Liquidity in Bond Markets

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Chapter 1

Introduction

1.1 Motivation

Bond markets are an important source of financing for private and public companies as well as for sovereigns. The total volume of outstanding bonds worldwide amounts to about 85 trillion USD and exceeds both the annual world GDP (72 trillion USD) and the market capitalization of global stock markets (53 trillion USD).[1] For all investors in bond markets, the recent financial crisis has highlighted the tremendous importance of liquidity risk management. Since investors demand higher future expected returns for illiquid assets, deteriorating liquidity directly transfers to a decrease of a bond's fair value. Therefore, as liquidity dried up in many markets during the financial crisis, investors not only faced large trading costs, but additionally suffered losses due to increased yield spreads of their bonds. Dick-Nielsen, Feldhütter, and Lando (2012) estimate that after the collapse of Lehman Brothers, the median yield spread component due to illiquidity for speculative U.S. corporate bonds increased to a peak of about 10% compared to less than 1% before the crisis. The decreasing value of illiquid bonds thus contributed to the capital losses of financial institutions and amplified the financial crisis (see, e.g., Brunnermeier, 2009). For that reason, an effective liquidity risk management requires both the quantification of liquidity and an understanding of its impact on bond prices.

However, measuring liquidity in bond markets is difficult. As bond trading is not centrally organized on exchanges like it is the case for stocks, intraday data on quotes is not available. Therefore, trading cost measures like intraday effective bid-ask spreads (i.e.,

[1]Data for 2012, see Bank of International Settlements (2013), International Monetary Fund (2013), and World Bank (2013).

the average difference of trade prices from quoted mid prices), which are established as the standard choice in equity markets (see, e.g., Lee, 1993; Hasbrouck, 2009), cannot be computed for bond markets. Rather, many researchers either design their own liquidity measures based on transaction data or simply use liquidity proxies developed and tested in stock markets. Due to the different market structure of decentralized over-the-counter (OTC) markets compared to centralized exchange markets, it is unclear whether these measures originally developed and tested in stock markets work in bond markets as well. Moreover, the lack of a comparative analysis of the different liquidity measures makes the choice of a particular measure often arbitrary. Such benchmarking studies are available for exchange traded stocks (see, e.g., Goyenko, Holden, and Trzcinka, 2009; Fong, Holden, and Trzcinka, 2010) or commodity futures (see Marshall, Nguyen, and Visaltanachoti, 2012), but it is unclear how their results transfer to decentralized OTC bond markets. In this thesis, we implement and empirically compare all the different liquidity measures used in the literature so far. Our goal is to provide guidance to researchers and practitioners by answering the question which liquidity measure is best suited in a particular situation.

Regarding the impact of illiquidity on asset prices, there are also fundamental differences between bonds and stocks. First, bonds offer investors the opportunity to wait until maturity and thereby avoid transaction costs. This opportunity impacts the value of liquidity and leads to a relation between the time until a bond's maturity and its yield spread due to illiquidity. Second, contrary to stock markets, bond issuers usually have multiple bonds outstanding. These securities are often very close substitutes with respect to credit risk and other characteristics and differ only in their maturity. Therefore, price discounts due to illiquidity should be analyzed from an aggregate perspective and not on an individual security level. Both reasons point to employing the term structure of liquidity premia as a natural means to describe the impact of illiquidity on bond prices.

From a risk management perspective, it is important to understand how this term structure of liquidity premia is related to fundamental economic factors. Although there are studies that analyze such a dependence (see, e.g., Longstaff, 2004; Kempf, Korn, and Uhrig-Homburg, 2012), it remains an open question how the influence changes in a crisis compared to normal times. Since the relationship between liquidity and bond prices is strongly dependent on the economic environment (see Acharya, Amihud, and Bharath, 2013), it is likely that the influence of fundamental factors on the term structure of liquidity premia is also different in crisis times. If this is the case, calibrating risk management models in normal times could potentially misjudge illiquidity risk. We analyze this issue by empirically investigating the difference in the zero coupon yield curves of two bond classes that only differ in their liquidity within a regime-switching model.

Since bonds of different maturities from the same issuer are often close substitutes, dependencies between different maturities, which are one of the constituting characteristics of bond markets, are of great importance. However, existing theoretical papers either analyze liquidity premia of bonds with infinite maturity (see, e.g., Vayanos and Vila, 1999; Huang, 2003) or look at bonds of one single maturity but do not allow for dependencies between different maturities (see, e.g., Feldhütter, 2012). There is also little consensus even on the most fundamental question: What is the shape of the term structure of liquidity premia in equilibrium? Empirically, the term structure is found to be decreasing (Ericsson and Renault, 2006), increasing (Dick-Nielsen, Feldhütter, and Lando, 2012), or U-shaped (Longstaff, 2004). Moreover, the literature offers no explanation for the puzzling observation that bonds with very short and long maturities are rarely traded, while there is an active market for bonds with intermediate maturities (Elton and Green, 1998; Hotchkiss, Warga, and Jostova, 2002). In this thesis, we provide a theoretical framework that allows investors with different investment horizons to simultaneously trade in bonds with different maturities. With this framework, we are able to unify many previous empirical results.

1.2 Structure of the Thesis

This thesis is structured as follows:

In Chapter 2, which is based on the working paper Schestag, Schuster, and Uhrig-Homburg (2014), we analyze how to best measure bond liquidity. We first implement and compare eight high-frequency liquidity measures based on a full trade record for the U.S. corporate bond market. We establish that these high-frequency transaction cost and price impact measures are highly correlated with each other. Based on this result, we take the high-frequency measures as benchmarks and test whether they are connected to a battery of transaction cost and price impact proxies that only need daily information and thus can be computed much more efficiently. We include those liquidity proxies that have been frequently used in the literature (see, e.g., Chen, Lesmond, and Wei, 2007; Goyenko, Subrahmanyam, and Ukhov, 2011; Lin, Wang, and Wu, 2013) but also some measures that have been only applied in stock markets so far (see, e.g., Goyenko, Holden, and Trzcinka, 2009; Hasbrouck, 2009; Holden, 2009; Fong, Holden, and Trzcinka, 2010; Corwin and Schultz, 2012). We analyze the ability of liquidity proxies to capture time series and cross sectional variations of transaction cost and price impact benchmarks as well as the magnitude of transaction costs.

After having established how to best quantify liquidity, we analyze in Chapters 3 and 4 how illiquidity is priced. In Chapter 3, which builds on the working paper Schuster and Uhrig-Homburg (2014), we study the term structure of liquidity premia as the difference between the zero coupon yield curves of German government bonds (BUNDs) and government guaranteed bonds issued by a German federal agency (Kreditanstalt für Wiederaufbau, KfW). Both bond segments differ in their liquidity, but bear the same default risk. In this clean setting, we empirically analyze how liquidity related risk premia behave in different economic regimes. Moreover, we identify drivers of different parts of the term structure of liquidity premia and analyze their relations in crisis and non-crisis times.

In Chapter 4, which is based on the working paper Schuster, Uhrig-Homburg, and Trapp (2014), we propose a parsimonious equilibrium model where investors that differ in their investment horizons trade bonds of different maturities. Our model predicts different shapes of the term structure for liquidity premia computed from bid and ask prices and explains the well documented aging effect (see, e.g., Alexander, Edwards, and Ferri, 2000; Edwards, Harris, and Piwowar, 2007), i.e., the observation that other things equal, old bonds trade less frequently than newly issued bonds. We test and confirm our theoretical predictions using data for U.S. corporate bonds.

Chapter 5 summarizes the main results of the thesis and gives a concise outlook on possible future research questions.

Chapter 2

Measuring Liquidity in Bond Markets

2.1 Introduction

As pointed out in Chapter 1, there is no consensus how to adequately measure bond liquidity. The goal of this chapter is to evaluate and comprehensively benchmark the different liquidity measures used in the literature so far. We first implement and compare eight high-frequency liquidity measures based on full intraday information from TRACE, which contains a full trade record for the U.S. corporate bond market. Due to the OTC nature of the bond market, this database does not contain data on quotes and it is thus not possible to calculate intraday effective bid-ask spreads, which are standard in equity markets. As a consequence, researchers in bond markets have developed a multitude of different liquidity measures and, so far, there is no comprehensive analysis of whether these measures work equally well. We therefore first have to establish that these high-frequency transaction cost and price impact measures are highly correlated with each other. Since this is the case, we can take the high-frequency measures as benchmarks and test, whether they are connected to a total of 23 transaction cost and price impact proxies. These proxies only need daily information, that can be easily downloaded, e.g., from Bloomberg. To find out whether the daily proxies actually measure intraday transaction costs and price impact, we run various tests analyzing the ability of liquidity proxies to capture time series and cross sectional variations of transaction cost and price impact benchmarks as well as the correct scale of transaction costs. Whereas the ability to capture liquidity differences is especially important for asset pricing studies, many studies that analyze, e.g., trading strategies or portfolio allocations depend on the magnitude of transaction costs. The correct scale of transaction costs is also most important for all dealers and investors trading bonds. Our observation period spans the time since the full implementation of TRACE from October 1, 2004 to September 30, 2012.

Our results show that most of our liquidity proxies indeed capture transaction costs. Time series and average cross sectional correlations between benchmark measures and proxies are on average even higher than in Goyenko, Holden, and Trzcinka (2009) for the stock market. The best measures in our competitions are the bid-ask spread estimator derived from high and low prices developed by Corwin and Schultz (2012), Hasbrouck's (2009) Gibbs measure, and the widely used Roll (1984) measure. The former two also estimate the magnitude of transaction costs very precisely. All three measures perform very consistently for different liquidity portfolios and different subperiods. In contrast to the stock market, the effective tick size measure is not in the winning group, which is most likely due to the OTC nature of bond markets. Measures based on quoted bid-ask spreads from Bloomberg are generally suited to capture effective transaction costs as well. Thereby, measures based on executable quotes do better than proxies based on the Bloomberg Generic bid-ask spread, which is probably the most widely employed daily liquidity measure (e.g., Longstaff, Mithal, and Neis, 2005; Chen, Lesmond, and Wei, 2007; Bao, Pan, and Wang, 2011). In one subperiod, this measure even yields a negative time series correlation with our liquidity benchmarks, presumably due to methodological changes by Bloomberg in the process to derive bid and ask prices.

For price impact, we find that the daily Amihud (2002) measure and the price impact version of the high-low measure (Corwin and Schultz, 2012; Goyenko, Holden, and Trzcinka, 2009) are best suited to proxy for the intraday price impact benchmarks. However, these results have to be interpreted with great caution since measuring price impact in decentralized OTC bond markets generally poses a conceptual problem. First, due to the decentralized market structure, it is unclear how information from a trade gets incorporated into subsequent prices. Second, without intraday quote data, the impact of a trade on the quoted midpoint cannot be observed (see Hasbrouck, 1991, who uses quote data to determine price impact). Third, effective transaction costs, especially for corporate and municipal bond markets and in sharp contrast to most other markets, decrease with trade size (see, e.g., Schultz, 2001; Harris and Piwowar, 2006; Edwards, Harris, and Piwowar, 2007). Thus, with the usual notion of static price impact as the first derivative of transaction cost with respect to size (see, e.g., Goyenko, Holden, and Trzcinka, 2009), price impact is negative. Nevertheless, intraday and daily measures of price impact are widely used in the literature (see, e.g., Mahanti et al., 2008; Dick-Nielsen, Feldhütter, and Lando, 2012; Lin, Wang, and Wu, 2013). Their value can be justified as they incorporate volume data and therefore capture an additional dimension of liquidity, which might be important, e.g., for asset pricing.

Our results imply that for many applications, especially if the focus is on average market liquidity, it might be sufficient to use liquidity measures based on daily data. The necessary daily closing, high, and low prices as well as trading volumes or bid-ask quotations can be downloaded conveniently, e.g., with Bloomberg. By employing daily liquidity proxies, researchers can circumvent the computationally intensive data handling and cleaning procedures resulting from using the full TRACE data set (which is by now more than 10 gigabytes large). Moreover, our results provide guidance for all markets where a trade reporting system is not implemented and therefore intraday data is not available which applies to all bond markets except the U.S. For these markets, we recommend using bid-ask spreads derived from executable quotations as data providers cannot distribute closing, high, and low prices, and daily trading volumes.

The remainder of this chapter is organized as follows. Section 2.2 describes the data used in our analyses. Section 2.3 presents the high-frequency benchmarks and low-frequency proxies. The testing methodology and our main findings regarding the measurement of bond liquidity are provided in Section 2.4. Section 2.5 concludes the chapter.

2.2 Data

To calculate our high-frequency liquidity benchmarks, our main source of data is the Trade Reporting and Compliance Engine (TRACE) established by the Financial Industry Regulatory Agency (FINRA) in July 2002. Since October 1, 2004, all OTC trades in the U.S. corporate bond market have to be reported to the TRACE database within a time window of 15 minutes. These reports contain the date, time, volume,[2] and price of a trade. Since November 2008, the data includes also information on whether the trade is a buy, a sell, or an interdealer transaction. Our sample ranges from October 1, 2004 to September 30, 2012. Following Bao, Pan, and Wang (2011), we only include bonds in our analysis that are active for at least one year during our period of observation and are traded on at least 75% of trading days during their lifespan. Bonds in default are only included up to

[2]The volume is capped at $5 million for investment grade and $1 million for non-investment grade bonds.

Table 2.1: **Summary statistics of the overall TRACE database and our sample** The table compares bond characteristics from all bonds reported to TRACE and our sample. Panel A describes the overall TRACE data set where duplicates, withdrawn entries, and corrections are already accounted for using the filters proposed by Dick-Nielsen (2009). Our sample consists of all non-defaulted bonds that are active for at least one year within our observation period and which are traded on at least 75% of the trading days during their lifespan (see Bao, Pan, and Wang, 2011). A comparison of Panel B and C illustrates the impact of our error correction filters described in the text. Data on bond characteristics, ratings, and outstanding amounts is obtained from Thomson Reuters and Bloomberg and averaged over the life of a bond before calculating summary statistics. The observation period is October 1, 2004 to September 30, 2012.

	Mean	$Q_{0.05}$	$Q_{0.25}$	$Q_{0.5}$	$Q_{0.75}$	$Q_{0.95}$	Not applicable
Panel A: All bonds in TRACE after removing duplicates, withdrawn entries, and corrections (see Dick-Nielsen, 2009)							
# Bonds	72571						
# Trades per bond	859.57	1	6	47	355	3922	-
Amount outstanding (in mn. USD)	213.2	0.65	4.34	25	203.47	1000	6375
Annual turnover (in %)	57.2	0.78	9.64	25.15	52.01	164.9	6375
Time to maturity at issuance	8.8759	1.1333	3.0278	6.0000	10.0194	29.75	6387
Numerical Rating (1: AAA,..., 22: D)	6.2207	1	2.7791	5.5	8.6667	15.376	15969
Panel B: Selected bonds after removing duplicates, withdrawn entries, and corrections (see Dick-Nielsen, 2009)							
# Bonds	3494						
# Trades per bond	10449.07	1511	3458	6889.5	12593	30635	-
Amount outstanding (in mn. USD)	987.37	167.43	500	750	1250	2500	-
Annual turnover (in %)	90.27	33.89	56.72	75.94	104.85	190.01	-
Time to maturity at issuance	10.6891	3.0055	4.9639	9.7889	10.0333	30.0028	20
Numerical Rating (1: AAA,..., 22: D)	8.0567	2.4029	5.1591	7.1582	10.2935	15.8338	25
Panel C: Final sample (3494 bonds) after applying all error correction filters							
# Trades per bond	10002.55	1374	3274	6531	12060	29481	-
Annual turnover (in %)	86.23	32.42	55	73.21	98.46	183.04	-

three months before the default date to prevent an impact of abnormal trading behavior around and after the default event. To account for duplicates, withdrawn and corrected trade entries, we apply the procedures described in Dick-Nielsen (2009). Additionally, we exclude trades under special conditions and delete records of trades taking place on a holiday and before the bond's origination or after it has been fully repaid. Further, we use the median and reversal filters introduced by Edwards, Harris, and Piwowar (2007) to eliminate extreme outliers and erroneous entries. After all these corrections, our final TRACE data sample consists of 3,494 bonds and 34,948,920 single trades, which is about 56% of all trades reported to TRACE.

Table 2.1 gives a brief summary of the characteristics of the bonds in our sample (Panel B) and compares them to the overall TRACE database (Panel A). Compared to the full TRACE data set, our bonds are on average more actively traded and have higher amounts outstanding.[3] Further, for time to maturity, the upper quantiles $Q_{0.75}$ and $Q_{0.95}$

[3] This implies that one should be cautious to transfer our results to very illiquid bonds. However, it

are very similar, whereas the lower quantiles are higher in our sample due to the fact that we demand a bond to be active for at least one year. In our sample, ratings are on average about two notches worse than for the overall TRACE database. Panel C shows the changing values when the error filters are applied. The decline in the number of trades per bond and annual turnover is only marginal.

In addition to TRACE data, some of our high-frequency liquidity measures depend also on a bond's fair market valuation. The Markit Group Limited, a leading global financial information service, collects price information from more than 30 dealers to compute a composite price and ensures its quality by running multiple data cleaning procedures on the contributed inputs (see Markit Group Limited, 2013). Therefore, we follow, e.g., Friewald, Jankowitsch, and Subrahmanyam (2012) and use Markit composite prices as fair market valuations. In total, Markit provides composite prices for 3,143 bonds in our TRACE sample.

To calculate our daily liquidity proxies, we use daily end of day, high, and low prices, trading volumes, and bid-ask quotations from Bloomberg. End of day prices as well as high and low prices and volumes are downloaded from Bloomberg's TRACE pricing source. Again, we correct the data using the above median and reversal filters, and delete days outside a bond's lifespan. Additionally, we delete data for days where the total trading volume is more than half of the total amount outstanding. We do not correct for discrepancies between our daily data and our intraday TRACE data set, as this would also not be possible for someone only employing our daily proxies. We thus fully separate our daily data from our intraday data set. End of day, high, and low prices as well as volumes are available for all 3,494 bonds in our sample. Volume data is only available in Bloomberg since March 29, 2005.

Daily bid-ask quotes are downloaded using the Bloomberg Generic Quote (BGN) and Composite Bloomberg Bond Trade (CBBT) pricing sources. Amongst others, Bao, Pan, and Wang (2011), Chen, Lesmond, and Wei (2007), and Longstaff, Mithal, and Neis (2005) employ BGN bid-ask spreads on the U.S. corporate bond market. BGN prices are computed as a weighted average of quotes from participating dealers and include indicative and executable quotes. Similarly, the CBBT pricing source also provides average bid-ask quotes, but is only based on executable quotes that are listed on Bloomberg's trading

would hardly make sense to construct intraday liquidity benchmarks for bonds that trade on average, e.g., one time per day. On the other hand, our robustness checks for different liquidity portfolios show that liquidity proxies, if anything, perform better in the more illiquid than in the more liquid portfolios. This is also found by Goyenko, Holden, and Trzcinka (2009) for stocks.

platform.[4] In addition to these already aggregated pricing sources, we use Bloomberg quotes contributed from 228 investment firms and exchanges where some of our bonds are listed. We eliminate all entries for BGN, CBBT, and single dealers' quotes with an ask quote larger than the corresponding bid quote and days outside the bond's lifespan. For the single dealers' quotes, we additionally delete stale prices,[5] duplicate time series, prices that would allow for arbitrage with Bloomberg's consensus prices, and quotes with corresponding bid-ask spreads that are more than ten times larger than the median bid-ask spread for that bond or all bonds in the sample on that day. After these corrections, we find BGN, CBBT, and single dealers' quotes for 3,480, 3,145, and 3,463 bonds, respectively.

For the daily proxies that need a fair market valuation as input, we use composite prices from the BGN pricing source. We do not use Markit composite prices, which we use for our intraday benchmarks, to ensure a full separation of our intraday and our daily data set. For proxies that need a reference index, we obtain price data for the FINRA-BLP Active Investment Grade US Corporate Bond Price Index and FINRA-BLP Active High Yield US Corporate Bond Price Index from Bloomberg. These indexes are based on the most frequently traded fixed coupon bonds in the TRACE database with rebalancing taking place each month (for details, see Financial Industry Regulatory Authority and Bloomberg, 2007).

Data on bond characteristics, historical ratings from Moody's, Standard & Poor's, and Fitch, and outstanding amounts are obtained from Thomson Reuters and Bloomberg.

2.3 Liquidity Measures

In this section, we briefly describe our high-frequency liquidity benchmarks as well as all liquidity proxies. We compute eight high-frequency measures calculated from our TRACE intraday sample and 23 liquidity proxies based on daily price, quote, and volume data collected from Bloomberg. For all bonds, we then compare high-frequency benchmarks with low-frequency proxies on a monthly basis. Following Goyenko, Holden, and Trzcinka (2009), we distinguish between spread and price impact measures and mainly compare our results within these subgroups.

[4]The exact methods of calculation are not disclosed by Bloomberg. CBBT prices are available since November 1, 2004.

[5]Specifically, we delete a quotation if it is stale for more than five business days (except, the respective BGN price does not change either, indicating that the fundamental value of the bond stays identical).

2.3.1 High-Frequency Benchmarks

As there is not one broadly accepted transaction cost benchmark in bond markets, we implement and compare all commonly used high-frequency transaction cost measures. Thereby, we slightly modify some of the measures so that they are able to fully exploit the information in our data set. As an example, we simplify the rather complex methods in Schultz (2001) or Edwards, Harris, and Piwowar (2007) to compute a fair reference price for a bond by using Markit composite prices. Some of our proxies have been initially defined on an absolute, and some on a relative (i.e., in percent of trade prices) level. As we also want to compare the scale of two measures, we standardize all benchmarks and proxies to measure relative transaction costs. For price impact, we calculate the intraday Amihud (2002) measure and a modified version of Hasbrouck's (2009) lambda.

2.3.1.1 Spread Benchmarks

Roundtrip Transaction Costs: Feldhütter (2012) develops an approach to compute roundtrip transaction costs based on trade prices. He argues that bonds are often traded with multiple trades taking place in a short time frame with identical trade volumes. Hence, it is safe to assume that dealers are undertaking what he calls imputed roundtrip trades (IRT) to coordinate buys and sells of investors. We aggregate all trades per bond with the same volumes that occur within a 15 minute time window to an IRT. We then compute the absolute effective spread estimator as the doubled difference between the lowest and highest trade prices for each IRT.[6] To get a relative spread proxy, we divide the roundtrip transaction costs by the mean of the maximum and minimum prices. A bond's monthly liquidity measure is then obtained as the average from all IRTs in a month.

Inter-Quartile Range: Han and Zhou (2007) and Pu (2009) use the inter-quartile range of trade prices as a bid-ask spread estimator. They divide the difference between the 75th percentile P_t^{75th} and the 25th percentile P_t^{25th} of intraday trade prices on day t by the

[6] Feldhütter (2012) notes that in his sample, only 4% of the imputed roundtrips after November 2008 consist of one single buy and one single sell transaction. However, nearly 90% include one buy or sell trade combined with one or two interdealer trades. Therefore, the doubled difference should be used to estimate bid-ask spreads. As in Feldhütter (2012), we exclude an IRT when the difference between the highest and lowest price is zero.

average trade price $\overline{P_t}$ of that day:

$$\text{B\_IQR} = \frac{P_t^{75th} - P_t^{25th}}{\overline{P_t}}. \tag{2.1}$$

This measure is similar to measures based on the price range (see also Han and Zhou, 2007), but is less sensitive to outliers. We calculate B_IQR for each day that has at least three observations and use the monthly mean as our monthly liquidity benchmark.

Roll: Friewald, Jankowitsch, and Subrahmanyam (2012) and Dick-Nielsen, Feldhütter, and Lando (2012) employ an intraday version of the Roll (1984) estimator for effective spreads. We adapt their measure to relative bid-ask spreads:

$$\text{B\_Roll} = \begin{cases} 2\sqrt{-Cov(r_i, r_{i-1})} & \text{if } Cov(r_i, r_{i-1}) < 0, \\ 0 & \text{otherwise,} \end{cases} \tag{2.2}$$

where $r_i = \frac{P_i - P_{i-1}}{P_{i-1}}$ is the return of the ith trade.

Adjusted Schultz: Schultz (2001) estimates transaction costs in the corporate bond market by running the following regression on a data set including a trade side indicator:

$$\Delta_i = \alpha_0 + \alpha_1 \cdot D_i^{Buy} + \epsilon_i. \tag{2.3}$$

For each trade i, D_i^{Buy} is a dummy variable indicating whether the trade is a buy ($D_i^{Buy} = 1$) or a sell ($D_i^{Buy} = 0$) and Δ_i is the difference of the trade price and the asset's bid quote.[7] The parameters estimated are α_0 and α_1 with the latter being an estimator for the effective bid-ask spread. We refine his model to get a relative spread proxy and to include information from interdealer trades, exploiting our full TRACE data set:

$$\Delta_i^{rel} = \alpha_0^{rel} + \alpha_1^{rel} \cdot D_i + \epsilon_i, \tag{2.4}$$

where Δ_i^{rel} is the relative deviation of the trade price from the bond's mid quote. We use the Markit composite price as proxy for the mid quote and adjust the dummy variable as follows:

$$D_i = \begin{cases} 1 & \text{if trade } i \text{ is a buy,} \\ 0 & \text{if trade } i \text{ is an interdealer trade,} \\ -1 & \text{if trade } i \text{ is a sell.} \end{cases} \tag{2.5}$$

[7] Schultz estimates bid quotes using a three-step procedure. For further details, see Schultz (2001).

We exclude the possibility of getting negative bid-ask spreads with the constraint $\alpha_1^{rel} \geq 0$. Our monthly bid-ask spread estimate is then obtained by first running the regression in Equation (2.4) on all trades in a given month, including only days on which a composite price is available. Second, as α_1^{rel} only estimates the *half* spread, we use B_AdjustedSchultz $= 2 \cdot \hat{\alpha}_1^{rel}$ and calculate this measure for the time when a buy/sell indicator is available from November 2008 onwards.

Adjusted EHP: Edwards, Harris, and Piwowar (2007) and Harris and Piwowar (2006) use an econometric model to estimate transaction costs utilizing the difference between returns from trade prices and returns from (unobserved) 'true values'. We proxy for the bond's unobserved true value return r_i^V with the log return of the Markit composite price $r_i^{Composite}$:

$$r_i^V = r_i^{Composite} + \epsilon_i. \tag{2.6}$$

In the spirit of Edwards, Harris, and Piwowar (2007), the error in the measurement of the unobserved true value return ϵ_i between trades $i-1$ and i is proportional to the time between both trades ΔT_i. Since we only observe $r_i^{Composite} \neq 0$ if trades $i-1$ and i take place on different days, we additionally distinguish between overnight and intraday error variance:

$$\sigma_{\epsilon_i}^2 = \sigma_{overnight}^2 \cdot D_{overnight} + \sigma_{intraday}^2 \cdot (1 - D_{overnight}) \cdot \Delta T_i, \tag{2.7}$$

where $D_{overnight}$ has value one if r_i^V is an overnight return and zero otherwise. Our modified version of their model for the observed (log) return $r_i^{Obs} = \log\left(\frac{P_i}{P_{i-1}}\right)$ is then given by:

$$r_i^{Obs} - r_i^{Composite} = c \cdot (D_i - D_{i-1}) + \eta_i, \tag{2.8}$$

where D_i is defined as in Equation (2.5) and η_i is an error term with mean zero and a variance σ_i^2 that is the sum of error variances from the measurement of unobserved true value returns ($\sigma_{\epsilon_i}^2$), transaction costs (σ_c^2), and interdealer price concessions (σ_δ^2):

$$\sigma_i^2 = \sigma_{\epsilon_i}^2 + (2 - D_i^{Int}) \cdot \sigma_c^2 + D_i^{Int} \cdot \sigma_\delta^2. \tag{2.9}$$

The variable D_i^{Int} equals zero, one, or two, when zero, one, or two of trades $i-1$ and i are interdealer trades. We estimate c from Equation (2.8) with the iterated weighted least-squares method for each bond and month, again starting in November 2008.[8] Since

[8] Following Harris and Piwowar (2006), we estimate the variance Equation (2.9) in a pooled regression over the whole sample. Since our simplified model has less parameters than the original model, we only need two observations (with different D_i) to identify the regression for a given bond/month combination. Further, we stop the iteration, if the maximum difference in c for our bonds is less than 10^{-6} between two

c only measures the half spread, we double the estimates to get our benchmark measure B_EHP.

Price Dispersion: Jankowitsch, Nashikkar, and Subrahmanyam (2011) introduce the following liquidity measure:

$$d = \sqrt{\frac{1}{\sum_{i=1}^{N} Q_i} \sum_{i=1}^{N} (P_i - m)^2 Q_i}. \tag{2.10}$$

This measure gives the daily dispersion of all N intraday trading prices P_i from the market-wide consensus price m. The higher this dispersion, the higher are the trading costs for investors. The authors develop a market microstructure model in which dealers' inventory risk and investors' search costs in addition to transaction costs are the drivers of price dispersion. Due to these additional cost components, the scale of this benchmark cannot be directly compared to the other transaction cost measures. A transaction's volume Q_i is used as a weighting factor, because it is assumed that dispersion in large trades reveals more information. We use a modified version of this measure for relative dispersions and double it to get an estimate for the effective spread.

$$\text{B\_PriceDispersion} = 2 \cdot \sqrt{\frac{1}{\sum_{i=1}^{N} Q_i} \sum_{i=1}^{N} \left(\frac{P_i - m}{m}\right)^2 Q_i}. \tag{2.11}$$

We calculate this benchmark for each day with at least one trade if a composite price m is available in Markit. Monthly measures are obtained as the mean of daily measures.

2.3.1.2 Price Impact Benchmarks

Amihud: Dick-Nielsen, Feldhütter, and Lando (2012) utilize the Amihud (2002) price impact measure on intraday TRACE data. We apply their approach and define the monthly price impact measure as

$$\text{B\_Amihud} = \frac{1}{N} \sum_{i=1}^{N} \frac{|r_i|}{Q_i}, \tag{2.12}$$

where N is the number of consecutive returns r_i in a sample month and Q_i is the volume (in USD) of trade i.

Lambda: Hasbrouck (2009) uses the estimator for λ in the following regression as high-

iteration steps. For more details on the regression method, see Harris and Piwowar (2006).

frequency price impact measure for equities:

$$r_\tau = \lambda \cdot \text{sign}(Q_\tau)\sqrt{|Q_\tau|} + \epsilon_\tau, \tag{2.13}$$

where r_τ is the stock's return and Q_τ is the signed traded dollar volume within the five-minute period τ. We adjust this measure to our TRACE data as follows. First, we do not restrict our intraday returns to originate within five-minute intervals, because price influences of single trades circulate much slower in decentralized OTC markets than it is the case for centrally cleared equity markets. Second, we do not need to accumulate traded volumes, since every return included in our regression arises from adjacent transactions. We therefore run the adjusted regression

$$r_i = \lambda \cdot D_i\sqrt{Q_i} + \epsilon_i, \tag{2.14}$$

where D_i is defined as in Equation (2.5). In the estimation of Regression (2.14), we preclude negative price impact by imposing $\lambda \geq 0$ and we exclude all overnight returns. B_Lambda = λ is computed starting in November 2008 for the lack of a buy/sell side indicator beforehand. Note, that both price impact benchmarks B_Amihud and B_Lambda also capture components of the bid-ask spread as larger bid-ask spreads increase price fluctuations.

2.3.2 Low-Frequency Proxies

We implement a total of 23 low-frequency liquidity proxies in our analysis, whereof eleven estimate effective spread and twelve measure price impact. Measures, which need end of day trade prices, are calculated using the Bloomberg TRACE pricing source. For measures on quotes, we utilize BGN, CBBT, or single dealers' quotes from Bloomberg. To ensure robustness of our results and to allow for fair comparisons between our proxies, we only calculate a measure for months with at least eight observations.

2.3.2.1 Spread Proxies

Our bid-ask spread proxies that only need end of day prices can be classified in the following sub-groups. First, we use two proxies that employ the negative auto-covariance of trade prices based on the idea of Roll (1984) and refined by Hasbrouck (2009) with his Gibbs measure. Second, our Effective-Tick proxy estimates bid-ask spreads based on price clustering. Third, we use two proxies that utilize the number of zero returns based on the paper of Lesmond, Ogden, and Trzcinka (1999) and enhanced by Fong, Holden, and Trzcinka (2010). Fourth, our high-low proxy developed by Corwin and Schultz (2012) filters bid-ask spreads out of daily high- and low prices which are also available in Bloomberg. Regarding the proxies that need information on quotes, we calculate quoted bid-ask spreads from three different pricing sources available in Bloomberg. Our last two proxies measure quote dispersion based on the idea of the price dispersion benchmark.

Roll: Analogously to our benchmark measure, we compute the daily Roll (1984) proxy as

$$\text{P\_Roll} = \begin{cases} 2\sqrt{-Cov(r_t, r_{t-1})} & \text{if } Cov(r_t, r_{t-1}) < 0, \\ 0 & \text{otherwise,} \end{cases} \tag{2.15}$$

where r_t is the return on day t.

Gibbs: Hasbrouck (2009) estimates the effective half-spread by using a Bayesian Gibbs sampler on the following model:

$$r_t = c \cdot \Delta D_t + \beta^m \cdot r_t^m + \epsilon_t. \tag{2.16}$$

Equation (2.16) adds a market factor m with return r_t^m on day t to the otherwise identical Roll (1984) approach with r_t being the return of the bond, D_t a sell side indicator[9] and c the half-spread. For m, we use the FINRA-BLP Active Investment Grade index, if in month t, the observed bond is rated investment grade on average (using the ratings from Moody's, Standard & Poor's, and Fitch) and the FINRA-BLP Active High Yield index, otherwise. We estimate c using the standard Bayesian normal regression model for each month and bond and double it to get our effective spread proxy P_Gibbs.[10]

Effective Tick: Goyenko, Holden, and Trzcinka (2009) together with Holden (2009) develop an effective spread proxy, called Effective Tick, that captures price clustering.[11]

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[9] $D_t = 1$ for a buy and $D_t = -1$ for a sell. Information on D_t is not needed, since D_t is estimated using the Gibbs sampler.

[10] We thank Joel Hasbrouck for sharing his programming code on `http://pages.stern.nyu.edu/~jhasbrou/Research/GibbsCurrent/Programs/RollGibbsLibrary02.sas` and use his code in our analysis. As priors for c, β^m, and σ_ϵ^2, we use normal distributions $N(\mu = 0.01, \sigma^2 = 0.01^2)$, $N(1, 1)$, and the inverted gamma $IG(\alpha = 10^{-12}, \beta = 10^{-12})$, respectively (see, e.g., Marshall, Nguyen, and Visaltanachoti, 2012). We run the sampler for 10,000 sweeps and discard the first 2,000 as burn-in period. For consistency with the original paper and mathematical tractability, we use log returns for this measure.

[11] Holden (2009) also develops another measure based on price clustering, called Holden. Like Marshall, Nguyen, and Visaltanachoti (2012), we do not incorporate this measure as it is very computationally intensive and therefore not widely used.

They assume the clustering of trade prices, e.g., at whole dollars, half-dollars, or quarters, to be determined by spread size, which enables them to compute the spread probabilities for a given set of possible mutually exclusive effective spreads s_j with $j = 1, 2, ..., J$. Their spread proxy is then defined as a probability-weighted average of all possible spreads s_j:

$$\text{P\_EffectiveTick} = \sum_{j=1}^{J} \frac{\hat{\gamma}_j \cdot s_j}{\overline{P}}, \tag{2.17}$$

where $\overline{P}$ is the average trade price in the observation period and $\hat{\gamma}_j$ is the (constrained) probability to trade at the jth spread (see Appendix A.1 for details on the calculation of $\hat{\gamma}_j$). Based on a histogram of the digits after the decimal point, we employ eight possible spread sizes, namely $s_1 = 0.001$, $s_2 = 0.01$, $s_3 = 0.05$, $s_4 = 0.1$, $s_5 = 0.125$, $s_6 = 0.25$, $s_7 = 0.5$, and $s_8 = 1$ and calculate P_EffectiveTick for each month and bond.

Zeros and FHT: Lesmond, Ogden, and Trzcinka (1999) use the proportion of zero return days as a measure of liquidity for equity markets. They argue that zero volume days (and thus zero return days) are more likely in less liquid stocks. We compute their measure on a monthly basis with T as the number of trading days in a month:

$$\text{P\_Zeros} = \frac{\#\text{ of zero return days}}{T}. \tag{2.18}$$

Fong, Holden, and Trzcinka (2010) establish a new effective spread proxy based on the Zeros measure.[12] In their framework, symmetric transaction costs of $S/2$ lead to observed returns of

$$R = \begin{cases} R^* + \frac{S}{2} & \text{if } R^* < -\frac{S}{2}, \\ 0 & \text{if } -\frac{S}{2} < R^* < \frac{S}{2}, \\ R^* - \frac{S}{2} & \text{if } \frac{S}{2} < R^*, \end{cases} \tag{2.19}$$

where R^* is the unobserved true value return which they assume to be normally distributed with mean zero and variance σ^2. Hence, they equate the theoretical probability of a zero return with its empirical frequency, measured via P_Zeros. Solving for the spread S, they get:

$$\text{P\_FHT} = S = 2 \cdot \sigma \cdot \Phi^{-1}\left(\frac{1 + \text{P\_Zeros}}{2}\right), \tag{2.20}$$

where Φ^{-1} is the inverse of the cumulative standard normal distribution. We compute a

[12] They find their measure to be simpler and perform better than LOT Mixed and Y-split, which also utilize zero returns. Therefore, we do not include these measures in our analysis.

bond's σ for each month and then calculate P_FHT.

High-Low Spread Estimator: Corwin and Schultz (2012) approximate bid-ask spreads for stocks based on high and low prices. They argue that daily high prices are likely to result from buy orders and low prices correspond to sell orders. Therefore, the ratio between those two reflects both the security's variance and the bid-ask spread as well. To separate these two components, the authors employ the high-low ratio on consecutive days. The variance component should be proportional to time, whereas the bid-ask spread should be constant. With this, their effective spread proxy is

$$\text{P\_HighLow} = \frac{2(e^{\alpha} - 1)}{1 + e^{\alpha}}, \tag{2.21}$$

where

$$\alpha = \frac{\sqrt{2\beta} - \sqrt{\beta}}{3 - 2\sqrt{2}} - \sqrt{\frac{\gamma}{3 - 2\sqrt{2}}}, \tag{2.22}$$

$$\beta = \sum_{j=0}^{1} \left(\ln \left(\frac{H_{t+j}}{L_{t+j}} \right) \right)^2, \tag{2.23}$$

$$\gamma = \left(\ln \left(\frac{H_{t,t+1}}{L_{t,t+1}} \right) \right)^2. \tag{2.24}$$

H_t (L_t) is the highest (lowest) price on day t and $H_{t,t+1}$ ($L_{t,t+1}$) is the highest (lowest) price on two consecutive days t and $t+1$. Again, we take the mean of the daily values in a month to get a monthly spread proxy for each bond.[13]

Quoted Spreads: We use quoted bid-ask spreads from the various Bloomberg pricing sources described in Section 2.2 as effective spread estimators. Let B_t and A_t be the bid and ask quotes for a given bond and day t. We get our daily relative spread estimate for the pricing sources s = BGN and s = CBBT as

$$\text{P\_Spread\_s} = \frac{A_t - B_t}{\frac{1}{2}(B_t + A_t)}. \tag{2.25}$$

Additionally, we calculate the daily means of the single dealers' bid and ask quotes and replace B_t and A_t, respectively, to get P_Spread_Mean. Monthly liquidity proxies are again obtained as the mean of daily bid-ask spreads.

Quote Dispersion: We apply the idea of our benchmark price dispersion measure to

[13]We adjust our data for the implicit assumptions made in the derivation of the measure (for details, see Corwin and Schultz, 2012, Sections 2.1 - 2.3). If the proxy is negative for a day, we follow Corwin and Schultz (2012) and set it to zero.

daily bid-ask quotes. Garbade and Silber (1976) show that it is reasonable to approximate the dispersion of trade prices via the dispersion of quotes. We measure the daily quote dispersion for a given bond as follows:

$$\text{P\_QuoteDispersion} = 2 \cdot \sqrt{\frac{1}{2N} \sum_{i=1}^{N} \left(\left(\frac{B_i - m}{m} \right)^2 + \left(\frac{A_i - m}{m} \right)^2 \right)}, \tag{2.26}$$

where N is the number of available pricing sources and B_i and A_i are the Bloomberg bid and ask quotes of dealer i. In contrast to the above B_PriceDispersion, we do not use Markit data for the consensus market valuation m, but employ the Bloomberg BGN composite price to fully separate our high- and low-frequency input data sets. Furthermore, we use the daily average mid quote, calculated from all single dealers' bid and ask quotes available, as a proxy for the market valuation m to get an additional effective spread estimator P_QDmid.[14] Again, our monthly liquidity proxies correspond to the means of daily values for P_QuoteDispersion and P_QDmid.

2.3.2.2 Price-Impact Proxies

Amihud: As Amihud (2002) originally developed his measure for end of day data, we include it as a price impact proxy in our analysis:

$$\text{P\_Amihud} = \frac{1}{N} \sum_{t=1}^{N} \frac{|r_t|}{Q_t}, \tag{2.27}$$

where N is the number of positive-volume days in a given month, r_t the return and Q_t the traded dollar volume on day t, respectively. As volume data is only available in Bloomberg since March 29, 2005, all price impact proxies are calculated from April 2005 onwards.

Extended Amihud: Goyenko, Holden, and Trzcinka (2009) derive extended Amihud measures which for every relative effective spread proxy sp and average daily dollar volume $\overline{Q}$ in the period under observation are defined as

$$\text{P\_PI\_sp} = \frac{sp}{\overline{Q}}. \tag{2.28}$$

[14] Garbade (1978) shows that the market valuation and the average mid-quote are in fact different from each other, because dealers account for inventory. To account for the degree of freedom we lose when calculating mid prices from dealer quotes, we use $2N - 1$ instead of $2N$ in the denominator of Equation (2.26).

We make use of their findings and compute monthly price impact liquidity measures for our Roll, Gibbs, Effective Tick (sp = ET), FHT, High-Low (sp = HL), Quoted Spreads, and Quote Dispersion (sp = QD and sp = QDmid) proxies.

Pastor and Stambaugh: Pastor and Stambaugh (2003) develop a measure for price impact based on price reversals for the equity market. It is given by the estimator for γ in the following regression:

$$r^e_{t+1} = \theta + \phi \cdot r_t + \gamma \cdot \text{sign}(r^e_t) \cdot Q_t + \epsilon_t, \tag{2.29}$$

where r^e_t is the asset's excess return over a market index, r_t is the asset's return and Q_t is the traded dollar volume on day t. For our market index, we use the same method as for the Gibbs proxy to assign Bloomberg's Investment Grade or High Yield index, respectively. γ should be negative and a larger price impact comes along with a larger absolute value. As all of our liquidity measures assign larger (positive) values to more illiquid bonds, we define P_PastorStambaugh $= -\gamma$ and expect it to be positively correlated with the other measures in this study.

2.4 Results

We first present descriptive statistics for each measure in Section 2.4.1, assess the consistency of our liquidity benchmarks by comparing them with each other in Section 2.4.2, and then compare our transaction cost and price impact proxies with the respective benchmarks in Sections 2.4.3 and 2.4.4. Tests based on time series and average cross sectional correlations allow us to analyze whether our proxies are useful for asset pricing. For our transaction cost proxies, we additionally run two tests analyzing to what extent the proxies are able to capture the correct scale of the benchmarks. In various robustness checks, we assess the consistency of our results for different liquidity portfolios and subperiods. Since in the literature, proxies like Amihud's (2002) price impact measure or the Pastor and Stambaugh (2003) measure are often used to proxy for liquidity in general, we compare these measures not only to price impact, but also to transaction cost benchmarks.

2.4.1 Descriptive Statistics

Table 2.2 shows descriptive summary statistics for our monthly high-frequency and low-frequency liquidity measures. Benchmarks depending on a buy/sell indicator are calculated for the last 47 sample months, whereas the remaining ones span our complete observation period of 96 months. For our effective spread high-frequency benchmarks in Panel A, with the exception of the price dispersion measure, we get average effective spread estimates between 0.93% and 1.23%. This means that a roundtrip trade for bonds of $100,000 on average leads to transaction costs between $930 and $1,230. B_Roundtrip and B_Roll yield very similar results in all categories. Although B_AdjustedSchultz and B_EHP are calculated only from November 1, 2008 onwards, they are on average of comparable size compared to the other measures. The different behavior of the price dispersion benchmark compared to the other transaction cost benchmarks is obvious as it produces by far the largest spread estimates. This result confirms findings of Jankowitsch, Nashikkar, and Subrahmanyam (2011) that price dispersion contains additional cost components over and above direct transaction costs. For this reason, the price dispersion benchmark essentially constitutes a separate category (in addition to pure transaction cost and price impact benchmarks). Nevertheless, for the sake of brevity, we discuss this measure together with the other transaction cost estimators.

Looking at our low-frequency spread proxies, the best measure regarding the magnitude of our benchmarks seems to be P_HighLow with an average value of 0.9738%, followed by P_Gibbs. Proxies calculated from executable quotes (P_Spread_CBBT, P_Spread_Mean, and both quote dispersion measures) are generally larger than effective spread benchmarks since many trades occur inside the bid-ask spread (see, e.g., Petersen and Fialkowski, 1994). The average P_Spread_BGN amounts to less than a third of its CBBT counterpart, indicating that the BGN does not give the magnitude of the bid-ask spread correctly. Bloomberg seems to have realized this shortcoming and presumably changed its methodology in March 2011 when bid-ask spreads for the BGN measure increased sharply. Because of this structural break, we restrict the observation period for P_Spread_BGN and P_PI_Spread_BGN to the time before February 28, 2011. P_EffectiveTick shows a very small mean of about 0.09% which is due to the fact that although price clustering is clearly present in our data, the majority of trades does take place at an 1/1000 price tick, leading to an assumed spread of $0.001 for these trades.[15] Also P_FHT does not seem to accurately capture the

[15] In bond markets, price clustering takes place both at the price and the yield level. However, the effective tick measure only captures clustering at the price level which might be one of the reasons this measure does not grasp the magnitude of bid-ask spreads correctly.

