SIMULATIONS OF THE MICROBUNCHING INSTABILITY AT ANKA USING A VLASOV-FOKKER-PLANCK SOLVER*

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Abstract

In order to produce coherent synchrotron radiation (CSR) the ANKA light source is operated frequently in short bunch mode. It is known that during this procedure strong self fields caused by high electron densities can amplify initial density fluctuations and thus lead to microbunching. The build-up of those substructures is accompanied by bursting radiation which provides higher radiation power for the users. Damping and diffusion due to incoherent radiation smoothens the bunch shape again and hence lead to periodic or chaotic bursting cycles. The evolution of the electron bunch density under the influence of self fields can be described by the Vlasov-Fokker-Plank (VFP) equation. We present results from a numerical solution of the VFP-equation for parameters used in standard short bunch mode at ANKA.

INTRODUCTION

The light source ANKA can provide CSR in the THz frequency range in either low-power constant emission or high-power radiation bursts, depending on the users' requirements. The threshold between the two modes depends on the bunch length and the electron beam current [1].

The longitudinal beam dynamics leading to a bursting behavior have been simulated for various storage rings [2, 3] using a Vlasov-solver with the wake field from CSR. These simulations show that bunch length, shape and centroid position vary as the bunch undergoes the bursting cycle. However, the detailed dynamics of these phenomena strongly depends on the ring and beam parameters in question.

LONGITUDINAL BEAM DYNAMICS

The evolution of the phase space density $f(q, p, t)$ can be described by the 1D VFP equation, including the force F due to the longitudinal impedance, radiation damping and diffusion [2]:

$$\frac{\partial f}{\partial \tau} + p\frac{\partial f}{\partial q} - [q + I_c F(q, f, \tau)]\frac{\partial f}{\partial p} = \frac{2}{\omega_s t_d}\frac{\partial}{\partial p}\left(pf + \frac{\partial f}{\partial p}\right) \quad (1)$$

where q and p are the normalized longitudinal phase space variables, t_d the damping time and ω_s the angular synchrotron frequency. The time is given as multiple of a synchrotron period: $\tau = \omega_s t$. The impact of the force is proportional to the bunch current which is included in the parameter $I_c = e^2 N\omega_0/(2\pi\omega_s\sigma_E)$, with the number of electrons N, the angular revolution frequency ω_0 and the energy spread σ_E.

This model takes the linear rf focusing into account and describes the storage ring as having a continuous curvature with radius R without straights. The electron beam is described by a purely longitudinal line density and all transverse motion is neglected. However, for a finite impedance a finite beam height of δh is required.

The collective force depends on the actual electron distribution and the impedance $Z(n)$:

$$F(q, f) = -\omega_0 \sum_n \exp(inq\sigma_z/R)Z(n)\lambda_n, \quad (2)$$

which causes the CSR with spectrum

$$P_{coh}(n, t) \propto ReZ(n)|\lambda_n(t)|^2. \quad (3)$$

Here λ_n is the Fourier transform of the longitudinal charge density. This implies that for a given impedance the spectrum will be extended to higher frequencies if the initial Gaussian distribution is distorted or shows substructures.

In our model we use the impedance treating the vacuum chamber as two perfectly conducting infinite parallel plates [4] with separation h which suppress radiation with wavelength longer than $\lambda_{thr} = \sqrt{4h^3/R}$.

NUMERICAL SOLUTION OF THE VFP EQUATION

The left hand side of eq. (1) describes the motion due to kinetic energy and rf potential and the impact of the collective force caused by CSR. Using operator splitting, this Vlasov part is treated separately by a (discretized) Perron-Frobenius operator, which has the advantage that the corresponding map guaranties (approximate) charge conservation. The Fokker-Planck term (r.h.s. of eq. (1)) describes the longterm dynamics of damping and diffusion and can be handled using difference quotients.

The longitudinal phase space density is given on a Cartesian grid, starting with a Gaussian distribution in q and p. The inverse map for single particle motion over a short time step is a rotation in phase space, followed by a kick due to CSR. The update of the phase space density requires the evaluation of the previous density at off-grid points, which is accomplished by Lagrangian bi-cubic interpolation using 16 surrounding points.

* Work supported by the Initiative and Networking Fund of the Helmholtz Association under contract number VH-NG-320

CHOICE OF PARAMETERS

Initial tests varying mesh size and granularity showed that a good charge conservation can be assured by choosing a quadratic grid with 400×400 points, ranging up to $|q| = |p| = 8$. It is worth noting that a refinement of the mesh does not improve the stability of the numerical treatment. Apparently a coarser mesh smoothens the distribution and prevents the build-up of dominant substructures on the distribution due to numerical noise. A time step of $0.002/2\pi$ of a synchrotron period was found to be a good trade-off, trying to decrease computation time while providing a stable numerical treatment.

