High Water Marks in Hedge Fund Management Contracts

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1 General Introduction

The fast-growing, extremely diverse and opaque hedge fund industry reached a record high of US $2.13 trillion total assets under management in April 2012.[1] According to rankings in *Absolute Return+Alpha*[2] the 25 top-earning hedge fund managers earned altogether US $14.4 billion in 2011. "Despite the industry's overall recent poor performance, investors haven't shied away ... Since the financial crisis, investors have been drawn to hedge funds because they have the ability to bet on all types of markets and don't simply expect stocks to move up."[3]

Hedge funds are primarily private partnerships or investment funds open only to large and sophisticated investors. The hedge fund industry is exempt from the mandatory registration with the US Securities and Exchange Comission (SEC) under the *Investment Company Act of 1940* which is intended to regulate investment companies such as mutual funds.[4] This regulatory exemption allows hedge funds to avoid the record-keeping requirements and substantial disclosure. Hedge funds use access to the capital markets via private placement without any constraints to register their shares. Therefore, managers have maximum flexibility in their portfolio choices and can employ an enormous variety of strategies. In this context Ineichen (2003) states: "Hedge funds leverage the capital they invest by buying securities on margin and engaging in collateralized borrowing. Better-known funds can buy structured products without putting up capital initially, but must make a succession of premium payments when the market in those securities trades up or down. In addition, some hedge funds negotiate secured credit lines with their banks, and some relative-value funds may even obtain unsecured credit lines." Hedge fund strategies vary enormously, but the main aim of most hedge funds is to raise and attempt to preserve sufficient capital in order to generate positive absolute returns under all possible market conditions.

Unfortunately, as an outsider or potential investor in a hedge fund, it is hard to receive reliable data about the funds' returns or to evaluate their performance. It is a well-known

[1] "Hedge-Fund Assets Rise to Record Level", Juliet Chung, The Wall Street Journal, June, 14th, 2012

[2] Published in March, 29th, 2012

[3] "Hedge Funds are Betting on Disaster", Maureen Farrell, CNN Money, August, 23th, 2012

[4] The act placed restrictions on the activities in which investment companies are allowed to engage. For example, it forbade short selling. It required investment companies to file financial disclosure and set limits on the fees they are allowed to charge; it also required that investment companies with more than a certain number of investors register with the SEC. (Source: *Morgan, Lewis & Bockius, LLP, 2005)*

fact that hedge funds are reluctant to provide detailed information on their returns and investment strategies. One possibility to obtain some information is to use the hedge fund indices, that are constructed by using different available data and heterogenous selection standards. However, problems can be caused by potential biases that can be included in the data sources. According to Fung and Hsieh (2000) the organizational structure of hedge funds, as private and often offshore vehicles, makes data collection a much more onerous task, amplifying the impact of performance measurement biases. Indeed, hedge fund indices show recent hedge fund performance within some degree of error, but at least they help investors and hedge fund outsiders reasonably to characterize the directionality of hedge fund performance.

Nevertheless, hedge fund performance data alone is not sufficiently informative for the outsiders, can even prove misleading, especially if they cover only a short term. Performance over a period of a few years or even months could sometimes tell more about the manager's luck than skill. And past performance alone has never provided a reliable prediction of future success. All the more reason for the investors to identify a skillful hedge fund manager. The investors expect better funds prospects and profitable allocation of their investment if they believe to invest in a fund that is managed by an experienced manager with required qualifications in her own special strategy.

It is generally thought that hedge fund returns are provided by the managers' skills, their depth knowledge of the fund's strategy and their holdings: "It is possible to claim that skillful managers outside their previous employers investment bank or fund management operation prefer to manage portfolios that best show their skills and for which they get best paid. An investment in a hedge fund is really an investment in a manager and the specialized talent she possesses to capture profits from a unique strategy."[5] In addition, the American economist and hedge fund manager Sanford J. Grossman argues that a "fund's return will be no better than its management and the economic environment in which it produces its product" and that the performance of a hedge fund depends on the underlying investment strategy and manager's talent to implement this strategy. Gray and Kern (2009) find overwhelming evidence that hedge fund managers have stockpicking skills. The authors provide evidence that the hedge fund managers in their sample cannot only identify outperforming stocks on average, but they are also able to distinguish the best and the worst of the outperforming stocks.

For private persons the most common way to enter the world of hedge fund investing

[5] "Hedge Funds Today: Talent Required", Sanford J., Grossman, The Wall Street Journal, September, 29th, 2005

is to qualify as a private investor. "There are two basic categories of private investors: accredited investors, who need a net worth of more than US $1 million [or an annual salary of at least US $200,000]; and qualified purchasers, who need to have at least US $5 million in investment assets not including a primary residence or any property used for a business."[6] Companies and institutional investors, that have at least US $25 million in investment assets, have also the possibility to contribute their capital to a hedge fund by generally qualifing as qualified purchasers. In order to overcome the registration in the US *Securities and Exchange Commission* and *Securities Exchange Act of 1934*, the hedge fund managers are legally allowed to accept either not more than 99 investors, where only 35 can be non-accredited, or not more than 1,999 qualified purchasers.

Hedge funds are popular investments, and investors are very competitive to contribute their money to the fund. "They arrive every week, in ones and twos and groups of 10, some of them coming straight from Sao Paulo's Guarulhos International Airport. These investors head for the dark-wood halls of Credit Suisse Hedging-Griffo as supplicants, asking to put their millions of dollars into one of the world's top-performing hedge funds."[7] But once the fund is fully invested, the hedge fund investors have to accept severe contractual share restrictions and, thus, to deal with poor liquidity.

Hedge funds typically accept capital contributions at the beginning of each period and allow investors to withdraw capital at specified periodic intervals. To do this, the investors must provide written notice to the hedge fund manager in advance of the permitted redemption date. Some funds impose in their contractual agreements a so called "lock-up" which is a time-specified period, during which a new investor is restricted from redeeming from the fund.[8] If the investor decides to withdraw the capital after the lock-up-period, in most cases she has to pay very high fees for doing so. Additionally, in order to retain the balance of the investors' capital, the hedge fund manager may have a contractually accorded authority to process only a portion of a redemption request by specifing a limitation on what percentage of capital may flow out of a fund, known as a "gate".[9]

[6]"Hedge Fund Investing 101", Lynn Sherman, Forbes, July, 15th, 2000

[7]"Brazil Hedge Funds Beat U.S. Competitors by Investing in Bonds", Bloomberg Markets Magazine, June 7th, 2011

[8]According to Belmont (2011) written notice has to be provided by the investors somtimes 90 or even 180 days prior to the redemption date. A typical lock-up period is one to two years.

[9]Ang and Bollen (2009) compute that the cost of lock-up provisions and withdrawal suspensions can be significant for investors. Lock-up provisions and gates vary with the liquidity of investments. For example, in FrontPoint Partners' FrontPoint-SJC Direct Lending Fund investors' funds are locked up for five years. (Source: *Financial Times Online, "FrontPoint raises $1bn for new fund", January 7, 2011.*)

The contract that specifies contractual obligations for the hedge fund manager and her investors sets not only the fund's status with regulators and limits on capital withdrawals. It also specifies the managerial involvement and fees that the fund's manager charges and how they are calculated. The first interesting contractual feature is that the managers of most hedge funds invest large amounts of own money in their fund. Another feature is that the contracts typically specify a guaranteed management fee between 0.5% and 2.5% of funds' assets under management and a fee based on a percentage of profits, known as a performance fee, that can vary between 10% and 25% (compare Figure 1).

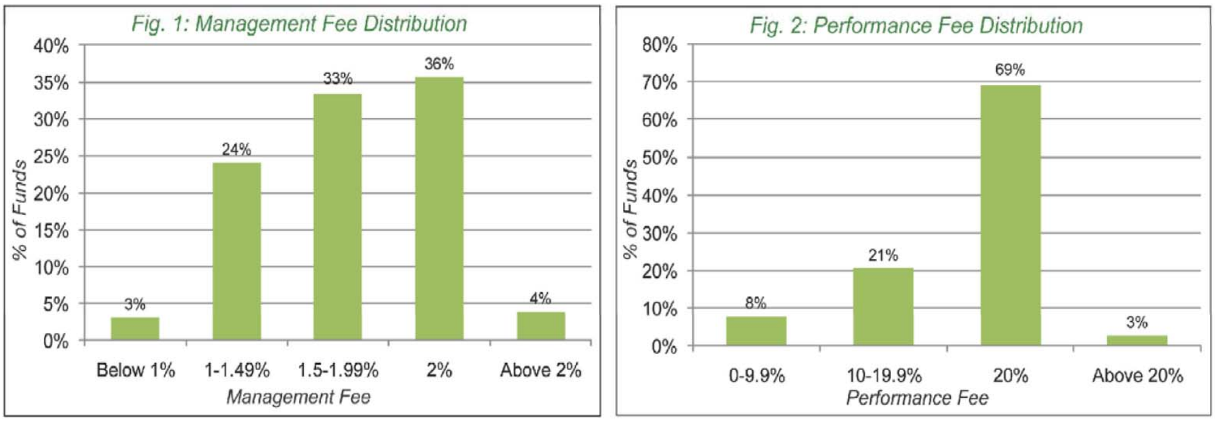

Figure 1: Management Fee and Performance Fee Distributions of Hedge Funds.
More than two-thirds of funds have management fees of 1.5% to 2%. Nearly 70% of the hedge funds maintain a performance fee of 20%.
Source: "Does "2 and 20" Still Exist? Results of Preqin's Hedge Fund Terms & Conditions Survey-Fees Special Report, July 2009.

Brown, Goetzmann and Ibbotson (1999) suggest that the management fee is designed to cover the hedge fund manager's cost of the operating expenses and portfolio switching. Fung (2011), on the contrary, argues that the management fee is one of the main components in hedge fund total compensation, when considering factors that incentivize the management. The performance (or incentive) fee is calculated as a percentage of the differential between the funds new net profits earned in the last period and some hurdle rate - in the majority of cases - the high water mark. A high water mark is the historic maximum of the fund net asset value's previously seen at the end of one of the past periods. The use of such kind of incentive contracts in the hedge fund industry foreshadows that the manager's reward depends not only on the recent performance level and on time, to which the fee is paid, but also on changes of the fund's past performance and the path[10],

[10] The manager's payoff, depending at any time on the high-water mark, which is related to the maximum asset value achieved over time, is commonly understood as a path dependent payoff. Compare for example Goetzmann, Ingesroll, Ross (2003).

on which the fund reached the current level. According to Agarwal, Daniel,

Naik (2011) performance based compensation contracts with high water mark provisions provide managers with a call option. The manager receives the performance based fee only if the fund value increases above a given maximum asset value achieved over time, that is called by Panageas, Westerfield (2009) the "strike price". When the hedge fund loses value, the level on high water mark retains unchanged.

Implications of High Water Marks

Economists have provided suggestions for the role of the high water mark in hedge fund management compensation contracts. But yet, it does not exist an unified theory on whether the high water mark represents an optimal contractual solution between the hedge fund manager and her investors. Many economists are thinking about the high water mark as a form of insurance for investors: a hedge fund manager who first created high returns on investment and afterwards loses a part of that capital cannot receive performance fee payment until the loss has been made up. Further, the high water mark prevent a manager from being paid twice for the same gains of the fund.

A broad field of research deals with increased risk-taking by the hedge fund manager and tries to answer the question whether the high water mark provision in the hedge fund manager's compensation contracts leads to more risk-taking or not. Hodder and Jackwerth (2007) and Chakraborty and Ray (2008) show theoretically that the manager's risk-taking crucially depends on a fund's remaining life span. The authors of the first paper model a situation in which the manager with the short-term perspective is willing to take added risks only if the fund's value is below the high water mark. In line with this, Chakraborty and Ray (2008) develop a model which predicts hedge fund managers' behavior by generally more risk-taking especially when the fund is below the high water mark. To the contrary, Panageas and Westerfield (2009) find that even a risk-neutral manager does not have additional incentives for higher risk-taking due to the high water mark provision. Also empirical evidence is mixed regarding the question of whether high water mark contracts boost managers' risk-taking behavior. Ray (2009), for example, finds that as soon a hedge fund falls below its high water mark the future expected Sharpe ratio decreases. Additionally, these effects are strongest for funds that are closer to the high water mark threshold. Brown, Goetzmann and Park (2001) and Aragon and Nanda (2009), on the other hand, show that the changes in risk are not conditional upon distance from the high water mark threshold.

Another implication of high water marks, discussed in the literature, provides prospects for overcoming the problems associated with adverse selection. Theoretical results in a multi-period model by Aragon and Qian (2010) suggest that a contract between managers and investors which include high water mark provisions can be optimal and allow highly skilled managers to signal their quality to the investors and, thus, to reduce the costs of adverse selection. Aragon and Qian (2010) empirically support their findings and show that high water mark provisions are more often used by smaller funds and funds with shorter track records.

This thesis ties on and extends previous results from literature primarily examining incentives in management contracts with high water mark provision. Thereby, it is supposed that the incentives set by contracts with high water marks are not limited to the existing approaches covered by recent research. Two additional conceptions will be developed.

Firstly, a common observation in the hedge fund market is that well performing funds shut down business for reasons not verifiable to outside observers. This leads to the question whether the contract with high water mark provision influences the managers' funds closing decisions. Secondly, the suggestion arises, especially due to the weak regulatory oversight in the hedge fund industry, hedge fund managers might have little constraints to misreport the funds recent performance, if doing so is beneficial to the manager. Especially, the contract that specifies high water mark provision may set different incentive to managers compared to contracts without high water marks.

Research Questions and Structure

The objective of the thesis is to point out and analyze additional features of the high water marks in hedge fund management compensation contracts which have not been considered in the literature so far.

Empirical evidence reveals that investors in a hedge fund are not able to play a monitoring role for the hedge fund manager. Thus, the question about the economic benefits and effectiveness from the incentives set by hedge fund management compensation contract remains legitimate. By assuming the fund manager to be better informed about the fund's investment strategy and fund's prospects the manager is assumed to have a bargaining power over the contract design and fund's decision making.

Using approaches of asymmetric information we compare the incentive effects on hedge fund managers depending on the structure of the fund manager's compensation contract. The basic question behind the both theoretical models presented in this thesis is whether decisions caused by the incentive contract with the high water mark provision are beneficial or even optimal for the manager and for the fund's investors if the hedge fund manager is assumed to be better informed about the uncertain funds prospects.

In the following chapter we develop a rationale for the inclusion of the high water mark provision to facilitate efficient closing of the hedge fund by their managers. Management compensation contracts that include the high water marks specify lower expected fees after periods of negative performance when fund closing may be warranted. Our approach implies that by using the contract with the high water mark provision the fund's manager has incentives to close the fund more quickly upon periods of poor performance than if the contract does not include a high water mark. If the fund with a high water mark provision decides to continue after the periods of poor performance, the performance levels on an after-fee basis in the following periods are expected to be superior to comparable funds employing a period performance fee.

"Academic research on hedge fund performance readily admits to biases in commercially available data ... These commercial databases have been the primary data source used by academics and regulators to study hedge funds. Yet, the voluntarily nature of the disclosure decision creates a host of biases that affect inferences on hedge fund performance and risk."[11] In the third chapter we discuss the hedge fund data biases and the problems that they cause in the field of empirical research methods.

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[11] Aiken, Clifford and Ellis (2010)

The fourth chapter studies hedge fund managers' incentives for return smoothing that can be caused by the contract with the high water mark provision. We show theoretically that managers of funds with high water mark provisions have stronger incentives to underreport positive actual returns than managers of funds whose management contracts specify period performance fees. Additionally, managers with high water marks have strong incentives to overreport negative returns when doing so prohibits outflows. The reason for this is that reporting, for example, a zero return rather than the actual negative one does not affect the high water mark and therefore is inconsequential for future fees. This pattern is not observed for managers whose fee income is determined by the fund's performance in each period.

2 Time to Wind Down: Closing Decisions and High Water Marks in Hedge Fund Management Contracts

This section[12] provides a rationale for the inclusion of high water mark provisions in hedge fund management contracts. When hedge fund managers are better informed about future fund profitability than investors, contracts including high water marks provide the fund managers with better incentives to efficiently close the fund than contracts with linear performance fees. The model implies that funds with high water marks tend to close more frequently upon periods of poor performance than their period performance fee counterparts. If, however, such funds with high water mark arrangement decide to continue, their after-fee performance is expected to be superior to comparable funds employing period performance fees. The model is also consistent with empirical evidence that high water marks are more common in smaller funds and funds run by managers without extensive track records.

2.1 Introduction

The literature on dynamic incentive provision typically proposes contracts that specify compensation that is based on outcomes during the time period the manager can affect these outcomes. For example, it is argued that a manager should be compensated for a period's activities exclusively based on that period's outcomes rather than on previous periods' results as this may distort current incentives (see, for example, Holmström and Milgrom, 1987). Hedge fund management contracts typically violate this property in that the performance component of the management contract specifies that a performance fee is based on the fund value at the end of a given period relative to the fund value's historic maximum rather than that at the beginning of the period.[13] This fee structure is frequently referred to as a performance fee with a high water mark provision. In addition to performance fees, most hedge fund management contracts also specify a so called management fee that is based on the value of assets under management.

[12]The research presented in this section was performed in cooperation with Martin E. Ruckes; see Ruckes and Sevostiyanova (2012a)

[13]In the comprehensive sample of Agarwal et al. (2009), 80.1 percent of hedge funds display such a fee structure.

The historic fund value and the fund value at the beginning of a period differ only when periods of losses have occurred in the past. Then, a high water mark provision has two effects compared to an otherwise identically structured fee based on period performance: 1) it reduces the expected fee amount paid to the hedge fund manager as a fee is applied to a smaller base and 2) it introduces a convexity in the fee structure as a fee is only paid on period performance above a strictly positive level. In a seminal contribution, Panageas and Westerfield (2009) argue that these properties of a high water mark provision may have desirable incentive effects in a dynamic context. They show that under a high water mark contract, a risk neutral hedge fund manager displays risk averse behavior provided that the fund's horizon is sufficiently long. In case of a long fund horizon, a large share of the fund manager's expected income stems from future fees. As future fees are reduced when the fund's value is below its historic maximum, the manager tries to avoid reaching such states. He will do so by limiting the risk of the fund's holdings.

When the fund's horizon is short, however, a high water mark provision's implications for managerial risk taking are much less clear. Specifically, a high water mark may lead to excessive risk taking by the hedge fund manager caused by the convexity of the fee structure (Hodder and Jackwerth, 2007, and Chakraborty and Ray, 2008). Indeed, the average life span of a hedge fund is rather short. According to Malkiel and Saha (2005), annual hedge fund attrition rates in the years 1994 to 2003 have been below 10 percent only in one of the years.[14]

In this paper we argue that high water mark provisions have desirable incentive effects especially for hedge funds with limited but uncertain horizons. They do so because high water mark provisions facilitate the efficient closing of hedge funds by their managers.[15]

The profitability of a hedge fund's strategy changes over time. At any point in time the hedge fund's manager is typically in a better position than investors to identify whether the prospects of the fund's strategy warrant the fund's continuation. For example, while investors may have to infer the quality of a fund's strategy from recent performance its manager possesses in depth knowledge of the fund's strategy and holdings. Combined with a close following of the markets relevant to the strategy this typically allows her to

[14] Chan et al. (2005) report that for their sample of hedge fund liquidations "... half of all liquidated funds never reached their fourth anniversary." Note that fund liquidation does not necessarily mean failure; see Liang and Park (2010).

[15] The notion that fund closings are frequently instigated by fund management, is reflected, for example, in the closing announcement of Atticus Global Fund: "This decision will come as a surprise to most of you, especially given that we have received redemptions of less than 5% of capital ..."

better assess whether the recent performance tends to be temporary in nature or indicates a permanent change of fund prospects. "The fund, which looked after \$1.9 billion at its peak, faced the prospect of spending the next few years trying to claw its way back to pre-crisis asset levels. Instead the founders decided to shut the fund and give investors their money back ... For investors, it is generally a good thing if underperforming managers are returning cash and not milking them for fees."[16]

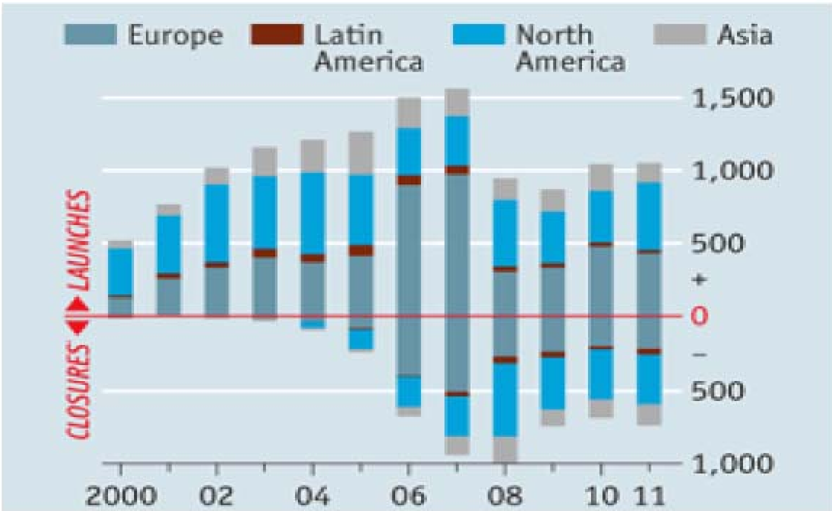

Figure 2: Life and death hedge funds in the time frame 2000-2011.
Source: "Drowning in High Water Hell?", The Economist, February 24th, 2012.

A fund manager's incentives to close the fund are not necessarily aligned with those of investors. Especially because negative fee payments are normally impossible to enforce, even funds with poor prospects may generate significantly positive expected fees for the manager. Thus, a management contract that leads to efficient fund closing needs to specify low expected fees in circumstances in which fund closure may be efficient. Since this is typically the case when recent performance has been poor, the high water mark's effect of reducing expected fees in these situations improves managerial incentives to close the fund. Anticipating a more efficient fund closing decision increases investors' willingness to provide capital to the fund and in turn tends to increase expected fees for the fund manager. The property that a management contract with high water mark generates relatively low expected fees when fund performance has been poor, allows the manager to set a relatively high performance fee rate. Doing so mitigates a second type of incentive problem. Because the manager does not fully participate in the value gains of a fund, she may close it even when the fund has performed well and fund prospects are intact. A

[16] "Drowning in High Water Hell?", The Economist, February 24th, 2012.

high performance fee rate implies high expected future fees when the fund value is at its historic maximum, which leads to a low probability of fund closing.

We present a model that formally characterizes the above argument and show that linear performance fee contracts with high water marks dominate those with period performance fees when fee levels are set endogenously. Our approach implies that funds with high water marks tend to close with a higher probability upon periods of poor performance than their period performance fee counterparts. If, however, such funds with high water mark arrangements decide to continue, their after-fee performance levels are expected to be superior to comparable funds employing period performance fees.

In our model, performance fee levels are set to optimize managerial incentives whereas management fees are typically used by the fund manager to extract rents. Optimal management fees are lower if the performance fee structure contains a high water mark than if it does not. In the former case, the non-negativity constraint of the management fee may even be binding. Then, the performance fee serves also as the instrument for manager to extract rents. When the probability of a deterioration of the fund's prospects upon poor performance is sufficiently low, a contract with period performance fee is even preferred over that with high water mark. Given that small funds and funds run by managers that lack extensive track records can be associated with relatively high probabilities of deterioration of prospects such funds are expected to more frequently employ performance fees with high water mark provisions.

Related Literature

Aragon and Qian (2010) provide a rationale for the inclusion of high water mark provisions in hedge fund management contracts based on *ex ante* asymmetric information. Hedge fund managers attempt to credibly signal their quality by offering a contract that pays lower expected fees when performance is poor. As a contract containing a high water mark tends to imply a lower fee for several periods, it is particularly well suited to be used as a signaling device. Aragon and Qian (2010) show that high water marks can reduce excessive closing caused by investor redemptions. In contrast, our approach focuses on the closing decision by fund management and argues that high water marks not only reduces excessive continuation by fund managers upon poor performance but also excessive termination after strong returns.[17] Also, in the model presented here an informational asymmetry

[17] While the latter benefit of high water marks may appear of less economic consequence, cases of fund closings by managers after strong results do exist. For example, Andrew Lahde, founder of Lahde Capital, decided to close down his funds and to return money to investors after a return of 870 percent the previous year ["Hedge fund returns money", Financial Times Online, September 22nd, 2008].

arises after the contract is signed rather than beforehand. Deuskar, Wang, Wu and Nguyen (2011) document in their empirical study that new unestablished hedge funds tend to charge higher incentive and lower management fees. Such an initial fee structure appears to be chosen by managers with superior skills and to predict better performance of the fund with a higher survival rate. The empirical evidence suggests that the contract in such form is adjusted to reflect beliefs updating about managerials skills on the funds past performance. Deuskar, Wang, Wu and Nguyen (2011) find that the hedge funds that face the choice between lowering the fee payment to the manager and fund closure after the poor performance, tend more likely to close the fund down.

In our approach, the use of high water mark provisions plays a significant role in hedge funds' closing decisions. Empirical studies confirm the impact of high water marks on closure rates of funds. Brown, Goetzmann and Park (2001), Aragon and Nanda (2009) as well as Ray (2009) document that hedge funds whose value is further below their high water marks close at higher rates. Anecdotal evidence also points towards the influence of future expected fees on closing decisions: "Most funds close down because it does not pay their managers to continue, not because their performance has been disastrous."[18] Particularly the high water mark contract component is thought to be responsible for this behavior: "[The fact that they are still below their peak performance] has lead many hedge funds to wind down rather than attempt to claw their way back to the point at which they can earn performance fees."[19] Liang and Park (2010) find that hedge funds with high water marks tend to close more quickly upon bad performance.

The remainder of the section is organized as follows. Subsection 2.2 presents the model. Subsection 2.3 is concerned with the optimal contract design when performance fee levels are set to optimize managerial incentives and management fees are used by the fund manager to extract rents. In subsection 2.4, additional empirical predictions related to fund closing, after-fee fund performance and the level of management fees are derived. Subsection 2.5 studies the parameter set in which the manager employs the performance fee also to extract rents. In subsection 2.6 we provide some robustness analysis by allowing for intermittent capital redemptions and capital contributions by the fund manager. Subsection 2.7 concludes this chapter.

[18] "Hedge podge", Economist, February 16th, 2008.

[19] "Atticus closes flagship fund", Financial Times Online, August 11th, 2009.

2.2 A Simple Model

We describe a stylized two-period model of hedge fund management contracting. During the first period, information about the quality of the fund's strategy is revealed which may lead to a subsequent closing of the fund.

Fund Manager and Investor

Consider an investment manager who has an idea for an investment strategy with a time horizon of two periods. The investment strategy is limited in scale: cash returns are linear in initial investment, but any initial amount above $V_0 = 1$ cannot be invested profitably. The manager does not have financial wealth of her own and needs to raise capital from an investor to implement her investment strategy. There exists an outside investor who has one unit of capital to invest. Alternatively to operating a hedge fund in the second period, the manager has a valuable opportunity to obtain an outside income of ω from working in a different occupation, if the fund is inactive. This outside income ω is privately observed by the manager at the beginning of the second period. Ex ante, ω is distributed according to a cumulative distribution function $F(\omega)$ with density $f(\omega)$ on a support of $[0, \omega_{max}]$.

The manager is assumed to have the entire bargaining power vis a vis the investor and thus to make a take-it-or-leave-it offer to the investor. Both parties are risk neutral and the risk free interest rate is zero.

Characteristics of Investment Strategy and Beliefs

Implementing the manager's strategy implies that in each period, the invested amount yields either a positive return $R^H > 0$ or a negative return $R^L < 0$. Consequently, fund values after the first period are either $V^H \equiv 1 + R^H$ or $V^L \equiv 1 + R^L$ and after the second period $V^{HH} \equiv (1 + R^H)^2$, $V^{LL} \equiv (1 + R^L)^2$, or $V^{LH} \equiv (1 + R^L)(1 + R^H) \equiv V^{HL}$, where the first and second superscripts indicate the realized return in the first and second periods, respectively. Returns are costlessly verifiable. At the time of contracting at date 0 the investor and the manager are symmetrically informed about the probabilities for both positive and negative returns of the fund in the first period $p \in (0, 1)$ and $(1 - p)$, respectively. It holds that

$$\eta := pR^H + (1 - p)R^L > 0, \qquad \text{(A1)}$$

i.e. the investment strategy is profitable in the first period.

Investing in the first period generates information about expected second-period returns.

While a positive first-period return (state H) is uninformative for second-period prospects, a negative first-period return tends to be associated with a deterioration of expected returns.[20] Concretely, given a negative first-period return, one of two return distributions may materialize: one where the first-period return is a purely temporary phenomenon and the probability of a positive second-period return remains at p (state L°), and one where the first-period return is indicative for second-period prospects and the probability of a positive second-period return decreases to $p - \varepsilon$ (state L^{-}), with $\varepsilon \in (0, p)$. As we are interested in whether fund closing takes place efficiently, we assume that the lowest probability of a positive return in the second-period, $p - \varepsilon$ implies a negative expected surplus:

$$(p-\varepsilon)R^{H} + (1-p+\varepsilon)R^{L} < 0. \qquad \text{(A2)}$$

Assumption (A2) implies that it is efficient to close the fund if the investment strategy's prospects have deteriorated irrespective of the manager's realisation of outside option. A change in prospects occurs with probability $1-\theta$ in case of a negative first-period return (see Figure 3).

We also assume that it is efficient to continue the fund after a positive first-period return:

$$\left(1+R^{H}\right)\eta \geq \omega_{max}. \qquad \text{(A3)}$$

While assumption (A3) appears natural, it illustrates a central economic notion underlying the model. When the fund has been doing well, investors want to see the fund continued. In that case, the only possibly relevant distorting behavior by the manager is to close the fund with positive probability.

Due to the intimate knowledge of her own investment strategy and close observation of market development, the manager observes the true return distribution arising at date 1. The investor is unable to observe the true return distribution and is only informed about the first-period return of the fund. Upon observing a positive first-period return, the investor's probability for a positive second-period return remains at p and the probability of a positive second-period return is $p - \varepsilon(1-\theta)$ when he observes a negative first-period return.

