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Complete Hierarchical Cut-Clustering: A Case Study on Modularity and Expansion

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Complete Hierarchical Cut-Clustering: A Case Study on Modularity and Expansion*

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Abstract. We present a simple and efficient method for constructing a cut-clustering hierarchy as introduced by Flake et al. Cut-clusterings excel by a clearly indicated membership of the vertices to the clusters due to strong connections inside the clusters compared to only weak connections outside. Their coarseness depends on a parameter that provides a quality guarantee in terms of expansion, which is NP-hard to compute. In this work we introduce a parametric search approach that guarantees the completeness of the resulting hierarchy and supersedes the necessity of choosing feasible parameter values in advance or applying a binary search to find those values. Our method is easy to implement and in a brief running time experiment it turns out to be significantly faster than a binary search. We further investigate the resulting clusterings with respect to modularity, a quality measure widely used in practice. In this context we propose a parameter-free approach that helps to estimate how well a graph can be generally clustered by cut-clusterings. With due regard to this estimation the algorithm of Flake et al. competes surprisingly well with respect to reliable reference clusterings, although it is not designed to optimize modularity. Further experiments focusing on the given guarantee on expansion exhibit that the actual expansion is even better than guaranteed and compared to trivial bounds the guarantee constitutes a true gain of knowledge.

1 Introduction

Clustering a graph means finding internally dense subgraphs, called clusters, that are only loosely connected to the remainder of the graph. The growing interest in graph clustering during the last decades has been driven by applications in physics and biology as well as sociology and many other fields. Attempts of formalizing the properties that characterize a set of good clusters resulted in a variety of different quality measures, which still affect the design of algorithms. Flake et al. [3], however, postulate a different approach. They search for clusterings where the membership of the vertices to a cluster is clearly indicated by strong connections inside the cluster while the connections to other clusters are weaker, a condition not expressible by any of the previous measures. This is what we call a *tight* clustering in the following. The strict behavior of tight clusterings—not clearly assigned vertices remain unclustered—is desirable whenever it is essential that ambiguous cases are interpreted by human experts as for example

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in sociology applications. Flake et al. introduce an algorithm that exploits properties of minimum s-t-cuts in order to find a tight clustering depending on a parameter. This parameter controls the coarseness of the resulting clustering and constitutes a guarantee on intra-cluster expansion, a common quality index, which is hard to compute. Different parameter values result in at most $n-1$ different clusterings, which form a hierarchy. Having a hierarchy of tight clusterings at hand, it is then possible to choose the best clustering with respect to any quality measure that suits a particular application.

Our Contribution. We characterize four different types of tight clusterings and introduce the problem Tight Clustering, which asks for an optimal tight clustering of a designated type with respect to a given quality measure. In this light we investigate the behavior of the cut-clustering algorithm of Flake et al. We develop a simple parametric search approach for efficiently constructing a complete cut-clustering hierarchy and provide a brief running time experiment in which our method outperforms a naive binary search, which contrariwise gives no guarantee on finding all different clusterings in the hierarchy. We further conduct a comparative analysis of the modularity values reachable by cut-clusterings. As reference we use a greedy modularity-based approach. This, however, is not restricted to tight clusterings and thus benefits from a larger search space. In order to assess the results as fair as possible we, hence, introduce a parameter-free method that on the other hand helps to exclude the existence of tight clusterings in instances of special shape. Our experiments demonstrate that the algorithm of Flake et al. competes surprisingly well with respect to the objective of Tight Clustering and modularity, and we reveal that the restriction to tight clusterings not necessarily causes a substantial loss of modularity. Finally, we compare the guaranteed intra-cluster expansion of the cut-clusterings to the expansion of the modularity-based references. Since expansion is hard to compute, we consider lower bounds. Our study gives evidence that trivial bounds do not match up to the given guarantee, and an analysis of special non-trivial bounds further indicates that also the true expansion of the cut-clusterings surpasses the modularity-based references.

Related Work. The cut-clustering algorithm of Flake et al. [3], CutC as a shorthand, is the protagonist in our work. The notion of tight clusterings is introduced in [1], however, there the clusters are called web-communities. The implementation of the modularity-based reference algorithm, which is a greedy approach based on vertex moves [2], we took from Lisowski [15]. The notion of modularity was introduces in [10], Montgolfier et al. [7] study the asymptotic behavior of modularity in selected graph classes, which helps us to explain some outlier results in our experiments. Apart from these, there is a huge number of publications on clustering algorithms and quality measures, for an overview see [8]. In our proofs we finally exploit some insights and lemmas on minimum s-t-cuts that date back to Gomory and Hu [4] and Gusfield [5].

2 Preliminaries

Throughout this work we consider a simple, undirected, weighted graph $G = (V, E, c)$ with vertex set V, edge set E and a non-negative edge cost function c. In unweighted graphs we count each edge by one. We denote the number of vertices (edges) by $n := |V|$ ($m := |E|$) and the costs of a set $E' \subseteq E$ by $c(E') := \sum_{e \in E'} c(e)$. Whenever we consider the degree $\deg(v)$ of $v \in V$, we implicitly mean the sum of all edge costs incident to v.

Table 1. Overview of different types of tight subgraphs.

A subgraph $S \subseteq V$ is a			WC	SC	sSC	ES
WC	$\forall u \in S$	$c(\{u\}, S \setminus \{u\}) > c(\{u\}, V \setminus S)$	x			
SC	$\exists s \in S : \forall U \subset S, s \notin U$	$c(U, S \setminus U) \geq c(U, V \setminus S)$		x		
strict SC	$\exists s \in S : \forall U \subset S, s \notin U$	$c(U, S \setminus U) > c(U, V \setminus S)$		x	x	
ES	$\forall U \subset S$	$c(U, S \setminus U) > c(U, V \setminus S)$	x	x	x	x

With $S, T \subset V$ we write $c(S,T)$ for the costs of all edges having one endpoint in S and one in T. If S, T induce a cut in G, $c(S,T)$ describes the costs of this cut. Let (S,T) denote a minimum s-t-cut, $s \in S$, $t \in T$. The cut (S,T) is called the *community cut* of s with respect to t if $|S|$ is minimum for all minimum s-t-cuts in G. Set S is the unique *community* of s while s is a *representative* of S, denoted by $r(S)$, not necessarily unique.

We reserve the term *node* for compound vertices of abstracted graphs, which may contain several basic vertices. A *contraction* by $C \subseteq V$ means replacing the set C in G by a single node and leaving this node adjacent to all former adjacencies u of vertices in C, with costs equal to the sum of all former edges between C and u.

Our understanding of a *clustering* $\mathcal{C}(G)$ is a partition of V into subsets $C^1, \ldots, C^k$, which define vertex-induced subgraphs, called *clusters*. A cluster is called *trivial* if it corresponds to a connected component. A vertex that forms a non-trivial singleton cluster we consider as *unclustered*. A clustering is *trivial* if it consists of trivial clusters or if $k = n$, i.e., all vertices are unclustered. A *hierarchy of clusterings* is a sequence $\mathcal{C}_1(G) \leq \cdots \leq \mathcal{C}_r(G)$ such that $\mathcal{C}_i(G) \leq \mathcal{C}_j(G)$ implies that each cluster in $\mathcal{C}_i(G)$ is a subset of a cluster in $\mathcal{C}_j(G)$. We say $\mathcal{C}_i(G) \leq \mathcal{C}_j(G)$ are *hierarchically nested*. Furthermore, we distinguish four types of *tight* clusterings: Those consisting of *web-communities* (WC), *source-communities* (SC), *strict source-communities* and *extreme sets* (ES). Table 1 gives exact definitions of the notions of the *tight* subgraphs in the clusterings and an overview in which other types a particular type is nested. SCs are characterized by the following (for a proof see App. A).

