

ECONOMIC OPTIMIZATION OF COMPACT HEAT EXCHANGERS

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ABSTRACT

Rough estimation of an economically optimal flow velocity still seems to be general engineering practice in heat exchanger design. The more rational alternative, a tediously detailed economic optimization, may not often be chosen because of excessive engineering costs. An intermediate path is shown in the present paper. Using some simplifying assumptions, a dimensionless function $FC(Re)$ may be derived, which is proportional to the sum of the annual costs of investment C_I (assumed to be proportional to the surface area) and the costs of operation C_O (proportional to the pumping power required). The minimum of this function at the optimal Reynolds number ($Re=Re_{opt}$) depends on the type of heat exchanger chosen, and on a new dimensionless quantity, called Re_{eco} , that contains all necessary economic input parameters. The minimum can be easily found by standard methods. The question of an economically optimal efficiency ϵ_{opt} can also be answered in a simple way. So the influence of changing economic situations may be easily taken into account even at an early stage of heat exchanger design.

ECONOMIC FLUID VELOCITY

The economic design of heat transfer equipment usually starts with assuming a value of the flow velocity, which is thought to be close to an economic optimum value. In some textbooks (e.g. in (Martin,1992)) one may find ranges of recommended flow velocities of say

$$0.2 \text{ m/s} < w_{\text{liquid}} < 2.0 \text{ m/s}, \text{ and}$$

$$5 \text{ m/s} < w_{\text{gas, atmospheric pressure}} < 50 \text{ m/s}.$$

Usually these flow velocities are roughly chosen depending on the individual experience of the designer. A more rational alternative to this rough engineering practice would be the detailed economic optimization of each heat

exchanger during design. An example of such a detailed step-by-step procedure is given in (Martin,1992) for a double-pipe heat exchanger.

Many authors, as e.g., Gregorig (1959), forty years ago, or, more recently, Hewitt and Pugh (1998) have tried to improve and to simplify the economic design of heat exchangers. In these sources, and especially in the later, more extended version of Gregorig's book (Gregorig, 1973), one may find a number of additional references on the topic. The present paper suggests a solution, which may be useful in the early stages of heat exchanger design. More or less experienced guessing of a value for the flow velocity should be replaced by calculation, based on a rational approach (see also: Martin, 1998). A simple explicit formula for the optimal flow velocity will be derived, which is very easy to apply. Taking the relevant economic parameters into account, the most economic cross-sectional area of an apparatus may thus be found.

ASSUMPTIONS

The **annual costs of investment** C_I or capital costs are taken to be proportional to the surface area A and the amortization a^* (of say 10%/year).

$$C_I = C_A A \cdot a^* \quad (1)$$

The price per unit area C_A will of course depend on the type of apparatus, on the material needed, and on the size (i.e., on the surface area A) itself. It is generally well known, that the price of equipment does not linearly increase with its size. So Eq. (1) should be regarded as a linearization of a more appropriate empirical power law ($C = \text{const}A^n$), with an exponent n less than one. If the prices of heat exchangers of a given type for different sizes are known, one may find $C_A = C_{A0} (A/A_0)^{(n-1)}$, to account for the depressive increase of price with equipment size.

The **costs of operation** C_O will be taken to be proportional to the pumping power required to overcome the flow resistances in the exchanger

$$C_O = k_{el}\tau(1+x)\frac{\Delta p V_t}{\eta_p} \quad (2)$$

The following variables do have an influence on the costs of operation: the price of electrical energy k_{el} , the hours of operation per year τ , the factor x accounting for the pumping power required on the other side of the heat exchanger (for symmetrical operation, x would be equal one in a plate heat exchanger for example), the pressure drop Δp , the volumetric flowrate V_t , and last but not least, the efficiency of the pump (or fan) η_p .

In many cases, C_O from Eq. (2) will be the main part of the cost of operation, at least for a heat exchanger with process fluids on both sides, *i.e.*, not a heater or a cooler in the sense of the pinch-point-method of energy integration. For heaters and coolers the (thermal) energy cost of the utilities will usually be much more important.