Table 2.2: **Descriptive statistics for benchmark and proxy liquidity measures**
Benchmark liquidity measures are calculated from high-frequency transaction data reported to TRACE from October 1, 2004 to September 30, 2012. Low-frequency proxies are computed based on daily price, volume, and quote data provided by Bloomberg. We calculate monthly measures as described in Section 2.3. B_AdjustedSchultz, B_EHP, and B_Lambda are only available since November 2008, resulting in 47 observation months. The remaining benchmarks are calculated for all 96 sample months. P_Spread_CBBT is calculated for the last 95 sample months. P_Amihud, P_PastorStambaugh, and all other price impact proxies, except P_PI_Spread_BGN, start in April 2005 and span 90 observation months. Both measures linked to the Bloomberg BGN quote are only calculated until February 2011, resulting in 77 and 71 observation months for P_Spread_BGN and P_PI_Spread_BGN, respectively. All other low-frequency proxies are calculated for each of the 96 months in our observation period.

	Unit	Mean	Std.dev.	$Q_{0.05}$	$Q_{0.25}$	Median	$Q_{0.75}$	$Q_{0.95}$	N
Panel A: Effective spread measures									
Benchmarks									
B_Roundtrip	%	1.2299	0.8966	0.2478	0.5999	1.0056	1.6124	2.9893	164674
B_IQR	%	0.9717	0.8413	0.1697	0.3918	0.7049	1.2749	2.6912	165205
B_Roll	%	1.1882	0.9512	0.2362	0.5636	0.942	1.5388	2.9914	165500
B_AdjustedSchultz	%	1.2280	1.2005	0.156	0.4723	0.8836	1.5971	3.3972	72468
B_EHP	%	0.9316	0.8047	0.1329	0.3734	0.688	1.2405	2.5665	72468
B_PriceDispersion	%	1.8064	3.0239	0.2681	0.6244	1.084	1.9473	5.3214	125209
Proxies									
P_Roll	%	1.7686	1.6676	0.1339	0.6799	1.3102	2.3325	4.9447	163304
P_Gibbs	%	1.4529	1.0405	0.2552	0.6714	1.2016	2.0006	3.4130	163293
P_EffectiveTick	%	0.0945	0.1656	0.001	0.005	0.0231	0.1320	0.3772	163304
P_Zeros	%	11.876	13.410	0	0	8.6957	19.047	40	163304
P_FHT	%	0.4497	1.0503	0	0	0.1731	0.5035	1.7416	163298
P_HighLow	%	0.9738	1.0781	0.1201	0.3183	0.6372	1.2729	2.8294	163304
P_Spread_BGN	%	0.4550	0.2834	0.0918	0.2495	0.4049	0.5931	0.9869	111416
P_Spread_CBBT	%	1.7769	2.2315	0.2726	0.6333	1.2016	2.1293	4.9934	103374
P_Spread_Mean	%	1.8585	2.1100	0.2404	0.704	1.337	2.2519	5.0584	133228
P_QDmid	%	2.6350	2.9794	0.3339	0.9931	1.8845	3.2069	7.1887	133228
P_QuoteDispersion	%	2.4660	2.7381	0.3331	0.9538	1.7875	3.0058	6.6539	122652
Panel B: Price impact measures									
Benchmarks									
B_Amihud	10^{-6}	0.8308	2.0311	0.0389	0.1851	0.408	0.8752	2.7558	165616
B_Lambda	10^{-6}	10.980	35.557	−0.0001	1.0944	2.8767	7.7279	50.758	97990
Proxies									
P_Amihud	10^{-6}	0.2207	1.0308	0.0017	0.0101	0.0399	0.1552	0.8775	156216
P_PI_Roll	10^{-6}	0.0511	0.5089	0.0002	0.0021	0.006	0.0189	0.2059	156216
P_PI_Gibbs	10^{-6}	0.0365	0.3352	0.0006	0.0021	0.0056	0.0164	0.1662	156205
P_PI_ET	10^{-6}	0.0029	0.0312	0	0	0.0001	0.0009	0.0095	156216
P_PI_FHT	10^{-6}	0.0257	0.3703	0	0	0.0007	0.0041	0.0684	156210
P_PI_HL	10^{-6}	0.0285	0.3053	0.0003	0.0011	0.0031	0.0093	0.1069	156216
P_PI_Spread_BGN	10^{-6}	0.0089	0.0332	0.0002	0.0007	0.0018	0.0048	0.0370	104257
P_PI_Spread_CBBT	10^{-6}	0.0168	0.0914	0.0006	0.0019	0.0048	0.0125	0.0585	100126
P_PI_Spread_Mean	10^{-6}	0.0286	0.1731	0.0006	0.0023	0.0059	0.0161	0.0949	128516
P_PI_QD	10^{-6}	0.0322	0.1864	0.0008	0.003	0.0076	0.0195	0.1028	118176
P_PI_QDmid	10^{-6}	0.0402	0.2393	0.0008	0.0032	0.0084	0.0229	0.1350	128516
P_PastorStambaugh	10^{-6}	0.0009	0.1454	−0.0076	−0.0005	0	0.0004	0.0080	156210

scale of the effective spread when compared to the benchmark measures.

Panel B compares the different price impact measures. As discussed in the introduction, measuring price impact in the U.S. corporate bond market poses a conceptual problem. Most importantly, the fact that transaction costs are smaller for larger trades (see, e.g., Edwards, Harris, and Piwowar, 2007), implies that a static definition of price impact as the 'first derivative of the effective spread with respect to order size' (see, e.g., Goyenko, Holden, and Trzcinka, 2009) leads to a negative price impact (which is precluded in the definition of most of the price impact measures). However, price impact measures are used in the literature and provide additional value as they depend on both effective transaction costs and trading volume. Therefore, we include them in our correlation analyses, but refrain from interpreting their magnitudes.

2.4.2 High-Frequency Benchmarks Correlation Analysis

Figure 2.1 presents the evolution of our high-frequency bid-ask spread benchmarks. Following Goyenko, Holden, and Trzcinka (2009), we compute the monthly average for each measure as the equally weighted mean across all bonds. We find a high level of co-movement between all measures. With the exception of price dispersion, which besides measuring transaction costs also captures inventory risk and search costs (see Jankowitsch, Nashikkar, and Subrahmanyam, 2011), all benchmarks are always located within a narrow range. Consistently, we find lowest levels of liquidity in the months that followed the default of Lehman Brothers in September 2008.

Next, we determine time series correlations based on the monthly means and provide our results in Panel A of Table 2.3. Regardless of whether effective spread measures, price impact measures, or a combination of both is considered, all of our benchmarks show high, positive, and significant pairwise time series correlations.[16] The smallest value is 0.8739 for B_PriceDispersion and B_Roundtrip and the highest correlation of 0.9978 comes from B_AdjustedSchultz and B_EHP.

Panel B shows average cross sectional correlations. We determine cross sectional correlations for each month across all bonds, transform them using Fisher's Z, compute the mean of the transformed values and re-transform the results (see, e.g., Fama and MacBeth,

[16] In this study, we test whether time series correlations ρ are significantly different from zero at the 5% level, using the test statistic $t = \rho \cdot \sqrt{\frac{n-2}{1-\rho^2}}$ (see, e.g., Swinscow, 1997) with n as the sample size. Under the null hypothesis, t is t-distributed with $n-2$ degrees of freedom.

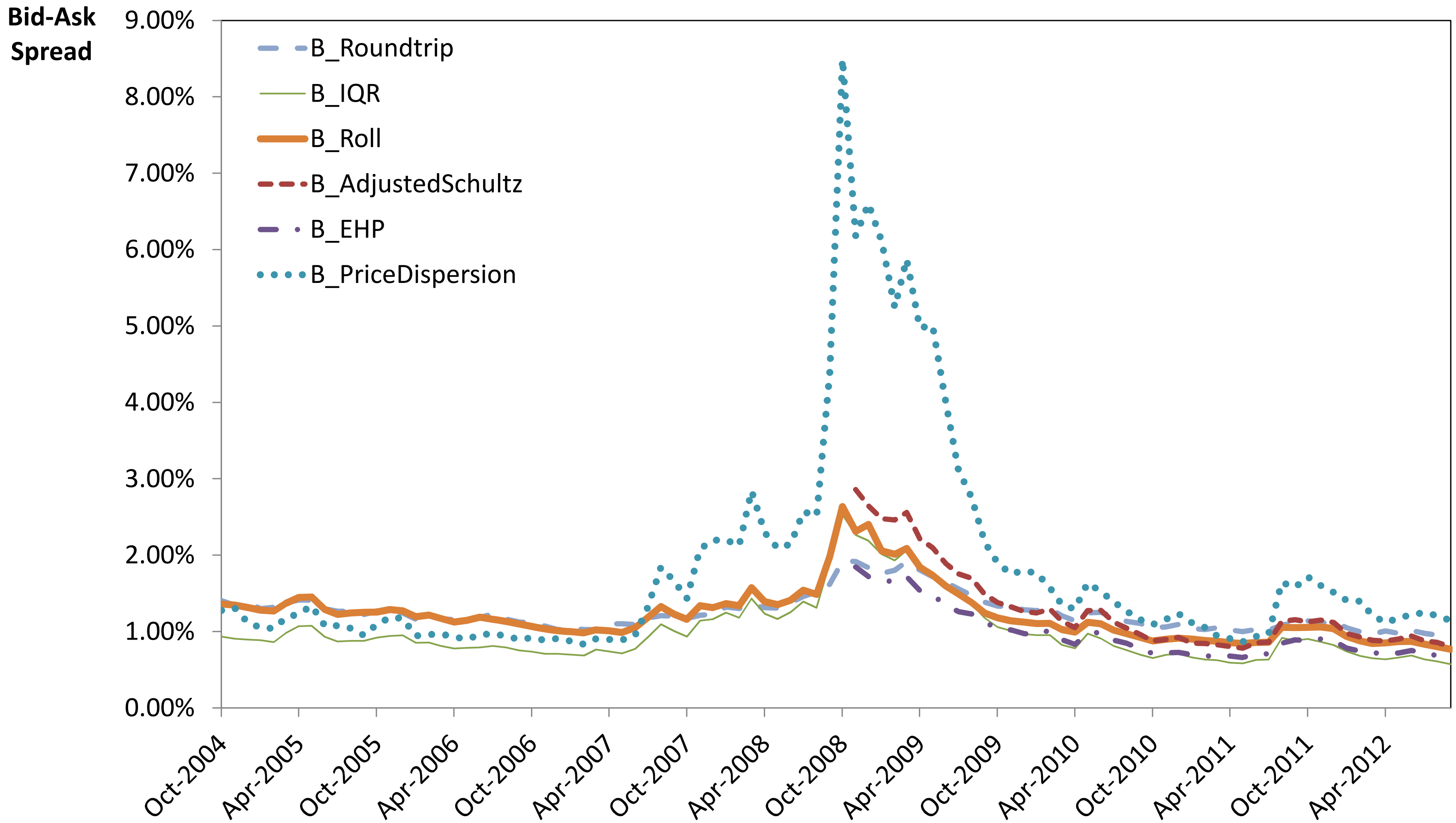

Figure 2.1: **Time series of monthly effective spread benchmarks**

Monthly high-frequency benchmarks are calculated from intraday TRACE data from October 1, 2004 to September 30, 2012. All measures are described in Section 2.3. B_AdjustedSchultz and B_EHP are calculated since November 2008, resulting in 47 observation months. The remaining benchmarks are calculated for all 96 sample months.

Table 2.3: **Monthly benchmark comparison**
Monthly high-frequency benchmarks are calculated from intraday TRACE data from October 1, 2004 to September 30, 2012. B_AdjustedSchultz, B_EHP, and B_Lambda are calculated since November 2008, resulting in 47 observation months. The remaining benchmarks are calculated for all 96 sample months. Bold numbers are statistically significant at the 5% level.

	B_Round-trip	B_IQR	B_Roll	B_Adjusted-Schultz	B_EHP	B_Price-Dispersion	B_Amihud
Panel A: Time series correlations							
B_IQR	**0.9494**						
B_Roll	**0.9533**	**0.9689**					
B_AdjustedSchultz	**0.9823**	**0.9975**	**0.9937**				
B_EHP	**0.9887**	**0.9951**	**0.9874**	**0.9978**			
B_PriceDispersion	**0.8739**	**0.9595**	**0.8991**	**0.9811**	**0.9721**		
B_Amihud	**0.9298**	**0.9652**	**0.9638**	**0.9773**	**0.9667**	**0.9392**	
B_Lambda	**0.935**	**0.9621**	**0.9729**	**0.9709**	**0.9597**	**0.9484**	**0.9775**
Panel B: Average cross sectional correlations							
B_IQR	**0.7465**						
B_Roll	**0.7775**	**0.8247**					
B_AdjustedSchultz	**0.7209**	**0.8186**	**0.7751**				
B_EHP	**0.7875**	**0.7941**	**0.8215**	**0.9103**			
B_PriceDispersion	**0.5627**	**0.6815**	**0.6337**	**0.6277**	**0.5814**		
B_Amihud	**0.5242**	**0.5832**	**0.5386**	**0.5412**	**0.4896**	**0.4595**	
B_Lambda	**0.5232**	**0.5031**	**0.5267**	**0.5721**	**0.5801**	**0.5201**	**0.4463**

1973). As in Goyenko, Holden, and Trzcinka (2009), average cross sectional correlations of our benchmark measures do not reach the level of Panel A, but they are always positive and significantly different from zero.[17] Also, they have a wider bandwidth than the time series correlations, ranging from 0.4463 to 0.9103. The highest value comes again from our adjusted versions of the Schultz (2001) and Edwards, Harris, and Piwowar (2007) measures, which is not surprising considering their similarities in design. We find that all correlations within the group of spread benchmarks are higher than the correlation of the two price impact benchmarks.[18]

[17] We test average cross sectional correlations for significance at the 5% level by running a t-test with a Newey-West correction of four lags (see, e.g., Goyenko, Holden, and Trzcinka, 2009).

[18] Friewald, Jankowitsch, and Subrahmanyam (2012) also conduct a correlation analysis for three of our high-frequency benchmarks. In contrast to our findings, their correlations are much lower, e.g., only 0.20 for the correlation between Roll and price dispersion. Possible reasons for the different results are (i) that the authors compute liquidity measures on a weekly rather than monthly basis leading to more noise, (ii) that they include also very illiquid bonds in their analysis with presumably very little trade data leading to large outliers, and (iii) that they compute correlations on a bond-week level, whereas we compute time series correlations aggregated over all bonds and average cross sectional correlations. However, if we calculate correlations on an individual bond-month level, we get, e.g., 0.51 for the Roll vs. price dispersion measure compared to the 0.20 they find.

2.4.3 Monthly Spread Proxy Results

2.4.3.1 Correlation Analysis and Prediction Errors

Figure 2.2 presents the evolution of our monthly low-frequency spread estimates together with one of our benchmark measures B_Roll. In the financial crisis, our quote based measures increase most strongly, which confirms results of Petersen and Fialkowski (1994) that in stock markets, effective spreads increase only up to 22% of the increase of quoted spreads. It seems that in times of stress, dealers protect themselves against further deteriorating conditions by quoting extremely wide spreads that do not represent actual transaction costs. The evolution of P_FHT closely resembles our quoted bid-ask-spread measures, indicating that the use of zero return days leads to a good approximation of quoted spreads.

Table 2.4 shows the main findings of our analysis regarding the question which daily spread proxies are best suited to approximate intraday effective bid-ask spreads. We also include P_Zeros, P_Amihud, and P_PastorStambaugh as these measures are often used to capture liquidity in general. Panel A reports time series correlations of each high-frequency spread benchmark with all low-frequency spread proxies and therefore analyzes the ability of our proxies to capture liquidity dynamics over time. Drawn through boxes mark the best-performing proxy for each benchmark and values significantly different from zero are written in boldface. Dashed boxes identify correlations which are insignificantly different from the best correlation in the same row. We test whether two measures are significantly different from each other at the 5% level by applying Steiger's Z test. Again, we find very high levels of correlation for all benchmarks and spread estimates and also for P_Amihud. P_Zeros and the Pastor and Stambaugh (2003) measures are clearly outperformed by the other proxies. The highest correlations with our high-frequency benchmarks are mostly found for P_Roll and P_Gibbs which are both based on the auto-covariance of daily trade prices. The high-low measure also performs quite well. From the quoted spread proxies, those based on executable quotes (P_Spread_CBBT) and on the average of all dealers' quotes (P_Spread_Mean) clearly dominate the Bloomberg Generic Quote (P_Spread_BGN).

Panel B provides our average cross sectional correlations results to analyze the ability of our proxies to capture liquidity differences between bonds. We test for significant differences between the proxies by computing the differences in monthly cross sectional correlations and t-testing their mean.[19] Again, the level of correlations is lower than for

[19]For calculating the mean, Fisher's Z is applied. Standard errors are computed with a Newey-West

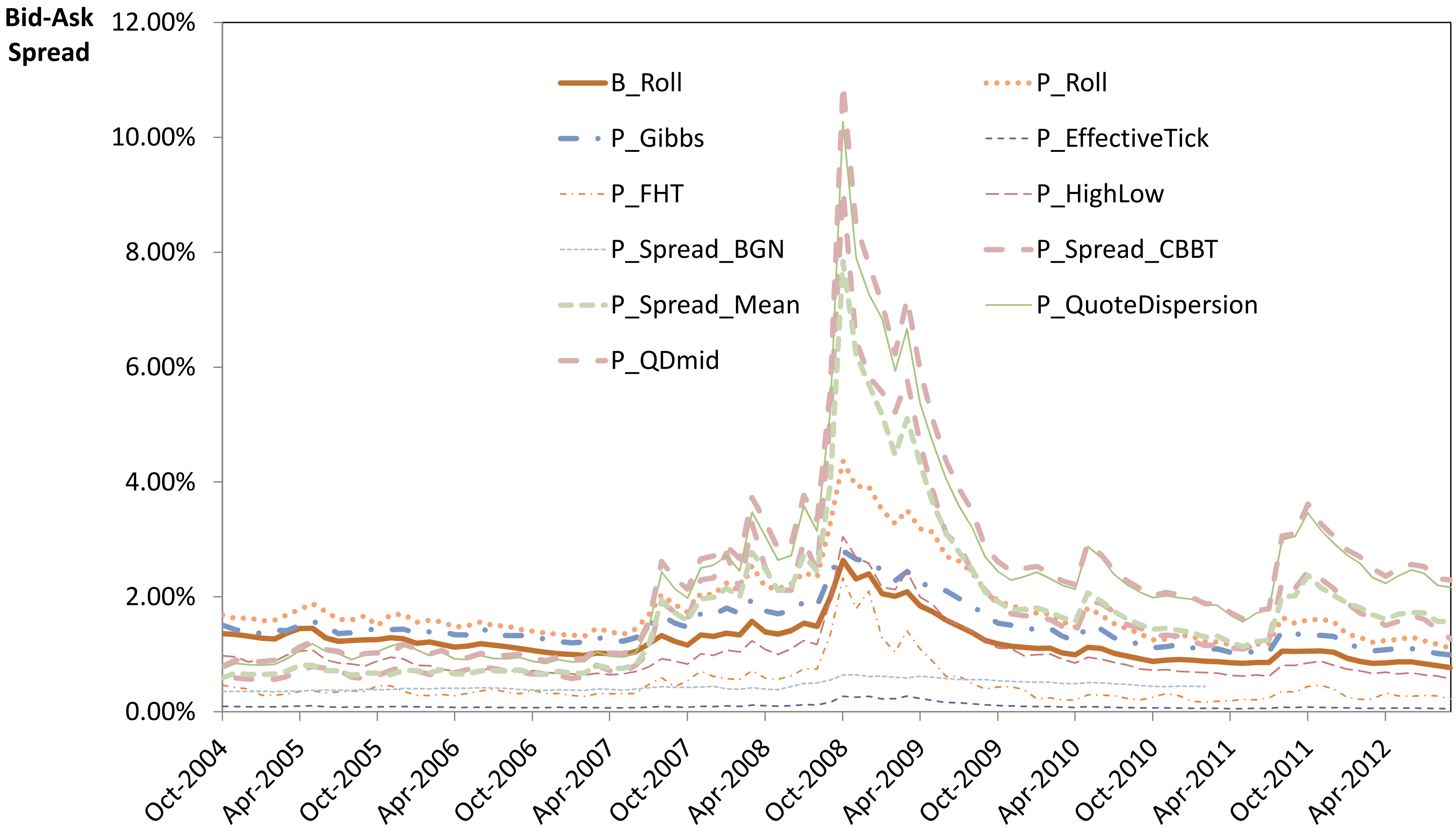

Figure 2.2: **Time series of monthly spread proxies**

Monthly low-frequency spread proxies are calculated from daily Bloomberg price, volume, and quote data from October 1, 2004 to September 30, 2012. B_Roll is computed based on intraday TRACE data. All measures are described in Section 2.3. P_Spread_BGN is only computed until February 2011 and spans the first 77 sample months. P_Spread_CBBT is not available for the first sample month, resulting in 95 observation months. The remaining proxies and B_Roll are calculated for all 96 months in the observation period.

Table 2.4: **Monthly spread proxies compared to spread benchmarks**

Monthly high-frequency benchmarks are calculated from intraday TRACE data from October 1, 2004 to September 30, 2012. Monthly low-frequency proxies are computed based on daily price, volume, and quote data provided by Bloomberg. All measures are described in Section 2.3. B_AdjustedSchultz and B_EHP are calculated since November 2008, resulting in 47 observation months. The remaining benchmarks are calculated for all 96 sample months. P_Spread_BGN is only computed until February 2011 and spans the first 77 sample months. P_Spread_CBBT is not available for the first sample month, resulting in 95 observation months. Both price impact measures start in April 2005 spanning the last 90 sample months. The remaining proxies are calculated for all 96 months in the observation period. Bold numbers are statistically significant at the 5% level. Drawn through boxes give the best value in a row and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	P_Roll	P_Gibbs	P_Effective-Tick	P_Zeros	P_FHT	P_HighLow	P_Spread-BGN	P_Spread-CBBT	P_Spread-Mean	P_Quote-Dispersion	P_QDmid	P_Amihud	P_Pastor-Stambaugh
Panel A: Time series correlations													
B_Roundtrip	**0.943**	**0.9486**	**0.9429**	**0.278**	**0.8356**	**0.938**	**0.7241**	**0.7768**	**0.7551**	**0.7311**	**0.7405**	**0.8986**	**0.3351**
B_IQR	**0.9978**	**0.9896**	**0.9682**	**0.4135**	**0.9331**	**0.9782**	**0.7607**	**0.9034**	**0.8851**	**0.8652**	**0.8729**	**0.9493**	**0.3897**
B_Roll	**0.9657**	**0.9708**	**0.9405**	**0.4525**	**0.9269**	**0.9471**	**0.6207**	**0.8159**	**0.7839**	**0.7609**	**0.769**	**0.9662**	**0.4358**
B_AdjustedSchultz	**0.9961**	**0.9936**	**0.9879**	**0.3906**	**0.9281**	**0.994**	**0.9512**	**0.9652**	**0.9665**	**0.954**	**0.9586**	**0.9571**	**0.3189**
B_EHP	**0.9934**	**0.9944**	**0.9828**	**0.3598**	**0.908**	**0.9884**	**0.9646**	**0.9519**	**0.952**	**0.9385**	**0.9437**	**0.9401**	**0.2973**
B_PriceDispersion	**0.9625**	**0.9222**	**0.9614**	**0.3761**	**0.9415**	**0.9769**	**0.8201**	**0.9673**	**0.9626**	**0.9514**	**0.9562**	**0.9111**	**0.435**
Panel B: Average cross sectional correlations													
B_Roundtrip	**0.567**	**0.5422**	**0.33**	0.0513	**0.3578**	**0.6979**	**0.4357**	**0.4661**	**0.4969**	**0.4636**	**0.4681**	**0.3027**	0.0088
B_IQR	**0.7019**	**0.6731**	**0.4255**	**0.0983**	**0.4656**	**0.7571**	**0.5349**	**0.6483**	**0.6385**	**0.5883**	**0.5977**	**0.3845**	0.0081
B_Roll	**0.6738**	**0.6426**	**0.3635**	**0.0832**	**0.4416**	**0.7273**	**0.5187**	**0.5883**	**0.5948**	**0.5533**	**0.5573**	**0.3622**	0.0075
B_AdjustedSchultz	**0.6443**	**0.6039**	**0.3236**	−0.01	**0.2992**	**0.7294**	**0.4201**	**0.5529**	**0.5361**	**0.5229**	**0.5179**	**0.2738**	**−0.0249**
B_EHP	**0.6053**	**0.5766**	**0.2548**	−0.0584	**0.2198**	**0.7589**	**0.3729**	**0.4918**	**0.4904**	**0.4892**	**0.4812**	**0.1951**	−0.0073
B_PriceDispersion	**0.5474**	**0.4715**	**0.3429**	**0.0845**	**0.3802**	**0.555**	**0.429**	**0.5572**	**0.5484**	**0.5043**	**0.5037**	**0.309**	0.0069
Panel C: Mean bias													
B_Roundtrip	**0.0054**	**0.0022**	**−0.0113**		**−0.0078**	**−0.0025**	**−0.0078**	**0.0069**	**0.0071**	**0.0133**	**0.0149**		
B_IQR	**0.008**	**0.0048**	**−0.0088**		**−0.0052**	0.0000	**−0.0056**	**0.01**	**0.0098**	**0.016**	**0.0175**		
B_Roll	**0.0058**	**0.0027**	**−0.0109**		**−0.0074**	**−0.0021**	**−0.0077**	**0.0077**	**0.0077**	**0.0139**	**0.0155**		
B_AdjustedSchultz	**0.0035**	**0.0008**	**−0.0114**		**−0.009**	**−0.0029**	**−0.0092**	**0.0083**	**0.0085**	**0.0163**	**0.0173**		
B_EHP	**0.0065**	**0.0037**	**−0.0085**		**−0.006**	**0.0001**	**−0.0055**	**0.0107**	**0.0113**	**0.0191**	**0.0201**		
B_PriceDispersion	**−0.0021**	**−0.0046**	**−0.0171**		**−0.0144**	**−0.0089**	**−0.0146**	**0.0025**	**0.0008**	**0.0076**	**0.0084**		
Panel D: Root mean squared error (RMSE)													
B_Roundtrip	0.0149	0.0093	0.0141		0.0137	0.0089	0.0109	0.0216	0.0202	0.0283	0.031		
B_IQR	0.0146	0.0088	0.0117		0.0111	**0.0072**	0.0093	0.0211	0.0195	0.0282	0.0308		
B_Roll	0.0139	**0.0084**	0.0138		0.0125	**0.0083**	0.0111	0.0207	0.0193	0.0276	0.0303		
B_AdjustedSchultz	0.0121	**0.0093**	0.016		0.0146	**0.0093**	0.0155	0.0195	0.0186	0.0272	0.0296		
B_EHP	0.0135	0.0085	0.0114		0.0114	**0.0077**	0.0096	0.0218	0.0211	0.0302	0.0327		
B_PriceDispersion	**0.0254**	**0.0269**	0.0335		0.0302	**0.0269**	0.0329	**0.0196**	**0.023**	0.025	0.0278		

the time series case,[20] but still most figures are significantly different from zero. Medium and high values are obtained for the Roll, Gibbs, High-Low, and quote based proxies, whereas P_EffectiveTick, P_Zeros, P_FHT, P_Amihud, and P_PastorStambaugh do not seem to capture cross sectional variation well. P_HighLow clearly dominates this analysis, showing the highest correlations for all benchmarks except B_PriceDispersion for which it is insignificantly different from the winning proxy P_CBBT.

To examine the capability of our proxies to capture the correct magnitude of our benchmarks, we report two metrics for the prediction errors: mean bias in Panel C and root mean squared error (RMSE) in Panel D. As before, we cannot compare the scales of our spread estimators with P_Zeros, P_Amihud, and P_PastorStambaugh.

We find mean biases for all benchmarks ranging from 0.0000 to 0.0201 in absolute terms. Especially the Roll, Gibbs, FHT, High-Low, and quoted spread proxies yield small biases, but with the exception of one mean bias, all are significantly different from zero.[21] P_Gibbs and P_HighLow show the smallest errors for all benchmark measures except B_PriceDispersion which is best captured by the average of dealers' quotes P_Spread_Mean. Panel D paints a similar picture when it comes to RMSEs with P_Gibbs and P_HighLow again winning all benchmarks except B_PriceDispersion. We also test whether a proxy significantly better captures the scale of a benchmark compared to using the benchmark's sample mean (values written in boldface). We find that only the best proxies P_HighLow and P_Gibbs outperform the mean when it comes to capturing the scale of many benchmarks.[22]

2.4.3.2 Robustness Checks

We perform robustness checks to assess the performance of our measures in different market conditions and to explore their consistency for more liquid and illiquid bonds. Additionally, we analyze their behavior when calculating them on an annual instead of a

correction of four lags (see, e.g., Goyenko, Holden, and Trzcinka, 2009).

[20]Goyenko, Holden, and Trzcinka (2009) note that part of this difference may result from a diversification effect when forming portfolios in the time series analysis.

[21]We test the mean bias for significance with a t-test and for significant differences between the measures by running a paired t-test.

[22]For this analysis, we compute a benchmark's sample mean over all bonds and the whole observation period when the respective proxy is available. We then use the U-Statistic from Theil (1966) to test whether the RMSE is significantly smaller when using the proxy to predict the benchmark compared to using the benchmark's sample mean. To test for significant differences between the RMSEs for each proxy, we employ a paired t-test (see, e.g., Goyenko, Holden, and Trzcinka, 2009).

monthly basis.[23]

Table 2.5 shows time series and cross sectional correlations for three subperiods, spanning around the financial crisis that started in 2007. Our pre-crisis period begins on October 1, 2004 and ends at March 31, 2007. We follow Dick-Nielsen, Feldhütter, and Lando (2012) and set the beginning of the financial crisis to April 1, 2007. We define its end as December 31, 2009. Hence, the final period is January 1, 2010 to September 30, 2012. Our modified versions of the Schultz (2001) and Edwards, Harris, and Piwowar (2007) measures can only be computed after November 1, 2008. Hence, the pre-crisis period is missing for them. Time series correlations in Panel A for nearly all benchmark-proxy combinations are lowest before and highest during the crisis.

Whereas the price based spread proxies only show low variations between the periods, measures based on quoted spreads perform poorly in the pre-crisis period, even yielding some negative correlations. Thus, the latter do not respond consistently in different market situations.[24] Measures based on zero returns (P_Zeros and P_FHT) also behave badly pre- and post-crisis as the number of zero returns is only positively correlated during the crisis period with all benchmarks. The best measures in our main analysis perform very consistently during all periods. So the two measures based on the auto-covariance of daily prices (P_Roll and P_Gibbs) as well as P_HighLow almost always produce the highest correlations.

Panel B provides the results for the subperiods with regard to average cross sectional correlations. Here, correlations do not vary as much between the periods as in Panel A and there is no clear direction of variation. Also the magnitudes are very similar to those in Table 2.4. Again, P_HighLow is the clear winner for all benchmarks except the price dispersion measures. In the last two subperiods, price dispersion is best captured by the quoted CBBT spread confirming results from our main analysis.

In the spirit of Goyenko, Holden, and Trzcinka (2009), Table 2.6 presents two additional robustness checks and assesses the consistency of our measures for more liquid and illiquid bonds. In Panel A, we form portfolios of equal size ranked by liquidity, which is measured

[23] In additional robustness checks, we show that results do not change qualitatively when weighting observations with the number of trades in TRACE in a month. Moreover, we experiment with the methodology how to handle the situation when a measure cannot be calculated for a bond in a given month due to missing data. In our main analyses, we ignore such a missing observation when averaging over all bonds. Alternative ways to handle such a situation, like the use of a default value or the confinement of our analyses to bonds where we can always calculate all benchmarks and proxies, do not change our findings. All results are available upon request from the authors.

[24] Another reason for the bad performance of our quoted spread proxies could be a possible change in Bloomberg's methodology in computing and distributing bid and ask prices.

Table 2.5: **Subperiod analysis: spread proxies**

Monthly high-frequency benchmarks are calculated from intraday TRACE data from October 1, 2004 to September 30, 2012. Monthly low-frequency proxies are computed based on daily price, volume, and quote data provided by Bloomberg. All measures are described in Section 2.3. The pre-crisis period is October 1, 2004 to March 31, 2007. The crisis period is defined as April 1, 2007 until December 31, 2009. The post-crisis period is January 1, 2010 until September 30, 2012. B_AdjustedSchultz and B_EHP are calculated since November 2008. The remaining benchmarks are calculated for all 96 sample months. P_Spread_BGN is only computed for the pre-crisis and crisis period. P_Spread_CBBT is only available since November 2004. Both price impact measures start in April 2005. The remaining proxies are calculated for all 96 months in the observation period. Bold numbers are statistically significant at the 5% level. Drawn through boxes give the best value in a row and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	P_Roll	P_Gibbs	P_Effective-Tick	P_Zeros	P_FHT	P_HighLow	P_Spread-BGN	P_Spread-CBBT	P_Spread-Mean	P_Quote-Dispersion	P_QDmid	P_Amihud	P_Pastor-Stambaugh
Panel A: Time series correlations													
B_Roundtrip													
Pre-crisis	**0.8725**	**0.8575**	**0.8573**	−0.1878	**0.4995**	**0.9301**	**−0.5111**	−0.0542	−0.0197	0.0793	0.0745	**0.8295**	0.0281
Crisis	**0.9579**	**0.949**	**0.976**	0.0152	**0.8373**	**0.9642**	**0.8929**	**0.914**	**0.9336**	**0.934**	**0.9374**	**0.8675**	0.3114
Post-crisis	**0.9008**	**0.9056**	**0.922**	**−0.4515**	0.1803	**0.9539**		**0.3872**	**0.4305**	**0.3558**	0.3303	**0.5457**	0.0164
B_IQR													
Pre-crisis	**0.9466**	**0.8875**	**0.9348**	−0.2208	**0.4502**	**0.9669**	−0.3495	0.1732	0.238	**0.4423**	**0.4197**	**0.8064**	0.0139
Crisis	**0.9986**	**0.9952**	**0.9615**	0.2105	**0.9266**	**0.9837**	**0.814**	**0.9796**	**0.9827**	**0.9824**	**0.9838**	**0.9322**	**0.3743**
Post-crisis	**0.987**	**0.9886**	**0.9663**	−0.2064	**0.4927**	**0.9411**		**0.7403**	**0.7601**	**0.72**	**0.6979**	**0.7898**	0.0238
B_Roll													
Pre-crisis	**0.9376**	**0.9077**	**0.9182**	−0.2324	**0.4722**	**0.9621**	**−0.4207**	0.04	0.0881	0.2487	0.2326	**0.8415**	0.0399
Crisis	**0.9891**	**0.9829**	**0.9442**	0.3178	**0.9657**	**0.9732**	**0.7564**	**0.982**	**0.9841**	**0.9834**	**0.9835**	**0.9644**	**0.4509**
Post-crisis	**0.9843**	**0.9884**	**0.9469**	−0.2665	**0.4409**	**0.9548**		**0.6736**	**0.6787**	**0.6402**	**0.6127**	**0.7321**	0.016
B_AdjustedSchultz													
Crisis	**0.9899**	**0.9862**	**0.9756**	**0.6618**	**0.9232**	**0.9932**	**0.9345**	**0.993**	**0.9899**	**0.9911**	**0.9924**	**0.9433**	0.2878
Post-crisis	**0.9797**	**0.975**	**0.9632**	−0.2265	**0.4443**	**0.9536**		**0.7031**	**0.7393**	**0.6829**	**0.6645**	**0.7622**	0.0174
B_EHP													
Crisis	**0.9884**	**0.987**	**0.978**	**0.6272**	**0.9054**	**0.9889**	**0.9399**	**0.9907**	**0.9841**	**0.9859**	**0.9877**	**0.9263**	0.2574
Post-crisis	**0.9674**	**0.961**	**0.9541**	−0.2468	**0.4138**	**0.9608**		**0.668**	**0.703**	**0.6415**	**0.6238**	**0.7236**	0.0397
B_PriceDispersion													
Pre-crisis	**0.8323**	**0.8038**	**0.8965**	0.0491	**0.6184**	**0.948**	**−0.4979**	0.0964	0.0873	0.233	0.1912	**0.7219**	−0.1238
Crisis	**0.985**	**0.969**	**0.9697**	0.1976	**0.9322**	**0.9875**	**0.8272**	**0.9779**	**0.9865**	**0.9877**	**0.9884**	**0.9153**	**0.4118**
Post-crisis	**0.8887**	**0.8741**	**0.85**	−0.0075	**0.5993**	**0.8147**		**0.8657**	**0.9102**	**0.8772**	**0.8706**	**0.8593**	−0.0304

Table 2.5 continued

	P_Roll	P_Gibbs	P_Effective-Tick	P_Zeros	P_FHT	P_HighLow	P_Spread-BGN	P_Spread-CBBT	P_Spread-Mean	P_Quote-Dispersion	P_QDmid	P_Amihud	P_Pastor-Stambaugh
Panel B: Average cross sectional correlations													
B_Roundtrip													
Pre-crisis	**0.6129**	**0.5422**	**0.4133**	**0.0899**	**0.4386**	**0.7196**	**0.499**	**0.4703**	**0.5574**	**0.4775**	**0.4944**	**0.3968**	0.0006
Crisis	**0.4992**	**0.5174**	**0.2843**	**0.1442**	**0.3897**	**0.6314**	**0.3939**	**0.4511**	**0.5061**	**0.4997**	**0.4968**	**0.3084**	**0.0309**
Post-crisis	**0.5874**	**0.5661**	**0.2962**	**−0.0776**	**0.2448**	**0.7366**		**0.4772**	**0.4273**	**0.4127**	**0.413**	**0.2241**	−0.0073
B_IQR													
Pre-crisis	**0.7452**	**0.6579**	**0.4903**	**0.1004**	**0.5024**	**0.813**	**0.6095**	**0.6107**	**0.6719**	**0.5635**	**0.5835**	**0.4426**	0.0108
Crisis	**0.6617**	**0.6791**	**0.4046**	**0.1778**	**0.497**	**0.7191**	**0.4954**	**0.6643**	**0.6558**	**0.645**	**0.647**	**0.3575**	0.0255
Post-crisis	**0.698**	**0.6807**	**0.3842**	0.0155	**0.3969**	**0.7351**		**0.6635**	**0.5867**	**0.5493**	**0.5574**	**0.3675**	−0.0112
B_Roll													
Pre-crisis	**0.7308**	**0.6582**	**0.4197**	**0.0804**	**0.4938**	**0.78**	**0.5913**	**0.5577**	**0.6324**	**0.535**	**0.551**	**0.4556**	0.0046
Crisis	**0.6154**	**0.6169**	**0.3595**	**0.1926**	**0.4866**	**0.6561**	**0.4732**	**0.6112**	**0.6116**	**0.6**	**0.6009**	**0.3607**	0.0319
Post-crisis	**0.6724**	**0.6532**	**0.3142**	−0.0258	**0.3419**	**0.7392**		**0.5912**	**0.5397**	**0.5203**	**0.5166**	**0.2908**	−0.0149
B_AdjustedSchultz													
Crisis	**0.5613**	**0.5707**	**0.3382**	**0.146**	**0.3581**	**0.5899**	**0.3311**	**0.5507**	**0.5397**	**0.5624**	**0.5381**	**0.3118**	−0.0012
Post-crisis	**0.6756**	**0.6173**	**0.3174**	**−0.0764**	**0.2734**	**0.7753**		**0.5539**	**0.5345**	**0.5054**	**0.5092**	**0.2575**	**−0.035**
B_EHP													
Crisis	**0.5291**	**0.5589**	**0.2522**	0.076	**0.2728**	**0.6119**	**0.2749**	**0.47**	**0.4954**	**0.5171**	**0.4978**	**0.2304**	0.0119
Post-crisis	**0.6347**	**0.5839**	**0.2558**	**−0.115**	**0.1968**	**0.8052**		**0.5009**	**0.4883**	**0.477**	**0.4741**	**0.1799**	−0.0155
B_PriceDispersion													
Pre-crisis	**0.6482**	**0.4987**	**0.5018**	**0.0831**	**0.4355**	**0.6739**	**0.5778**	**0.6241**	**0.6414**	**0.5272**	**0.543**	**0.3542**	0.0006
Crisis	**0.4478**	**0.4252**	**0.2585**	**0.1444**	**0.3834**	**0.4578**	**0.2979**	**0.5018**	**0.4766**	**0.49**	**0.4716**	**0.3017**	0.0503
Post-crisis	**0.5401**	**0.4913**	**0.2671**	**0.0254**	**0.3241**	**0.5241**		**0.5478**	**0.5244**	**0.4973**	**0.4983**	**0.2827**	−0.032

Table 2.6: **Portfolio analysis: spread proxies**

Monthly high-frequency benchmarks are calculated from intraday TRACE data from October 1, 2004 to September 30, 2012. Monthly low-frequency proxies are computed based on daily price, volume, and quote data provided by Bloomberg. All measures are described in Section 2.3. B_AdjustedSchultz and B_EHP are calculated since November 2008, resulting in 47 observation months. The remaining benchmarks are calculated for all 96 sample months. P_Spread_BGN is only computed until February 28, 2011 and spans the first 77 sample months. P_Spread_CBBT is not available for the first sample month, resulting in 95 observation months. Both price impact measures start in April 2005 spanning the last 90 sample months. The remaining proxies are calculated for all 96 months in the observation period. Portfolios are equally weighted and stratified by the level of liquidity implied by the respective benchmark or the number of trades in the bond. As the number of a portfolio increases, its liquidity declines. The number of monthly trades is obtained from TRACE. Bold numbers are statistically significant at the 5% level. Drawn through boxes give the best value in a row and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	P_Roll	P_Gibbs	P_Effective-Tick	P_Zeros	P_FHT	P_HighLow	P_Spread-BGN	P_Spread-CBBT	P_Spread-Mean	P_Quote-Dispersion	P_QDmid	P_Amihud	P_Pastor-Stambaugh
Panel A: Time series correlations based on equally weighted portfolios ranked by the respective benchmark liquidity measure (1: most liquid, 10: least liquid)													
B_Roundtrip													
Portfolio 1	**0.733**	**0.6888**	**0.7518**	**−0.4543**	**0.4378**	**0.7743**	**0.7732**	**0.6929**	**0.6036**	**0.528**	**0.5694**	**0.4111**	0.1437
Portfolio 2	**0.8687**	**0.878**	**0.8482**	**−0.3421**	**0.6523**	**0.8852**	**0.8448**	**0.7731**	**0.7243**	**0.6939**	**0.7033**	**0.6053**	−0.062
Portfolio 5	**0.9227**	**0.9333**	**0.9111**	0.1146	**0.7872**	**0.9445**	**0.7998**	**0.8234**	**0.8104**	**0.7857**	**0.7926**	**0.7295**	0.0859
Portfolio 9	**0.9298**	**0.9328**	**0.8981**	**0.6549**	**0.8019**	**0.9013**	**0.4929**	**0.7342**	**0.7194**	**0.7257**	**0.7283**	**0.8919**	**0.4136**
Portfolio 10	**0.9169**	**0.8777**	**0.8589**	**0.8223**	**0.819**	**0.8666**	**0.4733**	**0.7263**	**0.7319**	**0.7429**	**0.7415**	**0.8849**	**0.3662**
B_IQR													
Portfolio 1	**0.9627**	**0.9121**	**0.8649**	0.1859	**0.8197**	**0.9543**	**0.8288**	**0.9499**	**0.9298**	**0.9035**	**0.9114**	**0.6607**	−0.0746
Portfolio 2	**0.9808**	**0.9495**	**0.952**	0.1617	**0.8415**	**0.971**	**0.8351**	**0.9566**	**0.931**	**0.9031**	**0.9121**	**0.7554**	**0.3268**
Portfolio 5	**0.9849**	**0.9834**	**0.9465**	0.1408	**0.8145**	**0.977**	**0.7652**	**0.9415**	**0.9033**	**0.876**	**0.8831**	**0.8314**	**0.2189**
Portfolio 9	**0.9854**	**0.9645**	**0.9322**	**0.7027**	**0.9365**	**0.9584**	**0.5348**	**0.9019**	**0.8871**	**0.8889**	**0.8928**	**0.9201**	**0.3854**
Portfolio 10	**0.9558**	**0.8598**	**0.8987**	**0.7958**	**0.8855**	**0.9265**	**0.5536**	**0.8694**	**0.844**	**0.86**	**0.8505**	**0.9258**	**0.3534**
B_Roll													
Portfolio 1	**0.6587**	**0.6536**	**0.6255**	−0.1742	**0.5477**	**0.6396**	**0.5624**	**0.5174**	**0.4927**	**0.4705**	**0.4734**	**0.5805**	0.0692
Portfolio 2	**0.9523**	**0.9615**	**0.9104**	−0.1877	**0.815**	**0.947**	**0.69**	**0.841**	**0.7737**	**0.7408**	**0.743**	**0.669**	−0.1426
Portfolio 5	**0.9695**	**0.9788**	**0.899**	0.0231	**0.8266**	**0.9533**	**0.6635**	**0.8627**	**0.8103**	**0.7784**	**0.7846**	**0.8366**	0.1833
Portfolio 9	**0.9533**	**0.9444**	**0.9226**	**0.7656**	**0.919**	**0.9216**	**0.391**	**0.8321**	**0.7984**	**0.8184**	**0.817**	**0.9244**	−0.1873
Portfolio 10	**0.9334**	**0.7678**	**0.8533**	**0.8702**	**0.9048**	**0.9137**	**0.4299**	**0.784**	**0.7893**	**0.7908**	**0.7917**	**0.9285**	**0.5535**
B_AdjustedSchultz													
Portfolio 1	**0.8197**	**0.8518**	**0.7979**	0.0038	**0.7009**	**0.7607**	**0.8752**	**0.7268**	**0.7659**	**0.7424**	**0.7515**	**0.6745**	**0.3607**
Portfolio 2	**0.9836**	**0.9801**	**0.9181**	−0.1923	**0.7885**	**0.9813**	**0.9372**	**0.9286**	**0.943**	**0.9261**	**0.9296**	**0.8335**	0.0467
Portfolio 5	**0.9916**	**0.9921**	**0.9647**	−0.2164	**0.8207**	**0.9948**	**0.8369**	**0.9541**	**0.9443**	**0.9304**	**0.9308**	**0.8247**	0.0707
Portfolio 9	**0.994**	**0.9916**	**0.9643**	**0.5965**	**0.923**	**0.9904**	0.3632	**0.9743**	**0.9687**	**0.9524**	**0.9617**	**0.9035**	0.0876
Portfolio 10	**0.9687**	**0.9078**	**0.9761**	**0.9321**	**0.969**	**0.9766**	**0.3887**	**0.9862**	**0.9862**	**0.9779**	**0.9795**	**0.9645**	0.1091