Lemma 1. *A set $S \subset V$ is a source-community of a vertex $s \in S$ if and only if there exists a set $T \subset V$ such that $(S, V \setminus S)$ is a minimum s-T-cut in $G = (V, E, c)$. We call s the representative $r(S)$ of S.*

Remark 1. For an SC S it holds $c(S, V \setminus S) \leq deg(s)$. Otherwise it would be $c(S \setminus \{s\}, S) > c(S \setminus \{s\}, V \setminus S)$. Furthermore, let $S \subset V$ denote a community of s with respect to t. Then, due to the uniqueness and minimality of S, $V \setminus S$ is the unique (inclusion)maximal SC of t with $s \notin V \setminus S$, and S is a strict SC of s.

Tight Clustering. Tight Clustering is the problem of finding a tight clustering of maximum quality. Regarding the different types of tight clusters in Table 1 and the variety of existing quality indices, Tight Clustering encompasses a whole family of problems. In this work, we focus on modularity as objective quality measure, and we require the clusters to be at least source-communities, as SCs are closely related to minimum s-t-cuts, which can be efficiently calculated. In contrast, decomposing a graph into k web-communities is NP-hard [3], whereas all extreme sets can be computed in

Table 2. Overview of intra-cluster expansion bounds.

cut expansion $\Psi(S, C \setminus S)$:	$\frac{c(S,C\setminus S)}{\min\{\lvert S\rvert, \lvert C\setminus S\rvert\}}$	guarantee $\Psi_g(\mathcal{C})$:	by parameter
trivial lower bound $\Psi_\ell(C)$:	$\frac{c(A,C\setminus A)}{\lfloor \lvert C\rvert /2 \rfloor}$	non-trivial bound $\Psi_a(\mathcal{C})$:	$\min_{C\in\mathcal{C}} \Psi_a(C)$

$O(nm + n^2 \log n)$ time [6]. The latter are either nested or disjoint. Thus, for ESs Tight Clustering can be efficiently solved if the quality index is easy to calculate. We are interested in how well CutC approximates a solution of Tight Clustering with respect to modularity and SCs.

Quality Measures. A *quality measure* for clusterings is a mapping to real numbers. Depending on the measure, either high or low values correspond to high quality. The measures considered in this work, modularity and intra-cluster expansion, indicate high quality by high values.

Modularity bases on the total edge costs covered by clusters. The values range between -0.5 and 1 and express the significance of a given clustering compared to a random clustering. Formally, the modularity $\mathcal{M}(\mathcal{C})$ of a clustering $\mathcal{C}$ is defined as $\mathcal{M}(\mathcal{C}) := \sum_{C\in\mathcal{C}} c(E_C)/c(E) - \sum_{C\in\mathcal{C}} (\sum_{v\in C} deg(v))^2/4c(E)^2$, where E_C denotes the set of edges with both endpoints in C.

The intra-cluster expansion of a clustering derives from the expansion defined for cuts. The expansion $\Psi(S, C \setminus S)$ of a cut $(S, C \setminus S)$ in a cluster C evaluates the ratio of the costs and the size of the cut and is given in Table 2. The expansion $\Psi(C)$ of a cluster equals the minimum expansion of all cuts in C. The intra-cluster expansion $\Psi(\mathcal{C})$ of a clustering finally is the minimum expansion of all clusters in $\mathcal{C}$. Note that expansion is not defined for singleton clusters. Thus, only non-singleton clusters count for $\Psi(\mathcal{C})$. Trivial clusterings consisting of singleton clusters are omitted in the respective experiments. Computing $\Psi(C)$ is known to be NP-hard, however, a trivial lower bound $\Psi_\ell(C)$ can be easily determined from any global minimum cut $(A, C \setminus A)$ (see Table 2). The expansion of such a minimum cut further constitutes an upper bound on $\Psi(C)$. We denote this by $\Psi_u(C)$. In our experiments we compare the analog trivial bounds $\Psi_\ell(\mathcal{C})$ and $\Psi_u(\mathcal{C})$ to the guarantee given by the parameter of CutC, which we denote by $\Psi_g(\mathcal{C})$. We further consider an alternative non-trivial lower bound $\Psi_a(\mathcal{C})$ resulting from individually applying CutC to the subgraphs induced by the clusters. For those subgraphs we want CutC to return the trivial clustering that consists of the whole subgraph. This can be reached by accordingly choosing the parameter of CutC, which controls the coarseness. The chosen parameter value then constitutes a non-trivial bound $\Psi_a(C)$ on the expansion of the considered cluster/subgraph. Since this method considers the clusters as independent instances ignoring the edges between the clusters, the resulting bound $\Psi_a(\mathcal{C})$ often lies above $\Psi_g(\mathcal{C})$.

3 The Algorithms

In this section we review the parametric cut-clustering approach of Flake et al. and introduce a simple method for efficiently computing all different clusterings in the parameter range. We further give a short idea of the parameter-free approach we use to exclude the existence of tight clusterings in some particular cases.

3.1 Basic and Hierarchical Cut-Clustering

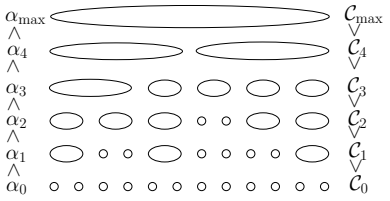


Fig. 1. Clustering hierarchy by CutC. Note that $\alpha_{\max} < \alpha_0$, $\mathcal{C}_{\max} > \mathcal{C}_0$.

The basic CutC algorithm of Flake et al. works as follows: Given a graph G and parameter $\alpha > 0$, as a preprocessing step augment G by inserting an artificial vertex t and connecting t to each vertex in G by an edge of costs α. Denote the resulting graph by $G_\alpha = (V_\alpha, E_\alpha, c_\alpha)$. Then apply CutC by iterating V and computing a community with respect to t for each vertex not yet contained in a community. Since communities are either disjoint or nested we finally get a set of (inclusion)maximal communities decomposing V. We call such a decomposition a cut-clustering. It inherits the uniqueness of the communities and, according to Remark 1, embodies a tight clustering $\mathcal{C}$ of strict SCs. Furthermore it holds $\Psi(\mathcal{C}) \geq \alpha$. Applying CutC iteratively with decreasing parameter values yields a hierarchy of at most n different clusterings (cp. Fig. 1). Note that for α_0 equal to the maximum edge costs in G CutC returns the trivial clustering consisting of singletons, while $\alpha_{\max} = 0$ yields the connected components.

Algorithm 1 describes a naive hierarchical approach, which exploits the hierarchical nesting property in order to shrink the next instance by contracting the previous clusters (line 5). The crucial point with this approach, however, is the choice of α. If we choose the next value too high we get the previous clustering again, which implies unnecessary effort. If we choose the next value too low we possibly miss a meaningful clustering. Flake et al. propose a binary search approach for the choice of α, however, this necessitates a discretization of the parameter range and still does not prevent missing clusterings. That is why we introduce a simple parametric search approach for constructing a complete hierarchy.

