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High-energy limit of quantum electrodynamics beyond Sudakov approximation

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ABSTRACT

We study the high-energy behavior of the scattering amplitudes in quantum electrodynamics beyond the leading order of the small electron mass expansion in the leading logarithmic approximation. In contrast to the Sudakov logarithms, the mass-suppressed double-logarithmic radiative corrections are induced by a soft electron pair exchange and result in enhancement of the power-suppressed contribution, which dominates the amplitudes at extremely high energies. Possible applications of our result to the analysis of the high-energy processes in quantum chromodynamics is also discussed.

In a renowned paper [1] V.V. Sudakov derived the leading asymptotic behavior of an electron scattering amplitude in quantum electrodynamics (QED) at high energy. It is determined by the "Sudakov" radiative corrections, which include the second power of the large logarithm of the electron mass m_e divided by a characteristic momentum transfer of the process per each power of the fine structure constant α. Sudakov double logarithms exponentiate and result in a strong universal suppression of any electron scattering amplitude with a fixed number of emitted photons in the limit when all the kinematic invariants of the process are large. This result plays a fundamental role in particle physics. Within different approaches it has been extended to the nonabelian gauge theories and to the subleading logarithms [2–7], which is crucial for a wide class of applications from deep inelastic scattering to Drell–Yan processes and the Higgs boson production. At the same time no significant progress has been achieved in the study of the logarithmically enhanced corrections to the subleading contributions suppressed by a power of electron mass at high energies. However, the power-suppressed contributions are of great interest. They can become asymptotically dominant at very high energies due to Sudakov suppression of the leading terms. At the intermediate energies the power corrections in many cases are phenomenologically important [8–11]. Moreover, in contrast to the Euclidean operator product expansion [12] or nonrelativistic threshold dynamics [13] very little is known about the general all-order structure of the large logarithms beyond the leading-power approximation in the high-energy limit, which is a real challenge for the effective field theory approach. This problem is now actively discussed in various contexts (see *e.g.* [14–19]). In this Letter we make the first step toward the solution of the problem and generalize the result of Ref. [1] to the leading power-suppressed contribution. We present a detailed analysis of the electron scattering in the external field and later discuss the extension of the result to more complex processes.

The amplitude $\mathcal{F}$ of the electron scattering in an external field can be parameterized in the standard way by the Dirac and Pauli form factors

$$\mathcal{F} = \bar{\psi}(p_1)\left(\gamma_\mu F_1 + \frac{i\sigma_{\mu\nu}q^\nu}{2m_e}F_2\right)\psi(p_2). \tag{1}$$

The Pauli form factor F_2 does not contribute in the approximation discussed in this Letter and we mainly focus on the high-energy behavior of the Dirac form factor F_1. We consider the limit of the on-shell electron $p_1^2 = p_2^2 = m_e^2$ and the large Euclidean momentum transfer $Q^2 = -(p_2 - p_1)^2$ when the ratio $\rho \equiv m_e^2/Q^2$ is positive and small. The Dirac form factor can then be expanded in an asymptotic series in ρ

$$F_1 = S_\lambda \sum_{n=0}^{\infty} \rho^n F_1^{(n)}, \tag{2}$$

where $F_1^{(n)}$ are given by the power series in α with the coefficients depending on ρ only logarithmically. The factor $S_\lambda = \exp\left[-\frac{\alpha}{2\pi}B(\rho)\ln\left(\lambda^2/m_e^2\right)\right]$ with $B(\rho) = \ln\rho + \mathcal{O}(1)$ accounts for the universal singular dependence of the amplitude on the auxiliary photon mass λ introduced to regulate the infrared diver-

