewart, 2002; Chaib-draa, 2002; Jakeman et al., 2010]. In the context of strategic decision-making it can be argued that a (purely) probabilistic conceptualisation of uncertainty may not do justice to the problems involved [Hansson, 1996]. First, there are situations in which not all (relevant) possible outcomes of an alternative are known. Second, frequentistic probabilities trace uncertainty back to frequencies and *repetitions.* In rare events or situations characterised by their uniqueness, it is doubtful whether recourse to the frequency of phenomena in similar situations offers much insight into the uncertainties involved [Lempert et al., 2006; March and Shapira, 1987; Sigel et al., 2010; Wright and Goodwin, 2009]. Bayesian approaches are prone to similar problems. Many studies have suggested that intuitive and unguided decision-making is subject to many *inconsistencies and biases.* For instance, the concept of decision frames has been introduced to take into account the norms, habits and personal characteristics of the decision makers as well as the formulation of the problem, which both control the framing [Tversky, 1981]. It has been shown that the evaluation of sets of probabilities and results can produce significant shifts of preference if the same problem is framed in different ways [Küberger, 1998]. It has also been argued that decision makers do not behave in the same way under risky gains and losses [Fishburn, 1984; March, 1988].

2.2.2.3. Fuzzy Logic and Fuzzy Inference: Principles

The applicability of probability theory in decision making relies inter alia on the availability of well-defined sets of events. To deal with information for which there is no sharp transition from membership to non-membership *fuzzy sets* have been introduced [Zadeh, 1975]. The fields of application for which fuzzy decision support is most useful are characterised by imprecisions and lacking knowledge. Using fuzzy sets, vague and imprecise information (such as "*much larger than ...*" in contrast to "*more than* a certain quantity *larger*" or "*strongly influencing*" in contrast to an exact quantification of influence) can be modelled.

A *fuzzy subset* $\tilde{A}$ of a set X is defined as a non-empty subset

$$\tilde{A} := \{(x, \mu_A(x)) : x \in X\} \subseteq X \times [0,1]$$

for a *membership function* $\mu_A : X \to [0,1]$ [Zadeh, 1975]. The common practice of referring to fuzzy subsets as fuzzy sets is adopted from now on.

The following basic definitions facilitate describing and handling fuzzy sets. For illustrations of all concepts, see Figure 2.2. The α-level set or *α-cut* of a fuzzy set $\tilde{A}$ on Ω is defined as [Williams and Steele, 2002]

$$A_\alpha = \{x \in X : \mu_A(\omega) \geq \alpha\} \text{ for } \alpha \in (0,1].$$

Since α-cuts are intervals, their use enables working with interval arithmetic rather than actual membership functions μ_A. Figure 2.2 shows an $\alpha = 0.6$-cut.

The *support* of a fuzzy set $\tilde{A}$ is the area where the elements of X are said to have some degree of membership to $\tilde{A}$, cf. Figure 2.2. The support of a $\tilde{A}$ is defined as

$$\text{supp}\left(\tilde{A}\right) = \bigcup_{\alpha > 0} \tilde{A}_\alpha = \{x \in X : \mu_A x) > 0\}.$$

The *kernel* of a fuzzy set $\tilde{A}$ is the area of maximum membership [Bellman and Zadeh, 1970]:

$$\ker\left(\tilde{A}\right) = \mathbb{1}_1\left(\mu_A(x)\right) = \{\omega \in \Omega : \mu_A(\omega) = 1\}.$$

In most decision-making domains a mix of imprecise numeric information, upon which linguistic variables are defined, and purely linguistic variables co-exist. An example of the first type of information is cost, which can be quali-

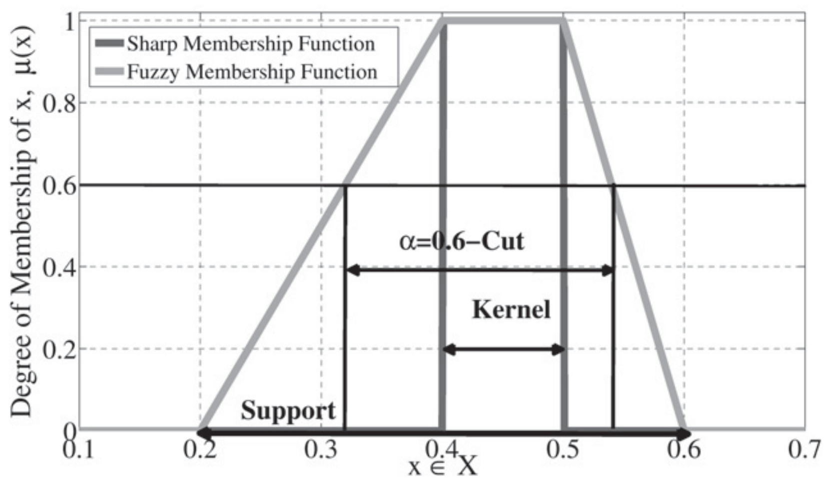


Figure 2.2.: *Trapezoidal Fuzzy Set. Example representing* "approximately between a and b" *compared with a sharp set* $[a, b]$*. The set's kernel, its support and an* α*-cut are shown.*

fied by terms like "expensive" or "cheap", while terms such as "beneficial" or "detrimental", for which there is no formal measurement scale, belong to the second type of information. According to Zadeh [1975], the theory of fuzzy sets facilitates constructing a conceptual framework for a systematic treatment of vagueness and imprecision in human reasoning in both qualitative and quantitative ways.

There are two particular forms of fuzzy sets that facilitate the elicitation of fuzzy sets: triangular and trapezoidal fuzzy sets [Williams and Steele, 2002]. A trapezoidal fuzzy set $\tilde{A}$ is characterised uniquely by an ordered quadruple of real numbers $\{a_1, a_2, a_3, a_4\}$ with $a_1 \leq a_2 \leq a_3 \leq a4$ such that $\text{supp}(\tilde{A}) = [a_1, a_4]$ and $\ker(\tilde{A}) = [a_2, a_3]$, see Figure 2.2.

2.2.2.4. Fuzzy Multi-Attribute Decision Support

Using fuzzy sets and variables it becomes possible to quantify imprecise goals, constraints or preferences. Accordingly, various fuzzy MADM techniques to account for imprecisions at several phases of the decision-making process have been developed.

In real-world decision problems, assessing the *attributes' scores* x_{kj} requires a number of expert judgements [Mendonça et al., 2007]. Often, it is (at least partly) impossible to capture these judgements with crisp sets or to model their likelihood with probabilistic techniques [David and Rongda, 1991]. When the assessments made are (partly) qualitative and vague, membership functions modelling the performance of a_k in an attribute j can be developed [Yager, 1977]. This is particularly useful when the assessments x_{kj} are made by a group of experts who have to come to a consensus [Chou et al., 2008].

Furthermore, the *inter-criteria weights* w_j^{al} expressing the importance attached to each specific attribute or criterion j (cf. Section 2.1.3) can be modelled by linguistic expressions [Chen and Klein, 1997; Chou et al., 2008], as they represent a degree of importance, from the most desirable features to the least ones [Ribeiro, 1996]. If the attribute scores x_{kj} are real numbers the fuzzy weights $\tilde{w}_j^{al}$ are defuzzified[4] to scalars $w_j^{al} \in [0,1]$ for the aggregation and ranking of results [Chou et al., 2008].

In case both weights and scores are expressed as fuzzy numbers, the corresponding fuzzy operators $\oplus$ and $\otimes$ can be used for a weighted additive aggregation [Buckley, 1985]. Alternatively, more general aggregation methods can be applied: as initially suggested by Bellman and Zadeh [1970], the degree of achieving attribute j, $j = 1, \ldots, n$ can be represented by fuzzy sets $\tilde{X}_j$ where $\tilde{X}_{kj}$ indicates the degree to which alternative a_k succeeds in j. Then, an overall decision function $\tilde{D}$ combining the results and the preferences of the decision makers must be identified. In terms of fuzzy logic, that is:

$$\tilde{D} = F\left(\tilde{X}_1, \ldots, \tilde{X}_n\right),$$

where $\tilde{D}$ is a fuzzy subset of $\tilde{X} = \left(\tilde{X}_1, \ldots, \tilde{X}_j\right)$ and F is a fuzzy objective function. Using this function, the optimal alternative a^* is such that [Bellman and Zadeh, 1970]

$$\tilde{D}\left(a^*\right) = \max_{a \in A} \left\{\tilde{D}(a)\right\}.$$

The form of the objective function F depends on the decision makers' preferences. Usually, decision makers seek to optimise the results with respect to all criteria [Clemen and Reilly, 1999]. Therefore, the logical connective *"and"*, which in the realm of fuzzy logic can be represented by strict triangular (or t-)norm operators [Dubois and Prade, 1985], is chosen for an appropriate formulation of F:

$$\tilde{D} = \tilde{C}_1 \cap \ldots \cap \tilde{C}_n \qquad \text{and} \qquad \tilde{D}(a) = \tilde{C}_1(a) \top \ldots \top \tilde{C}_n,$$

[4] A comparison of defuzzifying methods has been provided by Chen and Klein [1997].

where $\top$ is any t-norm operator [Yager, 1991]. In the simplest form, this corresponds to [Bellman and Zadeh, 1970; Yager, 1978]

$$\tilde{D}_{\top=\min} = \bigcap_{j=1}^{n} \tilde{C}_j \qquad \tilde{D}_{\top=\min}(a) = \min_j \left\{\tilde{C}_j(a)\right\} \qquad \text{or}$$

$$\tilde{D}_{\top=\prod} = \prod_{j=1}^{n} \tilde{C}_j \qquad \tilde{D}_{\top=\prod}(a) = \prod_{j=1}^{n} \tilde{C}_j(a) \quad .$$

To take into account relative importances among the attributes, it has been suggested to use crisp weights w_j and to adapt the decision function $\tilde{D}$ such that [Yager, 1977]:

$$\tilde{D}(a) = \top \left(\tilde{C}_1(a)\right)^{w_1} \top \ldots \top \left(\tilde{C}_n(a)\right)^{w_n} ,$$

where again $\top$ denotes any t-norm operator.

The application of fuzzy techniques in decision support has been criticised for several reasons. First, it has been argued that fuzzy sets cannot avoid the problem of requiring (precise) judgemental input by the use of a set of membership functions, because this must be defined precisely, too [French, 1984]. Furthermore, it has been doubted whether it is useful to integrate imprecision and ambiguity into a (normative) decision support system [French, 1984, 1995]. While the use of fuzziness for descriptive models of ambiguity and imprecision (e.g., those present in a third party's statements) is uncontested, the fact that fuzzy models for decision-making lack normative underpinning hampers their acceptance as tools for decision support systems [French et al., 2009].

2.3. Scenarios for Reasoning under Severe Uncertainty

The dilemma for decision makers confronted with complex situations where uncertainties are profound is the following: the more complex the problem is, the more the need for systematic and formalised support (computational and analytic tools, data, and statistics) increases. The use of formal quantitative approaches to handle uncertainty as introduced in the Section 2.2 forces the decision makers (or involved experts and analysts) to make a series of assumptions on the likelihood of the information they produce [Lempert et al., 2002]. A possibility to deal with fundamental and non-quantifiable uncertainties is the use of scenarios [Bunn and Salo, 1993].

2.3.1. Typologies of Scenario-Based Techniques

Originally, scenarios were developed as a sense-making tool for challenging tunnel vision [Roubelat, 2000; Schoemaker, 1995]. In that sense, scenarios aim at shifting paradigms and overcoming the tendency to extrapolate past or present trends into the future without considering structural shifts or discontinuities [Jetter and Schweinfort, 2011]. Generally, a scenario describes a situation and a plausible future development [Pomerol, 1998; Di Domenica et al., 2007].

Along with the increasing pace of change in economy and science, scenarios as a means to forecast and plan have become more and more popular [Bradfield et al., 2005; Huss, 1988; Jetter and Schweinfort, 2011; Mason and Herman, 2003; Varum and Melo, 2010]. Over the time, a variety of terms, such as "*planning*", "*thinking*", "*analysis*", and "*building*" have become commonly attached to the word "scenario" in literature [Varum and Melo, 2010]. Along with the spread in terminology, literature reveals a large number of different and sometimes conflicting definitions, characteristics, principles, and methodological ideas about scenarios [Bradfield et al., 2005]. Several scenario typologies have been developed to handle this diversity. A scenario typology proposed by van Notten et al. [2003] characterises scenarios according to the key features *goal, process design,* and *content.*

Goal: exploration of possible futures vs. targeted decision support. *Exploration* covers raising situation awareness, learning, and the stimulation of creative thinking [Schwartz, 1991; van der Heijden, 2007]. Explorative scenarios are elaborated to explicitly take into account profound and structural changes. Often, explorative methods focus on the organisational learning process while the quality and reliability of the resulting scenarios is less important [Bradfield et al., 2005]. One can furthermore distinguish *external scenarios* focusing exclusively on variables beyond the control of the decision makers and *strategic scenarios* describing a range of possible consequences of a decision [Börjeson et al., 2006]. Strategic scenarios take into account internal and external factors. In *decision support* scenarios are used as a means to examine paths to futures that vary with respect to their desirability [van Notten, 2006]. Normative scenarios allow for expressing preferences for particular paths and making decisions accordingly (preferable futures). Normative *preserving scenarios* aim at finding out how a certain target can be efficiently met (e.g., with some kind of optimisation). In *transforming scenarios* the starting point is a target that cannot be reached if current trends continue, and a trend break is necessary [Börjeson et al., 2006].

This approach typically results in a number of target-fulfilling images of the future development showing what changes are needed to reach the targets.

Process design: model-based or analytical vs. intuitive methods. This category distinguishes different methodological aspects of the scenario construction. Model-based or *analytical* approaches involving the quantification of identified uncertainties were among the earliest methods for scenario development [van Notten, 2006]. These approaches comprise computer simulations [Edwards, 1996; Morgan and Henrion, 1990] and scenario construction through research and literature analysis [Kuhlmann, 2001]. In contrast, *intuitive* approaches rely on knowledge and insights of experts. Creative techniques such as the construction of narrative scenarios in workshops are an example for the intuitive approach [Schwartz, 1991; van der Heijden, 2007].

Content: complex vs. simple scenarios. The content of the scenarios can be *complex,* or *simple* and limited in scope, e.g., focusing on a niche aspect of the situation.

This typology classifies the scenario planning process in general, as it comprises the purpose of the scenario building as well as the methods and tools used. Yet, due to its universal applicability, this typology lacks precision in the description of the specific scenario techniques used. Another scenario typology developed by Bishop et al. [2007] focuses on the specifically on the **techniques** used for scenario construction and analysis. Table 2.1 summarises the main features of the scenario variants that have been identified.

2.3.2. Problem Structuring Techniques: Networks for Reasoning under Uncertainty

As scenarios need to take into account interdependencies between different aspects of the situation, this section reviews a number of problem structuring techniques. In the field of Artificial Intelligence (AI) most problem structuring techniques rely on graph theoretical considerations [French et al., 2009]. One of the best-known AI technologies are *experts systems.* These are computer-based systems that assimilate the knowledge (via a *knowledge base)* and reasoning processes (via an *inference engine)* used by (human) experts to solve equal or similar decision problems [Turban and Aronson, 1997; Turban and Watkins, 1986]. Although the approach to scenario construction of this thesis does *not* standardise

Table 2.1.: *Techniques for Scenario Construction and Analysis According to Bishop et al. [2007]*

Type	**Characterisation**	**Examples and literature**
Judgemental Technique	Scenario construction relies on judgements of experts and stakeholders, little formalised	[Schnaars, 1987]
Baseline Scenario Technique	Construction of *one* scenario by analysing prevailing trends that are extrapolated into the future	Manoa [Curry and Schultz, 2009]
Elaboration of Fixed Scenarios	Scenario elaboration: detailing and shaping a set of predetermined basic scenarios	Incasting [Bonnett and Olson, 1993], SRI matrix [Huss and Honton, 1987]
Event Sequences	Definition of scenarios as chains of events with associated probabilities	Probability trees [Covaliu and Oliver, 1995; Heitsch and Römisch, 2009; Høyland and Wallace, 2001]
Backcasting	Definition of an envisioned future, investigation of paths resulting in the desired end state	Horizon Mission Methodology [Anderson, 1996; Höjer, 1998], Future mapping [Mason and Herman, 2003]
Dimensions of uncertainty	Scenario construction on basis of the most important sources of uncertainty	Morphological field analysis [Godet and Roubelat, 1996; Ritchey, 2006]
Cross Impact Analysis	Descriptions of plausible futures combined with a quantification of likelihood	SMIC [Duperrin and Godet, 1975; Godet, 2000]
Modelling techniques	Quantification of interdependencies between the most relevant variables, partly used to calculate the value of an objective function	Trend Impact Analysis [Gordon, 1994], Sensitivity Analysis [Campolongo et al., 2007; Saltelli et al., 2008], Dynamic Scenarios [Ward and Schriefer, 1997; Winch, 1999]

the reasoning by prescribing certain inference mechanisms, it relies on graph theoretical concepts to represent a scenario's structure.

2.3.2.1. Causal Relations and Interdependencies

A number of problem structuring techniques explicitly modelling the relations between different variables have been developed. A common feature of these approaches is the representation of the problem as a graph $\mathcal{G} = \langle STV, E \rangle$, where each variable $tv_j \in STV$ is depicted as a vertex, whereas the relations between the variables, which can be of statistical or causal nature, are represented as edges $e_i \in E$.

The nature of relations represented by E is a means to represent the impact of a variable's value on the state of any other variable within the network. In general, two events A and B are called *statistically related* if the probability of their joint occurrence $P(A \cap B)$ is *not* equal to the product of their individual probabilities $P(A) \cdot P(B)$ [Sayre, 1977]. Thus, if A is statistically related to B, then B is equally related to A, i.e., statistical relations are symmetric.

In decision-making it is, however, important to display asymmetric cause-effect relations, as the decision makers need to know what *impact* the manipulation of certain factors has [Krynski and Tenenbaum, 2007]. That is why displaying the relations between causes or means and effects or ends as a network of *directed edges* supports decision makers in identifying and evaluating alternatives [Eden, 2004].

A number of different network techniques corresponding to different principles of reasoning under uncertainty has been developed. These are discussed in the following.

2.3.2.2. Bayesian Networks

Bayesian Networks describe interactions between variables in terms of conditional probability distributions [Pearl, 2009]. Basically, a Bayesian Network is a directed acyclic graph $\mathcal{G} = \langle STV, E \rangle$ together with an associated set of probability tables. The variables in STV represent uncertain variables and the edges in E represent the causal or relevance relations between them [Fenton et al., 2004]. There is a probability table for each variable providing the probabilities of each value the variable can take. For variables without predecessors in the graph, the table contains the marginal probabilities, while for variables with predecessors it specifies conditional probabilities for each combination of values of the predecessor variables [Pearl, 2009].

Although in some cases good calibration in judgemental probability assessments have been demonstrated [Murphy and Medin, 1985], often eliciting probabilities can be problematic. It has been argued that the reason for high-quality probability judgements lies in the availability of sufficiently rich and accurate data or models and in the existence of a rapid feedback loop between prediction and outcome [Bolger and Wright, 1994]. In strategic decision-making, there is usually no self contained model or expert system covering the necessary domain knowledge for all eventualities [Dugdale, 1996]. Furthermore, the quality of the available data is usually heterogeneous [Pender, 2001]. Additionally, the time until the impact of a decision can be observed is rather long, and the complex interdependencies between all relevant factors make it difficult to attribute a consequence clearly to the decision made. Thus, the aforementioned conditions for accurate probability judgements are violated. Lastly, there the elicitation of probabilities can be prone to cognitive biases: although there may be little knowledge to develop a probability judgement, the experts may be overconfident at the same time [Fischhoff, 1975]. Furthermore, the arising results can be counter-intuitive and hard to understand; a problem that is common for all probabilistic techniques [Ben-Haim, 2004; Tversky and Kahneman, 1974].

2.3.2.3. Causal Maps and Fuzzy Cognitive Maps

Causal Maps are a discursive problem structuring technique representing interlinked variables in a network of causes and their effects. Causal Maps can be used in situations when variables can only be valued qualitatively, e.g., by labels indicating the direction of influence. Causal Maps capture the experts' world views and can be elicited either through interviews that are later transcribed into Causal Maps [Fiol and Huff, 1992; Warren, 1995] or through self-guided mapping approaches, such as Bougon's Self-Q-Technique [Ambrosini and Bowman, 2001; Bougon, 1983] or the Group Support Systems introduced by Sheetz et al. [1994]. A strictly qualitative analysis, however, has drawbacks. Particularly, Causal Maps can suffer from indeterminacy when a variable is influenced by an equal number of reinforcing and depleting edges [Axelrod, 1976]. Furthermore, it is difficult to make sense of large and complex maps [Jetter and Schweinfort, 2011].

If more information about the type and strength of influence is available, this can be modelled by a more fine graduation of labels, e.g., varying from strong positive to strong adverse influence [Montibeller and Belton, 2006], or by fuzzy membership functions, which lead to *Fuzzy Cognitive Maps* [Peña et al., 2008]. By using fuzzy inference mechanisms [Jetter and Schweinfort, 2011], the latter allow for modelling not only the state of a system at a given time, but also the system's behaviour over a specific period [Kosko, 1986].

For Causal Maps, the influence that the variables exert on each other can be modelled by causal inference mechanisms [Montibeller and Belton, 2006, 2009] defining the impact of a variable's value on a goal variable for each path through the network. Montibeller and Belton [2006] discusses several operators to calculate these impacts.

Bayesian Networks and Causal Maps are represented by acyclic graphs. Contrarily, Fuzzy Cognitive Maps contain feedback loops. Temporal aspects play a crucial role in Fuzzy Cognitive Maps: given a set of initial values at a time t_0 for each vertex within the map (captured in the state vector $sv(t_0)$), the development of the system is assessed by combining an incidence matrix W weighted with the causal relations' strengths (W_{ij} in $[-1, 1]$ assigned to each edge e_{ij}) and a fuzzy transformation F [Kosko, 1986]. The system's state at a time $t_0 + 1$ is then derived from its state at t_0 by setting $sv(t_0 + 1) = F\big(sv(t_0) \cdot W\big)$ [Peña et al., 2008; Yu and Tzeng, 2006]. The state of the system in further time steps $t_0 + i$, $i \in \mathbb{N}$, can be assessed by iteratively applying this procedure.

2.3.2.4. Applicability of Network Techniques for Scenario-Building

Scenarios are descriptions of a situation and how it may unfold into the future. They should be collaboratively built by people with different expertises and backgrounds who are likely to have different mental models and can challenge each others' world views [Goodier et al., 2010; Roubelat, 2000, 2006]. Methods for building scenarios cover soft processes of discussion, formal forecasting techniques and sophisticated computer-based modelling [Brauers and Weber, 1988; Breitman et al., 2005; Helbing and Kühnert, 2003; Tapio, 2003].

One relatively simple technique to help groups build scenarios is based on Causal Maps. These maps have long been used as a means to elicit the world views of multiple experts, facilitate discussion and challenge and improve mental models [Goodier et al., 2010]. Usually, Causal Maps are constructed by interviewing individuals or discussing with teams of experts and decision makers important change drivers and possible consequences of a change of these drivers [Eden, 2004; Warren, 1995]. Yet, large and complex Causal Maps are difficult to understand and analyse [Jetter and Schweinfort, 2011]. Furthermore, the underlying cause-effect relations remain qualitative [Montibeller et al., 2006; Montibeller and Belton, 2009; Ram et al., 2011], and the elicitation process requires time and availability of all experts involved [Belton and Stewart, 2002; Eden, 1992a, 2004]. This is clearly infeasible when the experts contributing to the scenario construction are geographically dispersed and time is bounded. Moreover, although the inherent informality of discursive scenarios facilitates their use with little support, their systematic construction and analysis is made difficult [Breitman et al., 2005].

Other approaches rely on standardised reasoning mechanisms. They elicit expert-knowledge a priori to build scenarios. State-charts [Glinz, 1995] describe complex discrete-event systems on basis of dependencies and transitions and include hierarchy, concurrence and communication between single elements [Harel, 1987]. Interaction networks [Helbing and Kühnert, 2003] and Fuzzy Cognitive Maps [Espinosa-Paredes et al., 2008; Jetter and Schweinfort, 2011] integrate feedback loops and indirect effects into the scenario building process. Other approaches use probabilistic DAGs for the representation of scenarios; these include risk graphs combining fault and event trees [Braendeland et al., 2010] and Bayesian Networks [Nadkarni and Shenoy, 2001]. Each of these approaches requires that the mechanisms for assessing the consequences of different scenarios can be standardised and elicited a priori. This is problematic in a highly dynamic environment or in case of rare events [Pavlin et al., 2009b; Wright and Goodwin, 2009].

In summary, Bayesian Networks, Fuzzy Cognitive Maps and Causal Maps assess the impact of a decision alternative a_i by one paradigm for handling uncertainty and the respective inference mechanisms. In large and complex problems, however, typically information of diverse qualities coexist [Dohnal, 1992; Peña et al., 2008]. For some information, statistics or accurate expert judgements may be available allowing for the deduction of probability distributions and the construction of Bayesian Networks. For other variables there may be vague and imprecise specifications which can be represented by fuzzy logic. Yet in other cases, information may be more sparse or even completely lacking. None of the systems presented allows for handling these diverse types of uncertainty at a time. Yet, to give a clear and transparent picture of the consequences of a decision, it is crucial to handle systematically different types and qualities of information without oversimplifying the situation description.

2.4. Summary and Discussion

This chapter provided the background for the research of this thesis. Firstly, a number of MCDA methods were described. Secondly, techniques for decision support under uncertainty, with a focus on probabilistic, fuzzy and scenario-based techniques, their respective requirements and limitations were discussed.

Due to its acceptance in the field of strategic decision-making, *Multi-Attribute Value Theory* (MAVT) is the MCDA method applied in this thesis. It facilitates the consideration of trade-offs and the decision makers' preferences. Various methods to provide support during the problem structuring phase have been developed [Fry et al., 1996; Keeney et al., 1979; von Winterfeldt and Edwards, 1986]. Still, the problem structuring takes time and requires some effort from the decision makers. The approach presented in this thesis is suitable for two types of situations. Firstly, it can be used for situations which allow for some degree of standardisation enabling the use of template attribute trees that are defined a priori by the decision makers (potentially) involved. This approach is applicable when the goals remain essentially unaffected for a certain class of decision problems. Secondly, this approach can be used to support strategic decision-making when there is enough time to elicit the attribute tree and the preferences.

To handle the uncertainties present, a number of techniques providing decision support were explored. *Probabilistic* and *fuzzy techniques* differ greatly in philosophy and assumptions. Which method is more appropriate depends on which set of assumptions seems most valid for a given situation, the available information and the expert assessing it. Each of these techniques handles uncertainty and evaluates alternatives by a single reasoning principle and the respective inference mechanisms. In large and complex problems, however, information of diverse qualities typically coexist [Dohnal, 1992; Pender, 2001]: for some information, rich statistics or accurate expert judgements may be available allowing for the deduction of probability distributions. For other variables there may be vague and imprecise (fuzzy) specifications. Yet in other cases, information on the likelihood of an event may even be lacking. Furthermore, as the situation develops, additional knowledge is acquired changing the type and quality of information available and offering new possibilities to (re-)assess and evaluate alternatives [Pender, 2001].

Scenario-Based Reasoning (SBR) offers the possibility to consider and discuss several possible situation developments regardless of their likelihood. Scenarios are a tool to help decision makers recognize, consider and reflect on the uncertainties they face. Ideally, scenarios support decision makers in making better sense of changes in their environment, spotting early warning signals and refining perceptions of existing or emerging problems and corresponding problem solving strategies [Volkery and Ribeiro, 2009]. Moreover, scenarios can facilitate conflict management between diverging preferences and value judgements by helping finding common ground for future action. While there are different approaches for constructing and using scenarios, this thesis focuses on the systematic aspects of scenario planning and analysis: a scenario describes the current state of a system and its future development by means of a set of interlinked impact variables. This is further detailed in Chapter 3.

Decision makers have used both scenarios and MAVT for evaluating alternatives for a long time, but due to their complexity their use of both approaches has been limited to long-term decision-making tasks [Jarke et al., 1998]. Scenario construction processes in strategic decision-making usually draw on multidisciplinary knowledge bases incorporating different disciplines such as natural, physical and social sciences, medicine, politics and ethics [Kiker et al., 2005]. Accordingly, experts from various disciplines need to contribute to the construction of reliable scenarios, and the need for a systematic approach to scenario construction arises (cf. objective O.1). Even if the scenarios are constructed in a distributed manner, each expert needs some time to assess the particular part of the scenario he is responsible for. Therefore, the system this thesis proposes is not designed for ad-hoc decisions, but for strategic decision-making in complex situations.

Despite of the advantages of using scenarios, the generation of a multitude of scenarios increases the amount of information that the experts and decision makers need to process and take into account. The use of scenarios can even exacerbate the decision makers' problem, when scenarios are not accompanied by further guidance and/or analysis tools. Existing scenario planning tools are, however, not suitable for assessing scenario quality (cf. objective O.3) and do not fully support evaluating scenarios through a comparison process [Durbach and Stewart, 2003; Ahmed et al., 2010]. Although recently a few approaches evaluating scenarios with respect to multiple goals have been developed Diakoulaki and Karangelis [2007]; Hites et al. [2006]; Montibeller and Belton [2006], none of these approaches *systematically integrates* scenario generation and the evaluation of alternatives. This thesis shows that in the novel integrated SBR & MCDA framework, MCDA provides a rationale for constructing **decision-relevant scenarios.** In this manner, the problem structuring approach of MCDA is used as a means to structure and manage information processing. This enables reducing information overload of the experts involved in the scenario construction process and the decision makers, to whom the final results are presented (cf. Section 5.1). Additionally, this thesis introduces an evaluation process for comparison of instances of homogeneous and heterogeneous scenarios that enables the recipients to identify the most suitable and plausible scenario for the problem at hand (see Chapter 8).

This thesis explores the possibilities of an integrated SBR & MCDA approach for the evaluation of decision alternatives (cf. Section 5.3). This approach integrates explicitly the decision makers' risk aversion and allows for balancing the risks and chances an alternative offers according to their preferences. In this manner the **robustness** of a decision gains in importance: rather than selecting the best evaluated alternative for one particular scenario, the integrated SBR & MCDA approach supports the choice of alternatives that perform sufficiently well for a variety of scenarios.[5]

[5] Note that this agrees with the definition of robustness used by Matos [2007] and Vincke [1999], where an alternative is termed *robust* if a minimum required threshold performance for a set of criteria is reached for all considered cases. In an SBR framework, the "cases" correspond to the scenarios considered.

3. Formalisation of Scenario-Based Reasoning: A Graph-Theoretical Approach

> Everything is vague to a degree you do not realise
> until you have tried to make it precise.
> (Bertrand Russell)

This chapter is dedicated to a formalisation of the scenario concept. The focus is directed on requirements each *single scenario* must fulfil to ensure first that the scenario construction in distributed settings is feasible (structural requirements), and second that the scenarios generated are acceptable for their recipients (requirements with regard to content). Furthermore, requirements to ensure that the *set of scenarios* is a valid basis for the sense- or decision-making are established.

For single scenarios, the structural requirements are designed in such a way that relevant information is collected, combined and processed into meaningful scenarios, i.e., they ensure the achievement of *objective O.1.* With regard to content, each single scenario must offer a plausible explanation based on causal logic of how the situation unfolds from the past or present to the future [Wright and Goodwin, 2009]. Therefore, important requirements for the scenario's acceptability are its internal correctness, plausibility, consistency and coherence. The requirements for sets of scenarios are imposed to ensure the comparability of results by requiring inter alia that all scenarios considered are based on the same initial information and must have a certain coherence to enable sound decision-making. The combination of requirements for both single scenarios and sets of scenarios facilitate the operationalisation of *objectives O.2* and *O.3* (construction of purposeful and acceptable scenarios).

This chapter first specifies the principles of Scenario-Based Reasoning (SBR) applied throughout this thesis. Second, single scenarios are defined formally using graph theoretical concepts. The developed framework allows the (internal) qualities of a scenario to be defined and in this manner facilitates the analysis of scenario correctness, plausibility, coherence and consistency. Third, sets of scenarios as well as relationships between individual scenarios and sets of scenarios are defined such that the stage is set to capture scenario construction processes, which include *scenario continuation* and the newly developed method of *scenario merging* facilitating distributed scenario construction.

3.1. Principles of Scenario-Based Reasoning

In Section 2.3.1 it was shown that there is a multitude of approaches for generating, analysing and exploiting scenarios. To distinguish the approach developed within this thesis from other scenario-based approaches, it is referred to as **Scenario-Based Reasoning (SBR)**. This section briefly outlines the main features of scenarios and SBR as applied throughout this thesis and highlights some requirements for the usability of scenarios as a basis for decision support.

In essence this thesis understands scenarios as well-structured, dynamic stories that capture key uncertainties about a system's future. Scenarios do not only capture expected or most likely futures (such as baseline scenario techniques assume, see Section 2.3.1), nor do they necessarily quantify the likelihood of any scenario (such as done in Cross Impact Analyses or in the scenarios derived from modelling techniques, see Section 2.3.1). Rather, this thesis uses scenarios as a means to challenge the imagination of decision makers; essentially, scenarios aim at overturning existing mental models and regimes of thought [Wack, 1985; Schnaars, 1987].

The SBR process should be *purposeful:* scenarios are introduced in environments of uncertainty where there is a *need for action*, *prioritisation*, or *making decisions* [Chermack, 2004]. SBR employs the use of imaginary future scenarios to help decision makers think about the main uncertainties they face and the drivers for change; scenarios reveal the implications of current trajectories and facilitate devising strategies to cope with uncertainty [Montibeller and Belton, 2006]. Each individual scenario itself is a purposeful story describing the situation and one possible development into the future. In this way, multiple scenarios offer the possibility to take into account several situation developments, which may be considered regardless of their concrete likelihood. Therefore, SBR is particularly suitable for reasoning in situations when the likelihood or probability of an event cannot be quantified, i.e., in situations of *severe uncertainty* [Ben-Haim, 2000], such as often encountered in (environmental) emergency management [Wright and Goodwin, 2009]. Two distinct purposes for a scenario can be identified.

Scenarios can be used for **raising situation awareness,** and for exploring the situation in general. The description of the situation and its future developments should provide a general overview and answer a set of important questions that facilitate real-time decision-making or the development of feasible alternatives or strategies in longer term decision-making that can, in a second step, be assessed using Scenario-Based Multi-Criteria Decision Analysis (see below). This use of scenarios is referred to as *Scenario-Based Sense-Making (SBR & SM).*

Scenarios can be tailored for a **decision problem.** The aim is assessing the consequences or implications of implementing certain alternatives. The scenarios cover not only factors that can be influenced by the decision makers, but a broad range of variables that include factors beyond the control of the decision makers. As this thesis focuses on multi-criteria decision problems and uses MCDA techniques to solve them, this application of scenarios is referred to as *Scenario-Based Multi-Criteria Decision Analysis (SBR & MCDA)* in the following.

As both sense- and decision-making in involve many parties having different views, objectives and responsibilities [Geldermann et al., 2009; Hämäläinen et al., 2000; Mustajoki et al., 2007], this thesis defines and distinguishes the following prototypical roles.

Scenario recipients are the actors, for which the scenarios are generated. The recipients specify the purpose of the scenario construction and the key variables on which they need information. In SBR & MCDA, the scenario recipients correspond to the decision makers.

Stakeholders share or perceive that they share the impact arising from the decision [Bertsch, 2008]. Therefore, they claim that their perceptions should be taken into account. In SBR & MCDA, their preferences and objectives are considered in the problem structuring phase. In SBR & SM stakeholders can participate in the definition of relevant variables enabling situation assessments (cf. Section 4.1.3).

Experts provide economic, engineering, scientific, environmental and other professional advice [Bertsch, 2008]. They assess the situation development and the consequences of the choice of an alternative with respect to their domain.

For both SBR & SM and SBR & MCDA the acceptance of scenarios is an important issues, as scenarios are designed explicitly to challenge the imagination and world views of the actors involved. Particularly when scenarios seem to be unlikely, some important requirements for a scenario's *acceptance* need to be guaranteed [Schnaars, 1987; Wright and Goodwin, 2009].

- *Plausibility:* not going beyond the realm of possibility.
- *Coherence:* making causal links explaining *why* a scenario arises explicit.
- *Consistency:* being unambiguous (no conflicts between the states of any set of variables in a scenario).

To ensure that these properties are fulfilled on a structural level, this thesis develops scenario construction methods that avoid time consuming ex post filtering procedures (cf. Section 5.1). To further substantiate the concept of acceptance, notions of *trust* and *credibility* have been developed. Five basic determinants for a scenario's credibility have been identified [Schoemaker, 1993; Selin, 2006]. These comprise the credibility of...

- *sources* is associated with the credibility of experts who contribute to the scenario construction,
- *content* refers to the strength and reliability of the data and information that make up the scenarios,
- the *scenario construction methodology,*
- the *narrative:* scenarios gain discursive power from their storied character and the use of compelling metaphors,
- the *channel* or dissemination referring to the range of distribution to the presenters of the scenarios and the context of the scenario presentation.

These issues need to be taken into account in the scenario construction process. In Section 5.2 it is shown, in which way and in how far the newly developed scenario construction process ensures that certain standards with respect to each of the credibility determinants are met.

3.2. Formalisation of Single Scenarios

Section 2.3 has shown that scenarios are powerful tools for preparing organisations for unexpected events, and for helping to overcome the inertia of conventional thinking. Furthermore, scenarios have some useful features for the evaluation of alternatives. Each scenario represents one element out of a complex, intermingled, dynamic and opaque set of possible developments. Scenarios are bounded, consisting of a limited number of states, events, actions and consequences. Finally, scenarios are coherent as their elements are causally related. On the whole, scenarios are generally perceived as transparent and easily understandable. Section 2.3.1 showed that the term *scenario* may refer to different concepts in different schools of thought. To describe scenarios, their properties and requirements for SBR a rigorous formal framework is developed in the following. This formalisation provides a flexible structure that clarifies the concepts used and allows tools and techniques from graph theory and AI to be applied. This enables specifying the requirements that a scenario must meet in a precise and strict way.

The four key concepts, which constitute each scenario, are briefly explained in the following. The *variables,* their *values* and *value operations* thereon provide the basic means to establish the **content** of a scenario: the description of a situation, its development and its consequences. To facilitate scenario construction and comparison, an approach to make variables unique and their possible values constrained is applied. Each value is further qualified via its *status,* which captures meta-information on the quality of information represented by the value. Particularly, the statuses allow for reflecting different types of uncertainty. In this manner, the achievement of objective O.1 will be ensured. In addition, a relation on the variables is defined. This relation specifies for each variable how its value depends on other variables' values. This *dependency relation* is later exploited in a number of ways, e.g., for determining interdependencies between scenarios (see Section 3.4), as a basis for the scenario generation (see Section 5.2) or to assess the duration and effort of scenario construction (see Section 8.3).

3.2.1. Typed Variables

The basic features for describing a situation and its development are captured by a *set of typed variables* STV, which contains all relevant variables that have an impact on the situation and its development. Here, it is assumed that STV is countable, i.e., $STV = \{tv_j\}_{j \in J}$, where $J \subseteq \mathbb{N}$. All variables used are assumed to be typed, i.e., each variable tv_j is defined by $tv_j = \langle Name(tv_j), Type(tv_j) \rangle$. The actual value of the variable is defined separately (see Section 3.2.2).

Name: The operator $Name(tv_j)$ refers to the identifier or name of a variable $tv_j \in STV$. Each variable's name is unique across all scenarios:

$$Name(tv_{j_i}) = Name(tv_{j_k}) \Leftrightarrow tv_{j_i} = tv_{j_k}.$$

Type: The type of a variable $tv_j \in STV$, denoted $Type^*(tv_j)$ refers to the class of values that may be assigned to the variable (e.g., real numbers, a function, plain text, xml files, RDF structures, .jpg images).

3.2.2. Values of Variables

The value $V(tv_j)$ of a variable $tv_j \in STV$ is one element within its range[6]. $Range(V(tv_j))$ is not constrained except for $Range(V(tv_j)) \subseteq Type^*(tv_j)$, the co-domain of $V(tv_j, x)$, [Rudin, 1986]. The operator

$$\begin{array}{rccl} V: & X & \to & Range(V(tv_j)) \\ & x & \mapsto & V(tv_j) \end{array}$$

maps tv_j to one element in its range. $x \in X$ represents the information on basis of which tv_j's value is determined.

[6] Following Rudin [1986], this thesis understands the range of the function V assigning a value to the variable tv_j in the following way:

$$Range(V(tv_j)) = \big\{ y : \quad \text{there exists an } x \text{ in the domain of } Range(V(tv_j)) \text{ such that } y = V(tv_j, x) \big\},$$

where x is information necessary to determine tv_j's value. Therefore,

$$Range(V(tv_j)) \subseteq Type^*(tv_j).$$

The default value for a variable $tv_j \in STV$ is $V^{def}(tv_j) = \infty$.[7] To ensure the well-definedness, $Type^*(tv_j)$ is expanded, and for all $tv_j \in STV$ the type of tv_j is defined as

$$Type(tv_j) = Type^*(tv_j) \cup \{\infty\}.$$

The value of a variable tv_j can depend on time. In this case, $V(tv_j) = V^t(tv_j)$, where t denotes the time that has passed since a fixed t_0. The dependence on t may be modelled in a continuous or discrete manner. For reasons of brevity and clarity, this thesis sticks to the notation $V(tv_j)$ and makes the time dependence explicit whenever it is relevant.

Sets of (single) values of variables: it is necessary to consider sets of variables and their values to describe how the values of the variables can be combined. To this end, the set sv is defined: for each variable $tv_j \in STV$ a single value is in the set sv. Using sv, this thesis does not refer to singletons, but rather to a set containing one and only one value per variable. For instance, $sv(tv_{j_k}, tv_{j_l}) = \{V_i(tv_{j_k}), V_i(tv_{j_l})\}$ is correct, but $sv(tv_{j_k}, tv_{j_l}) = \{V_{i_1}(tv_{j_k}), V_{i_2}(tv_{j_k})\}$, $V_{i_1}(tv_{j_k}) \neq V_{i_2}(tv_{j_k})$ is not. For each subset $\tau \subseteq STV$, according sets $V(\tau)$ of values of variables can be defined by $V_k(\tau) = \bigcup_{tv_j \in \tau} \{V_k(tv_j)\}$, where V_k is one (unique) value of tv_j.

3.2.3. Status Values of Variables

For all variables $tv_j \in STV$ meta-information describing the availability and quality of the assessment $V(tv_j)$ is determined. The **status** of a variable allows for keeping track of the determinacy of a variable; it specifies whether a variable has already been assessed by an expert, whether the necessary information to determine the value of a variable is available and whether the value is uncertain or confirmed.

The function $status(tv_j)$ has a set of predefined values it can take.

Not assessed: reserved as a default value to denote that tv_j does not have any value assigned to it.

Unknown: the possible values of tv_j cannot be assessed given the information available (at present).

[7] Here, ∞ corresponds to the infinitely far away point and should not be included in $Type^*$, as it is assumed that ∞ cannot be reached given constraints in time and other restrictions. This technique corresponds to the one-point compactification often applied for topological spaces [Munkres, 2000].

Not assessable: the possible values of tv_j cannot be assessed given the set of variables STV that is currently considered. In this case, the integration of further information (by extending STV) may resolve the problem.

Deterministic: one unique value has been judged certain enough to represent the set of possible values. This value may nevertheless be prone to (sufficiently small) perturbations. This definition is justified when following Nikolaidis [2005], who defines certainty as the condition of knowing everything *necessary* to choose the alternative whose outcome is most preferred.

Uncertain: the value of tv_j can *not* be determined with (sufficient) certainty. Yet, it may happen that despite $status\,(tv_j) = uncertain$, there is (at present) just one value assigned to tv_j. In case the principle used to capture the uncertainty can be made explicit, the following three sub-categories of the status *uncertain* can be distinguished [?].

- **Probabilistic or Bayesian** *(uncertain-B):* a (conditional) probability distribution for the value of a variable $V\,(tv_j)$ can be specified.
- **Fuzzy** *(uncertain-F):* the information on $V\,(tv_j)$ is vague and imprecise and can be modelled using fuzzy logic.
- **Limiting** *(uncertain-L):* the set of possible values of tv_j can be limited to a set $\tilde{R} \subset Range\,(V\,(tv_j))$. Yet, no information on the likelihood of the occurrence of $V\,(tv_j)$ is available.

For the statuses uncertain-B and uncertain-F the respective quantifications (distributions or membership functions) are made explicit and provided as annotations to the status.

Later on, certain subsets of variables need to be distinguished with respect to their statuses. The set NA of not (yet) assessed variables is:

$$STV \supseteq NA = \big\{ tv_j \in STV : \quad status\,(tv_j) \in \{not\ assessed,\ unknown,\ not\ assessable\} \big\}$$

It holds:

$$tv_j \in NA \Rightarrow V\,(tv_j) = \infty,$$

i.e., for the variables that are not (yet) assessed, the value is set to the default value ∞.

Furthermore, the uncertain variables play an important role in scenario construction and management. The set U of variables in STV whose value is qualified as uncertain is denoted

$$U = \Big\{tv_j \in STV : \; status\,(tv_j) \in \{uncertain,\; uncertain\text{-}B,\; uncertain\text{-}F,\; uncertain\text{-}L\}\Big\}. \tag{3.1}$$

For a set of variables $\tau \subseteq STV$, $status(\tau)$ is defined as

$$status(\tau) = \bigcup_{tv_j \in \tau} status\,(tv_j)\,.$$

3.2.4. Interdependence of Variables

The dependency relations between variables defined in this section allow for representing how (values of) variables depend on each other. This facilitates constructing a network graph that represents the relations between variables. For each $tv_j \in STV$, the variables tv_j directly depends on and the variables tv_j directly influences are identified. This analysis serves to build a graph of interdependencies. Let tv_j, tv_k be two variables in STV.

The **directly depends on** relation dd indicates whether tv_k depends directly on tv_j:

$$dd\,(tv_k, tv_j) = \begin{cases} 1, & \text{if } tv_k \text{ depends directly on } tv_j \text{ and } tv_k \neq tv_j \\ 0, & \text{otherwise.} \end{cases}$$

Particularly, dd is not reflexive, i.e., $dd\,(tv_j, tv_j) = 0$.

The relation dd allows defining the set of variables $tv_j \in STV$, on which tv_k depends directly:

$$\widetilde{\Psi}\,(tv_k) = \{tv_j \in STV : \; dd\,(tv_k, tv_j) = 1\}\,.$$

If $\widetilde{\Psi}\,(tv_k) = \varnothing$, tv_k is said to be *independent*.

The **directly influences** relation di is defined as:

$$di\,(tv_j, tv_k) = \begin{cases} 1, & \text{if } tv_j \text{ directly influences } tv_k \text{ and } tv_j \neq tv_k \\ 0, & \text{otherwise.} \end{cases}$$

Again, di is not reflexive. The set of variables influenced directly by tv_j is:

$$\widetilde{\Theta}(tv_j) = \{tv_k \in STV : \text{ and } di(tv_j, tv_k) = 1\}.$$

From the definition follows:

$$dd(tv_k, tv_j) = di(tv_j, tv_k).$$

Thus,

$$tv_k \in \widetilde{\Theta}(tv_j) \Leftrightarrow tv_j \in \widetilde{\Psi}(tv_k).$$

The rationale for defining the relations dd and di is not fixed a priori but adapted flexibly to the problem at hand. The relations are elicited from some (external) authorities and experts, who construct the scenario structures (cf. Section 5.1 for the theoretical framework and Chapter 9 for an example).

The dependency relations allow the definition of **paths** through the network of interdependencies: A set

$$\begin{aligned} P_{DI}(tv_j, tv_k) \quad = \quad & \Big\{\big(tv_j = tv_{n(1)}, tv_{n(2)}, \dots, tv_{n(m-1)}, tv_{n(m)} = tv_k\big) : \\ & \quad tv_{n(i+1)} \in DI\left(tv_{n(i)}\right) \ \forall i = 1, \dots, m-1\Big\} \end{aligned}$$

is called a (directed) path from tv_j to tv_k. The set $N = \{n(1), \dots, n(m)\} \subset \mathbb{N}$, $m \in \mathbb{N}$, $n(i) \leq n(i+1)\ \forall i = 1, \dots, m-1$ specifies the indices of variables on the path. The dd and di relations do not forbid multiple paths $P(tv_j, tv_k)$ from tv_j to tv_k, and loops can occur in the network.

The above relations allow for determining for each tv_j the set of variables, on which it depends , and the set of variables that it influences. The set of variables $\Psi(tv_j)$, on which tv_j **depends** (both directly and indirectly), is defined as:

$$tv_k \in \Psi(tv_j) \iff \exists P_{DI}(tv_k, tv_j).$$

Analogously, the set of variables $\Theta(tv_j)$ that tv_j **influences** (directly and indirectly) is:

$$tv_k \in \Theta(tv_j) \iff \exists P_{DI}(tv_j, tv_k).$$

That means Ψ and Θ are the transitive closures of di and dd [Purdom, 1970].

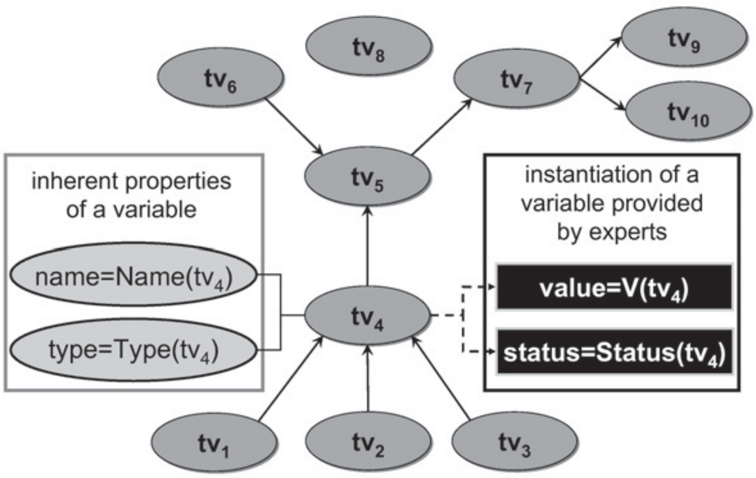


Figure 3.1.: *Representing a Scenario as a Network Graph*

3.2.5. Formal Scenario Definition

Using the definitions from the previous sections, the stage is set to define a scenario S_i as a tuple

$$S_i = \langle STV_i, sv_i, status_i, DI_i \rangle .$$

- $STV_i \subseteq STV$, i.e., the set of variables considered within the scenario S_i is well-defined within the given framework.
- $sv_i = \bigcup_{tv_j \in STV_i} \{V_i(tv_j)\}$, i.e., in S one value per variable is considered. Later, it is shown in how far the scenario generation is tailored to ensure the consistency of these values as far as possible (cf. Section 5.2).
- $status_i = \bigcup_{tv_j \in STV_i} \{status_i(tv_j)\}$, i.e., in S_i one status per variable is considered. For each variable tv_j, this status characterises the information incorporated within $V_i(tv_j)$.
- $DI_i = DI_i(STV_i) = \bigcup_{tv_j \in STV_i} \{DI_i(tv_j)\}$, i.e., DI_i is a set of dependency structures for each variable in STV_i.

It has already been indicated that the default value of all variables is ∞. The default status is *not assessed*. The default value and status are adapted as soon as an assessment of the variable's (possible) value(s) and status(es) are available. These definitions ensure that also in situations when the scenario lacks completeness, e.g., when the values and statuses of all or some variable are undetermined or when dependencies cannot be identified (i.e., $DI_i(tv_j) = \varnothing$ for all $tv_j \in STV_i$), the scenario itself is well-defined.

Using these definitions, a scenario S_i's structure can be represented by a directed graph $\mathcal{G}_i$, cf. Figure 3.1. The vertices of $\mathcal{G}_i$ represent the variables $STV_i = \{tv_j\}_{j \in J \subset \mathbb{N}}$ (each tv_j is of name $Name(tv_j)$ and type $Type(tv_j)$). Each tv_j is assigned a value $V_i(tv_j)$ and a status $status_i(tv_j)$. An edge from a vertex tv_j to tv_k means that tv_j influences tv_k directly. The definition of a scenario does not require that each variable depends on or influences other variables, i.e., there may also be isolated variables in the graph (see tv_8 in Figure 3.1).

This representation clarifies that tools from graph theory can be used to describe and exploit the dependencies of variables within a scenario S_i. For a graph with vertices $STV_i = \{tv_j\}_{j \in J}$ and edges

$$E_i = \left\{e_{j(l),j(l+1)}\right\}_{l=1}^{M} = \left\{\left(tv_{j(l)}, tv_{j(l+1)}\right)\right\}_{l=1}^{M}$$

(i.e., the edge $e^i_{j(l),j(l+1)}$ leaves $tv_{j(l)}$ and arrives at $tv_{j(l+1)}$, indicating that $tv_{j(l)}$ directly influences $tv_{j(l+1)}$), an **Incidence Matrix** $Inc^i \in \mathbb{R}^{N \times M}$, which has one row per vertex and one column per edge, can be defined. Let $e_{j(l),j(l+1)}$ be the l^{th} edge, then:

$$\mathrm{Inc}^i_{m,l} = \begin{cases} 1 & \text{if } m = j(l), \\ -1 & \text{if } m = j(l+1), \\ 0 & \text{otherwise.} \end{cases}$$

The incidence matrix is unique for a graph (and thereby, for the scenario S_i's structure as defined by STV_i and DI_i) up to a permutation of rows and columns (corresponding to re-ordering the vertices and edges). Table 3.1 shows the incidence matrix for the dependencies of the example scenario's structure shown in Figure 3.1.

The scenario building process, which is explained in detail in Chapter 5, relies on a distributed approach, in which heterogeneous processing protocols including human reasoning and automated algorithms are combined to meaningful workflows [Comes et al., 2009b, 2010a,d]. Experts (humans or automated systems) define their (reasoning) capabilities in terms of a task they can perform *(service)* and in terms of information this task requires. Although input and output are formalised, the algorithms used can be chosen freely and adapted to the situation. In terms of scenarios, the services correspond to determining the values of the variables present in the scenario: the task of an expert is to provide a value and status for the variable he is responsible for.

Table 3.1.: *Incidence Matrix Describing the Dependencies Shown in Figure 3.1*

vertex/edge	$e_{1,4}$	$e_{2,4}$	$e_{3,4}$	$e_{4,5}$	$e_{5,6}$	$e_{5,7}$	$e_{7,9}$	$e_{7,10}$
tv_1	1	0	0	0	0	0	0	0
tv_2	0	1	0	0	0	0	0	0
tv_3	0	0	1	0	0	0	0	0
tv_4	-1	-1	-1	1	0	0	0	0
tv_5	0	0	0	-1	1	-1	0	0
tv_6	0	0	0	0	0	1	0	0
tv_7	0	0	0	0	-1	0	1	1
tv_8	0	0	0	0	0	0	0	0
tv_9	0	0	0	0	0	0	-1	0
tv_{10}	0	0	0	0	0	0	0	-1

3.2.6. Comparison of Values

Before scenarios are further qualified, the values of the variables need some consideration: under uncertainty the expert determining the value of $tv_j \in STV_i$ is allowed to pass on several values. While in the deterministic case the function $V\left(tv_j\right)$ has the form

$$\begin{aligned} V:\ X &\to Range\left(V\left(tv_j\right)\right) \\ x &\mapsto V\left(tv_j\right), \end{aligned}$$

under uncertainty, it holds:

$$\begin{aligned} V\ :X &\to \left\{Range\left(V\left(tv_j\right)\right), \ldots, Range\left(V\left(tv_j\right)\right)\right\} \\ x &\mapsto \left\{V_1\left(tv_j\right), \ldots, V_n\left(tv_j\right)\right\}, \end{aligned}$$

where $X = \left\{Type\left(tv_{j_1}\right), \ldots, Type\left(tv_{j_M}\right)\right\}$ and $x = \left\{V_i\left(tv_{j_1}\right), \ldots, V_i\left(tv_{j_M}\right)\right\}$ represents in both cases the input information an expert processes to determine the value(s) of tv_j. In this thesis, this information corresponds to the values of tv_j's direct predecessors $\widetilde{\Theta}\left(tv_j\right)$, cf. Section 5.2.

Throughout this section let S_{i_1} and S_{i_2} be two scenarios, tv_j a variable in $STV_{i_1} \cap STV_{i_2}$ and $V_{i_1}\left(tv_j\right) \in sv_{i_1}$ and $V_{i_2}\left(tv_i\right)) \in sv_{i_2}$ the value of tv_j in S_{i_1} and S_{i_2} respectively. To compare the values of such a variable tv_j, first, definitions of value operations are required. Then, it is possible to define the distance of $V_{i_1}\left(tv_i\right)$ and $V_{i_2}\left(tv_j\right)$. On basis of this distance, comparisons that characterise the *similarity of scenarios* (see Chapter 7)–a key topic in scenario management (see Chapter 8)– can be made.

3.2.6.1. Value Operations

The values of a variable tv_j can be compared and combined by the operators detailed in the following. The actual semantics, i.e., the precise definition of the operators, depends inter alia on $Type(tv_j)$. The operators map input of the same type to a new value. For simplicity's sake this thesis does not consider additional transformation or mapping operators to assist in comparing values of different types (e.g., postal location to GPS coordinates, imaginary numbers to polar coordinates). The following operations are distinguished.

Equality: $equal(V_{i_1}(tv_j), V_{i_2}(tv_j))$ has as result either $true$ (if both values are equal) or $false$.

Aggregation: $V_{i_1}(tv_j)\ \&\ \delta$ where the & is an aggregation operator combines $V_{i_1}(tv_j)$ and $\delta \in Type(tv_j)$ to one value. If $Type(tv_j)$ is closed under &, this value is again of type $Type(tv_j)$. For instance, for values of type $\mathbb{N}$ the addition operator can be employed. An image may have another image imposed, such as imposing a plume-shape over a city-map. For each type $Type$, the 0-element is defined as the element, for which holds: for all $V \in Type$: $equal(V\&0,\ V)$.

3.2.6.2. Distance of Values

The *distance* of two values is defined as a metric $dist_j(V_{i_1}(tv_j), V_{i_2}(tv_j))$ assigning to every pair of elements $(V_{i_1}(tv_j), V_{i_2}(tv_j))$ of a common type $Type_j$ a non-negative number, i.e.:

$$dist_j: \quad Type_j \times Type_j \to [0, \infty).$$

Let $V_{i_1}(tv_j), V_{i_2}(tv_j)$ and $V_{i_3}(tv_j)$ be values of type $Type_j$ (e.g., the values of a variable tv_j in three scenarios $S_{i_1}, S_{i_2}, S_{i_3}$). Being a metric, the function $dist_j$ is required to satisfy the following conditions [Werner, 2000]:

$$\begin{aligned}
dist_j(V_{i_1}(tv_j), V_{i_2}(tv_j)) &\geq 0, \\
dist_j(V_{i_1}(tv_j), V_{i_2}(tv_j)) &= 0 \Rightarrow equal(V_{i_1}(tv_j), V_{i_2}(tv_j)), \\
dist_{Type_i}(V_m, V_n) &= dist_{Type_i}(V_n, V_m), \\
dist_j(V_{i_1}(tv_j), V_{i_3}(tv_j)) &\leq dist_j(V_{i_1}(tv_j), V_{i_2}(tv_j)) \\
&\quad + dist_j(V_{i_2}(tv_j), V_{i_3}(tv_j)),
\end{aligned}$$

For illustrative purposes, a number of possible approaches to define the distance depending on the type of the variable is detailed.

If $Type_j$ is a **normed vector space** $(Type_j, \|.\|_j)$, the metric $dist_j$ can simply be defined by setting

$$dist_j(V_{i_1}(tv_j), V_{i_2}(tv_j)) = \|V_{i_1}(tv_j), V_{i_2}(tv_j)\|_j .$$

In this case, the metric $dist_j$ is said to be induced by the norm $\|.\|_j$. Particularly, for all V that are δ-similar to V_m it holds

$$dist_j(V_{i_1}(tv_j), V_{i_2}(tv_j)) = \|V_{i_1}(tv_j) - V_{i_2}(tv_j)\|_j = \|\delta\|_j .$$

If $Type_j$ is not a normed vector space, other metrics can be used. To compare text strings and to determine their distance, the **Levenshtein distance** that measures the difference between two strings by the minimum number of edits necessary to transform one string into another (allowed operations: insertion, deletion, substitution) can be used [Navarro, 2001]. The **discrete metric** can be defined simply on basis of the equal operation by setting

$$dist(V_{i_1}(tv_j), V) = \begin{cases} 1, & \text{if } V_{i_1}(tv_j) = V, \\ 0, & \text{if } V_{i_1}(tv_j) \neq V \end{cases} \tag{3.2}$$

3.2.6.3. Similarity of Values

Similarity plays a fundamental role in theories of knowledge and decision-making, as it serves as an organizing principle, by which objects and concepts can be classified, and generalisations can be made [Tversky, 1977]. This thesis uses the concept of similarity to group scenarios to classes. These classes provide the basis for scenario management (see Chapter 8).

$\delta\text{-}similar\left(V_{i_1}\left(tv_j\right), V\right)$ has as result either *true* or *false*, based on the property that given δ of $Type\left(tv_j\right)$ equality holds for one value V^* in the δ-neighbourhood B_δ of $V_{i_1}\left(tv_j\right)$. $B_\delta\left(V_{i_1}\left(tv_j\right)\right)$ is defined as

$$B_\delta\left(V_{i_1}\left(tv_j\right)\right) = \left\{\tilde{V} \in Type\left(tv_j\right) :\ dist_j\left(V_{i_1}\left(tv_j\right)\&\tilde{V},\ V_{i_1}\left(tv_j\right)\&\delta\right) = 0\right\},$$

where $dist_j$ is a metric on $Type\left(tv_j\right)$. $V_{i_1}\left(tv_j\right)$ and V are called δ-similar if and only if

$$\exists V^* \in B_\delta\left(V_{i_1}\left(tv_j\right)\right) :\ equal\left(V, V^*\right).$$

For $\delta = 0$ (the 0-element in $Type\left(tv_j\right)$), δ-similarity is equivalent to equality of values.

Next, the notion of **weak similarity** is developed. This is a novel type of similarity targeted towards defining similarity with respect to the evaluation taking into account the decision makers preferences. Assuming that $Type_j$ is a topological vector space its weak topology is defined on its continuous dual space $Type_j^*$, which consists of all continuous functionals[8] $f : Type_j \rightarrow \mathbb{R}$ [Rudin, 1991]. This is particularly important, as evaluation functions f used in MADM are usually functionals. If the decision problem is operationalised by means of N attributes, it can be written as $f : Type_{att_1} \times \ldots \times Type_{att_N} \rightarrow \mathbb{R}$ [Stewart, 1992]. The rationale behind is that the set of real numbers $\mathbb{R}$ with comparison operators $<$ and $>$ is totally ordered. Thus, the use of functionals f facilitates ranking $\left(V_1\left(att_1\right), \ldots, V_1\left(att_N\right)\right), \ldots, \left(V_m\left(att_1\right), \ldots, V_m\left(att_N\right)\right)$, where $V_i\left(att_j\right)$ represents the value of attribute att_j in scenario S_i. Weak types of similarity offer a possibility to assess the similarity of values *after* an evaluation function has been applied.

ε^ω-similar$\left(V_{i_1}\left(tv_j\right), V_{i_2}\left(tv_j\right)\right)$ **(weak-ε-similarity)**: has as result either true or false, based on the property that, for a given $\varepsilon \in \mathbb{R}\ \exists \tilde{f} :\ Type_i \rightarrow \mathbb{R}$, such that

$$\left\|\tilde{f}\left(V_{i_1}\left(tv_j\right)\ \&\ V_{i_2}\left(tv_j\right)\right) - \tilde{f}\left(V_{i_1}\left(tv_j\right)\right)\right\| \leq \epsilon.$$

[8] More precisely, the functionals f must be continuous with respect to the strong topology over $Type_j$.

$(f,\varepsilon)^{\omega}$-similar$(V_{i_1}(tv_j), V_{i_2}(tv_j))$ **((f,ε)-similarity)** has as result either true or false based on the property that for a given $\varepsilon \in \mathbb{R}$ and *predefined* $f : Type_j \to \mathbb{R}$ it holds

$$\| f(V_{i_1}(tv_j) \;\&\; V_{i_2}(tv_j)) - f(V_{i_1}(tv_j)) \| \leq \epsilon.$$

3.3. Characterising Basic Properties of Single Scenarios

Based on the definition of a scenario S_i in the previous section it becomes possible to characterise a scenario according to some basic properties that it should fulfil. The concepts developed in this section will be the basis for achieving objective O.1: the processing of information into *meaningful scenarios.* The *well-definedness, structural correctness* and *connectivity* mainly deal with technical and structural properties. The concepts of *completeness* and *relevance* ensure that a scenario contains relevant information that supports the scenario recipients. These concepts provide the basis for achieving objective O.2: construction of purposeful scenarios that are tailored to the recipients needs. Finally, *plausibility, coherence* and *consistency* are requirements for a scenario's acceptance. These concepts contribute to achieving objective O.3: the construction of acceptable and credible scenarios.

3.3.1. Structural Properties

3.3.1.1. Well-Definedness

Well-defined scenarios use variables, values and statuses that are understood by their readers. During the scenario building phase, terms and references that may be unfamiliar to the recipients or are unique to the situation in question need to be defined. Conflicts and ambiguities in key definitions must be addressed and resolved [Alspaugh and Antón, 2008].

While the assessment of well-definedness is beyond the scope of this formalisation, as it requires knowledge on the experts' understanding of the terms and concepts used, the construction of a scenario (in terms of variables and dependences) (see Chapter 5.1) is designed in a participatory manner to make sure that all actors involved understand the information they receive and provide their output in a style and format that can be understood by further experts.

3.3.1.2. Structural Correctness

The structural correctness of a scenario depends on the unambiguity of values assigned to its variables as well as on the appropriateness of the dependence structure. The concepts of *within scenario unambiguity* and *acyclicity* facilitate the identification and pruning non-correct scenarios. To define the concept of **unambiguity,** all variants of ambiguity that may arise during the scenario building process are defined.

A scenario S_i is termed **value ambiguous** with respect to tv_j if $tv_j \in STV_i$ is assigned two or more values in sv_i, i.e.,

$$\exists tv_j \in STV_i : \{V_{i_1}(tv_j), V_{i_2}(tv_j)\} \in sv_i \text{ and } V_{i_1}(tv_j) \neq V_{i_2}(tv_j).$$

S_i is **within scenario value unambiguous** if there is no variable $tv_j \in STV_i$, with respect to which it is value ambiguous.

Similarly, one defines **within scenario status (un-)ambiguity:** S_i is status ambiguous with respect to tv_j if $tv_j \in STV_i$ is assigned two or more statuses in $status_i$. S_i is **within scenario status unambiguous** if there is only one status for each variable, i.e.

$$\begin{aligned} \forall tv_j \in STV_i : \quad & status_{i_1}(tv_j) \in status_i \wedge status_{i_2}(tv_j) \in status_i \\ \Rightarrow \quad & status_{i_1}(tv_j) = status_{i_2}(tv_j). \end{aligned}$$

Consequently, a scenario S_i is labelled **unambiguous** if each variable $tv_j \in STV_i$ is assigned exactly one value $V_i(tv_j) \in sv_i$ and one status $status_i(tv_j) \in status_i$. Stated differently, S_i is unambiguous if it is within scenario value and status unambiguous.

Concerning the dependency structure DI, each scenario is designed to capture chains of causes and effects. Therefore, within this thesis each scenario is required to be represented by a directed *acyclic* graph (DAG). The directionality is guaranteed by the definition of the dependency relation. Furthermore, a scenario can be represented by an **acyclic** graph if for any $tv_j, tv_k \in STV_i$:

$$\exists P_{DI_i}(tv_j, tv_k) \quad \Rightarrow \quad \nexists P_{DI_i}(tv_k, tv_j).$$

That means, when representing the structure of S_i as a network graph $\mathcal{G}_i$ (cf. Figure 3.1), no (directed) cycles[9] are allowed.[10]

Finally, a scenario S_i is said to be **structurally correct** if it is both unambiguous and acyclic.

3.3.1.3. Connectivity and Independence

The definitions on the topic of **connectivity** allow the definition of a notion of **independence,** which is tailored for scenarios. Both, connectivity and (in-)dependence are used for a precise description of the scenario construction process and important requirements thereof (cf. Chapter 5.1) as well as an analysis of the importance of a variable for scenario updating (cf. Chapter 8.3). The concepts introduced in this section are adopted and refined from standard definition in graph theory [Bollobás, 1998; Diestel, 2005].

Two variables $tv_j, tv_k \in STV_i$ are called **neighbours** (or adjacent), if $tv_j \in \widetilde{\Psi}_i(tv_k)$ or $tv_k \in \widetilde{\Psi}_i(tv_j)$, where $\widetilde{\Psi}_i$ is derived from DI_i. Referring to the graphical representation, denote $E_i = \{e_{j,k}\}$ the set of edges capturing DI_i and STV_i the set of vertices. tv_j, tv_k are neighbours if and only if there is an edge connecting them (i.e., $e_{j,k}$ or $e_{k,j} \in E_i$).

As the graph $\mathcal{G}_i = (STV_i, E_i)$ is directed, it is additionally possible, to define for each vertex $tv_j \in STV_i$ its in- and out-neighbourhoods or the sets of **direct predecessors** and **successors** $\widetilde{\Psi}_S(tv_i)$ and $\widetilde{\Theta}_S(tv_i)$, where

$$\begin{aligned} \widetilde{\Psi}_S(tv_i) &= \{tv_j \in STV_S : e_{j,i} \in E_S\}, \\ \widetilde{\Theta}_S(tv_i) &= \{tv_l \in STV_S : e_{i,l} \in E_S\}. \end{aligned}$$

On basis of these definitions, it is possible to further characterise the structural properties of a variable: tv_j with

- $\widetilde{\Psi}_i(tv_j) = \widetilde{\Theta}_i(tv_j) = \emptyset$ is called **isolated,**
- $\widetilde{\Psi}_i(tv_j) = \emptyset$ and $\widetilde{\Theta}_i(tv_j) \neq \emptyset$ is called a **source variable,**
- $\widetilde{\Psi}_i(tv_j) \neq \emptyset$ and $\widetilde{\Theta}_i(tv_j) = \emptyset$ is called a **sink variable.**

[9] A directed cycle is a path such that the start and end vertex are the same, and all edges connecting the vertices are oriented in the same direction.

[10] Although the graph $\mathcal{G}_i$ is required to be acyclic, loops, i.e., paths such that the start and end vertex are the same, but the edges are not oriented in the same direction, can occur.

The valency or **degree** of tv_j, $d_i(tv_j)$, is equal to the number of tv_j's neighbours. As $\mathcal{G}_i$ is directed, the in-degree $d_i^-(tv_j) = \left|\widetilde{\Psi}_i(tv_j)\right|$ and the out-degree $d_i^+(tv_j) = \left|\widetilde{\Theta}_i(tv_j)\right|$ can be distinguished.

The above definitions describe the structural properties of a scenario S_i. S_i is called **connected** if for any two variables $tv_j, tv_k \in STV_i$, there is a path $P_{DI_i}(tv_j, tv_k)$ or a path $P_{DI_i}(tv_k, tv_j)$ consisting of edges in E_i from tv_j to tv_k or vice versa.

Next, notions of (in-)dependence for scenarios can be defined. Usually ,in graph theory pairwise non-adjacent variables tv_j and tv_k are called **independent**, i.e., $tv_j \notin \widetilde{\Psi}(tv_k) \cup \widetilde{\Theta}(tv_k)$ [Diestel, 2005]. In this sense, independence is a form of non-adjacency. In SBR, however, edges represent dependence. To capture indirect dependencies, a more rigorous notion of independence is useful. Two variables $tv_j, tv_k \in STV_i$ are called **independent in $\mathbf{S_i}$,** if there is no (directed) path $P_{DI_i}(tv_j, tv_k)$ or $P_{DI_i}(tv_k, tv_j)$ connecting them in DI_i. In other words, tv_j and tv_k are independent if and only if they are disconnected.

To analyse the **independence of scenarios,** the above concept is generalised. Consider the scenarios S_i, S_{i_1} and S_{i_2} with $STV_{i_1} \subseteq STV_i$ and $STV_{i_2} \subseteq STV_2$. S_{i_1} and S_{i_2} are called independent in S_i, if and only if

$$\begin{array}{llcl} & STV_{i_1} \cap STV_{i_2} & = & \varnothing \\ \text{and} & \nexists\, e_{j,k} = (tv_j, tv_k): & & tv_j \in STV_{i_1}, tv_j \in STV_{i_2} \text{ and } e_{j,k} \in E_i, \end{array}$$

where E_i is the set of edges representing DI_i.

3.3.2. Scenario Content: Information Requirements

In this and the subsequent section, the content of a scenario becomes the centre of interest. In this section, the information a scenario is required to contain to support the decision-makers is qualified. In Section 3.3.3, the main requirements for a scenario's acceptance are defined.

3.3.2.1. Relevance

The nature of time constrained, large and complex decision problems requires experts and decision-makers to conduct their tasks in stressful situations involving information overload [Schaafstal et al., 2001]. To decide which information to provide to whom, **relevance** is one of the most fundamental issues.

Relevance has been defined as a twofold concept consisting of *logical relevance* and *utility* [Cook, 1975; Cooper, 1971; Cuadra and Katter, 1967; Wilson, 1973]. **Logical relevance** concerns the question *"whether or not a piece of information on a subject has some topical bearing on the information need"* [Cooper, 1971]. **Utility,** on the contrary, is a user-centred concept for evaluating *"the ultimate usefulness of the piece of information to the user"* [Cooper, 1971]. To determine whether or not a piece of information is relevant, each expert's and decision-maker's set of open problems and unanswered questions need to be examined.

Following Walker et al. [2003], the set of relevant variables is defined as the set of variables that produce change within the system under scrutiny. An analysis of the impact of changing a variable's value is a means to characterise the variable's relevance. The approaches developed in Sections 7.1.2.2 and 7.4 allow for determining the variables and values that are the most **impact relevant** by identifying those variables for which a change in value causes the greatest change or the greatest variation in the evaluation of alternatives.

On a **local level,** relevance is captured within the structure of a scenario S_i as it is assumed that the for each variable $tv_j \in STV_i$, the set of its direct predecessors $\widetilde{\Psi}\left(tv_j\right)$ (or rather the information on the value of each of these variables) is relevant to determine $V_i\left(tv_j\right)$. Section 5.1 explains that this concept of relevance is elicited from experts assessing tv_j's value. If the experts are able to specify the relative importance of each $tv_k \in \widetilde{\Psi}\left(tv_j\right)$, it is possible to distinguish several layers of relevance for each variable, e.g., varying from *compulsory* to *interesting* information.

Additionally, it is required that the problem or situation the scenario describes is relevant to the recipients. A scenario is called **backdrop relevant** if the description of the situation reflects the problem under scrutiny, and the *purpose* of the SBR corresponds to the questions relevant to the scenario recipients. These properties must be shared by a set of scenarios constructed to solve a given problem. Later, this type of relevance is operationalised by the concept of a *backdrop* (see Section 4.1.2).

Assuming that the scenario recipients are able to specify which variables are relevant to their purpose, these variables are captured in a set $FOCUS = \left\{tv_j^F\right\}_{j \in J(FOCUS)}$, cf. Section 4.1.3. A scenario S_i is termed **focus relevant,** if it allows for capturing all relevant information for the given purpose, i.e., $FOCUS \subseteq STV_i$. If the recipients are a heterogeneous group of experts, SoE, each of which specifies a set of focus variables $FOCUS_E$, then S_i is focus relevant, if $\bigcup_{E \in SoE} FOCUS_E \subseteq STV_i$.

3.3.2.2. Completeness

It is important to provide decision-makers with complete and accurate information to base their decision on [Hickey et al., 1999]. A complete scenario gives detailed definitions of the alternative a_l implemented and its (relevant) consequences [Pohl et al., 1997]. Yet, scenarios are abstractions and necessarily incomplete. Therefore, it is essential to systematically analyse what information a scenario *must* contain under different contingencies.

A scenario S_i is termed to be **complete** if it is *structurally correct* and if all variables $tv_j \in STV_i$ have been assigned a value. According to Section 3.2.3 this means:

$$\forall\, tv_j \in STV_i :\; V_i\,(tv_j) \neq \infty \qquad \text{(and } tv_j \notin NA\text{)}.$$

A scenario is considered as **incomplete** if at least one variable has the default value, i.e.,

$$\exists\, tv_j \in STV_i :\; V_i\,(tv_j) = \infty \qquad \text{(and } tv_j \in NA\text{)}.$$

To investigate whether it is *possible* to determine the values and statuses of all variables in a scenario given constraints in time and regarding the availability of experts, the notion of **assessable scenarios** is defined. A scenario S_i is termed assessable if all of for all $tv_j \in STV_i$ it holds

$$status_i\,(tv_j) \notin \{\text{“}unknown\text{”},\ \text{“}not\ available\text{”}\}\,.$$

According to these definitions complete scenarios are a (not necessarily equal) subset of the assessable scenarios.

3.3.3. Scenario Content: Congruity

As this thesis uses scenarios as basis for supporting decision-makers, the scenarios need to be acceptable. Three main requirements for a scenario's acceptance have been identified: *plausibility, coherence,* and *consistency* [Brewer, 2007; Schnaars and Ziamou, 2001; Schoemaker, 1993; Schwartz, 1991]. Rigorous definitions of these concepts are provided in this section.

3.3.3.1. Plausibility

Plausibility is related to the coherence of concepts with prior knowledge [Connell and Keane, 2004, 2006]. This definition presumes that something is plausible if it is consistent with what is known to have occurred in the past.

A scenario is said to be plausible, if it "*does not go beyond the realm of probability*" [Beck, 1982]. The set of scenarios should, however, cover unusual situations and events that are of low likelihood. In the formalisation used in this thesis, the plausibility of scenarios refers to the plausibility of (combinations of) values of the variables. Yet, the actual semantics are beyond the scope of the definitions provided, and plausibility ultimately rests on the (individual) judgements contained in the rationale and evidence (i.e., in the reasoning) that allows for an assessment of each variable's (present or future) value [Fahey, 1997]. A requirement for an expert providing a value of a variable is that he himself regards the value as plausible given the values of the direct predecessor vertices. The *status* of a variable allows for exploring the boundaries of plausibility and acceptability of each scenario in more depth, if the experts can provide likelihood assessments for each value (see Chapter 8.2.2). Still, plausibility itself is a subjective concept. Although the framework developed in this thesis provides support to the scenarios' recipients and according filtering mechanisms, it should be recognised that ultimately the decision-makers need to decide, which scenarios are plausible to them and which are not.

3.3.3.2. Coherence

A scenario is coherent if it represents the dynamic interplay of variables, showing how the variables interconnect and influence each other [Wright and Goodwin, 2009]. Coherence is a fundamental requirement, as it provides the conceptual foundation for the interpretation of results and validates the use of distribution or decomposition in the modelling [Harries, 2003]. While consistency is a judgement on the content of a scenario (see Section 3.3.3.3), coherence is imposed by the logic of the particular scenario construction approach [Bunn and Salo, 1993]. The following types of scenario coherence are distinguished.

A scenario S_i is said to be **interdependency coherent** if all relevant relations for determining the value of each $tv_j \in STV_i$ are captured in DI_i.

Conceptual coherence is achieved when the variables are processed according to coherent reasoning principles (e.g., rules of a theory that is used to assess a variable's value) [Connell and Keane, 2004; Murphy and Medin, 1985].

While the first type of coherence cannot be checked within the scenario formalisation, as it relies heavily on local domain knowledge, the latter can be checked if the scenarios are fully conceptualised (see Chapter 4.1.6).

3.3.3.3. Consistency

Consistency can be understood as information that is non-contradictory. For scenarios, consistency is achieved, when the values of different variables do not conflict with each other or with background information [Bunn and Salo, 1993]. Beyond this general definition, this thesis distinguishes the following types of consistency.

Value consistency: S_i is said to be *value consistent*, if for all variables $tv_j \in STV_i$ and for all subsets $\tau \subseteq STV_i$ with $tv_j \notin \tau$, $V_i(tv_j)$ is consistent with $\bigcup_{tv_k \in \tau} V_i(tv_k)$. In the simplest case, when $\tau = \{tv_k\}$, this means that $V_i(tv_j)$ and $V_i(tv_k)$ are not contradictory. The general definition above tightens this statement by requiring additionally that any combination of values of any set of variables in STV_i and $V_i(tv_j)$ are not contradictory.

Assuming that all relevant interdependencies are taken into account correctly (i.e., no edges are lacking and the direction of each edge represents the direction of influence correctly), this requirement is equivalent to the following (relaxed) formulation: S_i is value consistent if $\forall tv_j \in STV_i$ and $\forall \tau_{\widetilde{\Psi}} \subseteq \widetilde{\Psi}_{DI_i}(tv_j)$ it holds that $V_i(tv_j)$ is consistent with $\bigcup_{tv_k \in \tau_{\Psi}} V_i(tv_k)$.

The assessment of value consistency, however, relies on an understanding of the system and requires specific domain knowledge and expertise. Therefore, there is no general mechanism defined to ascertain value consistency beyond relying on the expertise used to assess the values of each variable.

Observation consistency: this type of consistency refers to the consistency of a scenario S_i with the (available information on the) actual situation development (e.g., measurements and observations). Whenever the value of $tv_j \in STV_i$ can be measured or observed, the discrepancy between $V_i(tv_j)$ and the observed value $V^*(tv_j)$ can be determined by any metric applicable to $Type(tv_j)$ (cf. Section 3.2.6.2). This gap is particularly relevant to scenario pruning (cf. Section 8.2) and updating (cf. Section 8.3).

3.4. Sets of Scenarios: Exploring the Space of Possibilities

Scenario-Based Reasoning never relies on a single scenario [Heugens and van Oosterhout, 2001]. Rather, by varying the values of each uncertain variable a multitude of scenarios is generated to answer questions about the situation and its development. This section defines sets of scenarios and a number of useful properties that allow comparisons across scenarios to be made. The concepts developed in this section provide the basis for achieving objective O.4, by building sets of scenarios for each alternative $a_l \in A$ that are the basis for evaluating the alternatives in A.

A **set of scenarios** is defined as $SS = \{S_i\}_{i \in I}$. $S_i = \langle STV_i, sv_i, status_i, DI_i \rangle$ is defined as in section 3.2 for all $i \in I$, $I \subseteq \mathbb{N}$ is an index set allowing each scenario $S_i \in SS$ to be identified uniquely. The mere bundling of scenarios to a set does not imply any relations or dependencies among the scenarios $S_i \in SS$. To warrant that meaningful sets of values are provided, section 3.5 defines relations among scenarios.

On the basis of the set of scenarios SS it becomes possible to make statements on the values of variables that are present in all or some scenarios in SS. Per definition, a variable is allowed to take only a single value in a scenario. Different values that a variable tv_j may have in different scenarios express uncertainty on its value. This uncertainty may reflect the uncertainty on $V(tv_j)$ itself, or uncertainty on one or more of its predecessor variables' values $V(tv_{j_k})$: $tv_{j_k} \in \Psi(tv_j)$. In the following, two characterisations of a variable's values in a set of scenarios are provided.

3.4.1. Sets of SS-Possible Values for a Variable

Given a set of scenarios SS it is possible to collect for each $tv_j \in \bigcup_{i \in I} STV_i$ all its values in the set of SS-possible values for tv_j, $SPV_{SS}(tv_j)$. Given the set $SS = \{S_i\}_{i \in I}$, denote SS_j the subset of SS where tv_j is present, i.e.,

$$SS_j = \{S_i \in SS : \ tv_j \in STV_i\}$$

The set $SPV_{SS}(tv_j)$ is the union of values that tv_j has been assigned in each scenario in $S_{j_i} \in SS_j$. Let $I_j \subseteq I$ be the index set of scenarios in SS_j, then

$$SPV_{SS}(tv_j) = \bigcup_{i \in I_j} \left\{ V_i(tv_j) : \; tv_j \notin \mathrm{NA} \right\}.$$

Furthermore, it is assumed that all values are well-defined, i.e., each $V_i(tv_j) \in Type_j$ and $SPV_{SS}(tv_j) \subseteq Range(V(tv_j))$. By definition, $SPV_{SS}(tv_j)$ is a set, i.e., if for $S_k, S_l \in SS_j$ it holds $V_k(tv_j) = V_l(tv_j)$, then the value $V_k(tv_j)$ appears only once in $SPV_{SS}(tv_j)$.

3.4.2. Degree of Diversity

Given a set of scenarios SS, the **degree of diversity** of a variable tv_j, $div_{SS}(tv_j)$, denotes the number of different values for tv_j present in SS:

$$div_{SS}(tv_j) = |SPV_{SS}(tv_j)|.$$

An upper bound of the degree of diversity of a variable is the number of scenarios in SS, i.e., $div_{SS}(tv_j) \leq |SS|$.

By definition of $SPV_{SS}(tv_j)$, the default value ∞ is not in $SPV_{SS}(tv_j)$, and the following properties can be deduced from $div_{SS}(tv_j)$:

- $div_{SS}(tv_j) = 0$ implies that tv_j has not been assigned a value in any $S_i \in SS$.
- If $div_{SS}(tv_j) > 1$ there is uncertainty about the value of tv_j. Yet, the contrary is not true: if $div_{SS}(tv_j) = 1$ it does not necessarily follow that tv_j's value is deterministic.

According to the statuses of the variables, a further statement can be made: if tv_j^{det} with $status_i(tv_j) =$"*deterministic*" for $S_i \in SS$, then for all $S_k \in SS$ with

$$\begin{aligned} tv_j^{det} &\in STV_k, \\ STV_k \cap \widetilde{\Psi}_i\left(tv_j^{det}\right) &= \widetilde{\Psi}_i\left(tv_j^{det}\right), \\ V_k\left(\widetilde{\Psi}_i\left(tv_j^{det}\right)\right) &= V_i\left(\widetilde{\Psi}_i\left(tv_j^{det}\right)\right), \end{aligned}$$

it must hold:

$$V_k\left(tv_j^{det}\right) = V_i\left(tv_j^{det}\right).$$

3.5. Defining Inter-Scenario Relations

On the basis of sets of scenarios it is possible to define relations between scenarios. A particularly useful relation the sub-/super-scenario relation. Based on this relation scenario extensions can be described (cf. section 3.6.2), which enable the definition of requirements for scenario construction and the documentation of the path of scenario construction.

3.5.1. Sources and Cuts of Scenarios

The graph theoretical scenario definition permits to track how a scenario can be constituted from two (smaller) scenarios. This is useful to determine how scenarios are related (e.g., to express that one scenario is an extension of another).

3.5.1.1. Source Variables

Source variables are a means to trace the origin of a scenario. Furthermore, they facilitate defining the system's boundaries. $tv_j \in STV_i$ is called an $\mathbf{S_i}$ **source variable** if and only if tv_j does *not* depend on any other variable in STV_i. Let DI_i be expressed by the incidence matrix $Inc_i \in \mathbb{R}^{N\times M}$ (i.e., $|STV_i| = n$ and $|E_i| = M$). A variable $tv_j \in STV_i$ is an S_i-source-variable if and only if

$$\mathrm{Inc}^i_{k,l} \geq 0 \qquad \forall\ l = 1, ..., M$$

where $E_i = \{e^i_l\}$, $l = 1, \ldots, M$ is the labelling chosen for the set of edges.

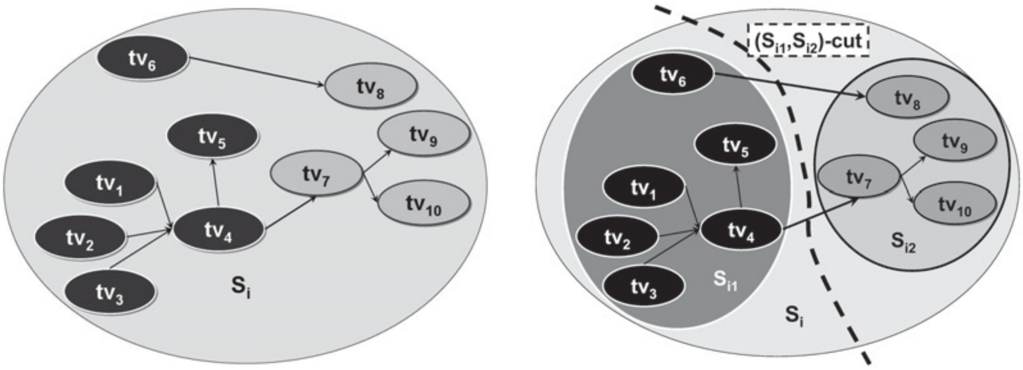
The relation is-S_i-source has a result either true or false. It holds:

$$\text{is-}S_i\text{-source}\,(tv_j) = \begin{cases} 0, & \text{if } tv_j \notin STV_i \\ 0, & \text{if } tv_j \in STV_i \ \wedge\ \exists r \in \{1, \ldots, M\} :\ \mathrm{Inc}^i_{k,r} = -1 \\ 1, & \text{otherwise.} \end{cases}$$

The set of S_i source variables, $SOURCE_i \subseteq STV_i$, is defined by:

$$SOURCE\,(S_i) = \{tv_j \in STV_i :\ \text{is-}S_i\text{-source}\,(tv_j) = 1\}$$

When S_i is represented graphically, the variables in $SOURCE_i$ are characterised by not having any incoming (directed) edges.



(a) Scenario S_i (b) Scenario S_i and S_i-Cut

Figure 3.2.: *Scenario Cuts: Example of an S_i-Cut*

To qualify the well-definedness of the scenario construction, the following definition is needed: a scenario S_i is **source originated** $\Leftrightarrow SOURCE_i \neq \emptyset$.

$SOURCE_i$ is a set of variables not a scenario itself. However, a scenario S_{SOURCE_i} that serves as a starting point for scenario generation (cf. section 5.2) can be created by defining

$$S_{SOURCE_i} = \left\langle SOURCE_i, sv_i\left(SOURCE_i\right), status_i\left(SOURCE_i\right), DI_{isol}\right\rangle \quad \text{with}$$

$$\begin{aligned} sv_i\left(SOURCE_i\right) &= \left\{V_i\left(tv_j\right) \in sv_i : \ tv_j \in SOURCE_i\right\} \\ status_i\left(SOURCE_i\right) &= \left\{status_i\left(tv_j\right) \in status_i : \ tv_j \in SOURCE_i\right\} \\ DI_{isol} &= 0^{|SOURCE_i| \times 1}, \end{aligned} \tag{3.3}$$

If for a variable $tv_j \in SOURCE_i$ no value (status) is available, its value (status) is set to its default value ∞ (“*not assessed*"). The last line of equation 3.3 signifies that the independence of all vertices in the source set is described by a 0-vector of length $|SOURCE_i|$.

