On Sensor Scheduling in Case of Unreliable Communication

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Abstract: This paper deals with the linear discrete-time sensor scheduling problem in unreliable communication networks. In case of the common assumption of an error-free communication, the sensor scheduling problem, where one sensor from a sensor network is selected for measuring at a specific time instant so that the estimation errors are minimized, can be solved off-line by extensive tree search. For the more realistic scenario, where communication is unreliable, a scheduling approach using a prioritization list for the sensors is proposed that leads to a minimization of the estimation error by selecting the most beneficial sensor on-line. To lower the computational demand for the priority list calculation, a novel optimal pruning approach is introduced.

1 Introduction

For sensor networks, where a large number of sensors is used, the so-called *sensor scheduling* is of paramount importance, since selectively activating the sensors saves limited resources like energy or communication bandwidth. Besides that, determining the best possible state estimate of the system observed by the sensor network is essential. Instead of treating each sensor independently, global sensor scheduling schemes permit improved estimation results [RB02]. In case of an error-free information transmission between the sensors, i.e., when no information gets lost, the optimal sensor schedule for linear systems observed by linear sensors corrupted by Gaussian noise can be determined off-line and independently of the measurements, where the optimality criterion or cost function is to minimize the state covariance of the system [MPD67]. Especially for sensor networks, where wireless communication is typical, the error-free communication assumption is too optimistic. The proposed method extends classical approaches for the sensor scheduling problem as it takes unreliable communication explicitly into account. Here, a prioritization list is constructed based on the optimal sensor schedule of the individual sensors. With a given prioritization list, selecting valuable sensors is possible even if some sensors are currently unavailable due to unreliable communication.

The next section gives a short introduction to sensor scheduling. The remainder of the paper is structured as follows: In Section 3, the calculation of the priority list with optimal pruning is described. The effect of priority list scheduling is demonstrated in Section 4 by simulations. The paper closes with conclusions and an outlook to future work.

2 Problem Formulation

This paper focuses on estimating the state $\underline{\boldsymbol{x}}_k$ of a linear dynamic system by means of a sensor network at discrete time steps $k = 0 \; 1 \qquad N$, where N is the estimation time horizon. To describe the system behavior, the linear stochastic discrete-time system equation

$$\underline{\boldsymbol{x}}_{k+1} = \mathbf{A}_k \underline{\boldsymbol{x}}_k + \mathbf{B}_k \underline{\boldsymbol{w}}_k$$

is used. Here, $\mathbf{A}_k \quad \mathbb{R}^{(n \times n)}$ and $\mathbf{B}_k \quad \mathbb{R}^{(n \times m)}$ are real-valued matrices, $\underline{\boldsymbol{w}}_k$ is white Gaussian noise with covariance matrix $\mathbf{C}_k^w$, and the initial state vector $\underline{\boldsymbol{x}}_0$ is also Gaussian with mean $\underline{x}_0$ and covariance matrix $\mathbf{C}_0^x$. This equation can be used e.g. for modeling a distributed phenomenon that is observed via a sensor network [SRH06].

For updating the state estimate, measurements obtained by S sensors are used. Each sensor $i = 1 \qquad S$ is described by the linear stochastic discrete-time measurement equation

$$\underline{y}_k^i = \mathbf{H}_k^i \underline{\boldsymbol{x}}_k + \underline{\boldsymbol{v}}_k^i$$

where $\underline{y}_k^i \quad \mathbb{R}^s$ is the current measurement, $\mathbf{H}_k^i \quad \mathbb{R}^{(s \times n)}$ is the real-valued measurement matrix, and $\underline{\boldsymbol{v}}_k^i$ is zero-mean white Gaussian noise with covariance matrix $\mathbf{C}_k^{(v,i)}$ affecting sensor i.

Assuming that each sensor node knows the measurement matrix as well as the noise vector of any other sensor and that the current estimate $\underline{x}_k$ with covariance matrix $\mathbf{C}_k^x$ of $\underline{\boldsymbol{x}}_k$ can be transmitted in an error-free manner over the sensor network, the sensor scheduling problem can be optimally solved by an extensive tree-search [MPD67]. If sensor i takes the measurement at time step k, the covariance evolves according to the recursive algebraic Riccati equation

$$\mathbf{C}_{k+1}^x = \mathbf{A}_k \mathbf{C}_k^x \mathbf{A}_k^{\mathrm{T}} + \mathbf{B}_k \mathbf{C}_k^w \mathbf{B}_k^{\mathrm{T}} - \mathbf{A}_k \mathbf{K}_k^i \mathbf{H}_k^i \mathbf{C}_k^x \mathbf{A}_k^{\mathrm{T}} \tag{1}$$

