*3rd European Thermal Sciences Conference 2000*
E.W.P. Hahne, W. Heidemann and K. Spindler (Editors)


# CALCULATION OF HEAT TRANSFER FROM PRESSURE DROP IN TUBE BUNDLES

Holger Martin and Volker Gnielinski

Universität Karlsruhe (TH), Thermische Verfahrenstechnik, D-76128 Karlsruhe

**ABSTRACT**

A new type of analogy between pressure drop and heat transfer has been discovered, that may be used in chevron-type plate heat exchangers, in packed beds, and in tube bundles. It is based on the „generalized Lévêque equation“. Available experimental data on heat transfer in tube bundles in crossflow, both inline, and staggered arrangements, have been tested in greater detail. Using an empirical correlation for pressure drop in these arrangements from the literature that has been successfully tested against a large number of experimental pressure drop data, we found that the heat transfer data collected earlier could be very well represented from the pressure drop correlation and the generalized Lévêque equation.
The fraction $x_f$ of the total pressure drop that is due to fluid friction only, and therefore related to the average shear rate at the surface, turned out to be a constant over the whole range of Reynolds numbers. The new method has the advantage, that apart from the empirical pressure drop correlation, the heat or mass transfer coefficients are obtained from an equation based on theory. The method results in a heat transfer prediction for bundles with more than two rows of tubes, which is at least as good as the existing empirical heat transfer correlations that do not make use of an analogy.

## 1. STATE OF THE ART

Heat transfer in tube bundles in cross flow is a classical problem of process heat transfer which has been investigated by many researchers already in the first half of the 20th century. The present state of the art can be found in the handbooks, like the Heat Exchanger Design Handbook [1] and the VDI-Heat Atlas [2]. In these handbooks, the heat transfer coefficient between the outer cylindrical surfaces of a tube bundle and a fluid flowing through the bundle in cross flow is calculated from the corresponding equations for the Nusselt number of a single (row of ) cylinder(s) in cross flow, multiplied by an empirically correlated arrangement factor $f_a(a, b, type)$, which was found to be a function of the type of bundle, i.e. inline or staggered arrangements, and of the transverse and longitudinal pitch ratios $a$ and $b$ respectively. The pitches $ad$ and $bd$ are defined as the distances of two adjacent tube centerlines of the bundle in a direction perpendicular to the main flow direction (lateral, $ad$) and in flow direction (longitudinal, $bd$), where $d$ is the outer diameter of the tubes. The equations to calculate $\mathrm{Nu}_{\mathrm{bundle}}(\mathrm{Re}, \mathrm{Pr}, a, b, type)$ as recommended in the Heat Exchanger Design Handbook [1] and in the VDI-Heat Atlas [1] have been developed by the second author in 1978 (reference [3] is the English translation of an original paper in German). In this paper [3], the experimental data then available from the literature had been used to find out the appropriate arrangement factors $f_a$. The method was tested against a large number of experimental data from the 20 sources as given here in Table 1. The comparison between the correlation (curve) and the experimental data (symbols) was shown graphically in reference [3] in eight figures containing up to fourteen single curves of a $(\mathrm{Nu}_{\mathrm{bundle}}/f_a)$ vs. Re plot with Pr as a parameter. From these figures one can find out that this state of the art method [1, 2, 3] fits the data for inline bundles with an acceptable degree of approximation in the whole range of the experimental variables. The data for staggered bundles are also well represented for gases (air mainly), while for the liquids (see Fig. 8 in [3]) the data systematically tend to give higher values with increasing Prandtl and decreasing Reynolds numbers.

## 2. THE LÉVÊQUE-ANALOGY

A new type of analogy between pressure drop and heat transfer has been discovered, that may be used in chevron-type plate heat exchangers [5], in packed beds, and in tube bundles [6]. It is based on the „generalized Lévêque equation“.

$$\mathrm{Nu}/\mathrm{Pr}^{1/3} = \mathrm{Sh}/\mathrm{Sc}^{1/3} = 0.404\,(x_f\,\mathrm{Re}^2\,d_h/L)^{1/3} \qquad (1)$$

Available experimental data on heat transfer in tube bundles in crossflow, both inline, and staggered arrangements, have been tested in greater detail. Using the empirical correlation for pressure drop in these arrangements by *Gaddis and Gnielinski* [4] that has been successfully tested against a large number of experimental pressure drop data, we found that the heat transfer data for bundles with more than 2 tube rows, collected by *Gnielinski* [3] could be very well represented from the pressure drop correlation and the generalized Lévêque equation.
When using the pressure drop correlation by *Gaddis and Gnielinski* [4], it has to be taken into account, that they defined their coefficient $x(=x_{\mathrm{total}})$ as the pressure drop per number of main resistances (=number of rows of tubes $N$) divided by the dynamic pressure $(r u_0^2/2)$ using the velocity in the narrowest cross section (see APPENDIX, 9.1).

