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THE LÉVÊQUE-ANALOGY

or

HOW TO PREDICT HEAT AND MASS TRANSFER FROM FLUID FRICTION

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ABSTRACT

Ten years ago a new type of analogy between frictional pressure drop and heat transfer has been discovered, that may be used for single cylinders and spheres in cross flow, in tube bundles, crossed-rod matrices, packed beds, and other periodic arrangements of solids in a fluid flow. It is based on the „Generalized Lévêque Equation (GLE)“, which allows to calculate heat or mass transfer coefficients – or the corresponding Nusselt and Sherwood numbers – from frictional pressure drop or friction forces in place of the flow rates or Reynolds numbers. The new method is not only applicable to internal flow with a periodic arrangement of solid surfaces, as proven in previous work, it can also be used in external flow situations. This is shown here for a single sphere as well as for a single cylinder in cross flow. The successful application of the GLE also in cases of external flow seems to confirm that this new type of analogy has a broad range of applications. It gives us a better understanding of the interrelation between fluid flow and heat or mass transfer in general, and it gave us prediction methods, which are in better agreement with experimental data from many different sources, than previously existing empirical correlations.

NOMENCLATURE

A	area (m^2)
c_D	drag coefficient, external flow
c_F	friction factor (< c_D), external flow
c_N	friction factor constant (Newton range), external flow
c_p	heat capacity at const. pressure (J/(kg K))
d	diameter (m)
D_{ij}	diffusion coefficient (m^2/s)
f	Fanning friction factor, f=ξ/4, internal flow
Hg	Hagen number $=(\rho \Delta p/\Delta z) d^3/\eta^2$
L	length (m)
Lq	Lévêque number $=2x_f \,\mathrm{Hg}\, d_h/L$
Nu	Nusselt number $=\alpha d/\lambda$
p	pressure (Pa)
Pr	Prandtl number $=\eta c_p/\lambda$
Re	Reynolds number $=ud/\nu$
Sc	Schmidt number $=\nu/D_{ij}$
Sh	Sherwood number $=\beta_{ij} d/D_{ij}$
Tu	turbulence intensity (%)
u	flow velocity in empty cross section (m/s)
V	volume (m^3)
x_f	frictional fraction of total pressure drop
x, y	coordinates (m)
z	coordinate in flow direction (m)
Greek Letters	
α	heat transfer coefficient (W/(m^2 K))
β_{ij}	mass transfer coefficient (m/s)
δ	boundary layer thickness (m)
λ	thermal conductivity (W/(m K))
η	viscosity (Pas)
ν	kinematic viscosity (m²/s)
ξ	(Darcy) friction factor = 4f, internal flow
ρ	density (kg/m^3)
ψ	void fraction
Subscripts	
0	limiting value (Re→0)
f	friction
h	hydraulic (diameter)
min	minimum
p	particle (nonspherical), equivalent sphere
s	solid
ψ	refering to the velocity in the bed

GENERALIZED LÉVÊQUE EQUATION

The following equation has been termed the Generalized Lévêque Equation (GLE) in (Martin, 2002)

$$Nu/Pr^{1/3} = Sh/Sc^{1/3} = 0.4038(2x_f \, Hg \, d_h/L)^{1/3} \quad (1)$$

where the dimensionless numbers (Nu, Pr, Sh, Sc) are defined as usual, while the Hagen number is $Hg=(\xi/2)Re^2$, or $Hg=2fRe^2$, with the Darcy- or Fanning friction factors, ξ or f, respectively, is a dimensionless number proportional to the pressure gradient ($\Delta p/\Delta z$) and does not contain a flow velocity. The hydraulic diameter d_h is defined as 4 times the cross sectional area divided by the circumference of the flow channel, and L is the length in the direction of flow.

The Darcy (or Fanning) friction factors $\xi=((\Delta p/\Delta z)d/[(\rho/2)u^2]$ ($f = \xi/4$) are proportional to pressure gradient times tube diameter d divided by the stagnation pressure $(\rho/2)u^2$. Substituting ξ (or f) from the Hagen-Poiseuille law for fully developed laminar tube flow, $\xi=64/Re$, ($f=16/Re$), or simpler $Hg = 32Re$, in eqn. (1) yields the classical form of Lévêque's equation:

$$Nu=1.615(Re\, Pr\, d/L)^{1/3} \quad (2)$$

as it is usually found in the textbooks. This equation has been theoretically derived for the first time in André Lévêques thesis (Lévêque, 1928, pp. 283-287). The choice of the characteristic length to be used in Nu, Sh, Re and ξ is arbitrary as both $Nu/Pr^{1/3}$ (or $Sh/Sc^{1/3}$) and $(Hg)^{1/3}$ contain this length with the same power of one. Of course the same (arbitrary) length has to be used in Nu, Sh, and Hg. The same consistency ought to be maintained in the use of the characteristic velocities in both the Reynolds number Re and the friction factor ξ (=4f). In the product $(\xi/2)Re^2$ this velocity cancels. So the heat or mass transfer coefficients predicted from the GLE do not depend on flow velocities, but only on the pressure gradient, the physical properties and the geometric ratio d_h/L.