[20] Assuming that a positive first-period return tends to be associated with an improvement of prospects does not affect the results.

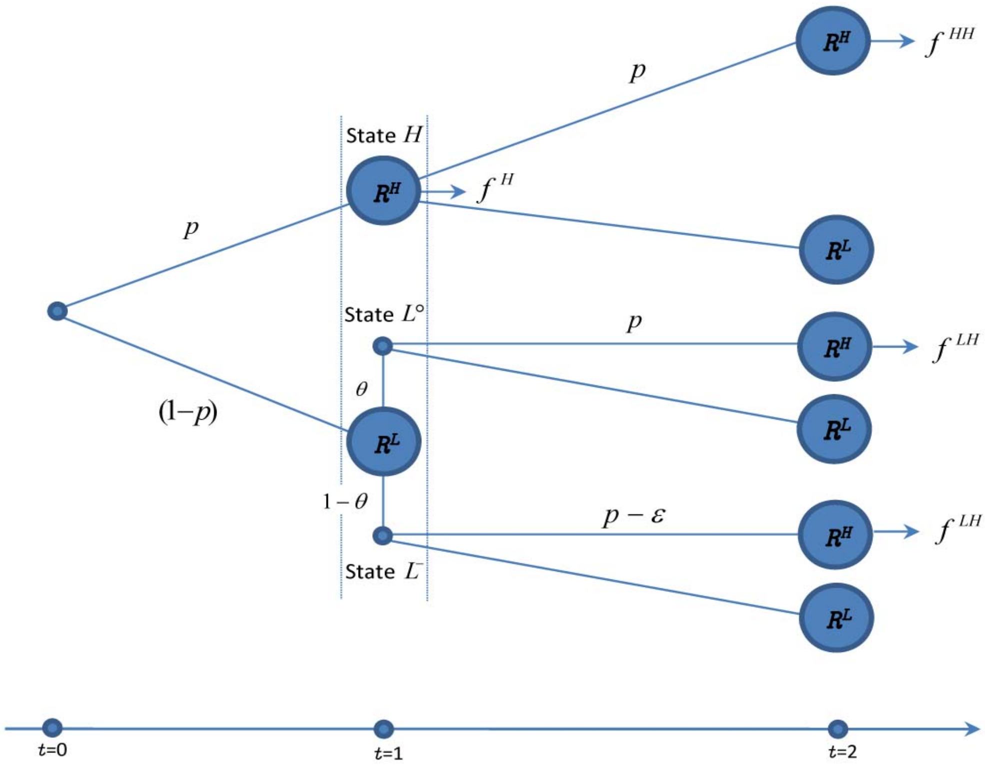

Figure 3: Return probabilities.
Ex ante view of states H, $L°$ and L^-, success probabilities in the first and second periods and corresponding fees in all outcomes after positive period return R^H.

Compensation and Fund Closing

While ω characterizes the manager's income during the second period if the fund is inactive,[21] the manager is compensated in the form of ex ante specified fees during active periods. The fees can be made contingent on the fund's performance. Due to the manager's limited wealth, fees cannot be negative in any period. This implies that after a fee has been paid out, they are unaccessible to the investor in later periods.

We focus on two performance-based compensation arrangements:

- a *period performance fee*, where at the end of each period the manager receives a constant fraction of the fund's value gain during the period and nothing when the fund loses value during the period,
- a *performance fee with a high water mark provision*, where at the end of each period

[21]Introducing a positive outside income in the first period, does not affect the results as long as the fund is still launched.

the manager receives a constant fraction of the fund's value gain during the period relative to the fund's historic maximum value and nothing when the fund's value is below its historic maximum.

In addition to the performance fee, the manager charges a one-time management fee $k \geq 0$.[22] In the following, we assume the non-negativity constraint to be not binding in the optimal contract. Allowing for a fixed management fee in that way enables us to separate the incentive effects of performance fees from the expected level of compensation. We examine the case in which the inequality is binding in subsection 2.5.

As either of the two performance-sensitive arrangements specifies a payment of zero to the manager in case of a negative return during the period, at maximum three payments have to be specified. The contractually agreed performance-based payment upon a positive first-period return is denoted by f^H. Second-period performance-based payments are dependent on first-period returns and are denoted by f^{HH} and f^{LH}, with the first and second superscripts denoting the first-period and second-period returns, respectively (see Figure 3). To simplify the analysis, it is assumed that the investor pays any fees separately from the fund to the manager.[23]

Due to the inalienability of human capital, the manager cannot be forced to continue the fund after period 1. Thus, the manager can close the fund at that date. It is not possible to specify a fee that is contingent on the manager's decision to close the fund. [24] Alternatively to the manager, the investor is able to effectively close the fund at date 1 if he is allowed to withdraw her capital from the fund. While hedge funds typically allow investors to withdraw capital, many funds impose material restrictions on redemptions. For example, "lock-up provisions" specify the time period that an investor has to at least leave his capital in the fund for and "gates" limit the amount of funds that can

[22]In practice, management fees are frequently paid periodically as a fraction of the assets under management. One purpose of the management fee is to cover a fund's operational expenses. For example, the investors in the funds Citadel Kensington Global Strategies and Citadel Wellington bear all the funds' expenses directly in place of paying a management fee [see "Citadel Discusses Fees, Redemptions," *Wall Street Journal Online*, September 10th, 2010]. Findings by Deuskar et al. (2011), however, indicate that a fund's improvement in perceived quality tends to allow it to increase its management fee.

[23]As long as the fund's assets are sufficiently liquid, assuming that the cash to pay the fees are generated by liquidating the corresponding part of the fund's assets does not change the results.

[24]Such a fee creates an additional moral hazard problem in that investors may withdraw funds strategically to preempt fund closing by the manager.

be withdrawn within a certain time span at the investor and/or the fund level.[25] In the following, we analyze a situation in which the investor is not permitted to withdraw capital from the fund. In subection 2.6, we discuss if it can be optimal to allow the investor to close the fund by redeeming his capital.

2.3 The Fund Management Contract

The sequence of events is as follows (see Figure 4). At date 0 the contract is signed and investors provide financial capital. The fund manager invests this capital according to her identified strategy. At date 1 the first-period return is observed by all parties and the fee to the manager is paid as specified in the fund management contract. Then the manager learns about her expected outside income ω and decides whether to continue the fund or close it. If the fund is closed, all assets are liquidated at no cost and the proceeds are paid to the investor. If the fund remains alive, assets are used according to the investment strategy. At date 2, an alive fund's return is observed, its assets are costlessly liquidated and the proceeds distributed to the investor. The contractually agreed fee is paid to the manager. If the fund is closed at date 1, the manager receives her outside income ω at date 2.

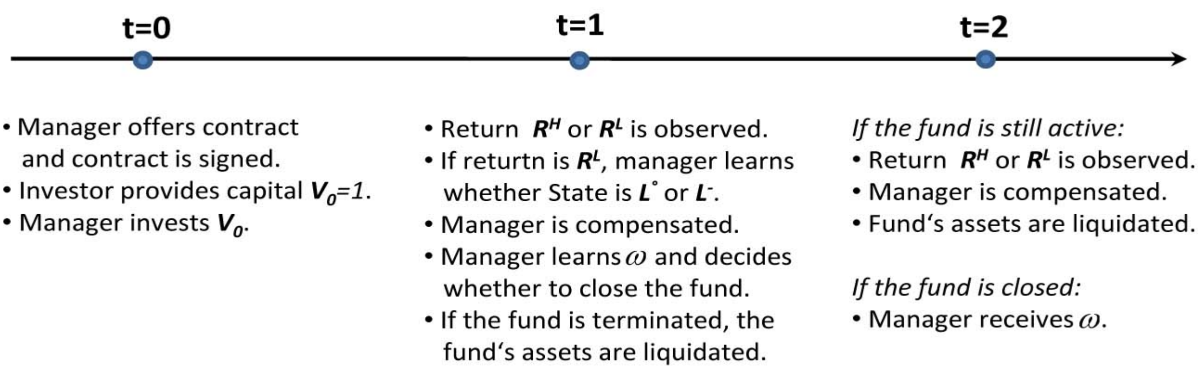

Figure 4: Sequence of events .

[25] Ang and Bollen (2009) compute that the cost of lockup provisions and withdrawal suspensions can be significant for investors. Lock-up provisions and gates vary significantly with the liquidity of investments. For example, in FrontPoint Partners' FrontPoint-SJC Direct Lending Fund investors' funds are locked up for five years. ["FrontPoint raises $1bn for new fund", Financial Times Online, January 7, 2011.]

Analysis

We first describe the fund manager's optimization problem independent of the specific structures of the performance fee discussed above. In this analysis, we represent the performance fee structure in the fund management contract by $\mathcal{A}$. Subsequently, we compare the outcomes when using a period performance fee and a performance fee with high water mark.

As the participation constraint of the investor can be satisfied by adjusting the fixed performance fee, k, the performance fee structure serves two potential conflicts of interest between investor and manager with respect to closing the fund. There is a potential incentive for the manager to continue the fund even though doing so is not in the interest of the investor, because she does not explicitly participate in losses the fund suffers. The only way the performance fee arrangement is able to control this incentive is by specifying relatively low expected future fees in the relevant states. The incentive for excessive continuation is present in state L^{-} and possibly in state L°, because negative first-period returns tend to be the consequence of a worsening of fund prospects. There is also a potential incentive to close the fund even though the investor would like to see it continued. This is, because the manager participates only with a certain fraction in the expected value gains of the fund. The contract can mitigate this incentive by offering relatively high expected fees in the relevant states. The incentive for excessive closing is present in state H and possibly in state L°.

To identify the optimal fund management contract, we first describe the manager's closing decision at date 1 and the investor's participation constraint as well as the manager's objective function in general.

The manager decides whether or not to close the fund at date 1 based on the realization of her outside income ω. She closes the fund whenever her outside income in period 2 equals or exceeds her expected second-period fee income from operating the fund. For any given fee structure and each of the three states at date 1, H, L° and L^{-}, there is a level of ω above which the manager closes the fund. Those cutoff levels depend on the performance fee arrangement are denoted by $\omega^{H}\left(\mathcal{A}\right)$, $\omega^{L^{\circ}}\left(\mathcal{A}\right)$ and $\omega^{L^{-}}\left(\mathcal{A}\right)$. Given our possible fee structures, it holds $\omega^{H}\left(\mathcal{A}\right) \geq \omega^{L^{\circ}}\left(\mathcal{A}\right) \geq \omega^{L^{-}}\left(\mathcal{A}\right)$. Then, the manager's closing decision can be characterized as follows: At date 1, the manager

$$\begin{array}{lll} \text{never closes} & \text{for} & \omega < \omega^{L^-} (\mathcal{A}) \\ \text{closes iff } prob(R^H) = p - \varepsilon & \text{for} & \omega \geq \omega^{L^-} (\mathcal{A}) \\ \text{closes iff first period return is } R^L \text{ and } prob(R^H) = p & \text{for} & \omega \geq \omega^{L^\circ} (\mathcal{A}) \\ \text{always closes} & \text{for} & \omega \geq \omega^{H} (\mathcal{A}). \end{array}$$

The investor's participation constraint depends on his anticipation of the manager's closing decision. The investor's participation constraint depends on the fund's performance and both the management fee, k, as well as the performance fee arrangement. The performance fee arrangement affects the investors payoff not only through fee payments to the manager but also via the manager's closing choice at date 1. We drop the descriptor $(\mathcal{A})$ for brevity. The investor's participation constraint is then given by

$$\begin{aligned} & V_0 \leq -k + \\ & +p\Big(- f^H + F(\omega^H)(p(V^{HH} - f^{HH}) + (1-p)V^{HL}) + (1 - F(\omega^H))V^H\Big) + \\ & +(1-p)\theta\Big(F(\omega^{L^\circ})\,(p(V^{LH} - f^{LH}) + (1-p)V^{LL}) + (1 - F(\omega^{L^\circ}))V^L\Big) + \\ & +(1-p)(1-\theta)\Big(F(\omega^{L^-})((p-\varepsilon)(V^{LH} - f^{LH}) + (1-p+\varepsilon)V^{LL}) + (1 - F(\omega^{L^-}))V^L\Big). \end{aligned} \tag{1}$$

While the first line of (1) contains the fixed management fee to be paid to the manager, lines 2 to 4 describe investor's payoffs in the three states weighted with the probabilities with which the states occur. Each payoff depends on the fee structure both directly and indirectly via the fee structure's impact on the manager's closing decision. In equilibrium, the manager will set the management fee, k, to its maximum value provided that the investor is willing to provide capital. Therefore, the investor just breaks even in equilibrium and (1) is fulfilled with equality.

Because the fund manager is able to appropriate the entire rent, she maximizes the expected surplus generated by the fund's investments. The surplus also takes into account the manager's income outside the fund. The expected surplus varies with the manager's closing decisions at date 1, represented by $\omega^H (\mathcal{A})$, $\omega^{L^\circ} (\mathcal{A})$ and $\omega^{L^-} (\mathcal{A})$. By assumption (A1), η denotes the fund's expected return if the success probability is p, the expected surplus, $S(\omega^H, \omega^{L^\circ}, \omega^{L^-})$, can be written as

$$\begin{aligned} & S(\omega^H, \omega^{L^\circ}, \omega^{L^-}) = -1 - \mathbb{E}(\omega) + \\ & +p\Big(F(\omega^H)(1+R^H)(\eta+1) + (1 - F(\omega^H))(1 + R^H + \mathbb{E}(\omega|\omega \geq \omega^H))\Big) + \\ & +(1-p)\theta\Big(F(\omega^{L^\circ})(1+R^L)(\eta+1) + (1 - F(\omega^{L^\circ}))(1 + R^L + \mathbb{E}(\omega|\omega \geq \omega^{L^\circ}))\Big) + \\ & +(1-p)(1-\theta)\Big(F(\omega^{L^-})(1+R^L)(\eta+1-\varepsilon(R^H - R^L)) + (1 - F(\omega^{L^-}))(1 + R^L + \mathbb{E}(\omega|\omega \geq \omega^{L^-}))\Big). \end{aligned} \tag{2}$$

Optimal Contracting without High Water Mark

First we consider the case in which the manager selects a period performance fee such that the performance-based fee in each period amounts to a constant additional fraction $a \geq 0$ of the gain in fund value during the period and nothing in case of a decrease in fund value. If the first-period fund return is positive, the manager receives a performance fee of $f^H = a(V^H - V_0) = aR^H$ at the end of the first period. If the second-period return is positive, the fee depends on the fund's first period return. In case of a positive first-period return, the manager's second-period fee is $f^{HH} = a(V^{HH} - V^H) = a(1 + R^H)R^H$ upon a positive second-period return. This implies that for the level on manager's expected outside income equal to $apR^H(1 + R^H) := \omega^H(a)$ the manager is indifferent between managing the fund in the second period and launching his expected outside income. In case of a negative first-period return, the second-period fee is $f^{LH} = a(V^{LH} - V^L) = a(1 + R^L)R^H$.[26] Thus, in state L° the manager closes the fund for ω larger than $apR^H(1 + R^L) := \omega^{L^\circ}(a)$ and in state L^- for ω above $a(p - \varepsilon)R^H(1 + R^L) =: \omega^{L^-}(a)$.

Optimal Contracting with High Water Mark

Consider now a fee structure that specifies a linear performance fee, $\widetilde{a} \geq 0$, with a high water mark provision. A high water mark specifies that a performance fee is based on the difference between the fund's value at the end of the period and the historic maximum of fund values provided that this difference is positive. Because the fund value at date 0, $V_0 = 1$ is (trivially) the historic maximum of fund values, the fee level $f^H = \widetilde{a}R^H$ is the same as under a period performance fee. The same applies to V^H and therefore $f^{HH} = \widetilde{a}(1 + R^H)R^H$. If the first-period return is negative, the historic maximum of fund values remains its initial value $V_0 = 1$. This implies that with a high water mark f^{LH} is structurally different from its period performance fee counterpart. It is given by $f^{LH} = \max\{0, \widetilde{a}(V^{LH} - 1)\}$, which we assume to be strictly positive. Thus, $f^{LH} = \widetilde{a}((1 + R^L)(1 + R^H) - 1) = \widetilde{a}(R^H(1 + R^L) + R^L) > 0$. For a given value of $\widetilde{a} > 0$, f^{LH} in case of a performance fee with high water mark is strictly smaller than that in the absence of a high water mark.

The corresponding closing thresholds for ω are defined as follows:

$$\omega^H(\widetilde{a}) := \widetilde{a}pR^H(1 + R^H),$$

$$\omega^{L^\circ}(\widetilde{a}) := \widetilde{a}p(R^L + R^H(1 + R^L)),$$

$$\omega^{L^-}(\widetilde{a}) := \widetilde{a}(p - \varepsilon)(R^L + R^H(1 + R^L)).$$

[26] Note that there is a convexity in the fee structure despite the seemingly linear contract.

Due to its smaller base, the fee percentage with high water mark $\widetilde{a}$ can be larger than its period performance counterpart a without inducing the manager to continue the fund in states L° and L^{-}. For a given percentage fee, the manager's optimal closing policy in state H is identical in both performance fee regimes, as the structure of the relevant fee, f^{HH} is not affected by a high water mark.

Comparing the manager's incentive constraints and the resulting expected total surplus levels of the manager between the contracts with and without a high water mark yields a central result:

Proposition 1 *The optimal contract with a high water mark provision yields at least as high a payoff to the fund manager as the optimal contract with a period performance fee.*

Proof: See Appendix A.1.

A contract with a high water mark (weakly) dominates a contract with a period performance fee. The formal argument for this is as follows: By selecting an appropriate fee level, a high water mark contract is able to generate an identical closing policy in the downward states L° and L^{-} as any given contract with period performance. The fee percentage of the high water mark contract is higher than that of its period performance counterpart. This typically reduces the manager's incentive to close upon a positive first-period return. Only if the optimal period performance contract implies the continuation of the fund with probability one in state H, is it possible that the two types of contracts yield the same payoff to the manager.

Uniform Distribution of Outside Income

To make the benefits of a performance fee with high water mark more transparent, we now assume that the fund manager's outside income is uniformly distributed, i.e. that the manager's outside income ω has cumulative distribution function $F(\omega) = \frac{\omega}{\omega_{max}}$ with density $f(\omega) = \frac{1}{\omega_{max}}$ on $[0, \omega_{max}]$. To allow for a relatively wide spectrum of outside income levels and to simplify the analysis, we also assume that $\omega_{max} = \left(1 + R^{H}\right)\eta$ (compare (A3)).

The following proposition presents the optimal fee choice dependent on the investor's participation constraint (1) and the manager's closing decision:

Proposition 2 *When the manager's outside income in period 2 is uniformly distributed on* $\left[0, \left(1+R^H\right)\eta\right]$*, the optimal contract with a high water mark provision is given by* $\left(\widetilde{a}^*, \widetilde{k}^*\right)$ *with*

$$\widetilde{a}^* = \frac{p^2R^H(1+R^H)^2\eta + (1-p)(1+R^L)(R^L+R^H(1+R^L))\left(p\theta\eta + (p-\varepsilon)(1-\theta)(\eta - \varepsilon(R^H - R^L))\right)}{p^3(R^H(1+R^H))^2 + (1-p)(R^L+R^H(1+R^L))^2\left(p^2\theta + (p-\varepsilon)^2(1-\theta)\right)}$$

and

$$\widetilde{k}^* = \eta - \widetilde{a}^* pR^H.$$

The optimal contract with a high water mark provision specifies a higher performance fee parameter and a lower management fee than the optimal contract with a period performance fee, (a^*, k^*)*.*

From an ex ante perspective, the optimal contract with a high water mark leads to a higher probability of closing in state L^-*, to a lower probability of closing in state* H*, and to a strictly larger payoff to the hedge fund manager than the optimal contract with a period performance fee.*

Proof: See Appendix A.2.

Compared to the optimal contract with a period performance fee, the optimal contract with a high water mark reduces excessive continuation by generating a lower expected fee from continuing the fund when closing is efficient. The fund manager is compensated for this reduction in expected fees by larger fee payment state H. Because in that state closing is not efficient, the contract with high water mark further improves efficiency buy curbing the manager's incentive to close the fund too often. The manager indeed strictly prefers a contract with high water mark to one without whenever it affects her closing decision. A high water mark arrangement is more efficiently able to utilize the superior

information of the manager about fund prospects. Thus, the model identifies a rationale for including high water mark provisions in hedge fund management contracts based on closing considerations.

For a numerical example, Figure 5 displays the expected surplus of a hedge fund under both performance fee structures as a function of the performance fee parameter. It illustrates that for any fixed level of the performance fee parameter, the expected surplus with high water mark is larger than with period performance fee. The figure also shows that

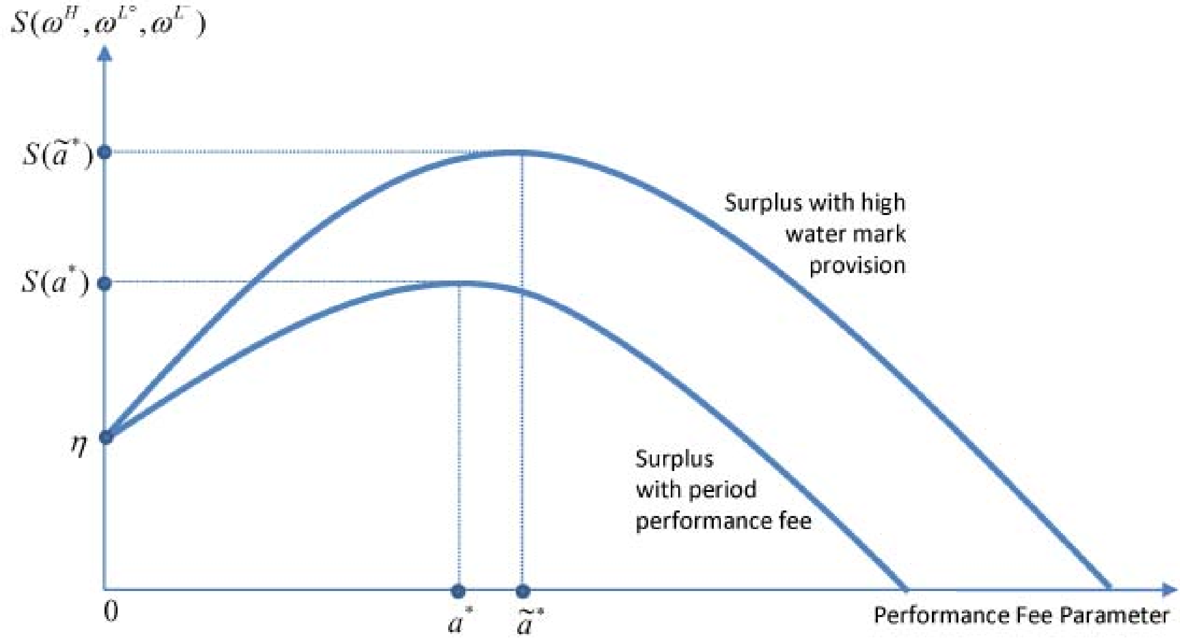

Figure 5: Expected fund surplus in different performance fee regimes.
The surplus-function with high water mark provision has an absolute maximum value $\tilde{a}^*$ and surplus-value $S(\tilde{a}^*)$, that are larger than the corresponding absolute maximum value a^* and surplus-value $S(a^*)$ of the surplus-function with period performance fee.

the optimal performance fee parameter is higher with high water mark than with period performance fee.

Adjusting the fixed management fee allows the manager to design the performance fee structure in a way to optimize incentives. Because the structure of the performance fee with high water mark is better suited to align incentives between the investor and the fund manager, compensation via a management fee is lower than when a period performance fee is used. [27]

The numerical examples in Table 1 show that the optimal contract with period performance fee includes a significantly lower performance fee parameter than the optimal contract with high water mark. The latter is better able to control both the incentive to excessively close in the upward state and the incentive to excessively continue upon a deterioration of the fund's prospects. The contract with high water mark generates, however, excessive closing in the downward state when fund prospects remain intact.

[27]Theoretical papers on the use of high water marks tend to ignore management fees. One notable exception is Lan, Wang and Yang (2011) who find that the management fee discourages risk taking by the fund manager.

	Parameter: ***R^H=7%, R^L=-6.5%, p=0.6, ε=0.3, ϴ=0.3*** *Pr.*(State *H*)=*0.6*, *Pr.*(State *L°*)=*0.12*, *Pr.*(State *L^-*)=*0.28*		Parameter: ***R^H=5%, R^L=-4%, p=0.6, ε=0.2, ϴ=0.2*** *Pr.*(State *H*)=*0.6*, *Pr.*(State *L°*)=*0.08*, *Pr.* (State *L^-*)=*0.32*	
	Period Performance Fee	Performance Fee with High Water Mark	Period Performance Fee	Performance Fee with High Water Mark
	a^* **=26.99%** k^* **=0.47%** $S(a^*)$**=1.92%**	$\tilde{a}^*$ **=38.06%** k^* **=0.001%** $S(\tilde{a}^*)$**=2.11%**	a^* **=36.58%** k^* **=0.3%** $S(a^*)$**=1.75%**	$\tilde{a}^*$ **=46.47%** k^* **=0.006%** $S(\tilde{a}^*)$**=1.84%**
	Closing Probabilities in States (Efficient Probabilities in parentheses):			
H	***29.14% (0%)***	***0.084% (0%)***	***21.62% (0%)***	***0.41% (0%)***
L°	***38.08% (12.62%)***	***99.39% (12.62%)***	***28.33% (8.57%)***	***84.83% (8.57%)***
L^-	***69.04% (100%)***	***99.79% (100%)***	***52.22% (100%)***	***89.93% (100%)***

Table 1: Numerical Examples.
Two different parameter combinations and the corresponding levels of performance fee, management fee, fund's expected surplus and closing probabilities in states *H*, *L°* and *L^-* in the contracts with period performance fee and performance fee with a high water mark provision, respectively.

This holds both relative to the efficient closing policy and relative to the policy generated by the contract with period performance fee. As in this state in the middle of the spectrum the difference in surplus from continuation compared to closing are relatively small, the misaligned incentives between investor and manager are less consequential than the benefits of a high water mark structure in the extreme states.

2.4 Implications

The model provides a number of implications on closing behavior, performance and contract design.

a) *Funds with high water marks tend to close more frequently upon negative performance.*

Corollary 1 *When the manager's outside income in period* 2 *is uniformly distributed on* $\left[0, \left(1+R^H\right)\eta\right]$*, the the closing probability upon a negative first-period return is higher for the optimal contract with a high water mark provision than for the optimal contract with a period performance fee.*

Proof: See Appendix A.3.1.

The use of high water mark provisions improves hedge funds' closing decisions. After periods of poor performance, high water marks reduce excessive fund continuation.[28] Empirical studies confirm the impact of high water marks on closure rates of funds. Liang and Park (2010) find that hedge funds with high water marks tend to close more quickly upon bad performance.[29]

b) *Funds with high water marks tend to outperform funds without high water marks after periods of poor performance.*

Corollary 2 *Suppose the manager's outside income in period* 2 *is uniformly distributed on* $\left[0, \left(1+R^H\right)\eta\right]$*. Then, conditional on fund continuation after a negative first-period return, the expected second-period after-fee return of a fund with a high water mark provision exceeds that of a fund with a period performance fee.*

Proof: See Appendix A.3.2.

Contracts with high water marks provide improved incentives for closing by specifying lower expected fees when the fund is under water. Thus, hedge funds with high water marks tend to have better after-fee performance when returns have (recently) been poor relative to otherwise comparable funds with period performance fees.

[28]This implication contrasts with the prediction in Aragon and Qian (2010) that high water marks reduce the probability of fund closing upon negative returns, because higher expected after-fee fund returns reduce investors' incentives to withdraw capital.

[29]The authors, however, don't explicitly test for the statistical difference between the two parameter estimates.

c) *In funds with high water mark contracts a higher expected performance tends to be associated with higher management fees.*

Corollary 3 *Suppose the manager's outside income in period 2 is uniformly distributed on $\left[0, \left(1+R^H\right)\eta\right]$ and the contract contains a high water mark provision. Then, an increase in the level of the positive return, R^H, leads to an increase in the management fee, $\widetilde{k}^*$.*

Proof: See Appendix A.3.3.

The level of the management fee is fund managers' instrument to extract the surplus the fund generates. As the magnitude of a positive return, R^H, increases, the management fee increases as well.[30] Deuskar et al. (2011) find that successful funds tend to increase their management fees suggesting that these increases of the management fees reflect higher return expectations.

2.5 Extension

Non-negativity of the management fee and probability of performance deterioration

In the model described above, the performance fee is set to align closing incentives of the fund manager with those of the investor whereas the adjustment of the management fee allows the fund manager to extract the expected surplus. This is possible, because we focus on the parameter space for which the optimal contract specifies a positive management fee. If, however, the requirement that the management fee be non-negative becomes binding, the specified performance fee affects the expected level of cash returns to the investor. Concretely, only a suboptimally low performance fee from an incentive point of view satisfies the investor's participation constraint. In the following, we examine this case maintaining the assumption of the uniform distribution of outside income on $\left[0, \left(1+R^H\right)\eta\right]$ and show that it is present when the likelihood of the deterioration of the fund's prospects is relatively small.

[30] A change in parameters not only affects the magnitude of the fund's expected return but also the optimal performance fee rate and therefore the expected fee income appropriated by the performance fee component alone. Thus, although other surplus-increasing parameter changes, such as increases in R^L or p tend to be associated with higher management fees, there are parameter combinations such that this is not the case.

Proposition 2 and Appendix A.2 reveal that the restriction on the management fee, $k \geq 0$, is binding if and only if the optimal performance fee rate derived in subsection 2.3 exceeds $\frac{\eta}{pR^H}$.[31] It turns out that the optimal performance fee parameter of a contract with period performance fee never exceeds that value (see Appendix A.4). Therefore, we focus in the following on the derivation of the high water mark performance fee parameter, $\widetilde{a}^*$. Due to the convexity of the optimization problem, the new optimal contract has the form $(\widetilde{a}^*, k = 0)$.

