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THE GENERALIZED LÉVÊQUE EQUATION AND ITS USE TO PREDICT HEAT OR MASS TRANSFER FROM FLUID FRICTION

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ABSTRACT

A new type of analogy between frictional pressure drop and heat transfer has been discovered that may be used in chevron-type plate heat exchangers, in tube bundles, in crossed-rod matrices, and in many other internal flow situations. It is based on the Generalized Lévêque Equation (GLE). This is a generalization of the well known asymptotic solution for thermally developing, hydrodynamically developed tube flow, which was first derived by A. Lévêque in 1928. Nusselt or Sherwood numbers turn out to be proportional to the cubic root of the frictional pressure drop ($\xi \mathrm{Re}^2$) under these conditions.
It was found that using a friction factor $\xi = x_f \cdot \xi_{total}$ leads to a very reasonable agreement between the analogy predictions and the experimental results. The fraction x_f of the total pressure drop coefficient ξ_{total} that is due to fluid friction only, turned out to be a constant over the whole range of Reynolds numbers in many cases. For the packed beds of spheres, Ergun's equation, or more appropriate equations from the literature may be used to calculate the total pressure drop.
The new method can also be used in external flow situations, not only for internal flow as shown so far. This is demonstrated here for a single sphere as well as for a single cylinder in cross flow. In these cases, however, the frictional fraction x_f of the total drag coefficient is not a constant over the range of Reynolds numbers. Nevertheless it is easily obtained from standard correlations of drag coefficients. The successful application of the GLE also in cases of external flow seems to confirm, that this new type of analogy has a broad range of applications and may lead to a better understanding of the interrelation between fluid flow and heat or mass transfer in general.

THE GENERALIZED LÉVÊQUE EQUATION

The following equation has been termed the Generalized Lévêque Equation (GLE) in [1]

$$\mathrm{Nu}/\mathrm{Pr}^{1/3} = \mathrm{Sh}/\mathrm{Sc}^{1/3} = 0.403755(x\,\mathrm{Re}^2 d_h/L)^{1/3} \quad (1)$$

where the dimensionless numbers (Nu, Pr, Sh, Sc, Re), named after Wilhelm Nusselt, Ludwig Prandtl, Thomas K. Sherwood, Ernst Schmidt, and Osborne Reynolds, are defined as usual in the heat and mass transfer literature, d_h is the hydraulic diameter, defined as 4 times the cross sectional area divided by the circumference of the flow channel, and L is the length in the direction of flow. The friction factor $\xi=(\Delta p/\Delta z)d/[(r/2)u^2]$ is defined as pressure gradient $(\Delta p/\Delta z)$ times tube diameter d divided by the stagnation pressure $(r/2)u^2$. Substituting ξ from the Hagen-Poiseuille law for fully developed laminar tube flow, ξ=64/Re, eqn. (1) yields the classical form of Lévêque's equation:

$$\mathrm{Nu}=1.615(\mathrm{Re}\,\mathrm{Pr}\,d/L)^{1/3} \quad (2)$$

as it is usually found in the textbooks. This equation has been theoretically derived for the first time in André Lévêques thesis [2], pp. 283-287. The choice of the characteristic length (d) to be used in Nu, Sh, Re and ξ is arbitrary as both $\mathrm{Nu}/\mathrm{Pr}^{1/3}$ (or $\mathrm{Sh}/\mathrm{Sc}^{1/3}$) and $(\xi\,\mathrm{Re}^2)^{1/3}$ contain this length with the same power of one. Of course the same (arbitrary) length has to be used in Nu, Sh, Re and ξ. The same consistency ought to be maintained in the use of the characteristic velocities in both the Reynolds number Re and the friction factor ξ. In the product $\xi \mathrm{Re}^2$ this velocity cancels. So the heat or mass transfer coefficients predicted from the GLE do not depend on flow velocities, but only on the pressure gradient, the physical properties and the geometric ratio d_h/L.

The generalization in eqn. (1) is to be seen in the fact, that the equation in this form may also be applied to turbulent flow, as long as the thermal boundary layer remains within the viscous sublayer. This idea seems to have been first suggested independently both by Bankston and McEligot [3] and by Schlünder [4]. These authors, however, suggested the use of eqn. (1), or its equivalent, only for the asymptotic behaviour in the entry region of a circular duct. In the meantime it has been shown, that the GLE is in fact applicable to a number of other problems of practical interest, like the cross-corrugated channels of chevron-type plate heat exchangers [1], tube bundles [5], and crossed rod matrices [6].

HEAT TRANSFER IN TUBE BUNDLES

Heat transfer in tube bundles in cross flow is a classical problem of process heat transfer which has been investigated by many researchers already in the first half of the 20th century. The present state of the art can be found in the handbooks, like the H E D H [7] and the VDI-Heat Atlas [8]. In these handbooks, the heat transfer coefficient between the outer cylindrical surfaces of a tube bundle and a fluid flowing through the bundle in cross flow is calculated from the corresponding equations for the Nusselt number of a single (row of) cylinder(s) in cross flow, multiplied by an empirically correlated arrangement factor $f_a(a, b, \text{type})$, which was found to be a function of the type of bundle, i.e. inline or staggered arrangements, and of the transverse and longitudinal pitch ratios a and b respectively. The pitches ad and bd are defined as the distances of two adjacent tube centerlines of the bundle in a direction perpendicular to the main flow direction (lateral, ad) and in flow direction (longitudinal, bd), where d is the outer diameter of the tubes. The equations to calculate $Nu_{bundle}(Re, Pr, a, b, \text{type})$ as recommended in the H E D H and in the VDI-Heat Atlas have been developed by Gnielinski [9]. In this paper the experimental data then available from the literature had been used to find out the appropriate arrangement factors fa. The method was tested against a large number of experimental data from 20 sources. The comparison between the correlation (curve) and the experimental data (symbols) was shown graphically by Gnielinski [9] in eight figures containing up to fourteen single curves of a (Nu_{bundle}/f_a) vs. Re plot with Pr as a parameter. From these figures one can find out that this state of the art method fits the data for inline bundles with an acceptable degree of approximation in the whole range of the experimental variables. The data for staggered bundles are also well represented for gases (air mainly), while for the liquids (see Fig. 8 in [9]) the data systematically tend to give higher values with increasing Prandtl and decreasing Reynolds numbers.

