Survivability Models for the Assessment of Smart Grid Distribution Automation Network Designs

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ABSTRACT

Smart grids are fostering a paradigm shift in the realm of power distribution systems. Whereas traditionally different components of the power distribution system have been provided and analyzed by different teams through different lenses, smart grids require a unified and holistic approach that takes into consideration the interplay of communication reliability, energy backup, distribution automation topology, energy storage and intelligent features such as automated failure detection, isolation and restoration (FDIR) and demand response.

In this paper, we present an analytical model and metrics for the survivability assessment of the distribution power grid network. The proposed metrics extend the system average interruption duration index (SAIDI), accounting for the fact that after a failure the energy demand and supply will vary over time during a multi-step recovery process. The analytical model used to compute the proposed metrics is built on top of three design principles: state space factorization, state aggregation and initial state conditioning. Using these principles, we reduce a Markov chain model with large state space cardinality to a set of much simpler models that are amenable to analytical treatment and efficient numerical solution. In the special case where demand response is not integrated with FDIR, we provide closed form solutions to the metrics of interest, such as the mean time to repair a given set of sections.

We have evaluated the presented model using data from a real power distribution grid and we have found that survivability of distribution power grids can be improved by the integration of the demand response feature with automated FDIR approaches. Our empirical results indicate the importance of quantifying survivability to support investment decisions at different parts of the power grid distribution network.

Categories and Subject Descriptors

C.4 [**Performance of Systems**]: Modelling Techniques

Keywords

Survivability; Transient Analysis; Smart Grid; Fault Tolerance

1. INTRODUCTION

Information and communication technologies (ICT) are being deployed to the distribution power grid to facilitate the management of energy demand and supply. The automatic management of customer consumption in response to variations in supply or as a result of power failures is referred to as *demand response*. The automatic detection, isolation and restoration of failures is known as *distribution automation* (DA). The automation of the smart grid brings novel challenges to the power grid engineers, such as the assessment of the tradeoffs involved to accurately engineer the power distribution reliability.

The introduction of automation (intelligence) to power distribution networks has created a need for the holistic assessment of the distribution network. The distribution network reliability is a function of the correct operation of several architecture artifacts such as electrical power components, telecommunications, distribution network topology, failure detection isolation and restoration, demand response and distributed generation and storage. The automation of power distribution requires a more integrated perspective across these domains. However, in the current mode of operation they are still being engineered separately.

Traditionally, the reliability of power systems has been quantified using average metrics, such as the system average interruption index (SAIDI). SAIDI is used by public service commissions in the United States to assess utilities' compliance with the commission rules. It was developed to track manual restoration times, and according to Standard 166-1998, the median value for North American utilities is roughly one and a half hours. In smart grid networks, power failure and restoration events will have a finer level of granularity, due to the deployment of reclosers, which isolate faulty sections, and demand side management system activities, such as distributed generators and demand response application systems. Therefore, there is a need to extend the SAIDI metric, and to develop new models and tools for the accurate computation of customer interruption indexes *after power failure events occur*, even if the occurrence of such events is rare. The *survivability* of a mission-critical application is the ability of the system to continue functioning during and after a failure or disturbance [17].

In [14] we presented a proposal for a common analysis framework to support the survivability analysis of distribution automation using extensions of the IEC-standardized common information model. The paper presented a case study of the application of the proposed method to the survivability analysis of a simple distribution automation network that was derived from a real power distribution network. In [19] we have evaluated the impact of available active and reactive power supply after a section failure on the distributed automation survivability metric and we derived closed-form expressions for certain survivability related metrics.

In this paper we present an analytical model to assess the survivability of distributed automation power grids and to predict SAIDI and related metrics as a function of different system parameters related to communications, distributed generation, demand response and other smart grid features. We use a performability model to capture how the system recovers from a failure. Our model accounts for the fact that the topology is sectionalized. Given a failure in section i, our key insight is to aggregate the sections of the network that may be fed by backup sources into a single node, denoted by $i+$. This aggregation allows us to efficiently quantify *transient metrics* of the network after a failure, also referred to as *survivability metrics*. For example, our model allows us to compute how the *energy not supplied* (ENS) after a failure varies over time as a function of the available backup power, the demand response application and of the state of the information and communication network.

After a power failure event, some power grid areas of the network may experience restoration times of the order of magnitude of minutes, while other power grid areas may require hours for the manual repair events to take place. Our model allows for the accurate assessment of the power grid network survivability by tracking the time-dependent state of the system under study.

The main contributions of this paper are the following.

Survivability model: We present a Markov chain model that supports the survivability assessment of power grid metrics accounting for the sectionalizing of distribution automation topology, the available excess power, the unreliability of the telecommunications network and the interaction with the demand response application. Our model can be generated and solved in a cost-efficient manner.

Implications of system integration: We bring awareness to the importance of accurate holistic power engineering that considers the interactions between telecommunications reliability and the reliability benefits of integration with other distribution automation features, such as the integration of failure recovery with demand response. In particular, we show that if demand response can be activated after a failure occurs, the reliability of the system significantly increases.

Extension of the SAIDI metric to support distributed automation: We present an extension of the SAIDI metric that captures the dynamic nature of the smart-grid by taking into account the number of customers impacted by the service interruption, the service impact of the interruption (*e.g.,* Energy not Supplied) and the duration of the recovery period. We use the analytical solution of the survivability model to capture the time spent in each state during the recovery period and the reward associated with each state to capture the service impact of the interruption.

The outline of this paper is as follows. In Section 2 we present a survey of the related literature. In Section 3 we present an overview of demand response and failure detection isolation and restoration applications. In Section 4 we introduce the survivability metrics that can be derived from our model. We present the model used in this paper in Section 5. The analysis of our empirical results is presented in Section 6. Section 7 presents our conclusions and suggestions for future research.

2. LITERATURE REVIEW

The available literature on power systems reliability is extensive [4, 8, 1]. Recently, researchers have studied how to improve power systems reliability with smart grid techniques [27, 22]. To our knowledge, our work is the first to assess survivability metrics of power systems accounting for the implications of electromechanical and computer-based strategies to address failures in an integrated manner.

Elmakias [8] presents a review of computational methods in power system reliability. The focus of the review is on the application of Markov models to reliability assessment. To address failures in the distribution system, the author studied a number of approaches such as the reduction of main feeder line length, the introduction of sectionalizer switches, and the automatic connection of backup power supply to sections isolated by a failure. The analysis focuses on steady state metrics, whereas in this paper our focus is on studying the system after a failure occurs. Conditioning the initial state to be a failure state is important in order to evaluate metrics such as the mean energy not supplied until recovery.

The impact of adding Distributed Generation (DG) as a backup source in a power system has been studied in [27, 29, 26, 28]. Wassem [27] and Zou *et al.* [29] analyzed the impact of DG placement on SAIDI, comparing several main feeder topologies. They concluded that placing the DG source at the end of the main feeder line provided the best improvement in power reliability. Wang *et al.* [26] propose an analytical model to evaluate system reliability. They use the model to obtain the placement and sizing of DG's that maximizes power reliability. An optimization approach for the optimal sizing of DG's is also presented by Zhang *et al.* [28]. These works are related to ours, as these results can be used to obtain the probability that a backup source can supply energy to the affected sections.