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gences [20]. In the double-logarithmic approximation the leading term is given by the Sudakov exponent $F_1^{(0)} = e^{-x}$, with $x = \frac{\alpha}{4\pi}\ln^2\rho$ [21]. Let us outline our approach for the analysis of the power-suppressed double logarithmic contributions. We use the expansion by regions method [22–24] to get a systematic expansion of the Feynman integrals in ρ. In this method the coefficients $F_1^{(n)}$ are given by the sum over contributions of different virtual momentum regions. Each contribution is represented by a Feynman integral which in general is divergent. These spurious divergences result from the process of scale separation and have to be dimensionally regulated. The singular terms cancel out in the sum of all regions but can be used to determine the logarithmic contributions to $F_1^{(n)}$. The double logarithmic contributions are determined by the leading singular behavior of the integrals and can be found by the method developed in Refs. [1,21,25]. Though the method is blind to the power corrections, it can be applied in this case since the expansion by regions provides the integrals which are *homogeneous* in the expansion parameter. Sudakov logarithms are produced by the soft virtual photons, which are collinear to either p_1 or p_2. We have found that such a configuration of virtual momenta does not produce double logarithms in the first order in ρ. This observation agrees with the analysis [26] of the cusp anomalous dimension, which determines the double-logarithmic corrections to the light-like Wilson line with a cusp. For the large cusp angle corresponding to the limit $\rho \to 0$ from the result of Ref. [26] one gets

$$\Gamma_{cusp} = -\frac{\alpha}{\pi}\ln\rho\left(1 + \mathcal{O}(\rho^2)\right), \tag{3}$$

with vanishing first-order term in ρ. Nevertheless, the $\mathcal{O}(\rho)$ double-logarithmic contribution does exist but originates from completely different virtual momentum configuration described below. Let us consider an electron propagator $S = \frac{\not p_i - \not l + m_e}{(p_i - l)^2 - m_e^2}$, where l is the momentum of a virtual photon with the propagator $D = \frac{-g_{\mu\nu}}{l^2-\lambda^2}$. In the soft-photon limit $l \to 0$ the electron propagator becomes eikonal $S \approx -\frac{\not p_i + m_e}{2p_i l}$ and develops a collinear singularity when l is parallel to p_i. Alternatively, we may consider the soft-electron limit $l' \to 0$, where $l' = p_i - l$. Then the electron propagator becomes scalar $S \approx \frac{m_e}{l'^2 - m_e^2}$ while the photon propagator becomes eikonal $D \approx \frac{g_{\mu\nu}}{2p_i l' - m_e^2 + \lambda^2}$. Thus the roles of the electron and photon propagators are exchanged. The existence of non-Sudakov double-logarithmic contributions due to soft electron exchange has actually been known for a long time [25,27]. However in our case this virtual momentum configuration does not produce a double-logarithmic contribution in one loop because the momentum shift distorts the eikonal structure of the second electron propagator and removes the soft singularity at small l' necessary to get the second power of the large logarithm. This may be avoided only in the two-loop diagram of nonplanar topology, Fig. 1(a). After shifting the photon virtual momenta by p_1 and p_2 the diagram can be twisted into the shape of Fig. 1(b), (c) with soft electron pair exchange between the eikonal lines. The corresponding contribution has an explicit suppression factor m_e^2 from two soft electron propagators. Hence the integration over the virtual momenta can be performed in the leading order of the small electron mass expansion. Note that in the case under consideration the electron mass regulates both soft and collinear divergences and we can put $\lambda = 0$. The calculation is conveniently performed by using the light-cone coordinates where $p_1 \approx p_{1-}$ and $p_2 \approx p_{2+}$. In this representation only the interaction of the transverse photons to soft electrons is not mass-suppressed and we can use $\frac{g_{kl}^{\perp}}{2p_i l}$ for the eikonal photon propagators. To get the double-logarithmic part of the correction

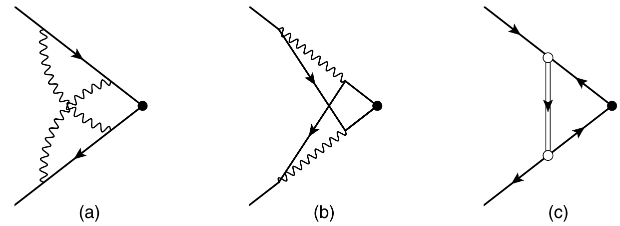

Fig. 1. Different representations of the two-loop Feynman diagram giving the leading power-suppressed double-logarithmic contribution. In figure (c) the double line arrow represents the soft electron pair propagator and the empty blobs represent the nonlocal interaction of the soft electron pair to the eikonal electrons and positrons.