Table 2.6 continued

	P_Roll	P_Gibbs	P_Effective-Tick	P_Zeros	P_FHT	P_HighLow	P_Spread-BGN	P_Spread-CBBT	P_Spread-Mean	P_Quote-Dispersion	P_QDmid	P_Amihud	P_Pastor-Stambaugh
B_EHP													
Portfolio 1	**0.9257**	**0.9357**	**0.8821**	−0.0424	**0.8163**	**0.8935**	**0.9387**	**0.8734**	**0.8574**	**0.8357**	**0.8462**	**0.7445**	0.0071
Portfolio 2	**0.9898**	**0.9843**	**0.9439**	0.0013	**0.8806**	**0.9844**	**0.9376**	**0.9281**	**0.9535**	**0.9404**	**0.9442**	**0.7725**	0.2747
Portfolio 5	**0.9873**	**0.9865**	**0.9712**	−0.112	**0.8014**	**0.9937**	**0.872**	**0.9473**	**0.9403**	**0.9267**	**0.9279**	**0.8595**	−0.2616
Portfolio 9	**0.988**	**0.9901**	**0.9359**	**0.5907**	**0.8891**	**0.9854**	**0.4469**	**0.9477**	**0.9464**	**0.9349**	**0.937**	**0.8861**	−0.0002
Portfolio 10	**0.9822**	**0.9546**	**0.9487**	**0.8796**	**0.9139**	**0.9674**	0.0177	**0.9715**	**0.9614**	**0.9507**	**0.9557**	**0.8848**	**0.3672**
B_PriceDispersion													
Portfolio 1	**0.9642**	**0.9219**	**0.8882**	0.0286	**0.8731**	**0.978**	**0.7984**	**0.9636**	**0.963**	**0.945**	**0.952**	**0.8014**	0.0355
Portfolio 2	**0.9734**	**0.9256**	**0.9475**	0.1425	**0.8857**	**0.9817**	**0.8378**	**0.9747**	**0.9688**	**0.9524**	**0.958**	**0.7427**	**−0.2657**
Portfolio 5	**0.9803**	**0.9544**	**0.9606**	0.1098	**0.8794**	**0.9805**	**0.7647**	**0.9812**	**0.9664**	**0.9505**	**0.9545**	**0.8415**	0.1325
Portfolio 9	**0.933**	**0.8783**	**0.8961**	**0.5419**	**0.928**	**0.9551**	**0.6**	**0.9539**	**0.963**	**0.9566**	**0.9618**	**0.8675**	**0.4983**
Portfolio 10	**0.9027**	**0.7283**	**0.8306**	**0.7399**	**0.9408**	**0.9003**	**0.2342**	**0.9319**	**0.958**	**0.9481**	**0.9542**	**0.8651**	**0.4121**

Table 2.6 continued

	P_Roll	P_Gibbs	P_Effective-Tick	P_Zeros	P_FHT	P_HighLow	P_Spread-BGN	P_Spread-CBBT	P_Spread-Mean	P_Quote-Dispersion	P_QDmid	P_Amihud	P_Pastor-Stambaugh
Panel B: Time series correlations based on equally weighted portfolios ranked by the number of trades in the bond (1: highest number of tr., 10: lowest number of tr.)													
B_Roundtrip													
Portfolio 1	0.5808	0.5234	0.4176	0.1015	0.3148	0.8022	0.4096	0.4825	0.4579	0.4858	0.4837	0.0245	0.1541
Portfolio 2	0.7806	0.7628	0.5559	0.1483	0.4606	0.8548	0.5938	0.7085	0.6773	0.674	0.6704	0.2376	0.0226
Portfolio 5	0.9074	0.9096	0.8779	0.0909	0.6505	0.8974	0.7566	0.7175	0.7066	0.6902	0.6944	0.6005	−0.0384
Portfolio 9	0.9356	0.9468	0.8906	0.4794	0.8637	0.9172	0.7615	0.7895	0.7589	0.7333	0.7459	0.9087	0.27
Portfolio 10	0.9568	0.9389	0.9125	0.5644	0.9075	0.9326	0.8337	0.8171	0.8084	0.7928	0.8006	0.9165	0.3772
B_IQR													
Portfolio 1	0.9513	0.9357	0.5386	0.2822	0.694	0.9015	0.0026	0.8325	0.7777	0.7656	0.7692	0.4662	0.0529
Portfolio 2	0.9812	0.9768	0.7205	0.2671	0.7263	0.9178	0.3294	0.8692	0.8254	0.795	0.8029	0.5818	0.0155
Portfolio 5	0.9912	0.9797	0.9703	0.2841	0.8413	0.9788	0.7688	0.9007	0.8906	0.8762	0.8795	0.7361	0.0299
Portfolio 9	0.9933	0.9737	0.954	0.5083	0.9372	0.9706	0.8843	0.9114	0.8982	0.8799	0.8896	0.9414	0.2721
Portfolio 10	0.9924	0.963	0.9435	0.5416	0.9382	0.9564	0.906	0.8951	0.8972	0.8859	0.8926	0.9241	0.3708
B_Roll													
Portfolio 1	0.8017	0.8394	0.7261	0.3501	0.7062	0.8249	0.0223	0.5848	0.4812	0.5151	0.504	0.404	0.2427
Portfolio 2	0.8934	0.9177	0.832	0.313	0.7331	0.8361	0.1556	0.7027	0.624	0.5977	0.6037	0.6447	0.0146
Portfolio 5	0.9465	0.9508	0.9035	0.2754	0.7934	0.9202	0.6154	0.7834	0.7395	0.7244	0.7244	0.7862	0.0535
Portfolio 9	0.9702	0.964	0.9147	0.5586	0.9248	0.9553	0.7906	0.85	0.8231	0.8018	0.8119	0.9539	0.2454
Portfolio 10	0.9529	0.9165	0.8999	0.6041	0.9761	0.9809	0.793	0.8613	0.8941	0.8759	0.8876	0.9278	0.5659
B_AdjustedSchultz													
Portfolio 1	0.8976	0.9133	0.7333	0.0484	0.4693	0.9246	−0.4315	0.9105	0.9173	0.8743	0.8909	0.3479	−0.0023
Portfolio 2	0.975	0.9723	0.8282	0.1352	0.631	0.9793	−0.0932	0.9454	0.9339	0.9059	0.9105	0.5068	0.2043
Portfolio 5	0.9899	0.9854	0.9654	0.1138	0.8028	0.9841	0.8609	0.9456	0.9524	0.9408	0.9448	0.7724	−0.2568
Portfolio 9	0.9921	0.9747	0.9745	0.6227	0.9503	0.9794	0.9648	0.9603	0.9697	0.9621	0.9669	0.9517	0.355
Portfolio 10	0.9706	0.9421	0.9628	0.7439	0.9371	0.9633	0.913	0.9146	0.9728	0.9679	0.9711	0.9724	0.2624
B_EHP													
Portfolio 1	0.838	0.8533	0.7084	0.0651	0.4396	0.8744	−0.3477	0.8485	0.8554	0.8369	0.8501	0.2702	−0.0358
Portfolio 2	0.9583	0.9575	0.8223	0.1166	0.6003	0.966	−0.0395	0.9285	0.9165	0.8943	0.8968	0.4698	0.1762
Portfolio 5	0.983	0.9796	0.9568	0.0494	0.754	0.9805	0.8581	0.9178	0.9222	0.9109	0.9147	0.7227	−0.2681
Portfolio 9	0.9906	0.9827	0.9622	0.5891	0.9332	0.9703	0.9655	0.9442	0.9529	0.9411	0.9497	0.9398	0.3452
Portfolio 10	0.9792	0.9522	0.9644	0.733	0.9404	0.9651	0.9255	0.9199	0.9737	0.9678	0.9723	0.9559	0.2732
B_PriceDispersion													
Portfolio 1	0.9283	0.8457	0.2765	0.1715	0.5369	0.8926	0.1481	0.9606	0.9403	0.8901	0.9042	0.4175	−0.1169
Portfolio 2	0.9336	0.8925	0.545	0.1938	0.6435	0.9512	0.5092	0.9544	0.9311	0.9087	0.9135	0.3921	0.0256
Portfolio 5	0.9427	0.9102	0.9407	0.2786	0.843	0.9595	0.7809	0.9612	0.9578	0.9483	0.951	0.6474	−0.0057
Portfolio 9	0.9438	0.8872	0.9594	0.4506	0.9338	0.9577	0.8939	0.9605	0.9571	0.9496	0.955	0.9045	0.2609
Portfolio 10	0.9417	0.8645	0.9258	0.5339	0.9735	0.9782	0.8528	0.9194	0.9627	0.957	0.9586	0.9398	0.5299

by the respective benchmark. We then calculate means of the benchmark and proxies within each portfolio. Finally, we calculate time series correlations for each portfolio.

We find high levels of correlations for most of the benchmark-proxy combinations in Portfolios 2-10. For the most part, our measures show highest correlations in the middle portfolios and lower figures on the edges. The decline is more severe for the most liquid bonds in Portfolio 1, which confirms results by Goyenko, Holden, and Trzcinka (2009) for stocks that liquidity measurement is more challenging when liquidity is high. The majority of wins is claimed again by our auto-covariance measures P_Roll and P_Gibbs, as well as by P_HighLow. The measures based on quoted spreads from pricing sources CBBT and all dealers' quotes (P_Spread_CBBT and P_Spread_Mean) also perform quite well, especially for the price dispersion benchmark. Proxies based on zero returns (P_Zeros and P_FHT) always perform best in Portfolio 9 or 10, which confirms the intuition that they work well for illiquid bonds with many zero returns.

In Panel B, bonds are allocated to portfolios by the number of monthly trades. Again, Portfolio 1 contains the most liquid and Portfolio 10 the most illiquid bonds. The results confirm our findings in Panel A. Again, most of the proxies have their lowest correlations in the most liquid portfolio. The outcome for our effective spread estimators is basically the same as before. We find that they are the most consistent proxies, taking their maximum in the small and middle trade size portfolios with somewhat lower correlations in Portfolio 1. B_PriceDispersion aside, our benchmarks are best captured by P_Roll, P_Gibbs, and P_HighLow.

In Table 2.7, we perform the same analysis as in Table 2.4 but aggregate benchmark and proxy liquidity measures on an annual instead of a monthly basis. Panel A shows extremely high time series correlations especially for P_Roll, P_Gibbs, and P_HighLow. Part of this result might be due to the fact that correlations are biased to one if the number of observations is small which is a side effect for time series correlations when aggregating annually. Average cross sectional correlations in Panel B are also higher than in the monthly analysis, but not to the same extent as for time series correlations. Contrary to the monthly results where the high-low bid-ask spread estimator clearly performed best, most of the wins in the annual comparisons are claimed by Hasbrouck's (2009) Gibbs measure.[25] Panels C and D paint a very similar picture for the prediction errors compared to our main analysis in Table 2.4 with the high-low and Gibbs measure splitting their

[25]The better performance of the Gibbs measure in the annual compared to the monthly sample is in line with Goyenko, Holden, and Trzcinka (2009) for the stock market. Hasbrouck (2009) also comments on problems that arise when calculating his measure on a monthly basis.

Table 2.7: **Annual spread proxies compared to spread benchmarks**

Annual high-frequency benchmarks are calculated from intraday TRACE data from January 1, 2005 to September 30, 2012. Annual low-frequency proxies are computed based on daily price, volume, and quote data provided by Bloomberg. All measures are described in Section 2.3. B_AdjustedSchultz and B_EHP are calculated since January 2009, resulting in four observation years. The remaining benchmarks are calculated for all eight sample years. P_Spread_BGN is only computed until December 2010 and spans the first six sample years. Both price impact measures start in January 2006 spanning the last seven sample years. The remaining proxies are calculated for all eight years in the observation period. Bold numbers are statistically significant at the 5% level. Drawn through boxes give the best value in a row and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	P_Roll	P_Gibbs	P_Effective-Tick	P_Zeros	P_FHT	P_HighLow	P_Spread-BGN	P_Spread-CBBT	P_Spread-Mean	P_Quote-Dispersion	P_QDmid	P_Amihud	P_Pastor-Stambaugh
Panel A: Time series correlations													
B_Roundtrip	**0.9609**	**0.9682**	**0.9673**	0.2003	**0.8255**	**0.9661**	0.7789	**0.764**	**0.7657**	**0.7211**	**0.7465**	**0.9033**	0.5677
B_IQR	**0.9987**	**0.9932**	**0.9693**	0.436	**0.9484**	**0.9904**	0.6874	**0.8658**	**0.8559**	**0.8052**	**0.8363**	**0.9744**	0.7173
B_Roll	**0.9651**	**0.9575**	**0.9192**	0.4883	**0.9234**	**0.9311**	0.4879	**0.728**	**0.7098**	0.6423	0.6847	**0.9883**	**0.7585**
B_AdjustedSchultz	**0.999**	**0.9986**	**0.9987**	0.037	**0.9881**	**0.9991**		**0.9846**	**0.9833**	**0.9652**	**0.9724**	**0.9904**	−0.5782
B_EHP	**0.99999**	**0.9999**	**0.9998**	−0.0026	**0.9813**	**0.9998**		**0.9768**	**0.9756**	**0.9545**	**0.9625**	**0.9842**	−0.5914
B_PriceDispersion	**0.9508**	**0.9494**	**0.9502**	0.3181	**0.8921**	**0.9766**	**0.8289**	**0.9483**	**0.9492**	**0.9244**	**0.9393**	**0.9155**	0.6233
Panel B: Average cross sectional correlations													
B_Roundtrip	**0.7701**	**0.797**	**0.4073**	0.038	**0.4427**	**0.8202**	**0.5293**	**0.5634**	**0.5593**	**0.5501**	**0.5436**	**0.4176**	0.0566
B_IQR	**0.8906**	**0.9067**	**0.5072**	0.0989	**0.5544**	**0.8284**	**0.6195**	**0.7184**	**0.6942**	**0.658**	**0.6677**	**0.5187**	0.0409
B_Roll	**0.8594**	**0.8809**	**0.4416**	0.0963	**0.5317**	**0.8223**	**0.6184**	**0.6756**	**0.662**	**0.6424**	**0.64**	**0.4641**	0.0563
B_AdjustedSchultz	**0.8317**	**0.864**	**0.375**	−0.0291	**0.362**	**0.786**	0.4695	**0.5975**	**0.604**	**0.6109**	**0.601**	**0.4191**	−0.0315
B_EHP	**0.8048**	**0.8449**	**0.303**	−0.0876	**0.2759**	**0.8308**	**0.4478**	**0.5477**	**0.5676**	**0.5995**	**0.5756**	**0.3263**	−0.0015
B_PriceDispersion	**0.7484**	**0.7373**	**0.3846**	**0.1081**	**0.4759**	**0.6467**	**0.5073**	**0.6167**	**0.5947**	**0.564**	**0.5645**	**0.4875**	0.0444
Panel C: Mean bias													
B_Roundtrip	**0.0057**	**0.0009**	**−0.0111**		**−0.0072**	**−0.0027**	**−0.0084**	**0.008**	**0.0065**	**0.0127**	**0.0142**		
B_IQR	**0.0085**	**0.0037**	**−0.0083**		**−0.0044**	**0.0001**	**−0.0059**	**0.0111**	**0.0093**	**0.0155**	**0.0171**		
B_Roll	**0.0059**	**0.0011**	**−0.0108**		**−0.0069**	**−0.0025**	**−0.0085**	**0.0084**	**0.0068**	**0.013**	**0.0145**		
B_AdjustedSchultz	**0.0033**	**−0.001**	**−0.0113**		**−0.0089**	**−0.0037**	**−0.0103**	**0.0074**	**0.0068**	**0.0142**	**0.015**		
B_EHP	**0.0065**	**0.0023**	**−0.0081**		**−0.0057**	**−0.0005**	**−0.0057**	**0.0105**	**0.01**	**0.0174**	**0.0183**		
B_PriceDispersion	**−0.001**	**−0.0057**	**−0.0166**		**−0.0135**	**−0.0087**	**−0.0155**	**0.0025**	**0.0005**	**0.0068**	**0.008**		
Panel D: Root mean squared error (RMSE)													
B_Roundtrip	0.0099	**0.0059**	0.0132		0.0112	**0.006**	0.0108	0.0187	0.0156	0.0219	0.0248		
B_IQR	0.0108	**0.0058**	0.0104		0.0083	**0.0046**	0.0086	0.019	0.0157	0.0228	0.0255		
B_Roll	0.0091	**0.0049**	0.0129		0.0103	**0.0057**	0.0108	0.0179	0.0148	0.0214	0.0242		
B_AdjustedSchultz	**0.0068**	**0.0056**	0.0152		0.013	**0.0076**	0.0158	0.015	0.0124	0.0195	0.021		
B_EHP	0.009	**0.005**	0.0104		0.009	**0.0045**	0.0088	0.0177	0.0149	0.0227	0.0243		
B_PriceDispersion	**0.0169**	**0.0185**	0.0272		0.0235	**0.0201**	0.0292	**0.0164**	**0.0179**	**0.0202**	**0.0216**		

wins.

Despite of the completely different market design of decentralized bond markets compared to centralized stock markets, low-frequency spread proxies developed mainly for stock markets generally approximate our high-frequency bond market liquidity benchmarks very well. Especially the two measures based on the auto-covariance of trade prices, P_Roll and P_Gibbs as well as P_HighLow show high, consistent, and robust results in time series and average cross sectional correlations. In contrast, only P_HighLow and P_Gibbs are able to measure the scale of transaction costs correctly, resulting in low mean biases and RMSEs. P_HighLow claims the most wins for the monthly comparisons, whereas P_Gibbs performs especially well when aggregating annually. Measures based on zero returns (P_Zeros and P_FHT) suffer from small time series correlations when bonds are more liquid. Our quote based measures show small time series correlations before the crisis which might indicate some consistency problems. However, the quoted spreads from Bloomberg's CBBT pricing source and average dealers' quotes often give the best results in capturing price dispersion. For quoted spreads, these two pricing sources generally perform better than quotes from Bloomberg's Generic Quote (BGN). P_PastorStambaugh is clearly dominated by the other proxies. Although P_Amihud has high time series correlations with our benchmarks, it does not do a good job considering cross sectional correlations and performs somewhat worse for highly liquid bonds and in the pre-crisis period. In comparison to stock markets, our measure based on price clustering (P_Effective-Tick) performs worse which is probably due to the different market design of the bond market.

2.4.4 Monthly Price Impact Results

Table 2.8 presents our main findings for the low-frequency price impact proxies. Panel A shows time series correlations based on the monthly means of our benchmarks and proxies. Again, we find high and significant values for all benchmark-proxy combinations. Quote based measures seem to work better when it comes to capturing B_Lambda, whereas for the other proxies, results for the two benchmarks are very similar. Both high-frequency benchmarks are best captured by the low frequency Amihud (2002) measure. In contrast to that, the price impact proxy of Pastor and Stambaugh (2003) performs worst.

Panel B reports average cross sectional evidence. P_PastorStambaugh aside, which is again clearly dominated by all other proxies, all average correlations are significant, ranging from 0.3642 to 0.7428. With the exception of P_Amihud, which captures B_Amihud best, our low-frequency proxies seem to grasp B_Lambda better than B_Amihud. The price

Table 2.8: **Monthly price impact proxies compared to price impact benchmarks**

Monthly high-frequency benchmarks are calculated from intraday TRACE data from October 1, 2004 to September 30, 2012. Monthly low-frequency proxies are computed based on daily price, volume, and quote data provided by Bloomberg. All measures are described in Section 2.3. B_Lambda is calculated since November 2008, resulting in 47 observation months. B_Amihud is calculated for all 96 sample months. Due to the availability of volume data in Bloomberg, all proxies start in April 2005. P_PI_Spread_BGN is only computed until February 2011 and spans 71 sample months. The remaining proxies are calculated for the last 90 months in the observation period. Bold numbers are statistically significant at the 5% level. Drawn through boxes give the best value in a row and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	P_Amihud	P_PI-Roll	P_PI-Gibbs	P_PI-ET	P_PI-FHT	P_PI-HL	P_PI-Spread-BGN	P_PI-Spread-CBBT	P_PI-Spread-Mean	P_PI-QD	P_PI-Qdmid	P_Pastor-Stambaugh
Panel A: Time series correlations												
B_Amihud	**0.9757**	**0.9269**	**0.8433**	**0.9431**	**0.9168**	**0.9675**	**0.601**	**0.7599**	**0.9099**	**0.9082**	**0.9064**	**0.4835**
B_Lambda	**0.9789**	**0.9021**	**0.8262**	**0.8798**	**0.8392**	**0.9189**	**0.849**	**0.8537**	**0.942**	**0.9427**	**0.9393**	**0.3245**
Panel B: Average cross sectional correlations												
B_Amihud	**0.5702**	**0.5049**	**0.538**	**0.415**	**0.4291**	**0.5538**	**0.4852**	**0.3653**	**0.4336**	**0.4181**	**0.4331**	−0.0017
B_Lambda	**0.3642**	**0.6607**	**0.6985**	**0.5307**	**0.5203**	**0.7428**	**0.7082**	**0.6154**	**0.6415**	**0.6242**	**0.6419**	−0.0333

impact adaptation of the high-low spread proxy P_PI_HL yields the highest correlation for B_Lambda. In contrast to the time series results, the correlation of P_Amihud with B_Lambda is relatively low.

In Tables 2.9, 2.10, and 2.11 we perform robustness checks regarding the consistency of our price impact measures in different subperiods, for more liquid and illiquid bonds, and when aggregating measures yearly instead of monthly. Due to the conceptual problems in the interpretation of price impact measures in the corporate bond market (see the introduction), we refrain from discussing these tables in detail but instead directly summarize the price impact evidence including the robustness checks.

We find that time series and average cross sectional correlations between our benchmark and proxy measures are high and significant for the most part. P_Amihud and P_PI_HL clearly show up as winners in our analysis. The former wins both time series competitions and is significantly better than the remaining proxies for B_Lambda. Cross sectional correlations are best captured by P_Amihud and P_PI_HL dependent on the chosen benchmark. Both measures, but especially P_PI_HL, are robust against changes in market or bond liquidity (see Tables 2.9 and 2.10), being in the leading group in most of our robustness tests and showing relatively small variations in correlations. Quote based price impact proxies, like their spread proxy counterparts, show high variations for different subperiods, even yielding some unreasonable correlations prior to the financial crisis. In all analyses, the Pastor and Stambaugh (2003) measure is consistently dominated by all other measures.

2.5 Conclusion

In this chapter, we address the issue of how to best measure liquidity in OTC bond markets. Goyenko, Holden, and Trzcinka (2009) answer this question for the stock market, but due to the fundamental differences in market structure, it is highly questionable whether their results are valid for bond markets. To provide guidance for researchers and practitioners, we adapt their empirical methodology on the U.S. corporate bond market. With the help of the TRACE database, which provides a complete trade record for this market after October 1, 2004, we calculate a total of eight monthly high-frequency benchmark liquidity measures. We compare them to 23 liquidity proxies that only need daily data (last price, volume, high- and low price, or bid-ask quotes) which can be conveniently downloaded, e.g., from Bloomberg.

Table 2.9: **Subperiod analysis: price impact proxies**

Monthly high-frequency benchmarks are calculated from intraday TRACE data from October 1, 2004 to September 30, 2012. Monthly low-frequency proxies are computed based on daily price, volume, and quote data provided by Bloomberg. All measures are described in Section 2.3. The pre-crisis period is October 1, 2004 until March 31, 2007. The crisis period is defined as April 1, 2007 until December 31, 2009. The post-crisis period is January 1, 2010 until September 30, 2012. B_Lambda is calculated since November 2008. B_Amihud is calculated for all 96 sample months. Due to the availability of volume data in Bloomberg, all proxies start in April 2005. P_PI_Spread_BGN is only computed for the pre-crisis and crisis period. The remaining proxies are calculated for the last 90 months in the observation period. Bold numbers are statistically significant at the 5% level. Drawn through boxes give the best value in a row and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	P_Amihud	P_PI-Roll	P_PI-Gibbs	P_PI-ET	P_PI-FHT	P_PI-HL	P_PI-Spread-BGN	P_PI-Spread-CBBT	P_PI-Spread-Mean	P_PI-QD	P_PI-Qdmid	P_Pastor-Stambaugh
Panel A: Time series correlations												
B_Amihud												
Pre-crisis	**0.8546**	**0.7343**	**0.6817**	**0.5273**	**0.4569**	**0.8726**	−0.2555	−0.3939	**−0.4581**	−0.3923	**−0.4125**	0.088
Crisis	**0.9731**	**0.9015**	**0.7607**	**0.9293**	**0.9045**	**0.9567**	**0.4599**	**0.809**	**0.9377**	**0.9118**	**0.9381**	**0.486**
Post-crisis	**0.5128**	**0.7477**	**0.7115**	**0.8297**	0.3084	**0.8148**		−0.1378	**0.3476**	**0.7749**	0.2943	0.0145
B_Lambda												
Crisis	**0.9662**	**0.7338**	0.4521	**0.7377**	**0.7248**	**0.8107**	**0.6659**	**0.9646**	**0.9017**	**0.8915**	**0.8979**	0.2689
Post-crisis	**0.7159**	**0.8999**	**0.8725**	**0.8421**	**0.5803**	**0.9022**		0.1648	**0.5893**	**0.8569**	**0.5532**	−0.0118
Panel B: Average cross sectional correlations												
B_Amihud												
Pre-crisis	**0.6006**	**0.6096**	**0.6222**	**0.4746**	**0.5113**	**0.6498**	**0.5344**	**0.3076**	**0.4482**	**0.4378**	**0.4481**	0.0015
Crisis	**0.7015**	**0.6096**	**0.6717**	**0.5287**	**0.5467**	**0.6783**	**0.4889**	**0.3704**	**0.4693**	**0.4709**	**0.4689**	0.0165
Post-crisis	**0.3729**	**0.2844**	**0.2879**	**0.2363**	**0.2233**	**0.3019**		**0.4006**	**0.3852**	**0.3469**	**0.3845**	−0.0221
B_Lambda												
Crisis	**0.4785**	**0.4957**	**0.5865**	**0.5109**	**0.5024**	**0.5259**	**0.6739**	**0.5649**	**0.6122**	**0.6239**	**0.6218**	−0.0772
Post-crisis	**0.3119**	**0.7164**	**0.7378**	**0.5389**	**0.5278**	**0.8057**		**0.6355**	**0.6534**	**0.6242**	**0.6503**	−0.0146

Table 2.10: **Portfolio analysis: price impact proxies**

Monthly high-frequency benchmarks are calculated from intraday TRACE data from October 1, 2004 to September 30, 2012. Monthly low-frequency proxies are computed based on daily price, volume, and quote data provided by Bloomberg. All measures are described in Section 2.3. B_Lambda is calculated since November 2008, resulting in 47 observation months. B_Amihud is calculated for all 96 sample months. Due to the availability of volume data in Bloomberg, all proxies start in April 2005. P_PI_Spread_BGN is only computed until February 2011 and spans 71 sample months. The remaining proxies are calculated for the last 90 months in the observation period. Portfolios are equally weighted and stratified by the level of liquidity implied by the respective benchmark or the number of trades in the bond. As the number of a portfolio increases, its liquidity declines. The number of monthly trades is obtained from TRACE. Bold numbers are statistically significant at the 5% level. Drawn through boxes give the best value in a row and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	P_Amihud	P_PI-Roll	P_PI-Gibbs	P_PI-ET	P_PI-FHT	P_PI-HL	P_PI-Spread-BGN	P_PI-Spread-CBBT	P_PI-Spread-Mean	P_PI-QD	P_PI-Qdmid	P_Pastor-Stambaugh
Panel A: Time series correlations based on equally weighted portfolios ranked by the respective benchmark liquidity measure (1: most liquid, 10: least liquid)												
B_Amihud												
Portfolio 1	**0.7767**	**0.7995**	**0.8339**	**0.54**	**0.7581**	**0.7418**	**0.7145**	**0.7196**	**0.6981**	**0.6956**	**0.698**	−0.189
Portfolio 2	**0.7915**	**0.8398**	**0.7934**	**0.4151**	**0.6963**	**0.886**	**0.5518**	**0.7137**	**0.6621**	**0.6392**	**0.6397**	**−0.4804**
Portfolio 5	**0.8053**	**0.8851**	**0.8343**	**0.7001**	**0.7882**	**0.9186**	**0.4029**	**0.7702**	**0.7759**	**0.7729**	**0.7534**	−0.2067
Portfolio 9	**0.9735**	**0.9427**	**0.912**	**0.9322**	**0.9305**	**0.9581**	**0.3232**	**0.7858**	**0.8757**	**0.8499**	**0.8757**	**0.4362**
Portfolio 10	**0.9756**	**0.8551**	**0.704**	**0.9507**	**0.9127**	**0.9575**	**0.6242**	**0.6783**	**0.9313**	**0.9246**	**0.931**	**0.4946**
B_Lambda												
Portfolio 1	**0.2957**	0.1251	0.2731	**0.2913**	0.0585	0.1633	**0.5376**	**0.4974**	**0.5347**	**0.5292**	**0.5322**	−0.051
Portfolio 2	**0.9018**	**0.5516**	**0.8886**	**0.5597**	**0.3958**	**0.6517**	**0.8193**	**0.8956**	**0.8883**	**0.9016**	**0.8706**	**0.3357**
Portfolio 5	**0.9116**	**0.9685**	**0.9491**	**0.9408**	**0.9068**	**0.9764**	**0.7928**	**0.9404**	**0.9366**	**0.9406**	**0.9201**	**−0.2936**
Portfolio 9	**0.9307**	**0.9358**	**0.9453**	**0.9465**	**0.9142**	**0.9137**	**0.8796**	**0.8744**	**0.9311**	**0.9257**	**0.9321**	**0.4389**
Portfolio 10	**0.9661**	**0.8406**	**0.7529**	**0.8941**	**0.9048**	**0.9252**	**0.7996**	**0.8464**	**0.9385**	**0.9195**	**0.9395**	0.0508
B_Amihud												
Portfolio 1	**0.5419**	**0.6035**	**0.5588**	**0.4116**	**0.5469**	**0.5626**	0.103	**0.5035**	**0.4813**	**0.4417**	**0.4108**	0.1402
Portfolio 2	**0.7294**	**0.5784**	**0.5567**	**0.4726**	**0.5979**	**0.6712**	0.0757	**0.5729**	**0.5689**	**0.5921**	**0.5434**	0.0546
Portfolio 5	**0.8017**	**0.6319**	**0.557**	**0.6693**	**0.7206**	**0.7655**	0.1519	**0.7659**	**0.7859**	**0.8166**	**0.7782**	0.0079
Portfolio 9	**0.9592**	**0.8783**	**0.7922**	**0.934**	**0.9074**	**0.9524**	**0.6406**	**0.5646**	**0.8671**	**0.8303**	**0.8641**	**0.2869**
Portfolio 10	**0.9679**	**0.8398**	**0.7523**	**0.9531**	**0.9318**	**0.951**	**0.7662**	**0.6603**	**0.9187**	**0.9081**	**0.9168**	**0.5565**
B_Lambda												
Portfolio 1	**0.3437**	**0.6119**	**0.5512**	**0.5365**	**0.4363**	**0.6431**	**−0.4173**	**0.7177**	**0.713**	**0.7441**	**0.7707**	0.0319
Portfolio 2	**0.6116**	**0.7067**	**0.6493**	**0.5258**	**0.464**	**0.7868**	0.0454	**0.7539**	**0.7443**	**0.7929**	**0.7317**	**0.3518**
Portfolio 5	**0.7681**	**0.8011**	**0.6798**	**0.8637**	**0.7543**	**0.8187**	**0.5609**	**0.6742**	**0.8091**	**0.796**	**0.7998**	**−0.3763**
Portfolio 9	**0.9704**	**0.891**	**0.8556**	**0.9126**	**0.8926**	**0.9497**	**0.8851**	**0.8013**	**0.8851**	**0.8606**	**0.8775**	0.2799
Portfolio 10	**0.96**	**0.769**	**0.6638**	**0.8505**	**0.8137**	**0.8349**	**0.7535**	**0.7915**	**0.8721**	**0.8194**	**0.8668**	**0.2982**

Table 2.11: **Annual price impact proxies compared to price impact benchmarks**

Annual high-frequency benchmarks are calculated from intraday TRACE data from January 1, 2005 to September 30, 2012. Annual low-frequency proxies are computed based on daily price, volume, and quote data provided by Bloomberg. All measures are described in Section 2.3. B_Lambda is calculated since January 2009, resulting in four observation years. B_Amihud is calculated for all eight sample years. Due to the availability of volume data in Bloomberg, all proxies start in January 2006. P_PI_Spread_BGN is only computed until December 2010 and spans five sample years. The remaining proxies are calculated for the last seven years in the observation period. Bold numbers are statistically significant at the 5% level. Drawn through boxes give the best value in a row and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	P_Amihud	P_PI-Roll	P_PI-Gibbs	P_PI-ET	P_PI-FHT	P_PI-HL	P_PI-Spread-BGN	P_PI-Spread-CBBT	P_PI-Spread-Mean	P_PI-QD	P_PI-Qdmid	P_Pastor-Stambaugh
Panel A: Time series correlations												
B_Amihud	**0.9791**	**0.8626**	**0.8917**	**0.9539**	**0.9796**	**0.9836**	0.8258	0.7112	**0.9502**	**0.9582**	**0.9488**	0.6911
B_Lambda	**0.9806**	**0.9841**	**0.9838**	**0.9871**	**0.9762**	**0.9861**		0.6303	**0.9776**	**0.9908**	**0.9756**	−0.5863
Panel B: Average cross sectional correlations												
B_Amihud	**0.7123**	**0.5485**	**0.5708**	**0.4813**	**0.5197**	**0.5996**	**0.6219**	**0.5158**	**0.5829**	**0.5877**	**0.5826**	0.0437
B_Lambda	**0.4393**	**0.8033**	**0.8088**	**0.6847**	**0.615**	**0.8693**	**0.7878**	**0.7629**	**0.7498**	**0.7188**	**0.7504**	−0.0017

We find that although most studies in corporate bond markets use their own approach to measure liquidity, results of previous studies using high-frequency bid-ask spread measures should be robust regarding the chosen approach as all of these measures are highly correlated.

Second, we test whether low-frequency liquidity proxies that only need daily data are useful to measure intraday transaction costs and price impact. We provide clear evidence that most of our proxies are able to capture variations in transaction costs on both a time series and cross sectional level. In contrast, only two of the low-frequency proxies precisely estimate the magnitude of transaction costs. Consistent with Goyenko, Holden, and Trzcinka (2009) for the stock market, intraday price impact is somewhat harder to grasp than transaction costs. Nevertheless, most of our price impact proxies are able to capture time series and cross sectional variations of price impact benchmarks. For some of our transaction cost and price impact proxies, the ability to capture the features of our high-frequency benchmarks varies over time and in different levels of market and bond liquidity. However, the group of dominant measures mostly stays the same.

When estimating effective spread, we find that, with the exception of our price dispersion benchmark, low-frequency proxies calculated from Bloomberg's TRACE pricing source, rather than quote data, are the superior choice. The proxies developed for stock markets by Corwin and Schultz (2012), Hasbrouck (2009), and Roll (1984) give high time series and cross sectional correlations and also perform well in our robustness tests. Further, the first two show low prediction errors. If computational costs are crucial, we recommend Roll's measure because of its low data and processing demands. In a monthly setting, the high-low bid-ask spread estimator of Corwin and Schultz is preferred over Hasbrouck's Gibbs measure because of its overall better performance and lower computational requirements. When aggregating liquidity annually, the Gibbs measure performs slightly better than the high-low estimator. If daily data on trades is not available, quoted bid-ask spreads distributed in Bloomberg are also suited to capture intraday liquidity. It is important to note, however, that one should prefer data from executable quotes (pricing source: CBBT) over Bloomberg's Generic bid-ask spread (pricing source: BGN). BGN bid-ask spreads, although useful in capturing time series and cross sectional variations, do not provide a correct scale of transaction costs. Quote based measures also do have some consistency problems especially for the time before 2007.

For price impact, the daily Amihud (2002) measure and the price impact version of the high-low bid-ask spread estimator win in the most categories with the Amihud (2002) measure performing better in the time series and the high-low estimator being superior in capturing cross sectional variations. Especially the high-low price impact measure, but also the Amihud (2002) low frequency proxy, give consistent and robust results for different levels of market and bond liquidity. However, the latter needs less data input and preprocessing in comparison. The Pastor and Stambaugh (2003) price impact Gamma, on the other hand, is inferior to merely all other proxies. Although price impact measures are employed in the bond market literature, we only recommend to use them with great caution. The reason is that larger sizes trade at better prices in many bond markets. This induces a negative price impact component and leads to difficulties in the interpretation of price impact measures.

Summarizing, our results in this chapter provide clear evidence that the reduction in data requirements and computational burden, when using low-frequency proxies over high-frequency liquidity measures, for many applications might outweigh the small losses in accuracy for the U.S. corporate bond market.

Chapter 3

The Term Structure of Liquidity Premia Conditional on the Economic Environment

3.1 Introduction

After having established how to best measure liquidity in the previous chapter, we now study the pricing implications of illiquidity for bonds of different maturity. For that, we analyze the term structure of liquidity premia as the difference between the zero coupon yield curves of two bond segments that differ only in their liquidity. Although, it is consensus in the literature that a large part of the yield spread compensates investors for the illiquidity of a bond (see, e.g., Longstaff, Mithal, and Neis, 2005), disentangling illiquidity related risk premia from other systematic factors such as default risk is in most papers subject to strong assumptions. In contrast, German government bonds (BUNDs) and government guaranteed bonds issued by the German federal agency Kreditanstalt für Wiederaufbau (KfW) provide a near-ideal setting to study liquidity premia. Both constitute major bond market segments in the eurozone with a sufficient number of bonds from both issuers in all maturity segments. Since KfW bonds are explicitly guaranteed by the German government, they bear effectively the same default risk as government bonds. However, trading volumes and bid-ask spreads indicate that they are less liquid.

In this clean setting, we study the following three questions based on 15 years of data from 1996 to 2010: How do liquidity related risk premia behave in different economic regimes? What are the drivers of different parts of the term structure of liquidity premia? And are relations between fundamental factors and liquidity premia different in crisis and non-crisis times? All three questions are relevant for investors and issuers active in global bond markets as an increase of the liquidity premium directly translates into a price decrease of a bond.[26]

Three main results emerge from the analysis. First, our regime-switching approach identifies the 1998 bailout of Long Term Capital Management (LTCM), the period after the burst of the dot-com bubble as well as the financial crisis starting in summer 2007 as liquidity stress periods in the European bond market. Within these stress periods, the extra yield to maturity of an illiquid agency bond compared to a liquid government bond is on average around 25 bps, compared to 15 bps in normal times. The increase is most prevalent at the short end, where premia, e.g., for two year bonds reach all-time highs of more than 120 bps after the collapse of Lehman Brothers. Thus, term structures of liquidity premia in times of stress are often strongly downward sloping as predicted by theoretical models when probabilities to sell are above their long-term mean (see Ericsson and Renault, 2006) or when aggregate liquidity shocks are more likely (see Feldhütter, 2012).

Second, we find that liquidity premia are highly dependent on the global availability of arbitrage capital as well as on foreign flows into bond markets. As suggested by the recent literature on slow moving capital, the supply of liquidity depends on sophisticated arbitrageurs providing liquidity for investors (see, e.g., Gromb and Vayanos, 2010). On the other hand, demand for liquidity only drives short-term liquidity premia.

Third and probably most importantly, we find that none of our economic drivers plays a major role in explaining premia in normal times. In contrast, a decrease in the available arbitrage capital directly translates into higher liquidity premia in crisis periods. This result goes well together with the theoretical insights of Brunnermeier and Pedersen (2009) that the impact of changes of speculator capital on liquidity is significantly stronger when funding is scarce and the system is in stress. Additionally, the regime-switching dependency of short-term liquidity premia on liquidity demand translates into an additional source of risk for short-term bonds. This risk factor mirrors the increasing demand for short-term and highly liquid bonds during liquidity stress periods. As as direct result,

[26] As a concrete example, insurance companies currently face the challenge to mark-to-market their hold-to-maturity assets in the light of Solvency II. In this regard, the European Insurance CFO/CRO Forum stresses that there is a lack of research regarding the term structure of liquidity premia (see European Insurance CFO Forum and CRO Forum (2010), p. 55).

short-term premia in crisis times increase disproportionately leading to pronounced term structure effects. The highly non-linear influence of fundamentals on liquidity premia implies that calibrating, e.g., risk management models in normal times, where liquidity premia are largely invariant to changes in fundamentals, heavily underestimates the systematic component of liquidity risk.

Our study relates to several strands of literature. First, papers like Longstaff (2004), Koziol and Sauerbier (2007), Bühler and Vonhoff (2011), and Kempf, Korn, and Uhrig-Homburg (2012) study the term structure of liquidity premia for Refcorp and Treasury bonds as well as for German Pfandbriefe. All of these studies do not pursue a conditional approach, rather they analyze 'average' effects over the whole business cycle. Second, Brunnermeier (2009), Dick-Nielsen, Feldhütter, and Lando (2012), and Acharya, Amihud, and Bharath (2013) analyze the different behavior of liquidity during stress and normal times. In contrast to these papers, we study the pricing implications across different maturity segments and do not rely on strong assumptions regarding the separation between liquidity and credit risk. Finally, Goyenko, Subrahmanyam, and Ukhov (2011) study the term structure of Treasury market illiquidity. Although their focus is on bond market trading cost measured via bid-ask spreads instead of yield differentials, their findings of increasing illiquidity in recessions is pretty much consistent with our insights on the premium side.

The remainder of the chapter is structured as follows: Section 3.2 extracts the term structure of liquidity premia and provides insights on its characteristics. In Section 3.3, we analyze economic determinants of liquidity premia of different maturities conditional on the economic environment in a regime-switching model. Section 3.4 performs several robustness checks. Section 3.4 concludes the chapter.

3.2 Liquidity Premia

The interpretation of the KfW-BUND bond yield spread curve as the term structure of liquidity premia (see also, e.g., Schwarz, 2010; Monfort and Renne, 2011, 2013) depends on two key assumptions: that federal agency bonds are less liquid than government bonds and that credit risk in both segments is exactly the same. We first provide evidence for both of these assumptions and describe our data set. We then discuss the economics of the KfW-BUND yield spread and provide first evidence of its regime-switching nature.

3.2.1 Data and Term Structure Estimation

We estimate zero coupon yield curves for German government bonds (BUNDs) and bonds of the Kreditanstalt für Wiederaufbau (KfW) using the parametric Nelson-Siegel approach (see, e.g., Kempf, Korn, and Uhrig-Homburg, 2012). KfW is a promotional bank owned by the German government and federal states. It was founded in 1948 to further the reconstruction of the German economy after World War II. Today, it serves as the leading financier of small and medium size enterprises, provides credit to retail customers for subsidized projects (e.g., improving energy efficiency of buildings), and acts on behalf of the German government on special tasks. The government of Germany guarantees the continuation of KfW through a maintenance obligation ('Anstaltslast'). In addition, all KfW bonds are explicitly guaranteed by the German government. Thus, they bear effectively the same default risk as government bonds. This is recognized, e.g., by the U.S. economic magazine 'Global Finance' honoring KfW as the world's safest bank in 2009, 2010, 2011, and 2012. Rating reports and conversations with analysts from different rating agencies suggest that they apply a 'credit substitution approach', equalizing the rating of KfW with that of the Federal Republic of Germany. Analysts state that only an extremely unlikely change of the legal framework would break down the direct link between the ratings of the two segments. Therefore, it is safe to assume that credit risk in both segments is identical. Moreover, KfWs and BUNDs are zero weighted in determining capital requirements within the Basel regulations, and are also identical in their tax treatment.