TOTAL COST FUNCTION

Starting from these assumptions one can easily show, that the total cost, *i.e.*, the sum $C = C_I + C_O$ measured in an appropriate currency unit per year [ACU/year], when nondimensionalized ($FC = C/C_N$) by

$$C_N = C_A a^* \rho c_p V_t \frac{d}{\lambda} NTU \quad (3)$$

leads to a relatively simple cost function

$$FC(Re) = \left(1 + (1+x)\frac{f}{2} \left(\frac{Re}{Re_{eco}} \right)^3 \right) \frac{1}{Nu_{ov}} \quad (4)$$

where the volume flowrate V_t , density, specific heat capacity, and conductivity (r, c_p, I) of the process fluid on one side (chosen for the design), the diameter d , and the Number of Transfer Units NTU have only to be known if absolute values in [ACU/year] are required. In Eq. (4) $Re = wd/v$, $f(Re)$ is the Fanning friction factor, the “velocity” in the so called “economic Reynolds number” Re_{eco} is to be calculated from the specific economic parameters and the fluid density ρ :

$$Re_{eco} = \left[\frac{C_A a^* \eta_p}{k_{el} \tau \cdot \rho} \right]^{1/3} \frac{d}{v} \quad (5)$$

The term Nu_{ov} stands for a dimensionless overall heat transfer coefficient $Nu_{ov} = kd/\lambda$. The latter can be expressed as

$$\frac{1}{Nu_{ov}} = \frac{1+y}{Nu} + R_{w,f}^* \quad (6)$$

where the factor y stands for a dimensionless transfer resistance of the other side (just as x had been introduced in Eq. (2) to account for the pumping power on the other side) and $R_{w,f}^*$ includes the resistances of the solid wall, and fouling resistances if necessary. As a first example, Fig. 1 shows the results of an optimization of this kind applied to the chevron-type plate heat exchangers, using the equations given in Martin (1996) for pressure drop and heat transfer. These equations are given in a comprehensive form in the *Appendix*. Varying the angle ϕ of the corrugation pattern from 30° to 80° , measured against the main flow direction, the pressure drop at a fixed flowrate would increase by a factor of about 20. So with increasing angle, the economically optimal Reynolds number varies from about $Re_{opt}(30^\circ) = 4300$ to $Re_{opt}(80^\circ) = 1500$. The total cost function FC from Eq. (4) has an absolute minimum at an angle of about 60° .

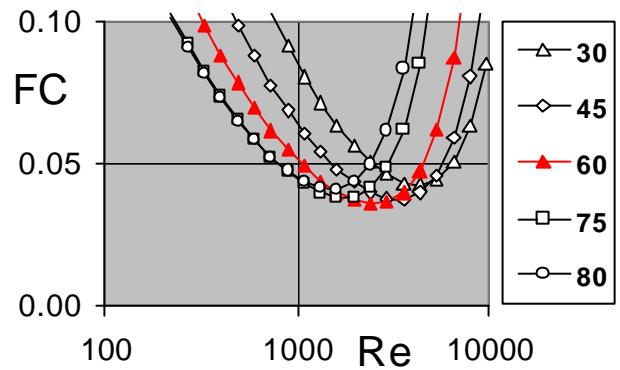


Fig. 1 Total Cost Function FC vs Re for Chevron-Type Plate Heat Exchangers, Parameter: Chevron angle, $\phi = 30^\circ, 45^\circ, 60^\circ, 75^\circ, 80^\circ$ (see Table 1)

This can be seen better in Fig. 2, where the values of FC_{min} from Fig. 1 are plotted versus ϕ . The numerical values used in this optimization are listed in Table 1.

The curves in Fig.1, showing the total cost function FC vs. Re , *i.e.*, the sum of C_I (falling with Re) and C_O (rising with Re) clearly show the minima of the total costs. The Reynolds numbers at these minima are the economically optimal ones. They can be found graphically or by standard procedures.

From Eq. (1) we can see, that increasing price per m^2 of heat transfer surface C_A , and increasing interest rates (*i.e.*, a^* increasing) would lift the falling branch of the curve, *i.e.*, move the optimum velocity to higher values.

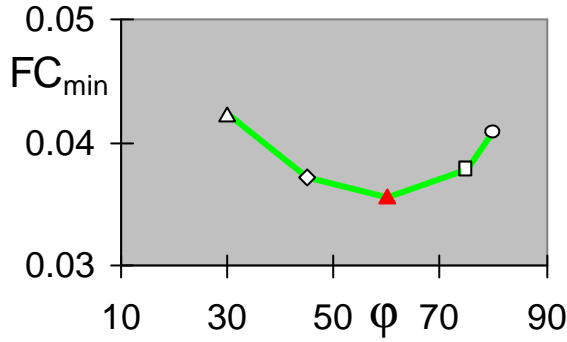


Fig. 2 Minima of the Total Cost Function FC_{min} vs Chevron-Angle ϕ

Equation (2) tells us that higher price of electrical energy k_e , higher number of hours of operation τ (max: 8760 h/year), and lower pump efficiency η_p would lift the rising branch of the curve, and thus move the optimum velocity to lower values.

