Modelling of motorcycle movements in mixed traffic conditions

Freddy Antony Mullakkal Babu
Indian Institute of Technology Bombay
Mumbai – 400 076, India
email: fredavincister@gmail.com

Peter Vortish
Karlsruhe Institute of Technology
Karlsruhe-76131, Germany
Email: peter.vortisch@kit.edu

Tom V Mathew (Corresponding Author)
Indian Institute of Technology Bombay
Mumbai – 400 076, India
email: tvm@civil.iitb.ac.in

4615 Words + (6 Figures + 3 Tables)*250 = 6865 Words

1st August, 2014

ABSTRACT

There exists limited literature that explains the movement of two-wheelers in mixed traffic conditions, where the traffic behaviour is characterised by different vehicle types moving together without much lane adherence, collision avoidance to neighbouring vehicles, response to infrastructure boundaries and following behaviour with respect to multiple lead vehicles. The study aims at filling this gap by proposing a microscopic simulation model exclusively for two-wheeler movement in a mixed traffic environment. The theories of the social force model and the intelligent driver model have been adapted and employed for this purpose. The model performance is assessed using individual trajectory comparison between simulation and observation, and the ability to qualitatively simulate naturalistic two-wheeler behaviour through a test scenario. The simulation results show that the model could visually represent the two-wheeler behaviour in mixed traffic scenarios.

INTRODUCTION

Motorcycles are an important component of the mixed traffic vehicle population which exists predominantly in Asian countries. High manoeuvrability, door to door accessibility, smaller parking space, low maintenance and fuel cost makes motorcycles more appealing for the low or middle income families. Simulation of motorcycle movements is therefore important in the field of traffic safety and capacity analysis (1). The motorcycle, being the smallest motorised vehicle type, would require simulation at a microscopic scale for understanding its riding behaviour in various traffic scenarios. Micro simulation in general is well established in design and testing of control strategies (2). Study on the basic characteristics of motorcycle flow, like the speed-flow relationship, headway distribution etc. was done by (3) and (4) on the macroscopic level. Cellular automata models were also developed to model mixed traffic flow including motorcycles (5).

Cho and Wu (6) suggested a model for the lateral position of two-wheelers based on relative positions of the influential surrounding vehicles. The problem lies in the fact that in Indian conditions the definition of surrounding vehicles as described in the paper may not hold. Also the idea of maintaining the mid-point of the nearby lateral space does not account for the safety distance from the infrastructural boundary and the surrounding vehicles. Long (7) proposed a model based on safety distance from the neighbour vehicle. A car-following model was applied to motorcycles by Lan et al. (5) but the lateral movements were described poorly. Lee (8) suggested an agent based motorcycle model which used a multinomial choice model for path choice, however the assumption of virtual lanes in this model has a practical difficulty in deciding the virtual lane width. Some more models try to explain lane changing behaviour of the vehicles using the concept of virtual lanes (9). This model however had inaccurate results for motorcycles.

Most of these models fail to replicate the characteristic behaviour of motorcycles in mixed traffic conditions: filtering through the traffic, tailgating (following the lead vehicle along a lateral edge) the lead vehicle, maintain safety distances from neighbour vehicles, swerving and frequent lateral movements (7). The proposed model tries to fill this gap. It aims at developing a microscopic behavioural model for naturalistic two-wheeler movement in a mixed traffic environment. The theories of the social force model (10) and the intelligent driver model (IDM) (11) have been adapted and employed for this purpose. The model deals with two-wheelers in a midblock section in uncongested situations with cars as neighbour vehicles. It does not attempt to model the vehicle dynamics in all details.

MODEL FRAMEWORK

The general structure of the model is explained in figure 1. The neighbour vehicles are identified using a perception line logic and are passed over as input for the lateral and longitudinal movement models. The lateral movement model is based on the concept of social force (12) and the longitudinal movement is modelled using the Intelligent Driver Model. (11). The output of these models, i.e. acceleration and the raw angle, is then adjusted subject to certain manoeuvrability constraints of the motorcycle. The updated position of the motorcycle for the next time interval is calculated and the cycle continues.