The new value of the period performance fee rate with a high water mark provision, $\widetilde{a}^*_{k=0}$, can be derived from the investor's participation constraint (1) and solves the following equation:[32]

$$
\begin{aligned}
&-(\widetilde{a}^*_{k=0})^2 \frac{(1-p)}{\omega_{max}} (R^L + R^H(1+R^L))^2 (p^2\theta + (p-\varepsilon)^2(1-\theta)) + \\
&+\widetilde{a}^*_{k=0} \frac{(1-p)}{\omega_{max}} (1+R^L)(R^L + R^H(1+R^L)) \Big(p\theta\eta + (p-\varepsilon)(1-\theta)(\eta - \varepsilon(R^H - R^L))\Big) + \\
&+(1+p(1+R^H))(\eta - \widetilde{a}^*_{k=0} pR^H) = 0.
\end{aligned} \tag{3}
$$

The performance fee rate $\widetilde{a}^*_{k=0}$ is smaller than the expected surplus maximizing performance fee rate $\widetilde{a}^*$ derived in Proposition 2.

To gain a better understanding of the possible states in which the performance fee with high water mark provision $\widetilde{a}^*$ increases beyond the threshold $\frac{\eta}{pR^H}$ and its consequences, we relate it to the model parameter θ, which characterizes the probability of a deterioration of the fund's prospects. Recall that in case of a negative first-period return the deterioration of prospects occurs with probability $1-\theta$.

Analyzing the new corresponding optimal contract and expected fund surplus leads to the following result:

Proposition 3 *Suppose the manager's outside income in period 2 is uniformly distributed on $\left[0, \left(1 + R^H\right)\eta\right]$. The surplus maximizing performance fee rate $\widetilde{a}^*$ is decreasing in θ.*

There exists a critical value for θ, $\theta^\circ \in (0, 1)$, below which the management fee restriction, $k \geq 0$, is binding.

[31] Note that if a performance fee parameter larger than $\frac{\eta}{pR^H}$ is chosen, the probability of fund continuation in state H is equal to one. Thus, a further increase in the performance fee parameter does not affect the closing probability in that state.

[32] The lengthy closed form solution of (3) is presented in Appendix A.4.

There is a critical value of θ, $\widehat{\theta} \in (\theta^{\circ}, 1)$, *below which the optimal contract with period performance fee leads to a strictly higher expected surplus than the optimal contract with a high water mark provision.*

Proof: See Appendix A4.

For a given performance fee rate, a high water mark provision specifies a larger difference in fee income for the manager across states. While this is always beneficial from an incentive standpoint, the additional restriction of a non-negative management fee may significantly impair the manager's desirability of a high water mark provision but not that of a period performance fee. There exist circumstances in which a contract with period performance fee leads to a larger expected surplus and is therefore preferred to a contract with a high water mark. Hence, the choice of a period performance fee may not only be determined by the absence of signaling considerations as shown in Aragon and Qian (2010); it may also be a consequence of the more severe effects of limited liability restrictions on contracts with high water marks.

	Period Performance Fee	Performance Fee with High Water Mark
θ =0.1	a^* =29.62% k^* =0.27% $S(a^*)$ =0.92%	$\widetilde{a}^*$ =42.70% $\widetilde{k}^*$ =0.031% $S(\widetilde{a}^*)$ =1.03%
θ =0.3	a^* =33.92% k^* =0.19% $S(a^*)$ =0.98%	$\widetilde{a}^*$ =43.82% $\widetilde{k}^*$ =0.011% $S(\widetilde{a}^*)$ =1.04%
θ =0.5	a^* =37.52% k^* =0.12% $S(a^*)$=1.03%	$\widetilde{a}^*$ =44.63% $\widetilde{k}^*$ =0.00% $S(\widetilde{a}^*)$ =1.05%
θ =0.7	a^* =40.60% k^* =0.07% $S(a^*)$ =1.10%	$\widetilde{a}^*$ =45.01% $\widetilde{k}^*$ =0.00% $S(\widetilde{a}^*)$ =1.07%
θ =0.9	a^* =43.25% k^* =0.02% $S(a^*)$=1.16%	$\widetilde{a}^*$ =45.51% $\widetilde{k}^*$ =0.00% $S(\widetilde{a}^*)$ =1.08%

Table 2: Numerical Examples. (Parameters: R^H=3%, R^L=-2.5%, p=0.6, ε=0. 4)
With increasing θ, performance fee parameters a^*, $\widetilde{a}^*$ and expected surplus $S(a^*)$, $S(\widetilde{a}^*)$ increase; at the same time the management fees k^*, $\widetilde{k}^*$ decrease. For $\theta > \hat{\theta}$ the expected surplus in the optimal contract with period performance fee $S(a^*)$ is larger than its counter-part $S(\widetilde{a}^*)$.

Contracts with period performance fees are preferred only if the probabilities of the deterioration of funds' prospects are relatively low. It appears reasonable to assume that investors assign significant probabilities of downward adjustments of fund prospects to small funds or those run by managers that lack extensive track records. Thus, the derived result is consistent with findings by Aragon and Qian (2010) that those types of funds more commonly employ high water mark provisions.

2.6 Robustness of the Results

Intermittent Redemption by the Investor

So far, we have abstracted from allowing the investor to withdraw funds after period 1. Given the linear investment technology, the investor either wants to redeem all or none of his funds. Thus, allowing for the intermittent redemption of funds, the investor has the opportunity to effectively close the fund. In the following, we discuss some of the main aspects of including the investor's option to redeem funds intermittently in the fund management contract. We do this maintaining the assumption of a uniform distribution of outside income as given in subsection 2.3.

First, note that it is never superior to allow the investor to redeem capital intermittently when the fee structure is designed in a way that the option is never exercised. Doing so only introduces additional restrictions.

If there are circumstances in which the investor closes the fund, he does so only upon a negative first-period return and under inferior information than the manager. His information is inferior in two ways: the investor cannot distinguish between states L° and L^{-}, and also is not informed about the realization of ω.

The investor leaves his capital in the fund if doing so increases his expected cash flows. In case of negative first-period return, fund withdrawal yields the investor a cash flow of the date-1 value of the fund, which is given by the fund's gross value, V^L.

Consider the case that the investor withdraws his capital from the fund with certainty after a negative first-period return. Note that in this case only the fees f^H and f^{HH} are paid with positive probability. Because these fee payments are independent of whether the contract specifies a period performance fee or contains a high water mark provision, no discrimination between these two performance fee structures is necessary. The investor's break even constraint in this case is given by:

$$
\begin{aligned}
&V_0 \leq -k + \\
&+p\Big(-f^H + F(\omega^H)(p(V^{HH} - f^{HH}) + (1-p)V^{HL}) + (1-F(\omega^H))V^H\Big) + \\
&+(1-p)V^L.
\end{aligned}
$$

Based on this constraint, the maximal level of performance fee parameter a that the investor is willing to accept is obtained if k is set to zero and is equal to $\frac{\eta}{pR^H}$.[33]

The manager anticipates intermittent redemption and consequently the fund's closing by the investor after a negative first-period return. Then, fund's expected surplus is:

$$
\begin{aligned}
&S^{rd}(\omega^H) = -1 - \mathbb{E}(\omega) + \\
&\quad +p\Big(F(\omega^H)(1+R^H)(\eta+1) + (1-F(\omega^H))(1+R^H+\mathbb{E}(\omega|\omega \geq \omega^H))\Big) + \\
&\quad +(1-p)V^L \\
\Leftrightarrow\ &S^{rd}(\omega^H) = \eta - \mathbb{E}(\omega) + p\Big(F(\omega^H)(1+R^H)\eta + (1-F(\omega^H))\mathbb{E}(\omega|\omega \geq \omega^H)\Big).
\end{aligned}
$$

The manager maximizes the expected fund surplus with respect to her optimal closing policy in state H, which is a function of the performance parameter a. Given assumption (A3), the expected surplus in state H is at least as high as the maximal level of her outside opportunity ω_{max}. Thus, the manager chooses the maximum possible continuation probability $F(\omega^H)$ equal to 1. Then, the optimal performance fee parameter is equal to $a^* = \frac{\eta}{pR^H}$.

Now we are in a position to compare the expected surplus with intermittent redemption by the investor to the one generated by the contract derived in Proposition 2.

Proposition 4 *When the manager's outside income in period* 2 *is uniformly distributed on* $\left[0, \left(1 + R^H\right) \eta\right]$*, the optimal contract with intermittent redemption by the investor leads to a strictly lower expected surplus than the optimal contract without intermittent redemption.*

Proof: See Appendix A5.

[33] For the proof see Appendix A.5.

Given that the manager has private information, granting intermittent redemption rights to the investor is not optimal. Thus, the model implies that intermittent redemption rights are typically used for reasons other than increasing the efficiency of the fund's closing policy. They may, for example, be included because of liquidity needs by hedge fund investors.

Capital Contribution by the Manager

So far, it has been assumed that the manager does not invest own financial wealth in the fund. Actually, hedge fund managers typically do invest their own capital in the fund.[34] The following arguments introduce the case in which the manager possesses financial wealth of $A > 0$ and contributes it to the fund. The initial investment amount that can be invested profitably is $V_0 = A + Y \equiv 1$, where A is the part indicates the manager's contribution and $Y = 1 - A$ investor's, respectively.

If the invested amount yields a positive return $R^H > 0$, the fund's value increases after the first period to V^H. We can distinguish between the manager's $A(1 + R^H)$ and the investor's $(1 - A)(1 + R^H)$ shares, respectively. Analogous is the wealth development after a negative first-period return $R^L < 0$ with decreasing value V^L and the manager's $A(1 + R^L)$ and the investor's $(1 - A)(1 + R^L)$ shares, respectively. This allocation of the share proportions between both parties is also kept constant in the second period. As described in the basic model the investor still has the same participation constraint (1). The modification is that he is now interested in changes in the portion of $(1 - A)V_0$.

The manager's decision whether to operate the fund after the first period or to close it depends on the realization of her outside income ω. In the new model setting the cutoff levels of ω depend not only on the period performance fee arrangement but also on the fund's fraction that was generated by the manager's investment A. Taking into account the adjusted values of $\omega^H, \omega^\circ, \omega^{L^-}$, the manager's closing decision and the expected surplus $S(\omega^H, \omega^\circ, \omega^{L^-})$ remain unaffected, as described (2) in the basic model.

Recall that the optimal contract in case $A = 0$, that was described in proposition 2, is given by $(\widetilde{a}^*, \widetilde{k}^*)$. After comparing the expected total fund surplus levels, with respect to the manager's incentive constraint between the contracts with the period performance

[34] Agarwal et al. (2009), report that "Discussions with industry practitioners suggest that often the manager reinvests all of the incentive fees earned back into the fund." Thus, they calculate the manager's coinvestment as the cumulative value of the incentive fee reinvested together with the returns earned on it.

fee and the period performance fee with a high water mark provision, we can state the following result:

Proposition 5 *When the manager contributes financial wealth $A > 0$ to the fund investment and her outside income in period 2 is uniformly distributed on $\left[0, \left(1+R^H\right)\eta\right]$ she chooses the optimal contract with the high water mark provision $(\widetilde{a}^*, \widetilde{k}_A^*)$. The optimal performance fee rate $\widetilde{a}^*$ does not depend on A. The optimal management fee $\widetilde{k}_A^*$ and the expected total fund surplus $S(\omega^H, \omega^{L^\circ}, \omega^{L^-})$ increase with increasing A.*

From an ex ante perspective, the increase in financial wealth $A > 0$ leads to a lower probability of closing in state H, to a lower closing probability in state L° and to higher closing probability in state L^-.

Proof: See Appendix A6.

Thus the manager investing her own capital in the fund brings about further alignment of interests. It is obvious to see that the higher the financial contribution the higher the expected loss in the case of low return realization. In order to prevent expected loss the manager's incentive to close the fund as efficiently as possible in each of the states increases with the amount of her investment in the fund. Findings by Agarwal et al. (2009), show that higher levels of managerial ownership, in the funds which use incentive contracts with inclusion of high-water mark provision, are associated with superior performance. The numerical examples in Table 3 show the changes of closing probabilities in different states as the managers capital contribution increases.

Because the optimal performance fee rate $\widetilde{a}^*$ is independent of whether the manager contributes financial wealth or not, the main results of Proposition 2 do not change.

2.7 Conclusion

In this section we studied the choice between two different types of performance fee structures in hedge fund management contracts: fees based solely on the performance during the preceding period and fees based on the performance relative to the historical fund value maximum. It provides a rationale for the inclusion of the latter, so-called high water mark provisions, based on the argument that such structures facilitate efficient fund closing. Significant levels of expected fees in states that potentially warrant fund closing provide incentives for fund managers to continue the fund even when doing so is inefficient. Management contracts with high water mark provisions specify lower expected fees after

	Closing Probabilities in States (Efficient Probabilities in parentheses):		
	H	***L°***	***L⁻***
A=0	***0.414% (0%)***	***84.825% (8.57%)***	***89.93% (100%)***
A=1%	***0.410%***	***84.062%***	***90.246%***
A=10%	***0.373%***	***77.200%***	***93.507%***
A=20%	***0.331%***	***69.574%***	***97.131%***

Table 3: Numerical Examples.(Parameters: R^H**=5%,** R^L**=-4%, p=0.6, ε=0.2, Θ=0.2**). The table schows change in the fund's continuation probabilities with increasing level on managerial capital contri-bution *A* if the manager's optimal contract includes management fee and period performance fee with a high water mark provision .

periods of negative performance when fund closing may be warranted. In equilibrium, managers receive higher fee rates and thus higher compensation in case of a continuously positive value development of the fund.

Our approach implies that funds with high water marks tend to close more quickly upon periods of poor performance than their period performance fee counterparts. If, however, such funds with high water mark arrangements decide to continue, their performance levels on an after-fee basis are expected to be superior to comparable funds employing period performance fees. The model is also consistent with empirical evidence that high water marks are more common in smaller funds and funds run by managers without extensive track records.

3 Hedge Fund Database Biases

For investors in a hedge fund or fund of hedge funds it can be very hard or even impossible to obtain a “true” or even “fair” information about the hedge fund’s performance. There are not many ways of collecting information about levels of the hedge fund performance as an outsider (and even as an accepted investor in a hedge fund). One possibility is to use the hedge fund indices, that are constructed by using different available data and heterogen selection standards. The reason for this inhomogeneity is the absence of a fully representative hedge fund data base, that could cover the data of all active hedge funds. The same problem is faced by researchers in empirical research. For empirical research on hedge funds the availability of high-quality data is the determining factor. The empirical literature on the hedge fund reported returns shows a large variety of different results due to different data samples.

Due to lack of the regulatory oversight hedge funds are not required to report their returns or any other information, such as size of managed assets, the investment strategy, pursued by the manager, or the portfolio composition. A unique, comprehensive and publicly accessible database containing the track records of all, especially active hedge funds simply does not exist. The publicly available hedge fund data contain voluntarily disclosured monthly investment performance of some hedge funds. Many empirical studies use as a main data source for their analysis the largest hedge fund data providers TASS, CTA, HFR or CISDM. The first problem the researcher has to deal with is the need of combining the data from various data sources in order to collect sufficient data sample. Through the wide usage of these databases by researchers and practitionals the hedge funds’ can exploit this status to convey some specific messages or signals to the audience. Thus, the empirical estimates of hedge funds performance are sometimes overstated and come from biased data sources. Brown, Goetzmann, Ibbotson, and Ross(1992), Fung and Hsieh (2000), Liang (2000), Jorion and Schwarz (2010) and Edwards and Caglayan (2001) cover these well-known data biases extensively in the hedge fund literature.

Survivorship bias

The most common and easily fixable data bias in a hedge fund study is the survivorship bias. Survivorship bias occurs when the database does not include the returns of hedge funds that have stopped reporting their performance during the observable period. The academic literature estimates that survivorship bias increases returns from 0.16% to 6.67%, p.a. depending on the observation period, but there exists wide disagreement in explaning why hedge funds stop reporting their returns to the data gathering services.

On one hand, many academic studies argue that hedge funds which perform poorly during the observed periods relative to the other funds stop reporting in order to hide the actual losses on their investments and to avoid harming their reputation (see, for example, Malkiel and Saha, 2005) or capital outflows. Thus, when analysing track records of hedge funds it can happen that the sample of current funds will include only those that have been successful in the past, while some hedge funds were closed because of a poor performance. In this case, survivorship bias causes reported hedge fund performance in the database to appear higher (and in the most cases better) than the true actual average hedge fund performance. On the other hand, hedge funds may stop reporting their performance because they have already collected sufficient capital contribution for their investment strategy and therefore are not interested in attracting more investors.

Instant History or Backfill Bias

Another possible problem associated with the hedge fund data is named instant history bias. Instant history or backfill returns occur when a hedge fund is added to a database but has been operating for many periods of time before making first report to this database. The academic literature has produced several estimates of the instant history bias on performance, which range between 0.05% and 4.35% p.a. Many hedge fund strategies, for example, may be running and generating returns for a while before they are offered to potential investors, to see whether the strategy is successful. Then, the manager can decide to report the data to a commercial database. In this case, the database includes the historical data and the past hedge fund's performance from when the hedge fund was not part of the database. Hedge funds that are unable to generate high performance and good track records with their strategy are unwilling to disclose their history to the potential investors. The backfill bias occurs because often only the managers with good hedge fund past performance are the ones who want to be included in a database.

Self-Selection Bias

Self-selection bias occurs if only funds with good performance report their performance to a database. This effect can create an upward bias, which can be limited due to the fact, that hedge funds with continuously good performance stop sometimes their reportings as they have reached an optimal size of assets under management and, thus, do not need to attract more investors.

According to Aiken, Clifford and Ellis (2010) there exist quite a number of other biases that could affect databases. As they point out: "Funds have some discretion as to the timing of their reports to the databases. ...In most cases, the fund has up to 3 months to file its monthly return. ... A fund with poor performance in a given month may have the incentive to delay reporting, increase the funds risk, and hope for a better outcome in the next month. If the strategy works, both monthly returns are listed. If it does not work, the fund never reports either return to the database. A similar version of this bias occurs when a previously delisted fund is allowed to rejoin the live funds file. If a fund is willing to fill all gaps in its time-series of returns, it is allowed to rejoin the database. As it is likely that only funds that performed well during their delisting period will re-list, these features of the commercial data will impart a further upward bias on the return data."

The academic literature proposes several suggestions for overcoming problems associated with biases in hedge fund indices. For instance, using fund of hedge fund indices to estimate the performance of the hedge fund market leads to the results that are less likely to be affected by issues such as survivorship bias or backfilling bias (see Fung and Hsieh (2000)). The track records of funds of hedge funds seems to be almost free of the many biases contained in databases of individual funds. Another idea suggests to use the database that contains information on when hedge funds actually joined the database (for example Hedge Fund Research (HFR) database contains this information). Deleting all the the backfill observations in the selected data set can help to reduce or even to eliminate the backfill bias. Ben-David, Franzoni, Landier and Moussawi (2011) combine in their empirical study a list of hedge funds (by Thomson-Reuters), mandatory institutional quarterly portfolio holdings reports (13F), and information about hedge fund characteristics and performance (TASS) in the conviction that the 13F filings are not affected by the self-selection and survivorship bias. Agarwal, Daniel and Naik (2011) exclude the first two years' data for each fund from their empirical analysis to tackle backfilling bias.

Nevertheless, the above described problems caused by database biases should be considered while trying to measure the hedge fund's performance. Fung and Hsieh (2004) mean that "existing hedge fund indices, while helpful in providing investors with an idea on the current progress of the industry on average, offer little clues to ... questions [of asset allocation and performance measurement]."

In the following chapter we argue that additionally to the problems associated with the biased hedge fund data the investors in a hedge fund can additionally make a mistake while trying to measure and evaluate the hedge fund's performance using available data. Namely, by disregarding the hedge fund managers contractual provisions. We develop a general theoretical framework to describe the managerial incentives that can be caused by two different contracts: that include performance fee with the high water mark provision or period peformance fee. We can show that the hedge fund voluntary reported performance is strictly dependent on the specification of the contractual agreement between the fund manager and the fund investors. In this context, we are paying close attention to the role of high water mark provisions in the hedge fund management compensation contract.

4 Performance Smoothing of Hedge Funds

This section[35] shows that performance fees with high water marks cause hedge fund managers to smooth performance. High water marks provide both the incentive to underreport returns in good times as well as to overreport returns during difficult periods. In good times, reporting a high return increases the high water mark and diminishes fees from future fund flows. During difficult periods, when the fund is under water, overreporting has little consequences on future fees income, because doing so does not effect the basis for future fees.

4.1 Introduction

Hedge fund returns appear to be smoother than the returns of mutual funds or common stocks. Compared, for example, with the S&P 500's monthly returns in the period from January 2009 to November 2011 (compare Figure 6), monthly hedge fund returns are much less volatile. For quite some time, both investors and researchers have been suspecting that at least part of this return smoothness is due to the reporting practices of many hedge funds. For example, according to De Souza and Gokcan (2004): "...there is a high degree of serial correlation in most hedge fund strategy monthly returns, which causes excess smoothness in their return series. This excess smoothness typically leads investors to understate both the true volatility of these strategies and their correlation with traditional asset classes and will significantly overstate the true Sharpe ratios." One way hedge fund managers indeed appear to be able to massage their results is by using their discretion in valuing illiquid assets. Many hedge fund strategies include investments in illiquid assets in their portfolios and in the most of the cases it is not easy to determine the net value of the assets under management. For less liquid or tradable assets there are frequently no market prices available. The manager has the opportunity to linearly extrapolate the approximate price between two observable prices or to use smoothed broker (dealer) quotes. Empirically, Green (2010) documents "patterns in hedge fund returns that suggest that reporting manipulation is significant and pervasive for hedge funds with discretion in valuing their portfolios of illiquid assets." One other way of affecting the reported value of assets under management is by deliberately reporting false values of assets in their portfolios as suggested by the results of Cici, Kempf and Puetz (2011). This may be facilitated by a lack of regulatory oversight compared to other investment management

[35]The research presented in this section was performed in cooperation with Martin E. Ruckes; see Ruckes and Sevostiyanova (2012b)

vehicles such as mutual funds.

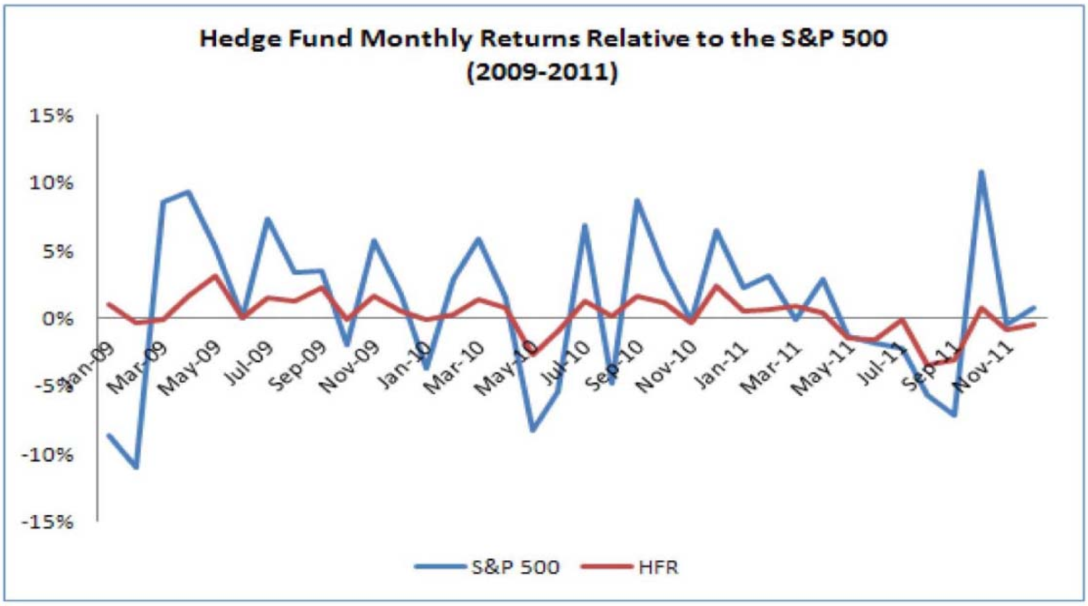

Figure 6: Hedge fund monthly returns relative to the S&P 500 in the period 2009-2011. Source: Lawbitrage, Law & Finance by Houman Shadab (January 25, 2012).

The smoothing of earnings is a pervasive phenomenon in many economies not only among financial but also non-financial firms. One goal of earnings smoothing is to reduce income stream fluctuations in order to conceal relevant information that can be used to measure corporate performance, to assess managerial ability or to predict the future earnings. For example, De Fond and Park's (1996) empirical analysis supports the notion that issues of job security provide powerful incentives for managers to smoothen reported income levels. Managers appear to be "borrowing" earnings from the future for current use, if actual earnings are poor but expected earnings are high. Conversely, managers "save" current earnings for future use, if the current performance is high but expected future performance is poor. Fudenberg and Tirole (1995) show that even managers whose firms have substantial earnings today and expect high earnings in the future may have an incentive to smooth earnings. This is the case when future earnings are still uncertain and the negative consequences of reporting low future earnings is significant for the manager. We build on Fudenberg and Tirole's (1995) insight and show that standard features in hedge fund management contracts can have a profound impact on income smoothing, because they affect the manager's benefits of reporting income streams with low volatilities. Specifically, we compare contracts that pay a performance fee to the fund manager whenever the reported value of the fund's assets increases relative to that at the beginning of a period (period performance fee) versus a performance fee that is paid only if the reported asset value of the fund exceeds its historic maximum (performance fee with high water mark).

When reporting the value of their funds' assets, hedge fund managers that maximize expected fee payments take into account both the performance fee generated by the report and the expected fund flow resulting from the report. The expected fund flow affects future fees from managing the fund. When a negative return in the future leads to significant outflows of capital, a manager with a positive current return may report a lower one in order to avoid having to report a negative return in the future. This is the case even when reporting a positive return today increases the managers perceived ability to investors provided that ability depreciates sufficiently quickly.

While the structure of the performance fee of a fund is inconsequential for fund flows themselves it affects managers financial consequences from these flows. Consider, for example, a fund with a performance fee with high water mark where the manager observes a positive return that, if reported truthfully, raises the high water mark for future periods. In case of future inflows the manager does not financially benefit from these unless the fund's asset value exceeds the new high water mark. This is different if the manager decides to report a lower than the actual return. Then she benefits more from future inflows, because of a lower high water mark. This is different for a manager who operates an otherwise identical fund with a period performance fee. Given that the historically maximal fund value is irrelevant for fund fees, the described consideration does not affect the manager's reporting decision. As a consequence, managers of funds with performance fees with high water marks have stronger incentives to underreport positive actual returns than managers of funds whose management contracts specify period performance fees. Similarly, managers with high water marks have strong incentives to overreport negative returns when doing so prohibits outflows. The reason is that reporting, say, a zero return rather than the actual negative one does not affect the high water mark and therefore is inconsequential for future fees. Again, this is different for managers whose fee income is determined by the performance in each period. Reporting a zero return rather that the actual negative one affects future returns negatively as the coming period's return is based on this period's reported asset value. In sum, fund managers whose performance is measured against their funds' high water marks have stronger incentives to report muted returns than those of funds specifying period performance fees. Our result is therefore consistent with empirical evidence in Green (2010), who finds more significant smoothing by funds whose management contracts contain high water mark provisions. One implication of this finding is that researchers are ill-advised to compare the performance of funds with high water mark fee structures to those with period performance fees based on

Sharpe ratios. For example, Aggarwal, Daniel and Naik (2009) find significantly higher Sharpe ratios of funds with high water marks. At least part of this difference may be due to a higher level of strategically motivated return smoothing by managers of funds with high water marks. Another implication of our model is that learning about the manager's quality and a fund's future prospects occurs more rapidly in funds that do not include high water marks in their management contracts.

Related Literature

Theory.

As mentioned above, our model uses the general framework of Fudenberg and Tirole (1995) as a basis. An important foundation in their, and our, model is that managers cannot permanently misreport their returns. If they provide a biased performance report today, future reports will have to be biased in the opposite direction. The authors study a manger who is concerned with job security and ignore any aspects of explicit performance-based contracting. Our model incorporates explicit performance-sensitive contracts that are commonly used in the hedge fund industry. We argue that option-like compensation contracts such as those that include high water marks are likely to induce their manager's incentives for smoothing their returns. In addition, our approach incorporates the effects of fund flows on managers' decisions to smoothen income.[36]

Acharia and Lambrecht (2012) present an alternative approach to income smoothing. They argue that managers fear a ratchet effect when reporting high earnings: investors increase their expectations of future earnings if the firm presents high current earnings. In order to avoid disappointments, managers are hesitant to report high earnings levels truthfully. This setting assumes relatively limited levels of information by investors and appears to be better suited for firms with individual investors rather than sophisticated institutional investors as is the case for hedge funds.

Jylhä (2011) studies the hedge fund manager's motives to misreport the funds returns. The author can show three new extensions for the empirical literature on hedge fund return misreporting, namely: misreporting is more prevalent in funds with capital flows that are strongly dependent on past performance, it is more prevalent in young funds, and more prevalent in times of capital outflows. The results presented by Jylhä (2011) suggest also the idea, that the hedge fund manager tries to represent the fund to appear as more attractive to the investors than it is in reality by misreporting her returns. On the contrary to our approach Jylhä (2011) assumes in his model the true hedge fund returns to be randomly drawn from a normal distribution and completely independent of any action taken by the manager. Thus, the fund manager in this approach can only misreport the true returns to the outsiders. The true fund returns in our model are assumed to be dependent on manager's skills and the quality of her investment strategy. The main difference in our model compared to Jylhä (2011) is that we describe smoothing of the

[36]For the hedge fund manager, losing her job is equivalent to the withdrawal of the entire capital by investors. Insofar, our model derives the manager's cost of losing her job endogenously as the expected value of forgone fees.

reported returns to be costly for the manager. Henceforth, the manager has to bear the costs of smoothing in the subsequent period. Another important difference is that the hedge fund manager in the model of Jylhä (2011) is compensated only via management fee, which sets completely different incentives compared to the performance fee chosen in our model.