Our data set consists of weekly closing prices for BUNDs and KfW bonds from the Frankfurt Stock exchange from February 14th, 1996 to September 29th, 2010. During this period, there are always a sufficient number of bonds from both issuers in all maturity segments available. Closing prices either result from trades or are determined in an auction like mechanism. We only include those KfW bonds that are well comparable to BUNDs: plain vanilla fixed coupon bonds with annual coupon payments that are exchange-tradable and denominated in Euro. Table 3.1 gives an overview of all bonds in our sample.

In contrast to their identical credit risk, the two segments differ in their liquidity due to the about eleven times higher outstanding total volume and the more than three times higher average issue size of BUNDs compared to KfW bonds (see Table 3.1). Regarding trading volume and turnover, KfW collects information from 25 banks trading in its bonds and shares the data with us. From July 2011 to June 2012, traded volume in Euro denominated KfW bonds amounts to 127.4 billion EUR compared to 5.4 trillion EUR

Table 3.1: **Summary statistics for KfW bonds and BUNDs**
This table shows summary statistics for the bonds included in the sample. The observation period is February 14th, 1996 to September 29th, 2010.

	Kreditanstalt für Wiederaufbau (KfW)	German government bonds (BUND)
Number of bonds	68	227
Average time to maturity at issue date (in years)	6.09	7.52
Average coupon (in %)	4.13	4.90
Average issuing volume (incl. all reopenings) (in bn EUR)	2.99	9.83
Total volume (in bn EUR)	203	2 231

in BUNDs in 2012. KfW bonds are approximately turned over once per year, compared to a turnover of five for BUNDs. In contrast to the U.S. bond market, where trading activity concentrates on just issued on-the-run bonds, trading in KfW bonds is distributed relatively equally on all maturities (see also, e.g., Ejsing and Sihvonen, 2009, who find the on-the-run status to have only negligible influence on liquidity for BUNDs). Thus, we do not separate between on-the-run and off-the-run bonds. Note also that although KfWs and BUNDs are both accepted by the European Central Bank (ECB) as collateral for repo transactions, ECB divides securities in liquidity categories. KfWs are in the second highest category, whereas BUNDs are in the highest.[27]

The estimation of the zero coupon yield curves for both BUND $y_t^{BUND}(T)$ (where T denotes the time to maturity) and KfW $y_t^{KfW}(T)$ using the Nelson-Siegel approach is detailed in the Appendix B.1. The term structure of liquidity premia at time t is then obtained as the difference between the two estimated Nelson-Siegel curves

$$illiq_t(T) = y_t^{KfW}(T) - y_t^{BUND}(T). \tag{3.1}$$

[27]This leads to small additional haircuts for KfWs of up to 2% for the longest maturities. KfWs and BUNDs are both accepted by the Federal Reserve for discount window loans with the same margin haircuts. The Bank of England accepts KfWs only as 'wider collateral' which can be used for long-term open market operations and the discount window facility, whereas BUNDs are accepted for all monetary policy operations.

3.2.2 Economics of the KfW-BUND Yield Spread

An obvious question is why yield differences between KfW bonds and BUNDs are not arbitraged away. The KfW-BUND yield spread is in this aspect in line with the Refcorp-Treasury spread (see Longstaff, 2004), the spread between Treasury bonds and inflation-swapped TIPS issues (see Fleckenstein, Longstaff, and Lustig, 2013), or the spread between Treasuries of different liquidity (see Fontaine and Garcia, 2012; Banerjee and Graveline, 2013). Although the arbitrage strategy of buying a KfW bond and short-selling a government bond of the same maturity (and rolling over the respective repo positions) is riskless in theory, in practice this strategy is costly, it consumes capital, and it is risky in the short run. The strategy is costly as, in addition to direct transaction costs, repo rates for BUNDs are lower than for KfW bonds (see Banerjee and Graveline, 2013, for a related discussion on Treasury markets). It consumes capital as margins of KfW bonds and BUNDs are different (see footnote 27) and it is risky, at least in the short-run, as the yield spread could widen (see Liu and Longstaff, 2004, for a formal discussion).

Thus, besides interpreting our yield-spread as a liquidity measure, it could also be interpreted as a measure of textbook arbitrage in a frictionless market. In contrast to the yield-spreads discussed above, but also in contrast to a recent measure of price deviation in the yield curve of Treasuries (see Hu, Pan, and Wang, 2013), we are able to derive a full term structure of our measure. Each point on this term structure can be interpreted as an arbitrage risk premium, i.e., an ex-ante return of a textbook arbitrage strategy in the respective maturity. It therefore not only incorporates the severeness of frictions, but also market expectations regarding these frictions for different time horizons, as well as risk premia. To back up this interpretation, we regress yearly excess returns $rx_{t+1}(T)$ of T-year KfW bonds over T-year BUNDs on our liquidity premia of three maturities and their principal components (see, e.g., Cochrane and Piazzesi, 2005) for maturities T of two, five, and ten years:

$$rx_{t+1}(T) = \gamma_0 + \gamma_1 \cdot illiq_t(2) + \gamma_2 \cdot illiq_t(5) + \gamma_3 \cdot illiq_t(10) + \epsilon_{t+1}^T, \tag{3.2}$$

$$rx_{t+1}(T) = \gamma_0 + \gamma_1 \cdot PCilliq1_t + \gamma_2 \cdot PCilliq2_t + \gamma_3 \cdot PCilliq3_t + \epsilon_{t+1}^T. \tag{3.3}$$

The large R^2s in Table 3.2 show that our measure has indeed explanatory power for excess returns. In the regression of excess returns on liquidity premia in Equation (3.2), the premium with the maturity that equals the maturity of the excess return is always significant at the 1% level. When we use principal components instead of liquidity premia,

Table 3.2: **Excess return regressions**
In this table, we regress yearly excess returns $rx_{t+1}(T)$ of T-year KfW bond over T-year BUNDs on liquidity premia and their principal components. $rx_{t+1}(T)$ is the yearly excess return from buying a T-year KfW bond in t, financing it with a T-year BUND and liquidating the position one year later, i.e., $rx_{t+1}(T) \equiv r_{t+1}^{KfW}(T) - r_{t+1}^{BUND}(T)$. Log returns are defined, e.g., for KfW bonds, as $r_{t+1}^{KfW}(T) \equiv p_{t+1}^{KfW}(T-1) - p_t^{KfW}(T)$, where $p_t(T)$ denotes the log price of a T-year discount bond that can be derived directly from the estimated yield curves of KfW bonds and BUNDs. $illiq_t(T)$ refers to T-year liquidity premia. $PCilliq1_t$, $PCilliq2_t$, and $PCilliq3_t$, denote the first, second, and third principal component derived from the two, five, and ten year liquidity premia. Standard errors are calculated using a Newey-West correction with 18 lags and are given in parentheses. The observation period is from February 1996 to September 2010 (164 monthly observations).

	$rx_{t+1}(T) = \gamma_0 + \gamma_1 \cdot illiq_t(2) + \gamma_2 \cdot illiq_t(5) + \gamma_3 \cdot illiq_t(10) + \epsilon_{t+1}^T$			$rx_{t+1}(T) = \gamma_0 \cdot const + \gamma_1 \cdot PCilliq1_t + \gamma_2 \cdot PCilliq2_t + \gamma_3 \cdot PCilliq3_t + \epsilon_{t+1}^T$		
	Short T=2 yr.	Medium T=5 yr.	Long T=10 yr.	Short T=2 yr.	Medium T=5 yr.	Long T=10 yr.
γ_0	−0.0019**	−0.0041*	−0.0095*	0.0014*	0.0014	0.0013
	(0.0006)	(0.0017)	(0.0039)	(0.0006)	(0.0014)	(0.0029)
γ_1	0.0114**	−0.0112	−0.0192	0.0017**	0.0028**	0.0052**
	(0.0039)	(0.0097)	(0.0192)	(0.0001)	(0.0004)	(0.0009)
γ_2	0.0084	0.0459**	−0.0003	−0.002**	0.0015	0.0117**
	(0.0063)	(0.0154)	(0.0362)	(0.0006)	(0.0012)	(0.0023)
γ_3	−0.0035	−0.0025	0.0798**	0.0004	0.0068*	−0.0042
	(0.0038)	(0.0053)	(0.0183)	(0.0013)	(0.0029)	(0.007)
adj. R^2	0.5473	0.396	0.3701	0.5473	0.396	0.3701

the first principal component is always significant. Interestingly, the second principal component is significant with oposite signs for two and ten year excess returns which confirms its interpretation as the slope of the term structure of risk premia.[28]

To provide further support that liquidity premia are driven by liquidity differences between the two segments, we analyze the time series relation between liquidity premia and corresponding transaction cost measures for our three maturity segments in more detail. In contrast to the U.S. corporate bond market, the last price of a trade on a day, but also high- and low prices for a given German government or agency bond are not publicly

[28]As in Cochrane and Piazzesi (2005), we use month-end instead of weekly data to reduce overlapping of yearly excess returns. R^2s are slightly lower when we exclude the financial crisis and are larger when only incorporating the crisis. The results are robust when using lagged liquidity premia (see Cochrane and Piazzesi, 2005) or when calculating excess returns of T-year KfW bonds over 1-year BUNDs (instead of T-year BUNDs).

available. Therefore, we cannot use transaction cost measures based on trade prices like Roll (1984), Hasbrouck's (2009) Gibbs measure, or the high-low spread estimator from Corwin and Schultz (2012), which performed best in our analyses in Chapter 2. From the quote based measures, we cannot use bid-ask spreads calculated from CBBT prices as this pricing source is not available before 2004. Based on the results in Chapter 2, we therefore use average bid-ask spreads from all dealers quoting prices in Bloomberg. As Table 2.4 shows, average bid-ask spreads from all dealers (P_Spread-Mean) have higher time series and cross sectional correlations with our benchmark measures than bid-ask spreads from Bloomberg's Generic pricing source (P_Spread-BGN) and the quote dispersion measures (P_QuoteDispersion and P_QDmid). Especially the high time series correlations between bid-ask spreads from all dealers' quotes and our intraday transaction cost benchmarks in Table 2.4 confirm that this liquidity measure captures liquidity dynamics well. We use the average bid-ask spreads of all dealers' quotes to estimate a 'term structure of bid-ask spreads' (for details, see Appendix B.2).

Figure 3.1 presents the evolution of the two, five, and ten year liquidity premia over time together with the respective quoted bid-ask spread differences between KfW bonds and BUNDs. For all three maturity segments, the figure shows a remarkable connection between bid-ask spread differences and liquidity premia. Their large unconditional correlations of 0.89 for two years, 0.85 for five, and 0.81 for ten years maturity provide clear evidence in favor of liquidity-driven yield spreads. But despite of the large unconditional correlations, when looking only at the period before the beginning of the subprime crisis from August 2001 to June 2007, correlations are much lower with 0.01 for two years, 0.16 for five years, and with -0.11 for ten years even (insignificantly) negative. In contrast, in the crisis period since June 2007 up to the end of the observation period, correlations are between 0.8 and 0.85 for all three series. This observation confirms findings of Acharya, Amihud, and Bharath (2013) that the impact of liquidity on bond prices is primarily relevant in times of stress. Therefore, in our analysis of the economic drivers of liquidity premia in Section 3.3, we use a regime-switching model allowing for different sensitivities during stress and normal times.

Liquidity premia of maturity two, five, and ten years (as well as their first principal component) are all significantly negatively correlated with German GDP. More importantly, the slope of the term structure of liquidity premia, i.e., the difference between ten year and two year liquidity premia, is significantly positively correlated with GDP. Thus, the shape of the term structure of liquidity premia is related to the economic environment. To further analyze the different shapes over time, Figure 3.2 plots the slope of

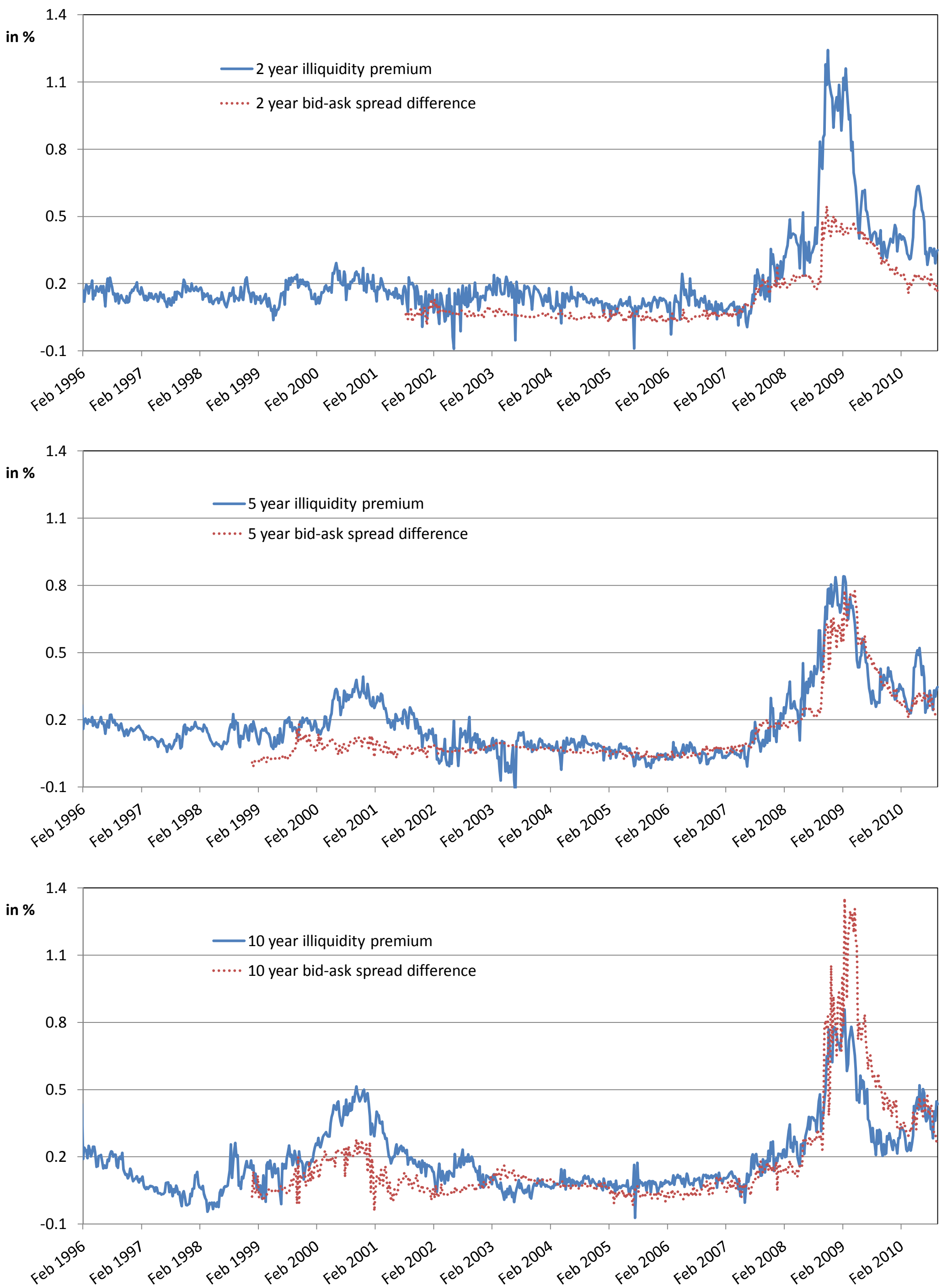

Figure 3.1: **Liquidity premia and quoted bid-ask spreads**
This figure shows the development of liquidity premia (solid lines) and quoted relative bid-ask spread differences between KfW bonds and BUNDs (dotted lines) over time. The upper graph depicts a time to maturity of two years, the middle graph provides five years, and the lower graph ten years time to maturity. Quoted bid-ask spread differences can only be calculated since January 6th, 1999 for five and ten years and since August 22nd, 2001 for two years maturity. The observation period is from February 14th, 1996 to September 29th, 2010 (764 weekly observations).

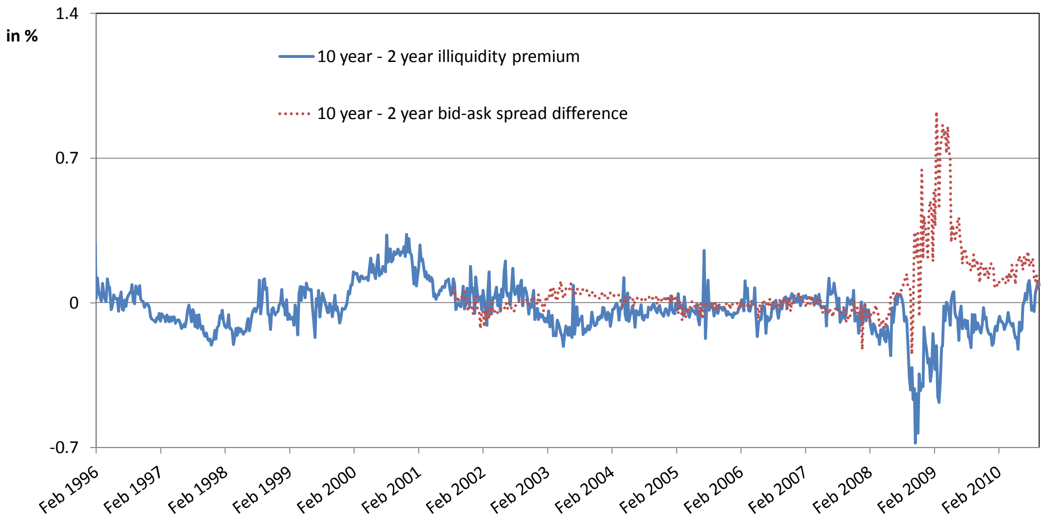

Figure 3.2: **Slope of the term structure of liquidity premia and bid-ask spread differences**
This figure shows the development of the slopes of the term structures of liquidity premia (solid line) and quoted relative bid-ask spread differences between KfW bonds and BUNDs (dotted line) over time. The slope of quoted bid-ask spread differences can only be calculated since August 22nd, 2001. The observation period is from February 14th, 1996 to September 29th, 2010 (764 weekly observations).

the term structure of liquidity premia together with the slope of the term structure of bid-ask spreads. Whereas the slope of the term structure of liquidity premia is negative during the financial crisis and also after the burst of the dot-com bubble, the slope of bid-ask spread differences between KfW bonds and BUNDs is positive during these times. The fact that rising term structures of bid-ask spreads do not simply transfer to rising term structures of liquidity premia, together with the close connection of bid-ask spreads and liquidity premia observed in Figure 3.1, suggests that there is no simple one-to-one relation between bid-ask spreads and liquidity premia. Rather, more fundamental factors, that potentially impact liquidity premia of different maturities differently, seem to be at play. We will explore this question further in the next section and more generally analyze how liquidity premia depend on global demand and supply of liquidity.

3.3 Term Structure Dynamics: A Conditional Approach

We are now ready to study the drivers of the term structure of liquidity premia in a regime-switching model in order to analyze the influence of liquidity supply and demand on liquidity premia of different maturities conditional on the economic environment.[29]

3.3.1 Economic Factors Driving Liquidity Premia

We measure the supply of liquidity with the global availability of arbitrage capital. As argued, e.g., by Duffie (2010), Gromb and Vayanos (2010), and many other papers in the context of the current discussion on slow moving capital, the provision of liquidity depends on sophisticated arbitrageurs providing liquidity to less sophisticated investors. Hu, Pan, and Wang (2013) proxy for the global availability of arbitrage capital with a measure of noise in the yields of U.S. Treasuries. They argue that in episodes of low liquidity, a shortage of arbitrage capital allows prices to fluctuate more freely relative to the yield curve. Moreover, they show that their measure is more informative on liquidity conditions than other liquidity measures like the bid-ask spread. The interpretation of this measure is also consistent with Vayanos and Vila's (2009) notion of arbitrageurs that arbitrage away yield differences between different maturities. We use data on the noise measure available on Jun Pan's website.

We control for possible differences in the global versus local availability of arbitrage capital with data on foreign flows into German bond markets. Deutsche Bundesbank calculates net fund flows of foreign investors into bonds from public and non-public issuers, where the KfW is classified as a non-public issuer. We expect an inflow of capital in bonds from public issuers to increase the liquidity difference between KfW and BUND and therefore widen liquidity premia. On the other hand, a capital inflow into bonds from non-public issuers might decrease the liquidity difference and thus lower liquidity premia. Data on net foreign flows (in EUR) is available from Deutsche Bundesbank and we deflate the data with the consumer price index.

In addition to these market wide proxies, bond specific liquidity differences between the

[29]The regime-switching behavior of liquidity premia is also supported by Chow tests rejecting the null hypothesis of parameter constancy in autoregressive models of different lag length for a wide range of break dates and all three series.

two segments are proxied by differences in the outstanding volume that is freely available for trading. For this, we construct measures of the amount of bonds outstanding of the representative two, five, and ten year KfW bond relative to the outstanding amount of the corresponding German government bond (for details, see Appendix B.3). As can be seen in Table 3.3, the average outstanding volume of the representative KfW bond is about 20% of the volume of its BUND counterpart.

We measure demand for liquidity with proxies for future trading needs and market wide risk premia. As a measure for information flowing into the market and thus for future trading needs, we select the benchmark volatility index for the German stock market (VDAX New). VDAX New is calculated by Deutsche Börse from options on futures on the DAX. As a measure of global financial uncertainty, which is closely linked to future liquidation needs, we use the TED spread. Brunnermeier (2009) points out that in times of higher uncertainty in the banking system, the risk of unsecured loans rises which in turn leads to higher LIBOR rates. Additionally, in times of higher uncertainty the value of first rate collateral rises pushing down T-Bill rates and widening the TED spread further. As a proxy for market wide risk premia or required returns, we select the dividend yield of the German stock market index DAX. Cochrane (2011) points out that variations in the market wide dividend yield reflect changes in risk premia rather than changes in future dividend growth (see also, e.g., Gârleanu and Pedersen, 2011). The dividend yield is calculated by Bloomberg under the assumption that for all 30 constituents of the DAX, realized dividends in the year before the observation date are paid as an infinite annuity. It is available after May 7th, 1997. We use the first principal component of VDAX New, TED spread, and dividend yield as our aggregate measure for liquidity demand.[30]

As we want to separate the effects of liquidity supply and demand, we do not use bid-ask spreads in our main analysis. Bid-ask spreads could be interpreted as price for immediate liquidity provision, and are thus influenced by both demand and supply of liquidity. However, we use bid-ask spreads in the robustness section for a shortened observation period as they are available only since August 2001 for all maturities.

As an additional explanatory variable, we include a measure for the market wide credit spread to control for perceived credit risk. We use the spread between the Bloomberg index for the yield of BBB rated industrial USD bonds and the corresponding AA index. Note

[30]It is not possible to separate the impacts of trading needs and risk premia, since both TED spread and VDAX New, besides being measures for trading needs, are also expected to be sensitive to an increase in risk premia. Moreover, the dividend yield is supposed to rise in uncertain times (with high future liquidation needs) due to declining stock prices.

Table 3.3: **Summary statistics of explanatory variables**
This table shows summary statistics for the variables included in the analysis. $Noise$ refers to a measure of noise in U.S. Treasury yields (in bps, see Hu, Pan, and Wang, 2013). $ForeignPub$ ($ForeignNonPub$) refers to the capital inflow into German bonds from public (non-public) issuers deflated with the consumer price index (in trillions of EUR, prices of 2005). $Volume(T)$ measures the outstanding volume of the representative KfW bond compared to its BUND counterpart with T years to maturity and $LiqDemand$ is the first principal component of the VDAX New, the TED spread, and the dividend yield of the DAX. $Credit(T)$ refers to the spread between the Bloomberg indices for AA rated corporate bonds and BBB rated corporate bonds (in percentage points). The observation period is May 7th, 1997 to September 29th, 2010 (700 weekly observations).

	Mean	Standard Deviation	Minimum	Median	Maximum
$Noise$	3.5557	2.7249	1.0424	2.8817	20.4675
$ForeignPub$	0.0036	0.0077	-0.0151	0.0032	0.0284
$ForeignNonPub$	0.0047	0.0083	-0.0241	0.0055	0.0223
$Volume(2)$	0.2117	0.0787	0.0838	0.2108	0.3782
$Volume(5)$	0.2014	0.0544	0.0923	0.2051	0.3031
$Volume(10)$	0.2452	0.0624	0.1324	0.2456	0.3763
$LiqDemand$	0	1.2989	-1.6635	-0.2836	7.8201
$Credit(2)$	0.7344	0.4583	0.1500	0.5992	2.3919
$Credit(5)$	0.7783	0.4309	0.1400	0.6947	2.4948
$Credit(10)$	0.7955	0.3577	0.3308	0.7293	2.4632

that we cannot utilize credit spreads of EUR bonds since these are available only after August 2001. The credit spread indexes are available with different maturities and we use the index with the corresponding time to maturity in the regression equations for short-, medium-, and long-term liquidity premia. Table 3.3 shows summary statistics for our explanatory variables.

3.3.2 Methodology

Most authors put liquidity stress periods on a level with financial crises and rely on exceptional events to identify them (see, e.g., Chordia, Sarkar, and Subrahmanyam, 2005).[31] In contrast, we endogenously identify liquidity stress periods by means of the Markov regime-switching model first proposed by Hamilton (1989). So essentially the data tell us, when the system is likely to be in the stress regime. We also check the robustness of our

[31]See also Barrell et al. (2010) for a short discussion of the problems of exogenous crisis identification.

results using exogenously specified financial crisis periods in Section 3.4

To analyze the different behavior of liquidity premia during liquidity stress periods and normal times, we estimate a two-regime AR model for two, five, and ten year liquidity premia and augment it with the economic drivers discussed in Section 3.3.1 An autoregressive model in levels is used to capture level relations between liquidity premia and our explanatory factors (see also Kempf, Korn, and Uhrig-Homburg, 2012).[32]

$$
\begin{aligned}
illiq_t(2) = a_{0,s}^{2y} + \sum_{i=1}^{p} \left(b_{i,s}^{2y} illiq_{t-i}(2) \right) + a_{1,s}^{2y} Noise_t + a_{2,s}^{2y} ForeignPub_t \\
+ a_{3,s}^{2y} ForeignNonPub_t + a_{4,s}^{2y} Volume_t(2) + a_{5,s}^{2y} LiqDemand_t \\
+ a_{6,s}^{2y} Credit_t(2) + \epsilon_{s,t}^{2y},
\end{aligned} \tag{3.4}
$$

$$
\begin{aligned}
illiq_t(5) = a_{0,s}^{5y} + \sum_{i=1}^{p} \left(b_{i,s}^{5y} illiq_{t-i}(5) \right) + a_{1,s}^{5y} Noise_t + a_{2,s}^{5y} ForeignPub_t \\
+ a_{3,s}^{5y} ForeignNonPub_t + a_{4,s}^{5y} Volume_t(5) + a_{5,s}^{5y} LiqDemand_t \\
+ a_{6,s}^{5y} Credit_t(5) + \epsilon_{s,t}^{5y},
\end{aligned} \tag{3.5}
$$

$$
\begin{aligned}
illiq_t(10) = a_{0,s}^{10y} + \sum_{i=1}^{p} \left(b_{i,s}^{10y} illiq_{t-i}(10) \right) + a_{1,s}^{10y} Noise_t + a_{2,s}^{10y} ForeignPub_t \\
+ a_{3,s}^{10y} ForeignNonPub_t + a_{4,s}^{10y} Volume_t(10) + a_{5,s}^{10y} LiqDemand_t \\
+ a_{6,s}^{10y} Credit_t(10) + \epsilon_{s,t}^{10y},
\end{aligned} \tag{3.6}
$$

where the state $s \in \{1, 2\}$ follows a homogeneous Markov chain with constant transition probabilities[33]

$$
\begin{aligned}
P(s_t = 1 | s_{t-1} = 1) = p_{1,1}, \\
P(s_t = 2 | s_{t-1} = 2) = p_{2,2}.
\end{aligned} \tag{3.7}
$$

The vector of error terms $(\epsilon_{s,t}^{2y}, \epsilon_{s,t}^{5y}, \epsilon_{s,t}^{10y})$ is multi-normally distributed with mean zero and

[32] Augmented Dickey Fuller (ADF) tests for the time series of liquidity premia reject the non-stationary hypothesis for the time period before the beginning of the financial crisis in June 2007 for all three time series of liquidity premia. If we include the whole time period, non-stationarity can only be rejected for two and five year premia. As these results are inconclusive regarding the stationarity of the variables, lagged values of the endogenous variables ensure that asymptotic distributions of the regression coefficients maintain their standard form (see Sims, Stock, and Watson, 1990).

[33] Although switching probabilities are assumed to be constant, the probability to be in the stress regime depends on the economic environment as illustrated in Section 3.3.3, Figure 3.3. See also Acharya, Amihud, and Bharath (2013) for a related discussion.

variance-covariance matrix Ω_s where

$$\Omega_s = \begin{pmatrix} (\sigma_s^{2y})^2 & \rho_s^{2y,5y} \cdot \sigma_s^{2y} \cdot \sigma_s^{5y} & \rho_s^{2y,10y} \cdot \sigma_s^{2y} \cdot \sigma_s^{10y} \\ \rho_s^{2y,5y} \cdot \sigma_s^{2y} \cdot \sigma_s^{5y} & (\sigma_s^{5y})^2 & \rho_s^{5y,10y} \cdot \sigma_s^{5y} \cdot \sigma_s^{10y} \\ \rho_s^{2y,10y} \cdot \sigma_s^{2y} \cdot \sigma_s^{10y} & \rho_s^{5y,10y} \cdot \sigma_s^{5y} \cdot \sigma_s^{10y} & (\sigma_s^{10y})^2 \end{pmatrix}. \tag{3.8}$$

We select this flexible variance-covariance matrix to allow for heteroskedasticity between the two regimes. Also, correlations of the error terms of different segments can be regime-switching. The model is estimated along the lines described in Hamilton (1994) using the expectation-maximization (EM) algorithm to maximize the log-likelihood function.

3.3.3 Results

Table 3.4 gives the estimation results of Model (3.4)-(3.8) with $p = 2$ lags. Due to the availability of data for our exogenous variables, the observation period is May 7th, 1997 to September 29th, 2010. We choose a lag-length of two to allow for a possible influence of past changes of liquidity premia, but results are generally robust if we use different lag lengths. We first present results regarding regime identification and average term structures of liquidity premia within the two regimes. We then discuss the results regarding our economic drivers.

The estimation of the parameters delivers the probability of the system being in the stress regime for each date in the sample. This probability is plotted in Figure 3.3. Stress periods can often be associated with economic events that might be causal for poor liquidity. So the 1998 bailout of LTCM, the period after the burst of the dot-com bubble as well as the financial crisis starting in summer 2007 are all identified as liquidity stress periods.

Figure 3.4 shows average term structures of liquidity premia in both regimes. A clear separation in the two regimes can be recognized. Whereas in the non-stress regime (regime 1), on average the extra yield to maturity of an illiquid KfW bond compared to the liquid BUND is around 15 bps, this liquidity premium nearly doubles in the stress regime (regime 2). Additionally, the standard deviation of the innovations is about two to three times larger in the stress regime. The shape of the term structure is slightly U-shaped in both regimes, but the decreasing part is much more pronounced in the stress regime due to large short-term liquidity premia. This result confirms our observation from Figure 3.2 that within crisis periods, the slope of the term-structure is negative.

Table 3.4: **Estimation results for Markov regime-switching AR model with exogenous variables**
This table shows the results of the maximum likelihood estimation of Model (3.4)-(3.8) with $p = 2$. White's (1982) standard errors are given in parentheses. *, ** indicate significance at the 5% or 1% level. The observation period is May 7th, 1997 to September 29th, 2010.

	Regime 1 (normal times)			**Regime 2 (stress regime)**		
	Short T=2 yr.	Medium T=5 yr.	Long T=10 yr.	Short T=2 yr.	Medium T=5 yr.	Long T=10 yr.
mean illiq. premium (T)	16.6 bps	14.1 bps	15.1 bps	31.3 bps	24.5 bps	25.3 bps
σ	0.0255	0.022	0.0246	0.0695	0.0584	0.0551
	(0.0024)	(0.0017)	(0.0018)	(0.0047)	(0.0041)	(0.0035)
correlation parameters	$\rho_1^{2y,10y}$	$\rho_1^{2y,5y}$	$\rho_1^{5y,10y}$	$\rho_2^{2y,10y}$	$\rho_2^{2y,5y}$	$\rho_2^{5y,10y}$
	0.0915	0.6702	0.3899	0.171	0.4801	0.4008
	(0.0702)	(0.0537)	(0.0626)	(0.0702)	(0.0691)	(0.0605)
transition probabilities		$p_{1,1}$			$p_{2,2}$	
		0.907			0.8483	
		(0.0231)			(0.0495)	
$Constant$	0.0047	0.0043	0.0073	0.0564**	0.0651**	0.0127
	(0.0069)	(0.0065)	(0.0112)	(0.021)	(0.0235)	(0.034)
$illiq_{t-1}(T)$	0.6728**	0.7594**	0.7009**	0.5224**	0.5404**	0.5181**
	(0.0916)	(0.1003)	(0.0705)	(0.0766)	(0.081)	(0.0851)
$illiq_{t-2}(T)$	0.2607**	0.186	0.2659**	0.2207**	0.2095**	0.2871**
	(0.0859)	(0.1089)	(0.072)	(0.0589)	(0.0686)	(0.0757)
$Noise_t$	0.001	0.0012	0.0013	0.006	0.005*	0.0061*
	(0.002)	(0.0019)	(0.0022)	(0.0032)	(0.0023)	(0.0029)
$ForeignPub_t$	0.1586	0.2585	0.3128	0.6017	1.0487	0.9702*
	(0.2261)	(0.1807)	(0.218)	(0.6955)	(0.548)	(0.4921)
$ForeignNonPub_t$	−0.0129	−0.0454	−0.066	−2.2655**	−2.0112**	−1.34*
	(0.2472)	(0.2069)	(0.2482)	(0.7953)	(0.6731)	(0.643)
$Volume_t(T)$	0.0013	0.0159	−0.0097	0.0529	−0.1515	0.0595
	(0.023)	(0.0262)	(0.041)	(0.0658)	(0.0803)	(0.1164)
$LiqDemand_t$	0.0023	0.0037	0.0003	0.031**	0.0115	0.0034
	(0.0038)	(0.0033)	(0.0037)	(0.0079)	(0.0061)	(0.0049)
$Credit_t(T)$	0.0058	−0.0039	−0.0077	−0.0342*	−0.0005	−0.0094
	(0.0099)	(0.01)	(0.0119)	(0.0152)	(0.0163)	(0.0205)
	Log-Likelihood		N	AIC	BIC	
	4196.85		700	-8257.7	-7873.52	

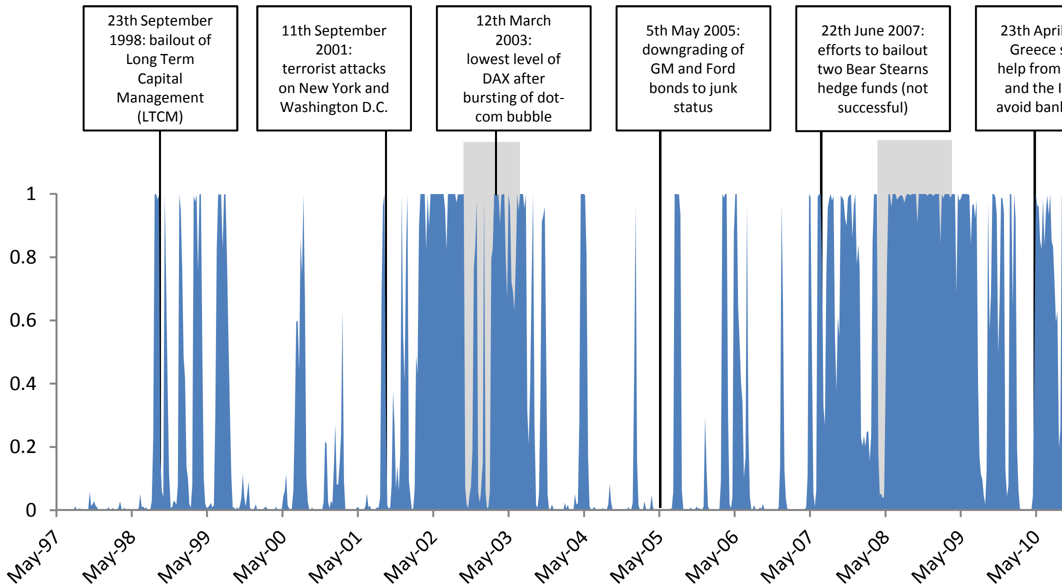

Figure 3.3: **Endogenously derived probability to be in the stress regime**
This figure shows smoothed probabilities of being in the stress regime (regime 2) estimated from the Markov regime-switching Model (3.4)-(3.8) with two lags. Additionally, events anecdotally linked to financial stress or low liquidity are marked. Recessions, defined as at least two consecutive quarters of negative real GDP growth in Germany (Q4 2002 – Q2 2003, and Q2 2008 – Q1 2009) are shaded.

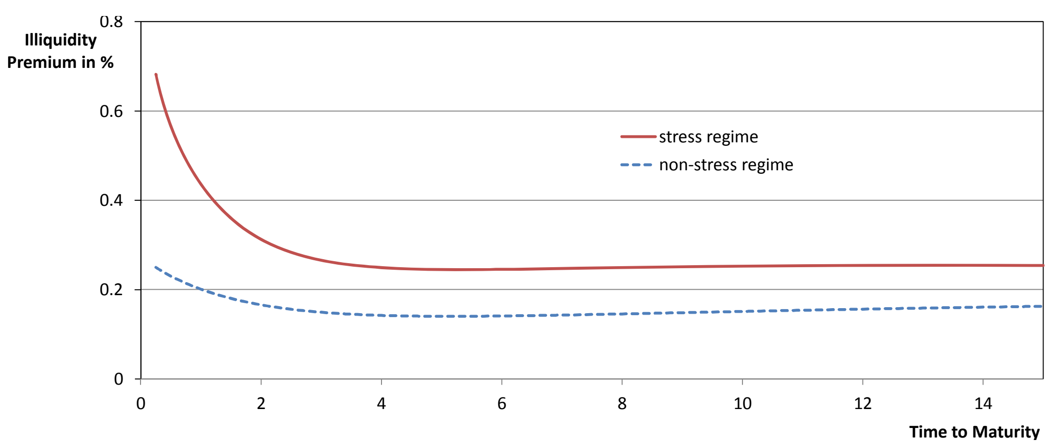

Figure 3.4: **Shapes of liquidity premia in different regimes**
This figure shows the shapes of the term structure of liquidity premia in the stress regime (solid line) and in the non-stress regime (dashed line). The average term structure of liquidity premia in one regime is calculated by weighting the term structure of each day with the probability to be in that regime on that date (see Figure 3.3).

We now discuss our main results concerning the explanatory variables in Table 3.4. First, the large and significant values of the lagged parameters for all maturities and both regimes show the high persistence of liquidity premia over time.

Second, liquidity supply proxied by the global availability of arbitrage capital as well as capital inflows into the German bond market is only significant in the stress regime. More available arbitrage capital proxied by lower noise in U.S. Treasury prices as well as capital inflows into bonds from non-public issuers (the KfW is classified by Bundesbank as a non-public issuer) lead to lower liquidity premia. On the contrary, more available arbitrage capital in the market for public bonds increases the KfW-BUND spread. To illustrate the economic significance, we look at a 10 billion Euro re-allocation of foreigners from public to non-public bond markets. Such a re-allocation leads to a decrease of, e.g., the ten year KfW-BUND spread of 2.3 bps. In Table 3.5, we analyze the regime-switching behavior of the sensitivities. The null hypotheses of identical influence of liquidity supply in both regimes (i.e., $H_0 : a^{2y}_{j,s=1} = a^{2y}_{j,s=2}$, $a^{5y}_{j,s=1} = a^{5y}_{j,s=2}$, and $a^{10y}_{j,s=1} = a^{10y}_{j,s=2}$ for $j \in \{1, 2, 3\}$) can only be rejected for capital inflow into bonds of non-public issuers and short- and medium-term premia, but parameter estimates for all nine liquidity supply parameters are clearly higher in the stress regime. The more important influence of liquidity supply in stressful times is consistent with Brunnermeier and Pedersen's (2009) result that 'the effect of speculator capital on market liquidity is highly nonlinear: a marginal change in capital has a small effect when speculators are far from their constraints, but a large effect when speculators are close to their constraints'.

The measured impact of capital inflows into the market for bonds of non-public issuers is well comparable to the effect of ECB's 2009 Covered Bond Purchase Programme (CBPP) on the covered bond market. Purchases of covered bonds in 2009 and 2010 amount to 60 billion Euro and tighten the spread of covered bonds on average by 12 bps (see Beirne, Dalitz, Ejsing, Grothe, Manganelli, Monar, Sahel, Sušec, Tapking, and Vong, 2011). This translates to a sensitivity of 2 bps per 10 billion Euro and approximately fits our sensitivity for medium-term liquidity premia of 2.0112 (% per trillion Euro = bps per 10 billion Euro).[34]

Third, liquidity differences, proxied by the fraction of outstanding volume of the representative KfW bond compared to its BUND counterpart, do not have any significant influence. A similar result is also observed for the Pfandbrief market by Kempf, Korn,

[34]The total outstanding notional volume of bonds from non-public issuers in prices of 2005 amounts to on average 1.7 trillion Euro during our observation period and is somewhat larger than the total outstanding volume of covered bonds of approximately 1.1 trillion Euro.

Table 3.5: **Regime-switching behavior of economic determinants**
This table shows the differences of the parameter estimates between the two regimes. The null hypothesis H_0 is that parameter estimates are identical in both regimes, i.e., the difference is 0. The Wald chi-squared statistics $W = (R\hat{\alpha} - r)'(R\hat{V}R')^{-1}(R\hat{\alpha} - r)'$ are given in square brackets, where R and r define the hypotheses for the parameter vector α. *, ** indicate rejection of H_0 at the 5% or 1% level. The observation period is May 7th, 1997 to September 29th, 2010.

	Differences between regimes		
	Short T=2 yr.	Medium T=5 yr.	Long T=10 yr.
$Constant : a_{0,s=2} - a_{0,s=1}$	0.0517^{*}	0.0608^{*}	0.0054
	[5.5435]	[6.3438]	[0.0236]
$illiq_{t-1}(T) : b_{1,s=2} - b_{1,s=1}$	−0.1504	−0.219	−0.1828
	[1.1564]	[2.0708]	[2.0653]
$illiq_{t-2}(T) : b_{2,s=2} - b_{2,s=1}$	−0.04	0.0235	0.0212
	[0.1139]	[0.0235]	[0.0308]
$Noise_t : a_{1,s=2} - a_{1,s=1}$	0.005	0.0038	0.0048
	[1.5277]	[1.3964]	[1.6014]
$ForeignPub_t : a_{2,s=2} - a_{2,s=1}$	0.4431	0.7902	0.6574
	[0.3352]	[1.8265]	[1.4862]
$ForeignNonPub_t : a_{3,s=2} - a_{3,s=1}$	-2.2526^{**}	-1.9658^{**}	−1.274
	[6.826]	[7.5632]	[3.194]
$Volume_t(T) : a_{4,s=2} - a_{4,s=1}$	0.0516	-0.1674^{*}	0.0692
	[0.5212]	[3.9848]	[0.279]
$LiqDemand_t : a_{5,s=2} - a_{5,s=1}$	0.0287^{**}	0.0078	0.0031
	[9.4873]	[1.0987]	[0.2247]
$Credit_t(T) : a_{6,s=2} - a_{6,s=1}$	-0.04^{*}	0.0034	−0.0017
	[4.6878]	[0.0275]	[0.0045]

and Uhrig-Homburg (2012). A possible explanation the authors provide is that perceived liquidity differences do not change with each issued bond but are rather static. Another explanation could be that the value investors attribute to liquidity increases with less available liquid BUNDs due to the law of supply and demand (see Krishnamurthy and Vissing-Jorgensen, 2010). If this is the case, there are two opposite effects from an increase of the BUND volume. First, the relative liquidity of KfW bonds decreases leading to an increase in liquidity premia. Second, the value of liquidity decreases which should decrease liquidity premia.

Fourth, coefficients on liquidity demand suggest that investors concentrate on short-term bonds when disengaging from illiquid securities in times of stress. The effect is both statistically and economically significant. A one standard deviation shock of our variable on liquidity demand leads to an impact on the two year liquidity premium of 4 bps in the stress regime. This is about 13% of the average premium in this regime. Moreover, the regime-switching impact of liquidity demand on the short end is confirmed by Table 3.5. In contrast, coefficients are insignificant both in normal times and for longer maturities. These results are particularly interesting as on the one hand, they constitute pronounced term-structure effects. On the other hand, they help to identify flight-to-liquidity periods that coincide with stressful periods. The increased demand for short-term and highly liquid BUNDs within these periods leads first to a strongly increased level of the short-term KfW-BUND yield spread (see Figure 3.4). Second, amplified by the increased wariness to bear risks within stress periods, effects stemming from liquidity demand become more important. This result also explains the opposite behavior of the slopes of the term structures of liquidity premia and bid-ask spreads in Figure 3.2: In stressful times, especially long-term bid-ask spreads increase strongly (presumably due to liquidity providers demanding larger compensations for the increased interest rate risk in long-term bonds). On the premium side, the effect is much more pronounced for short-term bonds with low risk due to flight to liquidity effects that drive up short-term yield difference between liquid BUNDs and illiquid KfW bonds.

Credit risk seems to have no effect on liquidity premia. Like in Longstaff (2004) for Refcorp bonds or in Kempf, Korn, and Uhrig-Homburg (2012) for the Pfandbrief market, parameter estimates are mostly negative and insignificant (for all but one maturity). For the one maturity, where credit risk is significant at the 5% level, the sign is negative. If credit spreads had an influence on the KfW-BUND spread, we would expect the parameter estimates to be positive. Note that our analysis does not simply imply that credit risk and liquidity premia are unrelated. Instead, unreported results show that economy wide credit spreads have significant explanatory power for the probability to be in the liquidity stress regime.