3.5.1.2. Scenario Cuts

To determine whether the scenario construction process follows the order from causes to effects (as denoted by the variable dependencies DI_i), the definition of **scenario cuts** is helpful. In graph theory, a cut is defined as a partition of the vertices of a graph into two disjoint subsets. The cut set is the set of edges whose end points are in different subsets of the partition [Bollobás, 1998].

To adapt this concept for the scenario framework, consider $S_i \in SS$. An (S_{i_1}, S_{i_2})-cut is defined as a partition of S_i into two disjoint scenarios S_{i_1} and S_{i_2} such that:

$$
\begin{aligned}
STV_{i_1} \cap STV_{i_2} &= \varnothing && \text{and} & STV_{i_1} \cup STV_{i_2} &= STV_i \\
sv_{i_1} &= sv_i\,(STV_{i_1}) && \text{and} & sv_{i_2} &= sv_i\,(STV_{i_1}) \\
status_{i_1} &= status_i\,(STV_{i_2}) && \text{and} & status_{i_2} &= status_i\,(STV_{i_2}) \\
DI_{i_1} &= DI_i|_{STV_{i_1}} && \text{and} & DI_{i_2} &= DI_i|_{STV_{i_2}}\,.
\end{aligned}
$$

The notation $DI_i|_{STV_{i_1}}$ means that for the scenario S_{i_1} the edges E_{i_1} are determined by adopting the edges E_i that connect vertices tv_j, tv_k which are *both* in STV_{i_1}. While $sv_i = sv_{i_1} \cup sv_{i_2}$ and $status_i = status_{i_1} \cup status_{i_2}$, E_i can not necessarily be completely constituted from $E_{i_1} \cup E_{i_2}$, as this union excludes the edges in DI_i which connect a vertex in STV_{i_1} and a vertex in STV_{i_2}.

The cut can be defined by specifying a partition (STV_{i_1}, STV_{i_2}) of the set of vertices STV_i. Alternatively, it can be defined via a set of edges: the cut set $C = \{e_{j,k}\}$ specifies the edges $e_{j,k} = (tv_j, tv_k)$ in DI_i with $tv_j \in STV_{i_1}$, $tv_k \in STV_{i_2}$. The definition of the cut set of edges corresponds to deleting the edges that connect STV_{i_1} and STV_{i_2} in S_i. Figure 3.2 shows an example: in Figure 3.2(a) the scenario S_i is defined. Figure 3.2(b) shows the cut defined by the set $C = \{e_{6,8}, e_{4,7}\}$ and the arising scenarios S_{i_1}, S_{i_2}.

To formalise the intuitive concept that variables in a scenario S_{i_2} depend on variables in S_{i_1} (without having the reverse or interdependencies of both scenarios), a further definition is given: an S_i-cut (S_{i_1}, S_{i_2}) is called a Parent-Children S_i Cut **(PC-S$_\mathbf{i}$-cut)** for the cut set $C = \{e_{j,k}\}$ if and only if

$$
\begin{aligned}
\forall\ e_{j,k} = (tv_j, tv_k) \in C: &\qquad tv_j \in S_{i_1} \text{ and } tv_k \in S_{i_2} \\
\nexists\ e_{k,j} = (tv_k, tv_j) \in C: &\qquad tv_k \in S_{i_1} \text{ and } tv_j \in S_{i_2}, \\
\forall tv_i \in S_i: &\qquad \widetilde{\Psi}\,(tv_i) \cap STV_{i_1} = \varnothing \text{ and } \widetilde{\Psi}\,(tv_i) \cap STV_{i_2} = \widetilde{\Psi}\,(tv_i) \\
\text{or} &\qquad \widetilde{\Psi}\,(tv_i) \cap STV_{i_2} = \varnothing \text{ and } \widetilde{\Psi}\,(tv_i) \cap STV_{i_1} = \widetilde{\Psi}\,(tv_i)\,.
\end{aligned}
$$

The last condition ensures that either $\widetilde{\Psi}\,(tv_i)$ is completely contained in STV_{i_1} or completely contained in STV_{i_2}. For example, the Cut (S_{i_1}, S_{i_2}) shown in Figure 3.2 represents a PC-S_i-cut (S_{i_1}, S_{i_2}). An S_i-Cut (S_{j_1}, S_{j_2}), which is no

PC-S_i-Cut (for the same S_i as shown in Figure 3.2) is defined by the sets of vertices $STV_{j_1} = \{tv_1, tv_2\}$ and $STV_{j_1} = \{tv_3, \ldots, tv_{10}\}$ as

$$\begin{aligned} & \{tv_1, tv_2\} &= \widetilde{\Psi}(tv_4) \cap STV_{j_1} \neq & \ \widetilde{\Psi}(tv_4) \\ \text{and} \qquad & \{tv_3\} &= \widetilde{\Psi}(tv_4) \cap STV_{j_2} \neq & \ \widetilde{\Psi}(tv_4) \,. \end{aligned}$$

3.5.2. Nested Scenarios and Scenario Dependencies

By using scenario cuts, situations when a scenario is a **sub-scenario** of another scenario can be defined allowing the *dependencies* among and the *connectedness* of scenarios to be expressed. This framework enables an operationalisation of the notion of sub-scenarios[11] and facilitates the comparison of scenarios. This is an important prerequisite for the implementation of the scenario generation process (cf. section 5.1).

3.5.2.1. Sub-Scenario Relations

The relation $\mathbf{S_{sub}}$ **is-sub-scenario-of** $\mathbf{S_{super}}$ or $S_{sub} \subset S_{super}$ expresses that scenario S_{sub} is part of scenario S_{super}. For the purposes of this thesis, this relation is defined in a strict manner, i.e., S_{super} must have at least one additional variable as compared to S_{sub}. Denote

$$\begin{aligned} S_{sub} &= \langle STV_{sub}, sv_{sub}, status_{sub}, DI_{sub} \rangle \\ \text{and} \qquad S_{super} &= \langle STV_{super}, sv_{super}, status_{super}, DI_{super} \rangle \,. \end{aligned}$$

S_{sub} is a sub-scenario of S_{super} if and only if there is an S_{super}-cut (S_{sub}, S_2), where $STV_2 = STV_{super} \setminus STV_{sub}$, i.e.,

$$\begin{aligned} S_{sub} \subset S_{super} \quad \Leftrightarrow \qquad & & STV_{sub} &\subset STV_{super} \,, \\ & \text{and} & sv_{sub} &= sv_{super}(STV_{sub}) \,, \\ & \text{and} & status_{sub} &= status_{super}(STV_{sub}) \,, \\ & \text{and} & DI_{sub} &= DI_{super}|_{STV_{sub}} \,. \end{aligned}$$

That means, $S_{sub} \subset S_{super}$ given that

- STV_{sub} is a proper subset of STV_{super},

[11] Although these equal parts of scenarios can also be called *episodes* [Alspaugh and Antón, 2008; do Prado Leite et al., 2000], this thesis uses the term *sub-scenario*.

- the sets sv_{sub} and $status_{sub}$ are, analogously, proper subsets of sv_{super} and $status_{super}$ respectively,
- the set of dependencies in DI_{sub} may be a subset of or equal to DI_{super}: $DI_{sub} \subseteq DI_{super}$.

To clarify the latter condition, consider the situation where S_{super} differs from S_{sub} in an isolated variable in $STV_{super} \setminus STV_{sub}$. Although the sets of edges E_{super} and E_{sub} are equal, the incidence matrices of DI_{sub} and DI_{super} differ nevertheless, as their structure does not only reflect the edges in the graph $\mathcal{G} = \langle STV, E \rangle$ but also the number of vertices.

The sub-scenario relation allows for sets of **nested scenarios** $S_1 \subset S_2 \subset \ldots \subset S_n$ to be created.

According to the refinement of cuts to PC-cuts, one can define a *Parent-Child sub-scenario relation:* $S_{sub} \subset_{PC} S_{super}$ if and only if there is PC-S_{super}-cut (S_{sub}, S_2), where $STV_2 = STV_{super} \setminus STV_{sub}$. That means, if $S_{sub} \subset_{PC} S_{super}$ the information on the variables influencing the source variables of S_2 is contained in S_{sub}. Therefore, $S_{sub} \subset_{PC} S_{super}$ is important for the definition of a well-defined scenario construction workflow. Analogue to the (PC-)sub-scenario relation, one can define $S_{super} \supset S_{sub}$ as the inverse of the (PC-)sub-scenario relation:

$$\begin{aligned} S_{super} \supset S_{sub} \quad &\Leftrightarrow \quad S_{sub} \subset S_{super}, \\ S_{super} \supset_{PC} S_{sub} \quad &\Leftrightarrow \quad S_{sub} \subset_{PC} S_{super}. \end{aligned}$$

3.5.2.2. Connectedness of Scenarios

On the basis of the sub-scenario relation, the dependence of two scenarios can be further characterised by an analysis of their **connectedness:** S_{i_1} is *connected* to S_{i_2}, denoted $S_{i_1} \to S_{i_2}$, if and only if $S_{i_1} \subset S_{i_2}$ and

$$\forall\ tv_j \in STV_{i_2} \setminus STV_{i_1} : \exists tv_k \in \mathrm{STV}_{i_1} : \exists P_{DI_{i_2}}(tv_k, tv_j) \lor P_{DI_{i_2}}(tv_j, tv_k).$$

Hence, $S_{i_1} \to S_{i_2}$ if and only if there is an S_{i_2}-cut $(STV_{i_1}, STV_{i_2} \setminus STV_{i_1})$ with non-empty cut set $C = \{e_{k,j}\}$. All edges $e_{k_l,j_m} = (tv_{k_l}, tv_{j_m}) \in C$ are such that $tv_{k_l} \in STV_{i_1}$ and $tv_{j_m} \in STV_{i_2} \setminus STV_{i_1}$ or vice versa.

For the connectedness of two scenarios it is *not* required that the scenarios are internally connected, i.e., it is not required that for any two variables in STV_{i_1} or STV_{i_2}, there is a path in DI_{i_1} (or DI_{i_2}) connecting them. For instance,

S_{i_1} in Figure 3.2 is not completely internally connected, as there is an S_{i_1}-cut $(\{tv_1, \ldots, tv_5\}, \{tv_6\})$, for which the cut set $C = \varnothing$. However, S_{i_1} and S_{i_2} are connected (and form the connected scenario S_i).

3.6. Completion of Scenarios: Scenario Continuation

The aim of this and the subsequent section is to define methods allowing for capturing the construction of scenarios in form and content. In general, two possibilities arise. First, the scenario can be **continued** or **extended** by adding further information to the existing network of cause-effect chains. The added sub-scenarios describe how the events contained in the scenario so far influence further variables, or which future events they may cause. Second, (incomplete) scenarios can be **merged.** The merging of scenarios corresponds to combining two incomplete descriptions or overlaying two incomplete networks to gain a better understanding of the situation and its development. While the second approach is described in section 3.7, this section is dedicated to the continuous types of scenario completion.

3.6.1. Scenario Tying and Scenario Continuation

This section shows how a scenario can be **continued.** The scenario formalisation allows for procedures and mechanisms to be implemented that ensure that each continued scenario fulfils certain quality requirements.

3.6.1.1. Scenario Tying

To determine whether the scenario continuation is justified and well-defined, first, the concept of scenario tying is explained. Scenario tying is a structural relation indicating that there are variables $\{tv_k\}_{k \in K}$ depending on variables $\{tv_j\}_{j \in J}$ within a given set of scenarios.

Let $SS_{tie} = \left\{S_{i_l^2}\right\}_{i_l^2 \in I}$ be a set of scenarios, and denote $STV_{tie} = \bigcup_{i \in I} STV_i$. SS_{tie} **is tied to** a scenario completely assessed scenario S_{i^1} if and only if for all $S_{i_l^2} \in SS_{tie}$ it holds:

$$
\begin{aligned}
& S_{i_1} \subset S_{i_l^2} \\
\text{and} \quad & \exists tv_k \in STV_{i_l^2} \setminus STV_{i_1} \text{ and } tv_j \in STV_{i_l^2} \cap STV_{i^1} : \\
& \qquad \exists\, P_{DI_{i_l^2}}(tv_k, tv_j) \text{ or } \exists P_{DI_{i_l^2}}(tv_j, tv_k).
\end{aligned}
$$

While all variables in STV_{i_1} need to be assessed by definition, it is possible that for all or some variables in $tv_j \in STV_{tie} \setminus STV_{i_l^2}$ and all or some scenarios $S_{i_l^2} \in SS_{tie}$, it holds $status_{i_l^2}(tv_j) \in NA$, and therefore it may be that $V_{i_l^2}(tv_j) = \infty$.

3.6.1.2. Scenario Continuation

Scenario continuation is a relation ensuring that a scenario is not only continued on a structural level but also contains further information. Let $SS_{cont} = \left\{S_{i_l^1}\right\}_{i_l^1 \in I}$ be a set of scenarios, and denote $STV_{cont} = \bigcup_{i \in I} STV_{i_l^1}$. SS_{cont} continues S_{i_2}, denoted $S_{i_2} \hookrightarrow SS_{cont}$, if and only if for all $S_{i_l^1} \in SS_{cont}$ it holds:

$$
\begin{aligned}
& S_{i_l^1} \text{ is tied to } S_{i_2} \\
\text{and} \quad & \exists\, tv_j \in STV_{i_2} \setminus STV_{i_l^1} : \; status_{i_2}(tv_j) \neq \text{“}not\ assessed\text{”}.
\end{aligned}
$$

Scenario continuation expresses that on the basis of scenario S_{i_2} a set of scenarios SS_{cont} is developed such that each scenario $S_{i_l^1}$ in SS_{cont} contains at least one additional variable that has been given a value (unequal to the default value). In that sense, the term *continuation* is justified.

If SS_{cont} continues a scenario S_r, then S_r is called a **Root** of SS_{cont}. A set SS_{cont} can have multiple roots: each scenario S_r, to which the set SS_{cont} is tied and which fulfils for all $S_{i_l^1} \in SS_{cont}$: $\exists tv_j \in STV_{i_l^1} \setminus STV_r$: $status_{i_l^1}(tv_j) \neq$ “$not\ assessed$” is a root of SS_{cont}.

To clarify the relation of *SOURCE* (cf. section 3.5.1) and *Root*, it is important to keep in mind that the source of a scenario is defined as a set of variables, whereas the Root is a scenario itself. However, the following statement can be made: if there is Root (scenario) S_r of SS_{cont} that is source originated, then each $S_{i_l^1} \in SS_{cont}$ is source originated. To prove this property, consider $S_{i_l^1} \in SS_{cont}$

with Root S_r. As S_r is source originated, there is an S_r-cut (S_{SOURCE_r}, S_2). S_{SOURCE_r} is the scenario constructed from $SOURCE_r$ as described in equation 3.3. Since by definition $S_r \subset S_{i_l^1}$, it follows that $STV_{SOURCE_r} \subset STV_{i_l^1}$, so there is an $S_{i_l^1}$-cut (S_{SOURCE_r}, S_3). As all variables in $SOURCE_r$ are isolated in S_{SOURCE_r} (by definition of S_{SOURCE_r}, cf. equation 3.3), $SOURCE_{i_l^1} \neq \emptyset$ and $S_{i_l^1}$ is source originated.

To ensure that the scenario continuation respects interdependencies between variables, a specific type of scenario continuation is defined: $S_j \hookrightarrow SS_{cont}$ is called **connected,** if each $S_{i_l^1} \in SS_{cont}$ is connected to S_{i_2}.

Using the above definitions, it is possible to define a type of scenario continuation which fulfils some desired properties, the **well-defined scenario continuation.** A *connected* scenario continuation $S_{i_2} \hookrightarrow SS_{cont}$ is *well-defined* if and only if

1. $\forall S_{i_l^1} \in SS_{cont} : S_{i_2} \subset_{PC} S_{i_l^1}$,
2. $\forall tv_j^{SOURCE_{i_l^1}} \in SOURCE_{i_l^1} : \nexists\, S_{i_3} : S_{i_2} \hookrightarrow SS_{cont}$ with $tv_j^{SOURCE_{i_l^1}} \in STV_{i_3}$ and $tv_j^{SOURCE_{i_l^1}} \notin SOURCE_{i_2}$.

The first condition ensures that the continuation of scenarios follows the order from causes to effects or means to ends. The second condition guarantees that there is no other variable in the same backdrop, for which the source variables depend on further variables' values.

To determine the source variables for the set of scenarios SS_{cont} let $SS_r = \{S_{r_j}\}_{j \in J}$ be the set of source-originated roots, for which SS_{cont} is a well-defined scenario continuation. Setting

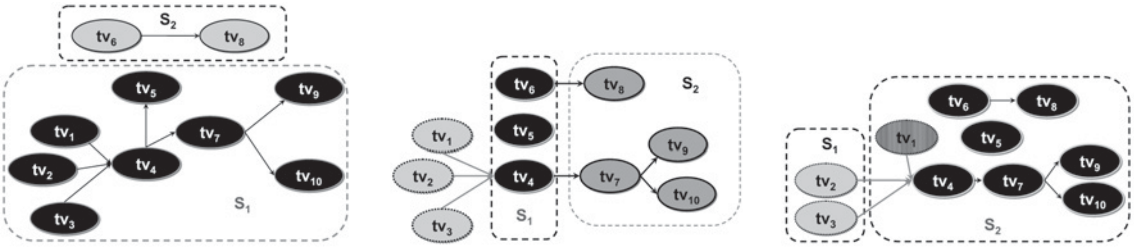
$$SOURCE_{SS_{cont}} = \bigcup_{j \in J} SOURCE_{r_j}$$

$SOURCE_{SS_{cont}}$ is well-defined and non-empty according to the above remark on the relation of $SOURCE$ and Root. Additionally,

$$SOURCE_{SS_{cont}} \in \bigcup_{S_{r_j} \in SS_r} STV_{r_j},$$

and all vertices in

$$\Psi_{SS_{cont}} := \left\{ tv_j \in \bigcup_{S_{r_j} \in SS_r} STV_{r_j} : tv_j \notin SOURCE_{SS_{cont}} \right\}$$



(a) Lacking Connectedness (b) Incompleteness of *SOURCE* (c) Violation of PC-condition

Figure 3.3.: *Examples of Not Well-Defined Scenario Continuations*

are dependent on at least one of the variables in $SOURCE_{SS_{cont}}$. Therefore, the scenario $S_{SOURCE_{SS_{cont}}}$ constructed from $SOURCE_{SS_{cont}}$ as described in equation 3.3 can be understood as the *minimum root scenario* of SS_{cont}.

The following examples illustrate the various types of scenario continuation: Figure 3.3 presents three cases, for which the continuation is not well defined. In each example shown, $S_{cont} = S_1 \cup S_2$ is a continuation of S_1. First, Figure 3.3(a) shows a situation, where the continuation is not connected, as there is no edge from any variable in STV_1 to a variable in STV_2. In the second example (cf. Figure 3.3(b)), the continuation is not well-defined, as S_1 does not contain $SOURCE_{S_{cont}}$ completely: there is a scenario S_3, for which variables tv_1, tv_2 and tv_3 are source variables relevant to determine tv_4. Yet, tv_1, tv_2 and tv_3 are not in STV_1. According to the definition, the continuation is not well-defined. In the third case (cf. Figure 3.3(c)), S_1 is not a parent-child sub-scenario of S_{cont}:[12] tv_4 directly depends on three variables (tv_1, tv_2 and tv_3). The PC-property requires that all direct predecessors of tv_4 must be in STV_1. Yet, $tv_1 \in \widetilde{\Psi}\,(tv_4) \cap STV_2$, and the continuation is not well-defined. Contrarily, Figure 3.4 shows an example of a well-defined scenario continuation: $S_1 \subset_{PC} S_{cont} = S_1 \cup S_2$ and $SOURCE_{S_{cont}} = SOURCE_{S_1}$.

3.6.2. Scenario Extension

While in the previous section mostly structural issues were considered, this section focuses on the content of scenarios, and particularly on the role of values. The multiplicity of values reflects uncertainty by generating *multiple* scenarios.

[12]This requirement ensures that the scenario is built successively and that it is only continued when all relevant information is available.

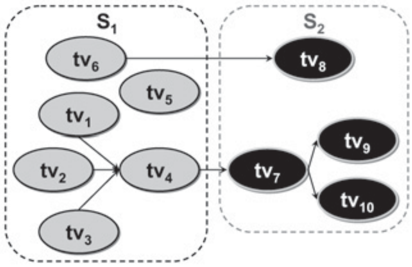


Figure 3.4.: *A Well-Defined Scenario Continuation*

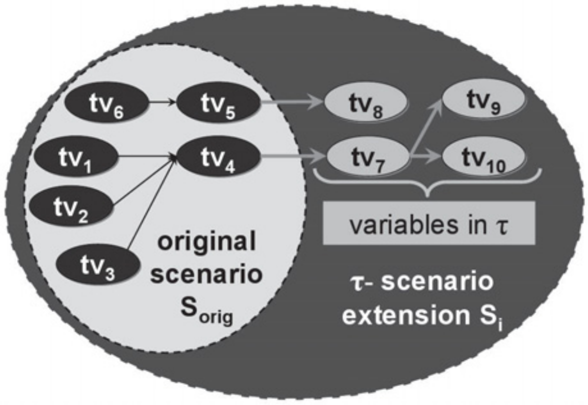


Figure 3.5.: *τ-Scenario Extension Example: S_i Extends S_{orig}*

To operationalise this feature, the relation *extends,* which is a special case of the scenario continuation relation, is defined.

As a prerequisite for the general concept, a special class of scenario extensions is defined now. The τ **scenario extension** of a scenario S_{orig} is defined as a tuple $\langle ESS_\tau, S_{orig}, \tau\rangle$, where the set of scenarios $ESS_\tau = \{S_i\}_{i \in I}$ extends the single scenario S_{orig}. It is required that $\tau \subseteq STV_i \ \forall S_i \in ESS$, and $\forall tv_j \in \tau$ it must hold $tv_j \notin STV_{orig}$, cf. Figure 3.5.

The scenarios in ESS_τ are considered an extension of the scenario S_{orig} τ if there is a $tv_{j^*} \in \tau$, for which $|SPV_{ESS_\tau}(tv_{j^*})| > 1$. That means, there is a multiplicity of possible values for at least one variable in τ.

For the $\langle ESS_\tau, S_{orig}, \tau\rangle$, the following conditions hold:

- S_{orig} is a sub-scenario of each $S_i \in ESS_\tau$,
- the union of the variables in STV_{orig} and τ is equal to the variables in STV_i for all $S_i \in ESS_\tau$, i.e.,

$$STV_i = STV_{orig} \cup \tau \ \forall i = 1, \dots, n,$$

- as the scenario extension is a special case of the continuation, for each $S_i \in ESS_\tau$ there is a $tv_j \in STV_i$, for which $status_i(tv_t) \neq$ "*not assessed*",
- as the scenarios in ESS_τ are distinct with respect to their values, it holds

$$\forall S_{i_k}, S_{i_l} \in ESS_\tau : \exists tv_{j(k,l)} \in \tau : V_{i_k}\left(tv_{j(k,l)}\right) \neq V_{i_l}\left(tv_{j(k,l)}\right).$$

The set of scenarios ESS_τ that extend S_{orig} is a means to handle uncertainty on the values of variables in τ: for each scenario in ESS_τ exactly one value for all uncertain variables is present. This implies that the number of scenarios that τ-extend S_{orig} can be calculated based on the degree of diversity of the variables in τ:[13]

$$|ESS_\tau| = \prod_{tv_j \in \tau} div_{ESS_\tau}(tv_j).$$

The general scenario extension covers all scenario extensions for a given scenario S_{orig}. Denote $\{\tau_i\}_{i \in I}$ the set of all sets of variables, for which $\tau_i \cap STV_{orig} = \emptyset$, and which allow for extending S_{orig} via $\langle ESS_{\tau_i}, S_{orig}, \tau_i\rangle$. The scenario extension is defined[14] as a tuple $\langle ESS, S_{orig}\rangle$ by the union of all τ_i scenario extensions:

$$\langle ESS, S_{orig}\rangle = \bigcup_{i \in I} \langle ESS_{\tau_i}, S_{orig}, \tau_i\rangle.$$

As the (τ-)scenario extension is a special case of scenario continuation, the concepts of connectedness and well-definedness of the extension as well as the Root can be adopted from verbatim from the scenario continuation.

The relation extends is defined by a set of scenarios ESS that extend a single scenario S_{orig}. It is, however, impossible to have for a set of scenarios SS_{orig} (with $|SS_{orig}| > 1$) an analogue relation extends-set $\langle ESS, SS_{orig}\rangle$. The definition of extends is such that S_{orig} must be a sub-scenario of each scenario in ESS. This implies that each scenarios in SS_{orig} must be a sub-scenario of every scenario in ESS. As the only structural difference between STV_{orig} and STV_i is defined by τ all scenarios in SS_{orig} are equal, and $|SS_{orig}| = 1$.

It is impossible to complete a scenario with an isolated vertex using the scenario continuation and extension procedures. These situations are covered by the **scenario merging approach,** see Section 3.7.

[13] As each variable $tv_k \in \tau$ is required to have a value unequal to the default value, $div(tv_k) \geq 1$ for all $tv_k \in \tau$.

[14] ESS depends on the available experts willing to provide information to the system in a certain time frame, see also Section 8.3.2.

3.6.3. Scenario Extension Paths

For the purpose of defining a well-structured scenario extension tree allowing for keeping track of the way the scenarios have been constructed (cf. Section 3.6.4), this section defines the concept of a **scenario extension path.** A scenario extension path $P_{SSE}(S_1, S_n)$ from S_1 to S_n requires the connectedness of all scenarios, and entails that one or more extends relations are present such that S_n is an extension of S_1. Given a set of scenario extensions, $SSE = \bigcup_{i \in I} \langle ESS_{\tau_i}, S_j, \tau_i \rangle$, the relation *is-path-$SSE$* (S_1, S_n) has a result either *true* or *false.* In the simplest case, S_1 and S_2 are directly related via a tv_j-continues or $\{tv_j\}$-extends relation in SSE:

$$\begin{aligned} & \text{is-path-}SSE\,(S_1, S_2, tv_j) = true \\ \Leftrightarrow \quad & \exists tv_j \in STV_2 : \; tv_j \notin STV_1 \\ & \quad \text{and} \quad \exists tv_k \in STV_1 : \; \exists P_{DI_{S_2}}(tv_k, tv_j) \\ & \quad \text{and} \quad tv_j \notin NA. \end{aligned}$$

For the general case, the relation is-path is defined recursively.

$$\begin{aligned} & \text{is-path-}SSE\,(S_1, S_n) = true \\ \Leftrightarrow & \exists (S_1, \ldots, S_n) \text{ and } (tv_{1_i}, \ldots, tv_{n_i}) \in (STV_1, \ldots, STV_n) : \\ & \prod_{i=1}^{n-1} \text{is-path-}SSE\,(S_i, S_{i+1}, tv_{i+1}) = true \end{aligned}$$

By definition each S_i, $i < n$ must be a sub-scenario of S_n:

$$\text{is-path-}SSE\,(S_1, S_n) = \text{true} \qquad \Rightarrow \qquad S_i \subset S_n \;\forall i < n.$$

By definition of the extends relation at each step there must be at least one variable for which the scenarios constructed multi-furcate[15], i.e., for $1 \le i \le n-1$ there is a scenario S_i, for which $\exists tv_{j_k} \in STV_i$: $tv_{j_k} \notin STV_{i-1}$, and a scenario S_{i_2}: *is-path-SSE* (S_{i-1}, S_{i_2})=*true,* where S_{i-1} is the scenario, from which both S_i and S_{i_2} were directly extended, such that $tv_{j_k} \in STV_{i_2}$ and $V_i(tv_{j_k}) \neq V_{i_2}(tv_{j_k})$.

Scenario extension path equivalence classes are defined to ensure that there is one unique scenario extension path from any scenario S_i to a given super-

[15] While a *bi-furcation* refers to a binary division, the scenario splits up not necessarily into two but multiple scenarios.

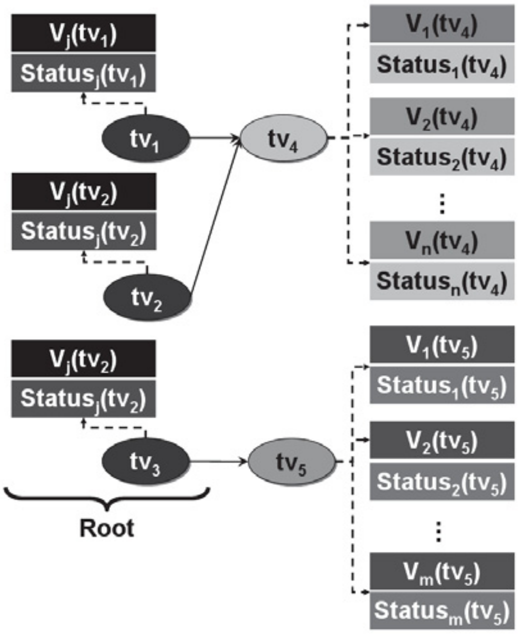


Figure 3.6.: *Scenario Extension Equivalence Classes Represented As a Network Graph. Example: Scenario* Root *is extended by tv_4 and tv_5.*

scenario S_i^{super} that is connected to S_i: let S_{i_1} and S_{i_2} be two scenarios constructed by means of two extension paths $P_{SSE}(S_{r_1}, S_{i_1})$ and $P_{SSE}(S_{r_2}, S_{i_2})$, where S_{r_1} and S_{r_2} are the respective root scenarios. The paths $P_{SSE}(S_{r_1}, S_{i_1})$ and $P_{SSE}(S_{r_2}, S_{i_2})$ are *equivalent* if and only if

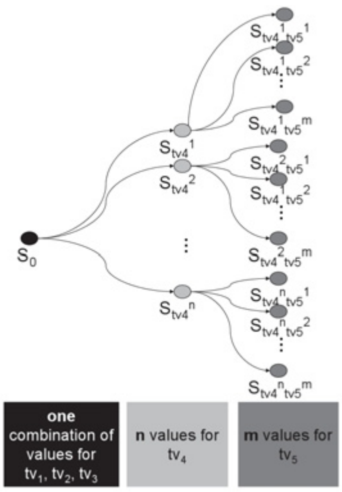
1. $S_{r_1} = S_{r_2}$ and $S_{i_1} = S_{i_2}$,
2. $\forall S_{i_k} \in P_{SSE}(S_{r_1}, S_{i_1}) : \exists S_{i_l} \in P_{SSE}(S_{r_2}, S_{i_2}) : \exists tv_{j_2} \in STV_{i_l}, tv_{j_1} \in STV_{i_k} : \exists\, P_{DI_{i_l}}(tv_{j_1}, tv_{j_2})$.

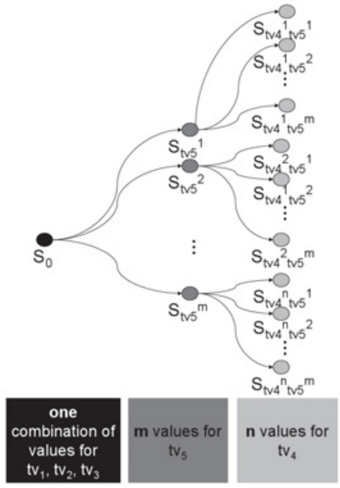
While the first two conditions ensure the equality[16] root scenarios, the last condition guarantees the convertibility of paths: from any intermediate step in $P_{SSE}(S_{r_1}, S_{i_1})$, it is possible to “switch path", and to continue the scenario extension via $P_{SSE}(S_{r_2}, S_{i_2})$.

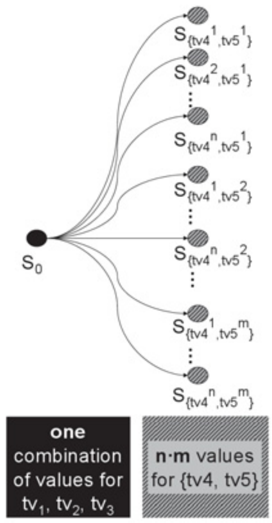
Consider, e.g., the situation shown in Figure 3.6. Both, tv_4 and tv_5 depend on a common root scenario *Root* but not on each other. A multitude of possible scenario extension trees arises depending on the sequence in which the values and statuses of the variables are determined. The left and middle column of table 3.2 show two of the possibly arising scenario extension paths. This repre-

[16]Scenario equality means that the tuples constituting the scenarios are equal with respect to each component. Here, $STV_{r_1} = STV_{r_2}$, $sv_{r_1} = sv_{r_2}$, $status_{r_1} = status_{r_2}$ and $DI_{r_1} = DI_{r_2}$.

Table 3.2.: *Scenario Extension Paths and Resulting Equivalence Class*







$\{ET_1\}$: paths, where first values for tv_4, then values for tv_5 are determined	$\{ET_2\}$: paths, where first values for tv_5, then values for tv_4 are determined	Resulting scenario extension equivalence classes

sentation is not exhaustive, as each possible sequence of determining one of the n possible values of tv_4 and one of the m possible values of tv_5 leads to another scenario extension tree. By using equivalence classes of scenario extensions the *order*, in which the values and statuses of the variables are determined, can be neglected. Thus, not tuples

$$\begin{aligned}\langle V_i(tv_4), V_i(tv_5)\rangle &\neq \langle V_i(tv_5), V_i(tv_5)\rangle \\ \langle status_i(tv_4), status_i(tv_5)\rangle &\neq \langle status_i(tv_5), status_i(tv_5)\rangle\end{aligned}$$

but **sets** of possible values and statuses for variables that do not influence each other are considered. As

$$\begin{aligned}\{V_i(tv_4), V_i(tv_5)\} &= \{V_i(tv_5), V_i(tv_5)\} \\ \{status_i(tv_4), status_i(tv_5)\} &= \{status_i(tv_5), status_i(tv_5)\},\end{aligned}$$

the *order* of values of the variables that are not interdependent on is not taken into account in the equivalence class, cf. right column in table 3.2.

$[P_{SSE}(S_1, S_n)]$ denotes a scenario extension path equivalence class. Each scenario S_i on the path is represented by $[S_i] = \langle STV_i, [sv_i], [status_i], DI_i \rangle$. The set of variables as well as the dependence structure remain the same as for scenarios. Stated differently, the structure, represented by the graphs, remains unaffected by the construction of scenario extension path equivalence classes. Contrarily, $[sv_i]$ and $[status_i]$ are ordered tuples of unordered sets.

By using scenario extension path equivalence classes, paths $[P_{SSE}(S_1, S_n)]$ from any root scenario S_1 to a scenario S_n can be considered *unique* by identifying the (completely assessed) intermediate scenarios in $[S_i]$ (for $2 \leq i \leq n-1$, as S_1 and S_n are single scenarios by definition). The well-definedness of $[P_{SSE}(S_1, S_n)]$ requires not connectedness of scenarios, but connectedness of $[S_i]$ and $[S_{i+1}]$ for all $i = 1, \ldots, n-1$.

The definitions of sub- and super-scenario equivalence classes and scenario equivalence class extensions can be adopted verbatim from the single scenario case. Similarly, other definitions from scenario extension paths are adapted for the equivalence classes. *is-path-*$[SSE(S_1, S_n)]$ has a result either true or false.

Simple paths: $[S_i]$ and $[S_{i+1}]$ are directly related via at least one τ_i-continues or τ_i-extends relation in SSE. τ_i is, however, not necessarily a single variable, but can also be a set of variables. Therefore, the following adaptations become necessary:

$$\begin{aligned}
& & & \text{is-path-}[SSE(S_i, S_{i+1}, \tau_i)] = \text{true} \\
\Leftrightarrow & & & \exists \tau_i \in STV_{i+1} : \tau_i \cap STV_i = \emptyset \\
& & \text{and} \quad & \forall\, tv_k \in \tau_i :\ \exists tv_s \in STV_i :\ P_{DI_{i+1}}(tv_s, tv_k) \\
& & \text{and} \quad & \forall\, tv_k \in \tau_i,\ \forall S_{i+1} \in [S_{i+1}] :\ tv_k \notin NA.
\end{aligned}$$

Generally, $[P_{SSE}(S_1, S_n)]$ is defined recursively.

$$\begin{aligned}
& \text{is-path-}[SSE(S_1, S_n)] = \text{true} \\
\Leftrightarrow \quad & \exists (\tau_1, \ldots, \tau_n) :\ \text{is-path-}[SSE(S_i, S_{i+1}, \tau_i)] = true\ \forall i = 1, \ldots, n-1.
\end{aligned}$$

Notation. For reasons of brevity and clarity, from here on this thesis keeps on referring to “scenario extension paths” instead of “scenario extension path equivalence classes". The notation in brackets $[S_i]$ for the intermediated stages is omitted, and from now on $S_i = \langle STV_i, [sv_i], [status_i], DI \rangle$, where

$$\begin{aligned}
[sv_i] &= \left\langle \left\{ V_i\left(tv_1^1\right), \ldots, V_i\left(tv_{m_1}^1\right) \right\}, \ldots, \left\{ V_i\left(tv_1^n\right), \ldots, V_i\left(tv_{m_n}^n\right) \right\} \right\rangle \\
\text{and } [status_i] &= \left\langle \left\{ status_i\left(tv_1^1\right), \ldots, status_i\left(tv_{m_1}^1\right) \right\}, \ldots \right. \\
&\qquad \left. \ldots, \left\{ status_i\left(tv_1^n\right), \ldots, status_i\left(tv_{m_n}^n\right) \right\} \right\rangle.
\end{aligned}$$

3.6.4. Scenario Extension Trees

A set of scenario extensions can form a **scenario extension tree.** Each tree has at least one root. The scenario extension tree $et\left\langle S_r, SSE, \{ESS_{\tau_i}\}_{i\in I} \right\rangle$ is defined by a root scenario S_r and a set of scenarios SSE, where each $S_i \in SSE$ has been derived via scenario extension from S_r. That means, for each $S_i \in SSE$ there is a set of scenarios ESS_{τ_i} with $S_{\tau_i} \subseteq S_i$ for all $S_{\tau_i} \in ESS_{\tau_i}$, and S_i is derived from S_r via $\langle ESS_{\tau_i}, S_r, \tau_i \rangle$.

The relation *is-et* $\left(\left\langle S_r, S_i, \{ESS_{\tau_i}\}_{i\in I} \right\rangle\right)$ allows for checking whether or not S_i has been derived from S_r. It has a result either *true* or *false.* To determine, if *is-et* $\left(\left\langle S_r, S_i, \{ESS_{\tau_i}\}_{i\in I} \right\rangle\right) = true$, the scenario set SS_{all}, which contains all possible extensions of S_r, is built:

$$SS_{all} = \bigcup_{i\in I} ESS_{\tau_i},$$

Denote $SS_{tree} = SSall \setminus S_r$. To warrant the **tree structure,** the following issues need to be guaranteed.

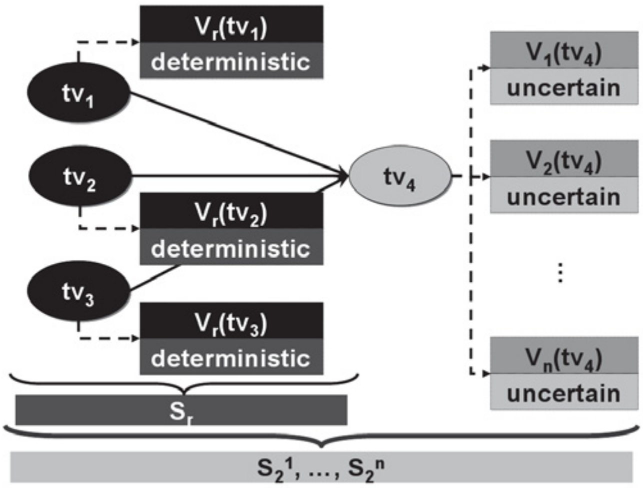
Acyclicity: for all scenarios in $S_i \in SS_{all}$, there is no path from S_i to itself:

$$\forall S_i \in SS_{all} : \nexists\, P_{SS_{all}}\left(S_i, S_i\right).$$

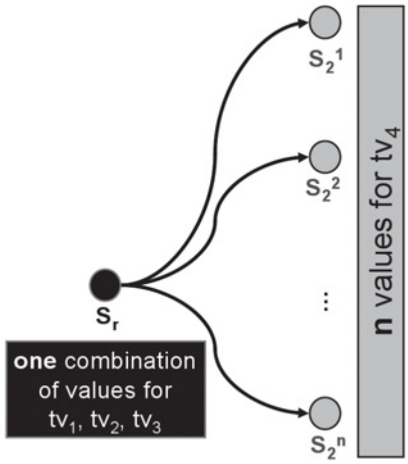
Branch structure: if there is a path $P_{SS_{all}}\left(S_1, S_n\right)$ that was generated from S_1 via a path of scenario extensions $\langle ESS_{\tau_2}, S_2, \tau_2 \rangle, \ldots, \langle ESS_{\tau_n}, S_n, \tau_n \rangle$, where each $S_i \in P_{SS_{all}}\left(S_1, S_n\right)$ is in ESS_{τ_i} $(i > 1)$, there is *no other path* $P_{SS_{all}}\left(S_1, S_n\right)$ from S_1 to S_n using an extension ESS^* not in $\{ESS_{\tau_2}, \ldots, ESS_{\tau_n}\}$.

Root connectedness: For each scenario S_i (except the root scenario S_r itself) there must be a path connecting the root to it:

$$\forall S_i \in SS_{tree} : \ \exists P_{SS_{all}}\left(S_r, S_i\right).$$



(a) Network Graph Representation



(b) Scenario Extension Tree Representation

Figure 3.7.: *Example of a Scenario Extension*

If $et\left\langle S_r, SS_{tree}, \{ESS_{\tau_i}\}_{i\in I}\right\rangle$ fulfils these properties,

$$is - et\left\langle S_r, SS_{tree}, \{ESS_{\tau_i}\}_{i\in I}\right\rangle = true.$$

By definition of scenario extension paths all scenarios in $P_{SSall}(S_r, S_i)$ are connected for any S_i. This implies that for each $tv_{j_k} \in STV_i \setminus STV_r$ and $tv_{j_1} \in STV_i \cap STV_r$ there is an ordered tuple of variables $TSTV_r^i = \langle tv_{j_1}, \ldots, tv_{j_k}\rangle$, $tv_{j_l} \in STV_i$ for all $l = 1, \ldots, k$, such that $\exists P_{DI_i}(tv_{j_1}, tv_{j_k})$. STV_r is said to be *connected to* tv_{j_k} in DI_i.

A brief example illustrates the notion of scenario extension trees. Consider a scenario S_r, where

$$\begin{aligned}
STV_r &= \{tv_1, tv_2, tv_3\}, \\
sv_r &= \{V_r(tv_1), V_r(tv_2), V_r(tv_3)\}, \\
status_r &= \{\text{“}deterministic\text{”}, \text{“}deterministic\text{”}, \text{“}deterministic\text{”}\}, \\
DI_r &= 0^{3\times 3}.
\end{aligned}$$

S_r consists of three isolated variables, each of which is assigned a single certain value. S_r is extended by a variable tv_4, whose status is *“uncertain"* and for which n possible values $V_1(tv_4), \ldots, V_n(tv_4)$ are determined. This can be represented as in Figure 3.7: the left side (cf. Figure 3.7(a)) shows the representation of the scenario as a network graph allowing for capturing the interdependencies of variables, and the right side (cf. Figure 3.7(b)) shows the scenario extension tree that represents the path of the scenario construction.

3.6.5. Scenario Extension Forests

A **scenario extension forest** is a set of scenario extension trees. Let $SS_r = \{S_r\}_{r \in R}$ be a set of root scenarios. It is required that from each $S_r \in SS_r$ a set of scenarios has been constructed via scenario extension. Hence, for each $S_r \in SS_r$, a scenario extension tree can be defined. The scenario extension forest SEF is defined as the tuple $SEF = \langle SS_r, SSET \rangle$, with

- $SS_r = \{S_r\}_{r \in R}$, where for any two scenarios S_{r_k} and $Root_{r_l}$ $(k \neq l)$ there is a difference in the sets sv_{r_k} and sv_{r_l}, i.e.,

$$\begin{array}{lrcl} & STV_{r_k} & \neq & STV_{r_l} \\ \text{or} \qquad & \exists tv_j \in STV_{r_k} \cap STV_{r_l} : V_{r_k}(tv_j) & \neq & V_{r_l}(tv_j). \end{array}$$

- $SSET$ is the union of all possible scenario extension trees given SS_r:

$$SSET = \bigcup_{i \in I} SS_i^{all} = \bigcup_{i \in I} \bigcup_{j \in J(i)} \mathrm{ESS}_i^{\tau_j},$$

  where SS_i^{all} contains all possible extensions $ESS_i^{\tau_j}$ $(j \in J(i))$ of S_{r_i}.

As $S_{r_i} \in et\langle S_{r_i}, ., . \rangle$, it follows $SEF \neq \emptyset$. Furthermore, as $S_{r_i} \neq S_{r_k}$ for all $i \neq k$ (see above, each scenario is part of one and only one tree. For each scenario $S_k \in \bigcup_{i \in I} \mathrm{SS}_i^{\mathrm{all}}$ it holds:

$$\exists^{=1} S_{r_{i^*}} : S_k \in et\left\langle S_{r_{i^*}}, SSE_{i^*}, \{ESS_{i^*}^{\tau_j}\}_{j \in J(i^*)} \right\rangle.$$

Scenario extension forests allow for following the construction of scenarios generated via scenario extensions from multiple root scenarios at a time. This is useful for scenario management, as in uncertain situations scenarios need to be constructed from a multiplicity of root scenarios S_r incorporating different assessments of the situation. Furthermore, in SBR & MCDA problems, one (set of) root scenario(s) per alternative arises, emphasizing the need for considering scenario extension forests.

3.7. Completion of Scenarios: Scenario Merging

In distributed reasoning frameworks, there can be an asymmetry in the time and effort required to determine values for different paths from the $SOURCE$ variables to one of the focus variables which are crucial for the scenario recipients (cf. Section 4.1.3). Reasons as to why this asymmetry arises may be that the assessment of some piece of information requires excessive analysis, or is computational expensive, an expert may only be available after a certain time, or some important basic information is not (yet) available.

In these cases, it can be helpful to not follow the extension tree structure layer by layer, but to start constructing scenarios on the basis of the information and expertise available. To this end, the novel **scenario merging procedure** presented in this section has been developed. This procedure enables bottlenecks to be identified and avoided and allows intermediate results to be achieved as fast as possible. Additionally, scenario merging is a means to facilitate scenario updates. Scenario updates modify the scenario according to a change in the information. Scenario merging enables the re-use of those parts of the scenario that were *not* affected by the new information. These parts are combined with the updated parts of the scenario (see Section 8.3.2).

The properties of the scenario resulting from merging two scenarios S_{i_1} and S_{i_2} depend on the internal structure and the content of both S_{i_1} and S_{i_2}. Ambiguous and cyclic scenarios may be generated. To ensure that the scenarios presented to the scenario recipients fulfil certain minimum requirements with respect to their structural and contextual correctness, the merged scenarios need to be filtered. To this end, they are classified into several sets. The filtering of scenarios for further scenario construction or presentation to the recipients can be more or less restrictive, depending on the recipients' preferences. If getting a broad spectrum of scenarios is important to them, and unambiguity of values or statuses is not required, they can specify that they are willing to accept scenarios which have a certain degree of ambiguity (cf. Section 8.2.2.2). The classification of scenarios into disjoint subsets of all scenarios arising by the scenario procedure allows the recipients to state their preferences by means of specification of some parameters. A similar process (selection of parameters) can also facilitate scenario selection (cf. Section 8.1).

3.7.1. Prerequisites: Classification of Scenarios

This section defines some prerequisites allowing for deducing basic requirements that ensure the usefulness of merging two scenarios, and classifying the resulting scenarios according to their degree of well-definedness.

3.7.1.1. Mergeable Pairs of Scenarios

Denote $et = et\left\langle S_r, SSE, \{ESS^{\tau_i}\}_{i \in I}\right\rangle$ a scenario extension tree, and let $P_{SSE}\left(S_r, S_{\max}\right)$ be a scenario extension path in et. A scenario $S_{\max} \in SSE$ is said to be a **maximum scenario extension** of S_r if and only if for all $S_i \in SSE$ with

$$\begin{aligned} STV_{\max} &\subseteq STV_i, \\ V_{\max}\left(tv_j\right) &= V_i\left(tv_j\right)\ \forall tv_j \in STV_{\max} \qquad \text{and} \\ status_{\max}\left(tv_j\right) &= status_i\left(tv_j\right)\ \forall tv_j \in STV_{\max} \end{aligned}$$

it holds $S_{max} = S_i$. Merging only maximum scenario extensions guarantees that the resulting scenario contains as much information as possible.

The following definition of *mergeable scenarios* specifies which pairs of scenarios are suitable for being combined. It is tested whether the combination of both scenarios enables the assessment of a variable's value that cannot be determined given information in only one of the scenarios. The scenario merging procedure is solely applied to pairs of mergeable scenarios.

Let $et_1 = et\left\langle S_{r_1}, SSE_1, \{ESS_1^{\tau_i}\}_{i \in I(1)}\right\rangle$, $et_2 = et\left\langle S_{r_2}, SSE_2, \{ESS_2^{\tau_i}\}_{i \in I(2)}\right\rangle$ be two scenario extension trees, $S_{r_1} \neq S_{r_2}$, and let $S_{\max_1}$ and $S_{\max_2}$ be two maximum extensions of S_{r_1} and S_{r_2} respectively. $S_{\max_1}$ and $S_{\max_2}$ are **mergeable,** if there are a variable tv^* and two scenarios S_1^* and S_2^* *tied to* $S_{\max_1}$ and $S_{\max_2}$ respectively such that

$$\begin{aligned} tv^* &\notin STV_{\max_1} \cup STV_{\max_2} \\ tv^* &\in STV_1^* \cap STV_2^* \\ \exists\, tv_j^1 \in STV_{\max_1} &: \exists\, P_{DI_1^*}\left(tv_j^1, tv^*\right) \\ \exists\, tv_k^2 \in STV_{\max_2} &: \exists\, P_{DI_2^*}\left(tv_k^2, tv^*\right). \end{aligned}$$

To determine the set of possible values of tv^*, $SPV\left(tv^*\right)$, the scenarios $S_{\max_1}$ and $S_{\max_2}$ are not sufficiently rich. Therefore, they are merged to a new scenario. If all scenarios stemming from et_1 and et_2 are source originated, the scenarios resulting from the scenario merging are also source originated by following the procedure defined in Section 3.7.2.

3.7.1.2. Classification of Not Well-Defined Scenarios

Before defining the scenario merging procedure, a classification scheme is developed facilitating the definition of requirements for acceptable merged scenarios. Let S_1 and S_2 be two mergeable scenarios. Let SMS be the set of merged scenarios created from S_1 and S_2. To characterise the scenarios arising, some subsets of SMS need to be considered, each of which reflects a potential conflict that can occur during the merging procedure. It can happen that the scenarios arising are ambiguous (see Section 3.3.1.2) with respect to some variables' values or statuses. Furthermore, cyclic scenarios may arise.

Value ambiguous merged scenarios: VAS. The resulting scenario $S^i_{1,2} \in VAS$ is value ambiguous, i.e.,

$$\exists tv_j \in STV_{(1,2)^i} : V_{(1,2)^*}(tv_j) = \{V_1(tv_j), V_2(tv_j)\},$$

where $V_1(tv_j) \neq V_2(tv_j)$

Value unambiguous merged scenarios: VUS. This set collects all $S^i_{1,2} \in SMS$, which are value unambiguous. According to the definition of VAS,

$$VUS = SMS \setminus VAS.$$

Status ambiguous merged scenarios: SAS. Each scenario $S^i_{1,2} \in SAS$ is status ambiguous, i.e.,

$$\exists tv_j \in STV_{(1,2)^*} : V_{(1,2)^*}(tv_j) = \{status_1(tv_j), status_2(tv_j)\},$$

where $status_1(tv_j) \neq status_2(tv_j)$.

As from the status ambiguity of a scenario it does not necessarily follow that it is also value ambiguous and vice versa, it is necessary to consider both sets VAS and SAS.

Status unambiguous merged scenarios: SUS. This set contains all $S^i_{1,2} \in SMS$, which are status-unambiguous. According to the definition that means:

$$SUS = SMS \setminus SAS.$$

Cyclic merged scenarios: CS. Each scenario $S^i_{1,2} \in CS$ is cyclic, i.e.,

$$\exists tv_j, tv_k \in STV_{(1,2)^i} : \exists P_{DI_{S^i_{1,2}}} (tv_j, tv_k) \wedge \exists P_{DI_{S^i_{1,2}}} (tv_j, tv_k) .$$

Acyclic merged scenarios: ACS. This set combines all $S^i_{1,2} \in SMS$ which are acyclic. According to the definition of CS:

$$ACS = SMS \setminus CS.$$

Structurally correct merged scenarios: SCS. Each scenario $S^i_{1,2} \in SCS$ is structurally correct. According to Section 3.3.1 this means that $S^i_{1,2}$ is within scenario value and status unambiguous andacyclic. Stated differently

$$S^i_{1,2} \in SCS \Leftrightarrow S^i_{1,2} \in SMS \setminus \{VAS \cup SAS \cup CS\} = VUS \cap SUS \cap ACS.$$

The sets VAS, SAS, and CS are not necessarily disjoint. To assess the degree of not well-definedness, the number of conflicts or requirement violations is determined. To this purpose, for each of the sets VAS, SAS, CS, the number of variables (or, for CS: pairs of variables) for which the conflict exists is assessed.

$\mathbf{VAS_{N(VA)}}$: define $VATV = \left\{tv_1, \ldots, tv_{N(VA)}\right\}$. Scenario $S^i_{1,2} \in SMS$ is value ambiguous of degree $N(VA)$ if $VATV$ is the minimum set, for which holds:

$$\forall tv_j \in STV_{(1,2)^i} \setminus VATV : \exists^{=1} V_{(1,2)^i} (tv_j) \in sv_{(1,2)^i} .$$

$\mathbf{SAS_{N(SA)}}$: define $SATV = \left\{tv_1, \ldots, tv_{N(SA)}\right\}$. Scenario $S^i_{1,2} \in SMS$ is status ambiguous of degree $N(SA)$ if $SATV$ is the minimum set for which $\forall tv_j \in STV_{(1,2)^i} \setminus SATV$:

$$\begin{aligned} & status_1 (tv_j) \in status_{(1,2)^i} \ \wedge \ status_2 (tv_j) \in status_{(1,2)^i} \\ \Rightarrow \quad & status_1 (tv_j) = status_2 (tv_j) . \end{aligned}$$

$\mathbf{CS_{N(Cyc)}}$: define $CSTV = \left\{tv_1, \ldots, tv_{N(Cyc)}\right\}$. Let $E\left(S^i_{1,2}\right) = \{e_{j,k}\} = \{(tv_j, tv_k)\}$ be the set of edges represented in $DI_{S^i_{1,2}}$. Set

$$\begin{aligned} NCSTV &= STV \setminus CSTV \\ \text{and} \quad E(NCSTV) &= \left\{e_{j,k} \in E\left(S^i_{1,2}\right) : tv_j \notin CSTV \wedge tv_k \in CSTV\right\} . \end{aligned}$$

Let DI_{NCSTV} be the dependency structure captured in an incidence matrix Inc_{NCSTV} that contains only rows for all vertices in $NCSTV$ and columns for the edges in $E(NCSTV)$. That means, compared to the original incidence matrix $Inc_{S^i_{1,2}}$ the rows r_i representing vertices in $CSTV$ and the columns k where at least one entry in row r_i is not equal to 0 are deleted. A scenario $S^i_{1,2} \in SMS$ is cyclic of degree $N(Cyc)$ if $CSTV$ is the minimum set for which the graph represented by Inc_{NCSTV} constructed as described above is acyclic.

These sets are used to determine the acceptability of merged scenarios. The scenario recipients can specify their requirements with respect to the scenario quality. Then, it is assessed which of the merged scenarios are sufficiently unambiguous to be passed on to further experts, and which need to be pruned. This issue is addressed in Section 8.2.2.

3.7.2. Scenario Merging Procedure

On the basis of the prerequisites specified in the previous Section 3.7.1 the scenario merging procedure, a new technique for distributed scenario construction, can be described. To this end, let et_1 and et_2 be two scenario extension trees, and denote $SS^i_{\max_1}$ and $SS^j_{\max_2}$ two sets of mergeable (maximum) scenarios drawn from et_1 and et_2 respectively. Define $List\left(SS^i_{\max_1}, SS^j_{\max_2}\right) = \{(i,j) : i \in I_1, j \in I_2\}$ the list of possible scenario combinations.

Step 1: Start.
Let S^1_i be a scenario in $SS^i_{\max_1}$ and S^2_j a scenario in $SS^j_{\max_2}$. Choose a pair (i,j) from $List\left(SS^i_{\max_1}, SS^j_{\max_2}\right)$, signifying that the combination of scenarios $\left(S^1_i, S^2_j\right)$ has not been investigated yet.

Step 2: Scenario merging.
To combine S^1_i and S^2_j to a new scenario $S_{(i,j)^{1,2}}$ the set of variables, the values and statuses for each of these variables and their interdependencies must be determined on the basis of S^1_i and S^2_j:

Set of typed variables: $STV_{(i,j)^{1,2}} = STV^1_i \cup STV^2_j$.

Values: denote $STV_{i^1 \cap j^2}$ the intersection of STV^1_i and STV^2_j. If there are $N \geq 1$ variables $tv^1_k, \ldots, tv^N_k \in STV_{(i,j)^{1,2}}$ with conflicting values $V_{i_1}\left(tv^l_k\right) \neq V_{j_2}\left(tv^l_k\right), \forall l \in \{1, \ldots, N\}$, S^1_i and S^2_j *cannot* be merged to a value unambiguous scenario (see Section 3.3.1). In this case, both values for all tv_k^l $(l = 1, \ldots, N)$ are admitted, i.e.,

$$sv_{(i,j)^{1,2}} \ni V_{(i,j)^{1,2}}\left(tv_k^l\right) = \left\{V_{i_1}\left(tv_k^l\right), V_{i_1}\left(tv_k^l\right)\right\} \qquad (l = 1, \ldots, N).$$

If for all $tv_l \in STV_{i^1 \cap j^2} \smallsetminus \left\{tv_k^1, \ldots, tv_k^N\right\}$ it holds $V_{i_1}(tv_l) = V_{j_2}(tv_l)$, $S_{(i,j)^{1,2}}$ is assigned to the set of value-ambiguous scenarios of degree N, VAS_N and the non-ambiguous values are determined for the remaining $tv_l \in STV_{i^1 \cap j^2}$:

$$sv_{(i,j)^{1,2}} \ni V_{(i,j)^{1,2}}(tv_l) = V_{i_1}(tv_l) = V_{i_2}(tv_l).$$

Otherwise, i.e., if for all $tv_l \in STV_{i^1 \cap j^2}$ it holds that $V_{i_1}(tv_l) = V_{i_2}(tv_l)$, then the value of all $tv_l \in STV_{i^1 \cap j^2}$ is set to

$$sv_{(i,j)^{1,2}} \ni V_{(i,j)^{1,2}}(tv_l) = V_{i_1}(tv_l) = V_{j_2}(tv_l)$$

and $S_{(i,j)^{1,2}}$ is assigned to the set of value-unambiguous scenarios, VUS.

Having established how to merge scenarios with respect to the values of the $tv_k \in STV_{i^1 \cap j^2}$, now the remaining variables' values in $STV_i^1 \smallsetminus STV_j^2$ and $STV_j^2 \smallsetminus STV_i^1$ need to be addressed. For these variables the original values are adopted:

$$\begin{aligned} sv_{(i,j)^{1,2}} \ni V_{(i,j)^{1,2}}\left(tv_k^1\right) &= V_{i_1}\left(tv_k^1\right) \ \forall\ tv_k^1 \in STV_i^1 \smallsetminus STV_j^2 \\ sv_{(i,j)^{1,2}} \ni V_{(i,j)^{1,2}}\left(tv_k^2\right) &= V_{j_2}\left(tv_k^2\right) \ \forall\ tv_k^2 \in STV_j^2 \smallsetminus STV_i^1. \end{aligned}$$

Finally, the value of tv^* is set to the default value:[17]

$$V_{(i,j)^{1,2}}\left(tv_k^1\right) = \infty.$$

Statuses: Again, the starting point is determining the status of all variables in $STV_{i^1 \cap j^2}$ by adopting the status from S_i^1 and S_j^2 in the following way: Assuming that there are $M \geq 1$ variables $tv_k^1, \ldots, tv_k^M \in STV_{i^1 \cap j^2}$ with conflicting statuses, i.e., $\forall l = 1, \ldots, M\}$

$$status_{i_1}\left(tv_k^l\right) \neq status_{j_2}\left(tv_k^l\right),$$

[17] tv^* is the variable which motivated the scenario merging, as there are scenarios, for which the determination of $V(tv^*)$ requires information on scenarios from both et_1 and et_2.

S_{i_1} and S_{j_2} are merged to a status ambiguous scenario. In this case, both statuses for all tv_k^l ($l = 1, \ldots, M$) are admitted, i.e., for $l = 1, \ldots, M$

$$status_{(i,j)^{1,2}} \ni status_{(i,j)^{1,2}}\left(tv_k^l\right) = \left\{status_{i_1}\left(tv_k^l\right), status_{i_1}\left(tv_k^l\right)\right\}.$$

If for all $tv_k \in STV_{i^1 \cap j^2} \setminus \left\{tv_k^1, \ldots, tv_k^M\right\}$, $status_{i_1}(tv_k) = status_{j_2}(tv_k)$, $S_{(i,j)^{1,2}}$ is assigned to the set of status-ambiguous of scenarios of degree M, SAS_M.

If for all $tv_k \in STV_{i^1 \cap j^2}$ it holds $status_{i_1}(tv_k) = status_{i_2}(tv_k)$, the status of all $tv_k \in STV_{i^1 \cap j^2}$ is set to

$$status_{(i,j)^{1,2}} \ni status_{(i,j)^{1,2}}(tv_k) = status_{i_1}(tv_k) = status_{j_2}(tv_k)$$

and $S_{(i,j)^{1,2}}$ is assigned to the set of status-unambiguous scenarios, SUS. For the variables $tv_k^1 \in STV_i^1 \setminus STV_j^2$, the original statuses are adopted, and analogously, one procedes for the variables $tv_k^2 \in STV_j^2 \setminus STV_i^1$.

The status of tv^* is set to the default status: $status_{(i,j)^{1,2}}(tv^*)$ = “not assigned".

Dependence: A new incidence matrix $Inc_{(i,j)^{1,2}}$ representing $DI_{(i,j)^{1,2}}$ for all variables in $STV_{(i,j)^{1,2}}$ can be created by combining the incidences matrices Inc_i^1 for DI_i^1 and Inc_j^2 for DI_j^2 (cf. Section 3.2.5). To determine the incidence matrix $Inc_{(i,j)^{1,2}}$, first the order of vertices $tv_k \in STV_{(i,j)^{1,2}}$ and edges $e \in E_{(i,j)^{1,2}}$ must be determined. To this end, consider the following (disjoint) partitions of the set of vertices:

$$\begin{aligned} STV_{i^1 \cap j^2} &= \left\{tv_k \in STV_i^1 \cap STV_j^2\right\} \\ STV_{i^1 \setminus j^2} &= \left\{tv_k \in STV_i^1 \setminus STV_j^2\right\} \\ STV_{j^2 \setminus i^1} &= \left\{tv_k \in STV_j^2 \setminus STV_i^1\right\} \end{aligned}$$

To detect inconsistencies and cycles consider the following sets of edges:

$$\begin{aligned} DI_{i^1 \cap j^2} &= \left\{e : e \in DI_i^1 \cap DI_j^2\right\}, \\ DI_{i^1 \setminus j^2} &= \left\{e : e \in DI_i^1 \setminus DI_j^2\right\}, \\ DI^*_{i^1 \setminus j^2} &= \left\{e : e_{k,l} = (tv_k, tv_l) \in DI_{i^1 \setminus j^2} : tv_k, tv_l \in STV_{i^1 \setminus j^2}\right\} \subseteq DI_{i^1 \setminus j^2} \\ DI^{pot-cyc}_{i^1 \setminus j^2} &= = DI_{i^1 \setminus j^2} \setminus DI^*_{i^1 \setminus j^2} \end{aligned}$$

and analogously for $DI_{j^2 \setminus i^1}$, $DI^*_{j^2 \setminus i^1}$, $DI^{pot-cyc}_{j^2 \setminus i^1}$.

To detect cycles that may arise when merging S_i^1 and S_j^2, the set DI^{cyc} of paths $P(tv_k, tv_l) = \{e_{k,j_1}, \ldots, e_{j_n,l}\}$ in DI_i^1, which can be closed to a cycle by adding a path $P(tv_l, tv_k)$ in DI_j^2 is formed:

$$DI^{cyc} = \left\{ P_{DI_i^1}(tv_k, tv_l) : \exists P_{DI_j^2}(tv_l, tv_k) \right\}.$$

$P(tv_l, tv_k)$ can be any path connecting tv_l and tv_k in DI_j^2. Particularly, it is not required that the path $P_{DI_j^2}(tv_l, tv_k) = \{e_1, \ldots, e_Z\}$ corresponds to a path in the opposite direction $\{e_Z, \ldots, e_1\}$ in DI_i^1. Furthermore, according to the definition of above sets cycles can only occur by merging the potential cycle sets $DI_{i^1 \setminus j^2}^{pot-cyc}$ and $DI_{j^2 \setminus i^1}^{pot-cyc}$.

If $|DI^{cyc}| = L$, $S_{(i,j)^{1,2}}$ is assigned to the set of cyclic scenarios of degree L, CS_L. If $DI^{cyc} = \varnothing$, then $S_{(i,j)^{1,2}}$ is assigned to the set of acyclic scenarios ACS.

To define an incidence matrix $Inc_{(i,j)^{1,2}}$ representing $DI_{(i,j)^{1,2}}$ without loss of generality the following labelling is used:

$$\begin{aligned} STV_{i^1 \cap j^2} &= \{tv_1, \ldots, tv_{n_1}\}, \\ STV_{i^1 \setminus j^2} &= \{tv_{n_1+1}, \ldots, tv_{n_2}\} \\ \text{and} \quad STV_{i^1 \setminus j^2} &= \{tv_{n_2+1}, \ldots, tv_N\} \end{aligned}$$

is the ordering of variables chosen and

$$\begin{aligned} DI_{i^1 \cap j^2} &= \{e_1, \ldots, e_{m_1}\}, \\ DI^*_{i^1 \setminus j^2} &= \{e_{m_1+1}, \ldots, e_{m_2}\}, \\ DI^*_{j^2 \setminus i^1} &= \{e_{m_2+1}, \ldots, e_{m_3}\}, \\ DI_{i^1 \setminus j^2}^{pot-cyc} &= \{e_{m_3+1}, \ldots, e_{m_4}\} \\ \text{and} \quad DI_{i^1 \setminus j^2}^{pot-cyc} &= \{e_{m_4+1}, \ldots, e_M\} \end{aligned}$$

the ordering of edges.

Using these definitions, an incidence matrix $Inc_{(i,j)^{1,2}}$ can be defined by completing the incidence matrix by ordering the vertices $STV_i^1 \cup STV_j^2$ as in (1) and the edges in $DI_i^1 \cup DI_j^2$ as specified in (2). Then, the columns from Inc_i^1 and Inc_j^2 can be copied to the new scheme (adding eventually rows with zeros, if a vertex is not part of the respective scenario).

Summarised, the merged scenario $S(i,j)^{1,2}$ is completed and assigned to

- the set of value ambiguous or value unambiguous scenarios,

- the set of status ambiguous or status unambiguous scenarios,
- the set of cyclic or acyclic scenarios.

$S_{(i,j)^{1,2}}$ is structurally correct if and only if $S_{(i,j)^{1,2}}$ is value unambiguous, status unambiguous and acyclic.

Step 3: Maintaining Traceability.
After having merged S_i^1 and S_j^2, the scenario extension trees et_1 and et_2 need to be considered. Let $P_{SSE_1}\left(S_{i_0}^1, S_i^1\right)$ be the scenario extension path within et_1, from which S_i^1 was constructed. Furthermore, as S_i^1 is source-originated, assume that the root scenario $S_{i_0}^1$ is the scenario, for which $STV_{i_0}^1 = SOURCE_i^1$. Such a scenario exists, as S_i^1 is source originated by the definition of mergeable extension trees. Analogously, let $P_{SSE_2}\left(S_{j0}^2, S_j^2\right)$ be the scenario extension path within et_2, connecting $S_{j_0}^2$ with $STV_{j_0}^2 = SOURCE\left(S_j^2\right)$ to the scenario S_j^2.

Both paths are combined to a new scenario extension path

$$P_{SSE_1,SSE_2}\left(\left(S_{i_0}^1, S_i^1\right), \left(S_{j0}^2, S_j^2\right)\right) = \left\{P_{SSE_1}\left(S_{i_0}^1, S_i^1\right), P_{SSE_2}\left(S_{j_0}^2, S_j^2\right)\right\}.$$

By this construction, the paths are not merged arbitrarily, but kept separately reflecting the fact that S_i^1 and S_j^2 were constructed independently.

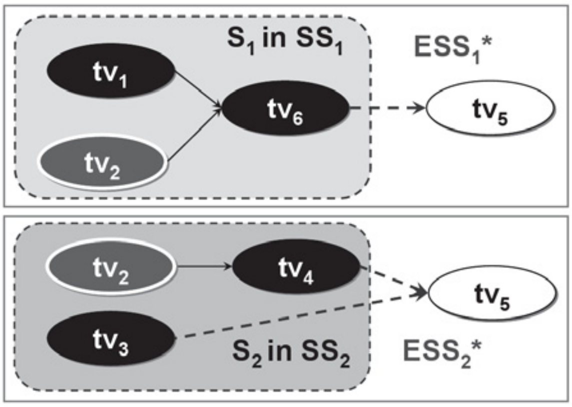
Step 4: Iteration.
Finally, the pair (i_1, j_2) is deleted from the list of scenarios to be merged, i.e.,

$$List_{new}\left(SS_{\max_1}^i, SS_{\max_2}^j\right) = \{(i,j) : i \in I_1, j \in I_2\} \setminus \{(i_1, j_2)\}.$$

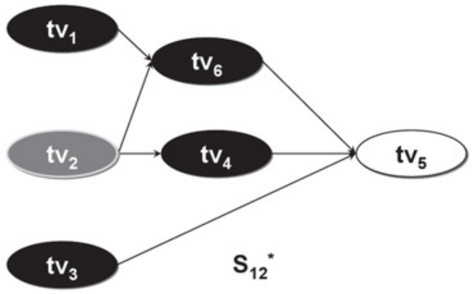
Next, another combination (i_1^*, j_2^*) still on $List_{new}\left(SS_{\max_1}^i, SS_{\max_2}^j\right)$ is chosen and the scenario procedure giving rise to $S_{(i,j)^{1,2}}^*$ is followed. This is done until

$$List_new\left(SS_{\max_1}^i, SS_{\max_2}^j\right) = \varnothing.$$

To illustrate the scenario merging procedure consider the following small example: given is the situation as shown in Figure 3.8(a). There are two sets of maximum scenarios SS_1 and SS_2 with root scenarios $S_{i_0}^1$ and $S_{j_0}^2$, where $STV_{i_0}^1 = \{tv_1, tv_2\} = SOURCE_{i_0}^1$ and $STV_{j_0}^2 = \{tv_2, tv_3\} = SOURCE_{j_0}^2$ respectively. Consider (source-originated) scenarios S_1 and S_2, which are both tied to tv_5. According to the definition, they are mergeable and allow for determining the possible value(s) of tv_5. The resulting set of scenario $S_{1,2}^*$ can be represented graphically as a network such as shown in Figure 3.8(b).



(a) Scenarios to be Merged



(b) Resulting scenario

Figure 3.8.: *Merging Two Scenarios*

3.8. Summary

This chapter provided a formal definition of single scenarios and sets of scenarios. Basically, a scenario is a set of variables (of a certain type), which are assigned a value and a status. In addition, the dependency relation states how values of variables depend on other variables' values. Graphically, a scenario is represented as a network of interdependent variables. This formalisation provides the framework for the construction of meaningful scenarios in a distributed approach. Particularly, the concept of *status* allows different types of uncertainties to be integrated into the scenario. In summary, the formalisation satisfies the achievement of objective O.1.

On the basis of this definition, further characterisations and **requirements on a single scenario's properties** have been defined. Table 3.3 summarises which basic determinants for a scenario's credibility and acceptance (as defined in Section 3.1) can be inspected by means of the scenario formalisation. For those features that can not be addressed directly, table 3.3 indicates how they can be warranted.[18] Thus, the formalisation provided enables an operationalisation of quality requirements and therefore facilitates achieving objective O.3.

The formalisation of **sets of scenarios** enables the use of scenarios to capture uncertainty on a variable's value in a well-structured manner. Additionally, this

[18]The identification of best timely available experts is not the focus of this thesis. Yet, it should be noted that negotiation protocols ensuring that the best timely available expertise is identified and used for the scenario building have been developed [Pavlin et al., 2009b,a].

Table 3.3.: *Determinants of Scenario Credibility*

Determinant of Credibility	**Consideration**
Credibility of sources	Identification of best timely available expertise
Credibility of the narrative	Well-definedness Structural correctness Relevance Completeness (Conceptual) coherence
Credibility of content	Plausibility: support of scenario recipients Consistency
Credibility of the scenario construction	Connectivity Relevance (Conceptual) coherence
Credibility of channel of dissemination	Use of MCDA visualisation methods Presentation of results by a trained facilitator

formalisation represents and exploits the **relationships of scenarios** and sets of scenarios for managing the (dynamic) scenario construction. This is an important feature, as the evolution of scenarios reflects the availability of more and more information or a better understanding of the situation. Changes of available information are manifested as additions to, subtractions from, or changes to both the scenario content and its structure. Currently available scenario representations are deficient in providing a mechanism in which scenarios can be created and managed. The newly developed techniques for **scenario continuation, extension** and **merging** capture these processes. Particularly, the scenario merging procedure allows for partially complete scenarios to be combined that have been developed independently and in that manner facilitates *distributed* scenario construction.

4. A Framework for Sound Scenario-Based Reasoning

> Never try to walk across a river
> just because it has an average depth of four feet.
> (Milton Friedman)

This chapter is dedicated to the assessment of the scenarios' quality with respect to the needs and requirements of the scenario recipients: the scenarios built must be purposeful and credible (objectives O.2 and O.3). Objective O.2 is addressed in Section 4.1, where a framework for ensuring that the scenarios contain the information the recipients require is developed. Section 4.3 provides a set of criteria for evaluating a scenario's quality with respect to its content. Particularly, the issues of plausibility, coherence, consistency and reliability are addressed. In this manner, the foundation for achieving objective O.3 is laid.

4.1. Structural Requirements on Sets of Scenarios for Sound Scenario-Based Reasoning

This section is dedicated to mechanisms ensuring that the scenarios serve the recipients' information needs *(objective O.2).* Some important concepts need to be made explicit to characterise the requirements a set of scenarios must fulfil on a structural level to ensure that it serves its purpose. The idea of *decision alternatives* in MCDA has already been discussed in Section 2.1. Here, the notion of alternatives in the realm of SBR & MCDA and the requirements for sound scenario-based decision support are explored. The following five concepts have been newly developed within this thesis. Beyond making the required information explicit, these concepts facilitate the implementation of both SBR & SM and SBR & MCDA in distributed settings. The *backdrop* captures background information necessary to solve the problem considered. It characterises the problem, the purpose of the SBR activity and the basic conditions, under which the SBR activity takes place. The set of *focus variables* provides a description of variables that must be addressed by the scenarios. In this manner, it serves as a means to ensure the relevance of the scenarios and to forestall infinite scenario building. The set of *seed variables* allows possible starting points for the scenario construction to be defined. The *initial situation* provides a set of description of (possibly uncertain) initial situations, on the basis of which the scenarios must be determined. Finally, the *conceptualisation* is a means to make explicit by whom and based on which techniques the values of the variables are determined.

The definitions in the following sections are mostly given for the general case, where the scenarios can be used to raise situation awareness (SBR & SM) or for a specific decision-making problem (SBR & MCDA). Whenever necessary, further specifications and adaptations for the application of scenarios SBR & MCDA are made. But first, consider the particular role of alternatives in SBR & MCDA.

4.1.1. Decision Alternatives

The purpose of SBR & MCDA is the selection of one alternative out of a finite set of feasible options. Hence, the alternatives that need to be evaluated play a crucial role in SBR & MCDA. Using the scenario formalisation, the set of alternatives $A = \{a_1, \dots, a_k\}$ is represented by means of a variable tv_A, where $Range\left(V\left(tv_A\right)\right) = A$.

As all alternatives need to be evaluated, it is mandatory that for each $a_l \in A$ ($l = 1, \dots, k$) at least one scenario $S\left(a_l\right)$ is constructed. To ensure that this requirement is met, the choice of the value $V\left(tv_A\right)$ is part of the start of the scenario generation. Setting $V_{SS(a_l)}\left(tv_A\right) = a_l$ ($l = 1, \dots, k$) the construction of a set of scenarios $SS\left(a_l\right)$ for each alternative is initiated.

4.1.2. Backdrop

The backdrop classifies scenarios describing the same situation and serving the same purpose to one common category. The backdrop can be interpreted as a scenery for the scenarios developed. It provides background information essential for understanding the dynamics of each scenario. Additionally, the backdrop contains meta-information on the requirements for the scenario building (e.g., the time available until the scenarios must be completed, requirements on the scenarios' quality).

This thesis distinguishes three types of backdrops according to the information they provide. The minimal or *M-backdrop* contains mandatory information without which a sound SBR is impossible. The *backdrop* contains further information for tailoring the scenarios and the SBR process to the recipients' needs. Finally, the *MCDA-backdrop* provides further information mandatory for SBR & MCDA.

A minimal or **M-backdrop** comprises descriptions of

- the (sense-making or decision) *problem* at hand (including the time and, if relevant, the physical location, e.g., an incident description),
- the *purpose* of the scenario construction (e.g., general sense-making or a specific decision problem to be solved),
- the decision makers or *scenario recipients.*

Beyond the content of an M-backdrop, the **backdrop** may contain

- *further information about the problem* such as past, ongoing or upcoming decisions that influence the problem or the resources available,
- *constraints* for scenario building and *requirements* each scenario must fulfil, e.g., the time frame for the scenario building process along with a description of the completeness of this process, the connectedness of scenarios to ensure coherence, the maintenance of a valid initial situation description and a focus of relevant variables (cf. Sections 4.1.5 and 4.1.3 respectively), the minimum required degree of reliability (cf. Section 4.3.4), a set of indicators for scenario selection (cf. Section 8.1.4) and the maximum acceptable degree of *ambiguity* (cf. Section 8.2).

In SBR & MCDA, an **(M-)MCDA-backdrop** must be used. Beyond the information in the (M-)backdrop it must contain the following information on the decision problem:

- a set of feasible *alternatives* A to be evaluated,
- an *attribute tree* providing the hierarchical structure of goals,
- the *preferences* of the decision makers (value functions and weights),
- the *aggregation methods* to apply.

4.1.3. Focus Variables

The concept of *focus variables* is used to express that the values of certain variables *must* be included in the completed scenarios. In this manner, the set of variables $FOCUS$ can be employed to prevent infinite expansion of a scenario:

when all variables in $FOCUS$ have been assigned a value, the scenario is considered to contain sufficient information for the recipients given the specific purpose of the scenario construction.

The set $FOCUS$ depends on the situation at hand, i.e., on the purpose of the SBR process and the information needs of the scenarios' recipients. Therefore, $FOCUS$ depends on the backdrop B: $FOCUS = FOCUS(B)$. For reasons of brevity and clarity, this thesis sticks to the short notation $FOCUS$ without explicitly referring to the dependence on B.

A scenario S_i is called *focussed* if it includes all variables specified in $FOCUS$, i.e., $FOCUS \subseteq STV_i$[19]. The relation $is\text{-}focussed\,(S_i)$ has as result *true* or *false.* $is\text{-}focussed \,:\; SS \rightarrow \{0,1\}$, where SS is a set of scenarios, $S_i \in SS$. The $is\text{-}focussed$ relation can be generalised for sets of scenarios. The relation $is\text{-}focussed(SS)$ has as result either true or false. Assuming that all scenarios in $SS = \{S_i\}_{i \in I}$ share the same set of focus variables, $is\text{-}focussed(SS) = true$ if each $S_i \in SS$ is focussed with respect to $FOCUS$:

$$is\text{-}focussed_j\,(SS) = \prod_{i \in I} is\text{-}focussed_j\;(S_i)\,.$$

Chapter 6 introduces mechanisms that guarantee that sets of scenarios used for evaluating common decision problems share the same $FOCUS$.

The set of variables $FOCUS = \left\{tv_j^F\right\}_{j \in J(FOCUS)}$, where $J(FOCUS)$ is an index set characterising the variables in $FOCUS$, fulfils the role of focussing scenario construction (via extension or merging) by requiring that all variables in $FOCUS$ must be assigned a value $\neq \infty$. A scenario is considered *focus complete*, if all variables in $FOCUS$ are assessed and assigned a value. Therefore, S_i is termed focus complete if and only if

$$\begin{array}{lrcl} & is\text{-}focussed\,(S_i) & = & true \\ \text{and} \qquad & \forall\; tv_j^F \in FOCUS:\; V_i\left(tv_j^F\right) & \neq & \infty. \end{array}$$

A set of scenarios SS sharing the same $FOCUS$ is termed focus complete if and only if each $S_i \in SS$ is focus complete.

[19] Equality is allowed for the sets $FOCUS$ and STV_i. In this case, all variables in the scenarios are required and their values must be presented to the recipients.

4.1.4. Seed Variables

The set of seed variables $SEED = SEED(B)$ defined for a given backdrop B contains variables that can be independent from information about any other variable's value. The purpose of $SEED$ is the definition of the *boundaries* of the system under consideration. As such, $SEED$ is part of the approach to prevent infinite expansion of a scenario network $\mathcal{G}_i$ by identifying the starting points for the scenario construction. The $SEED$ variables' values may be based on measurements, observations or expert assessments.

$SEED = SEED(B)$ is defined as a set $SEED = \left\{tv_j^{SEED}\right\}_{j\in J(SEED)}$. Each variable tv_j^{SEED} is characterised by the fact that there is a configuration of the reasoning framework (taking into account the information in B), for which the variable's value does *not* depend on the value of any other variable covered in the reasoning framework. Formally, this means that for $tv_j^{SEED} \in SEED$ there is a set of well-defined and focussed scenarios $SS = \{S_i\}_{i\in I(B)}$ that can be constructed respecting B such that for all $S_i \in SS$ and for all $tv_j^{SEED} \in SEED \cap STV_i$ it holds:

$$\Psi_i\left(tv_j^{SEED}\right) = \widetilde{\Psi}_i\left(tv_j^{SEED}\right) = \varnothing.$$

When representing the scenarios $S_i \in SS$ graphically, the variables $tv_j^{SEED} \in SEED$ are characterised by *not* having any incoming edges.

MCDA-Seed Variables. When faced with an SBR & MCDA problem, the concept of $SEED$ must be adapted in the following way: the set of MCDA-seed variables $SEED_{MCDA} = SEED_{MCDA}(B)$ consists of

- the set of alternatives A, represented by $SPV(tv_A)$, see Section 4.1.1.
- the set of variables $SEED_A$, where $tv_j^{SEED_A} \in SEED_A$ requires that $tv_j^{SEED_A} \neq tv_A$ and each $tv_j^{SEED_A} \in SEED_A$ does not depend on any variable *besides* possibly tv_A.[20]

[20]This dependence may vary with different values that tv_A can take.

Summarised, $SEED_{MCDA} = tv_A \cup SEED_A$. For each variable $tv_j^{SEED_A} \in SEED_{MCDA}$, there is a set of well-defined and focussed scenarios $SS(a_l)$ constructed to evaluate a_l given B such that for all $S_i \in SS(a_l)$:

$$\begin{aligned} tv_A &\in STV_i \\ V_i(tv_A) &= a_l \\ \widetilde{\Psi}\left(tv_j^{SEED_A}\right) &\in \{\varnothing, \{tv_A\}\}. \end{aligned}$$

Representing S_i graphically, the variables $tv_j^{SEED_A} \in SEED_{MCDA}$ are characterised by not having any incoming edges besides potentially an incoming edge from tv_A.

To clarify the relation of $SEED$ and $Source$, recall that a set of scenarios SS has a **valid source** if all variables within the source of the root scenario belong to the set of seed variables (MCDA-seed variables for SBR & MCDA problems). Again, the relation $is\text{-}valid\text{-}source(SS)$ has a result either *true* or *false*- $is\text{-}valid\text{-}source(SS) = true$ if and only if

$$\begin{aligned} &SOURCE \subseteq SEED && \text{for SBR \& SM} \\ &SOURCE \subseteq SEED_{MCDA} && \text{for SBR \& MCDA} \end{aligned}$$

4.1.5. Initial Situation Description

The initial situation description $INIT$ contains information that must be addressed by the set of scenarios constructed: for some variables sets of possible values are provided. The initial situation is not a scenario by itself, as for each variable multiple possible values may be provided, and it does not contain information on the variables' dependencies and statuses. The initial situation has a number of uses, which include requiring that specific information is taken into account during the scenario construction. Formally, $INIT$ is defined as the tuple $INIT = \langle STV_{init}, SSPV_{init} \rangle$, where

- $STV_{init} = \left\{tv_j^{init}\right\}_{j \in J(INIT)}$ is a set of variables capturing the information that is required to be taken into account.
- $SSPV_j^{init} \subseteq Range\left(V\left(tv_i^{init}\right)\right)$ associates to each $tv_j^{init} \in STV_{init}$ a finite set of possible values. It is required that for all $tv_j^{init} \in STV_{init}$

$SPV\left(tv_j^{init}\right) \neq \varnothing$. These values reflect the actual information on tv_j^{init}'s value that is required to be taken into account. Finally,

$$SSPV_{init} = \bigcup_{i \in I^{init}} SPV\left(tv_i^{init}\right).$$

Based on the initial situation description $INIT$, a set of scenarios SS^{init} is constructed by using scenarios $S_i^{init} \in SS^{init}$ as root scenarios for further scenario construction. The scenarios S_i^{init} are defined by setting $N = |STV_{init}|$,

$$S_i^{init} = \left\langle STV_{init}, sv_i^{init}, status_i^{init}, 0^{1\times N}\right\rangle,$$

where sv_i^{init} is a combination of values

$$\left(V_{i_{j_1}}\left(tv_{j_1}\right), \ldots, V_{i_{j_N}}\left(tv_{j_N}\right)\right) \in SPV\left(tv_{j_1}^{init}\right) \times \ldots \times SPV\left(tv_{j_N}^{init}\right),$$

$status_i^{init}$ is an according combination of statuses and DI^{init} is the 0-vector of length N, as it is assumed that no dependencies between the values in $SSPV_{init}$ exist. This independence assumption justifies also the combination of values.

The idea of using $INIT$ to construct scenarios taking into account specific pieces of information is the basis for defining *relevant initial situation descriptions.* If the purpose of SBR can be operationalised by the request to determine possible combinations of values for a set of variables $FOCUS$ the set of relevant variables within $INIT$, called $INIT^{rel}$, can be determined by

$$STV^{init_{rel}} = \left\{tv_j^{init} \in STV^{init} : \exists S_i \wedge \exists tv_k^F \in FOCUS : \exists P_{DI_j}\left(tv_j^{init}, tv_k^F\right)\right\},$$

where S_i is any scenario serving the purpose defined.

A scenario S_i is said to have a *valid initial situation* if

$$\begin{array}{lrcl} & STV_i \cap STV^{init_{rel}} & = & STV^{init_{rel}} \\ \text{and} & V_i\left(tv_j^{init}\right) & \in & SPV^{init}\left(\mathrm{tv}_j^{init}\right), \end{array}$$

i.e., S_i includes all variables in $INIT^{rel}$ and the value these variables take are required to be present in the set of scenarios. The relation $is-valid-INIT^{rel}\left(S_i\right)$ has as result either *true* or *false.*

A set of scenarios SS has a valid initial situation, if each of the scenarios in the set has a valid initial situation for the same initial situation $INIT$. *is-valid-*$INIT^{rel}(SS)$ has as result either true or false and

$$\textit{is-valid-}INIT^{rel}(SS) = \prod_{i \in I} \textit{is-valid-}INIT^{rel}(S_i).$$

In general, the relation between the source of S_i and $INIT$ (or $INIT^{rel}$) is not predetermined. Particularly, it may happen that $SOURCE(S_i) \notin INIT$ and $INIT \notin SOURCE(S_i)$. To check whether or not a scenario has been constructed from an appropriate starting point, the validity considerations for the source and the initial situation are combined in the following way: S_i has a **valid origin** if and only if

$$\begin{array}{lrcl} & \textit{valid-source}(S_i) & = & true \\ \text{and} & \textit{is-valid-}INIT^{rel}(S_i) & = & true. \end{array}$$

For each scenario S_i the relation $\textit{is-valid-origin}(S_i)$ has as a result either true or false. For sets of scenarios SS, this relation is represented by the function $\textit{valid-origin}: \mathcal{P}(SS) \to \{0,1\}$.[21] A set of scenarios SS has a valid origin if and only if for each $S_i \in SS$: $\textit{valid-origin}(S_i) = true$.

4.1.6. Conceptualisation

When building scenarios in distributed reasoning frameworks, it is useful to make explicit *by whom* and *how* a value of a variable is determined. Such traceability enhances trust and confidence in the information contained in a set of scenarios [Howick et al., 2008]. Particularly, the availability of more credible experts or of information derived by more credible concepts can be taken into account in scenario updating (cf. Section 8.3). In addition, it can be used as a basis for scenario pruning (cf. Section 8.2.2).

Following Laurence and Margolis [1999] this thesis understands **concepts** as representations, whose "*structure consists in their relations to other concepts specified by their embedding theories*" [Laurence and Margolis, 1999]. The conceptualisation of a variable is understood as the reasoning principle, method or theory used to determine its value. A concept can, e.g., refer to the expertise used or

[21] $\mathcal{P}(SS)$ is the powerset of SS.

on the different tools and algorithms one and the same expert may use to determine a variable's value. For example, in emergency management an expert may have to choose a guideline to determine the hazard potential of the concentration of a chemical. Some values are more accurate or more widely accepted than others, however, these may not be available for all chemical compounds [Cox, 1994]. In other cases (e.g., determination of a plume shape) some concepts may provide more accurate results, but the assessment of the values using these concepts might be more time consuming. These issues are crucial for scenario updating (cf. Section 8.3.2).