To our knowledge, Dutta and Fan (2012) is the only paper that also incorporates compensation contracting into their approach to income smoothing. The authors derive an optimal contract when earnings manipulation is possible. Given that their paper is not specifically designed to study the behavior of hedge fund managers, they do not look at typically used contract clauses and changes in firm size generated by inflows and outflows of capital.

Empirics.

Many studies document a positive serial correlation in the self-reported hedge fund returns as a result of deliberate misrreporting. Bollen and Pool (2008) argue in their empirical analysis that the structure of hedge fund incentive contracts and the competitive nature of the industry gives hedge fund managers stronger incentives to overreport losses than underreport gains. Bollen and Pool (2008) call this behavior "conditional smoothing". The authors construct and test a statistical model to screen fraudulent smoothing in the hedge fund returns and can show that a hedge fund manager tend smooth losses more than gains, which results in higher serial correlation when funds perform poorly. Bollen and Pool (2008) make it possible with their model to detect deliberate cheating in hedge fund returns. The authors argue that for some funds the most likely reason for return smoothing is simply fraud, by which the hedge fund managers misreport the true returns in order to reduce the fund's return volatility and thus to achieve higher Sharpe ratios. Also, in their second empirical analysis, Bollen and Pool (2009) find a discontinuity in the distributions of monthly hedge fund returns, pooled across funds and over time, due to prevalence of misreporting or more precisely, due to temporarily overstated returns.

In order to make their hedge funds more attractive for existing or potential investors or in order to prevent capital outflows the incentive for manipulating returns can be enormous for the manager. Ben-David, Franzoni, Landier and Moussawi (2011) present evidence that hedge funds engage in manipulation of stock prices for reporting purposes. The authors show that these manipulations are more attractive for funds that are better diversified, funds that have experienced poor performance in the last period or funds with a very high recent performance with the goal to attract investors' attention.

Getmansky, Lo, and Makarov (2004) document also a very high serial correlation of reported hedge fund returns. The authors offer many possible explanations for the presence of this serial correlation. They suggests the serial correlation may be a proxy of illiquidity in hedge fund investments. Hedge funds often use different share restrictions in their contracts such as lock-up periods or redemption periods for fund withdrawal. Such share restrictions avoid short-term capital outflows and make it possible for managers to invest in illiquid assets more easily. In order to evaluate the illiquid assets in their portfolios hedge fund managers can for example obtain value estimates from brokers who simply extrapolate past market values. Getmansky, Lo, and Makarov (2004) suggest as a second possible explanation for the presence of high serial correlation the purposeful managerial smoothing of contemporaneous and lagged returns in hedge funds. Lack of transparency and regulation and also the special fee structure allow the hedge fund managers to misreport their returns in order to charge higher fees.

Agarwal, Daniel, and Naik (2011) find a positive relation between contractual provisions (high water marks and long restriction periods on investor redemptions) and performance. They can show that returns are significantly higher in December compared to the rest of the year. This findings suggest that the hedge fund manager revise their returns upwards in order to earn higher fees.

Green (2010) analyzes the discontinuity in the distribution of the hedge fund reported returns. He uses two proxies for the variation in managers' incentives to manipulate reported returns: whether a fund has a high water mark provision and whether excess market returns are negative. Green (2010) is able to show that high water marks and restriction periods are positively associated with fund returns for quantiles of the return distribution with losses and small-to-moderate gains. In contrast to Agarwal, Daniel, and Naik (2011), the results from Green (2010) support the idea that contractual provisions are negatively associated with returns for quantiles with moderate-to-large gains. The most interesting finding suggests that hedge funds with high water mark provisions and funds in their first two years of reporting to a public database show the greatest discontinuity in the distribution of reported returns.

This section proceeds as follows. In the next subsection we introduce the economic environment of the model and set up the optimal reporting problem. In subsection 4.3 we provide an theoretical discussion of the structure of the compensation contract, which can be choosen between the contract with the period peformance fee and the contract with the high water mark provision. Subsections 4.4 and 4.5 contain our analysis of the optimal reporting choice and derivation of the equilibria, if the contract cpecifies the period performance fee and if the contract specifies performance fee with the high water mark provision, respectivelly. Subsection 4.6 compares findings of the both previous sections. In subssection 4.7 we present an extended model version, and a final subsection of this chapter contains concluding remarks.

4.2 The Basic Model

We consider a risk-neutral investment manager who has an idea for an investment strategy with a time horizon of three periods. The manager does not have financial wealth of her own, and must therefore borrow an initial amount $V_0 = 1$ from outside investors to implement her investment strategy. There exists a pool of outside investors with sufficient wealth to invest. Outside investors are risk-neutral.[37] The interest rate in the economy is normalized to zero.

Characteristics of the Investment Strategy

In the model there are four points of time $t = i$, $i \in \{0, 1, 2, 3\}$. At time $t = 0$ the manager and the investors are symmetrically informed about the fund's prospects and investors decide whether to make an initial investment of $V_0 = 1$. If investment took place, implementing the manager's investment strategy implies that per period the invested amount generates either a positive (gross) return $u > 1$ or a negative (gross) return $d = 1/u < 1$ (see Figure 7). If the first-period return is u, we will call the manager being of "u-type". If the first-period return is d, we will call the manager being of "d-type". We assume in our model that at each point in time only the latest performance of the fund is informative for the fund's prospects in the following period. More precisely,

- if the fund's current performance is positive, its return is equal to $u - 1 > 0$, the manager's probability for a positive return in the following period is equal to $p \in (0, 1)$,
- if the fund's current performance is negative, its return is equal to $d - 1 < 0$, the manager's probability for a positive return in the following period is equal to $p\varepsilon$, with $\varepsilon \in (\frac{1-p}{p}, 1)$. [38]

[37]We will exclude the possibility of income smoothing that may result from investors' risk aversion, because risk avers investors prefer a less volatile return pattern.

[38]By such definition of ε we first assume, that the manager's success probability in the following period decreases once she experienced losses, thus $\varepsilon < 1$. Second, we assume the decreased success probability

Given the assumption that the future expected profit depends only on the current return and not on the return levels in the past we explicitly assume the process governing return changes to be first order Markov. Consequently, the term about the manager's type refers only to her first-period return u or d and not to her success probability in the following periods. To see the economic reasoning for this assumption consider the possibility of the manager's ability for portfolio management and her professional skills that may change over time. Additionally, the market can adopt the idea of manager's investment strategy, making it less profitable over time. Poor performance can be considered as a "loss by accident" without any information for the future performance, but can also be indicative for the future performance.

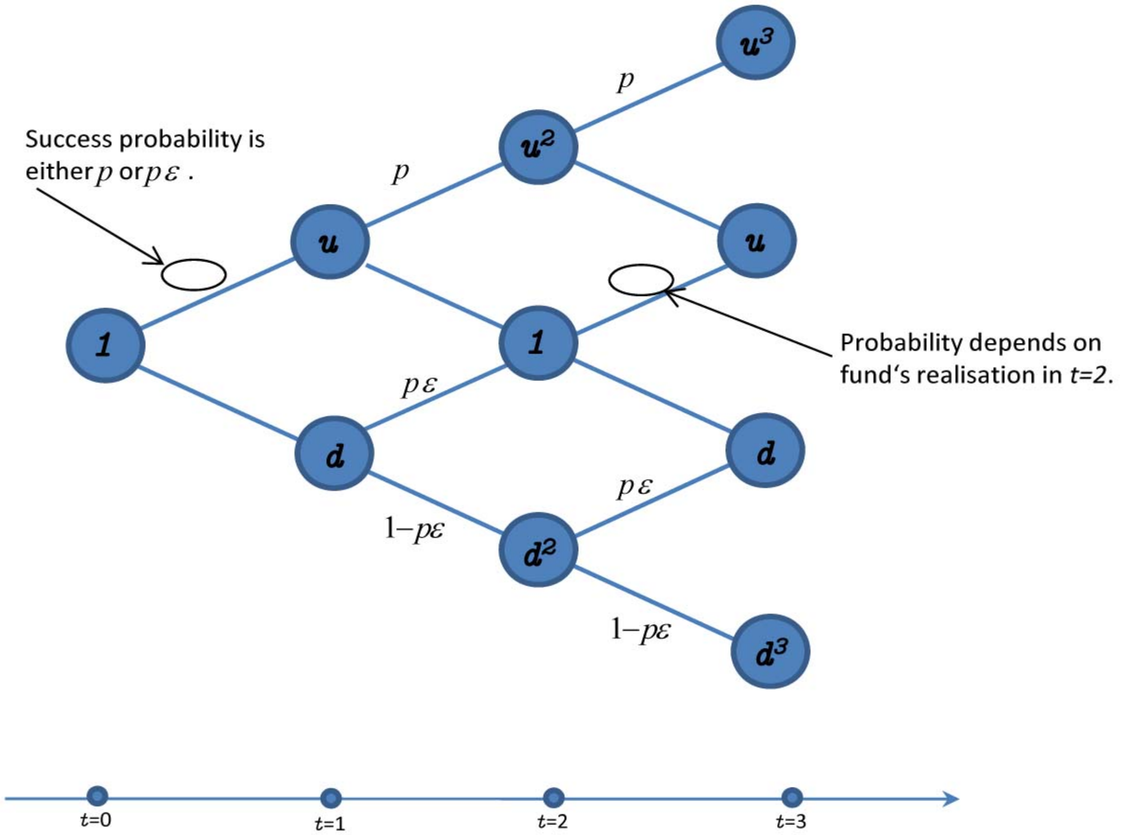

Figure 7: Return probabilities.
Success probabilities in three periods and all outcomes .

The hedge fund's returns generated are observed only by the manager but not by the investors. The investors are assumed, however, to be able to observe the actual value of

---

of $p\varepsilon$ to be still larger than the smallest possible failure probability, $p\varepsilon > 1-p$, in order to create positive probability for the manager to be able to recover past losses in the future periods.

assets under management in $t = 2$ and $t = 3$, but not after the first period.[39] The value of assets under management in $t = 1$ can be $V_1 \in \{u, d\}$. At the end of the second period, the value is $V_2 \in \{u^2, 1, d\}$ and at the end of the last period, in $t = 3$, the fund's value of assets under management can be $V_3 \in \{u^3, u, d, d^3\}$.

At the beginning of the first period there is no current return and, thus, neither the manager nor investors know exactly the fund's prospects in the first period.[40] Ex-ante, there exists a mass $\upsilon \in (0, 1)$ of managers with a first-period probability for a positive return of p. Analogously, with the mass $1-\upsilon$ managers have a first-period probability for a positive return of $p\varepsilon$. υ is common knowledge. It maintains that the manager's and the investors' prior belief of the manager's success probability in the first period to be equal p is given by:

$$\mu := \upsilon p + (1 - \upsilon)p\varepsilon. \qquad \text{(A1)}$$

To ensure that it is optimal to launch the fund we assume additionally that the investment strategy is profitable in the first period:

$$\mu u + (1 - \mu)d \geq 1. \qquad \text{(A2)}$$

We also assume that the investors are willing to invest in the fund.

Manager's Report in $t = 1$

The only value of assets under management unobservable to the investors is that at date 1. Thus, the new value V_1 in $t = 1$ is the manager's private information and specifies her type. As stated above, the second and the third period values, V_2 and V_3, are perfectly observable for both parties. After observing her current performance and, thus, her type in $t = 1$ the manager reports to the investors.

By privately observing the first-period asset value the manager has the opportunity to misreport it in $t = 1$. v_1 denotes the manager's reported asset value. When realizing $V_1 = u$ the manager can either report the true value $v_1 = u$ to the investors or report the lower value $v_1 = 1$, thus, underreporting the first-period return. Analogously, when realizing $V_1 = d$ the manager can either report the true value $v_1 = d$ to the investors or

[39]We adopt the assumption first presented by Fudenberg/Tirole (1995), in order to simplify the future calculation. The more general assumption about the unobservability of the second-period return on the investors' side does not change the results.

[40]Note, there is no adverse selection problem.

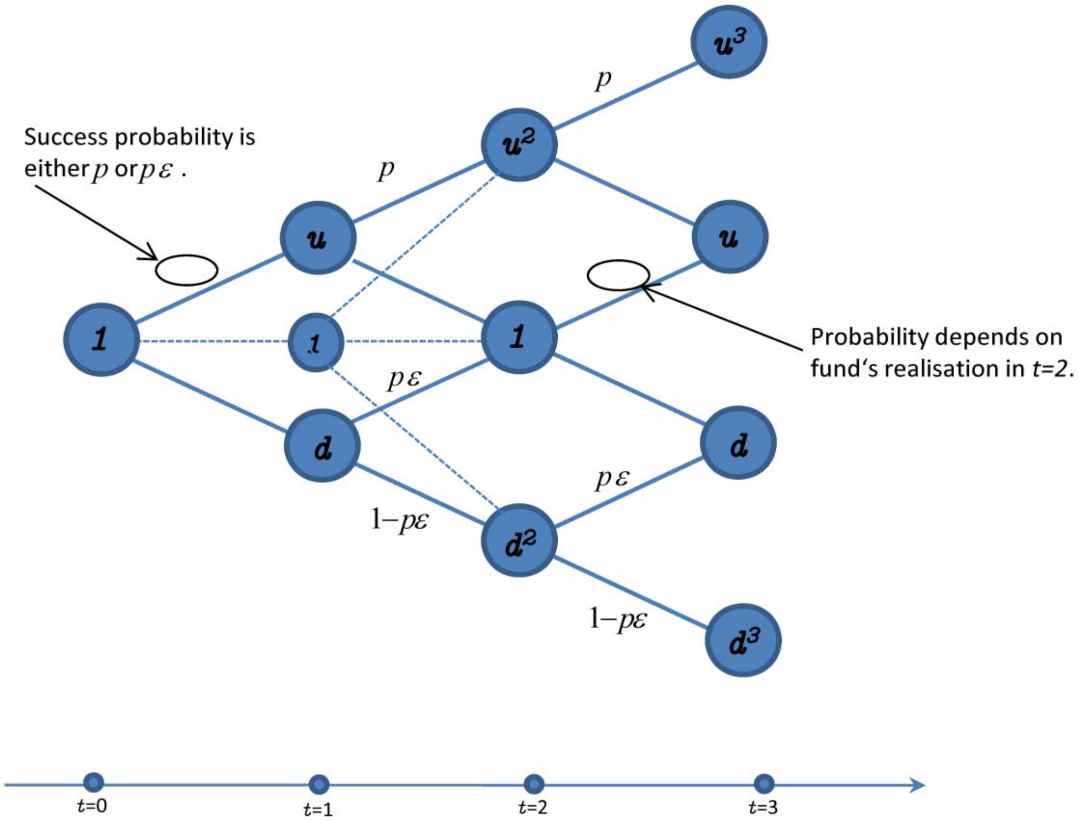

Figure 8: Return probabilities and managerial reporting flexibility in *t=1*.
After observing fund's first-period return, manager has the choice between the thruthful reporting of $v_1 \in \{u,d\}$ and misreporting, by announcing $v_1 = 1$. In *t=1* investors' information is the manager's reported value of assets under management at the end of the first period $v_1 \in \{u,1,d\}$.

report the higher value of $v_1 = 1$, thus, overreporting the first-period return (see Figure 8). Thus, we assume, that the manager is unable to report $v_1 = d$ if the true value of assets equals $V_1 = u$ and the manager is unable to report $v_1 = u$ if the true value of assets equals $V_1 = d$. By limiting the level of misreporting to a value that falls between the two possible values of $V_1 \in \{u, d\}$, we implicitly assume that there are significant falsification costs for the manager to deviate too much from the true asset value. For example, beyond a certain degree of misreporting it requires the manager to hold a suboptimal portfolio, possibly of illiquid securities, that allows her to report the desired asset value.

At time $t = 2$ the manager observes the second-period realization $i \in \{u, d\}$. We assume that the manager is unable to manipulate her reported return for the second time. Thus, the investors observe the true new value of assets under management $V_2 \in \{u^2, 1, d^2\}$. This assumption has the important implication that overreporting (underreporting) the true value V_1 affects the reported return in the second period negatively (positively).

Possible Changes in Net Asset Value

The initial net asset value of the hedge fund is the investors' capital contribution of one unit in $t = 1$. The value of assets under management changes over time by the way of realized investment returns. Additionally, the investors who invest in the hedge fund can contribute or withdraw money from the fund depending on the reported asset value at date 1 and subsequently the observed value of assets under management. In $t = 1$ we assume the investors' to withdraw capital, if they assume the expected fund's performance to be insufficient. In $t = 2$ we assume the investors' either contribute or withdraw capital, based on their assumption of the expected performance of the fund.

Manager's Compensation

There are two ways hedge fund managers typically profit from her investment strategy. Charging a fee that is proportion to the value of the assets under management (management fee) and, additionally, receiving a percentage of the fund's increase in asset value (performance fee). Hedge funds commonly use one of two types of performance fee: the manager either participates in any of the fund's value gains or, alternatively, the losses experienced by the fund in prior periods must first be recouped by compensating gains before further performance fees are paid. Due to the manager's limited liability, fees usually cannot be negative in any period. This implies that after a fee has been paid out, they are unaccessible to the investors in later periods.

We take as exogenous the two following different compensation contracts:

- a *period performance fee* ($f \geq 0$) that is paid out to the manager at the end of each period as a constant fraction of the fund's value gain during the period and not when the fund loses value during the period,
- a *performance fee with a high water mark provision* ($h \geq 0$) that is paid out to the manager at the end of each period as a constant fraction of the fund's value gain during the period relative to the fund's historic maximum value and not when the fund's value is below its historic maximum.

In both compensation contracts the underlying principle is that the manager is rewarded for her performance, which is calculated as the increase in net asset value of the fund (in the second contract - there is a high water mark above which these performance fees apply). Thus, the value of assets under management is the valuation basis for the level of the manager's fee. Consequently, capital withdrawals and capital inflows may play a key role in managerial incentives while deciding on which report is optimal for her in $t = 1$.

To simplify the analysis, we first normalize the management fee to zero. It is also assumed that the investor pays any fees separately from the fund to the manager.[41]

Investors' Beliefs and Responses

The investors' information in $t = 1$ is the manager's reported value of assets under management at the end of the first period $v_1 \in \{u, 1, d\}$. Observing a report of $v_1 \in \{u, d\}$, the investors know the value realization in the first period with certainty and, therefore, know also the manager's type. The only one state that can lead to an asymmetry of information is the one in which the manager's report in $t = 1$ is $v_1 = 1$. The investors know that the true fund value is either u or d, but are unable to verify the true value. Based on the information of the reported first-period return, investors update their prior belief μ about the manager's type according to the Bayes' rule and, thus, her probability of a positive return in the following period. Contingent on their observation and updated belief, the investors decide whether to withdraw the entire capital from the fund or not. For a sufficiently low belief probability of investors to face a u-type manager the capital outflow occurs with probability $\lambda_1 \in (0, 1)$. We denote the critical belief *below* which withdrawals can occur as $\overline{\mu}_1 \in (0, \mu)$. This implies that if the investors assess the probability of facing a u-type manager at least as high as at the outset, they do not withdraw capital. If, however, this probability is sufficiently small, a withdrawal occurs with probability λ_1.

At time $t = 2$ investors observe the true second-period value of assets under management $V_2 \in \{u^2, 1, d^2\}$. Based on the observed value and the new belief about the manager's type and, thus, her probability of a positive return in the following period, the investors respond either with an additional capital contribution of $\Delta = 1$ with probability $\eta \in [0, 1]$ or the withdrawal of the entire capital with probability λ_2. Recall that the descriptor u-type refers to the fund's return in period 1. Therefore $V_2 = 1$ implies that the second-period return of a u-type manager is negative. Thus, at date 2 a high probability of facing a u-type implies a relatively low probability of a positive return in period 3. Thus, there exists a critical investor belief about the probability of facing a u-type *above* which a withdrawal of funds can occur. This critical belief is denoted by $\overline{\mu}_2 \in (\mu, 1)$. Consistent with our assumption on $\overline{\mu}_1$, $\overline{\mu}_2 > \mu$ implies that in the absence of learning about the manager's type investors do not withdraw their capital. We assume that λ_2 is signifi-

[41] As long as the fund's assets are sufficiently liquid, assuming that the cash to pay the fees are generated by liquidating the corresponding part of the fund's assets does not change the results.

cantly larger than the probability of withdrawal at the preceding date, λ_1. This implies that investors sufficiently strongly penalize negative performance after periods of good performance. This is necessary in models of this type to create a sufficient importance of second-period performance for first-period reporting (for a discussion of this issue see Fudenberg/Tirole,1995).

Sequence of Events

The sequence of events is summarized in Figure 9. At date 0 the contract is signed and investors provide financial capital of $V_0 = 1$ to the fund. The fund manager invests that capital amount according to her investment strategy. At date 1 the first-period return $i \in \{u, d\}$ is observed only by the manager. Based on the observed first-period return the manager learns about her type and decides whether to report her true realization of i, $i \in \{u, d\}$, or to misreport the return, by announcing $v_1 = 1$. The fee is paid to the manager based on the reported value of assets under management $v_1 \in \{i, 1\}$, $i \in \{u, d\}$, as specified in the fund's management compensation contract. Investors observe the manager's reported value of assets under management $v_1 \in \{i, 1\}$, $i \in \{u, d\}$. If investors observe or with sufficiently large probability believe that the first-period return is negative and, thus, the manager's type is d, they decide to withdraw their capital with probability λ_1. In the remaining cases there is no capital outflow after the first period. If the fund is closed, all assets are liquidated at no cost and the proceeds are paid to the investors.

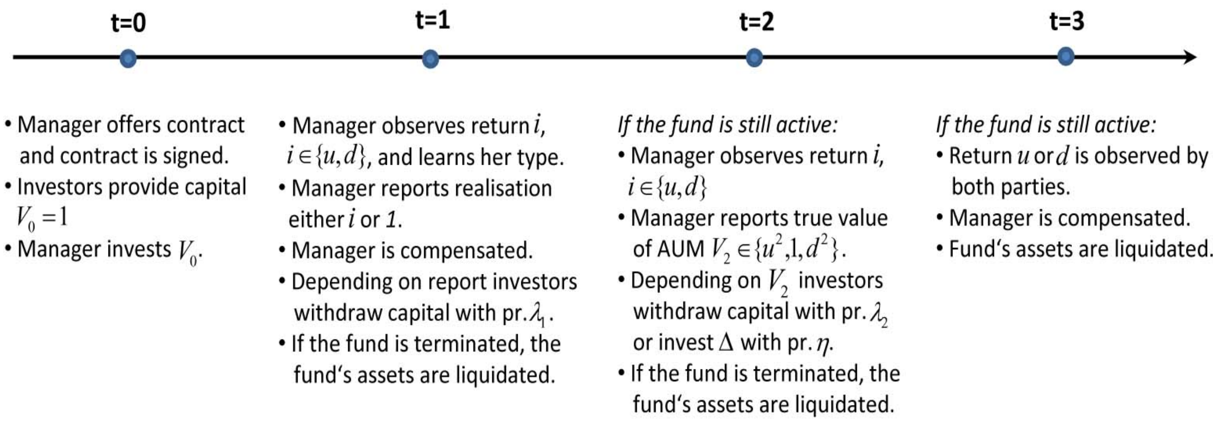

Figure 9: Sequence of events.

If the fund remains alive, assets $V_1 \in \{u, d\}$ are used according to the investment strategy in the following period. At date 2, the fund's new true value of assets under management $V_2 \in \{u^2, 1, d^2\}$ is observed by all parties. The fee is paid to the manager based on the value V_2 as specified in the fund management compensation contract. For the second time, the investors decide whether to leave the fund or not. If investors observe or at least with sufficiently large probability believe to experience a decrease in the value of assets under management compared to the previous period $V_2 < V_1$, they withdraw the investment with the probability λ_2. In the remaining cases they invest an additional capital amount of 1 in the fund with probability η. If the fund is closed, all assets are liquidated at no cost and the proceeds are paid to the investors.

At date 3, if the fund is still alive, the third fund's return is observed by all parties, its assets are costlessly liquidated and the proceeds are distributed to the investors. The contractually agreed fee is paid to the manager.

4.3 Analysis: Period-Performance Fee

First we study the manager's reporting and investors' investment decisions when the fund management contract specifies a period performance fee. The analysis uses the concept of backward induction, whereby the manager and the investors reason backward from the end of the third period to the beginning of the first period in order to determine which choices are rational at each stage.

Investors Beliefs and Responses in the Third Period

In $t = 2$ the are three possible values of assets under management, $V_2 \in \{u^2, 1, d^2\}$. V_2 is observed by both parties . There exist a total of 7 different reported fund "histories", that can describe the development of the reported fund asset values up to date 2. By history we denote the tuple of first- and second-period reported asset values: $\{v_1, V_2\}$.

Observing the value of assets under management of $V_2 = u^2$ the investors can face either a reported fund history of $\{v_1 = u,\ V_2 = u^2\}$ or $\{v_1 = 1,\ V_2 = u^2\}$. Based on one of the two possible reported fund histories the investors' update their beliefs about the manager's type and her probability of a positive return in the next period. Investors' beliefs are defined by a conditional probability: $\alpha(u|i, u^2)$, $i \in \{u, 1\}$, the probability that, conditional on observing the reported fund history of $\{v_1 \in \{u, 1\}, V_2 = u^2\}$ the manager is of u-type and, thus, have realized a positive return of u in the first period. If investors are certain to face a u-type manager the investors expect a probability of p for a positive return in the following period. However, the investors know that the asset value in the first period cannot be 1. As a consequence, the value of $V_2 = u^2$ is sufficiently informative for the investors, so, that their belief is identical upon observing the history of $\{v_1 = u,\ V_2 = u^2\}$ or $\{v_1 = 1,\ V_2 = u^2\}$ in this state:

$$
\begin{aligned}
\alpha(u|u,u^2) &= \frac{P(V_2\!=\!u^2|V_1\!=\!u)P(V_1\!=\!u)}{P(V_2\!=\!u^2|V_1\!=\!u)P(V_1\!=\!u)+P(V_2\!=\!u^2|V_1\!=\!d)P(V_1\!=\!d)}\\
&= \frac{p\mu}{p\mu+0(1-\mu)}=1,\\
&\Leftrightarrow\ \alpha(u|1,u^2)\equiv 1.
\end{aligned}
$$

By observing in $t=2$ an increase in the fund's asset value over the last period the investors respond with an additional capital contribution of 1 with probability η. Conditional on the investors' belief of $\alpha(u|u,u^2)=1$ or $\alpha(u|1,u^2)=1$, the u-type manager's expected period performance fee payment in the third period is given by:

$$
E^u(f|u^2,1)=fp(u-1)(u^2+\eta).
$$

For a value of assets under management equal to $V_2\!=\!d^2$ in $t\!=\!2$, the investors can observe a reported fund histories of $\{v_1=d,\ V_2=d^2\}$ or $\{v_1=1,\ V_2=d^2\}$. Like in the previous case, the negative return in the last-period and the knowledge of impossibility to obtain a zero return in the first period are sufficiently informative for the investors to learn the manager's type. For the reported fund's history of $\{V_1\in\{d,1\},\ V_2=d^2\}$, the investors believe with probability $\alpha(u|i,d^2)$, $i\in\{d,1\}$, that the manager's type is u is:

$$
\begin{aligned}
\alpha(u|1,d^2) &= \frac{P(V_2\!=\!d^2|V_1\!=\!u)P(V_1\!=\!u)}{P(V_2\!=\!d^2|V_1\!=\!u)P(V_1\!=\!u)+P(V_2\!=\!d^2|V_1\!=\!d)P(V_1\!=\!d)}\\
&= \frac{0\mu}{0\mu+(1-p\varepsilon)(1-\mu)}=0.\\
&\Leftrightarrow\ \alpha(u|d,d^2)\equiv 0.
\end{aligned}
$$

Therefore, the investors are sure, that the fund manager is a d-type manager and has performed poorly in both periods. The investors assess the next-period probability of a positive return to be $p\varepsilon$ and respond with capital withdrawals with probability λ_2. Anticipating the possible capital outflow in $t=2$ the d-type manager's expected period performance fee payment in the third period, conditional on the investors' belief of $\alpha(u|i,d^2)=0$, $i\in\{d,1\}$, is given by:

$$
E^d(f|d^2,0)=f(1-\lambda_2)p\varepsilon(d-d^2).
$$

The third possible value of assets under management that the investors can observe in $t = 2$ is $V_2 = 1$. There are three potential reported fund's histories, that can lead to this outcome.

- The investors may observe a fund history of $\{v_1 = u,\ V_2 = 1\}$. Then the investors know that the fund performed well in the first period but experienced a loss in the second period. Based on this information the investors believe with probability $\alpha(u|u,1) \equiv 1$ that they face a u-type manager. Because of last period's loss, the probability of a positive return in the following period is $p\varepsilon$. As a response, the investors withdraw their capital with probability λ_2. Contingent on investors' belief of $\alpha(u|u,1) \equiv 1$ and the possible capital outflow, the u-type manager's expected period performance fee payment equals:

$$E^u(f|1,1)) = f(1-\lambda_2)p\varepsilon(u-1).$$

- The investors' may observe a reported fund history of $\{v_1 = 1,\ V_2 = 1\}$. In this state the investors are unable to distinguish whether the true first-period return was positive, equal to u, or negative, equal to d. Generally, the investors place a probability $\alpha(u|1,1) \in [0,1]$ on the fund's manager being u typed.

  For sufficiently low probabilities to face a u-type manager, $\alpha(u|1,1) < \overline{\mu}_2$, investors assign a sufficiently high probability for a positive second-period return – and therefore also the third-period return – that they are willing to contribute additional capital of 1 with probability η to the fund. For $\alpha(u|1,1) \geq \overline{\mu}_2$, investors respond with the capital outflow with probability λ_2.