Table 1 Economic Optimization of Plate Heat Exchangers - numerical input, and results. Equations used: see Appendix.

Pr	3	x	1	R*_{wf}	0.003
Re_{eco}	3000	y	1		
j	30°	45°	60°	75°	80°
Re_{opt}	4287	3334	2518	1750	1517
FC_{min}	0.0423	0.0372	0.0356	0.0378	0.0408

So far the method (Eqs. (1) through (6)) had already been presented by Martin (1998). In the following, a shortcut solution will be derived, that allows for an explicit closed-form calculation of the optimal fluid velocity.

THE SHORTCUT SOLUTION

The Fanning friction factor f ($= \xi/4$) and the overall Nusselt number Nu_{ov} may be approximated by simple power laws in many practical cases.

$$f = c_F Re^{-n} \quad (7)$$

$$Nu_{ov} = c_h(Pr) Re^m \quad (8)$$

Here the factors c_F and c_h , as well as the exponents n and m are constants. The exponent m in Eq. (8) may be chosen a little bit smaller than the corresponding exponent in an

equation for the Nusselt number of one side, as the overall heat transfer coefficient contains a wall (and fouling) resistance, which do not depend on the flow rate. So if the Reynolds exponent in the Dittus-Boelter equation for turbulent tube flow is 0.8, one may take m in Eq. (8) to be about 0.7 or 0.6, depending on the relative importance of the wall resistance. The total cost function FC from Eq. (4), with Eqs. (7) and (8) leads to a relatively simple function of the Reynolds number $F^*(Re) = FC \cdot c_h$.

$$F^* = Re^{-m} + \left(\frac{(1+x)c_F}{2Re_{eco}^3} \right) Re^{3-n-m} \quad (9)$$

The derivative of F^* with respect to Re , when put equal to zero, yields an explicit formula to calculate the optimal Reynolds number, $w_{opt}d/v$, or the optimal flow velocity:

$$w_{opt} = \frac{v}{d} \cdot \left(\frac{m \cdot 2Re_{eco}^3}{(3-n-m)(1+x)c_F} \right)^{1/(3-n)} \quad (10)$$

It is clear that the factor $c_h(Pr)$ has no influence on the value of the optimal flow velocity. From Eq. (10) with Eq. (5) for Re_{eco} , one can find that the optimal flow velocity, under the above mentioned assumptions, depends on the exponents (n, m) of the friction and (overall) heat transfer laws, and on two physical properties of the fluid (the density ρ , and the dynamic viscosity $\mu = \nu\rho$). The diameter d of the channel has an effect on w_{opt} , too.

$$w_{opt} \propto \left(\frac{m}{3-n-m} d^n \cdot \mu^{-n} \rho^{-(1-n)} \right)^{1/(3-n)} \quad (11)$$

Using the well-known Blasius equation for turbulent tube flow, $n = 1/4$, and the optimal flow velocity depends on the tube diameter and on the viscosity with the weak exponents $1/11$, or $-1/11$ respectively, while the fluid density enters with an exponent of $-3/11$. For fully developed laminar tube flow, $n = 1$, *i.e.*, the density has no effect on w_{opt} , while d and μ enter with exponents of $1/2$ and $-1/2$.

Similar approximate explicit solutions of the economic optimization problem had been found much earlier (see *e.g.*, Gregorig, 1959), but they seem to have been greatly ignored in industrial practice.

EXAMPLES

Using typical sets of input data for Eqs. (5) and (10) the results may be compared to the values known from experience.