Overall, our results confirm the prediction of the theoretical literature (see, e.g., Brunnermeier and Pedersen, 2009) that the impact of changes in fundamentals on liquidity premia is significantly stronger when the system is in stress. Moreover, flight-to-liquidity effects contribute to declining term structures of liquidity premia in times of stress. Thus, calibrating, e.g., risk management models in normal times, when the influence of fundamentals on liquidity premia is weak, strongly underestimates the contribution of illiquidity to systematic risk and might systematically misjudge term structure effects.

3.4 Robustness

In this section, we perform several robustness checks. Most importantly, we use bid-ask spreads to proxy for the bonds' liquidity instead of our proxies for liquidity supply and demand. Additionally, we check the robustness against our regime identification methodology.[35]

Bid-ask spreads, as the price of immediate liquidity provision, are influenced by both liquidity supply and demand. Since our objective was to separate both effects on liquidity premia, we could not employ bid-ask spreads in Section 3.3 However, to validate our finding that the influence of economic factors on liquidity premia is much more pronounced in times of economic stress, we substitute our proxies for liquidity supply and demand through quoted bid-ask spread differences (see Figure 3.1) and re-estimate Model (3.4)-(3.8).

Table 3.6 presents the results for the shortened observation period since August 22nd, 2001 for which bid-ask spreads are available on a continuous basis. Bid-ask spread differences are significant for all maturities only in liquidity stress periods. The insignificance of bid-ask spread differences in normal times confirms the low correlations of bid-ask spreads and liquidity premia before the financial crisis discussed in Section 3.2.2

Next, we control for the mechanism to identify liquidity stress periods. As Boldin (1996) argues, the regime identification in Markov regime-switching models is sometimes

[35]In further unreported robustness checks, we control for the level of interest rates as well as specialness of BUNDs, look at the influence of the principal component analysis on our results, and exclude the time after the collapse of Lehman Brothers until the end of 2008 from the estimation. In all cases, our main results are qualitatively unchanged. The results are available from the authors upon request.

Table 3.6: **Robustness check: bid-ask spread**
This table shows the results of the maximum likelihood estimation of Model (3.4)-(3.8) with $p = 2$ with $BidAskDifr_t(T)$ instead of the proxies for liquidity supply and demand. $BidAskDifr_t(T)$ refers to quoted relative bid-ask spread differences between KfW bonds and BUNDs (see also Figure 3.1). White's (1982) standard errors are given in parentheses. *, ** indicate significance at the 5% or 1% level. The observation period is August 22nd, 2001 to September 30th, 2010.

	Regime 1 (normal times)			**Regime 2 (stress regime)**		
	Short T=2 yr.	Medium T=5 yr.	Long T=10 yr.	Short T=2 yr.	Medium T=5 yr.	Long T=10 yr.
mean illiq. premium (T)	16.0 bps	10.4 bps	12.0 bps	33.1 bps	25.1 bps	25.8 bps
σ	0.022	0.0171	0.0166	0.0767	0.06	0.053
	(0.0025)	(0.0013)	(0.0017)	(0.0055)	(0.004)	(0.0036)
correlation parameters	$\rho_1^{2y,10y}$	$\rho_1^{2y,5y}$	$\rho_1^{5y,10y}$	$\rho_2^{2y,10y}$	$\rho_2^{2y,5y}$	$\rho_2^{5y,10y}$
	−0.0682	0.5958	0.1445	0.2148	0.5014	0.4358
	(0.1268)	(0.0793)	(0.0896)	(0.0685)	(0.0671)	(0.0589)
transition probabilities		$p_{1,1}$			$p_{2,2}$	
		0.8372			0.8535	
		(0.04)			(0.0366)	
Constant	0.0102**	0.0016	0.0112	0.0041	0.0099	0.0099
	(0.0035)	(0.0034)	(0.0062)	(0.0105)	(0.0084)	(0.0119)
$illiq_{t-1}(T)$	0.6923**	0.7446**	0.5949**	0.6141**	0.5565**	0.5211**
	(0.082)	(0.0731)	(0.0633)	(0.0712)	(0.0731)	(0.0683)
$illiq_{t-2}(T)$	0.1372**	0.1109	0.2783**	0.2619**	0.2243**	0.3053**
	(0.0451)	(0.0763)	(0.0455)	(0.062)	(0.0684)	(0.0786)
$BidAskDifr_t$	0.13	0.0046	−0.0072	0.1965**	0.2448**	0.0825*
	(0.076)	(0.0267)	(0.0259)	(0.0681)	(0.0525)	(0.0323)
$Credit_t(T)$	0.0059	0.015	0.0003	0.0032	−0.0056	0.0173
	(0.0081)	(0.0087)	(0.0106)	(0.0136)	(0.0148)	(0.0178)
	Log-Likelihood		N	AIC	BIC	
	2741.12		476	-5394.23	-5162.62	

vulnerable to relatively small changes in the data. To rule out such an effect in our study, we perform our analysis (a) with a new data set with month-end liquidity premia and explanatory variables and (b) by exogenously specifying crisis and non-crisis periods.

The monthly analysis with the new data set in Table 3.7 confirms our main results. In the stress regime, the influence of liquidity supply proxied by the noise in U.S. Treasury prices and the two flow variables is significant in the expected direction in six out of nine cases. In contrast, in normal times, only three parameter-maturity combinations yield significant results. Additionally, all but one of the estimated sensitivities are larger in the stress regime. Although liquidity demand is now significant also for short-maturities in normal times, the influence is about three times larger during times of economic stress.

For the exogenous crises specification, we define the LTCM crisis as the time between June and October 1998 (see Acharya and Pedersen, 2005). The beginning of the dot-com stress period is dated on May 10th, 2001, the day the ECB started cutting back interest rates. As the end of the crisis, we select the end of the 2002-03 recession in Germany in June 2003. The financial crisis starts in June 2007 when two of Bear Stearn's hedge funds ran into trouble and transforms into the European debt crisis which lasts up to the end of the observation period.

The results in Table 3.8 confirm our main findings. In crisis times, less available arbitrage capital proxied by more noise in U.S. Treasury prices significantly increases medium- and long-term liquidity premia. Additionally, the foreign flow variables are significant in the expected direction in three out of six cases. Again, liquidity demand is only significant in stress periods and for short-term maturities. In normal times, none of the explanatory variables are significant. Although conclusions do not change when using exogenously defined crisis dates, our approach to endogenously identify stress periods does not require to assume that liquidity stress periods and financial crises fully coincide.

3.5 Conclusion

In this chapter, we extract the term structure of liquidity premia from the spread between two bond classes differing only in their liquidity. The availability of a data set of homogeneous bonds spanning a large time to maturity segment over a long period of time allows us to quantify the term structure of liquidity premia without strong assumptions regarding the separation of credit and liquidity risk. We analyze this term structure in a setting allowing for a different behavior during stressful and normal periods. We find that

Table 3.7: **Robustness check: monthly analysis**
This table shows the results of the maximum likelihood estimation of Model (3.4-(3.8) with $p = 2$. White's (1982) standard errors are given in parentheses. *, ** indicate significance at the 5% or 1% level. The observation period is May 31th, 1997 to September 30th, 2010.

	Regime 1 (normal times)			**Regime 2 (stress regime)**		
	Short T=2 yr.	Medium T=5 yr.	Long T=10 yr.	Short T=2 yr.	Medium T=5 yr.	Long T=10 yr.
mean illiq. premium (T)	13.2 bps	10.2 bps	10.3 bps	41.0 bps	35.4 bps	37.8 bps
σ	0.0331	0.0298	0.0414	0.0725	0.0593	0.0636
	(0.0031)	(0.0034)	(0.0044)	(0.0084)	(0.0048)	(0.0053)
correlation parameters	$\rho_1^{2y,10y}$	$\rho_1^{2y,5y}$	$\rho_1^{5y,10y}$	$\rho_2^{2y,10y}$	$\rho_2^{2y,5y}$	$\rho_2^{5y,10y}$
	−0.0064	0.1121	0.0674	−0.1371	0.4163	0.6444
	(0.1084)	(0.1291)	(0.0897)	(0.2188)	(0.1653)	(0.0931)
transition probabilities		$p_{1,1}$			$p_{2,2}$	
		0.9614			0.9375	
		(0.0171)			(0.0407)	
Constant	0.0636**	0.0251	−0.0559	0.0771	0.1733**	0.2018**
	(0.0208)	(0.0175)	(0.0317)	(0.0836)	(0.0647)	(0.063)
$illiq_{t-1}(T)$	0.5203**	0.3846**	0.373**	0.5199**	0.3381*	0.4194**
	(0.108)	(0.1138)	(0.1223)	(0.1151)	(0.1483)	(0.1179)
$illiq_{t-2}(T)$	0.1202	0.166	0.0924	−0.3214**	−0.0315	0.111
	(0.125)	(0.0871)	(0.0721)	(0.1062)	(0.1214)	(0.1077)
$Noise_t$	0.0019	0.0178**	0.0134**	0.0103	0.0139**	0.0213*
	(0.0046)	(0.0047)	(0.0046)	(0.0095)	(0.0052)	(0.009)
$ForeignPub_t$	0.3677	0.7525*	0.7661	3.5481**	1.7063	1.5629
	(0.3814)	(0.3378)	(0.4954)	(1.3459)	(1.0037)	(1.2025)
$ForeignNonPub_t$	−0.158	−0.3314	0.1837	−3.2908	−2.878*	−3.2047**
	(0.5604)	(0.3728)	(1.0312)	(1.7255)	(1.4405)	(1.1282)
$Volume_t(T)$	0.0423	0.0484	0.2581	0.4814	−0.3251	−0.2809*
	(0.0572)	(0.0677)	(0.1326)	(0.3362)	(0.3588)	(0.1392)
$LiqDemand_t$	0.0237**	0.0133	0.0067	0.063*	0.011	−0.0074
	(0.0077)	(0.0071)	(0.0066)	(0.0278)	(0.0125)	(0.0232)
$Credit_t(T)$	−0.0394	−0.0585*	0.0168	0.0441	0.0518	−0.0427
	(0.0265)	(0.0281)	(0.0317)	(0.0418)	(0.0495)	(0.0592)
	Log-Likelihood		N	AIC	BIC	
	847.46		161	-1558.91	-1274.67	

Table 3.8: **Robustness check: exogenous crises specification**
This table shows the results of the estimation of Model (3.4) - (3.6) with exogenously specified stress periods (LTCM-crisis: June, 1st to October 31st, 1998; burst of dot-com bubble: May 10th, 2001 to June 30th, 2003; financial and subsequent European debt crisis: June 1st, 2007 to September 29th, 2010). Newey and West's (1987) standard errors with five lags are given in parentheses. *, ** indicate significance at the 5% or 1% level. The total observation period is May 7th, 1997 to September 29th, 2010.

	Non-Crisis			**Crisis**		
	Short T=2 yr.	Medium T=5 yr.	Long T=10 yr.	Short T=2 yr.	Medium T=5 yr.	Long T=10 yr.
mean illiq. premium(T)	14.1 bps	12.0 bps	13.5 bps	32.6 bps	25.7 bps	26.0 bps
$Constant$	0.049^{**}	0.0042	−0.0045	0.0567^{*}	0.0311	−0.021
	(0.0127)	(0.0054)	(0.0123)	(0.0235)	(0.017)	(0.0209)
$illiq_{t-1}(T)$	0.3952^{**}	0.756^{**}	0.641^{**}	0.5886^{**}	0.6702^{**}	0.6055^{**}
	(0.0646)	(0.0496)	(0.0657)	(0.0682)	(0.0637)	(0.0668)
$illiq_{t-2}(T)$	0.2637^{**}	0.1694^{**}	0.3197^{**}	0.2225^{**}	0.1972^{**}	0.2115^{**}
	(0.0511)	(0.0513)	(0.0618)	(0.0592)	(0.0649)	(0.0663)
$Noise_t$	0.0015	0.0021	0.0032	0.0039	0.0039^{*}	0.006^{*}
	(0.0023)	(0.0019)	(0.0022)	(0.0029)	(0.0017)	(0.0024)
$ForeignPub_t$	−0.0282	0.1365	0.2895	0.2765	0.8315^{*}	0.9995^{**}
	(0.2262)	(0.1556)	(0.2153)	(0.4874)	(0.3915)	(0.3363)
$ForeignNonPub_t$	−0.1056	−0.2627	−0.2689	−1.0905	−0.8993	-0.9107^{*}
	(0.246)	(0.1633)	(0.2578)	(0.6211)	(0.4778)	(0.3753)
$Volume_t(T)$	−0.0368	−0.0065	−0.0077	−0.0963	−0.0452	0.1847^{**}
	(0.0276)	(0.0361)	(0.0521)	(0.0799)	(0.0695)	(0.0691)
$LiqDemand_t$	0.0083	0.0024	−0.0019	0.0207^{**}	0.007	0.0037
	(0.0046)	(0.0042)	(0.004)	(0.0055)	(0.0037)	(0.0031)
$Credit_t(T)$	0.0164	0.0061	0.0047	−0.0056	−0.0114	−0.0167
	(0.0115)	(0.012)	(0.012)	(0.0151)	(0.0124)	(0.0174)
adj. R^2	0.5393	0.8978	0.9267	0.9465	0.9332	0.9347

the term structure of liquidity premia varies over time and is strongly dependent on the general financial and economic situation. The availability of arbitrage capital influences all maturities, whereas our measure for liquidity demand only impacts short-term maturities. The regression coefficients display a significant impact only in the stress regime.

Our findings imply that systematic liquidity risk is prone to be underestimated. Through its regime-switching behavior, the illiquidity discount increases sharply when the general state of the economy is bad. Additionally, the sensitivity of liquidity premia to fundamentals increases in stressful periods. Ignoring one of these two channels systematically underestimates liquidity risk. From the issuer's perspective, our results show that in times of stress, it is even more important to optimize the liquidity of an issue. This is particularly true in the light of systematically different term structure effects during normal times and liquidity stress periods.

Chapter 4

A Heterogeneous Agents Equilibrium Model for the Term Structures of Liquidity Premia and Trading Volume

4.1 Introduction

After exploring the term structure of liquidity premia empirically in crisis and non-crisis times in the previous chapter, we now derive an equilibrium model for this term structure as well as for trading volumes. The model provides a new perspective on our previous results and at the same time helps to unify the different empirical outcomes regarding the shape of the term structure of liquidity premia of other papers. Our unified framework also explains the empirically observed hump-shaped term structure of trading volume and the well-documented aging effect (see, e.g., Alexander, Edwards, and Ferri, 2000; Edwards, Harris, and Piwowar, 2007): other things equal, old bonds trade less frequently than newly issued bonds.

In our model, agents with heterogeneous investment horizons trade bonds with a continuum of different maturities in a market with two simple frictions: transaction costs and shocks to investors' time preference parameter. If a preference shock occurs, the investor faces the trade-off between the cost (in terms of utility) of awaiting the asset's maturity, which is higher for long-term bonds, and the bid-ask spread charged by an exogenous market maker or dealer. Prior to the preference shock, the investor determines her optimal portfolio allocation by comparing the higher return earned when holding a long-term bond until the maturity date to the higher expected costs of selling this asset in case of a potential preference shock. Due to these investor-specific endogenous decisions on the portfolio composition and bonds' decreasing time to maturity, we obtain spill-overs from the short to the long end of the term structure. This agrees with the empirical evidence on liquidity transmission between different maturity segments by Goyenko, Subrahmanyam, and Ukhov (2011).

Our model offers four key testable predictions. First, assets with very short maturities are traded less frequently, as are assets with long maturities. The first effect arises because investors prefer the disutility from waiting to paying the bid-ask spread when maturity is short. As only investors with low preference shock probabilities hold assets with long maturities, these assets are rarely traded as well. Second, since these low preference shock investors still hold a proportion of aged (formerly long-term, but now short-term) bonds, our model endogenously explains the well-documented aging effect. We believe that ours is the first equilibrium model to explain the impact of aging on trading volume via a simple transaction cost friction. Third, liquidity premia in bond yields computed from ask prices are negligible for short maturities, and increase for longer maturities. The increasing term structure arises, even for constant bid-ask spreads, because the disutility from waiting increases with maturity. For longer maturities, the term structure flattens out as investors with low probabilities of preference shocks dominate. Fourth, liquidity premia from bid yields depend on the term structure of bid-ask spreads. If transaction costs do not depend on the bond's maturity, short-term liquidity premia are large, then decrease and flatten out at longer maturities. If transaction costs are increasing in maturity, the term structure takes on a U-shape.

We verify these key model predictions empirically using transaction data for highly rated U.S. corporate bonds from the TRACE database. The results of multiple regressions confirm the intuition from our equilibrium model. Transaction volume is hump-shaped and securities are traded less frequently as they age. Liquidity premia computed from ask prices are monotonously increasing with a decreasing slope. Liquidity premia computed from bid prices are U-shaped with significant liquidity premia even for very short maturities.

Our model also sheds new light on our previous results on the average shape of the term structure of liquidity premia in crisis and non-crisis times. Due to the availability of data and to enhance comparisons with other empirical studies, we used closing prices to calculate liquidity premia in Chapter 3. As closing prices are a mixture of bid and ask prices, the on average U-shaped term structure of liquidity premia of KfW bonds is in line with the model predictions. Additionally, the more inverse shape of the term structure in crisis times can be attributed to an increased demand for liquidity within the model as discussed in Section 4.3.2.

Our study adds to several strands of literature. Ericsson and Renault (2006) model the liquidity shock for assets with different maturities as the jump of a Poisson process that forces investors to sell their entire portfolio to the market maker, who charges a proportional spread. Liquidity premia are downward-sloping because only current illiquidity affects asset prices, and because investors have the option to sell assets early to the market maker at favorable conditions. Kempf, Korn, and Uhrig-Homburg (2012) extend this analysis by modeling the intensity of the Poisson process as a mean-reverting process. In this setting, liquidity premia depend on the difference between the average and the current probability of a liquidity shock, and can exhibit a number of different shapes. In contrast to these papers, we allow investors to trade-off the transaction costs when selling immediately versus the disutility from awaiting the bond's maturity. By endogenizing investors' trading decisions in bonds of different maturities, our model provides an equilibrium-based explanation for spill-overs of liquidity shocks between different ends of the maturity range.

Feldhütter (2012) is most closely related to our study, since he considers an investor's optimal decision to a holding cost shock. Search costs allow market makers to charge a spread, which results in a difference between the asset's fundamental value and its bid price. However, Feldhütter (2012) abstracts from aging as in his model, bonds mature randomly with a rate of $\frac{1}{T}$. Additionally, his model cannot accommodate any spill-over effects between maturities because bonds of different maturities T are not considered *simultaneously*.

Besides supporting the equilibrium model predictions, our results provide an explanation for the variation in the term structures found in previous empirical studies. Studies that document a decreasing term structure (Amihud and Mendelson, 1991; Ericsson and Renault, 2006) or a U-shaped term structure (Longstaff, 2004) use mid quotes or ask quotes net of a spread component such as brokerage costs. In contrast, Dick-Nielsen, Feldhütter, and Lando (2012) find an increasing term structure for the U.S. corporate bond market computed from average quarter-end prices. However, average trade prices in this market are dominated by buy transactions as the numbers of observations in our Table 4.5 document. Hump-shaped (Koziol and Sauerbier, 2007) or variable term structures

(Kempf, Korn, and Uhrig-Homburg, 2012; Feldhütter, 2012) arise from a varying mixture of bid and ask prices. Hence, consistent with our theoretical predictions, the shape of the liquidity term structure is crucially driven by whether most transactions occur at the dealer's bid or ask price.

Last, our study contributes to the growing literature on asset pricing in heterogeneous agents models. Similar to Beber, Driessen, and Tuijp (2012), we study optimal portfolio choice of heterogeneous investors faced with exogenous transaction costs in a stationary equilibrium setting. Duffie, Gârleanu, and Pedersen (2005), Vayanos and Wang (2007), and Weill (2008) endogenize transactions costs through search costs and bargaining power. None of these studies, however, can address the relation between maturity and liquidity as they do not simultaneously consider assets with different finite maturities.

The remainder of the chapter is structured as follows. We introduce our model setup and derive the equilibrium in Section 4.2. In Section 4.3, we display the resulting turnover and liquidity term structure. Section 4.4 describes the data used for the empirical tests of our model predictions, and discusses the results. We check the robustness of our empirical results in Section 4.5. Section 4.6 summarizes and concludes.

4.2 Model Setup

4.2.1 Assets and Investors

We consider a continuous-time model with an infinite horizon and cash as the numéraire. There are two types of assets: the money-market account, which is in infinite supply and which pays a constant non-negative return r, and a continuum of illiquid zero-coupon bonds with time to maturity between 0 and $T_{\max}$. Bonds are perfectly divisible and pay one unit of the numéraire at maturity. Each bond is characterized by its initial maturity at issuance $T_{\text{init}} \leq T_{\max}$, and bonds of each initial maturity are issued with rate a. Thus, in steady state, for each T_{init}, there are in total $a \cdot T_{\text{init}}$ bonds outstanding, and equally distributed with respect to their remaining time to maturity T in $(0, T_{\text{init}}]$. Hence, total outstanding volume of all bonds amounts to $\int_0^{T_{\max}} a \cdot T_{\text{init}} \, dT_{\text{init}} = \frac{1}{2} \cdot a \cdot (T_{\max})^2$. Note that with the assumption of a given issuance rate a, we take maturity dispersion as given. This assumption is supported for example by firms managing rollover or funding liquidity risk by spreading out the maturity of their debt (Choi, Hackbarth, and Zechner, 2013; Norden, Roosenboom, and Wang, 2013).[36]

There are three types of agents, one unit measure of short-horizon investors (type S), one unit measure of long-horizon investors (type L), and dealers who act as market makers. Dealers continuously quote competitive bid and ask prices at which they stand ready to trade. As in Amihud and Mendelson (1986), dealers are compensated for providing liquidity by a bid-ask spread $s(T)$ dependent on the (remaining) time to maturity T.[37] Hence, dealers quote an ask price $P^{\text{ask}}(T) = P\,(T)$ and a bid price $P^{\text{bid}}(T) = (1 - s\,(T)) \cdot P\,(T)$ for a bond with remaining maturity T. Investors are risk-neutral, i.e., they discount all cash flows with the risk-free rate r, and each investor is infinitesimally small. Type-i investors have aggregate wealth W_i and utility from consumption $U_i\,(c)\,,\ i \in \{S, L\}$. An investor can consume cash she either receives from the money-market account or as the proceeds from sold or matured bonds.

4.2.2 The Liquidity Shock

The liquidity shock arises in our model as follows. Each investor experiences a single preference shock with Poisson rate $\lambda_i,\ \ i \in \{S, L\}$, that increases her time preference rate from r to $r + b > r$. We can economically interpret this event as a funding shock that leads to an incentive for the investor to reduce her positions (see Brunnermeier and Pedersen, 2009).[38] Hence, total utility from consumption for an infinitesimally small investor of group i, given that a liquidity shock occurs at time $\tilde{T}_i$, is given by $U_i\,(c) = \int_0^{\tilde{T}_i} e^{-r \cdot t} c_t\, dt + \int_{\tilde{T}_i}^{\infty} e^{-r \cdot \tilde{T}_i - (r+b) \cdot \left(t - \tilde{T}_i\right)} c_t\, dt$, i.e., consumption discounted at the rate r prior to the shock and at $r + b$ after the shock. Since a short-horizon investor expects to experience an earlier preference shock than a long-horizon investor, we define that $\lambda_S > \lambda_L$. As a consequence, marginal utility of holding an illiquid bond is larger for long-horizon investors than for short-horizon investors for bonds of all maturities.

At the preference shock, the investor decides for each bond whether to sell it to the

[36]In contrast, Greenwood, Hanson, and Stein (2010) endogenize maturity structure with the argument that firms absorb supply shocks induced by changes in sovereign debt structure by varying the maturity of the bonds they issue both cross-sectionally and over time. He and Xiong (2012) take liquidity as exogenously given, and endogenize the choice of debt maturity and a firm's default decision. He and Milbradt (2012) endogenize liquidity, but only consider homogeneous investors.

[37]As bid-ask spreads $s(T)$ only depend on the remaining maturity T, bonds with the same time to maturity (but different initial maturities T_{init}) are used interchangeably by investors and dealers.

[38]Feldhütter (2012) or Duffie, Gârleanu, and Pedersen (2005) obtain a similar effect through an increased holding cost for the bond.

market maker at the bid price and consume the proceeds, or to hold the bond despite the increased time preference rate. It is intuitive that bonds with shorter maturity lead to less disutility from waiting, as disutility approaches zero for $T \to 0$. Therefore, investors will never sell bonds with very short maturities prematurely. We denote the maturity for which an investor is indifferent between selling the bond (left-hand side of Equation (4.1)) and holding it until maturity (right-hand side) by τ. τ therefore satisfies

$$P(\tau) \cdot (1 - s(\tau)) = e^{-(r+b)\cdot\tau} \tag{4.1}$$

and is identical for both investors.

4.2.3 The Investors' Optimization Problem

We consider a steady-state equilibrium where type-i investors experience preference shocks with a rate λ_i. Such a shock leads to an incentive for an investor to unwind her portfolio and exit the market. Investors having experienced a preference shock are replaced by new investors such that aggregate wealth from each investor group W_i remains constant.[39] As neither aggregate wealth nor the supply of bonds change over time, prices of bonds for a given time to maturity are constant over time. Note, however, that for aggregate wealth to remain constant, the wealth of any investor group cannot grow with a higher rate than members of the respective investor group leave the market. It turns out that in equilibrium, $r + b$ is an absolute upper bound for the growth rate of wealth. To ensure that constant aggregate wealth exists, we therefore assume $r + b < \lambda_L < \lambda_S$.

From the investors' risk neutrality and the corresponding additive structure of the expected utility function, it directly follows that investors either want to invest nothing or the maximum amount possible in a particular maturity (see, also, e.g., Feldhütter, 2012). Therefore, an investor who initially chooses to invest in a bond of some maturity T, again invests in a bond with this maturity if her old bond matures – or, in other words, the investor's willingness to invest in a particular maturity does not depend on her wealth or her portfolio holdings in other maturities. Hence, each investor chooses an initially optimal allocation strategy when she first enters the market and has no incentive

[39] As investors are infinitesimally small, the distribution of the investors' age remains constant over time in steady state. An investor's age determines how long she was able to collect liquidity premia and risk-free returns and therefore her individual wealth gains. Hence, as all newly arriving investors have an identical capital endowment, the constant distribution of investors' age directly leads to constant aggregate wealth.

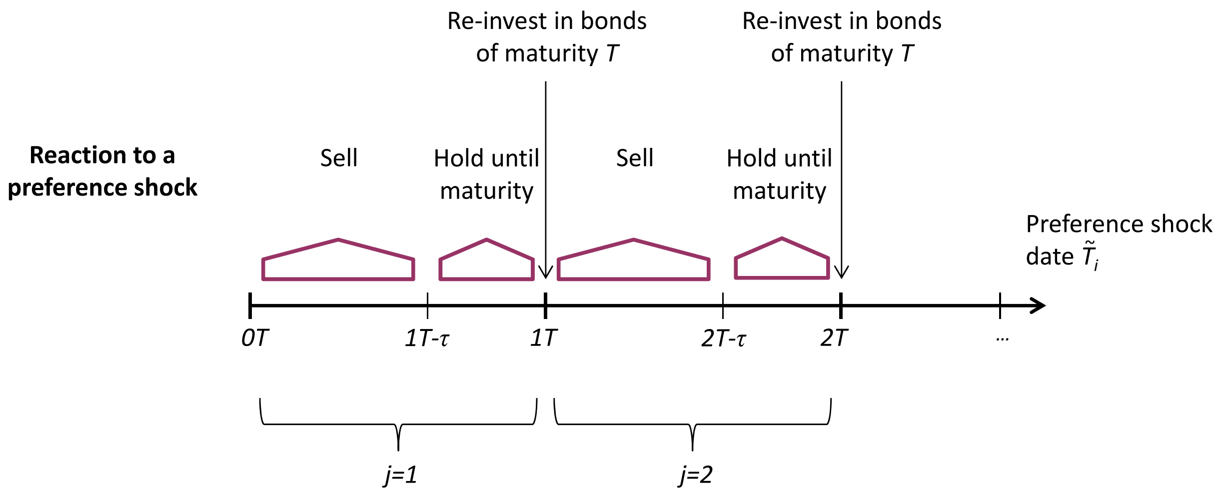

Figure 4.1: **Investor's decision problem**
The figure presents the trade-off within the investor's decision problem and her optimal reaction to a preference shock for a bond with given maturity T. At time 0, the investor buys the bond at a price $P(T)$. If a preference shock occurs between 0 and $T - \tau$, the disutility from awaiting the bond's maturity is larger than the disutility from selling the bond. Hence, the investor sells the bond prior to maturity. If a preference shock hits the investor between $T-\tau$ and T, disutility from awaiting the bond's maturity is smaller than the disutility from selling the bond. Hence, the investor holds the bond until maturity and realizes the bond's notional value. If the investor does not experience a preference shock until time T, she again invests into a bond with maturity T. Since the price of the bond $P(T)$ is smaller than the notional value 1, she realizes a wealth gain $1 - P(T)$ and her individual bond position grows over time. We denote the number of investment rounds during which an investor (re-)invests in bonds of maturity T by j.

to change her portfolio prior to a preference shock. For a given bond maturity T, we display the investor's decision problem in Figure 4.1.

Figure 4.1 also incorporates two implicit assumptions that we use in deriving the investor's optimization problem. First, we assume that in the case of a preference shock, it is either optimal to immediately sell the bond or hold it until maturity. Second, we assume that it is never optimal to sell bonds when no preference shock occurred. In Section 4.2.4 and in Appendix C.4, we discuss these assumptions in more detail and derive general conditions, under which investors have no incentive to deviate from them.

Since neither prices nor aggregate wealth changes over time, it is sufficient to consider the decision problem at time $t = 0$, where each type-i investor maximizes her expected utility $E[U_i(c)]$ from consumption by choosing the amount of money X_i invested into the money market account ($X_i\,(0)$) and into bonds with maturity T between 0 and $T_{\max}$

$(X_i(T))$. Short sales are not allowed, such that $X_i(T) \geq 0,\ \forall T \in [0, T_{\max}]$. Hence, a type-$i$ investor solves the following optimization problem:

$$\begin{aligned}
\max_{X_i}\ E\Bigg\{ & \int_0^{T_{\max}} X_i(T) \cdot \sum_{j=1}^{\infty} \frac{1}{P(T)^j} \cdot (1 - s(T \cdot j - \tilde{T}_i)) \\
& \qquad \cdot P(T \cdot j - \tilde{T}_i) \cdot e^{-r \cdot \tilde{T}_i} \cdot \mathbb{1}_{\{T \cdot (j-1) < \tilde{T}_i < T \cdot j - \min(\tau, T)\}}\, dT \\
& + \int_0^{T_{\max}} X_i(T) \cdot \sum_{j=1}^{\infty} \frac{1}{P(T)^j} \cdot e^{-r \cdot \tilde{T}_i - (r+b) \cdot (T \cdot j - \tilde{T}_i)} \cdot \mathbb{1}_{\{T \cdot j - \min(\tau, T) \leq \tilde{T}_i \leq T \cdot j\}}\, dT \\
& + X_i(0) \Bigg\}. \qquad (4.2)
\end{aligned}$$

The first summand in Expression (4.2) denotes utility of consumption from bonds which the investor sells to the dealer at the bid price $(1 - s(T \cdot j - \tilde{T}_i)) \cdot P(T \cdot j - \tilde{T}_i)$ immediately after a preference shock. The amount invested in bonds $X_i(T) \cdot \frac{1}{P(T)^j}$ grows for as many investment rounds j as the investor (re-)invests in the bond until the preference shock and thereby in each round collects the price difference between the notional value of the bond and the price of the bond $P(T)$ (see Figure 4.1). The second summand gives the utility of consumption from bonds which the investor holds after the preference shock until their maturity date. The third summand measures the utility from cash invested in the money market account.

The investor's budget constraint is given by $W_i = \int_0^{T_{\max}} X_i(T)\, dT + X_i(0)$. Simplifying Expression (4.2), taking expectations, and replacing $X_i(0)$ via the budget constraint yields the following optimization problem:

$$\begin{aligned}
\max_{X_i} \Bigg\{ & \int_0^{T_{\max}} X_i(T) \cdot \frac{\lambda_i \cdot e^{\lambda_i \cdot T}}{P(T) \cdot e^{r \cdot T} \cdot e^{\lambda_i \cdot T} - 1} \cdot \int_{\min(\tau, T)}^{T} P(x) \cdot e^{r \cdot x} \cdot (1 - s(x)) \cdot e^{-\lambda_i \cdot (T - x)}\, dx\, dT \\
& + \int_0^{T_{\max}} X_i(T) \cdot \frac{\lambda_i \cdot (1 - e^{(\lambda_i - b) \cdot \min(\tau, T)})}{(1 - P(T) \cdot e^{r \cdot T} \cdot e^{\lambda_i \cdot T}) \cdot (\lambda_i - b)}\, dT \\
& + W_i - \int_0^{T_{\max}} X_i(T)\, dT \Bigg\}. \qquad (4.3)
\end{aligned}$$

Taking partial derivatives with respect to each $X_i(T)$ yields the marginal utility of holding bonds with maturity T for a type-i investor:

$$\frac{\partial E[U_i(c)]}{\partial X_i(T)} = \frac{\lambda_i \cdot e^{\lambda_i \cdot T}}{P(T) \cdot e^{r \cdot T} \cdot e^{\lambda_i \cdot T} - 1} \cdot \int_{\min(\tau,T)}^{T} P(x) \cdot e^{r \cdot x} \cdot (1 - s(x)) \cdot e^{-\lambda_i \cdot (T-x)} \, dx + \frac{\lambda_i \cdot (1 - e^{(\lambda_i - b) \cdot \min(\tau,T)})}{(1 - P(T) \cdot e^{r \cdot T} \cdot e^{\lambda_i \cdot T}) \cdot (\lambda_i - b)} - 1 =: \Delta_i(T). \tag{4.4}$$

The fact that marginal utility does not depend on X_i simplifies our analysis of the equilibrium allocation. As investors are indifferent between all bonds they invest in, the marginal utility of these bonds must be equal. Otherwise, given prices cannot be equilibrium prices. As marginal utility does not depend on X_i, it is sufficient to consider whether an investor buys a bond at all. Given that the investor buys the bond, she is indifferent on how she distributes her wealth across all bonds she invests in.

Equation (4.4) also shows that the time preference rate r which applies prior to the liquidity shock does not affect the investor's optimization problem. To see why, we rewrite bond prices as $P(T) = e^{-r \cdot T} \cdot Q(T)$. Here, $Q(T)$ is the discount of an illiquid bond compared to the price of a perfectly liquid bond $e^{-r \cdot T}$. Substituting $Q(T) = e^{r \cdot T} \cdot P(T)$ into Equations (4.1) and (4.4) would lead to an identical optimization problem independent of r. To simplify notation, we therefore set $r = 0$ in the following analysis.

4.2.4 Equilibrium Mechanism

Our model can be viewed as a continuous modification of a linear exchange model (see Gale, 1960) for which unique solutions exist. The equilibrium mechanism is similar to the ones in Amihud and Mendelson (1986) and Beber, Driessen, and Tuijp (2012). We first derive prices for a particular equilibrium allocation, then analyze whether market clearing holds for these prices, show that no investor has an incentive to deviate from the allocation for the given bond prices, and last demonstrate that the implicit assumptions used in formulating the investor's optimizations problem hold.

Note that marginal utility for holding bonds is larger for long-horizon investors than for short-horizon investors, whereas the marginal utility of the money-market account is equal for both. Hence, we are able to exclude the allocation that short-horizon investors buy bonds and at the same time, long-horizon investors invest in the money-market account. We therefore focus on the most general remaining allocation: both short- and long-horizon investors hold bonds, and short-horizon investors additionally invest in the money-market account.[40]

When both short- and long-horizon investors hold bonds and short-horizon investors additionally invest in the money-market account, we show in Appendix C.2 that for the derived equilibrium prices there exists a maturity limit T_{lim} such that long-horizon investors buy only bonds with maturity between T_{lim} and T_{max}, and short-horizon investors buy only bonds with maturity between 0 and T_{lim}. This case corresponds to a clientele effect. The equilibrium conditions for short-horizon (S) and long-horizon (L) investors are then given by

$$\Delta_S(T) = 0 \qquad \text{for all } T \in (0, T_{\text{lim}}], \tag{4.5}$$

$$\Delta_L(T) = \Delta_L(T_{\text{lim}}) \qquad \text{for all } T \in (T_{\text{lim}}, T_{\text{max}}]. \tag{4.6}$$

Short-horizon investors are indifferent between holding bonds with a maturity up until T_{lim} and the money-market account, long-horizon investors are indifferent between buying bonds with maturities between T_{lim} and T_{max}. For given limiting maturities τ and T_{lim}, the conditions in Equations (4.5) and (4.6) lead to closed-form solutions for $P(T)$. We summarize these results in Proposition 1, which we prove in Appendices C.1 and C.2.

Proposition 1. (Equilibrium prices and clientele effect)
For constant or monotonously increasing bid-ask spreads $s(T)$ with $0 < s(T) < 1$, prices of illiquid bonds $P(T)$ are given in closed form

$$P(T) = \begin{cases} \frac{b \cdot e^{-\lambda_S \cdot T} - \lambda_S \cdot e^{-b \cdot T}}{b - \lambda_S}, & \text{if } T \leq \min(\tau, T_{\text{lim}}) \\ e^{-\int_\tau^T \lambda_S \cdot s(x)\, dx} \cdot P(\tau), & \text{if } \tau < T \leq T_{\text{lim}} \\ e^{-\int_{T_{\text{lim}}}^T \frac{(\Delta_L(T_{\text{lim}}) + s(x)) \cdot \lambda_L}{1 + \Delta_L(T_{\text{lim}})}\, dx} \cdot P(T_{\text{lim}}), & \text{if } \tau < T_{\text{lim}} < T \\ e^{-T \cdot \lambda_L} \cdot \left(1 - \frac{\lambda_L \cdot \left(1 - e^{T \cdot (\lambda_L - b)}\right)}{(1 + \Delta_L(T_{\text{lim}})) \cdot (\lambda_L - b)}\right), & \text{if } T_{\text{lim}} < T \leq \tau \\ e^{-\int_\tau^T \frac{(\Delta_L(T_{\text{lim}}) + s(x)) \cdot \lambda_L}{1 + \Delta_L(T_{\text{lim}})}\, dx} \cdot P(\tau), & \text{if } T_{\text{lim}} \leq \tau < T \end{cases} \tag{4.7}$$

In equilibrium, there arises a clientele effect that leads to long-horizon investors investing only in bonds with $T > T_{\text{lim}}$ and short-horizon investors investing in short-term bonds with $T \leq T_{\text{lim}}$.

[40] The case where short-horizon investors only invest in the money-market account and long-horizon investors invest in bonds and the money-market account is discussed in Footnote 41. We do not consider the degenerate allocations where the wealth of long-horizon investors or both investor groups exactly equals total bond supply. These cases are only valid for rather narrow combinations of total wealth and bond supply. Additionally, prices would strongly depend on total wealth which is hard to quantify empirically.

Both maturity limits τ and T_{lim} in Proposition (1) themselves depend on bond prices (because τ solves Equation (4.1), and because the wealth of long-horizon investors must be sufficient to buy all bonds with maturities larger than T_{lim}). Hence, we use an iterative algorithm to simultaneously derive equilibrium prices $P(T)$ and critical maturities τ and T_{lim} for a given parameter set (λ_L, λ_S, W_L, W_S, a, b, $T_{\max}$) and bid-ask spread function $s(T)$. From initial values of τ and T_{lim}, we use Proposition 1 to derive bond prices $P(T)$. We then use Equation (4.1) to compute a new τ. To determine a new T_{lim}, we use a market clearing argument for long-horizon investors that we formally derive in Appendix C.3.

Given the derived equilibrium prices, we analyze whether market clearing holds and check whether the allocation of bonds is as assumed. Both requirements are satisfied if aggregate wealth of both investor types exceeds total bond supply (left inequality), but on the other hand, total wealth of long-horizon investors alone does not suffice to buy all bonds (right inequality):

$$W_S + W_L > \int_0^{T_{\max}} P(T) \cdot \int_T^{T_{\max}} a \, dT_{\text{init}} \, dT > W_L. \tag{4.8}$$

The right inequality of (4.8) is automatically satisfied if the condition to determine T_{lim} (see Appendix C.3) yields a $T_{\text{lim}} \in (0, T_{\max})$. By inserting the closed form solutions for $P(T)$ from Proposition 1 for a given parameter set, it is easy to verify the left inequality of (4.8).[41]

Additionally, we analyze whether the two implicit assumptions used in formulating the investor's optimization problem hold, i.e., we check that for $T > \tau$, it is always optimal to immediately sell the bond if an investor experiences a preference shock and second, that no investor has an incentive to sell bonds prematurely without having experienced a preference shock. We formalize these assumptions in Appendix C.4 and show that for constant bid-ask spreads $s(T) = s$, they are always satisfied. For an arbitrary bid-ask spread function $s(T)$, we show that the first assumption always holds if bid-ask spreads do not 'grow too strongly' with maturity, since investors might otherwise postpone selling the bond until the bid-ask spread has decreased sufficiently (for a formal condition, see Appendix C.4). For arbitrary bid-ask spreads $s(T)$, the condition in Appendix C.4 for the second assumption has to be verified by plugging in prices $P(T)$ from Proposition 1.

[41] If the long-horizon investors' wealth alone is sufficient to buy all bonds, prices are as in Proposition 1 with $T_{\text{lim}} = 0$. The market clearing condition is then given as $W_L > \int_0^{T_{\max}} P(T) \cdot \int_T^{T_{\max}} a \, dT_{\text{init}} \, dT$. However, this case is not very interesting as short-horizon investors do not play a role.

Summarizing the results of this section, we have derived the equilibrium prices for the allocation where the wealth of both investor types is required to absorb total bond supply. An endogenous clientele effect arises that causes long-horizon investors to invest exclusively in long-term bonds, whereas short-horizon investors prefer short-term bonds. This result holds even if bid-ask spreads are identical for all bonds and is thus a more general case of the clientele effect in Amihud and Mendelson (1986). In analogy with their paper, short-term bonds are more "liquid" since they offer the opportunity to liquidate a portfolio without paying transaction costs more quickly. In contrast, such a result is not obtained if liquidity shocks are modeled as, e.g., in Ericsson and Renault (2006). In their paper, when investors experience a liquidity shock, they are forced to sell a bond immediately. Hence, in a continuous time setting, a liquidity shock and the maturity of a bond never coincide and there is no advantage from investing in short-term bonds.

4.3 Trading Volume and Liquidity Term Structure

4.3.1 Trading Volume

We first present the predictions of our model regarding the relations between trading volume, maturity, and age. These relations crucially depend on three intuitive effects. First, bonds of short maturities are not sold prematurely, since the disutility from awaiting maturity is low. Second, the clientele effect (short-horizon investors with strong trading needs only hold short-term bonds) translates into lower trading volumes for bonds with longer maturities. The first and second effect lead to a hump-shaped relation between maturity and trading volume. Third, an aged formerly long-term but now short-term bond is still partially locked up in the portfolios of long-horizon investors. This leads to a lower trading volume of this bond compared to a young short-term bond. We are not aware of any other model that is both able to endogenously derive relations between maturity, age, and trading volume and predict term structures of liquidity premia. The predictions regarding trading volume are summarized in the following proposition, which we prove in Appendix C.5. We focus on seller initiated turnover as a proxy for trading volume for two reasons. First, total trading volume equals seller-initiated trading volume plus trading volume from issuing activities in the primary market, which is exogenous in our setting. Second, we look at turnover, i.e., trading volume in percent of the outstanding volume for each maturity since the outstanding volume of short-term bonds exceeds that of long-term bonds due to the latter's aging.

Proposition 2. (Trading volume)
Consider the case that $\tau < T_{\text{lim}}$.

1. *Seller initiated turnover is hump-shaped in the time to maturity T, i.e., it is zero for $T < \tau$ and equals λ_L for $T > T_{\text{lim}}$. For T with $\tau < T < T_{\text{lim}}$, seller initiated turnover exceeds λ_L.*

2. *For two bonds 1 and 2 that both have a remaining maturity T with $\tau < T < T_{\text{lim}}$, but a different initial maturity $T_{\text{init},1} < T_{\text{lim}}$ and $T_{\text{init},2} > T_{\text{lim}}$, seller initiated turnover is higher for the younger bond 1 compared to the older bond 2.*

In the (less interesting) case that $T_{\text{lim}} \leq \tau$, short-horizon investors never sell bonds prematurely, and seller-initiated turnover is determined by long-horizon investors only. Hence, seller-initiated turnover is zero for $T < \tau$, and equals λ_L for $T > \tau$. Then, no aging effect arises.