When using the pressure drop correlation by Gaddis & Gnielinski [10], it has to be taken into account, that they defined their coefficient $\xi = \xi_{total}$ as the pressure drop per number of main resistances (=number of rows of tubes N) divided by the dynamic pressure $(ru_0^2/2)$ using the velocity in the narrowest cross section.

As the pressure gradient $(\Delta p/\Delta z)$ at the heated surface is needed, the values can be used directly for b>1, while they have to be divided by b for b<1.

The hydraulic diameter for the tube bundles was calculated as

$$d_h=((4a/p)-1)d \qquad (\text{for } b>1) \qquad (3)$$

and

$$d_h=((4ab/\pi)-1)d \qquad (\text{for } b<1) \qquad (4)$$

where a and b are the lateral and the longitudinal pitch ratios respectively, and d is the outer diameter of the tubes. Longitudinal pitches of b<1 are only possible for staggered bundles. The length L in the generalized Lévêque equation has been taken as the longitudinal or diagonal pitches, respectively

$$L = bd \quad (\text{for inline tube bundles}) \qquad (5)$$

and

$$L = cd \quad (\text{for staggered bundles}) \qquad (6)$$

where c is the diagonal pitch ratio $c=((a/2)^2+b^2)^{0.5}$.

It was found that using $\xi_f = x_f \cdot \xi$ leads to a very reasonable agreement between the GLE predictions and the experimental results. The fraction x_f of the total pressure drop coefficient ξ, that is due to fluid friction only, and therefore related to the average shear rate at the surface, turned out to be a constant over the whole range of Reynolds numbers. This fraction was found to be close to 0.5 for tube bundles. The optimized values where found to be x_f=0.54 for inline tube bundles, and x_f =0.46 for staggered bundles.

COMPARISON OF THE TWO METHODS

A detailed comparison of the state of the art method (Gnielinski, [9]), here denoted as: HEDH, with the new method, Eqn. (1): GLE, to calculate heat (and mass) transfer in tube bundles has been carried out by Martin & Gnielinski [5]. The results are shown in Table 1. For the inline bundles, the state of the art method (HEDH), based on empirical correlations for the arrangement factors $f_a(a, b, \text{type})$ that have to be multiplied to the equations for the Nusselt numbers of single cylinders in cross flow, is slightly better than the new method (especially so in the range of low Reynolds numbers. If the Reynolds numbers are greater than 500, both methods are roughly equal in their accuracy. For the staggered bundles the new method is definitely better in its RMS deviations, especially for the data obtained with liquids, where the state of the art method (HEDH) may lead to larger discrepancies in the range of higher Prandtl and lower Reynolds numbers.

Table 1 Comparison of data collected by Gnielinski [9] with both methods (GLE with x_f=0.5 in all cases).

inline tube bundles		HEDH	GLE
liquids		state of the art	new
669 data	RMS:	22.8%	24.6%
gas			
937 data	RMS:	14.6%	15.3%
all data			
1606 (a>1.05)	RMS:	**18.5%**	**19.7%***

staggered tube bundles		HEDH	GLE
liquids		state of the art	new
705 data	RMS:	45.4%	16.6%
gas			
752 data	RMS:	17.5%	10.5%
all data			
1457	RMS:	**34.0%**	**13.8%**

**** this deviation was improved by a correction to* 18.2%** ***even for a bigger number of data (a>1.02, see next section)***

IMPROVING THE NEW METHOD FOR INLINE TUBE BUNDLES

In case of the inline bundles the experimental data show a tendency towards lower values in the range of Re_0 below about 500. This can of course not be seen from Table 1, but from the figures in reference [5]. Here the new method (GLE) tends to overpredict the experimental results.

A reason for this may be seen in the fact that the inline bundles tend to be parallel channels, especially so for small longitudinal pitches. In this case, a laminar flow may not show periodically repeated developing boundary layers with the short length $L=bd$, as assumed in Eq. (4).

This was the starting point for an empirical improvement of the new method. It is suggested to use a simple correction function $C(Re_0)$ to compensate for the overprediction of the GLE at low Reynolds numbers, with ($0.50 < C < 1$):

$$C=[(Re_0+1)/(Re_0+1000)]^{0.10} \quad (7)$$

This is equivalent to taking a length L in the GLE, which is the same as given in eq. (4) for sufficiently high Reynolds numbers, while this length increases to $L_{Corr}=bd/C^3$. The correction C tends to unity for large Re, the smallest value of C is obtained in the limit as Re tends to zero. The data for inline tube bundles can be predicted from the GLE, using a best fit value of x_f=0.59, with an RMS-deviation of only 18.2%. With the correction, the new method is slightly better than the purely empirical state of the art method (see under HEDH in Table 1) even for the inline bundles. This is shown in Fig. 1, where Nu/C was plotted versus the argument

$$Lq = x_f \xi \, Re^2 \, Pr \, d_h/L \quad (8)$$

of the GLE. The simple straight line prediction log(Nu/C)=1/3log(Lq) is found to be valid over the whole range of 8 decades in the “Lévêque number”, Lq, which is proportional to the pressure drop. Compared to our earlier results in [5] and Table 1, the number of data (1609 for inline bundles) has been increased by 88 to 1694 including those data with very narrow lateral pitch ratios (a>1.02), that had been excluded in the state of the art method (HEDH). For the staggered bundles the GLE, without any correction, had already been shown to be much better than the state of the art method (see Table 1). This good agreement between the prediction of the empirical pressure drop correlation of Gaddis & Gnielinski [7, 8, 10] via the GLE and the total amount of experimental data from the literature may be seen as an encouragement to further study this new kind of analogy between momentum and heat (or mass) transfer.

