## Introduction

We aim for the full waveform inversion (FWI) of recorded shallow seismic surface waves. The inversion of shallow seismic surface waves is very attractive for geotechnical investigations because they can be easily excited by a hammer blow and have a high sensitivity to the shear wave velocity in the first few meters of the subsurface. Furthermore, surface waves have a better signal to noise ratio compared to body waves and can be used to investigate sites with low velocity zones which cannot be done with refracted body waves. There are established methods to invert surface waves (e. g. inversion of dispersion curves or wavefield spectra (Forbriger, 2003)) but all these methods assume 1D subsurface structures. To overcome this limitation we apply FWI to shallow seismic surface waves. Romdhane et al. (2011) have shown a first successful application of a FWI of surface waves which show the high potential of this method. In shallow seismic field data the effects of viscoelastic damping are significant. We normally observe $Q$ values between 10 and 50. As we aim to invert field data we investigate to which degree we have to consider viscoelasticity during the FWI. We present two investigations. Firstly, we show the comparison of field data with synthetic viscoelastic and pure elastic forward modelings. Secondly, we discuss inversion results for simulated viscoelastic observations with $Q = 20$. These data are inverted with a FWI using elastic forward modeling and viscoelastic modeling with $Q =$20, 25, and 10.

## Comparison of field data with modeled data

For the comparison with recorded data we use a field dataset acquired on a predominantly depth dependent subsurface structure at Rheinstetten near Karlsruhe (Germany). We infer a 1D model of the subsurface by an inversion of wavefield spectra (Forbriger, 2003). This model is used to calculate purely elastic and viscoelastic wavefields with a 2D Finite Difference algorithm in the time domain (Bohlen, 2002). Viscoelasticity is implemented by a generalized standard linear solid and we use three relaxation mechanisms to model an almost constant $Q$ factor of 20 in the frequency band 10 Hz-70 Hz. The synthetic wavefields differ clearly as the viscoelasticity causes a distance and frequency dependent damping and additional phase velocity dispersion of the signals which are not present in the elastic wavefields. To compare the 2D synthetic seismograms with the recorded data (acquired with a hammer blow) we first have to transform the recorded wavefield to the corresponding wavefield of a line source. This is done with a transformation suggested by Amundsen and Reitan (1994) which is exact for a 1D subsurface structure. Furthermore, we determine an optimized source time function by a deconvolution in the frequency domain (Forbriger, 2003) which is convolved with the modeled data.

Figure 1(a) shows the comparison of the recorded data with the synthetic viscoelastic data and the purely elastic data. Both wavefields match the recorded data quite well but they differ in some aspects. The amplitude decay with offset of the fundamental mode is better fitted by the viscoelastic data (time interval 0 s-0.15 s). For the elastic wavefield the optimized source time function acts as a low pass filter (see Figure 1(b)) and eliminates the high frequencies which are already damped in the recorded data at middle and large offsets. However, the source time function can only act as a frequency dependent filter (not distance dependent). Therefore, the high frequencies are also no more present in the traces at small offsets (see first trace in Figure 1(a)) where the bandwidth of the elastic seismograms are slightly too small compared to the bandwidth of the field data and the viscoelastically modeled data.

## Inversion tests with simulated observations

We run inversion tests with simulated observations to investigate the importance of $Q$ for the inversion of shear wave models by FWI. As true model (see Figure 2(b)) we use the 1D model derived from the field dataset already used in the previous section. The model contains a steep gradient in the topmost meter, below the gradient decreases. Such a gradient in the first meters of the shear wave velocity model is typical for unconsolidated sediments (Bachrach et al., 2000) and is often observed in shallow seismics. This gradient causes a long ringing in the elastically modeled data. Therefore, we investigate to which degree we have to consider viscoelasticity in the inversion to infer this gradient. To illustrate