The generalization in eqn. (1), as compared to eqn. (2), means, that in this form it may also be applied to turbulent flow, as long as the thermal boundary layer remains within the viscous sublayer. This idea was first suggested by both (Bankston and McEligot, 1970) and (Schlünder , 1970).

However, these authors suggested the use of eqn. (1) only for the entry region of a circular duct. In the last ten years it has been shown that the GLE is in fact applicable to a number of other problems of practical interest, like the cross-corrugated channels of chevron-type plate heat exchangers (Martin, 1996), tube bundles (Martin, 2002), and crossed rod matrices (Nanda et. al., 2000) and (Martin, 2002).

This new method can also be used in external flow situations, (Martin, 2002a) not only for internal flow. The present paper will clearly show the applicability of the GLE for both external and internal flows in a more detailed comparison with experimental data from the literature.

EXTERNAL FLOW

Single sphere in cross flow

The friction factor c_F of a sphere in cross flow is plotted in Figure 1.together with the (in general better known) total drag coefficient c_D. Total drag coefficients for a sphere in crossflow are well theoretically known from Stokes' law in the low Reynolds number range: $c_D = 24/Re$ ($Re < 1$). In this "creeping-flow" limit, Stokes' explicit solution of the flow field also yields the friction factor, the part of the total force, that occurs due to frictional forces at the surface alone: $c_F = 16/Re$ ($Re < 1$) The frictional fraction $x_f = c_F / c_D$ in this limit equals 2/3 from Stokes theory.

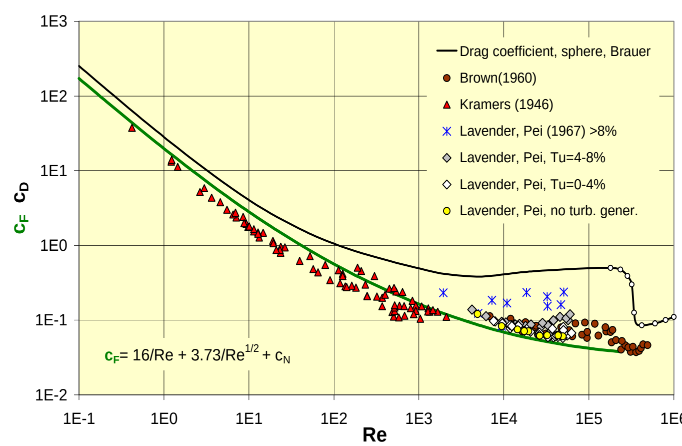

Figure 1: Drag Coefficient $\mathbf{c_D}$ and Friction Factor $\mathbf{c_F}$ of a Sphere in Cross Flow vs. Reynolds Number **Re**.

In the higher Reynolds number ranges, however, the friction factor is not well known, because there is no theoretical solution available, and it is rather difficult to measure the frictional fraction separately. From the literature, it is known, however, that the friction factor goes down to a few percent of the total drag only in the "Newton"-range, where c_D is nearly constant. The fraction x_f therefore must change from 2/3 at low Reynolds numbers to a much smaller value at Reynolds numbers in the range of 10^5. If we use the empirical formula for the drag coefficient as given by (Brauer, 1973) the whole range from Stokes' law up to the critical Reynolds number is covered:

$$c_D = 24/Re + 3.73/Re^{1/2} + (0.49 - 483Re^{1/2}/(10^5 + 0.3Re^{3/2})) \quad (3)$$

The third term gives the observed minimum and maximum behaviour around the nearly constant value of about $c_{D,} = 0.44$ in the Newton range ($2x10^3 < Re < 2x10^5$).

Using the theoretical limiting value of 2/3 for the Stokes range and replacing the third term by an unknown, smaller constant c_N; one can write a tentative equation for the friction factor as:

$$c_F = 16/Re + 3.73/Re^{1/2} + c_N \qquad (4)$$

The second term (boundary layer range) has not been changed, for simplicity. Now the generalized Lévêque equation for a sphere in cross flow has been used in the form:

$$Nu = 2 + 0.4038\ (c_F\ Re^2\ Pr)^{1/3} \qquad (5)$$

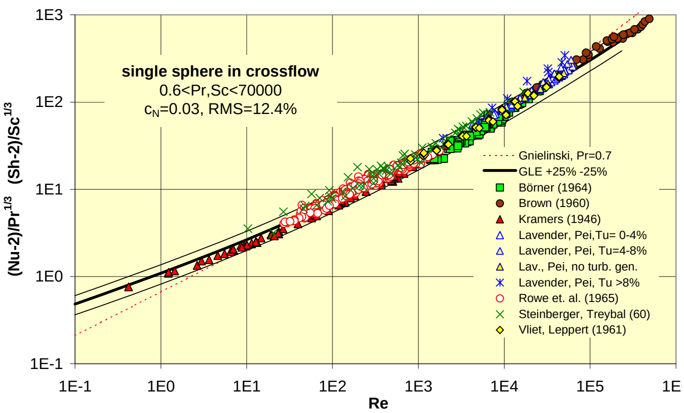

Figure 2: $(Nu-2)/Pr^{1/3}$ and $(Sh-2)/Sc^{1/3}$ vs. Reynolds Number Re, from the GLE, with $c_N = 0.03$.