Looking at a limited set of data, especially those, where pressure drop and heat transfer have been measured in the same equipment, the agreement may be shown to be even better, if the measured friction factors are used in place of a pressure drop correlation.

This is shown in Fig. 2 for the data of Kays, London & Lo [11] for six different staggered tube bundles in cross flow of air.

These data, when evaluated with their original friction factors measured in the same equipment show an RMS-deviation of only 3.9% against the GLE prediction, when using x_f=0.46 as has been found as the optimum value for the 1457 data for staggered bundles in combination with the pressure drop correlation of Gaddis & Gnielinski [7, 8, 10] in reference [5].

CROSSED ROD MATRICES

Crossed rod matrices, i.e. periodic arrangements of parallel solid metal rods of diameter d, with a regular lateral pitch ad, every second layer turned by 90° and the layers touching each other, have been used in compact heat exchangers. Experimental friction factor and heat transfer data for such inline, as well as staggered, and random arrangements, are to be found among many other compact heat transfer surfaces in the well known book by Kays & London [12]. Recently, Nanda, Das, & Martin [6] have shown, that these data also can be predicted from the GLE concept. In that paper, however, only a limited number of the data from Kays & London [12] have been shown in a graphical comparison. Here in Fig. 3 the total amount of data from table 10-10 in Compact Heat Exchangers has been compared to a slightly more general form of the GLE. As the flow in the crossed rod matrices is three-dimensional, as in a packed bed, the length L has been taken as an average length of a flow path, $L/d=(V/V_s)^n$, with $V_s/V=(1-\psi)$, ψ=void fraction, which is

$$\psi=1-\pi/(4a) \quad (9)$$

for the matrices with the rods touching each other. The hydraulic diameter is

$$d_h=d\,\psi/(1-\psi) \quad (10)$$

and therefore the term d_h/L in the GLE becomes

$$d_h/L=\psi/(1-\psi)^{(1-n)} \quad (11)$$

where n should be 1/2 in case of the cylindical rods, as it has been taken to be 1/3 for the beds of spherical particles. If both, x_f and n are taken as fitting parameters, the best fit is found for x_f =0.782, n=0.924, leading to an RMS-deviation of only 4.16%.

Using the more logical n=1/2 in d_h/L gives the same result with x_f as a function of voidage: $x_f=0.782(1-\psi)^{0.424}$.

The version published by Nanda, Das & Martin [6] corresponds to x_f =0.822, n=1, with a slightly higher RMS-deviation of 4.48%.

With n=1/2 and a constant average value of x_f =0.457 one finds a somewhat larger deviation, RMS=6.64%.

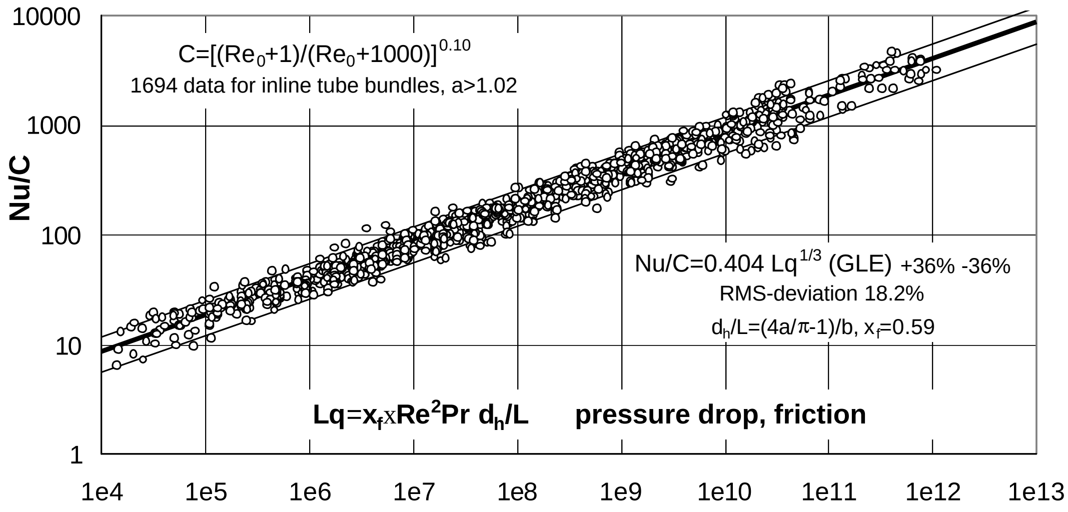

Fig. 1 Heat transfer, Nu/C, to the tubes of inline bundles in cross flow vs. frictional pressure drop, Lq. Correction $C(Re_0)$ to account for increasing length L of repeated developing boundary layers in the Generalized Lévêque Equation beyond the axial pitch *bd* for lower Reynolds numbers (0.5 < C < 1).