We illustrate the relation between maturity and trading volume with the help of a baseline parameter specification in Figure 4.2. In this specification, bid-ask spreads are 0.3% for all maturities T, short-horizon investors experience preference shocks with a rate of $\lambda_S = 0.6$, i.e., they experience on average one preference shock every 20 months. Long-horizon investors experience half as many shocks compared to short-horizon investors ($\lambda_L = 0.3$).[42] b equals 2%, i.e., if a shock arises, investors' time preference rate increases by 2% which can be thought of as the additional borrowing cost in excess to the risk-free rate. In Figure 4.2, the solid line presents seller initiated turnover aggregated over all bonds, i.e., volume from trades triggered by investors who sell their bonds prematurely, divided by the total outstanding volume of all bonds with the respective maturity:

$$\text{Turnover}(T) = \frac{\mathbb{1}_{\{T>\tau\}} \cdot \int_{T}^{T_{\max}} a \cdot \sum_{i=S,L} Y_i\left(T, T_{\text{init}}, T_{\text{lim}}\right) \cdot \lambda_i \, dT_{\text{init}}}{\int_{T}^{T_{\max}} a \, dT_{\text{init}}}, \tag{4.9}$$

where in the numerator and the denominator, we integrate over all bonds with initial maturity T_{init} and remaining maturity T that are held by both investor types. $Y_i(T, T_{\text{init}}, T_{\text{lim}})$ denotes the fraction of bonds with remaining maturity T and initial maturity T_{init} held in the portfolios of type-i investors (where $Y_S(T, T_{\text{init}}, T_{\text{lim}}) = 1 - Y_L(T, T_{\text{init}}, T_{\text{lim}})$ and

[42] Our parameter values are well comparable to Feldhütter (2012) who estimates for the U.S. corporate bond market that investors experience a preference shock once every three years.

$Y_L(T, T_{\text{init}}, T_{\text{lim}})$ is formally defined in Equation (C.31) in Appendix C.3). This fraction is multiplied with the rate at which preference shocks arrive. The denominator gives the total volume of all bonds with remaining maturity T and initial maturity T_{init} between T and $T_{\max}$. The entire fraction is multiplied by $\mathbb{1}_{\{T>\tau\}}$, since investors who experience a preference shock only sell bonds with maturity $T > \tau$.

The dependence of trading volume on the distribution of bonds over the portfolios of short- and long-horizon investors leads to the endogenous aging effect (second part of Proposition 2) which we illustrate in Figure 4.2. Bonds with initial maturity $T_{\text{init}} < T_{\text{lim}}$ (dotted line in Figure 4.2) are only held by short-horizon investors. These investors sell the bonds when experiencing a preference shock if the remaining maturity T is larger than τ. This leads to a turnover of these bonds which equals λ_S for $T > \tau$ and drops to zero for $T < \tau$.

The same intuition applies for long-horizon investors and bonds with initial maturity $T_{\text{init}} > T_{\text{lim}}$ (dashed line in Figure 4.2) when these bonds have a remaining maturity $T > T_{\text{lim}}$. If they reach a remaining time to maturity T below T_{lim}, both long- and short-horizon investors sell the bonds if they experience a preference shock (for $T > \tau$), but only short-horizon investors purchase the bonds. Hence, when bonds reach a remaining time to maturity below T_{lim}, they gradually move into the portfolios of short-horizon investors who suffer preference shocks with a higher rate. Therefore, the turnover increases for decreasing maturity until at τ, it drops again to zero. As a direct consequence, a bond with remaining maturity $T < T_{\text{lim}}$ has a lower turnover if its initial maturity was larger than T_{lim} (the bond is older), compared to a younger bond with initial maturity $T_{\text{init}} < T_{\text{lim}}$.

The solid line in Figure 4.2 shows turnover for all bonds. It can be viewed as a weighted average of the other two lines, where the weights equal the proportion of bonds of remaining maturity T that have an initial maturity below or above T_{lim}. Our model predictions are consistent with the aging effect discussed in Warga (1992) and empirically documented, e.g., in Fontaine and Garcia (2012) for U.S. Treasuries and Hotchkiss, Warga, and Jostova (2002) for corporate bonds.

4.3.2 The Term Structure of Liquidity Premia

To demonstrate the effect of illiquidity on the term structure of interest rates, we depict liquidity premia computed from both ask prices $P^{\text{ask}}(T) = P(T)$ and bid prices $P^{\text{bid}}(T) = (1 - s(T)) \cdot P(T)$. Liquidity premia are then defined as the bond yield minus the risk free

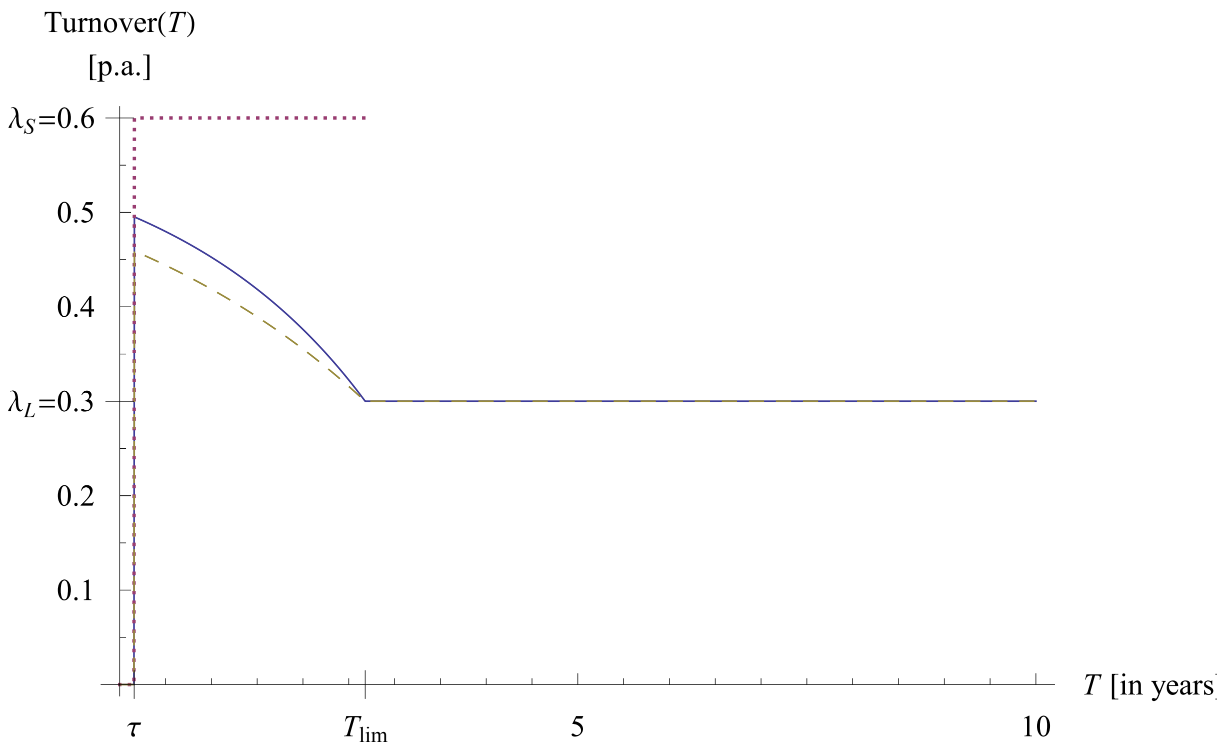

Figure 4.2: **Seller initiated turnover – hump-shaped trading volume and aging effect**
The figure presents seller initiated turnover for the baseline case where the rate at which preference shocks occur λ equals 0.6 for short-horizon investors and 0.3 for long-horizon investors, the time preference rate increases from 0 to $b = 0.02$ if a preference shock occurs, the bid-ask spread s equals 0.003, the maximum bond maturity $T_{\max}$ equals 10 years, both investor types have the same aggregate wealth W of 1, and for each initial maturity, bonds are issued with a rate of $a = 0.025$, leading to a total bond supply of 1.25. In the resulting equilibrium allocation, short-horizon investors invest in bonds with maturities up to $T_{\text{lim}} = 2.68$ years, and only bonds with a maturity higher than $\tau = 0.16$ years are sold if a preference shock arises. The dotted line presents turnover of bonds with initial maturity $T_{\text{init}} < T_{\text{lim}}$, the dashed line depicts turnover of bonds with initial maturity $T_{\text{init}} > T_{\text{lim}}$, and the solid line aggregates turnover over all bonds.

rate r, i.e.,

$$\mathrm{Illiq}^{\mathrm{ask}}(T) = -\frac{\log\left(P^{\mathrm{ask}}(T)\right)}{T} - r = -\frac{\log\left(P(T)\right)}{T} - r,$$
$$\mathrm{Illiq}^{\mathrm{bid}}(T) = -\frac{\log\left(P^{\mathrm{bid}}(T)\right)}{T} - r = -\frac{\log\left(1 - s(T)\right)}{T} - \frac{\log\left(P(T)\right)}{T} - r. \tag{4.10}$$

In the following, we again set $r = 0$ for ease of exposition. The formulas for liquidity premia can be interpreted as distributing the "liquidity discount" over the time to maturity T. Bid premia are increased in addition by bid-ask spreads $s(T)$, which are distributed over T, as $s(T) \approx -\log(1 - s(T))$ for small $s(T)$. We summarize our model predictions regarding the term structure of liquidity premia in Proposition 3, which we prove in Appendix C.5.

Proposition 3. (Term structure of liquidity premia)

1. *The term structure of liquidity premia from ask prices* $\mathrm{Illiq}^{\mathrm{ask}}(T)$ *is monotonously increasing in time to maturity* T *for all* T *and goes to zero for* $T \to 0$. *The term structure flattens at* T_{lim}, *i.e.,*
$$\lim_{T \uparrow T_{\mathrm{lim}}} \left(\mathrm{Illiq}^{\mathrm{ask}}(T)\right)' > \lim_{T \downarrow T_{\mathrm{lim}}} \left(\mathrm{Illiq}^{\mathrm{ask}}(T)\right)'. \tag{4.11}$$
2. *The term structure of liquidity premia from bid prices* $\mathrm{Illiq}^{\mathrm{bid}}(T)$ *is decreasing in* T *at the short end.*

The predictions in Proposition 3 are illustrated in Figure 4.3 for constant and in Figure 4.4 for monotonously increasing bid-ask spreads. First, ask premia $\mathrm{Illiq}^{\mathrm{ask}}(T)$ always go to zero for $T \to 0$ as the disutility from awaiting the bond's maturity vanishes in the case of a preference shock. Second, in Figure 4.3, the ask term structure $\mathrm{Illiq}^{\mathrm{ask}}(T)$ flattens out quickly. Since the slope is already close to zero for $T \uparrow T_{\mathrm{lim}}$, the small kink at T_{lim} is hard to detect as $\mathrm{Illiq}^{\mathrm{ask}}(T)$ cannot decrease in time to maturity. Otherwise, long-horizon investors would invest in bonds with shorter maturities. In Figure 4.4, ask liquidity premia increase more strongly for longer maturities as expected trading costs increase due to the increasing term structure of bid-ask spreads $s(T)$. Hence, the kink at T_{lim} becomes more apparent. Bid-premia $\mathrm{Illiq}^{\mathrm{bid}}(T)$ always exhibit an inverse shape.

Apart from these predictions of Proposition 3, Figure 4.3 and Figure 4.4 allow us to make two additional observations. First, for constant bid-ask spreads, bid liquidity premia also flatten out for longer maturities because the fixed bid-ask spread is distributed over a longer time period.[43] Second, for the empirically relevant case of bid-ask spreads increasing

[43] A recent working paper of Huang et al. (2013) confirms our model predictions empirically. The

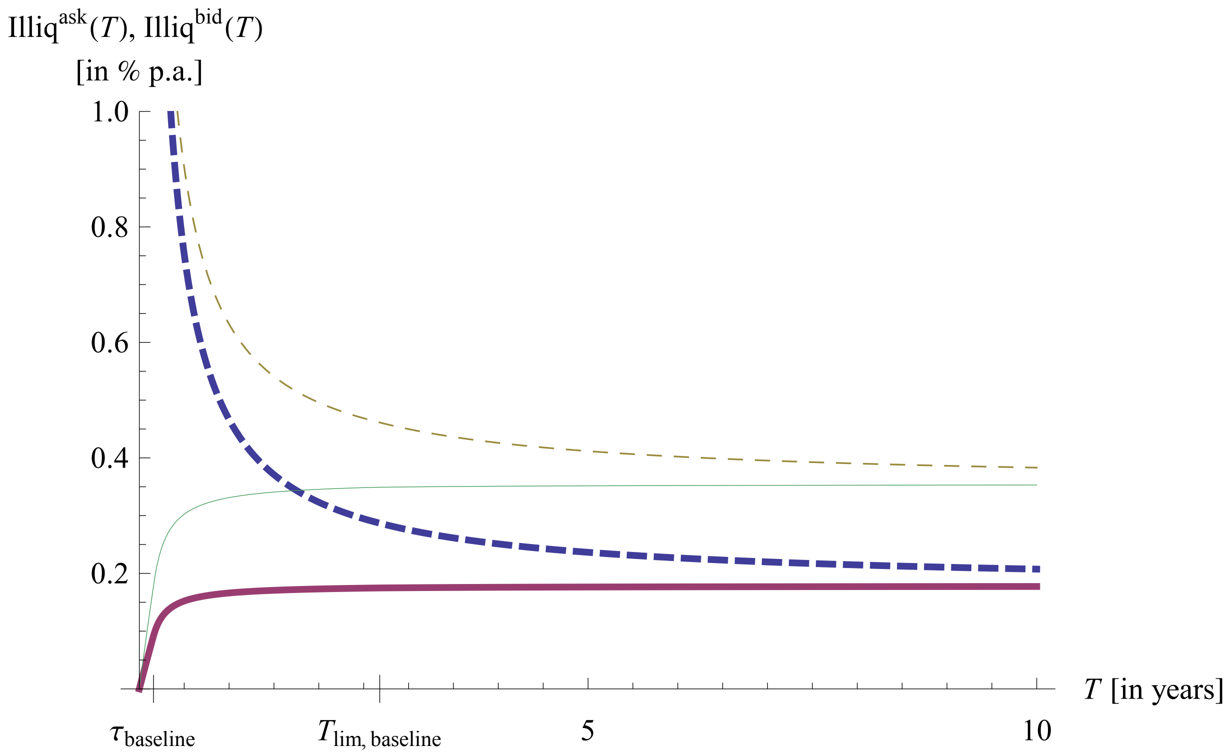

Figure 4.3: **Liquidity premia – baseline case and spill-over effects**
The figure presents liquidity premia for the baseline case (thick lines) where the rate at which preference shocks occur λ equals 0.6 for short-horizon investors and 0.3 for long-horizon investors, the time preference rate increases from 0 to $b = 0.02$ if a preference shock occurs, the bid-ask spread $s(T)$ equals 0.003 for all maturities, the maximum bond maturity T_{max} equals 10 years, both investor types have the same aggregate wealth W of 1, and for each initial maturity, bonds are issued with a rate of $a = 0.025$, leading to a total bond supply of 1.25. In the resulting equilibrium allocation, short-horizon investors invest in bonds with maturities up to $T_{\text{lim, baseline}} = 2.68$ years, and only bonds with a maturity higher than $\tau_{\text{baseline}} = 0.16$ years are sold if a preference shock arises. Thin lines present liquidity premia for the case of higher liquidity demand for short-horizon investors ($\lambda_S = 1.2$). All other parameters are identical to the baseline case. For this specification, critical maturities $\tau_{\lambda_S=1.2} = 0.17$ and $T_{\text{lim, }\lambda_S=1.2} = 2.63$ only change marginally compared to the baseline specification. Solid lines depict $\text{Illiq}^{\text{ask}}(T)$, dashed lines show $\text{Illiq}^{\text{bid}}(T)$.

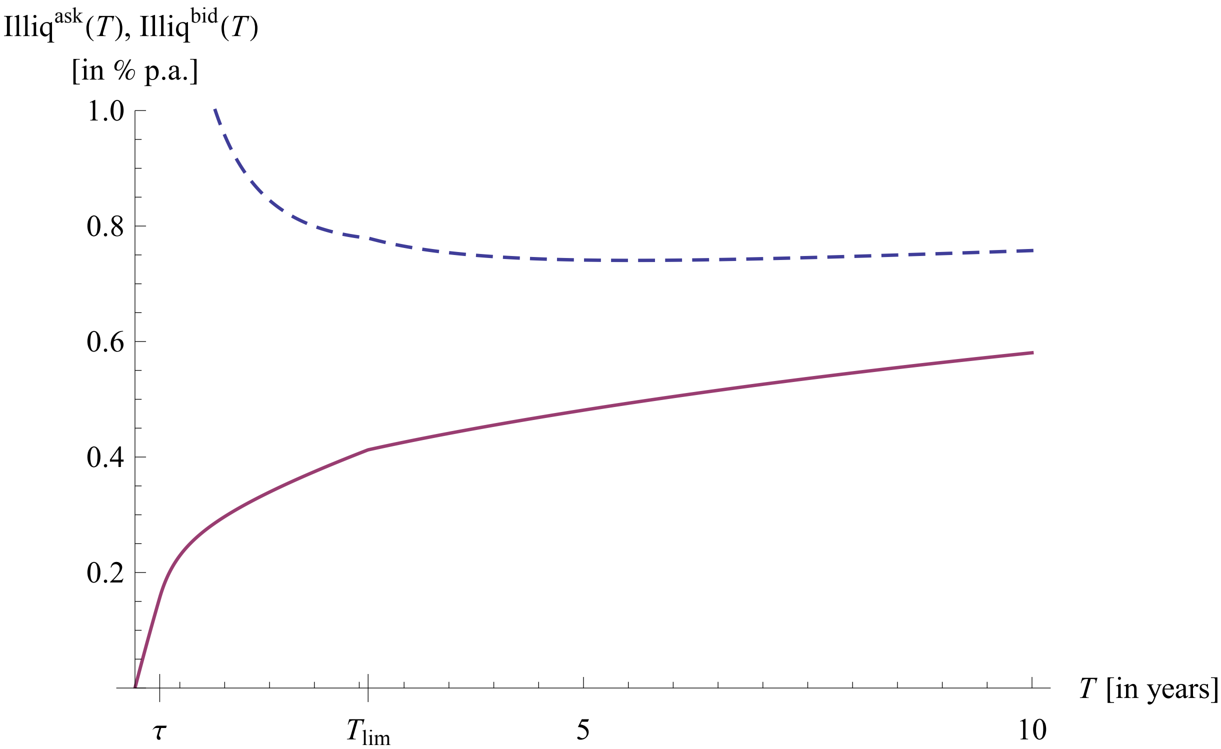

Figure 4.4: **Liquidity premia – bond-specific spreads increase in maturity**
The figure presents liquidity premia for the case where we have calibrated bid-ask spreads to observed prices: $s(T) = 0.00446 + 0.01868 \cdot \left(1 - e^{-0.1205 \cdot T}\right)$ (see also Section 4.4.2 and Figure 4.5). The rate at which preference shocks occur λ equals 0.6 for short-horizon investors and 0.3 for long-horizon investors, the time preference rate increases from 0 to $b = 0.02$ if a preference shock occurs, the maximum bond maturity $T_{\max}$ equals 10 years, both investor types have the same aggregate wealth W of 1, and for each initial maturity, bonds are issued with a rate of $a = 0.025$, leading to a total bond supply of 1.25. In the resulting equilibrium allocation, short-horizon investors invest in bonds with maturities up to $T_{\lim} = 2.6$ years, and only bonds with a maturity higher than $\tau = 0.28$ years are sold if a preference shock arises. The solid line depicts $\text{Illiq}^{\text{ask}}(T)$, the dashed line shows $\text{Illiq}^{\text{bid}}(T)$.

in maturity, bid liquidity premia increase at the long end. Liquidity premia for long-term bonds therefore reflect the shape of the bid-ask spread curve, which results in increasing term structure of liquidity for ask and U-shaped ones for bid liquidity premia.

Finally, Figure 4.3 also illustrates a spill-over effect of short-horizon investors' liquidity demand on long-term premia. For the thin lines, all parameters are identical as before, except λ_S which is twice as large as in the baseline case. Although only short-horizon investors', who hold bonds with maturities smaller than T_{lim}, are affected by this change, liquidity premia of all maturities increase. The reason for this spill-over is the same as above: long-horizon investors would prefer short-term bonds over long-term bonds if long-term ask liquidity premia were lower than short-term premia.

Taking into account that empirically observed bond yield spreads are typically computed from mid prices and incorporate a liquidity component, our model can also shed light on the credit spread puzzle. This puzzle refers to the observation that empirically observed bond yield spreads are too high, especially at the short end, compared to what structural models à la Merton (1974) can explain, see, e.g., Huang and Huang (2012). If we average ask and bid prices to compute mid liquidity premia in our framework, we get an inverse shape with large premia for very short maturities: in our baseline specification, we obtain about 185 bps for one month time to maturity.

In summary, the model predicts four main testable effects. First, seller initiated turnover is hump shaped. Second, for bonds which have identical maturity T but a different age, the older bond has lower or equal seller initiated turnover compared to the younger bond. Third, liquidity premia computed from ask prices are monotonously increasing in maturity at the short end, and, depending on the shape of the bid-ask spreads $s(T)$, either flatten out or keep increasing for longer maturities. Fourth, liquidity premia computed from bid prices are monotonously decreasing at the short end and, again depending on the shape of $s(T)$, flatten out or start increasing for longer maturities.

Before explicitly testing our model predictions using data from the U.S. corporate bond market, we now discuss the empirical results from Chapter 3 on the shape of liquidity premia during crisis and non-crisis times through the lenses of our equilibrium model. To enhance comparisons with previous empirical studies and most importantly due to missing data on bid and ask prices during our observation period, we used closing prices from the Frankfurt stock exchange to calculate liquidity premia in Chapter 3. These

authors find that long-horizon investors on average hold more illiquid bonds (with higher liquidity premia), but demand less compensation for less liquid bonds than short-horizon investors would. These empirical results correspond to our clientele effect and the flattening term structure of liquidity premia.

closing prices either result from a buy or a sell trade or are determined in an auction like setting. Therefore, the resulting term structure of liquidity premia can be interpreted as a weighted average computed from the term structures of bid and ask prices where the weightings are the probabilities for a buy or sell, respectively.

Through the lenses of our model, the reason for the more inverse shape of the term structure of 'closing price' liquidity premia in crises times is a mixture of two mechanisms. First, in crisis times, the incentive to sell in the case of a preference shock modeled though the increase in the time preference rate b is larger (since the alternatives to get funding become more expensive). This lowers τ and thus shrinks the maturity range where ask liquidity premia are small. Second, in crisis times, the probability for a particular trade to result from a sell order might be higher due to increased liquidity needs. Thus, the term structure of closing price liquidity premia is closer to the bid curve which results in a more inverse shape. Both mechanisms are in line with the empirically found increased impact of liquidity demand at the short end.

4.4 Empirical Analysis

In this section, we verify the four main predictions from our equilibrium model using empirical data. Our model forecasts non-linear relations between maturity T and bid and ask liquidity premia. More formally, it predicts that the sensitivity of liquidity premia on time to maturity T is different for short- and long-term bonds. To test these relations, we employ piecewise linear regressions that explicitly allow for such a different sensitivity for maturities below and above a breakpoint y. Regressions are of the form

$$\begin{aligned}\mathrm{Illiq}^{\mathrm{ask}}(T) &= \alpha^a + \beta_1^a \cdot \mathbb{1}_{\{T\leq y\}} \cdot (T-y) + \beta_2^a \cdot \mathbb{1}_{\{T>y\}} \cdot (T-y) + \gamma^a \cdot \mathrm{Controls} + \varepsilon,\\ \mathrm{Illiq}^{\mathrm{bid}}(T) &= \alpha^b + \beta_1^b \cdot \mathbb{1}_{\{T\leq y\}} \cdot (T-y) + \beta_2^b \cdot \mathbb{1}_{\{T>y\}} \cdot (T-y) + \gamma^b \cdot \mathrm{Controls} + \varepsilon, \quad (4.12)\end{aligned}$$

where $\mathrm{Illiq}^{\mathrm{bid}}(T)$ ($\mathrm{Illiq}^{\mathrm{ask}}(T)$) is the liquidity premium computed from bid (ask) prices, T is the time to maturity for which the liquidity premium applies, and ε is an error term. We explore a wide range of possible breakpoints y between 3 months and 3 years and do not endogenously derive an optimal breakpoint to avoid overfitting. If our hypotheses regarding the liquidity term structures are confirmed, we expect the following behavior. For ask premia, we should find positive and significant estimates for β_1^a and β_2^a as the slope of the ask liquidity premium term structure is positive for all maturities. Because our model predicts a flattening term structure, we expect β_1^a to be larger than β_2^a. For bid premia, we should find significant negative estimates for β_1^b. The shape of the bid term structure at the long end depends on the shape of bid-ask spreads $s(T)$, which we will show to be strongly increasing in the time to maturity T for our data set. Therefore, we expect β_2^b to be positive.

A similar intuition holds for trading volume. There, we use a regression of the form

$$\text{Turnover}(T) = \alpha + \beta_1 \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T - y) + \beta_2 \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \beta_3 \cdot \text{Age} + \gamma \cdot \text{Controls} + \varepsilon, \tag{4.13}$$

and expect positive and significant estimates for the slope at the short end β_1 and negative and significant estimates of β_2. We expect a significant and negative estimate for β_3 since our model predicts a negative relation between the age of a bond and trading volume.

4.4.1 Data

We use bond transaction data from TRACE (Trade Reporting and Compliance Engine) to test the predictions of our model. This database was introduced by the Financial Industry Regulatory Agency (FINRA) to increase transparency in the U.S. corporate bond market. It lists information concerning secondary market transactions of U.S. corporate bonds, e.g., actual trade prices, yields resulting from these prices, and trade sizes. Trades have to be reported since July 1, 2002 and reporting requirements have been tightened over three phases. According to Goldstein and Hotchkiss (2012), transaction information for nearly all U.S. corporate bonds is disseminated in TRACE since October 1, 2004. For that reason, the time period of our study starts on October 1, 2004 and ends on December 31, 2011.

We calculate bid and ask liquidity premia based on yields of investment grade bonds adjusted for riskless interest rates and credit risk. Turnover is based on trading volume and outstanding amounts. Table 4.1 summarizes the data selection procedure and the number of observations for our final sample and the subsamples used in our robustness checks in Section 4.5. We start with filtering out erroneous trades as described in Dick-Nielsen (2009). For the remaining bonds, we collect information on the bond's maturity, coupon, and other features from Reuters and Bloomberg using the bond's CUSIP. We drop all bonds which are not plain vanilla fixed rate bonds without any extra rights. We also collect the rating history from Reuters and drop all observations for bonds on days where fewer than two rating agencies (S&P's, Moody's, Fitch) report an investment-grade rating. We exclude private placements, bonds with more than 30 years remaining time to maturity, and all bonds that are not classified as senior unsecured in the Markit database. For the analysis of liquidity premia, we follow Dick-Nielsen (2009) and additionally drop all transactions with non-standard prices, e.g., prices which explicitly include a commission or only apply for specific settlement conditions.

We then collect the transaction yield, price, and volume, and the reporting date and time. To differentiate between bid and ask liquidity premia, we require bid and ask bond yields. As this information is only provided for transactions starting from November 1, 2008 in TRACE, we use the methodology of Feldhütter (2012), which we also employed for the roundtrip transaction cost measure in Section 2.3.1.1, to calculate bid and ask yields. As discussed in Section 2.3.1.1, a large part of the trades reported in TRACE are part of so called imputed roundtrip trades (IRTs), i.e., pre-matched arrangements where a customer trades a bond with a dealer and the dealer (in the case of a sell) resells it to another dealer (who possibly again sells it). As over 90% of these IRTs are either dealer-seller or dealer-buyer arrangements, we interpret the difference between the highest and lowest yield within an IRT as the half-spread. This half-spread is then added to the midpoint to get the bid-yield and deducted from the midpoint to get the ask-yield.[44] In a robustness analysis for the subperiod where TRACE information on bid and asks is available, we explore whether the identification procedure has an impact on our results.

To determine liquidity premia, we adjust for risk-free interest rates and for credit risk. We derive the risk-free rate that would apply to a specific bond using Treasury yields. We collect daily constant-maturity yields from the Federal Reserve's H-15 release, and derive a full term structure using linear interpolation. In a robustness check, we employ swap rates instead of Treasury yields as the risk-free rate.[45] As a measure of credit risk, we collect a time series of daily credit default swap (CDS) mid quotes of all available maturities for each bond issuer from Markit and again derive a full term structure by interpolating between the available maturities.[46] We then compute a bond's ask (bid) liquidity premium as the difference between the bond's ask (bid) yield and the sum of the

[44] The midpoint of the IRT equals the midpoint of either the bid or the ask and the price at which dealers trade among each other. Therefore, in about half of the cases we overestimate the real mid and in the other half, we underestimate the mid. Hence, on average, there should not be any bias from this methodology.

[45] We use the USD swap curves available via Bloomberg for maturities larger than three months and extend the curve at the short end by linearly interpolating the 6-months rate with USD LIBOR rates for a maturity of 1 and 3 months. We are cautious to account for the different day count conventions in swap and LIBOR markets.

[46] Since the shortest available maturity for CDS quotes is six months, we have to extrapolate the term structure of CDS premia at the short end.

Table 4.1: **Sample description**

The table presents the procedure used to arrive at the final samples employed in our main analysis and in the robustness checks in Section 4.5, the number of trades, the number of bonds, and the traded notional value in billion USD. The sample period is from October 1, 2004 to December 31, 2011.

	Number of trades	Number of bonds	Traded notional value (in bn. USD)
All trades within the TRACE database	57,192,955	65,900	19,901
Subtotal after filtering out erroneous trades with the procedures described in Dick-Nielsen (2009)	53,738,948	65,183	17,558
Subtotal after removing bonds with missing information (in Bloomberg, Reuters, or Markit), callable bonds (incl. make-whole call provisions), bonds with remaining time to maturity of more than 30 years, puttable bonds, bonds with sinking funds, zero coupon bonds, convertible bonds, bonds with variable coupon payments, bonds with other non-standard cash flow or coupon structures, issues which do not have an investment grade rating from at least two rating agencies (i.e., Moody's, S&P, or Fitch) at the trading date, bonds which are not classified as senior unsecured, private placements **(sample used to calculate turnover)**	13,544,882	10,327	3,910
Excluding trades under non-standard terms (e.g., special settlement or sale conditions) and trades where the price explicitly includes a commission (see Dick-Nielsen, 2009)	12,695,725	10,305	3,532
Imputed Roundtrip Trades (IRTs) (see Feldhütter, 2012)	5,983,293 (within 2,458,325 distinct IRTs)	9,768	1,188
Main sample: IRTs matched with respective Constant Maturity Treasury (CMT) yield and CDS premium, no government guarantee	5,167,485 (2,124,932 IRTs)	8,576	952
Sample swap-implied liquidity premia (Section 4.5.1): IRTs matched with respective swap rate and CDS premium, no government guarantee	5,202,564 (2,139,337 IRTs)	8,590	973
Sample AAA before financial crisis (Section 4.5.3): IRTs in AAA bonds matched with respective CMT yield, until Mar. 2007	131,420 (56,514 IRTs)	344	28
Sample TRACE-identified ask trades since Nov. 2008 (Section 4.5.2): dealer is seller, no interdealer trades, matched with respective CMT yield and CDS premium, no government guarantee	1,908,478	3,660	432
Sample TRACE-identified bid trades since Nov. 2008 (Section 4.5.2): dealer is buyer, no interdealer trades, matched with respective CMT yield and CDS premium, no government guarantee	1,153,607	3,671	378

risk-free yield and the CDS premium. For the turnover analysis and as a control variable, we collect the history of outstanding notional amounts for each bond from Reuters. To prevent an impact of outliers on our results, we winsorize liquidity premia and turnover at the 1% and 99% quantile.

4.4.2 Liquidity Premia Analysis

Our model predictions for the long end of the term structure of liquidity premia depend on the shape of the term structure of bid-ask spreads. Therefore, we first calibrate a parametric form for $s(T)$ to our data set. Using non-linear least squares, we minimize the sum of squared errors ϵ_i in the following equation:[47]

$$s_i(T_i) = a^{\text{bid-ask}} + b^{\text{bid-ask}} \cdot \left(1 - e^{-c^{\text{bid-ask}} \cdot T_i}\right) + \epsilon_i, \tag{4.14}$$

where bid-ask spreads $s_i(T_i)$ for each IRT i in a bond with maturity T_i are computed using the roundtrip transaction cost measure (see Feldhütter, 2012, and Section 2.3.1.1). We winsorize bid-ask spreads at the 1% and 99% quantile. Figure 4.5 presents the calibrated function $s(T)$ together with average bid-ask spreads for monthly time to maturity buckets and shows three important properties of bid-ask spreads. First, bid-ask spreads are non-zero even for securities with very short maturities, which corresponds to a fixed component of transaction cost. Second, bid-ask spreads increase in maturity. Third, the slope of the bid-ask spread term structure decreases for large maturities.[48]

We next present an overview over the average term structure of ask and bid liquidity premia together with the respective model predictions. Visual inspection of Figure 4.6 suggests that our main hypotheses regarding liquidity premia hold for the full sample. Ask liquidity premia go through the origin and are mostly increasing in maturity, while bid liquidity premia exhibit a U-shape. At the long end, both bid and ask premia slightly increase with maturity.

We now formally explore the effect of maturity on liquidity premia in bond bid and ask yields and estimate Equation (4.12). As control variables, we use age, outstanding amount, and credit risk effects not captured by subtracting the CDS premium via the

[47]Since the bond-specific spread should be limited between 0 and 1, a range of simple functions such as a linear form $s(T) = a^{\text{bid-ask}} + b^{\text{bid-ask}} \cdot T$ or its exponential counterpart $s(T) = a^{\text{bid-ask}} + b^{\text{bid-ask}} \cdot \left(e^{c^{\text{bid-ask}} \cdot T}\right)$ for $c^{\text{bid-ask}} > 0$ are not suitable for all possible $T_{\max}$.

[48]We also find these properties when looking at TRACE-identified bid and ask trades.

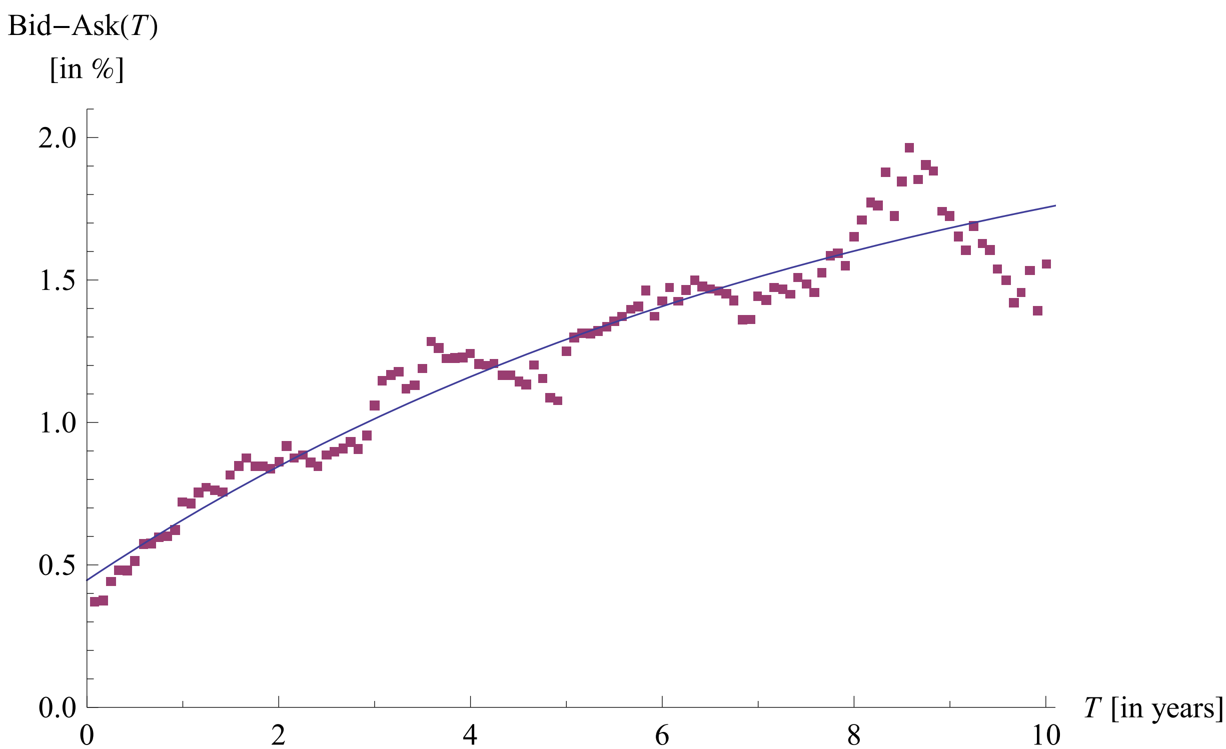

Figure 4.5: **Empirical term structure of bid-ask spreads**
The figure presents the average term structure of proportional bid-ask spreads (squares) together with the calibrated bid-ask spread function $s(T) = 0.00446 + 0.01868 \cdot \left(1 - e^{-0.1205 \cdot T}\right)$ (solid line). To calculate bid-ask spreads, we use the roundtrip transaction cost measure (for details, see Feldhütter, 2012, or Section 2.3.1.1). The depicted average spread for a given maturity is computed as the mean spread across all bonds of a given maturity. The sample period is from October 1, 2004 to December 31, 2011.

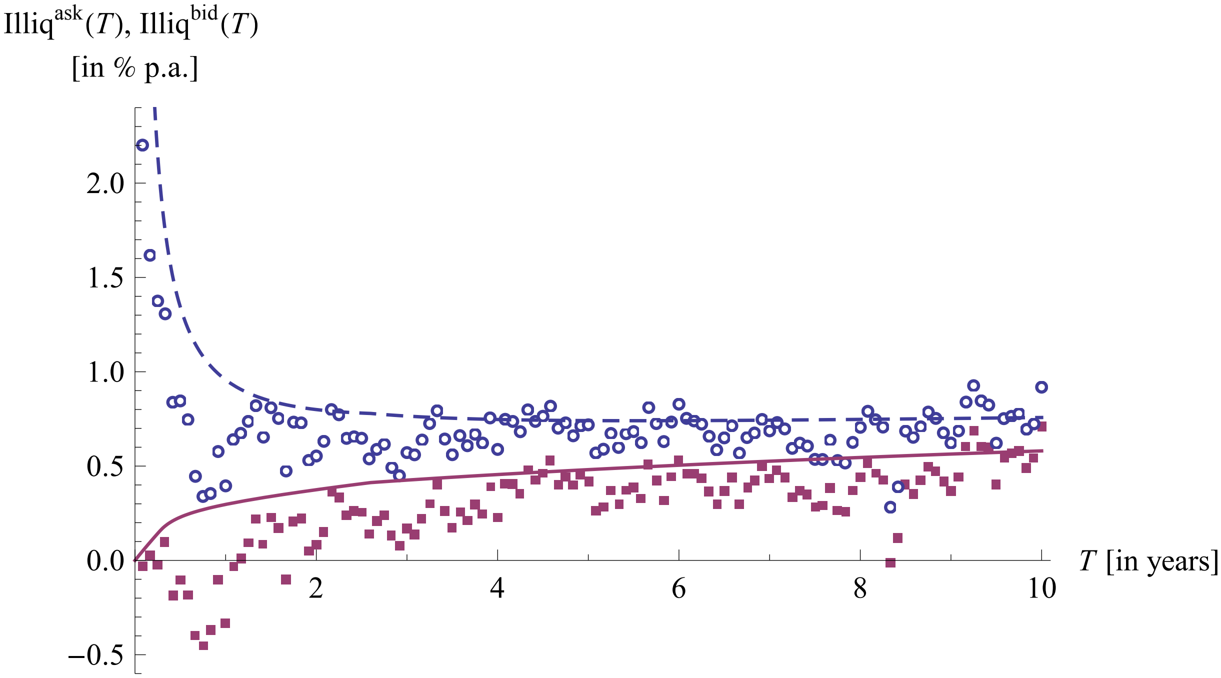

Figure 4.6: **Empirical term structure of ask and bid liquidity premia**
The figure presents the average term structure of ask and bid liquidity premia together with the predictions of our model (see Figure 4.4). The liquidity premium for a given bond is determined as the TRACE-reported bond yield minus the interpolated constant-maturity Treasury yield and the interpolated CDS premium from Markit. Bid and ask yields are calculated using the methodology of Feldhütter (2012). The depicted average liquidity premium for a given maturity is computed as the mean liquidity premium across all bonds of a given maturity. Squares indicate average ask liquidity premia, circles show average bid liquidity premia. The solid line depicts model implied $\text{Illiq}^{\text{ask}}(T)$, the dashed line shows model implied $\text{Illiq}^{\text{bid}}(T)$. The sample period is from October 1, 2004 to December 31, 2011.

numerical rating (where AAA (D) corresponds to a rating of 1 (22)). We control for age and the outstanding amount as, e.g., Edwards, Harris, and Piwowar (2007) report a dependence of transaction costs on age and outstanding volume which is not directly captured by our model. We expect a positive coefficient estimate for age and credit risk, and a negative estimate for the outstanding amount. We use standard errors clustered by firm as suggested by Petersen (2009) and include month fixed and firm fixed effects. As the exact location of the limiting maturities depends on unobservable factors like the wealth of investors, we test our predictions for a range of exogenously specified short-term breakpoints y. Note, however, that the predictions do not imply the U-shape in bid premia and the kink in the slope for ask premia for all specifications of y. The results of the regression are given in Table 4.2.

Table 4.2 confirms our hypotheses regarding liquidity premia. For ask liquidity premia, the estimates for the slope at the short end, β_1^a, are always positive. They are significant in 5 out of 6 specifications of the breakpoint y. The estimates for the slope at the long end, β_2^a, are also always positive and significant. In 5 out of 6 specifications, the slope is significantly steeper at the short end. Overall, the results strongly support our model prediction that ask liquidity premia are more strongly increasing for shorter maturities, and flatten out for longer maturities.

Regarding the results for bid liquidity premia, we obtain negative and in 4 out of 6 cases significant estimates for the slope at the short end. The positive but insignificant estimate for the 3-year breakpoint agrees with the intuition from Figure 4.6 that the break between short and longer maturities occurs at maturities below three years. Consistent with the increasing bid-ask spreads, all estimates for the slope at the long end (except for the 3-year breakpoint) are significantly positive. Overall, bid liquidity premia exhibit a U-shape with a strongly negative slope for short maturities and a subsequent positive, but flatter, slope for longer maturities.

The impact of the control variables is also as expected. Age, in the one case where it is significant, has a positive impact on liquidity premia. Outstanding volume has a negative and significant impact in all cases. The numerical rating always has a positive impact that is significant in 11 out of 12 cases, which implies that credit risk and liquidity premia are positively correlated.

Overall, the results of the regression analysis confirm our model predictions. Ask liquidity premia are monotonously increasing with a decreasing slope, while bid liquidity premia are decreasing for short maturities and, due to increasing bid-ask spreads, increasing at

Table 4.2: **Regression of ask and bid liquidity premia on maturity**

The table presents the regression analysis of ask and bid liquidity premia on maturity and control variables for different breakpoints that separate the short end from longer maturities of the liquidity term structure:

$$\text{Illiq}^{\text{ask}}(T) = \alpha^a + \beta_1^a \cdot \mathbb{1}_{\{T\leq y\}} \cdot (T-y) + \beta_2^a \cdot \mathbb{1}_{\{T>y\}} \cdot (T-y) + \gamma^a \cdot \text{Controls} + \varepsilon,$$
$$\text{Illiq}^{\text{bid}}(T) = \alpha^b + \beta_1^b \cdot \mathbb{1}_{\{T\leq y\}} \cdot (T-y) + \beta_2^b \cdot \mathbb{1}_{\{T>y\}} \cdot (T-y) + \gamma^b \cdot \text{Controls} + \varepsilon,$$

where $\text{Illiq}^{\text{ask/bid}}(T)$ is the liquidity premium computed from the ask or bid yield for a given transaction minus the interpolated Treasury yield as a proxy for the risk-free rate and the CDS premium as a proxy for the credit risk premium, all in percentage points. The explanatory variables are the (remaining) time to maturity T (in years) minus the breakpoint y for $T \leq y$ and $T > y$, as well as the control variables age in years, the average numerical rating (Rating), and the natural logarithm of the outstanding amount (ln(Amt)). The breakpoints y equal three months, six months, nine months, one year, two years, and three years. We use firm and month fixed effects. Clustered standard errors at the firm level are presented in parentheses. The sample period is from October 1, 2004 to December 31, 2011. *, ** indicate significance at the 5% or 1% level.