Algorithm 1: HIERARCHICAL CUTC

Input: $\alpha_0 = \max\{c(e) | e \in E\}$, G_{α_0}
1 $\mathcal{C}_0(G) \leftarrow \{\{v\} \mid v \in V\}$; $i \leftarrow 0$
2 **forall the** $v \in V$ **do** $r(\{v\}) \leftarrow v$
3 **while** $\mathcal{C}_i(G)$ *differs from conn. comp.* **do**
4 choose $\alpha_{i+1} < \alpha_i$
5 $G'_{\alpha_{i+1}} \leftarrow$ contract $\mathcal{C}_i(G)$ in $G_{\alpha_{i+1}}$
6 $\mathcal{C}_{i+1}(G) \leftarrow$ CutC($G'_{\alpha_{i+1}}$)
7 $i \leftarrow i+1$
8 **return** $\mathcal{C}_0, \ldots, \mathcal{C}_i$

3.2 Simple Parametric Search Approach

Our simple approach for constructing a complete hierarchy of cut-clusterings exploits the properties of cut-cost functions. The *cut-cost function* ω_C of a set $C \in V$ is a linear function depending on α that represents the costs of cut $(C, V_\alpha \setminus C)$ in G_α.

$$\begin{aligned} \omega_C : \mathcal{R}_0^+ &\longrightarrow [c(C, V \setminus C), \infty) \subset \mathcal{R}_0^+ \\ \omega_C(\alpha) &:= c(C, V \setminus C) + |C|\, \alpha \end{aligned}$$

For two consecutive hierarchy levels $\mathcal{C}_i < \mathcal{C}_{i+1}$ we call $\hat{\alpha}$ the breakpoint if CutC returns $\mathcal{C}_i$ for $\hat{\alpha}$ and $\mathcal{C}_{i+1}$ for $\hat{\alpha} - \varepsilon$. By construction, each breakpoint is an intersection point of the cut-cost functions of two clusters $C_i \subset C_j$. Thus, the idea is to compute relevant intersection points and check if they yield new clusterings.

Theorem 1. *Let $\mathcal{C}_i < \mathcal{C}_j$ denote two different clusterings with parameter values $\alpha_i > \alpha_j$. In time $O(|\mathcal{C}_i|)$ a parameter value α_m with $\alpha_j < \alpha_m \leq \alpha_i$ can be computed such that 1) $\mathcal{C}_i \leq \mathcal{C}_m < \mathcal{C}_j$, and 2) $\mathcal{C}_m = \mathcal{C}_i$ implies that α_m is the breakpoint between $\mathcal{C}_i$ and $\mathcal{C}_j$.*

Proof. We observe that 1) two functions ω_{C_i} and ω_{C_j} with $C_i \subset C_j$ intersect, as by construction it is $c(C_j, V \setminus C_j) \leq c(C_i, V \setminus C_i)$ and $|C_j| > |C_i|$. For the intersection point α' it holds $\omega_{C_i}(\alpha) > \omega_{C_j}(\alpha)$ if $\alpha < \alpha'$ and $\omega_{C_i}(\alpha) < \omega_{C_j}(\alpha)$ if $\alpha > \alpha'$ (cp. Fig. 2). Consider 2) a cluster $C_j \in \mathcal{C}_j$, a child $C_i \in \mathcal{C}_i$ of C_j (i.e., $C_i \subset C_j$) and the intersection point α'_j of ω_{C_i} and ω_{C_j}. It is $\alpha'_j \leq \alpha_i$ as otherwise (by 1) $\omega_{C_j}(\alpha_i) < \omega_{C_i}(\alpha_i)$, and thus, $C_j \ni r(C_i)$ would be a cheaper community in G_{α_i}. If 3) $r(C_j) \in C_i$ it is $\alpha'_j > \alpha_j$. Otherwise (by 1) $\omega_{C_i}(\alpha_j) \leq \omega_{C_j}(\alpha_j)$, $|C_i| < |C_j|$, and C_i would be a smaller community in G_{α_j}.

To determine α_m let $\hat{\alpha}_j := \max_{C_i \subset C_j}\{\alpha'_j\}$, suppose $\hat{\alpha}_j := \infty$ if C_j is also a cluster on level $\mathcal{C}_i$. Finally define $\alpha_m := \min_{C_j \in \mathcal{C}_j}\{\hat{\alpha}_j\}$. Due to 1)–3), $\alpha_j < \alpha_m \leq \alpha_i$ and α_m can be computed in time $O(|\mathcal{C}_i|)$. Let C_m denote the result of CutC when applied to G_{α_m}.

Claim 1: $\mathcal{C}_m \neq \mathcal{C}_j$. Let $C_j \in \mathcal{C}_j$ denote a cluster with $\hat{\alpha}_j = \alpha_m$. It holds $\alpha_m \geq \alpha'_j$ for all children C_i of C_j and (by 1) $\omega_{C_i}(\alpha_m) \leq \omega_{C_j}(\alpha_m)$, $|C_i| < |C_j|$. This is, $C_j \notin \mathcal{C}_m$.

Claim 2: If $\mathcal{C}_m = \mathcal{C}_i$ then α_m is the breakpoint between $\mathcal{C}_i$ and $\mathcal{C}_j$. Let $C_i \in \mathcal{C}_m$ denote a child with $\alpha'_j = \hat{\alpha}_j$ of any cluster C_j. Due to the construction of α_m it is $\alpha'_j \geq \alpha_m$ and (by 1) $\omega_{C_j}(\alpha_m - \varepsilon) < \omega_{C_i}(\alpha_m - \varepsilon)$. For any set C' with $C_i \subset C' \subset C_j$ $\omega_{C'}$ also intersects ω_{C_j} in α'_j but with lower slope. Thus, C_j is the community of $r(C_i)$ in $G_{\alpha_m - \varepsilon}$.

Theorem 1 allows for a simple parametric search starting with the trivial clusterings $\mathcal{C}_0 < \mathcal{C}_{\max}$ ($\alpha_0 > \alpha_{\max}$). In contrast to a binary search on the discretized parameter range this approach definitely returns a complete hierarchy.

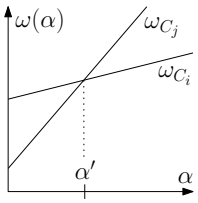


Fig. 2. Intersecting cut-cost functions.

Running time. The parametric search also outperforms the binary search on running times, since it calls CutC at most twice per level in the hierarchy. This yields a running time of $O((h-2)T(n))$ with h the number of levels and $T(n)$ the worst case running time for CutC, compared to $O(h\log(d)T(n))$ of the binary search, where $d \gg n$ is the number of discretization steps. Furthermore, the hierarchical nesting property still allows to contract the clusters on the lower level before applying CutC, and by scheduling the recursive calls carefully such that steps descending into the lower part are executed first we can even reuse the previously contracted structure.

As a proof of concept we conduct a brief experiment on running times of the parametric search and the binary search. The implementation was realized within the LEMON framework [14], version 1.2.1. The max-flow implementation provided by LEMON runs in $O(n^2\sqrt{m})$. For details on instances see Section 4. In order to discretize the continuous parameter range for the binary search, we use between 2^{10} and 2^{30} steps depending on the size of the instances. Comparing the resulting hierarchies to the complete ones confirms this discretization being detailed enough to find all levels for most of the instances. Nevertheless, we do not know how far from optimal our discretization is, although we tried to keep the number of steps low. The difficulty to determine a good discretization is one of the main drawbacks of the binary search approach. Both algorithms run on an AMD Opteron Processor 252 with 2.6 GHz and 16 GB RAM. Table 3 lists ascending CPU times of the parametric search without contraction (PasS).