  If the fund indeed performed well in the second period, the d-type manager's probability of a positive return is equal to p and her expected fee payment, contingent on the investors belief of $\alpha(u|1,1)$, equals:

$$E^d\Big(f|1,\alpha(u|u,1)\Big) = \begin{cases} fp(u-1)(1+\eta) & for \quad \alpha(u|u,1) < \overline{\mu}_2 \\ f(1-\lambda_2)p(u-1) & for \quad \alpha(u|u,1) \geq \overline{\mu}_2. \end{cases}$$

  If the fund performed poorly in the second period the u-type manager's probability of a positive return is equal to $p\varepsilon$ and her expected fee payment, contingent on the investors belief of $\alpha(1|1,1)$, equals:

$$E^u\Big(f|1,\alpha(u|1,1)\Big) = \begin{cases} fp\varepsilon(u-1)(1+\eta) & for \quad \alpha(u|1,1) < \overline{\mu}_2 \\ f(1-\lambda_2)p\varepsilon(u-1) & for \quad \alpha(u|1,1) \geq \overline{\mu}_2. \end{cases}$$

- The investors may observe a reported fund history of $\{v_1 = d,\ V_2 = 1\}$. Then the investors know that they face a d-type manager. Additionally, the investors know with certainty, that the fund performed well in the second period. Based on their belief of $\alpha(u|d,1) = 0$ the investors assign a probability of p for a positive return in the following period and respond with capital contribution of 1 with probability η. The d-type manager anticipates a possible capital contribution in this state and expects in the third period a value of period performance fee equal to:

$$E^d(f|1,0)) = fp(u-1)(1+\eta).$$

Investors Beliefs and Responses in the Second Period

In $t = 1$ the are two possible fund returns: a positive return of $V_1 = u$ and a negative return of $V_1 = d$, but there are three possible reports the manager can make, and, thus, the investors can observe: $v_1 \in \{u, 1, d\}$. Recall, that by the realization of u the manager is able to report $v_1 \in \{u, 1\}$ and by the realization of d the manager is able to report $v_1 \in \{1, d\}$.

By observing the reported return of $v_1 = u$ the investors receive a credible signal about the true fund performance and the fund's prospects for the following period. Namely, they assess the probability of a positive fund return in the second period to be p.

Given a first-period reported return of $v_1 = u$ and investors beliefs as described above, the u-type manager's expected performance fee in $t = 1$ is given by:

$$E^u(f|u,1) = f(u-1)+f\Big(p(u^2-u)+p^2(u-1)(u^2+\eta)+(1-p)(1-\lambda_2)p\varepsilon(u-1)\Big). \quad (4)$$

By observing a reported return of $v_1 = d$ the investors also receive a credible signal about the true fund performance and the fund's prospects for the following. Specifically, the investors are certain that $V_1 = d$ and that the probability for a positive return in the second period is $p\varepsilon$. As a response in $t = 1$ the investors withdraw their capital with probability λ_1. The fund is still alive in the second period with probability $(1-\lambda_1)$.

Then, the d-type manager's expected period performance fee at time $t = 1$ is given by:

$$E^d(f|d,0) = f(1-\lambda_1)\Big(p\varepsilon(1-d)+p^2\varepsilon(u-1)(1+\eta)+(1-p\varepsilon)(1-\lambda_2)p\varepsilon(d-d^2)\Big). \quad (5)$$

If investors observe a first-period reported value of assets under management equal to $v_1 = 1$ they know that the true return is either u or d, but are unable to distinguish whether the message originates from the u- or d-type manager.

Depending on investors' beliefs $\alpha(u|1) \in [0,1]$ about the manager's type and, therefore, her probability of gererating a positive return in the second period, the investors can respond either with a withdrawal of capital, which occur with probability λ_1, or not.

- If in $t = 1$ investors' with sufficiently high probability $\alpha(u|1) \geq \overline{\mu}_1$ believe, that the true first-period return is positive, they will stay with the fund and there is no capital outflow. In this situation the investors believe that it is relatively likely that they face a u-type manager who underreports her true return by announcing a lower value of assets under management of 1. If the manager in fact realized a return of u in the first period, her probability of a positive second period return is still p and her expected fee payment at time $t = 1$ by reporting $v_1 = 1$ is given by:

$$E^u\Big(f|1, \alpha(u|1) \geq \overline{\mu}_1\Big) = f\Big(p(u^2-1) + p^2(u-1)(u^2+\eta) + (1-p)(1-\lambda_2)p\varepsilon(u-1)\Big). \quad (6)$$

  If the manager in fact experienced a low last-period performance of d but overreported the value of assets under management, by announcing $v_1 = 1$, her probability of achieving a positive second-period return decreases to $p\varepsilon$. But given investors' belief of $\alpha(u|1) \geq \overline{\mu}_1$ the d-type manager does not expect a capital outflow and her expected fee payment in $t = 1$ by reporting $v_1 = 1$ is given by:

$$E^d\Big(f|1, \alpha(u|1) \geq \overline{\mu}_1\Big) = f\Big(p^2\varepsilon(1-\lambda_2)(u-1) + (1-p\varepsilon)(1-\lambda_2)p\varepsilon(d-d^2)\Big). \quad (7)$$

- Observing the reported return of $v_1 = 1$ investors may have a sufficiently low belief $\alpha(u|1) < \overline{\mu}_1$ that the manager is a u-type manager. As a response, the investors withdraw their capital with probability λ_1.
  If the manager in fact is u-type and realized a positive return of u, she has a high probability of a positive return of p in the second period. But by announcing $v_1 = 1$ and given investors' belief of $\alpha(u|1) < \overline{\mu}_1$, she is still affected by a possible capital outflow in $t = 1$. In this state the u-type manager's expected amount of period performance fee is equal to

$$
\begin{aligned}
E^u(f|1, \alpha(u|1) &< \overline{\mu}_1) = && (8)\\
&= f(1-\lambda_1)\Big(p(u^2-1)+p^2(u-1)(u^2+\eta)+(1-p)p\varepsilon(u-1)(1+\eta)\Big).
\end{aligned}
$$

If the manager in fact experienced a low first-period performance of d, she has a lower probability for a positive return in the second period of $p\varepsilon$. By announcing $v_1 = 1$, the d-type manager's expected period performance fee payment in $t = 1$, contingent on investors' belief of $\alpha(u|1) < \overline{\mu}_1$, is given by:

$$
E^d\Big(f|1, \alpha(u|1) < \overline{\mu}_1\Big) = f(1-\lambda_1)\Big(p^2\varepsilon(u-1)(1+\eta)+(1-p\varepsilon)(1-\lambda_2)p\varepsilon(d-d^2)\Big). \quad (9)
$$

4.4 Optimal Reporting Choice and Equilibria: Period Performance Fee

The findings discussed in the previous subsection describe an incomplete information signaling game. The informed player, the manager, has the choice between truthful reporting and misreporting of her first-period return and, thus, between revealing her true type or not. The manager chooses first her optimal reporting. Then her uninformed opponents, the investors, choose their optimal response based on updated beliefs about the true fund return, the manager's type and, thus, theprospects for the following period. Now we describe a systematic procedure to search for a Perfect Bayesian Equilibria in pure strategies.

Definition: In the model, the **Perfect Bayesian Equilibrium (PBE)** is a set of strategies and beliefs that satisfy each of the following three conditions:

1. The strategies of the manager and the investors are sequentially rational.

2. Based on the observed reported return investors update their prior beliefs about the manager's success probability in the following period according to Bayes' rule.

3. At the out-of-equilibrium information sets, beliefs are derived, using Bayes' rule, from the beliefs at the information sets that precede the information set in question and players' continuation strategies as implied by their equilibrium strategies, if possible.

In the equilibrium, the manager's expectations about the investors strategy is consistent with the investors' expectations about the manager's strategy. Each party chooses a best response to what it believes the other party will choose to do. Taking into account the manager's decision and the corresponding responses of the investors in each of the decisions nodes, we can analyze if the following four candidate strategy profiles constitute a Perfect Bayesian Equilibrium:

- *pooling(1,1)*: is a pooling strategy profile for the informed player, in which the manager independent of her first-period return announces $v_1 = 1$ in $t = 1$. Investors' optimal responses are contingent on their updated beliefs.
- *separating(u,d)*: is a separating strategy profile for the informed player, in which the u-type manager truthfully reports her first-period return by announcing $v_1 = u$, and the d-type manager truthfully reports her first-period return by announcing $v_1 = d$, in $t = 1$. Investors' optimal responses are contingent on their updated beliefs.
- *separating(u,1)*: is a separating strategy profile for the informed player, in which the u-type manager truthfully reports her positive first-period return by announcing $v_1 = u$ and the d-type manager overreports her negative first period return by announcing $v_1 = 1$. Investors' optimal responses are contingent on their updated beliefs.
- *separating(1,d)*: is a separating strategy profile for the informed player, in which the u-type manager underreports her positive first-period return of u by announcing $v_1 = 1$, and the d-type manager truthfully reports her negative first period return of d by announcing $v_1 = d$. Investors' optimal responses are contingent on their updated beliefs.

For each strategy profile we have now to calculate the investors' beliefs. Contingent on investors' beliefs we have to consider their optimal responses in each state.

We begin with the pooling strategy profile *pooling(1,1)*.

In a pooling equilibrium the manager reports $v_1 = 1$ independent of her type and, thus, whether her true first-period return was positive, equal to u, or negative, equal to d. Bayesian updating in $t = 1$, by observing a misreported value of $v_1 = 1$, implies that investors do not learn anything about the manager's type, her success probability and fund's prospects. Hence, the investors posterior belief $\alpha(u|1)$ coincides with their prior belief, μ. Given that investors were willing to invest capital into the fund in $t = 0$, we assume that in the absence of learning in a pooling equilibrium they will stay with the fund for at least one period longer. Thus, in *pooling(1,1)*, there is no capital outflow in $t = 1$.

In $t = 2$, if the investors observe $V_2 = u^2$ they are certain to face a u-type manager. By observing $V_2 = d^2$ they are certain to face a d-type manager. Only by observing $V_2 = 1$ the informational asymmetry is still unsolved. By observing $V_2 = 1$ investors are able to form beliefs that are more precise than those in $t = 1$. Because $V_2 = 1$ is a true value it is informative for investors. More precisely, after observing $\{v_1{=}1,\ V_2{=}1\}$, the investors' posterior belief about the u-type manager in $t = 2$ is equal to

$$\begin{aligned}\alpha(u|1,1) &= \frac{P(V_2{=}1|V_1{=}u)P(V_1{=}u)}{P(V_2{=}1|V_1{=}u)P(V_1{=}u)+P(V_2{=}1|V_1{=}d)P(V_1{=}d)} \\ &= \frac{(1-p)\mu}{(1-p)\mu + p\varepsilon(1-\mu)} < \mu.\end{aligned}$$

Investors' posterior belief of facing a u-type, $\alpha(u|1,1)$, is lower than their prior belief, μ. An increased probability of facing a d-type manager is good news for investors, because it is the d-type manager who realized a positive second-period return. Thus, investors expect a probability of p for a positive return in the third period and contribute additional capital of 1 with probability η to the fund.

Given the investors' optimal response, we can now determine the manager's optimal reporting choice, especially, whether the manager has an incentive to deviate from the previously described pooling strategy profile or not. The announcement of $v_1 \neq 1$ occurs off-the-equilibrium path according to the strategy profile *pooling(1,1)*, implying that the investors' off-the-equilibrium beliefs can be arbitrarily.

- If the true fund history is $\{V_1{=}u,\ V_2{=}1\}$ (implying that the manager is a u-type), her expected fee payment, given investors' beliefs in *pooling(1,1)* strategy profile, $\alpha(u|1) = \mu$ in $t = 1$ and $\alpha(u|1,1) < \overline{\mu}_2$ in $t = 2$, is described by:

$$\begin{aligned}E^u\Big(f|1,\mu,\alpha(u|1,1) \ < \ \overline{\mu}_2\Big) &= \qquad (10)\\ &= fp\Big(u^2-1+p(u-1)(u^2+\eta)+(1-p)\varepsilon(u-1)(1+\eta)\Big).\end{aligned}$$

  If the u-type manager deviates towards the truthful reporting of her first-period return, by announcing $v_1 = u$, she sends a credibly signal about her type to the investors. Her expected fee payment will be as described in (4). Comparing the u-type manager's expected fee payments given truthful reporting of $v_1 = u$ (4)

with the expected fees payment given underreported value of $v_1 = 1$ (10) leads to the following result:

$$E^u(f|u,1) \geq E^u\Big(f|1,\mu,\alpha(u|1,1) < \overline{\mu}_2\Big) \Leftrightarrow$$

$$\eta \leq \frac{1 - p\varepsilon\lambda_2}{p\varepsilon} := \eta^u \quad \Leftrightarrow \quad \lambda_2 \leq \frac{1 - p\varepsilon\eta}{p\varepsilon} := \lambda_2^u.$$

The value η^u defines the probability-weighted threshold amount of expected additional capital contribution below which the strong-type manager reports truthful her first-period return and above whom she has an incentive to underreport. The option to avoid the capital outflow and additionally to receive an expected amount of η contributed to the fund in $t = 2$, in the case of low second-period performance, is valuable to the u-type manager. The incentive to underreport the true first-period return and, thus, her type is higher for increasing levels for η.

Analogously, λ_2^u denotes an equivalent threshold value for the withdrawal probability in $t = 2$. More precisely, with increasing levels of λ_2 the u-type manager's incentive to pool with the d-type, by announcing $v_1 = 1$, increases. In order to prevent capital outflow in $t = 2$ in the case of poor second-period performance the u-type manager forgoes part of her period performance fee in $t = 1$.

Remark: As soon as the numerical values of λ_2^u or η^u are larger than 1, it means that the u-type manager reports the true first-period return of u with probability 1. By analyzing the values λ_2^u and η^u we can make an interesting observation about the success probability $p\varepsilon$. In order to incentivize the u-type manager to underreport her first-period return, the threshold probability λ_2^u (or η^u) has to be below 1, more precisely, due to $\eta \in [0,1]$, it holds:

$$1 \geq \eta^u = \frac{1 - p\varepsilon\lambda_2}{p\varepsilon}$$

$$\Leftrightarrow p\varepsilon \geq \frac{1}{1 + \lambda_2}.$$

In summary, the remark implies that the u-type manager has an incentive to underreport her first-period return for $\eta > \eta^u$ ($\lambda_2 > \lambda_2^u$) only if the probability of a positive return for the d-type manager, $p\varepsilon$, is sufficiently high $p\varepsilon \in [\frac{1}{1+\lambda_2}, 1)$.

- If the true fund history is $\{V_1 = d,\ V_2 = 1\}$ (implying that the manager is a d-type), her expected fee payment, given investors' beliefs in *pooling(1,1)* strategy profile, $\alpha(u|1) = \mu$ in $t = 1$ and $\alpha(u|1,1) < \overline{\mu}_2$ in $t = 2$, is described by:

$$E^d\Big(f|1,\mu,\alpha(u|1,1)<\overline{\mu}_2\Big)= \tag{11}$$
$$=f\Big(p^2\varepsilon(u-1)(1+\eta)+(1-p\varepsilon)(1-\lambda_2)p\varepsilon(d-d^2)\Big).$$

Consider now, that the d-type manager announces her true first-period return, $v_1 = d$. By doing so, the d-type manager credibly signals her type to the investors. Her expected fee payment by truthfully reporting her first-period return generates a belief investors of 0 to face a u-type manager. In this case the d-type manager's expected fee is described as in (5). Comparing the d-type manager's expected fee payment in both regimes leads to the following consideration:

$$E^d(f|d,0)\geq E^d\Big(f|1,\mu,\alpha(u|1,1)<\overline{\mu}_2\Big)\Leftrightarrow \tag{12}$$
$$\frac{1-\lambda_1}{\lambda_1}\geq\frac{p(u-1)(1+\eta)+(1-p\varepsilon)(1-\lambda_2)(d-d^2)}{1-d}$$

Given investors' beliefs in strategy profile *pooling(1,1)*, the d-type manager faces a trade-off between accepting capital outflow already after the first period, but receiving an expected amount of $(1-\lambda_1)p\varepsilon(1-d)$ in $t=2$ when truthfully reporting $v_1=d$, and avoiding any capital outflow in $t=1$, but also missing the expected fee amount by reporting $v_1=1$ in $t=2$. For very small values of λ_1 the d-type manager always reports her true first-period return $v_1=d$, especially for $\lambda_1=0$ we have the following result:

$$\lambda_1=0\Rightarrow\quad p\varepsilon(1-d)>0\Rightarrow$$
$$E^d(f|d,0)>E^d\Big(f|1,\mu,\alpha(u|1,1)<\overline{\mu}_2\Big).$$

In terms of λ_2 and η we can solve the inequality (12) with the following results:

$$\eta\leq\frac{(1-\lambda_1)(1-d)-\lambda_1(1-p\varepsilon)(1-\lambda_2)(d-d^2)}{\lambda_1p(u-1)}-1:=\eta^d$$

$$\Big(\Leftrightarrow\lambda_2\geq1-\frac{(1-\lambda_1)(1-d)-\lambda_1p(u-1)(1+\eta)}{\lambda_1(1-p\varepsilon)(d-d^2)}:=\lambda_2^d\Big).$$

After comparing the thresholds η^u and η^d (or, analogously λ_2^u and λ_2^d) we can find a critical value

$$\lambda_1^* := \frac{\varepsilon(1-d)}{u(1+p\varepsilon(1-\lambda_2))-p\varepsilon^2(1-\lambda_2)(d-d^2)+\varepsilon(1-d^2-p(1-\lambda_2)-\lambda_2(d-d^2))-1}$$

so that it holds: $\eta^d \geq \eta^u$ for all $\lambda_1 \leq \lambda_1^*$. Summarizing the considerations about the pooling strategy profile $(1,1)$ by given investors' beliefs of μ in $t=1$ and $\alpha(u|1,1) < \overline{\mu}_2$ about the u-type manager and corresponding responses in $t=1$ and $t=2$ leads to the following result:

Proposition 6 *Suppose the hedge fund management contract specifies a period performance fee, f. The pooling strategy profile* pooling(1,1) *can be supported as a PBE if and only if $p\varepsilon \geq \frac{1}{1+\lambda_2}$ and*

$$\eta > \begin{cases} \eta^u & for \quad \lambda_1 > \lambda_1^* \\ \eta^d & for \quad \lambda_1 \leq \lambda_1^*. \end{cases}$$

We now proceed with the separating strategy profile *separating(u,d)*.

In the strategy profile *separating(u,d)* the u-type manager reports her true first-period return of u by signaling her type to the investors. The d-type manager reports also her true first-period return of d. Observing the reported first-period returns of either u or d, the investors can perfectly distinguish between both manager types, thus, the informational asymmetry is resolved for all following periods. The u-type manager's expected period performance fee in this case is given by (4). The d-type manager's expected period performance fee in this situation is given by (5).

Now we analyze whether the u- or the d-type manager has an incentive to deviate from the truthful reporting in $t=1$. If investors observe a reported value of assets under management $v_1 = 1$, the investors must still update their belief of $\alpha(u|1)$. However, such reported value occurs off-the-equilibrium path according to the given strategy profile *separating(u,d)*. In consequence, the investors' belief can be arbitrarily specified, i.e., $\alpha(u|1) \in [0,1]$. In Appendix B.1 all possible expected rewards for the u- and the d-type manager are calculated, contingent on all possible beliefs in this case. We show the following result:

Proposition 7 *Suppose the hedge fund management contract specifies period performance fee, f. The separating strategy profile* separating(u,d) *can be supported as a PBE for*

$$\eta \leq \begin{cases} \eta^u & for \quad \lambda_1 > \lambda_1^* \\ \eta^d & for \quad \lambda_1 \leq \lambda_1^*, \end{cases}$$

if $p\varepsilon \geq \frac{1}{1+\lambda_2}$*, and for all* $\eta \in [0,1]$*, if* $p\varepsilon < \frac{1}{1+\lambda_2}$*.*

Proof: See Appendix B.1.

Figure 10 illustrates the results of Proposition 7: for high probabilities for capital outflow in $t = 1$, $\lambda_1 > \lambda_1^*$, the u-type manager has first an incentive to deviate towards misreporting for relatively high probabilities for additional capital contribution $\eta > \eta^u$. In this state the d-type manager will always mimic the u-type, as soon, as u-type misreports her first-period return. Thus, the strategy profile *pooling(1,1)* is a PBE as soon as the u-type manager misreports her first-period return (see the upper part of Figure 10).

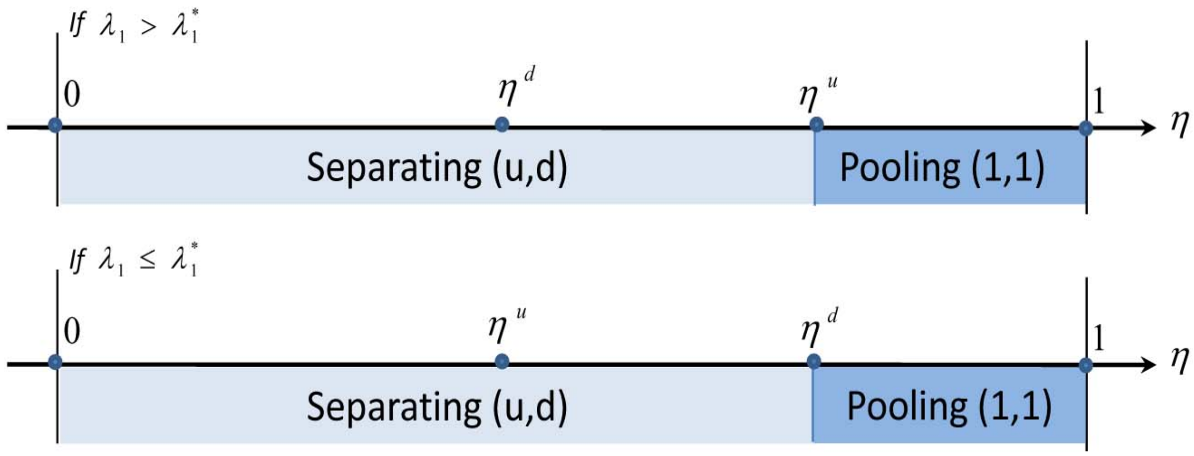

Figure 10: Perfect Bayesian equilibria depending on η and λ_1 if $p\varepsilon \geq 1/(1+\lambda_2)$ and the management compensation contract specifies the period performance fee.

With decreasing values of $\lambda_1 \leq \lambda_1^*$ the d-type manager's incentive for overreporting of her first-period return decreases. Thus, not until $\eta > \eta^d$ the d-type manager has an incentive to hide her true type by overreporting her first-period return. In this state for $\eta \leq \eta^d$ the u-type manager has also to reveal her type (see the lower part of Figure 10).

The separating strategy profiles *separating(u,1)* and *separating(1,d)* can not be supported as a PBE.

In the strategy profile separating $(u, 1)$ the u-type manager reports her true first-period return of u by signaling her type to the investors. Even though the d-type manager overreports her true first-period return, the investors nevertheless learn her type. The same happens in the strategy profile separating $(1, d)$. As soon as the d-type manager credibly signals her type to the investors perfectly distinguish between both manager types. Appendix B.2 shows that neither the *separating(u,1)* nor the *separating(1,d)* can be supported as a PBE.

If the hedge fund manager contract specifies a period performance fee, the manager's opportunity for misreporting is valuable only in certain parameter settings. A pooling equilibrium requires a sufficiently high probability of a positive return of the d-type manager as well as specific parameter levels for λ_1, λ_2 and η (see Figure 10).

4.5 Optimal Reporting Choice and Equilibria: Performance Fee with High Water Mark

Consider now a situation where the hedge fund management contract specifies a performance fee with high water mark. The fund's initial value $V_0 = 1$ is the first high water mark. The nodes $V_1 = u$, $V_2 = u^2$ and $V_3 = u^3$ in Figure 11 characterize further possible high water marks. In case of a loss in the first period, the only possible fund value that sets a new high water mark is $V_3 = u$ (see Figure 11).

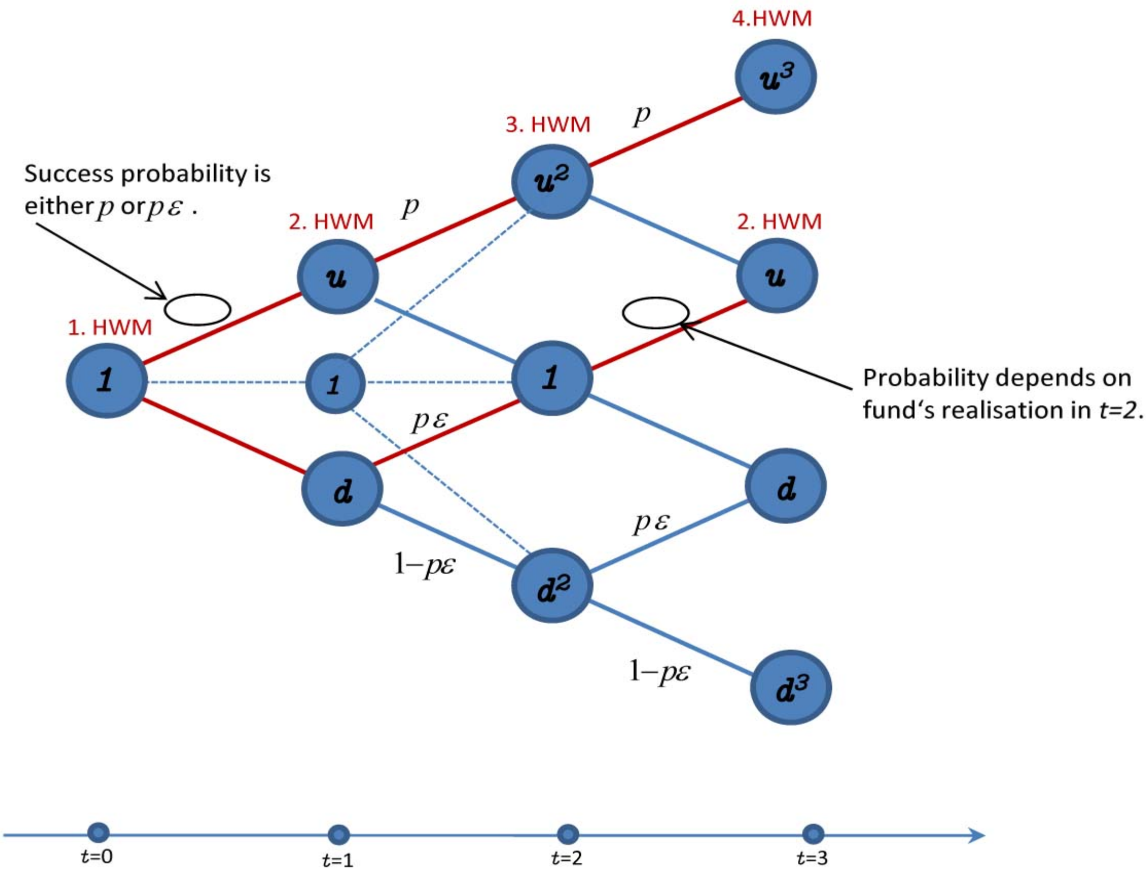

Figure 11: Possible high water marks.
Red lines denote the possible fund developments in which the new high water mark is reached.

u-Type Manager's Decision

The first positive fund's realisation $V_1 = u$ specifies the second high water mark and the manager's performance fee with the high water mark provision by truthful reporting of $v_1 = u$ equals to $h(u-1)$. By observing the new high water mark the investors identify the manager as a u-type manager and the informational asymmetry is resolved for the following periods. If investors observe the third high water mark $V_2 = u^2$ in $t = 2$ they contribute additional capital of η to the fund. If the second-period return is poor and the new value of assets under management drops to $V_2 = 1$, investors by observing the history $\{v_1 = u, V_2 = 1\}$ withdraw all of their capital with probability λ_2. The u-type manager's expected performance fee with the high water mark provision at time $t = 1$, given reported return of $v_1 = u$ and investors' belief of 1 on u-type manager, equals to:

$$E^u(h|u,1) = h(u-1) + hp\Big(u^2 - u + p(u-1)(u^2+\eta)\Big). \tag{13}$$

By anticipating a poor performance in the second period the u-type manager has an option to underreport the first-period return in order to prevent capital outflow in $t = 2$. The underreported fund's value of $v_1 = 1$ is uninformative for investors in $t = 1$. For a given investors belief $\alpha(u|1,1) < \overline{\mu}_2$ on the u-type the u-type manager is able not only to prevent capital outflow in $t = 2$, but additionally to get capital contribution of 1 with probability η. On the one hand, underreporting is costly for the u-type manager in this state, because he has to surrender the fees in value of $h(u-1)$. On the other hand, by performing well in the third period, the u-type manager is able to set a new high water mark and to receive a fee amount of $(1+\eta)(u-1)$. To deceide whether underreporting is beneficial in $t = 1$, the u-type manager compares her expected fee payment by thruthful reporting of $v_1 = u$ and by announcing $v_1 = 1$.

d-Type Manager's Decision

Observing a poor first-period realisation of $V_1 = d$ the d-type manager can thruthfully report $v_1 = d$. In this case capital outflow occures in $t = 1$ with probability λ_1. If investors observe the true manager's type in $t = 1$, there exist only one state in which the d-type manager is able to receive a positive fee amount, namely by realising $V_3 = u$ in the third period. By offering her true type to the investors the d-type manager' expected fee payment in $t = 1$ is given by:

$$E^d(h|d,0) = h(1-\lambda_1)p^2\varepsilon(1+\eta)(u-1). \tag{14}$$

By using the oppotunity of overreporting the d-type manager can take an advantage if investor's belief on the u-type is $\alpha(u|1,1) < \overline{\mu}_2$ in $t = 2$. Thus, by observing $V_2 = 1$ in $t = 2$ investors' are willing to contrubute additional capital of 1 with probability η to the fund, if they with sufficiently high probability beliefe to observe an increase in the value of assets under management during the second period.