where the term d_h/L from the internal flow equations has been replaced by $4A_c/A$, (with the flow cross-sectional area A_c and the surface area A) for the external flow cases, which turns out to be $4(\pi/4)d^2/(\pi d^2) = 1$ for the sphere. When putting c_F in the Stokes limit ($Re < 1$, $c_F = 16/Re$) in eqn. (5), we obtain:

$(Nu-2) = 1.017\ (Re\,Pr)^{1/3}$ for $Re<1$,

which is only less than 2.6% higher than the theoretical creeping flow limit

$(Nu - 2) = 0.9914(RePr)^{1/3}$

calculated from an integration of local values over the surface of the sphere. The Lévêque analogy in this limit obviously works very well. Using heat and mass transfer data from a number of different sources, the eqns. (4) and (5) have been used to fit $c_N = 0.03$ to make the analogy work in the whole range of Reynolds numbers.

Figure 2 shows more than 700 experimental data from 7 sources, that have been used already by Gnielinski in 1975 (see HEDH, 2.5.2-7 Fig. 9). Gnielinski's correlation is shown in Fig. 2 as the red broken curve (for Pr=0.7). The new GLE-method using eqns. (4) and (5) are obviously slightly closer to the data in the range of very low Reynolds numbers (Kramers' data, the red triangular symbols): The cubic root behavior for creeping flow is not represented in Gnielinski's formulae. Part of the scatter in Figs 1 and 2 may be caused by various levels of free stream turbulence in the equipment of different authors. In one these sources(see HEDH), Lavender & Pei, (1967), have systematically investigatet this additional parameter and its influence on heat transfer. The data obtained at the highest turbulence levels have been shown in Figs. 1 and 2 by the **X**-shaped symbols. They are always at the upper part of the band of scattering around the GLE-curve.

Figure 3 shows these data alone, with symbols in red, with no turbulence generator used, in white (for low turbulence levels, in grey for intermediate, and in black for the highest ones. The lines in this figure have been calculated from eqns. (4) and (5) with c_N equal to 0.04 (fitting to Tu = 0…4%), with c_N equal to 0.08 (fitting to Tu = 4...8%), and with c_N equal to 0.16 (fitting to Tu > 8%).

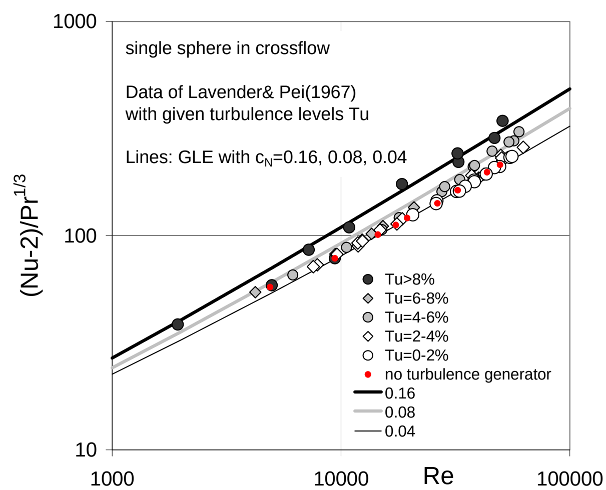

Figure 3: $(Nu-2)/Pr^{1/3}$ vs. Reynolds Number Re, Influence of Free Stream Turbulence Level, Tu. Lines: from the GLE, with $c_N = 0.04, 0.08, 0.16$.

The friction factor constant in Newton's range, c_N, seems to have a direct relation with the free stream turbulence level. So the GLE may also be used to correlate the influence of this additional parameter in a very simple and straightforward way.

In agreement with the experimental data, the turbulence level has a small, if not negligible effect on heat or mass transfer in the range of lower Reynolds numbers, the differences between the lines for $c_N = 0.04$ and 0.16 are much less at the Re = 1000 than at Re = 10000 or 100000.

If the turbulence level is not known, it is recommended to use $c_N = 0.3$, which has been determined from all data as the optimum value, that makes the RMS-deviation to a minimum.

Single cylinder in cross flow

There is no Stokes type solution for creeping flow around a single cylinder. Inertia can not be neglected completely as for the sphere. The known asymptotic solution due to Oseen has been approximated with sufficient accuracy together with the corresponding boundary layer, and Newton-range expressions by (Sucker and Brauer, 1975) as:

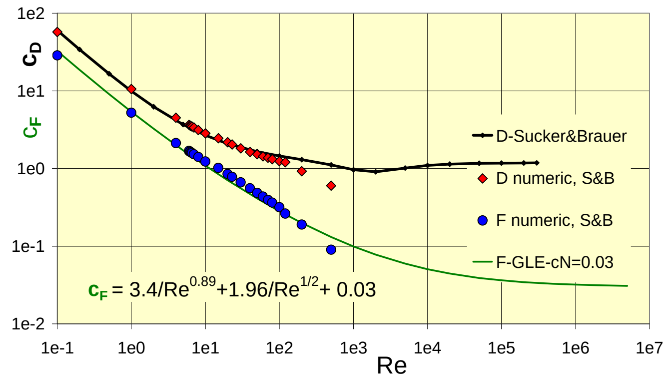

Figure 4: Drag Coefficient $\mathbf{c_D}$ and Friction Factor $\mathbf{c_F}$ of a Cylinder in Cross Flow vs. Reynolds Number **Re**.