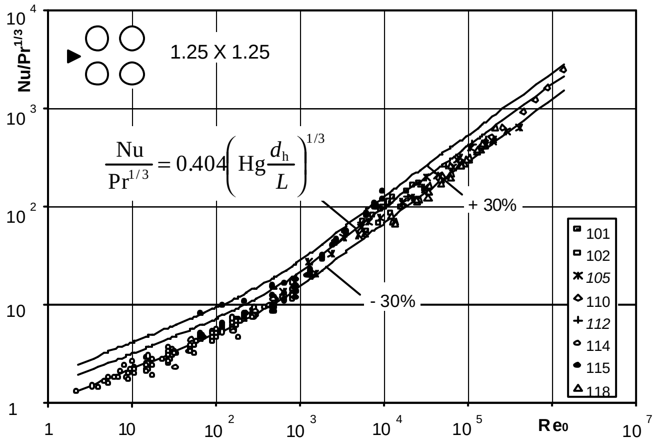


**Fig. 1** Heat Transfer Group Nu/Pr$^{1/3}$ vs. Reynolds-number, inline bundles $a$=1.25, $b$=1.25, for the number of data sources (at the symbols) see Table 1. Hg=($\xi$/2)Re$_0^2$.

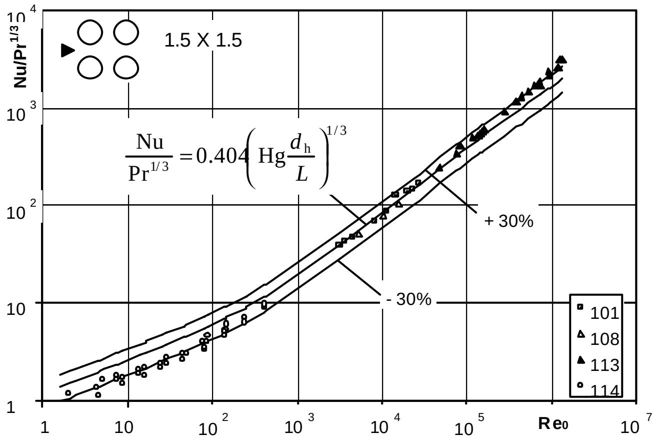


**Fig. 2** Heat Transfer Group Nu/Pr$^{1/3}$ vs. Reynolds-number, inline bundles $a$=1.5, $b$=1.5, for the number of data sources (at the symbols) see Table 1. Hg=($\xi$/2)Re$_0^2$.

As the pressure gradient ($Dp/Dz$) at the heated surface is needed, the values can be used directly for $b$>1, while they have to be divided by $b$ for $b$<1.
The hydraulic diameter for the tube bundles was calculated as

$$d_h=((4a/p)-1)d \qquad \text{(for } b>1) \qquad (2)$$

and

$$d_h=((4ab/p)-1)d \qquad \text{(for } b<1) \qquad (3)$$

where $a$ and $b$ are the lateral and the longitudinal pitch ratios respectively, and $d$ is the outer diameter of the tubes. Longitudinal pitches of $b$<1 are only possible for staggered bundles. The length $L$ in the generalized Lévêque equation has been taken as the longitudinal and diagonal pitch, respectively

$$L = bd \quad \text{(for inline bundles)} \qquad (4)$$

and

$$L = cd \quad \text{(for staggered bundles)} \qquad (5)$$

where $c$ is the diagonal pitch ratio

$$c=((a/2)^2+b^2)^{0.5}. \qquad (6)$$

It was found that using

$$x_f = x_f{\cdot}x \qquad (7)$$

leads to a very reasonable agreement between the analogy predictions and the experimental results. The fraction $x_f$ of the total pressure drop coefficient $x$, that is due to fluid friction only, and therefore related to the average shear rate at the surface, turned out to be a constant over the whole range of Reynolds numbers.
The heat transfer data had been collected by *Gnielinski* [3] from about twenty different sources. Fortunately, the collection of data had been conserved in a usable format, so it was possible to re-evaluate this experimental information.
The Figs. 1 through 4 show the dimensionless group Nu/Pr$^{1/3}$ plotted vs. the Reynolds number Re$_0$, for typical inline and staggered bundles. The curves in these figures are calculated from the generalized Lévêque-equation (1), with $d_h/L$ from Eqs. (2 –5), and the friction factor $x_f$ taken as $x$/2, with the total pressure drop coefficient $x$ calculated from the correlation of Gaddis and Gnielinski [4], which is also recommended in the handbooks [1, 2].