C_A	400 Euro/m ²	price/m ²
a^*	10%/year	amortization

η_P	0.5	pump efficiency
τ	6500 h/year	hours of operation
k_{el}	30 Euro/MWh	price of electrical energy
x	1	ratio of pumping powers

Water in the tubes of a shell-and-tube hx:

ρ	997 kg/m ³	density
ν	$8.93 \cdot 10^{-7}$ m ² /s	kinematic viscosity
d	12 mm	tube diameter
c_F	0.3164/4	turbulent tube flow
n	0.25	Blasius law
m	0.7	Re exponent of ov htc

$Re_{eco} = 6296 \quad Re_{opt} = 23734 \quad w_{opt} = 1.77 \text{ m/s}$

Air in the tubes of a shell-and-tube hx:

ρ	1.168 kg/m ³	density
ν	$1.58 \cdot 10^{-5}$ m ² /s	kinematic viscosity
d	12 mm	tube diameter
c_F	0.3164/4	turbulent tube flow
n	0.25	Blasius law
m	0.7	Re exponent of ov htc

$Re_{eco} = 3372 \quad Re_{opt} = 12009 \quad w_{opt} = 15.8 \text{ m/s}$

Water in a chevron-type plate hx:

ρ	997 kg/m ³	density
ν	$8.93 \cdot 10^{-7}$ m ² /s	kinematic viscosity
d	6 mm	hydraulic diameter
c_F	18.2/4	$\phi = 71^\circ$ (hard plate)*
n	0.25	as in Blasius law
m	0.6	Re exponent of ov htc

$Re_{eco} = 3148 \quad Re_{opt} = 2372 \quad w_{opt} = 0.35 \text{ m/s}$

Air in a chevron-type plate hx:

ρ	1.168 kg/m ³	density
ν	$1.58 \cdot 10^{-5}$ m ² /s	kinematic viscosity
d	6 mm	hydraulic diameter
c_F	18.2/4	$\phi = 71^\circ$ (hard plate)*
n	0.25	as in Blasius law*
m	0.6	Re exponent of ov htc

$Re_{eco} = 1687 \quad Re_{opt} = 1201 \quad w_{opt} = 3.17 \text{ m/s}$

* The values for $c_F = 18.2/4$ and $n = 0.25$ have been taken from (Martin, 1992, p.72, Fig. 2.29).

The results for water ($w_{opt} = 1.77 \text{ m/s}$) and air (15.8 m/s) respectively in a conventional shell-and-tube heat exchanger do in fact agree very well with the well-known recommended values for liquids (0.2 to 2.0 m/s) and for gases at normal pressure (5 to 50 m/s) as given at the beginning of the paper.

The considerably lower optimal velocities of only $w_{opt} = 0.35 \text{ m/s}$ for water (or 3.17 m/s for air) for a compact chevron-type plate heat exchanger are a result of the much higher flow resistance of this type of exchanger.

ECONOMICALLY OPTIMAL EFFICIENCIES

Once the cross-sectional area of the heat exchanger has been found from the given volume flowrate, and the optimal flow velocity, the remaining question is to fix the length, or the number of transfer units NTU. Usually, the efficiency ϵ of a heat exchanger is assumed to be given, when starting the design procedure. In that case NTU is also fixed, if the flow configuration has been chosen. Heat recovery, however, has an economic value opposite to the total costs C, which may be written as the savings, S:

$$S = S_{max} \epsilon \tag{12}$$

with

$$S_{max} = \rho c_p V_t (T_{h,in} - T_{c,in}) \tau k_{therm} \tag{13}$$

where the maximal possible savings are proportional to the maximal possible change of enthalpy flow, and k_{therm} is the price of thermal energy, often roughly estimated to be one third of the price of electrical energy ($k_{therm} = k_{el}/3$). The difference (S-C), with C as the total costs

$$C = C_{Aa}^* \rho c_p V_t (d/\lambda) NTU FC(Re, Re_{eco}, \dots) \tag{14}$$

should be maximized for optimal economic design. It is clear that under the assumptions made in this paper, the savings are proportional to the efficiency, while the cost are proportional to the NTU (see bold terms in Eqs. (12) and (14)). This idea has been brought forward only recently by Chawla (1999). The break even point, *i.e.* the situation, where the savings just equal the costs (S-C=0), naturally leads to a second dimensionless criterion (besides Re_{eco}) connected with the economic design of heat exchangers:

$$GT = \frac{\lambda (T_{h,in} - T_{c,in}) \tau \cdot k_{therm}}{d \cdot C_{Aa}^*} \tag{15}$$

which may be called a „thermal gain number“.