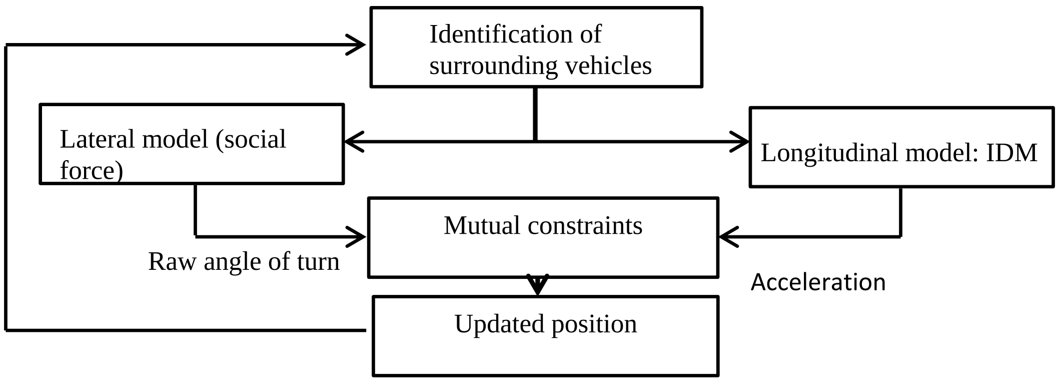

FIGURE 1: Model framework

Perception lines

In heterogeneous traffic, vehicles are placed at different lateral positions around the subject vehicle throughout the time. It is important to identify the surrounding vehicles that influence the movement of the motorcycle. Perception lines (PLs) are imaginary lines drawn from the front seat of the subject motorcycle outwards up to a distance of L_p

$$L_p = v\left(T + \frac{W_c}{\tan(\theta^m)}\right) + s^*_{long} \tag{1}$$

$W_c$is the width of a car (taken to be 1.8m),T is the desired time headway, θ^m is the maximum possible yaw angle at time t, V is the velocity of the motorcycle at time t, and s^*_{long} is the minimum longitudinal headway at rest. An individual PL is referred to using the anticlockwise angle it makes with the positive X axis as shown in figure 2. All the surrounding vehicles that first intercept a perception line are considered to be the neighbour vehicles. The vehicle that intersects the 90˚ PL first is assumed to be the direct lead vehicle. The positions of the those vehicles on the front 60˚ field of view closest on the left and right side of the lead vehicle are also identified as front left and front right vehicles respectively.

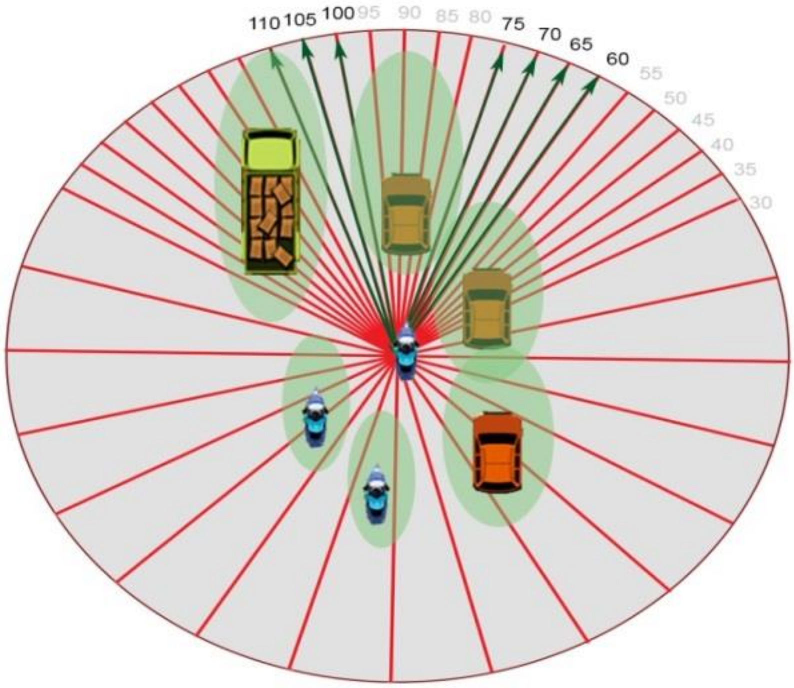

FIGURE 2: Representation of the perception lines

LATERAL MOVEMENT MODEL

The social force approach (10) was originally used to describe the movement of pedestrians. Later this model had also been employed for modelling motorcycle movement (12). Fellendorf et al. (13) also used the social force approach to model motor vehicle movement, however motorcycles were not considered in the study.

The social forces are measures of the instantaneous internal motivation to accelerate or change direction experienced by the driver to avoid collisions. The model assumes that the motorcycle prefers to remain in the state of no lateral movement in the absence of any force. The proposed model includes three types of social forces, namely:

1. A repulsive force from surrounding vehicles (F^t_{Ri}) leading to a lateral displacement ofΔY^t_{Ri}
2. A repulsive force from infrastructural boundaries (F^t_{Bi}), leading to a lateral displacement of ΔY^t_{Bi}
3. A repulsive force from the lateral gap in front (F^t_{Gi}), leading to a lateral displacement of ΔY^t_{Gi}

The total required lateral displacement as a reaction to the neighboring social forces is computed as

$$\Delta Y^t_i = \Delta Y^t_{Ri} + \Delta Y^t_{Bi} + \Delta Y^t_{Gi} \tag{2}$$

Force due to surrounding vehicle

The motorcycles generally tend to keep a safe lateral distance from the neighbouring vehicles. The closer they come beyond this perceived safe distance the more is the motivation to move away from the vehicle. The motivation to move away is further influenced by the relative velocity of the vehicle. The motorcycle tries to move more aggressively away from the neighbouring vehicle when it has a higher relative velocity. The region around the neighbour vehicle where the subject vehicle feels equally uncomfortable can be imagined as an ellipse. (7) The semi minor axis of this ellipse is b which is a constant for the given pair of subject vehicle and neighbour vehicle at a given point of time. When other vehicle comes closer to a target vehicle, the semi-minor axis of the ellipse becomes smaller and as a result, repulsive force gets bigger. The repulsive force F^t_{Rij}creates a lateral motivation which is formulated as shown in eq. (4).