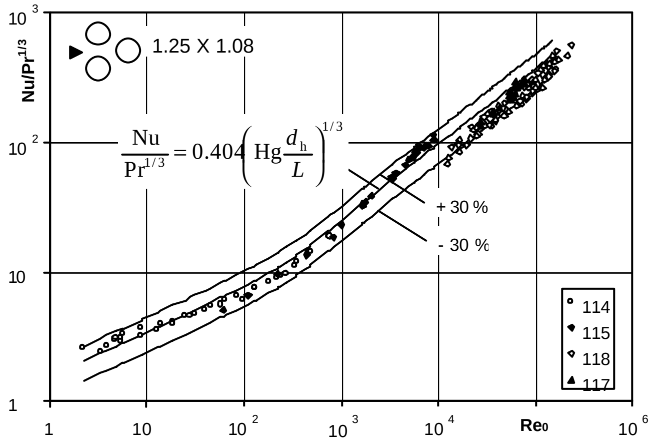


**Fig. 3** Heat Transfer Group Nu/Pr$^{1/3}$ vs. Reynolds-number, staggered bundles $a$=1.25, $b$=1.08, for the number of data sources (at the symbols) see Table 1. Hg=($\xi$/2)Re$_0^2$.

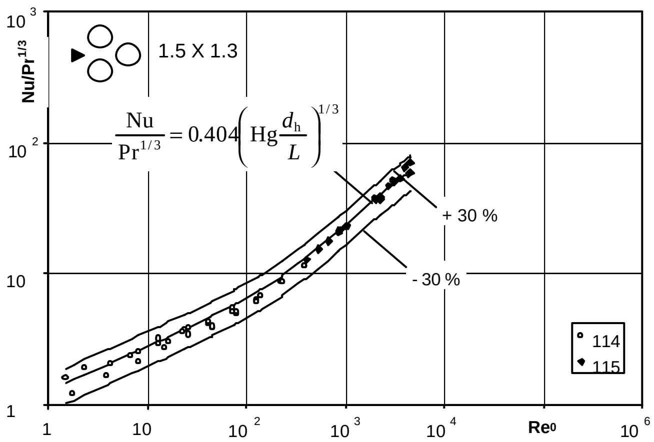


**Fig. 4** Heat Transfer Group Nu/Pr$^{1/3}$ vs. Reynolds-number, staggered bundles $a$=1.5, $b$=1.3, for the number of data sources (at the symbols) see Table 1. Hg=($\xi$/2)Re$_0^2$.

The total amount of 1606 data points for the inline tube bundles, when compared with the prediction of the generalized Lévêque equation, using the empirical correlation for $x$ in these arrangements by *Gaddis and Gnielinski* [4] by minimizing the root mean square (RMS)-deviation, leads to an optimum of $x_{f,,inline}$=0.54 with $RMS_{inline}$=19.5%. The same procedure, when used for the staggered tube bundles (1457 data points) results in $x_{f,,staggered}$=0.46 with a somewhat lower RMS-deviation of $RMS_{staggered}$=13.7%.

## 3. HEAT TRANSFER FROM PRESSURE DROP

Using the same average value of $x_f$ =0.5 in both cases does not change the RMS-deviations significantly, as the heat transfer coefficient depends on the cubic root of the friction factor ($RMS_{inline}$=19.7%, $RMS_{staggered}$=13.8%,). The effective fraction $x_f$ of the total pressure drop, which is due to friction only, has been obtained from the experimental data by calculating $x_f=(Nu_{exp}/Nu_{theor}(x_f=x))^3$, i.e. by dividing the experimental heat transfer coefficients by the theoretical ones from Eq. (1) with the friction factor equal to the total pressure drop coefficient as calculated by the Gaddis-Gnielinski-correlation [4] and taking the third power of this ratio. The results of this procedure are shown in the Figs. 5 and 6, where $x_f$ was plotted vs. the Reynolds number $Re_0$ (with the velocity in the narrowest cross section, and the outer tube diameter, as used in [4]). It may be easily seen, that the values so obtained for $x_f$ are nearly independent of $Re_0$ and close to the average value of 0.5 shown as a black horizontal line in these figures. The upper and lower gray parallel lines denote the 3 times 0.5 and 1/3 times 0.5 limits, corresponding to +44% and –31% of the mean Nusselt numbers.

The figures therefore may be also seen as a plot of $Nu_{exp}/Nu_{calc}$ if the cubic root of the ($x_f$/0.5) values on the vertical axis is taken. Three decades in the vertical axis have therefore been shown in about the same scale as one decade in the horizontal axis in these figures.

In case of the inline bundles the experimental values in Fig. 5 show a tendency towards lower values in the range of $Re_0$ below about 500. The same can be seen from Figs. 1 and 2. Here the new method (the generalized Lévêque equation) tends to overpredict the experimental results. A reason for this may be seen in the fact that the inline bundles tend to be parallel channels, especially so for small longitudinal pitches. In this case, a laminar flow may not show periodically repeated developing boundary layers with the short length $L=bd$, as assumed in Eq. (4). Maldistribution of fluid flow in parallel channels might be another reason for lower experimental values at low Reynolds numbers.