Janev [11] presents the implementation and evaluation of a power flow algorithm for power distribution grids with distributed generation. Specifically, the power flow approach presented in [11] was evaluated using an adaptation of a power distribution benchmark [23]. This benchmark employs the following types of renewable power generation equipment: PV, Wind, Small Hydro and Biomass.

Martins [18] presents a model for active distribution systems expansion planning that considers distributed generation together with traditional alternatives for distribution expansion such as rewiring, network reconfiguration and installation of protection devices. The authors evaluate different alternatives for distribution automation using average reliability metrics (SAIDI, SAIFI) and cost.

Brown [4, Section 2.2.5] presents a detailed discussion of the shortcomings of existing indices to assess reliability of power systems. In addition, the author proposes a novel metric, the system average interruption duration exceeding threshold, or SAIDET. In this paper, we argue that survivability metrics also play a key role in the assessment of smart grid networks.

Heegaard and Trivedi [10] studied the survivability of telecommunication systems. They presented a phase recovery model to capture the transient properties of the system after a failure. In this paper, we account for features that are specific to the smart grid domain and leverage such features for the efficient solution of the proposed model (see Section 5.1).

Performability metrics have been defined to measure the ability of a system to continue to operate after a failure but at (possibly) different performance levels [20, 21]. Performability is usually concerned with the quality of service provided that the system is operational. The initial system state is chosen accordingly. In this paper, in turn, our focus is on survivability metrics. In this case, the initial state of the system is set to a failure state, so survivability is "conditional performability" [16].

Keshav and Rosenberg [13] argued that concepts pioneered by the Internet are applicable to the design of smart grids, and suggest the initiation of a dialogue between the Internet community and the electrical grid research community. Our work is a product of such a dialogue [10, 25].

3. DISTRIBUTION SYSTEMS BACKGROUND

In this section we introduce some background on distribution systems, focusing on the aspects relevant to our model, namely 1) the demand response application and 2) fault detection, isolation and restoration. Currently, the two features are implemented by separate distribution automation systems, so we discuss the potential benefits of the integration of demand response and fault detection, isolation and restoration features. We start with a brief primer on distribution automation.

3.1 Distribution Automation Primer

The smartening of distribution networks can bring significant benefits to operators and customers, but will require considerably more effort than the smartening of transmission networks. Distribution networks have many more nodes to be instrumented and managed, and there is a need to meet stringent requirements for communication reliability. Distribution systems connect to nearly all electricity customers (excluding some large industrial customers that are connected directly to the transmission system). In addition, future distribution networks will become very complex with the introduction of new technologies, such as distributed generation and variable/dispatchable resources, and new load types, such as electric vehicles. Therefore, there is a need to quantitatively engineer and manage the distribution automation technology complexity and the associated costs, with the goal of optimizing the power grid to the benefit of all the stakeholders: power utilities, regulatory entities and customers.

The integration of smart grid related technology into the distribution side will lead to significant changes in the power system configuration. The current power grid distribution system is designed to meet the expected power load requirements. In the future, the distribution automation (DA) feature will be responsible for maintaining power reliability. Specifically, several alternatives will be available to reconfigure the power system distribution topology after a power event such as a failure. Two alternatives are: (1) depending on the load that is required to be met to recover from a power failure, demand response applications or distributed generators might be initiated, and (2) if several power distribution areas are interconnected through the use of tie line switches, spare power from one area can be used to meet the power demand from the failed area.

Therefore, the interconnection of several distribution areas and the introduction of distribution side energy management schemes like demand response, electric vehicle, energy storage and distributed generation can help manage power reliability by decreasing the mean fault clearing times and reducing the burden to be carried by the power protection devices.

3.2 Demand Response Application

Demand response are a set of the incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized. Demand response can be defined as the action taken by consumers to reduce electricity demand in response to price, monetary incentives, or utility directives so as to maintain reliable electric service or avoid high electricity prices.

A decade ago distribution side networks were considered mostly demand nodes, while now they can act both as a power generation source or a power demand sink. Bidirectional power flow can have a significant impact on the protection and reliability of the system. In addition, evolution of the nodal market creates a location based pricing mechanism, where the price is not only a function of the available energy but also depends on the congestion.

The introduction of demand response applications and the emergence of wholesale energy and reliability markets create new opportunities for demand-side resources by enabling customer loads to participate in the wholesale energy market. In addition, the application of demand response has the potential of enhancing power reliability and helping operators manage peak demand by reducing congestion on critical transmission lines.

3.3 Fault Detection, Isolation and Restoration

Fault detection, isolation and restoration is concerned with the detection of faults on the feeder line, determining the location of the fault as defined by the two feeder switches that determine the fault boundary, isolation of the faulty feeder section, and automated restoration of power to the feeder sections located outside the fault boundary, *i.e.,* the non-faulty feeder sections.

The granularity of fault detection, isolation and restoration depends on the type of switch/recloser used for dividing the feeder line into sections, and the availability of backup power to feed the healthy sections of the feeder line.

The time required for fault detection isolation and restoration depends on the level of automation implemented in the infrastructure deployed by the Utility to support the *FDIR feature*. Customers usually report outages 5-10 minutes after a fault occurrence. Power can be automatically restored to the healthy sections of the feeder line in about 2 minutes, when automated fault detection, isolation and restoration is implemented. When automated fault detection and isolation is not implemented, it may take up to 1 hour for power to be restored to the healthy sections of the feeder, because manual fault location and manual switching has to be performed. Repair of the faulty section of the feeder line may take 1 to 4 hours. When the feeder line is connected to a tie switch, the healthy parts of the feeder line can be powered by a secondary substation, after the faulty section is isolated. Table 1, which will be further discussed with the model presented in Section 5, summarizes the above numbers. The numbers in Table 1 reflect the multiple time scales at which repairs occur. In this paper, these numbers are set based on expert knowledge, but they could as well be adjusted based on time series or event logs.

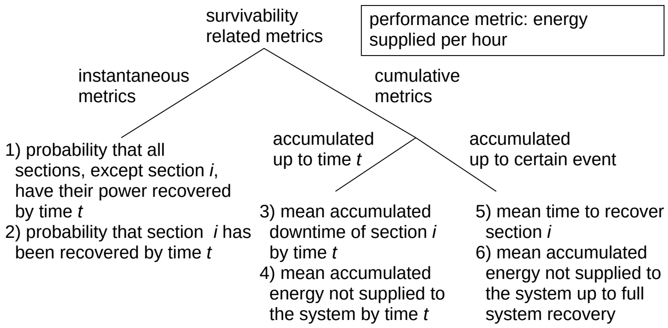


Figure 1: A taxonomy of survivability related metrics

3.4 Integration of Demand Response and Failure Recovery

In current distribution systems, demand response services are not integrated with failure recovery services. Usually, these two services are designed independently. Nonetheless, in this paper we consider the possibility of demand response being activated in response to a failure event with the aim of improving the power distribution survivability.

4. SURVIVABILITY METRICS

Survivability has been defined by ANSI as the transient performance of a system after an undesirable event [2]. The metrics used to quantify survivability vary according to applications, and depend on a number of factors such as the minimum level of performance necessary for the system to be considered functional, and the maximum acceptable outage duration of a system. Survivability metrics are transient metrics computed after the occurrence of a failure. In the remainder of this paper, time t refers to the time since a failure occurred and is measured in hours.