	Ask						Bid					
	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$
Constant	2.6982**	2.6981**	2.6995**	2.715**	2.854**	3.0569**	2.8296**	2.8456**	2.8466**	2.8283**	2.8221**	2.8626**
	(0.5489)	(0.5492)	(0.5503)	(0.5516)	(0.5597)	(0.5743)	(0.6339)	(0.6283)	(0.6264)	(0.6284)	(0.6337)	(0.6374)
$\mathbb{1}_{\{T\leq y\}} \cdot (T-y)$	2.665	1.2509*	0.8951**	0.7924**	0.4493**	0.2914**	−11.1866**	−2.9179**	−1.2532**	−0.5649**	−0.0184	0.0346
	(1.5043)	(0.5559)	(0.2927)	(0.2017)	(0.0874)	(0.0487)	(1.9409)	(0.6415)	(0.3206)	(0.2144)	(0.0878)	(0.0476)
$\mathbb{1}_{\{T>y\}} \cdot (T-y)$	0.0389**	0.0376**	0.036**	0.0333**	0.0234**	0.0163*	0.0171**	0.0192**	0.02**	0.0196**	0.0163*	0.0132
	(0.0057)	(0.0057)	(0.0058)	(0.0058)	(0.0067)	(0.0075)	(0.0061)	(0.0061)	(0.0061)	(0.0062)	(0.0071)	(0.0077)
Age [in years]	−0.0098	−0.0086	−0.007	−0.0047	0.0024	0.0059	0.0065	0.0046	0.0038	0.0044	0.0077	0.01*
	(0.0057)	(0.0057)	(0.0055)	(0.0054)	(0.0051)	(0.0051)	(0.005)	(0.0051)	(0.0051)	(0.005)	(0.005)	(0.005)
Rating	0.1223	0.1245*	0.1265*	0.1286*	0.1362*	0.1387*	0.2095**	0.2057**	0.2054**	0.2073**	0.2116**	0.2141**
	(0.0627)	(0.0626)	(0.0627)	(0.0629)	(0.0628)	(0.0627)	(0.068)	(0.0675)	(0.0674)	(0.0677)	(0.0683)	(0.0686)
ln(Amt)	−0.0862**	−0.0867**	−0.0875**	−0.0885**	−0.0922**	−0.0955**	−0.1057**	−0.1047**	−0.1042**	−0.1046**	−0.106**	−0.1073**
	(0.0103)	(0.0104)	(0.0105)	(0.0107)	(0.0117)	(0.0126)	(0.0129)	(0.0128)	(0.0126)	(0.0127)	(0.0132)	(0.0137)
Firm Fixed Effects	Yes											
Month Fixed Effects	Yes											
$\mathbb{1}_{\{T>y\}} \cdot (T-y)$	−2.6262	−1.2133*	−0.8592**	−0.7591**	−0.4259**	−0.2751**	11.2037**	2.9371**	1.2732**	0.5845**	0.0347	−0.0213
$-\mathbb{1}_{\{T\leq y\}} \cdot (T-y)$	(1.5054)	(0.5572)	(0.2941)	(0.2033)	(0.0908)	(0.0531)	(1.9434)	(0.6437)	(0.3227)	(0.2165)	(0.0911)	(0.0516)
N	2,124,932											
R^2	0.323	0.324	0.3254	0.3284	0.337	0.3394	0.3155	0.3168	0.3153	0.3127	0.3096	0.3096

the long end.

4.4.3 Turnover Analysis

To formally explore the hypotheses regarding trading volume, we consider two subsamples. First, we use all transactions available in TRACE, standardized with the outstanding amount of the bond under consideration. Second, we exclude bonds immediately around changes in their outstanding volume (through new issues, reopenings, and bond repurchases) since we do not consider these events in our model. When bonds are newly issued, they are often first held by dealers, who distribute them to clients and other dealers. Hence, the time interval around new issues of bonds might consist of multiple inter-dealer trades. We therefore exclude transactions two months prior to a new issue and six months following the issue, and denote this sample by Excl[-2,+6].[49] We now apply our piecewise regression approach with age as an additional explanatory variable according to Equation (4.13). The control variables we use are again the outstanding amount and credit risk. Since turnover cannot be calculated on a trade-by-trade basis, we aggregate traded volume for each bond and calendar month and compute average daily turnover to account for a different number of business days per month. The regression results are displayed in Table 4.3.

Table 4.3 confirms our model predictions regarding the hump-shaped turnover. For both subsamples, trading volume first increases strongly, since the factor loadings for the slope at the short end, β_1, are positive and significant in 10 out of 12 specifications. Following the breakpoint, trading volume decreases slowly, but the effect is only significant in the sample where we exclude bonds around changes in the outstanding amount. The negative loading for age in all specifications is consistent with our second model prediction regarding the aging effect: bonds which have been outstanding for a longer amount of time are traded less frequently.[50] The results for the outstanding amount are also as expected: bonds with a higher outstanding volume are on average more liquid, and thus display a higher trading volume.

[49] We exclude the time two months before changes in the amount outstanding mainly because of trades taking place in connection with bond repurchases that are typically announced about one month in advance.

[50] As discussed in Section 4.4.2, part of the effect could result from the dependence of bid-ask spreads on age, which is not explained in our model.

Table 4.3: **Regression of turnover on maturity and age**

The table presents the regression analysis of turnover for the two subsamples on maturity, age, and control variables for different breakpoints:

$$\text{Turnover}(T) = \alpha + \beta_1 \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T - y) + \beta_2 \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \beta_3 \cdot \text{Age} + \gamma \cdot \text{Controls} + \varepsilon,$$

where Turnover(T) is calculated as the average daily turnover for each bond and each calendar month. The left panel contains the regression results for the full sample, the right panel contains the regression results for the subsample that excludes bonds two months prior to and six months after changes in their outstanding amount. The explanatory variables are the (remaining) time to maturity T (in years) minus the breakpoint y for $T \leq y$ and $T > y$ as well as age (in years). The control variables are the average numerical rating (Rating) and the natural logarithm of the outstanding amount (ln(Amt)). The breakpoints y are given by three months, six months, nine months, one year, two years, and three years. We use month fixed effects. Clustered standard errors at the firm level are presented in parentheses. Parameter estimates and standard errors are multiplied by 1,000. The sample period is from October 1, 2004 to December 31, 2011. *, ** indicate significance at the 5% or 1% level.

	All						Excl[-2, +6]					
	$y = 0.25$	$y = 0.5$	$y = 0.75$	$y = 1$	$y = 2$	$y = 3$	$y = 0.25$	$y = 0.5$	$y = 0.75$	$y = 1$	$y = 2$	$y = 3$
Constant	−1.8381**	−1.8365**	−1.8366**	−1.8379**	−1.8455**	−1.8434**	−1.4082**	−1.408**	−1.4093**	−1.4117**	−1.4258**	−1.4348**
	(0.2947)	(0.295)	(0.2953)	(0.2956)	(0.2975)	(0.2992)	(0.2464)	(0.2468)	(0.2471)	(0.2475)	(0.2498)	(0.2523)
$\mathbb{1}_{\{T \leq y\}} \cdot (T - y)$	3.0653**	1.0484**	0.5219**	0.3**	0.0603*	0.0321	3.2045**	1.1245**	0.5734**	0.3364**	0.0684*	0.0258
	(0.1954)	(0.0922)	(0.0618)	(0.047)	(0.0268)	(0.0192)	(0.1875)	(0.0895)	(0.061)	(0.0469)	(0.0272)	(0.0196)
$\mathbb{1}_{\{T > y\}} \cdot (T - y)$	−0.0052	−0.0062	−0.0065	−0.0064	−0.005	−0.0055	−0.0137**	−0.015**	−0.0156**	−0.0157**	−0.0144**	−0.014**
	(0.0043)	(0.0043)	(0.0044)	(0.0044)	(0.0045)	(0.0048)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.0042)
Age [in years]	−0.0879**	−0.0874**	−0.0873**	−0.0875**	−0.0885**	−0.0886**	−0.0722**	−0.0717**	−0.0716**	−0.0717**	−0.0727**	−0.0731**
	(0.0074)	(0.0074)	(0.0074)	(0.0074)	(0.0075)	(0.0075)	(0.0055)	(0.0055)	(0.0055)	(0.0055)	(0.0056)	(0.0056)
Rating	0.0301	0.0302	0.0303	0.0304	0.0304	0.0306	0.0316*	0.0319*	0.032*	0.0321*	0.0321*	0.0321*
	(0.0171)	(0.0171)	(0.0171)	(0.0171)	(0.0172)	(0.0172)	(0.0159)	(0.0159)	(0.0158)	(0.0158)	(0.0159)	(0.0159)
ln(Amt)	0.1889**	0.1891**	0.1892**	0.1892**	0.189**	0.189**	0.1564**	0.1566**	0.1567**	0.1567**	0.1566**	0.1565**
	(0.016)	(0.016)	(0.016)	(0.016)	(0.0159)	(0.0159)	(0.0132)	(0.0132)	(0.0132)	(0.0132)	(0.0131)	(0.013)
Firm Fixed Effects	No											
Month Fixed Effects	Yes											
$\mathbb{1}_{\{T > y\}} \cdot (T - y)$ $-\mathbb{1}_{\{T \leq y\}} \cdot (T - y)$	−3.0705**	−1.0546**	−0.5284**	−0.3064**	−0.0652*	−0.0376	−3.2183**	−1.1395**	−0.589**	−0.3521**	−0.0828**	−0.0398
	(0.1965)	(0.0932)	(0.0627)	(0.0478)	(0.0279)	(0.0209)	(0.1886)	(0.0904)	(0.0617)	(0.0475)	(0.0281)	(0.021)
N	385,232						357,826					
R^2	0.05887	0.05843	0.05804	0.05772	0.05709	0.05704	0.04685	0.04638	0.04592	0.0455	0.04456	0.04436

4.5 Robustness

In the previous section, we compute ask and bid liquidity premia under three important assumptions. First, we use U.S. Treasury yields as the risk-free reference curve. Second, we identify bid and ask trades using the method of Feldhütter (2012). Third, we use CDS premia as a proxy for the credit risk premium.[51] In this section, we show that these assumptions do not affect the relation between liquidity premia and maturity by repeating our analysis for different subsamples. In Section 4.5.1, we repeat the regression analysis using swap rates instead of Treasury yields as a proxy of the risk-free interest rates. Section 4.5.2 contains the analysis for the TRACE subsample that includes information of whether a transaction occurred at the ask or at the bid price. In Section 4.5.3, we restrict liquidity premia to AAA bonds where the impact of credit risk on yield spreads is minimized and do not subtract the CDS premium.[52]

4.5.1 Swap Rates as Risk-Free Interest Rates

As mentioned in Section 4.4.1, instead of using Treasury yields, we also interpolate swap rates with maturities between one month and 30 years to obtain an alternative risk-free yield curve. Table 4.2 shows the results when we re-estimate Equation (4.12) using swap rates as the risk-free reference curve to calculate liquidity premia.

Table 4.4 shows that our estimation results are mostly unaffected by the use of swap rates as risk-free rates. For ask liquidity premia, the estimates for the slope at both the short and the long end are positive and significant in all specifications, with the estimates for the short end significantly exceeding those for the long end. For bid liquidity premia, the slope at the short end for a breakpoint of one year now also becomes insignificant, but remains negative. The impact of the rating control variable decreases, which may be due to the higher credit risk contained in swap rates. Our main conclusions, however, remain unaffected: ask liquidity premia increase in time to maturity, bid liquidity premia exhibit a U-shape.

[51]Since we also use daily ratings as a control variable, we already account for a possible correlation between credit risk and liquidity premia.

[52]In additional robustness checks, we restrict our dataset on transactions with a volume of $100,000 or more and only look at the time before the onset of the subprime crisis. All results confirm our hypotheses and are available upon request.

Table 4.4: **Regression of swap-implied ask and bid liquidity premia on maturity**
The table presents the regression analysis of swap-implied ask and bid liquidity premia on maturity and control variables for different breakpoints that separate the short end from longer maturities of the liquidity term structure:

$$\text{Illiq}^{\text{ask}}(T) = \alpha^a + \beta_1^a \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T-y) + \beta_2^a \cdot \mathbb{1}_{\{T>y\}} \cdot (T-y) + \gamma^a \cdot \text{Controls} + \varepsilon,$$
$$\text{Illiq}^{\text{bid}}(T) = \alpha^b + \beta_1^b \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T-y) + \beta_2^b \cdot \mathbb{1}_{\{T>y\}} \cdot (T-y) + \gamma^b \cdot \text{Controls} + \varepsilon,$$

where $\text{Illiq}^{\text{ask/bid}}(T)$ is the liquidity premium computed from the ask or bid yield for a given transaction minus the interpolated swap rate as a proxy for the risk-free rate and the CDS premium as a proxy for the credit risk premium, all in percentage points. The explanatory variables are the (remaining) time to maturity T (in years) minus the breakpoint y for $T \leq y$ and $T > y$, as well as the control variables age in years, the average numerical rating (Rating), and the natural logarithm of the outstanding amount (ln(Amt)). The breakpoints y equal three months, six months, nine months, one year, two years, and three years. We use firm and month fixed effects. Clustered standard errors at the firm level are presented in parentheses. The sample period is from October 1, 2004 to December 31, 2011. *, ** indicate significance at the 5% or 1% level.

	Ask						Bid					
	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$
Constant	2.3436**	2.3412**	2.3452**	2.3665**	2.5192**	2.7234**	2.4482**	2.4652**	2.4721**	2.4624**	2.4765**	2.5229**
	(0.5658)	(0.5652)	(0.5656)	(0.5658)	(0.571)	(0.5834)	(0.6382)	(0.6343)	(0.6327)	(0.6338)	(0.6358)	(0.637)
$\mathbb{1}_{\{T \leq y\}} \cdot (T-y)$	3.6781*	1.8134**	1.1554**	0.928**	0.4677**	0.2964**	−9.9741**	−2.3077**	−0.971**	−0.4158	0.0053	0.0427
	(1.5024)	(0.5886)	(0.3114)	(0.2133)	(0.0907)	(0.05)	(1.9065)	(0.6697)	(0.3379)	(0.225)	(0.0907)	(0.0489)
$\mathbb{1}_{\{T>y\}} \cdot (T-y)$	0.056**	0.0544**	0.0527**	0.0502**	0.0418**	0.0363**	0.0354**	0.0367**	0.0373**	0.037**	0.0347**	0.0329**
	(0.0059)	(0.0061)	(0.0062)	(0.0063)	(0.0073)	(0.0079)	(0.0068)	(0.0068)	(0.0069)	(0.007)	(0.0077)	(0.0081)
Age [in years]	−0.0127*	−0.011	−0.0092	−0.007	−0.0006	0.002	0.0036	0.0023	0.0017	0.0022	0.0046	0.0061
	(0.0062)	(0.0062)	(0.0061)	(0.0059)	(0.0058)	(0.0057)	(0.0053)	(0.0055)	(0.0055)	(0.0054)	(0.0055)	(0.0055)
Rating	0.1164	0.1195	0.1218	0.1236	0.1304*	0.1321*	0.2017**	0.199**	0.1987**	0.2003**	0.2035**	0.2052**
	(0.0629)	(0.0628)	(0.0628)	(0.0629)	(0.0626)	(0.0625)	(0.067)	(0.0667)	(0.0666)	(0.0668)	(0.0672)	(0.0674)
ln(Amt)	−0.0874**	−0.0881**	−0.089**	−0.0898**	−0.0929**	−0.0955**	−0.1064**	−0.1057**	−0.1054**	−0.1056**	−0.1066**	−0.1073**
	(0.011)	(0.0111)	(0.0113)	(0.0115)	(0.0125)	(0.0133)	(0.0139)	(0.0138)	(0.0138)	(0.0138)	(0.0142)	(0.0146)
Firm Fixed Effects	Yes											
Month Fixed Effects	Yes											
$\mathbb{1}_{\{T>y\}} \cdot (T-y)$ $-\mathbb{1}_{\{T \leq y\}} \cdot (T-y)$	−3.6221*	−1.7591**	−1.1027**	−0.8777**	−0.4258**	−0.2601**	10.0095**	2.3444**	1.0083**	0.4527*	0.0294	−0.0098
	(1.5041)	(0.5904)	(0.3132)	(0.2153)	(0.0943)	(0.0545)	(1.9096)	(0.6725)	(0.3405)	(0.2275)	(0.0942)	(0.0528)
N	2,139,337											
R^2	0.3214	0.3235	0.3253	0.3282	0.3349	0.3356	0.2985	0.2984	0.2974	0.2956	0.2938	0.2937

4.5.2 Identification of Bid and Ask Quotes

In this section, we focus on the impact of the method we use to identify ask and bid quotes. In the previous sections, we have followed Feldhütter (2012) to calculate bid and ask yields. We now exclusively focus on transactions between November 1, 2008 and December 31, 2011, for which TRACE indicates whether a transaction reported by a dealer occurred at the dealer's ask or bid quote. For these trades, we compute the liquidity premium by subtracting the interpolated Treasury yield and the CDS premium from the yield. We show the regression results where we only use TRACE-identified ask and bid transactions in Table 4.5.

As Table 4.5 shows, our main results hold for the subsample where bid and ask quotes are identified directly from TRACE-reported information. For ask premia, the coefficient estimate for the slope at the short end is always positive and significant, the slope at the long end is always positive and significant in 4 out of 6 cases, and significantly flatter than at the short end. For bid premia, the estimates for the short end are always negative and statistically significant in 4 out of 6 cases, the slope at the long end is, though not statistically significant, positive. Hence, our identification method does not affect the results materially.

4.5.3 Analysis of AAA Bonds

In our final robustness analysis, we analyze whether our results are sensitive to how we adjust the observed yield spreads for credit risk. To do so, we drop all transactions where the traded bond does not exhibit a AAA rating by at least two rating agencies on the transaction date. We also drop all transactions which occurred after March 31, 2007 since a AAA rating might not be indicative of negligible credit risk during the financial crisis. General Electric bonds, e.g., exhibited increasing yields long before the downgrade from AAA to AA+ by Standard&Poor's on March 12, 2009. We then interpret the difference between the bond's yield and the interpolated Treasury yield as a pure liquidity premium.[53] We explore the relation between liquidity premia and maturity for AAA rated bonds in Table 4.6. Since all bonds exhibit a AAA rating, we exclude rating as an explanatory variable.

Table 4.6 shows that our results are, if anything, stronger for the AAA sample than for

[53]In an alternative robustness check, we use agency bonds instead of AAA rated bonds. The results are virtually the same.

Table 4.5: **Regression of TRACE-identified ask and bid liquidity premia on maturity**
The table presents the regression analysis of ask and bid liquidity premia on maturity and control variables for different breakpoints that separate the short end from longer maturities of the liquidity term structure:

$$\text{Illiq}^{\text{ask}}(T) = \alpha^a + \beta_1^a \cdot \mathbb{1}_{\{T\leq y\}} \cdot (T-y) + \beta_2^a \cdot \mathbb{1}_{\{T>y\}} \cdot (T-y) + \gamma^a \cdot \text{Controls} + \varepsilon,$$
$$\text{Illiq}^{\text{bid}}(T) = \alpha^b + \beta_1^b \cdot \mathbb{1}_{\{T\leq y\}} \cdot (T-y) + \beta_2^b \cdot \mathbb{1}_{\{T>y\}} \cdot (T-y) + \gamma^b \cdot \text{Controls} + \varepsilon,$$

where $\text{Illiq}^{\text{ask/bid}}(T)$ is the liquidity premium computed from the ask or bid yield for a given transaction minus the interpolated Treasury yield as a proxy for the risk-free rate and the CDS premium as a proxy for the credit risk premium, all in percentage points. Bid and ask quotes are identified directly from TRACE-reported information. Hence, all observations are between November 1, 2008, and December 31, 2011. The explanatory variables are the (remaining) time to maturity T (in years) minus the breakpoint y for $T \leq y$ and $T > y$, as well as the control variables age in years, the average numerical rating (Rating), and the natural logarithm of the outstanding amount (ln(Amt)). The breakpoints y equal three months, six months, nine months, one year, two years, and three years. We use firm and month fixed effects. Clustered standard errors at the firm level are presented in parentheses. *, ** indicate significance at the 5% or 1% level.

	Ask						Bid					
	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$
Constant	−0.2156	−0.1168	−0.03	0.0677	0.2782	0.5318	0.0747	−0.0169	−0.0221	0.0015	0.0828	0.1256
	(1.2803)	(1.2828)	(1.2781)	(1.2588)	(1.2208)	(1.229)	(2.4159)	(2.4114)	(2.4166)	(2.4226)	(2.4355)	(2.4386)
$\mathbb{1}_{\{T\leq y\}} \cdot (T-y)$	8.1854**	3.8802**	2.542**	2.0449**	0.9296**	0.5337**	−11.1225**	−2.6877**	−1.1245**	−0.5227**	−0.0537	−0.0073
	(2.7247)	(1.0627)	(0.5405)	(0.3375)	(0.1262)	(0.0641)	(0.9474)	(0.3638)	(0.2126)	(0.1491)	(0.0649)	(0.0377)
$\mathbb{1}_{\{T>y\}} \cdot (T-y)$	0.0316**	0.0298**	0.0276**	0.0245*	0.0138	0.0061	0.0059	0.0067	0.007	0.0067	0.0056	0.005
	(0.0093)	(0.0094)	(0.0095)	(0.0096)	(0.0107)	(0.0111)	(0.0074)	(0.0074)	(0.0074)	(0.0074)	(0.008)	(0.0085)
Age [in years]	−0.0458**	−0.0419**	−0.0379**	−0.0329*	−0.02	−0.0136	0.0007	−0.0012	−0.0014	−0.0006	0.0022	0.0035
	(0.0141)	(0.0143)	(0.0143)	(0.0142)	(0.0143)	(0.0148)	(0.005)	(0.0049)	(0.0048)	(0.0048)	(0.0046)	(0.0046)
Rating	0.4232**	0.4227**	0.4228**	0.4256**	0.4568**	0.4659**	0.4684	0.4692	0.4694	0.4689	0.4671	0.4676
	(0.1431)	(0.1437)	(0.1434)	(0.1413)	(0.1346)	(0.1347)	(0.2407)	(0.241)	(0.2419)	(0.2426)	(0.2416)	(0.2409)
ln(Amt)	−0.0959**	−0.0988**	−0.1022**	−0.1059**	−0.1159**	−0.121**	−0.1628**	−0.1612**	−0.1609**	−0.1616**	−0.1641**	−0.165**
	(0.0259)	(0.0261)	(0.0262)	(0.0264)	(0.0271)	(0.0284)	(0.0289)	(0.0285)	(0.0281)	(0.0281)	(0.0286)	(0.0291)
Firm Fixed Effects							Yes					
Month Fixed Effects							Yes					
$\mathbb{1}_{\{T>y\}} \cdot (T-y)$	−8.1538**	−3.8505**	−2.5144**	−2.0204**	−0.9158**	−0.5276**	11.1284**	2.6945**	1.1315**	0.5294**	0.0593	0.0123
$-\mathbb{1}_{\{T\leq y\}} \cdot (T-y)$	(2.7281)	(1.0657)	(0.5435)	(0.3407)	(0.1321)	(0.0707)	(0.9499)	(0.365)	(0.2133)	(0.1501)	(0.0683)	(0.0418)
N	1,908,478						1,153,607					
R^2	0.4779	0.4833	0.4891	0.4983	0.5166	0.5177	0.4941	0.4934	0.4911	0.4886	0.4859	0.4857

Table 4.6: **Regression of ask and bid liquidity premia on maturity for AAA rated bonds**
The table presents the regression analysis of ask and bid liquidity premia on maturity and control variables for different breakpoints that separate the short end from longer maturities of the liquidity term structure:

$$\text{Illiq}^{\text{ask}}(T) = \alpha^a + \beta_1^a \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T - y) + \beta_2^a \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \gamma^a \cdot \text{Controls} + \varepsilon,$$
$$\text{Illiq}^{\text{bid}}(T) = \alpha^b + \beta_1^b \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T - y) + \beta_2^b \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \gamma^b \cdot \text{Controls} + \varepsilon,$$

where $\text{Illiq}^{\text{ask/bid}}(T)$ is the liquidity premium computed from the ask or bid yield of a AAA rated bond for a given transaction minus the interpolated Treasury yield as a proxy for the risk-free rate, both in percentage points. All observations are between October 1, 2004, and March 31, 2007. The explanatory variables are the (remaining) time to maturity T (in years) minus the breakpoint y for $T \leq y$ and $T > y$, as well as the control variables age in years and the natural logarithm of the outstanding amount (ln(Amt)). The breakpoints y equal three months, six months, nine months, one year, two years, and three years. We use firm and month fixed effects. Clustered standard errors at the firm level are presented in parentheses. *, ** indicate significance at the 5% or 1% level.

	Ask						Bid					
	$y = 0.25$	$y = 0.5$	$y = 0.75$	$y = 1$	$y = 2$	$y = 3$	$y = 0.25$	$y = 0.5$	$y = 0.75$	$y = 1$	$y = 2$	$y = 3$
Constant	−0.0597	−0.0354	−0.0039	0.0335	0.2053	0.3883	1.0089**	0.9844**	0.9776**	0.968**	0.9542**	0.9591**
	(0.3053)	(0.3013)	(0.2944)	(0.2879)	(0.262)	(0.2316)	(0.1567)	(0.1641)	(0.1696)	(0.1738)	(0.1898)	(0.2109)
$\mathbb{1}_{\{T \leq y\}} \cdot (T - y)$	0.1611	0.3071*	0.3109**	0.2754**	0.1805**	0.1409**	−4.9768**	−1.4072**	−0.6178**	−0.3364**	−0.0635**	−0.0149
	(0.3316)	(0.1152)	(0.0673)	(0.0468)	(0.0197)	(0.0109)	(0.308)	(0.0624)	(0.0245)	(0.0191)	(0.0112)	(0.0084)
$\mathbb{1}_{\{T > y\}} \cdot (T - y)$	0.0651**	0.0642**	0.0623**	0.0602**	0.0514**	0.0426**	0.0233**	0.0263**	0.0279**	0.029**	0.0307**	0.0312**
	(0.0132)	(0.0132)	(0.0132)	(0.0132)	(0.0129)	(0.0123)	(0.0051)	(0.0057)	(0.006)	(0.0064)	(0.008)	(0.01)
Age [in years]	0.0128*	0.0135*	0.0145*	0.0154*	0.0167**	0.0162**	0.0244**	0.0222**	0.0216**	0.0218**	0.0234**	0.0247**
	(0.006)	(0.0059)	(0.0059)	(0.0058)	(0.0058)	(0.0053)	(0.006)	(0.0057)	(0.0058)	(0.0058)	(0.0057)	(0.0061)
ln(Amt)	−0.0277**	−0.0278**	−0.028**	−0.028**	−0.0283**	−0.029**	−0.0514**	−0.0509**	−0.0511**	−0.0512**	−0.0513**	−0.0512**
	(0.001)	(0.001)	(0.0009)	(0.0008)	(0.0007)	(0.0006)	(0.0006)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0006)
Firm Fixed Effects	Yes											
Month Fixed Effects	Yes											
$\mathbb{1}_{\{T > y\}} \cdot (T - y)$	−0.096	−0.243	−0.2486**	−0.2152**	−0.1291**	−0.0983**	5.0001**	1.4335**	0.6457**	0.3654**	0.0942**	0.0461**
$-\mathbb{1}_{\{T \leq y\}} \cdot (T - y)$	(0.3349)	(0.1222)	(0.0761)	(0.0569)	(0.0302)	(0.021)	(0.3126)	(0.0677)	(0.0288)	(0.0226)	(0.0171)	(0.0164)
N	56,514											
R^2	0.3579	0.3589	0.3624	0.3664	0.3798	0.388	0.1963	0.2068	0.2015	0.1958	0.1829	0.1778

the entire sample. For ask liquidity premia, the estimates for β_1^a are always positive and significant in 5 out of 6 cases. The slope at the long end is always significantly positive and flatter than at the short end, and the control variables have the expected impact. Bid liquidity premia exhibit negative (positive) and significant estimates for the slope at the short (long) end in 5 out of 6 (all) specifications. The impact of the control variables is again as expected.

Summarizing the robustness section, we find that our results are robust against changes in the methodology to compute bid and ask liquidity premia.

4.6 Conclusion

In this chapter, we develop a parsimonious equilibrium model that generates a hump-shaped term structure of trading volume and, depending on whether we consider bid or ask prices, different shapes for the term structure of liquidity premia. Investors sell bonds of intermediate and long maturities because they experience a preference shock. Liquidity is supplied by an exogenous market maker who charges a positive spread. We then analyze liquidity premia of corporate bonds with a wide range of maturities, and show that the observed trading behavior and liquidity premia from ask and bid yields are consistent with our model predictions. The main conclusion from our analysis is that the different term structures can arise because of two frictions which are prevalent in bond markets: First, traders who provide liquidity charge a non-zero spread for bonds of all maturities. Second, investors differ with respect to their probability of experiencing a liquidity shock. Such a difference is obvious if we, for example, consider insurance companies who are unlikely to experience frequent liquidity shocks, and bond market funds which frequently experience cash outflows.

Our model also yields two central implications for market microstructure and financial stability. First, our model allows us to quantify the well-established price impact of any given bid-ask spread term structure for assets of different maturities. This is important because of two effects. First, artificially increasing transaction costs, especially at the short end such as through a fixed financial transaction tax, lead to uniformly higher required yields, and thus lower prices, for all bonds. Reversely, a decrease of transaction costs, e.g., via a subsidized dealer system, uniformly decreases yields and increases prices. Second, an increase (decrease) of transaction costs will shift the maturity limit below which investors do not sell bonds in spite of a preference shock to higher (lower) values. Therefore, our model predicts that higher bid-ask spreads can dry out the market for short-term securities.

The second important implication of our results concerns the interplay of liquidity and credit risk. As He and Xiong (2012) show, liquidity premia for corporate bonds can have a strong impact on the issuer's optimal default boundary. Hence, higher bid-ask spreads, which lead to higher liquidity premia, increase individual and aggregate credit risk. To the best of our knowledge, we are the first to also show that a higher probability of a liquidity shock for short-horizon investors also affects liquidity premia for long-term securities. Hence, even firms that issue long-term bonds might be affected by shocks to institutional investors who hold short-term debt to a similar extent as firms with short-term debt. This mechanism implies that liquidity risk management of investors with short investment horizons (e.g., liquidity buffers for banks under Basel III, or for mutual funds under the Investment Company Act) might increase financial stability for the entire economy.

Chapter 5

Summary and Outlook

In this thesis, we analyze how to measure and price illiquidity in bond markets.

In Chapter 2, we comprehensively compare all commonly employed liquidity measures based on intraday and daily data for the U.S. corporate bond market. We find that high-frequency measures based on intraday transaction data are very strongly correlated implying that previous results should be robust regarding the chosen measure. Most low-frequency proxies based on daily data generally also measure transaction costs and price impact well. When using a daily liquidity proxy, the best choices for effective transaction costs are the high-low spread estimator from Corwin and Schultz (2012), Roll (1984), and a measure based on Gibbs sampling introduced by Hasbrouck (2009). When only daily data on quotes is available, bid-ask spreads based on executable quotes should be preferred over data from Bloomberg's Generic Quote (BGN). To measure price impact, the adapted version of the high-low measure wins most comparisons, followed by the low-frequency Amihud (2002) measure. However, the interpretation of price impact in bond markets is difficult as larger sizes, in contrast to stock markets, often trade at more favorable prices.

Chapter 3 analyzes the term structure of liquidity premia as the difference between the yield curves of two major bond segments that are both government guaranteed but differ in their liquidity. We show that its characteristics strongly depend on the economic situation. In crisis times, liquidity premia are higher with the largest increase for short-term maturities. Moreover, their reaction to changes in fundamentals is only significant during stress: Premia of all maturities depend on the availability of arbitrage capital as a proxy for liquidity supply. In contrast, liquidity demand only impacts short maturities. Therefore, calibrating risk management models in normal times underestimates illiquidity risk and misjudges term structure effects.

In Chapter 4, we develop an equilibrium model to analyze the impact of market frictions on trading volume and liquidity premia for finite maturity assets when investors differ in their investment horizons. In equilibrium, short-horizon investors only invest in short-term bonds and illiquidity spills over from short-term to long-term maturities. The model predicts i) a hump-shaped relation between trading volume and maturity, ii) lower trading volumes of older compared to young assets, iii) an increasing liquidity term structure when considering ask prices, and iv) a liquidity term structure from bid prices that is decreasing or U-shaped. We verify these predictions empirically using data for U.S. corporate bonds.

Our results lead to interesting starting points for future research. Regarding the measurement of liquidity, it would be interesting to analyze other OTC markets, e.g., CDS or swap markets. However, in contrast to the U.S. corporate bond market, where transaction level data is publicly available, data for many other OTC markets is not available at all or only available on a proprietary basis.[54] It is, however, likely that the quest for transparency in response to the recent financial crisis leads to new data sources that could be exploited for future research endeavors in that direction.

With respect to the dependence of liquidity premia on the economic environment, it would be interesting to simultaneously model the yield curves of illiquid and liquid bonds within an integrated and arbitrage free macro finance model (see Ang and Ulrich, 2012, for a macro finance model with only liquid bonds and equity). Within such a model, the risk free yield curve and the term structure of liquidity premia could depend on forecasts for inflation and real output as well as central bank interventions modeled with the help of a Taylor (1993) rule. Another promising approach is to analyze dependencies of liquidity premia in different markets, e.g., U.S. and European bond and stock markets. The better understanding of spill-over effects between different markets could potentially help to curtail and dampen future liquidity stess periods and thus help to prevent global liquidity crises like the financial crisis of 2008.[55]

Regarding possible extensions of our equilibrium framework, it would be interesting to incorporate the risk of changing interest rates or credit risk into the model. For that, it would be reasonable to abandon the assumption of risk neutral investors. Both credit and interest rate risk could potentially impact short- and long-horizon investors differently and thus interact with our liquidity clientele effect. Further challenging extensions of

[54] So for example the Depository Trust & Clearing Corporation (DTCC) collects data on CDS transactions settled over their platform (see, e.g., Gündüz, Nasev, and Trapp, 2013).

[55] All of these issues are further explored within an ongoing project funded by the German Research Foundation (DFG). The working paper Schuster and Uhrig-Homburg (2014), which forms the basis for Chapter 3 of this thesis, is an outcome of this project.

our equilibrium model are to endogenize either bid-ask spreads or bond supply. On the one hand, a meaningful endogenization of bid-ask spreads would require to specify the optimization problem of dealers who set bid-ask spreads for different maturities dependent on their inventory risk or other costs. On the other hand, the endogenization of bond supply would require to model the decisions of bond issuers. Since they are the ones who finally have to pay for the liquidity premium, they have an incentive to issue those maturities with small liquidity premia. However, constraints like the matching of cash flows from investment projects or fixed costs of bond issuances might prevent them from issuing only bonds with very short maturities.

Appendix A

Additional Information on Liquidity Measures

A.1 Details on the Effective Tick Proxy

The (constrained) probabilities $\hat{\gamma}_j$ used in Equation (2.17) are calculated as follows:

$$\hat{\gamma}_j = \begin{cases} \min[\max\{U_j, 0\}, 1] & \text{if } j = 1, \\ \min\left[\max\{U_j, 0\}, 1 - \sum_{k=1}^{j-1} \hat{\gamma}_k\right] & \text{if } j = 2, 3, ..., J, \end{cases} \tag{A.1}$$

$$\text{with } U_j = \begin{cases} \frac{A_1}{B_1} \cdot F_1 & \text{if } j = 1, \\ \frac{A_j}{B_j} \cdot F_j - \sum_{k=1}^{j-1} \frac{O_{jk}}{B_k} \cdot F_k & \text{if } j = 2, 3, ..., J, \end{cases} \tag{A.2}$$

$$\text{and } F_j = \frac{N_j}{\sum_{j=1}^{J} N_j} \quad \text{for } j = 1, 2, ..., J. \tag{A.3}$$

F_j and N_j give the empirical probability and number of prices, respectively, belonging to the jth spread, where only positive-volume days are considered. With them, the *unconstrained* probabilities U_j are calculated as input for Equation (A.1). A_j gives the total number of possible trade prices corresponding to the jth spread. B_j is the number of so called special price increments, which are defined as prices that can be generated by the jth spread, but not by any larger spreads in the price grid. Finally, O_{jk} defines the number of price increments for the jth spread overlapping the price increments of spread k, but not overlapping the price increments of any spread between spreads j and k. The following table reports A_j, B_j, and O_{jk} for our price grid:[56]

j	Corresponding spread	A_j	B_j	O_{jk}
1	\$0.001	1000	896	
2	\$0.01	100	80	$O_{21} = 100$
3	\$0.05	20	8	$O_{31} = 0$, $O_{32} = 20$
4	\$0.1	10	8	$O_{41} = 0$, $O_{42} = 0$, $O_{43} = 10$
5	\$0.125	8	4	$O_{51} = 4$, $O_{52} = 0$, $O_{53} = 2$, $O_{54} = 2$
6	\$0.25	4	2	$O_{61} = 0$, $O_{62} = 0$, $O_{63} = 0$, $O_{64} = 0$, $O_{65} = 4$
7	\$0.5	2	1	$O_{71} = 0$, $O_{72} = 0$, $O_{73} = 0$, $O_{74} = 0$, $O_{75} = 0$, $O_{76} = 2$
8	\$1	1	1	$O_{81} = 0$, $O_{82} = 0$, $O_{83} = 0$, $O_{84} = 0$, $O_{85} = 0$, $O_{86} = 0$, $O_{87} = 1$

[56] Please refer to `www.kelley.iu.edu/cholden/examples.pdf` and Holden (2009) for further details.

Appendix B

Term Structure Estimations

B.1 Estimation of Zero Coupon Yield Curves

We estimate the term structure of zero coupon yields of BUNDs and KfW bonds using the Nelson and Siegel (1987) approach. Within this approach, the entire term structure information at time t is condensed in four parameters ($\beta_{0,t}$, $\beta_{1,t}$, $\beta_{2,t}$, τ_t). The zero coupon yield of bond class $i \in \{BUND, KfW\}$ at time t for time to maturity T is given as

$$y_t^i(T) = \beta_{0,t}^i + \beta_{1,t}^i \left(\frac{1 - e^{-\frac{T}{\tau_t^i}}}{\frac{T}{\tau_t^i}} \right) + \beta_{2,t}^i \left(\frac{1 - e^{-\frac{T}{\tau_t^i}}}{\frac{T}{\tau_t^i}} - e^{-\frac{T}{\tau_t^i}} \right). \tag{B.1}$$

We minimize the sum of squared differences between observed and Nelson-Siegel implied yields for both segments and each week. To make the β-factors of both BUNDs and KfWs directly comparable, we impose $\tau_t^{BUND} = \tau_t^{KfW} = \tau_t$ (see also Nelson and Siegel, 1987 or Kempf et al., 2012 who restrict τ to be constant over time t). To put equal weights on both segments for the estimation of the common τ_t, we weight the sum of squared yield differences with the inverse of the number of bonds in the respective bond class. We exclude bonds with time to maturity less than three months since for them small errors in the price would translate to large yield errors (see, e.g., Schich, 1997). Fitting errors are presented in Table B.1. Root mean square errors (RMSE) are in the same order of magnitude for all maturities and both segments and there is no systematic bias of yield curve estimates.

Table B.1: **Fitting errors**
This table shows the distribution of observations over different maturity segments, average root mean square errors (RMSE), and mean estimation errors of the yield curve estimation. Estimation errors are defined as $yld_{j,t}^{observed} - yld_{j,t}^{Nelson-Siegel}$ where $yld_{j,t}^{observed}$ is the observed yield of bond j (calculated from its closing price) and $yld_{j,t}^{Nelson-Siegel}$ is the theoretical yield one gets when discounting all cash flows of the bond with the respective KfW or BUND zero coupon yield (see Equation (B.1)). The maturity segments (short, medium, and long) are chosen around our benchmark maturities of two, five, and ten years. The observation period is February 14th, 1996 to September 29th, 2010.

	Distribution of observations		**Average RMSE**		**Mean estimation error**	
	KfW	BUND	KfW	BUND	KfW	BUND
Short: $T \leq 3.5$ yr.	49.2%	51.4%	8.7 bps	4.9 bps	−0.3 bps	0.6 bps
Medium: 3.5 yr. $< T \leq$ 7.5 yr.	30.2%	24.7%	5.0 bps	4.4 bps	0.7 bps	−1.2 bps
Long: 7.5 yr. $< T$	20.6%	24.0%	3.5 bps	6.7 bps	−0.8 bps	0.1 bps
All	100.0%	100.0%	7.3 bps	5.5 bps	0.0 bps	0.0 bps

B.2 Estimation of Term Structures of Bid-Ask Spreads

We compute the average relative bid-ask spread for each bond and each date from all quotations provided in Bloomberg. To calculate a time to maturity dependent measure, we estimate a linear relation between the duration of the bond and the bid-ask spread for each segment and each date. With the estimated ‘term structure of bid-ask spreads’, we are able to aggregate the information from all bonds of a segment in maturity dependent bid-ask spreads similarly as for the estimation of the term structure of liquidity premia. Bid-ask quotations are available only since 1999 for a majority of KfW bonds, two-sided quotes for bonds of durations less than two years are only available after August 22nd, 2001 on a continuous basis.

B.3 Maturity Dependent Outstanding Volumes

We calculate the measures of the amount of bonds outstanding of the representative two, five, and ten year KfW bond relative to the corresponding German government bond in three steps. First, we adjust for the effect that as bonds age, an increasing fraction of their issue amounts are locked away in the portfolios of buy-and-hold investors (see, e.g., Warga, 1992). Ejsing and Sihvonen (2009) estimate for German government bonds that the trading volume of an issue declines by eight percent each year. Therefore, we multiply the outstanding volume of each bond with $e^{-0.08 \cdot Age\ of\ the\ issue_t}$. Second, we aggregate the volume of all outstanding bonds from each segment into the outstanding volume of three representative bonds (with two, five, and ten years time to maturity). More precisely, we weight the volume of each bond with the influence it has on the zero coupon yield of the respective maturity. To measure this influence, we calculate the sensitivity of a small yield change of this bond on the zero coupon yield curve. The advantage of this weighting scheme is the independence from arbitrarily selected time to maturity bucket bounds. Additionally, it minimizes the time series variation resulting from bonds changing buckets. Third, we divide the volume of each representative KfW bond by that of the respective German government bond.

Appendix C

Equilibrium Model for Liquidity Premia and Trading Volume: Proofs and Derivations

As discussed in Chapter 4, we set $r = 0$ in all derivations to simplify notation. However, as discussed in the last paragraph of Section 4.2.3, our results generally hold also for positive interest rates.

C.1 Derivation of Equilibrium Prices

For given limiting maturities τ and T_{lim}, we derive equations for $P(T)$. In total, there are five different ranges: $T \leq \min(\tau, T_{\text{lim}})$, $\tau < T \leq T_{\text{lim}}$, $\tau < T_{\text{lim}} < T$, $T_{\text{lim}} < T \leq \tau$, and $T_{\text{lim}} < \tau < T$.

(i) For $T \leq \min(\tau, T_{\text{lim}})$, the integral term of Equation (4.4) is zero. Using the first order condition (4.5), we get

$$\Delta_S(T) = \frac{\lambda_S \cdot (1 - e^{(\lambda_S - b)\cdot T})}{(1 - P(T) \cdot e^{\lambda_S \cdot T}) \cdot (\lambda_S - b)} - 1 \stackrel{!}{=} 0. \tag{C.1}$$

Solving Condition (C.1) for $P(T)$ directly yields

$$P(T) = \frac{b \cdot e^{-\lambda_S \cdot T} - \lambda_S \cdot e^{-b \cdot T}}{b - \lambda_S} \qquad \text{for } T \leq \min(\tau, T_{\text{lim}}). \tag{C.2}$$

(ii) For $\tau < T \leq T_{\text{lim}}$, using again the first order condition (4.5), Equation (4.4) evaluates to

$$\Delta_S(T) = \frac{\lambda_S \cdot e^{\lambda_S \cdot T}}{P(T) \cdot e^{\lambda_S \cdot T} - 1} \cdot \int_\tau^T P(x) \cdot (1 - s(x)) \cdot e^{-\lambda_S \cdot (T-x)}\, dx + \frac{\lambda_S \cdot (1 - e^{(\lambda_S - b)\cdot\tau})}{(1 - P(T) \cdot e^{\lambda_S \cdot T}) \cdot (\lambda_S - b)} - 1 \stackrel{!}{=} 0. \tag{C.3}$$

The solution of this integral equation is given as

$$P(T) = e^{-\int_\tau^T \lambda_S \cdot s(x)\, dx} \cdot P(\tau) \qquad \text{for } \tau < T \leq T_{\text{lim}}, \tag{C.4}$$

which can be verified by plugging in (C.4) into (C.3). It is instructive to note that (C.4) corresponds to the market value of a defaultable bond with a default intensity λ_S and a "recovery-rate" of $(1 - s(T))$ when using the "recovery of market value assumption" in Duffie and Singleton (1999).

(iii) For $\tau < T_{\text{lim}} < T$, we insert Equation (4.4) into the first order condition for the long-horizon investors (4.6) and get

$$\Delta_L(T) = \frac{\lambda_L \cdot e^{\lambda_L \cdot T}}{P(T) \cdot e^{\lambda_L \cdot T} - 1} \cdot \int_\tau^T P(x) \cdot (1 - s(x)) \cdot e^{-\lambda_L \cdot (T-x)}\, dx + \frac{\lambda_L \cdot (1 - e^{(\lambda_L - b)\cdot\tau})}{(1 - P(T) \cdot e^{\lambda_L \cdot T}) \cdot (\lambda_L - b)} - 1 \stackrel{!}{=} \Delta_L(T_{\text{lim}}). \tag{C.5}$$

By plugging in

$$P(T) = e^{-\int_{T_{\text{lim}}}^T \frac{(\Delta_L(T_{\text{lim}}) + s(x)) \cdot \lambda_L}{1 + \Delta_L(T_{\text{lim}})}\, dx} \cdot P(T_{\text{lim}}) \qquad \text{for } \tau < T_{\text{lim}} < T \tag{C.6}$$

into (C.5), we show that (C.6) solves the integral equation.

(iv) For $T_{\text{lim}} < T \leq \tau$, we can ignore the first term of Equation (4.4) and then employ again the first order condition for the long-horizon investors (4.6) to get

$$\Delta_L(T) = \frac{\lambda_L \cdot (1 - e^{(\lambda_L - b)\cdot T})}{(1 - P(T) \cdot e^{\lambda_L \cdot T}) \cdot (\lambda_L - b)} - 1 \stackrel{!}{=} \Delta_L(T_{\text{lim}}). \tag{C.7}$$

Rearranging terms directly yields

$$P(T) = e^{-T \cdot \lambda_L} \cdot \left(1 - \frac{\lambda_L \cdot \left(1 - e^{T \cdot (\lambda_L - b)}\right)}{(1 + \Delta_L(T_{\text{lim}})) \cdot (\lambda_L - b)}\right) \qquad \text{for } T_{\text{lim}} < T \leq \tau. \tag{C.8}$$

(v) For $T_{\text{lim}} \leq \tau < T$, as in (iii), we obtain (C.5). Since $T_{\text{lim}} < \tau < T$, we get the solution

$$P(T) = e^{-\int_{\tau}^{T} \frac{(\Delta_L(T_{\text{lim}})+s(x))\cdot\lambda_L}{1+\Delta_L(T_{\text{lim}})}\,dx} \cdot P(\tau) \qquad \text{for } T_{\text{lim}} < \tau < T, \tag{C.9}$$

which we again verify by plugging it into (C.5), but now use $\Delta_L(T_{\text{lim}})$ from (C.7).