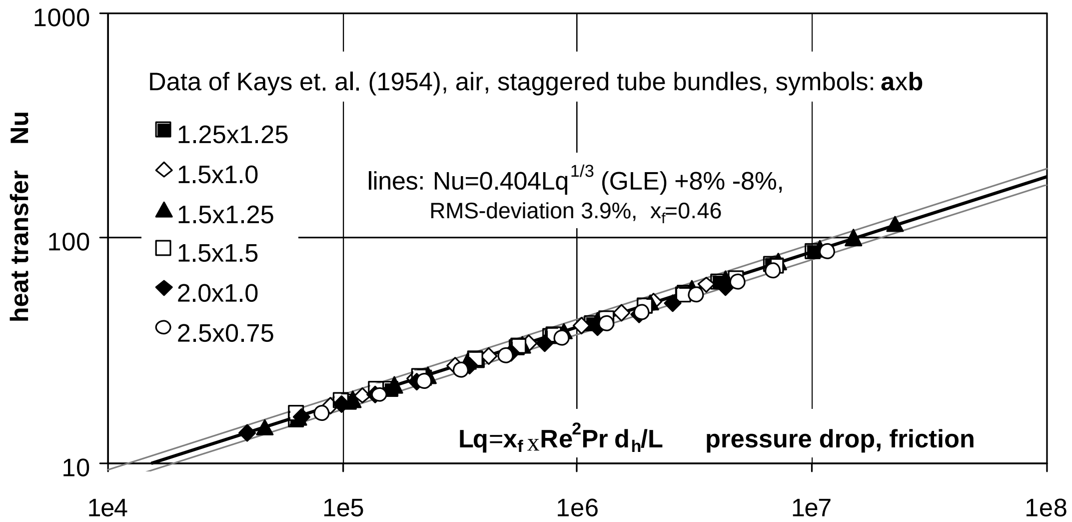

Fig. 2 Heat transfer, Nu, to the tubes of staggered bundles in cross flow vs. frictional pressure drop, Lq. Data of Kays, London, & Lo (1954) compared to the GLE with friction factors measured in the same equipment, x_f=0.46 as found for all staggered data by Martin & Gnielinski (2000).

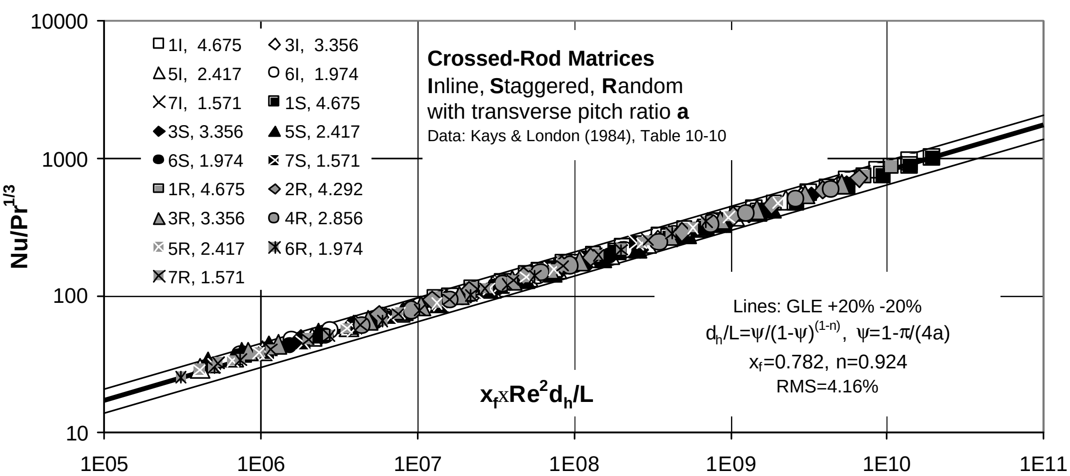

Fig. 3 Heat transfer, $Nu/Pr^{1/3}$ vs. frictional pressure drop, $Lq=x_f\xi Re^2 d_h/L$ with $d_h/L=\psi/(1-\psi)^{(1-n)}$ and $\psi=1-\pi/(4a)$, optimized values: (x_f=0.782, n=0.924), RMS=4.16%, ξ=4f, values of $Nu/Pr^{1/3}=(StPr^{2/3})Re$. Data: $StPr^{2/3}(Re)$, f(Re) from table 10-10, Kays & London (1984)

PACKED BEDS

Available experimental data on heat transfer in packed beds of spherical particles of diameter d had been collected and empirically correlated earlier by Gnielinski [13]. His correlation provides a simple way to test the GLE for its applicability in predicting packed bed heat or mass transfer from pressure drop. The hydraulic diameter is obtained from the well-known relationship

$$d_h = (2/3)\, d\, \psi/(1-\psi), \tag{12}$$

for a bed of spherical particles of diameter d and void fraction ψ. The length L in the generalized Lévêque equation optimization ($x_{f,opt}$=0.535 for these parameters). Again, the GLE works over the whole range of Reynolds numbers with a constant x_f.

Gnielinskis correlation has a Pr-dependency, which slightly deviates from the limiting 1/3 power law, so an intermediate value of Pr=100 has been chosen for this comparison. The deviation is practically negligible for higher Pr (or Sc) numbers, while for air Pr=0.7, the formal minimum value $Nu_{min}=2(1+1.5(1-\psi))$ in Gnielinskis correlation, based on the single sphere, would lead to greater deviations from the GLE. The data in this range, however, usually show lower values than those calculated from Gnielinskis equation, i. e. they are closer to the GLE prediction.