The range of validity of the Gaddis-Gnielinski correlation [4] for the pressure drop in tube bundles is given as $1 \le Re_0 \le 3.5\ 10^5$. The experimental data on pressure drop plotted in reference [4] show a minimum of $x$ at $Re_0$ around $2.5\ 10^5$ and an increase with $Re_0$ above this value (see Figs. 5, 6, and 10 in [4]). Extrapolation of the pressure drop correlation to Reynolds numbers above $3.5\ 10^5$ therefore leads to an underprediction of $Nu_{theor}$ or an increase in $x_f$ in Fig. 5. In case of the staggered bundles this effect would have been even more important. Therefore the values of $x$ from the Gaddis-Gnielinski correlation have been corrected in order to apply the new method even above $Re=3.5 \cdot 10^5$.

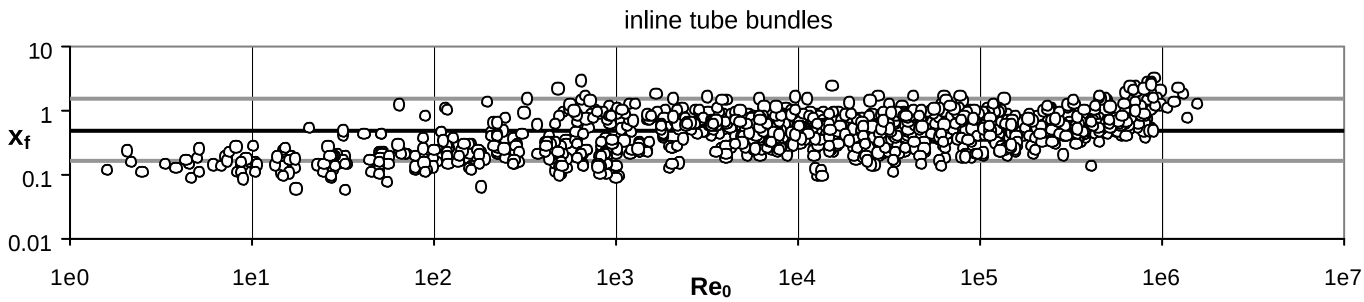


**Fig. 5** Fraction $x_f$ of the total pressure drop due to friction as calculated from all the experimental data for inline tube bundles (1606 data) vs. Reynolds number, $x_f = (Nu_{exp}/Nu_{theor}(x_f=x))^3$. Number of tube rows: $2 \le N \le 15$.

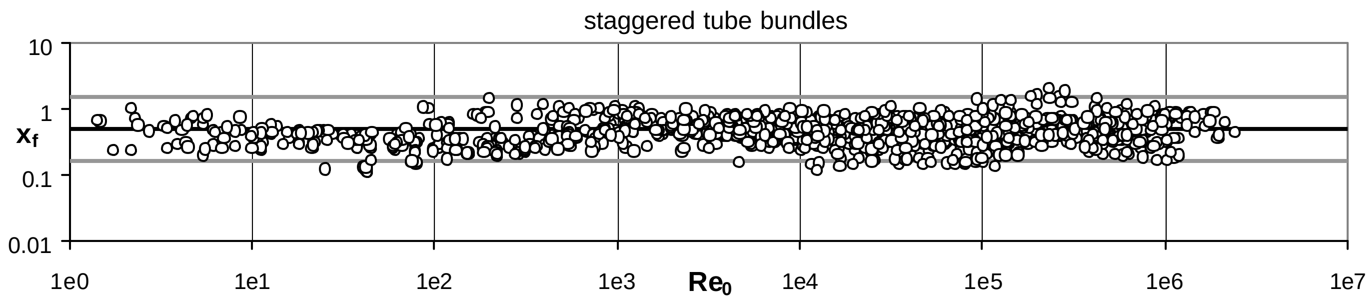


**Fig. 6** Fraction $x_f$ of the total pressure drop due to friction as calculated from all the experimental data for staggered tube bundles (1457 data) vs. Reynolds number, $x_f = (Nu_{exp}/Nu_{theor}(x_f=x))^3$. Number of tube rows: $4 \le N \le 80$.

For Re > 2.5 $10^5$ the pressure drop coefficients $x$ are calculated from

$$x = x_{\text{Gaddis-Gnielinski}}[1+(\mathrm{Re}_0 - 2.5 \cdot 10^5)/3.25 \cdot 10^5]. \qquad (8)$$

Using this empirical correction for the staggered bundles makes it possible to apply the new method even in the range of Reynolds numbers above the minimum of ξ up to the highest Reynolds numbers used in the heat transfer data of about $\mathrm{Re}_0 = 3 \cdot 10^6$.