Equilibria

First we investigate whether the strategy profile *pooling(1,1)* can be supported as PBE. As discussed in the basic model in the strategy profile *pooling(1,1)* both manager types are reporting $v_1 = 1$ at the end of the first period. Investors have a belief of μ in $t = 1$ and a belief of $\alpha(u|1,1)<\overline{\mu}_2$ in $t = 2$ that the manager's type is u. Thus, the investors do not withdraw any capital at the end of the first and the second periods, but additionally contribute capital of 1 with probability η in expectation in $t = 2$. Contingent on the investors' beliefs and their responses in the given strategy profile, the u-type manager's expected fee payment in *pooling(1,1)* is given by:

$$E^u\Big(h|1,\mu,\alpha(u|1,1)<\overline{\mu}_2\Big) = hp\Big(u^2-1+p(u^2+\eta)(u-1)+(1-p)\varepsilon(1+\eta)(u-1)\Big). \qquad (15)$$

Comparing the u-type manager's expected fees with the high water mark provision given truthful reporting (13) and underreporting (15) in the strategy profile *pooling(1,1)* leads to the following threshold:

$$\begin{aligned} E^u(h|u,1) &= h(u-1)+hp\Big(u^2-u+p(u-1)(u^2+\eta)\Big) \\ &\geq E^u\Big(h|1,\mu,\alpha(u|1,1)<\overline{\mu}_2\Big) \Leftrightarrow \\ &\qquad \eta \leq \frac{1-p\varepsilon}{p\varepsilon} := \widetilde{\eta}^u. \end{aligned}$$

Note, that the u-type manager's decision does not depend on the withdrawal probability λ_2. Additionally, in order to obtain correct numerical values for $\widetilde{\eta}^u \in [0,1]$ it has to hold $p\varepsilon \in [\frac{1}{2}, 1]$.

Analogously, contingent on the investors' beliefs and their responses in the strategy profile *pooling(1,1)*, the d-type manager's expected fee payment is given by:

$$E^d(h|1,\mu,\alpha(u|1,1)<\overline{\mu}_2) = hp^2\varepsilon(1+\eta)(u-1). \qquad (16)$$

Comparing the d-type manager's expected fees with the high water mark provision given truthful reporting (14) and underreporting (16) in the strategy profile *pooling(1,1)* shows, that the d-type manager has never an incentive to deviate from her strategy in strategy profile *pooling(1,1)*:

$$\begin{aligned} E^d(h|d,0) &= h(1-\lambda_1)p^2\varepsilon(1+\eta)(u-1) \\ &\geq E^d\Big(h|1,\mu,\alpha(u|1,1)<\mu\Big) \Leftrightarrow \end{aligned}$$

$$(1-\lambda_1) \geq 1 \longrightarrow \exists \quad only\ for \quad \lambda_1 \equiv 0.$$

In summary, the strategy profile *pooling(1,1)* can be supported as PBE for $\eta \in [\widetilde{\eta}^u, 1]$ if $p\varepsilon \in [\frac{1}{2}, 1]$.

In Appendix B.3 we prove whether remaining candidate strategy profiles can be supported as PBE. The following proposition summarizes the findings.

Proposition 8 *Suppose the hedge fund management contract specifies a period performance fee with high water mark provision, h. For all $\lambda_1, \lambda_2 \in (0,1)$ only the pooling strategy profile* pooling(1,1) *can be supported as a PBE for*

$$\eta \in (\widetilde{\eta}^u, 1),$$

if $p\varepsilon > 0.5$. If $p\varepsilon \leq 0.5$ only the separating strategy profiles separating(u,d) *and* separating(u,1) *can be supported as a PBE for all $\eta \in [0,1]$.*

Proof: See Appendix B.3.

The threshold $\widetilde{\eta}^u$ that defines whether the separating or the pooling equilibrium occurs in the market depends only on the u-type manager's decision and not on the d-type's. The d-type manager prefers to overreport her first-period return as soon as the investors beliefs on the u-type in $t = 2$ by observing the reported value of 1 to the second time is smaller than their prior belief of μ. In this situation it is irrelevant for the d-type manager whether there is capital outflow in the first period or not. The only one valuable option for her is the investors capital contribution η in $t = 2$.

4.6 Comparing Equilibria

Corollary 4 *The hedge fund manager, independent of her type and the type of the contract, has a greater incentive for misreporting of her first-period return with increasing values of $p\varepsilon$.*
If the contract specifies the period performance fee, the manager' incentive for misreporting of her first-period return increases with increasing values of λ_1 and λ_2.

Proof: See Appendix B.4.1.

The first part of Corollary 4 is intuitive, since the expected management compensation level as a percetage of expected future profits should be sufficiently large to incentivise the manager for misreporting. The u-type manager is willing to hide her type only, if her expected fee payment by achieving $V_3 = u$ after the poor second-period performance is high enough to recover her due fees. The d-type manager's achiving of the state $V_2 = 1$ is also more likely, if her success peobability $p\varepsilon$ increases. The second part of Corollary 4 says that in order to overcome capital outflow in $t = 1$ and $t = 2$ the manager misreports her first-period return for high levels of λ_1 and λ_2. More interesting fact is that if the contract specifies performance fee with the high water mark provision, the withdrawal probability λ_2 do not play such a great role in the manager's decision making. Additionally, as soon as the probability λ_1 is positive, the d-type manager is always preffering to pool with the u-type.

Comparing Propositions 6, 7 and 8 yields the central result of our model.

Proposition 9 *There exist parameters where return smoothing occurs when the hedge fund management contract specifies a performance fee with high water mark, but return smoothing is absent for a management contract with period performance fee. The opposite does not occur.*

Proof: See Appendix B.4.2.

Comparing the cutoff's for η, it is easy to see that if the management compensation contract specifies the period performance fee the threshold value η^u is larger that the counterpart $\widetilde{\eta}^u$ if the contract specifies the performance fee with the high water mark provision:

$$\frac{1-\lambda_2 p\varepsilon}{p\varepsilon} = \eta^u \geq \widetilde{\eta}^u = \frac{1-p\varepsilon}{p\varepsilon}$$
$$\Leftrightarrow 1 \geq \lambda_2.$$

In other words, the difference between the both parameters, η^u and $\widetilde{\eta}^u$, decreases with increasing λ_2, especially for $\lambda_2 \equiv 1$ we have: $\eta^u = \widetilde{\eta}^u$. Furthermore, the managerial optimal reporting choice in $t = 1$ does not depend on the withdrawal probabilities λ_1 and λ_2 if the manager's compensation contract specifies performance fee with the high water mark provision. To make the futher discussion more intuitive, we use a numerical example presented in Table 4.

	Corresponding Equilibria if the Contract Specifies	
	Period Performance fee	**Performance Fee With High Water Mark Provision**
$p\varepsilon = 56\%$	$\eta^u = 1.486 > 1 \Rightarrow$ only ***Separating(u,d)*** for all $\eta \in [0,1]$	$\widetilde{\eta}^u = 0.786 \Rightarrow$ ***Pooling(1,1)*** for $\eta \in (0.786,1)$
$p\varepsilon = 64\%$	$\eta^u = 1.263 > 1 \Rightarrow$ only ***Separating(u,d)*** for all $\eta \in [0,1]$	$\widetilde{\eta}^u = 0.563 \Rightarrow$ ***Pooling(1,1)*** for $\eta \in (0.536,1)$
$p\varepsilon = 72\%$	$\eta^u = 1.089 > 1 \Rightarrow$ only ***Separating(u,d)*** for all $\eta \in [0,1]$	$\widetilde{\eta}^u = 0.389 \Rightarrow$ ***Pooling(1,1)*** for $\eta \in (0.389,1)$
$p\varepsilon = 81\%$	$\eta^u = 0.935 \Rightarrow$ ***Pooling(1,1)*** for $\eta \in (0.935,1)$	$\widetilde{\eta}^u = 0.235 \Rightarrow$ ***Pooling(1,1)*** for $\eta \in (0.235,1)$

Table 4: Numerical Examples. (Parameters: $u = 1.05, \nu = 0.6, \lambda_2 = 0.3, \lambda_2 \geq \lambda_1 > \lambda_1^*$.)
The table schows the equilibria depending on the level of the expected probability weighted additional capital contribution η. With increasing values on$p\varepsilon$ the manager of each type has stronger incentives for misreporting. (We choose with intent $\lambda_2 \geq \lambda_1 > \lambda_1^*$, for smaller values on λ_1 there exist only ***Separating(u,d)*** if the contract specifies the period performance fee.)

If the contract specifies the period performance fee, in the numerical example (see Table 4) the pooling equilibrium occurs for $p\varepsilon \geq \frac{1}{1+\lambda_2} = 0.77$ (shown in Proposition 6). If the contract specifies performance fee with the high water mark provision, the pooling equilibrium occurs already for smaller values $p\varepsilon \geq 0.5$ (shown in Proposition 8). With increasing values on $p\varepsilon$ the needful expected amount η to incentivise the manager for misreporting decreases.

Proposition 9 shows that management contracts with high water marks are more vulnerable to managerial return smoothing than those with period performance fees. The reason for this is that both types of managers have stronger incentives to report $v_1 = 1$ when they receive fees based on value gains relative to the historic fund maximum. Note that the incentive to smooth returns does not depend on the magnitude of the fees, f and h, Thus, even if h were to be higher than f – because period performance fees are paid in more instances that performance fees with high water mark – the incentives to smooth returns do not change.

To see the economic reasoning for Proposition 9 consider first a u-type manager whose management contract specifies a performance fee with high water mark. If she reports V_1 truthfully, this raises the high water mark for future periods. In case of future inflows the manager does not financially benefit from these unless the fund's asset value exceeds the new high water mark. This is different if the manager decides to report a lower than the actual return. Then she benefits more from future inflows, because of a lower high water mark. This is different for a manager who operates an otherwise identical fund with a period performance fee. Given that the historically maximal fund value is irrelevant for fund fees, the described consideration does not affect the manager's reporting decision. As a consequence, managers of funds with performance fees with high water marks have stronger incentives to underreport positive actual returns than managers of funds whose management contracts specify period performance fees.

Managers with high water marks also have stronger incentives to overreport negative returns when doing so prohibits subsequent outflows. The reason is that reporting a zero return rather than the actual negative one does not affect the high water mark and therefore is inconsequential for future fees. Again, this is different for managers whose fee income is determined by the performance in each period. Reporting a zero return rather that the actual negative one affects future returns negatively as the coming period's return is based on this period's reported asset value. In sum, fund managers whose performance is measured against their funds' high water marks have a stronger incentives to report smuted returns than those of funds specifying period performance fees.

Corollary 5 *If the management compensation contract specifies performance fee with the high water mark provision h and the withdrawal probability λ_2 is smaller than 1, smoothing of the hedge fund reported returns occurs already for smaller success probabilities $p\varepsilon$ as if the contract specifies the period performance fee f.*

Proof: See Appendix B.4.3.

4.7 Extended Model Version

The basic model considers not many posibilities to achieve a high water mark, especially, if the first-period return was low equal to d. If the contract specifies performance fee with the high water mark provision, the d-type manager is able to receive a positive fee amount only in the third period by achieving a value of assets under management equal to $V_3 = u$. Also the u-type manager does not have possibility to receive a positive fee amount once the second-period return was low.

Consider an extension of the basic model in the following way. Denote by $R^H > 0$ the fund's positive net return, so the positive gross return equals to $u := 1+R^H$. Analoguously, denote by $R^L < 0$ the negative fund's return, so the negative gross return equals to $d := 1+R^L$. (Note, that the assumption $u = 1/d$ can be still satisfied for some parameters R^H and R^L but is not longer general.)

Now there are several possibilities to achieve new high water marks in additional states compared to the basic model (see Figure 12). At first consider the u-type manager: after the truthful reporting of $v_1 = u$ the new high water mark is set. If the u-type manager is additionally successfull in the second and also in the third periods, she recives twice the performance fee with the high water mark provision, exactly as in the basic model. Once the u-type manager underperforms in the second period there is now a new possibility to set a high water mark, if the manager succeeds in the third period. In this state the new value of assets under management equals to $V_3 = u^2 d = (1 + R^H)^2(1 + R^L)$. The new high water mark is set if $V_3 > V_1$, this condition is satisfied for

$$\Big(R^L + R^H(1 + R^L)\Big) > 0. \qquad \text{(A3)}$$

The second-period return of $V_2 = ud = (1 + R^H)(1 + R^L)$ is also a new high water mark relative to $V_0 = 1$, if (A3) is still satisfied. Consider now the d-type manager: after the truthful reporting $V_1 = d$ or overeporting $V_1 = 1$ the new high water mark is set if $V_2 = ud > 1$ and, thus, (A3) is satisfied. The third high water mark is possible, if the d-type manager is also successfull in the third period by generating $V_3 = u^2 d$, which value is larger than the last high water mark $V_2 = ud$.

Investors' beliefs and managerial incentives for misreporting of her first-period return do not change compared to the basic model. If the management compensation contract specifies the period performance fee f, the modified thresholds[42], that are crucial for

[42]For proofs see Appendix B.5.

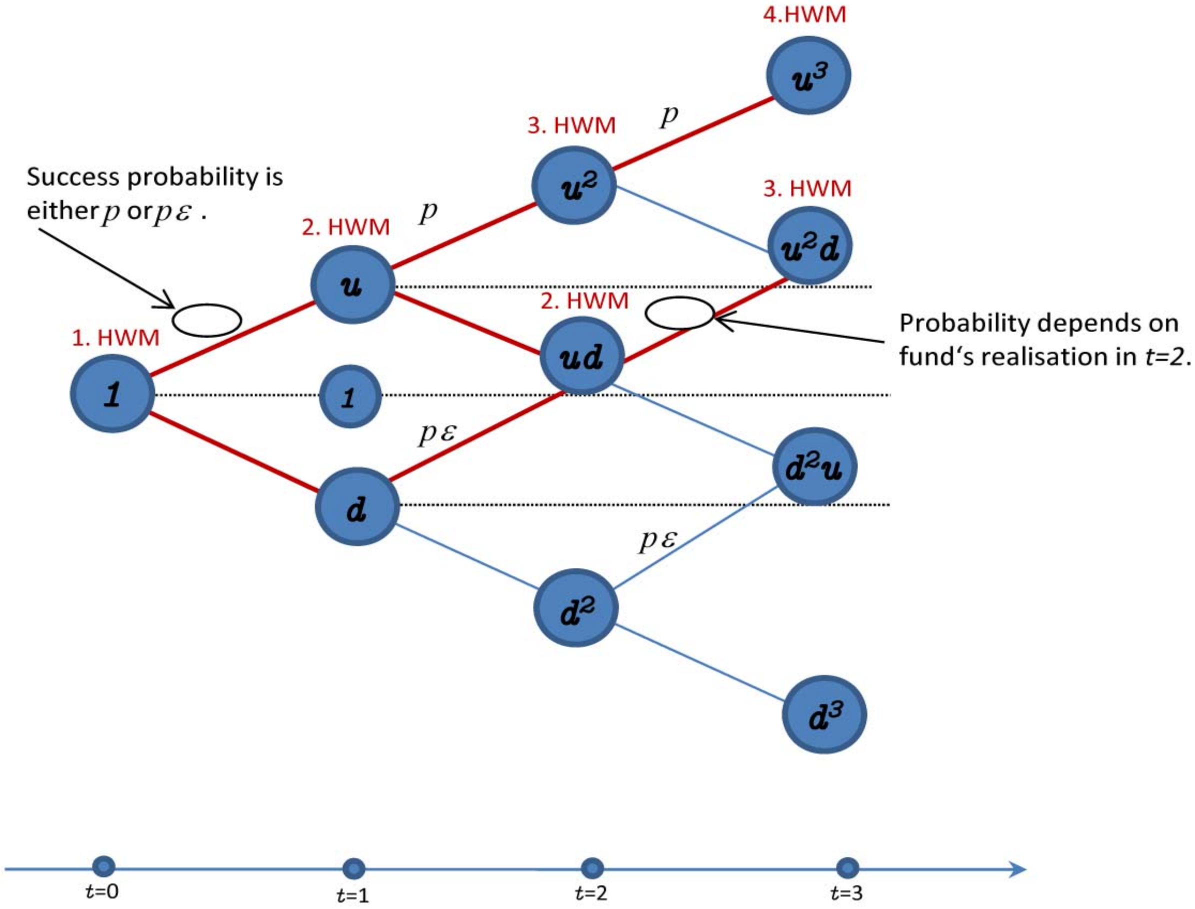

Figure 12: Return probabilities.
The high water mark can be achieved in more different states compared to the basic model .

determining the Perfect Bayesian Equilibria, are

$$\eta^u_{new} = \frac{1-\lambda_2 p\varepsilon ud}{p\varepsilon} \quad \Leftrightarrow \quad \lambda^u_{2\,new} = \frac{1-p\varepsilon\eta}{p\varepsilon ud} \qquad and \quad p\varepsilon \geq \frac{1}{1+\lambda_2 ud}. \tag{17}$$

The new threshold for λ_1 is given by

$$\lambda^*_{1\,new} := \frac{\varepsilon d}{1+\varepsilon d(1+d)+p\varepsilon d(1-\lambda_2)(u-d\varepsilon)+\varepsilon d^2\lambda_2}.$$

If the management compensation contract specifies the period performance fee with a high water mark provision h, the Perfect Bayesian Equilibria remain also alike in the basic model with the following thresholds:

$$\widetilde{\eta}^u_{new} = \frac{u(1-d)(1-p\varepsilon)-\lambda_2 p\varepsilon u(ud-1)}{p\varepsilon(u-1)}$$

$$\Leftrightarrow \quad \widetilde{\lambda}^u_{2\,new} = \frac{u(1-d)(1-p\varepsilon)-p\varepsilon\eta(u-1)}{p\varepsilon u(ud-1)} \quad \textit{and} \quad p\varepsilon \geq \frac{u(1-d)}{u(1-d)+\lambda_2 u(ud-1)+u-1}.$$

We now can better analyse how the change in expected returns affect managerial optimal reporting policy. Recall, that if the contract cpecifies period performance fee, the strategy profile *pooling (1,1)* is a PBE for $\eta \in [\eta^u_{new}, 1]$. Analogously, if the contract specifies performance fee with high water mark provision, the strategy profile *pooling (1,1)* is a PBE for $\eta \in [\widetilde{\eta}^u_{new}, 1]$.

We begin with numerical examples that provide some intuition about the changes in η^u_{new} and $\widetilde{\eta}^u_{new}$ for varying values on $p\varepsilon$, λ_2, R^H and R^L.

a) *Numerical Example:* η^u_{new} *and* $\widetilde{\eta}^u_{new}$ *as functions of* λ_2 *and* $p\varepsilon$*.*

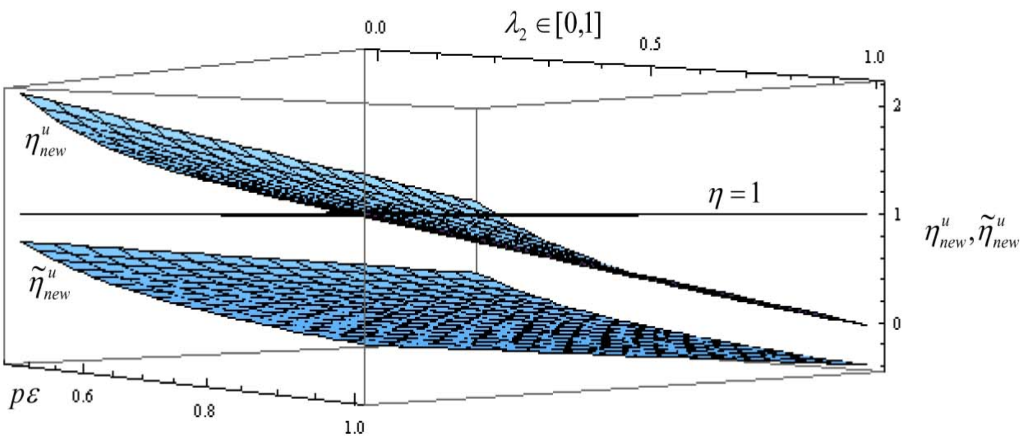

Figure 13: Difference between threshold values $\eta^u_{new}, \widetilde{\eta}^u_{new}$ as a function of λ_2 and $p\varepsilon$.

Depicted on the vertical axis is the analytic difference between the probability weighted values of additional capital contribution if the contract specifies period performance fee η^u_{new} (upper graph) and performance fee with the high water mark provision $\widetilde{\eta}^u_{new}$ (lower graph) depending on the lower success probability $p\varepsilon$ and the capital withdrawal probability in the second period λ_2.
Parameters are set to the following values: $\eta^u_{new}, \widetilde{\eta}^u_{new} \in [0,1], \lambda_2 \in [0,1], p\varepsilon \in [0.45,1], u = 1.05 (\Leftrightarrow R^H = 5\%)$, $d = 0.97 (\Leftrightarrow R^L = -3\%)$.

Suppose the parameters to be set to the following values: $R^H = 5\%$, which means that the fund's positive gross return is set to the value $u = 1.05$, and $R^L = -3\%$, which means that the fund's negative gross return is set to the value $d = 0.97$. The withdrawal probability in the second period λ_2 can take all values between 0 and 1. Use the result from above (17) to see that the values for $p\varepsilon$ are not permitted to be to small, more precise, it holds: $p\varepsilon \in [0.45, 1]$.

η^u_{new} and $\widetilde{\eta}^u_{new}$ as functions of λ_2 and $p\varepsilon$ are presented in Figure 13. On the vertical axis is shown the analytical difference between the probability weighted values of additional capital contribution if the contract specifies period performance fee η^u_{new} and performance fee with the high water mark provision $\widetilde{\eta}^u_{new}$. If we additionaly consider that $\eta \leq 1$, it is obviously in our example that PBE *pooling(1,1)* exists for all values $p\varepsilon \in [0.45, 1]$, $\lambda_2 \in [0, 1]$ (above the graph of $\widetilde{\eta}^u_{new}$) if the contract specifies performance fee with the high water mark provision. If the contract specifies period performance fee (compare the graph of η^u_{new}), the PBE *pooling(1,1)* does not exist for high values on $p\varepsilon$ and λ_2. In general, the example shows the same result like the basic model (compare Proposition 9), that managerial incentive for misreporting increases with increasing values on $p\varepsilon$ and λ_2. [43]

b) *Numerical Example:* η^u_{new} *and* $\widetilde{\eta}^u_{new}$ *as functions of* R^H *and* R^L.

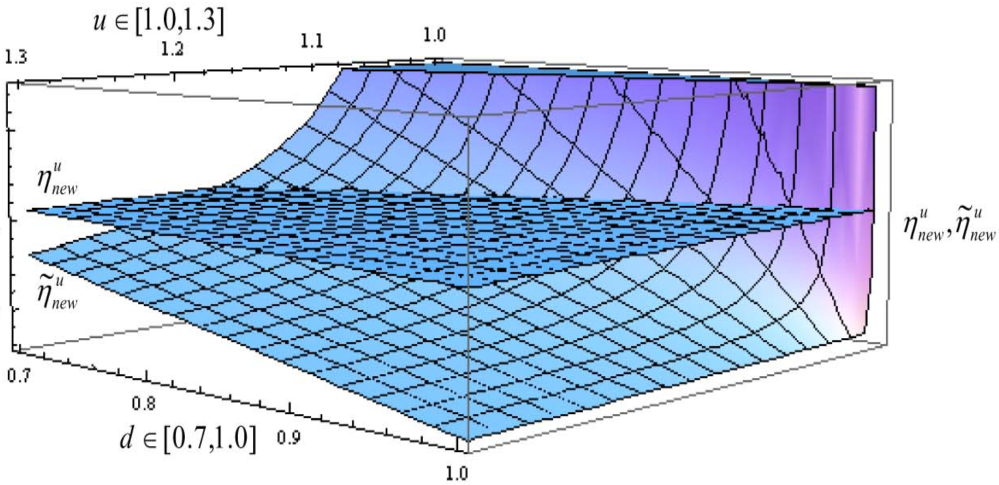

Figure 14: Difference between threshold values $\eta^u_{new}, \widetilde{\eta}^u_{new}$ as a function of u and d.

Depicted on the vertical axis is the analytic difference between the probability weighted values of additional capital contribution if the contract specifies period performance fee η^u_{new} (midle graph) and performance fee with the high water mark provision $\widetilde{\eta}^u_{new}$ depending on the positive gross return u and the negative gross return d.
Parameters are set to the following values: $\eta^u_{new}, \widetilde{\eta}^u_{new} \in [0,1], \lambda_2 = 0.3, p\varepsilon = 0.6, u \in [1.0,1.3], d \in [0.7,1.0]$.

Consider now η^u_{new} and $\widetilde{\eta}^u_{new}$ as functions of R^H and R^L. The remaining parameters are set to the following values: the withdrawal probability λ_2 in $t = 2$ equals 30%, the decreased success probability $p\varepsilon$ equals 60%. We assume additionally the positive net return, R^H, varying between 0 and 30% ($u \in [1.0, 1.3]$), the negative net return, R^L, between -30% and 0 ($d \in [0.7, 1.0]$). Figure 14 demonstrates the numerical example. Remarkable is the sensitvity of the function $\widetilde{\eta}^u_{new}(R^H, R^L)$.

[43]For general proof see Appendix B.4

4.8 Concluding Remarks

In this chapter we compared contracts that pay a performance fee to the hedge fund manager whenever the reported value of the fund's assets increases relative to that at the beginning of a period (period performance fee) versus a performance fee that is paid only if the reported asset value of the fund exceeds its historic maximum (performance fee with high water mark). We can show that standard features in hedge fund management contracts can have a profound impact on income smoothing, because they affect the manager's benefits of reporting income streams with low volatilities. When reporting the value of their funds' assets, hedge fund managers that maximize expected fee payments take into account both the performance fee generated by the report and the expected fund flow resulting from the report.

The main result shows that managers of funds with performance fees with high water marks have stronger incentives for both, underreporting positive actual returns and over-reporting negative actual returns when doing so prohibits outflows.

The smoothing of hedge fund returns requires that not only funds with poor performance overreport the value of their portfolio but also that funds with solid performance are willing to pool with them. The latter is the case when the expected increase in fee income of reporting solid performance todayis outweighed by the negative effects on fee levels of potentially having to report poor performance in the future. We show that this relationship holds more commonly for fund managers whose performance fees are paid only if the reported value of the fund's holdings exceeds its historic maximum.

The smoothness of the reported fund returns typically leads investors to understate both the true volatility of these strategies and their correlation with traditional asset classes. As a concequence, the investors are unable to detect suitable methods for performance and risk measurement.

5 General Conclusion

This thesis compares two different types of performance fee structures in hedge fund management compensation contracts under information asymmetry. We differ between two incentive structures stipulated between the hedge fund manager and the investors: firstly, fees based solely on the performance during the preceding period and, secondly, fees based on the performance relative to the historical fund value maximum - the high water mark. We find that the type of the contract fundamentally influences both, the manager's and the investors' optimal incentive structures.

In the first theoretical model we assume the hedge fund manager to be better informed about the future fund profitability than the investors. We assume, additionally, that the fund manager's incentives to close the hedge fund are not necessarily aligned with those of the investors. The hedge fund management contract that leads to the efficient closing should specify low expected fees in circumstances in which fund closure may be efficient. This is typically the case when the recent performance has been poor. We develop the argument about the efficient fund closing and can further show that the compensation arrangement between the hedge fund manager and the investors' optimally includes the high water mark provision opposed to the contract which employs period performance fee.

The crucial benefit of the high water mark provision contract is that it reduces the manager's expected performance fees in a state of poor realized returns. This enhances the manager's incentives to efficiently close the fund before realizing potential losses. But also for the investors a compensation contract that specifies a high water mark provision is beneficial, since they anticipate the manager's more efficient fund closing. Given this anticipation we can show that the investors have an increased willingness to provide capital to the hedge fund. However, in periods of poor fund performance the investors may refuse to withdraw their investment from the fund.

Finally, in the context of the optimal closing policy, the efficient incentive effects for high water mark contracts have their limits. Since the manager does not fully participate in value gains, under some conditions she may close the fund in states where it has still intact prospects.

The second main chapter of this thesis models a three period theoretical framework where managers report about their earnings, which can result in return smoothing. There are two different types of managers in the market who either realize positive or negative first-period returns. Depending on the realized first-period return the managers learn about their type. We assume this to be hidden information, observable only to the manager. This scenario sets the manager into the position to either truly report or misreport her actual earnings.

We explicitly model managerial incentive problems contingent on the choice of the compensation contract and recognize that the high water mark provision contract is more vulnerable to managerial return smoothing, compared to the contract with period performance fee, and therefore can imply inefficiencies on the investors' side. The reason for this is that both types of managers have stronger incentives to misreport their first-period return when they receive fees based on value gains relative to the historic fund maximum.

In line with current empirical literature we also find that managers under high water mark contracts have stronger incentives to overreport negative returns, when doing so prohibits subsequent outflows. Consequently, in case the manager reports smoothed reported fund returns, the investors have to accept the inefficiency, which results in slow learning about the manager's quality and therefore the fund's prospects.

An interesting implication of this result is that the smoothed reported fund returns in turn influence the investors' withdrawal policy. Since the investors are unable to learn about the manager's quality and the fund's prospects, they cannot efficiently decide whether to withdraw their capital or to make an additional investment.

The effects of high water marks in hedge fund management contract design may not only be restricted to the factors covered in this thesis, such as efficient fund closing or return smoothing. Further research may also take into account other influencing factors. For instance high water marks may also have material implications on the manager's incentives concerning portfolio choice or redemption policy.

Appendix A

We use the following notation for all proofs:

The indicator function of a subset A of a some set X is a function $\mathbb{I}_A : X \to \{0,1\}$ defined as $\mathbb{I}_A(x) := \begin{cases} 1 & if \quad x \in A \\ 0 & if \quad x \notin A. \end{cases}$

$\phi_{\mathbb{I}} := p^2 R^H (1+R^H)^2 \eta + (1-p)(1+R^L)(R^L \mathbb{I} + R^H (1+R^L)) \Big(p\theta\eta + (p-\varepsilon)(1-\theta)(\eta - \varepsilon(R^H - R^L))\Big)$

$\varphi_{\mathbb{I}} := p^3 (R^H (1+R^H))^2 + (1-p)(R^L \mathbb{I} + R^H (1+R^L))^2 \Big(p^2\theta + (p-\varepsilon)^2 (1-\theta)\Big)$

$\xi := p(1+R^H)^2 + (1-p)(1+R^L)^2$

For parameters $R^H > 0$, $R^L \in (\frac{-pR^H}{1-p}, \frac{-(p-\varepsilon)R^H}{1-p+\varepsilon})$, [44] $p \in (0,1)$, $\varepsilon \in (0,p)$ and $\theta \in (0,1)$ it is straightforward that $\phi_{\mathbb{I}}, \varphi_{\mathbb{I}}, \xi > 0$.