Let $SS = \{S_i\}_{i \in I}$ be a set of scenarios. Denote $\mathfrak{C}^*_{SS} = \mathfrak{C}^*_{SS}(B)$ the concepts to determine the value of any variable $tv_j \in \bigcup_{i \in I} STV_i$. These concepts depend on the backrop B, as B contains information on the framing and context of the problem to solve and allows the experts available and relevant to be determined. That means,

$$\mathfrak{C}^*_{SS} = \bigcup_{i \in I} \mathfrak{C}\left(STV_i\right) = \bigcup_{i \in I} \bigcup_{j \in J(i)} \mathfrak{C}\left(tv_j\right),$$

where $STV_i = \{tv_j\}_{j \in J(i)}$ and $\mathfrak{C}\left(STV_i\right)$ is the set of concepts available to determine the value of each $tv_j \in STV_i$ (denoted $\mathfrak{C}\left(tv_j\right)$). To consider situations, in which the concepts available to determine a variables' value are not known, a default value is defined by setting $\mathfrak{C}_{default}\left(tv_j\right) = \text{“}unknown\text{”}$. Finally,

$$\mathfrak{C}_{SS} = \mathfrak{C}^*_{SS} \cup \{\text{“}unknown\text{”}\}.$$

A variable $tv_j \in STV_i$ is termed *conceptualised*, if the concept used for determining its value is known, i.e.,

$$\begin{array}{rrcl} & tv_j & \notin & NA \\ \text{and} & \mathfrak{C}\left(tv_j\right) & \neq & \text{“}unknown\text{”} \end{array}$$

A scenario S_i is conceptualised, if each $tv_j \in STV_i$ is conceptualised. Particularly, this implies that for all $tv_j \in STV_i$, a value has been determined and $tv_j \notin NA$. A set of scenarios $SS = \{S_i\}_{i \in I}$ is conceptualised if and only if each $S_i \in SS$ is conceptualised.

4.2. A Structural Typology of Scenario-Based Reasoning

This novel SBR typology presented in this section characterises SBR activities by the extent, to which the set of scenarios SS considered adheres to certain structural properties and relations. No assumptions are made about rationales and reasoning process used for the construction of scenarios. Rather the types of SBR are defined in terms of the **scenarios' structural properties** making use of the graph-theoretical formalisation of scenarios and, particularly, the structural framework and definitions provided in the previous Section 4.1.

Based on the needs of the scenario recipients, one or more types of SBR could be of use, while others may not include sufficient information. The structural requirements on SBR for both sense- and decision-making are based on this typology of SBR, which describes the characterisations of SBR followed by an overview of (possible) types.

An SBR activity can be classified according to the following properties:

(not) deducible: if the scenarios considered have a valid source, the SBR activity is called deducible;

(not) framed: framing refers to the fact that all scenarios $S_i \in SS$ share one backdrop, which allows them to be related to a common purpose;

(not) grounded: grounding entails that the scenarios have a shared initial situation description;

(not) focussed: focussing requires that all scenarios in SS share a common set of focus variables;

(not) traceable: the traceability is ensured if all steps of the scenario construction are made transparent. This includes the availability of a valid scenario network tree covering continuation and extension of scenarios, and the documentation of merging procedures, where for each merged scenario, a valid scenario network tree must be available;

(not) conceptualised: conceptualisation refers to the traceability of concepts used to determine the variables' values.

The first two features ensure that the set of scenarios can be constructed within the given setting and that the construction paths are kept track of. The traceability is particularly important for scenario management (see Chapter 7). Framing, grounding and the maintenance of the focus ensure that the scenarios constructed are suitable and useful for the purpose at hand. Finally, the conceptualisation enhances the credibility of scenarios and facilitates scenario updating, as certain characteristics (such as time for updating or sensitivity of results to changes in input) depend heavily on the concepts used (cf. Section 6.2.2.2).

On the basis of these six SBR characteristics, which can be present or not, an extensive list of types of SBR can be built: given the binary nature of each of the features, $2^6 = 64$ types of SBR can be defined.

For the purposes of this thesis, only the three richest and most informative are considered useful:

Minimal SBR: for both sense- and decision-making it is of paramount importance that the scenarios $S_i \in SS$ are relevant and suitable for the problem at hand. Minimal SBR collects all properties that a set of scenarios must have for sound SBR. Therefore, minimal SBR is defined as SBR, which is

- *deducible:* all scenarios in SS must share the same backdrop $SEED$,
- *framed:* all scenarios in SS must share the same backdrop B,
- *grounded:* all scenarios in SS emanate from the same valid initial situation $INIT$,
- *focussed:* all scenarios in SS share the same set $FOCUS$.

Framed deducible focussed grounded SBR considers scenarios as defined in Chapter 3. It is important that all scenarios considered contain a valid set of variable dependencies. These dependencies enhance the credibility of the scenario as a narrative (as causes and effects are made explicit, see Section 3.1). Although the variables' dependencies can not be guaranteed to be of causal nature, their directed acyclic nature is suitable to represent any causal relationship. Furthermore, the information on dependencies allows the decision makers to gain insights on paths and potential bottlenecks in the scenario construction phase.

Framed scenarios ensure that the all scenarios considered have been constructed for a common backdrop and incorporate a valid origin. Deducible scenarios facilitate scenario construction in distributed reasoning frameworks and allow the system boundaries to be defined. Focussed scenarios ensure that the reasoning addresses the same variables. This is a mandatory requirement to guarantee the comparability of results. Grounded scenarios guarantee that reasoning is about the same situation (e.g., the same incident) and that it aims at the same purpose (e.g., a particular decision problem).

Traceable SBR: in addition to minimal SBR, it is useful to have traceability among scenarios (on scenario extension, continuation and merging). This feature enables the development and implementation of new techniques for scenario management. Therefore, traceable SBR is SBR, which is *minimal* and *traceable:* for all scenarios in SS it must be possible to retrace the path of their construction by means of extension trees or descriptions of merging procedures.

The additional information on how (focus incomplete) *scenarios interrelate* includes the tracking of scenario continuation as well as scenario merging and enables retracing the scenario construction. This feature offers scenario management possibilities such as scenario selection and pruning (cf. Chapter 8). This is very useful, for example, in curbing the combinatorial increase in numbers of scenarios. Yet, traceable SBR is qualified as desirable, as there may be other—less systematic and efficient—mechanisms to scenario selection and pruning.

Full SBR includes additional information on the reasoning principles used to determine values for variables. Full SBR is SBR, which is *traceable* and *conceptualised:* for each scenario $S_i \in SS$ and each $tv_j \in STV_i$, the concepts used to determine its value must be available.

This feature enables the management of the distributed reasoning framework involving multiple experts and concepts. This requirement is important to enhance the acceptance and credibility of a scenario, as the sources of information are made transparent. Additionally, it enhances the consistency of a set of scenarios and allows for the implementation of a novel approach of scenario pruning and reducing inconsistencies, which may arise from scenario merging (cf. Section 3.7). Finally, it facilitates the assessment of the impact of the change in a variables' value (via sensitivity assessment, cf. Section 6.2.2.2) as well as an assessment of the duration of the revision of a scenario, which is a requirement for efficient scenario updating (see Section 8.3). Full SBR is also qualified as desirable, as it supports a number of desirable features which are, however, not essential to perform SBR.

4.3. Requirements on Contents of Sets of Scenarios for Sound Scenario-Based Reasoning

In the scenario construction phase, multiple scenarios are generated. Having addressed the key features of single scenarios in section 3.3 and the structural features of SBR for strategic decision support in section 4.2, this section is dedicated to the characterisation of the required content of sets of scenarios enabling sound SBR. This section aims at highlighting the qualities a set of scenarios must have so that it is suitable for a given purpose. Namely, this section defines the *consistency, coherence, completeness, reliability* and *balance* of sets of scenarios. Each of these definitions makes use of the formalisation and systematic representation of scenarios that has been developed in Chapter 3.

Some requirements regarding the scenarios' content have been formulated in literature [Schoemaker, 1993; Godet, 2000], and some key features of sets of scenarios have been assessed in different frameworks [Alspaugh and Antón, 2008; Harries, 2003]. Nevertheless, a systematic assessment of the quality of scenario sets with respect to their content has been lacking [Alspaugh and Antón, 2008; Harries, 2003; Jarke et al., 1998], although it is a key factor in the use of scenarios. The novel systematic definitions provided below remedy this deficiency. This framework enables the identification of the scenarios that are the most *relevant* given the recipients' preferences. Thus, it allows requirements for sound SBR to be specified and provides the basis for scenario management (cf. Chapter 8).

Throughout this section, let $SS = \{S_i\}_{i \in I}$ be a set of scenarios and S_{i_k}, S_{i_l} two scenarios in SS.

4.3.1. Consistency

In a consistent set of scenarios, the single scenarios support and reinforce each other [Alspaugh and Antón, 2008]. Furthermore, consistency of a set of scenarios is an important requirement to ensure the comparability of scenarios. There are different types of consistency that need to be distinguished.

Scenario consistency: each $S_i \in SS$ is consistent (cf. Section 3.3.3).

Consistency of variables: for common terms and concepts, all scenarios $S_i \in SS$, where this term or concept is addressed, use the same variables tv_j to refer to that term. For example, if the variable $tv_j \in STV_i$ has the name $name_i(tv_j)$ in S_i, then it is must not have a different name in any other scenario in SS if SS is consistent with respect to the set of variables.

Backdrop consistency ensures that the scenarios in SS were built for the same purpose and reflect the same situation, i.e., all scenarios $S_i \in SS$ share one backdrop B.

Focus consistency ensures the comparability of results, i.e., all scenarios $S_i \in SS$ share one unique $FOCUS$. For SBR & MCDA, focus consistency entails that the set $FOCUS$ equals the set of attributes.

Context consistency ensures that the information within the backdrop reflect the actual situation and problem at hand correctly. For SBR & MCDA, backdrop consistency requires that the additional information concerning the decision problem (e.g., the set of alternatives that need to be evaluated, the attribute tree's structure and the preferences) reflect the decision-makers' (current) understanding of the problem at hand and their preferences.

Conceptual consistency ensures that the same reasoning principles (e.g., experts referred to, algorithms) have been used to generate each scenario $S_i \in SS$. SS is conceptually consistent if for any S_{i_k}, $S_{i_l} \in SS$ and $tv_j \in STV_{i_k} \cap STV_{i_l}$ the source of information providing $V(tv_j)$ is equal. If S_{i_k} and S_{i_l} are completely conceptualised, then they are conceptual consistent if and only if the concepts for the variables $tv_j \in STV_{i_k} \cap STV_{i_l}$ are equal.

4.3.2. Coherence

The following types of coherence are distinguished.

Intra-scenario coherence: each $S_i \in SS$ is coherent (cf. Section 3.3.3).

Inter-scenario coherence: for each $S_i \in SS$ the scenario construction is traceable, i.e., the path from the root scenario S_r to S_i can be reconstructed. Additionally, for any non-maximum scenario S_{sub} the scenarios in SS sharing S_{sub} as a sub-scenario can be identified. In this manner, the interdependencies of scenarios in SS can be analysed.

SS is coherent, if it is both intra- and inter-scenario coherent.

4.3.3. Completeness

A set of scenarios is complete if it covers decision-makers' (and possibly stakeholders') needs and "*explores the issues and trade-offs that if left unresolved would cause more delay or rework later*" [Alspaugh and Antón, 2008]. This definition makes explicit that completeness is a subjective concept that is intangible from the perspective of the formalisation. Yet, the use of B, $SEED$, $INIT$ and $FOCUS$ provides the means for the recipients to make their needs explicit. The subsequent formalisation allows for the achievement thereof, given that sufficient expertise is available to determine values for all variables.

Completeness in SBR & SM:

SS is considered *requirement complete*, if each $S_i \in SS$ has a backdrop B, a valid initial situation $INIT$ and a set of focus variables $FOCUS$, with respect to which it is focus complete.

SS is *concept complete*, if SS is requirement complete and each $S_i \in SS$ is conceptualised.

Completeness in SBR & MCDA: it is required that for each alternative $a_l \in A$ at least one scenario $S_i(a_l) \in SS$. Therefore, the notion of completeness must be expanded for MCDA & SBR problems.

SS is *MCDA requirement complete*, if SS is requirement complete, where the backdrop B is an MCDA-backdrop and the set of focus variables corresponds to the set of attributes specified in B and $SPV_{SS}(tv_A) = A$. The latter condition ensures that all alternatives are evaluated.

SS is *MCDA concept complete*, if it is MCDA requirement complete and each $S_i \in SS$ is conceptualised.

4.3.4. Scenario Uncertainty and Reliability

In the SBR approach developed the *multiplicity of scenarios* grows with growing *number of uncertain variables* $tv_j \in U$ and growing *degree of diversity* for each uncertain variable, $div_{SS}(tv_j)$. A large number of scenarios $|SS_{i_k}|$ ($SS_{i_k} \subseteq SS$), in which values $V_k(\tau) \subseteq sv_{i_k}$ for a set of variables $\tau \subseteq \bigcap_{S_{i_k} \in SS_{i_k}} STV_{i_k}$ and all $S_{i_k} \in SS_{i_k}$ are present, does not signify that these values $V_{i_k}(tv_j)$, $tv_j \in \tau$, are more likely or reliable than other values $V_{i_l}(tv_j)$ ($k \neq l$). Rather, the repetition of values in $V_k(\tau)$ may reflect structural issues. As intuitively (corresponding to frequentist approaches to probability), a high number of realisations may be confused with a high likelihood of these realisations, the need for an explicit assessment of the **reliability** of a scenario (or a set of scenarios) arises.

The next part of this section shows how an uncertainty assessment of scenarios is usually performed and which conditions need to be fulfilled to justify this approach. Subsequently, a novel framework assessing for assessing the reliability of a scenario is developed.

4.3.4.1. Standard Approaches Characterising Scenario Uncertainty

The main aim of SBR is to support decision-making under uncertainty. A standard way of doing so is to select the alternative which has the largest expected utility [Berleant et al., 2005]. To compute the expected utility of each alternative, it is necessary to assign to each scenario a probability [Kluyver and Moskowitz, 1984]. Furthermore, providing a likelihood estimate facilitates the assessment of the seriousness of the implied impacts. Without any assessment, experts and decision-makers are likely to work out (implicit) probability assignments for themselves leading often to irritation or not well-justified assumptions [Schneider, 2001].

Scenario uncertainty can manifest itself in various ways. The most important are [Walker et al., 2003]

1. uncertainty about the *relevant developments* (corresponding here to uncertainty about STV_i and DI_i) or
2. uncertainty about the *values* sv_i due to different underlying assumptions and *reasoning principles* (e.g., two experts having access to the same information provide different values for one variable),
3. *inherent* uncertainty about the *values* sv_i (e.g., a single expert provides several possible values for a variable).

While the first two types are usually not considered in the assessment of scenario probabilities [Walker et al., 2003], there are approaches to assess the uncertainty of a scenario by means of the probability of each variable's possible values [Godet and Roubelat, 1996; Mahesh and Moskowitz, 1990]: each scenario S_i is considered a distinct event in the set of all possible scenarios Ω containing only information on the variables in STV_i. Assuming that $|STV_i| = n$ it holds that $\Omega = \Omega_1 \times \ldots \times \Omega_n$, where Ω_j represents the possible realisations of tv_j given the realisations of all variables $tv_{j_k} \in STV_i \setminus \{tv_j\}$. For each $tv_{j_k} \in STV_i$, $Range\left(V\left(tv_{j_k}\right)\right) \supseteq \Omega_{j_k} \supseteq SPV\left(tv_{j_k}\right)$.

This approach has several limitations. The difficulties of determining or eliciting probabilities have been discussed in section 2.2.2.1. Furthermore, in practice, the set of constructed scenarios SS usually not fully covers Ω [Tonn, 2005], as for each (complete) scenario S_i the combination of values represented in sv_i is not necessarily exhaustive, i.e., $SPV_{SS}\left(tv_j\right) \neq \Omega_j$. It may, e.g., be that for a variable tv_j from a continuous set of possible values Ω_j, N_j (most relevant) discrete values are chosen (and constitute $SPV_{SS}\left(tv_j\right)$). In this case, even if it was possible to determine a (continuous) probability distribution over Ω the probability of each value in $SPV_{SS}\left(tv_j\right)$ would be 0, as continuous distributions do not assign any mass to discrete points.

4.3.4.2. Scenario Reliability Assessment

Given the arguments provided above, this thesis proposes following a different approach that has been newly developed: the likelihood or probability of each scenario is *not* determined. Rather the (relative) **reliability** for $FOCUS$ taking on a certain set of values is determined. Denote

$$\begin{aligned} FOCUS &= \left\{tv_1^F, \ldots, tv_N^F\right\}, \\ V(FOCUS) &= \left\{V\left(tv_1^F\right), \ldots, V\left(tv_N^F\right)\right\}. \end{aligned}$$

This section aims at developing a function $reliability\,(V\,(FOCUS))$ relying on information that is represented within each scenario $S_i \in SS$. $SS = \{S_i\}_{i \in I}$ is a set of focus complete scenarios constructed for a common backdrop.

Step 1: Determine support of each valuation of FOCUS.
The set of valuations of $FOCUS$ in SS, $SPV_{SS}(FOCUS)$, is defined as

$$\begin{aligned} SPV_{SS}(FOCUS) = \Big\{&\left\{V\left(tv_1^F\right), \ldots, V\left(tv_N^F\right)\right\} : \\ &\exists S_i \in SS : \left(V\left(tv_1^F\right), \ldots, V\left(tv_N^F\right)\right) = V_i(FOCUS)\Big\}. \end{aligned}$$

$SPV_{SS}(FOCUS)$ is well-defined. As all scenarios in SS share the same backdrop, by definition $FOCUS_{i_k} = FOCUS_{i_l}$ for all $S_{i_k}, S_{i_l} \in SS$.

$SUPP_{SS}\,(V_i(FOCUS)) \subseteq SS$ is defined as a set of scenarios leading to the same values for the variables in $FOCUS$ as S_i. In that sense, they *support* the valuation of $FOCUS$ as in S_i.

To determine the support of each realisation of $FOCUS$ in an efficient manner, an index set I_{SUPP} is defined. Initially, one sets $\tilde{I}_{SUPP} = \emptyset$. Then, one chooses a scenario $S_{i_k} \in SS$, determines $SUPP_{SS}\,(V_i(FOCUS)) = \bigcup_{i \in I(i_k)} S_i$ and sets $\tilde{I}_{SUPP} = \tilde{I}_{SUPP} \cup \{I\,(i_k)\}$. Then, one continues choosing a scenario S_{i_2}, $i_2 \notin \tilde{I}_{SUPP}$, continues by determining $SUPP_{SS}\,(V_{i+1}(FOCUS))$ and adding the respective index set to I_{SUPP}. For each set $I\,(i_k)$, a representative i_k^* is chosen. One defines $I_{SUPP} = \bigcup_{I(i_k)} i_k^*$. In this manner, it is ensured that $SUPP_{SS}\,(V_{i_k}(FOCUS)) \cap SUPP_{SS}\,(V_{i_l}(FOCUS)) = \emptyset$ for all $i_k \neq i_l \in I_{SUPP}$. This process is continued until

$$\bigcup_{i \in I_{SUPP}} SUPP_{SS}\,(V_i(FOCUS)) = SS.$$

Step 2: Reliability of Each Scenario in $\mathbf{SUPP_{SS}}\left(\mathbf{V_i(FOCUS)}\right)$.
Next, the reliability of each scenario supporting the result $V_i(FOCUS)$ is assessed. Let $S_i \in SUPP_{SS}\left(V_i(FOCUS)\right)$ be the scenario under scrutiny. The reliability measure proposed consists of several components including the number of uncertain vertices, the number of linked uncertain variables and a subjective assessment of the scenario information based on the likelihood of some variables' values. For each concept, this section describes how it can be calculated and what the rationale for its role in the reliability assessment is.

Number of vertices in $\mathbf{S_i}$ influencing FOCUS: $\mathbf{N_i}$.
Denote $\Psi_i\left(FOCUS\right) = \bigcup_{j=1}^{N} \Psi_i\left(\mathrm{tv}_j^F\right)$ the predecessors $FOCUS$ in S_i. The number of variables that need to be determined for an assessment of $FOCUS$ in S_i, N_i, is defined as:

$$N_i = \left|\Psi_i\left(FOCUS\right)\right| + \left|\left(FOCUS\right)\right|. \tag{4.1}$$

If S_i contains solely variables that are relevant to determine the variables in $FOCUS$, $STV_i = \Psi_i\left(FOCUS\right) + FOCUS$ and $N_i = |STV_i|$. and N_i^U.
Number of *uncertain* vertices in $\mathbf{S_i}$ influencing FOCUS: $\mathbf{N_i^U}$.
N_i^U denotes the number of **uncertain** variables (cf. Chapter 3) that contribute to the assessment of $FOCUS$:

$$\begin{aligned} N_i^U &= \left|\Psi_i^U\left(FOCUS\right) \cup FOCUS^U\right| \\ &= \left|\{tv_j \in \Psi_i(FOCUS) \cup FOCUS : \; tv_j \in U_i\}\right|. \end{aligned} \tag{4.2}$$

To determine what influence a growing number N_i^U has on the reliability of a scenario, consider the following argument making use of *probability bounds.* These bounds are a means to evade uncertainty about the precise specifications of probability distributions of the variables' values, as they capture imperfect information [Ferson and Hajagos, 2004]. The rationale for using probability bounds is that a random variable is characterised by a distribution function that is not known precisely. A probability box (or *p-box*) consists of a pair of functions used to circumscribe an imprecisely known distribution. This *p-box* is identified with the class of probability distributions that lie entirely within these bounds [Yager, 1986]. In the discrete case, the *p-box* can be expressed in terms of the lower and upper bound of an event X's probability, denoted $\pi^l(X)$ and $\pi^r(X)$ respectively. In that manner probability boxes can be determined requiring much less effort in terms of data and computation than the generation of precise probability distributions [Horvitz et al., 1989]. This approach is particularly suitable when thresholds that reflect the *"probabilities of interest"* [Horvitz et al., 1989] can be defined. Probability bounds have been successfully applied in risk management [Ferson, 2000; Tucker and Ferson, 2003; Wu, 1994].

For the lower and upper bound of the probability that a variables in tv_j takes a value $V_i(tv_j)$, it holds $\pi^l(V_i(tv_j)) \leq 1$ and $\pi^r(V_i(tv_j)) \leq 1$. $V_i(FOCUS)$ depends on the realisations of all variables in $\Psi_i(FOCUS)$. A lower bound for its probability is

$$\begin{aligned} & \pi^l\Big(V_i(tv)_1), \ldots, V_i\left(tv_{N_i^U}\right)\Big) \\ \leq\ & \underbrace{\pi^l\Big(V_i(tv_1)\Big) \cdot \ldots \cdot \pi^l\Big(V_i\left(tv_{N_i^U-1}\right)\Big)}_{\geq 0} \cdot \underbrace{\pi^l\Big(V_i\left(tv_{N_i^U}\right)\Big)}_{\leq 1} \\ \leq\ & \pi^l\Big(V_i(tv_1)\Big) \cdot \ldots \cdot \pi^l\Big(V_i\left(tv_{N_i^U-1}\right)\Big). \end{aligned}$$

Stated differently, the lower bound of probability *increases,* when *less* uncertain variables are considered. As a similar argument holds for the upper bound, the reliability function is modelled such that $reliability(S_i)$ *decreases* in N_i^U.

Average (maximum) number of uncertain variables in SOURCE-FOCUS-paths in $\mathbf{S_i}$: $\mathbf{l_i}$ ($\mathbf{L_i}$).
The length of a path is defined as the number of edges that constitute the path, i.e., if the path is a tuple

$$P(tv_{j_k}, tv_{j_k}) = (e_{j_k,p_1}, \ldots, e_{p_m,j_l}) = ((tv_{j_k}, tv_{p_1}), \ldots, (tv_{p_m}, tv_{j_l})),$$

then $length(P(tv_k, tv_l)) = m$. Let $SOURCE_i = \left\{tv_1^S, \ldots, tv_M^S\right\}$ be the set of S_i's source variables. Denote

$$\begin{aligned} & P_i(SOURCE_i, FOCUS) \\ =\ & \{P_{DI_i}(tv_{j_k}, tv_{j_l}) : tv_{j_k} \in SOURCE_i \text{ and } tv_{j_l} \in FOCUS\} \end{aligned}$$

the set of paths in S_i connecting source to focus variables. Let $(\omega_1, \ldots, \omega_Z)$ be a labelling of paths, i.e., $P_i(SOURCE_i, FOCUS) = \{P_{\omega_\iota}\}_{\iota=1,\ldots,Z}$. Next, for each path P_{ω_ι} determine the set of edges that leave an uncertain variable:

$$\begin{aligned} E^U(P_{\omega_\iota}) = \Big\{ & e_1^U = (tv_{1^-}, tv_{1^+}), \ldots, e_N^U = (tv_{N^-}, tv_{N^+}) : \\ & e_i^U \in P_{\omega_\iota} \text{ and } tv_{i^-} \in U \Big\}. \end{aligned}$$

The average number of uncertain vertices in $SOURCE$-$FOCUS$-paths in S_i is

$$l_i = \frac{1}{Z} \sum_{\iota=1}^{Z} \left| E^U \left(P_{\omega_\iota} \right) \right| . \tag{4.3}$$

The maximum number of relevant uncertain vertices connected in S_i is

$$L_i = \max \sum_{\iota=1,\dots,Z} \left\{ \left| E^U \left(P_{\omega_\iota} \right) \right| \right\} . \tag{4.4}$$

Again, a rationale for the dependence of the reliability of S_i on l_i and L_i is provided. First, a realisation $V_i(FOCUS) = \left\{ V_i \left(tv_1^F \right), \dots, V_i \left(tv_1^F \right) \right\}$ is at most as reliable as the least reliable valuation $V_i \left(tv_{lr}^F \right)$ (cf. argument for N_i^U). As the values of the variables within each path P_{ω_ι} are dependent,

$$\pi^r \left(V_i \left(\Psi_i^U (FOCUS) \right) \cup V_i (FOCUS) \right)$$

is an upper bound for the probability of the realisation of $FOCUS$ as in S_i. The deterministic variables can be omitted, as the probability of them attaining the value as in scenario S_i is 1. As

$$\pi^r \left(V_i \left(\Psi_i^U (FOCUS) \right) \cup V_i (FOCUS) \right) \leq \pi^r \left(V_i \left(tv_1 \right) \cap \dots \cap V_i \left(tv_n \right) \right) ,$$

where $\left(\left(tv_1, tv_2 \right), \dots, \left(tv_{n-1}, tv_n \right) \right)$ describes any $SOURCE$-$FOCUS$-path, the reliability of the scenario realisation is at most as reliable as the realisation of any path from $SOURCE$ to $FOCUS$. Denote $tv_1^U, \dots, tv_{n^U}$ the uncertain vertices within a $SOURCE$-$FOCUS$-path. Then,

$$\pi^r \left(\bigcup_{j=1}^{n} V_i \left(tv_j \right) \right) = \pi^r \left(\bigcup_{j=1}^{n^U} V_i \left(tv_j \right) \right) .$$

By the laws of conditional probabilities for probability intervals [de Campos et al., 1995], it holds that

$$\begin{aligned}
& \pi^r \left(V_i\left(tv_j\right) \cap V_i\left(tv_k\right)\right) \\
= \; & \pi^r \left(V_i\left(tv_j\right) \middle| V_i\left(tv_k\right)\right) \\
& \cdot \left(\pi^r \left(V_i\left(tv_j\right) \cap V_i\left(tv_k\right)\right) + \pi^l \left(\overline{V_i\left(tv_j\right)} \cap V_i\left(tv_k\right)\right)\right) \\
\leq \; & \pi^r \left(V_i\left(tv_j\right)\right) \\
& \cdot \left(\pi^r \left(V_i\left(tv_j\right) \cap V_i\left(tv_k\right)\right) + \pi^r \left(\overline{V_i\left(tv_j\right)} \cap V_i\left(tv_k\right)\right)\right) \\
\leq \; & \pi^r \left(V_i\left(tv_j\right)\right) \cdot \pi^r \left(V_i\left(tv_k\right)\right).
\end{aligned}$$

Here, $\overline{V_i\left(tv_j\right)}$ denotes the event $V\left(tv_j\right) \neq V_i\left(tv_j\right)$. The expression in the last line corresponds to the joint probability distribution that would arise if the events $V_i\left(tv_j\right)$ and $V_i\left(tv_k\right)$ were independent. The more connections between uncertain relevant vertices there are, the less reliable the scenario is. Hence, the reliability of S_i *decreases* monotonously with growing l_i and L_i.

mu$^{\mathbf{r}}$-corroboration of scenario $\mathbf{S_i}$: $\mu^{\mathbf{r}}$-corr$_{\mathbf{i}}$

The general idea is that a valuation $V_i\left(tv_j\right)$ *corroborates* the correctness of sv_i if a *lower bound of its likelihood exceeds* μ^r. The number μ^r-*corr*$_i$ can be interpreted as the corroboration of S_i for the level μ^r. To explore the space of likelihood bounds best and to offer flexibility to the decision-makers, one defines a vector μ, with $0 \leq \mu^1 \leq \ldots \leq \mu^M \leq 1$, $M \geq 1$.

μ^r-*corr*$_i$ corresponds to the number of $tv_j \in \Psi_i(FOCUS) \cup FOCUS$, for which $\pi^l\left(V_i\left(tv_j\right)\right)$ can be assessed and exceeds the threshold value at level μ^r, but does *not* exceed the next level of likelihood μ^{r+1} $(1 \leq r \leq M-1)$. To calculate μ^r-*corr*$_i$ for all $tv_j \in STV_i$, for those variables $tv_j \in STV_{SU}$, for which an assessment $\pi^l\left(V_i\left(tv_j\right)\right)$ is *not* available, one defines $\pi^l\left(V_i\left(tv_j\right)\right) = 0$. Then,

$$\mu^r\text{-}Corr_i = \begin{cases} \left|\left\{tv_j \in STV_i : \mu^{r+1} > \pi^l\left(V_i\left(tv_j\right)\right) \geq \mu^r\right\}\right| & \text{if } r \leq M-1 \\ \left|\left\{tv_j \in STV_i : \pi^l\left(V_i\left(tv_j\right)\right) \geq \mu^r\right\}\right| & \text{if } r = M. \end{cases} \tag{4.5}$$

If $\mu^r = 1$, μ^r-$Corr_i$ corresponds to the number of variables within STV_i that are deterministic.

Relative $\mu^{\mathbf{r}}$-corroboration of scenario $\mathbf{S_i}$: $\mu^{\mathbf{r}}$-corr$_{\mathbf{i}}^{\mathbf{rel}}$.

While μ^r-*corr*$_i$ corresponds to the absolute number of $tv_j \in \Psi_i(FOCUS) \cup FOCUS$, for which $\pi^l\left(V_i\left(tv_j\right)\right) \geq \mu^r$, the *relative* corroboration corresponds to the *share* of uncertain vertices $tv_j \in \Psi_i^U(FOCUS) \cup FOCUS^U$,[22] for which $\pi^l(V_i(tv_j)) \geq \mu^r$. $\mu^r\text{-}corr_i^{rel}$ can be calculated by:

$$\mu^r\text{-}corr_i^{rel} = \frac{\mu^r\text{-}corr_i}{|\Psi_i^U(FOCUS)| + |FOCUS^U|} \in [0,1]. \quad (4.6)$$

ν^1 refutation of scenario S_i: ν^1-ref$_i$.

$\nu^l\text{-}ref_i$ complements the reasoning about the reliability of a scenario by not only taking into account values that support the reliability of S_i, but also those that make it less credible. The number $\nu^l\text{-}ref_i$ can be interpreted as the refutation of S_i. Similarly as for the corroboration, an assessment $V_i(tv_j)$ is said to *refute* S_i if the *upper bound* $\pi^r(V_i(tv_j))$ falls below ν^l.

Define a vector ν with $0 \leq \nu^1 \leq \ldots \leq \nu^N < \mu^1$. The last inequality signifies that if $\pi^l(V_i(tv_j)) \geq \mu^1$, this fact is supposed to support the reliability of S_i. As for $\pi^r(V_i(tv_j)) \leq \nu^N$ the credibility is corroborated, it is required that $\nu^N < \mu^1$, so that there is no situation, for which $\mu^1 \leq \pi^l(V_i(tv_j)) \leq \pi^r(V_i(tv_j)) \leq \nu^N$.

Formally, $\nu^l\text{-}ref_i$ is defined as the number of variables $tv_j \in STV_i$, for which $\pi^r(V_i(tv_j))$ can be assessed and falls in a certain interval $\left[\nu^{l-1}, \nu^l\right]$. To calculate $\nu^l\text{-}ref_i$, for those variables $tv_j \in STV_{SU}$, for which $\pi^r(V_i(tv_j))$ is not available, one defines $\pi^r(V_i(tv_j)) = 1$. Then, one determines

$$\nu^l\text{-}ref_i = \begin{cases} \left|\left\{tv_j \in STV_i:\ \nu^{l-1} < \pi_r(V_i(tv_j)) \leq \nu^l\right\}\right| & \text{if } l \geq 2 \\ \left|\left\{tv_j \in STV_i:\ \pi_r(V_i(tv_j)) \leq \nu^l\right\}\right| & \text{if } l = 1 \end{cases} \quad (4.7)$$

For $\nu^l = 0$, $\nu^l\text{-}ref_i = 0$ for all *plausible* scenarios S_i, as the plausibility of a scenario has been defined as not going beyond the realm of possibility (see Section 3.3.3). Therefore, for each assessed variable tv_j with value

$$V_i(tv_j):\ \exists\, \varepsilon = \varepsilon(V_i(tv_j)) > 0:\ \pi_r(V_i(tv_j)) \geq \varepsilon > 0.$$

Furthermore, in this framework, the lower and upper bounds of likelihood for the variables $tv_j \in STV_{SU}$, for which the upper and lower bounds of the probability of $V_i(tv_j)$ can not be determined, are 0 and 1 respectively, which is

[22] $FOCUS^U$ denotes the uncertain focus variables, i.e.,

$$FOCUS^U = \{tv_j \in FOCUS:\ tv_j \in U\}$$

.

in line with modelling ignorance in evidential reasoning approaches [Bergsten and Schubert, 1993; Ruspini et al., 1992; Xu and Smets, 1996].

Relative refutation of scenario $\mathbf{S_i}$: $\nu^{\mathbf{l}}$-$\mathbf{ref_i^{rel}}$.
As for the corroboration, after having determined the absolute number of variables discrediting the reliability of S_i, $\nu^l\text{-}ref_i^{rel}$ denotes the *relative refutation*, i.e., the share of $tv_j \in \Psi^U(FOCUS) \cup FOCUS^U$, for which $\pi^r(V_i(tv_j)) \in \left[\nu^{l-1}, \nu^l\right]$. $\nu^l\text{-}ref_i^{rel}$ can be calculated by:

$$\mu^l\text{-ref}_i^{rel} = \frac{\mu^l\text{-ref}_i}{|\Psi_i^U(\text{FOCUS})| + |\text{FOCUS}|} \in [0,1]. \tag{4.8}$$

On the basis of the properties captured in equations 4.1, 4.2, 4.3, 4.4, 4.5, 4.6 and 4.7, the *reliability of S_i* can be defined.

$$\begin{aligned} reliable: \ \mathbb{N}_0 \times \mathbb{N}_0 \times [0,1]^M \times [0,1]^N &\to [0,1] \\ (S_i, \mu, \nu) &\mapsto reliable_i(\mu, \nu) \\ \left(\text{where } 1 \geq \mu^r > \mu^{r-1} \geq \nu^l \forall r = 2, \ldots, M, \right. & \quad \left. 0 \leq \nu^l \leq \nu_{l+1} \ \forall l = 1, \ldots, N-1\right). \end{aligned}$$

Additionally, it is required that $reliable_i$ fulfils the following properties

- monotonously decreasing in N_i^U,
- monotonously decreasing in l_i or L_i,
- monotonously increasing in $\mu^r\text{-}corr_i^{rel}$ and
- monotonously decreasing in $\nu^r\text{-}ref_i^{rel}$.

The rationale for each of these properties is described above.

Additionally, $reliable_i(\mu, \nu)$ is normalised such that

$$\begin{aligned} \mu^M\text{-}corr_i^{rel} = 1 &\Rightarrow reliable_i(\mu, \nu) = 1 \\ \text{and} \quad \nu^1\text{-}ref_i^{rel} = 1 &\Rightarrow reliable_i(\mu, \nu) = 0. \end{aligned}$$

If the scenario is completely corroborated (signified by the fulfilment of the highest level of corroboration $\mu^M\text{-}corr_i^{rel} = 1$), then the scenario is reliable. Contrarily, if the scenario is completely refuted (as its quality falls below the lowest level of refutation $\nu^1\text{-}ref_i^{rel}$), the reliability is set to 0. By definition $0 \leq \nu^l < \mu^r \leq 1$ for all $l = 1, \ldots, N$ and $r = 1, \ldots, M$. Therefore

$$\begin{aligned} \mu^r\text{-}corr_i^{rel} = 1 &\Rightarrow \nu^l\text{-}ref_i^{rel} = 0 \\ \text{and} \quad nu^l\text{-}ref_i^{rel} = 1 &\Rightarrow mu^r\text{-}corr_i^{rel} = 0. \end{aligned}$$

Hence, above normalisation is well-defined.

In general, a great variety of different reliability functions fulfilling these properties can be constructed. For reasons of simplicity and transparency, this thesis propose a rather simple form, namely:

$$reliable_i(\mu,\nu) = \left(\left(\sum_{r=1}^{M} \omega_r^{\mu} \mu^{r}\text{-}corr_i\right)\left(1-\sum_{l=1}^{N}\omega_l^{\nu}\nu^{l}\text{-}refi\right)\right)^{\log\left(N_i^U + l_i^U\right)}, \tag{4.9}$$

where $\omega^{\mu} \in [0,1]^M$ and $\omega^{\nu} \in [0,1]^N$ are vectors of weights describing importance of exceeding a certain threshold for the corroboration or of falling below a certain threshold for the refutation.[23] Furthermore, both vectors are normalised, i.e., $\sum_{r=1}^{M}\omega_r^{\mu} = 1$ and $\sum_{l=1}^{N}\omega_l^{\nu} = 1$. As the reliability of a scenario depends on what the decision-makers perceive as reliable and credible, these preferences should be elicited from the decision-makers. To this end, standard preference elicitation methods can be applied [Belton and Stewart, 2002; Chen and Pu, 2004; Keeney et al., 1979].

A more cautious reliability assessment can ground the complexity component of the reliability, represented by the path length on the longest $SOURCE$-$FOCUS$ path, resulting in:

$$\widetilde{reliable}_i(\mu,\nu) = \left(\left(\sum_{r=1}^{M} \omega_r^{\mu} \mu^{r}\text{-}corr_i\right)\left(1-\sum_{l=1}^{N}\omega_l^{\nu}\nu^{l}\text{-}refi\right)\right)^{\log\left(N_i^U + L_i^U\right)}. \tag{4.10}$$

Step 3: Aggregation of reliability of all scenarios supporting $\mathbf{V_i(FOCUS)}$. Recall that $SPV_{SS}(FOCUS) = \{V_i(FOCUS)\}_{i\in I_SUPP}$. For each $V_i(FOCUS)$, $i \in I_SUPP$, the support set has been defined as $SUPP_{SS}(V_i(FOCUS)) = \{S_{\tilde{i}}\}_{\tilde{i}\in I_{SUPP(i)}}$ (see Step 1). In Step 2, for each such $S_{\tilde{i}}$ the reliability functions $reliable_{\tilde{i}}(\mu,\nu)$ and $\widetilde{reliable}_{\tilde{i}}(\mu,\nu)$ have been determined. On this basis, the reliability of $V_i(FOCUS)$ is defined as

$$reliability(V_i(FOCUS),\mu,\nu) = \sum_{\tilde{i}\in I_{SUPP(i)}} reliable_{\tilde{i}}(\mu,\nu), \tag{4.11}$$

$$reliability^{L}(V_i(FOCUS),\mu,\nu) = \sum_{\tilde{i}\in I_{SUPP(i)}} \widetilde{reliable}_{\tilde{i}}(\mu,\nu). \tag{4.12}$$

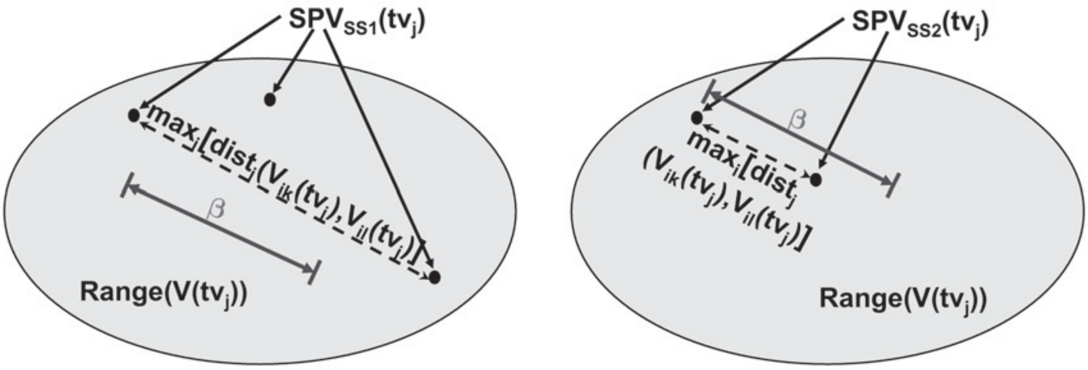
[23] Consequently, $reliable_i(\mu,\nu) = reliable_i(\mu,\omega^{\mu},\nu,\omega^{\mu})$. Yet, for reasons of brevity and clarity, the reference to the preferences ω^{μ} and ω^{ν} is omitted in the notation.

This novel approach to assessing the scenario reliability takes into account heterogeneous information on the likelihood of a variable's value. It overcomes the weaknesses of standard methods assigning probabilities to scenarios. Particularly, the assessment of upper and lower bounds for the probability of $V_i(tv_j)$ is much less demanding than the assessment of precise probabilities [Baudrit et al., 2007; Bier, 2004; de Campos et al., 1995; Dubois and Prade, 2003; Guo and Tanaka, 2010]. Additionally, this approach takes into account the preferences of the decision-makers and stakeholders for which bounds μ^r ($r = 1, \ldots, M$) they feel "*high enough*" to say that a certain variable's value corroborates a scenario (and its result) and which thresholds ν^l ($l = 1, \ldots, M$) are "*so low*" that falling below ν^l refutes the scenario's credibility. Furthermore, the definition of weights ω^μ, ω^ν allows for exploring the space of likelihoods by defining several degrees of severity or importance for the violation of a bound.

It should be kept in mind that the approach presented is *not* a probability assessment for a certain result, but rather a method *exploring the structural information* about each scenario as well as *several likelihood assessments* if these are available. It does not allow statements about the frequencies or probabilities of certain outcomes to be made. Rather, it is a means to support decision-makers who need to prioritise and weigh scenario results.

4.3.5. Balance

The set of scenarios SS presented to the decision-makers should represent a broad span of different developments. Therefore, it should be designed to capture a broad range of values for all uncertain factors, so that the decision-makers' (more or less) implicit assumptions on how the situation is at present, and how it will evolve are challenged [O'Brien, 2004]. To this end, a method is implemented to ensure that for each variable $tv_j \in STV_{SS}$ the set of possible values in SS, $SPV_{SS}(tv_j)$, represents $Range(V(tv_j))$ as good as possible (respecting restrictions on $|SPV_{SS}(tv_j)| \leq |SS|$, which is possibly much smaller than the magnitude of $Range(V(tv_j))$, cf. Section 8.1). Figure 4.1 illustrates the situation: while the (continuous) set $Range(V(tv_j))$ covers the grey area, the (discrete and finite) sets of possible values $SPV_{SS_1}(tv_j)$ and $SPV_{SS_2}(tv_j)$ are represented by black dots.



(a) SS_1: β-balanced in tv_j (b) SS_2: tv_j violates β-balance

Figure 4.1.: *Balanced Scenario Sets. Examples for a variable* tv_j.

A set of scenarios $SS = \{S_i\}_{i \in I}$ with variables $SSTV = \bigcup_{i \in I} STV_i$ is β-balanced if and only if

$$\min_{tv_j \in SSTV} \left(\max_{i_k, i_l \in I} \left[dist_j \left(V_{i_k} \left(tv_j \right), V_{i_l} \left(tv_j \right) \right) \right] \right) \geq \beta, \tag{4.13}$$

where $dist_j$ is a metric on $Type\left(tv_j\right)$, cf. Section 3.2.6.2.

The scenario sets SS_1 and SS_2 shown in figure 4.1 illustrate the concept of β-balance: for a variable tv_j the sets $SPV_{SS_1}\left(tv_j\right)$ and $SPV_{SS_2}\left(tv_j\right)$ are depicted in figure 4.1(a) and 4.1(b) respectively. Having chosen β as represented in both figures, it is checked whether the maximum distance between any pair of values $\left(V_{i_k}\left(tv_j\right), V_{i_l}\left(tv_j\right)\right)$) ($tv_j \in STV_{i_k} \cap STV_{i_l}$) for scenarios in $S_{i_k}, S_{i_l} \in SS_1$ in figure 4.1(a) and $S_{i_k}, S_{i_l} \in SS_2$ in figure 4.1(b) exceeds β. Apparently, SS_1 fulfils the requirement on tv_j. For SS_2, however, the condition is violated, and SS_2 is not β-balanced.

The value β must be chosen carefully. If a low value β is selected, the set of possible values is explored thoroughly, but the number of scenarios arising is likely to exceed the cognitive capacities of the human experts and decision-makers as well as the computational capacities of the automated reasoning systems involved. Contrarily, if the value β is chosen rather high, the number of scenarios can be limited to a size which is easily manageable. Yet, too few scenarios may be considered and important possible scenarios may be missed.

4.4. Summary

This chapter provided an approach for the specification of the recipients' (information) needs and requirements of the qualities of a set of scenarios. These specifications enable building purposeful scenarios that fulfil the recipients' quality requirements.[24] Therefore, this chapter provides the basis for achieving *objectives O.2* and *O.3*.

This chapter first characterised structural properties of sets of scenarios and types of SBR allowing for the definition of **structural requirements** for sound SBR & SM or SBR & MCDA. Multiple scenarios providing a wide range of plausible futures under different specified conditions must be generated to ensure that the scenarios provide a valid basis for sense- and decision-making. A rigorous framework allowing for assessing the mandatory and/or desired structural properties has been developed in this chapter. In this manner, it can be ensured that the scenarios' structural properties are acceptable to the recipients. Additionally, it is warranted that the scenarios contain the information the recipients need to make their decision(s). In this manner, objective O.2 is achieved. Furthermore, the structural requirements facilitate the implementation of SBR in distributed reasoning frameworks. The implementation is further supported by the model for information management tailored to those requirements and described in Chapter 6.

Additionally, the assessment of a **scenarios' quality with respect to its content** has been addressed. Stringent concepts of *consistency, coherence, completeness* have been developed. The concepts of scenario *reliability* and the assessment of the *balance* of a set of scenarios were operationalised on basis of the novel scenario formalisation. The contentual properties are prerequisites for scenario management. Particularly, the concepts of reliability and balance are used to determine the **relevance** of a scenario $S_i \in SS$ (cf. Chapter 8).

[24]The scenario building method is described in the next chapter.

5. Reasoning under Uncertainty: Integration of Scenarios and Multi-Attribute Value Theory

> In case of doubt, decide in favor of what is correct.
> (Karl Kraus)

This chapter presents a distributed approach to scenario building by taking into account information from various sources to assess the situation (SBR & SM) or to evaluate alternatives (SBR & MCDA) *(objective O.4)*. Furthermore, this chapter shows how the scenario building method ensures that the recipients' information needs are met. In this manner, this chapter completes the achievement of *objective O.2.* Regarding the requirements for *scenario acceptance (objective O.3),* this chapter explains how the scenario building process ensures that important scenario quality requirements (plausibility, coherence and consistency) are met. Further methods enabling a more detailed analysis of the scenarios built are introduced in Chapters 7 and 8, where the scenario quality and relevance is used as a means for managing and prioritising scenarios.

The scenario building is divided into a number of specific problems that are solved by experts with particular knowledge and skills. The approach to scenario building aims at establishing a cooperation between the best timely available experts (human experts and automated systems) constructing a set of scenarios to found the decision on.

The scenario building process consists of two phases. In the first phase, *Directed Acyclic Graphs* (DAGs) organising and structuring information processing and sharing such that a set of plausible, consistent and coherent scenarios arises are constructed (cf. Section 5.1). DAGs are particularly suitable to represent cause-effect chains [Galles and Pearl, 1997]. Therefore, this thesis refers to local DAGs that represent the experts' knowledge about the interdependence of variables or the mapping between input and output information as (local) *Causal Maps* (CMs) following e.g., Galles and Pearl [1997]; Goodier et al. [2010]. While sometimes, causal models are referred to as mental models of the decision-makers and experts involved [Butler et al., 2006] or to calculate the impact of an alternative on a set of attributes [Montibeller et al., 2008; Montibeller and Belton, 2009], this approach uses CMs as a means to structure the *flow of information*, proceeding from causes to effects. Particularly, *no* standardised causal inference or aggregation mechanisms are imposed.

Section 5.1.2 shows that by combining DAGs and techniques from MADM **Decision Maps** are created. Section 5.2 shows that the Decision Map framework facilitates *scenario generation*, the second phase of the scenario building process. In this phase, the scenarios are completed by adding successively information on their content (in terms of values and statuses). Particularly, MADM provides a rationale for constructing *decision-relevant scenarios.* In this manner, the problem structuring approach of MADM is used as a means to structure and manage information processing. This reduces the information overload of the experts involved in the scenario construction process and the decision-makers, to whom the final results are presented.

Section 5.3 is dedicated to the *evaluation* of alternatives and develops new methods for presenting the results of the SBR & MCDA process to the decision-makers. This is particularly important when the decision problem is large and complex. In this framework, the magnitude and complexity of the problem is reflected by a large number of scenarios and results and different degrees of granularity or aggregation levels, on which the information can be represented. This complexity reduced by *selecting* the most relevant scenarios to be presented in detail to the decision-makers or by an *aggregation* of all scenarios' results. To support the aggregation of results two novel approaches facilitating the elicitation of preferences reflecting the importance of each scenario are developed in Section 5.3.3. While the first method is based on the concept of *satsificing*, the second approach exploits the concept of the decision-makers' *risk attitude* and enables a quick and straightforward determination of scenario weights. Section 5.4 complements the multi-criteria evaluation by a risk assessment.

5.1. Distributed Determination of a Scenario's Structure

To represent the decision problem's structure and to manage the flow of information this thesis uses *directed acyclic graphs* (DAGs). The advantage of using DAGs is first that they facilitate the implementation of a distributed scenario generation framework. Second, they allow causal dependencies[25] between the variables to be captured. DAGs represent the problem's structure as a network, where a vertex corresponds to a variable tv_j, while a (directed) edge from tv_j to tv_k shows that tv_j influences tv_k *directly*. In contrast to Bayesian Networks [Pearl, 2009], it is *not* claimed that vertices, which are not directly connected, are independent. Contrarily, if there is a directed path $P_{DAG}(tv_j, tv_k)$ from tv_j to tv_k, tv_k depends on tv_j. If the length of the path $l(P_{DAG}(tv_i, tv_j)) \geq 2$, this influence is said to be indirect. Vertices tv_j and tv_k are independent if and only if there is no path $P_{DAG}(tv_j, tv_k)$ or $P_{DAG}(tv_k, tv_j)$ connecting them (cf. Section 3.3.1.3).

Raising situation awareness or assessing the consequences of alternatives in large and complex problems includes a range of tasks, varying in scope and complexity, as well as in their means of execution [Mendonca et al., 2006]. Therefore, the first step in the configuration of the DAG, which determines the scenarios' structure, is the identification of experts that can provide information on certain facets of the overall problem. To avoid overloading decision-makers and experts with irrelevant or redundant information and to ensure that all necessary information is passed on to the adequate experts, the problem needs to be structured so that information relevant to the purpose at hand can be determined. In the framework of this thesis, this means that the scenario recipients specify a set of variables $FOCUS$ that they need information on (cf. Section 4.1.3). In SBR & MCDA this set is complemented by a set of (feasible) alternatives A to be evaluated (cf. Section 4.1.1).

5.1.1. Scenario Construction

A distributed approach referring to locally-available expert knowledge is used to construct for a given backdrop B a DAG for a decision problem $D = D(B)$,

[25]For a precise definition of the underlying notion of causation [Dawid, 2002; Pearl, 1995].

called DAG_D in the following. The experts' (reasoning) capabilities are elicited in terms of the information they can provide (their *services*). If an expert cannot provide his service autonomously, but depends on information from further sources within the network of available expertise[26], he defines the information he needs as a prerequisite to provide his service *(required input).*

By using service discovery and collaboration protocols, the system successively identifies additional experts capable of providing the services required and connects all involved experts [Pavlin et al., 2009b]. The starting point of this process is the set of focus variables: experts capable of determining the focus variables' values are identified. These refer to their local CMs to indicate which information they need to provide their service. Successively, the experts are connected in a graph $\mathcal{G}_D = \mathcal{G}_D = \langle STV_D, E_D \rangle$ by merging all local CMs until a network allowing all variables' values to be determined is achieved [Comes et al., 2010a]. For SBR & MCDA problems, $\mathcal{G}_D = \mathcal{G}_D(a_i) = \langle STV_D(a_i), E_D(a_i) \rangle$. This process ensures that

- all available information the expert judged *relevant* to perform his task is provided to him,
- information that is irrelevant or redundant is hidden from him reducing the problem of *information overload,*
- the information the expert needs is provided in a type and format that can be understood and processed by him reducing the problem of *information inaccessibility.*

Technically, this is carried out by software agents, which also form the interface between the experts and the decision support system [Comes et al., 2010a].

The process ends when for each potentially arising DAG $\mathcal{G}_{D_i}$ all vertices in STV_i are *sufficiently connected.* That means, for any expert in the network determining the value of a variable $tv_j \in STV_i$ the required information $V(\tau) = V(\{tv_{k_1}, \ldots, tv_{k_\tau}\})$ is captured by a set of edges: $\tau \in \widetilde{Psi}_i(tv_j)$. This condition ensures that during the scenario generation phase (cf. Section 5.2), each expert is provided the information he requested. Particularly, for all variables tv_j with $\Psi(tv_j) = \varnothing$ it must hold: $tv_j \in SEED(B)$ (see Section 4.1.4).

If $\mathcal{G}_{D_i}$ cannot be completed, because a relevant piece of information can not be provided (in time), there are two possible approaches. Denote $\tau^{miss} =$

[26] The set of (human) experts and automated systems that can be addressed depends on the availability and willingness of experts to provide services for the specific incident.

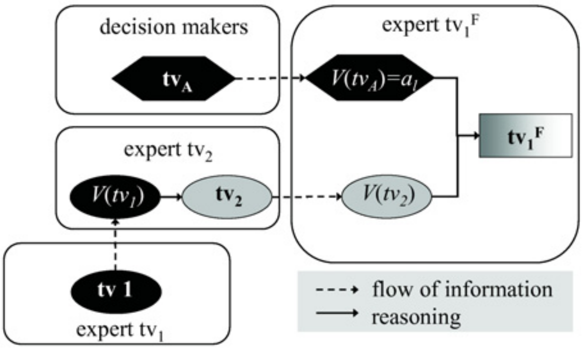


Figure 5.1.: *Configuration of $\mathcal{G}_D$ Based on Local CMs: Example for Focus Variable tv_1^F.*

$\left\{tv_1^{miss}, \ldots, tv_{M(\tau)}^{miss}\right\}$ the set of variables, for which information on the possible values can not be provided and $\widetilde{\Theta}\left(\tau^{miss}\right) = \left\{tv_1^{\Theta}, \ldots, tv_{M(\Theta)}^{\Theta}\right\}$ the set of variables, which depend directly on the information on $V\left(\tau^{miss}\right)$. Then, each of the experts responsible for determining $V\left(tv_i^{\Theta}\right)$, $i \in \{1, \ldots, M(\Theta)\}$ can be contacted and asked, if it was possible to determine $V\left(tv_i^{\Theta}\right)$ without any information on $V\left(\tau^{miss}\right)$, e.g., by providing less accurate values. If the information on τ^{miss} can not be provided *in time*, the decision-makers may be asked, if they are willing to wait until the assessment can be completed. Otherwise, the scenario generation can only be performed partly, providing values for just some of the focus variables (depending on the connectedness of the variables in τ^{miss}). By using the scenario merging procedure that has been developed in Section 3.7, partial scenarios can be valuable, as they can be combined easily and efficiently once, more information is available.

Figure 5.1 shows an example of the construction of a DAG $\mathcal{G}_D$, where the expert responsible to determine the focus variable tv_1^F states that he needs information on the alternative chosen ($tv_A = a_l$) as well as on the value of tv_2. Successively, the flows of information are determined, until for each path, a $SEED$ variable, which does not depend on any other information within the network of available experts, is reached. These are characterised as black vertices in Figure 5.1.

$\mathcal{G}_D$ integrates expert knowledge, information systems, algorithms and tools into a distributed system that allows $V(FOCUS)$ to be determined. In this manner, the required information is provided to the scenario recipients. By setting

$$\begin{aligned} STV_0^l &= STV_D(a_l) \\ sv_0^l &= V_0\left(STV_0^l\right) \\ status_0^l &= status_0\left(STV_0^l\right) \\ DI_0^l &= E(a_i), \end{aligned}$$

where for each $tv_j \in STV_0^l$

$$\begin{aligned} V_0(tv_j) &= \infty \\ \text{and} \qquad status_0(tv_j) &= \text{“}not\ assessed\text{”}, \end{aligned}$$

the basic scenario

$$S_0^l = \left\langle STV_0^l, sv_0^l, status_0^l, DI_0^l \right\rangle \tag{5.1}$$

that serves as the starting point for the generation of scenarios is defined.

5.1.2. Decision Maps: Integration of Scenarios and Multi-Attribute Value Theory

This section describes how the decision problem in SBR & MCDA can be structured by using Decision Maps that broaden the basis of the decision support system by merging the MADM attribute tree with a DAG$_D$ describing the problem D on behalf of interlinked *relevant variables.* This approach is applicable for decision problems, where a finite set of alternatives $A = \{a_1, \ldots, a_k\}$ needs to be evaluated in complex and uncertain environments.

The (overall) decision problem D needs to be structured, and the set $FOCUS$ must be determined to avoid overloading decision-makers and experts with irrelevant or redundant information while ensuring that all relevant information can be provided. In MAVT, this is achieved by developing an *attribute tree* (cf. right part of Figure 5.2). On the tree's lowest level, attributes operationalising each goal are established (cf. Section 2.1.3). Therefore,

$$FOCUS_D = \left\{tv_1^F, \ldots, tv_N^F\right\} = \{att_1, \ldots, att_N\}.$$

Additionally, the decision-makers' preferences are elicited (cf. Section 2.1.3) and captured in the backdrop B_D.

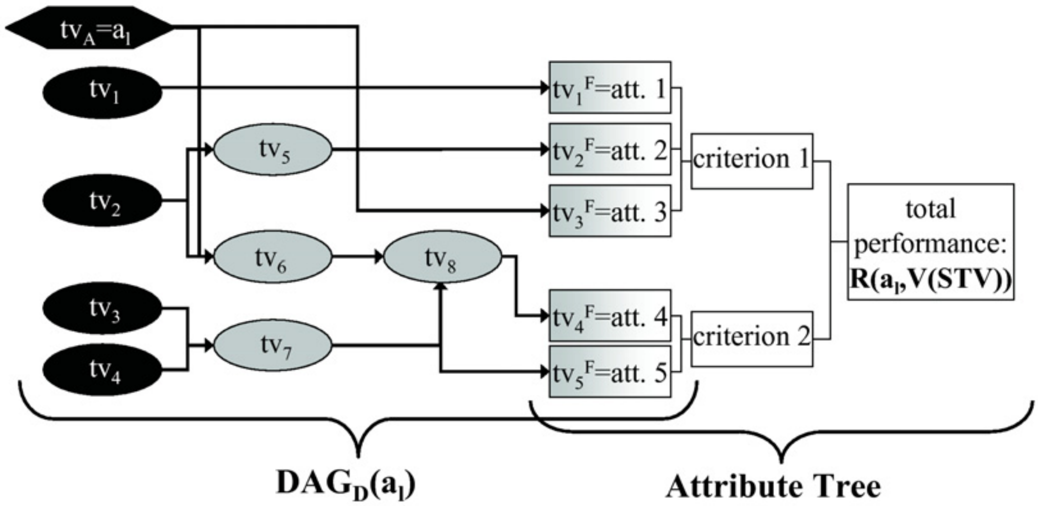


Figure 5.2.: *Merging $\mathcal{G}_D(a_l)$ and Attribute Tree into a Decision Map. Left side: $\mathcal{G}_D(a_l)$ with variables and their (causal) dependencies. Right side: attribute tree with hierarchical dependencies.*

By following the scenario construction procedure as described in Section 5.1, the problem structuring phase results in the **Decision Map,** cf. Figure 5.2. Each Decision Map consist of two parts: DAG$_D$ capturing relevant variables and their dependencies and an attribute tree representing the decision-makers perception of the evaluation problem and their preferences. As indicated in Section 4.1.1, for each $a_l \in A$ a DAG $\mathcal{G}_D(a_l)$ is constructed to generate the respective set of scenarios $SS(a_l)$. This reduces information overload, as for each alternative only the relevant aspects are covered. For instance, in emergency management, when a decision must be made whether to evacuate or shelter the population due to a chemical incident causing a plume (cf. Chapter 9), for the evacuation alternative, transportation infrastructure is crucial, while for the sheltering the building structure and the ventilation systems need to be considered. The attribute tree (identical for all $a_l \in A$) enables an assessment of the results whilst respecting multiple objectives and the decision-makers' preferences. The connection between both parts of the Decision Map is made via the attributes, which are part of both $\mathcal{G}_D(a_l)$ and the attribute tree.

As $\mathcal{G}_{D_i}(a_l) =: \langle STV_i, DI_i \rangle$ displays cause-effect chains, it is presumed to be chronologically ordered. let tv_{j_k} and $tv_{j_l} \in STV_i$ be two vertices. If $\exists P_{DI_i}\big(tv_{j_k}, tv_{j_l}\big)$, $V(tv_{j_k})$ at time t influences tv_{j_l} at a time $t + \Delta$, where $\Delta > 0$. The temporal structure of the $\mathcal{G}_D(a_l)$ allows loops to be eliminated [Nadkarni and

Shenoy, 2001]. It can therefore always be assumed that $\mathcal{G}_{D_i}^{t_r}(a_l)$ representing the problem's structure in a space of time $[t_r, t_{r+1})$ is a DAG. This is important if the scenarios constructed stretch far into the future and feedback between the variables has to be taken into account. In this case, a set of graphs $\left\{\mathcal{G}_{D_i}^{t_r}(a_l)\right\}_{r=0,\dots,R-1}$, where t_0 represents the first point in time considered and t_R the last.[27] For the attribute tree, the linear structure is ensured by the hierarchical order of the tree. By choosing the time steps appropriately, the structure of each Decision Map can therefore always be represented as a DAG, where possibly the results of different time spaces need to be aggregated in the end. This structure facilitates *distributed* or *local computation* of (intermediate) results. This is particularly important for those parts of the Decision Map that are solved by automated systems or submit to standardised inference mechanisms (e.g., Bayesian Networks being sub-graphs of the Decision Map).

5.2. Generation of Scenarios

The Decision Map configured as described in the previous sections is now used to generate a scenario set SS for assessing the situation or, in SBR & MCDA, scenario sets $SS(a_l)$ assessing the consequences of implementing each $a_l \in A$. The construction of the (general) set SS can be considered as a special case of the generation of $SS(a_l)$ (namely, for the case $A = \varnothing$ that is encountered in SBR & SM). Therefore, this section describes without loss of generality the scenario generation process by means of the generation of $SS(a_l)$.

For each $a_l \in A$ the corresponding scenario S_0^l, derived from $\mathcal{G}_D(a_l)$ as described in equation 5.1 is initialised by means of the initial situation description $INIT = INIT(MCDA\text{-}B)$ (cf. Section 4.1.5). Denote $STV_0^l \cap STV_{init} = J_{init} = \{j_1, \dots, j_K\}$. For each variable $tv_{j_k}^{init}$, $j_k \in J_{init}$, all values $\in SSPV_{init}$ need to be considered. Particularly, $(tv_A, SPV(tv_A) = A) \in INIT$ must be taken into account. By defining

$$S_i^{init,j_1,l} = \left\langle STV_i^{init,j_1,l}, sv_i^{init,j_1,l}, status_i^{init,j_1,l}, DI_i^{init,j_1,l} \right\rangle,$$

[27] $\mathcal{G}_{D_i}^{t_{R-1}}(a_l)$ is assumed to represent the space of time $[t_{R-1}, t_R)$.

where

$$
\begin{aligned}
STV_i^{init,j_1,l} &= STV_0^l \;\forall i = 1, \ldots n(j_k) \\
sv_i^{init,j_1,l} &= \begin{cases} V_i(tv_j) \;\forall i = 1, \ldots n(j_1) & \text{if } tv_j = tv_{j_1}^{init} \\ \infty & \text{otherwise,} \end{cases} \\
status_i^{init,j_1,l} &= \begin{cases} status_i(tv_j) \;\forall i = 1, \ldots n(j_k) & \text{if } tv_j = tv_{j_1}^{init} \\ \text{“}not\ assessed\text{”} & \text{otherwise,} \end{cases} \\
DI_i^{init,j_1,l} &= DI_0^l \;\forall i = 1, \ldots n(j_k)
\end{aligned}
\tag{5.2}
$$

the scenario initialisation starts with integrating the required values for $tv_{j_1}^{init}$ into the initial scenario S_0^l. In this manner, $n(j_1)$ scenarios arise that differ in the values (and possibly in the statuses) of $tv_{j_1}^{init}$. For *each* of these scenarios, the procedure is repeated and the possible values for $tv_{j_2}^{init}$ are integrated and so forth, until a set of scenarios $SS^{l,init}$ arises. As all values in $SSPV_{init}$ need to be combined, the number of scenarios in $SS^{l,init}$ is $\prod_{\iota=1}^{K} n(j_\iota)$.

Then, the scenario generation recurring to the network of experts captured in $\mathcal{G}_D(a_l)$ starts by determining the values of the $SEED_D = SEED_{MCDA,D}$ variables (cf. Section 4.1.4). For the independent vertices $tv_j^{SEED} \in SEED_D \setminus INIT$,[28] the responsible experts determine autonomously one or more values $V_i\left(tv_j^{SEED}\right)$ ($i = 1, \ldots, n\left(j^{SEED}\right)$) depending on the quality and accessibility of information as well as on the time available. Again, these values and the according statuses are integrated and combined to the scenarios $SS^{l,init}$, such that a set of scenarios $SS^{l,init,SEED}$, containing values and statuses for all variables in $SEED_D$ and $INIT_D$ arses.

After these initialisation steps, scenarios are further developed successively: let $S_{\Psi(tv_j)}^{\eta}$ be an incomplete scenario containing of values and statuses for all variables in $\Psi(tv_j)$, e.g.,

$$
\begin{aligned}
sv_\eta(\Psi(tv_j)) &= \{V_\eta(tv_{k_1}), \ldots, V_\eta(tv_{k_n})\}, \\
status_\eta(\Psi(tv_j)) &= \{status_\eta(tv_{k_1}), \ldots, status_\eta(tv_{k_n})\},
\end{aligned}
$$

for $|\Psi(tv_j)| = n$ and $\eta \in \{1, \ldots, N\}$, where N is the number of incomplete scenarios. Furthermore, $V_\eta(tv_j) = \infty$ and $status_\eta(tv_j) =$ “*not assessed*”.

[28] The variables in $INIT$ can be excluded here, as their values and statuses are already taken into account.

The expert responsible for determining tv_j's value is provided $sv_\eta\left(\widetilde{\Psi}\left(tv_j\right)\right) = \left\{V_\eta\left(tv_{k_j}\right)\ \forall tv_{k_j} \in \widetilde{\Psi}\left(tv_j\right)\right\}$.

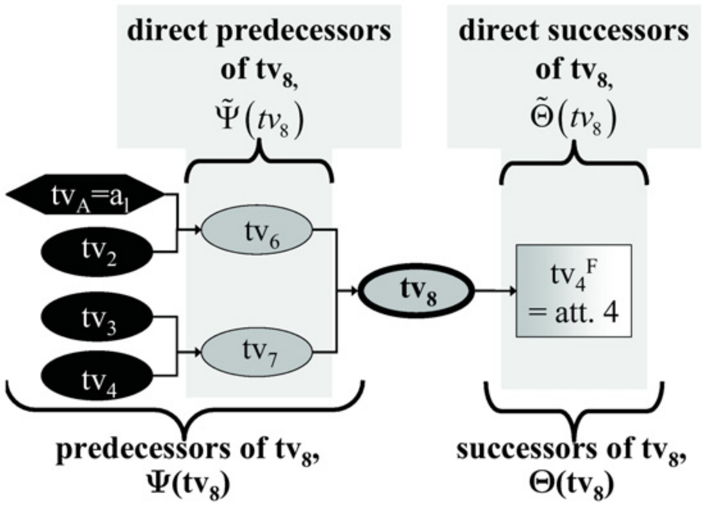


Figure 5.3.: *Structure of the Flow of Information: Example for tv_8.*

Figure 5.3 shows an example, where the scenario generation is performed for a part of $\mathcal{G}_D$ shown in Figure 5.2. Whilst assuming that $STV_{init} = \varnothing$, the scenario generation starts with determining the values for the $SEED$-variables, depicted in black. By using scenario extension equivalence classes, the step of determining these values and statuses falls into one equivalence class. Successively following the path from these $SEED$-variables to the $FOCUS$-variables (here, only $tv_4^F \in FOCUS$), the scenarios are completed. Although the information on the full scenarios (including values and statuses for *all* variables in STV) forms the scenario, each expert is only provided the values of the variables he judged relevant to perform his task. For example, in Figure 5.3 the expert responsible for tv_8 receives only combinations of values for tv_6 and tv_7 and no further, irrelevant or redundant information.

Given the values of the direct predecessor variables, the responsible expert determines the set of possible values $SPV_{S^\eta_{\Psi\left(tv_j\right)}}: V_l^{\eta,1}, \ldots, V_l^{\eta,\lambda_l}$ and a status for each of these values. In this manner, the scenario $S^\eta_{\Psi\left(tv_j\right)}$ multi-furcates to a new set of (possibly incomplete) scenarios $SS^{\eta,j}$. Thus, uncertainty is reflected in a multiplicity of values and statuses. Each of the previous instantiations of the scenario splits in a number of new (updated and more complete) scenarios.

Figure 5.4 refers to the example discussed throughout this section. It shows the multi-furcation of scenarios. Two sets of values for variables tv_6 and tv_7 with $|SPV_{SS}\left(tv_6\right)| = 3$ and $|SPV_{SS}\left(tv_7\right)| = 2$ need to be combined. As tv_6

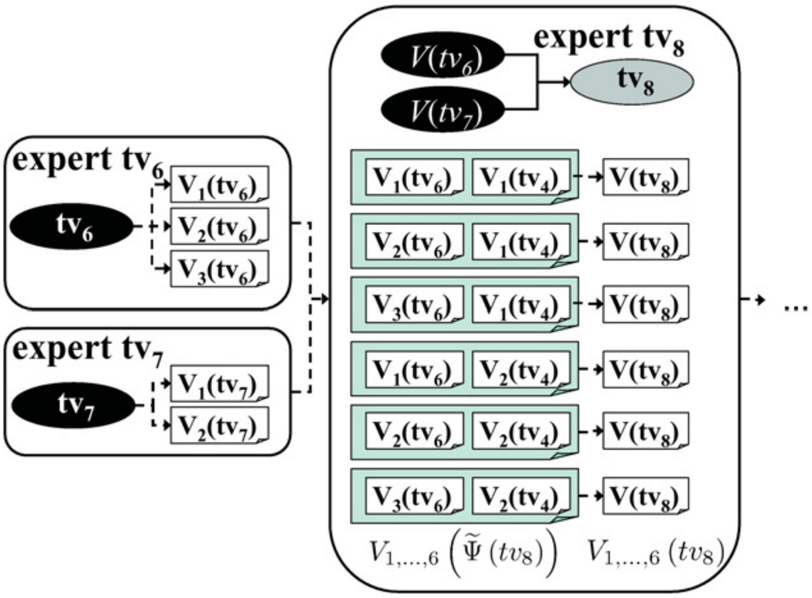


Figure 5.4.: *Example Scenario Multi-Furcation. (Causal) dependencies are represented as solid edges, flows of information as dashed edges. Values for variables tv_6 and tv_7 are combined to pairs $V_{\widetilde{\Psi}}(tv_8)$, each of which is used to determine $V(tv_8)$.*

and tv_7 are independent from each other all values are combined to $3 \cdot 2 = 6$ pairs $V_1\left(\widetilde{\Psi}(tv_8)\right), \ldots, V_6\left(\widetilde{\Psi}(tv_8)\right)$ as indicated in Figure 5.4. The expert determining $V(tv_8)$ is provided these pairs of possible values. As $status(tv_8) = deterministic$, he determines one value $V_i(tv_8)$ ($i = 1, \ldots, 6$) for each pair. These values are integrated into the scenarios (via scenario continuation). The scenario generation continues with the direct successors of tv_8.

The scenario generation is completed when each variable in the CM was assessed and is assigned at least one value. The arising set of scenarios can be understood as a way of expressing uncertainty in a set of values per variable, where each scenario contains a concerted set of values (and statuses).

5.3. Robust Decision Support: Scenario-Based Evaluation of Decision Alternatives

This section is dedicated to the evaluation of scenarios on the basis of MADM techniques. It is applicable for SBR & MCDA problems, for which an MADM attribute tree is available.

5.3.1. Evaluation of Scenarios

After the scenario generation, the alternatives are evaluated. To this end, the attribute tree is used. In the simplest case, all variables are deterministic, and only one scenario $S(a_l)$ with a corresponding set of attribute scores for each $a_l \in A$ is derived. Hence, standard MAVT techniques as described in Section 2.1.3 can be applied. If there are uncertain variables, a set of scenarios $SS(a_l)$, $|SS(a_l)| > 1$ is created $\forall a_l \in A$. Analogue to the deterministic case each scenario $S_i(a_l) \in SS(a_l)$ is evaluated using MAVT techniques. The computation of the attributes' values and their aggregation for each scenario results in an in-depth evaluation with respect to multiple criteria of each $S_i(a_l) \in SS$. Particularly, the overall performance $R(S_i(a_l))$ for each $S_i(a_l) \in SS(a_l)$ allows the scenarios $S_i(a_l)$ to be ranked.

A common approach in scenario planning is to present the worst and best scenarios to the decision-makers [Schnaars, 1987]. As a structured evaluation procedure is usually not part of scenario planning process [Durbach and Stewart, 2003], the selection of scenarios labelled "worst" and "best" is usually rather intuitive and highly demanding for the experts involved. By integrating scenario construction and evaluation, worst- and best-*evaluated* scenarios $S^w(a_l)$ and $S^b(a_l)$ can be identified for each alternative. Thus, the decision-makers' preferences are taken into account explicitly. This approach does neither refer to an intuitive definition of pessimistic or optimistic cases (as often done in scenario planning [Schnaars, 1987]) nor require measuring the dissimilarity or distance between scenarios by the difference in the values of the variables (which, in this thesis, corresponds to the values of the variables within each scenario) as in formative scenario analysis [Scholz and Tietje, 2002]. It makes the spread of evaluations according to the *decision-makers' preferences* and the worst-case performance for each alternative easily visible. In this manner, the approach presented supports decision-makers in choosing an alternative whose total performance (or performance in selected criteria) does not fall below a certain threshold τ, reflecting the minimal required performance. Thus the Decision Map approach facilitates **robust decision-making,** i.e. making a decision that performs sufficiently well for a set of scenarios [Ben-Haim, 2000] or that guarantees that a minimum performance is reached for all scenarios [Vincke, 1999].

5.3.2. Robust Decision-Making under Severe Uncertainty

Uncertainty is present throughout all phases of the decision-making process, i.e., from the construction of $\mathcal{G}_D$ and the generation of scenarios to the elicitation of preferences and, finally, the selection of an alternative. To address the problem of decision-making under severe uncertainty, where (given an alternative a_l) no information about the likelihood of a scenario $S_i(a_l)$ is available, it is widely suggested that the recommendations are derived from the **attitude towards risk** of the decision-makers [Acker, 1997; Liu, 2004; Woodward and Bishop, 1997; Yager, 2008]: for pessimistic decision-makers the alternative a_l, for which $\min_{i \in I(a_l)} R(S_i(a_l))$ is maximal is the most attractive. This behaviour is also referred to as *absolutely robust* [Kouvelis and Yu, 1997]. Contrarily, optimistic decision-makers focus on the best evaluated outcomes and choose the alternative, for which $\max_{i \in I(a_l)} R(S_i(a_l))$ is maximal.

It has already been mentioned that the integrated SBR & MCDA approach facilitates the identification of worst- and best-evaluated scenarios for an alternative a_l ($S^w(a_l)$ and $S^b(a_l)$). This supports purely optimistic and pessimistic decision-making. A method combining pessimistic and optimistic attitudes is the **Hurwicz** approach [Arrow and Hurwicz, 1977; Woodward and Bishop, 1997]; the total evaluation $R_H(a_l)$ for each alternative a_l is determined by a convex combination of the worst- and best-evaluated scenario's results:

$$R_H(a_l) = \alpha R(S^w(a_l)) + (1-\alpha) R\left(S^b(a_l)\right), \qquad \alpha \in [0,1].$$

Another approach for modelling risk attitudes between the extremes of optimistic and pessimistic decision-making is **neutral decision-making** [Acker, 1997] considering each scenario $S_i(a_l) \in SS(a_l)$ equally, resulting in an evaluation

$$R_n(a_l) = \frac{1}{n(a_l)} \sum_{j=1}^{n(a_l)} R(S_i(a_l)).$$

These methods reduce the complexity (represented by the number of scenarios and results to consider) by either the *selection* of the scenario results that are considered the most relevant or by the *aggregation* of all results. It has already been emphasised that the selection of worst- and best-evaluated scenarios is supported by the integrated SBR & MCDA framework. A drawback of selecting only a subset of $SS(a_l)$ for presentation to the decision-makers is that it does not convey the complete information available. Furthermore, decision-makers can be biased particularly by worst-evaluated scenarios [Hämäläinen et al., 2000]. Therefore, this thesis complements the selection of scenarios by an aggregation of results. To this end, the decision-makers' risk preferences are elicited. Although the Hurwicz approach reflects the decision-makers' risk preferences, it is standardised and prescriptive. Contrarily to preference elicitation methods in MADM, it does not facilitate a detailed elicitation of preferences and value judgements. Moreover, it takes into account only two scenarios. In the following a number of techniques for explicitly taking into account all scenarios and the risk attitudes and preferences for the importance of each scenario are developed.

5.3.3. Determining Inter-Scenario Preferences

This thesis proposes an approach based on MAVT techniques [Comes et al., 2009b] to take into account the performances of alternatives under a broad set of scenarios and to come to a more balanced decision. To this end, the decision-makers' preferences, which reflect the importance of each scenario on the basis of its evaluation, are elicited. These preferences are modelled as weights. The aggregation of the scenarios' performances $R(S_i(a_l))$ to the performance $R(a_l)$ of an alternative a_l can be achieved additively or multiplicatively depending on the preferences of the decision-makers (particularly, independence considerations). Additive aggregation (e.g., the Simple Additive Weighting (SAW) method) is prevalent due to its intuitive understandability that makes it easily accessible to those involved in the decision-making process [Belton and Stewart, 2002]. Furthermore, the sensitivity results to errors caused by the (undue) use of additive aggregation is significantly smaller than the sensitivity to incorrect modelling of the value functions [Stewart, 1996]. Therefore, SAW is applied here: for each $a_l \in A$ consider the set of scenarios $SS(a_l)$. The performance of a_l is:

$$R(a_l) = \sum_{S_i(a_l) \in SS(a_l)} \omega(S_i(a_l)) \cdot R(S_i(a_l)) . \tag{5.3}$$

It is not proposed to consider the performance $R(a_l)$ as a prescriptive "*right*" answer. This approach presented in this thesis rather aims at providing the decision-makers with a rationale supporting them to reason about the alternatives, their preferences and value judgements.

An obvious approach to determine the preferences is the adoption of preference elicitation techniques from MADM. Commonly used techniques to elicit the inter-criteria preferences are the Simple Multi-Attribute Rating Technique (SMART) [Edwards, 1977] or SWING [von Winterfeldt and Edwards, 1986]. Both are simple, easy to use and widely accepted methods based on ratio estimation [Mustajoki and Hämäläinen, 2005]. A requirement for their application is that the number of comparisons to be made is modest [Edwards, 1977]. For instance, Edwards [1977] admonishes to use not more than about eight dimensions.[29] Hence, the direct elicitation of scenario weights from the decision-makers becomes problematic if the set of scenarios is large. In this case, other approaches facilitating the elicitation process need to be implemented. In the following sections two new approaches to determine scenario weights requiring different skills and expertise from the decision-makers are presented.

5.3.3.1. Determining Inter-Scenario Preferences Based on the Concept of Satisficing

The first approach relies on an idea of robustness, which corresponds to the concept of satisficing developed by Simon [1979]. This concept is based on the assumption that decision-makers do not merely choose an alternative that maximizes the performance $R(a_l)$, but the one that *guarantees satisfactory performance.* Accordingly, a decision is called **robust,** if it ensures achieving a minimum performance R_{min} [Vincke, 1999] or if it ensures that specific sub-goals are reached (e.g., a minimum performance for a set of criteria or attributes) [Ben-Haim, 2000].

If the scenarios represent a broad span of possible futures, it may be that for all alternatives there are scenarios, in which some constraints are violated. In this case, this thesis assumes that the alternative that is *closest* to satisfactory performance in all constraints is to be chosen. This novel approach allows the risk preferences to vary between criteria to develop a measure for the distance to the satisfactory performance. For instance, in emergency management threats to human health and safety may be treated rather conservatively, while

[29] In standard MADM, these dimensions correspond to criteria. Here, the scenarios $S_i(a_l)$ should be regarded as dimensions.

the performance of criteria reflecting economic losses or resource use may be handled rather risk-neutral.[30]

The first step in this approach is the definition of thresholds t_ι^{risk} that should be achieved for a set of criteria or attributes $\left\{c_\iota : l \in C^{risk}\right\}$. Let $R_{c_\iota}\left(S_i\left(a_l\right)\right)$ be the performance of scenario $S_i\left(a_l\right)$ in criterion $c_\iota \in C^{risk}$. When $R_{c_\iota}\left(S_i\left(a_l\right)\right)$ violates a constraint (i.e., $R_{c_\iota}\left(S_i\left(a_l\right)\right) \leq t_\iota^{risk}$), the importance of $S_i\left(a_l\right)$ increases. Therefore, the weight $\omega\left(S_i\left(a_l\right)\right)$ of scenario $S_i\left(a_l\right)$ is raised according to a penalty-function θ_ι reflecting the decision-makers' risk aversion. Finally, the scenario weights are calculated as a deviation from equal weights. That means, the starting point for modelling the increasing relative importance of s is the assumption that all scenarios, which do not violate any constraint, are equally important.

Having normalised the attributes' scores with value functions and chosen weights w_i^k such that $\sum_{i=1}^{n(k)} w_i^k = 1$ at all abstraction levels k of the attribute tree (with $n(k)$ elements), $R_{c_\iota}\left(S_i\left(a_l\right)\right) \in [0,\ 1]$. A function $\theta_i\left(R_{c(j)}(S)\right)$ is introduced, to determine the importance of scenarios violating the constraint. (S is a scenario in the set SS.) θ_ι is required to have the following properties:

$$\begin{aligned}
\theta_\iota : [0,1] \rightarrow [0,1] \quad & \text{continuous,} \\
\theta_\iota\left(t_\iota^{risk}\right) = 0, & \\
\theta_\iota\left(R_{c_\iota}\left(S_\alpha\left(a_l\right)\right)\right) \geq \theta_\iota\left(R_{c_\iota}\left(S_\beta\left(a_l\right)\right)\right), & \\
& \text{if } t_\iota^{risk} - R_{c_\iota}\left(S_\alpha\left(a_l\right)\right) > t_\iota^{risk} - R_{c_\iota}\left(S_\beta\left(a_l\right)\right) > 0
\end{aligned} \tag{5.4}$$

where $S_\alpha\left(a_l\right), S_\beta\left(a_l\right) \in SS\left(a_l\right) : \; R_{c_\iota}\left(S_\alpha\left(a_l\right)\right) < t_\iota^{risk}$ and $R_{c_\iota}\left(S_\beta\left(a_l\right)\right) < t_i^{risk}$ violate the constraint in c_ι.

Absolute constraints (e.g., referring to past cases as benchmarks) can be used can be used to determine t_i^{risk}. Alternatively the average performance in c_ι, $\frac{1}{|SS|}\sum_{S_i \in SS} R_{c_\iota}(S)$ or p-quantiles (of the performance in c_ι) can be introduced to assess the *relative* performance of scenarios.

It can be important not to focus only on the risks but also on the *opportunities* an alternative offers [Matos, 2007]. To balance risks and opportunities, performance thresholds, which assign more importance to results *better* than the expected outcome t_κ^{opp} ($k \in C^{opp}$) are taken into account. (Again, if there

[30]As the risk aversion is used for assessing scenario weights, this thesis suggests using risk-neutral value functions for the normalisation of attributes (e.g., linear value functions).

is no information about a best guess, the average performance or a p-quantile can be used to determine t_κ^{opp}.) To this end, functions $\varphi_\kappa\left(R_{c_k}\left(S_j\left(a_l\right)\right)\right)$ rewarding an exceedance of the threshold t_κ^{opp} with higher scenario weights are determined. Analogue to the conditions 5.4 on θ_l, it is required that φ_κ is a continuous function with $\varphi_\kappa\left(t_\kappa^{opp}\right) = 0$. Let $S_\alpha\left(a_l\right), S_\beta\left(a_l\right) \in SS\left(a_l\right)$ be two scenarios exceeding t_κ^{opp} in criterion c_κ ($k \in C^{opp}$). For $R_{c_k}\left(S_\alpha\left(a_l\right)\right) - t_\kappa^{opp} > R_{c_l}\left(S_\beta\left(a_l\right)\right) - t_\kappa^{opp} > 0$, it holds:

$$\varphi_\kappa\left(R_{c(\kappa)}S^\alpha\right) \geq \varphi_\kappa\left(R_{c(\kappa)}S^\beta\right). \tag{5.5}$$

This approach combining risk and opportunity considerations reflects both the threat of falling below t_ι^{risk} and the opportunities of exceeding t_κ^{opp}. It facilitates furthermore evaluating negative and positive deviations from a goal differently through the explicit specification of θ_ι and φ_κ. This is likely to appeal to the users, as it has been shown that decision-makers in fact value risks and chances differently [Fischer et al., 1986].

If there is a criterion c_ι, for which both, risks and opportunities are taken into account (i.e., $i \in C^{risk} \cup C^{opp}$, and $t_\iota^{risk} \leq t_\iota^{opp}$ θ_l and φ_ι are combined to a single function $\xi_\iota\left(R_{c(i)}\left(S_i\left(a_l\right)\right)\right)$ for all $S_i\left(a_l\right) \in SS\left(a_l\right)$ is defined by:

$$\xi_\iota\left(R_{c_\iota}\left(S_i\left(a_l\right)\right)\right) = \begin{cases} \theta_\iota\left(R_{c_\iota}\left(S_i\left(a_l\right)\right)\right), & 0 \leq R_{c_\iota}\left(S_i\left(a_l\right)\right) < t_\iota^{risk} \\ 0, & t_\iota^{risk} \leq R_{c_\iota}\left(S_i\left(a_l\right)\right) < t_\iota^{opp} \\ \varphi_\iota\left(R_{c_\iota}\left(S_i\left(a_l\right)\right)\right), & t_\iota^{opp} \leq R_{c_\iota}\left(S_i\left(a_l\right)\right) \leq 1 \end{cases} \tag{5.6}$$

The conditions on θ_ι and φ_ι ensure that ξ_ι is continuous.

Figure 5.5 shows an example, where θ_ι and φ_ι were modelled by exponential functions (see formula 5.7). The parameters ρ determining the curvature were chosen equally for θ_ι and φ_ι (i.e., $\rho_1 = \rho_2$) by $\rho_{1,2}^\alpha = -0.1$ (solid line) and $\rho_{1,2}^\beta = 0.1$ (dashed line).

$$\xi_\iota\left(R_{c_\iota}\left(S_i\left(a_l\right)\right)\right) = \begin{cases} \frac{1-\exp\left(\left(R_{c_\iota}\left(S_i\left(a_l\right)\right)-t_\iota^{risk}\right)/\rho_1\right)}{1-\exp\left(-t_\iota^{risk}/\rho_1\right)}, & 0 \leq R_{c_\iota}\left(S_i\left(a_l\right)\right) < t_\iota^{risk} \\ 0, & t_\iota^{risk} \leq R_{c_\iota}\left(S_i\left(a_l\right)\right) < t_\iota^{opp} \\ \frac{1-\exp\left(\left(t_\iota^{opp}-R_{c_\iota}\left(S_i\left(a_l\right)\right)\right)/\rho_2\right)}{1-\exp\left(\left(t_\iota^{opp}-1\right)/\rho_1\right)}, & t_l^{opp} \leq R_{c_\iota}\left(S_i\left(a_l\right)\right) \leq 1 \end{cases} \tag{5.7}$$

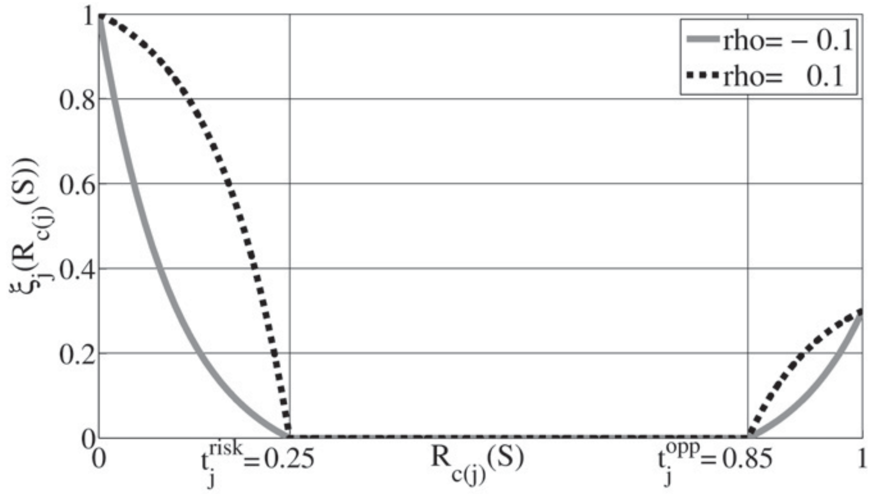


Figure 5.5.: *Relative Importance of Scenarios for Varying Results with respect to Criterion c_ι. $t_\iota^{risk} = 0.25$ and $t_\iota^{opp} = 0.85$ modelled by for varying ρ exponential functions, cf. equation 5.7.*

Let $\iota \in C$ be a criterion at abstraction level al. Setting

$$\xi_\iota\left(R_{c_\iota}\left(S_i\left(a_l\right)\right)\right) = \begin{cases} \xi_l\left(R_{c_l}\left(S_j\left(a_i\right)\right)\right), & l \in C^{risk} \cup C^{opp} \\ 0, & \text{otherwise} \end{cases}, \tag{5.8}$$

ξ is defined for the complete set of criteria C (at an abstraction level al).