C.2 Clientele Effect

In this section, we prove that for the derived equilibrium prices and constant or monotonously increasing bid-ask spreads $s(T)$ with $0 < s(T) < 1$, investors do not have any incentive to deviate from the assumed allocation of bonds, i.e., short-horizon investors only buy short-term bonds and long-horizon investors only invest in long-term bonds. This case corresponds to a clientele effect. It holds if there is a maturity T_{lim} such that short-horizon investors have no incentive to invest in bonds with longer maturity:

$$\Delta_S(T) < 0 \qquad \text{for all } T \in [T_{\text{lim}}, T_{\text{max}}], \tag{C.10}$$

and long-horizon investors have no incentive to invest in bonds with shorter maturity:

$$\Delta_L(T) < \Delta_L(T_{\text{lim}}) \qquad \text{for all } T \in (0, T_{\text{lim}}], \tag{C.11}$$

nor in cash, since they hold only bonds, i.e., $\Delta_L(T) > 0$. Long-horizon investors have higher marginal utility for all bonds than short-horizon investors, who have a marginal utility of 0 for bonds with maturity T_{lim}, therefore $\Delta_L(T_{\text{lim}}) > \Delta_S(T_{\text{lim}}) = 0$. Hence, the condition $\Delta_L(T_{\text{lim}}) > 0$ trivially holds.

Proof of Equation (C.11): We verify that $\Delta_L(T)$ is strictly monotonously increasing in T for $T \leq T_{\text{lim}}$ and arbitrary T_{lim}, i.e., $\Delta'_L(T) > 0$: For the case $T \leq \tau$, $\Delta_L(T)$ is given as

$$\Delta_L(T) = \frac{\lambda_L \cdot (1 - e^{(\lambda_L - b)\cdot T})}{(1 - e^{\lambda_L \cdot T} \cdot P(T)) \cdot (\lambda_L - b)} - 1. \tag{C.12}$$

By employing Equation (C.2) for $P(T)$, using $0 < b < \lambda_L < \lambda_S$ (see Section 4.2.3), and substituting $b = \lambda_L - c1$ and $\lambda_S = \lambda_L + c2$ with $c1, c2 > 0$ and $c1 < \lambda_L$, $\Delta'_L(T) > 0$

simplifies to

$$e^{(c1+c2)\cdot T}\cdot c1 + c2 > e^{c1\cdot T}\cdot (c1+c2). \tag{C.13}$$

(C.13) holds for all $T > 0$ since for $T = 0$, both sides are equal $(c1 + c2)$, and the first derivative with respect to T of the left-hand side of (C.13) is larger than that of the right-hand side, i.e.,

$$(c1+c2)\cdot c1\cdot e^{(c1+c2)\cdot T} > (c1+c2)\cdot c1\cdot e^{c1\cdot T} \tag{C.14}$$

which is always true since $c1, c2 > 0$.

For the second case with $T > \tau$, rearranging terms and using again $0 < b < \lambda_L < \lambda_S$, the condition $\Delta_L'(T) > 0$ simplifies to

$$(1-s(T))\cdot\left(e^{T\cdot\lambda_L}\cdot P(T)-1\right) - \left(\frac{\left(1-e^{(\lambda_L-b)\cdot\tau}\right)\cdot\left(-\lambda_L-\frac{P'(T)}{P(T)}\right)}{(\lambda_L-b)}\right.$$
$$\left.+\left(\int_\tau^T e^{-(T-x)\cdot\lambda_L}\cdot(1-s(x))\cdot P(x)\,dx\right)\cdot\left(e^{T\cdot\lambda_L}\cdot\lambda_L+e^{T\cdot\lambda_L}\cdot\frac{P'(T)}{P(T)}\right)\right) > 0. \tag{C.15}$$

We prove that (C.15) holds in two steps: In step (a), we show that (C.15) holds for $T \downarrow \tau$, i.e., we look at the right-sided limit of (C.15). In step (b), we then show that the first derivative with respect to T of the left-hand side of (C.15) is positive. For (a), rearranging Equation (4.1) yields[57]

$$s(\tau) = \frac{b\cdot\left(e^{(b-\lambda_S)\cdot\tau}-1\right)}{b\cdot e^{(b-\lambda_S)\cdot\tau}-\lambda_S}. \tag{C.16}$$

Using again our substitutions $b = \lambda_L - c1$ and $\lambda_S = \lambda_L + c2$ with $c1, c2 > 0$ and $c1 < \lambda_L$ and plugging in (C.16) we can simplify (C.15) to

$$e^{c2\cdot\tau}\cdot c1 + e^{-c1\cdot\tau}\cdot c2 - (c1+c2) > 0. \tag{C.17}$$

Again, it is easy to show that (C.17) holds for all $\tau > 0$ by verifying that its left-hand side equals 0 for $\tau \to 0$ and its first derivative with respect to τ is larger than 0.

[57]Note that for $T > \tau$, we implicitly assume that τ exists. If τ does not exist due to bid-ask spreads being to large, the already discussed case for $T \leq \tau$ applies for all T.

For (b), we rearrange (C.15) by employing (C.4) for $P(T)$ and substituting $g(T) = T \cdot \lambda_L - \int_\tau^T \lambda_S \cdot s(x)\,dx$ and $g'(T) = \lambda_L - \lambda_S \cdot s(T)$ to finally get

$$\frac{\left(e^{g(T)} \cdot P(\tau) - 1\right) \cdot (\lambda_S - \lambda_L)}{\lambda_S} + \left(\frac{e^{\lambda_L \cdot \tau} P(\tau) - 1}{\lambda_S} + \frac{e^{(\lambda_L - b)\cdot\tau} - 1}{b - \lambda_L} - \int_\tau^T \frac{e^{g(x)} \cdot P(\tau) \cdot (\lambda_S - \lambda_L)}{\lambda_S}\,dx\right) \cdot g'(T) > 0 \tag{C.18}$$

and it remains to show that the first derivative with respect to T of the left-hand side of (C.18) has to be positive:

$$\left(\frac{e^{\lambda_L \cdot \tau} \cdot P(\tau) - 1}{\lambda_S} + \frac{e^{(\lambda_L - b)\cdot\tau} - 1}{b - \lambda_L} - \int_\tau^T \frac{e^{g(x)} \cdot P(\tau) \cdot (\lambda_S - \lambda_L)}{\lambda_S}\,dx\right) \cdot g''(T) > 0. \tag{C.19}$$

As $g''(T) = -\lambda_S \cdot s'(T) \leq 0$ for monotonously increasing $s(T)$ and $-\int_\tau^T \frac{e^{g(x)} \cdot P(\tau) \cdot (\lambda_S - \lambda_L)}{\lambda_S}\,dx < 0$ (since all factors in the numerator of the integrand are positive), a sufficient condition for (C.19) to hold is that

$$\frac{e^{\lambda_L \cdot \tau} \cdot P(\tau) - 1}{\lambda_S} + \frac{e^{(\lambda_L - b)\cdot\tau} - 1}{b - \lambda_L} < 0. \tag{C.20}$$

Using once more our substitutions $b = \lambda_L - c1$ and $\lambda_S = \lambda_L + c2$ with $c1, c2 > 0$ and $c1 < \lambda_L$ and utilizing (C.2) for $P(\tau)$, (C.20) simplifies to

$$c1 \cdot (\lambda_L - c1) + e^{c2 \cdot \tau} \cdot \left(c1^2 - c1 \cdot \lambda_L + \left(e^{c1 \cdot \tau} - 1\right) \cdot c2 \cdot (c2 + \lambda_L)\right) > 0. \tag{C.21}$$

As before, it is easy to show that (C.21) holds for all $\tau > 0$ by verifying that its left-hand side equals 0 for $\tau \to 0$ and its first derivative with respect to τ is larger than 0.

Proof of Equation (C.10): Inequality (C.10) directly follows from $\Delta_L'(T) > 0$ for $T \leq T_{\text{lim}}$. To see this, assume that for some parameter set $(\lambda_S,\ \lambda_L,\ a,\ b,\ T_{\max})$ and given bid-ask spread function $s(T)$, the wealth of short-horizon investors is sufficient to buy all bonds and the wealth of long-horizon investors goes to zero ($W_L^* \to 0$), so that $T_{\text{lim}}^* \to T_{\max}$. Suppose now, that for the same parametrization $(\lambda_S,\ \lambda_L,\ a,\ b,\ T_{\max})$ and bid-ask spread function $s(T)$, the wealth of long-horizon investors $W_L^+ >> 0$, so that $T_{\text{lim}}^+ << T_{\max}$. Then it follows with the long-horizon investors' first order condition (4.6) that

$$\Delta_L^+(T) = \Delta_L^+(T_{\text{lim}}^+) \qquad \text{for all } T \in (T_{\text{lim}}^+, T_{\max}], \tag{C.22}$$

where we use (+) to indicate for which case of W^+/W^* $\Delta_L(T)$ applies. Moreover, it follows that

$$\Delta_L^+(T_{\text{lim}}^+) = \Delta_L^*(T_{\text{lim}}^+) \tag{C.23}$$

as $P(T_{\text{lim}}^+)$ is not affected from the choice of $T_{\text{lim}} \geq T_{\text{lim}}^+$ (dependent on τ, but independent of T_{lim}, either Equation (C.2) or (C.4) apply for $P(T)$). From the fact that $\Delta_L'(T) > 0$ for $T \leq T_{\text{lim}}$, we directly get

$$\Delta_L^*(T_{\text{lim}}^+) < \Delta_L^*(T) \qquad \text{for all } T \in (T_{\text{lim}}^+, T_{\text{lim}}^* = T_{\max}]. \tag{C.24}$$

Putting together (C.22)-(C.24), we get

$$\Delta_L^+(T) < \Delta_L^*(T) \qquad \text{for all } T \in (T_{\text{lim}}^+, T_{\max}]. \tag{C.25}$$

From the last Inequality (C.25), it directly follows that

$$P^+(T) > P^*(T) \qquad \text{for all } T \in (T_{\text{lim}}^+, T_{\max}] \tag{C.26}$$

since lower prices $P(T)$ directly result in higher marginal utilities due to higher wealth gains. Turning this argument around, we get

$$\Delta_S^+(T) < \Delta_S^*(T) \qquad \text{for all } T \in (T_{\text{lim}}^+, T_{\max}]. \tag{C.27}$$

Employing the short-horizon investors' first order condition (4.5)

$$\Delta_S^*(T) = 0 \qquad \text{for all } T \in (0, T_{\text{lim}}^* = T_{\max}], \tag{C.28}$$

it directly follows from (C.27) that

$$\Delta_S^+(T) < 0 \qquad \text{for all } T \in (T_{\text{lim}}^+, T_{\max}], \tag{C.29}$$

which equals Inequality (C.10) for $T_{\text{lim}} = T_{\text{lim}}^+$. □

C.3 Determination of T_{lim}

To determine T_{lim}, we exploit the market clearing condition for bonds with maturities $T_{\text{init}} \in (T_{\text{lim}}, T_{\text{max}}]$ that are held by long-horizon investors, i.e., we solve

$$W_L = \int_0^{T_{\text{max}}} P(T) \cdot \int_T^{T_{\text{max}}} a \cdot Y_L\left(T, T_{\text{init}}, T_{\text{lim}}\right) \, dT_{\text{init}} \, dT \tag{C.30}$$

for T_{lim}. Here, $Y_L(T, T_{\text{init}}, T_{\text{lim}})$ denotes the fraction of bonds with remaining maturity T and initial maturity T_{init} for a given T_{lim} that are held in the portfolios of long-horizon investors, i.e.,

$$Y_L\left(T, T_{\text{init}}, T_{\text{lim}}\right) = \begin{cases} 0, & \text{if } T, T_{\text{init}} \leq T_{\text{lim}} \\ e^{-\lambda_L \cdot (T_{\text{lim}} - T)}, & \text{if } T \leq T_{\text{lim}} \text{ and } T_{\text{init}} > T_{\text{lim}} \\ 1, & \text{if } T > T_{\text{lim}}. \end{cases} \tag{C.31}$$

For bonds with initial maturity $T_{\text{init}} > T_{\text{lim}}$ and current maturity $T \leq T_{\text{lim}}$, a fraction of $e^{-\lambda_L \cdot (T_{\text{lim}} - T)}$ is held by old long-horizon investors. Bonds with initial and current maturity smaller than T_{lim} are not held by long-horizon investors, bonds with current and initial maturity larger than T_{lim} are only held by long-horizon investors. To illustrate the mechanics of the market clearing argument for long-horizon investors, consider the extreme case of $W_L \to 0$. For Equation (C.30) to hold, $Y_L(T, T_{\text{init}}, T_{\text{lim}})$ has to be 0 for all T and T_{init}. Hence, $T_{\text{lim}} \to T_{\text{max}}$.

C.4 Optimal Investor Behavior

Bonds are sold immediately after a preference shock occurs if $T > \tau$: To formalize this requirement, we define the utility of an investor she receives if she sells a T-year bond d time periods after she experienced a preference shock:

$$f(d) = (1 - s(T - d)) \cdot P(T - d) \cdot e^{-b \cdot d}. \tag{C.32}$$

Bonds are always sold immediately, iff $f'(d) \leq 0$. For $\tau < T \leq T_{\text{lim}}$, this condition holds iff

$$s'(T-d) \leq (1 - s(T-d)) \cdot (b - \lambda_S \cdot s(T-d)), \tag{C.33}$$

i.e., if bid-ask spreads do not grow with maturity 'too strongly'. Note, that for constant bid-ask spreads $s(T) = s$, (C.33) always holds since $s'(T) = 0$, $s < 1$, and $b - \lambda_S \cdot s > 0$. The latter condition holds as inserting $b - \lambda_S \cdot s \leq 0$ into Equation (4.1) leads to a contradiction (i.e., τ would not exist). Condition (C.33) also ensures that Equation (4.1) cannot have more than one solution for τ.

For the other two relevant cases $T_{\text{lim}} \leq \tau < T$ and $\tau < T_{\text{lim}} < T$, it can be formally shown that $f'(d) < 0$ also holds when (C.33) applies. This follows intuitively from the clientele-effect since $P(T)$ decreases slower for increasing T when $T > T_{\text{lim}}$ compared to $T \leq T_{\text{lim}}$ (due to long-horizon investors demanding lower compensation for holding longer term bonds compared to short-horizon investors). Thus, the incentive to wait in the case of a preference shock is reduced, compared to $T \leq T_{\text{lim}}$ (since gains from increasing prices when the maturity decreases are smaller).

It is never optimal to sell bonds without preference shock: For short-horizon investors, this is intuitively clear since they are indifferent between all bonds with maturities between 0 and T_{lim}. Hence, selling one bond with $T \in (0, T_{\text{lim}}]$, paying the bid-ask spread $s(T)$, and buying another bond with $T_{\text{new}} \in (0, T_{\text{lim}}]$ can never be optimal. With the same argument, long-horizon investors can never have an incentive to sell bonds with maturity $T \geq T_{\text{lim}}$. For them, selling bonds with $T < T_{\text{lim}}$ without having experienced a preference shock can only be optimal if the marginal utility through the early reinvestment in a bond with maturity $T_{\text{new}} \in (T_{\text{lim}}, T_{\max}]$ plus the proceeds from selling the bond with maturity $T \in (0, T_{\text{lim}})$ is higher than the marginal utility from the later reinvestment (at maturity of the respective bond) plus the proceeds from the maturing bond if no preference shock occurs, or the proceeds from the optimal decision given that a preference shock occurs:

$$\begin{aligned}
(\Delta_L(T_{\text{lim}}) + 1) \cdot P(T) \cdot (1 - s(T)) &> Pr(\tilde{T}_L > T) \cdot (\Delta_L(T_{\text{lim}}) + 1) \\
&+ \int_0^{T-\min(T,\tau)} \underbrace{\lambda_L \cdot e^{-\lambda_L \cdot y}}_{\text{density function of the preference shock time}} \cdot (1 - s(T-y)) \cdot P(T-y)\, dy \\
&+ \int_{T-\min(T,\tau)}^{T} \lambda_L \cdot e^{-\lambda_L \cdot y} \cdot e^{-b\cdot(T-y)}\, dy.
\end{aligned} \tag{C.34}$$

Note that in deriving (C.34), we exploit the fact that marginal utility does not depend on the invested amount (see Equation (4.4)), i.e., the optimal investment of an amount z for a long-horizon investor leads to an expected utility of $(1 + \Delta_L(T_{\text{lim}})) \cdot z$. Rearranging Equation (C.34) shows that long-horizon investors have no incentive to sell bonds without having experienced a preference shock if

$$(1 + \Delta_L(T_{\text{lim}})) \cdot e^{-T\lambda_L} + \frac{e^{-T\cdot\lambda_L}\left(-1 + e^{(-b+\lambda_L)\cdot\min(T,\tau)}\right)\cdot\lambda_L}{-b+\lambda_L} + \int_{\min(T,\tau)}^{T} e^{(-T+x)\cdot\lambda_L}\cdot\lambda_L\cdot P(x)\cdot(1-s(x))\,dx - (1+\Delta_L(T_{\text{lim}}))\cdot P(T)\cdot(1-s(T)) > 0. \tag{C.35}$$

It can be formally shown that Condition (C.35) holds for $T \leq \tau$. This is intuitively clear since for $T < \tau$, a sell is not optimal even in the case of a preference shock. As b is an upper bound for the ask liquidity premium of an arbitrary maturity (and thus the maximum return a selling investor could gain from her new bonds), the incentive to sell is lower when no preference shock occurred. For constant bid-ask spreads $s(T) = s$, it can also never be optimal to sell prematurely for $\tau \leq T < T_{\text{lim}}$, as the relative wealth gain $-\frac{P'(T)}{P(T)}$ is higher than for $T > T_{\text{lim}}$. Since we have already shown that it is never optimal to sell prematurely for $T \leq \tau$ and $T \geq T_{\text{lim}}$, it can also not be optimal to sell during the time of highest wealth gains. In the most general case with increasing bid-ask spreads $s(T)$ and for $T \in (\tau, T_{\text{lim}})$, (C.35) has to be verified by plugging in prices $P(T)$ from Proposition 1.

C.5 Proof of Propositions 2 and 3

Proposition 2

The fact that seller initiated turnover is 0 for $T < \tau$ with $\tau > 0$ follows directly from Equation (4.1) as $P(\tau) \cdot (1 - s(\tau)) < 1$. The fact that seller initiated turnover is larger for $T < T_{\text{lim}}$ than for $T > T_{\text{lim}}$ if $\tau < T_{\text{lim}}$ is a direct consequence from the clientele effect. As elaborated in the main text, the second part of Proposition 2 also directly follows from the clientele effect proven in Appendix B. □

Proposition 3

$\mathbf{Illiq}^{\mathbf{ask}}(T)$ is monotonously increasing in T: To formalize this requirement, we calculate its first derivative with respect to T and show that it is greater or equal to zero, i.e.,

$$\left(\text{Illiq}^{\text{ask}}(T)\right)' = \frac{\log(P(T)}{T^2} - \frac{P'(T)}{T \cdot P(T)} \geq 0. \tag{C.36}$$

(i) For $T \leq \min(\tau, T_{\text{lim}})$, plugging in prices $P(T)$ from Equation (C.2) into (C.36) and multiplying with T^2 leads to the condition

$$b \cdot T + \frac{b \cdot e^{b \cdot T} \cdot T \cdot (b - \lambda_S)}{-b \cdot e^{b \cdot T} + e^{T \cdot \lambda_S} \cdot \lambda_S} + \log\left(\frac{b \cdot e^{-T \cdot \lambda_S} - e^{-b \cdot T} \cdot \lambda_S}{b - \lambda_S}\right) \geq 0. \tag{C.37}$$

(C.37) trivially holds for $T = 0$. Moreover, for the first derivative with respect to T of the left-hand side of (C.37) it holds

$$\frac{b \cdot e^{T \cdot (b + \lambda_S)} \cdot T \cdot (b - \lambda_S)^2 \cdot \lambda_S}{\left(b \cdot e^{b \cdot T} - e^{T \cdot \lambda_S} \cdot \lambda_S\right)^2} \geq 0 \tag{C.38}$$

such that (C.37) is true for all T.

(ii) For T with $\tau < T \leq T_{\text{lim}}$, multiplying (C.36) by T and exploiting the relation $\frac{P'(T)}{P(T)} = -s(T) \cdot \lambda_S$ from (C.4) as well as $-\frac{\log(P(T))}{T} = \text{Illiq}^{\text{ask}}(T)$ yields

$$\text{Illiq}^{\text{ask}}(T) \leq s(T) \cdot \lambda_S. \tag{C.39}$$

$s \cdot \lambda_S$ is the liquidity premium for the extreme case that bid-ask spreads s remain constant and investors are forced to immediately sell after a preference shock (see Equation (C.4)). As actual bid-ask spreads $s(T)$ can only decrease when the bond ages and investors have the option to wait until maturity, $s(T) \cdot \lambda_S$ is an upper bound for $\text{Illiq}^{\text{ask}}(T)$.

(iii) For T with $\tau < T_{\text{lim}} < T$, multiplying again (C.36) by T and exploiting the relation $\frac{P'(T)}{P(T)} = -\frac{(\Delta_L(T_{\text{lim}}) + s(T)) \cdot \lambda_L}{1 + \Delta_L(T_{\text{lim}})}$ from (C.6) yields

$$\text{Illiq}^{\text{ask}}(T) \leq \frac{(\Delta_L(T_{\text{lim}}) + s(T)) \cdot \lambda_L}{1 + \Delta_L(T_{\text{lim}})}. \tag{C.40}$$

$P^{\text{forced}}(T) = e^{-T \cdot \text{Illiq}^{\text{forced}}}$ with $\text{Illiq}^{\text{forced}} = \frac{(\Delta_L(T_{\text{lim}})+s)\cdot\lambda_L}{1+\Delta_L(T_{\text{lim}})}$ solves the indifference condition

$$\frac{\lambda_L \cdot e^{\lambda_L \cdot T}}{P^{\text{forced}}(T) \cdot e^{\lambda_L \cdot T} - 1} \cdot \int_0^T P^{\text{forced}}(x) \cdot (1-s) \cdot e^{-\lambda_L \cdot (T-x)}\, dx \stackrel{!}{=} \Delta_L(T_{\text{lim}}). \tag{C.41}$$

Therefore, $\text{Illiq}^{\text{forced}}$ can be interpreted as the liquidity premium long-horizon investors would demand for an artificial bond with the following characteristics: (a) only long-horizon investors are allowed to invest in this bond, (b) the bond has constant bid-ask spreads s, (c) investors are forced to immediately sell after a preference shock (see also (C.5)) . As actual bid-ask spreads $s(T)$ can only decrease when the bond ages, short-horizon investors are not excluded, and investors have the option to wait until maturity, $\frac{(\Delta_L(T_{\text{lim}})+s(T))\cdot\lambda_L}{1+\Delta_L(T_{\text{lim}})}$ is again an upper bound for $\text{Illiq}^{\text{ask}}(T)$.

The same reasoning as for (iii) applies also for our case (v), i.e., $T_{\text{lim}} \leq \tau < T$.

(iv) For the last case of T with $T_{\text{lim}} < T \leq \tau$, we exploit that $P(T)$ is continuously differentiable at $T = \tau$ (which can be shown using (C.8) and (C.9) for $P(T)$ as well as (4.1) solved for $s(\tau)$). If $P(T)$ is continuously differentiable at τ, $\left(\text{Illiq}^{\text{ask}}(T)\right)'$ is continuous at τ (see (C.36)). Since we have already shown that $\left(\text{Illiq}^{\text{ask}}(T)\right)'$ is larger or equal to zero for T with $T_{\text{lim}} \leq \tau < T$ (case (v)), $\left(\text{Illiq}^{\text{ask}}(T)\right)' \geq 0$ then also holds for $T = \tau$. To show that $\left(\text{Illiq}^{\text{ask}}(T)\right)' \geq 0$ for any T with $T_{\text{lim}} < T \leq \tau$, we introduce an artificial bid-ask spread function $\hat{s}(T) \leq s(T)$ such that the corresponding $\hat{\tau}$ that solves Equation (4.1) equals T. Now, we can again exploit case (v) with the artificial bid-ask spread function $\hat{s}(T)$, i.e., $\left(\widehat{\text{Illiq}}^{\text{ask}}(T)\right)' \geq 0$ for T with $T_{\text{lim}} \leq \hat{\tau} < T$. As prices do not depend on bid ask spreads when investors wait when experiencing a preference shock (see also Equation (4.7)), it holds that $P(T) = \hat{P}(T)$ for $T \leq \hat{\tau} < \tau$. Applying the same continuity argument as above for $\left(\widehat{\text{Illiq}}^{\text{ask}}(T)\right)'$ then proves the assertion for all $T (= \hat{\tau})$ with $T_{\text{lim}} < T \leq \tau$. □

$\text{Illiq}^{\text{ask}}(T)$ goes to zero for $T \to 0$: Applying l'Hôpital's rule and using (C.2) for $P(T)$) directly leads to $\lim_{T\to 0} \text{Illiq}^{\text{ask}}(T) = \lim_{T \to 0} \frac{-\log(P(T))}{T} = 0$. □

$\text{Illiq}^{\text{ask}}(T)$ flattens at T_{lim}: We prove condition (4.11) separately for $T_{\text{lim}} < \tau$, $T_{\text{lim}} = \tau$, and $T_{\text{lim}} > \tau$. For $T_{\text{lim}} < \tau$, using (C.2) and (C.8) for $P(T)$, (4.11) transforms to the condition

$$\frac{b \cdot e^{b \cdot T_{\text{lim}}} \cdot \left(b \cdot \left(e^{T_{\text{lim}} \cdot \lambda_L} - e^{T_{\text{lim}} \cdot \lambda_S}\right) - e^{T_{\text{lim}} \cdot \lambda_L} \cdot \lambda_S + e^{b \cdot T_{\text{lim}}} \cdot (\lambda_S - \lambda_L) + e^{T_{\text{lim}} \cdot \lambda_S} \cdot \lambda_L\right)}{\left(e^{b \cdot T_{\text{lim}}} - e^{T_{\text{lim}} \cdot \lambda_L}\right) \cdot T_{\text{lim}} \cdot \left(b \cdot e^{b \cdot T_{\text{lim}}} - e^{T_{\text{lim}} \cdot \lambda_S} \cdot \lambda_S\right)} > 0. \tag{C.42}$$

Exploiting that the denominator of (C.42) is positive and using our earlier substitution $b = \lambda_L - c1$ and $\lambda_S = \lambda_L + c2$ with $c1, c2 > 0$ and $c1 < \lambda_L$, (C.42) simplifies to

$$e^{T_{\text{lim}} \cdot c2} \cdot c1 + e^{-T_{\text{lim}} \cdot c1} \cdot c2 - c1 - c2 > 0. \tag{C.43}$$

We show that this condition holds by again verifying that the left-hand side of (C.43) equals 0 for $T_{\text{lim}} \to 0$, and its first derivative is strictly positive for $T_{\text{lim}} > 0$.

For $T_{\text{lim}} = \tau$, exactly the same line of arguments as for $T_{\text{lim}} < \tau$, but using (C.9) instead of (C.8) proves the assertion.

For $T_{\text{lim}} > \tau$, using (C.2) and (C.4), condition (4.11) evaluates to

$$s(T_{\text{lim}}) \cdot \lambda_S > \frac{\lambda_L \cdot (\Delta_L(T_{\text{lim}}) + s(T_{\text{lim}}))}{1 + \Delta_L(T_{\text{lim}})}, \tag{C.44}$$

which always holds due to the clientele effect. To see why, note that due to the clientele effect (see Equation (C.10)), short-horizon investors are not willing to invest in long-term bonds. Thus, for a fixed T, the price $P(T)$ is lower if T_{lim} is below T compared to a situation with T_{lim} above T. From that, it directly follows that the integrand in Equation (4.7) for $\tau < T \leq T_{\text{lim}}$ is larger than the integrand for $\tau < T_{\text{lim}} < T$, which directly implies (C.44). □

$\text{Illiq}^{\text{bid}}(T)$ is decreasing in T at the short end: We use (4.10) and (C.2) to calculate

$$\left(\text{Illiq}^{\text{bid}}(T)\right)' = \frac{T \cdot \left(b + \frac{b \cdot e^{b \cdot T} \cdot (b - \lambda_S)}{e^{T \cdot \lambda_S} \cdot \lambda_S - b \cdot e^{b \cdot T}} + \frac{s'(T)}{1 - s(T)}\right) + \log\left(\frac{(1 - s(T)) \cdot \left(b \cdot e^{-T \cdot \lambda_S} - e^{-b \cdot T} \cdot \lambda_S\right)}{b - \lambda_S}\right)}{T^2}. \tag{C.45}$$

Plugging in $T = 0$, the numerator of (C.45) evaluates to $\log(1 - s(0))$, which is strictly negative for $s(0) > 0$. Hence, $\lim_{T \to 0} \left(\text{Illiq}^{\text{bid}}(T)\right)' = -\infty$. □

Bibliography

Acharya, V. V., Y. Amihud, and S. Bharath, 2013, Liquidity Risk of Corporate Bond Returns: Conditional Approach, *Journal of Financial Economics 110,* 358–386.

Acharya, V. V. and L. H. Pedersen, 2005, Asset Pricing with Liquidity Risk, *Journal of Financial Economics 77,* 375–410.

Alexander, G. J., A. K. Edwards, and M. G. Ferri, 2000, The Determinants of Trading Volume of High-Yield Corporate Bonds, *Journal of Financial Markets 3,* 177–204.

Amihud, Y., 2002, Illiquidity and Stock Returns: Cross-Section and Time-Series Effects, *Journal of Financial Markets 5,* 31–56.

Amihud, Y. and H. Mendelson, 1986, Asset Pricing and the Bid-Ask Spread, *Journal of Financial Economics 17,* 223–249.

Amihud, Y. and H. Mendelson, 1991, Liquidity, Maturity, and the Yields on U.S. Treasury Securities, *Journal of Finance 46,* 1411–1425.

Ang, A. and M. Ulrich, 2012, Nominal Bonds, Real Bonds, and Equity, *Working Paper*.

Banerjee, S. and J. J. Graveline, 2013, The Cost of Short-Selling Liquid Securities, *The Journal of Finance 68* 637–664.

Bank of International Settlements, 2013, BIS Quarterly Review Detailed Statistical Annex, December 2013, data available on *http://www.bis.org/statistics/*.

Bao, J., J. Pan, and J. Wang, 2011, The Illiquidity of Corporate Bonds, *Journal of Finance 66,* 911–946.

Barrell, R., E. P. Davis, D. Karim, and I. Liadze, 2010, Bank Regulation, Property Prices and Early Warning Systems for Banking Crises in OECD Countries, *Journal of Banking and Finance 34,* 2255–2264.

Beber, A., J. Driessen, and P. Tuijp, 2012, Pricing Liquidity Risk with Heterogeneous Investment Horizons, *Working Paper*.

Beirne, J., L. Dalitz, J. Ejsing, M. Grothe, S. Manganelli, F. Monar, B. Sahel, M. Sušec, J. Tapking, and T. Vong, 2011, The Impact of the Eurosystem's Covered Bond Purchase Programme on the Primary and Secondary Markets, *ECB Occasional Paper Series No. 122*.

Boldin, M. D., 1996, A Check on the Robustness of Hamilton's Markov Switching Model Approach to the Economic Analysis of the Business Cycle, *Studies in Nonlinear Dynamcis & Econometrics 1,* 35–46.

Brunnermeier, M. K., 2009, Deciphering the Liquidity and Credit Crunch 2007-2008, *Journal of Economic Perspectives 23,* 77–100.

Brunnermeier, M. K. and L. H. Pedersen, 2009, Market Liquidity and Funding Liquidity, *Review of Financial Studies 22,* 2201–2238.

Bühler, W. and V. Vonhoff, 2011, Term Structure of Liquidity Premia in the U.S. Treasury Market, *Working Paper*.

Chen, L., D. A. Lesmond, and J. Wei, 2007, Corporate Yield Spreads and Bond Liquidity, *Journal of Finance 62,* 119–149.

Choi, J., D. Hackbarth, and J. Zechner, 2013, Granularity of Corporate Debt, *Working Paper*.

Chordia, T., A. Sarkar, and A. Subrahmanyam, 2005, An Empirical Analysis of Stock and Bond Market Liquidity, *Review of Financial Studies 18,* 85–129.

Cochrane, J. H, 2011, Presidential Address: Discount Rates, *Journal of Finance 66,* 1047–1108.

Cochrane, J. H. and M. Piazzesi, 2005, Bond Risk Premia, *American Economic Review 95* 138–160.

Corwin, S. A. and P. Schultz, 2012, A Simple Way to Estimate Bid-Ask Spreads from Daily High and Low Prices, *Journal of Finance 67,* 719–760.

Dick-Nielsen, J., 2009, Liquidity Biases in TRACE, *Journal of Fixed Income 19,* 43–55.

Dick-Nielsen, J., P. Feldhütter, and D. Lando, 2012, Corporate Bond Liquidity Before and After the Onset of the Subprime Crisis, *Journal of Financial Economics 103,* 471–492.

Duffie, D., 2010, Presidential Address: Asset Price Dynamics with Slow-Moving Capital, *The Journal of Finance 65* 1237–1267.

Duffie, D., N. Gârleanu, and L. H. Pedersen, 2005, Over-the-Counter Markets, *Econometrica 73,* 1815–1847.

Duffie, D. and K. J. Singleton, 1999, Modeling Term Structures of Defaultable Bonds, *Review of Financial Studies 12,* 687–720.

Edwards, A. K., L. E. Harris, and M. S. Piwowar, 2007, Corporate Bond Market Transaction Costs and Transparency, *Journal of Finance 62,* 1421–1451.

Ejsing, J. W. and J. Sihvonen, 2009, Liquidity Premia in German Government Bonds, *ECB Working Paper Series No. 1081.*

Elton, E. J. and T. C. Green, 1998, Tax and Liquidity Effects in Pricing Government Bonds, *Journal of Finance 53,* 1533–1562.

Ericsson, J. and O. Renault, 2006, Liquidity and Credit Risk, *Journal of Finance 61,* 2219–2250.

European Insurance CFO Forum and CRO Forum, 2010, QIS 5 Technical Specifications Risk-Free Interest Rates, *http://ec.europa.eu/internal_market/insurance/docs/solvency/qis5/cfo-forum-cro-forum-paper-risk-free-rates_en.pdf/*.

Fama, E. and J. MacBeth, 1973, Risk, Return, and Equilibrium: Empirical Tests, *Journal of Political Economy 81,* 607–636.

Feldhütter, P., 2012, The Same Bond at Different Prices: Identifying Search Frictions and Selling Pressure, *Review of Financial Studies 25,* 1155–1206.

Financial Industry Regulatory Authority and Bloomberg, 2007, FINRA - Bloomberg Active U.S. Corporate Bond Indices, *http://www.finra.org/web/groups/industry/@ip/@comp/@mt/documents/appsupportdocs/p015158.pdf*.

Fleckenstein, M., F. A. Longstaff, and H. Lustig, 2013, The TIPS-Treasury Bond Puzzle, *The Journal of Finance, forthcoming.*

Fong, K., C. Holden, and C. Trzcinka, 2010, How Well Can International Stock Liquidity Be Estimated?, *Working Paper*.

Fontaine, J.-S. and R. Garcia, 2012, Bond Liquidity Premia, *Review of Financial Studies 25,* 1207–1254.

Friewald, N., R. Jankowitsch, and M. G. Subrahmanyam, 2012, Illiquidity or Credit Deterioration: A Study of Liquidity in the US Corporate Bond Market During Financial crises, *Journal of Financial Economics 105,* 18–36.

Gale, D., 1960, The Theory of Linear Economic Models, *McGraw-Hill Book Company*.

Garbade, K. D., 1978, The Effect of Interdealer Brokerage on the Transactional Characteristics of Dealer Markets, *Journal of Business 51,* 477–498.

Garbade, K. D. and W. L. Silber, 1976, Price Dispersion in the Government Securities Market, *Journal of Political Economy 84,* 721–740.

Gârleanu, N. and L. H. Pedersen, 2011, Margin-Based Asset Pricing and Deviations from the Law of One Price, *Review of Financial Studies 24,* 1980–2022.

Goldstein, M. A. and E. S. Hotchkiss, 2012, Dealer Behavior and the Trading of Newly Issued Corporate Bonds, *Working Paper*.

Goyenko, R., A. Subrahmanyam, and A. Ukhov, 2011, The Term Structure of Bond Market Liquidity and Its Implications for Expected Bond Returns, *Journal of Financial and Quantitative Analysis 46,* 111–139.

Goyenko, R. Y., C. W. Holden, and C. A. Trzcinka, 2009, Do Liquidity Measures Measure Liquidity?, *Journal of Financial Economics 92,* 153–181.

Greenwood, R., S. Hanson, and J. Stein, 2010, A Gap-Filling Theory of Corporate Debt Maturity Choice, *Journal of Finance 65,* 993–128.

Gromb, D. and D. Vayanos, 2010, Limits of Arbitrage, *Annual Review of Financial Economics 2* 251–275.

Gündüz, Y., J. Nasev, and M. Trapp, 2013, The Price Impact of CDS Tradign, *Working Paper*.

Hamilton, J. D., 1989, A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, *Econometrica 57,* 357–384.

Hamilton, J. D., 1994, *Time Series Analysis*. (Princeton University Press).

Han, S. and H. Zhou, 2007, Nondefault Bond Spread and Market Trading Liquidity, *Working Paper*.

Harris, L. E. and M. S. Piwowar, 2006, Secondary Trading Costs in the Municipal Bond Market, *Journal of Finance 61,* 1361–1397.

Hasbrouck, J., 1991, Measuring the Information Content of Stock Trades, *Journal of Finance 46,* 179–207.

Hasbrouck, J., 2009, Trading Costs and Returns for U.S. Equities: Estimating Effective Costs from Daily Data, *Journal of Finance 65,* 1445–1477.

He, Z. and K. Milbradt, 2012, Endogenous Liquidity and Defaultable Bonds, *NBER Working Paper 18408*.

He, Z. and W. Xiong, 2012, Rollover Risk and Credit Risk, *Journal of Finance 67,* 391–429.

Holden, C., 2009, New Low-Frequency Liquidity Measures, *Journal of Financial Markets 12,* 778–813.

Hotchkiss, E. S., A. Warga, and G. Jostova, 2002, Determinants of Corporate Bond Trading: A Comprehensive Analysis, *Working Paper*.

Hu, G. X., J. Pan, and J. Wang, 2013, Noise as Information for Illiquidity, *The Journal of Finance, forthcoming*.

Huang, J.-Z. and M. Huang, 2012, How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?, *Review of Asset Pricing Studies 2,* 153–202.

Huang, J.-Z., Z. Sun, T. Yao, and T. Yu, 2013, Liquidity Premium in the Eye of the Beholder: An Analysis of the Clientele Effect in the Corporate Bond Market, *Working Paper*.

Huang, M., 2003, Liquidity Shocks and Equilibrium Liquidity Premia, *Journal of Economic Theory 109,* 104–129.

International Monetary Fund, 2013, World Economic Outlook Database, *http://www.imf.org/external/pubs/ft/weo/2013/02/weodata/download.aspx*,.

Jankowitsch, R., A. Nashikkar, and M. G. Subrahmanyam, 2011, Price Dispersion in OTC Markets: A New Measure of Liquidity, *Journal of Banking and Finance 35,* 343–357.

Kempf, A., O. Korn, and M. Uhrig-Homburg, 2012, The Term Structure of Illiquidity Premia, *Journal of Banking and Finance 36,* 1381–1391.

Koziol, C. and P. Sauerbier, 2007, Valuation of Bond Illiquidity: An Option-Theoretical Approach, *Journal of Fixed Income 16,* 81–107.

Krishnamurthy, A. and A. Vissing-Jorgensen, 2010, The Aggregate Demand for Treasury Debt, *Working Paper*.

Lee, C. M. C., 1993, Market Integration and Price Execution for NYSE-Listed Securities, *Journal of Finance 48,* 1009–1038.

Lesmond, D., J. Ogden, and C. Trzcinka, 1999, A New Estimate of Transaction Costs, *Review of Financial Studies 12,* 1113–1141.

Lin, H., J. Wang, and C. Wu, 2013, Liquidity Risk and Expected Corporate Bond Returns, *Journal of Financial Economics 99,* 628–650.

Liu, J. and F. A. Longstaff, 2004, Losing Money on Arbitrage: Optimal Dynamic Portfolio Choice in Markets with Arbitrage Opportunities, *The Review of Financial Studies 17* 611–641.

Longstaff, F. A., 2004, The Flight-to-Liquidity Premium in U.S. Treasury Bond Prices, *Journal of Business 77,* 511–526.

Longstaff, F. A., S. Mithal, and E. Neis, 2005, Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market, *Journal of Finance 60,* 2213–2253.

Mahanti, S., A. Nashikkar, M. Subrahmanyam, G. Chacko, and G. Mallik, 2008, Latent Liquidity: A New Measure of Liquidity, with an Application to Corporate Bonds, *Journal of Financial Economics 88,* 272–298.

Markit Group Limited, 2013, Products and Services - Markit Bonds, *https://www.markit.com/en/products/data/bonds/bonds.page*.

Marshall, B. R., N. H. Nguyen, and N. Visaltanachoti, 2012, Commodity Liquidity Measurement and Transaction Costs, *Review of Financial Studies 25,* 599–638.

Merton, R., 1974, On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance 29,* 449–470.

Monfort, A. and J.-P. Renne, 2011, Credit and Liquidity Risk in Euro-Area Sovereign Yield Curves, *Working Paper.*

Monfort, A. and J.-P. Renne, 2013, Default, Liquidity, and Crises: An Econometric Framework, *Journal of Financial Econometrics 11* 221–262.

Nelson, C. R. and A. F. Siegel, 1987, Parsimonious Modeling of Yield Curves, *Journal of Business 60,* 473–489.

Newey, W. K. and K. D. West, 1987, A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica 55,* 703–708.

Norden, L., P. Roosenboom, and T. Wang, 2013, Do Firms Spread Out Bond Maturity to Manage Their Funding Liquidity Risk?, *Working Paper.*

Pastor, L. and R. Stambaugh, 2003, Liquidity Risk and Expected Stock Returns, *Journal of Political Economy 111,* 642–685.

Petersen, M. A., 2009, Estimating Standard Errors in Finance Panel Data Sets: Comparing Approaches, *Review of Financial Studies 22,* 435–480.

Petersen, M. A. and D. Fialkowski, 1994, Posted versus Effective Spreads: Good Prices or Bad Quotes?, *Journal of Financial Economics 35,* 269–292.

Pu, X., 2009, Liquidity Commonality Across the Bond and CDS Markets, *The Journal of Fixed Income 19,* 26–39.

Roll, R., 1984, A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market., *Journal of Finance 39,* 1127–1139.

Schestag, R., P. Schuster, and M. Uhrig-Homburg, 2014, Measuring Liquidity in Bond Markets, *Working Paper.*

Schich, S. T., 1997, Estimating the German Term Structure, *Discussion paper 4/97, Economic Research Group of the Deutsche Bundesbank.*

Schultz, P., 2001, Corporate Bond Trading Costs: A Peek Behind the Curtain, *Journal of Finance 56,* 677–698.

Schuster, P. and M. Uhrig-Homburg, 2014, The Impact of Liquidity Supply and Demand on the Term Structure of Bond Market Illiquidity Premiums: A Conditional Approach, *Working Paper*.

Schuster, P., M. Uhrig-Homburg, and M. Trapp, 2014, A Heterogeneous Agents Equilibrium Model for the Term Structure of Bond Market Liquidity, *Working Paper*.

Schwarz, K., 2010, Mind the Gap: Disentangling Credit and Liquidity in Risk Spreads, *Working Paper*.

Sims, C. A., J. H. Stock, and M. W. Watson, 1990, Inference in Linear Time Series Models with some Unit Roots, *Econometrica 58* 113–144.

Swinscow, T., 1997, *Statistics at Square One*. (BMJ Publishing Group London) ninth edn.

Taylor, J. B., 1993, Discretion Versus Policy Rules in Practice, *Carnegie-Rochester Conference on Public Policy 39,* 195–214.

Theil, H., 1966, *Applied Economic Forecasts*. (North-Holland).

Vayanos, D. and J.-L. Vila, 1999, Equilibrium Interest Rate and Liquidity Premium, *Economic Theory 13* 509–539.

Vayanos, D. and J.-L. Vila, 2009, A Preferred-Habitat Model of the Term Structure of Interest Rates, *Working Paper*.

Vayanos, D. and T. Wang, 2007, Search and Endogenous Concentration of Liquidity in Asset Markets, *Journal of Economic Theory 136,* 66–104.

Warga, A., 1992, Bond Returns, Liquidity, and Missing Data, *Journal of Financial and Quantitative Analysis 27,* 605–617.

Weill, P.-O., 2008, Liquidity Premia in Dynamic Bargaining Markets, *Journal of Economic Theory 140,* 66–96.

White, H., 1982, Maximum Likelihood Estimation of Misspecified Models, *Econometrica 50,* 1–25.

World Bank, 2013, Market Capitalization of Listed Companies, *http://data.worldbank.org/indicator/CM.MKT.LCAP.CD*.