For the break-even situation, the Eqs. (12) to (15) require

$$\left(\frac{\epsilon}{NTU} \right)_0 = \Theta_0 = \frac{FC(Re, Re_{eco}, \dots)}{GT} \tag{16}$$

or $FC < GT$. Otherwise the installation of the heat exchanger would only lead to economic losses. The ratio of efficiency ϵ to NTU is the normalized mean temperature difference (NMTD), Θ (see Martin, 1992).

In general, the function to be maximized can be written:

$$S - C = S_{\max}(\varepsilon - \Theta_0 \text{NTU}). \quad (17)$$

For the important special case of a balanced counterflow heat exchanger, the relationships between NTU, and Θ with the efficiency ε are

$$\text{NTU} = \frac{\varepsilon}{1 - \varepsilon} \quad \Theta = 1 - \varepsilon \quad (18)$$

Using this in Eq. (17) to eliminate NTU, the resulting function of the efficiency is:

$$\frac{S - C}{S_{\max}} = \varepsilon - \Theta_0 \frac{\varepsilon}{1 - \varepsilon} \quad (19)$$

This function, as shown in Fig. 3, has a zero at $\varepsilon_0 = 1 - \Theta_0$, and at $\varepsilon = 0$, it has a maximum in between at:

$$\varepsilon_{\text{opt}} = 1 - \sqrt{\Theta_0} \quad (20)$$

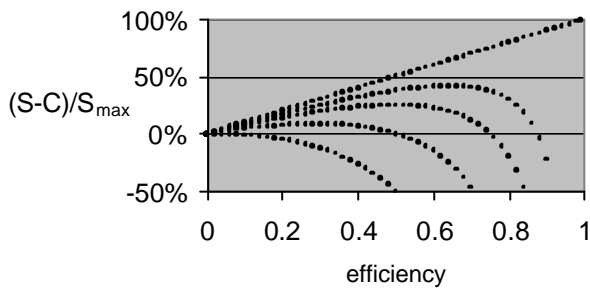


Fig. 3 The Savings-Costs-Function vs efficiency. Parameter: $\Theta_0 = 0, 0.125, 0.25, 0.5, 1.0$

The value of Θ_0 is to be obtained from Eq. (16) as the ratio of the total cost function FC, and the thermal gain number GT from Eq. (15).

CONCLUSIONS

The examples of the calculation of an economically optimal flow velocity for different types of heat exchangers have shown that the new method may be very easily applied even in an early stage of equipment design. The simplified method of optimization, with all economic data put together in the terms Re_{eco} , in Eq. (5), and GT in Eq. (15), make it easy to check how a higher price of material (increasing C_A), a lower price of electrical, k_{el} , and thermal energy k_{therm} , or a lower number of hours of operation per year τ , may change the optimal velocity and the optimum efficiency. The velocity determines the cross sectional area, while the

efficiency fixes the transfer surface area of the most economic design. The shortcut method may help to take economic considerations into account already in the early stages of heat exchanger design.

NOMENCLATURE

A	surface area of a heat exchanger, m
a^*	amortization, %/year
C	total costs, ACU/year (appropriate currency unit = EURO, USD, YEN)
C_A	price per unit area, ACU/m ²
C_I	annual costs of investment, ACU/year
C_N	normalization factor, Eq. (3), ACU/year
C_O	costs of operation, ACU/year
c_F	factor in Eq. (7), dimensionless
$c_h(\text{Pr})$	factor in Eq. (8), dimensionless
d	diameter, m
d_h	hydraulic diameter, m
f	Fanning friction factor, dimensionless
FC	total cost function, Eq. (4), dimensionless
GT	thermal gain number, Eq. (15), dimensionless
k	overall heat transfer coefficient, Wm ² K ⁻¹
k_{el}	price of electrical energy, ACU/MWh
m	exponent of Re in Eq. (8), dimensionless
n	exponent of Re in Eq. (7), dimensionless
NTU	number of transfer units, $\text{NTU} = kA/(\rho c_p V_t)$, dimensionless
Nu	Nusselt number, $\text{Nu} = \alpha d/\lambda$, dimensionless
Nu_{ov}	overall Nusselt number, $\text{Nu}_{\text{ov}} = kd/\lambda$
p	pressure, Pa
R_{wf}^*	dimensionless wall (plus fouling) resistance
Re	Reynolds number, $\text{Re} = wd/v$
Re_{eco}	defined in Eq. (5), dimensionless
S	savings by heat recovery, Eq. (12), ACU/year
S_{\max}	maximum savings, Eq. (13), ACU/year
T	temperature, K
$T_{h, \text{in}}$	inlet temperature of hot stream, K
$T_{c, \text{in}}$	inlet temperature of cold stream, K
V_t	volumetric flowrate, m ³ s ⁻¹
w	flow velocity, m/s
x	ratio of pumping powers
y	ratio of heat transfer resistances
α	heat transfer coefficient, Wm ² K ⁻¹
ε	efficiency, $\varepsilon = \text{NTU} \cdot \Theta$, dimensionless
Θ	normalized mean temperature difference (NMTD), $= \text{MTD}/(T_{h, \text{in}} - T_{c, \text{in}})$, dimensionless
η_P	pump (or blower) efficiency, dimensionless
λ	thermal conductivity, Wm ⁻¹ K ⁻¹
μ	viscosity, Pas
ν	kinematic viscosity, m ² s ⁻¹
ρ	density, kgm ⁻³
τ	hours of operation per year, h/year
φ	chevron-angle, °, measured against the main flow direction