Using ξ, weights $\omega\left(S_i\left(a_l\right)\right)$ reflecting the relative importance of each scenario $S_i\left(a_l\right) \in SS\left(a_l\right)$ can be deduced by setting

$$\omega\left(S_i\left(a_l\right)\right) = \mu + \sum_{\iota \in C} \xi_\iota\left(R_{c_\iota}\left(S_i\left(a_l\right)\right)\right), \qquad \mu > 0. \tag{5.9}$$

Particularly, $\omega\left(S_i\left(a_l\right)\right)$ take into account decision-makers' risk attitude. The weight of scenarios, which do not exceed any of the thresholds, is set to μ. The simplest way of choosing μ is using equal weights, i.e., $\mu = \frac{1}{|SS(a_l)|}$. The choice of the parameter μ has, however, a strong influence on $\omega\left(S_i\left(a_l\right)\right)$. Denote

$$\xi\left(S_i^{\max}\left(a_l\right)\right) = \max_{S_i(a_l) \in SS(a_l)} \sum_{\iota \in C} \xi_\iota\left(R\left(S_i\left(a_l\right)\right)\right)$$

the maximal value of the penalty function for all considered scenarios $S_i\left(a_l\right) \in SS\left(a_l\right)$. With decreasing $\frac{\mu}{\xi(S_i^{\max}(a_l))}$ the consideration of the penalty and reward functions becomes more and more important. Therefore, μ should be chosen carefully whilst taking into account the value $\xi\left(S_i^{\max}\left(a_l\right)\right)$.

If it is possible to determine the probability[31] or likelihood $p(S_i(a_l))$ for all $S_i(a_l) \in SS(a_l)$ and all $a_l \in A$ [Duperrin and Godet, 1975], the weights can be corrected by this factor, i.e., $\omega(S_i(a_l)) = p(S_i(a_l)) \left(\mu + \sum_{\iota \in C} \xi_\iota \left(R_{c_\iota}(S_i(a_l)) \right) \right)$.

Finally, the weights are normalised such that $\sum_{S_i(a_l) \in SS(a_l)} \omega(S_i(a_l)) = 1$. The performance of alternative a_l is

$$R(a_l) = \sum_{S_i(a_l) \in SS(a_l)} \omega(S_i(a_l)) \cdot R(S_i(a_l)) \,. \tag{5.10}$$

5.3.3.2. Determining Inter-Scenario Preferences Based on the Inclination Towards Risk

When it is necessary to aggregate numeric values to one single number, an aggregation function widely considered is the weighted arithmetic weighted mean (for general considerations including the quasi-arithmetic mean see, e.g., [Aczél, 1984], for applications in MCDA see [Belton and Stewart, 2002]).

An alternative aggregation function to the arithmetic weighted mean is the **Ordered Weighted Average** (OWA) operator [Yager, 2002, 2008]. For the aggregation of scenario evaluations define $R\left(\left\{S_1^{a_l}, \ldots, S_{n(a_l)}^{a_i}\right\}\right) =: R(SS(a_l))$. As in standard MAVT aggregation techniques, the OWA operator is a function mapping a finite set of values (each of which is in $[0,1]$) to $[0,1]$:

$$\begin{aligned} \mathrm{OWA}_\omega :\ & [0,1]^{n(a_l)} \to [0,1] \\ \mathrm{OWA}_\omega\left(S_1^{a_l}, \ldots, S_{n(a_l)}^{a_l}\right) &= \sum_{j=1}^{n(a_i)} \omega_j \cdot \sigma_j\left(R(SS(a_i))\right), \end{aligned} \tag{5.11}$$

$\omega_j \in [0,1]\ \forall\ j = 1, \ldots, n(a_i)$ and $\sum_{j=1}^{n(a_i)} \omega_j = 1$. Evidently, the main difference to the SAW method (cf. formula 5.3), is the use of the permutation operator $\sigma(R(SS(a_i)))$, which defines a permutation on the performances of a set of

[31] A controversy on handling uncertainty divides literature on Scenario-Based Reasoning into two camps: while some argue that users require guidance on likelihoods to use the scenarios and that assigning probabilities helps incorporate the best available expert information, others reply that scenario probability estimates suggest a misleading degree of certainty [Groves and Lempert, 2007; Parson et al., 2007]. While I do not take either side, I would like to describe how probabilistic information can be integrated in this framework.

scenarios such that $\sigma_j\left(R\left(SS\left(a_i\right)\right)\right)$ returns the result of the jth best evaluated scenario for an alternative a_i. Hence,

$$\sigma_1\left(R\left(SS\left(a_i\right)\right)\right)=\max_{S_j^{a_i}\in SS(a_i)}\left\{R\left(S_j\left(a_i\right)\right)\right\}$$

returns the result of the best evaluated scenario, while

$$\sigma_{n(a_i)}\left(R\left(SS\left(a_i\right)\right)\right)=\min_{S_j^{a_i}\in SS(a_i)}\left\{R\left(S_j\left(a_i\right)\right)\right\}$$

returns the result of the worst evaluated scenario for alternative a_i.

Comparing the weighted mean and the OWA operator, they share the property of mapping $n\left(a_i\right)$ numerical values to $\mathbb{R}$ according to a vector of weights $\omega\in\mathbb{R}^{n(a_i)}$. The main difference is the meaning assigned to ω: the weighted mean takes into account the *(relative) importance* of each criterion on a certain aggregation level [Keeney, 1996] regardless of its actual value. Contrarily, the OWA operator takes into account the importance of a value (of a criterion) in relation to other values and permits weighting the values in relation to their *ordering* [Torra, 1997]. That means, by means of the OWA operator the decision-makers can express their preferences in relation to the ranking of values of the criteria, but not the preferences between criteria [Valls et al., 2009]. In this way, it facilitates assigning more importance to certain subsets of the valuations. For instance, the weight of extreme values to the result can be diminished, increasing the influence of central values or vice versa [Torra, 1997].

Yager [1988] introduced two measures to characterise the weights ω: a measure of dispersion and the attitudinal character of the resulting OWA$_\omega$-operator. Although a number of *dispersion measures* characterising the OWA weights have been developed [Fullér, 2007; Xu, 2005], the *Shannon-entropy* is the most commonly used dispersion measure [Malczewski, 2005; Xu, 2005; Yager, 2008]. It is defined as (for the original definition and the properties of the operator see

[Shannon, 1948], for the first definition in the context of OWA opertors see [O'Hagan, 1988]):

$$
\begin{aligned}
e: [0,1]^{n(a_i)} &\rightarrow [0,1] \\
e(\omega) &= -\sum_{j=1}^{n(a_i)} \mathrm{ent}(\omega_j) \qquad (5.12) \\
\text{where} \qquad \mathrm{ent}(\omega_j) &= \begin{cases} \omega_j \cdot \ln(\omega_j), & \text{if } \omega_j > 0 \\ 0, & \text{if } \omega_j = 0 \end{cases}
\end{aligned}
$$

$e(\omega)$ essentially measures the degree to which all the arguments influence the result of the aggregation [O'Hagan, 1988; Yager, 1988]. Note that $e(\omega)$ is maximal for $\omega_j = \frac{1}{n(a_i)}$ for all $j = 1, \ldots, n(a_i)$ and minimal if $\exists j^* \in \{1, \ldots, n(a_i)\}$: $\omega_{j^*} = 1$ and $\omega_j = 0$ for all $j \neq j^*$. While in the first case, all values contribute equally, in the latter case essentially, only one argument influences the result of aggregation.

The *attitudinal character* measures the degree, to which the result of the OWA_ω operator resembles the logical OR[32], which has as a result the scenario with the best evaluation [Malczewski, 2005]. It is defined as

$$
\begin{aligned}
ac: [0,1]^{n(a_i)} &\rightarrow [0,1] \\
ac(\omega) &= \sum_{j=1}^{n(a_i)} \frac{n(a_i) - j}{n(a_i) - 1} \omega_j. \qquad (5.13)
\end{aligned}
$$

Interpreting ac in the context of well-established behavioural theories of inclination towards risk [March and Shapira, 1987; Rohrmann, 1998], ac can be interpreted as a measure of the degree of the decision-maker's optimism [Yager, 1988]. While risk averse decision-makers typically weigh negative outcomes highly, risk-taking decision-makers are likely to emphasize positive outcomes more. Risk attitudes can be represented on a continuum from risk aversion to risk seeking [Bodily, 1984; Malczewski, 2005]. Choosing $\omega^{opt} = [1, 0, \ldots, 0]$ captures, e.g., the *optimistic* decision-maker's attitude, who considers only the best evaluated scenario for each alternative ($ac(\omega^{opt}) = 1$), whereas $\omega^{pes} = [0, \ldots, 0, 1]$ models the *pessimistic* decision-maker's attitude considering only the worst evaluated scenario ($ac(\omega^{opt}) = 0$).

[32] The logical OR operator is equivalent to the max operator.

Using both entropy and attitudinal character, OWA weights can be obtained by solving the following problem:

$$\begin{aligned} & \text{maximize } e(\omega) \\ s.t. \quad & ac(\omega) = \alpha, \qquad \alpha \in [0,1], \\ & \sum_{j=1}^{n(a_i)} \omega_j = 1, \\ & \omega_j \in [0,1] \ \forall \ j = 1, \dots, n(a_i). \end{aligned} \tag{5.14}$$

The rationale behind is that for a given inclination to risk α, the influence of each entry ω_j on the overall result is maximised. Fullér and Majlender [2001] have solved this non-linear problem analytically facilitating the calculation of the optimal weighing vector ω^* modelling the decision-makers' inclination to risk.

Having determined the weights, the performance of an alternative a_i is:

$$R(a_i) = \mathrm{OWA}_{\omega^*}(R(SS(a_i))) = \sum_{j=1}^{n(a_i)} \omega_j^* \cdot \sigma_j(R(SS(a_i))). \tag{5.15}$$

5.4. Scenario-Based Risk Assessment

An important issue in decision-making under uncertainty is assessing the **risks** associated to a decision. Often, it is useful to complement the SBR by a risk assessment [Matos, 2007]. Before defining novel approaches to scenario-based risk assessment, it is necessary to give a concrete definition of the term risk as it is understood within this thesis.

5.4.1. Definition of Risk

A very broad definition considers risk as *"a chance of something bad happening"* [Fishburn, 1984]. Thus, risk is associated with **uncertainty of an event** and how sensitive the decision-makers are to the **impact** of this event. More precisely, risk can be defined as a measure of the likelihood and severity of adverse effects or the extent of loss [Haimes et al., 2002; Morgan and Henrion, 1990]. As the likelihood of an event and its consequences can not always be expressed in terms of probabilities and as risk assessment aims not at conveying a single number describing the risk, but to facilitate understanding the system under scrutiny and its interactions with its environment [Haimes et al., 2002], approaches for **scenario-based risk assessment** have been developed [Haimes et al., 2002; Kaplan, 1997; Kaplan and Garrick, 1981]. In these approaches, risk is defined as a set of tuples

$$Risk = \{\langle S_i, l_i, x_i \rangle\}_{i \in I},$$

where S_i is a scenario, l_i its likelihood, and x_i is a vector characterising S_i's consequences.

Table 5.1.: *Scenario-Based Risk Assessment: Definitions of Risk*

Focus Completeness	**Purpose of SBR**	**Risk**
Complete	SM	$\langle SS, \{V_i\,(FOCUS)\}_{i\in I}\rangle$
Complete	MCDA	$\langle SS, \{agg_D\,(V_i\,(FOCUS))\}_{i\in I}\rangle$
Incomplete	SM	$\langle SS, \{V_i\,(IND)\}_{i\in I}\rangle$
Incomplete	MCDA	$\langle SS, \{agg_D\,(V_i\,(IND))\}_{i\in I}\rangle$

Adapting this concept to the scenario framework that has been developed within this thesis, risk is understood as a function of a certain **set of scenarios** $SS = \{S_i\}_{i\in I}$), the **reliability** of each $S_i \in SS$ and the **severity of the damage** that results from each $S_i \in SS$. For focus complete scenarios, the latter is captured in SBR frameworks by $\{V_i(FOCUS)\}_{i\in I}$ and in SBR & MCDA by the evaluations $\{f_D\,(V_i(FOCUS))\}_{i\in I}$. In focus incomplete scenarios the indicator framework (cf. Section 7.4) enables a characterisation of the consequences. This thesis understands risk as a tuple that is defined according to the focus completeness of the set of scenarios $SS = \{S_i\}_{i\in I}$ and the SBR framework. The definitions are summarised in table 5.1.

Furthermore, if any of the variables $tv_j \in STV_i$ depends on the time t, the risk is a function of time. The risk function $R(t)$ is integrated to combine the risk assessments for the considered time span $[t_0 - T_1, t_0 + T_2]$:[33]

$$R_{t_0-T_1}^{t_0+T_2} = \int_{t_0-T_1}^{t_0+T_2} R(t)\mathrm{dt}, \qquad T_1, T_2 \geq 0.$$

If there is a discretisation τ of $[t_0 - T_1, t_0 + T_2]$, the integral $R(t)$ can be written as a sum:

$$R^{step}(t) = \sum_{[t_i,t_{i+1}):t_i,t_{i+1}\in\tau} R(t_i)\,\chi_{[t_i,t_{i+1})}(t).$$

5.4.2. Risk Indices

To make the concept of risk measurable, a number of risk indices have been developed. Mostly, some of the following risk indices are considered [Artzner et al., 1998; Bell, 1995; Fishburn, 1984; Miranda and Proenca, 1998]: probability, variance and expected value of losses, the worst-case value and the regret[34].

The use of **deviation based risk measures** like variance or regret reflect the fact that risk refers to a loss or damage, which can be represented on the basis of a benchmark. This benchmark is, e.g., the optimal possible path (in case of regret, [Miranda and Proenca, 1998]) or a predefined set of values for each variable considered reflecting the normal situation [Fishburn, 1984]. The **severity** of the loss is measured by the worst possible outcome or the expected value of losses.

Although not all of these indices can be adapted to the SBR framework, it is possible to characterise the risk represented by a set of scenarios SS via **worst-case considerations,** the **discrepancy of possible outcomes,** the **deviation of outcomes from benchmarks** and the **regret.** The first is a severity based risk

[33] Here, typically the time t_0 denotes the time when a critical event triggering the further development happens. As not only the developments *after* the event, but also the situation development beforehand (e.g., when preventive measures are investigated) are relevant, the time span is explicitly not restricted to $[t_0, t_0 + T]$.

[34] Usually, *regret* is defined as a function of alternative $a_l \in A$ and the evaluation of a_l's outcome given a state of the world S_i, $f_D(a_l, S_i)$. Then, one defines

$$regret(a_l, S_i) = \max_{a_l \in A} f_D(a_l, S_i) - f_D(a_l, S_i),$$

[Bell, 1982; Miranda and Proenca, 1998; Stoye, 2009]. Later, it is shown how this concept can be adapted for SBR.

measure, while the latter are deviation based. These indices are analysed in the following sections.

5.4.2.1. Worst-Case Considerations

One of the most intuitive risk assessment principles, which is applied in a variety of fields of application, considers worst-case scenarios [Korn, 2005; Matos, 2007; Rustem et al., 2000; Wright and Goodwin, 2009]. While in mono-criteria problems, the definition of a good or bad outcome is straightforward[35], trade-offs have to be made when multiple variables are used to assess the effect of a scenario [Stewart, 1992]. The integrated SBR & MCDA approach that has been developed in Section 5.3 provides the evaluation function f_D facilitating an evaluation according to the decision-makers' preferences.

$$S_w(SS) = \{S_{i^*} \in SS : \; f_D\left(V_{i^*}(FOCUS)\right) \leq f_D\left(V_i(FOCUS)\right) \;\forall S_i \in SS\}\,.$$

$S_w(SS)$ is not necessarily unique, and possibly $|S_w(SS)| > 1$.

As for each alternative $a_l \in A$, the subsets $SS\left(a_l\right) = \{S_i \in SS : \; tv_D = a_l\} \subset SS$ are generated (cf. Section 5.2), the worst-case scenarios given an alternative a_l can be identified by

$$\begin{aligned} S_w\left(a_l, SS\right) \quad = \quad & \big\{S_j \in SS\left(a_l, SS\right) : \\ & \quad f_D\left(V_j(FOCUS)\right) \leq f_D\left(V_i(FOCUS)\right) \;\forall S_i \in SS\left(a_l\right)\big\}. \end{aligned}$$

By means of the set

$$SS_w^A = \{S_w\left(a_l\right) : \; a_l \in A\}$$

the alternative with the best worst-case performance can be identified, i.e., the mini-max principle can be applied [Matos, 2007; Stoye, 2009].

In SBR & SM the identification of what is perceived as worst possible scenario out of the scenarios constructed can only be decided by the recipients. To facilitate the identification, for each $tv_j^F \in FOCUS$ of type $Type_j$, for which a norm $\| \,.\, \|_j$ can be defined such that $(Type_j, \| \,.\, \|_j)$ is a normed vector space, the sce-

[35] Here, the assumption is made that the type of the criterion allows the definition of a metric, which itself measures the performance.

narios with the lowest (for increasing preferences) and highest (for decreasing preferences) performance are identified by:

$$S_j^w(SS) = \begin{cases} S_i \in SS : \ V_i\left(tv_j^F\right) \leq V_k\left(tv_j^F\right) \ \forall S_k \in SS & \text{for } \textit{increasing} \text{ preferences} \\ S_i \in SS : \ V_i\left(tv_j^F\right) \geq V_k\left(tv_j^F\right) \ \forall S_k \in SS & \text{for } \textit{decreasing} \text{ preferences.} \end{cases}$$

5.4.2.2. Discrepancy of Possible Outcomes

In SBR & SM the discrepancy needs to be considered first with respect to each variable $tv_j^F \in FOCUS$ by

$$spread_j(SS) = \max_{S_i, S_k \in SS} \left\{ \widetilde{dist}_j \left(V_i\left(tv_j^F\right), V_k\left(tv_j^F\right)\right)\right\}.$$

From this spread for each variable, the total spread can be derived:

$$spread_F(SS) = \frac{1}{|FOCUS|} \sum_{tv_j^F \in FOCUS} spread_j(SS).$$

Here, it is assumed that the spreads for all variables are equally important. Otherwise, weights modelling the users' preferences can be used.

In MCDA & SBR frameworks, the evaluations can be used as a basis to determine the spread. Globally, the spread of evaluations is

$$spread_{Pref_D}(SS) = \max_{S_{i_k}, S_{i_l} \in SS} \left\{ \left| f_d\left(V_{i_k}\left(FOCUS\right)\right) - f_d\left(V_{i_l}\left(FOCUS\right)\right)\right| \right\}.$$

To compare alternatives, it is useful to determine the spread of evaluations for each $a_l \in A$

$$\begin{aligned} & spread_{Pref_D}\left(SS\left(a_l\right)\right) \\ = & \max_{S_{i_k}, S_{i_l} \in SS(a_l)} \left\{ \left| f_d\left(V_{i_k}\left(FOCUS\right)\right) - f_d\left(V_{i_l}\left(FOCUS\right)\right)\right| \right\}. \end{aligned}$$

While the first approach allows all possible developments and results to be considered, the latter represents the span of possible consequences of a decision. Furthermore, it holds:

$$0 \leq spread_{Pref_D}\left(SS\left(a_l\right)\right) \leq spread_{Pref_D}(SS) \leq 1.$$

5.4.2.3. Deviation of Results from Benchmarks

The deviation of scenario results from benchmarks (which is also the basis for robust decision-making, see Section 5.3.3.1) can be operationalised by the number of scenarios that violate a set of constraints [Matos, 2007]. Here, it is assumed again that types of the focus variables are normed vector spaces. Given a set of constraints $C = \{C_1, \ldots, C_N\}$, each of which is defined with respect to a set of focus variables $SFV_j = \left\{tv^F_{j_1}, \ldots, tv^F_{j_{n(j)}}\right\}$ ($j = 1, \ldots, |FOCUS|$), $SFV_j \cap SFV_k = \varnothing \; \forall \; j \neq k$. Each constraint $C_j = \left\{c_{j_1}, \ldots, c_{j_{n(j)}}\right\}$ has the form

$$V_i\left(tv^F_{j_k}\right) \geq c_{j_k} \; k = 1, \ldots, n(j) \; \forall \; S_i \in SS.$$

The set of C_j violating scenarios in SS is

$$Viol_j(SS) = \left\{S_i \in SS : \; \exists tv_{j_i} \in FOCUS \cap SFV_j \cap STV_i : \; V_i\left(tv^F_{j_i}\right) < c_{j_i}\right\}.$$

The set of constraint violating scenarios is

$$Viol(SS) = \bigcup_{j=1}^{N} Viol_j(SS).$$

Again, in SBR & MCDA it is useful to investigate the constraint violating scenarios per alternative $a_l \in A$. Therefore, one defines

$$\begin{aligned} Viol_j\left(SS\left(a_l\right)\right) &= \left\{S_i \in SS\left(a_l\right) : \right. \\ &\quad \left. \exists tv_{j_i} \in FOCUS \cap SFV_j \cap STV_i : \; V_i\left(tv^F_{j_i}\right) < c_{j_i}\right\} \\ \text{and } Viol\left(SS\left(a_l\right)\right) &= \bigcup_{j=1}^{N} Viol_j\left(SS\left(a_l\right)\right). \end{aligned}$$

Matos [2007] has suggested to use the **exposure**

$$\begin{aligned} exposure(SS) &= \frac{|Viol(SS)|}{|SS|} \\ \text{and} \quad exposure\left(SS\left(a_l\right)\right) &= \frac{\left|Viol\left(SS\left(a_l\right)\right)\right|}{|SS|} \end{aligned}$$

as a risk index. This approach corresponds to using equal weights $w_j = \frac{1}{|SS|}$ for each scenario violating at least one constraint C_j. In Section 5.3.3.1, a new method that takes into account the **severity** of the violation by means of functions $\theta_j\left(R_{c_j}\left(S_i\right)\right)$ ($j = 1, \ldots, N$), where $R_{c_j}\left(S_i\right)$ denotes the performance of

S_i in criterion c_j has been developed [Comes et al., 2010c]. For SBR & SM frameworks, this concept can be adapted by using the performance of S_i in the focus variables (namely, $V_i\left(tv_j^F\right)$) and according thresholds instead of performances in higher level criteria c_j. The severity of violation $SoViol(SS)$ or $SoViol\left(SS\left(a_l\right)\right)$ with respect to one constraint C_j can be assessed by:

$$\begin{aligned} SoViol_j(SS) &= \sum_{S_i \in Viol_j(SS)} \sum_{k=1}^{n(j)} \theta_j\left(R_{c_{j_k}}\left(S_i\right)\right) \\ \text{or} \qquad SoViol_j\left(SS\left(a_l\right)\right) &= \sum_{S_i \in Viol_j(SS(a_l))} \sum_{k=1}^{n(j)} \theta_j\left(R_{c_{j_k}}\left(S_i\right)\right). \end{aligned}$$

By definition of θ_j (see equation 5.4), $\theta_j \geq 0$. θ_j increases the more constraint c_j is violated. For normalisation purposes, define

$$\begin{aligned} SoViol_j^{\max}(SS) &= \max_{j=1,\dots,N} \left\{SoViol_j(SS)\right\} \\ \text{and} \qquad SoViol_j^{\max}\left(SS\left(a_l\right)\right) &= \max_{j=1,\dots,N} \left\{SoViol_j\left(SS\left(a_l\right)\right)\right\}. \end{aligned}$$

Then, one defines

$$\begin{aligned} \widetilde{SoViol_j(SS)} &= \begin{cases} 0, & \text{if } SoViol_j^{\max}(SS) = 0 \\ \frac{SoViol_j(SS)}{SoViol_j^{\max}(SS)}, & \text{otherwise} \end{cases} \\ \text{and } \widetilde{SoViol_j\left(SS\left(a_l\right)\right)} &= \begin{cases} 0, & \text{if } SoViol_j^{\max}\left(SS\left(a_l\right)\right) = 0 \\ \frac{SoViol_j(SS(a_l))}{SoViol_j^{\max}(SS(a_l))}, & \text{otherwise.} \end{cases} \end{aligned}$$

Finally, weights w_j ($j = 1, \dots, N$) reflecting the relative importance of each constraint C_j are defined by the scenario recipients such that $w_j \geq 0 \ \forall j = 1, \dots, N$ and $\sum_{j=1}^{N} w_j = 1$. Then, the **severity of violation** is defined as

$$\begin{aligned} SoViol_j(SS) &= \sum_{j=1}^{N} w_j \cdot \widetilde{SoViol_j(SS)} \in [0,1] \\ \text{and} \qquad SoViol_j\left(SS\left(a_l\right)\right) &= \sum_{j=1}^{N} w_j \cdot \widetilde{SoViol_j\left(SS\left(a_l\right)\right)} \in [0,1]. \end{aligned}$$

5.4.2.4. Concept of Regret

The **regret** of $a_l \in A$ is usually defined as the difference of the performance of a_l for a certain development of the uncertain factors that are beyond the influence of the decision-makers, S^{ext} and the performance of the best alternative given S^{ext} [Bell, 1982; Miranda and Proenca, 1998; Stoye, 2009]. The concept regret hence assumes that the uncertainty has been resolved (by a development S^{ext}), so that *ex post* the result of implementing a_l can be determined with certainty and contrasted with the best possible result for the same development of external random variables.

The combination of the development of external impact factors is often called "*scenario*" [Matos, 2007]. This notion of scenario is, however, not equivalent to the more extensive definition applied throughout this thesis (cf. Section 3). As the impact factors combined in S^{ext} are characterised by the fact that they defy the decision-makers' control, they are called *external impact factors* in this thesis.

As for the worst-case considerations (see Section 5.4.2.1), it is necessary to specify the quality of a certain set of valuations of focus variables. Denote $\left\{tv_k^F\right\}_{k=1,\dots,N} = FOCUS$. The type of each tvk^F is considered as a normed vector space. Denote

$$STV^{ext} = \left\{ tv_j \in \bigcap_{S_i \in SS} STV_i : \; tv_A \notin \Psi\left(tv_j\right) \wedge \exists tv_k^F \; : \; \exists P_{DI_i}\left(tv_j, tv_k^F\right) \right\}$$

the set of variables beyond the control of the decision-makers that do affect $FOCUS$. Given $S_{i^*} \in SS$ and $\varepsilon > 0$ denote

$$SS_{i^*}^{ext}(\varepsilon) = \left\{ S_i \in SS : \; \widetilde{dist}_j\left(V_i\left(tv_j\right), V_{i^*}\left(tv_j\right)\right) \leq \varepsilon \forall tv_j \in STV^{ext} \right\},$$

where $dist_j$ is a metric on $Type\left(tv_k\right)$, the ε similar scenarios with respect to STV^{ext}. Without loss of generality assume that the preferences with respect to tv_k^F are increasing ($k \in \{1, \dots, N\}$). The regret of S_{i^*} with respect to tv_k^F given $V_{i^*}\left(STV^{ext}\right)$ is

$$Regret\left(tv_k^F, SS, S_{i^*}, \varepsilon\right) = \max_{S_i \in SS_{i^*}^{ext}(\varepsilon)} \left\|V_i\left(tv_k^F\right)\right\|_j - \left\|V_{i^*}\left(tv_k^F\right)\right\|_k .$$

This definition broadens the standard definition of regret, as it does not restrict the considerations to scenarios that have *exactly* the same values $V\left(STV^{ext}\right)$.

Rather, all sufficiently similar situation developments are considered. (Apparently, choosing $\varepsilon = 0$, the standard regret can be calculated.)

In SBR & MCDA, where the evaluation function f_D is made explicit, the regret is calculated by

$$Regret\left(SS, S_{i^*}, f_D, \varepsilon\right) = \max_{S_i \in SS_{i^*}^{ext}(\varepsilon)} f_D\left(V_i\left(FOCUS\right)\right) - f_d\left(V_{i^*}\left(FOCUS\right)\right)$$

This definition allows the **minimax regret approach** to be implemented. This approach suggests selecting the alternative for which the maximum regret is minimal [Miranda and Proenca, 1998]. This approach is particularly suitable when the quality of the decisions is evaluated ex post [Matos, 2007].

5.5. Summary and Discussion

This chapter presented the **Decision Map approach** for scenario building and evaluation. Decision Maps integrate directed acyclic graphs (DAGs) used for scenario building and and attribute trees enabling the evaluation of scenarios.

5.5.1. Scenario Building

This chapter presented the use of DAGs for organising information such that a set of plausible, consistent and coherent scenarios can be generated. This approach uses **Causal Maps** as a means to structure the flow of information, proceeding from causes to effects. Particularly, *no* standardised causal inference or aggregation mechanisms are imposed. Nevertheless, it is important that the experts contributing to the scenario generation accept and perform their task of transforming specified input to specified output.

The Decision Map approach for scenario building can be positioned in between discursive scenario-based decision support techniques and expert systems. While the first require face-to-face meetings [Schoemaker, 1993; Scholz and Tietje, 2002] and are therefore time-consuming, the latter solve decision problems autonomously by using a (necessarily limited) model of the domain [Dugdale, 1996; Papamichail and French, 2005; Zimmermann, 1990]. The Decision Map approach is targeted at strategic complex decision problems, where expertise from several domains has to be brought together and time and availability of all or some experts is bounded. The use of discursive scenario techniques is constricted in these situations, as bounded time and availability of experts need to be respected. Furthermore, the Decision Map approach supports decision-makers in dynamic, highly varying and uncertain environments, and in situations that potentially involve rare event (e.g., due to a long time horizon). In these cases, the use of automated systems, which requires a vast, continuously updated case- or knowledge-base covering all eventualities, is problematic [Dugdale, 1996; Pavlin et al., 2009b].

The **scenario generation approach** ensures that all information necessary to determine each variable's possible values is provided to the responsible expert, as all direct dependencies are taken into account. The risk of information overload is reduced by avoiding that redundant or irrelevant information that was not judged necessary by the expert is passed on. Via the definition of $FOCUS$ and $INIT$ it is furthermore ensured that the scenarios contain information relevant to the recipients. In this manner, *objective O.2,* the generation of purposeful scenarios, is achieved.

Regarding the requirements for scenario acceptance and credibility *(objective O.3),* coherence is ensured as interdependencies are represented in the Decision Map. Consistency is ensured as far as possible given the information and expertise available, as all direct interdependencies are explicitly considered. The indirect interdependencies are integrated by conditioning each variable's value on the values of its (direct) predecessors. Finally, the plausibility of the scenarios depends on the experts available given time constraints and on the credibility of their judgements. Advanced negotiation protocols ensure that the best available expertise is identified ensuring that the scenarios' plausibility is as good as possible.

There might be trade-offs between the amount of information an expert can process in a given time and the accuracy of the information he determines and passes on. This concerns both the information regarding different aspects of the situation (represented by the number of direct predecessor variables) and the number of values per variable. The novel approaches to scenario management that are developed in the following chapters reveal these trade-offs and can help analysing which pieces of information are the most relevant and must be taken into account even when time is critical.

5.5.2. Evaluation of Scenarios: Robust Decision Support

This chapter presented multiple approaches to exploit DAGs for providing **robust decision support** (and, hence, to achieve *objective O.5)* by considering single scenarios or by aggregating all scenario results.

The consideration of single scenarios presented as plausible stories appeals to most decision-makers. The coupling of scenarios and attribute tree facilitates the **identification of worst- and best evaluated scenarios.** This approach explicitly takes into account the decision-makers' preferences. Comparing the values of the worst- and best-evaluated scenarios *across* alternatives may yield insights in the strengths and drawbacks of each alternative. It may, e.g., be that one alternative a_l performs particularly bad for assumptions on the future development of the environment, while others are less affected by thereof. In this case, understanding why the different performances arise, may result in refining a_l and successively develop a set of (more) robust alternatives.

An **additional aggregation step** for taking into account the performances of each alternative under varying assumptions is applied to avoid cognitive biases. To facilitate the elicitation of weights for each scenario, two novel approaches have been presented. Which method to choose, depends on the time available for the elicitation of weights (in general, the method based on satisficing require more time) and the expertise of the decision-makers in MCDA and related fields.

The first method is based on the concept of **satisficing.** The decision-makers need to define goals (or thresholds) that should be met in any scenario. If in a scenario S_j a threshold violated, a penalty function Ψ defined according to the decision-makers risk preferences is used to increase the weight of S_j compared to a basic weight μ. To avoid that the decision-makers focus only on risks and not on the chances an alternative may offer, an analogous approach for the opportunities has been introduced. This approach precisely models the decision-makers' preferences for scenario weights if they comply to the concept of satisficing and robustness introduced above. Although this approach reduces the workload for the decision-makers compared to direct weighting methods, it is quite demanding. The decision-makers need to specify the most important risk (opportunity) criteria and risk (opportunity) thresholds for the minimum required (desired) performance in each criterion. Moreover, they have to determine the penalty functions θ and φ defining the importance of violation as well as the basic weighting μ (as the penalty functions constitute a deviation from $\omega(S_j) = \mu \ \forall S_j \in SS(a_i)$.

The second approach determines weights by the definition of **inclination towards risk** α. In this approach, the decision-makers need to specify only one parameter (namely, α) and the scenario weights (as well as the final results) can be derived adopting the Ordered Weighted Average approach. As the parameter α is a rather abstract construct, it is suggested to perform sensitivity analyses before making a suggestion.

Finally, the **risk** associated to each alternative is analysed on the basis of the set of scenarios $SS(a_l)$. Several measures of risk applicable to scenarios have been developed. The decision-makers can choose one or more of these measures to complement the SBR & MCDA approach. In this manner *objective O.5* is fully achieved.

6. Management of Information on Scenarios

> Get your facts first, then you can distort them as you please.
> (Mark Twain)

While the previous chapters addressed the scenarios' content and structure, this chapter is dedicated to the management of information describing the context in which a scenario was built, and of information about the scenario building process. **Scenario information management** is a novel approach to the *information systems representation* of the scenario formalisation described in Chapter 3. In general, information systems are used to represent the structure and behaviour of systems [Falkenberg et al., 1996]. Information systems are therefore the basis for coordinated action *within* the system. In this manner, the management of scenarios that is described in Chapter 8 is enabled. Therefore, this chapter is the basis for achieving objective O.6: supporting decisions whilst respecting constraints in terms of time and availability, limited resources and capacities for information processing.

Furthermore, scenario information management ensures that scenarios remain valid (i.e., purposeful and acceptable), which is particularly important in dynamic and highly uncertain environments. Thereby, this chapter ensures that *objectives O.2* and *O.3* are met.

The scenario building process for a decision problem D comprises the construction of one or more graphs DAG_D (cf. Section 5.1) and the scenario generation, i.e., the assessment of values and statuses (cf. Section 5.2). As a scenario S_i is being built it undergoes changes. These changes represent gains in the problem's understanding [Breitman et al., 2005]. In this thesis, they are manifested in structural changes (i.e., changes in the STV_i and DI_i), and in additions to, subtractions from, or changes of the scenario content (represented by sv_i and $status_i$). On the whole, scenario building is a dynamic process based on relationships among scenarios. As *consistency* within single scenarios, among versions of scenarios, and among their changes must be maintained, it is necessary to **manage** and **control the building of scenarios** [Breitman et al., 2005; Jarke et al., 1998]. By the novel scenario continuation, extension and merging procedures (cf. Sections 3.6 and 3.7), scenarios multi-furcate, or merge with other scenarios, making overall understanding difficult and impeding the tracing.[36] In summary, there is the need to capture information on the sense- or decision-making process, the evolution of scenarios and to record authorship and responsibility for the scenarios' content [Breitman et al., 2005; do Prado Leite et al., 2000].

The intent of the newly developed scenario information management approach presented in this chapter is that the flexibility of scenario construction expressed by the formalisation is mirrored in the scenario information management while retaining the useful and desired properties of SBR & SM and SBR & MCDA. To this end, the scenario information management approach combines the distributed and decentralised scenario building with a (recipient-centred, centralised) component for scenario (information) management. The advantage of the approach presented is that the management of the processes of both SBR & SM and SBR & MCDA can more easily be operationalised.

This chapter starts by detailing the objectives of scenario information management (cf. Section 6.1). Then, concepts for scenario information management are developed. Firstly, **Scenario Information Bubbles** (SIBs) capturing all information relevant to a single scenario are introduced. Secondly, the concept of **context** is developed as a means to aggregate information bubbles of related scenarios (such as scenarios that extend each other). Thirdly, the concept of **purpose** is introduced as a means to relate contexts within a SBR task, such as a specific sense- or decision-making problem. Finally, SIB management and the problems arising when two sets of scenarios are merged are discussed. This chapter concludes with a discussion of implications for the implementation of the described scenario information management approach.

6.1. Objectives for Scenario Information Management

Scenario information management is intended to facilitate SBR. As such, it must provide sufficient information about scenarios, their relations, etc. All relations

[36] Following Jarke [1998], **traceability** is defined as the ability to describe and follow the evolution of a scenario, in both a forward and backward direction, ideally through the whole sense- or decision-making process. **Tracing** is the process that supplies and exploits these traces.

specified in Section 3.6 need to be made explicit in scenario information management combined with **traceability** in terms of expertise and time (i.e., which expert provided which information at what time).

The major objectives for scenario information management are:

Management of information within scenarios including

- the *characterisation of scenarios* with respect to their consistency, coherence, completeness and reliability, and
- the *characterisation of sets of scenarios* with respect to their coherence, consistency and balance

(cf. Section 4.3) for raising situation awareness or for the sound evaluation of alternatives.

Management of changes within a scenario S_i with respect to

- a change of the set of variables STV_i,
- a change of the value of a variable $V_i\,(tv_j)$,
- a change of its status $status_i\,(tv_j)$,
- a change of the dependency structure DI_i, as well as
- a change of conceptualisation $\mathfrak{C}_i$.

Keeping track of these changes facilitates scenario pruning and updating (cf. Sections 8.2 and 8.3).

Management of scenario dependencies including keeping track of the relations

- is (valid) source of,
- is sub-scenario of,
- equal episodes: has equal sub-scenario,
- is Parent-Child sub-scenario,
- is root of and
- has equal root

(cf. Section 3.5). Tracing these relations enables firstly (more) efficient scenario construction and updating. Secondly, it allows the possibilities of re-using partial scenarios for further problems to be investigated (cf. Section 8.3).

Detection of overlapping SBR activities, such as two decision problems about (aspects of) the same situation, cf. Section 10.2.4 for

- (more) efficient processing of information (as multiple uses of information for one SBR activity are detected), and

- the detection of possible decision conflicts (e.g., because of scarce resources, where one decision reduces the set of feasible options for another).

(Historical) inspection and auditing enabling

- accounting for decisions made,
- an after action analysis,
- training and
- the development of case-base.

To manage the information within one scenario, the novel concept of **SIBs** is developed in the next section. To group together scenarios that allow similar questions to be answered, the **context** and **purpose** of SBR will be introduced in the sections thereafter. Together, these new techniques allow all aims described above to be achieved.

6.2. Structuring Information within a Scenario: Scenario Information Bubbles

A Scenario Information Bubble (SIB) is a concept for representing all information in a scenario, as defined in Chapter 3 extended with some useful annotations. For the definition of SIBs the following assumptions are made.

Well-definedness: all variables $tv_j \in STV_i$ are uniquely defined (with name and type). Possible conflicts and ambiguities of definitions are resolved. Thus, the services of experts providing a (set of possible) value(s) for a variable are uniquely defined and identifiable.

Conceptualisability: each reasoning principle used to determine the value and status of a variable can be identified uniquely. The concept is identified with this principle (see Section 4.1.6). The finite set of possible concepts applicable given a certain backdrop B is denoted $\mathfrak{C}^*(B)$.

Timedness: it is possible to generate a finite set of time stamps $\mathfrak{T}^*_\Delta$ that can be partially ordered. That means, if two events A and B are assigned time stamp $timestamp_A, timestamp_B \in \mathfrak{T}^*_\Delta$ and $timestamp_A \leq timestamp_B$, then A precedes B. The partial order allows for the timely order's indeterminateness to be represented. Furthermore, the time stamps may be related to wall-clock time, allowing durations to be determined.

6.2.1. Minimal Scenario Information Bubbles

A minimal SIB is defined as follows. Let $SS = \{S_i\}_{i \in I}$ be a set of scenarios with a common backdrop B and a common focus $FOCUS$. Each SIB SIB_i^{SS} represents exactly one scenario $S_i \in SS$. Therefore, using the same index set I as for SS identifies each SIB uniquely. Figure 6.1 shows an example minimal SIB. Each SIB_i^{SS} contains *minimally* the following information.

Information on the scenario $S_i = \langle STV_i, sv_i, status_i, DI_i \rangle$.

Conceptualisations provide annotations specifying the concept used to determine the value of each variable $tv_j \in STV_i$. *concept* = *concept* $\left(SIB_i^{SS}, tv_j\right) \in \mathfrak{C}^*(B)$ represents, e.g., the expert (or group of experts) that provided the service resulting in a value (and status) for a variable. The default value of the concept *concept* $\left(SIB_i^{SS}, tv_j\right)$ = *concept*$^{SS}(tv_j)$ for each variable tv_j is set to $unknown$. Therefore, one defines $\mathfrak{C}(B) = \mathfrak{C}^*(B) \cup \{unknown\}$.

Time stamps for each variable tv_j specify *when* $V_i(tv_j)$ and $status_i(tv_j)$ were provided. The default time stamp for a variable tv_j, which has not been assessed yet within a given bubble SIB_i^{SS}, denoted $timestamp\left(SIB_i^{SS}, tv_j\right) := timestamp_i^{SS}(tv_j)$, is set to ∞. One defines $\mathfrak{T}_\Delta := \mathfrak{T}_\Delta^* \cup \{\infty\}$.

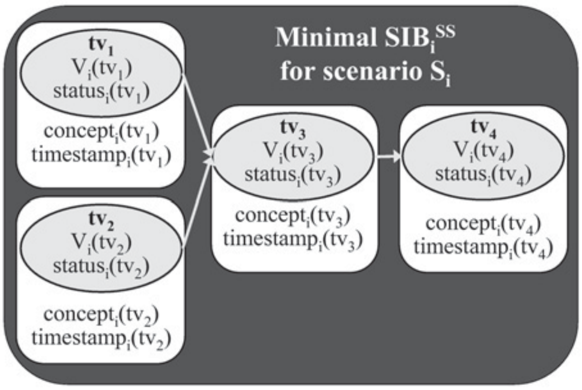


Figure 6.1.: *A Minimal Scenario Information Bubble*

6.2.2. Extended Scenario Information Bubbles

A minimal SIB can be extended with the following meta-information on the assessments of a variable's value. These include an evaluation of the **credibility of the concepts** used allowing the decision-makers' preferred sources of information to be taken into account, **sensitivity assessments** addressing the question of the magnitude of impact that a change in the predecessors' values on the value of a variable has, as well as an **effort assessment** referring to the effort necessary to provide a value for a variable. Although these pieces of information are not mandatory for SBR, they facilitate scenario management: while the credibility of concepts can support scenario pruning (cf. Section 8.2), the sensitivity analysis and the effort assessment provide helpful insights for scenario updating (cf. Section 8.3).

6.2.2.1. Evaluation of the Concept Used

As the credibility of sources that provided the values within a scenario is crucial for a scenario's acceptance [Schoemaker, 1993; Selin, 2006].The assessment of the quality of a concept (used to determine a value of a variable) is a means to keep track of the scenario's credibility. The notion of *concept* used in this thesis is very flexible (see Section 4.1.6): it can refer to an expert, or to the mechanisms and tools he uses as well as to automated reasoning systems (e.g., simulation models). If the scenarios' recipients are able to specify a (subjective) assessment of quality or credibility for each concept used in the scenario, this information can be represented in SIB_i^{SS}. This approach facilitates the construction of credible and trustworthy scenarios and allow rationales for scenario selection and pruning to be derived (see Sections 8.1 and 8.2).

6.2.2.2. Sensitivity Assessment

A sensitivity assessment can be used to complement the likelihood assessment $status_i(tv_j)$. While the likelihood quantifies the uncertainty in the result of the concept used (i.e., the value of the variable), the sensitivity analysis has the role of assessing the *strength* and *relevance* of the impact of variation in the input values (i.e., the values of the direct predecessor vertices $\widetilde{\Psi}(tv_j)$) on the value of tv_j. The sensitivity assessment can only be performed for the variables $tv_j \in STV_i$, for which $\widetilde{\Psi}(tv_j) \neq \emptyset$.

In sensitivity analyses, local and global types are distinguished [Saltelli et al., 2008]. For the purposes of this thesis, it is sufficient to perform a **local sensitivity assessment,** as the one scenario represented by SIB_i^{SS} is *not* supposed to represent $Range(V(tv_j))$. Rather, the set of scenarios built should be a means to explore the most relevant points of $Range(V(tv_j))$. There are two types of assessments that can be performed:

Sensitivity assessment independent of the direction. To choose an appropriate environment of the current assessment of values in sv_i, some definitions are needed: Let $\Psi_i^*(tv_j) \subseteq \widetilde{\Psi}(tv_j)$ be a non-empty subset of tv_j's direct predecessors, and denote

$$V_i\left(\Psi^*\right) = \left\{V_i\left(tv_{j_k}\right) :\ tv_{j_k} \in \Psi_i^*\left(tv_j\right)\right\}.$$

Let $dist_{j_k}$ be a metric on $Type\left(tv_{j_k}\right)$ for all $tv_{j_k} \in \Psi_i^*$ and define

$$\begin{aligned} NN_{i,j}^k \quad = \quad & \Big\{S_l \in SS :\ \exists tv_{j_k} \in \Psi_i^*\left(tv_j\right) \cap STV_l :\ dist_{j_k}\left(V_l\left(tv_{j_k}\right), V_i\left(tv_{j_k}\right)\right) \\ & \leq dist_{j_k}\left(V_m\left(tv_{j_k}\right), V_i\left(tv_{j_k}\right)\right)\ \forall\ S_m \in \mathrm{SS} :\ tv_{j_k} \in \mathrm{STV}_m\Big\}. \end{aligned}$$

This allows the environment $U_i^{SA}\left(tv_j\right)$ to be defined, where the sensitivity analysis for the given scenario will be performed. Let $x = (x_1, \ldots, x_K)$ be a vector, where x_{j_k} $(j_k = 1, \ldots, K)$ is of type $Type\left(tv_{j_k}\right)$, with

$$dist_{j_k}\left(V_i\left(tv_{j_k}\right), x_{j_k}\right) < dist_{j_k}\left(V_i\left(tv_{j_k}\right), V_l\left(tv_{j_k}\right)\right), \tag{6.1}$$

where $S_l \in NN_{i,j}^k$. Defining

$$DIST(x) = \sum_{k=1}^{K} dist_{j_k}\left(V_i\left(tv_{j_k}\right), x_{j_k}\right)$$

the sum of distances between each entry of x_{j_k} of x and the corresponding value $V_i\left(tv_{j_k}\right)$. By definition of x,

$$0 \leq DIST(x) \leq \sum_{k=1}^{K} dist_{j_k}\left(V_i\left(tv_{j_k}\right), V_l\left(tv_{j_k}\right)\right),$$

$S_k \in NN_{i,j}^k$ for all j_k.

The sensitivity of $V_i\left(tv_j\right)$ to changes in $V_i\left(\Psi_i^*\right)$ is determined for variations of Ψ_i^*, i.e. for all admissible vectors x parameters are varied simultaneously (*Variation In Combination* (VIC) sensitivity analysis [French, 2003]). By choosing Ψ_i^* with $|\Psi_i^*| = 1$, the option of varying parameters *One At a Time* (OAT sensitivity analysis [French, 2003]) is included. The variation may be accomplished analytically (e.g., via derivatives), numerically (e.g., by Monte Carlo or simulation methods) or by the judgement of a human expert. For an overview of techniques see, e.g., [Saltelli et al., 2008].

Direction-specific sensitivity assessment. There may be situations, when the assessment of a variable's value does not only depend on the magnitude of input variation, but also on its direction. This can be reflected by partitioning the possible variations for all admissible x (defined as in equation 6.1).

If $tv_{DP} = \{tv_{j_k} \in STV_i : tv_{j_k} \in \Psi_i^* (tv_j)\}$ is the set of variables considered and $Type_{DP} := Type\left(tv_1^{DP}\right) \times \ldots \times Type\left(tv_n^{DP}\right)$ is a vector space, then vectors for defining *critical* or *insensitive* directions can be specified to partition the environment of sensitivity analysis $U_i^{SA} (tv_j)$ into several sectors, where distinct analyses are performed (as in the independent case). In other cases, e.g., projections to sub-spaces of $Type_{DP}$ (e.g., the positive or negative semi-axes) can be used.

6.2.2.3. Duration and Effort of Assessment

The **duration** of a variable's value assessment (depending on the concept used) can be used as an estimate of how long it does take to determine a (new) value for tv_j given $V_i\left(\widetilde{Psi}_i (tv_j)\right)$ relying on $\mathfrak{C}_i (tv_j)$. This is relevant when the value of a variable has not been determined yet as well as for the question of scenario update (see Section 8.3.2) when the input information may have (significantly) changed. The duration of determining $V(tv_j)$ using $\mathfrak{C}_i (tv_j)$ is denoted $dur_i (tv_j)$. It is required that $0 < dur_i (tv_j) < \infty$.

Besides the duration of the assessment itself, experts can indicate their **degree of occupation** by using qualitative scales varying, e.g., from *overloaded* to *available without any qualification.* To operationalise these assessments, simple numerical scales and tables or fuzzy numbers can be used [Kuchta, 2001]. The degree of occupation of the expert determining $V(tv_j)$ is denoted $occ_i (tv_j)$. It is required that $0 < occ_i (tv_j) \leq 1$, where a value $occ_i (tv_j)$ close to 0 means that there are little restrictions in the availability of the experts, whereas a value close to 1 indicates that the experts are overloaded. In the latter situation, it may, e.g., be that the assessment is delayed because the expert needs to handle multiple problems in parallel.

Finally, the overall effort for assessing tv_j's value using $\mathfrak{C}_i (tv_j)$ can, e.g., be calculated by

$$ef_i (tv_j) = dur_i (tv_j) \cdot (1 + occ_i (tv_j)) . \tag{6.2}$$

By definition of both the duration and the degree of occupation, $0 < ef_i (tv_j) < \infty$. This approach assumes that the effort increases monotonically with an increase in effort and occupation. A very high workload ($occ_i(tv_j) = 1$) results in a doubling of the effort assessment. If the experts and decision-makers involved feel that this does not represent the actual effort correctly, naturally, weights w^{occ} and w^{dur} ($0 \leq w^{occ}, w^{dur} < \infty$) for modelling the respective importance of duration and occupation can be used.

6.3. Context

The concept of **context** is used to group together SIBs containing scenarios that have been constructed for the same situation and purpose. For a specific sense- or decision-making problem, multiple contexts may come into play. Thus, a context is a means to ensure consistency of scenarios with respect to an initial situation, and the values used therein.

Whenever the information incorporated by the scenarios changes significantly, a new context is created (e.g., scenario updates, see Section 8.3).

6.3.1. Minimal Context

The context is a means to group SIBs together including annotations of their dependencies. Let $SSIB_I = \{SIB_i\}_{i \in I}$ be a set of SIBs, each of which contains a scenario that has been constructed with a backdrop $Back_i$, a focus $FOCUS_i$, an initial situation $INIT_i$, a conceptualisation $\mathfrak{C}_i = \{concept_i(tv_j)\}$, and a set of time stamps $\mathfrak{T}_i = \{timestamp_i(tv_j)\}$. These SIBs are grouped to the set of contexts $CON_L = \{Con_l\}_{l \in L}$, where each SIB_i is assigned to exactly context Con_l. The set of contexts CON_L has the following properties:

- Each context $Con_l \in CON_L$ is uniquely identifiable.
- The function

$$\begin{aligned} context\text{-}of: \quad SSIB_I &\rightarrow CON_L \\ context\text{-}of\left(SIB_i^S S\right) &= Con_l \end{aligned}$$

  assigns each SIB its context.

- If two SIBs SIB_{i_1}, SIB_{i_2} share the same context Con_l (i.e., $context-of\,(SIB_{i_1}) = context-of\,(SIB_{i_2})$, then

$$\begin{aligned} Back_{i_1} &= Back_{i_2}, \\ FOCUS_{i_1} &= FOCUS_{i_2}, \\ INIT_{i_1} &= INIT_{i_2}. \end{aligned}$$

The notion of **conceptual contexts** is developed for representing an even stronger type of relatedness: S_{i_1} and S_{i_2} share the same minimal *and* conceptual context if they belong to the same context Con_l and for each $tv_j \in STV_{i_1} \cap STV_{i_2}$, it holds:

$$\begin{aligned} concept_{i_1}(tv_j) &= concept_{i_2}(tv_j) \\ timestamp_{i_1}(tv_j) &= timestamp_{i_2}(tv_j). \end{aligned}$$

A context groups scenarios that are constructed for the same purpose and relying on the same information and assumptions together. SIBs sharing the same minimal context have been constructed for the same sense-making or decision problem[37] relying on the same initial information. SIBs having in common both minimal and conceptual context and use the same concepts for determining the variables' values, and the values of the variables not in NA have been determined at the same time, as they contain the same time stamp of the information (i.e., one $SIB_{i_k}^{SS}$ is not more outdated or current than any other $SIB_{i_l}^{SS}$ belonging to the same context). The notion of context heavily depends on definitions of backdrop, initial situation and focus (cf. Sections 4.1.2, 4.1.3 and 4.1.5).

6.3.2. Extended Context

The extended context captures relations between SIBs and annotations thereof. Thus, it is the basis for SIB management (cf. Section 6.4). The description of SIB relations is facilitated as the $context-of$ relation classifies SIB_i's built for the same purpose and given the same information and assumptions together. To this end, some relations are defined on the lines of the continuation and extension relation for single scenarios (cf. Section 3.6):

[37]For SBR & MCDA problems, it is furthermore required that the same attribute tree and preferences are used.

Continues-in-context-relation: let SIB_i and SIB_j be two SIBs sharing a common context Con_l. Let S_i be the scenario represented in SIB_i and S_j the scenario represented in SIB_j. If

$$\begin{array}{rrcl} & S_i & \subset & S_j \\ \text{and} & status_i\,(tv_k) & \neq & \text{“}not\ assessed\text{”}\ \forall\ tv_k \in STV_j \setminus STV_i \end{array}$$

then S_j **continues** S_i in Con_l, or

$$S_i \overset{Con_l}{\Rightarrow} S_j.$$

Extends-in-context-relation: Let let SIB_i and $SIB_J = \{SIB_j\}_{j \in J}$ be an SIB and a set of SIBs respectively. Let all SIB_i, SIB_j $(j \in J)$ share a common context Con_l. Let S_i be the scenario represented in SIB_i and SS_J the set of scenarios represented in SIB_J. If for all S_{j_1}, S_{j_1} $(j_1, j_2 \in J)$ it holds that

$$\begin{array}{rcl} \{STV_{j_1} \setminus STV_i\} \cap \{STV_{j_2} \setminus STV_i\} & \neq & \varnothing, \\ \exists tv^* \in \{STV_{j_1} \setminus STV_i\} \cap \{STV_{j_2} \setminus STV_i\} :\ V_{j_1}\left(tv^*\right) & \neq & V_{j_2}\left(tv^*\right) \\ S_i & \overset{Con_l}{\Rightarrow} & S_{j_1}, \\ S_i & \overset{Con_l}{\Rightarrow} & S_{j_2} \end{array}$$

then SS_J extends S_i in Con_l.

Scenario interdependencies: given the above definitions, the SIBs adopt and represent all scenario interdependencies analogously given that the scenarios considered share the same context. This includes the relations

- is sub-scenario of,
- is PC-sub-scenario of,
- equal episodes: has equal sub-scenario,
- is root of,
- equal origin: has equal root and
- is (valid) source of.

Time issues: let S_j be a scenario that continues S_i in Con_l. Denote

$$T_{j \setminus i} = \{timestamp\,(tv_k) :\ tv_k \in STV_j \wedge tv_j \notin STV_i\}\,.$$

That means, $T_{j\smallsetminus i}$ is the set of time stamps of the continuation S_j of S_i in Con_l. Particularly, denote

$$T^{\max}_{j\smallsetminus i} = \overset{\preceq}{\max_{tv_k \in STV_j \smallsetminus STV_i}} \left(dist_T \left(timestamp^{\max}_i , timestamp\left(tv_k\right)\right)\right),$$

where

$$timestamp^{\max}_i = \overset{\preceq}{\max_{tv_l \in STV_i}} \left(timestamp\left(tv_l\right)\right)$$

is the maximum time stamp in S_i, i.e., it identifies the last value that was determined in S_i, as far as that can be determined given the partial order $\preceq$. $T^{\max}_{j\smallsetminus i}$ allows the timely distance between the completion of the determination of values in S_i and the last assignment in S_j to be determined. The metric $dist_T$ is any metric on the set of timestamps $\mathfrak{T}^*_j$ and for any $tv_l \in STV_j$, one defines $dist_T\left(timestamp\left(tv_l\right), \infty\right) := \infty$ for all $timestamp\left(tv_l\right)$ in $\mathfrak{T}^*_j$.

Extended contexts can only be realised for **traceable SBR** (cf. Section 4.2), which makes the scenario continuation and extension relations explicit for all partial scenarios, which eventually result in the set of (focus complete) scenarios that are the basis for the sense- or decision-making.

6.4. Scenario Information Bubble Management

Whenever a context Con_l changes (given a description of the initial problem ip and the according purpose P^{ip}_{ω}) new SIBs for that new context $Con_{\tilde{l}}$ are created. The reason for the creation of a new SIB must be made explicit (see above Is-update-of relations), and captured in a relation between the old and the new context.

For a finished SBR activity, there is always exactly one context, within which the reasoning was performed and completed. During any SBR activity, however, many contexts may have come into play. The history of contexts and their relations for each purpose can be captured and used for logging and after-action analysis.

6.5. Purpose

A **purpose** as represented in the backdrop B, present in each context Con_l, represents a specific Scenario-Based Reasoning activity, such as Scenario-Based

Multi-Criteria Decision Analysis (SBR & MCDA), sense-making, etc. Each purpose can involve multiple backdrops and thus, multiple contexts.

Given the description of the initial problem ip (e.g., a description of an incident in emergency management) as represented in a backdrop, a set of purposes for that initial problem is defined as $P^{ip} = \left\{P^{ip}_{\omega}\right\}_{\omega\in\Omega}$. Each purpose P^{ip}_{ω} has a unique identification $\omega \in \Omega$.

The purpose does not only contain a description of the problem to be analysed and solved (e.g., a particular decision problem), but it also provides information on the time available for solving this problem. This is represented as an upper limit $T^{\max}\left(P^{ip}_{\omega}\right)$. If no timely restrictions exist, one sets $T^{\max}\left(P^{ip}_{\omega}\right) = \infty$. Whenever a new purpose $P^{ip}_{\omega+1}$ is created, a new SIB arises. For each purpose, a number of minimum and/or extended contexts arise, which depends on

- the backdrop $Back_i$: as incident description and purpose remain unchanged, the variation in the backdrop is restricted to a variation of the recipients or decision-makers, and, for the SBR & MCDA problems, a variation of the attribute tree and preferences.
- the focus $FOCUS_i$,
- the initial situation description $INIT_i$,
- the conceptualisation $\mathfrak{C}_i$,
- the time stamps $\mathfrak{T}_i$.

A purpose may define relations among contexts (and SIBs), where a relation can be annotated according to the varying elements in the context:

- Is-update-of recipients
- Is-update-of preferences
- Is-update-of focus
- Is-update-of initial situation
- Is-update-of conceptualisation
- Is-update-of time stamps

The Is-update-of ... relations presumes that there is a timely structure of the variations in the underlying context elements.

6.6. Scenario Merging: Implications for Scenario Information Management

The scenario merging procedure combines two (possibly overlapping) sets of scenarios SS_1 and SS_2 that have been determined separately, see Section 3.7. This approach is particularly useful to develop quick intermediate results in case of bottlenecks or for scenario updating. Beyond the merging of scenarios themselves, for the scenario information management, it is also necessary to merge the respective SIBs.

Analogue to the prerequisites for the scenario merging procedure, at this point, some prerequisites need to be defined.

6.6.1. Mergeable Scenario Information Bubbles

Two SIBs SIB_{i_1} and SIB_{i_2} are called *mergeable*, if and only if the respective scenarios S_{i_1} and S_{i_2} share a common backdrop and focus, i.e., for all $S_{i_1} \in SS_1$ and $S_{i_2} \in SS_2$ it holds that

$$\begin{aligned} Back_{i_1} &= Back_{i_2} \\ \text{and} \qquad FOCUS_{i_1} &= FOCUS_{i_2}. \end{aligned}$$

Particularly, these conditions ensure that the scenarios merged have been constructed for a common purpose. Furthermore, it is required that for all pairs $(timestamp_A, timstamp_B)$, used for characterising timely information in both SIB_{i_1} and SIB_{i_2}, the partial ordering is stable, i.e.,

$$\begin{aligned} & \forall\ timestamp_A,\ timestamp_B \in \mathfrak{T}^*_\Delta\,(SIB_{i_1}) \cap \mathfrak{T}^*_\Delta\,(SIB_{i_2}) : \\ & timestamp_A \le timestamp_B \in \mathfrak{T}^*_\Delta\,(SIB_{i_1}) \\ \Leftrightarrow \quad & timestamp_A \le timestamp_B \in \mathfrak{T}^*_\Delta\,(SIB_{i_2})\,, \end{aligned}$$

where $\mathfrak{T}^*_\Delta\,(SIB_{i_1})$ and $\mathfrak{T}^*_\Delta\,(SIB_{i_2})$ denote the sets of non-infinite time stamps in SIB_{i_1} and SIB_{i_2} respectively.

6.6.2. Merging Scenario Information Bubbles

Beyond the merging the scenario themselves, both minimum and extended SIBs contain further information on the scenarios. Conflicts may arise when for the scenarios merged $S_{i_1} \in SIB_{i_1}$ and $S_{i_2} \in SIB_{i_1}$, as for example, different concepts are used to determine the value of a variable $tv_j \in STV_{i_1} \cap STV_{i_2}$. Similarly, even if the concepts coincide, it may also be that the time stamps of time when tv_j was determined are not equal. These conflicts will be discussed in Section 8.2.2.

6.7. Summary

The concepts for scenario information management, namely SIB, context, and purpose, allow SBR processes to efficiently be encoded. Each SIB, context, and purpose are uniquely identifiable, and as such can be fully described in one place, and referred to from other places. Given the relation among SIBs, it is possible to represent SIBs in an operationalisation efficiently, by not repeating information that is present in an earlier SIB that this SIB depends on, as long as these are in the same context.

Moreover, scenario information management enables the management of scenarios on the basis of a context. At any time, there is exactly one active context. Suppose that first, a context Con_1 exists, in which multiple scenarios are constructed for an initial situation description $INIT_1$ starting from time t_0. At time t_1, the initial situation description is updated. This update has such an impact that a new context Con_2 is started, to which the scenarios from Con_1 are copied (as far as possible, cf. Section 8.3 on scenario updating), and further extended and continued. Finally, at a time t_2, information becomes available, which invalidates the update at time t_1. It is now most suitable to abandon context Con_2, and continue with context Con_1 (reusing some scenarios and values if possible, cf. Section 8.3).

On the whole, this novel approach manages the information within scenarios by capturing information on the progress and status of the construction of (multiple) scenarios. This abstract information model is described to capture SBR with sufficient annotations to support its management (cf. Section 8). In this manner, it provides the basis for *objective O.6* (respecting constrained resources, capacities and limited time). It facilitates combining the decentralised scenario building process with a (centralised) scenario information management component that takes into account the decision-makers' preferences throughout all phases of the scenario building process. By orchestrating scenario emergence in this manner, the relevance of the scenarios for the purpose at hand is ensured.

Thus, *objectives O.2* and *O.3* are warranted from the start to the completion of the SBR process.

7. Formalisation of Principles for Scenario Management

Modest doubt is called the beacon of the wise.
(William Shakespeare)

This chapter describes principles for the management of scenarios in both SBR & SM and SBR & MCDA. These principles constitute the basis for scenario management (i.e., scenario selection, pruning or updating, see Chapter 8), which itself warrants that constraints in terms of the time for the decision-making, the bounded availability of experts, the limited resources and capacities for information processing are met *(objective O.6)*.

As indicated in Section 5.2, the issue of handling the potential combinatorial explosion of the number of scenarios[38] constructed is crucial for the implementation and application of the SBR approach. The advantages of controlling the combinatorics include lowering workload on humans and artificial systems. The approaches and formalisations developed in Chapter 3 are powerful tools that enable the application of a wide range of techniques from graph theory and AI network models for scenario management.

This chapter provides the basis for scenario management, which is further detailed in the next chapter, and is intended to make headway in controlling the (inherent) combinatorics of SBR using generic principles. For that purpose, various **(dis-)similarity measures** applicable at different stages of the scenario generation process for SBR & SM or SBR & MCDA are developed. On basis of these similarities, scenarios considered for the same purpose are arranged into **equivalence classes,** i.e., groups of *sufficiently similar* scenarios. Subsequently, the concepts of the **representativeness** and the **inaccuracy** of the arising sets are

[38]The number of scenarios grows combinatorially with a growing number of variables that are assigned more than one value.

considered. Both concepts, which are founded on the similarity of scenarios, have been developed to fit SBR & SM and SBR & MCDA.

7.1. Defining the Similarity of Scenarios

When comparing two scenarios S_{i_k} and S_{i_l}, one of the most fundamental questions is: are S_{i_k} and S_{i_l} equal? This is of particular interest for scenario management, for if S_{i_k} and S_{i_l} are equal, it is sufficient to consider only one of both scenarios. A natural and useful extension of equality is similarity: how "*close to equal*" are S_{i_k} and S_{i_l}? The approach to control the number of scenarios applied in this thesis is based on clustering scenarios that represent descriptions of possible situation developments that are similar in terms of their consequences.

In general, similarity serves as an "*organizing principle by which individuals classify objects, form concepts and make generalizations*" [Tversky, 1977]. Therefore, grouping together similar scenarios is likely to appeal to the users. (Dis-)similarity appears in different forms varying with the types of variables compared and the nature of comparison. Therefore, this section first reviews some concepts for scenario similarity that have been suggested. Subsequently, a novel similarity concepts targeted at the scenario recipients' needs is developed (cf. Section 7.2).

7.1.1. Tools and Concepts

This thesis adapts and extends approaches to defining similarity from Case-Based Reasoning (CBR), where the identification of the classes of situations, to which an event belongs (e.g., an incident in emergency management), is a key element [Núñez et al., 2004]. CBR is an approach to solving problems by referring to previous similar situations and by reusing information and knowledge acquired within that situation (which is called the **case**) [Aamodt and Plaza, 1994]. While scenarios represent stories about how the future could unfold, cases are stories about what happened in the *past* including information about how the problems was handled. Cases and scenarios share the property that they describe a situation and its future development. It is therefore justified to adapt CBR similarity concepts for measuring the similarity of scenarios.

To quantify the degree of resemblance between a pair of cases (c_a, c_b), c_a, $c_b \in \mathcal{C}$, where $\mathcal{C}$ is the set of cases, similarity measures,

$$similarity : \mathcal{C} \times \mathcal{C} \rightarrow [0,1]$$

that assign a number in $[0,1]$ expressing the degree of similarity between both cases to each pair (c_a, c_b), are constructed [Liao et al., 1998].

A difficulty in defining the similarity measures is, however, that in general there is a multitude of possible descriptions of an object. But when performing a similarity assessment, only a limited set of relevant features can be determined [Tversky, 1977]. A common characteristic of similarity measures is therefore the identification of some *decisive features* characterising the cases. The representation of an object as a collection of features requires therefore assessing what are the most relevant features for assessing similarity are in the given context.

7.1.2. Similarity Measures for Scenarios

This section adapts similarity measures from CBR to the SBR framework developed. Similarity measures with respect to several features and of different (functional) forms are reviewed briefly. A scenario S_i represents information on the variables in STV_i that were judged relevant to the given backdrop B_i as a set of values and statuses sv_i and $status_i$. The variables represent the **features,** and the values are their **specifications** allowing the similarity of two scenarios S_{i_k} and $S_{i_l} \in SS$ to be determined, where SS is a set of scenarios.

Similarity measures may be divided into **syntactic measures** that compare the representations of the two entities and **semantic measures** that compare the meaning of semantic representations [Miller and Charles, 1991]. As in both SBR & SM and SBR & MCDA there is little or no knowledge about the semantic structures used by the expert contributing to a scenario, this thesis focuses on syntactic similarity. The syntactic similarity is furthermore easier to understand than the semantic similarity, as the results of the similarity measurements map in an intuitive way to the scenarios and the extra labour of constructing a semantic structure is not needed [Alspaugh et al., 1999].

Following Lin [1998], the similarity measure constructed has to fulfil the following properties:

- The similarity between two scenarios S_{i_k} and S_{i_l} is related to their commonalities and differences. The more commonalities they share and the less difference they have, the more similar they are.
- The maximum similarity between S_{i_k} and S_{i_l} is reached when S_{i_k} and S_{i_l} are identical, no matter how much commonality they share (in absolute terms).

Various concepts for how the commonalities of scenarios can be defined are introduced in the next section.

The purpose of measuring the scenario similarity in this thesis is, ultimately, the selection of the most relevant scenarios for a given purpose. The relevant information for the scenarios' recipients is (according to the scenario formalisation provided in Section 4.1) captured in the values of the focus variables. Therefore, interdependencies (captured in DI_i) or meta-information on the likelihood of each value (captured in $status_i$) are not considered here.

7.1.2.1. Similarity Measures for Similarity of Sets of Variables

The similarity of two scenarios S_{i_k} and S_{i_l} with respect to their variables (called **STV similarity** in the following) is determined by comparing the sets STV_{i_k} and STV_{i_l}. This approach corresponds to Tversky's idea of *matching* [Tversky, 1977], where the similarity of two objects is a function of features that are *common* to both objects and of the features that *belong to one and only one* of them.

Let $|STV_{i_k} \cap STV_{i_l}| = N > 0$ and denote $STV_{i_k} \cup STV_{i_l} = \{tv_j\}_{j \in J}$. The STV similarity of S_{i_k} and S_{i_l}, termed $sim_{STV}(S_{i_k}, S_{i_l})$, is defined by

$$sim_{STV}(S_{i_k}, S_{i_l}) = 1 - dist_{STV}(S_{i_k}, S_{i_l}) = 1 - \sqrt{\frac{1}{N} \sum_{j \in J} dist\left(STV_{i_k}^j, STV_{i_l}^j\right)},$$

where

$$dist\left(STV_{i_k}^j, STV_{i_l}^j\right) = \begin{cases} 0, & \text{if} \quad tv_j \in STV_{i_k} \cap STV_{i_l} \\ 1, & \text{else.} \end{cases}$$

The metric $dist_{STV}(S_{i_k}, S_{i_l})$ (a modification of the discrete metric, cf. equation 3.2) is idempotent, as

$$dist_{STV}(S_{i_k}, S_{i_l}) = \left(dist_{STV}(S_{i_k}, S_{i_l})\right)^2 = dist_{STV}^2(S_{i_k}, S_{i_l}).$$

sim_{STV} represents a Euclidean distance, where all variables are assigned equal weights [Liao et al., 1998]. Additionally,

$$
\begin{aligned}
sim_{STV}\left(S_{i_k}, S_{i_l}\right) &= 1 - \sqrt{\frac{\sum_{j\in J} dist^2_{STV}\left(S^j_{i_k}, S^j_{i_l}\right)}{N}} \\
&= 1 - \sqrt{\frac{|STV_{i_k} \cap STV_{i_l}|}{|STV_{i_k} \cup STV_{i_k}|}}.
\end{aligned}
$$

STV similarity can be interpreted as a *measure of overlap* [Alspaugh et al., 1999], where the minimal similarity value of 0 indicates that S_{i_k} and S_{i_l} do not have any variables in common. The maximal STV similarity value of 1 indicates a complete overlap of variables.

7.1.2.2. Similarity Measures for Similarity of the Values of Variables

Beyond the structural difference between the scenarios it is also useful to determine the similarity with respect to the variables that both scenarios share. For $tv_j \in STV_{i_k} \cap STV_{i_l}$ it is possible to compare the values $V_{i_k}(tv_j)$ and $V_{i_l}(tv_j)$. This leads to the concept of **value similarity.** Define $M = |STV_{i_k} \cap STV_{i_l}|$ and denote $dist_j$ the selected metric on $Type(tv_j)$. The value similarity of S_{i_k} and S_{i_l} *with respect to* tv_j can be defined by a number of different metrics. The easiest case is, again, using the Euclidean metric:

$$
dist^{Euc}_j\left(S_{i_k}, S_{i_l}\right) = 1 - \sqrt{\frac{1}{M} \sum_{tv_j \in STV_{i_k} \cap STV_{i_l}} dist^2_i\left(V_{i_k}(tv_j), V_{i_l}(tv_j)\right)}.
$$

For defining value similarity in general it is useful to normalise the distance measure, as the values of the variables may be measured (and compared) on different scales and units. To this end, one defines:

$$
\begin{aligned}
d^{\min}_j &= \min_{S_{i_k}, S_{i_l} \in SS} \left\{dist_j\left(V_{i_k}(tv_j), V_{i_l}(tv_j)\right)\right\} \qquad \forall tv_j \in STV_{i_k} \cap STV_{i_l}, \\
d^{\max}_j &= \max_{S_{i_k}, S_{i_l} \in SS} \left\{dist_j\left(V_{i_k}(tv_j), V_{i_l}(tv_j)\right)\right\} \qquad \forall tv_j \in STV_{i_k} \cap STV_{i_l}.
\end{aligned}
$$

The *normalised Euclidean* **value similarity** of S_{i_k} and S_{i_l} is defined as:

$$\begin{aligned} sim_V^{Euc}\left(S_{i_k}, S_{i_l}\right) &= 1 - \widetilde{dist}_V^{Euc}\left(S_{i_k}, S_{i_l}\right) \\ &= 1 - \sqrt{\frac{1}{M} \sum_{tv_j \in STV_{i_k} \cap STV_{i_l}} \widetilde{dist}_j^2\left(V_{i_k}\left(tv_j\right), V_{i_l}\left(tv_j\right)\right)}, \end{aligned}$$

where $\widetilde{dist}_j$: $SPV\left(tv_j\right) \times SPV\left(tv_j\right) \to [0,1]$ is the normalised distance

$$\widetilde{dist}_j\left(V_{i_k}\left(tv_j\right), V_{i_l}\left(tv_j\right)\right) = \frac{dist_j\left(V_{i_k}\left(tv_j\right), V_{i_l}\left(tv_j\right)\right) - d_j^{\min}}{d_j^{\max} - d_j^{\min}}$$

$\widetilde{dist}_V^{Euc}\left(S_{i_k}, S_{i_l}\right)$ can be interpreted as a measure of dissimilarity between S_{i_k} and S_{i_l} with respect to their shared variables.

More generally, it is possible to assign importance weights ω_j to each variable $tv_j \in STV_{i_k} \cap STV_{i_l}$ representing the importance of feature tv_j. Agre [1995] suggests calculating these weights as the ratio of the number of cases containing this feature to the whole number of cases. Adapted to scenarios this approach corresponds to the following definition of weights:

$$\omega_j = \frac{\left|\left\{S_{i_k} \in SS : tv_j \in STV_{i_k}\right\}\right|}{|SS|} \in [0,1]. \tag{7.1}$$

Accordingly, the **weighted Euclidean value similarity** is:

$$\begin{aligned} sim_{\omega,V}^{Euc}\left(S_{i_k}, S_{i_l}\right) &= 1 - \widetilde{dist}_{\omega,V}^{Euc}\left(S_{i_k}, S_{i_l}\right) \\ &= 1 - \sqrt{\frac{1}{M} \sum_{i=1}^{M} \underbrace{\omega_j^2}_{\leq 1} \widetilde{dist}_j^2\left(V_{i_k}\left(tv_j\right), V_{i_l}\left(tv_j\right)\right)} \\ &\geq sim_V^{Euc}\left(S_{i_k}, S_{i_l}\right). \end{aligned} \tag{7.2}$$

In this approach, the multiplication of the dissimilarity with weights $\omega_j \in [0,1]$ (see equation 7.1), leads to a higher similarity measure than the approach relying on equal weights ($\omega_j = 1$ for all $tv_j \in STV_{i_k} \cap STV_{i_l}$).

Alternatively, one can use a **ratio model,** as introduced by Tversky [1977]. In this model, $STV_i \cap STV_j$ is partitioned to three sets

$$\begin{aligned} STV^{C}_{i_k \cap i_l} &= \left\{ tv_j \in STV_{i_k} \cap STV_{i_l} : dist_j \left(V_{i_k} \left(tv_j \right), V_{i_l} \left(tv_j \right) \right) \right) \leq \eta^{C} \right\}, \\ STV^{D}_{i_k \cap i_l} &= \left\{ tv_j \in STV_{i_k} \cap STV_{i_l} : dist_j \left(V_{i_k} \left(tv_j \right), V_{i_l} \left(tv_j \right) \right) \right) \geq \eta^{D} \right\}, \\ STV^{E}_{i_k \cap i_l} &= \left\{ tv_j \in STV_{i_k} \cap STV_{i_l} : \eta^{D} \geq dist_k \left(V_{i_k} \left(tv_j \right), V_{i_l} \left(tv_j \right) \right) \right) \geq \eta^{C} \right\}, \end{aligned}$$

Choosing the threshold values η^C and η^D such that

$$\begin{aligned} & \min_{S_{i_k}, S_{i_l} \in SS} \left\{ dist_j \left(V_{i_k} \left(tv_j \right), V_{i_l} \left(tv_j \right) \right) \right\} \\ \leq \quad & \eta^{C} \qquad < \eta^{D} \\ \leq \quad & \max_{S_{i_k}, S_{i_l} \in SS} \left\{ dist_j \left(V_{i_k} \left(tv_j \right), V_{i_l} \left(tv_j \right) \right) \right\}, \end{aligned}$$

a partition of $STV_{i_k} \cap STV_{i_l}$ is created. Next, one defines

- $N^C = \left| STV^{C}_{i_k \cap i_l} \right|$: the number of variables whose values are classified as **similar,**
- $N^{Diff} = \left| STV^{D}_{i_k \cap i_l} \right|$: the number of variables whose values are classified as **dissimilar.**

The value similarity according to the ratio model is defined by:

$$sim^{rat}_{V} \left(S_{i_k}, S_{i_l} \right) = \frac{\alpha \cdot N^C}{\alpha \cdot N^C + \beta \cdot N^D}, \tag{7.3}$$

where α, $\beta \geq 0$ and $\alpha + \beta \neq 0$ define weights for taking into account common and different features of the scenarios [Liao et al., 1998]. According to equation 7.3, $sim^{rat}_V = 0$ for $\alpha = 0$, while for $\beta = 0$ (and $\alpha \neq 0$), $sim^{rat}_V = 1$.

This approach relies on the recipients' ability and willingness to define the thresholds η^C and η^D for characterising what they perceive as sufficiently similar or sufficiently different. Furthermore, the recipients need to specify weights α and β for reflecting the importance of the common values versus the different values. This method has the advantage that it can handle errors of measurement or small perturbations by grouping these values into the same set.

7.2. An Approach To Scenario Similarity: Integrating STV and Value Similarity

As scenarios combine sets of variables and their values, this section defines an approach integrating STV and value similarity. The main challenge is to define the similarity of values in $STV_{i_k} \setminus STV_{i_l}$ and $STV_{i_l} \setminus STV_{i_k}$.

7.2.1. Existing Approaches Expanding Value Similarity Measures

This section reviews existing literature and considers some approaches dealing with *missing features* [Agre, 1995], i.e., approaches, which assign a value to the variables $tv_j \in \{STV_{i_k} \setminus STV_{i_l}\} \cup \{STV_{i_l} \setminus STV_{i_k}\}$, to determine the (dis-)similarity the dissimilarity of scenarios. Without loss of generality assume that $tv_j \in STV_{i_k} \setminus STV_{i_l}$. The simplest approach to defining the desired similarity measure is assigning one unique number to the distance of $V_{i_k}(tv_j)$ and any (unknown) value that tv_j might have taken if it was in STV_{i_l}. This distance is denoted $dist_j(V_{i_k}(tv_j), S_{i_l})$. It has been suggested to use

$$\begin{aligned} & dist_j^{Ricci}(V_{i_k}(tv_j), S_{i_l}) &=& \quad 0.5 \quad \forall tv_j \in STV_i \setminus STV_j \\ \text{or} \quad & dist_j^{Surma}(V_{i_k}(tv_j), S_{i_l}) &=& \quad 1 \quad \forall tv_j \in STV_i \setminus STV_j. \end{aligned}$$

Whilst $dist_j^{Ricci}$ was suggest by Ricci and Avesani [1995], $dist_j^{Surma}$ was proposed by [Surma and Vanhoof, 1995]. These approaches do, however, not take into account the possible values that tv_j might have in scenario S_{i_l}. Particularly, it is not assessed, which values in $Range(V(tv_j))$ are consistent with and plausible for sv_{i_l}.

An approach taking explicitly into account the concept of the **potential valuation** of a variable in scenario S_{i_l} has been developed by Agre [1995]. Denote

$$L_j = |SPV(tv_j)| = |\{V_i(tv_j) : S_i \in SS\}|$$

the number of possible values of tv_j present in SS. L_j is used to measure the variety of possible values of tv_j's values in SS. On basis of L_j, the distance of values in S_{i_k} and S_{i_l} with respect to tv_j can be determined by

$$dist_j^2\left(V_{i_k}\left(tv_{i_k}\right), S_{i_l}\right) = \frac{1}{L_j}\left(1 - \frac{1}{L_j}\right). \tag{7.4}$$

$dist_j^2\left(V_{i_k}\left(tv_j\right), S_{i_l}\right)$ is well-defined as $L_j \geq 1$. $dist_j\left(V_{i_k}\left(tv_j\right), S_{i_k}\right) \in [0,1)$ is monotonously increasing in L_j and according to equation 7.4 it holds:

$$\begin{array}{rrcll} & dist_j\left(V_{i_k}\left(tv_j\right), S_{i_k}\right) & = & 0, & \text{for } L_j = 1 \\ \text{and} & dist_j\left(V_{i_k}\left(tv_j\right), S_{i_k}\right) & \to & 1, & \text{for } L_j \to \infty. \end{array}$$

The rationale of equation 7.4 is that if there is only one unique possible value for a variable, this variable would have taken the same value in S_{i_l}, if tv_j had been considered in STV_{i_l}. For a growing number of possible values the potential dissimilarity with respect to the value that is assumed in scenario S_{i_k} increases, and is maximal for an infinite number of possible values. This approach does, however, neither take into account the total number of scenarios considered nor the status or likelihood of the possible values of tv_j. Additionally, perturbations and small variations in the values result in the same distances as values that vary greatly.

7.2.2. A Novel Approach to Scenario Similarity

A drawback of the approaches discussed above is that they do not take into account the structural differences of both scenarios: the fact that tv_j lacks in STV_{i_l} means that $V\left(tv_j\right)$ is irrelevant in S_{i_l}.[39] This thesis proposes another approach recognizing that scenarios are *purposeful* stories that are targeted towards determining the values of the focus variables. To this end, combining STV and value similarity are combined. Furthermore, to reduce complexity this novel approach projects the multi-dimensional space of the scenarios and their valuations to $\mathbb{R}$, where the totally ordered structure facilitates comparisons.

This section develops an approach for enabling the identification of equality or similarity between sub-scenarios. This approach answers the question

[39] More precisely, the fact $tv_j \notin STV_{i_l}$ means that for a focussed scenario S_{i_l}, tv_j has not been judged relevant to determine $V_{i_l}\left(FOCUS\right)$ yet. Nevertheless, it is considered *possible* to assign tv_j a value given sv_{i_l}.

whether or not two scenarios contain sequences of events that are equal or similar. To this end, several **classes of subsets** of the variables for further characterising the similarity of scenarios and for taking into account both structural and valuation aspects are developed. One denotes

$$\begin{aligned} S^{sub}_{SS,i_k} &= \{S_{i_m} \in SS : S_{i_m} \subseteq S_{i_k}\} \\ S^{sub}_{SS,i_l} &= \{S_{i_m} \in SS : S_{i_m} \subseteq S_{i_l}\}. \end{aligned}$$

Furthermore, one defines

$$STV^{share}_{i_k \cap i_l} = \left\{ tv_j : \exists S_{i_m} \in S^{sub}_{SS,i_k} : tv_j \in STV_{i_m} \cap \exists S_{i_n} \in S^{sub}_{SS,i_l} : tv_j \in STV_{i_n} \right\}.$$

Next, the sets of ι-similar valued variables are considered, where $\iota \in [0,1]$:

$$STV^{\iota}_{i_k \cap i_l} = \left\{ tv_j \in STV_{i_k} \cap STV_{i_l} : \widetilde{dist}_j \left(V_{i_k}(tv_j), V_{i_l}(tv_j) \right) \leq \iota \right\}.$$

For the purposes of this thesis, which refers to the values of the set of focus variables to assess a situation or to evaluate alternatives, it is necessary to consider sets of variables, i.e., to aggregate the (dis-)similarities. Therefore, sets of η-similar valued variables are defined. Let $\widetilde{\eta} \in [0, |STV_{i_k} \cap STV_{i_l}|]$ be a threshold, and define

$$\begin{aligned} \widetilde{STV}^{\tilde{\eta}}_{i_k \cap i_l} &= \Big\{ \{tv_{j_1}, \ldots, tv_{j_n}\} \in STV_{i_k} \cap STV_{i_l} : \\ &\qquad \sum_{m=1}^{n} \left(\widetilde{dist}_{j_m} \left(V_{i_k}(tv_{j_m}), V_{i_l}(tv_{j_m}) \right) \right) \leq \eta \Big\}. \end{aligned}$$

Next, the set of maximum sets of variables for which the value distance does not exceed $\widetilde{\eta}$ is defined:

$$\begin{aligned} & STV^{\max(\widetilde{\eta})}_{i_k \cap i_l} = \\ & \Big\{ \{tv_{j_1}, \ldots, tv_{j_n}\} \in \widetilde{STV}^{\tilde{\eta}}_{i \cap j} : \\ & \nexists\, tv_{j^*} \in STV_{i_k} \cap STV_{i_l}, tv_{j^*} \neq tv_{j_m} \ (m = 1, \ldots, n) : \\ & \widetilde{dist}_{j^*} \left(V_{i_k}(tv_{j^*}), V_{i_l}(tv_{j^*}) \right) + \sum_{m=1}^{n} \left(\widetilde{dist}_{j_m} \left(V_{i_k}(tv_{j_m}), V_{i_l}(tv_{j_m}) \right) \right) \leq \widetilde{\eta} \Big\}. \end{aligned}$$

Clearly, $\widetilde{STV}^{\tilde{\eta}}_{i_k \cap i_l} \subseteq STV^{\max(\tilde{\eta})}_{i_k \cap i_l}$. Moreover, from $tv_{j_m} \in \widetilde{STV}^{\tilde{\eta}}_{i_k \cap i_l}$ it does not follow $\widetilde{dist}_{j_m}\left(V_{i_k}\left(tv_{j_m}\right), V_{i_l}\left(tv_{j_m}\right)\right) \leq \frac{\eta}{n}$. Rather, it is possible to compensate for an exceedance of $\frac{\eta}{n}$ of tv_{j_m}, e.g., when

$$\widetilde{dist}_{j_m}\left(V_{i_k}\left(tv_{j_m}\right), V_{i_l}\left(tv_{j_m}\right)\right) = \frac{\eta}{n} + \Delta,$$

$\Delta > 0$, by requiring that the sum of the distance of the other thresholds fulfils

$$\sum_{\substack{o=1,\ldots,n \\ o \neq m}} \left(\widetilde{dist}_{j_o}\left(V_{i_k}\left(tv_{j_o}\right), V_{i_l}\left(tv_{j_o}\right)\right)\right) \leq \frac{n-1}{n}\,\tilde{\eta} - \Delta.$$

This complies with the compensatory approaches used in MADM (cf. Section 2.1.2). If for a variable $tv_j \in STV_{i_k} \cap STV_{i_l}$, $tv_j \notin \widetilde{STV}^{\tilde{\eta}}_{i_k \cap i_l}$, then

$$\widetilde{dist}_j\left(V_{i_k}\left(tv_j\right), V_{i_l}\left(tv_j\right)\right) > \tilde{\eta}.$$

Similar inequalities can also be defined for subsets of $STV_{i_k} \cap STV_{i_l}$ of magnitude M. Varying the parameter $\tilde{\eta}$ allows the exploration of the degree of difference between S_{i_k} and S_{i_l} with respect to the values of (sets of) variables. Finally, one denotes

$$\begin{aligned}
&\widetilde{STV}^{\eta}_{i_k \cap i_l} = \widetilde{STV}^{|STV| \cdot \tilde{\eta}}_{i_k \cap i_l} \\
= \quad & \Bigg\{ \{tv_{j_1}, \ldots, tv_{j_n}\} \in STV_{i_k} \cap STV_{i_l} : \\
& \qquad \sum_{m=1}^{n} \left(\widetilde{dist}_{j_m}\left(V_{i_k}\left(tv_{j_m}\right), V_{i_l}\left(tv_{j_m}\right)\right)\right) \leq \tilde{\eta} \cdot |STV| \Bigg\}.
\end{aligned}$$

Analogously, one defines

$$STV^{\eta}_{i_k \cap i_l} = STV^{|STV| \cdot \tilde{\eta}}_{i_k \cap i_l}.$$

In this manner, the specification of the parameter η is facilitated, as it is sufficient to consider $\eta \in [0, 1]$ (instead of $|STV_{i_k}|$ or $|STV_{i_k} \cap STV_{i_l}|$).