Survivability metrics are computed with respect to a measure of interest $\mathcal{M}$, also referred to as the *performance metric* [10]. In the realm of power systems, the performance metric $\mathcal{M}$ is the energy supplied per hour, measured in kilowatts. Assuming that $\mathcal{M}$ has value μ just before a failure occurs, the survivability behavior is quantified by attributes such as the relaxation time for the system to restore the value of $\mathcal{M}$ to μ. In this paper we compute metrics related to the relaxation time, focusing on the mean energy not supplied per hour after a failure occurs.

4.1 Metrics Taxonomy

Figure 1 shows the taxonomy of the survivability related metrics considered in this paper. We classify the metrics into two broad categories. Instantaneous metrics are transient metrics that capture the state of the system at time t. An example of an instantaneous metric is the probability that a given section i has been recovered by time t.

Cumulative metrics are obtained in our model by assigning *reward rates* to system states. A reward is gained per time unit in a state, as determined by the reward rate assigned to that state. The *accumulated reward* is the result of the accumulation of rewards since the failure up to time t or up to a certain event. The mean accumulated downtime of a given section by time t and the mean accumulated energy not supplied by time t are examples of cumulative metrics computed up to time t. The mean accumulated energy not supplied up to the full recovery of the system is an example of a cumulative metric computed up to a certain event occurs. The mean time to recover a given section is also an example of the latter class of metrics, where the accumulated reward in this case is the time itself, obtained by assigning a reward of one per time unit at every state. Other definitions of transient metrics can be found in [6]. In Section 6 we present the evaluation of the metrics described above as applied to the case study presented in this paper.

4.2 From SAIDI to Survivability Related Metrics

We now define and extend one of the key metrics of interest in the realm of power systems, the System Average Interruption Duration Index (SAIDI). SAIDI is an important measure of the power utility's ability to cope with recovery from failures. It is a measure of average customer impact of system interruptions as it computes the sum of customer interruption durations over the total number of customers [4].

Given a topology with C sections, let N be the total number of customers, let $N_{j,k}$ be the number of customers in the system impacted by the k-th failure at section j and let K_j be the number of failures at section j during a pre-established large observation period, $j = 1, \ldots, C$, $k = 1, \ldots, K_j$. Let $\varphi_{j,k}$ be the outage duration due to the k-th failure that occurred at section j, measured in hours. Let φ_j be the average outage duration due to all failures at section j. Let ϕ_j be the average number of failures at section j, during the same pre-established observation period. The observation period is usually assumed to be one year so that φ_j and ϕ_j are the annual average outage duration and number of failures, respectively.

The System Average Interruption Duration Index (SAIDI) is an important measure of the power utility's ability to cope with recovery from failures. It is a measure of average customer impact of system interruptions as it computes the sum of customer interruption durations over the total number of customers [24],

DEFINITION 4.1. *The SAIDI index is the average outage duration for each customer served,*

$$\mathsf{SAIDI} = \sum_{j=1}^{C} \sum_{k=1}^{K_j} \varphi_{j,k} \frac{N_{j,k}}{N} \tag{1}$$

After a failure, the energy not supplied will vary over time during a multi-step recovery process. Let $\{m_j(t), t \geq 0\}$ be a stochastic process in which the random variable $m_j(t)$ characterizes the energy not supplied per unit time, after a failure in section j, $j = 1, \ldots, C$, t units of time after the failure; $m_j(t)$ accounts for the effect of one single failure in section j. If a full system recovery occurs at time T, we set $m_j(t) = 0$ for $t \geq T$. Let $\overline{m}_j(t)$ be the mean value of $m_j(t)$. In the remainder of this paper, given a random variable Z we denote its mean by $\overline{Z}$.

Let $M_j(\tau)$ be the accumulated energy not supplied by time τ after a failure in section j, $j = 1, \ldots, C$,

$$\overline{M}_j(\tau) \quad = \quad \int_{t=0}^{\tau} \overline{m}_j(t)dt, \quad j = 1, \ldots, C \tag{2}$$

Note that the total energy demanded per unit time can also vary during recovery. This occurs, for instance, if demand response is integrated with failure recovery. Let $\{d_j(t), t \geq 0\}$ be a stochastic process in which the random variable $d_j(t)$ characterizes the total energy demanded per unit time at time t during the recovery from a failure in section j. Let $D_j(\tau)$ be the energy demanded over the first τ time units during the recovery from a failure in section j,

$$\overline{D}_j(\tau) \quad = \quad \int_{t=0}^{\tau} \overline{d}_j(t)dt, \quad j = 1, \ldots, C \tag{3}$$

Let ϕ_j be the expected number of failures at section j during a pre-established large observation period (typically one year). We define the extended SAIDI index (ESAIDI) as the outage duration accounting for the energy demanded and not supplied during the first τ units of time after a failure at a section, averaged over all sections,

DEFINITION 4.2. *The extended SAIDI index is given by*

$$\mathsf{ESAIDI}(\tau) = \sum_{j=1}^{C} \phi_j \tau \left(\frac{\overline{M}_j(\tau)}{\overline{D}_j(\tau)} \right) \tag{4}$$

The term inside parentheses in (4) is the fraction of the mean energy not supplied over the mean energy demanded by time τ after a failure. Note that we assumed that τ is a scalar value. Alternatively, let X_j be a random variable characterizing the time to full system recovery after a failure at section j, $j = 1, \ldots, C$, and $\mathbf{X} = (X_1, \ldots, X_C)$. Replacing τ in (4) by the corresponding mean recovery times yields

$$\mathsf{ESAIDI}(\mathbf{X}) = \sum_{j=1}^{C} \phi_j E[X_j] \left(\frac{E[\overline{M}_j(X_j)]}{E[\overline{D}_j(X_j)]} \right) \tag{5}$$

where

$$E[\overline{M}_j(X_j)] = \lim_{\tau \to \infty} \overline{M}_j(\tau) \tag{6}$$

Let $\overline{M}_j = E[\overline{M}_j(X_j)]$ and let N_j be the average number of customers affected by a failure at section j. The equality in (6) follows from the fact that if a full system recovery occurs at time T, $\overline{m}(t) = 0$ for $t \geq T$. Quantity $\overline{m}_j(t)$ is precisely that defined by ANSI [2] as *survivability*. Cumulative quantities such as $\overline{M}_j$ are defined as extension to the basic survivability measure and called Excess Loss due to Failures (ELF) [15].

Next, we show conditions according to which Definition 4.1 follows as a special case of Definition 4.2. To this goal, assume that the energy demanded per user per unit time is constant and equal to E, and that the number of customers affected by a failure at section j is also constant and equal to N_j. Therefore, $\overline{m}_j(t) = N_j E$ and $\overline{d}_j(t) = NE$. In addition, we also assume that the mean outage duration due to one failure at section j is τ, and the number of failures at section j is independent of the outage duration due to a failure at that section, $\varphi_j = E[X_j]\phi_j = \tau\phi_j$. Let $\widehat{\varphi}_j$ be the mean outage duration due to one failure at section j, $\widehat{\varphi}_j = \varphi_j/\phi_j$. Then, $\mathsf{ESAIDI}(\widehat{\boldsymbol{\varphi}}) = \mathsf{SAIDI}$, where $\widehat{\boldsymbol{\varphi}} = (\widehat{\varphi}_1, \ldots, \widehat{\varphi}_C)$.