Table 3. Running times of the parametric search without contraction (ParS) and the binary search with (BinS cont.) and without contraction (BinS). Instances sorted by CPU times of ParS. Times longer than six days are marked by *. See also Appendix D.

graph	n	m	h	ParS [m:s]	BinS cont. [fac]	BinS [fac]
celegans_metabolic	453	2025	8	0.300	7.620	8.380
celegansneural	297	2148	17	0.406	8.653	9.919
netscience	1589	2742	38	4.310	4.030	11.952
power	4941	6594	66	1:25.736	8.773	15.742
as-22july06	22963	48436	33	39:54.495	12.419	20.583
cond-mat	16726	47594	80	44:15.317	14.917	27.425
rgg_n_2_15	32768	160240	46	245:25.644	32.748	22.573
G_n_pin_pout	100000	501198	4	369:29.033	*	*
cond-mat-2005	40421	175691	82	652:32.163	*	21.446

The factors listed for the binary search with contraction (BinS cont.) and without (BinS) describe how much longer the applications run compared to ParS.

ParS outperforms both binary search approaches by a factor of four up to 32. However, the running time does not only depend on the input size but also on the number of different levels in the hierarchy. This effect can be nicely observed at the two last instances. Although G_n_pin_pout is the biggest graph in this list it takes less time to find four levels therein than constructing 82 levels in the smaller instance cond-mat-2005. The random geometric graph rgg_n_2_15 further demonstrates the impact of constant factors hidden in the asymptotic running time of BinS cont. Asymptotically both binary search approaches are comparable since contraction takes only linear time in terms of m, and is thus dominated by the max-flow computation. In practice BinS cont. performs at least 45 contractions for this instance, each merging only few vertices. Thus, the decreasing size of the graph does not offset the additional costs for contraction.

3.3 Parameter-Free Exclusion Approach

Our parameter-free exclusion approach (ParFree as a shorthand) aims at finding meaningful maximal SCs for most of the vertices in a given instance. If these maximal SCs are still small we conclude that there exists no coarser tight clustering of nice clusters.

ParFree considers all components separately. A maximal SCs is meaningful if it contains at most half of the vertices of the current component. Components that are smaller than half of the largest component in G become trivial clusters with an arbitrary vertex as representative, which is marked *green* in order to illustrate the special type of this cluster. Any other component H is decomposed into SCs by iterating the vertices in H in a non-increasing order by their weighted degrees. Thereby the SCs are marked with the help of further colors indicating the individual properties of the SCs.

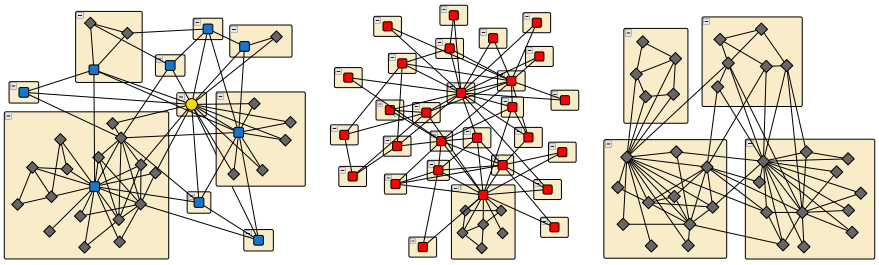
The first vertex designates the source s. The algorithm consecutively computes the community S of s with respect to the next vertex t that is not yet covered by an SC. If $|H \setminus S| \leq |H|/2$ and does not intersect any previously found SC, $H \setminus S$ is the maximal SC of t not containing s (cp. Remark 1). Thus t is marked *blue*. If $|H \setminus S| \leq |H|/2$ but intersects with another SC, $H \setminus S$ is replaced by a non-intersecting SC Q of t according to Gusfield [5] and Gomory and Hu [4] (see Lemma 3 in App. B). Since Q is no longer maximal, t is marked *red*. The representative of any other SC nested in Q becomes

uncolored again. If $|S| < |H|/2$ vertex t becomes the current source and S is an SC of s according to Remark 1; then s is also marked *red*. In the end all vertices are assigned to SCs apart from the source considered last. This source s is marked *orange*.

In a post-processing step the algorithm then searches for a meaningful maximal SC of s consisting of unclustered vertices. If such an SC is found s changes from orange to *yellow*. For a detailed description of the post processing and a proof of the following lemma see Appendix C.

Lemma 2. *Let v denote a blue or uncolored vertex in an SC Q. Then any coarser possibly existing SC Q' of v with $Q \subset Q'$ is also an SC of a red, orange or yellow vertex.*

This is, in any coarser clustering the number of further possibly existing SCs is bounded by the number of red, orange and yellow vertices. On the other hand, these vertices might still induce coarser SCs, which we do not know. Thus we call the number of red, orange and yellow vertices the *uncertainty* of a parfree-clustering. Figure 3(a) shows an example of uncertainty one.



(a) Parfree-clustering. (b) Cut-clustering. (c) Modularity-based clus.

Fig. 3. Different clusterings for karate. The parfree-clustering (a) consists of one yellow vertex (round) besides blue (rectangular) and uncolored ones (diamond shaped): 4 maximal non-singleton SCs of different size, 5 unclustered vertices, uncertainty is one. (b) Cut-clus: one small non-singleton cluster, 27 unclustered vertices. (c) mod-clus: 4 clusters of balanced size.

4 Experiments

For the experimental analysis we used several instances of the clustering testbed of the 10th DIMACS Implementation Challenge [12] as well as the protein interaction network bo_cluster published by Jeong et al. [13] and a snapshot of the linked wiki pages at www.dokuwiki.org (cp. Fig. 4 or Tab. 5 in App. E). Furthermore, we consider a large number of snapshots of the email-communication network of the Department of Informatics at KIT [11] (cp. Tab. 6 to 8 in App. E).

Modularity analysis. Our first experiment addresses the question how close cut-clusterings can get to a modularity-optimal tight clustering with respect to SCs, and which modularity values can be reached in general. Thus, we focus on the best cut-clusterings in the hierarchies with respect to this objective and compare them to reference clusterings of good modularity (mod-clusterings for short) generated with the help of a modularity-based greedy agglomerative approach [2], as computing a modularity-optimal clustering is NP-hard [9]. Figure 4 shows the results.