$$\Delta Y_{Rij} = \beta_R (b - b^o)^{-\rho_R} \tag{3}$$

$$b = \frac{\sqrt{\left(|\vec{r_i} - \vec{r_j}| + |\vec{r_i} - \vec{r_j} - (\vec{v_j} - \vec{v_i})\Delta t|\right)^2 - \left(|\vec{v_j} - \vec{v_i}|\Delta t\right)^2}}{2} \tag{4}$$

Where $\vec{r_i}$ and $\vec{r_j}$are the position vectors of the subject vehicle and neighbour vehicle and the distance between them at time t. Similarly $(\vec{v_j} - \vec{v_i})$ Δt are the velocity vectors. β_R is a multiplicative calibration constant.ρ_R is the exponential calibration constant and $b^o$is the additive constant used in the formulation of repulsive force.

Force due to infrastructural boundary

Similar to the repulsive force from the neighboring vehicles, the motorcycle perceives a repulsive force from the infrastructural boundaries (F^t_{Bi}). The resulting lateral displacement motivation is formulated in eq. (5)

$$\Delta Y_{Bi} = \beta_B {r_b}^{-\rho_B} \tag{5}$$

Where, $\beta_B$the multiplicative calibration is constant, ρ_B is the exponential calibration constant, r_b is the lateral distance towards the infrastructural boundary.

Force due to gap choice

Motorcyclists in mixed traffic streams often exhibit behaviors such as tailgating and filtering through the lateral gaps in uncongested conditions. The common attribute observed for these behaviors is the presence of a safe lateral gap ahead. The direct front vehicle is identified using the concept of perception lines. Also it is observed that the rider identifies the gap-ahead only if it exists in the gap search area ahead of it. The gap search area is the sector made by the 60˚ and 120˚ perception line with the rider as the center. The lateral positions of all the vehicles in the search area are analyzed. If the edge to edge distance between the nearest vehicle and the leader vehicle is more than twice the safe lateral distance (S^*_{lat}), then a gap is identified on that particular side.

Availability of gap on the left and right side defines the gap-scenario, which decides the accepted gap. This decision framework is illustrated in figure3. When there is only left gap, or right gap, or there is no gap, the decision is straight forward. However, when both the left and right gaps are available, a binomial choice model is used for the decision. The utility equations for the left gap (U_L) and the right gap (U_R) are formulated follows:

$$U_L = \alpha^1 g_L + \alpha^2 a_L + \alpha^3 c_L + \epsilon \,, \tag{6}$$

$$U_R = \alpha^1 g_R + \alpha^2 a_R + \alpha^3 c_R + \epsilon \,, \tag{7}$$

where g_L and g_R are the lateral distance to the corresponding overtaking position with respect to the front vehicle; a_L and a_R are the parameters of relative affinity towards left and right direction respectively; c_L and c_R are the lateral clearance available for the lead vehicle with the nearest vehicle on the left and right sides respectively; α^1, α^2 and α^3 are coefficients for the general terms; and ϵ is a rider specific random term which is Gumbel distributed. The lane affinity is a generally observed phenomenon on express ways. They are dummy variables incorporated to explain the lane affinity. It means that the motorcycle has higher probability to choose the gap in the lane to which it has higher affinity. The estimates α^1, α^2 andα^3 can be calculated using maximum likelihood method (ALOGIT software is used in the current study). The probabilities for choosing an alternative are calculated using the binomial logit model. The probability corresponding to each of the options is calculated. The left gap is chosen when, $U_L > U_R$. The probability of choosing the left gap P_L therefore is $P_L = Pr[U_L > U_R]$, by the binary logit model,

$$P_L = \frac{e^{U_L}}{e^{U_L} + e^{U_R}} \text{ and } P_R = \frac{e^{U_R}}{e^{U_L} + e^{U_R}} \tag{8}$$

The choice model returns the probabilistic choice of the rider for the corresponding time interval. Once the choice is identified, then the rider is subject to a new force in order to occupy the gap (F^t_{Gi}).