$\mathsf{ESAIDI}(\tau)$ is a function of ϕ_j, $\overline{D}_j(\tau)$ and $\overline{M}_j(\tau)$, $j = 1, \ldots, C$. ϕ_j is computed from an availability model while $\overline{D}_j(\tau)$ and $\overline{M}_j(\tau)$ are computed from a survivability model. In the remainder of this paper our focus will be on the survivability model, which allows us to compute the terms in parentheses in equations (4) and (5), as described in Section 5 and illustrated in a case study in Section 6.

5. SURVIVABILITY MODEL

In this section we present the model used to compute survivability metrics of power distribution systems. We describe the modeling challenges and design principles, followed by the model overview and the specific model instantiation used throughout the remainder of the paper.

5.1 Challenges and Design Principles

Survivability assessment of the power grid distribution topology is implemented by taking advantage of the power line design that uses sections for failure isolation.

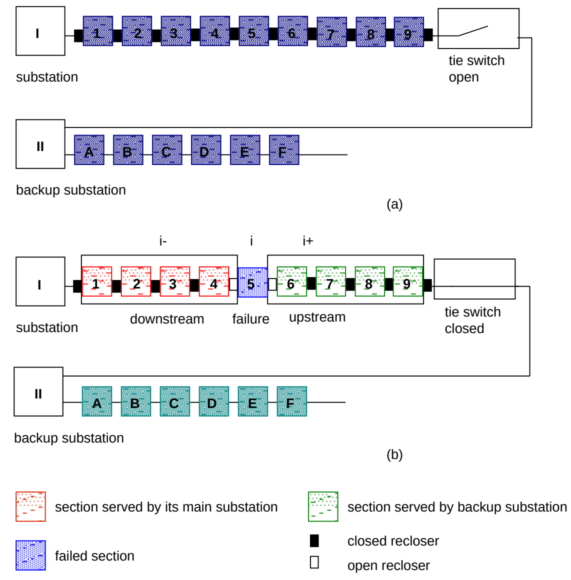


Figure 2: Failed section and its upstream and downstream

Initially, we attempted to characterize the individual behavior of each of the sections by conducting an exhaustive quantitative analysis accounting for the failure rates of each of the sections and their multiple possible states. During this initial model design phase we encountered the following challenges:

- capturing the state of each section individually leads to a large state space, as the number of states grows exponentially as a function of the number of sections;
- generating, storing, and solving a model with a very large state space is computationally challenging;
- accounting for component failures that occur at a much coarser level of granularity than the failure repair yields a model that is numerically hard to solve.

The methodology presented in this paper addressed the above challenges by relying on three key principles: (1) state space factorization; (2) state aggregation and (3) initial state conditioning.

5.1.1 *State Space Factorization*

Our methodology encompasses a set of models, where each model characterizes the system evolution after the failure of a given section. Given a topology with C sections, our methodology yields C models, where each model is tailored to the characteristics of the failed section. The advantages of such a state space factorization are:

$i)$ **flexibility:** having a model tailored to a given section enables us to capture specific details about the impacts of failures on that particular section;

$ii)$ **reduced complexity:** the computational complexity to compute the metrics of interest is reduced by considering a set of models as opposed to a single model with cardinality C times larger. Consider a distribution automation topology that has C sections, and let K be the number of states at which a section can be found. The computational complexity to solve the non-factorized model is $O(C^3K^3)$, using, for instance, the GTH solution method [9] to compute the steady

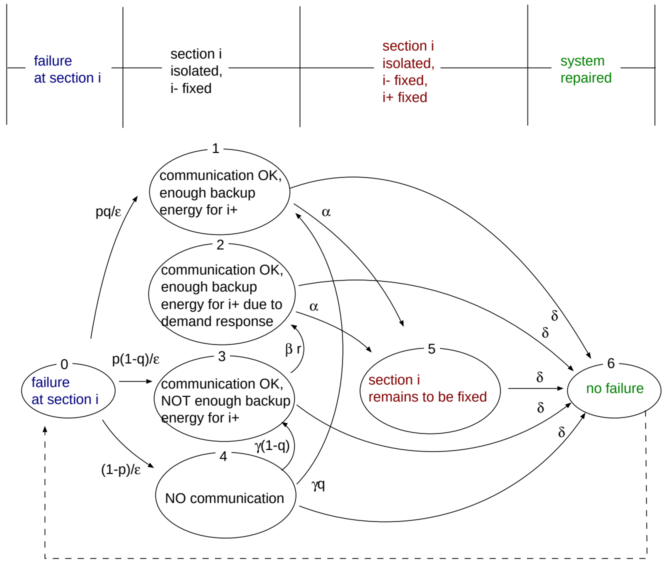


Figure 3: Phased recovery model

state probabilities of an associated ergodic model (described in the Appendix), without taking advantage of any possible special structure in the model. The decomposition approach, in contrast, requires the solution of C models, each one requiring $O(K^3)$ steps to be solved, which results in a complexity of $O(CK^3)$.

5.1.2 State Aggregation

One of the insights of this paper is the observation that after a failure of a given section the remaining sections of the distribution automation topology can be aggregated into groups of affected and non-affected sections. In the scenario considered in the remainder of this paper after the failure of section i, section i is isolated and the non-failed sections can be aggregated into two groups: the downstream sections that are aggregated into a set of sections $i-$ and are served by their original substation and the upstream sections that are aggregated into into a set of sections $i+$ and might be served by a backup substation, if enough backup power is available (see Figure 2). State aggregation yields significant reduction in the computational complexity required to obtain the desired metrics, since the system state space can be described in terms of the aggregated section states.

5.1.3 Initial State Conditioning

The computations of the metrics of interest are performed by assuming that the initial state is a failure state. Our models do not capture the failure *rates* of different components. Instead, the models are parameterized by using the *conditional probability* that specific system components are still operational after a specific section failure. In the remainder of this paper we will consider conditional probabilities to account for the probability that a substation backup power is able to supply isolated sections (q), the reliability of the telecommunications network (p) and the effectiveness of the demand response application (r).

5.2 Model Overview

Automatic and manual restoration events are initiated after a section failure event. The restoration process is a combination of electro-mechanical and computer-based events. In what follows, we describe the sequence of events initiated after the failure of section i.

The isolation of the failed section is automatically performed by *reclosers*, within 10-50 ms after the failure, and power is instantaneously restored to the *downstream* sections ($i-$). The *upstream* sections ($i+$) have their power restored depending on the following factors:

- **communication:** communication is needed for all failure detection, isolation and recovery operations. In particular, communication is used by the *supervisory control and data acquisition* (SCADA) system at a substation to detect failure location, recalculate flow and close the tie switch to feed the upstream sections ($i+$);
- **backup power:** sufficient spare backup power must be available at a backup substation;
- **demand response:** demand response applications can reduce the load in the system after a failure, increasing the probability that the available backup power is able to supply energy to the upstream sections.

Recall from Section 3.3 that, after a section failure, if the communication system is available and the backup power is able to restore energy to the upstream sections, it takes an average of 1-2 minutes to execute the automated restoration feature (see Table 1). If there is not enough available backup power for the restoration of the upstream sections, but communication is available, the demand response feature might be used to adjust the demand accordingly. When the demand response is effective, demand of sections $i+$ can be lowered to the target values within 15 minutes on average. Note that we do not explicitly model demand that is shifted to a later point in time (i.e. load shedding [3]). If the communication system is not available after the section failure, a 1 hour repair time is required for manual restoration of the communication system. This time is dominated by the time it takes for a truck to arrive at the failure site.