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APPENDIX

Friction factors for chevron-type plate and-frame heat exchangers

The equations for friction factors from Martin (1996, 1997) have been rewritten in a more comprehensive form (see Shah, 1998). These equations are based on a model of the flow pattern in the channels of these compact heat exchangers. The numerical constants in these equations have been fitted to experimental data from the literature. (for details, see Martin, 1996).

The Fanning friction factor $f(\text{Re})$ is obtained from the following set of equations

$$\frac{1}{\sqrt{f}} = \frac{\cos\phi}{\sqrt{0.045\tan\phi + 0.09\sin\phi + f_0/\cos\phi}} + \frac{1 - \cos\phi}{\sqrt{3.8f_1}} \quad (\text{A})$$

where

$$f_0 = \begin{cases} 16/\text{Re} & \text{for } \text{Re} < 2000 \\ (1.56 \ln \text{Re} - 3.0)^{-2} & \text{for } \text{Re} \geq 2000 \end{cases} \quad (\text{B})$$

$$f_1 = \begin{cases} \frac{149}{\text{Re}} + 0.9625 & \text{for } \text{Re} < 2000 \\ \frac{9.75}{\text{Re}^{0.289}} & \text{for } \text{Re} \geq 2000 \end{cases} \quad (\text{C})$$

The velocity w in the Reynolds number $\text{Re} = w d_h / \nu$ is defined as

$$w = \frac{V_t}{2 \hat{a} B_p}, \quad (\text{D})$$

with V_t as the volumetric flowrate, $2\hat{a}$ as the gap width of a channel, (\hat{a} is the amplitude of the sinusoidal corrugation), and B_p is the width of the plate between the gaskets.

The hydraulic diameter is defined as

$$d_h = \frac{4\hat{a}}{\Phi} \quad (\text{E})$$

with the area enhancement factor Φ , which can be approximately found from

$$X = \frac{2\pi\hat{a}}{\Lambda} \quad (\text{F})$$

$$\Phi(X) = \frac{1}{6} \left(1 + \sqrt{1 + X^2} + 4\sqrt{1 + X^2/2} \right) \quad (\text{G})$$

if the geometric parameters of the corrugation, *i.e.*, the amplitude \hat{a} , and the wavelength Λ are known.

The friction correlation of Eq. (A) is valid for the corrugation angle ϕ within $0-80^\circ$, and is accurate within -50% and $+100\%$.

Nusselt numbers for chevron-type plate and-frame heat exchangers

It has been shown in (Martin, 1996), that the generalized L ev eque equation may be used to obtain heat transfer data from pressure drop. For the industrial plate-and-frame heat exchangers, an empirically modified version of this theoretical result has been given as:

$$\text{Nu} = 0.205 \text{Pr}^{1/3} \left(\frac{\mu_m}{\mu_w} \right)^{1/6} \left(f \text{Re}^2 \sin 2\phi \right)^{0.374} \quad (\text{H})$$

This correlation is valid for the corrugation angle ϕ within $10-80^\circ$, and is accurate within $\pm 20\%$. Note that if Eq. (H) is used for gases, the viscosity correction term $(\mu_m/\mu_w)^{1/6}$ should be omitted from Eq. (H).

It is interesting to note that the product $f\text{Re}^2$ is proportional to the pressure drop.

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