Next, those subscenarios of both S_{i_k} and S_{i_l} that contain only *maximum* sets of variables that are (at least) η-similar are considered:

$$SS^{\eta}_{i_k^{\subseteq} \cap i_l^{\subseteq}} = \left\{ S_{i_p} \in S^{sub}_{SS, i_k} \cap S^{sub}_{SS, i_l} : STV_{i_p} \in STV^{\eta}_{i_k \cap i_l} \right\}.$$

It is not required that any of the subscenarios is connected to the source of S_{i_k} or S_{i_l}. Choosing $\eta = 0$, the set $SS^{\eta}_{i_k^{\subseteq} \cap i_l^{\subseteq}}$ contains the subscenarios with equal values, while for $\eta = |STV_{i_k} \cap STV_{i_l}|$, $SS^{\eta}_{i_k^{\subseteq} \cap i_l^{\subseteq}}$ contains all sub-scenarios which share the same variables (regardless of their values).

This approach identifies the **sufficiently similar episodes.** In this manner, the *traceability of differences* is enhanced by determining where the values of the scenarios did start to diverge, or where they did start to converge. As for the ratio model presented in Section 7.1.2.2, it is possible to define several *lattices of similarity* for considering more or less different values as *sufficiently similar.* In this manner, it becomes possible to avoid treating scenarios that are valued differently only because of minor perturbations (e.g., measurement errors) as different. Finally, this approach enables extending the considerations to special sub-scenarios that are particularly relevant in SBR and SBR & MCDA:

7.2.2.1. Source Similarity

For reasons of brevity, one denotes

$$S^{SOURCE}_{i_k} = \left\langle SOURCE_{i_k}, V_{i_k}\left(SOURCE_{i_k}\right), status_{i_k}\left(SOURCE_{i_k}\right), 0^{1\times\left|SOURCE_{i_k}\right|}\right\rangle.$$

Two source-originated scenarios S_{i_k} and S_{i_l} are η-$SOURCE$-similar if and only if

$$\begin{aligned} SOURCE_{i_k} &= SOURCE_{i_l}, \\ S^{SOURCE}_{i_k} &\in SS^{\eta}_{i_k{}^{\subseteq} \cap i_l{}^{\subseteq}}, \\ S^{SOURCE}_{i_l} &\in SS^{\eta}_{i_k{}^{\subseteq} \cap i_l{}^{\subseteq}}. \end{aligned}$$

η-$SOURCE$-similarity indicates that S_{i_k} and S_{i_l} have been constructed using η-similar information on the source variables.

7.2.2.2. Focus Similarity

Similarly to the approach for Source similarity, one defines

$$S^{FOCUS}_{i_k} = \left\langle FOCUS_{i_k}, V_{i_k}\left(FOCUS_{i_k}\right), status_{i_k}\left(FOCUS_{i_k}\right), 0^{1\times\left|FOCUS_{i_k}\right|}\right\rangle.$$

Two focussed scenarios S_{i_k} and S_{i_l} are η-$FOCUS$-similar if they share the same set of focus variables and the respective values are η-similar:

$$\begin{aligned} FOCUS_{i_k} &= FOCUS_{i_l}, \\ S_{i_k}^{FOCUS} &\in SS^{\eta}_{i_k^{\subseteq} \cap i_l^{\subseteq}}, \\ S_{i_l}^{FOCUS} &\in SS^{\eta}_{i_k^{\subseteq} \cap i_l^{\subseteq}}. \end{aligned}$$

η-$FOCUS$-similarity indicates that S_{i_k} and S_{i_l} result in η-similar valuations of the focus variables. This is particularly interesting as the $FOCUS$ represents the information that is presented to the recipients of the scenario. If the set of scenarios $SS = \{S_i\}_{i \in I}$, S_{i_k}, $S_{i_l} \in SS$ were generated for one (minimal) context Con_l, then by definition $FOCUS_{i_k} = FOCUS_{i_l}$ $\forall S_{i_k}, S_{i_l} \in SS$.

7.3. Equivalence Classes of Scenarios

Having established different concepts of scenario similarity, this section focuses on scenarios that are not only equal or similar with respect to some variables or values, but that can be considered as *interchangeable* in specific situations or under specific conditions. That means, this section analyses under which circumstances are two scenarios can be considered as **equivalent.** A particularly interesting equivalence is the relationship of operational equivalence [Alspaugh, 2002]. Two scenarios are *operationally equivalent* if the effects (given a certain purpose) of the events they describe are the same. A number of novel equivalence class approaches enabled by the scenario formalisation and based on the notion of similarity defined in the previous section is developed to operationalise this notion of equivalence.

The set $FOCUS$ has the role of gearing the scenario construction towards containing the information that is relevant to the recipients. $FOCUS$ contains those variables that must be assigned a value (cf. Section 4.1.3). Hence, the concept of **focus similarity** is particularly suitable for measuring the operational equivalence. While in SBR & SM frameworks it is necessary to rely on the similarity of the focus variables' values, in SBR & MCDA the effects of two different scenarios with respect to the assessment of alternatives can be operationalised by means of a MAVT evaluation. Both approaches are presented in this section.

Throughout this chapter, let $SS = \{S_i\}_{i \in I}$ be a set of scenarios sharing a given (minimal) context Con_l and let $FOCUS = \left\{tv_1^F, \ldots, tv_N^F\right\}$ be the set of focus variables.

7.3.1. Focus Equivalence of Scenarios

To define the equivalence relation denote $S_{i^*} \in SS$ a reference scenario, S_{i_k}, S_{i_l} two scenarios in SS and let η be a parameter in $[0,1]$. It is furthermore required that all $S_i \in SS$ are focus complete. The **FOCUS equivalence relation** is defined as

$$\begin{aligned} & S_{i_k} \sim^{\eta}_{V_{i^*}(FOCUS)} S_{i_l} \\ \Leftrightarrow \quad & S_{i_k} \text{ and } S_{i^*} \text{ are } \eta \; FOCUS \text{ similar and } S_{i_l} \text{ and } S_{i^*} \text{ are } \eta \; FOCUS \text{ similar.} \end{aligned} \tag{7.5}$$

Then, for all $\eta \in [0,1]$ the relation $\sim^{\eta}_{V_{i^*}(FOCUS)}$ is an equivalence relation, as it fulfils the following properties (for the definition of equivalence relations see [Werner, 2000]):

reflexivity: $S_{i_k} \sim^{\eta}_{V_{i^*}(FOCUS)} S_{i_k}$ for all $\eta \in [0,1]$,

symmetry: $S_{i_k} \sim^{\eta}_{V_{i^*}(FOCUS)} S_{i_l} \Rightarrow S_{i_l} \sim^{\eta}_{V_{i^*}(FOCUS)} S_{i_k}$ and

transitivity: $S_{i_k} \sim^{\eta}_{V_{i^*}(FOCUS)} S_{i_l}$ and $S_{i_l} \sim^{\eta}_{V_{i^*}(FOCUS)} S_{i_m} \Rightarrow S_{i_k} \sim^{\eta}_{V_{i^*}(FOCUS)} S_{i_m}$.

All properties follow immediately from the definition (cf. equation 7.5).

It is possible to define the relation $\sim^{\eta}_{V^*(FOCUS)}$ for any value $V^*(FOCUS) \in Range\left(V\left(tv_1^F\right)\right) \times \ldots \times Range\left(V\left(tv_N^F\right)\right)$. An arbitrary vector $V^*(FOCUS)$ may, however, not be admissible, because it does *not* represent a plausible and consistent combination of values. Furthermore, there may be plenty of combinations $(\eta, V^*(FOCUS))$ for which no scenario in SS is η focus similar to $V^*(FOCUS)$ resulting in multiple meaningless equivalence relations. Restricting the similarity relations to using an actual realisation $V_{i^*}(FOCUS)$ ($S_{i^*} \in SS$) as a reference valuation makes the comparison formulation of equivalence and the determination of the most relevant equivalence classes in scenario management more efficient, e.g., via clustering analysis [Martino and Chen, 1978; Tapio, 2003].

As $dist_j$ is a metric on $Type_j$, it follows that $dist_j$ and the derived metric $\widetilde{dist}_j$ fulfil the triangle inequality. Consequently:

$$\begin{aligned} & \widetilde{dist}_j\left(V_{i_k}\left(tv_j\right), V_{i_l}\left(tv_j\right)\right) \\ \leq\ & \widetilde{dist}_j\left(V_{i_k}\left(tv_j\right), V_{i^*}\left(tv_j\right)\right) + \widetilde{dist}_j\left(V_{i^*}\left(tv_j\right), V_{i_l}\left(tv_j\right)\right) \\ \leq\ & 2 \cdot \eta. \end{aligned}$$

If $S_{i_k} \sim^{\eta}_{V_{i^*}(FOCUS)} S_{i_l}$, then S_{i_k} and S_{i_l} are 2η focus similar.

The equivalence relation defined in equation 7.5 allows $V_{i^*}(FOCUS)$ scenario equivalence classes for all $S_{i^*} \in SS$ and $\eta \in [0,1]$ to be defined by

$$[S_{i^*}]_{\sim_{V(FOCUS)^\eta}} = \left\{ S_i \in SS : S_i \sim_{V_{i^*}(FOCUS)^\eta} S_{i^*} \right\}. \tag{7.6}$$

This equivalence class is well-defined and non-empty as S_{i^*} is η FOCUS similar to itself for all $\eta \in [0,1]$.

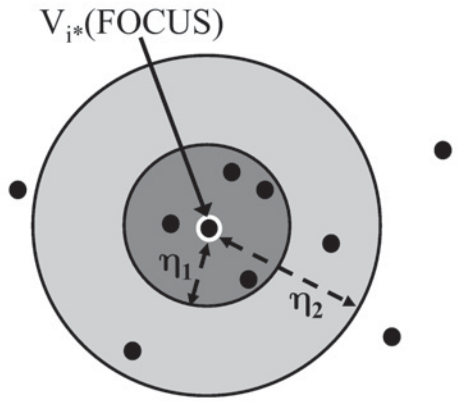
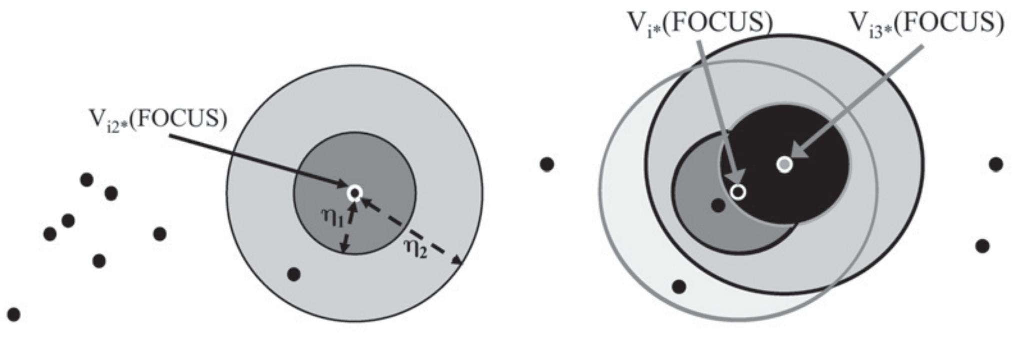


Figure 7.1.: *FOCUS Equivalence Classes of Scenarios. Example Determined On The Basis Of* $V_{i^*}(FOCUS)$ *For Two Values* $0 \geq \eta_1 \geq \eta_2$.

Figure 7.1 shows an example: the valuation of $FOCUS$ in S_{i^*} is projected to the plane and depicted by as the central (black) point. For all other scenarios $S_i \in SS$, $S_i \neq S_{i^*}$, $V_{i^*}(FOCUS)$ is projected to the same plane and represented by further black points. For the smaller value η_1 five scenarios (including S_{i^*} itself) are in the equivalence class $[S_{i^*}]_{\sim_{V(FOCUS)^{\eta_1}}}$, represented by the inner light grey circle. Contrarily, the equivalence class $[S_{i^*}]_{\sim_{V(FOCUS)^{\eta_2}}}$ contains seven scenarios (represented by all dots within the larger dark grey circle).

The scenarios within an equivalence class do not only vary with changing values of η, but also for changing reference scenarios S_{i^*}. Figure 7.2 shows possible consequences of shifting the reference scenario. Figure 7.2(a) (left side) shows $\left[S_{i_2^*}\right]_{\sim_{V(FOCUS)^{\eta_{1,2}}}}$, where even the broader equivalence class using parameter η_2 contains only two scenarios. The right side (cf. Figure 7.2(b)) shows overlapping equivalence classes, where even the reference scenario $S_{i_3^*}$ is in the η_1 equivalence classes of S_{i^*} and vice versa.



(a) Effect of Changing the Reference Scenario from S_{i^*} to $S_{i_2^*}$ (b) Overlapping Equivalence Classes

Figure 7.2.: *Effects on the Equivalence Class of a Change in the Reference Scenario*

7.3.2. Evaluation Equivalence of Scenarios

The definition of focus equivalence classes does not take into account how important reaching a certain performance in the focus variables (or a subset thereof) is, nor does it consider trade-offs between the importance of different focus variables. The SBR & MCDA enables equivalence classes to be defined according to the **distance of evaluations** whilst taking into account the preferences of the decision-makers explicitly.

Weak types of similarity allow values of $Type_j$ to be compared after the evaluation, which is defined by a functional $f : Type_{j_1} \times \ldots \times Type_{j_N} \rightarrow \mathbb{R}$ (cf. Section 3.2.6.2). In SBR & MCDA, the type and shape of the evaluation function $f = f_D$ for a given decision problem D captured in the context Con_l is determined by:

- a set of attributes $\{att_1, \ldots, att_N\} = \left\{tv_1^F, \ldots, tv_N^F\right\} = FOCUS$ for all scenarios built in Con_l;
- a set of intra-criteria preferences captured in a value function

$$v_D : SPV\left(tv_1^F\right) \times \ldots \times SPV(\left(tv_N^F\right) \rightarrow [0,1]^N,$$

which consists of one-dimensional value functions $v_D^j : tv_j^F \to [0,1]$ for each attribute tv_j^F;

- a set of inter-criteria preferences $\mathfrak{P}_D$ for defining an aggregation function $agg_D : [0,1]^N \to [0,1]$.

The evaluation function f_D is defined as

$$\begin{array}{rcl} f_D : SPV\left(tv_1^F\right) \times \ldots \times SPV(\left(tv_N^F\right) & \to & [0,1] \\ f_d\left(V_i\left(tv_1^F\right), \ldots, V_i\left(tv_N^F\right)\right) & = & agg_D \circ v_D\left(V_i\left(tv_1^F\right), \ldots, V_i\left(tv_N^F\right)\right) \end{array}$$

(for further details, cf. Section 2.1.3).

For a given $\varepsilon \in [0,1]$, two scenarios S_{i_k} and S_{i_l} are called ε **evaluation equivalent,** if there is a scenario S_{i^*}, with $V_{i^*}(att_1, \ldots, att_N) = V_{i^*}(FOCUS)$, such that

$$\begin{array}{rrcl} & \left\| f_D\left(V_{i_k}\left(tv_1^F, \ldots tv_N^F\right)\right) - f_D\left(V_{i^*}\left(tv_1^F, \ldots tv_N^F\right)\right)\right\|_D & \leq & \varepsilon \\ \text{and} & \left\| f_D\left(V_{i_l}\left(tv_1^F, \ldots tv_N^F\right)\right) - f_D\left(V_{i^*}\left(tv_1^F, \ldots tv_N^F\right)\right)\right\|_D & \leq & \varepsilon. \end{array}$$

The distance measure $\| \, . \, \|_D$ can be any metric on $\mathbb{R}$ can be adapted to the preferences of the decision-makers and captured in Con_l. By default, the Euclidean metric over $\mathbb{R}$

$$dist_{Euc}\left(v_{i_k}^j, v_{i_l}^j\right) = \sqrt{\left(v_{i_k}^j - v_{i_l}^j\right)^2} = \left|v_{i_k}^j - v_{i_l}^j\right|, \qquad v_{i_k}^j, v_{i_l}^j \in \mathbb{R}$$

is used.

The respective **evaluation equivalence classes** are defined by

$$\begin{array}{l} [S_{i^*}]_{\sim V(FOCUS)^\varepsilon}^{eval} \\ = \left\{S_{i_k} \in SS : \left\| f_d\left(V_{i_k}\left(tv_1^F, \ldots, tv_N^F\right)\right) - f_d\left(V_{i^*}\left(tv_1^F, \ldots, tv_N^F\right)\right)\right\| \leq \varepsilon\right\} \end{array} \tag{7.7}$$

7.4. Equivalence Classes for Focus Incomplete Scenarios

During the scenario generation process there may be situations where scenarios need to be selected or pruned *before* the values of all focus variables can be determined. In this case, it is useful to identify a set of variables that allows the focus variables' values to be assessed. These assessments provide the basis for defining equivalence classes for focus incomplete scenarios. Again, a distinction between SBR & SM and SBR & MCDA, where the latter takes into account the decision-makers' preferences, is made. The first step in defining these novel equivalence classes in both frameworks is the determination of the scenarios' similarity with respect to one $tv_k^F \in FOCUS$.

7.4.1. Indicator Similarity with Respect to Individual Focus Variables

First, for each variable $tv_k^F \in FOCUS$, a list of potential indicators $ind_j^k \in IND\left(tv_k^F\right)$ is determined. For each ind_j^k, the following properties must hold:

$$\exists \left\{S_{i_l}\right\}_{l=1,\ldots,L} = SS_L \subseteq SS : \; ind_j^k = tv_j \in STV_{i_l} \text{ and } L \geq 2 \tag{7.8}$$

$$V_{i_l}\left(tv_j\right) \neq \infty \; \forall S_{i_l} \in SS_L \tag{7.9}$$

$$\exists P_{DI_{i_l}}\left(ind_j^k, tv_k^F\right) \tag{7.10}$$

$$\text{and} \quad \exists i_l^* \in \{1,\ldots,L\} : \; V_{i_l^*}\left(tv_{j_m}\right) = \infty \; \forall tv_{j_m} \in \widetilde{\Theta}\left(tv_j\right) \cap P_{DI_{i_l^*}}\left(ind_j^k, tv_k^F\right). \tag{7.11}$$

Equations 7.8 and 7.9 ensure that there are at least two scenarios whose indicator similarity can be compared (otherwise, the equivalence class would consist of only one scenario and would not be useful for scenario management). The third condition (equation 7.10) indicates that ind_j^k influences $V\left(tv_k^F\right)$. The fourth condition (equation 7.11) ensures that the most significant indicators are used: as for all predecessors $tv_p \in \Psi\left(tv_j\right)$, which are not connected by another path to tv_k^F, the information relevant to determine $V\left(tv_k^F\right)$ is incorporated in $V_{i_l}\left(tv_j\right)$. That means, the assumption is made that the less vertices in $P_{DI_{i_l}}\left(ind_j^k, tv_k^F\right)$ or the more directly ind_j^k is linked to tv_k^F, the more significant $V\left(ind_j^k\right)$ is for assessing $V\left(tv_k^F\right)$. This last condition ensures also that whenever the value of the focus variable itself is available, this value is used (as it is supposed to be the best indicator of itself).

7.4.1.1. Assessment of Indicator Importance

Let ind_j^k be of type $Type_j$. Additionally requiring that there is a norm $\| \,.\, \|_j$ such that $\left(Type_j, \| \,.\, \|_j\right)$ is a normed vector space, **causality coefficients** [Lee et al., 1992; Zhang et al., 1989] indicating the *logical interdependence* [Lee et al., 1992] or the *direction of influence* [Kok, 2009; Kosko, 1986; Montibeller and Belton, 2006] are determined for each indicator:

$$cc\left(ind_j^k, tv_k^F\right) = \begin{cases} + & \text{if increase in } \left\|V\left(ind_j^k\right)\right\|_j \text{ augments } \left\|V\left(tv_k^F\right)\right\|_j, \\ 0 & \text{if influence of } V\left(ind_j^k\right) \text{ on } V\left(tv_k^F\right) \text{ is undetermined,} \\ - & \text{if increase } \left\|V\left(ind_j^k\right)\right\|_j \text{ weakens or decreases } \left\|V\left(tv_k^F\right)\right\|_j. \end{cases} \tag{7.12}$$

If $Type_j$ can not be considered as a vector space, it may be possible to derive (numerical) auxiliary indicators $aux - ind_j^k$ representing key features of ind_j^k. For example, in emergency management an indicator for a variable tv_j that represents a map with a plume, the size of the area covered by the plume (measured in km^2) at a certain time t could be used as an auxiliary indicator allowing conclusions on the number of people whose health is affected by an incident to be drawn. These auxiliary indicators must be directly deducible from ind_j^k without involving any further expertise. For reasons of brevity and readability, this thesis does not distinguish between ind_j^k and $aux - ind_j^k$ in the following text. Beyond the direction of influence (+ or −), it may be possible to specify the *importance* or *degree of influence.* Then, the overall *weighted influence* can be captured by $w^{ind_j^k} \in [-1, 1]$.

If for all vertices tv_j in $P_{DI_{i_l}}\left(ind_j^k, tv_k^F\right) =: \{(tv_j, tv_{j+1})\}_{j\in J}$ the direction of influence of tv_j on its successor tv_{j+1} in the path, denoted $cc\,(tv_j, tv_{j+1})$, as well as a sensitivity assessment $SA\,(tv_j, tv_{j+1})$ ($0 \leq SA\,(tv_j, tv_{j+1}) \leq 1$ [Hamby, 1994; Helton, 1994; Saltelli et al., 2008]) indicating the degree of influence of $V\,(tv_j)$ on $V\,(tv_{j+1})$ are available[40], then the *overall influence* can be assessed by

$$\begin{aligned} w^{ind_j^k} &= \prod_{j\in J} cc\,(tv_j, tv_{j+1})\, SA\,(tv_j, tv_{j+1}) \\ &= \prod_{j\in J^+} SA\,(tv_j, tv_{j+1}) \cdot \prod_{j\in J^-} (-1) \cdot SA\,(tv_j, tv_{j+1}), \end{aligned}$$

[40]For instance, the sensitivity assessment can be captured in the extended context Con_d, cf. Section 6.2.2.2.

where

$$\begin{aligned} P_{DI_{i_l}}\left(ind_j^k, tv_k^F\right) &= \{e_j\}_{j\in J} = \{(tv_j, tv_{j+1})\}_{j\in J} \\ M^+ &= \left\{tv_j : e_j \in P_{DI_{i_l}}\left(ind_j^k, tv_k^F\right) \wedge cc\left(tv_j, tv_{j+1}\right) = +\right\} \\ M^- &= \left\{tv_j : e_j \in P_{DI_{i_l}}\left(ind_j^k, tv_k^F\right) \wedge cc\left(tv_j, tv_{j+1}\right) = -\right\}. \end{aligned}$$

For all tv_j crossed by $P_{DI_{i_l}}\left(ind_j^k, tv_k^F\right)$, it is assumed that $SA\left(e_j\right) \neq 0$ and $cc\left(e_j\right) = cc\left(tv_j, tv_{j+1}\right) \neq 0$. The first assumption is justified as the connectedness of tv_j and tv_{j+1} in S_{i_l} signifies that $V\left(tv_j\right)$ influences $V\left(tv_{j+1}\right)$. The aggregation method chosen is conform to the common (fuzzy) cognitive mapping approaches, where the weight of indirect influences is determined by their product [Jetter and Schweinfort, 2011; Kok, 2009; Wu and Lee, 2007].

If sensitivity indices are not available for some edges e_j in $P_{DI_{i_l}}\left(ind_j^k, tv_k^F\right)$, an upper and a lower bound for $w^{ind_j^k}$, denoted $w_+^{ind_j^k}$ and $w_-^{ind_j^k}$ respectively, can be determined. To this end, one defines

$$\begin{aligned} STV_P &= \left\{tv_j : e_j = (tv_j, tv_{j+1}) \in P_{DI_l}\left(ind_j^k, tv_k^F\right)\right\} \\ \text{and} \qquad STV_P^{NA} &= \left\{tv_j \in STV_P : \nexists\, SA\left(e_j\right)\right\}. \end{aligned}$$

Then, one sets the upper bound of the sensitivity of tv_{j+1} to changes in tv_j to $SA^r\left(e_j\right) = 1$ and the lower bound $SA^l\left(e_j\right) = 0$ for each such edge e_j in STV_P^{NA}. Finally, if $\exists n \in \mathbb{N}_0$ with $|M^-| = (2 \cdot n) - 1$ (i.e., $w^{ind_j^k} \leq 0$), a lower bound $w_-^{ind_j^k}$ can be determined by

$$\begin{aligned} w_-^{ind_j^k} &= -\prod_{j\in M\setminus STV_P^{NA}} SA\left(e_j\right) \cdot \underbrace{\prod_{j\in J\cap STV_P^{NA}} SA^r\left(e_j\right)}_{=1} \\ &= -\prod_{j\in J\setminus STV_P^{NA}} SA\left(e_j\right). \end{aligned}$$

Overall, $w_-^{ind_j^k} \leq w^{ind_j^k} \leq 0$.

Analogously, if $\exists n \in \mathbb{N}_0$ with $|M^-| = (2 \cdot n)$ (i.e., $w^{ind_j^k} \geq 0$) an upper bound of the $w_+^{ind_j^k}$ is given by:

$$w_+^{ind_j^k} = \prod_{j\in J\setminus STV_P^{NA}} SA\left(e_j\right).$$

Thus, $w_{+}^{ind_j^k} \geq w^{ind_j^k} \geq 0$.

The quality of this assessment depends not only on the quality of each sensitivity assessment $SA(e_j)$, but also on the length of $P_{DI_{i_l}}\left(ind_j^k, tv_k^F\right)$, as the multiplicative aggregation methods are rather sensitive to errors or perturbations [Stewart, 1996].

If a sensitivity assessment is not available (or too rough, as information on various edges e_j is lacking or J is too large), or if no scenario $S_{i_l} \in SS$ contains a path $P\left(tv_j, tv_k^F\right)$ for any variable $tv_j \in STV_{i_l}$ with $V_{i_l}(tv_j) \neq \infty$, the (human) experts responsible for determining $V\left(tv_k^F\right)$ can be asked to provide an assessment of the degree of influence. Often, human experts prefer expressing the importance of influence in qualitative terms (such as "*very high influence*", "*weak influence*" etc.) [Montibeller and Belton, 2006]. These scales can be mapped to Fuzzy numbers [Kosko, 1986; Wu and Lee, 2007]. Again, the assessments can be captured in Fuzzy numbers or numerical values $w^{ind_j^k}$.

The sets

$$\begin{aligned} IND_k^+ &= \bigcup_j \left\{ind_j^k \in IND\left(tv_k^F\right) : cc\left(ind_j^k, tv_k^F\right) = +\right\} \cup \left\{ind_j^k : w^{ind_j^k} > 0\right\}, \\ IND_k^- &= \bigcup_j \left\{ind_j^k \in IND\left(tv_k^F\right) : cc\left(ind_j^k, tv_k^F\right) = -\right\} \cup \left\{ind_j^k : w^{ind_j^k} < 0\right\} \end{aligned}$$

represent the supporting and depleting indicators for $V\left(tv_k^F\right)$. Denoting

$$\begin{aligned} IND_k\left(S_{i_k}, S_{i_l}\right) &= IND_k \cap STV_{i_k} \cap STV_{i_l} \\ IND_k^+\left(S_{i_k}, S_{i_l}\right) &= IND_k^+ \cap STV_{i_k} \cap STV_{i_l} \\ \text{and} \quad IND_k^-\left(S_{i_k}, S_{i_l}\right) &= IND_k^- \cap STV_{i_k} \cap STV_{i_l} \end{aligned}$$

for both types of assessment methods, a partial order of the influence of indicators $\preceq_{Ind}$ can be achieved.[41] This property is used for a selection of indicators, which can be necessary in case the set of available indicators is large.

In general, both the direction of influence $cc\left(ind_j^k, tv_k^F\right)$ and the weight of influence $w^{ind_j^k}$ may depend on the scenario considered, i.e., for a scenario S_l, where $ind_j^k = tv_j \in STV_{i_l}$, $cc\left(ind_j^k, tv_k^F\right) = cc_l\left(ind_j^k, tv_k^F\right)$ and $w^{ind_j^k} = w_l^{ind_j^k}$.

[41] While the comparison of crisp numbers is straightforward, for the comparison of Fuzzy numbers, various defuzzification or direct comparisons can be used [Cheng, 1998; Kim and Park, 1990; Sengupta and Pal, 2000].

Let $inf_k^l\left(ind_j^k\right)$ the influence of the indicator ind_j^k on tv_k^F in S_l. To determine the influence across two scenarios $S_{i_l}, S_{i_k}, tv_j \in STV_{i_l} \cap STV_{i_k}$, define

$$inf_k^{i_l,i_k}\left(ind_j^k\right) = \max_{S_{i_l},S_{i_k}}^{\preceq_{Ind}} \left\{inf_k^{i_l}\left(ind_j^k\right), inf_k^{i_k}\left(ind_j^k\right)\right\}.$$

Using the maximum operator ensures that the influence of an indicator is not underestimated for either scenario. The rationale behind is that it is more important to consider indicators that are very influential for at least one out of two scenarios than indicators which are of intermediate influence for both.

7.4.1.2. Indicator Equivalence

Two scenarios S_{i_l} and S_{i_k} are called (μ,η)-$IND^k\left(S_{i_l}, S_{i_k}\right)$ equivalent ($\mu \geq 0$, $0 \leq \eta \leq 1$) if and only if S_{i_l} and S_{i_k} have at least μ indicators for tv_k^F in common and the distance of the values of the μ most important indicators is smaller or equal to η. To capture this equivalence relation more precisely, denote for each indicators $ind_m^{k,i_l,i_k} \in IND_k\left(S_{i_l}, S_{i_k}\right)$ and for each the sets $IND_{k,i_l,i_k}^{\prec}\left(ind_m^{k,i_l,i_k}\right)$ of more influential indicators by

$$\begin{aligned} & IND_{k,i_l,i_k}^{\prec}\left(ind_m^{k,i_l,i_k}\right) \\ = \quad & \left\{ind_j^k \in IND^k\left(S_{i_k}, S_{i_l}\right) : \; inf_k^{i_k,i_l}\left(ind_j^{k,i_k,i_l}\right) \preceq_{Ind} inf_k^{i_k,i_l}\left(ind_m^{k,i_k,i_l}\right)\right\}. \end{aligned}$$

Then, $\left|IND_{k,i_k,i_l}^{\prec}\left(ind_m^{k,i_k,i_l}\right)\right|$ allows for establishing an ordering of the most important or influential indices in $IND_k\left(S_{i_k}, S_{i_l}\right)$:

$$\begin{aligned} & ind_1^{k,i_l,i_k}, \ldots, ind_M^{k,i_l,i_k}, \quad \text{where } m \geq \mu \\ \text{and } & \left|IND_{k,i_l,i_k}^{\prec}\left(ind_1^{k,i_l,i_k}\right)\right| \leq \ldots \leq \left|IND_{k,i_l,i_k}^{\prec}\left(ind_M^{k,i_l,i_k}\right)\right|. \end{aligned} \tag{7.13}$$

It is required that η equivalence must hold for $ind_1^{k,i_l,i_k}, \ldots, ind_\mu^{k,i_l,i_k}$, i.e.,

$$\forall\; l = 1, \ldots, \mu : \; \widetilde{dist}_l\left(V_{i_l}\left(ind_l^{k,i_l,i_k}\right), V_{i_k}\left(ind_l^{k,i_l,i_k}\right)\right) \leq \eta,$$

where the distance measure $dist_l$ (on the basis of which the normalised metrics $\widetilde{dist}_l$ is determined) is the metric induced by the norm $\| \,.\, \|_l$ over the vector space $Type_l$[42].

[42]That means $dist_l(x,y) = \|x-y\|_l$ for all x, $y \in Type_l$.

Using this definition, it holds: S_{i_k} and S_{i_l} are (μ,η)-$IND^k\left(S_{i_k},S_{i_l}\right)$ equivalent if and only if

$$\begin{aligned} & \left|IND_k\left(S_{i_k},S_{i_l}\right)\right| \geq \mu \\ \text{and}\qquad & S_{i_k} \in \left[S_{i_k}\right]_{\sim_{V\left(ind_1^{k,i_k,i_l},\ldots,ind_m^{k,i_k,i_l}\right)^{\eta}}}, \end{aligned}$$

where

$$\begin{aligned} & \left[S_{i_k}\right]_{\sim_{V\left(ind_1^{k,i_k,i_l},\ldots,ind_m^{k,i_k,i_l}\right)^{\eta}}} \\ = \quad & \left\{S_i \in SS : \widetilde{dist}_l\left(V_i\left(ind_l^{k,i_k,i_l}\right), V_{i_k}\left(ind_l^{k,i_k,i_l}\right)\right) \leq \eta \ \forall\ l = 1,\ldots,\mu\right\}. \end{aligned}$$

7.4.2. Indicator Equivalence Classes

After having established the notion of similarity of two with respect to a single variable $tv_k^F \in FOCUS$, the next step in constructing equivalence classes of focus incomplete scenarios is to determine the **similarity with respect to the set of focus variables.** The aim of building scenario equivalence classes is to develop sets of scenarios $SS_i \subseteq SS$ that are similar with respect to their **effect** or **impact on the situation** (in SBR & SM) or the **evaluation of alternatives** (in SBR & MCDA). For both situations the key information is captured in $V(FOCUS)$. The following prerequisites are assumed to be determined for each variable $tv_k^F \in FOCUS^{rel}$:

- the set $IND_k\left(S_{i_k},S_{i_l}\right)$
- for all $IND_k\left(S_{i_k},S_{i_l}\right) \neq \emptyset$, the parameter $\mu\left(tv_k^F\right)$
- the $\mu\left(tv_k^F\right)$ most important indicators (cf. equation 7.13).

In this manner, a partition of the indicator set $IND_k\left(S_{i_k},S_{i_l}\right)$ is achieved:

$$IND_k\left(S_{i_k},S_{i_l}\right) = IND_k^{\mu}\left(S_{i_k},S_{i_l}\right) \cup IND_k^{Res}\left(S_{i_k},S_{i_l}\right),$$

where $IND_k^{\mu}\left(S_{i_k},S_{i_l}\right) \cap IND_k^{Res}\left(S_{i_k},S_{i_l}\right) = \emptyset$.

Thereby, for all $ind_l^{k,i_k,i_l} \in IND_k^{\mu}\left(S_{i_k},S_{i_l}\right)$, $ind_m^{k,i_k,i_l} \in IND_k^{Res}\left(S_{i_k},S_{i_l}\right)$ it holds:

$$inf_k^{i_k,i_l}\left(ind_j^{k,i_k,i_l}\right) \preceq_{Ind} inf_k^{i_k,i_l}\left(ind_m^{k,i_k,i_l}\right).$$

Furthermore, $\left|IND_k^{\mu}\left(S_{i_k},S_{i_l}\right|\right) = \mu\left(tv_k^F\right)$.

One denotes $IND_k^{\mu}(S_{i_k}, S_{i_l}) = \left\{ind_l^{k,i_k,i_l}\right\}_{l=1,\dots,\mu}$, where each $V\left(ind_l^{k,i_k,i_l}\right)$ is of $Type_l$. $Type_l$ is assigned the metric $dist_l$, and

$$\mathcal{K} = \bigcup_{tv_k^F \in FOCUS} \{IND_k(S_{i_k}, S_{i_l}) : IND_k(S_{i_k}, S_{i_l}) \neq \varnothing\}.$$

The overall distance of S_{i_k} and S_{i_l} with respect to their $FOCUS$ indicators is derived from the p-norm by defining

$$dist_{p,i_k,i_l}^{F,\mu} = \begin{cases} \left(\sum\limits_{k\in\mathcal{K}} \sum\limits_{l=1}^{\mu(k)} \left(\widetilde{dist}_l(i_k, i_l)\right)^p\right)^{\frac{1}{p}} & \text{for } 1 \leq p < \infty \\ \max\limits_{k\in\mathcal{K}} \max\limits_{l=1,\dots,\mu(i)} \left|\widetilde{dist}_l(i_k, i_l)\right| & \text{for } p = \infty, \end{cases} \tag{7.14}$$

where

$$\widetilde{dist}_l(i_k, i_l) = \widetilde{dist}_l\left(V_{i_k}\left(ind_l^{k,i_k,i_l}\right), V_{i_l}\left(ind_l^{k,i_k,i_l}\right)\right).$$

As usual, $\widetilde{dist}_l$ is the normalised distance measure

$$\widetilde{dist}_l(i_k, i_l) = \frac{dist_l\left(V_{i_k}\left(ind_l^{k,i_k,i_l}\right), V_{i_l}\left(ind_l^{k,i_k,i_l}\right)\right)}{\max\limits_{S_i\in SS_k} dist_l\left(V_i\left(ind_l^{k,i_k,i_l}\right), V_{i_l}\left(ind_l^{k,i_k,i_l}\right)\right)},$$

where $SS_k = \left\{S_i \in SS : ind_l^{k,i_k,i_l} = tv_l \in STV_i\right\}$.

To facilitate comparisons $dist_{p,i_k,i_l}^{F,\mu}$ is normalised by using two weighting vectors $w_p^l\left(ind_l^{k,i_k,i_l}\right)$, $w_p^k\left(tv_k^F\right)$ that respect the following conditions

$$\begin{aligned} \sum_{l=1}^{\mu(k)} \left(w_p^l\left(ind_l^{k,i_k,i_l}\right)^p\right)^{\frac{1}{p}} &= 1 \\ \text{and} \qquad \left(\sum_{k\in\mathcal{K}} \left(w_p^k\left(tv_k^F\right)\right)^p\right)^{\frac{1}{p}} &= 1. \end{aligned}$$

By setting

$$dist_{p,i_k,i_l}^{F,\mu,w} = \begin{cases} \left(\sum\limits_{k\in\mathcal{K}} \sum\limits_{l=1}^{\mu(k)} \left(w_p^k\left(tv_k^F\right) w_p^l\left(ind_l^{k,i_k,i_l}\right) \widetilde{dist}_l(i_k, i_l)\right)^p\right)^{\frac{1}{p}} & , 1 \leq p < \infty \\ dist_{p,i_k,i_l}^{F,\mu} & , p = \infty \end{cases} \tag{7.15}$$

a normalised distance measure is attained. As default values equal weights can be used, i.e.,

$$\begin{aligned} w_p^k\left(tv_k^F\right) &= \frac{1}{|\mathcal{K}|} \\ w_p^l\left(ind_l^{k,i,j}\right) &= \frac{1}{\mu(k)} \qquad \forall\ k \in \mathcal{K}. \end{aligned}$$

For the indicators $ind_l = ind_l^{k_1,\ldots,k_N,i_k,i_l}$, which are relevant to assess N focus variables, for $1 \leq p < \infty$ the distance $\widetilde{dist}_l\left(V_{i_k}\left(ind_l\right), V_{i_l}\left(ind_l\right)\right)$ is taken into account N times whilst reflecting the higher impact of ind_l on $V(FOCUS)$. For $p = 1$, ind_l is summed N times. The number of focus variables, for which ind_l's value is relevant, can be interpreted as a weight of importance of ind_l. Contrarily, for $p = \infty$ the distance with respect to ind_l may not be taken into account at all (namely, if $\left(V_{i_k}\left(ind_l\right), V_{i_k}\left(ind_l\right)\right)$ is never the pair of values for which the maximum occurs). In summary, the parameter p has a considerable influence on the similarity measure and should be chosen carefully.

For constructing **indicator equivalence classes** for a set of scenarios $SS = \{S_i\}_{i \in I}$, the assessment of similarity needs to be made on basis of indicators that are common (and assessed) for all $S_i \in SS$. That means the similarity must be assessed not only for $IND_k\left(S_i, S_j\right)$, but for the sets of *common* indicators. Denote $S_{i^*} \in SS$ a scenario. Then, one defines

$$IND_k\left(SS\right) = \bigcap_{\substack{S_i \in SS \\ S_i \neq S_{i^*}}} IND_k\left(S_i, S_{i^*}\right)$$

Requiring that all indicators are in $IND_k\left(SS\right)$, S_i is in the η-$V_{i^*}\left(IND\right)$ equivalence class of S_{i^*} for a given set SS, if and only if

$$dist_{p,i,i^*}^{F,\mu,w} \leq \eta.$$

The summation covers only the set $IND_k\left(SS\right) \subseteq IND$, i.e., the indicators present and relevant to all scenarios $S_i \in SS$. Finally, the η-$V_{i^*}\left(IND\right)$ **equivalence class** of S_{i^*} is defined by:

$$\left[S_{i^*}\right]_{\sim_\eta V_{i^*}(IND)} = \left\{S_i \in SS :\ dist_{p,i,i^*}^{F,\mu,w} \leq \eta.\right\}$$

7.4.3. Evaluation of Indicator Equivalence Classes

In SBR & MCDA frameworks, the effect of a scenario is measured by a set of attributes and it holds: $FOCUS = \{att_1, \dots, att_N\}$. The values $V_i(FOCUS)$ for all $S_i \in SS$ serve as a basis for the evaluation (cf. Section 5.3), which is performed by a function

$$\begin{aligned} f_D : SPV\left(tv_1^F\right) \times \dots \times SPV\left(tv_N^F\right) &\rightarrow [0,1] \\ f_D\left(V_i\left(tv_1^F\right), \dots, V_i\left(tv_N^F\right)\right) &= agg_D \circ v_D\left(V_i\left(tv_1^F\right), \dots, V_i\left(tv_N^F\right)\right). \end{aligned}$$

In MAVT, the MCDA technique applied here, the aggregation is usually performed by the **simple additive weighing** (SAW) method [Belton and Stewart, 2002]. The advantages of additive aggregation techniques are that they are easy to understand and more robust to errors than multiplicative techniques [Stewart, 1996]. Therefore, the SAW method is the aggregation technique chosen within the framework of this thesis.

Let the attribute tree (cf. Section 2.1.3) for the decision problem D at hand have M abstraction levels of criteria, sub-criteria, etc. Furthermore, let $w_k^j = w_k^j(D)$ ($k = 1, \dots, N$, $j = 1, \dots, M$) be the weight, with which the (normalised) value of attribute k (= tv_k^F) is multiplied at level j. Then, the total weight, attribute k is multiplied with, is

$$w_k^D = \prod_{j=1}^{M} w_k^j. \tag{7.16}$$

The weights w_k^D reflect the *overall* relative importance of attribute k for the decision problem D, whereas w_k^j capture the relative importance at a certain *abstraction level*. Therefore, w_k^D can be used to reflect as the decision-makers' preference function and adopted to model the relative importance of a set of indicators $IND\left(tv_k^F\right) = IND\left(att_k\right) = \{Ind_{k_1}, \dots, Ind_{k_m}\}$ compared to the other indicator sets $IND\left(tv_{k_l}^F\right)$, $k \neq k_l$, $IND\left(tv_k^F\right), IND\left(tv_{k_l}^F\right) \neq \emptyset \forall k_l \in \mathcal{K}$.

The distance of two scenarios S_{i_k} and S_{i_l} with respect to $FOCUS$ given the **currently available information** and taking into account the **decision-makers' inter-criteria preferences** is:

$$pref_D - dist^{F,\mu,w}_{p,i_k,i_l} =$$
$$\begin{cases} \left(\sum\limits_{k \in \mathcal{K}} \sum\limits_{j=1}^{\mu(k)} \left(w^D_k w^l_p \left(ind^{k,i_k,i_l}_l \right) \widetilde{dist}_l \left(i_k, i_l \right) \right)^p \right)^{\frac{1}{p}} & , 1 \leq p < \infty \\ dist^{F,\mu}_{k,i_k,i_l} & , p = \infty, \end{cases} \tag{7.17}$$

where w^D_k is defined as in equation 7.16. This distance measure is normalised and takes values between 0 and 1. Therefore, a normalisation with value functions v_D is omitted.

Finally, scenario S_i is in the η-$pref_D\left(V_{i^*}(IND)\right)$ **equivalence class** of S_{i^*} if and only if

$$pref_D - dist^{F,\mu,w}_{p,i,i^*} \leq \eta.$$

Accordingly, the η-$pref_D\left(V_{i^*}(IND)\right)$ equivalence class of S_{i^*} is defined as

$$\left[S_{i^*}\right]_{\sim_{\eta - V_{i^*}(IND)}} = \left\{ S_i \in SS : \; pref_D - dist^{F,\mu,w}_{p,i,i^*} \leq \eta \right\}.$$

7.5. Defining the Representativeness of Scenario Equivalence Classes

One of the main purposes of scenarios is to stimulate discussion and to overcome too narrow reasoning frames [Schoemaker, 1995]. More precisely, a complete set of scenarios should contain information on both best guess or standard scenarios and atypical, extreme cases. Therefore, the fact that some scenarios are considered *more typical* than others should be taken into account [Lesot and Bouchon-Meunier, 2004]. After having developed equivalence classes based on the focus variables' values (or indicators thereof), it becomes possible to define the **representativeness** of each scenario equivalence class.

The equivalence classes may be of different size and contain scenarios of varying plausibility and reliability. For instance, one class may consist of multiple unreliable scenarios, while another class consists of just one highly reliable scenario. Therefore, it is crucial to determine *how many* scenarios support a certain result and *how reliable* each of these scenarios is.

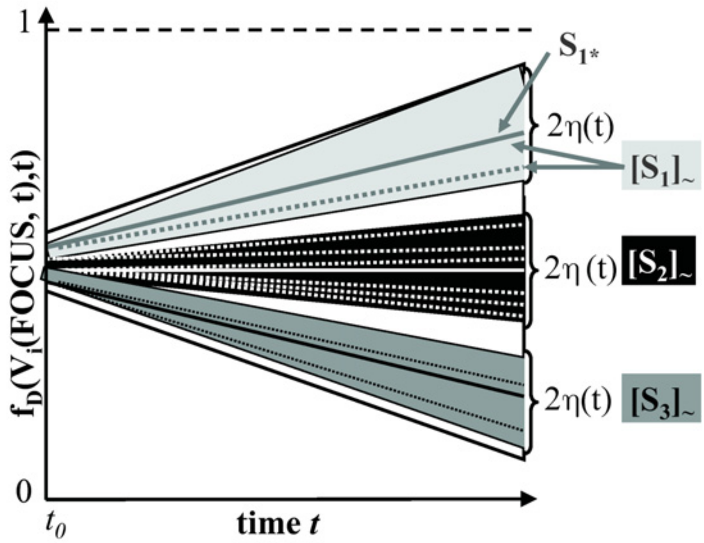


Figure 7.3.: *The Concept of Representativeness: Three Exemplary Scenario Equivalence Classes*

Figure 7.3 illustrates these questions. It shows the development of the evaluation of a set of scenarios over time represented in a *scenario trumpet*. The starting point of SBR & MCDA is t_0. All possible future states of the system are represented in the trumpet. The upper and lower margins represent the best and worst evaluated possible states at each time step t. Figure 7.3 shows how the set of scenarios is partitioned into three scenario equivalence classes $[S_1]_\sim$, $[S_2]_\sim$ and $[S_3]_\sim$ (each class is depicted as a set of dotted and solid lines) with representatives S_{1^*}, S_{2^*} and S_{3^*} (depicted by the solid lines). The equivalence classes consist of the scenarios generated for decision problem D. It may be that there are more admissible scenarios that describe how the situation may evolve. In Figure 7.3 these can be in both, one of the sectors around $[S_1]_\sim$, $[S_2]_\sim$ and $[S_3]_\sim$ or between the outermost edges of the scenario trumpet representing all possible situation developments.

While the definition of maximum accepted difference η is equal for all classes, the number of scenarios captured in each class diverges considerably. Furthermore, in Figure 7.3, $V_i(FOCUS) = V_i(FOCUS, t)$ and $f_D = f_D(t)$ and $\eta = \eta(t)$, where t denotes the time that has passed since a time t_0. This reflects that both the value of focus variables and the evaluation of these values may vary in time. While the variation can occur due to more or changing information on the scenarios, the latter reflects a change in the decision-makers' preferences. In these situations, it is useful to adapt the parameter η can in parallel to the dissimilarity of the scenarios, which increases as time passes.

This thesis does not use an absolute notion of representativeness, but defines the concept of **relative representativeness.** The representativeness of an equivalence class is understood the share of reliability it represents compared to the overall representativeness of SS. If there is only one equivalence class $[S_i]_\sim$ representing *all* scenarios that have been built, its relative representativeness, denoted $Rep\left([S_i]_\sim\right)$, equals 1. Contrarily, if an equivalence class $[S_i]_\sim$ is empty, then $Rep\left([S_i]_\sim\right) = 0$.

Let $SSEC = \left\{[S_i]_\sim\right\}_{i\in I}$ be a set of scenario equivalence classes with $SS = \dot{\bigcup}_{i\in I}\left\{[S_i]_\sim\right\}$.[43] Without information on the reliability of each scenario, the relative representativeness of each equivalence class $[S_i]_\sim$ can be defined by

$$Rep\left([S_i]_\sim\right) = \frac{\left|[S_i]_\sim\right|}{|SS|} \tag{7.18}$$

Beyond this simple approach consider cases, where an assessment of each or some scenario's reliability can be made. The reliability of a scenario S_i has been defined (see equation 4.9) as

$$reliable_i\left(\mu,\nu\right) = \left(\left(\sum_{r=1}^{M}\omega_r^\mu \mu^r\text{-}corr_i\right)\left(1-\sum_{l=1}^{N}\omega_l^\nu \nu^l\text{-}ref_i\right)\right)^{\log\left(N_i^U + l_i^U\right)},$$

where μ and ν are the threshold vectors of corroboration and refutation of scenarios, N_i^U is the number of uncertain vertices, and l_i^U is the average number of uncertain vertices in any path from $SOURCE_i$ to $FOCUS_i$.

For the scenarios that are not focus complete, the set $IND_SS(FOCUS)$ is used as an *auxiliary focus*. To perform the respective reliability and representativeness assessments, let S_i be a focus incomplete scenarios with

$$\begin{aligned} V_i\left(tv_j^{SOURCE_i}\right) &\neq \infty\ \forall tv_j^{SOURCE_i} \in SOURCE_i \\ V_i\left(ind_l^k\right) &\neq \infty\ \forall \text{indicators} l \text{of} tv_k^F \in FOCUS_i. \end{aligned}$$

Additionally, for any path $P_{DI_i}\left(ind_l^k, tv_k\right) := \left\{\left(tv_1^P, tv_2^P\right), \ldots, \left(tv_{n-1}^P, tv_n^P\right)\right\}$, it holds $V_i\left(tv_r\right) = \infty$ for all $r \neq 1$. Lastly, it is required that S_i is **indicator complete,** i.e., for any path connecting the source to one of the indicators, all variables have been assigned at least one value.

[43]The union is required to be disjoint, and therefore $\left\{[S_{i_k}]_\sim\right\} \cap \left\{[S_{i_l}]_\sim\right\} = \varnothing\ \forall i_k \neq i_l$.

One denotes

$$\begin{aligned}
STV_i^{aux} &= \left\{tv_j \in STV_i : \exists P_{DI_i}^j \left(tv_j^{Source}, ind_l^k\right) : tv_j \in P_{DI_i}^j\right\},\\
sv_i^{aux} &= \left\{V_i\left(tv_j\right) : tv_j \in STV_i^{aux}\right\},\\
status_i^{aux} &= \left\{status_i\left(tv_j\right) : tv_j \in STV_i^{aux}\right\},\\
DI_i^{aux} &= DI_i \setminus \left\{STV_i \setminus STV_i^{aux}\right\} = DI_i\Big|_{STV_i^{aux}}.
\end{aligned}$$

The scenario S_i^{aux} is defined by

$$\begin{aligned}
S_i^{aux} &= \left\langle STV_i^{aux}, sv_i^{aux}, stauts_i^{aux}, DI_i^{aux}\right\rangle,\\
SOURCE_i^{aux} &= SOURCE_i,\\
FOCUS_i^{aux} &= \bigcup_{k \in \mathcal{K}} ind_l^k.
\end{aligned}$$

The reliability analysis of a focus incomplete scenario can be performed by analysing the reliability of the (focus complete) scenario S_i^{aux}.

Next, one defines the **relative reliability of a scenario equivalence class** as

$$reliable\left(\left[S_{i^*}\right]_\sim, \mu, \nu\right) = \frac{\sum_{S_i \in \left[S_{i^*}\right]_\sim} reliable_i\left(\mu, \nu\right)}{\sum_{S_i \in SS} reliable_i\left(\mu, \nu\right)}. \tag{7.19}$$

The relative reliability, which takes values in $[0, 1]$, is used as a *corrective factor* for determining the representativeness of a scenario equivalence class (cf. equation 7.18):

$$Rep\left(\left[S_i\right]_\sim, \mu, \nu\right) = reliable\left(\left[S_{i^*}\right]_\sim, \mu, \nu\right) \frac{\left|\left[S_i\right]_\sim\right|}{|SS|}. \tag{7.20}$$

As the assessment of representativeness is founded on the reliability of equivalence classes (cf. equation 7.19), it decreases for growing μ and ν. The relative representativeness $Rep\left(\left[S_i\right]_\sim, \mu, \nu\right)$ does therefore not only depend on the equivalence classes (and the parameters chosen to define them), but also on the preferences of the scenarios' recipients, for what they feel sufficiently certain for corroboration and sufficiently uncertain for the refutation of a scenario.

7.6. Inaccuracy of Scenario Equivalence Classes

The representativeness of a scenario equivalence class $[S_{i^*}]_\sim$ increases with growing parameter η: the larger η is, the more scenarios in SS fulfil the requirement that the distance from S_{i^*} is smaller or equal to η. Contrarily, the **accuracy,** i.e., the precision of description within $[S_{i^*}]_\sim$, *decreases,* as with a growing magnitude $\left|[S_{i^*}]_\sim\right|$, for each variable $tv_j \in \bigcup S_i \in [S_{i^*}]_\sim STV_i$, $SPV_{[S_{i^*}]_\sim}(tv_j) = \left\{V_i(tv_j) : S_i \in [S_{i^*}]_\sim \text{ and } tv_j \in STV_i\right\}$ is monotonically increasing (though not necessarily strictly). Although the equivalence class may have been constructed on basis of a scenario $S_{i^*} \in [S_{i^*}]_\sim$, the inaccuracy of the class must not depend on its representative.[44]

7.6.1. Inaccuracy of Focus Complete Scenario Equivalence Classes in SBR & SM

First, consider the case that all scenarios S_i $(i \in I(i^*))$ are focus complete.The **inaccuracy** of the scenario equivalence class $[S_{i^*}]_\sim$ with respect to $V(FOCUS)$ is defined as

$$\begin{aligned} & inacc^{V(FOCUS)}\left([S_{i^*}]_\sim\right) \\ & = \sum_{tv_k^F \in FOCUS} \left(\max_{\substack{i_k, i_l \in I(i^*) \\ i_k \neq i_l}} \left\{ \widetilde{dist}_k \left(V_{i_k}\left(tv_k^F\right), V_{i_l}\left(tv_k^F\right)\right)\right\}\right). \end{aligned} \tag{7.21}$$

For each variable $tv_k^F \in FOCUS$,

$$\max_{\substack{i_k, i_l \in I(i^*) \\ i_k \neq i_l}} \left\{ \widetilde{dist}_k \left(V_{i_k}\left(tv_j^F\right), V_{i_l}\left(tv_k^F\right)\right)\right\}$$

is called the **spread** of values in $[S_{i^*}]_\sim$.

The rationale is that the scenario equivalence class inaccuracy represents the *maximum possible deviation of values* of any two scenarios in $[S_{i^*}]_\sim$ with respect to each $tv_k^F \in FOCUS$. Thus, the inaccuracy is independent of the represen-

[44]The *representative* of an equivalence class is the one and only one element of each equivalence class to represent the class. In this manner, the class representative allows the equivalence classes to uniquely be identified.

tative chosen. Assuming that $[S_{i^*}]_\sim$ has been constructed as η-$V(FOCUS)$ equivalence class with representative S_{i^*}, it holds

$$\sum_{tv_j^F \in FOCUS} \underbrace{\left(\max_{\substack{i_k, i_l \in I(i^*) \\ i_k \neq i_l}} \left\{ \widetilde{dist}_k \left(V_{i_k} \left(tv_k^F \right), V_{i_l} \left(tv_k^F \right) \right) \right\} \right)}_{\leq 2 \cdot \eta} \leq |FOCUS| \cdot 2\eta.$$

Therefore, $|FOCUS| \cdot 2\eta$ is an upper bound for the inaccuracy of $[S_{i^*}]_\sim$.

7.6.2. Inaccuracy of Focus Complete Scenario Equivalence Classes in SBR & MCDA

In SBR & MCDA the evaluation of the focus variables can be used to determine the inaccuracy. Assuming that the attribute tree has M abstraction levels, the weights for each attribute tv_k^F at each abstraction level are denoted w_k^j ($k = 1, \ldots, N$, $j = 1, \ldots, M$). The total weight of attribute k is (see equation 7.16) $w_k^D = \prod_{j=1}^M w_k^j$. v_D is the vector-valued value function capturing the intra-criteria preferences. $v_{D,k}$ maps the value of each attribute tv_k^F to $[0, 1]$. Using these weights, one defines the inaccuracy with respect to the evaluation of $V(FOCUS)$ by:

$$\begin{aligned}
& inacc_{pref_D}^{V(FOCUS)} \left([S_{i^*}]_\sim \right) \\
= & \sum_{tv_k^F \in FOCUS} \left(\max_{\substack{i_k, i_l \in I(i^*) \\ i_k \neq i_l}} \left| w_k^D \cdot \left(v_{D,k} \left(V_{i_k} \left(tv_k^F \right), V_{i_l} \left(tv_k^F \right) \right) \right) \right| \right) \\
= & w_k^D \cdot |FOCUS| \sum_{tv_k^F \in FOCUS} \left(\max_{\substack{i_k, i_l \in I(i^*) \\ i_k \neq i_l}} \left| v_{D,k} \left(V_{i_k} \left(tv_k^F \right), V_{i_l} \left(tv_k^F \right) \right) \right| \right).
\end{aligned} \tag{7.22}$$

7.6.3. Inaccuracy of Focus Incomplete Scenario Equivalence Classes in SBR & SM

For determining the inaccuracy of focus incomplete scenarios, this thesis refers once more to the indicator framework introduced in Section 7.4.

$$IND_{SS}(FOCUS) = \left\{ ind_j^k \right\}_{\substack{j=1,\ldots \mu(k) \\ k=1,\ldots |FOCUS|}}$$

denotes the indicators chosen as basis for the similarity assessment. Particularly, $V_i\left(ind_j^k\right) \neq \infty \forall S_i \in SS$. The **indicator inaccuracy** of $[S_{i^*}]_\sim$ is defined by

$$
\begin{aligned}
& inacc^{V(IND)}\left([S_{i^*}]_\sim\right) \\
&= \sum_{\substack{tv_k^F \in \\ FOCUS}} \sum_{\substack{tv_j \in \\ IND(tv_k^F)}} \left(\max_{\substack{i_k, i_l \in I(i^*) \\ i_k \neq i_l}} \left\{ \widetilde{dist}_j \left(V_{i_k}\left(tv_j\right), V_{i_l}\left(tv_j^F\right)\right)\right\}\right).
\end{aligned} \tag{7.23}
$$

7.6.4. Inaccuracy of Focus Incomplete Scenario Equivalence Classes in SBR & MCDA

For determining the potential gap in the evaluations of the scenarios within on equivalence class, this thesis uses the evaluation of the (weighted) indicator value spread as a basis. Again, the evaluation framework used in the previous sections is adopted. Accordingly, the evaluation indicator inaccuracy of with respect to one attribute tv_k^F is defined by

$$
\begin{aligned}
& inacc_{Pref_D}^{V\left(IND\left(tv_k^F\right)\right)}\left([S_{i^*}]_\sim\right) \\
&= \left(\sum_{l=1}^{\mu(k)} \left(w_p\left(ind_j^k\right) \cdot \max_{\substack{i_k, i_l \in I(i^*) \\ i \neq j}} \widetilde{dist}_j \left(V_{i_k}\left(ind_j^k\right), V_{i_l}\left(ind_j^k\right)\right)\right)^p \right)^{\frac{1}{p}},
\end{aligned}
$$

where $IND\left(tv_k^F\right) = \left\{ind_1^k, \ldots, ind_{\mu(k)}^k\right\}$ denotes the $\mu(k)$ most important indicators for attribute tv_k^F. The **evaluation indicator inaccuracy** is:

$$
inacc_{Pref_D}^{V(IND)}\left([S_{i^*}]_\sim\right) = \sum_{k \in \mathcal{K}} w_k^D \cdot inacc_{Pref_D}^{V\left(IND\left(tv_k^F\right)\right)}\left([S_{i^*}]_\sim\right).
$$

The weights w_k^D are defined as in equation 7.16.

7.6.5. Relative Inaccuracy of Focus Incomplete Scenario Equivalence Classes

Determining the **relative inaccuracy** of each $[S_i]_\sim$ in a set of disjoint scenario equivalence classes $SSEC = \left\{[S_i]_\sim\right\}_{i \in I}$ facilitates comparisons of inaccuracies across classes. Again, the type of the equivalence relation varies according to the purpose and completeness of information: for SBR & SM $FOCUS$ and indicator equivalence classes are considered, whereas for SBR & MCDA evaluation equivalence classes are used.

When scenarios are used for sense-making, one defines the partition

$$SSEC^{SM} = SSEC^{SM}_{V(FOCUS)} \uplus SSEC^{SM}_{V(IND)},$$

where $SSEC^{SM}_{V(FOCUS)}$ contains scenario equivalence classes of focus complete scenarios in $SSEC^{SM}$, while $SSEC^{SM}_{V(IND)}$ consists of scenario equivalence classes for focus incomplete scenarios. The total inaccuracy is defined as

$$\begin{aligned} inacc\left(SSEC^{SM}\right) &= \sum_{[S_i]_\sim \in SSEC^{SM}_{V(FOCUS)}} inacc^{V(FOCUS)}\left([S_i]_\sim\right) \\ &\quad + \sum_{[S_i]_\sim \in SSEC^{SM}_{V(Ind(FOCUS))}} inacc^{V(IND)}\left([S_i]_\sim\right). \end{aligned}$$

Similarly, for decision-making (DM), first partition

$$SSEC^{DM} = SSEC^{DM}_{V(FOCUS)} \uplus SSEC^{DM}_{V(IND)},$$

where $SSEC^{DM}_{V(FOCUS)}$ contains scenario equivalence classes of focus complete scenarios in $SSEC^{DM}$, while $SSEC^{SM}_{V(IND)}$ consists of scenario equivalence classes for focus incomplete scenarios. The total inaccuracy, $inacc\left(SSEC^{DM}\right)$, is hence defined as:

$$\begin{aligned} inacc\left(SSEC^{DM}\right) &= \sum_{[S_i]_\sim \in SSEC^{SM}_{V(FOCUS)}} inacc^{V(FOCUS)}_{Pref_D}\left([S_i]_\sim\right) \\ &\quad + \sum_{[S_i]_\sim \in SSEC^{SM}_{V(Ind(FOCUS))}} inacc^{V(IND)}_{Pref_D}\left([S_i]_\sim\right). \end{aligned}$$

If the total inaccuracy is 0, the inaccuracy of each scenario equivalence class as well as the relative inaccuracy is 0. Otherwise, for each $[S_i]_\sim$, the **relative inaccuracy** is derived by dividing its inaccuracy by the total inaccuracy.

Sense-making: for $[S_i]_\sim \in SSEC^{SM}$ the relative inaccuracy of $[S_i]_\sim$ is

$$inacc^{rel}\left([S_i]_\sim\right) = \begin{cases} \frac{inacc^{V(FOCUS)}\left([S_i]_\sim\right)}{inacc\left(\mathrm{SSEC}^{SM}\right)} & \text{for } [S_i]_\sim \in SSEC^{SM}_{V(FOCUS)} \\ \frac{inacc^{V(IND)}\left([S_i]_\sim\right)}{inacc\left(\mathrm{SSEC}^{SM}\right)} & \text{for } [S_i]_\sim \in SSEC^{SM}_{V(IND)}. \end{cases}$$

Decision-making: for $[S_i]_\sim \in SSEC^{DM}$ the relative inaccuracy of $[S_i]_\sim$ is

$$inacc^{rel}\left([S_i]_\sim\right) = \begin{cases} \frac{inacc_{Pref_D}^{V(FOCUS)}([S_i]_\sim)}{inacc_{Pref_D}(\mathrm{SSEC}^{DM})} & \text{for } [S_i]_\sim \in SEC_{V(FOCUS)}^{DM} \\ \frac{inacc_{Pref_D}^{V(IND)}([S_i]_\sim)}{inacc_{Pref_D}\left(\mathrm{SSEC}_{Pref_D}^{DM}\right)} & \text{for } [S_i]_\sim \in SSEC_{V(IND)}^{DM}. \end{cases}$$

$inacc^{rel}\left(\left[S_i^{SM}\right]_\sim\right)$ and $inacc^{rel}\left(\left[S_i^{DM}\right]_\sim\right) \in [0,1]$. The **relative accuracy** of $\left[S_i^{SM}\right]_\sim$ and $\left[S_i^{DM}\right]_\sim$ are defined by:

$$\begin{array}{rrcl} & acc^{rel}\left(\left[S_i^{SM}\right]_\sim\right) & = & 1 - inacc^{rel}\left(\left[S_i^{SM}\right]_\sim\right) \\ \text{and} & acc^{rel}\left(\left[S_i^{DM}\right]_\sim\right) & = & 1 - inacc^{rel}\left(\left[S_i^{DM}\right]_\sim\right). \end{array}$$

7.7. Summary

This section has provided the basic concepts for scenario management. By defining scenario equivalence classes and by analysing these classes' representativeness and inaccuracy, this chapter makes headway in characterising, clustering and ultimately selecting the most relevant scenarios.

Firstly, different concepts of **similarity** according to the purpose at hand and the information available have been developed. These approaches are targeted at grouping together scenarios which are similar with respect to the information the scenario recipients judged relevant to their purpose, namely the $FOCUS$. Furthermore a projection from the multi-dimensional space of the focus variables' values to $[0,1]$ has been defined by adapting techniques from MCDA. As $[0,1]$ with $>$ is a totally ordered space, this projection facilitates comparison of scenarios and the definition of **scenario equivalence classes.** Each such class groups together scenarios that are judged sufficiently similar with respect to their evaluation, i.e., according to preferences of the decision-makers.

Secondly, the concepts of **representativeness** and **inaccuracy** of the scenario equivalence classes were defined. Both concepts allow the scenario selection procedures to be tailored such that they correspond to the scenario recipients' needs.

8. Scenario Management

> A theory should be as simple as possible,
> but no simpler.
> (Albert Einstein)

The management of SBR & SM and SBR & MCDA processes involves trade-offs and decisions in terms of desired properties and constraints that must be met. These include a good exploration of the realm of credible situation developments *(objective O.3)* as well as the manageability of the information processed and the number of scenarios that are built *(objective O.6)*.

Chapter 7 has provided a formal way to analyse the similarity among scenarios. This has lead to the definition of scenario equivalence classes, which form the basis for two important scenario management operations: *scenario selection* and *scenario pruning.* The first section of this chapter is dedicated to **scenario selection.** It presents the principles, via which to arrive at a prioritisation for both the generation of scenarios and the presentation of the resulting (focus complete) scenarios to the recipients.

The subsequent section describes **scenario pruning.** It provides a systematic approach for assessing which scenarios are to be discontinued (*pruned*) such that only those scenarios which are valid and acceptable for the scenarios' recipients remain. Beyond pruning scenarios that contain information that has been proven wrong (Section 8.2.1), Section 8.2.2 addresses pruning with respect to ambiguities and cycles that may arise in the scenario merging procedure (cf. Section 3.7).

This chapter concludes with a section on **scenario updating.** It addresses the problem of how to deal with newly available information. Basically, a decision must be made whether the information is important enough to justify a re-generation of scenarios or not. If the need for scenario updating is acknowledged, the update may lead to the construction of new scenarios, the selection of different scenarios for being passed on as well as to the pruning of scenarios that have become unacceptable.

8.1. Scenario Selection

Ideally, the set of scenarios covers all possible situation developments [Haimes et al., 2002; Kaplan, 1997; Kaplan and Garrick, 1981]. In large and complex problems it is, however, often impossible to identify all admissible (i.e., plausible, consistent and coherent) scenarios from $Range\left(V\left(tv_1\right)\right)\times\ldots\times Range\left(V\left(tv_\Sigma\right)\right)$, where $\Sigma = \left|\bigcup_{S_i \in SS} STV_i\right|$, as complex problems preclude the description of interdependencies between the variables by a unique model (e.g., a system of equations). Particularly, no simple filtering approach (e.g., specification of boundary conditions) for identifying the admissible scenarios can be applied.

Yet, an important part of scenario management is controlling the potential combinatorial explosion of the number of scenarios arising. The concept of scenario selection developed in this section is based on the same idea as branch-and-bound algorithms [Lawler and Wood, 1966]: it uses the same metaphor of a tree-like structure (or forest of trees), in which a multitude of branches start, but early the most relevant branches are selected and extended further, possibly spreading into a set of new scenarios via extension (branching), while the others are cut at the status quo and not developed further (bounding)[45].

In this thesis, an approach for generating a finite set of admissible scenarios relying on (local) expertise has been developed (cf. Section 5.2). Yet, the number of scenarios arising grows exponentially with the number of uncertain variables and the values per uncertain variable. To avoid information overload and too high workload, approaches to filter the most relevant scenarios need to be implemented.

To operationalise the notion of **relevance,** several concepts that have been developed within this thesis are used. Particularly, it is important that the scenarios cover the set of admissible scenarios as good as possible. Therefore, the need for scenario equivalence classes grouping together sufficiently similar scenarios arises (cf. Section 7.3). The use of single scenarios representing each class (vs. using sets of scenarios) is supported by investigations on the perception of risk that confirmed a considerable difference in the decision attitude when ac-

[45]As these (focus incomplete) scenarios may be judged more relevant when further information is available, they are *not* pruned.

tual plausible consequences of the decision were presented instead of a more vague aggregated description [Benartzi and Thaler, 1999].

8.1.1. Determining the Number of Scenarios to be Selected

The maximum number of states that the expert determining the value of tv_j is allowed to pass on to further experts in DAG_D is denoted λ_j. The choice of λ_j is a compromise between uncertainty and ambiguity. This thesis proposes a new distributed approach to determine the number of scenario equivalence classes to be passed on: each expert responsible for determining the value of a variable tv_j asked to limit the multiplicity of the values of their output to a number λ_j.

The bound λ_j is determined by the experts responsible for $tv_{j_1}, \ldots, tv_{j_N} \in \widetilde{\Theta}(tv_j)$, i.e., by the experts responsible for assessing the variables, which are directly influenced by tv_j's value. Given a certain time T[46] they are allowed to spend for determining the values of the variables tv_{j_k}, these experts indicate in a number $\lambda_{j_k} = \lambda_{j_k}(T)$ how many different values tv_j they can handle within a given time T.

To facilitate this process, each expert should take into account the total number of (focus incomplete) scenarios $SS_i \subseteq SS$, he needs to consider. If for $tv_{j_k} \in \widetilde{\Theta}(tv_j)$ it holds

$$\left|\left\{tv_l \in \widetilde{\Psi}(tv_{j_k}) \cap U_{SS_i}\right\}\right| =: u_{j_k} > 1$$

(cf. equation 3.1 for a definition[47] of U_i), an upper bound for the number of partial scenarios to be processed in tv_{j_k} is $n_{j_k} = \prod_{l=1}^{u_{j_k}} \lambda_l$.

A straightforward way to determine λ_{j_k} from n_{j_k}, is choosing equal values, i.e., $\lambda_{j_k} = \left\lfloor \frac{b_{j_k}}{u_{j_k}} \right\rfloor$, where b_{j_k} is the maximum number of scenarios that can be processed given time constraints T. As this approach does *not* reflect the importance of considering a multiplicity of scenarios for each $tv_l \in \widetilde{\Psi}(tv_{j_k})$, ideally, each expert should take into account the impact that a change of the state of a node $j \in \widetilde{\Psi}(tv_{j_k}) \cap U_{SS_i}$ has on his output. If an extended context Con_d is avail-

[46] In general, T depends on the time available for the decision problem as well as on the complexity of the problem and the size of the network DAG_D, see Section 8.4.

[47] Here, U_{SS_i} is defined for the set of scenarios $SS_i \subseteq SS$ as $\bigcup_{S_i \in SS_i} U_i$. Furthermore, as $\widetilde{\Theta}$ depends on the structure of each $S_i \in SS_i$, one defines $\widetilde{\Theta}(tv_j) = \widetilde{\Theta}_{SS_i}(tv_j) = \bigcup_{S_i \in SS_i} \widetilde{\Theta}_i(tv_j)$, where $\widetilde{\Theta}_i(tv_j)$ is defined by the dependencies of DI_i.

able that contains a sensitivity assessment for the tv_{j_k} (cf. Section 6.2.2.2), this assessment can be exploited here: if the value $V(tv_{j_k})$ is sensitive to changes in $V(tv_l)$, λ_l should be larger than in case of robust results. Possibly, the experts can also specify directions of sensitivity (e.g., sensitive to increase, but robust for decrease). For automated systems, this assessment can be done via sensitivity analyses. Human experts can also use qualitative assessments based on their experience (e.g., *robust* or sensitive).

Having identified in this manner values $1 \le \lambda_{j_k}$ for all $tv_{j_k} \in \widetilde{\Theta}(tv_j)$, one sets

$$\lambda_j = \min_{tv_{j_k} \in \widetilde{\Theta}(tv_j)} \{\lambda_{j_k}\}. \tag{8.1}$$

If for a variable tv_j, $SPV_j(SS_i) > \lambda_j$, a mechanism for selecting the most relevant values (and ultimately, the most relevant scenarios) must be implemented.

8.1.2. Review of Scenario Selection Procedures

In literature, scenario selection procedures have only been discussed for formative approaches to SBR, as in the discursive SBR approaches, the number of scenarios arising is naturally limited by the the actors themselves and the shortage of time and resources: in discursive scenario planning, just a small set of scenarios, each of which is assigned a certain leitmotiv, is constructed [Schoemaker, 1993, 1995; O'Brien, 2004].

In formative scenario analysis, it is suggested to perform a cluster analysis combined with filtering techniques to select the most distinct sufficiently consistent scenarios from an exhaustive the set of scenarios [Götze and Henselmann, 2001; Spielmann et al., 2005; Tietje, 2005]. The notion of *distance* used is based on the distance of variables' values. In terms of the scenario formalisation in this thesis, this means that these approaches aim at selecting scenarios that explore the space of possible values for each variable $Range(V(tv_1)) \times \ldots \times Range(SPV(tv_\Sigma))$ as good as possible.

This approach has several drawbacks. First, comparisons of scenarios that do not share the same set of variables can not be made. Second, it does not guarantee that the scenarios which explore the space of focus variables best are chosen. To illustrate the second point, consider the following example from emergency management, where only the variables, for which the scenarios have different values, are specified in table 8.1.

Table 8.1.: *The Concept of Scenario Distance in Formative Scenario Analysis: An Example Application*

Scenario	Leak Size	Amount of chemical released	Weather situation	Presence of population	Number of children affected
S_1	small	medium	stable	average	0
S_2	medium	small	unstable	pessimistic	50
S_3	medium	small	unstable	pessimistic	500

In formative scenario analysis, it is suggested to use the *discrete metric* for each variable as a distance measure [Tietje, 2005]. Therefore, the the overall distance of two scenarios S_{i_k}, S_{i_l} is

$$dist_{FSA}\left(S_{i_k}, S_{i_l}\right) = \sum_{tv_j \in STV_{i_k} \cap STV_{i_l}} \mathbb{1}_{V_{i_k}(tv_j)} V_{i_l}\left(tv_j\right),$$

where $\mathbb{1}_y(x) = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{otherwise} \end{cases}$. The distance of the scenarios in table 8.1 is

$$\begin{aligned} & dist_{FSA}\left(S_1, S_2\right) &=& \; 5, \\ & dist_{FSA}\left(S_1, S_3\right) &=& \; 3 \\ \text{and} \quad & dist_{FSA}\left(S_2, S_3\right) &=& \; 4. \end{aligned}$$

If only two of the three scenarios S_1, S_2 and S_3 are allowed to be passed on, one would therefore select S_1 and S_2, as this choice maximizes the overall distance $dist_{FSA}$. However, S_1 and S_2 are rather similar with respect to the only focus variable tv_j^F present (*Number of children affected*). assuming that the metrics chosen on the latter variable is $|\ .\ |$, it holds

$$\begin{aligned} & dist\left(V_1\left(tv_j^F\right), V_2\left(tv_j^F\right)\right) &=& \; 50, \\ & dist\left(V_1\left(tv_j^F\right), V_3\left(tv_j^F\right)\right) &=& \; 500 \\ \text{and} \quad & dist\left(V_2\left(tv_j^F\right), V_3\left(tv_j^F\right)\right) &=& \; 450. \end{aligned}$$

Therefore, presenting S_1 and S_3 to the decision-makers would maximize the distance of scenarios with respect to the focus variables. Keeping in mind that the focus variables represent the factors the most relevant to the decision-makers, this thesis presumes that a broad variety of values in $FOCUS$ is most important to the decision-makers. Furthermore, the experts performing the scenario selection need to have knowledge about all realisations of all variables throughout all scenarios increasing their workload.

To overcome these drawbacks, this thesis proposes new and systematic approaches for **global** and **local scenario selection.** The first approach suitable to both, global and local selection, is based on the **dissimilarity** of the focus variables' values for SBR & SM and the difference of evaluations for SBR & MCDA. In fact, the idea is to use the evaluation function as a projection to the space $\mathbb{R}$, where a simple metrics (such as $|\ .\ |$) allows the scenarios that are the most distinct to be chosen. This approach ensures that the dissimilarity is measured with respect the variables that are considered the most important by the scenario recipients. By using ratio and not nominal scales problems such as measurement errors or imprecise judgements are avoided. The second approach facilitates **selecting scenarios locally** without any information on the values of further variables within the scenarios. This approach is useful in situations, when time is critical, or information is so sparse that an indicator framework for local scenario assessment can not be derived.

8.1.3. Scenario Selection for Focus Complete Scenarios: Global Scenario Selection

In both, SBR and SBR & MCDA, the first step in the scenario selection is the construction of λ scenario equivalence classes $SSEC = \left\{[S_i]_\sim\right\}_{i=1,\dots,\lambda}$. To this end, the parameters η, μ and ν are chosen such that each equivalence class has at least a certain minimum **representativeness** $Rep^{\min}$ (cf. Section 7.5), i.e.,

$$Rep\left([S_i]_\sim, \mu, \nu\right) \geq Rep^{\min}\ \forall\ [S_i]_\sim \in SSEC.$$

Furthermore, it is required that each $[S_i]_\sim$ does not exceed a certain maximum inaccuracy $inacc^{\max}$ (cf. Section 7.6 for a definition of inaccuracy for SBR and SBR & MCDA problems)

$$inacc\left([S_i]_\sim\right) \leq inacc^{\max} \forall\ [S_i]_\sim \in SSEC.$$

For further details on how to construct the equivalence classes using *Clustering techniques* see [Groves and Lempert, 2007; Bryant and Lempert, 2010].

Then, one chooses a representative S_{i^*} of each class such that

- in SBR & SM

$$\min_{\substack{S_{i^*} \in [S_i]_\sim \\ S_j \notin [S_i]_\sim}} \sum_{tv_k^F \in FOCUS} dist_k\left(V_{i^*}\left(tv_k^F\right), V_{i^*}\left(tv_k^F\right)\right);$$

- in SBR & MCDA

$$\min_{\substack{S_{i^*} \in [S_i]_\sim \\ S_j \notin [S_i]_\sim}} \left| f_D\left(V_{i^*}\left(FOCUS\right)\right) - f_D\left(V_j\left(FOCUS\right)\right)\right|$$

is maximised

8.1.4. Scenario Selection for Focus Incomplete Scenarios: An Indicator-Based Approach

For local scenario selection, a similar technique is used. Yet, the basis for the construction of scenario equivalence classes, are the equivalence classes based on **indicator similarity of scenarios.**

This novel approach allows the most relevant scenarios with respect to the decision to be made to be selected *before* having determined the actual values of the focus variables. To this end, λ_j equivalence classes as described in Section 7.4 are developed. Again, it is required that these classes do fulfil a certain minimum representativeness $Rep^{\min}$ and do not exceed a certain maximum inaccuracy $inacc^{\max}$ (analogue to the approach for global scenario selection, see Section 8.1.3).

Finally, a representative S_{i^*} of each class is chosen such that

$$\min_{\substack{S_{i^*}\in[S_i]_\sim \\ S_j\notin[S_i]_\sim}} \left(\sum_{\substack{tv_k^F\in \\ FOCUS}} \left(w_k \sum_{\substack{ind_j^k\in \\ IND\left(tv_k^F\right)}} dist_k\left(V_{i^*}\left(ind_j^k\right)\right), V_{i^*}\left(ind_k\right) \right)^p \right)^{\frac{1}{p}} \to \max!$$

The arising set of scenarios SS_{i^*} is then used as a basis for further scenario generation.

Further constraints for both the building of scenario equivalence classes and the selection of representatives can be that all values within $INIT$ must be respected and that the set of scenarios selected is β-balanced with respect to one or more variables tv_j. To ensure that the **initial situation** is respected completely, for each $tv_j^{init} \in INIT$, and each value $V_k\left(tv_j^{init}\right)$ in $SPV\left(tv_j^{init}\right) = \left\{V_1\left(tv_j^{init}\right), \ldots, V_N\left(tv_j^{init}\right)\right\}$ it can be required that there is at least one scenario $S_i \in SS_{i^*}$ with $V_i\left(tv_j^{init}\right) = V_k\left(tv_j^{init}\right)$. Naturally, this is only feasible if $\prod_{tv_j^{init}\in INIT}\left|SPV\left(tv_j^{init}\right)\right| \leq \lambda_j$. The guarantee of the β-**balance** is modelled as a further boundary condition in the clustering process.

The **weights** w_k can be chosen by the scenario recipients. In SBR & MCDA they can also be derived from the preferences of the decision-makers for the decision problem D (see equation 7.16). The default value in SBR & SM is $w_k = \frac{1}{|FOCUS|}$.

8.1.5. Scenario Selection for Focus Incomplete Scenarios: A Status-Based Approach

When it is not possible to determine scenario equivalence classes on the basis of the indicator framework (e.g., as there are too few indicators available, or as the time for an in-depth assessment is too short), a mechanism for *locally* selecting the values to be passed can be implemented. This novel mechanism is designed to ensure that the set of focus incomplete scenarios arising from this procedure, SS_j, explores the set possible developments as good as possible and contains the most likely ones.

Naturally, λ_j is an upper limit of $|SS_j|$, and each expert is free to provide $\tilde{\lambda}_j < \lambda_j$ assessments, if these reflect sufficiently the possible developments. Still, it is useful to develop some general guidelines that can be implemented easily

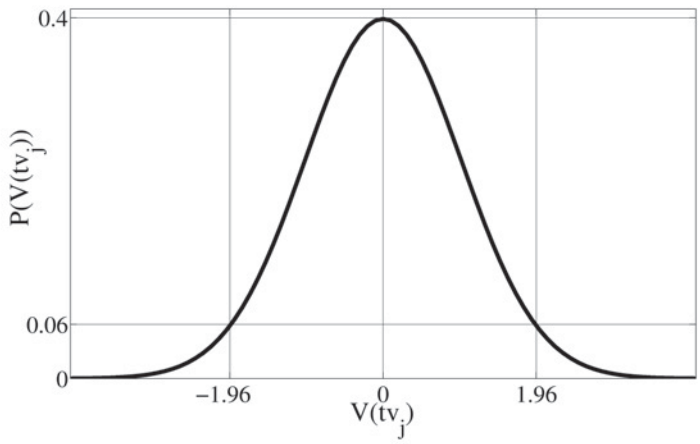


Figure 8.1.: *Selection of Values for a Standard Gaussian Variable tv_j. Example with $\lambda_j = 3$, quantiles $\alpha = 1 - \beta = 0.025$. $V^{\alpha}(tv_j) = -1.96$, $V^{\max}(tv_j) = 0$ and $V^{\beta}(tv_j) = 1.96$.*

for the involved automated systems. These ensure the consistency of the selection and can be adapted to the decision-makers' requirements (inter alia the *minimum required reliability of each piece of information*). Assume that $\lambda_j \geq 2$. Let S_i be the scenario, which is extended in tv_j (by adding a set of possible values and statuses for tv_j), i.e., $V_i(tv_j) = \infty$ and $V_i(tv_j) =$ "*not assessed*".

According to the type of present uncertainty and the paradigm for reasoning under uncertainty chosen by the expert responsible for tv_j (cf. Section 3.2.3), a number of cases need to be distinguished. As the problem of local scenario selection occurs only for $\left|SPV_j^{\mathfrak{C}_i}\right| > 1$, the deterministic case can be excluded.

Probabilistic: if for all $S_{j_k} \in SS_j$, $status_{j_k}(tv_j) =$"*uncertain-B*", **quantiles** are used to determine $SPV_{SS_j}(tv_j) = \{V_{j_1}(tv_j), \ldots, V_{j_\lambda}(tv_j)\}$. The values $V_j^{\alpha_l}(tv_j)$ and $V_j^{\beta_l}(tv_j)$ that correspond to α_l- and β_m-quantiles are passed on. Assuming that $V(tv_j)$ is a normed space, the probability that a scenario generates a score that is lower than $V_j^{\alpha_l}(tv_j)$ $(l = 1, \ldots, L)$ (higher than $V_j^{\beta_m}(tv_j)$, $(m = 1, \ldots, M$, where $L + M < \lambda_j))$ is smaller or equal than α_l $(1 - \beta_m)$. The choice of α_l and β_m (particularly for the extreme states x_j^1 and $x_j^{\lambda_j}$) depends on the minimum acceptable likelihood. But as it is advised not to restrict too much the possible scenarios by likelihood considerations [Schoemaker, 1993], α_1 and β_M should be chosen such that they are as close to 0 resp. 1 as acceptable. For each decision problem D these values are stored in the according backdrop $Back_D$.

In addition, the $\lambda_j - (L + M)$ most probable state(s) are selected. For instance, for $\lambda_j - (L + M) = 1$ one selects $V_j^{\max}(tv_j) = \arg\max p_j(V(tv_j))$, where $p_j(V(tv_j))$ is the (conditional) probability distribution of $V(tv_j)$, see Figure 8.1. If $V_j^{\max}(tv_j)$ is not unique, the state that is closest to the mean is selected.

Fuzzy: if for all $S_{j_k} \in SS_j$ $status_{j_k}(tv_j)$ ="*uncertain-F*", a similar approach making use of the underlying fuzzy membership function μ_j is implemented. Firstly, the scenario recipients define a threshold of the minimal acceptable membership $\gamma^{\min}$. This value is stored in $Back_D$. Secondly, analogue to the approach in the probabilistic case, the *extreme states* $V_j^{\alpha_1}(tv_j)$ and $V_j^{\beta_M}(tv_j)$ can be derived by considering the $\gamma^{\min}$-cut (cf. Section 2.2.2.3) and choosing

$$\begin{aligned} V_j^{\alpha_1}(tv_j) &= \inf\left\{V(tv_j) : \mu_j(V(tv_j)) \geq \gamma^{\min}\right\}, \\ \text{and} \quad V_j^{\beta_M}(tv_j) &= \sup\left\{V(tv_j) : \mu_j(V(tv_j)) \geq \gamma^{\min}\right\}. \end{aligned}$$

Further thresholds $\gamma_{\alpha_2}, \ldots, \gamma_{\alpha_L}$ and $\gamma_{\beta_2}, \ldots, \gamma_{\beta_M}$ $(L + M < \lambda_j)$ can be used to explore a broader variety of scenarios.

Additionally, the *mean* of the kernel[48] is used. If $\lambda_j - (M + L) > 1$ (i.e., the reliability of the prognosis is important), further states with $\mu_j(V(tv_j)) = 1$ that explore the kernel are passed on.

Severe uncertainty: if $status_{j_k}(tv_j)$ ="*uncertain-L*" for all $S_{j_k} \in SS_j$, the likelihood of any $V(tv_j)$ *cannot* be assessed. In this case, the *extreme scores* (i.e., $\inf\left(SPV_j^{\mathfrak{C}_i}\right)$ and $\sup\left(SPV_j^{\mathfrak{C}_i}\right)$) as well the score closest to the *median* can be selected. Alternatively, in case the expert providing the values of tv_j is human (as opposed to an automated system), he can be asked to select the values which are the most likely and relevant to him.

By using the scores determined as described above new (plausible and consistent) partial scenarios S_{j_i} $(i = 1, \ldots, \lambda_j)$ are generated from S_i whilst respecting the processing capacity of each expert and constraints in time.

8.2. Scenario Pruning

While scenario selection identifies and selects the most relevant scenarios, scenario pruning aims at removing scenarios that are not valid or acceptable for the scenario recipients. Therefore, scenario pruning reduces the number of

[48] The kernel is the region of maximum membership. In this framework, that means that for a fuzzy set $\tilde{A}$ characterised by a membership function $\mu_A : Type_j \to [0, 1]$, $\ker\left(\tilde{A}\right) = \{V(tv_j) : \mu_A(V(tv_j)) = 1\}$, cf. Section 2.2.2.3.

scenarios considered, while scenario selection does not. Scenarios that violate well-definedness, plausibility and consistency requirements (see Section 3.3.1 for the well-definedness, Section 3.3.3 for scenario plausibility and consistency and Section 4.3.1 for consistency of sets of scenarios) are pruned. This concerns both observation- and value-consistency. While the first refers to situations when scenarios prove to be erroneous, the latter can arise during the scenario merging procedure, when two scenarios that were built independently from each other are combined.

8.2.1. Pruning Irreliable Scenarios

The approach presented to pruning observation-inconsistent scenarios corresponds to the definition of the ν^l-refutation (see Section 4.3.4, particularly, equation 4.7): a scenario S_i is pruned from the set of scenarios SS, if there is a $tv_j \in STV_i$, for which an upper bound of the likelihood of $V_i\left(tv_j\right)$, denoted $\pi^r\left(V_i\left(tv_j\right)\right)$ can be assessed and falls below a certain threshold value $0 \leq \varepsilon \leq 1$. The threshold ε assigns the **minimum required likelihood** for each variable's value.