$$c_D = 6.8/Re^{0.89}+1.96/Re^{1/2} + + (1.18 - 1/(2500/Re + Re/1100)) \qquad (6)$$

From numerical calculations (for total drag, D, and friction, F, in Fig. 4) it is known, that the frictional fraction $x_f = 0.5$ in the low Reynolds number range, so we write, as in eqn. (4):

$$c_F = 3.4/Re^{0.89}+1.96/Re^{1/2} + c_N \qquad (7)$$

The GLE for the cylinder becomes :

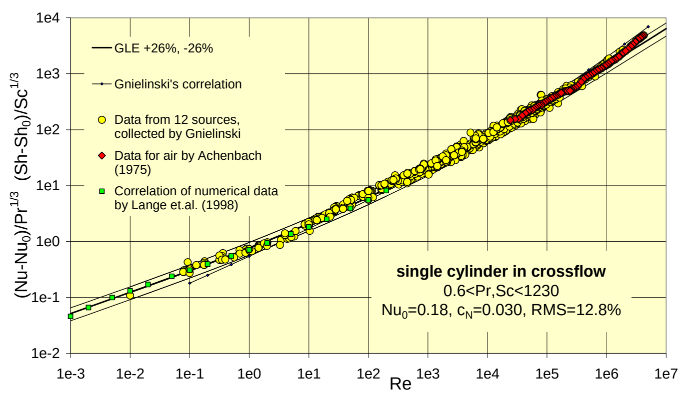

Figure 5: $(Nu-Nu_0)/Pr^{1/3}$ and $(Sh-Sh_0)/Sc^{1/3}$ vs. Reynolds number Re, from the GLE, with $c_N = 0.03$.

$$Nu = Nu_0 + 0.4038\,((4/\pi)\, c_F\, Re^2\, Pr)^{1/3} \qquad (8)$$

The factor $(4/\pi) = 4A_c/A$ has to be included as c_D is based on the cross-sectional area A_c=*dl*, and the surface area is $A = \pi dl$. The data shown in Fig. 4 as yellow circles are from 12 different sources, that have been collected by (Gnielinski, 1981) and compared with his correlation, based on an equation for the Nusselt number of a single sphere in cross flow, with a correction function of bed voidage. Gnielinski's equations are also recommended in the Heat Exchanger Design Handbook (HEDH) and the VDI-Heat Atlas. The red diamond symbols are data by (Achenbach, 1978), which were not included in Gnielinski's collection. They cover the range of the highest Reynolds numbers. The green square symbols are results of (Lange et. al., 1998), which have been obtained from an empirical correlation of numerically calculated Nusselt numbers for Reynolds numbers in the range $10^{-4} < Re < 200$. These numerical values are useful for hot wire anemometry. The limiting (minimum) Nusselt number Nu_0 for the cylinder has been given by Gnielinski to be Nu_{min}=0.3, with the characteristic length equal $L=(\pi/2)d$. Here, the Nusselt, Sherwood, and Reynolds numbers are defined with the diameter *d*, so Nu_0=0.191 would be the equivalent value. Comparison with the carefully calculated numerical data of (Lange et. al., 1998) resulted in a slightly lower optimal value of Nu_0=0.18 (in the range of $10^{-3} < Re < 10^7$). Equations (7) and (8) give a good and a simple correlation of the whole range in a single curve.

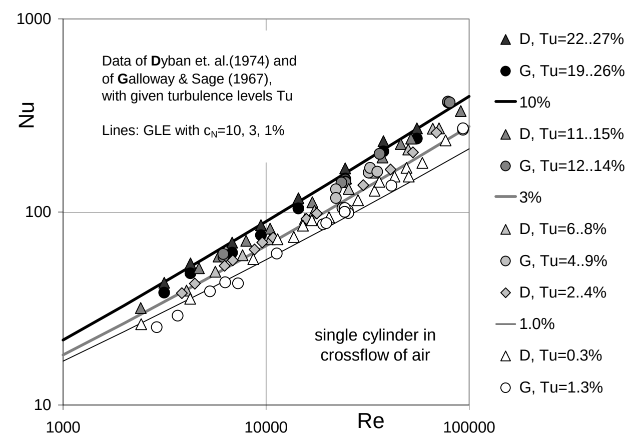

Figure 6: Nusselt vs. Reynolds numbers, Influence of Free Stream Turbulence. D=Dyban et. al., G=Galloway &Sage Lines from the GLE, with c_N= 0.10, 0.03, 0.01

Figure 6 shows Nusselt numbers taken from two of the sources, that have also been used in Fig. 5, and earlier by Gnielinski in 1975 (see HEDH, 2.5.2-6, Fig. 6 and Fig. 7). The results are quite similar to those shown in Fig. 3 for the sphere. The lines in this figure have been calculated from eqns. (4) and (5) with $c_N = 0.01$ (fitting to Tu = 0…2%), with $c_N = 0.03$ (fitting to Tu = 3...10%), c_N equal to 0.10 (fitting to Tu = 11..23%).