with $\mathbf{K}_k^i = \mathbf{C}_k^x (\mathbf{H}_k^i)^{\mathrm{T}} \left(\mathbf{H}_k^i \mathbf{C}_k^x (\mathbf{H}_k^i)^{\mathrm{T}} + \mathbf{C}_k^{(v,i)} \right)^{-1}$, as in the well-known Kalman filter. The optimal sensor sequence $u_{0:N}^* = \arg\min_{u_{0:N}} V(u_{0:N})$, when selecting one sensor per time step, results from minimizing the *cost function* or estimation error

$$V(u_{0:N}) = \sum_{n=0}^{N} g(\mathbf{C}_{n+1}^x) \Big|_{i=u_n} \tag{2}$$

with $\mathbf{C}_{n+1}^x$ according to (1), $g(\cdot)$ can be the trace or the determinant of $\mathbf{C}_{n+1}^x$, and u_n is the n-th element of $u_{0:N}$ indexing the sensor to be selected for measurement at time step n. The extension to multiple measurements per time step is straightforward [Kri02].

In sensor networks, communication is typically carried out over a wireless medium. Thus, the assumption of an error-free estimation transmission is no longer valid. The communication link between two sensors is unreliable, i.e., the packet containing the current estimation may be dropped. In literature, this effect has not been considered so far when scheduling sensors for measurement.

3 Priority List Sensor Scheduling

In the optimal sensor schedule $u^*_{0:N}$, the sensor to measure at time step k, $k \quad [0 \; N]$, is indexed by u_k. Under unreliable communication it is possible that the *optimal* sensor u_k is not available. Two possibilities arise: The measurement update for the current time step can be omitted or another sensor can be selected for measurement. In the following sections, we present a scheduling scheme that gives a practical solution to this problem.

3.1 Assumptions

First, some assumptions concerning the communication network are given. Each communication link between two distinct sensors either successfully or unsuccessfully transmits at time step k. Communication losses between two distinct sensors are uncorrelated over time. The probability of a communication loss is not known to the sensor nodes. A sensor schedule $u^*_{k:N}$ can be calculated in-between two consecutive time steps k and $k+1$.

3.2 Scheduling Scheme

The key idea of the proposed sensor scheduling approach is to provide a prioritization of the sensors. The sensor with the highest priority at time step $k+1$ is the first sensor of the sensor schedule with the overall minimum estimation error during time horizon N. The sensor with the second highest priority is the first sensor of the sensor schedule with the second lowest estimation error and so on. As illustrated in Fig. 1, at time step k the priority list for $S = 2$ sensors is calculated (framed by rounded box) by determining the optimal schedules $u^{*,1}_{k+1:N}$ and $u^{*,2}_{k+1:N}$ beginning with sensor $u_{k+1} = 1$ and $u_{k+1} = 2$, respectively. If the sensor schedule starting with $u_{k+1} = 1$ has the lowest cost, then sensor 1 is the sensor with highest priority and the priority list is $P_k = \; 1 \; 2$. Otherwise, sensor 2 is the sensor with the highest priority and the priority list is $P_k = \; 2 \; 1$.

In the proposed priority list scheduling algorithm for any time step k, three operations have to be performed:

Priority List Calculation For each sensor i its optimal sensor schedule $u^{*,i}_{k+1:N}$ with $u_{k+1} = i$ is calculated according to (2). Ranking the sensors in ascending order with respect to the cost function or estimation error $V(u^{*,i}_{k+1:N})$ yields the priority list P_k. All these calculations take place at sensor s, which was selected at time step $k-1$ for performing the measurement.

Reachability Check Sensor s broadcasts the priority list to the sensors of the sensor network. Sensors that received the list send an ACK back to s. Sensor s lists all responding sensors in the reachability list R_k.

Sensor Selection The sensor with highest priority in P_k that is listed in R_k is the best reachable sensor for performing the next measurement. Sensor s sends the current state estimate $\underline{x}_k$ and state covariance $\mathbf{C}^x_k$ to this sensor.