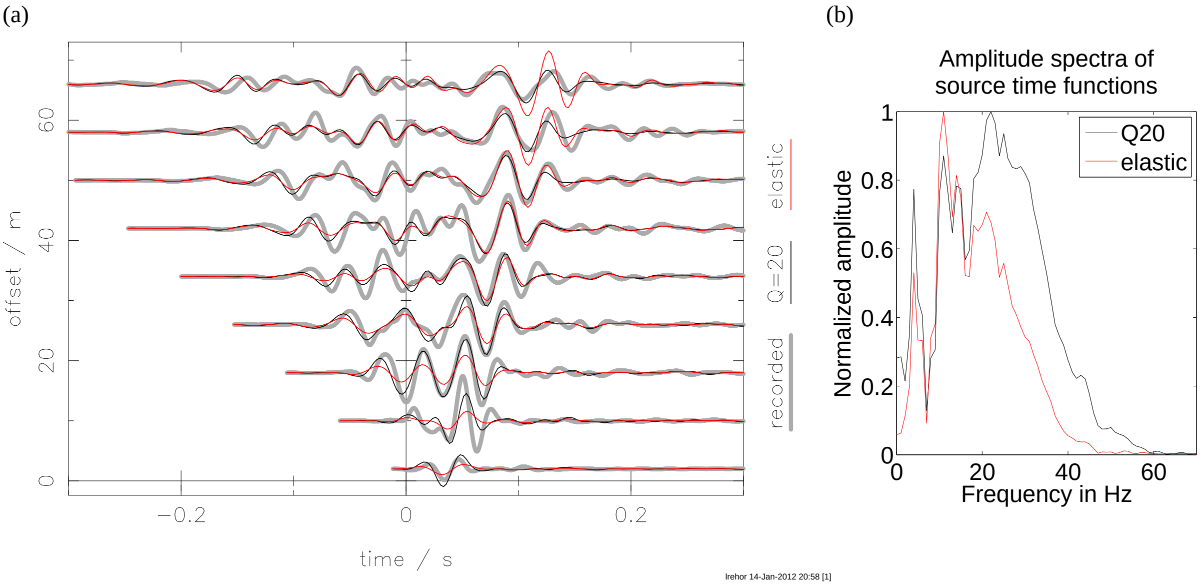

***Figure 1*** *(a) Comparison of field data (gray) with viscoelastic synthetic data with Q=20 (black) and purely elastic synthetic data (red). The synthetic data are convolved with the optimized source time function. The traces are multiplied by an offset dependent factor of* $(r/1\,\mathrm{m})^{0.7}$*. The time axis is reduced with a velocity of 170 m/s. (b) Comparison of the amplitude spectra of the source time functions.*

the differences between viscoelastically and elastically modeled data Figure 3 shows a comparison of wavefields which are all modeled for the true model. There are clear differences between the elastic and viscoelastic modeling. The elastic and viscoelastic wavefields differ mainly in amplitude because the variation in $Q$ is too small to produce significant phase differences (phase velocity dispersion caused by damping is 3.3 % for $Q = 20$, 2.6 % for $Q = 25$, and 6.5 % for $Q = 10$ in the considered frequency band).

For the inversion tests we use the data modeled with $Q = 20$ as simulated observed data. For these data we calculated a slant stack (see Figure 2(a)) which shows three well separated modes (the fundamental mode and two higher modes of Rayleigh waves). For the inversion we use eight vertical force sources with a spacing of 10 m and 63 receivers (vertical and radial component) which are located between the sources with a spacing of 1 m. The starting model is a linear gradient shown in Figure 2(c). We use a 2D FWI code developed by Köhn (2011). It uses the time domain adjoint method. The forward modeling is done with the Finite Difference (FD) method (Bohlen, 2002). As misfit function we use the global correlation norm suggested by Choi and Alkhalifah (2011) because it is not sensible to an amplitude decay with offset. Therefore, far and near offset traces similarly contribute to the misfit. We apply frequency filtering during the inversion by starting at 10 Hz and increasing the bandwidth up to 70 Hz in steps of 5 Hz. The gradients are preconditioned around the sources and the models are smoothed with a 2D median filter (filterlength 0.6 m). We invert for P-wave velocity $v_p$, S-wave velocity $v_s$ and density $\rho$. As the surface waves are most sensitive to the shear wave velocity we only show the results for this parameter. We do not invert for viscoelastic parameters like $Q$ values or relaxation frequencies. When we use viscoelastic forward modeling in the inversion we asume a constant a priori known $Q$ value implemented by three relaxation mechanisms in the FD modeling. We invert the viscoelastic simulated observations ($Q$=20) with an elastic inversion and inversions using viscoelastic modelings with $Q = 20$, 25, and 10. The results are shown in Figure 2(d). In all inversion results we observe a periodic pattern caused by the low number of sources used in this inversion. The best result is obtained with the correct $Q$ factor of 20. When we apply a purely elastic inversion we obtain artifacts in the vicinity of the sources and the inversion is not able to infer the steep gradient near the surface because the difference between elastic and viscoelastic data are largest at high frequencies. The velocities at very shallow depth are inferred from surface waves with high frequencies because of their low penetration depth. When we use a slightly wrong $Q$ factor ($Q = 25$ in our example) the result is very similar to the result using the correct