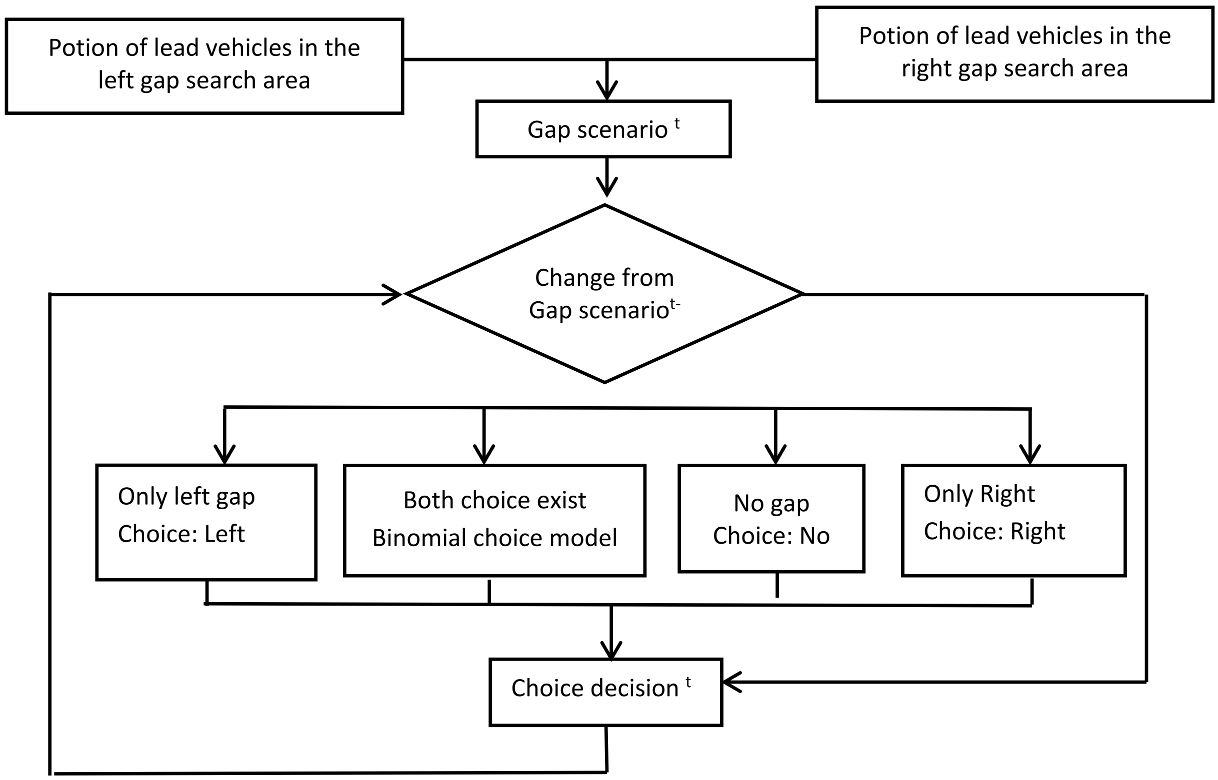

FIGURE 3: Flowchart for identification of gap scenario

Formulation of the attractive force due to lateral gap choice (F^t_{Gi}).

Once the gap is accepted, motorcycle perceives an attractive force driving it towards the accepted gap. The resulting lateral displacement motivation ΔY_{Gi} is formulated as shown in eq. (9).

$$\Delta Y_{Gi} = S\, f_\theta \tag{9}$$

Where, S is the coefficient of certainty and f_θ is the required lateral force parameter. The term S is ensures that the motorcycle is more certain to make the lateral movement toward the gap when it is along the corresponding lateral edge and has a maximum value of 1. The lateral force parameter is a measure of the required angle towards the comfortable lateral position.

$$S = \left(\beta_G G_{long}^t\right)^{-\rho_G G_{lat}^t} \tag{10}$$

$$f_\theta = vdt \tan(\theta_g^t) \tag{11}$$

$$\theta_g^t = \tan^{-1}\left(\frac{G_{lat}^t}{\max\left[S_{long,}^* G_{long}^t - S_{glong}^t\right]}\right) \tag{12}$$

$$S_{glong}^t = \frac{G_{lat}^t}{\tan\theta^m} \tag{13}$$

where, β_G is the multiplicative calibration is constant, ρ_G is the exponential calibration constant θ_g^t is the target angle of gap force, S_{glong}^t, is the safe longitudinal headway required for following along the edge of leader vehicle, G_{lat}^t is the lateral distance towards the following edge corresponding to front vehicle at time t, G_{long}^t is the longitudinal headway available with the front vehicle at time t, s_{long}^* is the safe longitudinal headway for motorcycle at stop position and θ^m is the maximum turn angle. The attractive force due to lead gap would not require any change in parameter values in order to be applied for different lead vehicle types. This is because the parameter values only depend on the geometric dimensions of the lead vehicle.