Although a high value ε reduces the set of scenarios to a greater extent, ε should be chosen carefully. The selection of ε makes the trade-off between exploring a wide range of scenarios and credibility considerations transparent and explicit. In general, scenarios are a means to stimulate people to evaluate and reassess their beliefs about the system [Brewer, 2007; Greeuw et al., 2000]. It has been argued that useful scenarios incorporate imaginative speculation and a wide range of possibilities [Peterson et al., 2003; van der Heijden, 2007]. Therefore, scenarios should not be restricted too much by likelihood considerations. Yet, the users should not be forced to consider too extreme scenarios that lack credibility [Schoemaker, 1993]. The believability of a single scenario may be undermined due to the low perceived probabilities associated with any single event or with the unlikely conjunction of events. While the first refers to the low likelihood that a variable tv_j takes a value $V_i\left(tv_j\right)$, the latter refers to a situation, when it is highly unlikely that a combination of values $\left(V_i\left(tv_{j_1}\right), \ldots, V_i\left(tv_{j_N}\right)\right)$, $tv_{j_1}, \ldots, tv_{j_N} \in STV_i$ occurs at a time. Summarised, ε should be chosen as close to 0 as acceptable to the decision-makers.

Firstly, for all $tv_j \in STV_i$, it is checked if $status_i\left(tv_j\right)$ allows **upper bounds for the probability** that tv_j takes $V_i\left(tv_j\right)$ to be determine. This bound is denoted $\pi^r(V_i(tvj))$. For those $tv_j \in STV_i$, for which such an upper bound *cannot* be determined, one sets $\pi^r(V_i(tvj)) = 1$.[49]

Secondly, the set of variables that are the basis for the pruning procedure is determined by:

$$STV_i \supseteq SPPTV_i = \{tv_j \in STV_i : \pi^r(V_i(tv_j)) < 1\}.$$

Thirdly, **an upper bound of each path's likelihood** is assessed. One denotes $P_{DI_i}\left(tv_k^{SOURCE}, tv_l^F\right) =: P_{k,l}$ from $tv_k^{SOURCE} \in SOURCE$ to $tv_l^F \in FOCUS$. To assess $P_{k,l}$'s likelihood, the set of vertices it passes is determined:

$$\begin{aligned} STV_i(P_{k,l}) \quad = \quad & \Big\{tv_j, tv_{j+1} \in STV_i : \exists P_{DI_i}\left(tv_k^{SOURCE}, tv_l^F\right) : \\ & e_j = (tv_j, tv_{j+1}) \in P_{DI_i}\left(tv_k^{SOURCE}, tv_l^F\right)\Big\}. \end{aligned}$$

On the basis of $STV_i(P_{k,l})$ an upper bound for the likelihood of

$$V(STV_i(P_{k,l})) = V_i(STV_i(P_{k,l}))$$

can be determined by considering the set

$$\left\{tv_j^{\pi^r_{k,l}}\right\}_{j=1,\ldots,M} = STV_i(P_{k,l}) \cap SPPTV_i.$$

For $\left\{tv_j^{\pi^r_{k,l}}\right\}_{j=1,\ldots,M}$ one estimates:

$$\begin{aligned} & \pi^r(V_i(P_{k,l})) \\ \leq \quad & \pi^r\left(V_i\left(tv_1^{\pi^r_{k,l}}\right), \ldots, V_i\left(tv_M^{\pi^r_{k,l}}\right)\right) \\ \leq \quad & \prod_{j=1}^{M} \pi^r\left(V_i\left(tv_j^{\pi^r_{k,l}}\right)\right). \end{aligned}$$

These considerations lead to the following **pruning function:**

$$prune(S_i) = \begin{cases} 1, & \text{if } \exists P_{k,l} : \prod_{j=1}^{M} \pi^r\left(V_i\left(tv_j^{\pi^r_{k,l}}\right)\right) \leq \varepsilon \\ 0, & \text{otherwise.} \end{cases} \tag{8.2}$$

[49] If the underlying distribution of upper probabilities is continuous, it can be discretised by using scenario equivalence classes, and by allowing a deviation of the variables' value that results in a total impact smaller or equal to h on the focus variables (in SBR) or on the evaluation (in SBR & MCDA).

If $prune(S_i) = prune(S_i, \varepsilon) = 1$, S_i is deemed unacceptable (according to the threshold ε) and pruned. This approach prunes scenarios, for which the value of a *single* variable is deemed not credible, and scenarios, for which a the *conjunction* of events is not acceptable.

The pruning function provides also **guidelines for the scenario generation** if the uncertainty can be quantified by means of upper and lower bounds to the probability: the values the experts assign to the variables must have an upper bound of probability greater than ε. Yet, pruning can also occur, when a scenario S_i contains a path that connects uncertain nodes $tv_1, \ldots, tv_M$, for which $\prod_{j=1}^{M} \pi^r(V_i(tv_j)) < \varepsilon$.

Furthermore, pruning can occur when *new information* is available. This may happen for two reasons: first, the information on a variable tv_j's value that was uncertain can become certain and available (e.g., as some measurements were taken). Second, information on variables, on which tv_j depends may change causing a change of the status of tv_j. In both cases, the pruning function is reassessed for all $S_i \in SS$ with $tv_j \in STV_i$. To assess the pruning function (see equation 8.2) only paths, which traverse tv_j need to be considered. If $prune(S_i) = 1$, S_i is deleted from the set of scenarios SS, and the new set $\widetilde{SS} = SS \setminus S_i$ is used as the basis for further scenario generation, sense- or decision-making.

8.2.2. Pruning Ambiguous and Cyclic Scenarios Arising in Scenario Merging

The scenario merging procedure described in Section 3.7 facilitates distributed scenario construction: partial scenario sets SS_1 and SS_2 can be developed independently from each other until the value of a variable tv^*, for which the expert needs information from both SS_1 and SS_2, needs to be determined (cf. Figure 8.2(a), where SS_1 and SS_2 need to be merged to determine the value of tv_8).

For the set of merged scenarios SMS structural problems of **ambiguity** and **cyclicity** can occur when some (partial) scenarios in $S_{i_1} \in SS_1$ and $S_{j_2} \in SS_2$ share some variables $tv_k \in STV_{i_1} \cap STV_{j_2}$. To deal with these problems, recall that the merged scenarios $S_{i_1,j_2} \in SMS$ are classified to the following sets (cf. Section 3.7):

- $VAS_{N(VA)}$ contains all $S_i \in SMS$ that are value ambiguous of degree $N(VA)$.
- $SAS_{N(SA)}$ contains all $S_i \in SMS$ that are status ambiguous of degree $N(SA)$.
- $CS_{N(Cyc)}$ contains all $S_i \in SMS$ that are cyclic of degree $N(Cyc)$.

By assuming that whenever (value or status) ambiguities in a set $ASTV$ occur, two (or more) different concepts $\mathfrak{C}_i\,(tv_k)$ for determining the value of each $tv_k \in ASTV$ were used[50], the simplest approach to resolve ambiguity and cyclicity is to determine which **concept** is favourable. That means, the recipients are asked to choose the experts that the scenario recipients deem the most trustworthy). A prerequisite for this approach is the conceptualisation of each scenario.

If $|STV_i|$ is large and numerous experts participate in the scenario construction this simple approach is hardly feasible. Furthermore, this approach requires in-depth knowledge of the scenarios' recipients about all available concepts and their quality. Therefore, in the following a different approach related to **structural issues** is proposed. Nevertheless, if for some variables, the scenarios' recipients specify a preferred source of information, it is recommended using this concept, as it enhances acceptance and trust of the scenarios used.

The formal scenario pruning procedure, which filters and passes on the scenarios that are acceptable to the recipients, is implemented on the basis of the **degree of value** and **status ambiguity,** and the degree of **cyclicity.** Ideally, the scenarios passed on are value- and status unambiguous and acyclic (see Section 3.3.1). There may, however, be situations, when most scenarios in SMS show some degree of ambiguity, and for reasons of time restrictions, also ambiguous scenarios need to be passed on. Furthermore, there may be problems such as measurement errors (i.e., a minor deviation in the values of a variable) leading to ambiguity. Therefore, the degree of value ambiguity needs to be complemented with an assessment of how severe this ambiguity is. This is achieved by using the *similarity framework* developed in Chapter 7.

[50] $\mathfrak{C}_l\,(tv_k)$ represents the expert(s) or the tools and algorithms these expert(s) apply to determine the value (and status) for the variable tv_k in scenario S_i (see Section 4.1.6 for the definition of concepts).

8.2.2.1. Value Ambiguity

To begin with, two methods are presented that enable the value ambiguities to be **resolved** if certain conditions are met, namely an assessment of value similarity and the definition of a set of favoured concepts. If this is not possible, the pruning function is used to determine if the scenario is acceptable.

One denotes S_{i_1} and S_{j_2} the scenarios that have been merged to the scenario $S_{i,j} \in SMS$. $VATV := \left\{tv_1^{i,j}, \ldots, tv_{N(VA)}^{i,j}\right\}$ is the set of values, for which $V_{i_1}\left(tv_k^{i,j}\right) \neq V_{j_2}\left(tv_k^{i,j}\right)$. To analyse if the ambiguities can be resolved, a **similarity assessment** is performed for all $tv_k^{i,j} \in VATV$. Let $\left(Type\left(tv_k^{i,j}\right), dist_k\right)$ be a metric space. The first step in the assessment is checking if there are scenarios $S_{i_1^*} \in SS_1$, $S_{j_2^*} \in SS_2$ that are more appropriate for merging. One denotes

$$\begin{aligned}
SS_{i_1} &= \Big\{S_m \in SS_1 : \Psi_{i_1}\left(tv_k^{i,j}\right) \subset STV_m \\
&\qquad \text{and } V_m\left(tv_l\right) = V_i\left(tv_l\right) \forall tv_l \in \Psi_{i_1}\left(tv_k^{i,j}\right)\Big\}, \\
SS_{j_2} &= \Big\{S_n \in SS_2 : \Psi_{j_2}\left(tv_k^{i,j}\right) \subset STV_n \\
&\qquad \text{and } V_n\left(tv_l\right) = V_j\left(tv_l\right) \forall tv_l \in \Psi_{j_2}\left(tv_k^{i,j}\right)\Big\},
\end{aligned}$$

where $\Psi_{i_1}\left(tv_k^{i,j}\right)$ is the set of (direct and indirect) predecessors of $tv_k^{i,j}$ in S_{i_1} and $\Psi_{j_2}\left(tv_k^{i,j}\right)$ is the set of (direct and indirect) predecessors of $tv_k^{i,j}$ in S_{j_2}. If there is a scenarios $S_{i_1^*} \in SS_{i_1}$ with

$$dist_k\left(V_{i^*}\left(tv_k^{i,j}\right), V_{i^*}\left(tv_k^{i,j}\right)\right) \leq dist_k\left(V_{i_1}\left(tv_k^{i,j}\right), V_{i^*}\left(tv_k^{i,j}\right)\right),$$

$S_{i_1^*}$ is considered more suitable for merging with S_{j_2}, and $S_{i,j}$ is pruned.

If there is no such scenario $S_{i_1^*}$, for all $tv_k^{i,j}$ in $VATV$ it is analysed whether the values can be reconciled. This is the case if all valued successor vertices of $tv_k^{i,j}$, i.e., the variables tv_l^{k+} within $\Theta\left(tv_k^{i,j}\right)$ are value consistent with both $V_{i_1}\left(tv_k^{i,j}\right)$ and $V_{j_2}\left(tv_k^{i,j}\right)$. Then, $V_{i_1}\left(tv_k^{i,j}\right)$ and $V_{j_2}\left(tv_k^{i,j}\right)$ are **value consistency similar.** Either value can be used, while the other value is pruned.

To determine whether $V_{i_1}\left(tv_k^{i,j}\right)$ and $V_{j_2}\left(tv_k^{i,j}\right)$ are value consistency similar, a possible approach is to ask the experts that provided the values of $tv_l^{k+} \in \Theta_{i_1}\left(tv_k^{i,j}\right)$ and $\Theta_{j_2}\left(tv_k^{i,j}\right)$ if the value they provided is still valid under the assumption that $V\left(tv_k^{i,j}\right) = V_{j_2}\left(tv_k^{i,j}\right)$ (for experts having contributed to S_{i_1}) or $V\left(tv_k^{i,j}\right) = V_{i_1}\left(tv_k^{i,j}\right)$ (for experts having contributed to S_{j_2}). If all experts agree, $V_{i_1}\left(tv_k^{i,j}\right)$ and $V_{j_2}\left(tv_k^{i,j}\right)$ are value consistency similar.

If $\left|\Theta_{i_1}\left(tv_k^{i,j}\right)\right|$ or $\left|\Theta_{j_2}\left(tv_k^{i,j}\right)\right|$ are large or if the experts are overstrained, this approach may not be applicable. In this case, the *sensitivity assessment* of the experts responsible for the variables in $\Theta_{i_1}\left(tv_k^{i,j}\right)$ and $\Theta_{j_2}\left(tv_k^{i,j}\right)$ can be consulted (see Section 6.2.2.2). $V_{i_1}\left(tv_k^{i,j}\right)$ and S_{j_2} ($V_{j_2}\left(tv_k^{i,j}\right)$ and S_{i_1}) are value consistency similar if for any path $P\left(tv_k^{i,j}, tv_l\right)$ in $\Theta_{i_1}\left(tv_k^{i,j}\right)$ (in $\Theta_{j_2}\left(tv_k^{i,j}\right)$), the sensitivity of all vertices to changes in its direct predecessor falls below a certain threshold ε_{SI}. Stated differently, for a path

$$P\left(tv_k^{i,j}, tv_l\right) := \left\{\left(tv_{k_1}, tv_{k_1+1}\right), \ldots, \left(tv_{k_L-1}, tv_{k_L}\right)\right\}$$

it holds

$$\max_{l=1,\ldots,L}\left\{SI_{\Delta l}\left(tv_{k,l+1}\right)\right\} < \varepsilon_{SI},$$

where $SI_{\Delta l}\left(tv_{k,l+1}\right)$ denotes the sensitivity (index) of $V\left(tv_{k,l+1}\right)$ to changes in $V\left(tv_{k,l}\right)$ (in direction Δ if $Type\left(tv_{k,l}\right)$ is a vector space[51]).

Another possibility is to resolve value ambiguity by **adopting the value of preferred concept:** assuming that $V_{i_1}\left(tv_k^{i,j}\right)$ ($V_{j_2}\left(tv_k^{i,j}\right)$) has been determined using $\mathfrak{C}_{i_1}\left(tv_k^{i,j}\right)$ ($\mathfrak{C}_{j_2}\left(tv_k^{i,j}\right)$), the recipients of the scenario can request using either concept. In this case, the values of the vertices that are not value consistency similar to the value stipulated need to be re-assessed. Yet, this approach may be time consuming and demand substantial effort from the experts involved (cf. Section 8.3 on scenario updating). Therefore, it should only be applied if the number of values that need to be recalculated is small and the assessment does not consume much time.

If it is not possible to resolve the value ambiguity, the scenario recipients are asked to specify the **maximum degree of value ambiguity** $E_{VA} \in \mathbb{N}_0$ that is acceptable. If $N(VA)\left(S_{i,j}\right) > E_{VA}$, $S_{i,j}$ is *pruned.*

8.2.2.2. Status Ambiguity

Analogue to the previous section, tv_k denotes the variable under scrutiny. As the focus is on status ambiguity, $status_{i_1}\left(tv_k\right) \neq status_{j_2}\left(tv_k\right)$. Status ambiguity is a concept, which is closely connected to value ambiguity. It is consequential that whenever two different values for tv_k occur in S_{i_1} and S_{j_2}, it can be that $status_{i_1}\left(tv_k\right) \neq status_{j_2}\left(tv_k\right)$. While the problem of value ambiguity has

[51]To this end, different sensitivity measures and indices need to be normalised to a common scale before ε_{SI} can be specified.

been discussed in the previous section, this section focuses on situations where $V_{i_1}(tv_k) = V_{j_2}(tv_k)$, but $status_{i_1}(tv_k) \neq status_{j_2}(tv_k)$.

Again, the first step is an assessment of the possibility to **resolve** the status ambiguity. The status of a variable is closely connected to the *concepts* $\mathfrak{C}_{i_1}(tv_k)$ and $\mathfrak{C}_{j_2}(tv_k)$ used and the *time* the variable's value was assessed $timestamp_{i_1}(tv_k) \in \mathfrak{T}_{i_1}$ and $timestamp_{j_2}(tv_k) \in \mathfrak{T}_{j_2}$ (cf. Section 6.2.1). Therefore, both can be used to resolve the status ambiguity.

If the scenario recipients are able to specify their **preferred concepts** $\mathfrak{C}^*(tv_k)$ from the set $\{\mathfrak{C}_{i_1}(tv_k), \mathfrak{C}_{j_2}(tv_k)\}$, $status^*(tv_k)$ is adopted for both scenarios.

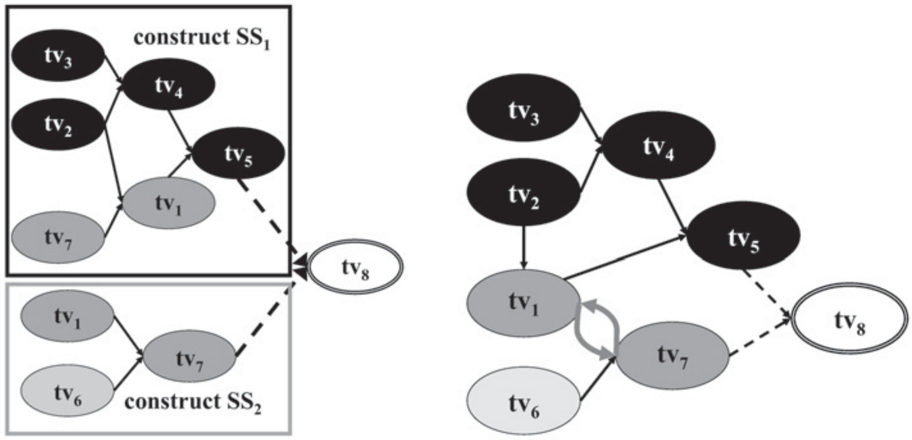
Alternatively, the recipients can also state their preference for the **most recent** assessment. The underlying rationale is that, usually, more or better information is available as time passes and that therefore, the status assessment becomes more and more accurate. To resolve the ambiguity, the time stamps $timestamp_{i_1}(tv_k)$ and $timestamp_{j_2}(tv_k)$ are used. If $timestamp_{i_1}(tv_k) \prec timestamp_{j_2}(tv_k)$, the value of tv_k in scenario S_{i_1} has been determined more recently (and vice versa). In this case, $status_{i_1}(tv_k)$ is adopted (analogously for $timestamp_{j_2}(tv_k) \prec timestamp_{i_1}(tv_k)$). Yet, the time stamps establish only for a *partial* ordering of $\mathfrak{T}$. In some cases, it may not be possible to identify the variable whose value was determined more recently.

In case the status ambiguity cannot be resolved for a set of variables $SATV$, the scenario recipients are asked to specify the **maximum degree of status ambiguity** $E_{SA} \in \mathbb{N}_0$ that is acceptable for them. Analogue to the approach for value ambiguity, $S_{i,j}$ is *pruned* whenever $N(SA)(S_{i,j}) > E_{SA}$.

8.2.2.3. Cyclicity

Cyclic scenarios may be internally consistent. The cyclic nature, however, forestalls scenario generation in distributed settings, which requires that the flow of information from one expert to another for each $SOURCE$-$FOCUS$-path is uniquely defined. Otherwise, several scenarios, all of which are structurally correct, can arise, violating transparency and coherence requirements. Therefore, if cyclicity can not be resolved, the scenarios must be pruned.

To **resolve cyclicity,** the approach below may be followed. Cyclic scenarios arise, as the expertise used for SS_1 and SS_2 refer to different local causal maps. Figure 8.2 illustrates the cyclicity problems: the left side (cf. Figure 8.2(a)) shows that in SS_1, the expert determining tv_1's value states that he needs information on $V(tv_7)$, which can be determined in SS_1 without further information on any other variable's value. Contrarily, in SS_2, $V(tv_1)$ can be determined by the responsible expert autonomously, while the expert responsible for tv_7 states that he needs information on $V(tv_1)$. Merging t$S_{i_1} \in SS_1$ and $S_{j_2} \in SS_2$ leads to the structure as represented in Figure 8.2(b). Particularly, there is a cycle between tv_1 and tv_7.



(a) Structure of Scenario Sets SS_1 and SS_2 Containing the Scenarios S_{i_1} and S_{j_2} to be Merged

(b) Resulting Structure after Merging S_{i_1} and S_{j_2} with Cycle between tv_1 and tv_7

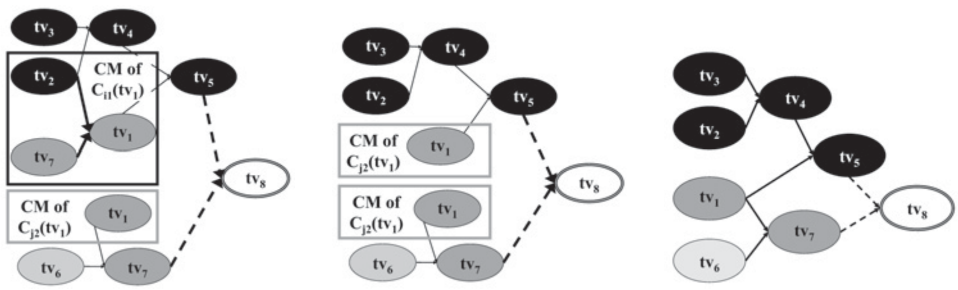
Figure 8.2.: *Cyclicity in Scenario Merging*

Resolving cyclicity amounts to choosing the preferred set of experts. Consider tv_{k_1}, $tv_{k_2} \in STV_{i_1} \cap STV_{j_2}$, for which there are paths $P_{DI_{i_1}}(tv_{k_1}, tv_{k_2})$ and $P_{DI_{j_2}}(tv_{k_1}, tv_{k_2})$. The merged scenario $S_{i,j}$ contains a cycle C_{k_1,k_2} between tv_{k_1} and tv_{k_2}. The cyclicity can be resolved if one edge in C_{k_1,k_2} can be deleted. As S_{i_1} and S_{j_2} are assumed to be structurally correct, they are both acyclic. In S_{i_1} tv_{k_1} can be determined without any information on tv_{k_2}[52] and vice versa for $tv_{k_2} \in S_{j_2}$. If the scenario recipients can agree that they prefer using the information determined using $\mathfrak{C}_{i_1}(tv_{k_1})$ or $\mathfrak{C}_{j_2}(tv_{k_2})$, the cycle can be resolved by adopting this expertise for both sets of scenarios and adapting the causal map accordingly.

Figures 8.3 and Figure 8.2 illustrate the described approach. Figure 8.2 shows that a cycle between tv_1 and tv_7 arises when merging two scenarios constructed by means of the structure depicted in Figure 8.3(a). The reason for this cyclic-

[52] Stated differently, $tv_{k_2} \notin \Psi_{i_1}(tv_{k_1})$, i.e., tv_{k_1}'s value is neither directly nor indirectly influenced by $V(tv_{k_2})$.

ity is that all scenarios in SS_1 contain the path $P(tv_7, tv_1) = \{(tv_7, tv_1)\}$, while the scenarios generated in SS_2 contain the path $P(tv_1, tv_7) = \{(tv_1, tv_7)\}$. Assume that the scenarios' recipients decide that they prefer using $\mathfrak{C}_{j_2}(tv_1)$ over $\mathfrak{C}_{i_1}(tv_7)$. In this case, the local causal map of the expert responsible for tv_1 in SS_2 is adopted for all scenarios in SS_1 (illustrated in Figure 8.3(b)). The cyclicity is resolved, and the scenarios $S_{i_1} \in SS_1$ and $S_{j_2} \in SS_2$ can be merged without giving rise to any cycles.



(a) Comparison of Local CMs for tv_1 (b) Adopting Local CM of j_2 (tv_1) for both Scenarios (c) Resulting Directed Acyclic Graph

Figure 8.3.: *Resolving Cyclicity*

A first requirement for the cyclicity resolving approach based on the **preferred expertise** is the complete conceptualisation of the scenarios. Secondly, the scenario recipients need to be able to specify which concept they prefer. Alternatively, the cyclicity can be resolved by choosing the **preferred value,** $V_{i_1}(tv_{k_l})$ and $V_{j_2}(tv_{k_l})$ ($l = 1$ or 2, reflecting the set of scenarios from which the expertise is adopted). To adopt a value, the scenarios S_{i_1} and S_{j_2} must be value consistency similar with respect to tv_k, or there must be enough time and effort available to re-do the scenario generation for the scenario, to which a new value has been assigned, or the arising value inconsistency must be acceptable to the users. If none of these prerequisites is fulfilled, $S_{i,j}$ is **pruned.**

8.2.3. Pruning of Scenario Information Bubbles

SIBs have been introduced as a means to manage the information within and about a scenario (cf. Section 6.2). Beyond the scenarios themselves, minimum SIBs contain information on the concepts used,and the time stamps. First, the approach for merging *Minimum SIBs* (cf. Section 6.2.1) for the (mergeable) scenarios S_{i_1} in SS_1 and S_{j_2} in SS_2 is explained. Second, this approach is extended to the general case of Extended SIBs (cf. Section 6.2.2).

Assume that SIB_{i_1} and SIB_{j_2} are two Minimum SIBs for the (mergeable) scenarios S_{i_1} in SS_1 and S_{j_2} in SS_2. SIB_{i_1} and SIB_{j_2} need to be merged to a common $SIB_{i,j}$ for representing the content of scenario $S_{i,j}$ that arose from the merging of S_{i_1} and S_{j_2}. Although ambiguities and (structural) incorrectness already have been discussed in Section 8.2.2, for the conceptualisation and the time stamps additional conflicts may arise.

Four cases are distinguished according to the presence of value and status ambiguities and the possibility to resolve the ambiguity.

Resolution of ambiguities by preferred expertise: if value or status ambiguity for a variable $tv_k \in STV_{i_1} \cap STV_{j_2}$ are resolved by adopting the value and/or status of the preferred concept $\mathfrak{C}^*(tv_k)$, the same approach is used for the resolution of conflicts for the conceptualisation and time stamps. Without loss of generality, assume that $\mathfrak{C}^*(tv_k) = \mathfrak{C}_{i_1}(tv_k)$. In this case,

$$\begin{array}{rrcl} & \mathfrak{C}(SIB_{i,j}, tv_k) & = & \mathfrak{C}(SIB_{i_1}, tv_k) \\ \text{and} & timestamp(SIB_{i,j}, tv_k) & = & timestamp(SIB_{i_1}, tv_k). \end{array}$$

Resolution of status ambiguities by currentness: if the recipients follow the rationale that the more recent assessment of a variable $STV_{i_1} \cap STV_{j_2} \ni tv_k$'s status is more accurate (assuming that there are no ambiguities within the values), this approach can be adopted for the resolution of conflicts for the conceptualisation and time stamps. Without loss of generality, $timestamp(SIB_{i_1}, tv_k) < timestamp(SIB_{j_2}, tv_k)$, i.e., the value and status of tv_k in scenario S_{j_2} has been more recently determined. Then,

$$\begin{array}{rrcl} & \mathfrak{C}(SIB_{i,j}, tv_k) & = & \mathfrak{C}(SIB_{j_2}, tv_k) \\ \text{and} & timestamp(SIB_{i,j}, tv_k) & = & timestamp(SIB_{j_2}, tv_k). \end{array}$$

Conflicting conceptualisations or time stamps: if $S_{i,j}$ is value and status unambiguous, it may be possible to resolve the ambiguities in conceptualisation and timestamps by

- determining the preferred concept or
- resolving conflicts by adopting the most recently used concepts and time stamps.

In this case, the acceptance of the scenario is not hampered by inconsistencies and ambiguities within the scenario itself.

Value or status ambiguous scenarios: if the scenario $S_{i,j}$ is value or status ambiguous, but still acceptable for the scenario recipients (i.e., it is not pruned, see Section 8.2.2.1), $SIB_{i,j}$ should represent the sources of these ambiguities. That means, that the conflicts residing in concepts or time stamps must not be resolved, but kept as a means of documentation within the $SIB_{i,j}$.

To prune **Extended Scenario SIBs,** recall that extended SIBs contain, additional to the Minimum SIBs' content, information on:

- a quality assessment of the concept *(q-o-c)* used along with an assessment of the reliability or acceptance for each concept *(a-o-c)* ;
- the duration of the determination of a variable's value *(dur)*
- an assessment of the sensitivity of a variable's value to changes of the direct predecessor vertices' values *(SI)*

(see Section 6.2.2). As for each variable $tv_k \in STV_{i_1} \cap STV_{j_2}$ all these issues depend on $\mathfrak{C}(tv_k)$, the information adopted for $SIB^{ext}i,j$ depends on the choice of concept.

If $\mathfrak{C}(SIB_{i,j}, tv_k) = \mathfrak{C}(SIB_{i_1}, tv_k)$,

$$\begin{aligned}
q\text{-}o\text{-}c\left(SIB^{ext}_{i,j}, tv_k\right) &= q\text{-}o\text{-}c\left(SIB^{ext}_{i_1}, tv_k\right),\\
a\text{-}o\text{-}c\left(SIB^{ext}_{i,j}, tv_k\right) &= a\text{-}o\text{-}c\left(SIB^{ext}_{i_1}, tv_k\right),\\
dur\left(SIB^{ext}_{i,j}, tv_k\right) &= dur\left(SIB^{ext}_{i_1}, tv_k\right)\\
\text{and} \qquad SI\left(SIB^{ext}_{i,j}, tv_k\right) &= SI\left(SIB^{ext}_{i_1}, tv_k\right).
\end{aligned}$$

The procedure for the case $\mathfrak{C}(SIB_{i,j}, tv_k) = \mathfrak{C}(SIB_{j_2}, tv_k)$ is analogue. If $\mathfrak{C}(SIB_{i,j}, tv_k) = \{\mathfrak{C}(SIB_{i_1}, tv_k), \mathfrak{C}(SIB_{j_2}, tv_k)\}$,

$$\begin{aligned}
q\text{-}o\text{-}c\left(SIB^{ext}_{i,j}, tv_k\right) &= \left\{q\text{-}o\text{-}c\left(SIB^{ext}_{i_1}, tv_k\right) q\text{-}o\text{-}c\left(SIB^{ext}_{j_2}, tv_k\right)\right\},\\
a\text{-}o\text{-}c\left(SIB^{ext}_{i,j}, tv_k\right) &= \left\{a\text{-}o\text{-}c\left(SIB^{ext}_{i_1}, tv_k\right) a\text{-}o\text{-}c\left(SIB^{ext}_{j_2}, tv_k\right)\right\},\\
dur\left(SIB^{ext}_{i,j}, tv_k\right) &= \left\{dur\left(SIB^{ext}_{j_2}, tv_k\right), dur\left(SIB^{ext}_{j_2}, tv_k\right)\right\},\\
SI\left(SIB^{ext}_{i,j}, tv_k\right) &= \left\{SI\left(SIB^{ext}_{i_1}, tv_k\right), SI\left(SIB^{ext}_{j_2}, tv_k\right)\right\}.
\end{aligned}$$

8.3. Scenario Updates

The environment is constantly changing; no expert or agency is able to consistently and correctly forecast the future [Mahmoud et al., 2009]. As the future unfolds into the present, scenarios need to be reviewed and assessed to determine whether they are still acceptable, if the current alternative(s) must be modified or if new alternatives need to be considered [Jarke et al., 1998]. Hence, continuous revisions and corrections of scenarios are necessary. To keep scenario updates manageable, the efficiency and responsiveness of scenario updates are crucial.

The formal approach to SBR that has been developed in this thesis has the advantage that (direct and indirect) dependencies are made explicit. In this manner, scenario updating is facilitated, as this novel approach does not necessitate a complete revision of all (focus complete or incomplete) scenarios. Rather, the parts of the scenarios that are affected by the change in the environment can be identified, and the update is restricted to these parts. By using the merging procedure (cf. Section 3.7), updated and stable parts of the scenario can be merged making the scenario update process more efficient than conducting a complete revision (which is the usual approach for scenario update, [Mahmoud et al., 2009]). Nonetheless, scenario updating requires time and input from the experts involved. The scenario pruning section already alluded to the trade-off between effort of the update (with respect to time and expertise required) and consistency (including value, status and observation consistency).

A scenario can be updated due to a **change in its structure,** due to a change of a variable's **value** or its **status.** These issues are handled in different subsections. As scenario updating is closely related to timing issues, from now on the scenario is be annotated with the time when it was determined. $S_i = S_i^t = \left\langle STV_i^t, sv_i^t, status_i^t, DI_i^t \right\rangle$.

8.3.1. Update of a Scenario's Structure

The structure of a scenario S_i^t, that is determined by the set of variables STV_i^t and the interdependencies DI_i^t between them is closely related to the set of **focus variables** $FOCUS_i^t$ and the **concepts** $\mathfrak{C}_i^t\,(tv_j)$ underlying the determination of the variables' values.

8.3.1.1. Update of Focus Variables

The scenario generation process is targeted towards providing the recipients with an assessment of the focus variables' values. A change in $FOCUS$ may have a major impact on the structure of the scenario. $FOCUS$ changes, whenever the variables that are considered expedient for the purpose at hand do change. While usually, in decision-making it is assumed that the focus variables are stable [Belton and Stewart, 2002; French, 1986], it is possible that a change in the group of scenario recipients or in the preferences and interests of the recipients necessitates adapting $FOCUS$. This is particularly important in case pre-defined templates for starting the scenario generation procedure quickly are used.

An update $FOCUS_i^t \to FOCUS_i^{t+T}$ is defined by:

$$update^{t+T}\left(FOCUS_i^t\right) = \left\{FOCUS_i^t \cup FOCUS_{new}\right\} \setminus FOCUS_{outdated},$$

where

$$\begin{array}{rrcl} & FOCUS_{new} \cap FOCUS_i^t & = & \varnothing \\ \text{and} & FOCUS_{outdated} \cap FOCUS_i^t & = & FOCUS_{outdated}. \end{array}$$

$FOCUS_{new}$ is the set of focus variables **added** to $FOCUS_i^t$. $FOCUS_{outdated}$ defines the set of focus variables **deleted,** as they are considered irrelevant now. The approach described below is defined such that *first* the new focus variables (and required information to determine their values) are added, and *second* the outdated variables (and the variables relevant only for determining their values) are deleted.

For each $tv_k^F \in FOCUS_{new}$, the set of variables it depends on (directly and indirectly), $\Psi\left(tv_k^F\right)$ is determined successively by the procedure described in Section 5.1. If for a variable $tv_{j+1}^{new} \in \Psi\left(tv_k^F\right)$ information on tv_j^{new} is required, one has to distinguish

- for the variables $tv_j^{new} \notin STV_i^t$, the (best timely available) experts capable of providing the required information are identified. These experts refer again to their local causal maps, to determine information on which further variables tv_{j-1} they require (cf. Section 5.1).

Starting with the initialisation

$$\begin{aligned} STV_i^{new} &= FOCUS_{new}, \\ sv_i^{new} &= \emptyset, \\ status_i^{new} &= \emptyset \\ \text{and} \qquad DI_i^{new} &= \emptyset, \end{aligned}$$

each time, information on a variable $tv_j^{new} \notin STV_i^t$ is required, one sets

$$\begin{aligned} STV_i^{new} &= STV_i^{new} \cup \left\{tv_j^{new}\right\}, \\ sv_i^{new} &= sv_i^{new} \cup V_i^{new}\left(tv_j^{new}\right) = sv_i^{new} \cup \{\infty\}, \\ status_i^{new} &= status_i^{new} \cup status_i^{new}\left(tv_j^{new}\right) \\ &= status_i^{new} \cup \{\text{“}not\ assessed\text{”}\} \\ \text{and} \qquad DI_i^{new} &= DI_i^{new} \cup \{e_{j,j+1}\} = \left(tv_j^{new}, tv_{j+1}^{new}\right) \end{aligned} \tag{8.3}$$

- for the variables $tv_j^{new} = tv_j \in STV_i^t$, one connects the experts determining $V_i^t\,(tv_j)$ with the expert determining $V\left(tv_{j+1}^{new}\right)$. One defines:

$$\text{and} \qquad DI_i^{new} = DI_i^{new} \cup \left\{e_{j,j+1} = \left(tv_j^{new}, tv_{j+1}^{new}\right)\right\},$$

while STV_i^{new} remains unaffected.

This procedure continues, until all variables tv_j, for which $\widetilde{\Psi}\,(tv_j) = \emptyset$ are in the set $SEED$, i.e., they can be determined autonomously.[53]

Subsequently, for each $tv_k^F \in FOCUS_{outdated}$, the set of predecessors that are not relevant to assess any other focus variable's values is determined:

$$\begin{aligned} &STV_i^{outdated} \\ &= \bigcup_{\substack{tv_k^F \in \\ FOCUS_{outdated}}} \Big\{tv_j \in \Psi_i^t\left(tv_k^F\right) \text{ and } \nexists\, tv_l^F \in FOCUS_i^t \cup FOCUS_{new} : \\ &\qquad tv_j \in \Psi_i^t\left(tv_k^F\right)\Big\}. \end{aligned} \tag{8.4}$$

[53] Section 5.1 provides a discussion on how to handle scenarios that do not fulfil this property.

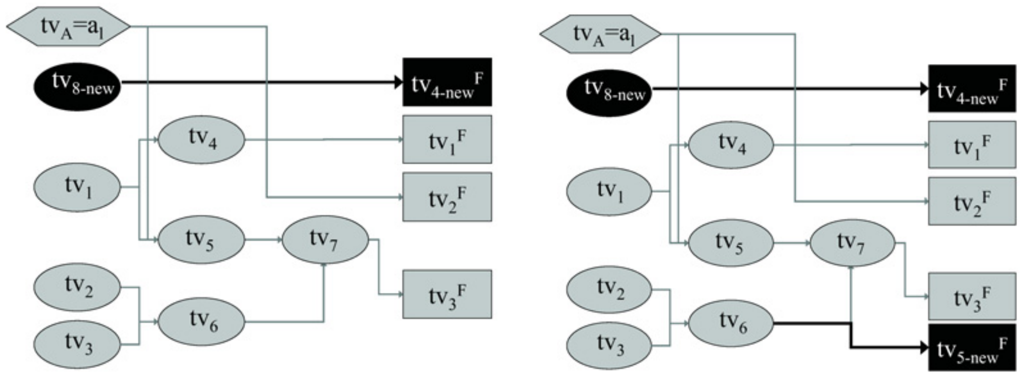
After having established the STV_i-outdated as non-relevant predecessors (with relevance to the remaining and new sets of focus variables), the set of irrelevant dependencies is determined, and one defines:

$$\begin{aligned}
sv_i^{outdated} &= \bigcup_{tv_j \in STV_{outdated}} \left\{ V_i^t\left(tv_j\right) \right\}, \\
status_i^{outdated} &= \bigcup_{tv_j \in STV_{outdated}} \left\{ status_i^t\left(tv_j\right) \right\}, \\
DI_i^{outdated} &= \bigcup_{tv_j \in STV_{outdated}} \Big\{ e \in DI_i^t : \exists tv_l \in STV_i^t : \\
&\qquad e = \left(tv_j, tv_l\right) \text{ or } e = \left(tv_l, tv_j\right) \Big\}.
\end{aligned} \tag{8.5}$$

The new scenario STV_i^{t+T} is defined by

$$\begin{aligned}
S_i^{t+T} &= \left\langle STV_i^{t+T}, sv_i^{t+T}, status_i^{t+T}, DI_i^{t+T} \right\rangle, \\
\text{where} \quad STV_i^{t+T} &= STV_i^t \cup STV_i^{new} \setminus STV_i^{outdated}, \\
sv_i^{t+T} &= sv_i^t \cup sv_i^{new} \setminus sv_i^{outdated}, \\
status_i^{t+T} &= status_i^t \cup status_i^{new} \setminus status_i^{outdated} \\
\text{and} \quad DI_i^{t+T} &= DI_i^t \cup DI_i^{new} \setminus DI_i^{outdated}.
\end{aligned}$$

If $STV_i^{new} \neq \varnothing$, S_i^{t+T} is **focus incomplete.** For providing the values desired to the scenario recipients, an update of the new scenario's values (and statuses) is needed, cf. Section 8.3.2.



(a) Adding a Focus Variable tv_{4-new}^F not Linked to any Variable in STV_i^t (b) Adding a Focus Variable tv_{5-new}^F Dependent on STV_i^t

Figure 8.4.: *FOCUS Update: Adding a Variable*

Figures 8.4 and 8.5 show examples of a $FOCUS$ update. While Figure 8.4 presents the effects of adding new variables to $FOCUS$, Figure 8.5 presents an example of the removal of an irrelevant focus variable. For both examples, the original scenario S_i^t is based in a set of variables

$$\begin{aligned} STV_i^t &= \left\{tv_1, \ldots, tv_z, tv_1^F, tv_2^F, tv_3^F\right\}, \\ \text{where} \quad FOCUS &= \left\{tv_1^F, tv_2^F, tv_3^F\right\}. \end{aligned}$$

In a first step, the variable $tv_{4-new}^F \in FOCUS(t+T)$ is added (cf. Figure 8.4(a)). The expert responsible for determining $V\left(tv_{4-new}^F\right)$ indicates that he needs information on a variable tv_{8-new} to provide his service. Therefore, both tv_{8-new} and tv_{4-new}^F are added to the set of typed variables STV_i^{t+T}. Additionally, the edge $\left(tv_{8-new}, tv_{4-new}^F\right)$ is added to the set of dependencies. As the expert responsible for tv_{8-new} can determine tv_{8-new}'s value autonomously, no further variables and edges need to be added to complete the update. In a second step, another focus variable $tv_{5-new}^F \in FOCUS(t+T)$ is added (see Figure 8.4(b)). This time, the expert responsible for determining tv_{5-new}^F's value requires information on tv_6's value. As $tv_6 \in STV_i^t$, it is sufficient to add tv_{5-new}^F and the edge from tv_6 to tv_{5-new}^F to the new scenario.

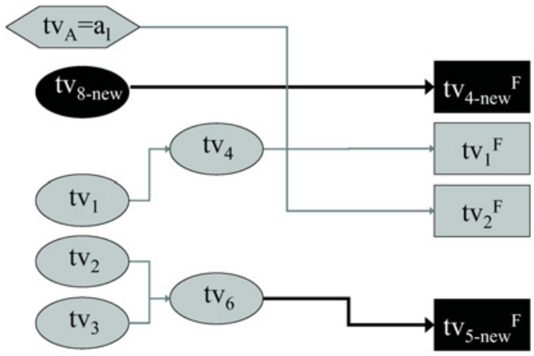


Figure 8.5.: *$FOCUS$ Update Example: Deleting the Irrelevant Focus Variable tv_3^F*

Lastly, tv_3^F is deleted (see Figure 8.4(b)). Now, all variables and dependencies only required for determining tv_3^F's value have been deleted from the scenario. The structure of the new scenario S_i^{t+T} is shown in Figure 8.4(b).

8.3.1.2. Update of Conceptualisation

Changing the **conceptualisation** of a variable $tv_j \in STV_i^t$, $\mathfrak{C}_i^t(tv_j) \rightarrow \mathfrak{C}_i^{t+T}(tv_j)$, $\mathfrak{C}_i^t(tv_j) \neq \mathfrak{C}_i^{t+T}(tv_j)$ has an effect on the scenarios' structure if the *local causal maps* of the original and the newly used concepts differ. As described above for the update of $FOCUS$ (see equation 8.3 for the variables and edges to be added and equations 8.4 and 8.5 for the irrelevant variables and edges), one determines the new scenario. Furthermore, one sets

$$V_i^{t+T}(tv_j) = \infty$$
$$\text{and} \qquad status_i^{t+T}(tv_j) = \text{“}not\ assessed\text{”},$$

as the value and status $V_i^t(tv_j)$ and $status_i^t(tv_j)$ were determined on the basis of $\mathfrak{C}_i^t(tv_j)$.

8.3.2. Value Update

If an expert provides a new value $V_i^{t+T}(tv_j)$ for a variable $tv_j \in STV_i^T$, a decision must be made, whether the change in the value is **sufficiently relevant** to justify a scenario update. This concerns both focus complete and incomplete scenarios. For the latter, it is assumed that there is a set of variables $\tau \in \Theta(tv_j)$, for which holds: $\forall tv_j \in \tau : V_i(tv_j) \neq \infty$. The scenario recipients must balance

- the **time** (still) **available** for the purpose at hand,
- the **time and effort necessary** for that update taking into account the duration of the update as well as the workload of the experts involved,
- the (projected) **effect** of adopting $V_i^{t+T}(tv_j)$ on the focus variables (or on their evaluation).

In case, a full update of the scenarios cannot be accomplished (e.g., as some experts are hardly available, or as some assessments are too time consuming), a decision must be made whether

- the **original** (consistent) **scenarios** are retained as a basis for the purpose at hand,
- a **partial update** (that may lead to inconsistency) is performed by updating only the branches of the scenario, for which an update is feasible given time and effort restrictions,
- some (ideally, the most relevant) scenarios are **selected** for updating.

8.3.2.1. Time and Effort Assessment

For the assessment of the time and effort necessary to perform a value update, all steps within the scenario generation and evaluation need to be respected (cf. sections 5.2 and 5.3).[54] Therefore, the time and effort depends on

- the **number of scenarios** that need to be updated,
- the **number of variables** tv_j in each STV_i, whose value must be updated and
- the **effort and duration** these updates require.

An upper limit that must not be exceeded is provided by considering the purpose P_ω^{ip} (cf. Section 6.5). The time available to solve the problem is denoted $T^{\max}\left(P_\omega^{ip}\right)$. As the results provided are not to be understood as prescriptions but as a support and guidance for the recipients, the analysis and interpretation of the results requires some additional time $T^{analysis}$. This is particularly true if this approach is applied in an iterative manner integrating feedback and refinements in each iteration. Therefore, an upper limit for the time for scenario (value) updating is $T^{\max}\left(P_\omega^{ip}\right) - T^{analysis}$.

If for a set of scenarios SS with $tv_j \in \bigcap_{S_i \in SS} STV_i$, $V_i^t\left(tv_j\right) \neq V_i^{t+T}\left(tv_j\right)$, the SIBs (cf. Section 6.2.2) provide an assessment of duration $dur_i\left(tv_k\right)$ (given the concepts applied $\mathfrak{C}_i\left(tv_j\right)$) for all $tv_{j_k} \in \Theta\left(tv_j\right)$, then the overall duration of the value update can be assessed by first determining the time for updating each scenario in SS. To this end, consider for $S_i \in SS$ and for all $tv_k^F \in FOCUS$ the paths from tv_j to tv_k^F:

$$P_{DI_i}\left(tv_j, tv_k^F\right) =: \left\{e_{j_1,j_2}, \ldots, e_{j_{n-1},j_n}\right\},$$

where $j_1, \ldots, j_n$ is a labelling of variables such that $e_{j_{l-1},j_l} = \left(tv_{j_{l-1}}, tv_{j_l}\right)$ and $tv_{j_1} = tv_j$, $tv_{j_n} = tv_k^F$. The **duration of a value update along that path** is

$$dur\left(P_{DI_i}\left(tv_j, tv_k^F\right)\right) = \sum_{l=2}^{n} dur_i\left(tv_{j_l}\right). \tag{8.6}$$

[54] As structural issues were discussed in the previous section, the process of construction of the underlying DAG_D (see Section 5.1) can be omitted.

The longest path P^L_{i,tv_j} from tv_j to $tv^F_k \in FOCUS$[55] can be determined by weighing the edges of the truncated graph $\mathcal{G}_{i,tv_j} = \left(STV_{i,tv_j}, DI_{i,tv_j}\right)$, where

$$\begin{aligned} STV_{i,tv_j} &= STV_i \cap \left(\{tv_j\} \cup \Theta\left(tv_j\right)\right), \\ DI_{i,tv_j} &= DI_i\left(STV_i \cap \left(\{tv_j\} \cup \Theta\left(tv_j\right)\right)\right). \end{aligned}$$

Each edge $e_{j_{i-1},j_i} \in DI_{i,tv_j}$ is assigned a weight $w_{j_{i-1},j_i} = dur_i\left(tv_{j_k}\right)$. As $\mathcal{G}_{i,tv_j}$ a weighted DAG, the longest path problem on $\mathcal{G}_{i,tv_j}$ can be solved in linear time using dynamic programming [Diestel, 2005].

As any two scenarios S_{i_k}, $S_{i_k} \in SS$ may have structural differences (i.e., $STV_{i_k} \neq STV_{i_l}$ or $DI_{i_k} \neq DI_{i_l}$), for the assessment of the total duration it is not sufficient to consider a mere summation of differences. Yet, an *upper bound of the duration* can be calculated by

$$\sum_{i \in I} dur\left(P^L_{i,tv_j}\right),$$

where the duration of following P^L_{i,tv_j} is calculated as indicated in equation 8.6.

To determine the total duration a weighted DAG $\mathcal{G}_{SS,j} = \left(STV_{SS,j}, DI_{SS,j}\right)$ is constructed from $\left\{\mathcal{G}_{i,tv_j}\right\}_{i \in I}$ by setting

$$\begin{aligned} & STV_{SS,tv_j} &=& \bigcup_{i \in I} STV_{i,tv_j} \\ \text{and} \quad & DI_{SS,tv_j} &=& \bigcup_{i \in I} DI_{i,tv_j}. \end{aligned}$$

The weights (or durations) w_{j_{i-1},j_i} of each edge $e_{j_{i-1},j_i} \in DI_{i,tv_j}$ are determined by summing up the durations that the assessment of $V\left(tv_{j_k}\right)$ throughout all $S_i \in SS$, where an assessment of $V\left(tv_{j_k}\right)$ is required: by

$$w_{j_{i-1},j_i} = w^{dur}_{j_{i-1},j_i} = \sum_{\substack{i \in I \\ e_{j_{i-1},j_i} \in DI_{i,tv_j}}} dur_i\left(tv_{j_l}\right).$$

The underlying assumption is that the assessment of values can only be performed *consecutively.* If parallel computation or reduction of time by the computation of multiple values can be achieved, this can be integrated in the deter-

[55] For all S_{i_k}, $S_{i_k} \in SS$: $FOCUS_{i_k} = FOCUS_{i_l} = FOCUS$ as the scenarios in SS are constructed for a common purpose.

mination of weights before determining the longest path through $\mathcal{G}_{SS,j}$, P^{L}_{SS,tv_j} can be determined. The duration of the value update of tv_j is

$$dur^{up}\left(tv_j\right) = dur\left(P^{L}_{i,tv_j}\right), \tag{8.7}$$

where, again, equation 8.6 is used for the calculation of the duration. Apparently, a complete update of SS (i.e., generating a set of focus complete scenarios S_i^{t+T} is only possible if

$$dur\left(P^{L}_{i,tv_j}\right) \leq T^{\max}\left(P^{ip}_{\omega}\right) - T^{analysis}.$$

Analogously, one proceeds if the assessment of duration is complemented by the degree of occupation of experts. Hence, the effort of the value update can be complements the duration weights by weights $w^{ef}_{j_{i-1},j_i}$ which represent the **effort** (cf. equation 6.2 for the determination of effort from duration and occupation assessments). As above, the weighted DAG $\mathcal{G}_{SS,tv_j}$ is constructed and the longest path with respect to the effort weights $P^{L,ef}_{SS,tv_j}$ is determined. The overall effort for the scenario update is

$$ef^{up}\left(tv_j\right) = ef\left(P^{L,ef}_{SS,tv_j}\right) = \sum_{e_{j_{i-1},j_i} \in P^{L,ef}_{SS,tv_j}} w^{ef}_{j_{i-1},j_i}.$$

In case, the SIBs do not contain any information on the duration of assessments and the occupation of experts, it is possible to use the number of variables whose value must be updated as an approximation of the duration. Again, first the underlying structure $\mathcal{G}_{SS,j}$ is determined. Then, one sets

$$w^{struct}_{j_{i-1},j_i} = \sum_{i \in I} \mathbb{1}_{DI_{i,tv_j}}\left(e_{j_{i-1},j_i}\right).$$

An **approximation of the effort** required for the value update is achieved by determining the longest path $P^{L,struct}_{SS,tv_j}$ with respect to the graph structure, and calculating

$$ef^{struct}\left(tv_j\right) = ef^{struct}\left(P^{L,struct}_{SS,tv_j}\right) = \sum_{e_{j_{i-1},j_i} \in P^{L,ef}_{SS,tv_j}} w^{struct}_{j_{i-1},j_i}.$$

8.3.2.2. Assessing the Impact of Value Updates

To determine the **impact** of changing a value on the focus variables, indicator similarity assessments are used (see Section 7.4). In this manner, the effect of a change in (one or more) variables' values on the values of the focus variables (or the evaluation thereof) can be determined for *focus incomplete* scenarios. The basis for this assessment is a set of indicator values $V_i\left(IND\left(tv_k^F\right)\right)$ that are defined for each $tv_k^F \in FOCUS$.

To apply this approach in the context of assessing the impact of updating $V_i(tv_j)$ for a scenario S_i, the scenario S_i^{valid} that contains only values that are still valid is defined by:

$$\begin{aligned} STV_i^{valid} &= STV_i, \\ sv_i^{valid} &= \bigcup_{tv_k \in STV_i} V_i^{valid}(tv_k), \\ status_i^{valid} &= \bigcup_{tv_k \in STV_i} status_i^{valid}(tv_k), \\ DI_i^{valid} &= DI_i \end{aligned} \tag{8.8}$$

with

$$V_i^{valid}(tv_k) = \begin{cases} V_i^t(tv_k) & \text{if } tv_k \neq tv_j \wedge tv_k \in STV_i \setminus \Psi_i(tv_j), \\ V_i^{t+T}(tv_k) & \text{if } tv_k = tv_j, \\ \infty, & \text{if } tv_k \in \Psi_i(tv_j), \end{cases}$$

$$status_i^{valid}(tv_k) = \begin{cases} status_i^t(tv_k) & \text{if } tv_k \neq tv_j \wedge tv_k \in STV_i \setminus \Psi_i(tv_j), \\ status_i^{t+T}(tv_k) & \text{if } tv_k = tv_j, \\ \text{“}not\ assessed\text{”}, & \text{if } tv_k \in \Psi_i(tv_j). \end{cases}$$

To achieve comparable assessments, it is useful to not use $V_i^t\left(FOCUS\right)$ (or the evaluation), but to construct a scenario S_i^{out,tv_j} as a basis for the comparison, which differs from S_i^{valid} only in the value and status of tv_j, i.e.,

$$\begin{aligned}
STV_i^{out,tv_j} &= STV_i^{valid}, \\
sv_i^{out,tv_j} &= sv_i^{valid} \cup V_i^t\left(tv_j\right) \setminus V_i^{t+T}\left(tv_j\right), \\
status_i^{out,tv_j} &= status_i^{valid} \cup status_i^t\left(tv_j\right) \setminus status_i^{t+T}\left(tv_j\right), \\
DI_i^{out,tv_j} &= DI_i^{valid}.
\end{aligned}$$

As the focus is on the consequences of the update, it is sufficient to perform the similarity assessment only for those indicator(s) ind_j^k on $tv_k^F \in FOCUS$ that are derived from the variables tv_j for which $V_i^t\left(tv_j\right) \neq V_i^{t+T}\left(tv_j\right)$. One denotes $IND_{update}^k = \left\{ind_j^k\right\}_{j\in J(k)}$ the set of these indicators and defines $IND_{update} = \bigcup_{tv_k^F \in FOCUS} IND_{update}^k$. For simplicity's sake, one sets:

$$\widetilde{dist}_j\left(V_i^{valid}\left(ind_j^k\right), V_i^{out,tv_j}\left(ind_j^k\right)\right) = \widetilde{dist}_{j,i,k}^{update}$$

Analogue to equation 7.15 one sets

$$impact_{p,t,t+T}^{F,j,w} = \\
\begin{cases}
\left(\sum\limits_{tv_k^F \in FOCUS} \sum\limits_{j\in J(k)} \left(w_p^h\left(ind_j^k\right)\widetilde{dist}_{j,i,k}^{update}\right)^p\right)^{\frac{1}{p}} & , 1 \leq p < \infty \\
\max\limits_{tv_k^F \in FOCUS} \max\limits_{j\in J(k)} \left|\widetilde{dist}_{j,i,k}^{update}\right| & , p = \infty.
\end{cases} \tag{8.9}$$

In the simplest case when a single variable tv_j requires a value update and when there is only one indicator ind_j^k for tv_j, the expression in equation 8.9 can be simplified to

$$impact_{p,t,t+T}^{F,j,w} = \begin{cases}
\left(\sum\limits_{tv_k^F \in FOCUS} \left(w_p^k\left(tv_k^F\right)\widetilde{dist}_{j,i,k}^{update}\right)^p\right)^{\frac{1}{p}} & , 1 \leq p < \infty \\
\max\limits_{tv_k^F \in FOCUS} \left|\widetilde{dist}_{j,i,k}^{update}\right| & , p = \infty.
\end{cases} \tag{8.10}$$

In this case the impact of changing tv_j's value is determined by the **distance of the original and the new value** in $t+T$ as well as the **number** and **importance**

of focus variables tv_k^F that are influenced by $V(tv_j)$. (The importance of tv_k^F is expressed by the weights $w_p^k\left(tv_k^F\right)$.)

For SBR & MCDA these weights are derived from the preferences of the decision-makers. Instead of eliciting $w_p^k\left(tv_k^F\right)$, one uses weights w_k^D elicited during the problem structuring phase (cf. equation 7.16).

8.3.2.3. Decision on Scenario Update Due to a Change of Value

The decision on whether or not to perform an update of scenarios due to newly available information on a variable's value is itself a multi-criteria decision problem: besides the objectives of *reducing the effort* and taking into account important changes in *impact, time and occupation constraints* need to be respected. One assumes that new information on J independent variables $tv_{j_1},\ldots,tv_{j_J} \in \bigcap_{i\in I} STV_i$ is available.[56] In this situation, a decision must be made, which of the values are updated.

To avoid an overly complex formalisation, here only the update of single variables' value is investigated. This allows for a ranking of the most useful updates to be established, which can in a second step be investigated in more depth. Hence, the problem can be formulated as an optimisation problem, where the alternatives are updating any of the variables tv_{j_k} (denoted a_k) or not performing an update at all (denoted a_0):

$$\begin{aligned} w^1 v_D^1\left(x_k^1(a_j)\right) + w^2 v_D^2\left(x_k^2(a_j)\right) &\rightarrow \text{max!} \\ \text{subject to} \qquad\qquad j &= 0,\ldots,J, \\ w_l &\geq 0 \qquad (l = 1,2), \\ w^1 + w^2 &= 1, \\ dur\left(P_{i,tv_j}^L\right) &\leq T^{\max}\left(P_\omega^{ip}\right) - T^{analysis}. \end{aligned}$$

Here, $x_k^1(a_j)$ denotes the score of the duration or effort assessment (depending on the information available), while $x_k^2(a_j)$ denotes the result of the impact assessment (depending on the purpose) for $j = 1,\ldots,J$. For $j = 0$ (i.e., the option of not performing any update), one defines $x_k^1(a_0) = x_k^2(a_0) = 0$. v_D^1 and v_D^2 are the respective value functions modelling the decision-makers' intra-criteria preferences. Here, the preferences for the effort are assumed to be decreasing

[56] The independence requirement ensures that there is no pair of variables tv_{j_k}, tv_{j_l}, for which $tv_{j_k} \in \Psi\left(tv_{j_l}\right)$ or vice versa.

(i.e., the lower the effort the better), while the preferences for the impact are supposed to be increasing (i.e., the bigger the difference, the more important it is to perform an update). The weights w^1, w^2 indicate the respective importance the objectives (cf. Section 2.1.3). In this manner, a ranking of feasible updates can be achieved.

If a_0 is the only feasible option, it is useful to analyse $impact^{F,j,w}_{p,t,t+T}$. If that impact exceeds a certain threshold (i.e., it is judged hihgly important), possible options to perform an update are partial updating or selection.

Partial Scenario Updating: the scenario updating procedure is started knowing that no consistent set of scenarios can be completed given time constraints. In this case, the risk that the arising scenarios are incomplete is accepted. In this case, a partial valuation $V_i^{t+T}(FOCUS)$ can be provided to the scenario recipients at time $T^{\max}\left(P_\omega^{ip}\right) - T^{analysis}$ case S_i^t. If S_i^t is focus complete and if consistency is considered negligible, one can decide to use

$$V_i^{t,t+T}\left(tv_k^F\right) = \begin{cases} V_i^t\left(tv_k^F\right) & \text{if } V_i^{t+T}\left(tv_k^F\right) = \infty, \\ V_i^{t+T}\left(tv_k^F\right) & \text{otherwise.} \end{cases}$$

and to provide $V_i^{t,t+T}(FOCUS) = \bigcup_{tv_k^F \in FOCUS} V_i^{t,t+T}\left(tv_k^F\right))$ to the scenario recipients as a basis for their further assessments.

Scenario Selection: the newly developed scenario selection procedure filters the scenarios considered most relevant from the set of scenarios SS to be updated (see Section 8.1). By updating only $SS_{select} \subset SS$ (with $|SS_{select}| < |SS|$), the duration of the update may be reduced sufficiently to determine a set of focus complete scenarios SS^{t+T}_{select}. To this end, the auxiliary (focus incomplete) scenarios as defined in equation 8.8 should be used as the basis for the indicator similarity assessment and the according definition of scenario equivalence classes.

8.3.3. Status Update

When an expert updates both the value and the status of a variable, and if the updated values are the basis for a scenario update as discussed in the previous section, the status all updated variables should be updated as well wherever necessary. Yet, there may be situations, when the expert sticks to the value(s) he provided, but may need to change their status (e.g., be, when more information corroborating a certain value $V_i^t(tv_j)$ becomes available or if $\mathfrak{C}_i(tv_j)$ the expert used to determine a variable's value changes.

As the status of the variable does not have an impact on any further experts contributing to the scenario generation, the most precise status should be adapted Here, it is assumed that the the most recent status, i.e., the assessment $status_i^{t+T}(tv_j)$ is not less precise than $status_i^t(tv_j)$. A **status update** is performed whenever a new status is available. Yet, the status does have an effect on the reliability assessment (cf. Section 4.3.4). Thus, a change in the status may also provoke the need to **prune** the scenario (see Section 8.2.1).

8.3.4. Update of Backdrop

Besides the scenarios and the information they contain, it may also be that the backdrop or the purpose of the scenario construction change. It may, e.g., be that the preferences of the scenario recipients may vary over time. In this case, the evaluation of scenarios (as described in Section 5.3) changes, with a potential impact not only on the scenario results themselves but also on the similarity assessment for SBR & MCDA problems, which makes use of the decision-makers' preferences. To deal with this problem before-hand, sensitivity analyses varying the intra- and inter-criteria preferences can be useful, as these ensure that the results are either stable for varying preferences within certain bounds, and allow for critical thresholds, whose exceedance causes a change in the ranking of alternatives or in the similarity of scenarios, to be determined.

8.4. Summary and Discussion

This chapter has provided novel methods for scenario management facilitated by the scenario formalisation, the scenario information management framework (cf. Chapter 6), and the concepts of scenario similarity and scenario equivalence classes (cf. Chapter 7. Scenario management facilitates the characterisation and selection of the most relevant scenarios, the pruning of scenarios, and the updating of scenarios.

Scenario management is essential for the usability of scenarios, for it allows the potential combinatorial explosion to be curbed, and it ensures the plausibility and relevance of the scenarios. The concept of **plausibility** refers not only to the correctness and topicality of information, but also to the credibility and acceptability of the information present within a scenario. **Relevance** comprises both the relevance of information for the decision and relevance with respect to the other scenarios considered for the same decision.

The presented scenario management approach combines the distributed and decentralised approach to scenario building, where experts chained together in a work flow successively generate the scenario, with a decision-centric scenario management component that provides an overview on the ongoing processes. It allows for controlling the number of scenarios by **selecting** and exploring only **relevant scenarios** and takes into account the constraints, requirements, and preferences of the decision-makers and the experts. Scenario management facilitates **pruning** of irrelevant or falsified scenarios ensuring that the quality of scenarios justifies their use as a basis for the decision to be made, and it allows for handling **scenario updates** with respect to all constituents of the scenarios and the SIBs.

In this manner, scenario management facilitates the adaptation of scenarios to the information available at later moments in time, which is of great importance in highly dynamic environments. Yet, it takes into account the relevance of the novel information and avoids the problem of infinite updating without ever establishing a (set of) focus complete scenario(s). Usually, scenario updating is a burdensome task, as in scenario analysis an update equals a complete revision of the scenario as a whole [Girod et al., 2009; Mahmoud et al., 2009]. The newly developed scenario formalisation that explicitly captures variable dependencies enables the use of procedures that facilitate (more) efficient scenario update. Particularly, the values and statuses of the variables that are *independent* from the updated variables can be **re-used** in the updated scenario. This approach saves time, effort of the experts involved and computation resources. The scenario merging procedure (see Section 3.7) ensures that the partial scenarios that are affected by the update and the partial scenarios that are re-used can be combined to scenarios that fulfil the ambiguity requirements for acceptance as specified by the scenario recipients (see Section 8.2.2).

Another newly opened area of application of value re-use is the identification of **similar decision** or sense-making problems, which share the same incident description (covered in the backdrop). In these cases, the the approach to scenario formalisation developed in this thesis (cf. Chapter 3) facilitates the identification of variables that are used for both problems. In this case, values that have already been determined for problem D_1 can be used as the $INIT$

for problem D_2 and vice versa. Furthermore, potential inconsistencies can be tracked.

One of the main aims of scenario management is to ensure that the results are available betimes so that (in SBR & MCDA) the decision can be made in due time. To assess the **time necessary to provide decision support,** all phases in the Decision Map approach, from the initial problem structuring to the final analyses (e.g., consensus building supported by sensitivity analyses), need to be considered. The next paragraphs discuss the required time and possible options to accelerate the processes for the problem structuring, the scenario building and the scenario evaluation phase.

The first phase in the Decision Map approach is the **problem structuring phase.** It encompasses the definition of the focus variables for Scenario-Based Sense-Making (SBR & SM), or the elicitation of the attribute tree and the preferences for SBR & MCDA, and the configuration of the CM. If the problem at hand allows some degree of standardisation of its evaluation (e.g., when the decision-makers' goals remain essentially unaffected for a set of decision problems), it is possible to accelerate the problem structuring phase by using templates (of the focus variables, the attribute tree or the preferences, depending on what the decision-makers perceive as appropriate). These templates must be adapted if changes and improvements are necessary. If the situation at hand follows certain patterns and the input and output each expert potentially contributing to the scenario construction provides can be defined a priori, the configuration of the Decision Map can be further expedited by using predefined (partial) DAG_Ds. Contrarily, if the decision problem is unique or if there is disagreement on the evaluation principles and preferences within the attribute tree and discussion is required to build a consensus, more time needs to be reserved for the problem structuring phase as well as for the analysis of results.

In the second phase, a set of scenarios (per alternative) is built **(scenario building phase).** The time required for this phase depends on the number of scenarios built as well as on the effort necessary to build each scenario. The maximum number of scenarios built depends itself on the number of variables in DAG_D and on the number of possible values per variable. If for a set of scenarios $SS = \bigcup_{i\in I} S_i$, $|\bigcup_{i\in I} STV_i| = n$, each $tv_j \in \bigcup_{i\in I} STV_i$ is assigned at most λ_j values, an upper bound for the magnitude of SS is $\prod_{j=1}^{n} j^{\lambda_j}$. Following the rationale of distributed information processing, it is each expert's responsibility to specify the number of values that he can handle within a given time for each of the input variables he uses. This approach also ensures that the set of scenarios is completed within a predefined time by allocating a time τ_j to each expert determining the value of a variable tv_j. By weighing each edge (tv_j, tv_k) from tv_j to one of its direct successors $tv_k \in \widetilde{\Theta}(tv_j)$, an upper bound for the time of scenario construction for each alternative corresponds to the length of longest path P^*: $l(P^*) = \sum_{(tv_j, tv_k) \in P^*} \tau_j$. There might be trade-offs between the amount of different types of information an expert can process and the accuracy of the information he determines. This concerns both the information regarding different aspects of the situation (represented by different vertices in the DAG) and the level of granularity of this information (ranging from very specific information to general trends). The Decision Map approach reveals these trade-offs and can help analysing which pieces of information are the most relevant and must be taken into account even when time is critical. Other pieces of information (or information on a higher level of detail) might only be processed if time is abundant.

Last, the scenarios are evaluated using techniques from MAVT **(scenario evaluation phase).** While the computation of the ranking of alternatives is negligible in terms of time required, time must be reserved for analysis and discussion of evaluations. The results of the SBR & MCDA process should not be understood as an imperative prescription but rather as support and guidance for the decision-makers. Therefore, the analysis and interpretation of the results requires some additional time (particularly, when the approach is applied in an iterative manner by allowing the integration of feedback and refinements).

9. Emergency Management Example

The purpose of computing is insight,
not numbers.
(Richard Hamming)

This chapter illustrates the Decision Map approach by means of a strategic emergency management example. In this domain, common characteristics include [French et al., 2005; Bertsch, 2008; Wright and Goodwin, 2009]:

- a finite set of feasible alternatives to choose from;
- the need to respect multiple conflicting objectives;
- the participation of multiple, locally dispersed decision-makers, experts and stakeholders, each of which has different knowledge, skills, competences and preferences;
- the need to integrate information of heterogeneous type, quality and (un-)certainty;
- the need to respect constraints in terms of time and bounded availability of experts and decision-makers;
- the need for transparency and documentation to achieve compliance and to answer possibly the need for justification.

On the whole, the need for distributed, timely, coherent and effective decision support as offered by the Decision Map approach arises. The features of distributed problem solving, the integration of multiple goals and the reduction of complexity by generating scenarios targeted at a MAVT evaluation effectuate transparency and understandability and keep the workload for all experts and decision-makers involved manageable.

9.1. Problem Description

The example describes a hypothetical chemical incident in Glostrup, Denmark. The underlying data and assumptions were elicited from experts and decision-makers working for the Danish Emergency Management Agency (DEMA). The incident involves the leakage of chlorine from a tank wagon.

The example is deliberately kept small to clearly explore the approaches by highlighting their main features. It has been extracted from an exhaustive use case that has been investigated with decision-makers from emergency management authorities (cf. [Comes et al., 2009a, 2010b; Conrado and Pavlin, 2010]).

9.1.1. Chemical Incidents Arising Due to the Transportation of Chlorine

Under standard conditions, the halogen chlorine (Cl_2) is a greenish-yellow gas. Chlorine is a powerful oxidant used in bleaching and disinfectants, and as an essential reagent in the chemical industry [Kleijn et al., 1997; Winder, 2001]. In the latter field of application, chlorine is mainly used as a chemical intermediate: many of the highest value chlorine chain products such as polycarbonate, polyurethanes and epoxy resins do not contain chlorine, but depend on it for their synthesis [European Communities, 2007]. In the European Union, due to the economic crisis the production of chlorine in 2009 decreased to 9.1 million tonnes (compared to 10.4 million tonnes produced in 2008) [Euro Chlor, 2009]. Germany remained the EU's largest chlorine producer in 2009, accounting for 43.5 % of European production, followed by Belgium and The Netherlands with 14.9 % [Euro Chlor, 2009].

Under standard conditions for temperature and pressure, chlorine is a toxic gas that irritates the respiratory system. The odour and irritant action provide some warning properties, although there is small margin of safety between olfactory detection and exceeding the exposure standard [Pötzsch, 2004; Winder, 2001]. Health effects due to chlorine exposure are related to the intensity and duration of exposure. An exposure may cause inflammation of the respiratory system and skin and severe corrosion to skin, eyes and respiratory tract at high concentrations. Furthermore, delayed fatal pulmonary oedema are possible [EPA, 2009]. The basic mechanism of toxicity is related to solubility of chlorine in water-based environments to form hydrochloric and hypochlorous acids, and subsequent ionisation [Winder, 2001]. These reactions will occur in the body, such as in the moist linings of airways [Winder, 2001; Wenck et al., 2007]. Chlorine has a time-weighted average exposure standard of 0.5 to 1 ppm and a short-term exposure limit of 1 to 3 ppm [EPA, 2009; European Commission, 2006; Pötzsch, 2004; Winder, 2001].

Exposure to chlorine has occurred in a number of situations, including as a chemical warfare agent, in industrial and domestic exposures, and as a result of accidents and spills [Winder, 2001]. One important source of risk stems from the **transportation of chlorine.** For instance, on November 11, 1979, following the derailing in the Toronto suburbs of several tank wagons carrying dangerous products (mainly propane gas and chlorine), the Canadian authorities decided to evacuate more than 200,000 people [Meslin, 1981]. In January 2005, in Graniteville, South Carolina, a train carrying three tanker cars of liquid chlorine was inadvertently switched onto an industrial spur, where it crashed into a parked locomotive [Buckley et al., 2007; Wenck et al., 2007]. The train derailed and one of the chlorine tankers was breached, releasing approximately 46 tons of chlorine immediately and an additional 14 tons over the next three days, until a patch could be applied [Wenck et al., 2007]. The dense and highly toxic cloud of chlorine gas that formed in the vicinity of the accident was responsible for nine fatalities and caused injuries to more than 500 others [Buckley et al., 2007].

Although the amount of chlorine transported in Europe by rail and road has halved during the past decade, still about 5 % of the chlorine produced is transported by road or rail [European Communities, 2007; Euro Chlor, 2009]. Of the 475.000 tonnes transported in 2009, about 70 % was shipped in bulk by rail with an average distance of 450 km [Euro Chlor, 2009]. Typically, chlorine is shipped and stored as a liquid in a container under pressure. Chlorine's boiling point is around -34° C at atmospheric pressure, but it can be liquefied at room temperature with pressures above eight atmospheres (in hPa) [Air Liquide, 2010]. One litre of chlorine liquid produces about 0.43 m^3 of chlorine gas at 25° C. The types of tanks authorisised for chlorine transportation are regulated for Germany in the Gefahrgutbeförderungsgesetz (GGBefG) [GGBefG, 2009]. It makes use of the UN Recommendations on the Transport of Dangerous Goods [United Nations, 2007]. The maximum-sized container tank shipped by rail is capable of holding 60 m^3 of liquid chlorine [Pötzsch, 2004; GGBefG, 2009]. At ambient temparatures (20° C), the pressure within such a tank is typically about 6.7 bar [Air Liquide, 2010; Pötzsch, 2004].

9.1.2. Decision Problem

It is assumed that a freight train derailed causing the leakage of chlorine from a ruptured tank wagon. The incident is imagined to happen in the early-morning hours in Glostrup, Copenhagen, Denmark. A responder unit specialised in dealing with hazardous material covers the rupture, thereby temporarily stabilizing the situation. Once emergency measures have been taken and the situation is stabilised, the provider of the chlorine is contacted, which agrees to provide a transportation tank into which the remaining chlorine, captured in temporary vessels of varying sizes, will be transferred. This action is planned to happen a few hours after situation stabilisation.

During this transfer, there is the risk of two types of *leakage:* a temporary vessel containing chlorine may tip over releasing a significant amount of chlorine in a short time (large leak), or the transfer hose used to connect the temporary vessel to the tank may get loose (small leak). Of course, there is also the possibility that no leak occurs, in which case the transfer is qualified as *successful*. Given these risks, a decision on the preventive measure to be applied must be made. The alternatives evaluated are: a_1 (do nothing), a_2 (sheltering of downwind areas) and a_3 (evacuation of the most critical areas, sheltering in the rest of the downwind areas).

As there is not only uncertainty about the success of the transfer but also about a number of other factors (e.g., weather situation, number of residents present) that can only partly be controlled by the decision-makers, and for which the information available varies in type and quality, the decision needs to be made under fundamental uncertainty. Furthermore, it is necessary to take interdependencies between different issues such as health and population, economic, ecological and social factors into account. To this end, different experts need to contribute to the decision process. Finally, although there is some time available to make the decision (as there is no immediate threat), the time for the decision-making is limited.

9.2. Configuration of the Decision Map

The relevant variables and their interdependencies for the presented decision problem can be determined based on the configuration of DAG_D as described in Section 5.1. Table 9.1 summarises the most important sets of variables and

Table 9.1.: *Variables and Their Roles in Scenario Building*

Variables	**Role**
$FOCUS = STV_{SEED}$	Variables, whose values must be included in the completed scenarios. A completion of $FOCUS$ is considered as necessary and sufficient for the completion of a scenario. Hence, $FOCUS$ is a means to steer the scenario building towards relevant information, and to prevent infinite scenario expansion.
$SEED = STV_{SEED}$	Definition of the boundaries of the system under consideration, $V(SEED)$ can be determined autonomously by the experts involved (i.e., without further information from within the network of experts), $SEED$ enables the identification of the starting points for the scenario generation
$INIT = \langle STV_{init}, SSPV_{init} \rangle$	Definition of information that *must* be addressed by the set of scenarios built by providing for some variables sets of possible values

their roles in the scenario building process. From a request for the determination of values for $FOCUS$, the resolution of tasks dependencies between experts configures DAG_D, which includes all relevant variables for decision problem D.

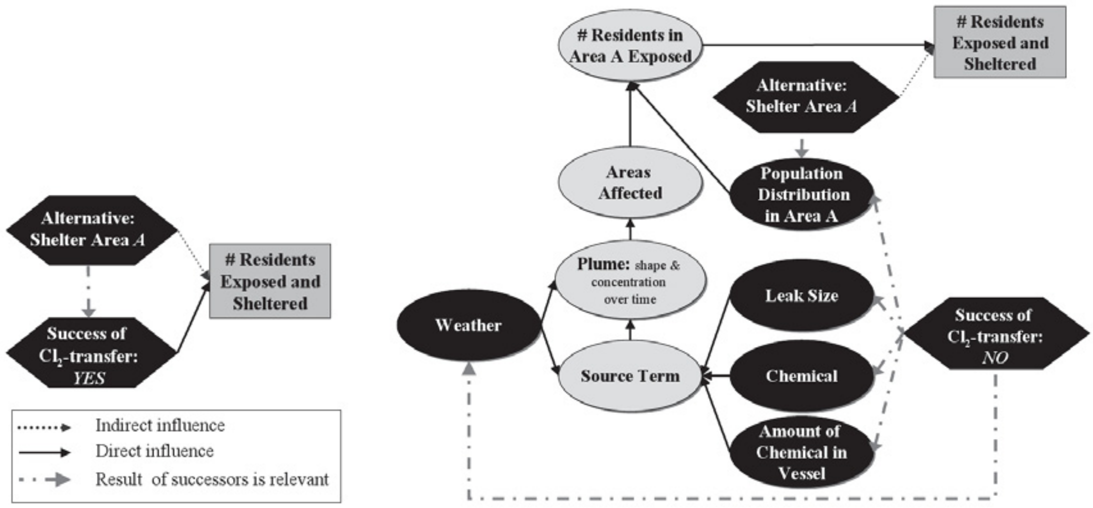
To structure the decision problem and to determine $FOCUS$, an **attribute tree** has been elicited in several workshops from potentially involved decision-makers. This tree includes the criteria *Health, Effort* and *Impact on Society,* which are broken down into 39 attributes. To illustrate the configuration of the Decision Map, this chapter focuses on health topics, and shows how the partial Decision Map for assessing the scores for the attribute *Number of residents exposed and sheltered* is constructed. Here, this attribute encompasses the number of residents present in the area affected, where the alternative *sheltering* is applied and where the concentration of the chemical exceeds a certain threshold. Although only a portion of the overall causal map is used, this example is rich enough to highlight relevant aspects while sufficiently constrained to not overcomplicate the example.

First the (best) expert able to provide information about the *Number of residents exposed* is identified. In the following, this attribute is referred to as tv_1^F. The expert determining $V\left(tv_1^F\right)$ indicates that he needs information on the alternative considered for implementation. Particularly, information on the areas, where sheltering is considered, is required. For a_1 (*Do Nothing*), the number of people sheltered is 0, and therefore, $V\left(tv_1^F\right)$ can be determined only on the basis of the alternative (represented by the variable tv_A) for all S_i with $tv_A = a_1$. In the examples shown in Figure 9.1, the alternative to be evaluated is a_2, and the area of sheltering is area $\mathcal{A}$. Additional information on the assumptions of the success of the transfer is crucial. If the transfer succeeds, no plume is created, and therefore the population is not exposed to the chlorine. Hence $V_{success}\left(tv_1^F\right) = 0$ (see Figure 9.1(b)). Contrarily, if the transfer fails and chlorine is released, further information is needed (see Figure 9.1(b)).

Continuing this process iteratively, DAG$_D$ expands until all paths start with a variable $tv_j \in SEED$. $SEED_{MCDA}(B)$ contains those variables, which provide information available relevant for the situation (as described in the MCDA-backdrop), for which there is an expert who does not rely on further information from within the framework (besides, potentially, the information on tv_A's value) to determine the variables' value(s), cf. Section 4.1.4. For the problem D, $SEED_D = \left\{tv_1^{SEED}, \ldots, tv_7^{SEED}\right\}$ where

$$\begin{aligned}
name\left(tv_1^{SEED}\right) &= \text{Chemical,}\\
name\left(tv_2^{SEED}\right) &= \text{Leak Size,}\\
name\left(tv_3^{SEED}\right) &= \text{Amount of Chemical in Vessel,}\\
name\left(tv_4^{SEED}\right) &= \text{Weather Information,}\\
name\left(tv_5^{SEED}\right) &= \text{Population Distribution in Area } A\text{,}\\
name\left(tv_6^{SEED}\right) &= \text{Success of Chlorine Transfer}\\
name\left(tv_7^{SEED}\right) &= \text{Decision Alternative.}
\end{aligned}$$

These are depicted as the dark vertices in Figure 9.1. Hence, both the alternative as well as the success of the chlorine transfer belong to the set of seed variables, although their values can neither be measured nor observed (as their value will only be realised in the future). The particular role of tv_6^{SEED} and tv_7^{SEED} whose value influences the structure of the graph is highlighted by representing them as diamonds in Figure 9.1. Furthermore, their influence on the other variables' relevance is represented by the dashed lines.



(a) No Chlorine Release (b) Chlorine Release

Figure 9.1.: *Possible Structures Of DAG_D. Example: determining* Number (#) of residents exposed and sheltered *for alternative a_2:* Shelter area A *depending on the* Success of Cl_2 transfer. *Focus variables represented in grey boxes, seed variables in black, variables whose value has an influence on the structure of DAG_D in diamonds.*

$FOCUS$ corresponds to the set of attributes, each of which must be assigned a score to evaluate the alternatives. For reasons of clarity and brevity in the example, only the attribute *Number of residents exposed and sheltered* is considered. For a valid evaluation, of course, the complete set of attributes must be considered, including information on various possible health effects, effort, and factors having am impact on the society (such as economic or ecological considerations).[57] For the example D shown here

$$\begin{aligned} FOCUS &= \left\{tv_1^F\right\}, \\ name\left(tv_1^F\right) &= \text{Number of residents exposed and sheltered.} \end{aligned}$$

Hence, the scenarios constructed in the example are focus complete, whenever $V\left(tv_1^F\right) \neq \infty$.

$INIT$ contains a set of variables, each of which is assigned a set of values to be investigated. $INIT$ serves for both representing the currently available information on the situation, e.g. variables that have been measured, as-

[57] For a description of the attributes see Section B.1, and for a presentation of the complete attribute tree see Section B.2.

sessed, calculated, etc., and for ensuring that certain values are considered (e.g., for legal reasons). Here, it includes the possible values of the alternatives $tv_A = tv_7^{SEED}$ and the possible realisations of the transfer success variable tv_6^{SEED}. Furthermore, it is assumed that the chemical as well as the sizes of the temporary vessels (that may cause the problem during transfer) are known and the types of leakage that may occur have been assessed.

$INIT_D$ contains the variables $tv_1^{INIT} = tv_1^{SEED}$, $tv_2^{INIT} = tv_2^{SEED}$, $tv_3^{INIT} = tv_6^{SEED}$and $tv_4^{INIT} = tv_7^{SEED} = tv_A$. The (sets of) values for each of these variables required to taken into account are

$$\begin{aligned}
sv^{INIT}\left(tv_1^{INIT}\right) &= \{Cl_2\}, \\
sv^{INIT}\left(tv_2^{INIT}\right) &= \{\text{small, medium, large}\}, \\
sv^{INIT}\left(tv_3^{INIT}\right) &= \{0,1\} \\
\text{and} \qquad sv^{INIT}\left(tv_4^{INIT}\right) &= \{a_1, a_2, a_3\}.
\end{aligned}$$

When the configuration of DAG_D is finished for the partial problem considered, the graph for the chlorine release scenarios can be represented as in Figure 9.2 (which is itself based on Figure 9.1(b)). The experts contributing to the scenario generation and their local causal maps are represented as well. For the vertices whose value is determined by the $INIT_D$, the underlying local causal maps are highlighted.

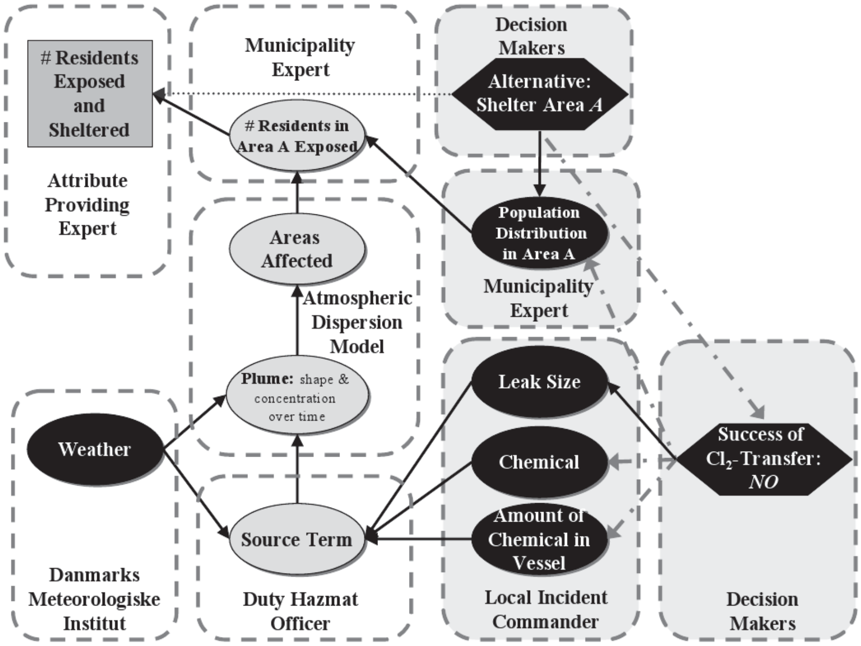


Figure 9.2.: *DAG_D and Collaborating Experts for the Chlorine Release Case*

9.3. Generation of Scenarios

After the configuration of DAG$_D$, scenarios are generated by processing information following the links established. First, the seed variables' values are determined. For the evaluation of *Do Nothing* (alternative a_1), the *Number of residents exposed and sheltered* is set to 0. For a_2 and a_3, which include both the sheltering of a certain region, a multitude of scenarios arises.

For instance, there is uncertainty about the transfer's success. According to the $INIT_D$, the variable *Success of transfer* is a binary, where both values *must* be considered. Therefore, two different sets of scenarios are generated. If *Success of transfer* is 1 (i.e., no chlorine is released), no residents are exposed and the *Number of residents exposed and sheltered* is set to 0. For the scenarios assuming that chlorine is released, the full graph DAG$_D$ (cf. Figure 9.2) needs to be considered. The role of the variables to be considered for the set of scenarios $SS\left(V\left(tv_6^{SEED}\right) = 0\right)$ is summarised in table 9.2.

Using the values represented in table 9.3 the scenario initialisation as described in Section 5.2 starts. For each combination i of values in $INIT$, a scenario S_i^{init} is derived (cf. equation 5.2). In this manner, $3 \cdot 3 \cdot 3 + 3 = 30$ initial scenarios are generated. On the basis of $SS^{init} = \left\{S_i^{init}\right\}_{i=1,\ldots,30}$ the scenario generation procedure successively determining the variables' values and sta-

Table 9.2.: *Initial Information for Starting the Scenario Generation Procedure for Chlorine Release Scenarios*

Variable	Name	$INIT_D$	$SOURCE_D$	$FOCUS_D$
tv_1^{SEED}	Chemical	**Yes**	**Yes**	No
tv_2^{SEED}	Leak size	**Yes**	**Yes**	No
tv_3^{SEED}	Amount of chemical in vessel	No	**Yes**	No
tv_4^{SEED}	Weather information	No	**Yes**	No
tv_5^{SEED}	Population in $\mathcal{A}$	No	**Yes**	No
tv_6^{SEED}	Transfer success	**Yes**	**Yes**	No
tv_6^{SEED}	Alternative	**Yes**	**Yes**	No
tv_1^F	Residents exposed and sheltered	No	No	**Yes**
tv_1	Source term	No	No	No
tv_2	Plume	No	No	No
tv_3	Areas affected	No	No	No
tv_4	Residents exposed in $\mathcal{A}$	No	No	No

tuses as described in Section 5.2 starts. To this end, focus incomplete scenarios are continued, extended or merged (cf. sections 3.6 and 3.7).

For instance, the expert determining the value of tv_3^{SEED} *(Amount of chemical left in the vessel)* states that there is still a *considerable amount* of the chemical left in the tank, which has a volume of 100 m^3. Apparently, the status of tv_3^{SEED} is $uncertain - F$ in all scenarios S_i, where $tv_3^{SEED} \in STV_i$. Here, the amount of chlorine left is modelled as a trapezoidal fuzzy number, cf. Figure 9.3. As the expert estimating the source term for chlorine release, which is the only direct successor of tv_3^{SEED}, specified that he can process up to three possible values for the amount of chlorine, $\lambda_3^{SEED} = 3$. An assessment which values to choose is made. As no indicators can be derived from tv_3^{SEED}, the scenario selection is based on tv_3^{SEED}'s status (cf. Section 8.1.5). The decision-makers defined a threshold of minimal acceptable membership $\gamma^{\min}$, which is contained in B. Let S_i^{init} be the initial scenario, which is extended in tv_1. To explore the of $Range\left(V\left(tv_3^{SEED}\right)\right)$, the extreme states $V_i^{\alpha_1}\left(tv_3^{SEED}\right)$ and $V_i^{\alpha_2}\left(tv_3^{SEED}\right)$ are chosen:

$$\begin{aligned} & V_i^{\alpha_1}\left(tv_3^{SEED}\right) &=& \inf\left\{V\left(tv_3^{SEED}\right):\ \mu\left(V\left(tv_3^{SEED}\right)\right) \geq \gamma^{\min}\right\} \\ \text{and} \quad & V_i^{\alpha_2}\left(tv_3^{SEED}\right) &=& \sup\left\{x : \mu(x) \geq \gamma^{\min}\right\}. \end{aligned}$$

Additionally, the mean of the kernel is used. As the threshold $\gamma^{\min}$ was set to 0.8, the values passed on are $V_1\left(tv_3^{SEED}\right) = 62$ m^3, $V_2\left(tv_3^{SEED}\right) = 72.5$ m^3 and $V_3\left(tv_3^{SEED}\right)^3 = 82$ m^3 (see Figure 9.3). For the leak sizes according to $INIT_D$, three values must be considered and passed on to the expert estimating the source term, who needs to consider each of the nine possible combinations of amount of chlorine and leak size.