Parallel flow over a flat plate

For the classical case of a flat plate in laminar flow parallel to its surface, Lévêque's idea had been used by (Schuh, 1953, see Schlichting, 1965, p. 262) to find a closed form temperature field solution for the limiting case of high Prandtl numbers. In this case the introduction of a similarity variable $\eta = y/\delta(x)$ results in a closed form solution for the local heat transfer coefficient which, with Blasius' solution for the local friction factor $c_F = 0.332/Re_x^{1/2}$ gives $Nu_x = 0.3387\, Re_x^{1/2}\, Pr^{1/3}$ ($Pr \rightarrow \infty$). The limiting solution for high Prandtl numbers is practically useful down to Prandtl numbers of slightly less than one (air, $Pr = 0.7$); the constant at $Pr = 1$ is 0.332, i. e. by about 2% lower than for the high Prandtl number limit. The well known boundary layer solution, therefore, can also be seen to be obtained directly by the Lévêque analogy.

INTERNAL FLOW

Packed beds of spherical solids

Available experimental data on heat transfer in packed beds of spherical particles of diameter d had been collected and empirically correlated earlier by Gnielinski (Gnielinski, 1981). His correlation provides a simple way to test the GLE for its applicability in predicting packed bed heat or mass transfer from pressure drop. The hydraulic diameter is obtained from the well-known relationship

$$d_h = (2/3)\, d\, \psi/(1-\psi) \tag{9}$$

for a bed of spherical particles of diameter d and void fraction ψ. The length L in the GLE has been taken as the average distance between two particles in the bed of spheres obtained from $L/d = (V/V_s)^{1/3}$, with the total bed volume V and the solids volume V_s

$$L = d\,/(1-\psi)^{1/3} \tag{10}$$

resulting in the geometric ratio in the GLE to be a function of the void fraction only:

$$d_h/L = (2/3)\, \psi/(1-\psi)^{2/3} \tag{11}$$

The total pressure drop can be calculated for example from Ergun's equation, which is found in many textbooks (Bird et. al., 1960, p. 200). This gives a relatively good agreement between the Nusselt numbers from the empirical packed bed equations by Gnielinski (Gnielinski, 1981), as recommended in some relevant handbooks, as the VDI-Heat Atlas and the Heat Exchanger Design Handbook. Recently we have reactivated Gnielinski's collection of experimental data from the literature, which contained data from 21 different sources and have added data from other sources, so that our collection on packed bed heat and mass transfer now covers data from 43 sources. The best results with pressure drop correlations from the literature have been obtained using Ergun's equation, which is written here with the Hagen number (proportional to the pressure drop):

$$Hg = Re\,[150(1-\psi) + 1.75Re]\,(1-\psi)/\psi^3 \tag{12}$$

Molerus' equation, (Molerus, 1993) which is based on the drag coefficient of a single sphere, and more complicated in it's structure, did not give better results. A slightly better result (25.4%, compared to 25.5% RMS) could be found using a modified Ergun's equation

$$Hg = (0.4/\psi)^{0.78}\, Re\,[150(1-\psi) + 1.75Re]\,(1-\psi)/\psi^3 \tag{13}$$

with the correction $(0.4/\psi)^{0.78}$, that has been successfully used in a different form of pressure drop correlation (VDI-Heat Atlas, 1993, p. Le1). The correction in equation (13) makes Ergun's equation more closely agree with experimental data for void fractions lower as well as higher than 0.4. The optimal values of the frictional fraction x_f to be used in eqn. (1) have been obtained by minimizing the sum of the squares of $ELOG = \lg Nu_{calc} - \lg Nu_{exp}$ (*i. e.* a linear regression in a log-log law). This method was found to be superior to the previously used one, where the RMS-value of the relative errors $ERC = (Nu_{calc} - Nu_{exp})/Nu_{calc}$ had been used to find the optimal x_f. The relative errors, whether we use ERC, or ERE, with Nu_{exp} in the denominator, give unsymmetric error distributions around the theoretical curve. The linear regression of the log-log-law, however, behaves symmetrical in this respect. The deviation (as given in Table 1 at the end of the text) is obtained from $RMSD = 10^{RMS(ELOG)} - 1$. These values are greater than the ones obtained earlier by minimizing the RMS of the relative errors based on the calculated values ERC.

The influence of void fraction ψ of the packed bed is relatively well represented by Ergun's equation. It can be improved by using the modified version, with the term $(0.4/\psi)^{0.78}$ multiplied. Obviously this modification has no effect for the vast majority of data with void fractions near 0.4.

Data for the densest packing with ψ=26%, those with void fractions of 32%, 50% ,63%, 78%, and 94% are shown in Fig.7 together with the lines calculated from GLE with the modified Ergun equation (eqn. (13)).

The number of data shown in Fig. 7 is 813, i.e. about 30% of the total amount collected – the remaining 70% of data with around 40% bed voidage have been left out for clarity. The lower broken line shows the heat or mass transfer values for a single sphere in cross flow (as calculated from eqns. (4) and (5) respectively) for comparison. Using the original Ergun equation would give qualitatively the same behaviour, but the deviations for voidages far from 0.4 would be somewhat greater.