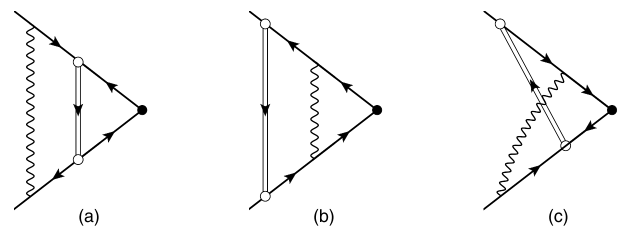

Fig. 2. Feynman diagrams contributing to the double-logarithmic correction factors $\phi^{a,b,c}$, Eq. (7).

we use Sudakov parametrization of a virtual photon momentum $l = up_1 + vp_2 + l_\perp$. After integrating over the transverse components $l_\perp$ we get the following representation of the two-loop power-suppressed form factor

$$F_1^{(1)}\Big|_{2\text{-loop}} = -4x^2 \int K(\eta_1, \eta_2, \xi_1, \xi_2)\mathrm{d}\eta_1\mathrm{d}\eta_2\mathrm{d}\xi_1\mathrm{d}\xi_2, \tag{4}$$

where $\eta = \ln v/\ln\rho$, $\xi = \ln u/\ln\rho$, the integration goes over the four-dimensional cube $0 < \eta_i, \xi_i < 1$, and the kernel

$$K(\eta_1, \eta_2, \xi_1, \xi_2) = \theta(1-\eta_1-\xi_1)\theta(1-\eta_2-\xi_2) \times \theta(\eta_2-\eta_1)\theta(\xi_1-\xi_2) \tag{5}$$

selects the kinematically allowed region of double-logarithmic integration. This gives $F_1^{(1)} = -\frac{x^2}{3} + \mathcal{O}(x^3)$, in agreement with [8,28]. The higher-order double-logarithmic corrections are generated in a usual way through the exchange of soft photons with the propagator $\frac{-g_{\mu\nu}}{l^2-\lambda^2}$. A key observation here is that an exchange of a soft photon between an eikonal and a soft electron line does not produce double logarithms. The reason for this is that such a loop is always separated from the second eikonal line by a scalar electron propagator, which does not communicate any information on the second external momentum. Hence the loop integral cannot depend on the scalar product $p_1 p_2$, which is the only large scale in the problem. Thus it is sufficient to consider only the diagrams of the topologies given in Fig. 2. By using the factorization properties of the soft photon contribution [1] after separating the singular factor S_λ we find the following representation of the all-order double-logarithmic result

$$F_1^{(1)} = -4x^2 \int \phi^b(\eta_1, \xi_2)\phi^c(\eta_1, \xi_1)\phi^c(\xi_2, \eta_2) \times \Big[\phi^a(\eta_2, \xi_1)K_1(\eta_1, \eta_2, \xi_1, \xi_2) + K_2(\eta_1, \eta_2, \xi_1, \xi_2)\Big]\mathrm{d}\eta_1\mathrm{d}\eta_2\mathrm{d}\xi_1\mathrm{d}\xi_2, \tag{6}$$

where the Sudakov correction factors corresponding to Fig. 2(a)–(c) are

Table 1
The normalized coefficients of the series (9) up to $n=7$.