Finally, section i may require manual repair, $e.g.$, to remove weather related damage and restore the damaged components to their original condition. After section i is repaired, if the upstream sections are still not recovered, these sections will be connected to the main substation through section i. The average time to manually repair a section is 4 hours.

5.3 Model Description

A Markov chain with rewards is used to model the phased recovery of the distribution automation network. The states of the model correspond to the different recovery phases at which the system might be found as shown in Figure 3. Each state is associated with a reward rate that corresponds, for instance, to the energy not supplied per hour or the number of customers not served per hour in that state. In this paper we assume that state residence times are exponentially distributed, which serves to illustrate our methodology in a simple setting. Future work consists of extending the model to allow for general distributions for the state residence times. The system states and the state rewards are described in the following subsections.

5.3.1 Phased Recovery Model

The phase recovery model is characterized by the following states and events. After a section failure the model is initialized in state 0. The residence time at state 0 corresponds to the time required for

Parameter	Description	Value
ϵ	mean time for recloser to isolate failed section	≈ 0
α	automatic restoration rate	30
β	demand response rate	4
γ	communication repair rate	1
δ	manual repair rate	1/4

Table 1: Model Parameters (rates are given in units of events/hour)

Variable	Description
C	number of sections
i	failed section
$i+$	upstream of section i (sections $\{i+1,\ldots,C\}$)
$i-$	downstream of section i (sections $\{1,\ldots,i-1\}$)
p	probability that communication works after failure
q	probability that backup power suffices to supply isolated sections
r	probability that demand response is effective after failure

Table 2: Table of Notation

the recloser to isolate the section, which takes an average of ϵ. As mentioned in Section 5.2, a recloser isolates a section within 10-50 ms after a failure, so in the remainder of this paper we assume $\epsilon = 0$. Let p be the probability that the communication network is still operational after a section failure and q be the probability that there is sufficient backup power to supply energy for sections $i+$. After the isolation of section i is completed the model transitions to one of three states:

1. with probability pq the model transitions to state 1, where the distribution network is amenable to automatic restoration,

2. with probability $1-p$, the model transitions to state 4, where the communication system requires manual repair, which occurs at rate γ,

3. with probability $p(1-q)$ the model transitions to state 3, where the effectiveness of demand response will determine if the system is amenable to automatic restoration.

At state 3, demand response takes place after a period of time with average duration $1/\beta$. Let r be the probability that demand response effectively reduces the load of the system to a level that is supported by the backup substation. In this case, the model transitions from state 3 to state 2 with rate βr. When the model is in states 1 or 2 the distribution network is amenable to automatic restoration, which occurs after a period of time with average duration $1/\alpha$. What distinguishes states 1 from state 2 is the fact that state 1 can be reached in one step transition after a failure, whereas state 2 is reached only after the successful activation of the demand response feature. Therefore, the state reward rates associated to states 1 and 2, such as the energy not supplied per hour at those states, are usually different. A manual repair of section i takes on average $1/\delta$ hours (and can occur while the system is in states 1-5). After a manual repair, the model transitions to state 6, which corresponds to a fully repaired system.

We now describe the computation of the survivability metric (Energy Not Supplied) by using the phased recovery model described in Figure 3. In each state of the model of Figure 3 we associate the energy not supplied per hour at that state, the state *reward rate*. Let $\pi_k(t)$ be the transient probability associated with state k and σ_k be the reward rate (*e.g.,* mean energy not supplied per hour) associated with state k, $k = 0,\ldots,6$. Let $L(t)$ be a random variable characterizing the reward accumulated by time t after a failure (*e.g.*, accumulated energy not supplied by time t). The mean reward accumulated by time t is

$$\overline{L}(t) = \sum_{k=0}^{6} \int_{y=0}^{t} \sigma_k \pi_k(y) dy \quad (7)$$

Let s_k be the residence time at state k before reaching state 6 (*i.e.,* up to full system recovery), $k = 0,\ldots,5$. Let L be a random variable characterizing the accumulated energy not supplied up to full system recovery. The mean reward accumulated up to full system recovery is

$$\overline{L} = \lim_{t\to\infty} \overline{L}(t) = \sum_{k=0}^{5} \sigma_k \overline{s}_k \quad (8)$$

Note that (7) is the mean energy not supplied in the interval $[0,t]$ after a failure, defined in (2), and (8) is the ELF measure defined in (6), with subscript j dropped since we consider a single failure. In the Appendix we show how to compute $\overline{s}_k$ and $\pi_k(t)$, and present their closed form solutions when $r = 0$ and $\epsilon = 0$.

5.4 Model Solutions

To compute the metrics of interest presented in Section 4 we used standard techniques for the solution of Markov chains. In the Appendix we show that if $r = 0$, that is, demand response is not enabled, closed form solutions can be derived for the probability distributions of states 1-6 of the Markov model shown in Figure 3. If $r > 0$ we can still find closed form solutions, although they cannot be written in a compact form. Therefore, when $r > 0$ we use standard Markov chain numerical methods to solve the model. For the computation of the average accumulated reward at a given point in time, we use the techniques based on uniformization implemented at the Tangram-II tool [5, 7].

6. ANALYSIS

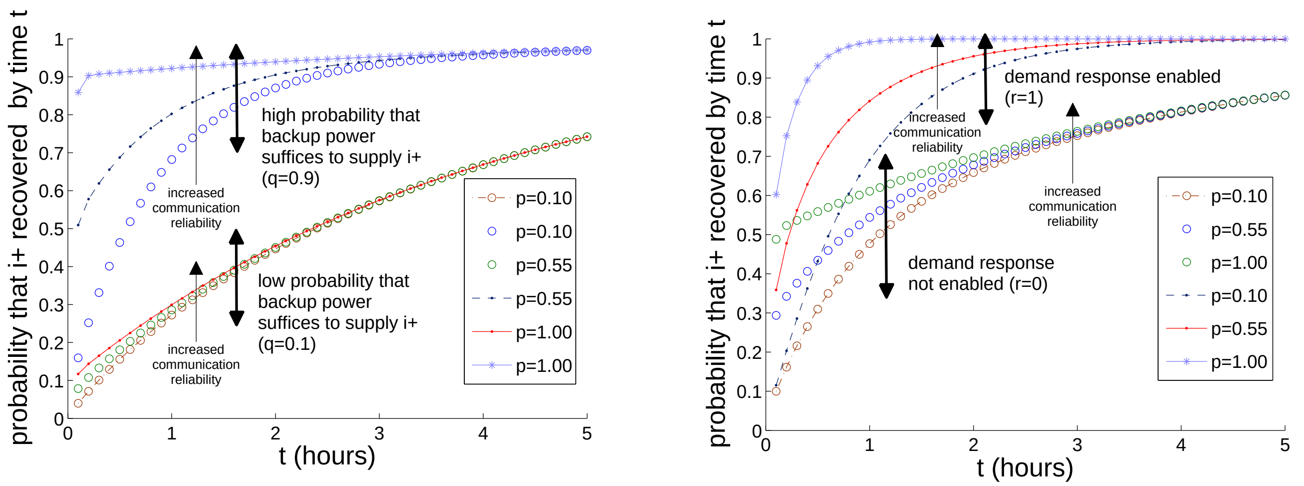
In this section we present the analysis of the empirical results obtained using the analytical modeling approach introduced in Section 5.