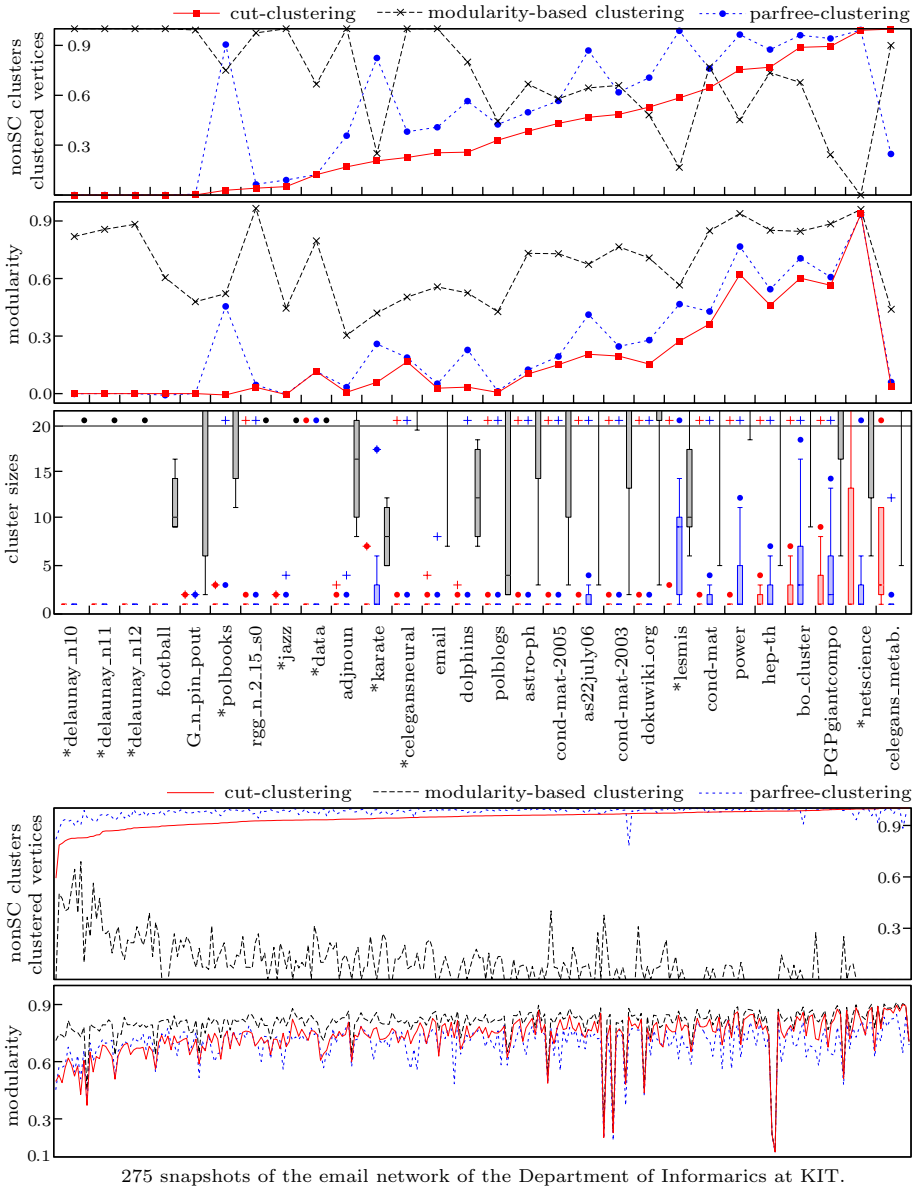


Fig. 4. Results of the modularity analysis of cut-clusterings, modularity-based clusterings and parfree-clusterings. The results for the email snapshots are displayed in the lower part, the upper part addresses the remaining instances. Instances where the uncertainty of the parfree-clustering is at most two are marked by *. In both parts the upper charts show the ratio of clustered vertices in the cut- and parfree-clusterings and the ratio of nontrivial clusters missing the SC-property in the modularity-based clusterings. For the upper instances the cluster sizes are shown by whisker-bars regarding cut-, parfree- and mod-clustering with maximum (+) and minimum (•) of the outliers. Note that values greater than 20 are printed at the edge of the displayed range. Due to the high number of email snapshots whisker-bars are omitted for those instances.

As expected the mod-clusterings are of higher modularity than the cut-clusterings and prefer clusters of decent size. In contrast the cut-clusterings are finer with several unclustered vertices (see also Fig. 3), which is due to the restriction to tight clusterings. Nevertheless, the modularity of the latter increases with the amount of clustered vertices and the size of the clusters.

The fact that for some instances CutC returns clusterings with a modularity much lower than the references, however, does not necessarily mean that the cut-clustering is far from the objective. The instance might just lack a tight clustering of better modularity. In order to find out if this is indeed the case we focus on the properties of the parfree-clustering. As all clusters in this clustering, apart from those counted by the uncertainty, constitute maximal SCs, small clusters and a low uncertainty indicate the absence of a coarser tight clustering, which might provide a higher modularity. In this case, it seems to be more appropriate to compare the modularity of the cut-clustering to the value reached by ParFree. The Delaunay triangulations are nice examples where the modularity gap between the cut-clustering and the mod-clustering is large, but the parfree-clustering consists only of singleton clusters and has a low uncertainty. Thus we can exclude the existence of a better tight clustering with high probability and suppose the cut-clustering, which is basically the same as the parfree-clustering, is close to a modularity-optimal tight clustering, although the reference clustering has a much higher modularity. Note that all clusters in the references of the Delaunay triangulations miss the SC-property. We derive the existence of such degenerated cases from the fact that the asymptotic modularity of some graph classes is provably high [7] whereas the same classes often lack any meaningful SC such that CutC has no chance to return a nontrivial clustering of good modularity. In this light CutC competes surprisingly well.

For the main part of the email snapshots and for the netscience graph CutC reaches modularity values very close to the references, which is rather unexpected since CutC is not designed to optimize modularity. We further observe that the absolute modularity values for these instances are quite high and the amount of clusters in the mod-clusterings that miss the SC-property decreases. The common trend of both modularity curves finally reveals that most instances are either difficult for both clustering approaches or for none of them. We conjecture that if there exists a tight clustering of good modularity, CutC most often finds it.

Expansion analysis. In order to answer the question if there is an advantage from knowing the quality guarantee given by the parameter of CutC, our second experiment compares this guarantee to the trivial lower bound Ψ_ℓ induced by global minimum cuts. We further study the non-trivial lower bound Ψ_a to get an idea on the exact expansion values. Recall Table 2 for an overview of the different intra-cluster expansion bounds. We consider the same cut-clusterings and mod-clusterings as before, however, we skip the Delaunay triangulations and the football graph, as for those instances CutC returns only singleton clusters. The results are given in Figure 5.

We observe that the trivial lower bound Ψ_ℓ in both clustering categories follows a similar trend and stays below the guarantee Ψ_g for most of the instances, which confirms the guarantee as a truly meaningful bound. A value that exceeds Ψ_g appears for example if all the non-singleton clusters are close to cliques of maximum edge costs. This yields a trivial bound close to two times the maximum edge costs, while Ψ_g is

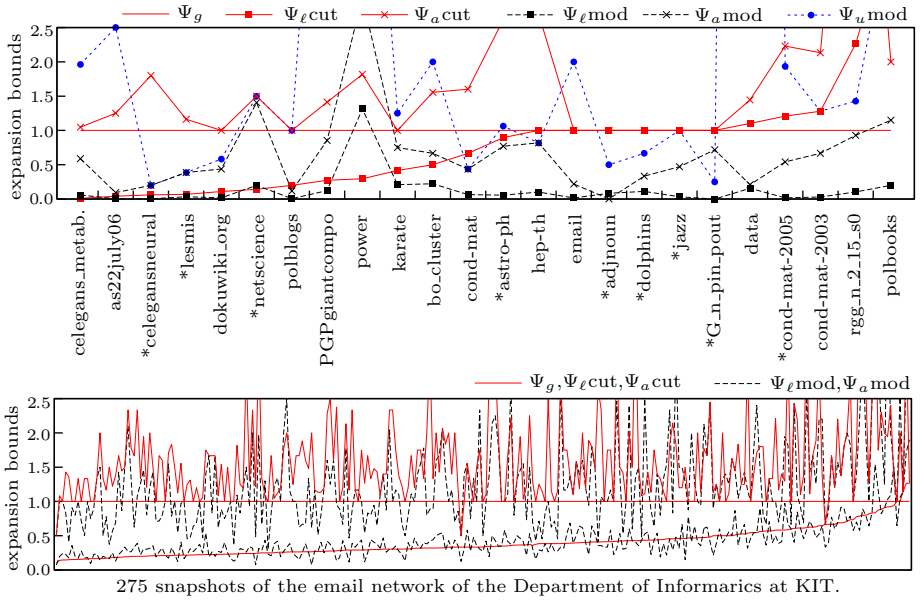


Fig. 5. Trivial and non-trivial bounds on the intra-cluster expansion of cut-clusterings and modularity-based clusterings. For definitions of these bounds recall Table 2. In both charts the guarantee Ψ_g is normalized to one, further values are displayed proportional. Instances where $\Psi_u$cut (not shown) meets the maximum lower bound in the cut-clustering are marked by *. For those instances the maximum lower bound equals the true intra-cluster expansion in the cut-clustering. In the lower chart $\Psi_\ell$cut is shown by the monotone curve different from one, $\Psi_\ell$mod is given by the dashed line close to $\Psi_\ell$cut; $\Psi_a$cut and $\Psi_a$mod are represented by the remaining solid and dashed line. For the sake of readability $\Psi_u$mod is omitted in the lower chart.