LONGITUDINAL MOVEMENT MODEL

The movement of a motorcycle on a road can be explained mainly by the turning angle and its acceleration behavior. This section deals with the estimation of acceleration characteristics of the motorcycle in different traffic scenario. The Intelligent driver model is a well-established car following model for cars in lane disciplined traffic. However, modifications have to be done to incorporate this model for explaining the motorcycle movement in mixed traffic scenarios. In the proposed model, the lead vehicle is identified based on the perception lines logic and a threshold lateral distance for the influence of lead vehicle has been introduced. Lead vehicles in this model are defined as all the vehicles that are visible to the rider in his 20° angle of vision. The applied IDM belongs to the class of deterministic follow-the-leader models like the optimal-velocity model of Treiber et al. (11).

$$\dot{v}_\imath = \frac{\partial v_i}{\partial t} = ac\left(1 - \left(\frac{v_i}{v^o}\right)^\delta - \left(\frac{s^*(v, \Delta v)}{G_{long}}\right)^2\right) \tag{14}$$

$$s^*(v, \Delta v) = s_{long}^* + vT + \frac{v * \Delta v}{2\sqrt{ac * dc}} \tag{15}$$

where, v^o is the velocity with which the vehicle would drive in free traffic condition, s_{long}^* is the minimum longitudinal distance that the motorcycle keep even at stand-still in a traffic jam, T is the desired time headway to the vehicle in front, ac is the maximum possible acceleration, dc is the comfortable braking deceleration and δ is the exponent coefficient.

Threshold for lead vehicle interaction

The motorcycle interacts with the lead vehicle in the following regime. Beyond this threshold of following regime it accelerates freely to acquire the desire speed. The threshold is defined as the function of velocity and lateral clearance.

$$v = 3.74 S_{lat}^* + 12.44 \tag{16}$$

This means that, a motorcycle moving with a velocity v will start to accelerate to its desired velocity, if a lateral clearance of at least S_{lat}^* is available with respect to all the lead vehicles. The equation was obtained from analyzing the data samples. The position of the subject vehicle where it was seen to move out of the interaction with the front vehicle was assumed to be the threshold. That is, if the subject vehicle accelerates while following a front vehicle, it is considered to be out of the interaction threshold. 40 sample points were obtained and linear regression was used to calculate the estimates. The values of coefficient of S^*_{lat} is estimated to be 3.74 and the estimate for the constant was 12.44 using linear regression with R^2=0.193. The p value for variable is 0.005 and the t stat is 2.98 which mean that there is a significant relationship between velocity and the threshold lateral headway.

Integration of the lateral and longitudinal behavior

The lateral and longitudinal behavior of a motorcycle is closely related to each other. Even though the lateral and longitudinal motivations are found separately, there are certain mutually imposed constraints. The turn angle that the rider applies should not exceed the maximum possible turn angle even if the lateral motivational forces require it to be more. The maximum turn angle (θ^m) made by the motorcycle is found to be a decreasing function of velocity at the point eq. (17).

$$\theta^m = \gamma_1 v + \gamma_2 \tag{17}$$

Here γ_1 and γ_2 are the two calibration parameters.

Position update

The position of the two wheeler is updated based the constraint of maximum turning angle. The equation for motion is eq.(18)

$$\begin{aligned} Y_i^t &= Y_i^{t-dt} + \max(\Delta Y_{Ri}^t + \Delta Y_{Bi}^t + \Delta Y_{Gi}^t,\ v_t dt \tan(\theta^m)) \\ X_i^t &= X_i^{t-dt} + v_t(\Delta t) + 0.5\, a_t(\Delta t)^2 \end{aligned} \tag{18}$$

The position of the motorcycle is updated and the next set of influential vehicles at time t + dt are identified by PL algorithm. The procedure is then repeated till the motorcycle reaches the end of the stretch.

RESULTS AND DISCUSSION

The proposed model is calibrated and validated with field data and a case study is presented to illustrate the performance of the model.

Data collection

The trajectory data set used in this study was collected on a section of the westbound direction of Eastern Expressway, Bhandup Mumbai from 8.30 AM to 11.00 AM. The data was collected using 3 video cameras that were mounted on a foot-over bridge. The entire section is approximately 650 m meters long and 18.3 m wide (5 lanes). The weather was clear with good visibility and pavement was dry during the data collection period. Furthermore, there were no incidents or events within the section during this period. The motorcycle constituted about 38% of the vehicular flow during the time of survey. The data extraction was done using the software- Traffic data extractor developed in IITB (14)

Model calibration

The proposed model could be broadly summarized as a combination of social force model and intelligent driver model put in the framework of perception lines logic. This means that the entire component models have to be calibrated separated for their parameters and the final model have to be checked for functioning harmoniously. The developed model contains various parameters. As the first step parameters such as visual range for perception lines etc. have been taken from related studies. Secondly fixing the above parameters, other parameters are calibrated using trial and error.

Calibration of measurable parameters

The field of view for the proposed perception lines logic (ø) is taken to be 120˚ from the related works (15). The exponential parameter (δ) used in IDM is not sufficiently sensitive and hence the original value, 4 given by Treiber et al. (11) is adopted. For the purpose of simulation, the road section (0 m to 18.3 m) is divided into 3 sections namely section 1 (0–5 m), section 2 (5-10 m) and section 3 10-18.3 m). a_L and a_R are the parameters of relative affinity towards left and right direction respectively. The values of these parameters were decided based on the average lane occupancy of the motorcycle for the given data. The value of the lane affinity taken for the purpose of simulation is as follows; a_L in section 3 is 0.1 and a_R in section 1 is 0.5 and the lane affinity has value 1 in all the remaining cases.