For the weather conditions, particularly for the wind direction, probabilistic techniques can be applied [Raskob et al., 2009]. Three possible meteorological conditions are passed on. Subsequently, the nine source terms estimated as well as the three meteorological conditions are used by an atmospheric transport and dispersion model to predict the according (27) plume shapes.

Again, the number of plume shapes need to be reduced. This time, nine plume shapes are allowed. To this end, the local scenario selection procedure based on indicator similarity is performed (cf. Section 8.1.4). The indicators chosen are: number of firms affected by the plume, number of residents affected by the plume, number of kindergartens affected by the plume. Using a geographic information system containing land register and further information

Table 9.3.: *Information for Determining SS^{init} for all Potentially Relevant Variables*

tv	Name	SPV_D	$status_D$	Depends on
tv_1^{SEED}	Chemical	Cl_2	deterministic	$\varnothing$
tv_2^{SEED}	Leak size	small, medium, large	uncertain	$\varnothing$
tv_3^{SEED}	Amount of chemical in vessel	∞	not assessed	$\varnothing$
tv_4^{SEED}	Weather	∞	not assessed	$\varnothing$
tv_5^{SEED}	Population in $\mathcal{A}$	∞	not assessed	$\varnothing$
tv_6^{SEED}	Transfer success	0, 1	uncertain	$\varnothing$
tv_7^{SEED}	Alternative	a_1, a_2, a_3	uncertain	$\varnothing$
tv_1^F	Residents exposed and sheltered	∞	not assessed	tv_4
tv_1	Source term	∞	not assessed	tv_1^{SEED}, ..., tv_4^{SEED}
tv_2	Plume	∞	not assessed	tv_4^{SEED}, tv_1
tv_3	Areas affected	∞	not assessed	tv_2
tv_4	Residents exposed in $\mathcal{A}$	∞	not assessed	tv_5^{SEED}, tv_3

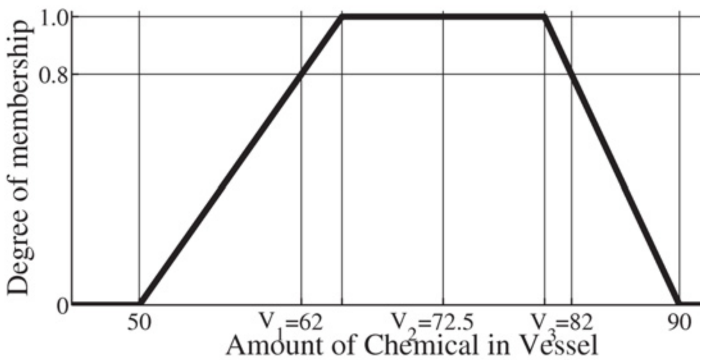


Figure 9.3.: *Local Scenario Selection Based on a Fuzzy Membership Function. Example modelling* "Considerable amount of chlorine left in the tank" *as a trapezoidal fuzzy number.*

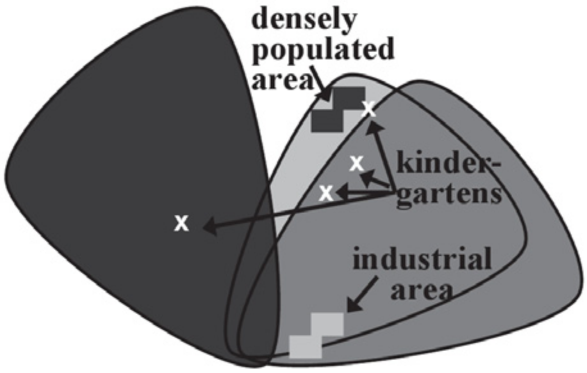


Figure 9.4.: *Indicator-Based Scenario Selection. Example of three plume shapes.*

from the municipality, these data can be derived from the plume shape and the underlying maps. For each plume shape, the indicator values are determined. In Figure 9.4, three plume shapes with information relevant for determining the indicator values are shown.

To compare the approach to scenario selection developed in this thesis with other approaches (cf. Section 8.1.2) assume that only two of the three plumes shapes in Figure 9.4 can be handed over to the next expert(s). Formative Scenario Analysis approaches use the discrete metrics as a measure of distance between scenarios [Tietje, 2005]. Therefore no distinction between the three plume shapes could be made. Other approaches introducing different measures of distance rely on the difference in the variables' values [Ahmed et al., 2010]. In this example, the plume shapes themselves would be used to introduce a metrics for assessing their difference.(e.g., the size of the area not covered by both, i.e., in $Size\left(\left(Plume_i \cup Plume_j\right) \setminus \left(Plume_i \cap Plume_j\right)\right)$). Using this notion of dis-

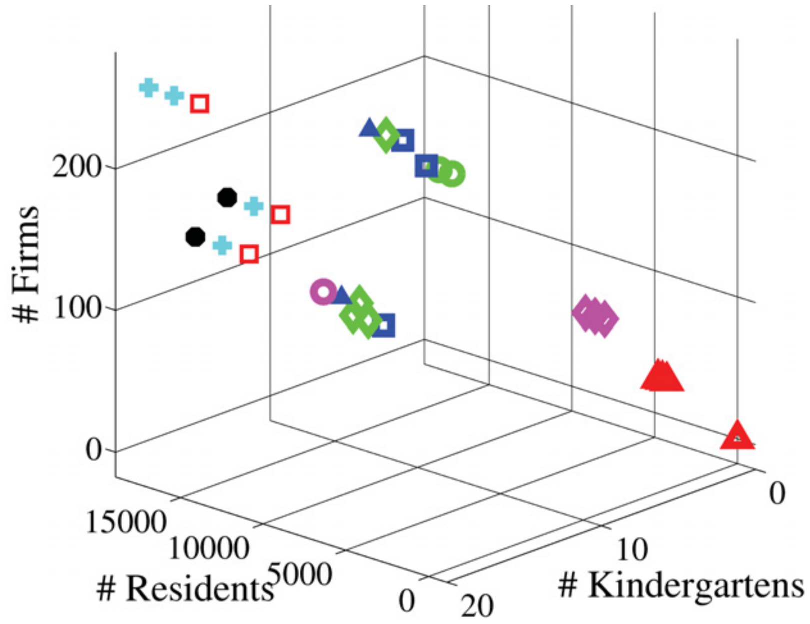


Figure 9.5.: *Example for Local Scenario Selection Based on Indicator Similarity: Resulting Scenario Equivalence Classes*

tance, the outer shapes would be passed on, as these areas have the smallest area of intersection (cf. Figure 9.4). The medium plume shape, however, threatens densely populated areas and some kindergartens, while for the left plume shape, less elements at risk are exposed, which is expressed in a minor presence of indicators.

To overcome these weaknesses, the novel approach based on an assessment of indicators (cf. Section 8.1) estimates the difference of scenarios with respect to the goals of the decision-makers (incorporated in the focus variables and their evaluation). Here, the indicators *Number of residents affected* (by the plume) and *Number of kindergartens affected* are both a means to assess attributes falling under the criteria *Health* and *Effort*. Similarly, *Number of residents affected* and *Number of firms affected* can be used to assess attributes contributing to the criterion *Impact on Society*.

By taking into account the inter-criteria preferences, weights for the importance of each indicator are derived (cf. equation 7.16). The weights are used to construct nine scenario equivalence classes, which are shown in Figure 9.5. In that figure, each class is assigned a different symbol, and each scenario within each class is plotted with respect to the three indicator values. Finally, a representative for each class is chosen, such that the distance to the next representatives is maximised for the whole set of scenarios. These representatives are then the basis for further scenario construction.

9.4. Evaluation of Alternatives

When all generated scenarios for all values in $INIT_D$ are focus complete, the attribute tree is used to perform the evaluation using techniques from MAVT as introduced in Section 2.1.3. Denote $SS(a_l)$ the set of scenarios constructed for an alternative $a_l \in A$. Calculating the attributes' values and aggregating them for each scenario results in an in-depth evaluation with respect to multiple objectives of each $S_i(a_l) \in SS$. Particularly, the overall performance $R(S_i(a_l))$ for each $S_i(a_l) \in SS(a_l)$ allows the scenarios $S_j(a_l)$ to be ranked.

As the number of scenarios is usually large, and decision-makers can only cope with about seven scenarios at a time [Miller, 1956], the complexity (the number of scenarios and results to consider) must be reduced by either **selecting scenario results** considered to be the most relevant or by **aggregation.** Section 5.3 showed that the selection of best and worst evaluated scenarios is supported by the integrated SBR & MCDA framework. Yet, a drawback of selecting only a subset of $SS(a_l)$ for presentation to the decision-makers is that it does not convey the full amount of information. Furthermore, decision-makers can be biased particularly by worst case scenarios [Hämäläinen et al., 2000]. Therefore, the selection of scenarios is accompanied by an aggregation of results. In the following, first, an example of the **global scenario** selection based on the newly developed concept of scenario equivalence classes is presented (cf. Section 7.3 for the definition of scenario equivalence classes and Section 8.1.3 for the approach to selection of focus complete scenarios). Then, the **aggregation of scenario results** is demonstrated. In Section 5.3, two approaches for explicitly taking into account the risk attitudes and preferences for the importance of each scenario have been developed. Both are demonstrated in this section.

9.4.1. Presenting Individual Scenario Results to the Decision-Makers

A common approach in scenario planning is to present the worst and best scenarios to the decision-makers [Schnaars, 1987]. As a structured evaluation procedure is usually not part of scenario planning process [Durbach and Stewart,

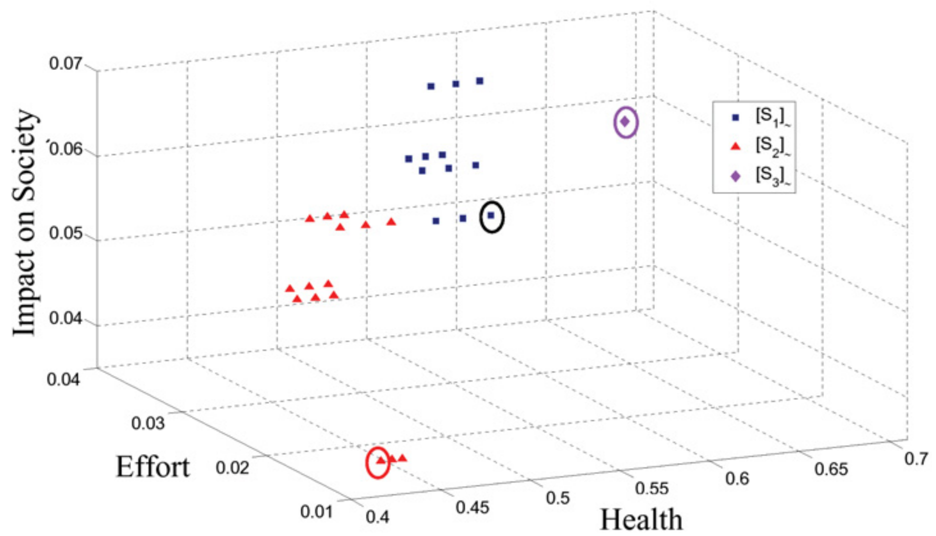


Figure 9.6.: *Scenario Equivalence Classes According to Evaluation. Representatives chosen per class are highlighted.*

2003], the selection of scenarios labelled "*worst*" and "*best*" is usually rather intuitive and highly demanding for the experts involved. By integrating scenario construction and evaluation, worst- and best-*evaluated* scenarios $S^w(a_l)$ and $S^b(a_l)$ can be identified for each alternative.

Moreover, the global scenario selection procedure can be used to come to a more fine-grained representation, when the decision-makers would like to see the results for more than two scenarios per alternative. Assume that the decision-makers would like to base their decision on three scenarios per alternative. Then, for each of the sets $SS(a_l)$ ($l = 1, 2, 3$), the global scenario selection procedure as developed in Section 8.1.3 is performed, and three scenario equivalence classes are built. For the set of scenarios $SS(a_3)$ (i.e., the evacuation scenarios), the arising equivalence classes are shown in Figure 9.6.

Finally, a **representative** of each class is chosen. For a_3, these representatives are highlighted with circles in Figure 9.6. For each of these representatives, an in-depth evaluation is represented to the decision-makers. For example, stacked bar charts as shown in Figure 9.7 present the overall performance of each alternative in each of the three classes, and the performance with respect to each criterion. The evacuation alternative shows the highest stability of results: for evacuation, the distance between the worst and best evaluated scenar-

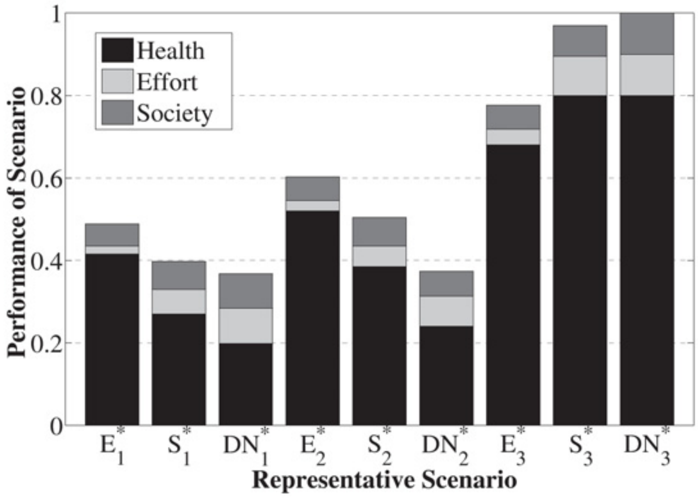


Figure 9.7.: *Performances of Scenario Equivalence Class Representatives for all Alternatives with respect to Criteria* Health, Effort *and (Impact on)* Society

ios' results, $R(E_3^*) - R(E_1^*)$ in Figure 9.7 is smaller the distances for sheltering $(R(S_3^*) - R(S_1^*))$ and do nothing $(R(DN_3^*) - R(DN_1^*))$.

$$\min_{i=1,2,3} R\left(E_i^*\right) > \min_{i=1,2,3} R\left(S_i^*\right),$$

$$\min_{i=1,2,3} R\left(E_i^*\right) > \min_{i=1,2,3} R\left(DN_i^*\right).$$

Therefore, a_3 is the most **robust** alternative. Alternative a_1 (do nothing), however, offers the **best chances,** as

$$\max_{i=1,2,3} R\left(DN_i^*\right) > \max_{i=1,2,3} R\left(E_i^*\right),$$

$$\max_{i=1,2,3} R\left(DN_i^*\right) > \max_{i=1,2,3} R\left(S_i^*\right).$$

By specifying that the good performance for a_1 occurs in the scenarios in which the transfer of chlorine succeeds, it becomes apparent that do nothing is the preferred alternative when the danger of chlorine release can be neglected.

9.4.2. Presenting Aggregated Results to the Decision-Makers

In Section 5.3 two novel methods for the elicitation of scenario preferences from the decision-makers have been developed. By using these methods, the evalu-

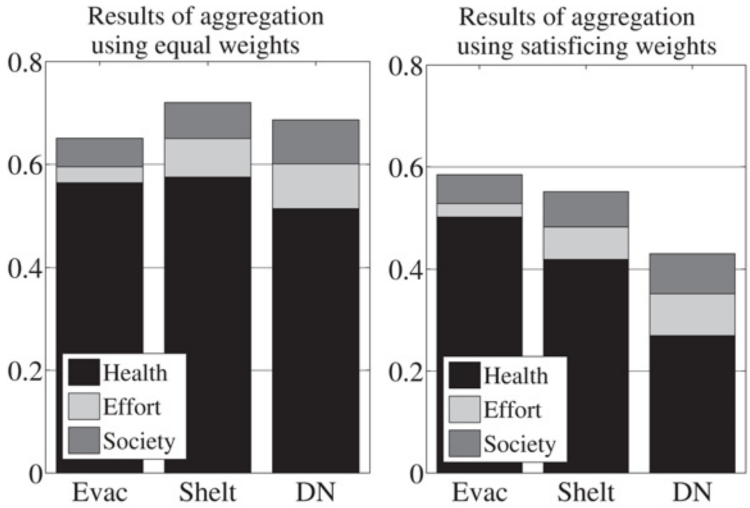


Figure 9.8.: *Aggregated Scenario Results For All Alternatives. Comparison of results for equal weights (left side) with results using weights determined with respect to criterion* Health *with* $t^{risk}_{Health} = 0.25$ *(right side).*

ation results can be aggregated whilst respecting the risk averseness of all involved actors. In the following, results for both methods are presented.

9.4.2.1. Aggregation of Results Based on the Satisficing Weights

To illustrate the technique for the elicitation of scenario preferences based on the concept of *satisficing* (see Section 5.3.3.1), assume that in sum, 28 focus complete scenarios per alternative were constructed. For this illustrative example, these comprise only uncertainties about the success of the transfer and the local incident commanders' uncertainty about the amount of chemical within the vessel as well as the size of the potential leak. Based on the preferences of users from the Danish Emergency Management Agency elicited during two workshops, all scenarios were evaluated. Yet, presenting all ($28 \cdot 3 = 84$) individual scenario results to the decision-makers is not considered useful.

Figure 9.8 shows a comparison of aggregated results. On the left side, equal scenario weights $\omega(S_j(a_i)) = 1/28$ ($i = 1, 2, 3$) were used to determine the total performance of a_1, a_2 and a_3. In this case, a_2 (sheltering) is the best evaluated alternative, followed by a_1 (do nothing). Alternative a_3 (evacuation) has the worst performance.

The introduction of a risk threshold $t^{risk}_{Health} = 0.25$ for criterion *Health* and the use of exponential penalty functions (see equation cf. equation 5.7) with $\rho = 0.1$

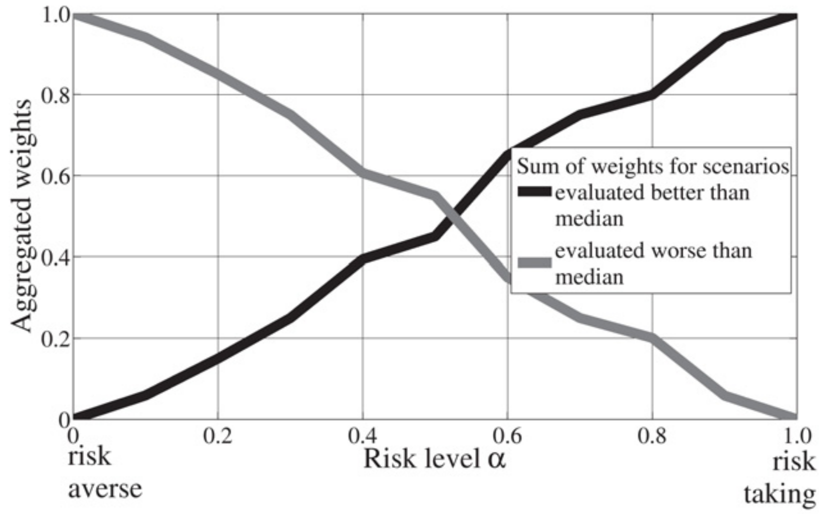


Figure 9.9.: *Sum of Weights for* Optimistic *and* Pessimistic *Scenarios (evaluated better/worse than median) Determined by Solving Problem 5.14*

(cf. dashed line in Figure 5.5) reduces the total performances (as the weights are generally reduced) and changes the ranking (see right part in Figure 9.8): a_3 (evacuation) performs best, followed by a_2 (sheltering) and a_1 (do nothing). The reason for this change is that most evacuation scenarios do not violate the constraint $t^{risk}_{Health} = 0.25$. Furthermore, the violations of both evacuation and sheltering scenarios are less severe than the violation for do nothing.

9.4.2.2. Aggregation of Results Based on Inclination Towards Risk Weights

To illustrate the technique of preference elicitation based on the inclination towards risk, where the decision-makers need to specify only one parameter, and the scenario weights are derived using the Ordered Weighted Average (OWA) operator (see Section 5.3.3.2, and particularly, equation 5.14 defining the problem to solve for determining the scenario weights).

Figure 9.9 shows the sum of weights for the "optimistic" scenarios (black increasing line), whose performance is better than the median of scenario performances, and "pessimistic" scenarios (grey decreasing line), whose performance is worse than the median of scenario performances. Apparently, the more risk taking the decision-makers are, the higher the weight for the optimistic and the lower the weight for the pessimistic scenarios.

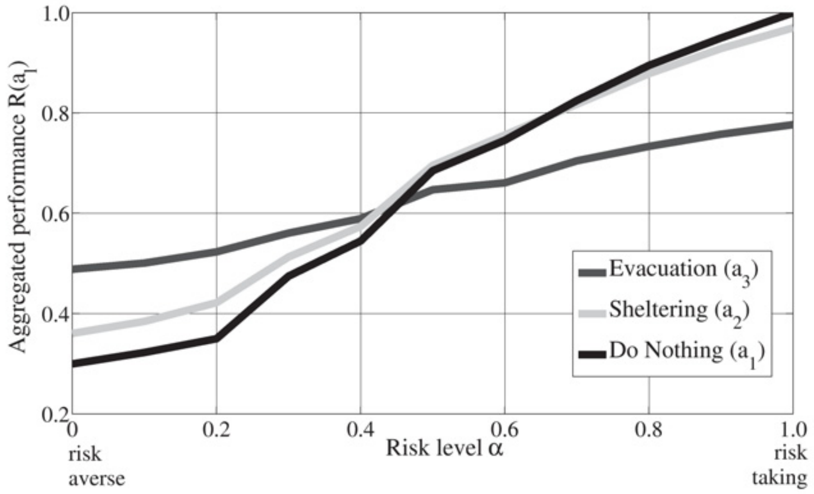


Figure 9.10.: *Aggregated Performances for Varying Risk Aversion Levels* α

Evaluating the alternatives a_1 to a_3 for varying inclinations towards risk ($\alpha \in [0, 1]$) yields the results shown in Figure 9.10: while risk averse decision-makers prefer alternative a_3 (evacuation), risk taking decision-makers tend to opt for alternative a_1 (do nothing). In the medium range, a_2 (sheltering) is the preferred alternative.

9.4.2.3. Correction of Scenario Weights by Reliability Assessment

The aggregation of scenario results is based on the assumption that the weights reflecting the importance of scenarios can be based on the scenarios' performance in one or more criteria. Yet, this approach does not reflect the *reliability* of results. In the Decision Map approach, the number of scenarios depends on the structural properties of the underlying network.

Given a set of graphs $\mathcal{G}_{SS} = \{\mathcal{G}_i\}_{i \in I} = \{STV_i, DI_i\}_{i \in I}$, from which the set of scenarios SS is derived, for each $\mathcal{G}_i$, the number of scenarios constructed on basis of $\mathcal{G}_i$ grows with

- the number of uncertain variables, i.e., the number of $tv_j \in STV_i$, for which $status\,(tv_j) =$ "$uncertain$" and
- the number of values per uncertain variable, i.e., $|SPV_i\,(tv_j)|$: $tv_j \in U$.

This characteristic does not correspond to the (frequentist) probabilistic approaches, where the relative frequency of occurrence does increase the probability of an event [Morgan and Henrion, 1990]. To avoid irritation of the decision-makers and misunderstandings, the newly developed reliability assessment presented in Section 4.3.4 can be used to balance the results.

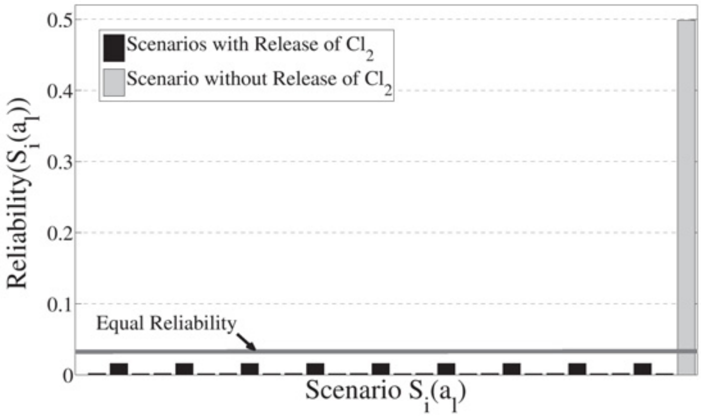


Figure 9.11.: *Reliability of Scenarios $S_i(a_l)$ with respect to the Structure of the Underlying Graphs and the Uncertainties in Equation 9.1 Contrasted to the Assumption of Equal Reliabilities*

Assume now that the variables with uncertain status are tv_6^{SEED} *(Transfer Success)*, tv_3^{SEED} *(Amount of chemical in vessel)*, tv_4^{SEED} *(Weather Information)* and tv_4 *(Residents exposed in Area $\mathcal{A}$)*, where the latter depends on assumptions about the presence of the residential population within area $\mathcal{A}$. Furthermore, an assessment of probability bounds is available for tv_6^{SEED} and tv_4^{SEED}. It holds

$$\begin{aligned} & \pi^l\left(V\left(tv_6^{SEED}\right)=1\right)=0.9 \\ \text{and} \qquad & \pi^l\left(V\left(tv_4^{SEED}\right)=standard\right)=0.9. \end{aligned} \tag{9.1}$$

That means, a lower bound of the probability that the transfer succeeds and no chlorine is released is 0.9. Equally, a lower bound for the probability that the weather corresponds to standard conditions is 0.9. Figure 9.1 shows the underlying graph structures for the scenarios with varying values for tv_6^{SEED}. While for $V\left(tv_6^{SEED}\right)=1$ only one uncertain variable is present in the arising scenarios $SS\left(a_l, V\left(tv_6^{SEED}\right)=1\right)$ namely, tv_6^{SEED}), for $SS\left(a_l, V\left(tv_6^{SEED}\right)=0\right)$, four uncertain variables influence the focus variable tv_1^F. The underlying parameters μ^r and ν^l (see equations 4.5 and 4.7) are set to $\mu^r=0.85$ and $\nu^l=0.3$. On the whole, the reliability assessment varies considerably between both sets of scenarios, cf. Figure 9.11 showing reliabilities for the sets of scenarios constructed for one alternative a_l. The differences between the scenarios in $SS\left(a_l, V\left(tv_6^{SEED}\right)=0\right)$ stem from the varying weather informations incorporated.

The results determined using the OWA-weights can be corrected by the reliability assessment. To this end, a weight $0 \le \omega_{Rel} \le 1$ is chosen. This weight

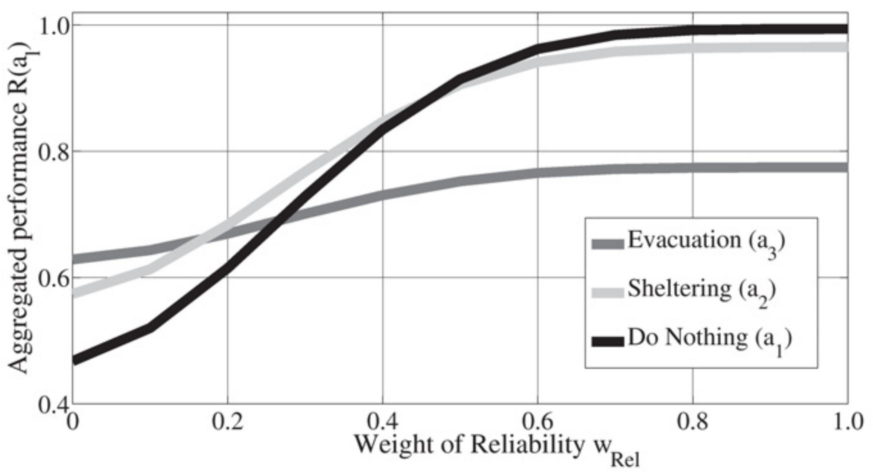


Figure 9.12.: *Aggregated Performances for Varying Reliability Weights* ω_{Rel}

reflects the importance of the reliability assessment compared to the assessment of scenario importance ω_{Imp} with respect to the performance in the criteria selected (which can be weights derived via the concept of satisficing or using the OWA-approach).

Then, the weight of each scenario $S_i(a_l) \in SS(a_l)$ is

$$\omega_{total}(S_i(a_l)) = (1 - \omega_{Rel})\,\omega_{Imp}(S_i(a_l)) + \omega_{Rel} reliability_{S_i(a_l)}\left(\mu^r, \nu^l\right). \tag{9.2}$$

On basis of equation 9.2, the overall performance of each alternative with respect to both its performance and its reliability can be assessed. Figure 9.12 shows an example, where for given ω_{Imp} derived via the OWA approach (with $\alpha = 0.2$), the weight of the reliability is varied.

As the scenarios, for which the transfer succeeds, are much more reliable than the scenarios, where the transfer fails and the chlorine is released, the performance of alternative a_1 *(Do Nothing)* increases with increasing reliability weights. Contrarily, when the performance (e.g., with respect to criterion health) is more important, and the reliability assessment is considered negligible, alternative a_3 *(Evacuation)* performs best (for the inclination towards risk $\alpha = 0.2$) chosen. In between, there is a realm, where alternative a_2 *(Sheltering)* has the best performance.

Finally, Figure 9.13 shows the results for varying the parameter determining the inclination towards risk of the decision-makers, α, and the weight for the reliability, ω_{Rel} at a time. Apparently, a_3 performs best for low values of the reliability weights and a rather risk averse attitude of the decision-makers.

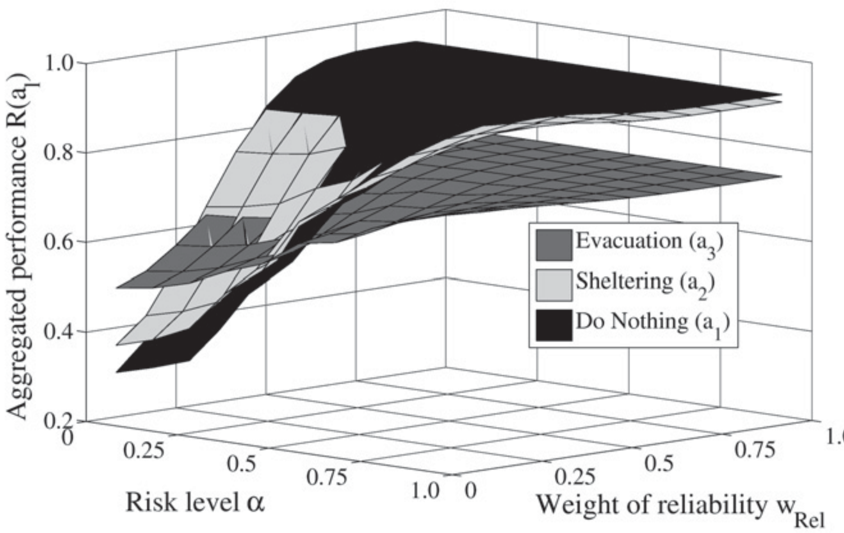


Figure 9.13.: *Aggregated Performances for Simultaneous Variation of Reliability Weight ω_{Rel} and Risk Aversion Level α*

Contrarily, a_1 is preferred by decision-makers, for whom the reliability of the scenarios is decisive and/or who show risk taking behaviour. Finally, alternative a_2 minimizes the regret, as it has—on average—the lowest distance to the alternative with the best performance.

10. Conclusions and Outlook

We dance around in a ring and suppose,
But the secret sits in the middle and knows.
(Robert Frost)

Contemporary industrial, economic, social and environmental systems are shaped by their complexity and uncertainty. Each of these systems can be characterised as a highly interlaced network. Hence, solving strategic decision problems requires a well-structured multidisciplinary approach, which takes into account the present uncertainties in an adequate manner. This thesis proposes a framework for decision support that respects multiple goals in complex situations susceptible to severe uncertainty, where the decision consists in choosing one of a small set of alternatives.

To achieve the aim of providing **robust decision support** whilst taking into account the chances and risks associated with each alternative, an approach combining Multi-Attribute Value Theory (MAVT) and Scenario-Based Reasoning (SBR) has been developed. MAVT facilitates making trade-offs between conflicting goals by taking into account the decision-makers' preferences in a transparent and well-structured way. Scenarios, which are understood as descriptions of the situation and its possible future developments, are used to consider uncertainties in an easily understandable way.

The formalisation represents each scenario as a directed acyclic graph (DAG) that captures the relevant impact factors, their values as variables and their mutual interdependencies as edges between the variables. The use of DAGs enables efficient distributed scenario generation. In this manner, expert knowledge from various domains can be integrated in the scenarios, although time and availability of experts may be limited. The scenario construction process is *efficient*, as it provides mechanisms to avoid the processing of irrelevant or redundant information. The scenario generation process is particularly suitable for large and complex situations that preclude standardised solutions, as it allows for the flexible adaptation of the reasoning principles used to the problem at hand and the information available.

The integration of MAVT and SBR is achieved by **Decision Maps** that link scenarios and their evaluation. More precisely, Decision Maps couple the DAGs for scenario generation with MAVT attribute trees, which permit the scenarios' evaluation. By the integration of MAVT and SBR, important features of each technique are established or strengthened: SBR is a means to ensure the robustness of the decision in MCDA, and on basis of MAVT it becomes possible to steer and manage the scenario construction process.

To ensure that the scenarios generated provide a valid basis for supporting decision-makers, novel approaches to **scenario management** have been developed. The scenario formalisation and evaluation methodology enables the design of a rigorous framework to specify and assess the requirements for the *quality* of single scenarios and sets of scenarios (e.g., in terms of correctness, coherence, consistency, credibility of experts, reliability, relevance, currentness and balance). On the basis of *relevance* and *reliability* assessments novel approaches that facilitate the control of the (potential) combinatorial explosion of the number of scenarios have been developed. At the same time, scenario management warrants that requirements on each scenario's quality are met. Scenarios are *pruned* when they are considered invalid, and *updated* whenever sufficiently relevant new information emerges.

On the whole, the Decision Map approach leads to a decision support system that takes into account the requirements of complex strategic decision-making: the integration of expert knowledge from different domains, the consideration of different types of uncertainties, the reduction of information overload, the compliance with scenario quality requirements, the assessment of alternatives with respect to multiple goals, the explicit elicitation of preferences to make trade-offs and the robustness of the recommended solution.

The remainder of this chapter is divided into two main secions. Section 10.1 shows how each of the identified objectives enabling **robust decision support for large and complex strategic decision problems** (cf. section 1.3) has been achieved. Section 10.2 points out how the findings can be extended methodologically and adapted to further classes of decision-making problems.

10.1. Key Findings and Conclusions

Decision-making in complex situations is a challenging task. Decision-makers need to identify and evaluate alternatives whilst taking into account multiple, in general at least partly conflicting objectives, although the information underlying the decision may be limited, uncertain or contradictory [Morgan and Henrion, 1990]. The availability and intelligibility of relevant information are crucial to provide reliable decision support. As the situation is complex, information from different sources must be acquired, structured, analysed, evaluated and distributed to the right experts at the right time in the right form.

To achieve the aim of robust and transparent decision support providing all relevant information to experts and decision-makers in situations of severe uncertainty, this thesis developed a three-step approach of determining scenario structures, scenario construction and evaluation. First, automated systems and multiple (human) experts from different fields are linked for information procurement, organisation and processing. The resulting networks of expertise structurally correspond to directed acyclic graphs (DAGs). Second, the DAGs are exploited to construct scenarios tailored for the decision problem at hand, whereby the goals and preferences of the decision-makers steer this process. In particular, the MAVT attribute tree provides a rationale for constructing *decision-relevant scenarios.* This approach enables the reduction of information overload of the experts involved in the scenario construction process and the decision-makers, to whom the final results are presented. Third, scenarios are used to support decision-making under uncertainty. The scenarios provide the basis for exploring the performances of alternatives under varying assumptions about how the future might unfold. This approach enables the identification of *robust* alternatives, i.e., it favours alternatives that performs sufficiently well for a variety of scenarios.

A number of objectives have been identified that must be attained to achieve the aim of robust Multi-Criteria Decision Support. These objectives include

- the combination and processing of different types of information into **meaningful scenarios,**
- the **relevance** and purposefulness of scenarios,
- the **acceptability** of scenarios,
- the **evaluation** of each alternative for varying scenarios,
- the analysis of each alternative's **robustness,** and

- the **manageability** of the scenario building and evaluation processes.

The remainder of this section briefly shows how each of these objectives has been achieved.

It is mandatory that different pieces of information collected are combined and processed into **meaningful scenarios** whilst taking into account different principles for capturing uncertainty in a well-structured manner. This thesis has presented a novel system supporting the collaborative processing of information by combining the (cognitive) capabilities of multiple human experts with automated reasoning processes, each contributing specific expertise and processing resources. Local Causal Maps (CMs) are used to organise and structure information processing and sharing. In this manner, the presented approach is particularly suitable for large and complex strategic decision problems, where expertise from several domains has to be brought together, time is limited, and the availability of all or some experts are bounded. As the work-flows are constructed at runtime, the Decision Map is particularly flexible and supports decision-makers in dynamic, highly varying and uncertain environments, as well as in situations that potentially involve rare events.

To ensure their **relevance,** the scenarios must be tailored to the recipients' information needs; they must address the problem at hand and answer the question(s) relevant for scenario recipients. The novel concepts of focus variables and initial situation descriptions have been developed to ensure that the scenarios comply with the recipients' information needs. The set of *focus variables* contains all variables that *must* be included in the scenarios. Considering the valuation of all focus variables necessary and sufficient for the completion of a scenario, the set of focus variables steers the scenario building process and prevents infinite scenario expansion. The *initial situation description* is a means to define information that must be included in the set of scenarios. It contains a set of variables and for each variable a set of values that must be considered.

To ensure the scenarios' **acceptability** and credibility, the recipients' quality requirements must be respected. *Scenario coherence, plausibility* and *consistency* are ensured locally. The experts contributing to the scenario construction are presumed to be able to specify their services in terms of local CMs. These local CMs are merged to a global DAG that allows the experts to be organized in an information processing work-flow. Each scenario arising from coherent local CMs is itself coherent, as the global DAG explicitly represents interdependencies. Concerning the plausibility, the presented approach assumes that each expert provides information to the best of his knowledge given (potential) constraints regarding the information and time available. By keeping track of the expertise and concepts used and by assessing the credibility of each concept, it is possible to qualify the plausibility of scenarios. For consistency, similar considerations hold. It is assumed that each expert is capable of specifying the factors that have an impact on his assessment and that the output of each expert agrees with the input he received.

The recipients' requirements also constitute a key element of scenario management. The newly developed approaches to assess a scenario's reliability and relevance facilitate scenario selection and updating. In scenario pruning, the recipients are asked to specify which requirements (in form and content) each scenario must fulfil as well as their preferred sources of information. On the whole, these novel approaches enable building scenarios that respect the scenario recipients' quality requirements.

The coupling of the DAG and the attribute tree facilitates the implementation of a distributed system taking into account information from various sources to **evaluate each alternative** with respect to multiple criteria for varying scenarios. This thesis has presented a new methodology that integrates scenario-based reasoning (SBR) and MCDA. The MCDA attributes serve as focus variables in the scenario building process. In this way, it is possible to prevent infinite scenario expansion and to identify *relevant* variables: a variable is deemed relevant when changing the variable's value has an impact on at least one attribute's score. Having determined a (focus complete) set of scenarios $SS(a_l)$ for each alternative $a_l \in A$, each scenario $S_i \in SS(a_l)$ can be evaluated using techniques from MAVT. This enables the application of several approaches for providing robust decision support.

To achieve the objective of providing **robust decision support,** this thesis uses two approaches. A scenario selection approach that presents only the most significant scenarios' results to the decision-makers is complemented by an aggregation approach that combines the results of *all* scenario evaluations.

Selection of the most significant scenarios: the Decision Map approach facilitates the identification of best and worst evaluated scenarios. This allows for the decision-makers' preferences to explicitly be taken into account. Comparing the values of the worst and best evaluated scenarios *across* alternatives yields insights into the strengths and drawbacks of each alternative. A better understanding of the reasons why or the circumstances under which different performances arise may result in the refinement of alternatives and enable the systematic development of a set of (more) robust alternatives.

Aggregation of the scenario evaluations: an additional aggregation step taking into account the performances of each alternative under varying scenarios is applied to avoid cognitive biases such as overconfidence or anchoring. To facilitate the elicitation of *scenario importance weights*, two novel methods have been developed. The first method is based on the concept of *satisficing*. The second method, which is less demanding in terms of the necessary specifications, is the determination of weights by eliciting the decision-makers' *inclination towards risk*. Which method to choose depends on the time available for the elicitation of weights and the expertise of the decision-makers.

While scenario importance weights reflect the risk associated with each scenario and the decision-makers' attitude towards risk, *scenario reliability weights* represent each scenario's reliability. Here, reliability is understood as a subjective concept that is founded on the preferences and beliefs of the decision-makers. Accordingly, scenario reliability is assessed on the basis of thresholds and requirements elicited from the decision-makers. Combining both scenario importance and reliability weights in the evaluation of alternatives allows for balancing risk and likelihood considerations. If there is uncertainty about the appropriate weighting, sensitivity analyses provide further support.

The scenario building and evaluation processes must be **manageable.** Constraints in terms of time available for the decision-making process, bounded availability of experts as well as limited resources and capacities for information processing must be respected. The newly developed approaches for scenario management (including scenario selection, pruning and updating) combine the distributed scenario building process with a decision-centric scenario management component. This component provides an overview of the ongoing processes and steers the scenario building.

Scenario management controls the number of scenarios being generated by *selecting* and further developing only the most relevant scenarios throughout all phases of the scenario generation process. To this end, the constraints, requirements and preferences of experts (in terms of capacities and effort) and decision-makers (in terms of time available for the decision-making, quality, relevance and reliability requirements) are taken into account. Besides scenario selection, *pruning* of irrelevant, incredible, structurally incorrect or falsified scenarios ensures that the quality of scenarios justifies their use as a basis for the decision to be made. Lastly, scenario management enables efficient scenario *updates.* The scenario update approach facilitates the adaptation of scenarios whenever sufficiently relevant new information emerges. This feature is of great importance in highly dynamic environments. Furthermore, scenario updating enables the adoption of information determined by experts or concepts that are considered to be more credible and reliable than the concepts originally used. The approach developed takes into account the relevance of the novel information and the time constraints for the decision-making. In this manner, it avoids the problem of infinite updating without ever establishing a complete (set of) scenario(s).

To assess the *time necessary to provide decision support,* all phases of the Decision Map approach–from the initial problem structuring to the final analyses–need to be considered. Following the rationale of distributed information processing, each expert specifies, for each of the input variables he uses, the number of values that he can process within a given time. This approach ensures that the set of scenarios is completed within a predefined time. Furthermore, potential trade-offs between the quantity of input information an expert can process and the accuracy of his output information are revealed.

10.2. Directions for Future Research

The Decision Map approach provides a basis for further exploration of distributed decision support under uncertainty. By researching SBR & SM and SBR & MCDA, the presented formalisations, concepts and methods have been developed and shaped. Thus, new research directions for further extending and enhancing SBR & SM and SBR & MCDA in both theory and practice have been opened up. The remainder of this section identifies a number of open aspects for future research. These cover structural aspects (in terms of warranting the acyclicity of DAG_D and the completion of scenarios), temporal aspects, sequential decision problems, the coordination of decision problems solved at several hierarchical levels and the representation of uncertainties.

10.2.1. Structural Aspects

10.2.1.1. Causality and Acyclicity

One of the foundations of SBR is based on the relations among the variables. Represented graphically, the scenarios' structure must correspond to a directed acyclic graph (DAG). The term (local) *Causal Map* has been used to refer to the use of local (causal) models, from which a DAG is constructed. To explain the concept of causality used in this thesis, note that it is usually assumed that if an event A causes B, then A happens before B, i.e., causal relations imply a temporal structure [Davidson, 1980]. This temporal structure can be used as an argument for why it is always possible that scenarios–consisting of cause-effect-chains–can always be represented as DAGs.

The direction of edges in the presented framework is, however, "*causal*" only in the following (weak) sense: if there is an edge from tv_j to tv_k, information on $V(tv_k)$ can be determined once the information on $V(tv_j)$ is known. That means, the causal relationship is not founded on a temporal but on an information-dependence structure. This concept of causality is often used in the AI community [Chaib-draa, 2002; Goodier et al., 2010; Lin and Wu, 2008; Montibeller and Belton, 2006, 2009; Wu and Lee, 2007]. For a centralised system, careful analysis of a domain can yield such knowledge on causation. But ensuring even this weak type of causality in a distributed system with heterogeneous experts is difficult as causality cannot be guaranteed nor enforced. Although the information-dependence structure may represent causality, it is unclear whether or not each expert's knowledge (or domain model) is causal.

The pragmatic stance of this thesis is the following: all experts are considered to provide local DAGs *(local Causal Maps)* based on their expertise. In these local Causal Maps a relation is made between input and output information, where the output can be determined given information on the input. This structure is sufficient to build a larger (global) Causal Map that is itself a DAG.

Finally, the scenario formalisation allows cycles to be detected. Some methods for severing these cycles based on the recipients' preferences for the concepts or values have been developed and presented in section 8.2.2. Nevertheless, further mechanisms to ensure that the overall Causal Map is acyclic need to be developed. In particular, approaches that are applicable while DAG_D is being constructed could improve the efficiency of the scenario building.

10.2.1.2. Completion

The SBR approach is targeted at providing consistent sets of possible values for a set of focus variables to the scenario recipients. There may, however, be cases when sufficient information to determine the values of the focus variables is not (or not timely) available (e.g., as an expert is not available or as some measurements have not yet been performed). In these cases, a decision must be made whether the recipients are provided the partial information on the focus variables momentarily available, whether heuristics are available enabling a rough assessment of the lacking information or whether it is most useful to wait until the required information becomes available. To support decision-makers in these situations, approaches for *partial MCDA evaluations* that either do explicitly emphasise the lack of completeness of information or use indicators (similar to the definitions provided in section 7.4) can be developed.

10.2.2. Temporal Aspects

An important aspect of scenarios, which has not been handled in full detail, concerns time. Each scenario is considered as a *story* about how a given situation may unfold; a concept which carries in it a strong temporal notion. Adopting approaches from literature theory, this thesis distinguishes the event described in the scenario and the time related to its description and narration. Accordingly, two concepts of time are used [Chatman, 1980]: *discourse time* (time covered by the scenario) and *narrative time* (time that the scenario recipients need to understand and evaluate the scenarios). Furthermore, the *scenario building time* is defined as the time the construction of DAG_D and the generation of scenario from DAG_D takes (depending on the time when an expert provides the value(s), captured in timestamps, cf. Section 6.2.1) and the clock-wall-time, which refers to the actual time. For a representation of the change of some variables' values over the discourse time of a scenario, see Figure 10.1.

For operationalisation purposes, the discourse time is discretised into *time slices*. Beyond the concept of *time stamps* (see Section 6.2.1) more advanced methods for discretisation, which facilitate an implementation (and may use the concept of *temporal causality*) can be investigated. According to the (time) preference of the scenario recipients, the values of the focus variables for all time slices or aggregated results for all slices can be determined.

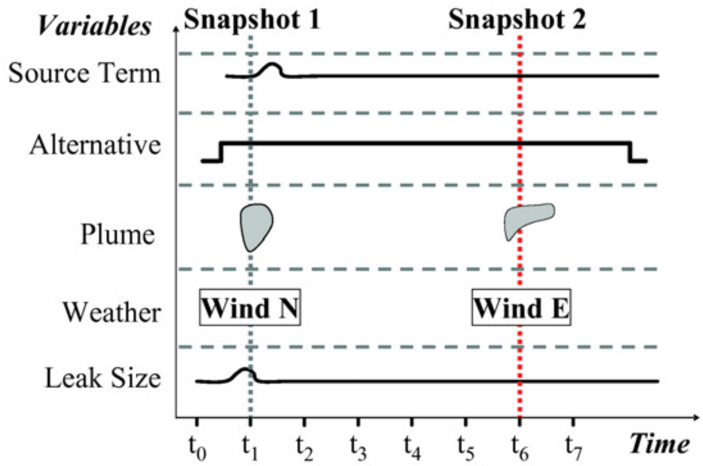


Figure 10.1.: *Discourse Time: Exemplary Illustration With Respect To A Set Of Typed Variables Extracted From The Example Presented In Chapter 9.*

The preferences of the decision-makers in SBR & MCDA may change with passing time. Although usually in MCDA, the stability of attributes and preferences is assumed [French et al., 2005], the preferences may change in the light of developing circumstances [Kornbluth, 1992]. While a change in focus variables is discussed in Section 8.3.1.1 and methods such as sensitivity analysis for testing the robustness of results and recommendations for varying preferences have been developed [Bertsch et al., 2007; Hiete et al., 2010], further methods explicitly integrating time-dependent preferences can be investigated.

Further aspects related to the scenario construction time have been addressed in the Section 8.3. These cover updates of the sets of variables and focus variables, the variables' values and statuses and their interdependencies.

10.2.3. Sequential Decision Problems

The approaches developed so far supports scenario-based decision-making for isolated, single decision problems. This corresponds to the standard approach in MCDA, where usually the assumption is made that all decisions are taken at one single point in time [Bertsch, 2008; Hiete et al., 2010]. The underlying assumption is that the alternatives (which can be mitigation measures a_l that should be implemented as soon as possible or strategies consisting of tuples $\langle a_l, t_l \rangle$ of measures a_l and the time of their respective implementation t_l) and the events that affect the decision can be foreseen modelled exhaustively. In strategic decision problems, however, knowledge is hardly perfect, and decisions are rarely completely uncoupled. Contrarily, decisions can most often be described as a nested series of interdependent decisions [French and Ríos-Insua, 2000], as each choice affects both the set alternatives available in the future and the performance of these alternatives [Keeney, 1982]. That means, the environment changes as a function of the decision-maker's actions and in response to (uncertain) environmental events. The latter concerns not only events that are beyond the control of the affected organisation, but also decisions within the organisation that refer to the same problem and can be harmonised (e.g., decisions made on different hierarchical levels). While this section discusses the possibilities of coordinating decisions over **time** to establish an optimal sequence of decisions, the coordination across several hierarchical levels is discussed in Section 10.2.4.

10.2.3.1. Sequential Decision-Making Techniques

Sequential Decision-Making (SDM) provides a framework facilitating decision-making in situations where a series of decisions needs to be made. Usually, it is assumed that the decision-maker possesses a set of beliefs about the situation and a set of performances for each alternative [Mookerjee and Mannino, 1997].

The two most popular methods to structure sequential decision problems are influence diagrams and decision trees [Clemen and Reilly, 1999]. Both are briefly presented in Annex A. Generally, influence diagrams and decision trees are tailored to solve decision problems that are founded on conditional probability models (cf. Section 2.3.2.2). Both methods are usually only applied for mono-criterion problems (e.g., minimising losses in capacity planning and pricing decisions [Balakrishnan and Sivaramakrishnan, 2001; Jensen et al., 2006] or maximising profit in production models [Antle, 1983]), as with each additional criterion that needs to be represented and computed, the complexity increases (exponentially for decision trees, linearly for Influence diagrams). The problems associated with using purely probabilistic techniques have been discussed in sections 2.2.2.1 and 2.3.2.2.

Another problem of both decision trees and influence diagrams is related to time management: usually, it is not known a priori how many decisions will need to be made until the problem is solved. Furthermore, it is often unclear at what point in time new information will be available. In this case, both influence diagrams and decision trees represent the integration of new information by a series of branches, where the chance vertex (representing the new information) is placed in a different position in each branch according to the time when the information is available. This increases the complexity and the computational effort necessary to solve the problem. Furthermore, there is no feature for representing series of decisions of different lengths efficiently. For that reason, new branches would need to be constructed, where each of the potential decisions is filled in with dummy alternatives (“*do nothing*"). This leads to incoherent sequences of decisions [Bielza et al., 2000].

10.2.3.2. Decision Maps for Supporting Sequential Decision-Making

The basic idea for SDM based on the Decision Map approach consists in determining **scenarios** targeted at solving sequential decision problems by exploiting the adaptability of the DAGs. The aim of this approach is to identify robust strategies $a^S = \{\langle a_1, t_1\rangle, \dots, \langle a_T, t_T\rangle\}$, where a_i denotes a (feasible) alternative and t_i the time of a_i's implementation.

As the number of strategies is overwhelmingly large, in the first step, experts and decision-makers determine successively which **alternatives** can be implemented in a time step t_j. At t_1, a set of feasible alternatives $\{a_1(t_1), \dots, a_l(t_1)\}$ respecting physical and organisational constraints at time t_1 is determined.

To analyse the feasibility of each individual tuple $\langle a_j, t_j\rangle$ and of combinations of tuples, experts are asked to assess for each alternative a_j which **conditions** must be fulfilled to apply the measure successfully (cf. Figure 10.2). In some cases, these conditions may depend on uncertain events beyond the control of the decision-makers. In emergency management, e.g., meteorological constraints can be crucial (dry weather, no frost, etc.). Furthermore, the feasibility may depend on the implementation of alternatives in other areas within the same organisation (e.g., resources required). Yet in other cases, the constraints may be imposed by the decision-makers themselves (e.g., budget constraints). The constraints may also depend on the measures *previously* implemented. By updating the constraints at each time step, the constraints can be modelled such that they depend only on the lastly implemented measure (i.e., a **Markov property** can be assumed, cf. Figure 10.2). For instance, in case material consumption or budget constraints for the period $[t_0, t_T]$ are imposed, in each step t_j ($j \in \{0, T\}$) the consumed material or the budget spent in $(t_{j-1}, t_j]$ is subtracted from the material or budget still allowed to be spent.

The time steps $[t_j, t_{j+1}]$ are not fixed a priori but adapted dynamically according to the alternative chosen. Assume the implementation of alternative a_j

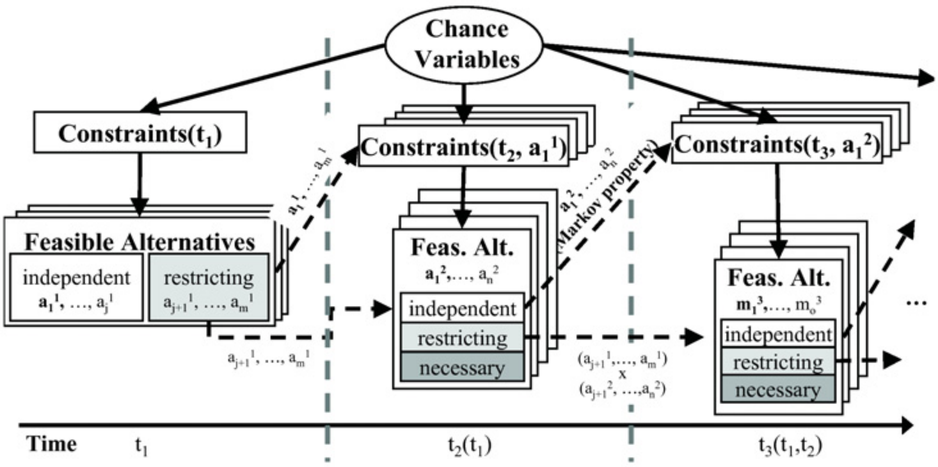


Figure 10.2.: *A Procedure To Compile Feasible Strategies*

starts at time t_j. Then, either t_{j+1} is the point in time, when the application of a_j is finished, or it marks a chance event that prevents the further application of a_j. In both cases, the interval $[t_j, t_{j+1}]$ represents the time interval during which a_j is being executed. If multiple measures are applied simultaneously, t_{j+1} represents the point in time, when at least one of the measures cannot be executed any more.

The alternatives are grouped to classes to control the combinatorial explosion of possible strategies (see Figure 10.2). To this end, alternatives are distinguished by the influence they exert on the set of feasible alternatives. If an alternative does not have a direct impact on the set of feasible alternatives, it is said to be independent. Contrarily, if an alternative precludes the implementation of some alternatives or makes their application useless, it is called restricting. For example the application of "*sheltering*" is independent, whereas the implementation of "*evacuation*" eliminates the possibility of sheltering. In other cases, the implementation of an alternative may result in the necessity of additional or auxiliary measures in the next or one of the future steps (e.g., waste disposal after scavenging a contaminated area). It is compulsory that a strategy includes all necessary measures. The process of strategy definition is finished when either the maximum time horizon of the decision-makers is reached or when the set of feasible alternatives is empty. Furthermore, the set of necessary measures must be empty.

Having determined the feasible strategies, the constraints and the interdependencies of alternatives, SDM can, in essence, be understood as an extension of scenario updating (cf. Section 8.3). Starting at a time t_1, for each feasible alter-

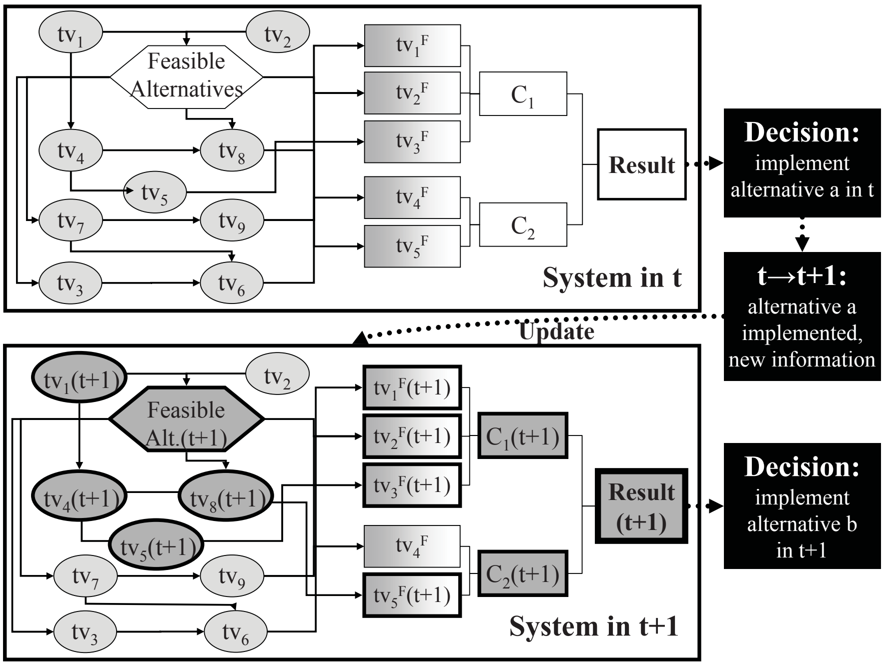


Figure 10.3.: *The Decision Map Approach For A Sequence Of Two Decisions*

native $a_j \in \{a_1(t_1), \ldots, a_l(t_1)\}$, a set of scenarios is generated and evaluated by the Decision Map approach (as detailed in Chapter 5).

Figure 10.3 shows an example: the decision consists in determining a sequence of two alternatives. Denote $SS(a_j(t_k)) = \{S_i(a_j(t_k))\}_{i\in I(a_j)}$ the set of alternatives constructed for evaluating alternative a_j at time t_k. The result of implementing a_j is determined. Then, the consequences of a_j on the values of the variables in $\bigcup_{i\in\bigcup_{a_m\in A\setminus a_j} I(a_m)} STV_i$ are determined. This equals an updating of values (cf. Section 8.3.2). Figure 10.3 shows the change of values in $t+1$ by the highlighted boxes. Possibly, also the structure of the scenario requires updating. This can be performed as described in Section 8.3.1. On basis of the novel values and dependencies, the new set of feasible alternatives at t_{k+1} is determined.[58] Then, the results of implementing all feasible alternatives $a_m(t_{k+1})$ are determined.

Figure 10.4 illustrates the SDM process on basis of the Decision Map approach: at each step t_i, first, the constraints and the set of feasible alternatives are determined. Then, each feasible alternative is evaluated. After having updated the constraints and the set of feasible alternatives, the next evaluation step (representing step t_{i+1}) is performed. This procedure is applied succes-

[58] It is assumed that the constraints can be derived from sv_i for all alternatives a_m and scenarios $S_i(a_m)$.

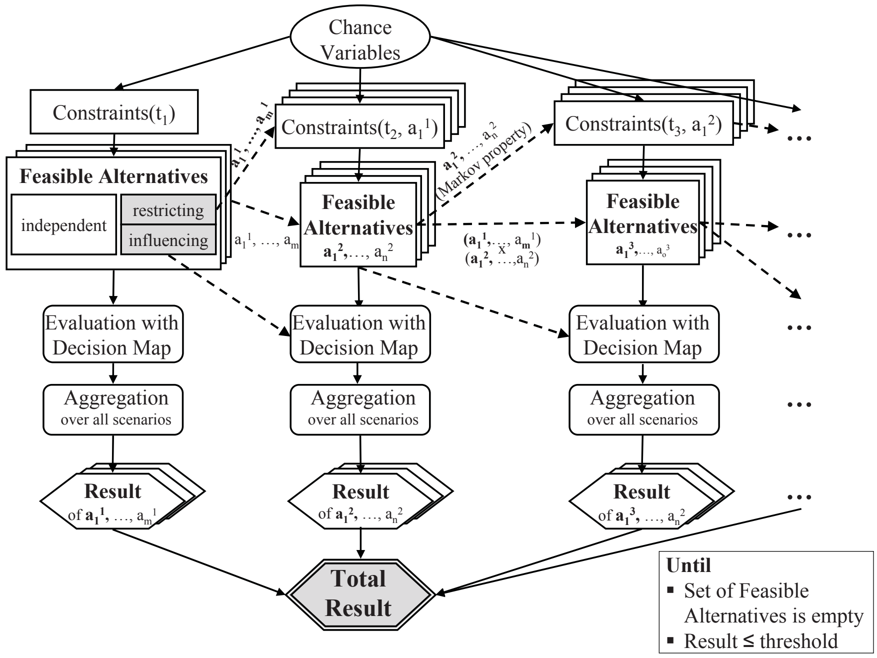


Figure 10.4.: The Sequential Decision-Making Process

sively until the set of feasible alternatives is empty or no amelioration of the situation can be achieved by implementing further alternatives.

In this manner, the Decision Map approach can be exploited analogue to the approach for scenario updating. Yet, methods to handle the potential combinatorial explosion of the strategies to be investigated must be developed. For instance, **filtering techniques** for determining promising combinations of results for further investigation [Wang and Zionts, 2008] could be used. To enhance the efficiency of filtering, the alternatives in A can be clustered to sets $A_1, \ldots, A_n$, $A_i \subset A$ for all $i = 1, \ldots, n$, where $A_i \cap A_j = \emptyset$ for all $i \neq j$, such that the sequences $a_{i_1}, a_{i_2}, \ldots, a_{i_n}$, $a_{i_j} \in A_j$ are promising (see Hiete et al. [2010] for an application of this technique for two-stage sequential decision problems in nuclear emergency management).

To investigate whether or not an alternative is promising, for example, the notion of **flexibility** can be used. The issue of flexibility arises when decision-makers have to choose between an alternative that limits the space of possible future actions and one that is amendable in the future. Intuitively, a reversible or flexible position should be preferred over a rigid solution that limits the space for manoeuvre. In the light of new or updated information, it can be appropriate or necessary to reconsider, re-evaluate and perhaps modify the as yet unimplemented steps of the strategy. If the possibility of adapting and revising the momentary strategy has not played a role in its specification, there may no longer be adequate residual flexibility, and desired outcomes may not be achievable. In this sense, flexibility limits the risks of an early commitment to an alternative when the value of this alternative is not known with certainty [Benjaafar et al., 1995]. Therefore, it can be useful to analyse whether the first steps of a strategy offer a high degree of flexibility and do not limit the space of manoeuvre. In the later steps of the strategy more and more restricting or irreversible alternatives can be implemented. Furthermore, the time steps and the discretisation methods applied require further investigation, as it may be that not only the sequence of the alternatives, but also the (wall-clock-)time of their application has an impact on their consequences.

10.2.4. Coordination of Parallel Decisions: Hierarchical Aspects

The formalisation of scenarios can help detecting dependent or parallel decisions via the detection of overlaps of related DAGs. If two sets of scenarios SS_{D_1} and SS_{D_2} share some variables tv_k (i.e., $\exists SS_{i_1} \subset SS_{D_1}$ and $SS_{j_2} \in SS_{D_2}$, for which $tv_k \in \bigcap_{i \in I_{i_1}} STV_i \cap \bigcap_{j \in J_{j_2}}$) or sub-scenarios, this overlap can be detected and exploited.

This detection has several potential advantages. First, if the scenarios in one of the sets are not (yet) completed and time is critical, the values and statuses determined in the other set of scenarios can be adopted by choosing the set $INIT$ accordingly (cf. Section 8.4). Second, if some of the variables in SS_{D_1} are **constraints** for implementing alternatives in SS_{D_2} (or vice versa), this problem can be detected by constructing sets of constraints, where $CONSTRAINT_{D_i} = \left\{\left(tv_j^C, V_i^C\left(tv_j^C\right)\right)\right\}_{j \in J}$ is defined for each decision problem D_i. This set specifies critical requirements and resources, potential bottlenecks in terms of a variable and a value which must not be exceeded. The exceedance is defined with respect to a relational operator $\geq_j$ which defines a total or partial order on $Range\left(V\left(tv_j\right)\right)$.

Assume that there are two scenarios S_{i_1} in SS_{D_1} and S_{j_2} in SS_{D_2} and a variable $tv_j^C \in STV_{i_1} \cap STV_{j_2}$. Let $\left(tv_j, V_2^C\left(tv_j\right)\right) \in CONSTRAINT_{D_2}$ be a constraint for decision problem D_2 and let $\geq_j$ be the relational operator on $Range\left(V\left(tv_j\right)\right)$ chosen. If $V_{i_1}\left(tv_j\right) \geq_j V_2^C\left(tv_j\right)$, then S_{i_1} defies S_{j_2}. Particularly, the alternative whose implementation is assumed in S_{j_2} cannot be used.

In these situations, the simplest approach is to alert and inform decision-makers working on D_1 and D_2 about the other decision. Possibly, the Decision Maps built for D_1 and D_2 can be merged (facilitated by the scenario merging procedure as described in Section 3.7). By defining tuples of alternatives $a_{i,j} = \left(a_i^1, a_j^2\right)$, where $a_i^1 \in A(D_1)$ and $a_j^2 \in A(D_2)$ are alternatives for decision problems in D_1 and D_2, a novel decision problem can be defined, which solves both problems simultaneously and takes into account their entanglement.

Yet, this problem requires further investigation, as time constraints for both decision problems need to be respected. Furthermore, also the hierarchical level of the decision may need to be taken into account, as it is likely that decisions on a higher level are required to outrank decisions made on a lower level.

10.2.5. Representation of Results and Uncertainties

One of the key issues when coping with uncertainties is the communication of their effects [Hiete et al., 2010]. Elicitation techniques and interfaces to support each expert's assessment need to be further developed. This concerns particularly the meta-information (e.g., the number of input an expert can process within a given times, the sensitivity of his results to change in input). An important requirement for the acceptance of the results of both SBR & SM and SBR & MCDA is the understandable and transparent representation of results. This includes **visualisation methods** as well as **textual language reports** that explain and justify the results. Both, visualisations and reports may contain information on the scenarios themselves as well as meta-information about the scenario (e.g., its reliability or the concepts used to determine the values in the scenario).

Exemplary ideas for visualisations of the scenarios $S_i \in SS$ and their evaluations are shown in figures 10.5 and 10.6. Both visualisations have been developed on basis of the example presented in Chapter 9; they are designed to represent the results of the SBR & MCDA evaluation of the alternatives *evacuation, sheltering* and *do nothing* [Neidhart et al., 2010].

Figure 10.5 shows a **ring chart**, where results for all scenarios are organised in a ring-shaped manner. Each ring refers to a variable tv_j common to all $S_i \in SS$ (i.e., $tv_j \in \bigcap_{i \in I} STV_i$). $SPV_{SS}(tv_j)$ is the set of possible values of tv_j covered by SS. For each $V_i(tv_j) \in SPV_{SS}(tv_j)$, the set of scenarios $SS(V_i(tv_j)) = \left\{S_{i_j}\right\}_{i_j \in I}(V_i(tv_j))$ is constructed, where for all $S_{i_j} \in SS(V_i(tv_j))$ it holds

$V_{i_j}(tv_j) = V_i(tv_j)$. As for each scenario S_i each variable $tv_j \in STV_i$ is assigned one unique value, $\bigcup_{V_i(tv_j) \in SPV_{SS}(tv_j)} SS(V_i(tv_j))$ is a partition of SS. Particularly,

$$SS(V_{i_k}(tv_j)) \cap SS(V_{i_l}(tv_j)) = \varnothing \quad \forall\, V_{i_k}(tv_j),\ V_{i_l}(tv_j) \in SPV_{SS}(tv_j).$$

For each such set of scenarios, another partition according to the alternative implemented in the respective scenarios is constructed:

$$SS(a_k, V_i(tv_j)) = \{S_{i_l} \in SS(V_{i_k}(tv_j)) : V_{i_l}(tv_A) = a_k\}.$$

By evaluating each scenario's result, assigning weights to each scenario $S_{i_l} \in SS(a_k, V_i(tv_j))$ and an according aggregation of scenario results as described in Section 5.3, the evaluation of each alternative $a_k \in A$ assuming $V_i(tv_j)$, $R(a_k, V_i(tv_j)) \in [0,1]$ is determined. These results are then used to rank the alternatives a_k.

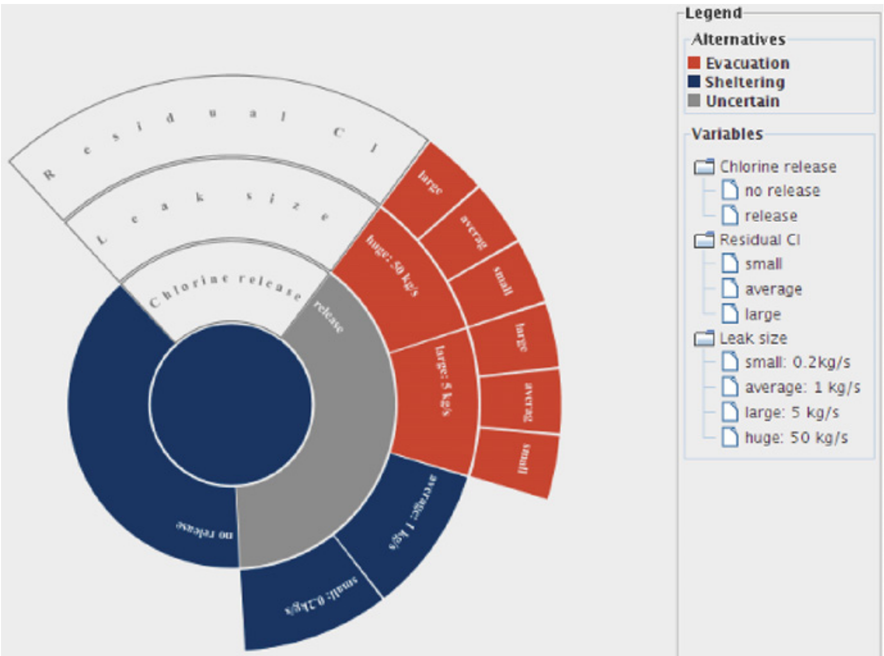


Figure 10.5.: *Ring Chart Visualisation Of Scenarios And Best Evaluated Alternatives. Example after aggregation of all scenarios' results sharing the denoted valuesevacuation in red, sheltering in blue)*

In the ring chart (cf. Figure 10.5), the best evaluated alternative for given values $V_i(tv_j)$ are represented by differently coloured segments. Each colour represents the alternative evaluated best for a given valuation of a set of variables. By further partitioning the scenario sets $S(a_k, V_i(tv_j))$ on basis of further variables $tv_{j_l} \in \bigcup_{i \in I} STV_i$, more and more rings can be added. This process can be performed in an interactive manner, where the decision-makers can deliberately select the scenarios and variable values they are interested in to investigate further, in this case the remaining set of scenarios is hidden.

The **geographical mapping** shown in Figure 10.6 follows a similar design rational as the ring charts discussed above. The variables used for partitioning SS must relate to the space or area and allow the best evaluated alternatives for certain situations to be represented on a map. Figure 10.6 shows the best evaluated alternatives for different meteorological conditions. While for wind to the north segments, alternative *evacuation* is most favourable, for other wind directions, *sheltering* is the alternative preferred.

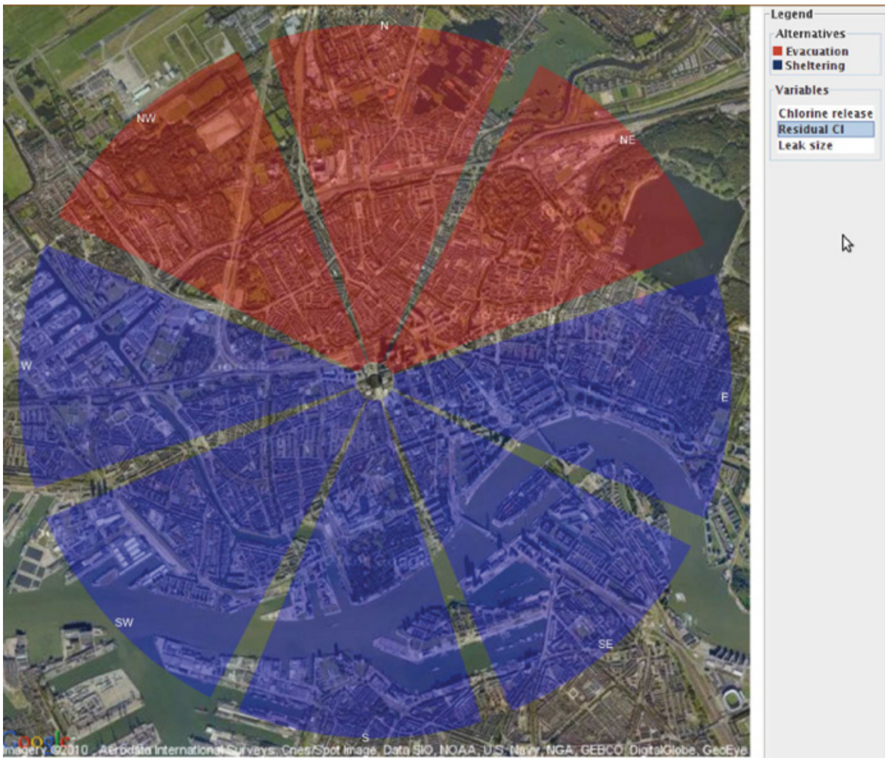


Figure 10.6.: *Geographic mapping showing best evaluated alternatives (evacuation in red, sheltering in blue) after aggregation of all scenarios' results for different wind directions*

Both visualisation techniques assess the evaluation of alternatives under varying assumptions about the future development as represented in the underlying scenarios. They facilitate understanding under *which conditions* and *why* certain alternatives show a good or bad performance. This allows the alternatives to be successively improved by combining alternatives which balance their respective drawbacks or enhance their respective strengths.

Explanation facilities contribute to positive user attitudes and improve user performance [Geldermann et al., 2009]. Originally, explanation systems have been developed to elucidate the conclusions of expert systems or artificial intelligence approaches [Reiter and Dale, 1997; Papamichail and French, 2003]. Natural language reports influence user perceptions such as trust, confidence and satisfaction and increase levels of acceptance and learning [Dhaliwal and Benbasat, 1996]. Particularly, decision-makers who are not familiar with MCDA visualisations are provided an easily understandable interpretation [Hiete et al., 2010]. On the basis of the developed framework for scenario characterisation, a start can be made to develop tools that automatically generate reports. In particular, the scenario reliability assessment developed in Section 4.3.4 and the specification of concepts used as described in Section 6.2.1 can provide the basis for generating the reports using natural language techniques as described by Reiter and Dale [1997].

Summary

Decision-makers responsible for establishing any organisation's strategy face a task characterised by complexity, uncertainty and ambiguity. A central feature of strategic decision problems is their lacking structure. Therefore, the need for well-structured and transparent support arises.

The Decision Map approach supports decision-makers facing large, complex and uncertain problems. The novel methodology developed in this thesis is particularly useful when the type and quality of information is heterogeneous, as it allows several principles for reasoning under uncertainty to be handled simultaneously. As the approach is a generic framework supporting strategic decision-making, it can be applied in various domains, e.g., environmental management, policy assessment or risk management.

The Decision Map approach is tailored for situations that defy standardised (automated) and discursive solutions. This approach allows the reasoning processes to be adapted to the problem at hand while respecting constraints in time and availability of experts and decision-makers. To facilitate decision-making in these situations, this thesis presents a method supporting distributed processing of information bringing together decision-makers and experts from various disciplines: the Decision Map integrates a directed acyclic graph (DAG) for scenario building and an attribute tree for scenario evaluation.

The use of DAGs facilitates **distributed scenario building** involving human experts and automated reasoning systems. Using DAGs that are constructed from local Causal Maps ensures that only relevant information is passed on to further experts avoiding time consuming filtering of redundant or irrelevant information and **reducing information overload.** To ensure that the scenarios respect the information needs of their recipients, the concepts of focus variables and relevant initial situation descriptions are developed. In this manner, it is ensured that **purposeful scenarios** complying with the scenario recipients' needs are built.

Additionally, the use of DAGs enables constructing plausible, consistent and coherent scenarios as far as possible given restrictions in time and availability of experts. The use of graph theoretical concepts facilitates the assessment and quantification of quality requirements such as plausibility, consistency, coherence and reliability for single scenarios as well as balance and accuracy for sets of scenarios. By implementing approaches for pruning scenarios that are considered invalid (e.g., as they are deemed not sufficiently credible), it is ensured that the scenarios built meet the recipients' **quality requirements.**

The collaboration of experts is based on their respective domain knowledge and information needs; the construction of DAGs does *not* refer to standardised procedures. Rather, the processing of information (the "*reasoning*") can be adapted flexibly to the problem at hand as well as to the information and time available. Uncertainty about a variable's value is expressed as a number of possible values that propagate through the Decision Map as (incomplete) scenarios: whenever an expert is uncertain about a variable's value (given the information about the direct predecessor variables' values), this uncertainty results in multi-furcation of scenarios. This approach to scenario construction facilitates coping with **heterogeneous types and qualities of information,** as it enables the flexible adaptation of reasoning principles to the information available. In this manner, it became possible to evade the risk of loosing information, which happens if methods with little requirements but offering less precision are imposed. For instance, modelling a problem in qualitative terms by a Causal Map [Montibeller and Belton, 2006] possibly available information on the probability of certain events is dismissed. Moreover, misjudgements and biases due to the application of too demanding techniques for handling uncertainty (occurring, e.g., when using probabilistic techniques without having sufficiently rich statistical information or reliable expert judgements) can be avoided.

The integrated Scenario-Based Multi-Criteria Decision Analysis is achieved by merging a DAG and an attribute tree to a Decision Map. The Decision Map facilitates **robust decision-making** respecting multiple criteria in a transparent and well-structured manner. This methodology enables taking into account the decision-makers' preferences with respect to their risk attitude, the reliability required and the importance of different goals. To support robust decision-making the most significant scenarios are selected and presented in detail to the decision-makers. An aggregation of all scenarios' results complements this approach and helps avoiding cognitive biases. To this end, aggregation techniques from Multi-Attribute Value Theory are adapted and two approaches for the elicitation of scenario importance weights are introduced. On the whole, this approach yields deeper insights into the decision situation than those provided by standard methods that base the evaluation on one (best guess or extreme) development.

The integration of scenario building and evaluation enables developing a novel framework for **scenario management.** Scenario management is based on an assessment of the relevance and reliability of each scenario and respects the information integrated in further scenarios as well as the specific needs and preferences of the scenario recipients. In this manner, (local and global) scenario selection, scenario pruning and scenario updating questions are based on systematic evaluations rather than on the users' intuition or on an abstract notion of distance for a set of scenarios. Scenario management enables respecting constraints such as the time for the decision-making, bounded availability of experts and limited resources for information processing. The scenario management approaches furthermore ensure that the scenarios remain valid and acceptable throughout the sense- or decision-making process.

The new Decision Map methodology facilitates efficient robust decision-making in complex situations where fundamental uncertainties prevail taking into account constraints with respect to the time and resources available. This transparent and well-structured approach respects and integrates the decision-makers' preferences throughout all phases of the decision support process and thus enhances acceptance and compliance to the decisions made.

Bibliography

A. Aamodt and E. Plaza. Case-based reasoning: foundational issues, methodological variations, and system approaches. *AI Communications*, 7(1):39–59, 1994.

M. H. Acker. Tempered Regrets Under Total Ignorance. *Theory and Decision*, 42(3):207–213, 1997.

J. Aczél. On weighted synthesis of judgements. *Aequationes Mathematicae*, 27(1):288–307, 1984.

G. Agre. KBS maintenance as learning two-tiered domain representation. In M. Veloso and A. Aamodt, editors, *Case-Based Reasoning Research and Development*, volume 1010 of *Lecture Notes in Computer Science*, pages 109–120. Springer Berlin Heidelberg, 1995.

D. M. Ahmed, D. Sundaram, and S. Piramuthu. Knowledge-based scenario management – process and support. *Decision Support Systems*, 49(4):507–520, 2010.

Air Liquide. Sicherheitsdatenblatt Chlor. Data according to RL 1907/2006/EG (REACH) 2-00, Air Liquide, 2010.

T. Akter and S. P. Simonovic. Aggregation of fuzzy views of a large number of stakeholders for multi-objective flood management decision-making. *Journal of Environmental Management*, 77(2):133–143, 2005.

T. A. Alspaugh. *Scenario networks and formalization for scenario management*. PhD thesis, North Carolina State University, Raleigh, 2002.

T. A. Alspaugh and A. I. Antón. Scenario support for effective requirements. *Information and Software Technology*, 50(3):198–220, 2008.

T. A. Alspaugh, A. I. Antón, T. Barnes, and B. W. Mott. An integrated scenario management strategy. In *Proceedings of the 4th IEEE International Symposium on Requirements Engineering*, pages 142–149, 1999.

V. Ambrosini and C. Bowman. Tacit Knowledge: Some Suggestions for Operationalization. *Journal of Management Studies*, 38(6):811–829, 2001.

J. Ananda and G. Herath. Evaluating public risk preferences in forest land-use choices using multi-attribute utility theory. *Ecological Economics*, 55(3):408–419, 2005.

J. Anderson. Leaps of the imagination: Interstellar flight and the Horizon Mission Methodology. *Journal of the British Interplanetary Society*, 49:15–20, 1996.

J. M. Antle. Sequential decision making in production models. *American Journal of Agricultural Economics*, 65(2):282–290, 1983.

K. J. Arrow and L. Hurwicz. *Studies in resource allocation processes*, chapter An Optimality Criterion for Decision-Making Under Ignorance, pages 463–474. Cambridge University Press, 1977.

P. Artzner, F. Delbaen, J.-M. Eber, and D. Heath. Coherent measures of risk. *Mathematical Finance*, 9(3):203–228, 1998.

T. Aven. *Foundations of Risk Analysis. A Knowledge- and Decision-Oriented Perspective*. John Wiley & Sons, New York, 2004.

R. M. Axelrod. *Structure of Decision: The Cognitive Maps of Political Elites*. Princeton University Press, Princeton, 1976.

H.-R. Bae, R. V. Grandhi, and R. A. Canfield. Epistemic uncertainty quantification techniques including evidence theory for large-scale structures. *Computers & Structures*, 82 (13–14):1101–1112, 2004.

A. Baklanov, J. Sorensen, S. C. Hoe, and B. Amstrup. Urban meteorological modelling for nuclear emergency preparedness. *Journal of Environmental Radioactivity*, 85(2-3): 154–170, 2006.

R. Balakrishnan and K. Sivaramakrishnan. Sequential solutions to capacity-planning and pricing decisions. *Contemporary Accounting Research*, 18(1):1–26, 2001.

C. Baudrit, I. Couso, and D. Dubois. Joint propagation of probability and possibility in risk analysis: Towards a formal framework. *International Journal of Approximate Reasoning*, 45(1):82–105, 2007.

P. W. Beck. Corporate planning for an uncertain future. *Long Range Planning*, 15(4):12–21, 1982.

T. Bedford and R. Cooke. *Probabilistic Risk Analysis: Foundations and Methods.* Cambridge University Press, Cambridge, 2001.

M. Behzadian, R. Kazemzadeh, A. Albadvi, and M. Aghdasi. PROMETHEE: A comprehensive literature review on methodologies and applications. *European Journal of Operational Research*, 200(1):198–215, 2010.

D. E. Bell. Regret in Decision Making under Uncertainty. *Operations Research*, 30(5):961–981, 1982.

D. E. Bell. Risk, Return, and Utility. *Management Science*, 41(1):23–30, 1995.

R. E. Bellman and L. A. Zadeh. Decision-making in a fuzzy environment. *Management Science*, 17(4):141–164, 1970.

V. Belton and J. Hodgkin. Facilitators, decision makers, D.I.Y. users: Is intelligent multicriteria decision support for all feasible or desirable? *European Journal of Operational Research*, 113(2):247–260, 1999.

V. Belton and T. Stewart. *Multiple Criteria Decision Analysis. An Integrated Approach.* Kluwer Academic Publishers, Boston, 2002.

Y. Ben-Haim. Robust rationality and decisions under severe uncertainty. *Journal of the Franklin Institute*, 337(2-3):171–199, 2000.

Y. Ben-Haim. Uncertainty, probability and information-gaps. *Reliability Engineering & System Safety*, 85(1-3):249–266, 2004.

S. Benartzi and R. H. Thaler. Risk Aversion or Myopia? Choices in Repeated Gambles and Retirement Investments. *Management Science*, 45(3):364–381, 1999.

S. Benjaafar, T. Morin, and J. Talavage. The strategic value of flexibility in sequential decision making. *European Journal of Operational Research*, 82(3):438–457, 1995.

U. Bergsten and J. Schubert. Dempster's rule for evidence ordered in a complete directed acyclic graph. *International Journal of Approximate Reasoning*, 9(1):37–73, 1993.

D. Berleant, S. Ferson, V. Kreinovich, and W. Lodwich. Combining interval and probabilistic uncertainty: foundations, algorithms, challenges – an overview. In *4th International Symposium on Imprecise Probabilities and Their Applications*, Pittsburgh, Pennsylvania, 2005.

V. Bertsch. *Uncertainty Handling in Multi Attribute Decision Support for Industrial Risk Management*. PhD thesis, Universität Karlsruhe (TH), Karlsruhe, 2008.

V. Bertsch, J. Geldermann, O. Rentz, and W. Raskob. Multi-criteria decision support and stakeholder involvement in emergency management. *International Journal of Emergency Management*, 3(2–3):114–130, 2006.

V. Bertsch, J. Geldermann, and O. Rentz. Preference Sensitivity Analyses for Multi-Attribute Decision Support. In *Operations Research Proceedings 2006*, pages 411–416. Springer Berlin Heidelberg, 2007.

C. Bielza and P. P. Shenoy. A Comparison of Graphical Techniques for Asymmetric Decision Problems. *Management Science*, 45(11):1552–1569, 1999.

C. Bielza, M. Gómez, S. Ríos-Insua, and J. A. del Pozo Fernández. Structural, elicitation and computational issues faced when solving complex decision making problems with influence diagrams. *Computers & Operations Research*, 27(7–8):725–740, 2000.

V. Bier. Implications of the research on expert overconfidence and dependence. *Reliability Engineering & System Safety*, 85(1-3):321–329, 2004.

P. Bishop, A. Hines, and T. Collins. The current state of scenario development: an overview of techniques. *Foresight*, 9(1):5–25, 2007.

S. E. Bodily. *Modern Decision Making: A Guide to Modeling With Decision Support Systems*. McGraw-Hill, 1984.

F. Bolger and G. Wright. Assessing the quality of expert judgment: Issues and analysis. *Decision Support Systems*, 11(1):1–24, 1994.

B. Bollobás. *Modern graph theory*. Springer, 1998.

T. W. Bonnett and R. L. Olson. Four scenarios of state government services and regulation in the year 2010. *Strategy & Leadership*, 22(4):26–33, 1993.

L. Börjeson, M. Höjer, K.-H. Dreborg, T. Ekvall, and G. Finnveden. Scenario types and techniques: Towards a user's guide. *Futures*, 38(7):723–739, 2006.

M. Bougon. Uncovering cognitive maps: the "Self-Q" technique. In *Beyond Method: Strategies for Social Research*. Sage Publications, 1983.

R. Bradfield, G. Wright, G. Burt, G. Cairns, and K. Van Der Heijden. The origins and evolution of scenario techniques in long range business planning. *Futures*, 37(8):795–812, 2005.

G. Braendeland, A. Refsdal, and K. Stolen. Modular analysis and modelling of risk scenarios with dependencies. *Journal of Systems and Software*, 83(10):1995–2013, 2010.

J. P. Brans and P. Vincke. A Preference Ranking Organisation Method: The PROMETHEE Method for Multiple Criteria Decision-Making. *Management Science*, 31(6):647–656, 1985.

J. Brauers and M. Weber. A new method of scenario analysis for strategic planning. *Journal of Forecasting*, 7(1):31–47, 1988.

K. K. Breitman, J. C. S. d. P. Leite, and D. M. Berry. Supporting scenario evolution. *Requirements Engineering*, 10(2):112–131, 2005.

G. Brewer. Inventing the future: scenarios, imagination, mastery and control. *Sustainability Science*, 2(2):159–177, 2007.

B. P. Bryant and R. J. Lempert. Thinking inside the box: A participatory, computer-assisted approach to scenario discovery. *Technological Forecasting and Social Change*, 77(1):34–49, 2010.

J. Buckley. Fuzzy decision making with data: Applications to statistics. *Fuzzy Sets and Systems*, 16(2):139–147, 1985. ISSN 0165-0114.

R. L. Buckley, C. H. Hunter, R. P. Addis, and M. J. Parker. Modeling Dispersion from Toxic Gas Released after a Train Collision in Graniteville, SC. *Journal of the Air & Waste Management Association*, 59(3):268–278, 2007.

D. Bunn and A. Salo. Forecasting with scenarios. *European Journal of Operational Research*, 68(3):291–303, 1993.

J. Butler, J. Jia, and J. Dyer. Simulation techniques for the sensitivity analysis of multi-criteria decision models. *European Journal of Operational Research*, 103(3):531–546, 1997.

J. C. Butler, J. S. Dyer, and J. Jia. Using Attributes to Predict Objectives in Preference Models. *Decision Analysis*, 3(2):100–116, 2006.

C. Camerer and M. Weber. Recent developments in modeling preferences: Uncertainty and ambiguity. *Journal of Risk and Uncertainty*, 5(4):325–370, 1992.

F. Campolongo, J. Cariboni, and A. Saltelli. An effective screening design for sensitivity analysis of large models. *Environmental Modelling & Software*, 22(10):1509–1518, 2007.