6.1 Setup Description

Our experimental setup is based on an adaptation of the data reported in [27], about the energy load in a number of sections in the US state of Virginia. Figure 2 illustrates the topology considered in our experiments. The topology consists of nine sections. The average number of customers and load per section, in KW, are shown in Table 3.

section	users	load	load amenable to demand response	net load
1	21	49.50	2.03	47.47
2	25	54.80	**0.00**	54.80
3	9	12.00	**4.07**	7.93
4	15	23.22	**11.58**	11.63
5	28	47.80	**0.00**	47.80
6	111	142.25	**15.28**	126.97
7	12	27.40	**0.00**	27.40
8	50	178.40	**0.00**	178.40
9	4	6.90	**1.40**	5.50
total	275	542.27	34.36	507.80

Table 3: Load per section (in KW)



(a) (b)

Figure 4: Probability of $i+$ being recovered by time t, varying a) p and q (no demand response enabled, $r = 0$); b) p and r (backup power suffices to supply $i+$ with probability 0.5, $q = 0.5$).

The energy supplied and not supplied per hour at each of the model states can be obtained from Table 3. We consider the worst case scenario in which section 1 fails, $i = 1$, maximizing the demand placed on the backup substation to supply the $i+$ sections. Table 4 shows the values of the rewards at the different model states. In states 0, 1, 3 and 4, the energy not supplied per hour is 542.27 KW, which corresponds to the average power demand placed on the distribution network. In state 2, the energy not supplied per hour decreases by 32.33 (sum of elements in bold in Table 3), to 509.94 KW, due to the activation of demand response. In state 5, the energy not supplied per hour is 49.5 KW as section 1 remains to be fixed. The energy supplied in state 5, in turn, depends on whether demand response was enabled. To simplify presentation, we set the energy supplied per hour in state 5 to its lower bound, 460.38 KW, assuming that demand response is always enabled at that state. Since the approximation has minimal impact in the results that follow, we proceed with our analysis under such simplification, noting that a straightforward extension of the model consists of splitting state 5 into two states, to account for whether demand response is enabled or not at state 5. Finally, we assume that the failed section 1 is not affected by demand response.

state	0 - 1	2	3 - 4	5	6
ES/h	0.00	0.00	0.00	460.38	542.27
ENS/h	542.27	509.94	542.27	49.50	0.00

Table 4: Rewards: lower bound on energy supplied per hour (ES/h) and energy not supplied per hour (ENS/h).

In the next subsections we evaluate how the metrics introduced in Section 4 vary as a function of the following parameters: the probability that the substation backup power is able to supply the isolated sections $i+$ (q), the reliability of the telecommunications network (p) and the effectiveness of the demand response application (r).

In what follows, we present experimental results for the time to recover (Section 6.2) and for the energy not supplied (Section 6.3). The results presented in Section 6.2 are independent of the rewards at different states and of the index of the failed section because in this experiment we did not include the capacity of the backup

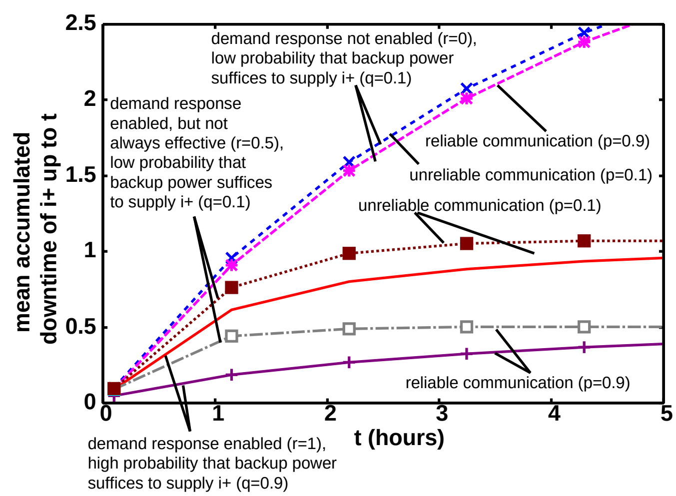


Figure 5: Mean accumulated downtime of $i+$.

stations as an exogenous parameter. Instead, we captured the substation backup capacity indirectly through the parameter q. In contrast, the results presented in Section 6.3 depend on the rewards and the index of the failed section as presented above.

6.2 Upstream Recovery Time

We consider the probability that the upstream $i+$ sections have recovered by time t. When $r = 0$, the expression of the probability that $i+$ has been recovered by t is given by (11) in the Appendix. Figure 4(a) shows how the probability that $i+$ has recovered increases over time. We observe that if q, the probability of the backup energy being sufficient to supply $i+$, is small, the communication infrastructure is ineffective to recover the $i+$ stations. In contrast, Figure 4(b) shows that if demand response is integrated with failure recovery ($r = 1$), the mean time for the system to recover decreases. Furthermore, with demand response, communication significantly impacts the probability that $i+$ has been recovered by time t. We observe that with demand response the probability that $i+$ has been recovered by time t is roughly 1 for t greater than 3. Otherwise, this probability does not reach 0.9 even after five hours.

Figure 5 shows the mean accumulated downtime of the upstream $i+$ sections as a function of the parameters evaluated in this study. If the evaluated parameter settings don't allow for the automated recovery of the $i+$ sections, the accumulated downtime will grow until a manual repair event occurs.

In contrast, if the evaluated parameter settings allow for automated recovery of the $i+$ sections the mean accumulated downtime will level off after a shorter period of time. For example, for the parameter values of $p = 0.9$, $q = 0.1$ and $r = 0.5$, Figure 5 shows that the mean accumulated downtime of $i+$ levels off after one hour, and never surpasses 0.5 hours.

Closely related to the mean accumulated downtime is the mean time to recover $i+$, shown in Figure 8. The mean time to recover decreases as p or q increase. As r increases from 0 to 0.5, the impact of q decreases because if demand response is integrated with failure recovery, demand response can reduce the load demand to the power grid after a section failure.

6.3 Quantifying Energy Not Supplied

In this subsection we present the computation of the accumulated energy not supplied, from the time of occurrence of a failure event up to time t. We use the rate rewards associated with each of the model states as described in Section 6.1. Figures 6(a) and 6(b) show the mean accumulated energy not supplied by time t, and the fraction of mean energy not supplied over mean energy demanded by time t, as a function of time. The former corresponds to $\overline{M}(t)$ (eq. (2)), and the latter corresponds to the term inside parentheses in the definition of ESAIDI(t), $\overline{M}(t)/\overline{D}(t)$ (eq. (4)). $\overline{M}(t)$ and $\overline{D}(t)$ are computed using (7), setting $reward_k$ to the mean energy not supplied per hour and the mean energy demanded per hour, respectively. If $q = 0.9$, that is, there is a high probability that the backup power suffices for sections $i+$, demand response does not have a significant impact on the energy not supplied, since the backup station is likely to support the additional load demand even in the absence of demand response. In contrast, if $q = 0.1$ demand response plays a key role, because sections $i+$ can be automatically restored when demand response is effective. The plots in Figures 6(a) and 6(b) also demonstrate the significant impact of integrated demand response on the mean accumulated energy not supplied when the probability that backup power suffices to supply $i+$ is low ($q = 0.1$).