From the results of these comparisons, especially from the fact, that $x_f$ turned out to be constant over the whole range of Reynolds numbers one can conclude, that as for the chevron type plate heat exchangers [5], for packed beds and for similar periodically arranged structures, the heat (and mass) transfer coefficients can be predicted from the pressure drop if the frictional fraction of the pressure drop is known. To show this graphically, the experimental data from the literature (see Table 1) have been plotted against the dimensionless group in the generalized Lévêque equation (1)

$$x \mathrm{Re}^2 \,\mathrm{Pr}\, d_h/L \qquad (9)$$

in Figs. 7 and 8. The product of the friction factor and the square of the Reynolds number does not contain a velocity, so the definition of the characteristic velocity in ξ and Re is arbitrary. It should be the same of course in both ξ and Re. This was the reason to use $\mathrm{Re}_0$ as in the Gaddis-Gnielinski correlation. The term

$$(x/2)\mathrm{Re}^2 = \mathrm{Hg} = (1/r)(Dp/Dz)d^3/\nu^2 \qquad (10)$$

is a dimensionless number that might be termed Hagen number Hg. It is related to the driving force of a flow. In case of a gradient of static pressure $(Dp/Dz) = gDr$, i.e. a bouyancy driven, natural convection flow, the Hagen number becomes equal to the Grashof number.

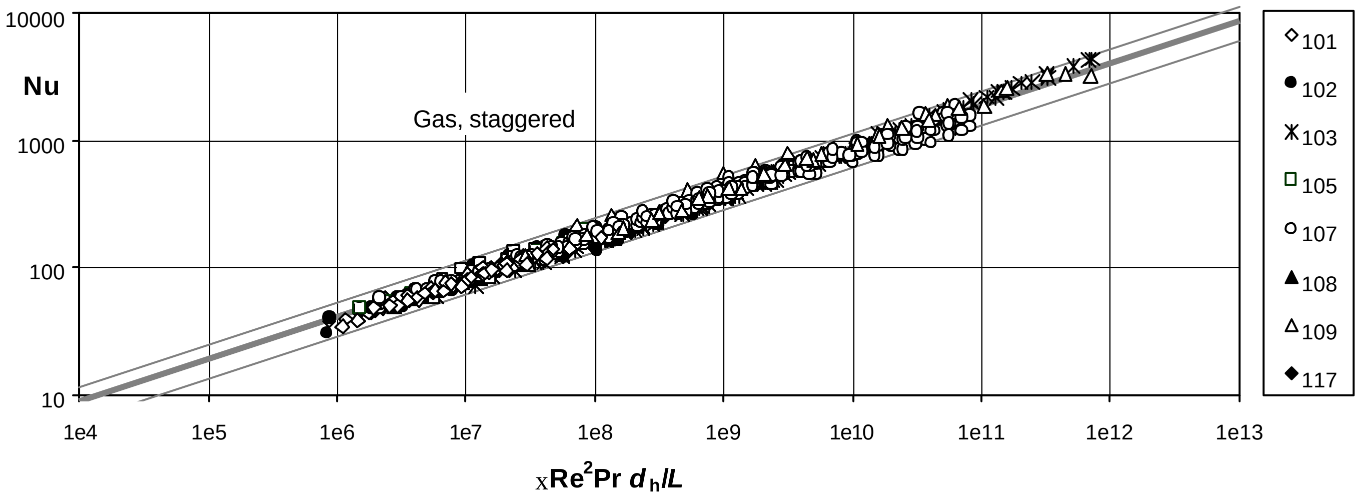


**Fig. 7** Nusselt number vs. the dimensionless group in the generalized Lévêque equation (proportional to *Dp/Dz*). Symbols: Data for gases in staggered tube bundles (numbers at the symbols denote the data source, see Table 1). Line: Generalized Lévêque equation (1).