For the simulations we used machine parameters which are typical for the low-momentum-compaction lattice during short bunch operation at ANKA (Tab. 1). For these machine settings the bursting stable threshold was measured in multibunch fill. The threshold current was found to be 0.15 mA per bunch which corresponds to $3.5 \cdot 10^8$ electrons [1]. In our simulations we varied the number of electrons from $3 \cdot 10^8$ to $20 \cdot 10^8$. For bunches with more than $13 \cdot 10^8$ particles the charge conservation became poor rapidly (losses $> 1\%$).

Table 1: ANKA Parameters

Beam energy	E	1.3 GeV
Momentum compaction	α	$4.5 \cdot 10^{-3}$
Natural bunch length	σ_z	1.5 mm
Synchrotron tune	$\omega_s/2\pi$	6.6 kHz

RESULTS AND DISCUSSION

In general, the behavior of the first 70 synchrotron periods can be described by a sharp instantaneous rise of the quantities emitted coherent power P_{coh}, bunch length σ_q, energy spread σ_p and oscillation amplitude of $\langle q \rangle$ and $\langle p \rangle$. Thereafter a settling process follows, which decreases all the latter mentioned properties until a stable but time varying pattern evolves. For high currents this initial rise in bunch length causes the main particle loss, later on the relative fluctuation of charge $\Delta Q/Q$ is less than 10^{-5}. As an example, Fig.1 (top) shows the coherent emission for the first 250 synchrotron periods and in Fig.1 bottom displays the details of the settled pattern.

Focusing on the longtime dynamics, three different behaviors, depending on the bunch charge, can be distinguished:

(A) For low currents only small oscillations in $\langle q \rangle, \langle p \rangle, \sigma_q, \sigma_p$ and P_{coh} due to the small collective force occur which completely vanish for the zero current limit. The small oscillation of bunch position is a dipole mode while the bunch length and radiation variation follow a quadrupole mode. The bunch profile is smooth but differs from a pure Gaussian since the collective force causes potential well distortion. The coherent spectrum is constant in time and does not exceed 4 cm^{-1} (Fig. 2 (a)).

(B) For $6 \cdot 10^8$ and more particles per bunch an additional oscillation appears with lower and decreasing frequency for increasing currents. The electron distribution shows time varying deformations and substructures (Fig. 3 top) which enlarges Fourier components with high frequencies. Accordingly the coherent spectrum extends to higher frequencies and varies with time (Fig. 2(b), (c)) which is the effect called bursting. The bunch length and energy spread show a sawtooth pattern which is typical for that kind of instability.

(C) For even higher currents this regular pattern gains more and more chaotic behavior. The substructures dominate the bunch profile (Fig. 3 bottom) and the spectrum broadens once more to higher frequencies (Fig. 2(d)).

Comparison with Measurements

At the ANKA IR1 beamline several measurements concerning spectra, bursting stable threshold and bursting emission characteristics were conducted. The first bursting radiation of a multibunch fill was measured at nearly half the current as seen in these single bunch simulations. This indicates that leading or following bunches can act on each other via the wake field. The observed temporal radiation characteristics (Fig. 1) are comparable with earlier measurements [5].

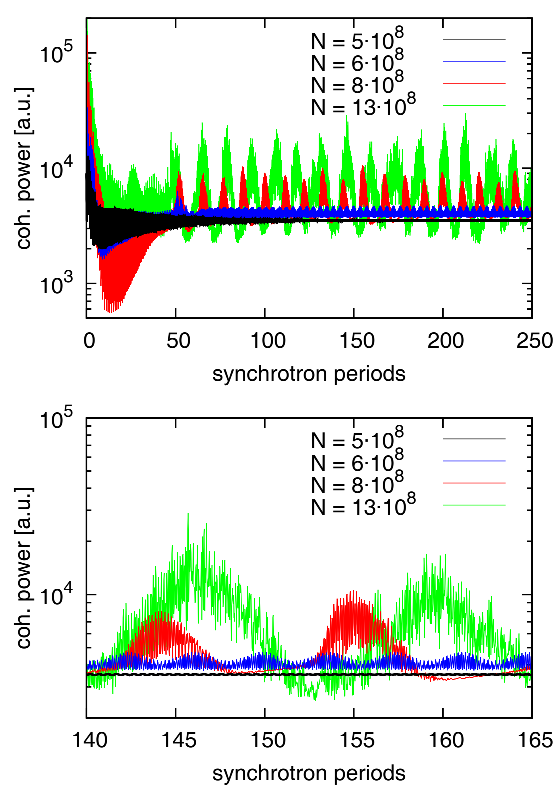
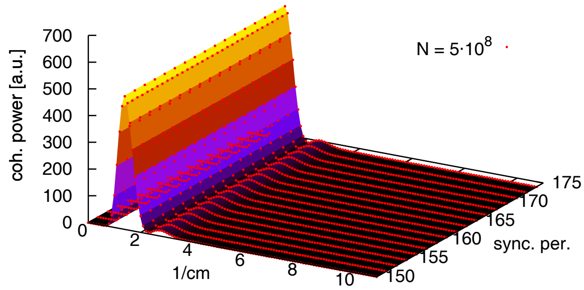
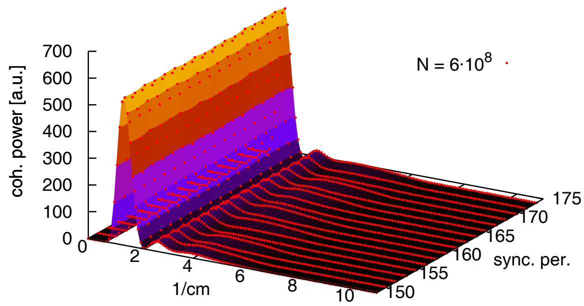