bounded by the maximum edge costs. On the other hand, the non-trivial lower bound Ψ_a for the cut-clusterings clearly outperforms the guarantee, and thus, reveals that the actual intra-cluster expansion in the cut-clusterings is even higher than guaranteed. At the same time it also exceeds the analog bound for the mod-clusterings, and hence, suggests that the cut-clusterings also outperform the expansion of the modularity-based reference clusterings. This becomes even a fact whenever the upper bound Ψ_u for the mod-clusterings drops below the maximum lower bound in the cut-clusterings, which indeed happens for some instances. This proves a truly better intra-cluster expansion on the latter. Finally, for some cut-clusterings we yet know the actual intra-cluster expansion, as the analog upper bound Ψ_u meets the lower bounds. The corresponding instances are marked by * in the upper chart of Figure 5, for the email snapshots the amount of those graphs is about 20%.

Conclusion. In this work we studied the behavior of the cut-clustering algorithm of Flake et al. [3] in the light of tight clusterings and the quality measures modularity and expansion. We introduced a characterization of different types of tight clusterings and gave a simple but efficient approach for constructing all different levels of tight clusterings in the cut-clustering hierarchy formed by different parameter values. Our new approach directly computes all breakpoints in the parameter range where new clus-

terings come up, and thus, outperforms binary search approaches in running time and accuracy. Our experiments further exhibited that, although it is not designed to optimize modularity, the cut-clustering algorithm fairly well combines the significance of tight clusterings with a good modularity and a guaranteed intra-cluster expansion that is often better than trivial bounds, provided that there exists a reasonable tight clustering in the given instance.

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Appendix

A Omitted Proof of Lemma 1

Lemma 1. *A set $S \subset V$ is a source-community of a vertex $s \in S$ if and only if there exists a set $T \subset V$ such that $(S, V \setminus S)$ is a minimum s-T-cut in $G = (V, E, c)$. We call s the representative $r(S)$ of S.*

Proof. Let $S \subset V$ denote an SC of a vertex $s \in S$. Then $(S, V \setminus S)$ is a minimum s-$(V \setminus S)$-cut. Otherwise, assume a cheaper s-$(V \setminus S)$-cut $(W, V \setminus W)$ with $s \in W$. It holds $W \subset S$ and $V \setminus S \subset V \setminus W$ and $c(W, S \setminus W) + c(W, V \setminus S) = c(W, V \setminus W) < c(S, V \setminus S) = c(W, V \setminus S) + c(S \setminus W, V \setminus S)$, i.e, $c(W, S \setminus W) < c(S \setminus W, V \setminus S)$. Thus, $U := S \setminus W \not\ni s$ contradicts the SC-definition. With the same argument induces each minimum s-T-cut an SC of s.

B Reviewing Gomory and Hu and Gusfield

Gomory and Hu [4] are the pioneers of the cut tree construction whereas Gusfield [5] introduced a nice simplification of the approach of Gomory and Hu avoiding contractions, which are pesky to implement. A *cut tree* $T(G) = (V, E_T, c_T)$ of a graph G is a tree on V and represents for any vertex pair $\{u, v\} \in \binom{V}{2}$ a minimum u-v-cut $\theta_{u,v}$ in G by the cheapest edge on the unique path between u and v in $T(G)$. Neither must this edge be unique, nor $T(G)$. An edge $e_T = \{u, v\}$ of $T(G)$ induces the cut $\theta_{u,v}$ in G by decomposing $T(G)$ into two connected components. The two cut-based algorithms CutC and ParFree considered in this work basically build partial cut-trees. Their correctness directly follows from the techniques introduced by Gomory and Hu and Gusfield. The most important Lemma in this context is the following:

Lemma 3. *Let $(Y, V \setminus Y)$ be a minimum x-y-cut in G, with $y \in Y$. Let $(H, V \setminus H)$ be a minimum u-v-cut, with $u, v \in V \setminus Y$ and $y \in H$. Then the cut $(Y \cup H, (V \setminus Y) \cap (V \setminus H))$ is also a minimum u-v-cut.*

This lemma allows to bend a minimum u-v-cut such that Y is not injured. The final shape of the bent cut depends on the side of the original cut containing y—roughly speaking, the cut is "deflected" by y. In the situation of ParFree where $H \setminus S$ intersects with a previously fond SC the bent cut induces a new non-intersecting SC of t.

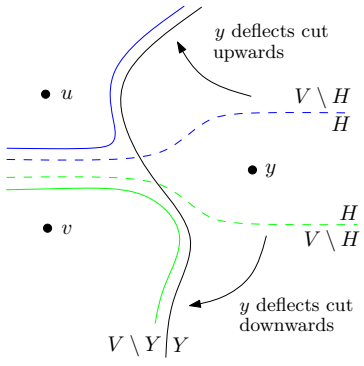

Fig. 6. Depending on y there are two different directions to which Lemma 3 bends the cut $(H, V \setminus H)$: upwards or downwards.

C Omitted Aspects of ParFree

If there are non-singleton SCs in the current component H after the first phase of ParFree the post processing contracts all non-singleton SCs to a node N. Then the community S of N with respect to the last remaining vertex s is calculated. According to Remark 1 $V \setminus S$ is the maximal SC of s that consists of unclustered vertices. We incorporate the found SC in the final parfree-clustering if $|V \setminus S| \leq |H|/2|$. In this case, s is colored yellow. Otherwise, s remains orange forming a singleton cluster.

Lemma 2. *Let v denote a blue or uncolored vertex in an SC Q. Then any coarser possibly existing SC Q' of v with $Q \subset Q'$ is also an SC of a red, orange or yellow vertex.*

Proof. Let v denote a blue vertex in an SC Q in a parfree-clustering of a component H. This is, there exists a red, orange or yellow vertex $s \in H \setminus Q$ such that $H \setminus Q$ is a community of s. According to Remark 1 there is no SC of v containing Q but not s. Thus, for any coarser SC $Q' \supset Q$ of v it is $s \in Q'$. By Lemma 1 $(Q', H \setminus Q')$ is a minimum v-T-cut for a set $T \subseteq H \setminus Q'$. Since $Q \subset Q'$ the cut $(Q, H \setminus Q)$ also separates v and T, thus $c(Q', H \setminus Q') \leq c(Q, H \setminus Q)$. If there was another cut cheaper than $c(Q', H \setminus Q')$ separating T and s, this cut would also separate s and v which contradicts the fact that $(Q, H \setminus Q)$ is a minimum v-s-cut. Hence $(Q', H \setminus Q')$ is also a minimum s-T-cut and Q' is an SC of s.

Let w denote an uncolored vertex in Q. We claim that the community of s with respect to w contains $H \setminus Q$, i.e., the maximal SC of w not containing s is in Q. Then, the previous arguments apply analogously. The claim is easy to see. Either $(Q, H \setminus Q)$ is also a minimum w-s-cut, then $H \setminus Q$ is also the community of s with respect to w. Or there exists a cheaper minimum w-s-cut $(W, H \setminus H)$, $w \in W$ that separates w and v. If $W \not\subset Q$, according to Lemma 3, Q could be reshaped contradicting the community $H \setminus Q$ of s with respect to v. Thus it is $W \subset Q$, particularly for the community $H \setminus W$ of s with respect to w.