A.1 Proof of Proposition 1. Consider a contract with the period performance fee rate $a > 0$ and a contract with a high water mark provision and performance fee rate $\widetilde{a} > 0$, so that the following condition is satisfied:

$$\widetilde{a} = a \underbrace{\frac{R^H(1+R^L)}{(R^L + R^H(1+R^L))}}_{>1, \quad since R^L < 0}.$$

Note, that the performance fee rate with a high water mark provision $\widetilde{a}$ is strictly larger than the corresponding period performance fee rate a in this setting. The manger's closing threshholds in state L^- are equal in both regimes: $\omega^{L^-}(a) \equiv \omega^{L^-}(\widetilde{a})$, and also in state L°:

$$\omega^{L^\circ}(\widetilde{a}) = a \frac{R^H(1+R^L)}{(R^L + R^H(1+R^L))} p(R^L + R^H(1+R^L)) = apR^H(1+R^L) = \omega^{L^\circ}(a).$$

Calculating the corresponding closing thresholds in state H yields:

$$\omega^H(\widetilde{a}) = \underbrace{apR^H(1+R^H)}_{=\omega^H(a)} \underbrace{\frac{R^H(1+R^L)}{(R^L + R^H(1+R^L))}}_{>1} > \omega^H(a)$$

[44] Follows from the assumptions (A1) and (A2).

Using (1) and comparing the fund's expected surplus under the contract with the period performance fee a and the contract with the performance fee with a high water mark provision $\widetilde{a}$ yields:

$$S(\omega^H,\omega^{L^\circ},\omega^{L^-})\,(\widetilde{a}) \geq S(\omega^H,\omega^{L^\circ},\omega^{L^-})(a)$$

$$\Leftrightarrow (1+R^H)\eta\Big(F(\omega^H(\widetilde{a}))-F(\omega^H(a))\Big)+(1-F(\omega^H(\widetilde{a})))\mathbb{E}(\omega|\omega\geq\omega^H(\widetilde{a}))-(1-F(\omega^H(a)))\mathbb{E}(\omega|\omega\geq\omega^H(a))\geq 0$$

$$\Leftrightarrow (1+R^H)\eta\underbrace{\Pr(\omega^H(a)\leq\omega<\omega^H(\widetilde{a}))}_{>0,\quad since\quad \omega^H(a)<\omega^H(\widetilde{a})}+\int_{\omega^H(\widetilde{a})}^{\omega_{max}}\omega f(\omega)d\omega-\int_{\omega^H(a)}^{\omega_{max}}\omega f(\omega)d\omega\geq 0$$

$$\Leftrightarrow \int_{\omega^H(a)}^{\omega^H(\widetilde{a})}\Big((1+R^H)\eta-\omega\Big)f(\omega)d\omega>0$$

The last integral is positive for all $\omega\in[0,\,\omega_{max}]$ with $\omega_{max}=(1+R^H)\eta$. ■

A.2 Proof of Proposition 2. Rewrite the equality (2)

$$S(\omega^H,\omega^{L^\circ},\omega^{L^-})=\eta+p\frac{\omega^H}{\omega_{max}}(1+R^H)\eta+(1-p)\theta\frac{\omega^{L^\circ}}{\omega_{max}}(1+R^L)\eta+(1-p)(1-\theta)\frac{\omega^{L^-}}{\omega_{max}}(1+R^L)(\eta-\varepsilon(R^H-R^L))$$

Using definitions for ω^H, ω^{L° and ω^{L^-} we receive:
$S(\omega^H(a),\omega^{L^\circ}(a),\omega^{L^-}(a))=\eta+\frac{a_{\mathbb{I}}\phi_{\mathbb{I}}}{\omega_{max}}+\frac{a_{\mathbb{I}}^2}{2\omega_{max}}$.
The expected surplus-function, as a parabola, is twice continuously differentiable at $a_{\mathbb{I}}$. The first derivative of the function equals to zero at the extrema:

$$a_{\mathbb{I}}=\frac{\phi_{\mathbb{I}}}{\varphi_{\mathbb{I}}}:=\begin{cases}\widetilde{a}^* & if\quad \mathbb{I}\equiv 1\\ a^* & if\quad \mathbb{I}\equiv 0\end{cases}$$

The second order condition yields

$$\frac{d^2S(\omega^H,\omega^{L^\circ},\omega^{L^-})}{da_{\mathbb{I}}}=\frac{-1}{\omega_{max}}\Big(p^3(R^H(1+R^H))^2+(1-p)(R^L\mathbb{I}+R^H(1+R^L))^2(p^2\theta+(p-\varepsilon)^2(1-\theta))\Big)$$

and is constant and negative for all parameters in the domain of the definition. This shows that the expected surplus has a unique global maximum at $a_{\mathbb{I}}$.

Because the investor must break even, a maximal possible amount that can be collected in $k_{\mathbb{I}}$ with respect to the investor's participation constraint (1) is equal to: $k_{\mathbb{I}}=\eta-a_{\mathbb{I}}pR^H$. Thus, the optimal contract with a high water mark provision is given by $\left(\widetilde{a}^*,\widetilde{k}^*\right)$ and optimal contract with period performance fee is given by (a^*,k^*) .

Lemma 1 *The performance fee rate a^* in the optimal contract with period performance fee is smaller than the performance fee rate $\widetilde{a}^*$ in the optimal contract with high water mark provision $\widetilde{a}^*$. The management fee k^* is larger than $\widetilde{k}^*$.*

Proof: We show that $\widetilde{a}^* = \frac{\phi_1}{\varphi_1} > \frac{\phi_0}{\varphi_0} = a^*$. Recall that $\varphi_{\mathbb{I}} > 0$, so we have

$$\begin{aligned}
0 &< \phi_1\varphi_0 - \phi_0\varphi_1 \\
&\Leftrightarrow \underbrace{(R^L+2R^H(1+R^L))}_{>R^H(1+R^L)}\Big(\theta+(1-\frac{\varepsilon}{p})^2(1-\theta)\Big) > R^H(1+R^L)\Big(\theta+(1-\frac{\varepsilon}{p})(1-\theta)(1-\frac{\varepsilon}{\eta}(R^H-R^L))\Big) \\
&\Leftrightarrow \theta+(1-\frac{\varepsilon}{p})^2(1-\theta) > \theta+(1-\frac{\varepsilon}{p})(1-\theta)(1-\frac{\varepsilon}{\eta}(R^H-R^L)) \\
&\Leftrightarrow (1-\frac{\varepsilon}{p}) > (1-\frac{\varepsilon}{\eta}(R^H-R^L)) \\
&\Leftrightarrow 0 \gneqq R^L \rightarrow True
\end{aligned}$$

With $\widetilde{a}^* > a^*$ we can argue: $\widetilde{a}^* pR^H > a^* pR^H \Leftrightarrow \eta - \widetilde{a}^* pR^H < \eta - a^* pR^H \Leftrightarrow \widetilde{k}^* < k^*$. For $k_{\mathbb{I}} \equiv 0$, we have $0 = \eta - a_{\mathbb{I}} pR^H \Leftrightarrow a_{\mathbb{I}} = \frac{\eta}{pR^H}$, thus the restriction on management fee, $k \geq 0$, is binding iff the optimal performance fee rate exceeds the value $\frac{\eta}{pR^H}$. ■

In the next step we show that from the ex ante perspective, the optimal contract with a high water mark provision leads to strictly lower closing probability in state H. Pr(state H) is equal to$1 - F(\omega^H(a))$, hence it is sufficient to show that:
$F(\omega^H(a^*)) = \frac{\omega^H(a^*)}{\omega_{max}} < \frac{\omega^H(\widetilde{a}^*)}{\omega_{max}} = F(\omega^H(\widetilde{a}^*)) \Leftrightarrow \omega^H(a^*) < \omega^H(\widetilde{a}^*)$:

$$\omega^H(\widetilde{a}^*) \equiv \omega^H(a^*\omega^H(\widetilde{a}^*) \equiv \widetilde{a}^* pR^H(1+R^H) > a^* pR^H(1+R^H) \equiv \omega^H(a^*).$$

We show now that from ex ante perspective, the optimal contract with a high water mark provision leads to higher closing probabilities in states L° and L^-. Proof for state L°:

$$\begin{aligned}
\omega^{L^\circ}(a^*) &> \omega^{L^\circ}(\widetilde{a}^*) \\
\Leftrightarrow a^* pR^H(1+R^L) &> \widetilde{a}^* p(R^L + R^H(1+R^L)) \\
\Leftrightarrow \phi_0\varphi_1 R^H(1+R^L) &> \phi_1\varphi_0(R^L + R^H(1+R^L)) \\
\Leftrightarrow \underbrace{R^H(1+R^L)}_{>0}\underbrace{(\phi_0\varphi_1 - \phi_1\varphi_0)}_{>0} &> \underbrace{\phi_0\varphi_1}_{>0}\underbrace{R^L}_{<0}
\end{aligned}$$

Analogously for state L^-:

$$\begin{aligned}
\omega^{L^-}(a^*) &> \omega^{L^-}(\widetilde{a}^*) \\
a^*(p-\varepsilon)R^H(1+R^L) &> \widetilde{a}^*(p-\varepsilon)(R^L + R^H(1+R^L))
\end{aligned}$$

use the same proof as in state L°. ■

A.3 Proofs of Corollaries 1, 2 and 3.

A.3.1 **Proof of Corollary 1**: With proof of Proposition 2 we have: $\omega^{L^\circ}(a^*) > \omega^{L^\circ}(\widetilde{a}^*)$. Thus, the closing probability upon a negative first-period return in state L° for the optimal contract with high water mark provision is larger than the corresponding probability for the optimal contract with period performance fee: $1 - F(\omega^{L^\circ}(\widetilde{a}^*)) > 1 - F(\omega^{L^\circ}(a^*))$.

Analogously we have $1 - F(\omega^{L^-}(\widetilde{a}^*)) > 1 - F(\omega^{L^-}(a^*))$ in state L^-. Thus, the weighted closing probability upon a negative first-period return for the optimal contract with high water mark provision is higher than the corresponing weighted closing probability for the optimal contract with period performance fee:

$$\theta(1 - F(\omega^{L^\circ}(\widetilde{a}^*))) + (1-\theta)(1 - F(\omega^{L^-}(\widetilde{a}^*))) > \theta(1 - F(\omega^{L^\circ}(a^*))) + (1-\theta)(1 - F(\omega^{L^-}(a^*)))$$

$$\Leftrightarrow \theta\Big(\underbrace{F(\omega^{L^\circ}(a^*)) - F(\omega^{L^\circ}(\widetilde{a}^*))}_{>0}\Big) + (1-\theta)\Big(\underbrace{F(\omega^{L^-}(a^*) - F(\omega^{L^-}(\widetilde{a}^*))}_{>0}\Big) > 0 \qquad ■$$

A.3.2 **Proof of Corollary 2**: In the first step we use Proposition 2 and Corollary 1 to see: $f^{LH}(a^*) = a^*R^H(1 + R^L) > \widetilde{a}^*(R^L + R^H(1+R^L)) = f^{LH}(\widetilde{a}^*)$. Conditional on fund continuation after a negative first-period return, the investor has a posterior belief to face state L° equal to $\theta F(\omega^{L^\circ})/(\theta F(\omega^{L^\circ}) + (1-\theta)F(\omega^{L^-}))$, that is independent of whether the contract specifies a high water mark provision or a period performance fee, because

$$\begin{aligned}
\frac{\theta\omega^{L^\circ}(\widetilde{a}^*)}{\theta\omega^{L^\circ}(\widetilde{a}^*)+(1-\theta)\omega^{L^-}(\widetilde{a}^*)} &= \frac{\theta\omega^{L^\circ}(a^*)}{\theta\omega^{L^\circ}(a^*)+(1-\theta)\omega^{L^-}(a^*)} \\
\Leftrightarrow \omega^{L^\circ}(\widetilde{a}^*)\omega^{L^-}(a^*) &= \omega^{L^\circ}(a^*)\omega^{L^-}(\widetilde{a}^*) \\
\Leftrightarrow \frac{p}{2p-\varepsilon} &= \frac{p}{2p-\varepsilon}.
\end{aligned}$$

In the second step we compare the expected fund's second-period after-fee return in both regimes:

$$\begin{aligned}
\frac{\theta F(\omega^{L^\circ}(\widetilde{a}^*))p(V^{LH} - \widetilde{a}^*(R^L+R^H(1+R^L)))}{\theta F(\omega^{L^\circ}(\widetilde{a}^*)) + (1-\theta)F(\omega^{L^-}(\widetilde{a}^*))} &> \frac{\theta F(\omega^{L^\circ}(a^*))p(V^{LH} - a^*R^H(1+R^L))}{\theta F(\omega^{L^\circ}(a^*))+(1-\theta)F(\omega^{L^-}(a^*))} \\
\Leftrightarrow V^{LH} - \widetilde{a}^*(R^L+R^H(1+R^L)) &> V^{LH} - a^*R^H(1+R^L) \\
\Leftrightarrow a^*R^H(1+R^L) &> \widetilde{a}^*(R^L+R^H(1+R^L)). \qquad ■
\end{aligned}$$

A.3.3 **Proof of Corollary 3**: The manager's outside income in period 2 is uniformly distributed on $\left[0, \left(1+R^H\right)\eta\right]$. Thus the manager chooses the optimal contract with the high water mark provision given by $\left(\widetilde{a}^* = \frac{\phi_1}{\varphi_1}, \widetilde{k}^* = \eta - \widetilde{a}^* pR^H\right)$. The management fee can be rewritten as $\widetilde{k}^* = pR^H + (1-p)R^L - \widetilde{a}^* pR^H = pR^H(1-\widetilde{a}^*) + (1-p)R^L$. Recall that $\widetilde{a}^*$ depends on R^H, so the first derivative of $\widetilde{k}^*$ equals:

$$\frac{d\widetilde{k}^*(R^H)}{dR^H} = \frac{d}{dR^H} pR^H(1-\widetilde{a}^*) + \frac{d}{dR^H}(1-p)R^L = p\Big(\underbrace{(1-\widetilde{a}^*) + \frac{R^H}{\varphi_1}(\widetilde{a}^* \frac{d\varphi_1}{dR^H} - \frac{d\phi_1}{dR^H})}_{>0}\Big) > 0$$

because $\widetilde{a}^* = \frac{\phi_1}{\varphi_1} < 1 < \frac{1-R^H \frac{d\phi_1}{dR^H}}{1-R^H \frac{d\varphi_1}{dR^H}}$ and $\frac{d\varphi_1}{dR^H} < \frac{d\phi_1}{dR^H}$. For a numerical example see Figure 4. ■

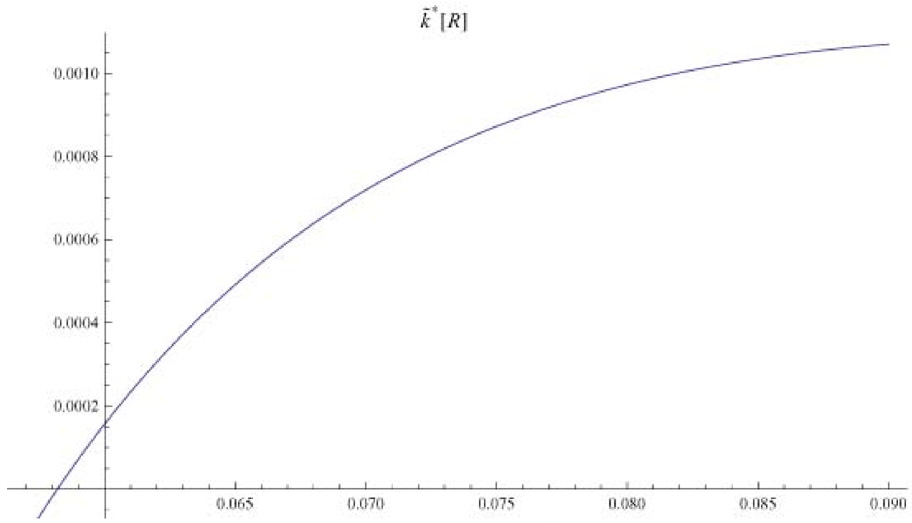

Figure 15: Numerical Example. Parameters: ***R^L=-5.5%, p=5.5, ε=0. 3, Θ=0.2***. Changes in management fee for varying values of ***R^H*** .

A.4 Proof of Proposition 3:

In the situation with high uncertainty about the fund's prospects after low first-period return associated with low θ, the both performance fee parameter a^* and $\widetilde{a}^*$ are below the boundary value $\frac{\eta}{pR^H}$. With increase on θ both parameter increase also:

$$\frac{da_{\mathbb{I}}(\theta)}{d\theta} = \frac{(1-p)\varepsilon(R^L\mathbb{1} + R^H(1+R^L))}{\varphi_{\mathbb{I}}^2}\Big(\underbrace{(1+R^L)(\eta + (p-\varepsilon)(R^H - R^L))\varphi_{\mathbb{I}} - (2p-\varepsilon)(R^L\mathbb{1} + R^H(1+R^L))\phi_{\mathbb{I}}}_{>0, \quad due \quad to \quad (R^L + R^H(1+R^L)) \geq 0}\Big).$$

That leads to the simultaneously decrease in $k_{\mathbb{I}}^* = \eta - a_{\mathbb{I}}^* pR^H$. For θ equal to 1 the maximal level on a^* equals

$$\lim_{\theta\to 1} a^* = \frac{p^2R^H(1+R^H)^2\eta+(1-p)(1+R^L)^2R^Hp\eta}{p^3(R^H(1+R^H))^2+(1-p)(R^H(1+R^L))^2p^2} = \frac{\eta}{pR^H}.$$

Additionally we conclude that $k^* = \eta - a^*pR^H = \eta - \frac{\eta}{pR^H}pR^H = 0$. Rewrite $\widetilde{a}^*$ in a new way:

$$\widetilde{a}^* = \frac{\eta}{pR^H}\Big(\underbrace{\frac{p(1+R^H)^2+(1-p)(1+R^L)(\frac{R^L}{R^H}+(1+R^L))(\theta+(1-\frac{\varepsilon}{p})(1-\theta)(1-\frac{\varepsilon(R^H-R^L)}{\eta}))}{p(1+R^H)^2+(1-p)(\frac{R^L}{R^H}+(1+R^L))^2(\theta+(1-\frac{\varepsilon}{p})^2(1-\theta))}}_{:=\Upsilon(\theta)}\Big).$$

The value of $\widetilde{a}^*$ is equal to $\frac{\eta}{pR^H}$ for $\Upsilon(\theta) \equiv 1$. Thus, the critical value $\theta^\circ \in (0,1)$, below which the management fee restriction, $k \geq 0$, is binding, satisfies the condition: $\Upsilon(\theta^\circ) \leq 1$. For $\Upsilon > 1$ and increasing $\theta > \theta^\circ$ the value of $\widetilde{a}^*$ exceeds $\frac{\eta}{pR^H}$ and for $\theta = 1$ the performance fee rate with high water mark provision $\widetilde{a}^*$equals to

$$\lim_{\theta\to 1}\widetilde{a}^* = \frac{\eta}{pR^H}\Big(\underbrace{\frac{\xi+(1-p)\frac{R^L}{R^H}(1+R^L)}{\xi+(1-p)\frac{R^L}{R^H}(\frac{R^L}{R^H}+2(1+R^L))}}_{>1,\quad due\quad to\quad (R^L+R^H(1+R^L))\geq 0}\Big).$$

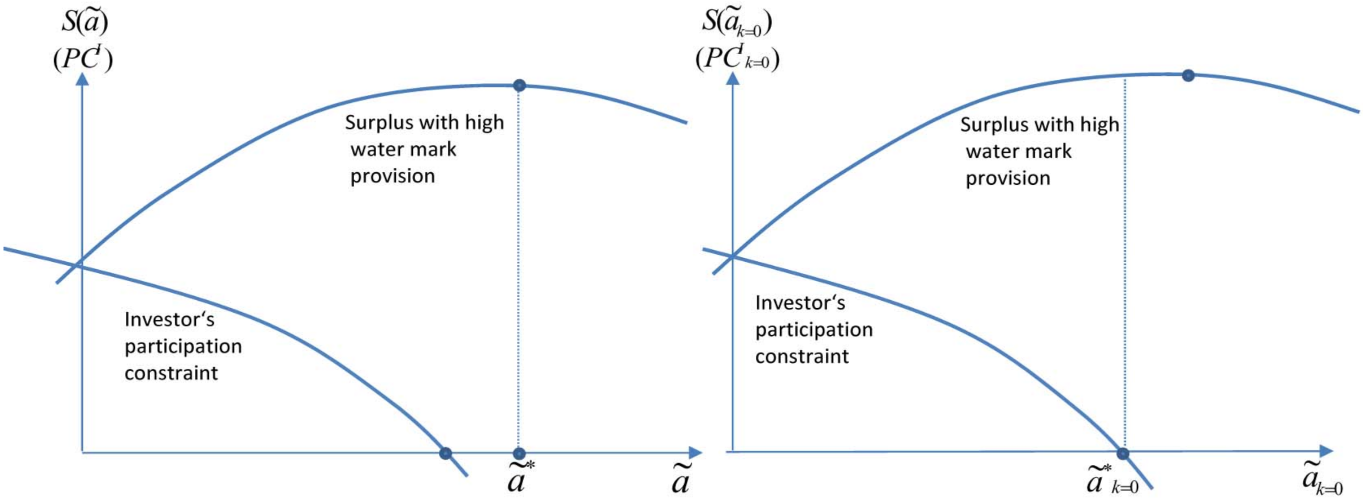

Figure 16: Expected fund surplus and investor's participation constraint in different performance fee regimes.

The investor's participation constraint with higher level of $\widetilde{a}^*$ can only be satisfied with negative value of $\widetilde{k}^*$ (compare Figure 16, first part). With the assumption about the non-negativity of the management fee, $\widetilde{k}^* \equiv 0$, for values of $\theta \in (\theta^\circ, 1)$ we calculate the new level of performance fee rate $\widetilde{a}^*_{k=0}$ using modified[45] investor's participation constraint (3). The solution can be described as:

$$\widetilde{a}^*_{k=0} = \frac{\Psi + \sqrt{\Psi^2 + 4(1-p)(R^L + R^H(1+R^L))^2(p^2\theta + (p-\varepsilon)^2(1-\theta))(1+p(1+R^H))\eta\omega_{max}}}{2(1-p)(R^L + R^H(1+R^L))^2(p^2\theta + (p-\varepsilon)^2(1-\theta))},$$

with $\Psi := (1-p)(1+R^L)(R^L + R^H(1+R^L))\Big(p\theta\eta + (p-\varepsilon)(1-\theta)(\eta - \varepsilon(R^H - R^L))\Big) - pR^H(1+p(1+R^H))$.

The new value of period performance fee rate $\widetilde{a}^*_{k=0}$ does not achieve the maximum of the surplus-function and is smaller than $\widetilde{a}^*$ (compare Figure 16, second part). For $\theta \geq \widehat{\theta} \in (\theta^\circ, 1)$ the value of the expected fund's surplus with respect to the new rate $\widetilde{a}^*_{k=0}$ is smaller than the counterpart surplus in the optimal contract with period performance fee. The difference increases for $\theta = 1$:

$$S(\widetilde{a}^*_{k=0}) - S(a^*) = \frac{\eta\xi}{2(1+R^H)}\Bigg(\underbrace{\frac{\Big(\xi + (1-p)\frac{R^L}{R^H}(1+R^L)\Big)^2}{\xi + (1-p)\frac{R^L}{R^H}(\frac{R^L}{R^H} + 2(1+R^L))}}_{<1} - 1\Bigg) < 0. \quad \blacksquare$$

Thus, the expected surplus as a function of θ is continuous at $\theta \in (0, 1)$, and $S(\widetilde{a}^*) - S(a^*) > 0$ for $\theta \in (0, \widehat{\theta})$ and $S(\widetilde{a}^*) - S(a^*) < 0$ for $\theta = 1$, we have a suffucuent condition for existence of $\widehat{\theta} \in (\theta^\circ, 1)$.

A.5 Proof of Proposition 4.

At first we recalculate the level on performance fee parameter a^* in the optimal contract with intermittent redemption by the investor, if the manager's outside income in period 2 is uniformly distributed on $\left[0, \left(1 + R^H\right)\eta\right]$, and show that the maximal level on a^* that the investor is willing to pay to the manager is equal to $\frac{\eta}{pR^H}$. Investor's participation constraint is:

$$V_0 \leq -k + p\Big(-f^H + F(\omega^H)(p(V^{HH} - f^{HH}) + (1-p)V^{HL}) + (1 - F(\omega^H))V^H\Big) + (1-p)V^L$$

$$\Longleftrightarrow \frac{a^2p^3(R^H)^2(1+R^H)}{\eta} - apR^H(1 - p(1+R^H)) - \eta + k \leq 0$$

$$a \leq \frac{\eta\sqrt{1 + p^2(1+R^H)^2 + \frac{2p}{\eta}(1+R^H)(\eta - 2k)} - (1 - p(1+R^H))}{2p^2R^H(1+R^H)}$$

[45]For performance fee rate larger than $\frac{\eta}{pR^H}$, the probability of fund continuation in state H is equal to one.

The value of the performance fee parameter a increases if k decreases, so it is optimal to set $k = 0$. That leads to the result

$$a = \frac{\eta}{pR^H}.$$

(The second analytical value $a = \frac{-\eta}{p^2R^H(1+R^H)} < 0$ is not feasible.)

We proof now that the optimal performance fee that maximizes the expected surplus with respect to the manager's optimal closing policy is also equal to $a^* = \frac{\eta}{pR^H}$:

$$\begin{aligned} S^{rd}(\omega) &= -1-\mathbb{E}(\omega)+p\Big(F(\omega^H)(1+R^H)(\eta+1)+(1-F(\omega^H))(1+R^H+\mathbb{E}(\omega|\omega\geq\omega^H))\Big)+(1-p)V^L \\ \Leftrightarrow\quad & \eta-\mathbb{E}(\omega)+p\Big(F(\omega^H)(1+R^H)\eta+(1-F(\omega^H))\mathbb{E}(\omega|\omega\geq\omega^H)\Big) \\ \Leftrightarrow\quad & \eta-\frac{(1-p)\omega_{max}}{2}+a\frac{p^2R^H(1+R^H)^2\eta}{\omega_{max}}-a^2\frac{p^3(R^H(1+R^H))^2}{2\omega_{max}}\longrightarrow Max \end{aligned}$$

$$\frac{dS(a)}{da}=\frac{p^2R^H(1+R^H)^2\eta}{\omega_{max}}-a\frac{p^3(R^H(1+R^H))^2}{\omega_{max}}=0\Rightarrow a^*=\frac{\eta}{pR^H}$$

The second order condition leads to $\frac{d^2S(a)}{da}<0$.

With use of propositions 1 and 2 it is sufficient to show that the expected surplus without redemptions $S(a^*)$, with the choice of the performance fee $a^* = \frac{\eta}{pR^H}$, is at least as high the expected surplus with redemptions by the investor $S^{rd}(a^*)$. This is equal to the consideration that the difference of both surpluses is positive:

$$\begin{aligned} S(a^*)-S^{rd}(a^*) = -V^L + & \\ & +\theta\Big(F(\omega^{L^\circ})V^L(\eta+1)+(1-F(\omega^{L^\circ}))\left(V^L+\mathbb{E}(\omega|\omega\geq\omega^{L^\circ})\right)\Big)+ \\ & +(1-\theta)\Big(F(\omega^{L^-})V^L(\eta+1-\varepsilon(R^H-R^L))+(1-F(\omega^{L^-}))\Big(V^L+\mathbb{E}(\omega|\omega\geq\omega^{L^-})\Big)\Big)>0 \\ \Leftrightarrow\quad & \frac{\eta(1+R^H)}{2}+\frac{(1+R^L)^2}{2p^2(1+R^H)}\Big(p^2\theta\eta+(p-\varepsilon)(1-\theta)\underbrace{(\eta(p+\varepsilon)-2p\varepsilon(R^H-R^L))}_{>0,\quad with\quad (A1)}\Big)>0 \quad \blacksquare \end{aligned}$$

A.6 Proof of Proposition 5: A first we consider the change in the model parameter, if $A > 0$.

Optimal Contract With a Period Performance Fee: Denote the performance fee by $a_A \geq 0$, a_A is charged on the fraction $(1-A)$. That leads to the following manager's period performance fees in each period, in the each of the states :

$f^H = a_A(1-A)R^H$, $f^{HH} = a_A(1-A)R^H(1+R^H)$ and $f^{LH} = a_A(1-A)R^H(1+R^L)R^H$.

Manager needs to consider not only the possible profits after the positive return but also the possible losses on her fraction A. This implies closing thresholds for ω equal to $(1+R^H)(a_A(1-A)pR^H + A\eta) = \omega^H(a_A)$ in state H, $(1+R^L)(a_A(1-A)pR^H + A\eta) = \omega^{L^\circ}(a_A)$ in state L° and $(1+R^L)(a_A(1-A)(p-\varepsilon)R^H + A(\eta - \varepsilon(R^H - R^L)) = \omega^{L^-}(a_A)$ in state L^-.

Optimal Contract With a High Water Mark Provision: Consider now the contract that specifies a linear performance fee, $\widetilde{a}_A \geq 0$, with a high water mark provision. At date 0 the historic maximum of the fund value $V_0 = 1$ describes the first high water mark, the fee level is $f^H = \widetilde{a}_A(1-A)R^H$. The same applies to V^H and therefore $f^{HH} = \widetilde{a}_A(1-A)(1+R^H)R^H$. If the first-period return is negative, the historic maximum of the fund value remains its initial value $V_0 = 1$. This implies that a high water mark provision is given by $f^{LH} = \widetilde{a}_A(1-A)(R^H(1+R^L) + R^L) > 0$.

The corresponding closing thresholds for ω are defined as follows:

$\omega^H(\widetilde{a}_A) = \widetilde{a}_A(1-A)pR^H(1+R^H) + A\eta(1+R^H)$,

$\omega^{L^\circ}(\widetilde{a}_A) = \widetilde{a}_A(1-A)p(R^H(1+R^L) + R^L) + A(1+R^L)\eta$,

$\omega^{L^-}(\widetilde{a}_A) = \widetilde{a}_A(1-A)(p-\varepsilon)(R^H(1+R^L) + R^L) + A(1+R^L)(\eta - \varepsilon(R^H - R^L))$.