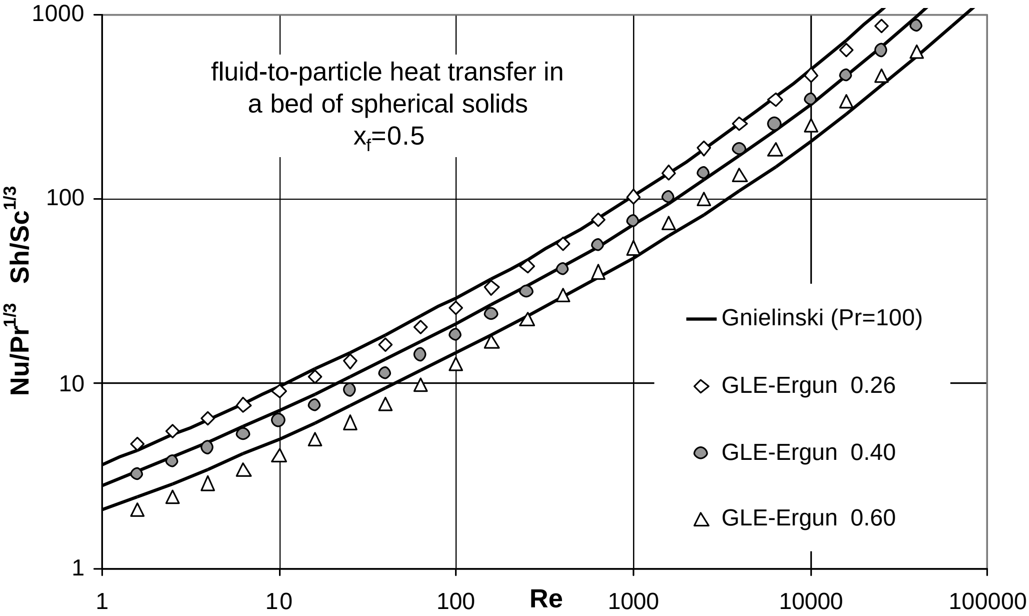

Fig. 4 Comparison between Gnielinski's correlation and the GLE with Ergun's eqn.

has been taken as the average distance between two particles in the bed of spheres obtained from $L/d=(V/V_s)^{1/3}$, with the total bed volume V and the solids volume V_s

$$L = d/(1-\psi)^{1/3}, \tag{13}$$

resulting in the geometric ratio in the GLE to be a function of the void fraction only:

$$d_h/L = (2/3)\psi/(1-\psi)^{2/3}. \tag{14}$$

The total friction factor ξ can be calculated for example from the Ergun equation, which is found in many textbooks, see, for example [14], p. 200). This gives a relatively good agreement between the Nusselt numbers from the empirical packed bed equations by Gnielinski [13], as recommended in some relevant handbooks, as the VDI-Heat Atlas [8] and the Heat Exchanger Design Handbook [7].

Figure 4 shows $Nu/Pr^{1/3}$ vs. Re, with the voidage ψ as parameter, calculated from Gnielinski's correlation as the full lines for ψ=0.26, 0.4, and 0.6 and from the GLE with the same parameter values, shown as the symbols. The frictional fraction x_f was found to be very close to x_f =0.5 in this case by

EXTERNAL FLOW

Heat transfer to a single sphere in cross-flow

In this case the Nusselt number is calculated from a generalized Lévêque equation (GLE) in the form

$$Nu_{GLE,sphere} = 2 + 0.404(c_F\, Re^2 Pr)^{1/3} \tag{15}$$

with a friction factor c_F - in place of $\xi\ d_h/L$ in eqn. (1) - calculated from the drag coefficient $c_D(Re)$ as given by Brauer [15], see Fig. 5, but with the "constant" term in the Newton range ($10^3 < Re < 10^5$) multiplied by an empirical factor x_N=0.085, and x_S=2/3 from Stokes law (Re <1). The term d_h/L is replaced by $4A_c/A$, (with the flow cross-sectional area A_c and the surface area A) for the external flow cases, which turns out to be $4(\pi/4)d^2/(\pi d^2)$ =1 for the sphere.

$$c_F = x_S(24/Re) + 3.73/Re^{1/2} + x_N(0.49 - 4830\, Re^{1/2}/(10^6+3Re^{3/2})) \tag{16}$$

The original drag coefficient (Brauer, [15]) is $c_D=c_F$(from eqn. (16), with x_S=1, x_N=1). Here two parameters x_S and x_N

might be used to find a suitable friction factor c_F from the total drag coefficient. Theoretically Stokes's law, valid in the creeping flow limit of very low Reynolds numbers, requires x_S to be 2/3. This theoretical value has been retained in the GLE, as it leads to $(Nu-2)=1.0174(RePr)^{1/3}$ for Re<1, which is only slightly higher than the theoretical creeping flow limit $(Nu-2)=0.9914(RePr)^{1/3}$ calculated from an integration of local values over the surface of the sphere. Therefore also in this case only one single fitting parameter (x_N) was needed to find an appropriate friction factor from the total drag coefficient.

Figure 6 shows Nu vs. Re from the correlations of Gnielinski [13] as the full diamond symbols, of Achenbach [16] as the open circles, of Whitaker [17] as the open triangles, and a correlation by Ihme et. al. [18] as the full circles, based on their numerical data and on numerical as well as experimental data of others together with the line representing the Generalized Lévêque Equation, eqn. (15), with the friction factor from eqn. (16). Gnielinski's correlation has been tested against a large number of experimental data from the literature and is recommended in the relevant handbooks like the VDI-Heat Atlas and the Heat Exchanger Design Handbook. From this comparison it can be seen, that the GLE is also applicable to the single sphere in cross flow. A more detailed comparison with experimental data is presently under preparation.