D Omitted Running Times

Table 4. Running times of the parametric search without contraction (ParS) and the binary search with (BinS cont.) and without contraction (BinS). Instances sorted by CPU times of ParS. Times longer than six days are marked by *.

graph	n	m	h	ParS [m:s]	BinS cont. [fac]	BinS [fac]
jazz	198	2742	3	0.062	7.726	7.871
celegans_metabolic	453	2025	8	0.300	7.620	8.380
celegansneural	297	2148	17	0.406	8.653	9.919
delaunay_n10	1024	3056	2	0.470	8.930	8.994
emailgraph550K_19	491	853	33	0.771	11.855	13.850
email	1133	5451	4	1.116	8.463	8.758
emailgraph550K_26	527	1046	38	1.231	10.434	11.957
delaunay_n11	2048	6127	2	1.792	9.256	8.893
netscience	1589	2742	38	4.310	4.030	11.952
bo_cluster	2114	2277	19	4.355	6.007	12.800
polblogs	1490	16715	7	4.493	11.560	12.097
delaunay_n12	4096	12264	2	7.226	10.914	9.870
data	2851	15093	4	11.506	8.021	9.620
dokuwiki_org	4416	12914	18	39.815	12.423	15.571
power	4941	6594	66	1:25.736	8.773	15.742
hep-th	8361	15751	56	6:26.213	7.373	18.183
PGPgiantcompo	10680	24316	94	13:25.121	6.463	18.575
as-22july06	22963	48436	33	39:54.495	12.419	20.583
cond-mat	16726	47594	80	44:15.317	14.917	27.425
astro-ph	16706	121251	60	98:25.791	21.843	24.825
rgg_n_2_15	32768	160240	46	245:25.644	32.748	22.573
cond-mat-2003	31163	120029	74	268:14.601	18.306	20.933
G_n_pin_pout	100000	501198	4	369:29.033	*	*
cond-mat-2005	40421	175691	82	652:32.163	*	21.446

E Tables of Tested Instances

Table 5. Testbed encompassing real-world networks and randomly generated graphs.

graph	n	m	graph	n	m
karate	34	78	dolphins	62	159
lesmis	77	254	polbooks	105	441
adjnoun	112	425	football	115	613
jazz	198	2742	celegansneural	297	2148
celegans_metabolic	453	2025	delaunay_n10	1024	3056
email	1133	5451	polblogs	1490	16715
netscience	1589	2742	delaunay_n11	2048	6127
bo_cluster	2114	2203	data	2851	15093
delaunay_n12	4096	12264	dokuwiki_org	4416	12914
power	4941	6594	hep-th	8361	15751
PGPgiantcompo	10680	24316	astro-ph	16706	121251
cond-mat	16726	47594	as-22july06	22963	48436
cond-mat-2003	31163	120029	rgg_n_2_15_s0	32768	160240
cond-mat-2005	40421	175691	G_n_pin_pout	100000	501198

Table 6. Snapshots of the email network of the Department of Informatics at KIT.

graph	n	m	graph	n	m
emailgraph550K_100.graph	304	453	emailgraph550K_101.graph	325	610
emailgraph550K_102.graph	314	513	emailgraph550K_103.graph	322	634
emailgraph550K_104.graph	288	445	emailgraph550K_105.graph	298	528
emailgraph550K_106.graph	310	498	emailgraph550K_107.graph	224	249
emailgraph550K_108.graph	296	507	emailgraph550K_109.graph	307	547
emailgraph550K_10.graph	345	654	emailgraph550K_110.graph	274	348
emailgraph550K_111.graph	213	227	emailgraph550K_112.graph	319	529
emailgraph550K_113.graph	326	569	emailgraph550K_114.graph	247	310
emailgraph550K_115.graph	325	581	emailgraph550K_116.graph	339	639
emailgraph550K_117.graph	255	298	emailgraph550K_118.graph	254	323
emailgraph550K_119.graph	338	629	emailgraph550K_11.graph	315	461
emailgraph550K_120.graph	325	570	emailgraph550K_121.graph	306	487
emailgraph550K_122.graph	291	448	emailgraph550K_123.graph	276	405
emailgraph550K_124.graph	337	614	emailgraph550K_125.graph	306	449
emailgraph550K_126.graph	250	314	emailgraph550K_127.graph	356	740
emailgraph550K_128.graph	304	481	emailgraph550K_129.graph	345	677
emailgraph550K_12.graph	309	456	emailgraph550K_130.graph	310	476
emailgraph550K_131.graph	293	434	emailgraph550K_132.graph	326	662
emailgraph550K_133.graph	248	294	emailgraph550K_134.graph	340	710
emailgraph550K_135.graph	238	282	emailgraph550K_136.graph	322	555
emailgraph550K_137.graph	221	274	emailgraph550K_138.graph	311	556
emailgraph550K_139.graph	195	212	emailgraph550K_13.graph	246	286
emailgraph550K_140.graph	295	482	emailgraph550K_141.graph	277	374
emailgraph550K_142.graph	207	231	emailgraph550K_143.graph	217	242
emailgraph550K_144.graph	206	196	emailgraph550K_145.graph	332	568
emailgraph550K_146.graph	261	330	emailgraph550K_147.graph	295	515
emailgraph550K_148.graph	311	520	emailgraph550K_149.graph	297	446
emailgraph550K_14.graph	459	893	emailgraph550K_150.graph	245	295
emailgraph550K_151.graph	44	23	emailgraph550K_152.graph	349	693
emailgraph550K_153.graph	257	336	emailgraph550K_154.graph	338	692
emailgraph550K_155.graph	279	372	emailgraph550K_156.graph	349	827
emailgraph550K_157.graph	274	344	emailgraph550K_158.graph	333	690
emailgraph550K_159.graph	287	399	emailgraph550K_15.graph	464	868
emailgraph550K_160.graph	353	652	emailgraph550K_161.graph	334	564
emailgraph550K_162.graph	345	683	emailgraph550K_163.graph	320	536
emailgraph550K_164.graph	366	822	emailgraph550K_165.graph	338	549
emailgraph550K_166.graph	242	316	emailgraph550K_167.graph	346	658
emailgraph550K_168.graph	299	465	emailgraph550K_169.graph	331	615
emailgraph550K_16.graph	491	883	emailgraph550K_170.graph	358	668
emailgraph550K_171.graph	64	45	emailgraph550K_172.graph	260	285
emailgraph550K_173.graph	90	68	emailgraph550K_174.graph	346	675
emailgraph550K_175.graph	244	280	emailgraph550K_176.graph	374	772
emailgraph550K_177.graph	258	328	emailgraph550K_178.graph	361	705
emailgraph550K_179.graph	261	338	emailgraph550K_17.graph	556	1171

Table 7. Snapshots of the email network of the Department of Informatics at KIT.