Calibration of non-measurable parameters

After fixing the measurable parameter, there is a total of 16 parameters remaining to be calibrated. These are optimized using as by trial and error. The values are tabulated in Table 1.

TABLE 1: Parameter values for the model

Coefficient	Eq.	Trial and error value
β_R(car)	(3)	0.91
β_R(motorcycle)	(3)	0.42
β_R(HCV)	(3)	5.5
β_R(auto)	(3)	0.6
ρ_R	(3)	0.80
b^o	(3)	0.83
β_B	(5)	0.235
ρ_B	(5)	0.246
β_G	(10)	0.01
ρ_G	(10)	-0.334
v^o	(14)	30
s^*_{long}	(14)	1m
ac	(14)	1 m/s^2
bc	(14)	3 m/s^2
δ	(14)	4
T	(14)	0.7 s

The coefficients of the binary choice model are calibrated from the data. The sample of 45 instances where the rider took a gap decision was extracted manually. In 29 instances the rider chose the left gap and in 14 instances the rider chose the right gap. The results of calibration using ALOGIT software is summarized in Table 2. The Rho – squared w.r.t to zero is 0.5349, and the Rho –squared w.r.t to constants is 0.5047.

TABLE 2: Estimates of choice model coefficients

Estimates	α^1	α^2	α^3
Mean	-0.82	1.767	0.5993
Std error	0.619	1.22	0.227
t stat	-3.1	1.4	2.6

Validation

To analyse the performance of the model, individual real world trajectories was extracted and then compared to the simulated trajectory in similar traffic scenario.

Individual trajectory simulation

The motorcycle trajectory was simulated keeping the original trajectories for the surrounding vehicle. Figure 4 illustrates an example of passing manoeuvre. The front vehicle in this case is a HCV. The values of the parameters were given as in described earlier. Different forces dominate with respect to the relative position of the motorcycle. Initially, the lateral movement towards the right was triggered due to the right gap identification. At a later point of time, the right gap became unavailable and the gap choice was changed. This made the motorcycle move towards its left. Lateral forces became insignificant as it reached the comfortable lateral edge position. The tailgating behaviour was observed here. Finally the longitudinal gap decreased, the repulsive force started to dominate and the vehicle was pushed further left.

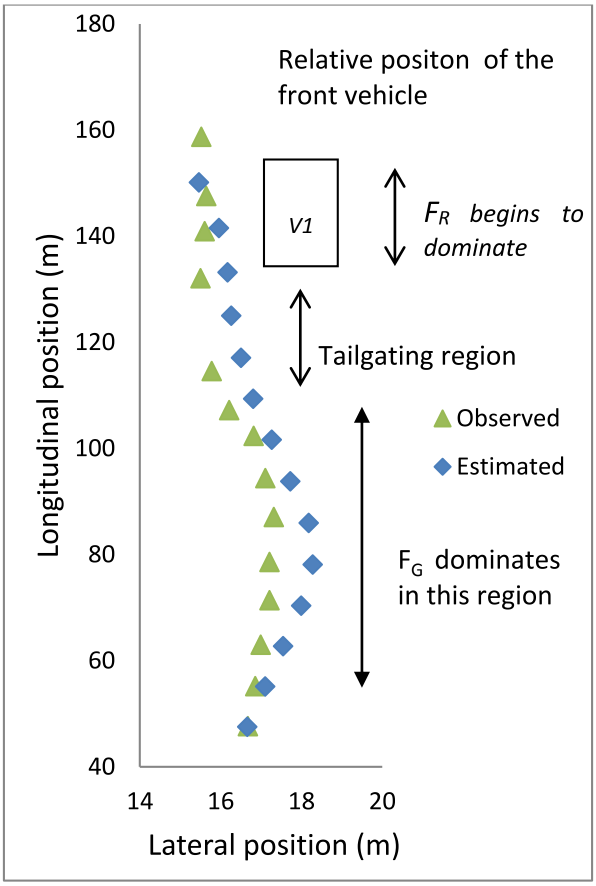

FIGURE 4: Individual trajectory Vs Observed trajectory - 1

The error measurement used here is MAPE (mean absolute percentage error). The MAPE values for lateral position are found to be 1.93% and longitudinal position is 3.71%. It has to be noted that, during this period 76.5% of lateral movement have been caused by the gap force. Individual trajectory simulation was carried out with different neighbor vehicle types. As explained earlier, the only parameter that would vary in lateral movement calculation while having different neighbor vehicle type isβ_R. Hence trajectories involving the interaction of motorcycle to different vehicle type was extracted and compared with simulated trajectories. The summarised MAPE values are given in Table 3.