The curves corresponding to $q = 0.9,\ r = 0$ and $q = 0.1,\ r = 1$ cross each other in Figures 6(a) and 6(b) because during the first moments after a failure, it is beneficial to have a high value of q independently of the value of r, as demand response takes an average of 15 minutes to become operational. After two hours the mean accumulated energy energy not supplied is smaller when $q = 0.1,\ r = 1$ as opposed to $q = 0.9,\ r = 0$.

Figure 7 shows $\overline{M}/E[\overline{D}(X)]$, the mean fraction of mean energy not supplied over mean energy demanded up to full system recovery (term inside parentheses in the definition of ESAIDI(X), eq. (5), with subscript j removed since we consider a single failure). $\overline{M}$ is computed using (8), for different values of p and r, with $q = 0.1$ ($E[\overline{D}(X)]$ is computed similarly). The figure indicates that even a slight increase in r yields a substantial decrease in the mean accumulated energy not supplied up to full system recovery. It also shows that communication reliability, p, becomes more relevant as r increases, because communication is needed for all automatic failure recovery operations, including the activation of demand response for failure recovery.

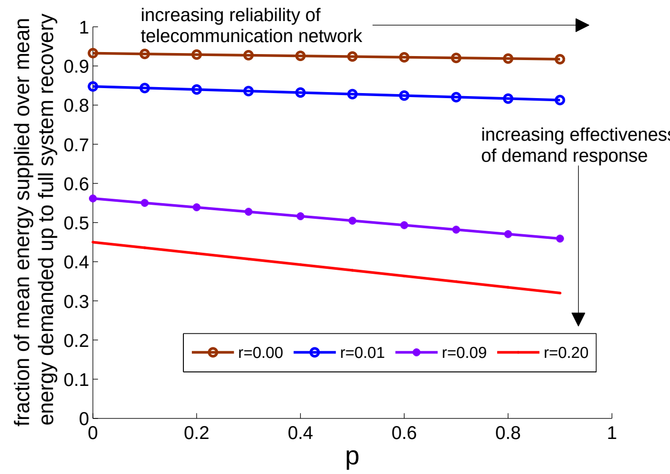


Figure 7: Fraction of mean energy not supplied over mean energy demanded up to full system recovery, $\overline{M}/E[\overline{D}(X)]$ ($q = 0.1$).

6.4 Discussion

With the presented results, investment decisions for the studied circuit can be based on survivability considerations. With the extended SAIDI as presented in (4), different options of how to improve reliability and survivability can be evaluated and compared to each other. For example, the effect of investing in the reliability of the communication infrastructure (*e.g.*, by adding redundancy) can be compared to investing in demand response mechanisms (*e.g.*, by subsidizing smart meters at the consumer).

7. CONCLUSION

In this paper we have argued, using analytical models, that new metrics are important to assess the quality of future intelligent power grids. Specifically, it was shown that survivability metrics can be used to drive a holistic engineering approach and are able to capture dynamic behavior of the smart grid after the occurrence of events of interest, *e.g.*, section failures.

We have proposed a methodology to assess the impact of different system parameters on the survivability of distribution automation power grids. Specifically, we have shown the interactions between three key parameters evaluated in this study and their impact on six survivability metrics. The empirical results obtained indicate that the integration of demand response with failure recovery can yield significant reductions in the amount of energy not supplied after a failure. In addition, the communications reliability parameter is most important when demand response is integrated. These results illustrate the need for holistic and accurate approaches to guide investment decisions on different parts of the network.

We believe that our work opens up several avenues for future exploration. Our models can serve to quantify the tradeoffs between investment cost and reliability gains. In face of such tradeoffs, one can devise algorithms to issue recommendations on how to invest on smart grids in light of prospective survivability gains. Specifically, the analytical solution of the survivability model enables the analytical optimization of the distribution automation investment given the costs of achieving a certain communication reliability (p), available backup power (q) or demand response effectiveness (r).

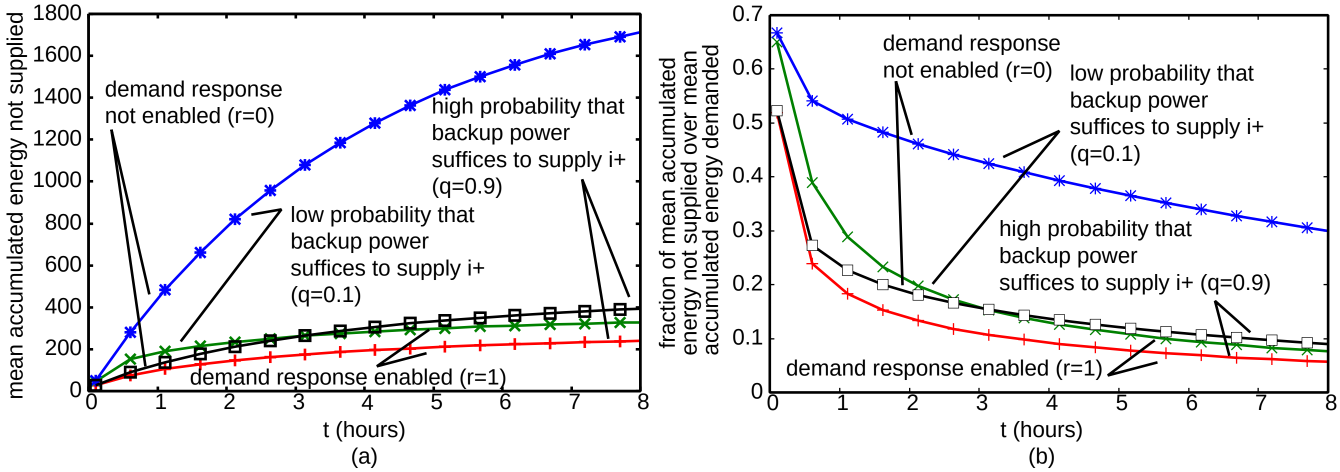


Figure 6: (a) Mean accumulated energy not supplied, by time t, $\overline{M}(t)$, and (b) fraction of mean accumulated energy not supplied over mean accumulated energy demanded, by time t, $\overline{M}(t)/\overline{D}(t)$ ($p = 0.9$).

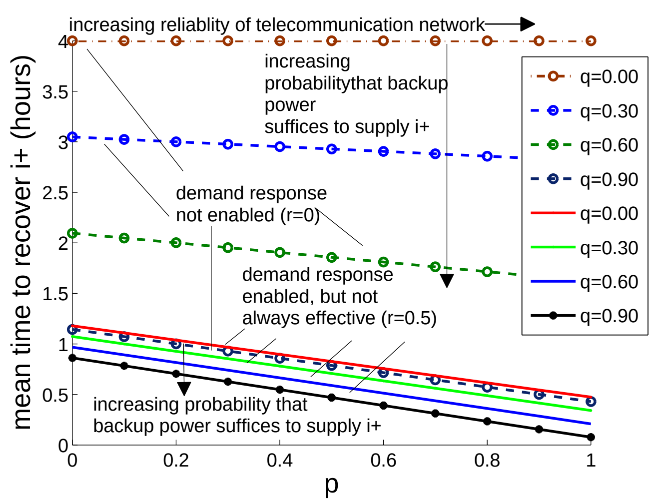


Figure 8: Mean time to recover $i+$, varying p, q and r. Dotted and solid lines correspond to demand response not enabled ($r = 0$) and enabled but not always effective ($r = 0.5$), respectively.