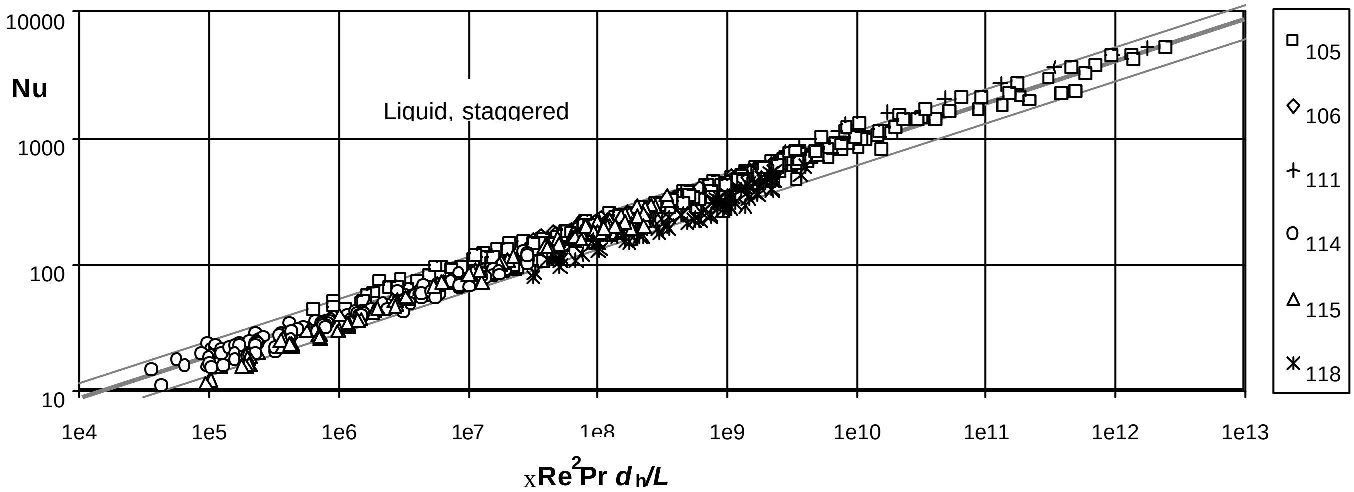


**Fig. 8** Nusselt number vs. the dimensionless group in the generalized Lévêque equation (proportional to *Dp/Dz*). Symbols: Data for liquids in staggered tube bundles (numbers at the symbols denote the data source, see Table 1). Line: Generalized Lévêque equation (1).

It can be seen that the generalized Lévêque equation represents all the data for staggered tube bundles very well over a range of ($HgPrd_h/L$) that covers eight decades. The upper and lower parallel lines to the cubic root law of the Lévêque equation are 1.3 and 0.7 times the theoretical curve from Eq. (1), with $x_f = x/2$.

The new method has the advantage, that apart from the empirical pressure drop correlation, the heat or mass transfer coefficients are obtained from an equation based on theory. The only one empirical factor $x_f$ turned out to be essentially constant over the whole range of Reynolds numbers and is given by the value $x_f = 1/2$. The method results in a heat transfer prediction, which is at least as good as the existing empirical heat transfer correlations that do not make use of an analogy.

**Table 1** Sources of Experimental Data for Heat Transfer in Tube Bundles (collected by Gnielinski [3])

| No. | Author(s), *Journal*, *etc*. Vol, pp, year |
|---|---|
| 101 | Pierson, O. L., *Trans. ASME*, 59, 563-572, 1937 |
| 102 | Huge, E. C., *Trans. ASME*, 59, 573-581, 1937 |
| 103 | Hammeke, K. E. et al., *Int. J. Heat Mass Transfer*, 10, 427-446, 1967 |
| 104 | Kays, W. M. et al., *Trans. ASME*, 76, 386-396, 1954 |
| 105 | Zhukauskas, A., in *Advances in Heat Transfer*, 8, 93-160, 1972 |
| 106 | Isachenko, V. P., *Izd. Akad. Nauk, SSSR Moscow*, MCL-222N, AD 261/110, 1959 |
| 107 | Niggeschmidt, W., Dr.-Ing.-Thesis (Diss.) TH Darmstadt 1975 |
| 108 | Bressler, R., *Forsch. Ing.-Wes*. 24, 90-103, 1958 |
| 109 | Stasiulevicius, J., P. Samoska, Lietuvos TSR Mokslu Akademijos Darbai, Ser. B, 4 (35), 77-81, 1963 |
| 110 | Samoska, P., J. Stasiulevicius, Lietuvos TSR Mokslu Akademijos Darbai, Ser. B, 4 (55), 133-144, 1968 |
| 111 | Samoska, P. et al., Lietuvos TSR Mokslu Akademijos Darbai, Ser. B, 3 (50), 105-113, 1967 |
| 112 | Makarevicius, V., A. Zhukauskas, Lietuvos TSR Mokslu Akademijos Darbai, Ser. B, 3 (26), 231-241, 1961 |
| 113 | Zhukauskas, A., Int. Seminar, ICHMT, Trogir, Yugosl., 1972 |
| 114 | Bergelin, O. P. et al., U of Delaware, Engng. Expt. Station, Bulletin No. 2, 1950 |
| 115 | Bergelin, O. P. et al., U of Delaware, Engng. Expt. Station, Bulletin No. 4, 1958 |
| 116 | Fairchild, H. N., C. P. Welch, Paper No. 61-WA-250 ASME Ann. Meeting, New York, N Y, 1961 |
| 117 | De Bartoli, et. al., *Nuclear Sci. Engng*., 1, 239-251, 1956 |
| 118 | Austin, A. A.,Ph.D.-Thesis, Carnegie Inst. of Technology, Dept. Chem. Engng., 1959 |
| 119 | Smyczek, H., J. Zablocki, *Inzyniera chemiczna*, 5, 143-159, 1975 |
| 120 | Scholz, F., *Chemie-Ing.-Techn*. 40, 988-995, 1968 |