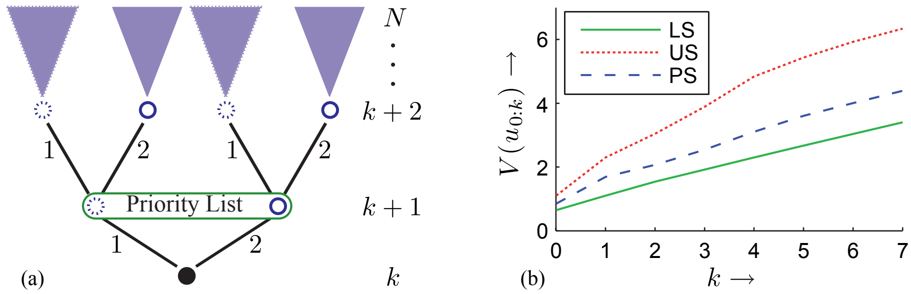

Figure 1: (a) Priority list determination for two sensors. (b) Simulation run for $N = 8$, where the blue, dashed line denotes the evolution of the estimation error of the proposed approach PS.

At time step $k + 1$, the operations described above are repeated until the end of the time horizon is reached. It is obvious that in case of an error-free communication, the sensor sequence resulting from the priority list approach is identical to the well-known solution neglecting communication constraints.

3.3 Optimal Pruning

Due to the fact that calculating the priority list requires searching each sub-tree of a sensor, naïve implementation is computationally demanding. Pruning techniques of search trees for sensor scheduling range from suboptimal methods, where conserving the best schedule is not guaranteed [GCHM04], to optimal methods, where eventually many complete schedules have to computed [CMPS06]. By employing the monotonic character of the Riccati equation (1), the computational demand can be drastically reduced by early pruning of paths that lead to suboptimal schedules. Comparing two paths leading from time step n to N with differing initial sensors i and j but otherwise identical sensors along the path, the path of sensor i can be pruned, if the following two conditions are satisfied:

1. $\mathbf{C}_{n+1}^{(x,i)} > \mathbf{C}_{n+1}^{(x,j)}$, where '>' implies that $\mathbf{C}_{n+1}^{(x,i)} - \mathbf{C}_{n+1}^{(x,j)}$ is positive definite,
2. $V(u_{0:n}^{i}) > V(u_{0:n}^{j})$.

Thus, with this novel pruning technique it is not necessary to calculate complete schedules to decide if early pruning is possible, while on the other hand conserving optimal schedules is guaranteed. Proofs and quantitative analyses are omitted due to space limitation.

4 Simulation Results

For simulation purposes, a sensor network with $S = 3$ sensor nodes is considered. A two-dimensional system is observed for 8 time steps ($N = 7$) and is characterized by

$\mathbf{A}_k = \mathbf{I}$, $\mathbf{B}_k = \mathbf{I}$, and $\mathbf{C}_k^w = 0\ 05\,\mathbf{I}$, where $\mathbf{I}$ is the identity matrix. Furthermore, $g(\cdot)$ in (2) is the trace function. Initially, the system state is $\underline{x}_0 = [0\ \ 0]^{\mathrm{T}}$ with covariance matrix $\mathbf{C}_0^x = 0\ 5\,\mathbf{I}$. The measurement and noise covariance matrices of the sensors are given by

$$\mathbf{H}_k^1 = 0\ 5\,\mathbf{I} \quad \mathbf{C}_k^{(v,1)} = 2\,\mathbf{I} \quad \mathbf{H}_k^2 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \quad \mathbf{C}_k^{(v,2)} = \mathbf{I} \quad \mathbf{H}_k^3 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \quad \mathbf{C}_k^{(v,3)} = 0\ 1\,\mathbf{I}$$

The communication error probability between sensor node 1 and 2 is $0\ 3$, between node 2 and 3 it is $0\ 5$, and between node 1 and 3 it is $0\ 7$. For comparison, two further sensor scheduling methods are used: The method denoted by US omits measurement updates when communication fails, while LS selects sensors under an error-free communication and thus provides the lower error bound. 10 Monte Carlo simulation runs are performed. In Fig. 1, one of these simulation runs is depicted. It is obvious that the prioritization used in the proposed approach (PS) significantly outperforms US, while being relatively close to the lower bound. According to this, the root means square error $\mathrm{RMS}_{\mathsf{PS}} = 0\ 727$ of PS with respect to the lower bound over all runs is lower than $\mathrm{RMS}_{\mathsf{US}} = 1\ 367$ of US.

5 Conclusions and Future Work

A novel sensor scheduling approach that explicitly considers unreliable communication has been presented. By priorizing individual sensors, the best reachable sensor for specific time instants can be selected for measurement. This approach can be extended in many ways. Especially weakening the assumptions in Section 3.1 is relevant for practical application, e.g. knowing the communication loss probability improves the estimation quality.

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