TABLE 3: MAPE estimates for individual trajectory simulation

Front vehicle	MAPE lateral (%)	MAPE longitudinal (%)
Heavy vehicle	2.83	3.56
Car	0.84	2.40
Motorcycle	1.41	3.15
Auto	5.30	4.03

Validation of the model for lane affinity

The lane affinity is a commonly observed phenomenon on Indian roads especially in uncongested conditions. To verify if the proposed model was capable of exhibiting lane affinity, the position of motorcycles after 20 sec of vehicle input was calculated. The vehicle input of the surrounding vehicles was predefined. The vehicle arrival model is taken to be negative exponential and the lateral distribution of the cars is assumed to be a skewed normal distribution with mean on 9m, resembling the field behavior. The lateral input distribution of the motorcycles is assumed to be a uniform probability function, which means that the motorcycles have equal probability of occupying any lateral position initially. The parameters extracted from the field are mean headway: 0.86 s for car and 1.75 for motorcycles, average speed: 12m/s. The results in figure 5 show that the motorcycle position after 20 sec was concentrated to particular lane position as it was seen on the field.

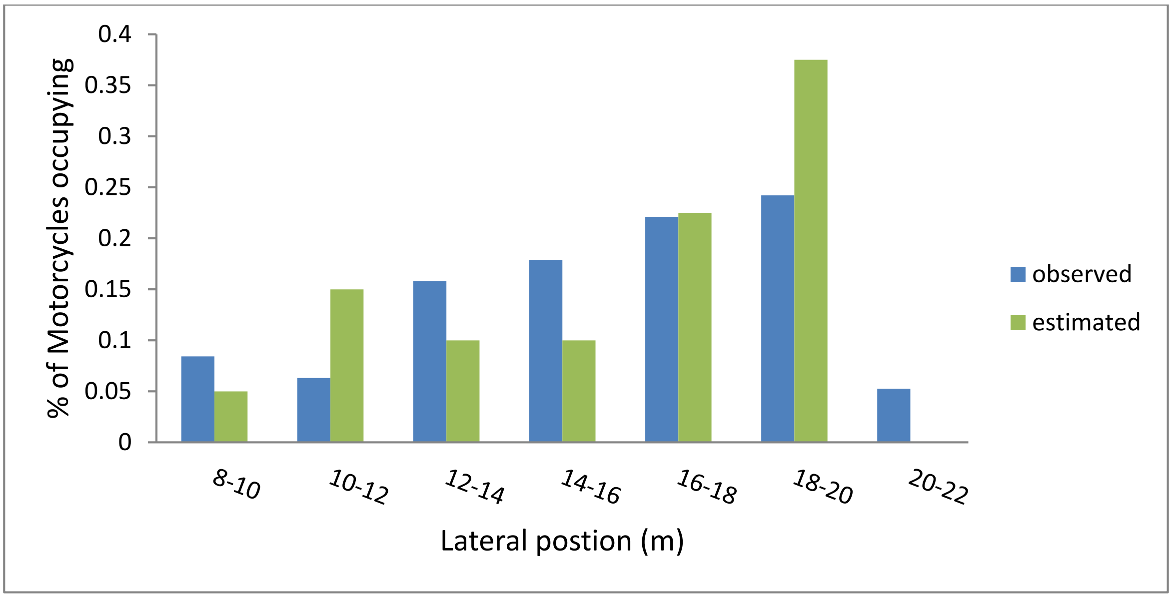

FIGURE 5: Lateral position distribution of estimated vs observed

Case study

In order to verify the ability of the model to exhibit naturalistic riding behaviour, a hypothetical situation resembling an uncongested highway was created. The behaviour of the neighbor vehicles denoted as V2 and V3 were predefined in terms of position and velocity. The trajectory of the motorcycle in these conditions was estimated using the proposed model. The estimated acceleration and the relative position of the motorcycle with respect to the neighboring vehicle were studied. It can be seen that initially the motorcycle decelerates as it did not have required lateral headway with vehicle 2. Later, when a safe lateral headway with respect to the V2 was achieved, the motorcycle began to accelerate. This demonstrates the ability of the model to simulate the naturalistic motorcycle behaviours while maintaining the safety distance from the neighbouring vehicle.