## 4. COMPARISON OF THE TWO METHODS

A detailed comparison of the two methods ($1^{st}$: state of the art: [1, 2, 3], $2^{nd}$: new Eq. (1) Lévêque) to calculate heat (and mass) transfer in tube bundles has been carried out. The results are shown in Table 2. For the inline bundles, the state of the art method, based on empirical correlations for the arrangement factors $f_a(a,b,\ type)$ that have to be multiplied to the equations for the Nusselt numbers of single cylinders in cross flow, is slightly better than the new method (especially so in the range of low Reynolds numbers, see Figs. 1, 2, and 5). If the Reynolds numbers are greater than 500, both methods are roughly equal in their accuracy. For the staggered bundles the new method is definitely better in its RMS deviations, especially for the data obtained with liquids, where the presently recommended method may lead to larger discrepancies in the range of higher Prandtl and lower Reynolds numbers.

**Table 2** Comparison of data from Table 1 with both methods

| | | | |
|---|---|---|---|
| **inline bundles** | | [1,2,3] | Lévêque |
| liquids | | | |
| 669 data | RMS: | 22.8% | 24.6% |
| gas | | | |
| 937 data | RMS: | 14.6% | 15.3% |
| all data | | | |
| **1606** (a>1,05) | RMS: | **18.5**% | **19.7**% |
| **staggered bundles** | | [1,2,3] | Lévêque |
| liquids | | | |
| 705 data | RMS: | 45.4% | 16.6% |
| gas | | | |
| 752 data | RMS: | 17.5% | 10.5% |
| all data | | | |
| **1457** | RMS: | **34.0**% | **13.8**% |

## 5. CONCLUSIONS

The new method to calculate heat and mass transfer from pressure drop in tube bundles, with a single adjustable parameter $x_f$, which has been found to be 0.5 in all cases can be recommended especially for the staggered tube bundles. In case of the inline bundles, the new method is slightly less accurate than the older one in the low Reynolds number range. If the calculation of pressure drop is additionally needed, the new method is even simpler to apply than the presently recommended one.

## 6. NOMENCLATURE

| | |
|---|---|
| $a$ | lateral pitch ratio =(pitch)/(tube diameter), 1 |
| $b$ | longitudinal pitch ratio, 1 |
| $c$ | diagonal pitch ratio, 1 |
| $c_p$ | heat capacity at const. pressure, J/(kg K) |
| $d$ | outer tube diameter, m |
| Hg | Hagen number, $Hg=(x/2)Re^2$, proportional to *Dp/Dz* |
| $L$ | length, m |
| $N$ | number of tube rows, 1 |
| Nu | Nusselt number, $(ad/l)_{cp}$ (see APPENDIX, 9.2) |
| $p$ | pressure, Pa |
| Pr | Prandtl number, $Pr=hc_p/l$ |
| Re | Reynolds number, $Re=ud/n$ |
| $u$ | flow velocity, m/s |
| $x_f$ | fraction of total pressure drop due to friction, 1 |
| $a$ | heat transfer coefficient, $W/(m^2 K)$ |
| $l$ | thermal conductivity, W/(m K) |
| $h$ | viscosity, Pas |
| $n$ | kinematic viscosity, $m^2/s$ |
| $x$ | friction factor, pressure drop coefficient, 1 |
| $r$ | density, $kg/m^3$ |
| subscripts | |
| cp | constant physical properties (see APPENDIX 9.2) |

f friction
h hydraulic (diameter)
0 refering to the narrowest cross section

## 7. REFERENCES

(see also Table 1 at the end of section 3)


1. Schlünder, E.U. (editor-in-chief), *Heat Exchanger Design Handbook*, Hemisphere Publ. Co., Washington, New York, London, 1983.
2. VDI-GVC (ed.), *VDI Heat Atlas*, VDI-Verlag, Düsseldorf, 1993, English translation of the 6th ed. of the German original: *VDI-Wärmeatlas* (now available in German: 8th ed., Springer Verlag, Heidelberg, 1997.
3. Gnielinski, V., Equations for calculating heat transfer in single tube rows and banks of tubes in transverse flow, *Int. Chem. Eng*. Vol.19, No. 3, pp. 380-390, 1979.
4. Gaddis, E.S. and Gnielinski, V.,. Pressure drop in cross flow across tube bundles, *Int. Chem. Eng*. Vol. 25, No.1, pp. 1-15, 1985.
5. Martin, H., A theoretical approach to predict the performance of chevron-type plate heat exchangers *Chem. Eng. Process*., vol. 35 pp. 301-310, 1996.
6. Martin, H., Gaddis, E. S., Gnielinski, V., Berechnung des Wärmeübergangs in querdurchströmten Rohrbündeln aus dem Druckverlust, *Chemie-Ing.-Techn*., Vol. 70, p. 1177, 1998 (abstract of a poster presentation in German).