graph	n	m	graph	n	m
emailgraph550K_180.graph	358	710	emailgraph550K_181.graph	355	622
emailgraph550K_182.graph	322	501	emailgraph550K_183.graph	289	408
emailgraph550K_184.graph	327	586	emailgraph550K_185.graph	306	450
emailgraph550K_186.graph	222	253	emailgraph550K_187.graph	339	617
emailgraph550K_188.graph	294	443	emailgraph550K_189.graph	290	405
emailgraph550K_18.graph	411	557	emailgraph550K_190.graph	342	609
emailgraph550K_191.graph	259	305	emailgraph550K_192.graph	340	618
emailgraph550K_193.graph	295	387	emailgraph550K_194.graph	231	250
emailgraph550K_195.graph	239	263	emailgraph550K_196.graph	363	820
emailgraph550K_197.graph	295	421	emailgraph550K_198.graph	381	777
emailgraph550K_199.graph	340	563	emailgraph550K_19.graph	491	853
emailgraph550K_1.graph	295	456	emailgraph550K_200.graph	361	698
emailgraph550K_201.graph	334	541	emailgraph550K_202.graph	340	577
emailgraph550K_203.graph	327	572	emailgraph550K_204.graph	291	401
emailgraph550K_205.graph	193	204	emailgraph550K_206.graph	341	579
emailgraph550K_207.graph	259	319	emailgraph550K_208.graph	331	510
emailgraph550K_209.graph	314	459	emailgraph550K_20.graph	509	1097
emailgraph550K_210.graph	354	684	emailgraph550K_211.graph	298	425
emailgraph550K_212.graph	340	579	emailgraph550K_213.graph	341	553
emailgraph550K_214.graph	344	694	emailgraph550K_215.graph	327	530
emailgraph550K_216.graph	361	691	emailgraph550K_217.graph	351	589
emailgraph550K_218.graph	249	284	emailgraph550K_219.graph	374	732
emailgraph550K_21.graph	310	278	emailgraph550K_220.graph	257	314
emailgraph550K_221.graph	338	582	emailgraph550K_222.graph	335	518
emailgraph550K_223.graph	258	269	emailgraph550K_224.graph	219	191
emailgraph550K_225.graph	324	515	emailgraph550K_226.graph	297	508
emailgraph550K_227.graph	207	200	emailgraph550K_228.graph	335	533
emailgraph550K_229.graph	278	351	emailgraph550K_22.graph	190	166
emailgraph550K_230.graph	268	286	emailgraph550K_231.graph	353	609
emailgraph550K_232.graph	325	460	emailgraph550K_233.graph	241	251
emailgraph550K_234.graph	351	684	emailgraph550K_235.graph	351	544
emailgraph550K_236.graph	365	643	emailgraph550K_237.graph	363	703
emailgraph550K_238.graph	331	463	emailgraph550K_239.graph	241	260
emailgraph550K_23.graph	393	608	emailgraph550K_240.graph	375	687
emailgraph550K_241.graph	330	494	emailgraph550K_242.graph	285	350
emailgraph550K_243.graph	268	310	emailgraph550K_244.graph	287	361
emailgraph550K_245.graph	274	328	emailgraph550K_246.graph	223	201
emailgraph550K_247.graph	223	201	emailgraph550K_248.graph	223	200
emailgraph550K_249.graph	331	476	emailgraph550K_24.graph	419	561
emailgraph550K_250.graph	314	404	emailgraph550K_251.graph	174	175
emailgraph550K_252.graph	319	425	emailgraph550K_253.graph	208	190
emailgraph550K_254.graph	199	193	emailgraph550K_255.graph	276	347
emailgraph550K_256.graph	301	425	emailgraph550K_257.graph	223	263
emailgraph550K_258.graph	288	364	emailgraph550K_259.graph	284	377
emailgraph550K_25.graph	533	999	emailgraph550K_260.graph	266	301
emailgraph550K_261.graph	283	354	emailgraph550K_262.graph	224	256

Table 8. Snapshots of the email network of the Department of Informatics at KIT.

graph	n	m	graph	n	m
emailgraph550K_263.graph	322	435	emailgraph550K_264.graph	321	506
emailgraph550K_265.graph	291	403	emailgraph550K_266.graph	280	326
emailgraph550K_267.graph	185	163	emailgraph550K_268.graph	169	152
emailgraph550K_269.graph	175	145	emailgraph550K_26.graph	527	1046
emailgraph550K_270.graph	278	348	emailgraph550K_271.graph	238	247
emailgraph550K_272.graph	268	351	emailgraph550K_273.graph	264	287
emailgraph550K_274.graph	249	272	emailgraph550K_275.graph	144	115
emailgraph550K_27.graph	346	578	emailgraph550K_28.graph	330	709
emailgraph550K_29.graph	282	383	emailgraph550K_2.graph	309	503
emailgraph550K_30.graph	310	612	emailgraph550K_31.graph	306	562
emailgraph550K_32.graph	319	651	emailgraph550K_33.graph	307	603
emailgraph550K_34.graph	224	303	emailgraph550K_35.graph	321	628
emailgraph550K_36.graph	272	455	emailgraph550K_37.graph	209	226
emailgraph550K_38.graph	222	237	emailgraph550K_39.graph	223	283
emailgraph550K_3.graph	287	428	emailgraph550K_40.graph	305	525
emailgraph550K_41.graph	290	480	emailgraph550K_42.graph	287	498
emailgraph550K_43.graph	313	526	emailgraph550K_44.graph	244	327
emailgraph550K_45.graph	188	196	emailgraph550K_46.graph	261	384
emailgraph550K_47.graph	207	229	emailgraph550K_48.graph	319	573
emailgraph550K_49.graph	289	451	emailgraph550K_4.graph	322	504
emailgraph550K_50.graph	278	388	emailgraph550K_51.graph	309	559
emailgraph550K_52.graph	265	379	emailgraph550K_53.graph	269	360
emailgraph550K_54.graph	314	543	emailgraph550K_55.graph	305	497
emailgraph550K_56.graph	202	223	emailgraph550K_57.graph	302	533
emailgraph550K_58.graph	303	549	emailgraph550K_59.graph	257	425
emailgraph550K_5.graph	331	647	emailgraph550K_60.graph	239	282
emailgraph550K_61.graph	298	495	emailgraph550K_62.graph	307	543
emailgraph550K_63.graph	323	606	emailgraph550K_64.graph	289	454
emailgraph550K_65.graph	276	434	emailgraph550K_66.graph	251	394
emailgraph550K_67.graph	252	355	emailgraph550K_68.graph	218	266
emailgraph550K_69.graph	181	181	emailgraph550K_6.graph	333	562
emailgraph550K_70.graph	283	457	emailgraph550K_71.graph	295	474
emailgraph550K_72.graph	250	367	emailgraph550K_73.graph	179	218
emailgraph550K_74.graph	273	469	emailgraph550K_75.graph	266	458
emailgraph550K_76.graph	302	599	emailgraph550K_77.graph	277	498
emailgraph550K_78.graph	280	414	emailgraph550K_79.graph	333	673
emailgraph550K_7.graph	429	826	emailgraph550K_80.graph	262	385
emailgraph550K_81.graph	289	421	emailgraph550K_82.gaph	327	628
emailgraph550K_83.graph	220	265	emailgraph550K_84.graph	324	581
emailgraph550K_85.graph	335	612	emailgraph550K_86.graph	276	343
emailgraph550K_87.graph	241	258	emailgraph550K_88.graph	343	628
emailgraph550K_89.graph	326	582	emailgraph550K_8.graph	296	414
emailgraph550K_90.graph	239	280	emailgraph550K_91.graph	328	564
emailgraph550K_92.graph	84	54	emailgraph550K_93.graph	322	519
emailgraph550K_94.graph	252	298	emailgraph550K_95.graph	326	559
emailgraph550K_96.graph	342	665	emailgraph550K_97.graph	300	459
emailgraph550K_98.graph	219	244	emailgraph550K_99.graph	331	634
emailgraph550K_9.graph	345	654			