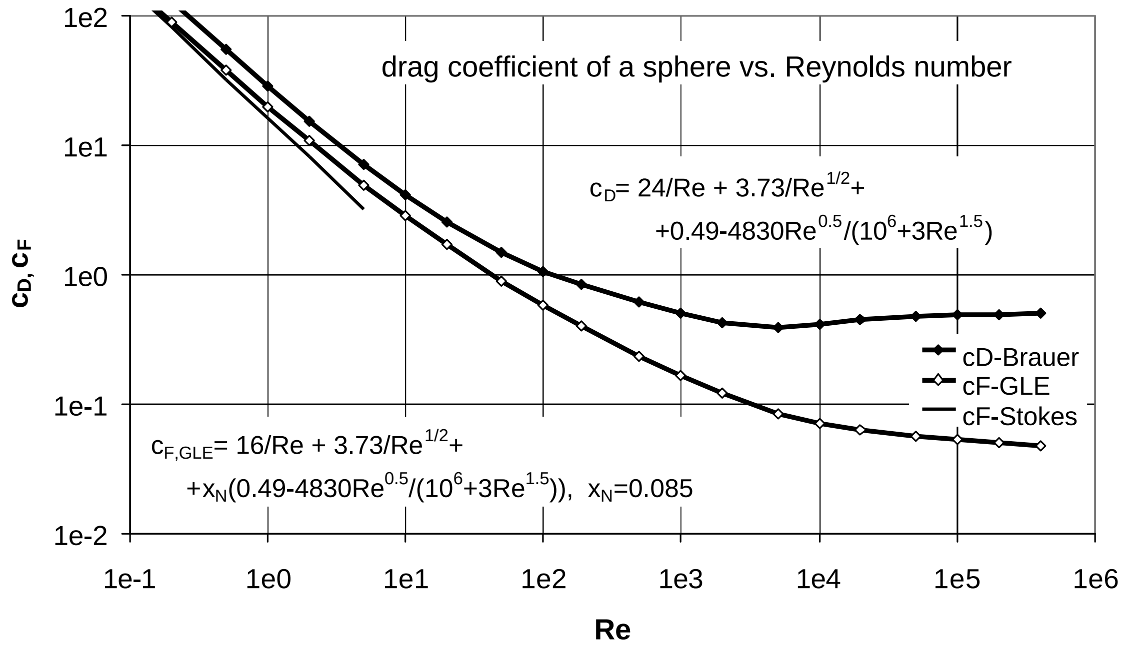

Fig. 5 Friction factor c_F of single sphere and total drag coefficient c_D

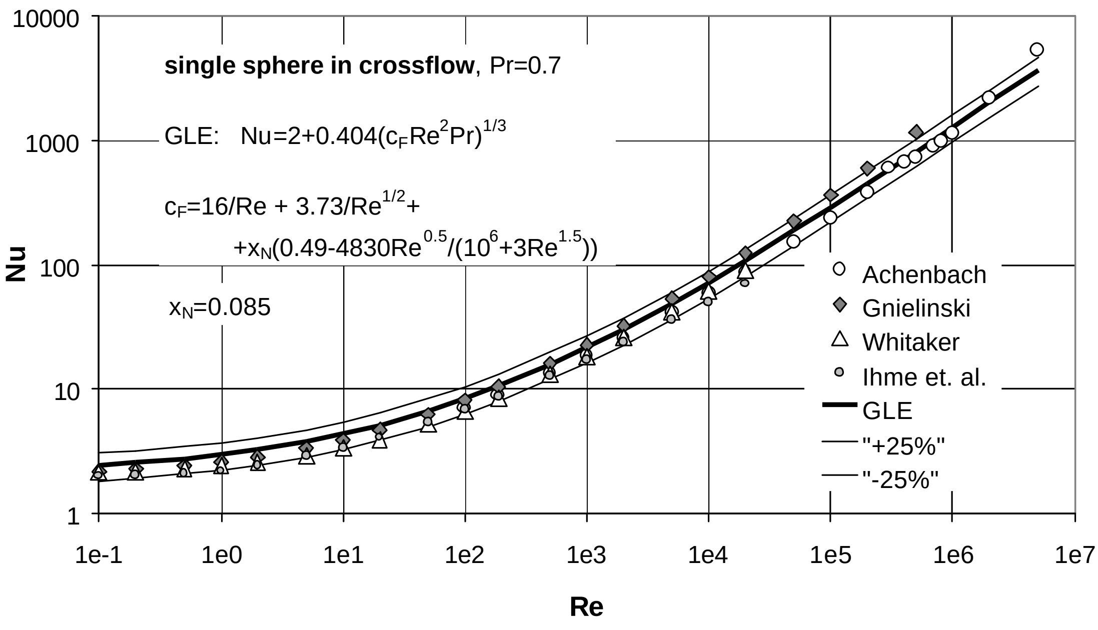

Fig. 6 Nusselt vs. Reynolds numbers for a single sphere in cross flow (Pr=0.7) Lines: GLE

Heat transfer to a single cylinder in cross-flow

The Nusselt number is calculated from the generalized Lévêque equation (GLE) in the form:

$$Nu_{GLE,cylinder}=0.18+0.404(c_F\,Re^2 Pr\,4/\pi)^{1/3}, \qquad (17)$$

with the friction factor c_F calculated from the formula for c_D from Sucker and Brauer [19] , see Fig. 7, where the „constant“ term for the Newton range (10^3 <Re< 10^5) has been multiplied by an empirical factor x_N=0.027,

$$c_F=x_S(6.8/Re^{0.89})+1.96/Re^{0.5}+ +x_N\,(1.18-1/(2500/Re+Re/1100)) \qquad (18)$$

whereas $x_S=c_F/c_D$ has been retained as the theoretical value x_S=0.5 for cylinders at low Re.