## 8. ACKNOWLEDGEMENTS

The authors want to express their gratitude to Matthias Senne, who carried out the tedious work of data processing within the frame of his „Seminararbeit“. Thanks are also due to Markus Nickolay, who assisted the first author in preparing some of the figures.

## 9. APPENDIX

### 9.1 The Gaddis-Gnielinski correlation [4] for pressure drop in tube bundles:

Definition of *x*: $x=2\Delta p/(N r\ u_0^2)$ (A1)

*N*= number of rows, $N=\Delta z/d$ (b>1) (A2)

$N=\Delta z/bd$ (b≤1) (A3)

If $u_{empty}$ is the fluid velocity in the empty cross section, the velocity in the narrowest cross section $u_0$ is:

$$u_0=u_{empty}\, a/(a-1) \quad \text{(A4)}$$

in most cases. It is, however,

$$u_0=u_{empty}\, a/[2(c-1)] \quad \text{(A5)}$$

in cases, where the narrowest cross section is in the diagonal for staggered bundles with $b< 0.5\cdot(2a+1)^{1/2}$.

For inline bundles:

$$x=x_{lam}+(x_{turb,i}+f_{n,t})[1-\exp(1-(\mathrm{Re}_0+1000)/2000)] \quad \text{(A6)}$$

and for staggered bundles:

$$x=x_{lam}+(x_{turb,s}+f_{n,t})[1-\exp(1-(\mathrm{Re}_0+200)/1000)] \quad \text{(A7)}$$

The laminar part of ξ is:

$$x_{lam}=280p[(b^{0.5}-0.6)^2+0.75]/(\boldsymbol{a}^{\mathbf{1.6}}(4ab-p)\mathrm{Re}_0) \quad \text{(A8)}$$

in most cases, but $\boldsymbol{a}^{\mathbf{1.6}}$ in the denominator has to be replaced by $\boldsymbol{c}^{\mathbf{1.6}}$ for staggered bundles with $b< 0.5\cdot(2a+1)^{1/2}$.

The turbulent contributions are:
for inline bundles

$$x_{turb,i}=f_{t,i}/\mathrm{Re}_0^{0.1b/a} \quad \text{(A9)}$$

$$f_{t,i}=[0.22+1.2(1-(0.94/b))^{0.6}/(a-0.85)^{1.3}]\cdot 10^{0.47(b/a-1.5)}+ +0.03(a-1)(b-1)$$

and for staggered bundles:

$$x_{turb,s}=f_{t,s}/\mathrm{Re}_0^{0.25} \quad \text{(A10)}$$

$$f_{t,s}=2.5+1.2/(a-0.85)^{1.08}+0.4(b/a-1)^3-0.01(a/b-1)^3$$

The term $f_{n,t}$ in Eqs.(A6, A7) accounts for the influence of inlet and outlet pressure losses.

$$f_{n,t}=(1/a^2)(1/N-1/10) \quad \text{for } 5\leq N \leq 10 \quad \text{(A11)}$$

and $$f_{n,t}=0 \quad \text{for } N>10 \quad \text{(A12)}$$

The term $(1/a^2)$ in Eq. (A11) is to be replaced by

$$[2(c-1)/a(a-1)]^2 \text{ in case of } b< 0.5\cdot(2a+1)^{1/2}.$$

This effect was not included in the calculation of $\xi_f$, we used $f_{n,t}=0$ irrespective of the number *N*, which was in the range $2 \leq N \leq 80$ in the heat transfer data used here.

### 9.2 Definition of the Nusselt numbers used here

The heat transfer coefficients α are defined as in references [1, 2, and 3] as the heat flux density divided by the logarithmic mean temperature difference between the fluid and the outer surface of the tubes. The physical properties are evaluated at the arithmetic mean $T_m$ between the fluid inlet and fluid outlet temperatures. In order to account for the effects of temperature dependent physical properties, the experimentally obtained heat transfer coefficients $\alpha_{exp}$ have been divided by the physical property correction terms K, that have been used in [3], and consequently also in the handbooks [1,2]:

$$\mathrm{Nu}=(ad/l)_{cp}=(a_{exp}d/l)/\mathrm{K} \quad \text{(A13)}$$

$\mathrm{K}=(\mathrm{Pr}/\mathrm{Pr}_w)^m$ for liquids $\mathrm{K}=(T_m/T_w)^{0.12}$ for gases

$m=0.25$ (for $\mathrm{Pr}/\mathrm{Pr}_w>1$) $m=0.11$ (for $\mathrm{Pr}/\mathrm{Pr}_w<1$)