n	1	2	3	4	5	6	7
$n!n^3c_n$	$\frac{7}{30}$	$\frac{68}{105}$	$\frac{509}{350}$	$\frac{992}{945}$	$\frac{32\,225}{29\,106}$	$\frac{208\,044}{175\,175}$	$\frac{3\,946\,313}{3\,281\,850}$

$$\begin{aligned}
&\phi^a(\eta,\xi)=\exp\left[-x(1-\eta-\xi)^2\right],\\
&\phi^b(\eta,\xi)=\exp\left[-2x\eta\xi\right],\\
&\phi^c(\eta,\xi)=\exp\left[x\eta\,(\eta+2\xi-2)\right],
\end{aligned}\tag{7}$$

respectively, and the new kernels read

$$\begin{aligned}
K_1(\eta_1,\eta_2,\xi_1,\xi_2)&=\theta(1-\eta_2-\xi_1)\theta(1-\eta_2-\xi_2)\\
&\quad\times\theta(\eta_2-\eta_1)\theta(\xi_1-\xi_2),\\
K_2(\eta_1,\eta_2,\xi_1,\xi_2)&=\theta(1-\eta_1-\xi_1)\theta(1-\eta_2-\xi_2)\\
&\quad\times\theta(\eta_2-\eta_1)\theta(\xi_1+\eta_2-1),
\end{aligned}\tag{8}$$

with $K=K_1+K_2$. We are not able to find the result for the four-fold integral (6) in a closed analytic form. However, the coefficients of the series

$$F_1^{(1)}=-\frac{x^2}{3}\left(1+\sum_{n=1}^{\infty}c_nx^n\right)\tag{9}$$

can in principle be analytically computed for any given n. The first seven coefficients of the series are listed in Table 1. At the same time in the large-n limit we get an approximate result

$$c_n=\frac{C}{n^3n!}\left[1+\mathcal{O}\left(\frac{1}{n^{1/2}}\right)\right]\tag{10}$$

where $C=1.1994\ldots$ is a numerical constant. Eqs. (9), (10) give the following asymptotic behavior of the form factor at large x

$$F_1^{(1)}\sim e^{x-\ln x}\left[-\frac{C}{3}+\mathcal{O}\left(\frac{1}{x^{1/2}}\right)\right],\tag{11}$$

i.e. the power-suppressed amplitude is *enhanced* by the double-logarithmic corrections. A similar effect has been observed before *e.g.* for the electron–muon backward scattering [25]. In the case under consideration the positive sign of the exponent may be related to a specific structure of the process with the soft electron pair exchange. Through the pair emission an ultrarelativistic electron is converted into an ultrarelativistic positron with approximately the same momentum but opposite electric charge, Fig. 1(c). As a result of the charge flip the double-logarithmic contribution of the topology of Fig. 2(c) has an opposite sign with respect to the one of Fig. 1(a), (b) and actually determines the behavior of the exponent in Eq. (11). It is interesting to compare the high-energy asymptotic behavior of the leading and subleading terms of Eq. (2). In the limit $\rho\to 0$ we get

$$F_1^{(0)}\sim\rho^{-\frac{\alpha}{4\pi}\ln\rho},\qquad \rho F_1^{(1)}\sim\rho^{1+\frac{\alpha}{4\pi}\ln\rho}.\tag{12}$$

Thus above the energy corresponding to $|\ln\rho|=\frac{2\pi}{\alpha}$ the originally power-suppressed term exceeds the leading contribution of $F_1^{(0)}$. Note that at this energy the pure QED running coupling $\alpha(Q^2)\approx 3\alpha$ and we are still in a weak coupling regime. However, this energy is too high to be phenomenologically relevant and this result is likely to be of pure theoretical interest.