Detailed models for communications, back-power and demand response are needed to validate that the desired values of p, q and r are achieved for the distribution automation network design.

Other future research topics include the extension of the presented methodology to account for more complex topologies, more complex failure trees (cascading failures) and other features that are required to be included in the modeling of industrial distribution networks.

8. ACKNOWLEDGEMENT

This work was supported in part by grants from CNPq, FAPERJ/ Brazil and Siemens Corporate Research, Inc and by the Swiss Research Foundation (SNF) as a Forschungskredit of the University of Zurich, grant no. 53510401.

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Appendix - Closed Form Solutions

Next, we present closed form solutions to the proposed model. We consider the case in which the time it takes for a recloser to isolate a section is zero, $\epsilon = 0$ (in practice, it is much smaller than all the other time intervals considered in the system) and demand response is not integrated with the failure recovery system, $r = 0$.

Let Y_k be the k-th visited state after a failure, $Y_0 = 0$. Let $P(Y_1 = j)$ be the system *initial condition* after the failure, which is determined by p and q, $P(Y_1 = 1) = pq$, $P(Y_1 = 3) = p(1-q)$ and $P(Y_1 = 4) = 1 - p$. Let $\pi_k(t)$ be the probability that the system is at state k at time t, $t \geq 0$. Then,

$$\pi_k(t) = \sum_{l=1}^{6} \pi_k(t|Y_1 = l)P(Y_1 = l), \quad k = 1, \ldots, 6 \qquad (9)$$

To compute $\pi_k(t|Y_1 = j)$ we note that there is at most one sample path from each state to every other. Let $\Upsilon_{i,j}$ be the sample path from state i to state j. Let $\Upsilon_{j,i}(k)$ be the k-th state in the path $\Upsilon_{j,i}$, $1 \leq k \leq |\Upsilon_{j,i}|$. Let $P(\Upsilon_{j,i})$ be the probability that $Y_k = \Upsilon_{j,i}(k)$, $k = 1, \ldots, |\Upsilon_{j,i}|$, conditioned on $Y_1 = j$. Then,

$$\pi_k(t|Y_1 = j) = \pi_k(t|Y_1 = j, \Upsilon_{j,i})P(\Upsilon_{j,i}) \qquad (10)$$

Substituting (10) into (9) yields closed form expressions for $\pi_k(t)$, $k = 0, \ldots, 6$,

$$\begin{aligned}
\pi_0(t) &= \pi_2(t) = 0 \\
\pi_1(t) &= e^{-(\alpha+\delta)t}pq + \left(e^{-(\gamma+\delta)t} - e^{-(\alpha+\delta)t}\right)\left(\frac{\gamma q(1-p)}{\alpha - \gamma}\right) \\
\pi_3(t) &= e^{-\delta t}p(1-q) + (1-q)(e^{\gamma t} - 1)e^{-(\delta+\gamma)t}(1-p) \\
\pi_4(t) &= e^{-(\delta+\gamma)t}(1-p) \\
\pi_5(t) &= (1 - e^{-\alpha t})e^{-\delta t}pq + \\
&\quad + \left(e^{-\delta t} - \frac{\alpha e^{-(\delta+\gamma)t} - \gamma e^{-(\delta+\alpha)t}}{\alpha - \gamma}\right) q(1-p) \\
\pi_6(t) &= 1 - e^{-t\delta}
\end{aligned}$$

The probability that $i+$ has been recovered by time t is $\pi_{5\cup 6}(t)$ and is given by

$$\pi_{5\cup 6}(t) = \pi_5(t) + \pi_6(t) \qquad (11)$$

We now compute the mean time to fix $i+$, *i.e.*, to reach states 5 or 6 ($\overline{M}$, the mean time to reach state 6, defined by (6), follows similarly). To this goal, we consider a Markov chain with state space Ω' and infinitesimal generator Q'_ψ, where ψ is a a real valued parameter. The states in Ω' are indexed by 1, 3, 4 and $\star$ and the infinitesimal generator Q' is obtained from Figure 3 after 1) replacing the transitions to states 5 and 6 by transitions to state $\star$ and 2) adding transitions from state $\star$ to states 1, 3 and 4 with rates ψpq, $\psi p(1-q)$ and $\psi(1-p)$, respectively,

$$Q'_\psi = \begin{array}{cc} \left[\begin{array}{llll} -(\alpha+\delta) & 0 & 0 & \alpha+\delta \\ 0 & -\delta & 0 & \delta \\ \gamma q & \gamma(1-q) & -(\delta+\gamma) & \delta \\ \psi pq & \psi p(1-q) & \psi(1-p) & -\psi \end{array} \right] & \begin{array}{l} \mathit{1} \\ \mathit{3} \\ \mathit{4} \\ \star \end{array} \end{array} \tag{12}$$

Let $\boldsymbol{\pi}'$ be the steady state solution of the MC above, satisfying $\boldsymbol{\pi}' Q'_\psi = 0$ and $\pi'_1 + \pi'_3 + \pi'_4 + \pi'_\star = 1$. Then,

$$\pi'_1 = \psi\delta q(\gamma + \delta p)/\Delta \tag{13}$$

$$\pi'_3 = \psi(\alpha+\delta)(1-q)(\gamma+\delta p)/\Delta \tag{14}$$

$$\pi'_4 = \psi\delta(\alpha+\delta)(1-p)/\Delta \tag{15}$$

$$\pi'_\star = \delta(\alpha+\delta)(\delta+\gamma)/\Delta \tag{16}$$

where Δ is a normalization constant to ensure $\pi'_1+\pi'_3+\pi'_4+\pi'_\star = 1$.

Let C be a random variable characterizing the time to reach state $\star$ immediately after leaving $\star$. This corresponds to the mean time to reach states 5 or 6 in the original Markov chain, starting from state 0. The mean time to repair $i+$ is given by $E[C]$. It follows from (13)-(16) and [12, Chapter III] that

$$E[C] = \psi^{-1}(1/\pi'_\star - 1) \tag{17}$$

(17) is used to obtain the curves in Figure 4.

Let $s_k(\{5,6\})$ be the mean time spent at state k before reaching states 5 or 6 from state 0 in the original Markov chain, $k = 1,3,4$. It follows from (13)–(17) that

$$\overline{s}_k(\{5,6\}) = \frac{\pi'_k}{\psi(\pi'_1+\pi'_3+\pi'_4)}\left(\frac{1}{\pi'_\star} - 1\right), \quad k = 1,3,4$$

Let $L(\{5,6\})$ be the mean accumulated reward up to reaching states 5 or 6 of the original Markov chain,

$$\overline{L}(\{5,6\}) = \sum_{i=1,3,4} \sigma_k \overline{s}_k(\{5,6\}) \tag{18}$$

In this appendix we have shown how to compute $\overline{s}_k(\{5,6\})$, $k = 1,3,4$ and $\overline{L}(\{5,6\})$. The mean time spent at state k before reaching state 6, $\overline{s}_k$, $k = 1,3,4,5$, and the mean accumulated reward up to reaching state 6, $\overline{L}$, defined by (8), follow similarly. An adaptation of (18) is used to obtain the curves in Figure 7.

Although in this appendix we assumed that $r = 0$ and $\epsilon = 0$, a similar approach is applicable when $r > 0$ or $\epsilon > 0$.