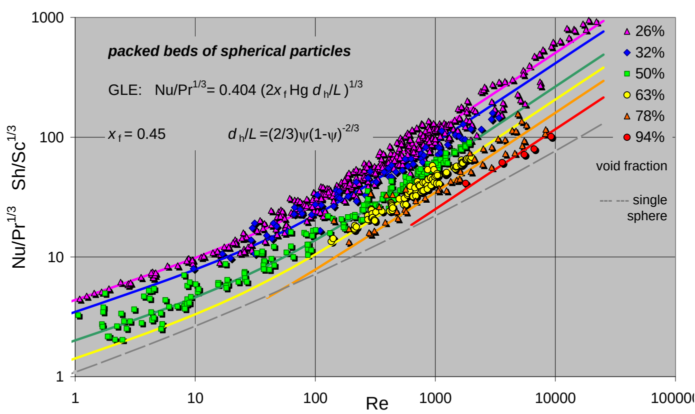

Figure 7: $Nu/Pr^{1/3}$, $Sh/Sc^{1/3}$ vs. Re with the bed void fraction as a parameter. Comparison of 813 experimental data with GLE.

One can see from Figure 8, that all the 2646 data for $Re_{\psi} Pr > 100$ and the whole range of void fractions $0.25 < \psi < 0.95$ are correctly represented by the new method and follow the 1/3 power-law of the GLE very well.

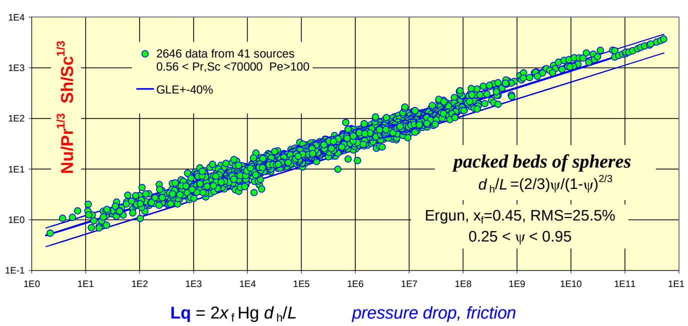

Figure 8: $Nu/Pr^{1/3}$, $Sh/Sc^{1/3}$ vs. Lq. Comparison of 2646 Experimental Data with the GLE including those from Fig. 5.

<u>*Packed beds of nonspherical solids*</u>

The number of data available in the literature for packings of nonspherical solids is much less than for spherical ones. We could evaluate 256 experimental data for cubes, cylinders, rings (hollow cylinders) and Berl saddles. The pressure drop correlations, based on Ergun's equation are usually recommended to be used with an equivalent diameter defined as

$$d_p = 6(V/A)_p \qquad (14)$$

i.e. the diameter of a sphere having the same ratio of volume V to surface area A of the particle in question. As Ergun's equation has proven to give very reasonable results, this definition has also been used in the calculation of heat or mass transfer from the GLE.

Under these assumptions, the hydraulic diameter d_h is the same as for a sphere (see eqn, (9)), and if the length L in the Lévêque equation is also taken as the average distance between two "equivalent spheres" in the packing (eqn. (10)), we end up with a simple and uniform calculation method, where eqn. (11) for d_h/L can also be used.

With Ergun's equation, eqn. (12) the calculation follows exactly the same route as that for spherical solids. From the data of (Glaser &Thodos, 1958) for cubes, those of (Wilke &Hougen, 1943), (Glaser &Thodos, 1958), and those of (Handley & Heggs, 1968) for cylinders, the data of (Taecker & Hougen, 1949), (Shulman et.al., 1955), (H. Glaser, 1955), for rings, as well as those of (Taecker & Hougen, 1949), of (Shulman and Margolis, 1957), for saddles, the optimal values of the frictional fraction x_f to be used in the GLE have been obtained by linear regression in a log-log law, as for the spherical solids, from the GLE with Ergun's equation for the total pressure drop (Martin, 2003).

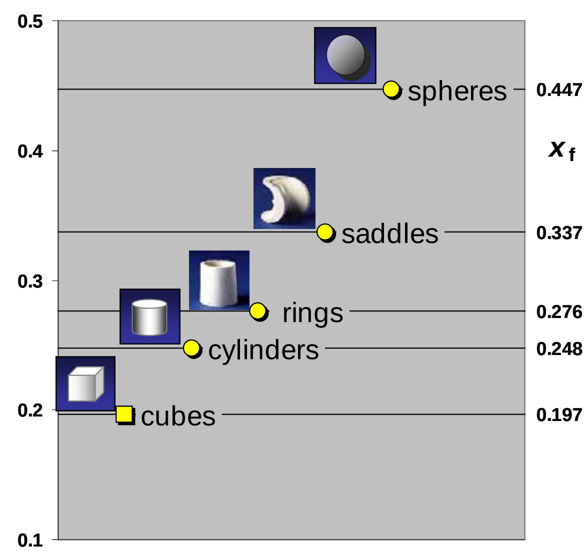

Figure 9: Frictional Fractions of Various Solid Particles in Packed Beds to be Used in Erguns Equation with the GLE.