B. Chaib-draa. Causal maps: theory, implementation, and practical applications in multi-agent environments. *IEEE Transactions on Knowledge and Data Engineering*, 14(6):1201–1217, 2002.

Y.-H. Chang and C.-H. Yeh. Evaluating airline competitiveness using multiattribute decision making. *Omega*, 29(5):405–415, 2001.

S. Chatman. *Story and Discourse: Narrative Structure in Fiction and Film*. Cornell University Press, Ithaca, NY, US, 1980.

C.-b. Chen and C. M. Klein. An efficient approach to solving fuzzy MADM problems. *Fuzzy Sets and Systems*, 88(1):51–67, 1997.

L. Chen and P. Pu. Survey of preference elicitation methods. Technical Report IC/2004/67, Ecole Politechnique Federale de Lausanne (EPFL), 2004.

C.-H. Cheng. A new approach for ranking fuzzy numbers by distance method. *Fuzzy Sets and Systems*, 95(3):307–317, 1998.

T. J. Chermack. Improving decision-making with scenario planning. *Futures*, 36(3):295–309, 2004.

S.-Y. Chou, Y.-H. Chang, and C.-Y. Shen. A fuzzy simple additive weighting system under group decision-making for facility location selection with objective/subjective attributes. *European Journal of Operational Research*, 189(1):132–145, 2008.

R. Clemen and T. Reilly. *Making Hard Decisions with DecisionTools*. Duxbury, Pacific Grove, CA, 1999.

T. Comes, M. Hiete, and N. Wijngaards. MCDA for distributed collaboration. DEMA use case: Application of MCDA & SBR. Technical report, Report within the EC FP7-ICT project Diadem (ref. no: 224318), 2009a.

T. Comes, M. Hiete, N. Wijngaards, and M. Kempen. Integrating Scenario-Based Reasoning into Multi-Criteria Decision Analysis. In J. Landgren and S. Jul, editors, *Proceedings of the 6th International Conference on Information Systems for Crisis Response and Management*, Gothenburg, 2009b.

T. Comes, C. Conrado, M. Hiete, M. Kamermans, G. Pavlin, and N. Wijngaards. An intelligent decision support system for decision making under uncertainty in distributed reasoning frameworks. In S. French, B. Tomaszewski, and C. Zobel, editors, *Proceedings of the 7th International Conference on Information Systems for Crisis Response and Management*, Seattle, 2010a.

T. Comes, C. Conrado, and N. Wijngaards. Principles of Scenario-Based Reasoning and Scenario-Based Multi-Criteria Decision-Making. Technical report, Report within the EC FP7-ICT project Diadem (ref. no: 224318), 2010b.

T. Comes, M. Hiete, and F. Schultmann. A decision support system for multi-criteria decision problems under severe uncertainty in longer-term emergency management. In C. H. Antunes and D. Ríos Insua, editors, *Proceedings of the 25th Mini EURO Conference on Uncertainty and Robustness in Planning and Decision Making*, Coimbra, 2010c.

T. Comes, M. Hiete, N. Wijngaards, and F. Schultmann. Enhancing Robustness in Multi-Criteria Decision-Making: A Scenario-Based Approach. In *Proceedings of the 2nd International Conference on Intelligent Networking and Collaborative Systems*, 2010d.

L. Connell and M. Keane. What plausibly affects plausibility? Concept coherence and distributional word coherence as factors influencing plausibility judgments. *Memory & Cognition*, 32(2):185–197, 2004.

L. Connell and M. T. Keane. A Model of Plausibility. *Cognitive Science*, 30(1):95–120, 2006.

C. Conrado and G. Pavlin. Demo Design and Prototype. Technical report, Report within the EC FP7-ICT project Diadem (ref. no: 224318), 2010.

K. H. Cook. A threshold model of relevance decisions. *Information Processing & Management*, 11(5-7):125–135, 1975.

W. S. Cooper. A definition of relevance for information retrieval. *Information Storage and Retrieval*, 7(1):19–37, 1971.

Z. Covaliu and R. M. Oliver. Representation and Solution of Decision Problems Using Sequential Decision Diagrams. *Management Science*, 41(12):1860–1881, 1995.

R. D. Cox. Decontamination and management of hazardous materials exposure victims in the emergency department. *Annals of Emergency Medicine*, 23(4):761–770, 1994.

C. Cuadra and R. Katter. Opening the black box of "relevance". *Journal of Documentation*, 23(4):291–303, 1967.

A. Curry and W. Schultz. Roads less travelled: Different methods, different futures. *Journal of Futures Studies*, 13(4):35–60, 2009.

A. David and Z. Rongda. An expert system with fuzzy sets for optimal planning. *IEEE Transactions on Power Systems*, 6(1):59 –65, 1991.

D. Davidson. Causal relations. In *Essays on Actions and Events*, pages 149–162. Oxford University Press, Oxford, 1980.

A. P. Dawid. Influence Diagrams for Causal Modelling and Inference. *International Statistical Review / Revue Internationale de Statistique*, 70(2):161–189, 2002.

L. de Campos, J. Huete, and S. Moral. Uncertainty management using probability intervals. In B. Bouchon-Meunier, R. Yager, and L. Zadeh, editors, *Advances in Intelligent Computing (IPMU '94)*, volume 945, pages 190–199. Springer Berlin Heidelberg, 1995.

N. Dechy, T. Bourdeaux, N. Ayrault, M.-A. Kordek, and J.-C. Le Coze. First lessons of the Toulouse ammonium nitrate disaster, 21st September 2001, AZF plant, France. *Journal of Hazardous Materials*, 111(1–3):131–138, 2004.

J. Dhaliwal and I. Benbasat. The Use and Effects of Knowledge-based System Explanations: Theoretical Foundations and a Framework for Empirical Evaluation. *Information Systems Research*, 7(3):342–362, 1996.

N. Di Domenica, G. Mitra, P. Valente, and G. Birbilis. Stochastic programming and scenario generation within a simulation framework: An information systems perspective. *Decision Support Systems*, 42(4):2197–2218, 2007.

D. Diakoulaki and F. Karangelis. Multi-criteria decision analysis and cost-benefit analysis of alternative scenarios for the power generation sector in Greece. *Renewable and Sustainable Energy Reviews*, 11(4):716–727, 2007.

M. Diehl and Y. Haimes. Influence diagrams with multiple objectives and tradeoff analysis. *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*, 34 (3):293–304, 2004.

R. Diestel. *Graph Theory*, volume 173 of *Graduate Texts in Mathematics*. Springer Berlin Heidelberg, 2005.

J. C. S. do Prado Leite, G. D. S. Hadad, J. H. Doorn, and G. N. Kaplan. A Scenario Construction Process. *Requirements Engineering*, 5(1):38–61, 2000.

M. Dohnal. Ignorance and uncertainty in reliability reasoning. *Microelectronics and Reliability*, 32(8):85–121, 1992.

M. Doumpos and C. Zopounidis. A multicriteria classification approach based on pairwise comparisons. *European Journal of Operational Research*, 158(2):378–389, 2004.

D. Draper. Assessment and Propagation of Model Uncertainty. *Journal of the Royal Statistical Society. Series B (Methodological)*, 57(1):45–97, 1995.

D. Dubois and H. Prade. A review of fuzzy set aggregation connectives. *Information Sciences*, 36(1-2):85–121, 1985.

D. Dubois and H. Prade. Possibility theory and its applications: a retrospective and prospective view. In *12th IEEE International Conference on Fuzzy Systems*, pages 5–11, 2003.

J. Dugdale. A cooperative problem-solver for investment management. *International Journal of Information Management*, 16(2):133–147, 1996.

J. C. Duperrin and M. Godet. SMIC 74–A method for constructing and ranking scenarios,. *Futures*, 7(4):302–312, 1975.

I. Durbach and T. Stewart. Integrating scenario planning and goal programming. *Journal of Multi-Criteria Decision Analysis*, 12(4-5):261–271, 2003.

C. Eden. On the Nature of Cognitive Maps. *Journal of Management Studies*, 29(3):261–265, 1992a.

C. Eden. Strategy Development as a Social Process. *Journal of Management Studies*, 29(6): 799–812, 1992b.

C. Eden. Analyzing cognitive maps to help structure issues or problems. *European Journal of Operational Research*, 159(3):673–686, 2004.

C. Eden and F. Ackermann. “Horses for courses”?: A stakeholder approach to the evaluation of GDSSs. *Group Decision and Negotiation*, 5(4):501–519, 1996.

P. N. Edwards. Global comprehensive models in politics and policymaking. *Climatic Change*, 32(2):149–161, 1996.

W. Edwards. How to use multiattribute utility measurement for social decisionmaking. *IEEE Transactions on Systems, Man and Cybernetics*, 7(5):326–340, 1977.

M. Enserink. After red mud flood, scientists try to halt wave of fear and rumors. *Science*, 330(6003):432–433, 2010.

EPA. Acute Exposure Guideline Levels (AEGLs). Technical report, US Environmental Protection Agency, 2009.

G. Espinosa-Paredes, A. Nuñez Carrera, A. Laureano-Cruces, A. Vázquez-Rodríguez, and E.-G. Espinosa-Martinez. Emergency management for a nuclear power plant using fuzzy cognitive maps. *Annals of Nuclear Energy*, 35(12):2387–2396, 2008.

Euro Chlor. Chlorine industry review. Technical report, Euro Chlor, 2009.

European Commission. Commission Directive 2006/15/EC–establishing a second list of indicative occupational exposure limit values in implementation of Council Directive 98/24/EC and amending Directives 91/322/EEC and 2000/39/EC. *Official Journal of the European Union*, 38:36–39, 2006.

European Communities. European Union Risk Assessment Report–Chlorine. Technical Report on Risk Assessment (Environment) 7782-50-5, European Communities, 2007.

L. Fahey. Industry Scenarios. In L. Fahey and R. Randall, editors, *Learning from the future: Competitive foresight scenarios*, pages 189–222. John Wiley & Sons, New York, 1997.

E. D. Falkenberg, W. Hesse, P. Lindgreen, B. E. Nilsson, J. L. H. Oei, C. Rolland, R. K. Stamper, F. J. M. Van Assche, A. A. Verrijn-Stuart, and K. Voss. A Framework of Information System Concepts. The FRISCO Report. Technical report, International Federation for Information Processing, Leiden, 1996.

N. Fenton and M. Neil. Making decisions: using bayesian nets and MCDA. *Knowledge-Based Systems*, 14(7):307–325, 2001.

N. Fenton, W. Marsh, M. Neil, P. Cates, S. Forey, and M. Tailor. Making resource decisions for software projects. In *Proceedings of the 26th International Conference on Software Engineering*, pages 397–406, 2004.

S. Ferson. What Monte Carlo methods cannot do. *Human and Ecological Risk Assessment: An International Journal*, 2(4):990–1007, 1996.

S. Ferson. Probability Bounds Analysis Solves the Problem of Incomplete Specification in Probabilistic Risk and Safety Assessments. In *ASCE Conference Proceedings*, volume 306, pages 16–16, Santa Barbara, 2000.

S. Ferson and J. G. Hajagos. Arithmetic with uncertain numbers: rigorous and (often) best possible answers. *Reliability Engineering & System Safety*, 85(1-3):135–152, 2004.

C. M. Fiol and A. S. Huff. Maps for managers: Where are we? Where do we go from here? *Journal of Management Studies*, 29(3):267–285, 1992.

G. W. Fischer, M. S. Kamlet, S. E. Fienberg, and D. Schkade. Risk Preferences for Gains and Losses in Multiple Objective Decision Making. *Management Science*, 32(9):1065–1086, 1986.

B. Fischhoff. Hindsight is not equal to foresight: The effect of outcome knowledge on judgment under uncertainty. *Journal of Experimental Psychology: Human Perception and Performance*, 1(3):288–299, 1975.

P. C. Fishburn. Utility theory. *Management Science*, 14(5):335–378, 1968.

P. C. Fishburn. Foundations of Risk Measurement. I. Risk As Probable Loss. *Management Science*, 30(4):396–406, 1984.

S. French. Fuzzy decision analysis: some criticisms. In H. J. Zimmerman, B. R. Gaines, and L. A. Zadeh, editors, *Fuzzy Sets and Decision Analysis*, TIMS Studies in the Management Sciences, pages 29–44. North-Holland, 1984.

S. French. *Decision Theory: An introduction of the Mathematics of Rationality*. Ellis Harwood Limited, Chichester, 1986.

S. French. Uncertainty and Imprecision: Modelling and Analysis. *The Journal of the Operational Research Society*, 46(1):70–79, 1995.

S. French. Multi-attribute decision support in the event of a nuclear accident. *Journal of Multi-Criteria Decision Analysis*, 5(1):39–57, 1996.

S. French. Modelling, making inferences and making decisions: The roles of sensitivity analysis. *TOP: An Official Journal of the Spanish Society of Statistics and Operations Research*, 11(2):229–251, 2003.

S. French and J. Geldermann. The varied contexts of environmental decision problems and their implications for decision support. *Environmental Science & Policy*, 8(4):378–391, 2005.

S. French and D. Ríos-Insua. *Statistical Decision Theory*, volume 9 of *Kendall's Library of Statistics*. Wiley, London, 2000.

S. French, T. Bedford, and E. Atherton. Supporting ALARP decision making by cost benefit analysis and multiattribute utility theory. *Journal of Risk Research*, 8(3):207–223, 2005.

S. French, J. Maule, and N. Papamichail. *Decision Behaviour, Analysis and Support*. Cambridge University Press, Cambridge, 2009.

M. Friedman and L. J. Savage. The expected-utility hypothesis and the measurability of utility. *The Journal of Political Economy*, 60(6):463–474, 1952.

P. C. Fry, D. B. Rinks, and J. L. Ringuest. Comparing the predictive validity of alternatively assessed multi-attribute preference models when relevant decision attributes are missing. *European Journal of Operational Research*, 94(3):599–609, 1996.

R. Fullér. On Obtaining OWA Operator Weights: A Sort Survey of Recent Developments. In *IEEE International Conference on Computational Cybernetics*, pages 241 –244, 2007.

R. Fullér and P. Majlender. An analytic approach for obtaining maximal entropy OWA operator weights. *Fuzzy Sets and Systems*, 124(1):53–57, 2001.

S. O. Funtowicz and J. R. Ravetz. *Uncertainty and Quality in Science for Policy.* Theory and Decision Library A. Kluwer Academic Publishers, Dordrecht, 1990.

D. Galles and J. Pearl. Axioms of causal relevance. *Artificial Intelligence*, 97(1–2):9–43, 1997.

S. I. Gass. Model World: The Great Debate-MAUT Versus AHP. *Interfaces*, 35(4):308–312, 2005.

J. Geldermann, V. Bertsch, and O. Rentz. Multi-Criteria Decision Support and Uncertainty Handling, Propagation and Visualisation for Emergency and Remediation Management. In H.-D. Haasis, H. Kopfer, and J. Schönberger, editors, *Operations Research Proceedings 2005*, Operations Research Proceedings, pages 755–760. Springer Berlin Heidelberg, 2006.

J. Geldermann, V. Bertsch, M. Treitz, S. French, K. N. Papamichail, and R. P. Hämäläinen. Multi-criteria decision support and evaluation of strategies for nuclear remediation management. *Omega*, 37(1):238–251, 2009.

GGBefG. Gesetz über die Beförderung gefährlicher Güter. *Bundesgesetzblatt*, I(3975):1774, 2009.

B. Girod, A. Wiek, H. Mieg, and M. Hulme. The evolution of the IPCC's emissions scenarios. *Environmental Science & Policy*, 12(2):103–118, 2009.

M. Glinz. An integrated formal model of scenarios based on statecharts. In W. Schäfer and P. Botella, editors, *European Software Engineering Conference*, volume 989 of *Lecture Notes in Computer Science*, pages 254–271. Springer, Berlin, Heidelberg, 1995.

M. Godet. The Art of Scenarios and Strategic Planning: Tools and Pitfalls. *Technological Forecasting and Social Change*, 65(1):3–22, 2000.

M. Godet and F. Roubelat. Creating the future: The use and misuse of scenarios. *Long Range Planning*, 29(2):164–171, 1996.

Y. Gong and J. Zhang. A Method for Fuzzy Multi-attribute Decision Making with Preference Information in the Form of Fuzzy Complementary Judgment Matrix. In *Fifth International Conference on Fuzzy Systems and Knowledge Discovery*, volume 3, pages 336–340, 2008.

C. Goodier, S. Austin, R. Soetanto, and A. Dainty. Causal mapping and scenario building with multiple organisations. *Futures*, 42(3):219–229, 2010.

P. Goodwin and G. Wright. Enhancing Strategy Evaluation in Scenario Planning: a Role for Decision Analysis. *Journal of Management Studies*, 38(1):1–16, 2001.

T. J. Gordon. Trend Impact Analysis. Technical report, AC/UNU Millennium Project, 1994.

U. Götze and K. Henselmann. *Risikomanagement*. Physica-Verlag, 2001.

S. Greeuw, M. van Asselt, J. Grosskruth, C. Storms, N. Rijkens-Klomp, D. Rothman, and J. Rotmans. Cloudy crystal balls: An assessment of recent European and global scenario studies and models. Expert's corner report: Prospects and scenarios 4, European Environment Agency, Copenhagen, 2000.

D. G. Groves and R. J. Lempert. A new analytic method for finding policy-relevant scenarios. *Global Environmental Change*, 17(1):73–85, 2007.

S. J. Guastello. *Human Factors Engineering and Ergonomics: A Systems Approach*. Lawrence Erlbaum Associates, Mahwah, 2006.

A. Guitouni and J.-M. Martel. Tentative guidelines to help choosing an appropriate MCDA method. *European Journal of Operational Research*, 109(2):501–521, 1998.

P. Guo and H. Tanaka. Decision making with interval probabilities. *European Journal of Operational Research*, 203(2):444–454, 2010.

Y. Y. Haimes, S. Kaplan, and J. H. Lambert. Risk Filtering, Ranking, and Management Framework Using Hierarchical Holographic Modeling. *Risk Analysis*, 22(2):383–398, 2002.

R. P. Hämäläinen, M. R. Lindstedt, and K. Sinkko. Multiattribute risk analysis in nuclear emergency management. *Risk Analysis*, 20(4):455–468, 2000.

D. M. Hamby. A review of techniques for parameter sensitivity analysis of environmental models. *Environmental Monitoring and Assessment*, 32(2):135–154, 1994.

S. O. Hansson. Decision making under great uncertainty. *Philosophy of the Social Sciences*, 26(3):369–386, 1996.

D. Harel. Statecharts: a visual formalism for complex systems. *Science of Computer Programming*, 8(3):231–274, 1987.

C. Harries. Correspondence to what? Coherence to what? What is good scenario-based decision making? *Technological Forecasting and Social Change*, 70(8):797–817, 2003.

H. Heitsch and W. Römisch. Scenario tree reduction for multistage stochastic programs. *Computational Management Science*, 6(2):117–133, 2009.

D. Helbing and C. Kühnert. Assessing interaction networks with applications to catastrophe dynamics and disaster management. *Physica A: Statistical Mechanics and its Applications*, 328(3-4):584–606, 2003.

J. C. Helton. Treatment of uncertainty in performance assessments for complex systems. *Risk Analysis*, 14(4):483–511, 1994.

E. G. Hertwich, T. E. McKone, and W. S. Pease. A systematic uncertainty analysis of an evaluative fate and exposure model. *Risk Analysis*, 20:439–454, 2000.

P. P. M. A. R. Heugens and J. van Oosterhout. To boldly go where no man has gone before: integrating cognitive and physical features in scenario studies. *Futures*, 33(10):861–872, 2001.

A. M. Hickey, D. L. Dean, and J. F. J. Nunamaker. Establishing a foundation for collaborative scenario elicitation. *ACM SIGMIS Database*, 30(3–4):92–110, 1999.

M. Hiete and M. Merz. An Indicator Framework to Assess the Vulnerability of Industrial Sectors against Indirect Disaster Losses. In J. Landgren and S. Jul, editors, *Proceedings of the 6th International Conference on Information Systems for Crisis Response and Management*, Gothenburg, 2009.

M. Hiete, V. Bertsch, T. Comes, F. Schultmann, and W. Raskob. Evaluation strategies for nuclear and radiological emergency and post-accident management. *Radioprotection*, 45(5):133–148, 2010.

R. Hites, Y. De Smet, N. Risse, M. Salazar-Neumann, and P. Vincke. About the applicability of MCDA to some robustness problems. *European Journal of Operational Research*, 174(1):322–332, 2006.

J. Hodgkin, V. Belton, and A. Koulouri. Supporting the intelligent MCDA user: A case study in multi-person multi-criteria decision support. *European Journal of Operational Research*, 160(1):172–189, 2005.

R. M. Hogarth and H. Kunreuther. Decision making under ignorance: Arguing with yourself. *Journal of Risk and Uncertainty*, 10(1):15–36, 1995.

M. Höjer. Transport telematics in urban systems – a backcasting delphi study. *Transportation Research Part D: Transport and Environment*, 3(6):445–463, 1998.

S. C. Hora. Aleatory and epistemic uncertainty in probability elicitation with an example from hazardous waste management. *Reliability Engenieering & System Safety*, 54(2–3): 2–3, 1996.

E. Horvitz, H. J. Suermondt, and G. F. Cooper. Bounded conditioning: Flexible inference for decisions under scarce resources. In *Fifth Workshop on Uncertainty in Artificial Intelligence*, pages 182–193, Windsor, 1989.

R. A. Howard. Decision Analysis: Practice and Promise. *Management Science*, 34(6):679–695, 1988.

S. Howick, C. Eden, F. Ackermann, and T. Williams. Building confidence in models for multiple audiences: The modelling cascade. *European Journal of Operational Research*, 186(3):1068–1083, 2008.

K. Høyland and S. W. Wallace. Generating scenario trees for multistage decision problems. *Management Science*, 47(2):295–295, 2001.

W. R. Huss. A move toward scenario analysis. *International Journal of Forecasting*, 4(3): 377–388, 1988.

W. R. Huss and E. J. Honton. Scenario planning–What style should you use? *Long Range Planning*, 20(4):21–29, 1987.

C. Hwang, S. Paidy, K. Yoon, and A. Masud. Mathematical programming with multiple objectives: A tutorial. *Computers & Operations Research*, 7(1-2):5–31, 1980.

E. Jacquet-Lagréze and Y. Siskos. Assessing a set of additive utility functions for multicriteria decision-making, the UTA method. *European Journal of Operational Research*, 10 (2):151–164, 1982.

E. Jacquet-Lagréze and Y. Siskos. Preference disaggregation: 20 years of MCDA experience. *European Journal of Operational Research*, 130(2):233–245, 2001.

J. Jakeman, M. Eldred, and D. Xiu. Numerical approach for quantification of epistemic uncertainty. *Journal of Computational Physics*, 229(12):4648–4663, 2010.

M. Jarke. Requirements tracing. *Communications of the ACM*, 41(12):32–36, 1998.

M. Jarke, X. T. Bui, and J. M. Carroll. Scenario management: An interdisciplinary approach. *Requirements Engineering*, 3(3):155–173, 1998.

F. V. Jensen, T. D. Nielsen, and P. P. Shenoy. Sequential influence diagrams: A unified asymmetry framework. *International Journal of Approximate Reasoning*, 42(1-2):101–118, 2006.

A. Jetter and W. Schweinfort. Building scenarios with Fuzzy Cognitive Maps: An exploratory study of solar energy. *Futures*, 43(1):52–66, 2011.

S. Kaplan. The words of risk analysis. *Risk Analysis*, 17(4):407–417, 1997.

S. Kaplan and B. J. Garrick. On the quantitative definition of risk. *Risk Analysis*, 1(1): 11–27, 1981.

R. L. Keeney. Utility independence and preferences for multiattributed consequences. *Operations Research*, 19(4):875–893, 1971.

R. L. Keeney. The art of assessing multiattribute utility functions. *Organizational Behavior and Human Performance*, 19(2):267–310, 1977.

R. L. Keeney. Decision Analysis: An Overview. *Operations Research*, 30(5):803–838, 1982.

R. L. Keeney. Value-focused thinking: Identifying decision opportunities and creating alternatives. *European Journal of Operational Research*, 92(3):537–549, 1996.

R. L. Keeney and H. Raiffa. *Decisions with Multiple Objectives*. John Wiley & Sons, New York, 1976.

R. L. Keeney, H. Raiffa, and D. W. Rajala. Decisions with multiple objectives: Preferences and value trade-offs. *IEEE Transactions on Systems, Man and Cybernetics*, 9(7):403–403, 1979.

J. M. Keynes. *A Treatise On Probability*. MacMillan, London, 1921.

F. I. Khan and S. A. Abbasi. Major accidents in process industries and an analysis of causes and consequences. *Journal of Loss Prevention in the Process Industries*, 12(5):361–378, 1999.

F. I. Khan and S. A. Abbasi. A criterion for developing credible accident scenarios for risk assessment. *Journal of Loss Prevention in the Process Industries*, 15(6):467–475, 2002.

G. A. Kiker, T. S. Bridges, A. Varghese, T. P. Seager, and I. Linkov. Application of multi-criteria decision analysis in environmental decision making. *Integrated Environmental Assessment and Management*, 1(2):95–108, 2005.

K. Kim and K. S. Park. Ranking fuzzy numbers with index of optimism. *Fuzzy Sets and Systems*, 35(2):143–150, 1990.

A. D. Kiureghian and O. Ditlevsen. Aleatory or epistemic? Does it matter? *Structural Safety*, 31(2):105–112, 2009.

K. Klamroth and K. Miettinen. Integrating approximation and interactive decision making in multicriteria optimization. *Operations Research*, 56(1):222–234, 2008.

R. Kleijn, A. Tukker, and E. van der Voet. Chlorine in the Netherlands, Part I, An Overview. *Journal of Industrial Ecology*, 1(1):95–116, 1997.

C. A. d. Kluyver and H. Moskowitz. Assessing Scenario Probabilities Via Interactive Goal Programming. *Management Science*, 30(3):273–278, 1984.

F. H. Knight. *Risk, Uncertainty, and Profit*. Houghton Mifflin Co., Boston, 1921.

K. Kok. The potential of Fuzzy Cognitive Maps for semi-quantitative scenario development, with an example from Brazil. *Global Environmental Change*, 19(1):122–133, 2009.

P. Korhonen, H. Moskowitz, and J. Wallenius. Multiple criteria decision support – A review. *European Journal of Operational Research*, 63(3):361–375, 1992.

R. Korn. Worst-case scenario investment for insurers. *Insurance: Mathematics and Economics*, 36(1):1–11, 2005.

J. S. H. Kornbluth. Dynamic multi-criteria decision making. *Journal of Multi-Criteria Decision Analysis*, 1(2):81–92, 1992.

B. Kosko. Fuzzy cognitive maps. *International Journal of Man-Machine Studies*, 24(1):65–75, 1986.

P. Kouvelis and G. Yu. *Robust Discrete Optimization and Its Applications*, volume 14. Kluwer Academic Publishers, Dordrecht, 1997.

T. Krynski and J. Tenenbaum. The role of causality in judgment under uncertainty. *Journal of Experimental Psychology: General*, 136(3):430–450, 2007.

A. Küberger. The influence of framing on risky decisions: A meta-analysis. *Organizational Behavior and Human Decision Processes*, 75(1):23–55, 1998.

D. Kuchta. Use of fuzzy numbers in project risk (criticality) assessment. *International Journal of Project Management*, 19(5):305–310, 2001.

S. Kuhlmann. Future governance of innovation policy in europe Ů three scenarios. *Research Policy*, 30(6):953–976, 2001.

S. Laurence and E. Margolis. Concepts and cognitive science. In E. Margolis and S. Laurence, editors, *Concepts: core readings*, pages 3–81. The MIT Press, 1999.

E. L. Lawler and D. E. Wood. Branch-And-Bound Methods: A Survey. *Operations Research*, 14(4):699–719, 1966.

S. Lee, J. Courtney Jr., and R. O'Keefe. A system for organizational learning using cognitive maps. *Omega*, 20(1):23–36, 1992.

R. Lempert, S. Popper, and S. Bankes. Confronting surprise. *Social Science Computer Review*, 20(4):420–440, 2002.

R. J. Lempert, D. G. Groves, S. W. Popper, and S. C. Bankes. A general, analytic method for generating robust strategies and narrative scenarios. *Management Science*, 52(4): 514–528, 2006.

M.-J. Lesot and B. Bouchon-Meunier. Cluster Characterization through a Representativity Measure. In H. Christiansen, M.-S. Hacid, T. Andreasen, and H. L. Larsen, editors, *Flexible Query Answering Systems*, volume 3055 of *Lecture Notes in Computer Science*, pages 446–458. Springer Berlin Heidelberg, 2004.

T. Liao, Z. Zhang, and C. Mount. Similarity measures for retrieval in case-based reasoning systems. *Applied Artificial Intelligence*, 12(4):267–288, 1998.

C.-J. Lin and W.-W. Wu. A causal analytical method for group decision-making under fuzzy environment. *Expert Systems with Applications*, 34(1):205–213, 2008.

D. Lin. An Information-Theoretic Definition of Similarity. In *Proceedings of the Fifteenth International Conference on Machine Learning*, pages 296–304. Morgan Kaufmann Publishers, 1998.

X. Liu. On the methods of decision making under uncertainty with probability information. *International Journal of Intelligent Systems*, 19(12):1217–1238, 2004.

D. R. Luce and H. Raiffa. *Games and Decisions: Introduction and Critical Survey*. Dover Publications, 1989.

S. Mahesh and H. Moskowitz. An information-maximizing interactive procedure for scenario probability elicitation. *Decision Sciences*, 21(3):533–550, 1990.

M. Mahmoud, Y. Liu, H. Hartmann, S. Stewart, T. Wagener, D. Semmens, R. Stewart, H. Gupta, D. Dominguez, F. Dominguez, D. Hulse, R. Letcher, B. Rashleigh, C. Smith, R. Street, J. Ticehurst, M. Twery, H. van Delden, R. Waldick, D. White, and L. Winter. A formal framework for scenario development in support of environmental decision-making. *Environmental Modelling & Software*, 24(7):798–808, 2009.

J. Malczewski. Integrating multicriteria analysis and geographic information systems: the ordered weighted averaging OWA approach. *International Journal of Environmental Technology and Management*, 6(13):7–19, 2005.

J. G. March. Variable risk preferences and adaptive aspirations. *Journal of Economic Behavior & Organization*, 9(1):5–24, 1988.

J. G. March and Z. Shapira. Managerial perspectives on risk and risk taking. *Management Science*, 33(11):1404–1418, 1987.

J. P. Martino and K.-L. Chen. Cluster analysis of cross impact model scenarios. *Technological Forecasting and Social Change*, 12(1):61–71, 1978.

D. H. Mason and J. Herman. Scenarios and strategies: making the scenario about the business. *Strategy & Leadership*, 31(1):23–31, 2003.

M. A. Matos. Decision under risk as a multicriteria problem. *European Journal of Operational Research*, 181(3):1516–1529, 2007.

I. Matzkevich and B. Abramson. Decision Analytic Networks in Artificial Intelligence. *Management Science*, 41(1):1–22, 1995.

J. C. Mc Carthy, P. G. Healey, P. Wright, and M. D. Harrison. Accountability of work activity in high-consequence work systems: human error in context. *International Journal of Human-Computer Studies*, 47(6):735–766, 1997.

T. L. McDaniels, R. S. Gregory, and D. Fields. Democratizing risk management: Successful public involvement in local water management decisions. *Risk Analysis*, 19(3):497–510, 1999.

D. Mendonca, G. E. Beroggi, D. van Gent, and W. A. Wallace. Designing gaming simulations for the assessment of group decision support systems in emergency response. *Safety Science*, 44(6):523–535, 2006.

D. Mendonça, T. Jefferson, and J. Harrald. Collaborative adhocracies and mix-and-match technologies in emergency management. *Communications of the ACM*, 50:44–49, 2007.

T. B. Meslin. Assessment and management of risk in the transport of dangerous materials: The case of chlorine transport in france. *Risk Analysis*, 1(2):137–141, 1981.

G. A. Miller. The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review*, 63:81–97, 1956.

G. A. Miller and W. G. Charles. Contextual correlates of semantic similarity. *Language and Cognitive Processes*, 6(1):1–28, 1991.

V. Miranda and L. Proenca. Why risk analysis outperforms probabilistic choice as the effective decision support paradigm for power system planning. *IEEE Transactions on Power Systems*, 13(2):643–648, 1998.

G. Montibeller and V. Belton. Causal maps and the evaluation of decision options—a review. *Journal of the Operational Research Society*, 57(57):779–791, 2006.

G. Montibeller and V. Belton. Qualitative operators for reasoning maps: Evaluating multi-criteria options with networks of reasons. *European Journal of Operational Research*, 195 (3):829–840, 2009.

G. Montibeller and A. Franco. Multi-criteria decision analysis for strategic decision making. In P. M. Pardalos, D. W. Hearn, C. Zopounidis, and P. M. Pardalos, editors, *Handbook of Multicriteria Analysis*, volume 103 of *Applied Optimization*, pages 25–48. Springer Berlin Heidelberg, 2010.

G. Montibeller, H. Gummer, and D. Tumidei. Combining scenario planning and multi-criteria decision analysis in practice. *Journal of Multi-Criteria Decision Analysis*, 14(1-3): 5–20, 2006.

G. Montibeller, V. Belton, F. Ackermann, and L. Ensslin. Reasoning maps for decision aid: an integrated approach for problem-structuring and multi-criteria evaluation. *Journal of the Operational Research Society*, 59(5):575–589, 2008.

V. Mookerjee and M. Mannino. Sequential decision models for expert system optimization. *IEEE Transactions on Knowledge and Data Engineering*, 9(5):675–687, 1997.

M. G. Morgan and M. Henrion. *Uncertainty: A Guide to Dealing with Uncertainty in Quantitative Risk and Policy Analysis*. Cambridge University Press, Cambridge, 1990.

J. Munkres. *Topology*. Prentice Hall, Upper Saddle River, 2nd edition edition, 2000.

G. L. Murphy and D. L. Medin. The role of theories in conceptual coherence. *Psychological Review*, 92(3):289–316, 1985.

J. Mustajoki and R. P. Hämäläinen. Decision Support by Interval SMART/SWING–Incorporating Imprecision in the SMART and SWING Methods. *Decision Sciences*, 36 (2):317–339, 2005.

J. Mustajoki, R. P. Hämäläinen, and K. Sinkko. Interactive computer support in decision conferencing: Two cases on off-site nuclear emergency management. *Decision Support Systems*, 42(4):2247–2260, 2007.

S. Nadkarni and P. P. Shenoy. A Bayesian network approach to making inferences in causal maps. *European Journal of Operational Research*, 128(3):479–498, 2001.

G. Navarro. A guided tour to approximate string matching. *ACM Comput. Surv.*, 33(1): 31–88, 2001.

T. Neidhart, T. Comes, and R. Pinchuk. Visualization and Human Machine Interaction. Technical report, Report within the EC FP7-ICT project Diadem (ref. no: 224318), 2010.

E. Nikolaidis. Types of uncertainty in design decision making. In E. Nikolaidis, D. M. Ghiocel, and S. Singha, editors, *Engineering Design Reliability Handbook*. CRC Press, New York, 2005.

H. Núñez, M. Sànchez-Marrè, U. Cortés, J. Comas, M. Martínez, I. Rodríguez-Roda, and M. Poch. A comparative study on the use of similarity measures in case-based reasoning to improve the classification of environmental system situations. *Environmental Modelling & Software*, 19(9):809–819, 2004.

W. L. Oberkampf, J. C. Helton, S. F. Joslyn, Cliff A. Wojtkiewicz, and S. Ferson. Challenge problems: uncertainty in system response given uncertain parameters. *Reliability Engenieering & System Safety*, 85(1–3):11–19, 2004.

F. A. O'Brien. Scenario planning—lessons for practice from teaching and learning. *European Journal of Operational Research*, 152(3):709–722, 2004.

M. O'Hagan. Aggregating template or rule antecedents in real-time expert systems with fuzzy set logic. In *Twenty-Second Asilomar Conference on Signals, Systems and Computers*, volume 2, pages 681–689, 1988.

K. Papamichail and S. French. Design and evaluation of an intelligent decision support system for nuclear emergencies. *Decision Support Systems*, 41(1):84–111, 2005.

K. N. Papamichail and S. French. Explaining and justifying the advice of a decision support system: a natural language generation approach. *Expert Systems with Applications*, 24(1):35–48, 2003.

E. Parson, V. Burkett, K. Fisher-Vanden, D. Keith, L. Mearns, H. Pitcher, C. Rosenzweig, and M. Webster. Global Change Scenarios: Their Development and Use. In *Sub-report 2.1B of Synthesis and Assessment Product 2.1 by the U.S. Climate Change Science Program and the Subcommittee on Global Change Research*. Department of Energy, Office of Biological & Environmental Research, Washington D.C., 2007.

E. Paté-Cornell. Risk and uncertainty analysis in government safety decisions. *Risk Analysis*, 22:633–646, 2002.

G. Pavlin, M. Kamermans, and M. Scafes. Dynamic process integration framework: Towards efficient information processing in complex distributed systems. In G. Papadopoulos and C. Badica, editors, *Intelligent Distributed Computing III*, volume 237 of *Studies in Computational Intelligence*, pages 161–174. Springer Berlin Heidelberg, 2009a.

G. Pavlin, N. Wijngaards, and K. Nieuwenhuis. Towards a single information space for environmental management through self-configuration of distributed information processing systems. In J. Hřebíček, J. Hradec, E. Pelikán, O. Mírovský, W. Pillmann, I. Holoubek, and T. Bandholtz, editors, *Proceedings of the European conference TOWARDS eENVIRONMENT. Opportunities of SEIS and SISE: Integrating Environmental Knowledge in Europe*, pages 94–103, Masaryk University, Brno, Czech Republic, 2009b.

A. Peña, H. Sossa, and A. Gutiérrez. Causal knowledge and reasoning by cognitive maps: Pursuing a holistic approach. *Expert Systems with Applications*, 35(1-2):2–18, 2008. ISSN 0957-4174.

J. Pearl. Causal Diagrams for Empirical Research. *Biometrika*, 82(4):669–688, 1995.

J. Pearl. *Causality: models, reasoning, and inference*. Cambridge University Press, Cambridge, 2009.

S. Pender. Managing incomplete knowledge: Why risk management is not sufficient. *International Journal of Project Management*, 19(2):79–87, 2001.

G. D. Peterson, G. S. Cumming, and S. R. Carpenter. Scenario Planning: a Tool for Conservation in an Uncertain World. *The Journal of the Society for Conservation Biology*, 17 (2):358–366, 2003.

M. Peterson. *An Introduction to Decision Theory*. Cambridge Introductions to Philosophy. Cambridge University Press, Cambridge, 2009.

L. D. Phillips. A theory of requisite decision models. *Acta Psychologica*, 56(1–3):29–48, 1984.

K. Pohl, K. Pohl, P. Haumer, and P. Haumer. Modelling contextual information about scenarios. In *Proceedings of the Third International Workshop on Requirements Engineering: Foundations of Software Quality*, pages 187–204, 1997.

J.-C. Pomerol. Scenario development and practical decision making under uncertainty: Application to requirements engineering. *Requirements Engineering*, 3(3):174–181, 1998.

M. Pötzsch. *Risikobewertung des Transports von Binnentankschiffen auf dem Rhein unter besonderer Beachtung des Gefahrgutrechts*. PhD thesis, Bergische Universität Wupptertal, 2004.

J. W. Pratt. Risk aversion in the small and in the large. *Econometrica*, 32(1/2):122–136, 1964.

P. Purdom. A transitive closure algorithm. *BIT Numerical Mathematics*, 10(1):76–94, 1970.

H. Raiffa. Preferences for multi-attributed alternatives. *Journal of Multi-Criteria Decision Analysis*, 14(4-6):115–157, 2006.

C. Ram, G. Montibeller, and A. Morton. Extending the use of scenario planning and MCDA for the evaluation of strategic options. *Journal of the Operational Research Society*, 62:817–829, 2011.

W. Raskob, F. Gering, and V. Bertsch. Approaches to visualisation of uncertainties to decision makers in an operational Decision Support System. In *Proceedings of the 6th International Conference on Information Systems for Crisis Response and Management*, Gothenburg, 2009.

J. C. Refsgaard, J. P. van der Sluijs, A. L. Højberg, and P. A. Vanrollegehem. Uncertainty in the environmental modelling process – A framework and guidance. *Environmental Modelling & Software*, 22(11):1543–1556, 2007.

H. M. Regan, Y. Ben-Haim, B. Langford, W. G. Wilson, P. Lundberg, S. J. Andelman, and M. A. Burgman. Robust Decision-Making Under Severe Uncertainty For Conservation Management. *Ecological Applications*, 15(4):1471–1477, 2005.

E. Reiter and R. Dale. Building applied natural language generation systems. *Natural Language Engineering*, 3(1):57–87, 1997.

R. A. Ribeiro. Fuzzy multiple attribute decision making: A review and new preference elicitation techniques. *Fuzzy Sets and Systems*, 78(2):155–181, 1996.

F. Ricci and P. Avesani. Learning a local similarity metric for case-based reasoning. In M. Veloso and A. Aamodt, editors, *Case-Based Reasoning Research and Development*, pages 301–312. Springer Berlin Heidelberg, 1995.

D. Ríos-Insua and S. French. A framework for sensitivity analysis in discrete multi-objective decision-making. *European Journal of Operational Research*, 54(2):176–190, 1991.

T. Ritchey. Problem structuring using computer-aided morphological analysis. *The Journal of the Operational Research Society*, 57(7):792–801, 2006.

B. Rohrmann. The risk notion: Epistemological and empirical considerations. In M. Stewart and R. Melchers, editors, *Integrated risk assessment : application and regulations. Proceedings of the third Conference on Integrated Risk Assessment*, pages 39–46. Balkema, Rotterdam, 1998.

F. Roubelat. Scenario Planning as a Networking Process. *Technological Forecasting and Social Change*, 65(1):99–112, 2000.

F. Roubelat. Scenarios to challenge strategic paradigms: Lessons from 2025. *Futures*, 38 (5):519–527, 2006.

W. D. Rowe. Understanding uncertainty. *Risk Analysis*, 14(5):743–750, 1994.

B. Roy. The outranking approach and the foundations of electre methods. *Theory and Decision*, 31(1):49–73, 1991.

B. Roy. Robustness in operational research and decision aiding: a multi-faceted issue. *European Journal of Operational Research*, 200(3):629–638, 2010.

W. Rudin. *Real and Complex Analysis*. McGraw-Hill Science/Engineering/Math, New York, 1986.

W. Rudin. *Functional Analysis*. McGraw-Hill Science/Engineering/Math, New York, 1991.

E. H. Ruspini, J. D. Lowrance, and T. M. Strat. Understanding evidential reasoning. *International Journal of Approximate Reasoning*, 6(3):401–424, 1992.

B. Rustem, R. G. Becker, and W. Marty. Robust min-max portfolio strategies for rival forecast and risk scenarios. *Journal of Economic Dynamics and Control*, 24(11-12):1591–1621, 2000.

A. Saltelli, M. Ratto, T. Andres, F. Campolongo, J. Cariboni, D. Gatelli, M. Saisana, and S. Tarantola. *Global Sensitivity Analysis: The Primer*. Wiley-Interscience, Chichester, 2008.

L. J. Savage. *The Foundations of Statistics*. Dover Publications, New York, 2nd revised edition, 1972.

K. M. Sayre. Statistical Models of Causal Relations. *Philosophy of Science*, 44(2):203–214, 1977.

A. M. Schaafstal, J. H. Johnston, and R. L. Oser. Training teams for emergency management. *Computers in Human Behavior*, 17(5-6):615–626, 2001.

S. Schnaars and P. L. Ziamou. The essentials of scenario writing. *Business Horizons*, 44(4): 25–31, 2001.

S. P. Schnaars. How to develop and use scenarios. *Long Range Planning*, 20(1):105–114, 1987.

S. H. Schneider. What is ’dangerous’ climate change? *Nature*, 411(6833):17–19, 2001.

P. Schoemaker. Scenario planning: A tool for strategic thinking. *Sloan Management Review*, 36(2):25–40, 1995.

P. J. H. Schoemaker. Multiple Scenario Development: Its Conceptual and Behavioral Foundation. *Strategic Management Journal*, 14(3):193–213, 1993.

R. W. Scholz and O. Tietje, editors. *Embedded Case Study Methods. Integrating Quantitative and Qualitative Knowledge*. Sage Publications, Thousand Oaks, 2002.

P. Schwartz. *The Art of the Long View: Planning for the Future in an Uncertain World*. John Wiley & Sons, New York, 1991.

C. Selin. Trust and the illusive force of scenarios. *Futures*, 38(1):1–14, 2006.

A. Sengupta and T. K. Pal. On comparing interval numbers. *European Journal of Operational Research*, 127(1):28–43, 2000.

R. D. Shachter. Evaluating Influence Diagrams. *Operations Research*, 34(6):871–882, 1986.

R. D. Shachter. Probabilistic Inference and Influence Diagrams. *Operations Research*, 36(4): 589–604, 1988.

G. Shafer. Conditional Probability. *International Statistical Review / Revue Internationale de Statistique*, 53(3):261–275, 1985.

C. E. Shannon. A mathematical theory of communication. *The Bell System Technical Journal*, 27:379–423, 623–656, 1948.

M. J. Shaw and M. S. Fox. Distributed artificial intelligence for group decision support : Integration of problem solving, coordination, and learning. *Decision Support Systems*, 9 (4):349–367, 1993.

S. D. Sheetz, D. P. Tegarden, K. A. Kozar, and I. Zigurs. A group support systems approach to cognitive mapping. *Journal of Management Information Systems*, 11(1):31–57, 1994.

P. P. Shenoy. A comparison of graphical techniques for decision analysis. *European Journal of Operational Research*, 78(1):1–21, 1994.

W. S. Shin and A. Ravindran. Interactive multiple objective optimization: Survey i – continuous case. *Computers & Operations Research*, 18(1):97–114, 1991.

K. Sigel, B. Klauer, and C. Pahl-Wostl. Conceptualising uncertainty in environmental decision-making: The example of the eu water framework directive. *Ecological Economics*, 69(3):502–510, 2010.

H. A. Simon. Rational Decision Making in Business Organizations. *The American Economic Review*, 69(4):493–513, 1979.

Y. Siskos and A. Spyridakos. Intelligent multicriteria decision support: Overview and perspectives. *European Journal of Operational Research*, 113(2):236–246, 1999.

Y. Siskos, A. Spyridakos, and D. Yannacopoulos. Using artificial intelligence and visual techniques into preference disaggregation analysis: The MIIDAS system. *European Journal of Operational Research*, 113(3):281–299, 1999.

J. P. v. d. Sluijs, M. Craye, S. Funtowicz, P. Kloprogge, J. Ravetz, and J. Risbey. Combining Quantitative and Qualitative Measures of Uncertainty in Model-Based Environmental Assessment: The NUSAP System. *Risk Analysis*, 25(2):481–492, 2005.

J. E. Smith, S. Holtzman, and J. E. Matheson. Structuring conditional relationships in influence diagrams. *Operations Research*, 41(2):280–297, 1993.

M. Spielmann, R. Scholz, O. Tietje, and P. d. Haan. Scenario Modelling in Prospective LCA of Transport Systems. Application of Formative Scenario Analysis. *The International Journal of Life Cycle Assessment*, 10(5):325–335, 2005.

D. Stainforth, M. Allen, E. Tredger, and L. Smith. Confidence, uncertainty and decision-support relevance in climate predictions. *Philosophical Transactions of the Royal Society A*, 365(1857):2145–2161, 2007.

T. J. Stewart. A critical survey on the status of multiple criteria decision making theory and practice. *Omega*, 20(5-6):569–586, 1992.

T. J. Stewart. Robustness of Additive Value Function Methods in MCDM. *Journal of Multi-Criteria Decision Analysis*, 5(4):301–309, 1996.

J. Stoye. Minimax regret treatment choice with finite samples. *Journal of Econometrics*, 151 (1):70–81, 2009.

J. Surma and K. Vanhoof. Integrating rules and cases for the classification task. In M. Veloso and A. Aamodt, editors, *Case-Based Reasoning Research and Development*, Lecture Notes in Computer Science, pages 325–334. Springer Berlin Heidelberg, 1995.

R. Swart, L. Bernstein, M. Ha-Duong, and A. Petersen. Agreeing to disagree: uncertainty management in assessing climate change, impacts and responses by the ipcc. *Climatic Change*, 92(1):1–29, 2009.

C. Tannert, H.-D. Elvers, and B. Jandrig. The ethics of uncertainty. In the light of possible dangers, research becomes a moral duty. *EMBO Report*, 8(10):892–896, 2007.

P. Tapio. Disaggregative policy Delphi: Using cluster analysis as a tool for systematic scenario formation. *Technological Forecasting and Social Change*, 70(1):83–101, 2003.

O. Tietje. Identification of a small reliable and efficient set of consistent scenarios. *European Journal of Operational Research*, 162(2):418–432, 2005.

B. Tonn. Imprecise probabilities and scenarios. *Futures*, 37(8):767–775, 2005.

V. Torra. The weighted owa operator. *International Journal of Intelligent Systems*, 12(2): 153–166, 1997.

W. T. Tucker and S. Ferson. Probability bounds analysis in environmental risk assessment. In *Applied Biomathematics*, Setauket, 2003.

E. Turban and J. Aronson. *Decision Support Systems and Intelligent Systems*. Prentice Hall, Upper Saddle River, 1997.

E. Turban and P. R. Watkins. Integrating expert systems and decision support systems. *MIS Quarterly*, 10(2):121–136, 1986.

A. Tversky. Features of similarity. *Psychological Review*, 84(4):327–352, 1977.

A. Tversky. The framing of decisions and the psychology of choice. *Science*, 211(4481):453 –458, 1981.

A. Tversky and D. Kahneman. Judgment under uncertainty: Heuristics and biases. *Science*, 185(4157):1124–1131, 1974.

A. Tversky and P. Wakker. Risk attitudes and decision weights. *Econometrica*, 63(6):1255–1280, 1995.

United Nations. Globally harmonized system of classification and labelling of chemicals. Technical report, United Nations, New York, Geneva, 2005.

United Nations. Recommendations on the Transport of Dangerous Goods. Model Regulations. Technical report, United Nations, New York and Geneva, 2007.

A. Valls, M. Batet, and E. M. López. Using expert's rules as background knowledge in the clusdm methodology. *European Journal of Operational Research*, 195(3):864–875, 2009.

M. B. A. van Asselt and J. Rotmans. Uncertainty in integrated assessment modelling. *Climatic Change*, 54(1):75–105, 2002.

K. van der Heijden. *Scenarios: The Art of Strategic Conversation*. Wiley, Chichester, 2007.

P. van der Keur, M. Brugnach, A. Dewulf, J. Refsgaard, P. Zorilla, M. Poolman, N. Isendahl, G. Raadgever, H. Henriksen, J. Warmink, M. Lamers, and J. Mysiak. Identifying uncertainty guidelines for supporting policy making in water management illustrated for upper guadiana and rhine basins. *Water Resources Management*, 24(1):1–38, 2010.

P. van Notten. Scenario development: a typology of approaches. In *Schooling for Tomorrow: Think Scenarios, Rethink Education*, volume 6, pages 66–92. OECD Education & Skills, 2006.

P. W. F. van Notten, J. Rotmans, M. B. A. van Asselt, and D. S. Rothman. An updated scenario typology. *Futures*, 35(5):423–443, 2003.

C. A. Varum and C. Melo. Directions in scenario planning literature - a review of the past decades. *Futures*, 42(4):355–369, 2010.

P. Vincke. Robust solutions and methods in decision-aid. *Journal of Multi-Criteria Decision Analysis*, 8(3):181–187, 1999.

A. Volkery and T. Ribeiro. Scenario planning in public policy: Understanding use, impacts and the role of institutional context factors. *Technological Forecasting and Social Change*, 76(9):1198–1207, 2009.

J. Von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior*. Princeton University Press, 1953.

D. von Winterfeldt and W. Edwards. *Decision Analysis and Behavioral Research*. Cambridge University Press, Cambridge, 1986.

P. Wack. Scenarios: uncharted waters ahead. *Harvard Business Review*, 63(5):73–89, 1985.

W. Walker, P. Harremoës, J. Rotmans, J. P. van der Sluijs, M. B. A. van Asselt, P. Janssen, and M. P. Krayer von Krauss. Defining uncertainty: A conceptual basis for uncertainty management in model-based decision support. *Integrated Assessment*, 4(1):5–17, 2003.

J. Wallenius, J. S. Dyer, P. C. Fishburn, R. E. Steuer, S. Zionts, and K. Deb. Multiple Criteria Decision Making, Multiattribute Utility Theory: Recent Accomplishments and What Lies Ahead. *Management Science*, 54(7):1336–1349, 2008.

P. Walley and T. L. Fine. Towards a frequentist theory of upper and lower probability. *The Annals of Statistics*, 10(3):741–761, 1982.

J. Wang and S. Zionts. Negotiating wisely: Considerations based on MCDM/MAUT. *European Journal of Operational Research*, 188(1):191–205, 2008.

E. Ward and A. E. Schriefer. Dynamic scenarios: systems thinking meets scenario planning. In *Learning from the Future: Competitive Foresight Scenarios*, pages 140–156. John Wiley & Sons, New York, 1997.

K. Warren. Exploring competitive futures using cognitive mapping. *Long Range Planning*, 28(5):10–21, 1995.

C. Wei, W. J. Rogers, and M. S. Mannan. Application of screening tools in the prevention of reactive chemical incidents. *Journal of Loss Prevention in the Process Industries*, 17(4): 261–269, 2004.

M. A. Wenck, D. Van Sickle, D. Drociuk, A. Belflower, C. Youngblood, D. Whisnant, R. Taylor, V. Rudnick, and J. Gibson. Rapid assessment of exposure to chlorine released from a train derailment and resulting health impact. *Public Health Reports*, 122 (6):784–792, 2007.

D. Werner. *Funktionalanalysis*. Springer, Berlin, 2000.

J. Williams and N. Steele. Difference, distance and similarity as a basis for fuzzy decision support based on prototypical decision classes. *Fuzzy Sets and Systems*, 131(1):35–46, 2002.

P. Wilson. Situational relevance. *Information Storage and Retrieval*, 9(8):457–471, 1973.

G. Winch. Dynamic visioning for dynamic environments. *The Journal of the Operational Research Society*, 50(4):354–361, 1999.

C. Winder. The toxicology of chlorine. *Environmental Research*, 85(2):105–114, 2001.

R. T. Woodward and R. C. Bishop. How to Decide When Experts Disagree: Uncertainty-Based Choice Rules in Environmental Policy. *Land Economics*, 73(4):492–507, 1997.

G. Wright and P. Goodwin. Decision making and planning under low levels of predictability: Enhancing the scenario method. *International Journal of Forecasting*, 25(4): 813–825, 2009.

W.-W. Wu and Y.-T. Lee. Developing global managers' competencies using the fuzzy DEMATEL method. *Expert Systems with Applications*, 32(2):499–507, 2007.

Y.-T. Wu. Computational methods for efficient structural reliability and reliability sensitivity analysis. *AIAA Journal*, 32:1717–1723, 1994.

J.-L. Wybo and H. Lonka. Emergency management and the information society: how to improve the synergy. *International Journal of Emergency Management*, 1:183–190, 2003.

H. Xu and P. Smets. Reasoning in evidential networks with conditional belief functions. *International Journal of Approximate Reasoning*, 14(2-3):155–185, 1996.

Z. Xu. An overview of methods for determining OWA weights. *International Journal of Intelligent Systems*, 20(8):843–865, 2005.

R. R. Yager. Multiple objective decision-making using fuzzy sets. *International Journal of Man-Machine Studies*, 9(4):375–382, 1977.

R. R. Yager. Fuzzy decision making including unequal objectives. *Fuzz*, 1(2):87–95, 1978.

R. R. Yager. Arithmetic and other operations on dempster-shafer structures. *International Journal of Man-Machine Studies*, 25(4):357–366, 1986.

R. R. Yager. On ordered weighted averaging aggregation operators in multicriteria decisionmaking. *IEEE Transactions on Systems, Man and Cybernetics*, 18(1):183–190, 1988.

R. R. Yager. Connectives and quantifiers in fuzzy sets. *Fuzzy Sets and Systems*, 40(1):39–75, 1991.

R. R. Yager. On the valuation of alternatives for decision-making under uncertainty. *International Journal of Intelligent Systems*, 17(7):687–707, 2002.

R. R. Yager. Decision Making Under Dempster-Shafer Uncertainties. In R. Yager and L. Liu, editors, *Classic Works of the Dempster-Shafer Theory of Belief Functions*, volume 219 of *Studies in Fuzziness and Soft Computing*, pages 619–632. Springer Berlin Heidelberg, 2008.

R. Yu and G.-H. Tzeng. A soft computing method for multi-criteria decision making with dependence and feedback. *Applied Mathematics and Computation*, 180(1):63–75, 2006.

L. A. Zadeh. Fuzzy logic and approximate reasoning. *Synthese*, 30(3):407–428, 1975.

W. Zhang, S. Chen, and J. Bezdek. Pool2: a generic system for cognitive map development and decision analysis. *IEEE Transactions on Systems, Man and Cybernetics*, 19(1):31–39, 1989.

H.-J. Zimmermann. Problems and tools to model uncertainty in expert- and decision support systems. *Mathematical and Computer Modelling*, 14:8–20, 1990.

H. J. Zimmermann. An application-oriented view of modeling uncertainty. *European Journal of Operational Research*, 122(2):190–198, 2000.

E. Zitzler, K. Deb, and L. Thiele. Comparison of multiobjective evolutionary algorithms: Empirical results. *Evolutionary Computation*, 8(2):173–195, 2000.

A. Graphical Representations of Sequential Decision-Making

To structure these situations two methods for graphical representation have been developed: influence diagrams and decision trees [Clemen and Reilly, 1999].

A.1. Decision Trees

Decision trees can be used to represent (small) series of decision problems. They are acyclic network graphs, where *alternatives* or *decision vertices* are represented by rectangles, *chance vertices* by ovals and *consequences* or *evaluation vertices* by diamonds (cf. Figure A.1). The edges emanating from the decision vertices represent the choices available to the decision makers, whereas the edges from a chance vertex represent the possible outcomes of the chance event. The evaluation vertices can be found at the end of each branch. The results comprise the consequences of each step are aggregated.

As decision trees are most often applied in probabilistic frameworks, usually expected utilities are used for the evaluation of each sequence of decisions. To calculate these utilities, for each chance vertex (conditional) probabilities for each possible event are determined.[59] One of the advantages of decision trees is that they are very well suited to represent the temporal order of the situation's development with the decisions that need to be made at each step. They are intuitive and easily understandable [Shenoy, 1994]. Following a branch of a tree from the root to a leaf vertex corresponds to the order in which the decisions are made or the outcomes of chance vertices are revealed to the decision maker. Thus, decision trees follow the cause-effect-reasoning approach as used in SBR.

[59] See [Clemen and Reilly, 1999] for techniques for deriving an optimal sequence of decisions from the probabilistic information.

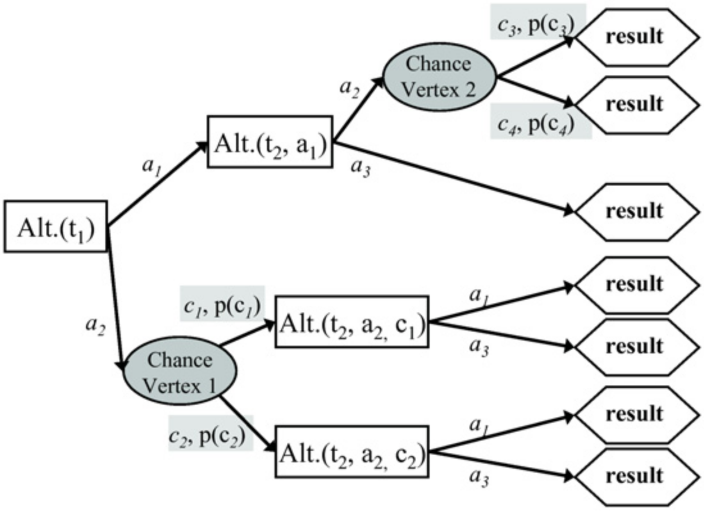


Figure A.1.: *A Decision Tree*

However, the representation of sequential decisions in a decision tree is limited to small problems, as the tree grows exponentially with every additional vertex (so called *bushy mess* phenomenon, [Bielza and Shenoy, 1999]). Furthermore, it is not possible to represent variables with continuous domains: since each possible state of the variable corresponds to a new scenario in the decision tree, an infinite number of scenarios (or branches) would have to be drawn. Therefore, variables with continuous range of values must be discretised and clustered to groups.

Furthermore, decision trees do not allow for representing dependencies in a structured way. Despite of them showing the temporal order of events and decisions, they do not depict dependencies between the variables' values or the flow of information [Shachter, 1986]. This impedes reasoning explicitly about interrelations and hampers coherence and forestalls fast detection of conditional independence (therefore, possibly redundant results are computed [Covaliu and Oliver, 1995].

An advantage of decision trees is that their use facilitates solving **asymmetric decision problems.** A decision problem is asymmetric, when the number of leaf vertices is less than the cardinality of the product of the state spaces of all chance and decision variables. Figure A.1 represents such an asymmetric problem, as the number of leaves (6) is smaller than the product of all possible outcomes of both decision variables (each has 2 possible values) and the chance vertex (2 possibles states, making a total of $2^3 = 8$). A particular aspect of asymmetric decision problems is that the next realisation of a chance vertex or the next decision depends on the past [Jensen et al., 2006]. In a decision tree this is represented by the states of the predecessor vertices.

Yet, there is not only dependence of the decision and observations on the past (structural asymmetry [Jensen et al., 2006]), but sometimes the order of decisions and observations is not clearly settled beforehand (order asymmetry). In these cases, this is part of the decision problem. Consider, for example the placement of chance vertices: As a decision tree represents a sequence of events, the placement of a chance vertex represents the point in time, when the outcome of the event is revealed to the decision makers. Similarly, to be able to determine the location of decision vertices, it must be known how many decisions need to be made, what their exact sequence is and at what point in time they need to be made. To integrate this type of uncertainty, new branches must be integrated in the decision tree, making it even more complex. For larger decision problems, this will make this technique prohibitive as all possible scenarios should be explicitly represented.

Finally, there is the general problem of using probabilistic techniques for strategic decision making as discussed, e.g., in chapters 2.2.2.1 and 2.3.2.2.

A.2. Influence Diagrams

Influence diagrams are graphical representations of decision problems that are at once easily understood representations of sequential decision problems and a formal description of the problem that can be treated numerically [Smith et al., 1993]. They give a more compact representation of the decision problem than decision trees, as their size (measured by the number of vertices) grows only linearly with each variable added [Covaliu and Oliver, 1995]. Since the variables' values are not explicitly represented in the graph, influence diagrams allow for the integration of variables with a continuous range of values. Furthermore, influence diagrams reveal the interdependence of decisions and chance vertices. Thus, they allow for exploring the reasons *why* a alternative has a better performance than another or identifying the influential factors responsible for a certain result.

In an influence diagram, four types of vertices are distinguished (cf. Figure A.2): Rectangles represent alternatives, ovals represent chance events, and diamonds stand for the consequences of a decision. As these consequences can not usually not be derived directly, rounded rectangles that stand for the calcu-

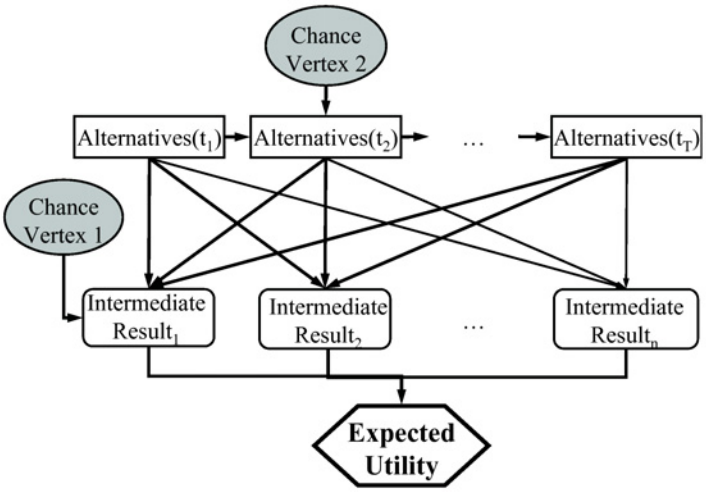


Figure A.2.: *An Influence Diagram*

lation of intermediate results. The values of intermediate results are assigned through algebraically determined relationships [Covaliu and Oliver, 1995].

The edges between the vertices can have two meanings: they are either influential or informational, depending on the context in which they are used [Matzkevich and Abramson, 1995]. Edges that point to a decision vertex are informational, i.e., they represent information available at the time the decision is made. In Figure A.2, only the information about previously made decisions is known with certainty to the decision makers. If an edge from a chance vertex points to a decision vertex, the chance vertex is resolved, and the decision makers do have the information about the actual value of the chance event before making their decision. All other edges represent relevance or influence.

Influence diagrams bring together several levels of problem descriptions. Looking at the edges and vertices only, the **structural, graphical** or the **level of relation** reveals the variables and relationships under scrutiny. This is particularly useful in the early phases of problem structuring, when knowledge about the question *how* the variables are related is vague. The **functional level** captures the probability distributions of the chance vertices and the sets of alternatives for the control vertices. The actual numbers associated with the distributions are specified on the **numerical level** [Diehl and Haimes, 2004].

Usually, in an influence map, there are deterministic and probabilistic vertices. All possible combinations of elements from the ranges of values of a variable's predecessors is the preimage of the functional form captured in the variable itself. Since Influence Diagrams have emerged from the Bayesian community, the uncertain relationships are usually given by marginal or condi-

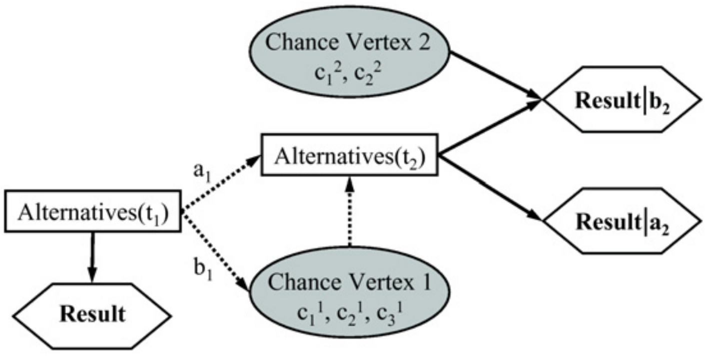


Figure A.3.: *A Sequential Influence Diagram*

tional probability distributions [Shachter, 1988]. Therefore, the evaluation in the performance vertex is usually based on the expected utility. Similar to decision trees, recursive techniques are used to solve influence diagrams [Shachter, 1986].

The relationship between decision trees and influence diagram has often been analysed [e.g., Howard, 1988; Shachter, 1988]. While an influence diagram can directly be converted to a decision tree, the opposite holds only for symmetric decision trees. In case of an asymmetric decision problem, dummy variables with the respective artificial states, degenerate distributions and value functions will be used to transform the asymmetric problem into a symmetric one [Smith et al., 1993]. However, this enhances the graphical as well as the computational complexity.

Another possibility to represent asymmetric problems is to complement the Influence Diagram with a Sequential Decision Diagram (SDD) [Covaliu and Oliver, 1995]. In the discrete case (when all variables have only take a discrete set of values), an SDD can be interpreted as a clustered decision tree, its paths show all possible scenarios in a compact way [Covaliu and Oliver, 1995]. To this end, all branches leading from one vertex (chance or decision) the same successor are merged. This has the advantage that the graphical representation remains clear. An edge in the SDD indicates that the next vertex will be reached *given* a certain configuration of its predecessor vertices. This configuration is usually written above each edge. However, one of the main problems of using SDDs consists in its problems with representing probability distributions: the distributions used in the influence diagrams and in the underlying formulation tables may differ. Furthermore, extensive preprocessing of the data may be necessary [Bielza et al., 2000].

Sequential Influence Diagrams (SIDs) are closely related to SDDs as they both encode structural asymmetry similarly using guard functions which describe the constraints under which a certain vertex is the next in the diagram [Jensen et al., 2006]. In Figure A.3 structural edges encoding the information precedence are depicted in dashed lines. The guards noted at the edge represent the condition under which the vertex the edge points to is the next one in this scenario. The solid edges encode the structure of the probability or evaluation model that is not temporarily structured.

Similarly to the remarks for the decision trees, information about the series of decisions must be available to construct an influence diagram, SDD or SID. This is problematic, as the temporal structure (e.g. the time, when new information is available) may not yet be clear or a part of the decision process.

B. Additional Data for the Emergency Management Example

The aim of this appendix is providing detailed information on the extensive use case that is the basis of the results presented in chapter 9. It elaborates the data, assumptions and calculations which were integrated into a Matlab software tool for the calculation of performances per scenario and the aggregation of results. The information for the use case has been elicited from experts of the Danish Emergency Management Agency, fire fighters and police in multiple workshops. For the atmospheric dispersion modelling, the software tool ARGOS [Baklanov et al., 2006] was used.

B.1. Descriptions of Variables for the Example

Tables B.1, B.2 and B.3 explain, some (uncertain) variables which determine the scenarios and provide descriptions of all attributes along with their types.

Table B.1.: *Explanation of Variables and Scales Used—Part I*

Variable	**Explanation**
Alternative	*Evac:* evacuation of area A_1, sheltering of area A_2
	Shelt: sheltering of areas A_1 and A_2
	DN: do nothing; no preventive measures are implemented
Chlorine release	Measured in kg (Integer)
Weather situation	0: Release of Cl_2 is irrelevant
	1: Release at 12 p.m.,
	2: Release at 6 p.m.,
	3: Release at 2 a.m.
att_1 # Residents to be evacuated	Integer
att_2: # Employees to be evacuated	Integer
att_3: # Nurseries, kindergartens and schools to be evacuated	Integer
att_4: # People in event places to be evacuated	Integer
att_5: Population density	Average number of people present in the area to be evacuated in [# people/km^2] (Integer)
att_6: Infrastructure capacity	Number of *vulnerable* people (e.g., ill in hospital) that can be carried out of the area to be evacuated in 1 h (Integer)
att_7: Population distribution	Modelled on basis of the distribution of age and the average size of households. 1: Easy to evacuate
	2: Average evacuation difficulty
	3: Hard to evacuate
att_8: Number of people present in retirement homes to be evacuated	Integer
att_9: Number of people present in hospitals to be evacuated	Integer
att_{10}: Infrastructure capacity	Number of *vulnerable* people (e.g., ill in hospital) that can be carried out of the area in 1 h (Integer)

Table B.2.: *Explanation of Variables and Scales Used—Part II*

Variable	Explanation
att_{11}: distance to next hospital	Kilometres (Real)
att_{12}: Number of residents exposed and sheltered	Integer
att_{13}: Number of employees exposed and sheltered	Integer
att_{14}: Compliance to sheltering	Modelled as a function of duration 0: Compliance can be assumed
	1: Compliance can be mostly assumed
	2: Compliance can be partly assumed
att_{15}: Building structure	Impermeability of residences and firms sheltered 0: Good
	1: Average
	2: Bad
att_{16}: Number of people present in nurseries and kindergartens sheltered	Integer
att_{17}: Number of people present in schools sheltered	Integer
att_{18}: Compliance of people in nurseries, kindergartens, and schools	As att_{14}, cf. table B.1
att_{19}: Building structure of nurseries, kindergartens, and schools	As att_{15}
att_{20}: Number of people in hospitals sheltered	Integer
att_{21}: Number of people in retirement homes sheltered	Integer
att_{22}: Compliance of vulnerable people (in hospitals and retirement homes)	As att_{14}, cf. table B.1
att_{23}: Building structure of hospitals and retirement homes	As att_{15}

Table B.3.: *Explanation of Variables and Scales Used—Part III*

Variable	Explanation
att_{24}: Number of people exposed and unsheltered	Integer
att_{25}: Number of people present in nurseries and kindergartens exposed and unsheltered	Integer
att_{26}: Number of people present in schools exposed and unsheltered	Integer
att_{27}: Number of people present in hospitals exposed and unsheltered	Integer
att_{28}: Number of people present in retirement homes exposed and unsheltered	Integer
att_{29}: Acute toxicity of the chemical	According Acute Toxicity Assessment for with respect to Health. Scale: categories 1 to 5, where category 1 refers to the most toxic chemical compounds [United Nations, 2005]
att_{30}: Maximum concentration	Exceedance of AEGL-threshold value in [%] [see EPA, 2009]
att_{31}: Health care staff required	Man-hours (Integer)
att_{32}: Fire fighters required	Man-hours (Integer)
att_{33}: Police resources required	Man-hours (Integer)
att_{34}: Cost of supplies and shelter for evacuated people	Euro (Real)
att_{35}: Cost of transportation	Euro (Real)
att_{36}: Number of firms that cannot operate	Integer
att_{37}: Number of train stations blocked	Integer
att_{38}: Length of highways and important roads blocked	Kilometres (Real)
att_{39}: Duration until measures are lifted and everyday life can be resumed	Hours (Integer)

B.2. Attribute Tree for the Example

The subsequent figures show how the attribute tree is constituted from health effects from evacuation (see figure B.1), health effects from exposure (of sheltered and unsheltered people, see figure B.2 and B.3 respectively), a threat assessment (see figure B.3), and assessments of effort and societal impact (see figure B.4). Additive aggregations are represented by solid lines, multiplicative aggregations by dashed lines. The weights elicited during several expert workshops are denoted at the respective lines. For the normalisation linear value functions were used for all attributes.

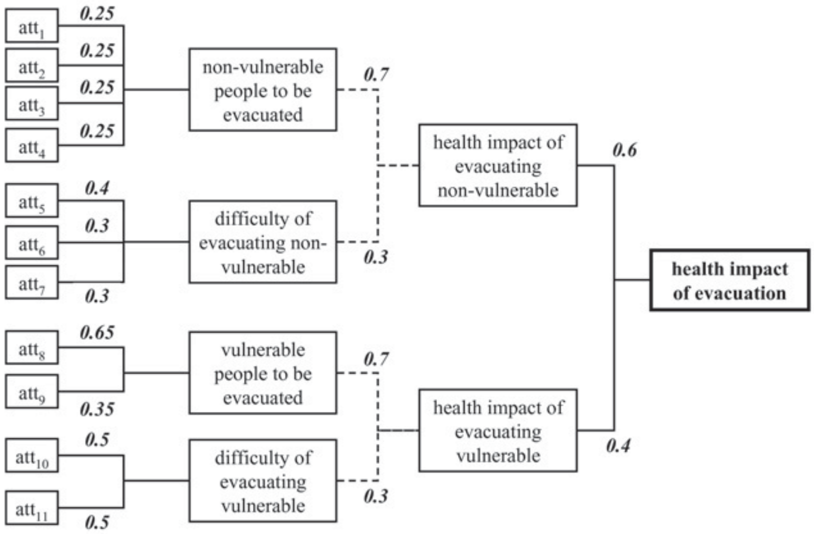


Figure B.1.: *Attribute Tree for Health Effects from Evacuation*

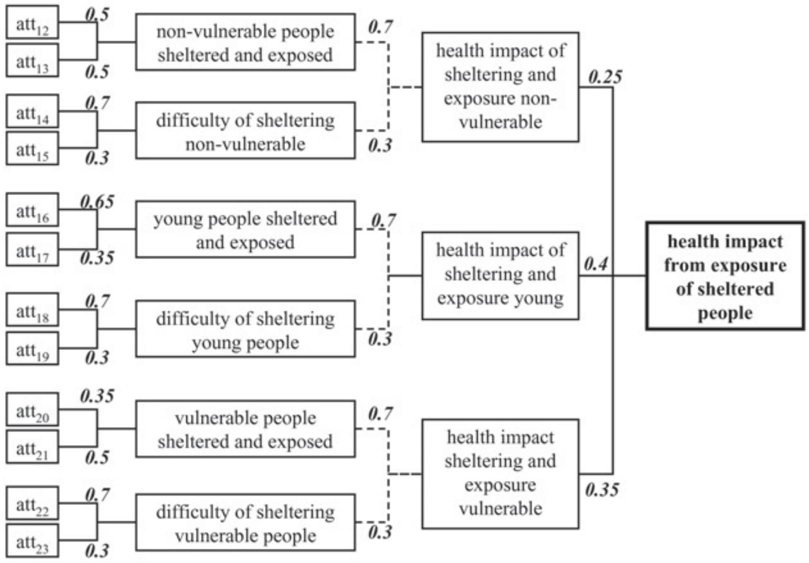


Figure B.2.: *Attribute tree for Health Effects from Exposure of Sheltered Population*

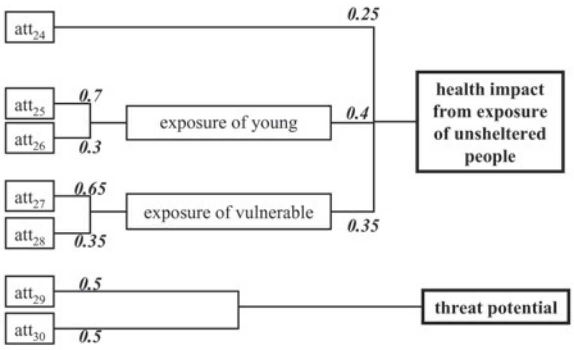


Figure B.3.: *Attribute Tree for Health Effects from Exposure of Unsheltered Population and Toxicity Assessment*

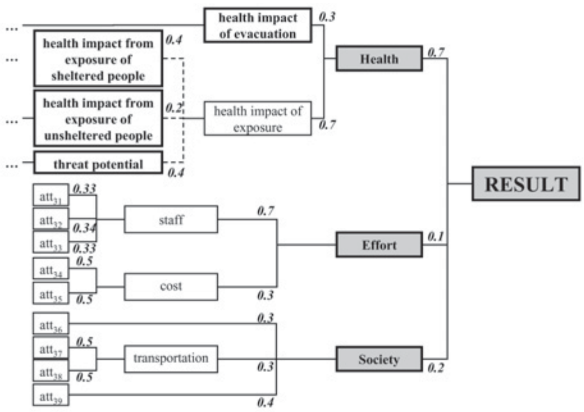


Figure B.4.: *Attribute tree for Determining the Overall Result with respect to Criteria Health, Effort and society. Further branches for health attributes are specified in figures B.1, B.2, and B.3*

B.3. Scenarios and Attribute Scores

The following tables specify the values of some uncertain variables considered in the scenarios and specify the respective attribute scores.

Table B.4.: *Scenarios and values for all attributes related to criterion* Health—*Part I*

Alt.	Evac	Shelt	DN	Evac	Shelt	DN	Evac	Shelt	DN	Evac
Cl_2 [kg]	0	0	0	500	500	500	1000	1000	1000	5000
Weather	0	0	0	1	1	1	1	1	1	1
att_1	230	0	0	230	0	0	230	0	0	230
att_2	2500	0	0	2000	0	0	2500	0	0	3000
att_3	0	0	0	0	0	0	0	0	0	0
att_4	0	0	0	0	0	0	0	0	0	0
att_5	300	300	300	300	300	300	300	300	300	300
att_6	200	200	200	200	200	200	200	200	200	200
att_7	2	2	2	2	2	2	2	2	2	2
att_8	0	0	0	0	0	0	0	0	0	0
att_9	0	0	0	0	0	0	0	0	0	0
att_{10}	20	20	20	20	20	20	20	20	20	20
att_{11}	15	15	15	15	15	15	15	15	15	18
att_{12}	0	0	0	2210	2400	0	4770	5000	0	6070
att_{13}	0	0	0	1000	1500	0	2000	2500	0	4500
att_{14}	0	0	0	0	0	0	1	1	1	1
att_{15}	1	1	1	1	1	1	1	1	1	1
att_{16}	0	0	0	110	110	0	165	165	0	388
att_{17}	0	0	0	0	0	0	0	0	0	0
att_{18}	0	0	0	0	0	0	1	1	1	1
att_{19}	1	1	1	1	1	1	1	1	1	1
att_{20}	0	0	0	0	0	0	1500	1500	0	1500
att_{21}	0	0	0	220	220	0	220	2200	0	220
att_{22}	0	0	0	0	0	0	0	0	0	0
att_{23}	1	1	1	1	1	1	1	1	1	1
att_{24}	0	0	0	0	0	2400	0	0	5000	5160
att_{25}	0	0	0	0	0	110	0	0	165	850
att_{26}	0	0	0	0	0	0	0	0	0	0
att_{27}	0	0	0	0	0	0	0	0	1500	0
att_{28}	0	0	0	0	0	0	0	0	0	
att_{29}	3	3	3	3	3	3	3	3	3	3
att_{30}	0	0	0	20	20	20	0	0	220	220

Table B.5.: *Scenarios and Values for all Attributes related to Criterion* Health—*Part II*

Alt.	Shelt	DN	Evac	Shelt	DN	Evac	Shelt	DN	Evac	Shelt
Cl_2 [kg]	5000	5000	500	500	500	1000	1000	1000	5000	5000
Weather	1	1	2	2	2	2	2	2	2	2
att_1	0	0	230	0	0	230	0	0	230	0
att_2	0	0	2500	0	0	3500	0	0	5000	0
att_3	0	0	0	0	0	0	0	0	0	0
att_4	0	0	0	0	0	0	0	0	0	0
att_5	300	300	300	300	300	300	300	300	300	300
att_6	200	200	200	200	200	200	200	200	200	200
att_7	2	2	2	2	2	2	2	2	2	2
att_8	0	0	0	0	0	0	0	0	0	0
att_9	0	0	0	0	0	0	0	0	0	0
att_{10}	20	20	20	20	20	20	20	20	20	20
att_{11}	15	15	15	15	15	15	15	15	15	15
att_{12}	6300	0	5370	5600	0	6070	6300	0	6070	6300
att_{13}	5000	0	7000	7500	0	7500	8000	0	7500	8000
att_{14}	1	1	0	0	0	1	1	1	2	2
att_{15}	1	1	1	1	1	1	1	1	1	1
att_{16}	385	0	385	385	0	385	385	0	385	385
att_{17}	0	0	0	0	0	0	0	0	0	0
att_{18}	1	1	0	0	0	0	0	0	0	0
att_{19}	1	1	1	1	1	1	1	1	1	1
att_{20}	1500	0	1500	1500	0	1500	1500	0	1500	1500
att_{21}	220	0	220	2200	0	220	220	0	220	220
att_{22}	0	0	0	0	0	0	0	0	0	0
att_{23}	1	1	1	1	1	1	1	1	1	1
att_{24}	5160	11460	0	0	5600	0	0	6300	1370	1370
att_{25}	220	605	0	0	385	0	0	385	55	55
att_{26}	850	850	0	0	0	0	0	0	425	425
att_{27}	0	1500	0	0	1500	0	0	1500	0	0
att_{28}	0	0	0	0	0	0	0	0	220	220
att_{29}	3	3	3	3	3	3	3	3	1	1
att_{30}	220	440	0	0	220	0	0	220	15	15

Table B.6.: *Scenarios and Values for all Attributes related to Criterion* Health—*Part III*

Alt.	DN	Evac	Shelt	DN	Evac	Shelt	DN	Evac	Shelt	DN
Cl_2 [kg]	5000	500	500	500	1000	1000	1000	5000	5000	5000
Weather	2	3	3	3	3	3	3	3	3	3
att_1	0	230	0	0	230	0	0	230	0	0
att_2	0	50	0	0	50	0	0	50	0	0
att_3	0	0	0	0	0	0	0	0	0	0
att_4	0	0	0	0	0	0	0	0	0	0
att_5	300	300	300	300	300	300	300	300	300	300
att_6	200	200	200	200	200	200	200	200	200	200
att_7	2	2	2	2	2	2	2	2	2	2
att_8	0	0	0	0	0	0	0	0	0	0
att_9	0	0	0	0	0	0	0	0	0	0
att_{10}	20	20	20	20	20	20	20	20	20	20
att_{11}	15	15	15	15	15	15	15	15	15	15
att_{12}	0	4970	5200	0	5770	6000	0	6070	6300	0
att_{13}	0	300	350	0	325	375	0	450	500	0
att_{14}	2	0	0	0	0	0	0	0	0	0
att_{15}	1	1	1	1	1	1	1	1	1	1
att_{16}	0	0	0	0	0	0	0	0	0	0
att_{17}	0	0	0	0	0	0	0	0	0	0
att_{18}	0	0	0	0	0	0	0	0	0	0
att_{19}	1	1	1	1	1	1	1	1	1	1
att_{20}	0	0	0	0	750	750	0	750	750	0
att_{21}	0	102	102	0	102	102	0	102	102	0
att_{22}	0	0	0	0	0	0	0	0	0	0
att_{23}	1	1	1	1	1	1	1	1	1	1
att_{24}	7670	0	0	5000	0	0	6000	900	900	7200
att_{25}	440	0	0	0	0	0	0	0	0	0
att_{26}	425	0	0	0	0	0	0	0	0	0
att_{27}	1500	0	0	0	0	0	750	0	0	750
att_{28}	440	0	0	102	0	0	102	0	0	102
att_{29}	1	1	1	1	1	1	1	1	1	1
att_{30}	15	25	25	25	25	25	25	25	25	25

Table B.7.: *Scenarios and Values for attributes related to criteria* Effort *and* Impact on Society—*Part I*

	Alternative	Evac	Shelt	DN	Evac	Shelt	DN	Evac	Shelt	DN	Evac	Shelt	DN	Evac	Shelt	DN
	Cl_2 [kg]	0	0	0	500	500	500	1000	1000	1000	5000	5000	5000	500	500	500
	Weather	0	0	0	1	1	1	1	1	1	1	1	1	2	2	2
Effort	att_{31}	100	10	0	115	15	0	125	25	0	200	100	0	115	15	0
	att_{32}	210	10	0	250	50	0	260	60	0	280	80	0	280	80	0
	att_{33}	210	10	0	250	50	0	260	60	0	280	80	0	280	80	0
	att_{34}	2000	0	0	2000	0	0	2000	0	0	2000	0	0	2000	0	0
	att_{35}	3000	0	0	3000	0	0	3000	0	0	3000	0	0	3000	0	0
Society	att_{36}	50	30	0	40	40	0	50	50	0	60	60	0	50	50	0
	att_{37}	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0
	att_{38}	25	20	0	25	20	0	25	20	0	25	20	0	25	20	0
	att_{39}	7	5	0	10	8	24	12	10	30	14	12	36	10	8	24

Table B.8.: *Scenarios and Values for Attributes related to Criteria* Effort *and* Impact on Society—*Part II*

	Alternative	Evac	Shelt	DN	Evac	Shelt	DN	Evac	Shelt	DN	Evac	Shelt	DN	Evac	Shelt	DN
	Cl_2 [kg]	1000	1000	1000	5000	5000	5000	500	500	500	1000	1000	1000	5000	5000	5000
	Weather	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3
Effort	att_{31}	125	25	0	250	150	0	115	15	0	125	25	0	140	40	0
	att_{32}	260	60	0	300	100	0	240	40	0	250	50	0	260	60	0
	att_{33}	260	60	0	300	100	0	240	40	0	250	50	0	260	60	0
	att_{34}	2000	0	0	2000	0	0	2000	0	0	2000	0	0	2000	0	0
	att_{35}	3000	0	0	3000	0	0	3500	0	0	3500	0	0	3500	0	0
Society	att_{36}	70	70	0	100	100	0	20	20	0	40	40	0	50	50	0
	att_{37}	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0
	att_{38}	25	20	0	25	20	0	25	20	0	25	20	0	25	20	0
	att_{39}	12	10	30	14	12	36	10	8	24	12	10	30	14	12	36

B.4. Evaluation Results for Selected Scenarios

Table B.9.: *Performance for Criteria* Health, Effort, *Impact on* Society *and Total* Result *for Selected Scenarios—Part I*

Health	Effort	Society	Result	Alt.	Cl_2 [kg]	Weather
0.68	0.04	0.06	0.78	Evac	0	0
0.80	0.09	0.08	0.97	Shelt	0	0
0.80	0.10	0.10	1.00	DN	0	0
0.53	0.03	0.06	0.62	Evac	500	1
0.49	0.07	0.07	0.63	Shelt	500	1
0.32	0.09	0.09	0.50	DN	500	1
0.46	0.03	0.06	0.55	Evac	1000	1
0.41	0.07	0.07	0.54	Shelt	1000	1
0.24	0.09	0.09	0.41	DN	1000	1
0.42	0.02	0.05	0.49	Evac	5000	1
0.27	0.06	0.07	0.40	Shelt	5000	1
0.20	0.09	0.08	0.37	DN	5000	1
0.46	0.02	0.06	0.55	Evac	500	2
0.34	0.06	0.07	0.47	Shelt	500	2
0.30	0.08	0.08	0.46	DN	500	2
0.45	0.03	0.05	0.53	Evac	1000	2
0.35	0.06	0.07	0.48	Shelt	1000	2
0.24	0.08	0.08	0.40	DN	1000	2

Table B.10.: *Performance for Criteria* Health, Effort, *Impact on* Society *and Total* Result *for Selected Scenarios—Part II*

Health	**Effort**	**Society**	**Result**	**Alt.**	**Cl_2 [kg]**	**Weather**
0.43	0.01	0.04	0.48	Evac	5000	2
0.33	0.06	0.06	0.45	Shelt	5000	2
0.18	0.08	0.07	0.33	DN	5000	2
0.54	0.03	0.07	0.64	Evac	500	3
0.43	0.06	0.08	0.56	Shelt	500	3
0.24	0.08	0.07	0.39	DN	500	3
0.53	0.03	0.06	0.61	Evac	1000	3
0.39	0.05	0.07	0.51	Shelt	1000	3
0.24	0.08	0.07	0.38	DN	1000	3
0.52	0.03	0.06	0.60	Evac	5000	3
0.39	0.05	0.07	0.50	Shelt	5000	3
0.24	0.07	0.06	0.37	DN	5000	3

Table B.11.: *Aggregated Results for Two Risk Aversion Levels α and Varying Reliability Weights w_{Rel}*

	w_{Rel} =	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\alpha = 0.1$	**Evac**	0.50	0.53	0.57	0.63	0.68	0.72	0.74	0.75	0.75	0.76	0.76
	Shelt	0.42	0.47	0.56	0.67	0.77	0.85	0.89	0.91	0.92	0.92	0.92
	DN	0.35	0.41	0.51	0.64	0.76	0.85	0.90	0.93	0.94	0.94	0.94
$\alpha = 0.9$	**Evac**	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76
	Shelt	0.93	0.93	0.93	0.93	0.93	0.92	0.92	0.92	0.92	0.92	0.92
	DN	0.95	0.95	0.95	0.95	0.94	0.94	0.94	0.94	0.94	0.94	0.94