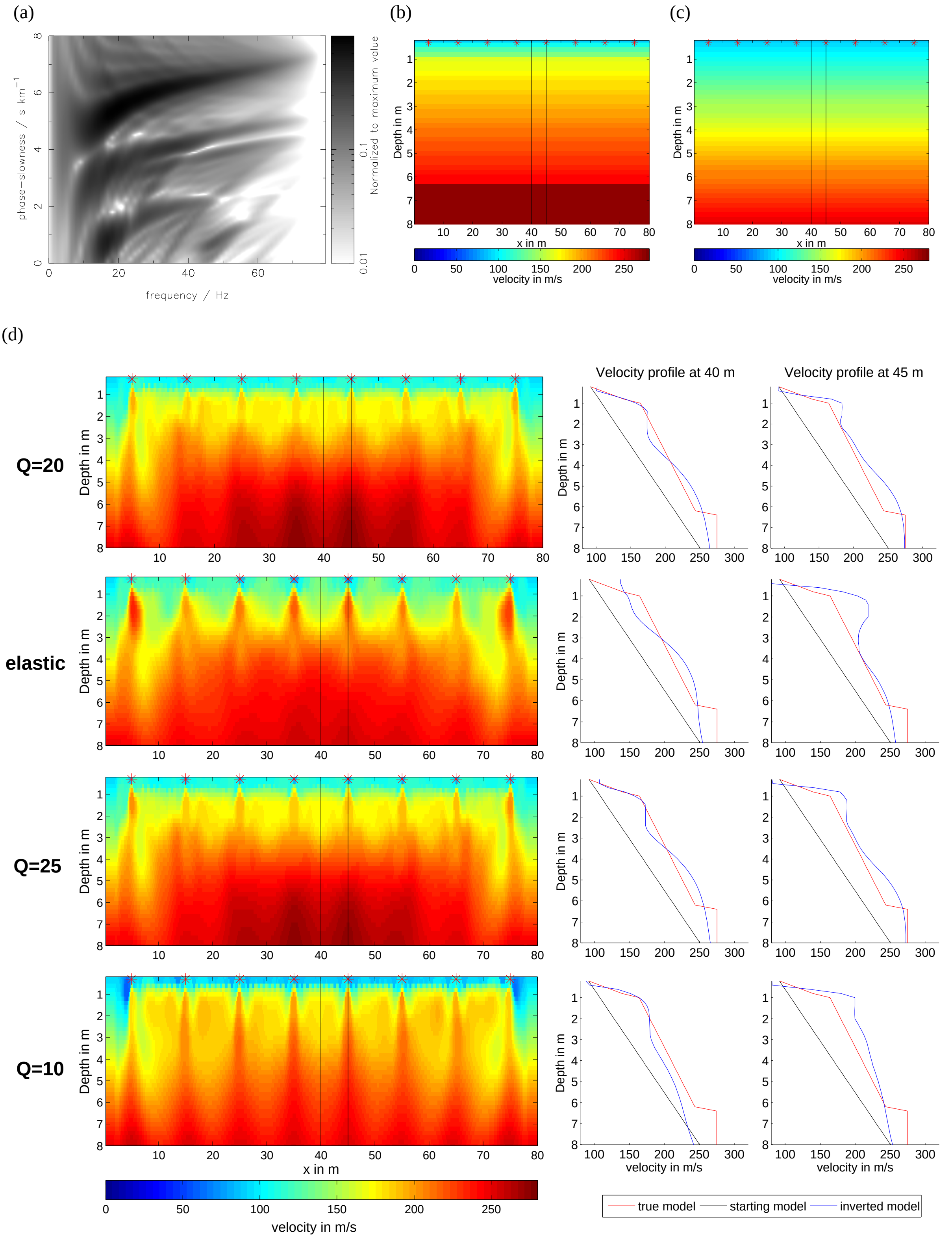

*Figure 2* *True model and synthetic inversion results. (a) shows a slant stack of the simulated observed data calculated with the true model. (b) shows the true* $v_s$ *model and (c) shows the starting* $v_s$ *model. The position of the sources are marked by the red stars. (d) shows the inversion results. On the left side the inverted* $v_s$ *models are plotted and on the right side two velocity profiles at* $x = 40\,m$ *and* $x = 45\,m$ *are shown. The location of the profiles is marked by the black lines in the models.*