Recall that the the optimal period performance fee, as calculated in Proposition 2, is given by $a_{\mathbb{I}} = \frac{\phi_{\mathbb{I}}}{\varphi_{\mathbb{I}}}$ for $A = 0$. Using new closing thresholds for ω fund's expected surplus (2) can be rewritten as

$$S(\omega^H, \omega^{L^\circ}, \omega^{L^-}) = \eta + \frac{a_{\mathbb{I}}(1-A)^2}{\omega_{max}}\phi_{\mathbb{I}} - \frac{a_{\mathbb{I}}^2(1-A)^2}{2\omega_{max}}\varphi_{\mathbb{I}} + \frac{A(2-A)}{2\omega_{max}}Const,$$

where $Const = p^2(1+R^H)^2\eta^2 + (1-p)(1+R^L)^2(p\eta^2 + (1-\theta)(\eta - \varepsilon(R^H - R^L))^2)$. The first and the second order conditions lead to the optimal performance fee parameter choice given by:

$$a_{\mathbb{I}}^* = \frac{\phi_{\mathbb{I}}}{\varphi_{\mathbb{I}}}.$$

With the proposition 2 it holds that $\widetilde{a}^* > a^*$, and also the fund's expected surplus with the choice of $\widetilde{a}^*$ is larger than the corresponding surplus with the choice of a^*.

Let us calculate the management fee $\widetilde{k}_A^*$ for $A > 0$. Using $\widetilde{a}^*$ given by $\widetilde{a}^* = \frac{\phi_1}{\varphi_1}$ and the investor's participation constraint (1) yields

$$\widetilde{k}^*_{A>0} = \frac{\eta - \widetilde{a}^* pR^H}{(1-A)} - \frac{A\widetilde{a}^*\phi_1}{(1-A)\omega_{max}} + \frac{A}{\omega_{max}}Const.$$

Note that for $A = 0$ the above equation becomes $\widetilde{k}^*_{A=0} = \eta - \widetilde{a}^* pR^H$ and presents the management fee that the manager chooses in the optimal contract without wealth contribution, that was calculated in proposition 2.

We now show that increasing in $A > 0$ results in increasing $S(\omega^H, \omega^{L^\circ}, \omega^{L^-})$ and $\widetilde{k}^*_A$. Examining the term

$$\frac{dS(\omega^H, \omega^{L^\circ}, \omega^{L^-})(\widetilde{a}^*)}{dA} = -\frac{(1-A)}{\omega_{max}}\frac{\phi_1^2}{\varphi_1} + \frac{(1-A)}{\omega_{max}}Const = \frac{(1-A)}{\omega_{max}}\underbrace{\Big(Const - \frac{\phi_1^2}{\varphi_1}\Big)}_{>0}$$

we observe that $\frac{dS(\omega^H, \omega^{L^\circ}, \omega^{L^-})}{dA} > 0$. Identical steps lead to $\frac{d\widetilde{k}^*_A}{dA} > 0$ since

$$\begin{aligned} \frac{\eta - \widetilde{a}^* pR^H}{(1-A)^2} + \frac{Const}{\omega_{max}} - \frac{\phi_1^2}{\varphi_1\omega_{max}} &= \\ \underbrace{\frac{\eta - \widetilde{a}^* pR^H}{(1-A)^2}}_{=\widetilde{k}^*_{A=0}>0} + \frac{1}{\omega_{max}}\underbrace{\Big(Const - \frac{\phi_1^2}{\varphi_1}\Big)}_{>0} &> 0 \end{aligned}$$

Note that the term $Const - \frac{\phi_1^2}{\varphi_1}$ is positive since

$$\begin{aligned} & Const - \frac{\phi_1^2}{\varphi_1} = \\ = \; & \frac{(1-\theta)(1-p)\varepsilon^2(R^H(1+R^L)R^L)^2\Big(\theta(1-p)(1+R^L)^2 + p(1+R^H)^2\Big)}{\varphi_1} > 0. \quad \blacksquare \end{aligned}$$

Proof of Proposition 5: Recall that the optimal contract with the high water mark provision in case $A = 0$ is given by $(\widetilde{a}^*, \widetilde{k}^*)$ with $\widetilde{a}^* =: \widetilde{a}^*_{A=0} = \frac{\phi_1}{\varphi_1}$ and

$\widetilde{k}^* =: \widetilde{k}^*_{A=0} = \eta - \frac{\phi_1}{\varphi_1}pR^H$ and is positive. We also use the previous denotation for the fund's expected return $\eta = pR^H + (1-p)R^L > 0$, where $R^L < 0$. The continuation probability in state H is given by $F(\omega^H(\widetilde{a}_A)) = \frac{\widetilde{a}(1-A)pR^H(1+R^H) + A\eta(1+R^H)}{\omega_{max}}$.

The continuation probability in state ω^H is given by $F(\omega^H(\widetilde{a}_A)) = \frac{(1+R^H)(\frac{\phi_1}{\varphi_1}(1-A)pR^H + A\eta)}{\omega_{max}}$. Rearranging gives:

$$F(\omega^{L^H}(\widetilde{a}_A^*)) = \underbrace{\frac{\phi_1}{\varphi_1 \omega_{max}} pR^H(1+R^H)}_{=F(\widetilde{a}_{A=0}^*)>0} + \tfrac{A}{\omega_{max}}(1+R^H)\underbrace{(\eta - \frac{\phi_1}{\varphi_1} pR^H)}_{=\widetilde{k}^*>0}$$

Identical consideration we have in states L° and L^-

$$F(\omega^{L^\circ}(\widetilde{a}_A^*)) = \underbrace{\frac{\phi_1}{\varphi_1 \omega_{max}} p(R^L + R^H(1+R^H))}_{=F(\widetilde{a}_{A=0}^*)>0} + \tfrac{A}{\omega_{max}}\Big((1+R^L)\underbrace{(\eta - \frac{\phi_1}{\varphi_1} pR^H)}_{=\widetilde{k}^*>0} \underbrace{- \frac{\phi_1}{\varphi_1} pR^L}_{>0}\Big)$$

$$F(\omega^{L^-}(\widetilde{a}_A^*)) = \underbrace{\frac{\phi_1}{\varphi_1 \omega_{max}} (p-\varepsilon)(R^L + R^H(1+R^H))}_{=F(\widetilde{a}_{A=0}^*)>0} -$$

$$- \tfrac{A}{\omega_{max}} \underbrace{\Big(\frac{\phi_1}{\varphi_1}(p-\varepsilon)(R^L + R^H(1+R^H)) + (1+R^L)(\eta - \varepsilon(R^H - R^L))\Big)}_{>0}. \qquad \blacksquare$$

Appendix B

Recall for all following proofs, that the critical belief *below* which withdrawals can occur in $t = 1$ is $\overline{\mu}_1 \in (0, \mu)$. The critical belief *above* which withdrawals of funds can occur in $t = 2$ is $\overline{\mu}_2 \in (\mu, 1)$.

B.1 Proof of Poposition 7

At first we prove that in state *separating(u,d)* there exist only one state in which the u-type manager has an incentive to report $v_1 = 1$.

If u-type manager deviates from the strategy profile *separating(u,d)* by reporting $v_1 = 1$ in $t = 1$, we have to consider all possible investors' beliefs $\alpha(u|1)$ in $t = 1$ and $\alpha(u|1,1)$ in $t = 2$ (if the second-period reported value of assets under managemenst will be $v_2 = 1$). The u-type manager's expected fee payments, contingent on all possible investors' beliefs in $t = 1, 2$ are described as follows:

$$E^u\Big(f|1, \alpha(u|1), \alpha(u|1,1)\Big) =$$

$$\begin{cases} E^u\Big(f|u,1\Big) - f(1-p)(u-1) & for \quad \alpha(u|1) \geq \overline{\mu}_1 \quad and \quad \alpha(u|1,1) \geq \overline{\mu}_2 \\ E^u\Big(f|1, \mu, \alpha(u|1,1) < \overline{\mu}_1\Big) & for \quad \alpha(u|1) \geq \overline{\mu}_1 \quad and \quad \alpha(u|1,1) < \overline{\mu}_2 \\ (1-\lambda_1)E^u\Big(f|1, \alpha(u|1) \geq \overline{\mu}_1\Big) & for \quad \alpha(u|1) < \overline{\mu}_1 \quad and \quad \alpha(u|1,1) \geq \overline{\mu}_2 \\ (1-\lambda_1)E^u\Big(f|1, \mu, \alpha(u|1,1) < \overline{\mu}_2\Big) & for \quad \alpha(u|1) < \overline{\mu}_1 \quad and \quad \alpha(u|1,1) < \overline{\mu}_2. \end{cases}$$

The value $E^u(f|u,1)$ is described as (4), the value $E^u\Big(f|1, \alpha(u|1) \geq \overline{\mu}_1\Big)$ as (6) and the value $E^u\Big(f|1, \mu, \alpha(u|1,1) < \overline{\mu}_2\Big)$ as (10). Thus, it appears that the u-type manager can only have an incentive to deviate towards underreporting of her first-period return, if the investors beliefs are $\alpha(u|1) \geq \overline{\mu}_1$ in $t = 1$ and $\alpha(u|1,1) < \overline{\mu}_2$ in $t = 2$. Exactly this beliefs investors have in the strategy profile *pooling(1,1)*. Based on the previous consideration we can conclude, that the u-type manager has an incentive to underreport her first-period return for $\eta \geq \eta^u$ or, equaivalently, for $\lambda_2 \geq \lambda_2^d$, if investors beliefs are as described in the strategy profile *pooling(1,1)*.

Analogously, given the strategy profile *separating(u,d)*, we consider the d-type manager's expected fee payments by deviating towards misreporting and by announcing $v_1 = 1$, contingent on all possible investors' leliefs in $t = 1, 2$:

$$E^d\Big(f|1, \alpha(u|1), \alpha(u|1,1)\Big) =$$

$$\begin{cases} E^d\Big(f|1,\alpha(u|1)\geq\overline{\mu}_1\Big) & for \quad \alpha(u|1)\geq\overline{\mu}_1 \quad and \quad \alpha(u|1,1)\geq\overline{\mu}_2 \\ E^d\Big(f|1,\mu,\alpha(u|1,1)<\overline{\mu}_2\Big) & for \quad \alpha(u|1)\geq\overline{\mu}_1 \quad and \quad \alpha(u|1,1)<\overline{\mu}_2 \\ (1-\lambda_1)E^d\Big(f|1,\alpha(u|1)\geq\overline{\mu}_1\Big) & for \quad \alpha(u|1)<\overline{\mu}_1 \quad and \quad \alpha(u|1,1)\geq\overline{\mu}_2 \\ (1-\lambda_1)E^d\Big(f|1,\alpha(u|1,1)<\overline{\mu}_2\Big) & for \quad \alpha(u|1)<\overline{\mu}_1 \quad and \quad \alpha(u|1,1)<\overline{\mu}_2. \end{cases}$$

The value $E^d\Big(f|1,\alpha(u|1)\geq\overline{\mu}_1\Big)$ is described as (7), the value $E^d\Big(f|1,\mu,\alpha(u|1,1)<\overline{\mu}_2\Big)$ as (11) and the value $E^d\Big(f|1,\alpha(u|1,1)<\overline{\mu}_2\Big)$ as (9). Comparing the d-type manager's expected fee payments shows that the d-type manager can only have an incentive to deviate towards overreporting of her first-period return if investors' beliefs are $\alpha(u|1)\geq\overline{\mu}_1$ in $t=1$ and $\alpha(u|1,1)<\overline{\mu}_2$ in $t=2$. This is exactly what the strategy *pooling(1,1)* describes. The d-type manager has an incentive to overreport her first-period return for $\eta\geq\eta^d$ or, equaivalently, for $\lambda_2<\lambda_2^d$, if investors' beliefs are as in the strategy profile *pooling(1,1)*.

In summary, the strategy profile *separating(u,d)* is an PBE for

$$0\leq\eta\leq\begin{cases}\eta^u & for \quad \lambda_1>\lambda_1^* \\ \eta^d & for \quad \lambda_1\leq\lambda_1^*.\end{cases} \qquad \blacksquare$$

B.2 Proof for the Nonexistence of *separating(u,1)* and *separating(1,d)* Equilibria in the Contract With the Period Performance Fee.

Consider at first the strategy profile *separating(u,1)*. Given the contract specifies the period performance fee, the d-type manager's expected fee payment in this state is given by $E^d\Big(f|1,\alpha(u|1)<\overline{\mu}_1\Big)$ (9). Deviating towards thruthful first-period reporting of $v_1=d$ yields for the d-type manager an expected fee payment of $E^d(f|d,0)$ given by (5). In both cases, independent on whether the d-type manager reports her true first-period return or overreports it, the investors have always a belief of 1 on the d-type in the predescribed strategy profile.

Comparing the both expected fee payments shows, that the d-type manager has never an incentive for underreporting, given investors' beliefs in the strategy profile *separating(u,1)*:

$$E^d\Big(f|1,\alpha(u|1)<\overline{\mu}_1\Big)>E^d(f|d,0)\Leftrightarrow 0>p\varepsilon(1-d)\longrightarrow\nexists,$$

thus, the separating strategy profile *separating(u,1)* can not be supported as a PBE, independent on whetrer the u-type manager has an incentive to deviate from the strategy profile or not.

Analogously, consider the separating strategy profile *separating(1,d)*. According to the definition of this strategy profile, by observing the reported return of $v_1 = 1$ the investors believe with probability 1 to face an u-type manager. By observing $v_1 = d$ they believe with probability 1 to face a d-type manager. The u-type manager's expected fee payment in strategy profile *separating(1,d)* is given by $E^u\Big(f|1, \alpha(u|1) \geq \overline{\mu}_1\Big)$ (6). Deviating towards thruthful fisrt-period reporting of $v_1 = u$ generates an expected fee payment of $E^u(f|u, 1)$ (4) for the u-type manager. Comparing both terms shows, that the u-type managerhas never an incentive for underreporting, given the investors' beliefs in the strategy profile *separating(1,d)*:

$$E^u\Big(f|1, \alpha(u|1) \geq \overline{\mu}_1\Big) > E^u(f|u, 1) \Leftrightarrow$$

$$p(u^2 - 1) > (u - 1) + p(u^2 - u) \Leftrightarrow p > 1 \longrightarrow \nexists,$$

thus, the strategy profile *separating(1,d)* can not be supported as a PBE, independent on whetrer the d-type manager has an incentive to deviate from the strategy profile or not. ■

B.3 Search for the Remaining PBE Equilibria if the Contract Specifies the Performance Fee With the High Water Mark Provision.

At first we proof, that the strategy profile *separating(u,d)* is a PBE.

If the u-type manager reports $v_1 = 1$ in the strategy profile *separating(u,d)*, investors' beliefs $\alpha(u|1)$ in $t = 1$ and $\alpha(u|1, 1)$ in $t = 2$ (if the second- period reported value of assets under managemenst is $V_2 = 1$) have to be specified. The u-type manager's expected fee payments, contingent on investors' beliefs are described as follows:

1) $E^u\Big(h|1, \alpha(u|1) \geq \overline{\mu}_1, \alpha(u|1, 1) \geq \overline{\mu}_2\Big) = hp\Big(u^2-1+p(u-1)(u^2+\eta)\Big) + h\Big((1-p)(1-\lambda_2)p\varepsilon(u-1)\Big)$
$< E^u(h|u, 1)$, since $1 \geq (1-\lambda_2)p\varepsilon$.

For the given investors' beliefs, $\alpha(u|1) \geq \overline{\mu}_1$ in $t = 1$ and $\alpha(u|1, 1) \geq \overline{\mu}_2$ in $t = 2$, the u-type manager reports $v_1 = u$, since the condition:

$$E^u(h|u, 1) \geq E^u\Big(h|1, \alpha(u|1) \geq \overline{\mu}_1, \alpha(u|1, 1) \geq \overline{\mu}_2\Big) \Leftrightarrow 1 \geq (1-\lambda_2)p\varepsilon$$

is always satisfied for each set of parameters.

2) $E^u\Big(h|1, \alpha(u|1) \geq \overline{\mu}_1, \alpha(u|1, 1) < \overline{\mu}_2\Big)$

described as in (15). For $\eta \geq \widetilde{\eta}^u$ the u-type manager has an incentive for underreporting.

3) $E^u\Big(h|1, \alpha(u|1) < \overline{\mu}_1, \alpha(u|1,1) \geq \overline{\mu}_2\Big) =$

$$= h(1-\lambda_1)\Big(p(u^2-1+p(u-1)(u^2+\eta))+(1-p)(1-\lambda_2)p\varepsilon(u-1)\Big).$$

The u-type manager's expected fee in this case is even smaller than in 1). Thus, for given investors' beliefs $\alpha(u|1) < \overline{\mu}_1$ in $t = 1$ and $\alpha(u|1,1) \geq \overline{\mu}_1$ in $t = 2$, the manager has never an incentive to underreport her first-period return.

4) $E^u\Big(h|1, \alpha(u|1) < \overline{\mu}_1, \alpha(u|1,1) < \overline{\mu}_2\Big) =$

$$= h(1-\lambda_1)\Big(p(u^2-1+p(u-1)(u^2+\eta))+(1-p)p\varepsilon(1+\eta)(u-1)\Big).$$

Consider the strategy profile *separating(u,d)*. The d-type manager's expected fee payments by deviating towards announcing $v_1 = 1$, contingent on all possible investors' beliefs in this state are given by:

1) $E^d\Big(h|1, \alpha(u|1) \geq \overline{\mu}_1, \alpha(u|1,1) \geq \overline{\mu}_2\Big) = hp^2\varepsilon(1-\lambda_2)(u-1) < E^d(h|d,0)$.
2) $E^d\Big(h|1, \alpha(u|1) \geq \overline{\mu}_1, \alpha(u|1,1) < \overline{\mu}_2\Big) = hp^2\varepsilon(1+\eta)(u-1) \geq E^d(h|d,0)$.
3) $E^d\Big(h|1, \alpha(u|1) < \overline{\mu}_1, \alpha(u|1,1) \geq \overline{\mu}_2\Big) = h(1-\lambda_1)p^2\varepsilon(1-\lambda_2)(u-1) < E^d(h|d,0)$.
4) $E^d\Big(h|1, \alpha(u|1) < \overline{\mu}_1, \alpha(u|1,1) < \overline{\mu}_2\Big) = h(1-\lambda_1)p^2\varepsilon(1+\eta)(u-1) \equiv E^d(h|d,0)$.

In summary, for $p\varepsilon \leq 0.5$ and $\eta \in [0, \eta^u]$ the state *separating(u,d)* is an PBE.

Proof of Proposition 8.

Consider now the separating strategy profile *separating(u,1)*. In this state in spite of observed misreporting in the first period the investors are completely informed about the menager's types. Thus, by observing the reported value of $V_1 = 1$ the investors' have a believ of 0 to face the u-type manager. The d-type manager's expected pees are given by:

$$E^d\Big(h|1, \alpha(u|1) < \overline{\mu}_1, \alpha(u|1,1) < \overline{\mu}_2\Big) = h(1-\lambda_1)p^2\varepsilon(1+\eta)(u-1) \equiv E^d(h|d,0).$$

Obviously, the d-type manager receives the same expected fees by thrutful reporting of $v_1 = d$ and overreporting of her first-period return in the given strategy profile. Using the previous consideration, we know that the u-type manager never deviates towards underreporting in the given strategy profile:

$$E^u\Big(h|1,\alpha(u|1)<\overline{\mu}_1,\alpha(u|1,1)<\overline{\mu}_2\Big)=h(1-\lambda_1)\Big(p(u^2-1+p(u-1)(u^2+\eta))+(1-p)p\varepsilon(1+\eta)(u-1)\Big).$$

Consider as last the separating strategy profile *separating(1,d)*. In this state the u-type manager underreports and the d-type manager thruthfully reports her first-period return. By observing the reported value of $v_1 = 1$ in $t = 1$ investors' belief on u-type manager is 1. Therefore, the u-type manager's expected fees in this state, $E^u\Big(h|1,\alpha(u|1)\geq \overline{\mu}_1,\alpha(u|1,1)\geq\overline{\mu}_2\Big)$ is always smaller than her expected fees by thruthful reporting, $E^u(h|u,1)$. Hence, the strategy profile *separating(1,d)* can not be supported as a PBE. ■

B.4 Proof of Corrolaries 4, 5 and Proposition 9

B.4.1 **Proof of Corollary 4**: If the contract specifies the period performance fee, we have the following consideration:

$$\frac{d\eta^s}{dp\varepsilon}=\frac{d(1-\lambda_2 p\varepsilon)/p\varepsilon}{dp\varepsilon}=-\frac{1}{p\varepsilon^2}<0.$$

With increasing values of $p\varepsilon$ the threshold η^u desreases. Consequently, the manager's incentive condition for misreporting is satisfied for larger numerical interval $\eta^u, 1$ and smaler values of η^u. Analogously we have: $\frac{d\eta^u}{d\lambda_2}=-1<0$.
If the contract specifies performance fee with the high water mark provision, the following condition are satisfied:

$$\frac{d\widetilde{\eta}^u}{dp\varepsilon}=\frac{d(1-p\varepsilon)/p\varepsilon}{dp\varepsilon}=-\frac{1}{p\varepsilon^2}<0.$$

B.4.2 **Proof of Proposition 9**: At first we use the Proposition 6 to define the restricted domain D_1 in which the strategy profile *pooling(1,1)* is a PBE, if the management contract specifies the period performance fee:

$$D_1 := \Big\{p\varepsilon,\eta,\lambda_1,\lambda_2 \mid p\varepsilon\in(\frac{1}{1+\lambda_2},1],\lambda_1>\lambda_1^*,\lambda_2\in[0,1],\eta\in(\eta^u,1]\Big\}.$$

As second we use the Proposition 8 to define the restricted domain D_2 in which the strategy profile *pooling(1,1)* is a PBE, if the management contract specifies performance fee with the high water mark provision:

$$D_2 := \Big\{p\varepsilon, \eta, \lambda_1, \lambda_2 \mid p\varepsilon \in (\frac{1}{2}, 1], \lambda_1 \in (0,1), \lambda_2 \in [0,1], \eta \in (\widetilde{\eta}^u, 1]\Big\}.$$

We consider additionally that $\widetilde{\eta}^u \leq \eta^u \Leftrightarrow (\eta^u, 1] \subset (\widetilde{\eta}^u, 1]$ to see that $D_1 \subset D_2$. In summary, there exist a non-empty (for numerical examples compare Table 4) subset of parameters

$$D_2 \setminus D_1 = \Big\{p\varepsilon, \eta, \lambda_1, \lambda_2 \mid p\varepsilon \in (\frac{1}{2}, \frac{1}{1+\lambda_2}), \lambda_1 \in (0,1), \lambda_2 \in [0,1], \eta \in (\widetilde{\eta}^u, \eta^u)\Big\}$$

for which *pooling(1,1)* equilibrium occurs only when the hedge fund management contract specifies performance fee with the high water mark provision. (To illustrate the consideration in a simplified way compare Figure 17)

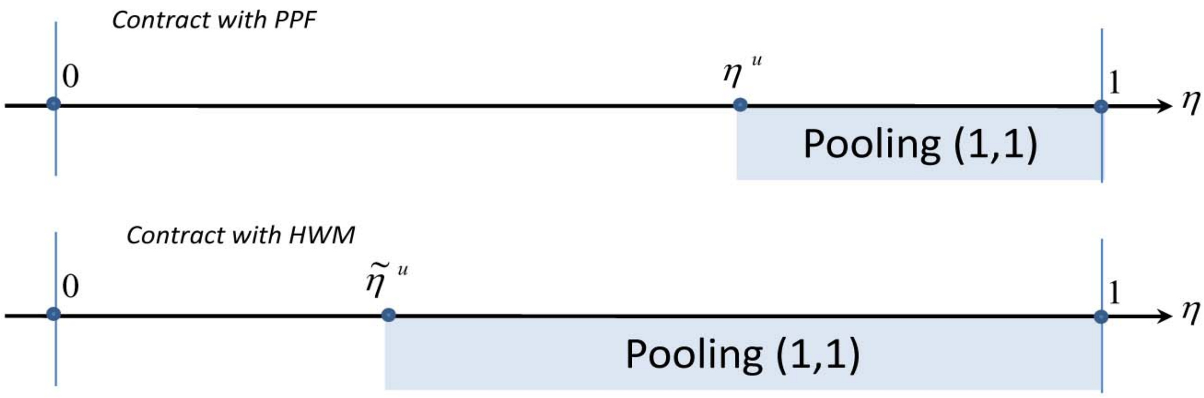

Figure 17: Perfect Bayesian equilibria depending on η^u if contract specifies period performance fee and $\widetilde{\eta}^u$ if contract specifies performance fee with the high water mark provision.

B.4.3 **Proof of Corollary 5**: If the contract specifies the period performance fee, the pooling equilibrium occurs for $p\varepsilon \geq \frac{1}{1+\lambda_2}$ (shown in Proposition 6). If the contract specifies performance fee with the high water mark provision, the pooling equilibrium occurs for $p\varepsilon \geq 0.5$ (as shown in Proposition 8). For $\lambda_2 < 1$ it is always satisfied $\frac{1}{1+\lambda_2} > \frac{1}{2}$. ■

B.5 New Thresholds in the Extended Model Version

We begin with the contract that specifies the period perfoemance fee. In the strategy profile *pooling(1,1)* the u-type manager's expected fee payment is described as follows:

$$E^u\Big(f|1, \mu, \alpha(u|1,1) < \overline{\mu}_2\Big) =$$

$$= fp\Big(u^2-1+p(u-1)(u^2+\eta)+(1-p)\varepsilon(u-1)(ud+\eta)\Big).$$

By reporting $v_1 = u$ the u-type manager credibly signals her type to the investors. In this state the u-type manager's expected fees are given by:

$$E^u(f|u,1) = f(u-1)+fp\Big(u^2-u+p(u-1)(u^2+\eta)+(1-p)(1-\lambda_2)\varepsilon ud(u-1)\Big).$$

The u-type manager has an insentive to deviate towards thruthful first-period reporting in $t = 1$ for the given investors' beliefs of 1 on the strong-type manager for:

$$E^u(f|u,1) \geq E^u\Big(f|1,\mu,\alpha(u|1,1) < \overline{\mu}_2\Big) \Leftrightarrow$$

$$\eta \leq \frac{1-p\varepsilon\lambda_2 ud}{p\varepsilon} := \eta^u_{new} \quad \Leftrightarrow \quad \lambda_2 \leq \frac{1-p\varepsilon\eta}{p\varepsilon ud} := \lambda^u_{2\,new}.$$

In order to receive values $\eta \in [0,1]$, it should be satisfied:

$$p\varepsilon \geq \frac{1}{1+\lambda_2 ud}.$$

d-type manager's expected fee payment in the strategy profile *pooling(1,1)* is given by:

$$E^d\Big(f|1,\mu,\alpha(u|1,1)<\overline{\mu}_2\Big) = fp\varepsilon\Big(p(u-1)(ud+\eta)+(1-p\varepsilon)(1-\lambda_2)d^2(u-1)\Big).$$

Given strategy profile *pooling(1,1)*. Thruthful reporting of $v_1 = d$ by the d-type manager leads to a credible signal about her type. The expected fees of d-type manager in this state is given by

$$E^d(f|d,0) = f(1-\lambda_1)p\varepsilon\Big(d(u-1)+p(ud+\eta\Delta)(u-1)+(1-p\varepsilon)(1-\lambda_2)d^2(u-1)\Big).$$

Comparing the d-type manager's expected fee payments by thruthful reporting of $v_1 = d$ and overreporting of her first-period return by announcing $v_1 = 1$ leads to the following thresholds:

$$E^d(f|d,0) \geq E^d\Big(f|1,\mu,\alpha(u|1,1)<\overline{\mu}_2\Big) \Leftrightarrow$$

$$\frac{1-\lambda_1}{\lambda_1} \geq \frac{p(ud+\eta)+(1-p\varepsilon)(1-\lambda_2)d^2}{d}$$

$$\Leftrightarrow \ \eta \leq \frac{(1-\lambda_1)d-\lambda_1\Big(pud+(1-p\varepsilon)(1-\lambda_2)d^2\Big)}{\lambda_1 p} := \eta\Delta^d_{new}$$

$$\Leftrightarrow\ \Big(\lambda_{2new}^{d}=\frac{(1-\lambda_1)d-\lambda_1(p(ud+\eta)+(1-p\varepsilon)d^2)}{\lambda_1(1-p\varepsilon)d^2}\Big).$$

Analogously, if the contract specifies performance fee with the high water mark provision, the u-type manager's expected fee payment in the strategy profile *pooling(1,1)* is given by:

$$E^u\Big(h|1,\mu,\alpha(u|1,1)<\overline{\mu}_2\Big)=$$
$$=h\Big(p(u^2-1)+p^2(u^2+\eta)(u-1)+(1-p)(ud-1)+(1-p)p\varepsilon(ud+\eta)(u-1)\Big).$$

By deviating towards thruthful reporting of $v_1=u$ the u-type manager's expected fee payment is given by:

$$E^u(h|u,1)=h(u-1)+hp\Big(u^2-u+p(u-1)(u^2+\eta)+(1-p)(1-\lambda_2)\varepsilon u(ud-1)\Big).$$

Comparing the u-type manager's expected fees in both regimes leads to the following thresholds:

$$E^u(h|u,1)\geq E^u\Big(h|1,\mu,\alpha(u|1,1)<\overline{\mu}_2\Big)\Leftrightarrow$$
$$\eta\leq\frac{u(1-d)(1-p\varepsilon)-\lambda_2 p\varepsilon u(ud-1)}{p\varepsilon(u-1)}:=\widetilde{\eta}_{new}^{u}$$
$$\Leftrightarrow\ \lambda_2\leq\frac{u(1-d)(1-p\varepsilon)-p\varepsilon\eta(u-1)}{p\varepsilon u(ud-1)}:=\widetilde{\lambda}_{2new}^{u}.$$

In order to receive values $\eta\in[0,1]$, it should be satisfied:

$$\varepsilon\geq\frac{u(1-d)}{u(1-d)+\lambda_2 u(ud-1)+u-1}.\qquad\blacksquare$$

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