It is interesting to find, that the values of the frictional fraction x_f that have been obtained from fitting the GLE to the data for beds of nonspherical particles (see Fig. 9) are all lower than the one for the spherical particles. The possible reason for these lower heat or mass transfer performance at the same pressure drop may be seen in a partly blockage of surface area by particles touching each other by plane faces (especially so for the cubes, cylinders and rings). If one calculates an "inactive surface fraction" as:

$$A_{inactive}/A = 1 - (x_{f\,opt} / x_{f\,sphere})^{1/3} \quad (14)$$

the following values are obtained:

bed particles	$x_{f\,opt}$	"inactive surface fraction"
cubes	0.197	**24%**
cylinders	0.248	**18%**
rings	0.276	**15%**
saddles	0.337	**9%**
spheres	0.447	**0%**

Tube bundles and crossed rod matrices

The GLE has also been successfully applied to the calculation of heat transfer in tube bundles. The final results of a comparison of more than 3000 experimental data collected by (Gnielinski, 1979) from 20 different sources, also listed in (Martin & Gnielinski, 2000), has been given in (Martin, 2002).

Data on crossed rod matrices from (Kays &London, 1984) have been also compared to the GLE, by (Nanda et. al., 2001), and more completely, by (Martin, 2002).

Plate heat exchangers of chevron-type

The GLE has been applied for the first time exactly ten years ago - the paper (Martin, 1996) carries the note: "Received 18 September 1995; accepted 6 October 1995" in the title section.

The cross corrugated channels formed by chevron-type plates are widely used in industrial heat exchangers. The strong influence of chevron angle on pressure drop – comparable to the influence of void fraction on pressure drop in packed beds – has been described in that paper by a physically reasonable, yet simple model equation. Three parameters in that model have in turn been fitted to some experimental data. Heat and mass transfer data from model plates, as well as from industrial heat exchanger plates have shown a Lévêque-type behaviour, that is a dependency on the cubic root of the pressure drop. The number of experimental data then available for that first application of the Lévêque analogy was relatively small. It might be interesting to test the GLE with more data from that area.

CONCLUSION

Table 1 shows a list of most problems, that have in the meantime been treated by the GLE approach. It contains the kind of problem (1st column), the number of data points from the literature (2nd column), that has been used to determine the frictional fraction x_f (3rd column) to be used as a factor to the total drag, or total pressure drop respectively, that contributes to heat or mass transfer via the Lévêque analogy.

Table 1 Number of data, frictional fraction, and RMS-deviations from HEDH, and GLE predictions

problem	data	x_{fopt}	HEDH	GLE
			RMS/%	**RMS/%**
Single Sphere	732	0.67...0.068	20	12
Single Cylinder	1036	0.50...0.030	26	13
EXTERNAL FLOW	1768		**23**	**13**
	data	x_{fopt}	HEDH	GLE
Packed Bed of	2902	x_{fopt}	**RMS/%**	**RMS/%**
Spheres	2646	**0.45**	31	26
Saddles	79	**0.34**	19	15
Rings	110	**0.28**	22	23
Cylinders	47	**0.25**	18	13
Cubes	20	**0.20**	31	15
Tube Bundle, Rod Matrix	3361			
inline	1694	**0.59**	19	18
staggered	1457	**0.46**	34	14
Crossed Rod Matrices	210	**0.46**		4
Plate Heat Exchangers	75	**0.50**		
INTERNAL FLOW	6338		**25**	**18**
TOTAL:	8106	**0.48**	**24**	**15**

The 4th and the 5th columns contain the root mean square (RMS) deviations of the data (Nu, or Sh), when compared to the state of the art methods as presently recommended by the Heat Exchanger Design Handbook (Schlünder, 1983), short HEDH, and the Generalized Lévêque Equation (GLE), eqn. (1), or (5), or (8), respectively. From the more than 8000 data tested so far, the number-weighted average x_f comes out to be 0.48, close to 0.5. The overall RMS-deviation can be reduced by the GLE, compared to the HEDH-methods from 24% to 15%. So its general application in these problems may be highly recommended.

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REFERENCES

Achenbach, E., 1974, "Heat transfer from smooth and rough surfaced cylinders in a cross-flow", Proc. 5th Int. Heat Transf. Conf., Tokyo, FC6.1, Vol. 5, pp. 229-233.

Achenbach, E., 1975, Table of data from (Achenbach, 1974), personally communicated to Dr. Gnielinski in 1975.

Achenbach, E., 1978, "Heat transfer from spheres up to Re = $6x10^6$", Proc. 6th Int. Heat Transfer Conference, Toronto, FC(b)-28, Vol. 5, pp. 341-346.

Bankston, C. A., and D. M. McEligot, 1970: "Turbulent and laminar heat transfer with varying properties in the entry region of circular ducts," Int. J. Heat Mass Transfer, vol. 13, pp. 319-344.

Bird, R.B., W.E. Stewart & E.N. Lightfoot, 1960, "Transport Phenomena," Wiley, New York, p. 364 & p. 399.

Brauer, H., 1973, „Impuls-, Stoff- und Wärmetransport durch die Grenzfläche kugelförmiger Partikeln," Chemie-Ing. Techn. vol.45, pp. 1099-1103.