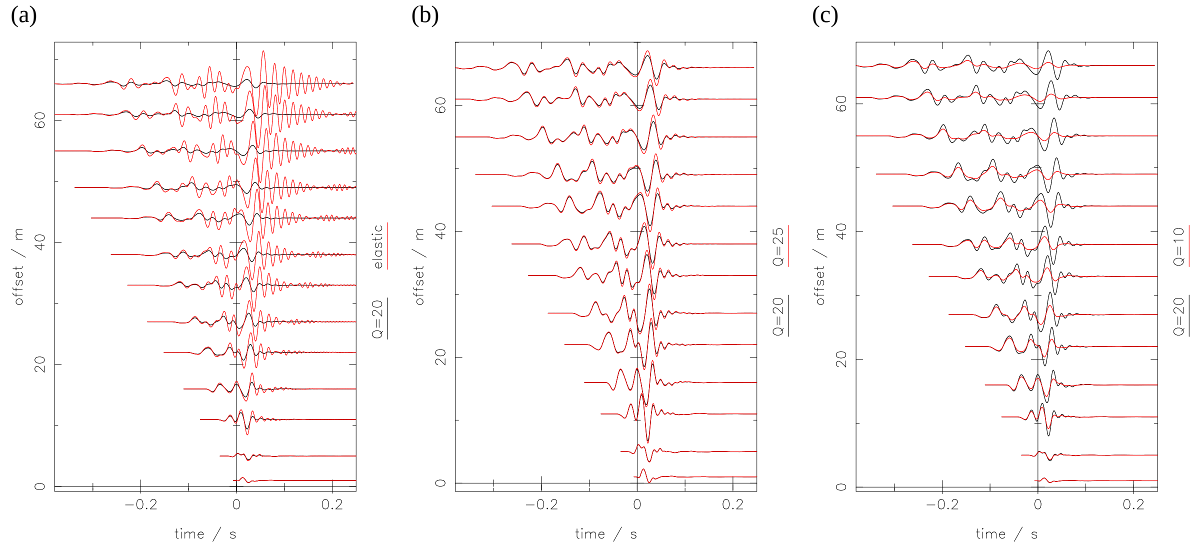

***Figure 3*** *Comparison of modeled viscoelastic data* ($Q = 20$) *with (a) purely elastic data, (b) viscoelastic data with* $Q = 25$ *and (c) with* $Q = 10$. *The traces are multiplied with an offset dependent factor of* $(r/1\,\mathrm{m})^{0.8}$ *for (a) and (b) and* $(r/1\,\mathrm{m})^{1.1}$ *for (c). The time axes are reduced with a velocity of 145 m/s.*

$Q$ factor. In the inversion result with $Q$=10 we again observe artifacts in the vicinity of the sources. However, in contrast to the elastic inversion result it is still possible to derive the steep gradient in the topmost meter between the sources (velocity profile at $x$=40 m).

## Conclusions and Outlook

The comparison of synthetic data with field data demonstrates that a significant portion of the residuals between elastically and viscoelastically modeled data can be compensated by the optimized source time function. However, the viscoelastic data match the field data better for the amplitude decay with offset of the fundamental mode and the near offset traces. Furthermore, the inversion tests with simulated observations show that we have to consider viscoelastic modeling during the FWI to resolve the very shallow shear wave velocity structure. This is not possible using purely elastic modelings in the FWI. Based on the result that the source time function can compensate differences in wavefields caused by viscoelasticity further inversion tests should show how an inversion of the source time function at each iteration step in the FWI can improve the results.

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## References

Amundsen, L. and Reitan, A. [1994] Transformation from 2-D to 3-D wave propagation for horizontally layered media. *Geophysics*, **59**(12), 1920–1926.

Bachrach, R., Dvorkin, J. and Nur, A.M. [2000] Seismic velocities and poisson's ratio of shallow unconsolidated sands. *Geophysics*, **65**(2), 559–564.

Bohlen, T. [2002] Parallel 3-D viscoelastic finite difference seismic modelling. *Comput. Geosci.*, **28**, 887–899.

Choi, Y. and Alkhalifah, T. [2011] Application of encoded multi-source waveform inversion to marine-streamer acquisition based on the global correlation. *Extended Abstracts, 73rd Conference and Technical Exhibition*, EAGE, F026.

Forbriger, T. [2003] Inversion of shallow-seismic wavefields: Part I and II. *Geophys. J. Int.*, **153**(3), 719–752.

Köhn, D. [2011] *Time domain 2D elastic full waveform tomography*. Ph.D. thesis, Christian-Albrechts-Universität zu Kiel.

Romdhane, G., Grandjean, G., Brossier, R., Rejiba, F., Operto, S. and Virieux, J. [2011] Shallow-structure characterization by 2D elastic full-waveform inversion. *Geophysics*, **76**(3), R81–R93.