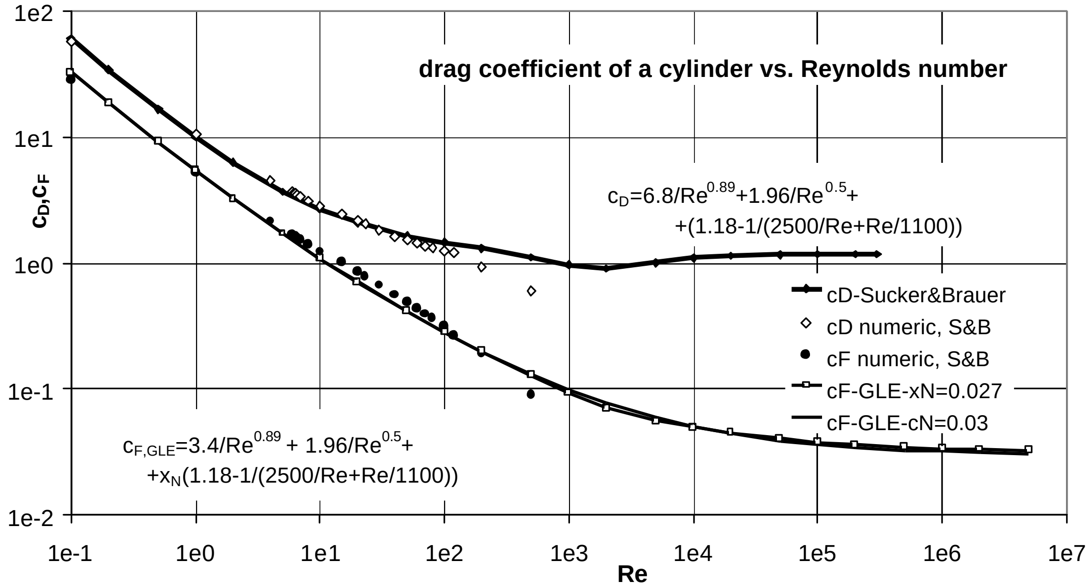

Fig. 7 Friction factor c_F of single cylinder and total drag coefficient c_D

The factor $(4/\pi)=4A_c/A$ (in place of d_h/L for internal flow) has to be included in eqn. (17) as c_D is based on the cross-sectional area $A_c=dl$. The surface area, however, is $A=\pi dl$. The original drag coefficient (Sucker and Brauer [19]) is $c_D=c_F$(from eqn. (18), with x_S=1, x_N=1). The weak function of Re in the “constant” term with x_N of eqn. (18) can be replaced without loss of accuracy by a real constant c_N, so the friction factor to be used in the GLE is simply: $c_F = 3.4/Re^{0.89}+1.96/Re^{1/2}+c_N$.

Figure 8 shows $(Nu-0.18)/Pr^{1/3}$ or $(Sh-0.18)/Sc^{1/3}$ vs. Re from the experimental data collected by Gnielinski [13] from 12 different sources, additionally the data for air by Achenbach [20, 21] at high Reynolds numbers, and a correlation by Lange et. al. [22] based on numerical data in the range 10^{-4}<Re<200, together with the line representing the Generalized Lévêque Equation, eqn. (17), with the friction

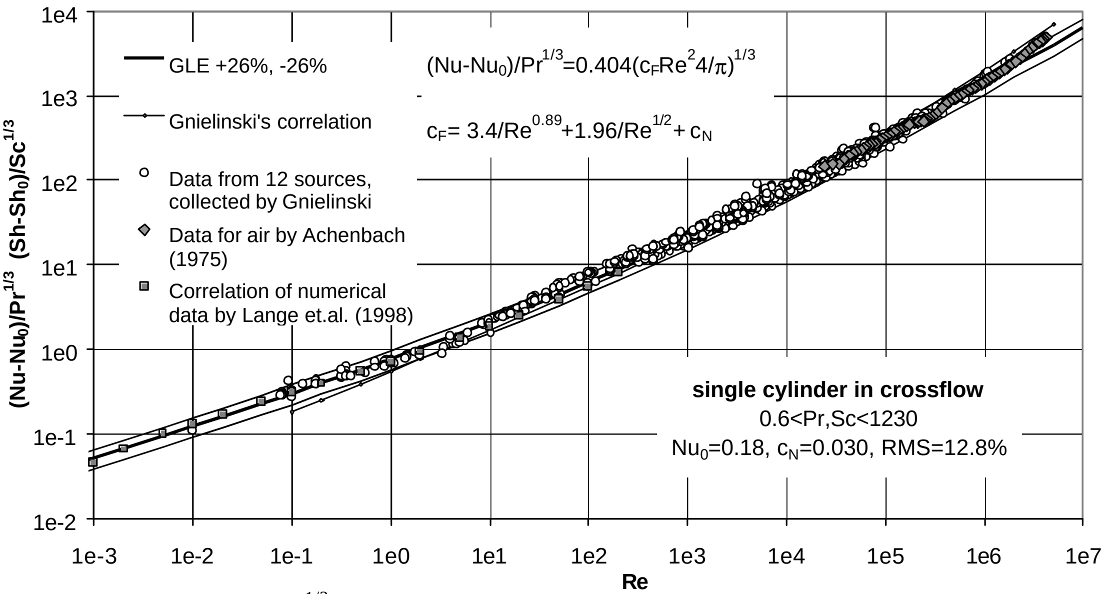

Fig. 8 $(Nu-0.18)/Pr^{1/3}$ vs. Reynolds numbers for a single cylinder in cross flow. Lines: GLE

factor from eqn. (18). The total amount of 1036 data - about 30% of these data are from Zukauskas [23] and his group - is represented by the GLE with an RMS-deviation of 12.8% in the range $10^{-3}<\mathrm{Re}<5\cdot 10^{6}$, 0.6<(Pr, Sc)<1230.