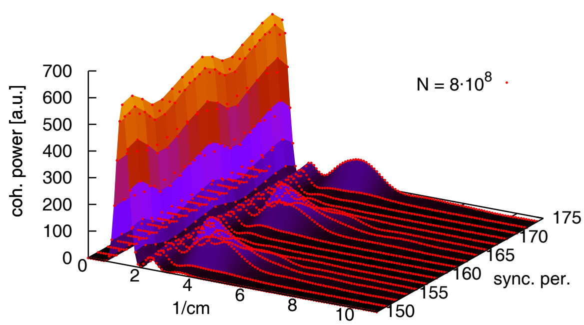
Figure 1: Time dependent emission of coherent radiation for different numbers of electrons per bunch. The bottom graph shows a close-up example of the longterm dynamics.



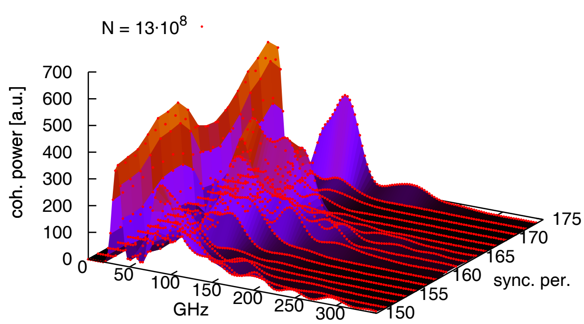
(a) Below busting threshold



(b) Arising bursting



(c) Periodic bursting



(d) Chaotic bursting

Figure 2: Power spectra for different numbers of particle per bunch.

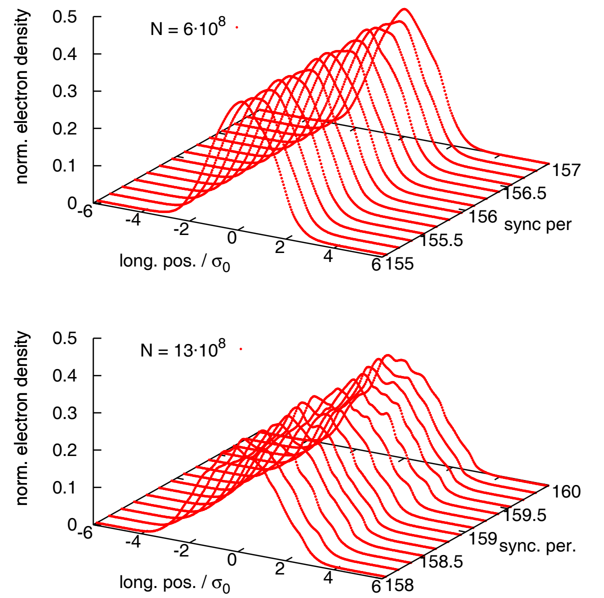


Figure 3: Normalized charge densities at different times. High current leads to time varying substructures, the so-called microbunching.

Latest measurements with a recently installed Martin-Puplett-Interferometer showed indication of a two-part spectrum as seen in the simulations: a low frequency part not changing significantly with current and a high frequency part vanishing in steady state emission.

ACKNOWLEDGMENT

We gratefully acknowledge Robert Warnock for help and valuable discussions.

Thanks to the Karlsruhe House of Young Scientitst (KHYS) for granting M. Klein's stay at the SLAC National Laboratory where this work has been accomplished.

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