According to a naive estimate based on the fixed-order power counting the double logarithmic approximation of $F_1^{(1)}$ is valid for $\alpha\ll x\ll 1/\alpha$, which covers the energy interval $1\ll|\ln\rho|\ll 1/\alpha$ sufficient for any practical applications. For higher energies corresponding to $x\sim 1/\alpha$ the subleading terms proportional to powers of $\alpha\ln\rho\sim 1$ have to be resummed to all orders. The naive estimate given above does not work for the leading term because of its exponential suppression and the double logarithmic approximation of $F_1^{(0)}$ is not applicable already for $1\ll x$. However a systematic resummation of the subleading logarithms [3,4] proves that Eq. (12) does describe the correct asymptotic behavior of $F_1^{(0)}$ up to $x\sim 1/\alpha$ as far as the effective coupling $\alpha(Q^2)$ remains small. At the same time for $1\ll x$ the subleading logarithms may in principle affect the asymptotic behavior of $F_1^{(1)}$. The relevant subleading terms are of the form $(\alpha\ln\rho)^nf_n(x)$, where $f_n(x)$ is an unknown function. Such a term would modify Eq. (12) only if at large x the function $f_n(x)$ grows at least exponentially faster than e^x. In this case the $\mathcal{O}(\rho)$ contribution start to dominate the form factor at even lower energy.

Unlike the Sudakov double logarithms, the leading power-suppressed double-logarithmic corrections depend not only on the charges of the initial and final states but also on the details of the scattering process. For example, the $\mathcal{O}(\rho)$ double-logarithmic corrections to the scalar form factor vanish to all orders in α due to a specific Lorentz and Dirac structure of the soft electron pair interaction with the eikonal electrons and positrons. A less trivial example is the Pauli form factor. The expansion of F_2 in ρ (*cf.* Eq. (2)) starts with the first order term $F_2^{(1)}$. In the double logarithmic approximation $F_2^{(1)}=0$ and for the leading mass correction from the soft electron pair exchange we obtain $F_2^{(2)}=4F_1^{(1)}$, in agreement with [8,28]. Thus the $\mathcal{O}(\rho)$ corrections are universally related to the soft electron pair exchange and can be obtained as a straightforward generalization of our analysis for more complicated processes such as Bhabha scattering, where only the leading result of the small electron mass expansion is available in two loops [29,30]. Moreover, up to two loops the structure of the $\mathcal{O}(\rho)$ double-logarithmic correction in quantum chromodynamics (QCD) is similar to the one in QED. In particular, the double-logarithmic power-suppressed term in two-loop corrections to the heavy-quark vector form factor differs from the QED result only by the $C_F^2-C_AC_F/2$ color factor of the diagram in Fig. 1. Thus our method can be applied to the calculation of the dominant two-loop power-suppressed corrections to the high-energy processes involving heavy quarks. For the energies ranging from approximately 10 to 100 times the heavy-quark mass we have $|\ln\rho|\gg 1$ and $\rho\ln^4\rho\sim 1$, *i.e.* the double-logarithmic terms saturate the power-suppressed contribution and are comparable in magnitude to the nonlogarithmic leading-power corrections in the strong coupling constant, which are phenomenologically significant. Beyond the two-loop approximation our result is not directly applicable to the QCD amplitudes since the eikonal gluons in Fig. 1(b) can radiate soft gluons producing additional double-logarithmic corrections. As a consequence, the leading power-suppressed double-logarithmic corrections to the heavy-quark vector form factor get a nonabelian contribution in every order of perturbation theory in contrast to the purely abelian Sudakov double logarithms.

To summarize, we have generalized the result of Sudakov [1] to the leading power-suppressed contribution. This is an important step towards a systematic renormalization group analysis of the high energy behavior of the gauge theory amplitudes beyond the leading power approximation. The leading power-suppressed double-logarithmic corrections reveal a few characteristic features which distinguish them from the Sudakov double logarithms. In particular, they are induced by a soft electron pair exchange and result in a strong enhancement of the power-suppressed contribution. In QCD our method can be used for the analysis of the high-energy processes involving heavy quarks up to two loops. Extending the analysis to the higher orders of perturbative QCD and to subleading logarithms is a very interesting problem which is beyond the scope of this Letter.

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