Glaser, H., 1955, „Instationäre Messung der Wärmeübertragung von Raschigringschüttungen," Chemie-Ing.-Techn. Vol. 27, pp 637-643.

Glaser, M. B. , G. Thodos, 1958, "Heat and momentum transfer in the flow of gases through packed beds," AIChEJ, vol.4, pp. 63-68.

Gnielinski, V., 1979,." Equations for calculating heat transfer in single tube rows and banks of tubes in transverse flow," Int. Chem. Eng., 19, 380-390.

Gnielinski, V., 1981, "Equations for the Calculation of Heat and Mass Transfer During Flow through Stationary Spherical Packings at Moderate and High Peclet Numbers," Int. Chem. Eng. vol. 21, pp. 378-383.

Handley, D., and P. J. Heggs, 1968, "Momentum and heat transfer mechanisms in regular shaped packings," Trans. Inst. Chem. Engrs. Vol. 46, pp 251-264.

Kays, W. M. and London, A. L., 1984, "Compact Heat Exchangers," (3rd ed.), Washington, Hemisphere.

Lange, C. F., F. Durst, M. Breuer, 1998, "Momentum and heat transfer from cylinders in laminar crossflow at $10^{-4} \leqq Re \leqq 200$", Int. J. Heat Mass Transf., 41, pp. 3409-3430.

Lévêque, A., 1928, "Les lois de la transmission de chaleur par convection," Ann. Mines, vol. 13, pp. 201-299, 305-362, 381-415.

Martin, H., 2003, "Heat Transfer in Packed Beds from Pressure Drop – An Application of the Lévêque Analogy–," Proc. 4th Int. Conf. on Compact Heat Exchangers, Fodele Beach, Crete Island, Greece, Sept 29-Oct 3, pp. 119-124.

Martin, H., 2002, "The Generalized Lévêque Equation and its practical use for the prediction of heat and mass transfer rates from pressure drop," Chem. Eng. Sci., vol. 57, pp. 3217-3223.

Martin, H., 2002a, "The Generalized Lévêque Equation (GLE) and its use to predict heat and mass transfer from fluid friction," Proc. 12th Int. Heat Transfer Conf., Grenoble, France, pp. 135-140.

Martin, H., and Gnielinski, V., 2000, "Calculation of heat transfer from pressure drop in tube bundles", in: E. W. P. Hahne et. al. (Eds.) Proceedings of the 3rd European Thermal Sciences Conf. 2000, pp. 1155-1160.

Martin, H., 1996, "A theoretical approach to predict the performance of chevron-type plate heat exchangers," Chem. Engng. Process., Vol. 35, pp. 301-310.

Molerus, O., 1993, "Principles of Flow in Disperse Systems," Chapman & Hall, London

Nanda, P., S. K. Das, and H. Martin, 2001, "Application of a New Analogy for Predicting Heat Transfer to Cross Rod Bundle Heat Exchanger Surfaces," Heat Transfer Engng., vol. 22, pp. 17-25.

Rowe, P. N., K. T. Claxton, 1965, "Heat and mass transfer from a single sphere to fluid flowing through an array," Trans. Inst. Chem. Engrs. Vol. 43, pp. 321-331.

Schlichting, H., 1965, „Grenzschichttheorie", 5. ed. Karlsuhe.

Schlünder E.U., 1983, (editor-in-chief) "HEDH - Heat Exchanger Design Handbook," Washington: Hemisphere.

Schlünder, E. U. , 1970, „Die wissenschaftliche Theorie der Wärmeübertragung -geschichtliche Entwicklung und heutiger Stand," Chemie-Ing.-Techn., vol.42, pp. 905-910.

Shulman, H., L., et. al., 1955, "Wetted and effective interfacial areas, gas and liquid phase mass transfer rates," AIChEJ, vol. 1, pp. 253-258.

Shulman, E., J. E. Margolis., 1957, "Performance of packed columns," AIChEJ, vol. 3, pp. 157-161.

Sucker, D., and H. Brauer, 1975, „Fluiddynamik bei quer angeströmten Zylindern," Wärme- und Stoffübertragung, vol. 8, pp. 149-158.

Taecker, R., O. Hougen, 1949, "Heat, mass transfer of gas film in flow of gases through comercial tower packings," Chem. Eng. Progr. vol. 45, pp. 188-193.

VDI Heat Atlas, 1993, VDI-GVC(ed.), Düsseldorf: VDI-Verlag, English translation of the 6th ed. of the German original, 1993, "VDI-Wärmeatlas"

Wilke, C., O. Hougen, 1943, "Mass transfer in the flow of gases through granular solids extended to low modified Reynolds numbers," Trans. Amer. Inst. Chem. Engrs. Vol. 41, pp. 445-451.

The 43 references used to compare the GLE-predictions to experimental data for packed beds of spherical particles is not included here. A complete list of these references may be obtained upon request from the author.

The references used earlier by Gnielinski to develop his well known equations for plate, cylinder, and sphere, as well as those for the packed beds are not again listed here. They are easily found in the corresponding sections of the Heat Exchanger Design Handbook (Schlünder, 1983), the VDI-Heat Atlas, 1993, or in its German original, the VDI-Wärmeatlas, presently available in the 9. edition 2002, soon in the 10th edition, 2006.