CONCLUSIONS

The most important result of these comparisons is the fact, that x_f turned out to be a constant over the whole range of Reynolds numbers for internal flow. Therefore, one can conclude, that as for the chevron type plate heat exchangers (Martin, 1996), for packed beds and for similar periodically arranged structures, the heat (and mass) transfer coefficients can be predicted from the pressure drop.

To show this graphically, the experimental data from the literature have been plotted against the dimensionless group in the Generalized Lévêque Equation (1).

The product of the friction factor and the square of the Reynolds number does not contain a velocity, so the definition of the characteristic velocity in ξ and Re is arbitrary. It should be the same of course in both ξ and Re. This was the reason to use Re_0 as in the Gaddis-Gnielinski correlation for the pressure drop in tube bundles. The term

$$(\xi/2)\mathrm{Re}^2 = \mathrm{Hg} = (1/r)(\Delta p/\Delta z)d^3/n^2 \qquad (19)$$

is a dimensionless number that may be termed Hagen number Hg. It is related to the driving force of a flow. In case of a gradient of static pressure $(\Delta p/\Delta z) = g\Delta\rho$, i.e. a bouyancy driven, natural convection flow, the Hagen number becomes equal to the Grashof number. Using Hg and Re in place of the various friction factors defined in the literature makes pressure drop vs. flowrate equations and figures more easy to understand.

The Hagen-Poiseuille law, a linear relationship between pressure drop and flowrate, when written in terms of the friction factor leads to the hyperbolic law ξ=64/Re, because the friction factor is defined with $(\rho/2)u^2$ as a reference pressure. With eqn. (19), however, the Hagen-Poiseuille law simply reads

$$\mathrm{Hg}=32\mathrm{Re} \qquad (20)$$

i.e. as the original linear relationship between pressure drop (Hg) and flowrate (Re). A “pressure drop number” (like Hg) has been suggested earlier by Steimle [24] under the label SK (from German: Strömungs-Kennzahl, i. e. flow number). He developed a direct relationship between Nu and SK or modified versions of that number. Steimles empirical power product functions $\mathrm{Nu}=c(\mathrm{SK.Pr})^m$ are in fact rather close to what follows from the GLE. The exponent m chosen by Steimle was somewhat greater than 1/3, as to be expected for a fully developed turbulent tube flow (m=0.37), but in the case of spacewise periodic structures, like tube bundles in cross flow, Steimle’s figures often show, that the slope of 1/3 (as from Lévêques theory) fits better than 0.37.

In case of the tube bundles for example it was found that the Generalized Lévêque Equation represents all the data very well over a range of “Lévêque numbers”

$$\mathrm{Lq} = (2x_f)\,\mathrm{Hg}\,\mathrm{Pr}\,d_h/L \qquad (21)$$

that covers eight decades. With x_f close to 0.5 for internal flow, the factor $(2x_f)$ in eq. (21) is typically close to unity.

The examples given to calculate heat or mass transfer via the Generalized Lévêque Equation from fluid friction, clearly demonstrate, that a large class of heat and mass transfer problems may be solved in that way. A number of so far empirical heat and mass transfer correlations might thus be replaced by the theoretical equation we originally owe to André Lévêque. Furthermore the new analogy may lead to a better understanding of the interrelation between fluid flow and heat or mass transfer in general.

NOMENCLATURE

Latin symbols

a	lateral pitch ratio =(pitch)/(tube diameter), 1
A	area, m^2
b	longitudinal pitch ratio, 1
c	diagonal pitch ratio, 1
c_p	heat capacity at const. pressure, J/(kg K)
c_D	drag coefficient, 1
c_F	friction factor, 1
d	outer tube diameter, m
Gr	Grashof number, $(g\Delta r/r)d^3/n^2$
Hg	Hagen number, $\mathrm{Hg}=(x/2)\mathrm{Re}^2$, proportional to $\Delta p/\Delta z$ see eqn. (19)
L	length, m
Lq	Lévêque number, see Eqs.(8, 21)
N	number of tube rows, 1
Nu	Nusselt number, ad/l
p	pressure, Pa
Pr	Prandtl number, $\mathrm{Pr}=hc_p/l$
Re	Reynolds number, $\mathrm{Re}=ud/n$
u	flow velocity, m/s
V	volume, m^3
x	coordinate in flow direction, m
x_f	fraction of total pressure drop due to friction, 1
y	distance from the wall, m
z	coordinate in flow direction, m

Greek symbols

a	heat transfer coefficient, W/(m^2 K)
l	thermal conductivity, W/(m K)
h	viscosity, Pas
n	kinematic viscosity, m²/s
x	friction factor, pressure drop coefficient, 1
r	density, kg/m^3

Subscripts

f	friction
h	hydraulic (diameter)
s	solid
0	refering to the narrowest cross section

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The present invited lecture is based on published material from the references* [1, 5] *and partly from the references* [25, 26], *to be published in 2002:

25. H. Martin, The Generalized Lévêque Equation and its practical use to predict heat and mass transfer rates from pressure drop, to be published in *Chem. Eng. Sci.* 2002. ***(first part on internal flow)***

26. H. Martin, The Generalized Lévêque Equation (GLE) and its use to predict heat and mass transfer from fluid friction, to be presented at the 12th Int. Heat Transfer Conference, Grenoble, 2002. ***(second part on external flow)***