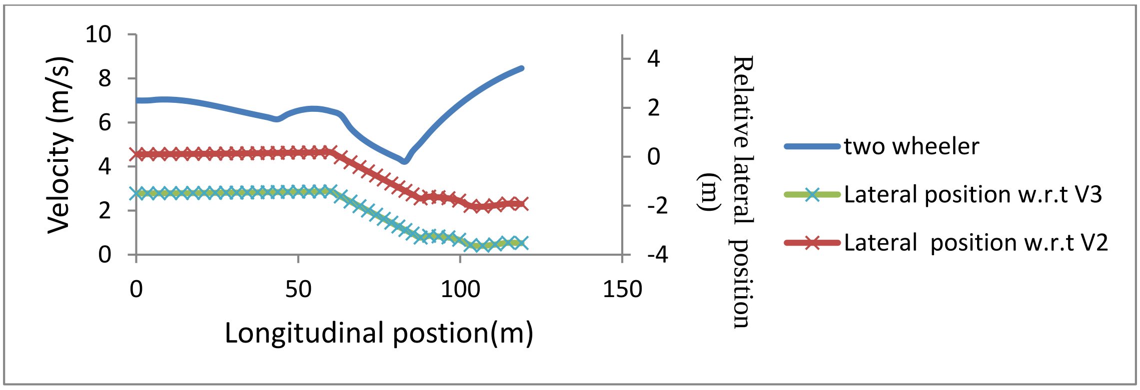

FIGURE 6: Velocity profile and relative lateral position of the test motorcycle

CONCLUSION

Microscopic motorcycle behaviour in mixed traffic condition is studied in this paper. The proposed modelling framework consists of a social force concept based lateral movement and an intelligent driver model (IDM) based longitudinal movement. The effect of surrounding vehicles and infrastructure is accounted by considering the gap available for the subject vehicle to move ahead. This models the turning angle of the motorcycles and subsequent lateral movement. The conventional intelligent driver model for longitudinal movement is a modified by redefining the leader vehicle and its influence regime. Both the lateral and longitudinal model was integrated to provide a continuous movement model. The specific contribution of this study is the development of a behaviourally sound and comprehensive model to simulate the movement of motorbikes in mixed traffic conditions. The proposed model is calibrated and validated using field data. It can be seen that the gap seeking behaviour of motorcycle plays an important role in its lateral behaviour. The MAPE error values for individual trajectory simulation were found to be reasonably low. Moreover the case study demonstrated the ability of the model to simulate naturalistic riding behaviour. The model could be extended to incorporate the behaviour of motorcycles at intersections, curved roads, and rider behaviour and pavement condition. Finally the model is presently meant only for motorcycles. However this could be extended to other motor vehicles

AKNOWLEDGEMENTS
The first author would like to thank German Academic Exchange Programme (DAAD) for supporting the project.

REFERENCES

1. Luk, J., and Johann Tay. *The use and application of microsimulation traffic models*. No. AP-R286/06. 2006.
2. Antonini, Gianluca, Santiago Venegas Martinez, Michel Bierlaire, and Jean Philippe Thiran. "Behavioral priors for detection and tracking of pedestrians in video sequences." *International Journal of Computer Vision* 69, no. 2 (2006), pp.159-180
3. Minh, Chu Cong, Kazushi Sano, and Shoji Matsumoto. "The speed, flow and headway analyses of motorcycle traffic." *Journal of the Eastern Asia Society for Transportation Studies* 6 (2005), pp. 1496-1508.
4. Kov, Monyrath, and Tetsuo Yai. "Traffic characteristics of motorcycle-dominated urban street considering the effect of light vehicle." *Journal of the Eastern Asia Society for Transportation Studies* 8 (2010), pp.1806-1821.
5. Lan, Lawrence W., and Chiung-Wen Chang. "Inhomogeneous cellular automata modeling for mixed traffic with cars and motorcycles." *Journal of Advanced Transportation* 39, no. 3 (2005),pp. 323-349.
6. Cho, Hung-Jung, and Yuh-Ting Wu. "Modeling and simulation of motorcycle traffic flow." In *Systems, Man and Cybernetics, 2004 IEEE International Conference on*, vol. 7, IEEE, 2004.pp. 6262-6267.
7. Kanda, Manabu, Kunio Takahashi, Yasuo Asakura, Daisuke Fukuda, and Tetsuo Shimizu. "A Concept of Safety Space for Describing Non-lane-based Movements of Motorcycles." PhD diss., Tokyo Metropolitan University, 2012.
8. Lee, Tzu-Chang. "An agent-based model to simulate motorcycle behaviour in mixed traffic flow." PhD diss., Imperial College London (University of London), 2007.
9. Mathew, Tom V., Caleb Ronald Munigety, and Ashutosh Bajpai. "A Strip-based Approach for Simulation of Mixed Traffic Conditions." *Journal of Computing in Civil Engineering* (2013).
10. Helbing, Dirk, and Peter Molnar. "Social force model for pedestrian dynamics."*Physical review E* 51, no. 5 (1995), pp. 4282.
11. Treiber, Martin, Ansgar Hennecke, and Dirk Helbing. "Congested traffic states in empirical observations and microscopic simulations." *Physical Review E* 62, no. 2 (2000), pp. 1805.
12. Helbing, Dirk, and Benno Tilch. "Generalized force model of traffic dynamics."*Physical Review E* 58, no. 1 (1998), pp.133.
13. Fellendorf, Martin, Robert Schönauer, and Weinan Huang. "Social force based vehicle model for two-dimensional spaces." In *Transportation Research Board 91st Annual Meeting*, no. 12-1347. (2012).
14. Vivek Vicraman, C. R. Munigety, T.V.Mathew,. Semi automated tools for traffic data extraction. *Proceedings of 93rd Annual Transportation Research Board Meeting*. Washington D C: Transportation research board. (2013).

15. Astarita, Vittorio, Vincenzo Giofré, Giuseppe Guido, and Alessandro Vitale. "Investigating road safety issues through a microsimulation model." *Procedia-Social and Behavioral Sciences* 20 (2011), pp. 226-235.